Statements

Statements : sentence that is either True or False. (not both)

eg: $2+2=4$: True , $2+2=5$: False

Non-statements: • Fridays are nice (subjective, ∴ not statement)

Compound statements

and: $\wedge$ or: $\vee$ not: $\neg$ or $\sim$

• several statements in sentence using logical connectives

eg: • My brother, John, is a student → statement 1

• $3+5=8$ → statement 2

statement 1 $\wedge$ statement 2 (and)

and : conjunction
or : disjunction

Truth Tables

and

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

or

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

xor

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

sign different.

Logical Equivalence ($\equiv$)

Given 2 statements p, q.

$p \wedge q = q \wedge p$ → logical equivalent.

$p \equiv \sim\sim p$ (not)

Tautologies & Contradictions

– construct truth table to determine tautology or contradiction

Tautology = All values T

Contradictn = All values F

Tautology: statement that always yield True.

eg: (or)

p	$\sim p$	$p \vee \sim p$
T	F	T
F	T	T

Contradiction: statement that always yield False (and)

p	$\sim p$	$p \wedge \sim p$
T	F	F
F	T	F

p	q	$\sim p$	$\sim q$	$\sim(p \vee \sim q)$	$\sim q \wedge p$
T	T	F	F	F	F
T	F	F	T	F	T
F	T	T	F	T	F
F	F	T	T	F	F

De Morgan's Law (Logic)

1. $\sim(p \wedge q) = \sim p \vee \sim q$ proof: using truth table
2. $\sim(p \vee q) = \sim p \wedge \sim q$.

Logical Equivalence laws

$p \wedge q \equiv q \wedge p$: commutative

$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$: associative

$p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$: distributive

$p \wedge t \equiv p$, t = tautology = All true.

$p \vee t \equiv t$

$p \wedge c \equiv c$, c = contradiction = All false

$\sim t \equiv c$

Conditional statements

if ... then ... eg: p : work hard, q = get paid. : if p (hypothesis) then ($\rightarrow$) q (conclusion).

then.

denoted : $p \rightarrow q$.

p	q	$p \rightarrow q$
T	T	T
F	T	T
$\boxed{T}$	$\boxed{F}$	$\boxed{F}$
F	F	T

← if work hard (hypothesis true) but no pay (conclusion false) : yields $p \rightarrow q$ = false

the rest will be true

Negation of conditional statements

$\boxed{p \to q \equiv \sim p \vee q}$

$\therefore \sim(p \to q) \equiv \sim(\sim p \vee q)$

$\equiv \sim(\sim p) \wedge \sim q$

$\boxed{\sim(p \to q) \equiv p \wedge \sim q}$ ← Note: After negation, if disappears.

Steps

1. identify p & Q
2. Apply relevant logic then sub in p & q for final result.

Contrapositive statement

$p \to q \equiv \sim q \to \sim p$

eg: if $\underbrace{\text{today tuesday}}_{p}$ then $\underbrace{\text{today work day}}_{q}$. $\equiv$ if not today work day then not today tuesday.

Converse & Inverse statements

$p \to q$: if p then q.

converse: $q \to p$: if q then p.

Inverse : $\sim p \to \sim q$

contrapositive: $\sim q \to \sim p$. hypothesis true, conclusion false yield false

p	q	~p	~q	p → q	q → p	~p → ~q	~q → ~p
T	T	F	F	T	T	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T
				original ≢	converse ≡	Inverse	contrapositive

Biconditional statement

$p \leftrightarrow q$

if p then q happens.

but if q then p happens.

eg: if <u>you study hard</u> then <u>you pass</u>.

if <u>you pass</u> then <u>you have studied hard</u>

p	q	$\boxed{p \leftrightarrow q}$
T	T	T
T	F	F
F	T	F
F	F	T

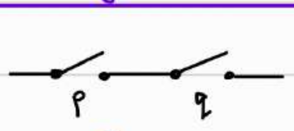
← T if p & q similar.

$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

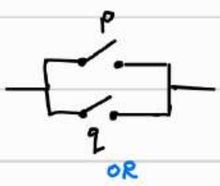
$\boxed{\begin{aligned} p \leftrightarrow q &\equiv (\sim p \vee q) \wedge (\sim q \vee p) \\ p \to q &\equiv \sim p \vee q \end{aligned}}$ ~ ↔ & → converted to ∨ & ∧

* Can use Truth table generator (online help tool)

Digital Logic Circuits



AND

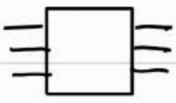


OR

0 - open
1 - closed } Bit : Binary digit

Black boxes & Gates

- Black box used to represent circuits (abstraction)
- only interested in inputs & outputs. (not interested in implementation)

inputs  Outputs

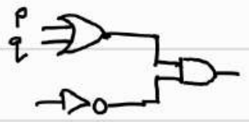
Basic Gates

AND
OR
NOT


Boolean Expressions

contain boolean variables & boolean connectives

True or False $\sim, \wedge, \vee$: not, and, or.



: $\boxed{(p \vee q) \wedge (\sim v)}$ - Boolean expression

Construct circuit from truth table

eg:

P	Q	r	output
F	F	F	F
F	F	T	F
F	T	F	F
F	T	T	F
T	F	F	T
T	F	T	T
T	T	F	F
T	T	T	T

Steps

1. identify all rows with output as T
2. using only AND, form required logic for these rows.

   eg:

	P	Q	r
$P \wedge \sim Q \wedge \sim r$ for	T	F	F
$P \wedge \sim Q \wedge r$ for	T	F	T
$P \wedge Q \wedge r$ for	T	T	T

3. connect boolean expressions formed in step 2 with OR

   eg: $(P \wedge \sim Q \wedge \sim r) \vee (P \wedge \sim Q \wedge r) \vee \ldots$

4. Draw circuit with reference to expression in step 3.

Equivalent Circuits

- if inputs & outputs are same (construct truth table to verify)
- can reduce complicated circuits to simpler equivalent.

NAND & NOR Gates

	A	B	C
0			
1	0	0	1

AND with NOT & OR with NOT

 NAND

 NOR

Notation: $|$ $P | Q$ $\downarrow$ $P \downarrow Q$

Shefer stroke   Pierce Arrow

P	Q	output
1	1	0
0	1	1
1	0	1
0	0	1

P	Q	output
1	1	0
0	1	0
1	0	0
0	0	1

Quantified statements - ALL

either T or F

All : $\forall$ suppose $A = \{1, 2, 3, 4, 5\}$

$\forall x \in A, \boxed{x > 0}$. ← known as universal statement

all elements in set A   s.t each element > 0

rule. true if all element satisfy rule.

universal quantifier.

Quantified statements - THERE EXIST

There Exist : $\exists$ suppose m = any integer
$\mathbb{Z}$ = set of all integers

$1^2 = 1$

$\exists\, m \in \mathbb{Z},\ m^2 = m$: True

Negation of Quantified Statements

$\exists$ to negate $\forall$.   eg: Q(x) = wear hat
A = set of all men

$\forall x \in A, Q(x)$, $\underline{\exists\, x \in A, \sim Q(x)}$ negation of that

$\forall$ to negate $\exists$ eg: $\exists\, x \in A, Q(x)$, $\forall x \in A, \sim Q(x)$