Sequences

- list of things with an order (not random).
- Every sequence has a rule / pattern

eg: 2, 4, 6, 8, 10, ... : Rule = $2n$, n = term., 6^{th} term = $2(6) = 12$

$X_n = 2n+3$, n = term number.

Arithmetic Sequences

- go between terms by Adding / subtracting a common value ← must be constant

eg: 3, 9, 15, 21   +6 is the common value.
+6 +6 +6   * For fixed common value

n^{th} term = $a + (n-1)d$. ← linear curve

a = first term. n = desired term number. (eg: 6^{th} position). $= 6$.

d = common term/value.

* if not fixed, then not arithmetic sequence.

Geometric sequence

10, 30, 90, 270 ...

1, 2, 4, 8, 16 ...

* if no common ratio, then not geometric sequence.

- increment / decrement no longer constant.
- go b/w terms by multiplying / dividing common ratio

X_n (n^{th} term) $= ar^{n-1}$, a = 1^{st} term of sequence.
r = common ratio
n = term no.

Given $a = 3$, $r = 2$., X_5 (5^{th} term) $= 3(2)^{5-1} = 48$.

Given $3^{rd} = \frac{63}{4}$, $6^{th} = \frac{1701}{32}$

$ar^2 = \frac{63}{4}$, $a = \frac{63}{4r^2}$

$ar^5 = \frac{1701}{32}$

$\frac{63}{4\cancel{r^2}} \cancel{r^5}^{3} = \frac{1701}{32}$

$r^3 = \frac{\cancel{4}^{1}}{63} \frac{1701}{\cancel{32}8}$

$r = 1.5$, $a = 7$

$X_5 = 7\left(\frac{3}{2}\right)^4$
$= \frac{567}{16}$.

∃ common difference (+ or −)

Partial sums of Arithmetic sequences

- Adding first n terms of Arithmetic sequence.

eg: Given: 1, 3, 5, 7, 9, 11 ...

Partial sum of $n = 4$: $1 + 3 + 5 + 7 = 16$.

- i.e partially summing arithmetic sequence.

partial sum $S_n = \frac{n}{2}\left(2a + (n-1)d\right)$. n = term number. (eg. 60^{th} term)
a = 1^{st} term.
d = common difference.

$S_n = n\left(\frac{a + a_n}{2}\right)$ last term. (eg: if sum to 10^{th} term, 10^{th} term is last)

∃ common ratio (* or /)

Partial sum of geometric sequence

- partially summing geometric sequence up to n terms

$S_n = \frac{a(1-r^n)}{1-r}$. a = 1^{st} term, r = common ratio, $n = n_{th}$ term. (eg. 10^{th} term $= 10$)

Series

series = $a + a_1 + a_2 + \ldots$ • Not all series sum to ∞.

• sum to ∞

convergence : sum to certain value. (values < 1)

divergence : sum to $\pm\infty$

Test for geometric series (converge or diverge)

if $|r| < 1$, series converges.

$S = \frac{a}{1-r}$ a = first value, r = common ratio $= \frac{a_n}{a_{n-1}}$

↑ converged value.

if $|r| > 1$, series diverges.