Computer Exercise 2
EL2520 Control Theory and Practice

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Minimum phase case

For the minimum phase case, the cross terms λ_{12} of the Relative Gain Array are negative when evaluated at $\omega = 0$. Hence we choose to pair u_1 and y_1,and u_2 and y_2. The controller $F(s)$ will thus be of the form:

$$F(s) = \begin{bmatrix} f_1(s) & 0 \\ 0 & f_2(s) \end{bmatrix}$$

where $f_i = K_i(1 + \dfrac{1}{T_i s})$, $i \in \{1, 2\}$. The computed controller was:

$$F(s) = \begin{bmatrix} \frac{9.904s+1.678}{5.904s} & 0 \\ 0 & \frac{12.87s+2.014}{6.391s} \end{bmatrix}$$

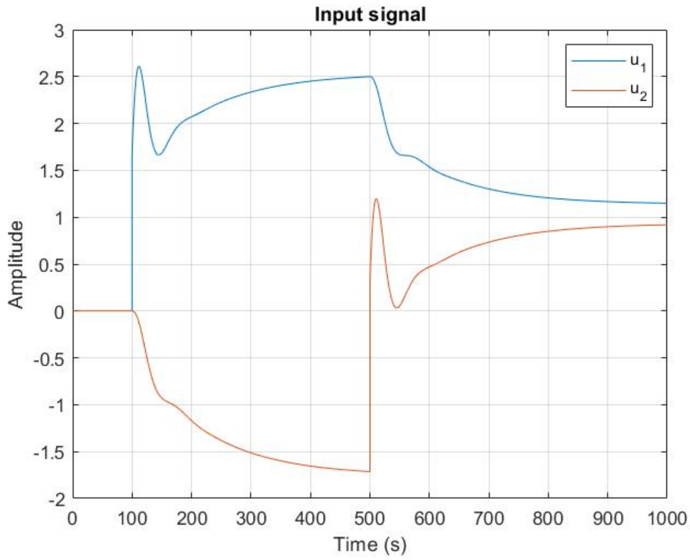

Figure 1: Control signal

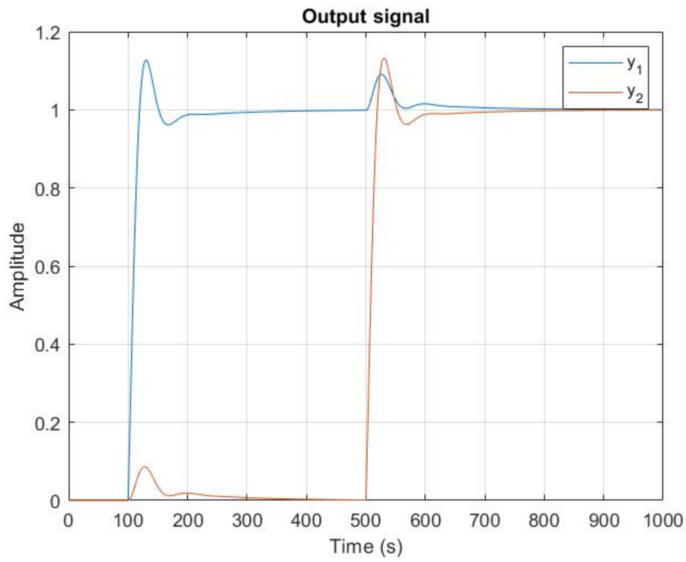

Figure 2: Output signal

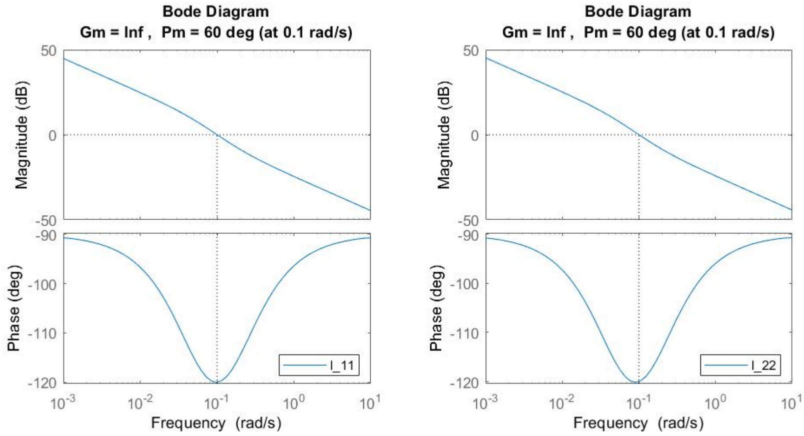

Figure 3: Bode diagram of the loop gain $L(s)$. $l_{11} = left$ and $l_{22} = right$

Is the controller good?
Looking at figure 2, control performance looks good. The system converges quickly after each step input and the loop gain satisfies both desired phase margin and cross-over frequency.

