```
clear
close all
```

Exercise 3.1.1. Calculate the transfer matrix G(s). Investigate each element of the matrix (Hint: G(1,1) extracts element (1,1)). Calculate the poles and zeros of the elements.

$$G(s) = C(s)(sI - A)^{-1}B + D = \frac{1}{det(sI - A)} * r(s)$$

```
sys_m = minphase
```

```
sys_m =

  A = 
             x1        x2        x3        x4
   x1  -0.05645         0   0.02572         0
   x2         0  -0.05187         0    0.0213
   x3         0         0  -0.02572         0
   x4         0         0         0   -0.0213

  B = 
            u1       u2
   x1    0.174        0
   x2        0   0.1506
   x3        0  0.09038
   x4   0.1044        0

  C = 
        x1   x2   x3   x4
   y1  0.2    0    0    0
   y2    0  0.2    0    0

  D = 
       u1  u2
   y1   0   0
   y2   0   0

Continuous-time state-space model.
```

```

% The system transfer matrix
G_m = tf(sys_m)
```

```
G_m =

  From input 1 to output...
         0.0348
   1:  -----------
       s + 0.05645

                 0.0004446
   2:  --------------------------
       s^2 + 0.07317 s + 0.001105

  From input 2 to output...
                 0.0004649
   1:  --------------------------
       s^2 + 0.08217 s + 0.001452
```

```
         0.03013
  2:  -----------
      s + 0.05187

Continuous-time transfer function.
```

```
% Poles of the individual elements
p_11 = pole(G_m(1,1))
```

```
p_11 = -0.0564
```

```
p_12 = pole(G_m(1,2))
```

```
p_12 = 2×1
   -0.0564
   -0.0257
```

```
p_21 = pole(G_m(2,1))
```

```
p_21 = 2×1
   -0.0519
   -0.0213
```

```
p_22 = pole(G_m(2,2))
```

```
p_22 = -0.0519
```

Exercise 3.1.2. Calculate the poles and zeros of the multivariable system. Do these imply any constraint on the achievable control performance?

```
% Poles of G
e_m = eig(sys_m.a)
```

```
e_m = 4×1
   -0.0564
   -0.0519
   -0.0257
   -0.0213
```

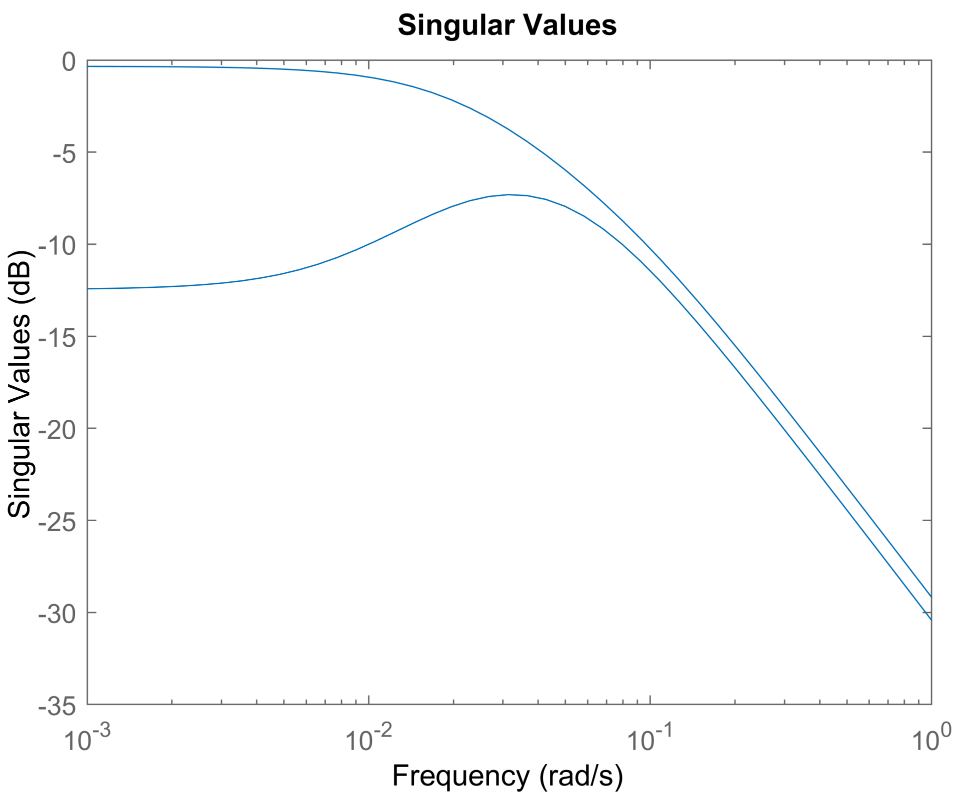
```
% Zeros of G
G_z_m = G_m(1,1) * G_m(2,2) - G_m(1,2) * G_m(2,1);

tzero(G_z_m)
```

```
ans = 4×1
   -0.0564
   -0.0519
   -0.0377
   -0.0093
```

No RHP poles or zeros that imply any restrictions.

Exercise 3.1.3. Investigate the largest and smallest singular values for the system at different frequencies. Calculate the H_∞norm of the system.

```
figure
sigma(sys_m.a, sys_m.b, sys_m.c, sys_m.d)
```




```
% Singular values at frequency 0
[U_m,S_m,V_m] = svd(evalfr(G_m,0))
```

```
U_m = 2×2
   -0.7003   -0.7139
   -0.7139    0.7003
S_m = 2×2
    0.9619         0
         0    0.2382
V_m = 2×2
   -0.7476   -0.6642
   -0.6642    0.7476
```

Exercise 3.1.4. Investigate the RGA of the system at frequency 0. What conclusions can we draw about the possibility of using decentralized control?

