Sets

A collection of things, can be anything

eg: collection of books ← considered a set

• objects in set known as elements

Math Notation:

• $A = \{1, a, \$, 4 \ldots\}$ ← elements in set; ← indicate infinitely long set; ← set braces

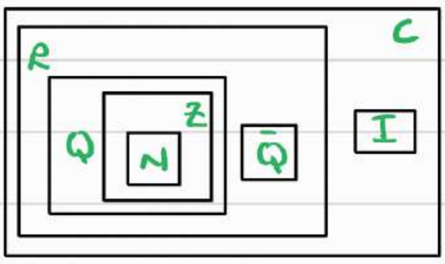
↑ Name of set (upper case convention)

• $\$ \in A$ ← belongs to

• $b \notin A$ ← Does not belong to

Number Sets

1. Natural Numbers set $\mathbb{N} = \{1, 2, 3 \ldots\}$ i.e all positive integer 1 to ∞
2. Integers $\mathbb{Z} = \{\ldots -2, -1, 0, 1, 2 \ldots\}$
3. Rational Numbers $\mathbb{Q}$ = fractions (terminating or recurring) : $\frac{4}{2}, \frac{5}{3}, \frac{2}{3}$ etc. (terminating, recurring)
4. Irrational Numbers $\bar{\mathbb{Q}}$: Non terminating & non recurring eg: $\pi, e, \sqrt{2}$
5. Real Numbers $\mathbb{R}$: Both Rational & Irrational numbers
6. Imaginary Numbers $\mathbb{I}$: $i = \sqrt{-1}$: sqrt of negative numbers. $\sqrt{-9} = 9\sqrt{-1} = 9i$
7. Complex Numbers $\mathbb{C}$: $a + ib$, a = Real, b = imaginary eg: $2 + 3i$



Set Equality

Axiom of Extension : sets are equal if unique elements are equal.

↳ order or duplicates does not matter

eg: $A = \{1, 3, 1, 5, 5, 3, 1\}$ $A = B$

$B = \{3, 1, 5\}$

Note : $\{0\} \neq 0$ (A set, A number)

Set Builder Notation

$\{x \mid x = y\}$ ← condition

All elements ↑; ↑ such that, S.t. or $\exists$ also same

eg: • $\{x \mid x > 0\}$

• $\{x \in \mathbb{R} \mid -2 < x < 5\}$ (Real No.)

• $\{x \in A \mid x > 0\}$

$A = [-5, -4, 2, 0, 1]$

• $\{x \in \mathbb{R} \mid x^2 = 4\} = \{-2, 2\}$

Types of Sets

1. Universal set $\mathbb{U}$: contains everything eg: $A = \{1, 2\}$, $B = \{3, 4\}$, $C = \{1, 2, 3, 4\}$ ← universal set
2. Empty (Null) set $\Phi \neq 0$ eg: $\{x \in \mathbb{N} \mid 3 < x < 4\}$
3. Singleton set : Single element within set
4. Finite set : $\{1, 2, 3\}$
5. Infinite set : $\{1, 2 \ldots\}$
6. Subset : part of a set $A = \{1, 2, 3\}$, $B = \{1, 2\}$ ← B is subset of A

Cardinal Number of set

• No. of distinct elements in set

$A = \{1, 2, 3\}$

$n(A) = 3$ (cardinal no.) distinct element

Equivalent Sets

• Cardinal number are same for sets

eg: $A = \{1, 2, 3\} \rightarrow n(A) = 3$

$B = \{10, 11, 12, 11, 10\} \rightarrow n(B) = 3$

$\therefore A \sim B$ (equivalent)

Subset

- A is subset of B ($A \subseteq B$) if every element of A also element in B
  - if $A = \{1,2\}$ & $B = \{1,2\}$ $A \subseteq B$ valid $= A \leq A$, $A = B$.
  - Empty set $\{\}$ is subset of every set

Proper subset

: Subset & $n(B) > n(A)$ $A \subset B$ proper subset

$1 \subset \{1, 2, 3\}$: invalid, 1 is number, not a set.

$\{2\} \in \{1, 2, 3\}$: False, $\{2\}$ is not an element inside $\{1, 2, 3\}$

$\{2\} \subseteq \{\{1\}, \{2\}\}$: False

Power sets

- set of all subsets of given set denoted $P(A)$ main set

eg: Given $A = \{1, 2, 3\}$.

$P(A) = \{\}, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{1,3\}, \{1,2,3\}$

Number of subsets $= 2^n$, n = No. of elements

Ordered Pairs

ordered n tuples

- order matters $(1,2) \neq (2,1)$ or $(1,2,3,4) \neq (1,2,4,3)$

ordered 2 tuple ordered 4 tuple

Cartesian Product

ordered pair.

