Functions

- defined r/s b/w independant variable (s) & dependant variable (s)

input (s) $\rightarrow$ $\boxed{\text{function}}$ $\rightarrow$ output (s)

↳ rules / algorithm.

Domain : set of inputs accepted by function w/o breaking it.

Range : Difference b/w highest & lowest value of output.

input (independant var.).

Notation : $y = f(x) = x^2$

↑ output (dependant var.)

↳ Rule to apply for function.

* All elements in domain must map to output, else, not a function.

eg: inputs outputs.

Not a fn since ∃ element not mapping to output

Domain

- To find domain, find cases that breaks fn.
- Then, domain = All $\mathbb{R}$ except values that break fn.

eg : Given $f(x) = \frac{1}{x-1}$, set $x - 1 = 0$

$x = 1$

∴ Domain = $\forall \mathbb{R}$ where $x \neq 1$. $(-\infty, 1) \cup [2, \infty)$

excluding

including

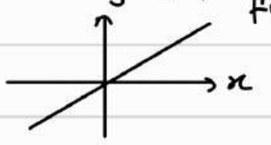
Range

- All possible outputs from a fn.

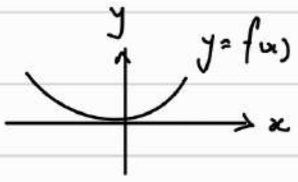
Graph

- plot of functions.

eg: $y = f(x) = x$



$f(x) = x$.



$y = f(x) = x^2$.

Domain from graph: All x - values.

Range from graph: All y - values.

} Including all real numbers in between

Function Composition

- Nested fn.

eg: $f(g(x))$. , $g(x) = x^3 - 5$, $f(x) = x^2$.

$f(g(x)) = (x^3 - 5)^2$.

$g(f(x)) = (x^2)^3 - 5$.

$f(g(x)) \equiv f \circ g(x)$, $g(f(x)) \equiv g \circ f(x)$.

symbol for of.

Function Combination.

- operation between 2 or more functions.

$+ \; - \; \times \; \div$

- can combine & simplify expression or take individual fn values then perform operation.

eg: $f(x) = x^2$, $g(x) = x^3$.

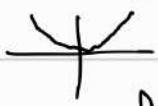
$f(x) \times g(x) = x^2 \times x^3 = x^5$

$f(2) \times g(2) = 2^5 = 2^2 \times 2^3$.

Even & odd functions

Even fn : graph symmetric about y-axis. : eg $f(x) = x^2$.

odd fn : graph symmetric about origin : eg $f(x) = x^3$

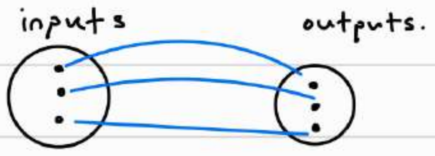


Even f^n : $f(x) = f(-x)$
odd f^n : $f(-x) = -f(x)$.

} if both conditions not satisfied, then <u>neither</u> even nor odd f^n.

One - to - one Functions.

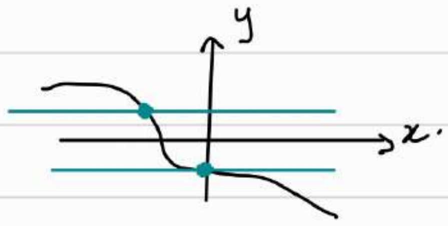
- No 2 elements in domain have same $f(x)$
- No 1 element in domain maps to ≥ 2 elements in range.



$f(x) = x^2$ is not 1 to 1.

$f(x) = x + 1$ is 1 to 1.

Horizontal line test : Drawn on graph, if $\exists$ 2 intersections, then f^n not 1 to 1.



Inverse Functions.

- Function that takes range value & return back to domain values.
- undo function.

eg: $f(7) = 15$, $g(15) = 7$.

* Function must be Bijective to find inverse.

$y = 3x - 2$, inverse = make x subject.

$x = \frac{y+2}{3}$ $\rightarrow$ $g(x) = \frac{x+2}{3}$

Polynomial Long division

$$\frac{4x^3 - 13x^2 + 2x - 7}{x^2 + 3x - 2}$$

$$= (4x - 25)(x^2 + 10x - 2) + 85x - 57$$

$$\begin{array}{r} 4x - 25 \\ x^2 + 3x - 2 \overline{\smash{)}\, 4x^3 - 13x^2 + 2x - 7} \\ -(4x^3 + 12x^2 - 8x) \\ \hline 0 - 25x^2 + 10x - 7 \\ -(\quad - 25x^2 - 75x + 50) \\ \hline 0 + 85x - 57. \end{array}$$