RGA = Relative Gain Array

pair u_i and y_j if RGA(G(ω_c)ij ~1

avoid paring of u_i and y_j if RGA(G(0))ij < 0

```
RGA = evalfr(G_m,0) .* inv(evalfr(G_m,0))'
```

```
RGA = 2×2
    1.5625   -0.5625
   -0.5625    1.5625
```

```
omega_c_m = 0.1;
i = sqrt(-1);
RGA = evalfr(G_m,i*omega_c_m) .* inv(evalfr(G_m,i*omega_c_m))'
```

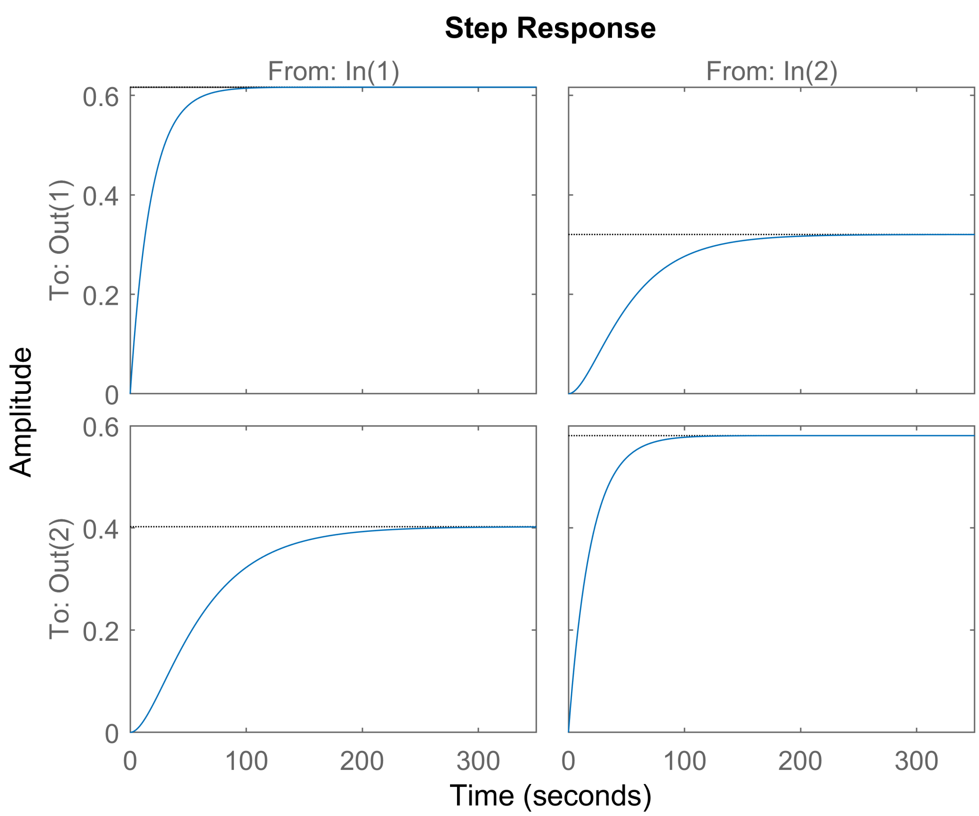
```
RGA = 2×2 complex
  -0.5013 - 0.8462i   0.0087 + 0.0162i
   0.0111 + 0.0147i  -0.5599 - 0.8086i
```

Here we can se that the cross terms λ_{12} are negative when evaluated at 0. Hence we choose not to pair u_1 and y_2 but instead u_1 with y_1 and u_2 with y_2.

Exercise 3.1.5. Plot the step response for one input at the time. Investigate the outputs: Is the system coupled? Is this in line with the properties of RGA?

- The system should be coupled since the cross terms in G(s) is non-zero.

```
figure
step(G_m)
```



In the step plot one can clearly see that a step in u_1 results in a response in y_2 and vice versa. This confirms the earlier belief.

Now solve the above problems above for the non-minimum phase case

Exercise 3.1.1.NP Calculate the transfer matrix G(s). Investigate each element of the matrix (Hint: G(1,1) extracts element (1,1)). Calculate the poles and zeros of the elements.

$$G(s) = C(s)(sI - A)^{-1}B + D = \frac{1}{det(sI - A)} * r(s)$$

```
sys_nm = nonminphase
```

```
sys_nm =

  A =
             x1        x2        x3        x4
   x1  -0.05106         0   0.08582         0
   x2         0  -0.04692         0   0.09089
   x3         0         0  -0.08582         0
   x4         0         0         0  -0.09089

  B =
            u1       u2
   x1   0.1044        0
   x2        0  0.09038
   x3        0   0.1506
   x4    0.174        0

  C =
         x1   x2   x3   x4
   y1   0.2    0    0    0
   y2     0  0.2    0    0

  D =
        u1  u2
   y1    0   0
   y2    0   0

Continuous-time state-space model.
```

```
% The system transfer matrix
G_nm = tf(sys_nm)
```

```
G_nm =

  From input 1 to output...
          0.02088
   1:  -----------
       s + 0.05106

                 0.003163
   2:  ------------------------
```

```
        s^2 + 0.1378 s + 0.004265

   From input 2 to output...
                0.002586
    1:  -------------------------
        s^2 + 0.1369 s + 0.004382

          0.01808
    2:  -----------
        s + 0.04692

  Continuous-time transfer function.
```

```
% Poles of the individual elements
p_11_nm = pole(G_nm(1,1))
```

```
p_11_nm = -0.0511
```

```
p_12_nm = pole(G_nm(1,2))
```

```
p_12_nm = 2×1
   -0.0858
   -0.0511
```

```
p_21_nm = pole(G_nm(2,1))
```

```
p_21_nm = 2×1
   -0.0909
   -0.0469
```

```
p_22_nm = pole(G_nm(2,2))
```

```
p_22_nm = -0.0469
```