$A \times B = \{(a,b) \mid a \in A \ \& \ b \in B\}$

cross product

eg: $A = \{1, 2\}$ $A \times B = \{(1,c), (2,d), (1,d), (2,c)\}$

$B = \{c, d\}$ $B \times A = \{(c,1), (d,2), (d,1), (c,2)\}$

$A \times B \neq B \times A$ (non-commutative)

$A \times B \times C = \{(a,b,c) \mid a \in A, b \in B, c \in C\}$.

$(A \times B) \times C \rightarrow A = \{1,2\}, B = \{c,d\}, C = \{x,y\}$.

$(A \times B) = \{(1,c), (1,d), (2,c), (2,d)\}$.

$(A \times B) \times C = \{((1,c),x), ((1,d),x), ((2,c),x), ((2,d),x), \ldots$ Same for $y\}$

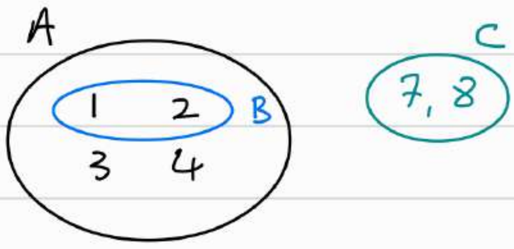
Venn Diagram

- To show r/s b/w sets
- Typically overlapping circles

$A = \{1, 2, 3, 4\}$

$B = \{1, 2\}$

$C = \{7, 8\}$

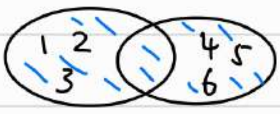


Set Operations (Union & Intersect)

- Union, denoted $\cup$ $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

$A = \{1, 2, 3\}$. $A \cup B = \{1, 2, 3, 4, 5, 6\}$

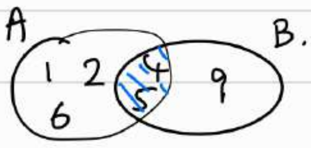
$B = \{4, 5, 6\}$.



* only unique value.
don't need to add duplicates.

• Intersect, denoted $\cap$ $\quad A \cap B = \{x \mid x \in A \text{ and } x \in B\}$.

$A = \{1, 2, 4, 5, 6\}$.
$B = \{4, 5, 9\}$
$A \cap B = \{4, 5\}$.



if no common, then empty set.

Properties of Union & Intersection

1. $A \cup B = B \cup A$, $A \cap B = B \cap A$: commutative
2. $(A \cup B) \cup C = A \cup (B \cup C)$: Associative
3. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$: Distributive.
4. $A \cup \{\} = A$, $A \cap \{\} = \{\}$
5. $A \cup U = U$

universal set.

Set Operations (Difference & Complements)

Difference:

$A = \{1, 2, 3, 4\}$
$B = \{4, 5, 6\}$
$A - B = \{1, 2, 3\}$. $\quad B - A = \{5, 6\}$.

elements in B but not in A.

All elements in A but not in B.

Complement: (Also known as prime: i.e $A^c \equiv A'$

$U = \{1, 2, 3, 4, 5, 6, 7\}$
$A = \{1, 2, 3\}$
$A^c = \{4, 5, 6, 7\}$

element in universal set but not in A.
(i.e everything not in A)

Properties of Difference & Complements

1. $A \cup A^c = U$
2. $(A^c)^c = A$
3. $U^c = \{\}$, $\{\}^c = U$
4. $A - B = A \cap B^c$

De Morgan's Law (set)

$(A \cup B)^c = A^c \cap B^c$
$(A \cap B)^c = A^c \cup B^c$

Partition of sets

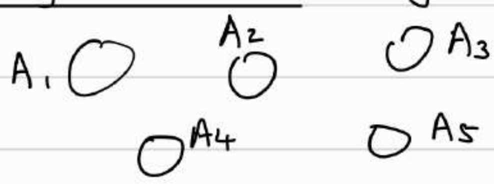
Disjoint sets: 2 sets without any element in common

i.e. $A \cap B = \phi$

empty set.

$A = \{1, 2, 3\}$
$B = \{4, 5, 6\}$ Disjoint

Mutually disjoint sets: Many sets but non have any element in common.



Partition of sets: •can be finite or infinite.

known as partitions of A.

$A = \{A_1, A_2, A_3\}$. where A_1, A_2, A_3 are all sets.

Mutually disjoint

eg: $A = \{1, 2, 3\}$.
$A_1 = \{1\}, A_2 = \{2\}, A_3 = \{3\}$ ← partitions