

## Number Theory

- Deals with properties & concepts of numbers.
  - even & odd
  - prime numbers
  - composite numbers
  - GCD, LCM

## Parity

- even or odd integer.  
 $\downarrow$   
 $2K$
  - $2K+1$ ,  $K = \text{integer}$ .
- 0 is even number since  $2 * 0 = 0$

$$4a^2b = 2(\underbrace{2a^2b}_{\text{integer}}) \therefore \text{Even number.}$$

- even  $\pm$  even = even
- even  $\pm$  odd = odd
- odd  $\pm$  odd = even
- even  $*$  even = even
- even  $*$  odd = even
- odd  $*$  odd = odd

## Divisibility

$$n, d \in \mathbb{Z} \quad d \neq 0$$

eg:  $6 \mid 18 = 3$   $\leftarrow K$   
 $\uparrow \quad \uparrow$   
 $d \quad n$

$$d \mid n \Leftrightarrow \exists K \text{ s.t. } n = dK$$

$$\overset{r}{d} \mid \overset{r}{n} \quad \text{if \& only if}$$

- All integers divisible by 1
  - last digit even divisible by 2.
- \* see more in divisibility notes

Other properties:

1) Given  $a, b, c \in \mathbb{Z}$ , if  $a \mid b$  &  $b \mid c$  then  $a \mid c$

2) Given  $a, b \in \mathbb{Z}$ , if  $a \mid b$  &  $b \mid a$ ,  $a$  not necessary =  $b$  eg,  $\frac{2}{-2}$ .

## Prime Numbers

- integer  $> 1$
  - divisible only by 1 & itself.
- \* 2 is the only even prime no.

eg: 2, 3, 5, 7, 11, 13...

## Fundamental theorem of Arithmetic

For integers  $> 1$ , number is either a prime number or it can be created by product of prime numbers.

2      3      4      5      6      7      8      ...      16      17  
prime prime  $2 \times 2$  prime  $2 \times 3$  prime  $2 \times 2 \times 2$   $2 \times 2 \times 2 \times 2$  prime.  
composite numbers

## Prime Factorization

Factors = The numbers multiplied together to get another number.

eg:  $10 = 2 \times 5$       1, 2, 5, 10 are factors of 10.  
 $1 \times 10$

Prime factors = factors but only prime numbers.

eg: Given 50:  $\begin{array}{r} 2 \mid 50 \\ 5 \mid 25 \\ 5 \mid 5 \\ 1 \end{array}$       2 & 5 are prime factors of 50.

(divide by smallest divisible prime)

eg:  $24 = 2 \times 2 \times 2 \times 3$   
 $42 = 2 \times 3 \times 7$

## Greatest common Divisor & Least Common Multiple

GCD: Find prime factors of numbers & take highest common prime.

LCM: List out multiples of numbers & identify LCM:

L eg:  $4 : 4, 8, 12, 16, \textcircled{20}$       or  $\begin{array}{l} 3 = 3 \\ 9 = 3 \times 3 \text{ \& most '3' } = 3 \times 3 \times 7 \\ 21 = 3 \times 7 \end{array}$   
 $10 : 10, \textcircled{20}$

LCM eg:  $12 : 2 \times 2 \times 3$        $80 : 2 \times 2 \times 2 \times 2 \times 5$        $\rightarrow \text{LCM} = 2 \times 2 \times 2 \times 2 \times 3 \times 5 = 240$

$$\boxed{\text{LCM}(a, b) = \frac{|a \cdot b|}{\text{GCD}(a, b)}}$$