Exercise 3.1.2.NP Calculate the poles and zeros of the multivariable system. Do these imply any constraint on the achievable control performance?

```
% Poles of G_nm
e_nm = eig(sys_nm.a)
```

```
e_nm = 4×1
   -0.0511
   -0.0469
   -0.0858
   -0.0909
```

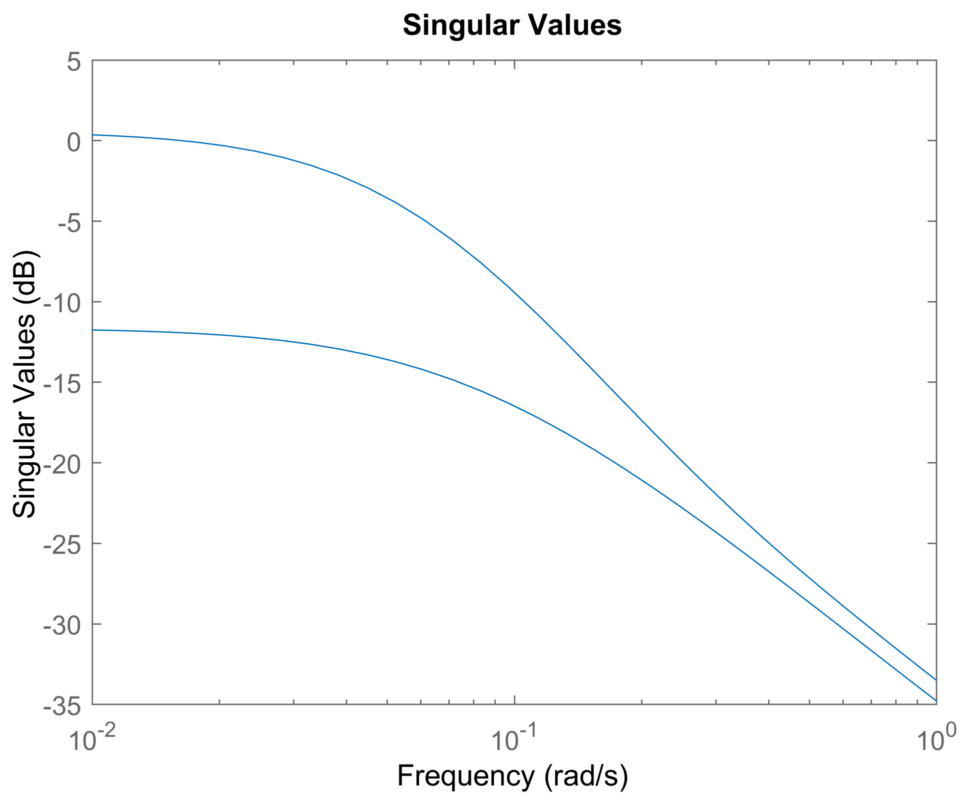
```
% Zeros of G
G_z_nm = G_nm(1,1) * G_nm(2,2) - G_nm(1,2) * G_nm(2,1);
tzero(G_z_nm)
```

```
ans = 4×1
   -0.2356
    0.0589
   -0.0511
   -0.0469
```

No RHP poles or zeros that imply any restrictions.

Exercise 3.1.3.NP Investigate the largest and smallest singular values for the system at different frequencies. Calculate the H_∞ norm of the system.

```
figure
sigma(sys_nm.a, sys_nm.b, sys_nm.c, sys_nm.d)
```




```
% Singular values at frequency 0
[U_nm,S_nm,V_nm] = svd(evalfr(G_nm,0))
```

```
U_nm = 2×2
   -0.6442   -0.7649
   -0.7649    0.6442
S_nm = 2×2
    1.0702         0
         0    0.2617
V_nm = 2×2
   -0.7762    0.6305
   -0.6305   -0.7762
```

Exercise 3.1.4.NP Investigate the RGA of the system at frequency 0. What conclusions can we draw about the possibility of using decentralized control?

