Combinatorics

- mathematics of counting things / arrange objects in certain condition.
  - ↳ different combinations.

A B C D etc.
B A C D
C A B D.

Factorials

$n! = n \cdot (n-1) \cdot \ldots \cdot 3 \cdot 2 \cdot 1$

eg: $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$0! = 1$

$\frac{16!}{14! \cdot 3!} = \frac{16 \cdot 15}{3 \cdot 2 \cdot 1} = 40.$ $\qquad \frac{(n+1)!}{(n-1)!} = \frac{(n+1) \cdot n \cdot \cancel{(n-1)!}}{\cancel{(n-1)!}} = n^2 + n$

Fundamental Counting Principle (Basic counting principle)

Suppose:

chicken salad	chips	water
Beef salad	fries	coke.
Salmon salad	onion rings	

3 choices × 3 choices × 2 choices = 18 different combinations.

Total no. of outcomes = a × b × c × ....

a, b, c, ... = ways of doing things.

* Events must not be dependant on each other.

if events are dependant: Addition principle must be used

eg: 2 pants 4 shirts

M	A
L.	B
	C
	D

$\boxed{\text{if wear M pants, cannot wear shirt D.}}$

Total outcomes = $(\underbrace{1}_{M} \times \overbrace{3}^{A,B,C}) + (\underbrace{1}_{L} \times \overbrace{4}^{A,B,C,D})$ = 7 outcomes possible.

Permutations

- how many ways to arrange something / satisfy particular condition.
- order is important. eg: lock combination (order matters) vs ingredients in bowl (order doesn't matter) ← combination.

case 1: repetition exist

safe lock : 4 digits $\underline{10}\ \underline{10}\ \underline{10}\ \underline{10}$

n^r, n = different choices. $= 10^4$ ← 4 repetitions.
r = repetition. ↳ 10 digits

case 2: repetition not allowed

$n!$, n = no. of choices.

- once choice used, total number of choices reduce by 1.

∴ $n!$

$P(n,r)\ \frac{n!}{(n-r)!}$, n = No. of choices / elements
r = No. of options selected.
eg: out of n choices, only r choices are selected.

Combinations

- like permutation but order does not matter.

eg: lottery, order of no. does not matter.

case 1: repetition not allowed

$$C(n,k) = \frac{n!}{k!\,(n-k)!}$$

n = No. of elements / choices

k = No. of things selected.

13 of 16 balls selected

case 2: repetition allowed

$$C(n,k) = \frac{(k+n-1)!}{k!\,(n-1)!}$$

eg: donut shop.

5 different types. n = 5

choose 3. k = 3.

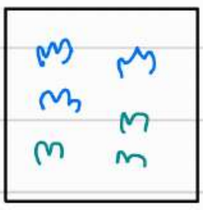
Total combination possible

Pigeonhole Principle

- used extensively in a lot of domains of mathematics.
- if n items put into m containers, n > m, at least one container must contain > 1 item.

eg 1. of usage:

Suppose cabinet with Blue & Green gloves.



Minimum no. of gloves to draw to get matching pair = 3.

eg 2: 366 people needed to get duplicate birthday

At least.

Pascal Triangle

counting numbers diagonal

triangular numbers.

- symmetric about center.

1	1
2	1 1
4	1 + 2 + 1
8	1 + 3 + 3 + 1
16	1 4 6 4 1
32	1 5 10 10 5 1

combination result from Pascal Triangle:

$$C(n,k) = \frac{n!}{k!\,(n-k)!} \quad \text{(repetition not allowed)}$$

suppose n = 4

k = 2

← n = 0

1

1 1

1 2 1

1 3 3 1

n = 4 → 1 4 6 4 1.

↑ k = 0 ↑ k = 2.

$$\frac{4!}{2!\,(4-2)!} = 6$$