RGA = Relative Gain Array

pair u_i and y_j if RGA(G(ω_c))ij ~1

avoid paring of u_i and y_j if RGA(G(0))ij < 0

```
RGA = evalfr(G_nm,0) .* inv(evalfr(G_nm,0))'
```

```
RGA = 2×2
   -0.5625    1.5625
    1.5625   -0.5625
```

```
omega_c_nm = 0.2;
i = sqrt(-1);
RGA = evalfr(G_nm,i*omega_c_nm) .* inv(evalfr(G_nm,i*omega_c_nm))'
```

```
RGA = 2×2 complex
  -0.5424 - 0.5065i   0.2506 + 0.2243i
   0.2499 + 0.2251i  -0.5617 - 0.4849i
```

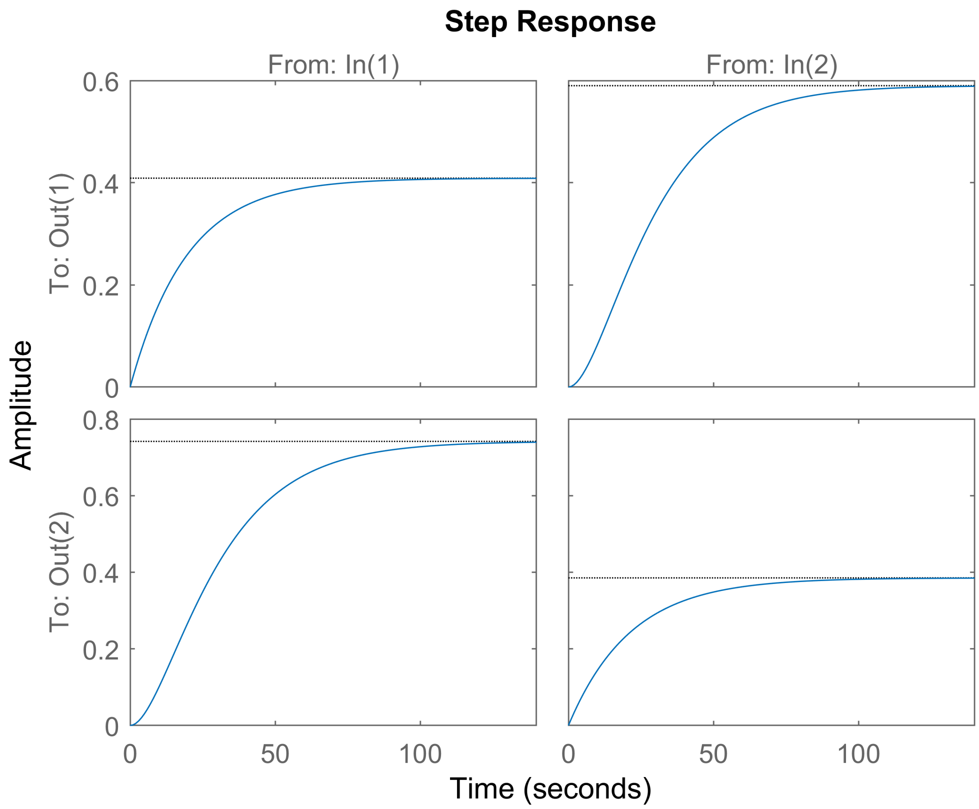
Here we can se that the cross termsλ_{12} are positive when evaluated at 0. Hence we choose to pair u_1 and y_2 instead of $u_1$with y_1 and u_2 with y_2.

Exercise 3.1.5.NP Plot the step response for one input at the time. Investigate the

outputs: Is the system coupled? Is this in line with the properties of RGA?

- The system should be coupled since the cross terms in G(s) is non-zero.

```
figure
step(G_nm)
```



In the step plot one can still clearly see that a step in u_1 results in a response in y_2 and vice versa. This once again confirms the earlier belief.

Exercise 3.1.6. Describe the most important differences between the two cases and

discuss how it affects the control performance.

The biggest difference is that in the non-minimum phase case the coupling is stronger and the input u_1 affects y_2 more than it affects y_1.

3.2 Decentralized control

Exercise 3.2.1. Design a decentralized controller by pairing inputs and outputs according

to the RGA analysis. The intended phase margin is 'm = ϕ_m/3 and the crossover

frequency !c is 0.1 rad/s for the minimum phase case and 0.02 rad/s for the

non-minimum phase case. (To make sure that the problem is correctly solved, investigate

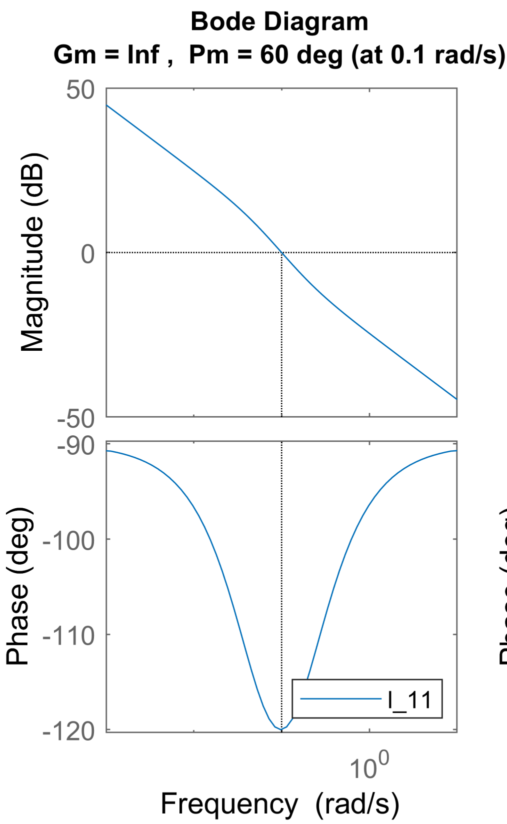
the Bode diagram of L.)

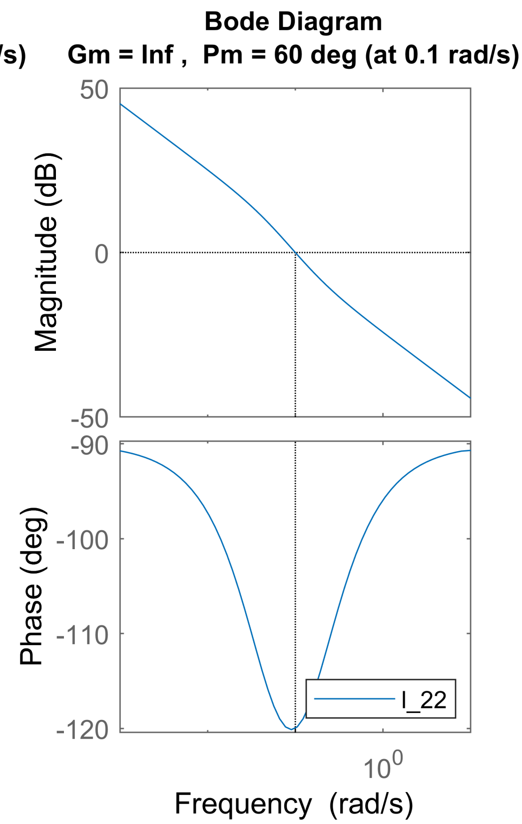
```
sys_m = minphase;

% Desired phase and crossover freq
pm = pi/3;
```

```
wc_m = 0.1;

s = tf('s');

G_m = tf(sys_m);

[~, ph_11_m] = bode(G_m(1,1), wc_m);
[~, ph_21_m] = bode(G_m(2,2), wc_m);

Ti_1_m = 1/wc_m*tan(-pi+pi/2+pm-ph_11_m*pi/180);
Ti_2_m = 1/wc_m*tan(-pi+pi/2+pm-ph_21_m*pi/180);

l11_m = G_m(1,1)*(1+1/(s*Ti_1_m));
l22_m = G_m(2,2)*(1+1/(s*Ti_2_m));

[l11_wc_m, ~] = bode(l11_m, wc_m);
[l22_wc_m, ~] = bode(l22_m, wc_m);

K1_m = 1/abs(l11_wc_m);
K2_m = 1/abs(l22_wc_m);

f1_m = K1_m*(1+1/(s*Ti_1_m));
f2_m = K2_m*(1+1/(s*Ti_2_m));

F_m = [f1_m, 0; 0, f2_m];
L_m = minreal(G_m*F_m);
```

Plot the results

```
figure()
subplot(1,2,1)
margin(L_m(1,1))
legend('l_{11}', 'Location', 'southeast')
subplot(1,2,2)
margin(L_m(2,2))
legend('l_{22}', 'Location', 'southeast')
```



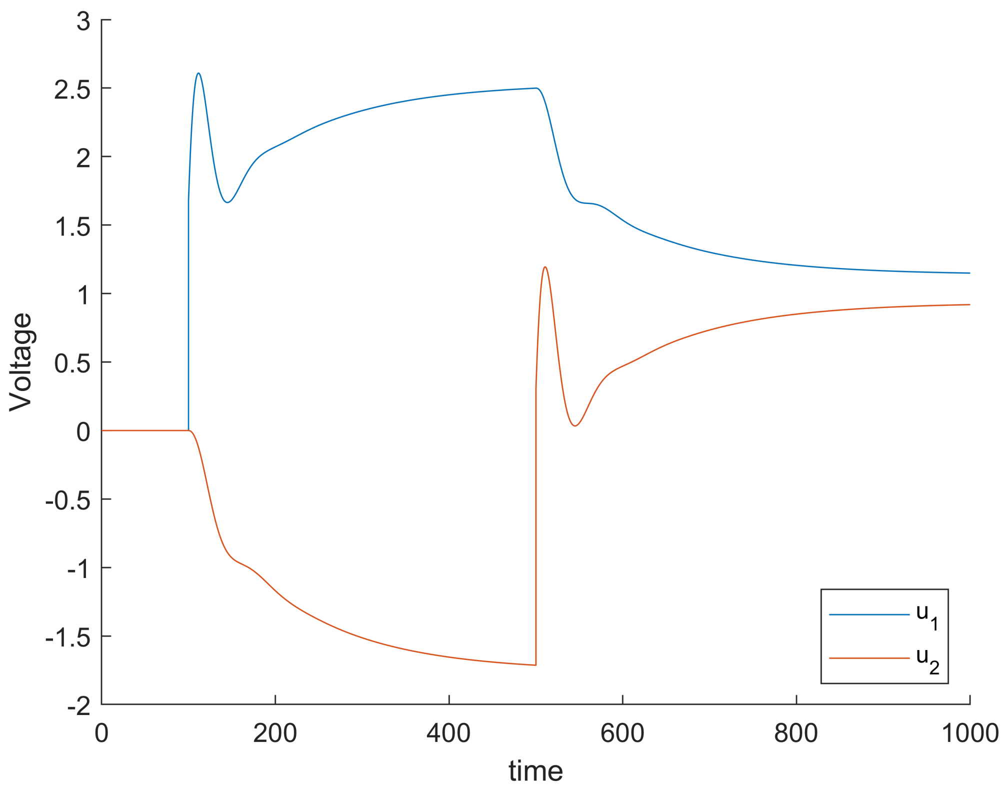



```
% Simulate system in simulink
F = F_m;
L = L_m;
G = G_m;
closedloop'
```

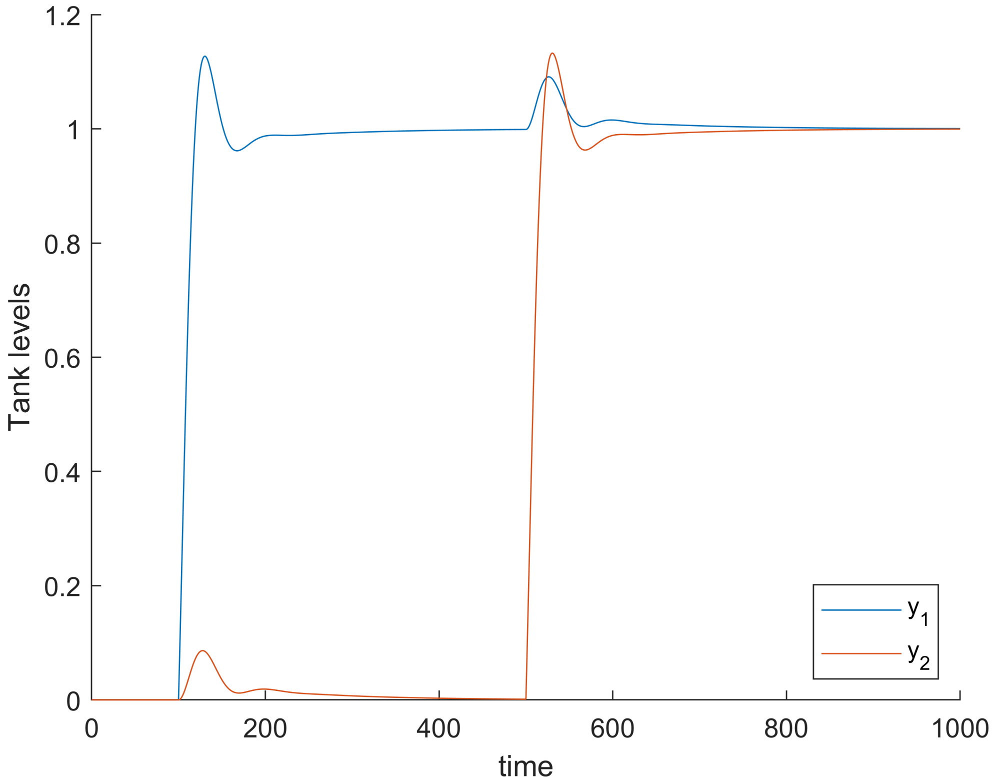
```
ans = 1×7
     8     0     0     0     0     0     2
```

```
sim('closedloop')
```

```
figure()
hold on
time_m = uout.Time;
u1_m =  uout.Data(:,1);
u2_m =  uout.Data(:,2);
plot(time_m, u1_m)
plot(time_m, u2_m)
legend('u_1', 'u_2','Location', 'southeast')
xlabel('time')
ylabel('Voltage')
hold off
```




```
figure()
hold on
y1_m =  yout.Data(:,1);
y2_m =  yout.Data(:,2);
plot(time_m, y1_m)
plot(time_m, y2_m)
legend('y_1', 'y_2','Location', 'southeast')
xlabel('time')
ylabel('Tank levels')
hold off
```




```
close_system
```

Exercise 3.2.2. Calculate the singular values of the sensitivity function

```
I = eye(2);
S_m = 1/(I+L_m);
T_m = I - S_m;

[U_S_m,S_S_m,V_S_m] = svd(evalfr(S_m,0))
```

```
U_S_m = 2×2
     1     0
     0     1
S_S_m = 2×2
     0     0
     0     0
V_S_m = 2×2
     1     0
     0     1
```

```
[U_T_m,S_T_m,V_T_m] = svd(evalfr(T_m,0))
```

```
U_T_m = 2×2
     1     0
     0     1
S_T_m = 2×2
     1     0
     0     1
V_T_m = 2×2
     1     0
```

```
    0     1
```

Yet again, now solve the above problems above for the non-minimum phase case

Exercise 3.2.1. Design a decentralized controller by pairing inputs and outputs according to the RGA analysis. The intended phase margin is 'm = ϕ_m/3 and the crossover frequency !c is 0.1 rad/s for the minimum phase case and 0.02 rad/s for the non-minimum phase case. (To make sure that the problem is correctly solved, investigate the Bode diagram of L.)

```
sys_nm = nonminphase
```

```
sys_nm =

  A =
             x1        x2        x3        x4
   x1  -0.05106         0   0.08582         0
   x2         0  -0.04692         0   0.09089
   x3         0         0  -0.08582         0
   x4         0         0         0  -0.09089

  B =
            u1       u2
   x1   0.1044        0
   x2        0  0.09038
   x3        0   0.1506
   x4    0.174        0

  C =
         x1   x2   x3   x4
   y1  0.2    0    0    0
   y2    0  0.2    0    0

  D =
       u1  u2
   y1   0   0
   y2   0   0

Continuous-time state-space model.
```

```
% Desired phase and crossover freq
pm = pi/3;
wc_nm = 0.02;

s = tf('s');

G_nm = tf(sys_nm);

[~, ph_12_nm] = bode(G_nm(1,2), wc_nm);
[~, ph_21_nm] = bode(G_nm(2,1), wc_nm);

Ti_1_nm = 1/wc_nm*tan(-pi+pi/2+pm-ph_12_nm*pi/180);
Ti_2_nm = 1/wc_nm*tan(-pi+pi/2+pm-ph_21_nm*pi/180);
```

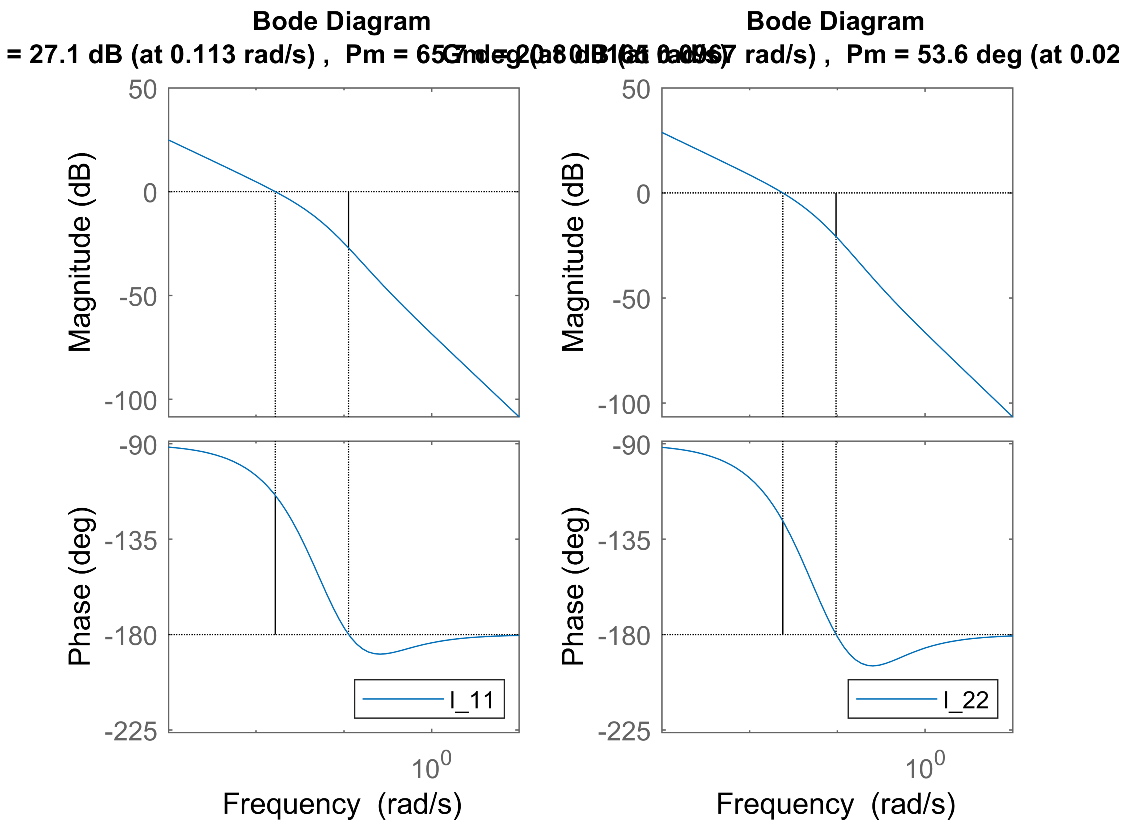
```
l12_nm = G_nm(1,2)*(1+1/(s*Ti_1_nm));
l21_nm = G_nm(2,1)*(1+1/(s*Ti_2_nm));

[l12_wc_nm, ~] = bode(l12_nm, wc_nm);
[l21_wc_nm, ~] = bode(l21_nm, wc_nm);

K1_nm = 1/abs(l12_wc_nm);
K2_nm = 1/abs(l21_wc_nm);

f1_nm = K1_nm*(1+1/(s*Ti_1_nm));
f2_nm = K2_nm*(1+1/(s*Ti_2_nm));

F_nm = [0, f1_nm; f2_nm, 0]
```

```
F_nm =

  From input 1 to output...
   1:  0

       0.6915 s + 0.1437
   2:  -----------------
            4.811 s

  From input 2 to output...
       0.5792 s + 0.1469
   1:  -----------------
            3.943 s

   2:  0

Continuous-time transfer function.
```

```
L_nm = minreal(G_nm*F_nm);
```

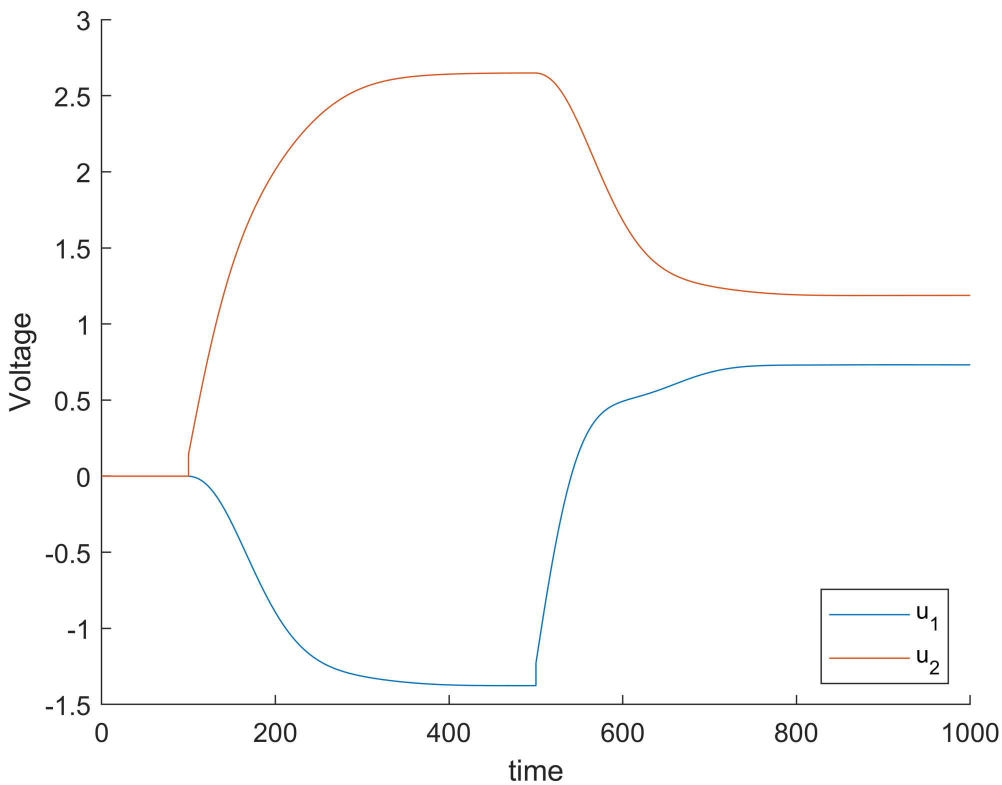
Plot the results

```
figure()
subplot(1,2,1)
margin(L_nm(1,1))
legend('l_{11}', 'Location', 'southeast')
subplot(1,2,2)
margin(L_nm(2,2))
legend('l_{22}', 'Location', 'southeast')
```

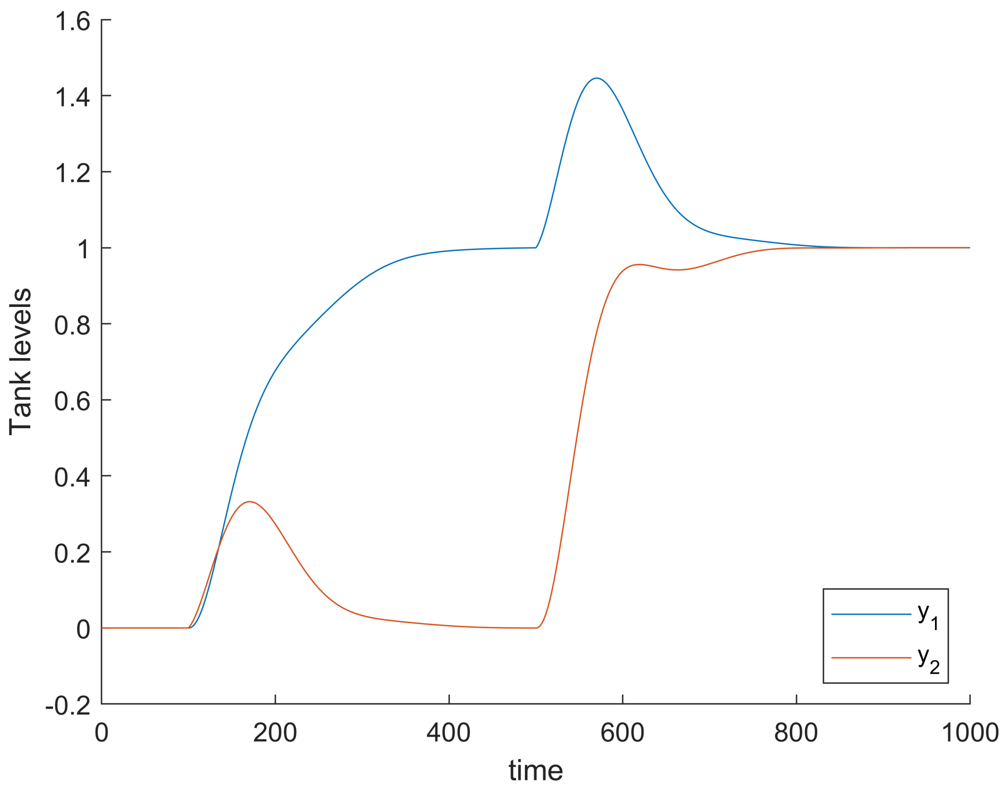



```
% Simulate system in simulink
F = F_nm;
L = L_nm;
G = G_nm;
closedloop
sim('closedloop')
```

```
figure()
hold on
time_nm = uout.Time;
u1_nm =  uout.Data(:,1);
u2_nm =  uout.Data(:,2);
plot(time_nm, u1_nm)
plot(time_nm, u2_nm)
legend('u_1', 'u_2','Location', 'southeast')
xlabel('time')
ylabel('Voltage')
hold off
```




```
figure()
hold on
y1_nm =  yout.Data(:,1);
y2_nm =  yout.Data(:,2);
plot(time_nm, y1_nm)
plot(time_nm, y2_nm)
legend('y_1', 'y_2','Location', 'southeast')
xlabel('time')
ylabel('Tank levels')
```




```
close_system
```

Exercise 3.2.2. Calculate the singular values of the sensitivity function

```
I = eye(2);
S_nm = 1/(I+L_nm);
T_nm = I - S_nm;

[U_S_nm,S_S_nm,V_S_nm] = svd(evalfr(S_nm,0))
```

```
U_S_nm = 2×2
     1     0
     0     1
S_S_nm = 2×2
     0     0
     0     0
V_S_nm = 2×2
     1     0
     0     1
```

```
[U_T_nm,S_T_nm,V_T_nm] = svd(evalfr(T_nm,0))
```

```
U_T_nm = 2×2
     1     0
     0     1
S_T_nm = 2×2
     1     0
     0     1
V_T_nm = 2×2
     1     0
```

0 1