Are the output signals coupled?
As the step response plots from u_1 to y_2 and from u_2 to y_1 in G(s) suggests, the signals are coupled. On top of that the cross terms g_{12} and g_{21} are non-zero which reinforces the conclusion.

Non-minimum phase case

For the non-minimum phase case, the terms λ_{11} and $\lambda 22$ of the Relative Gain Array are negative when evaluated at s = 0. Hence we choose to pair u_1 with y_2 instead, and u_2 with y_1. The controller $F(s)$ will thus be of the form:

$$F(s) = \begin{bmatrix} 0 & f_1(s) \\ f_2(s) & 0 \end{bmatrix}$$

where $f_i = K_i(1 + \dfrac{1}{T_i s})$, $i \in \{1, 2\}$.The computed controller was:

$$F(s) = \begin{bmatrix} 0 & \frac{0.6915s+0.1437}{4.811s} \\ \frac{0.5792s+0.1469}{3.943s} & 0 \end{bmatrix}$$

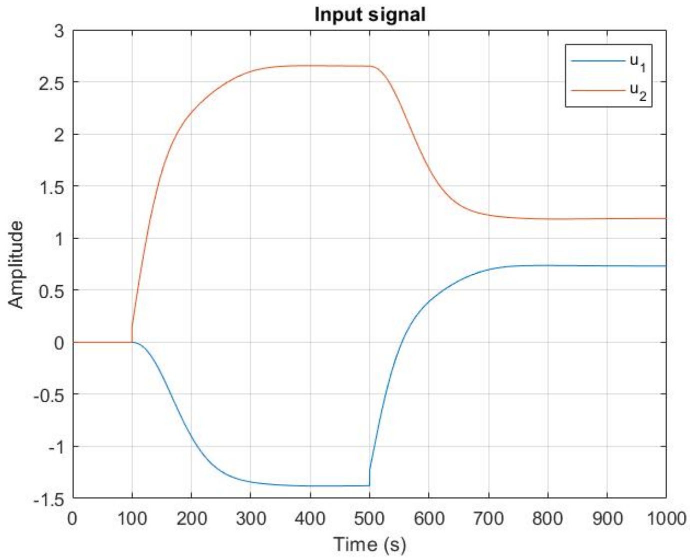

Figure 4: Control signal

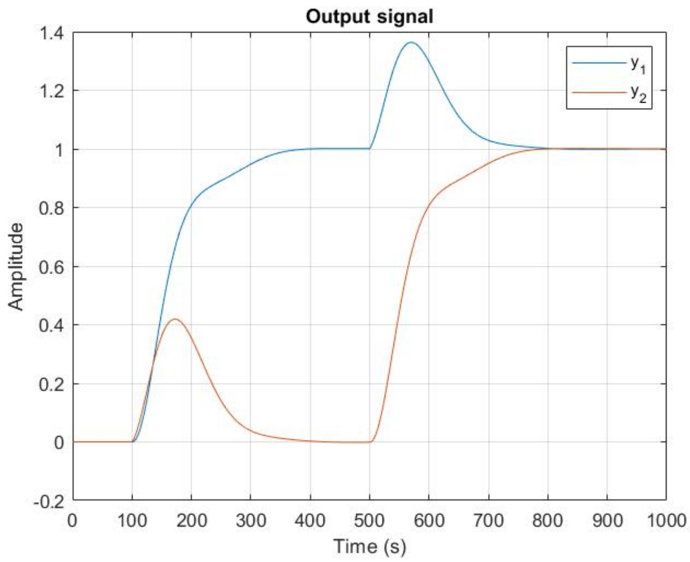

Figure 5: Output signal

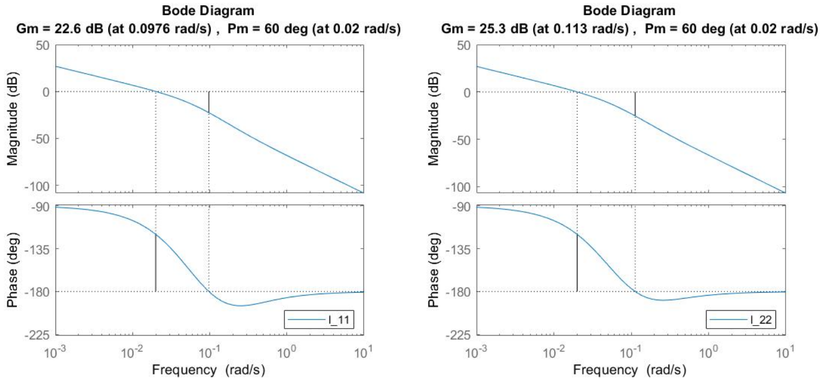

Figure 6: Bode diagram of the loop gain $L(s)$. $l_{11} = left$ and $l_{22} = right$

Is the controller good?
Comparing output signal plots from both cases it is clear that the control in the non-minimum phase performs worse. Performance is not very good. It is slow, but the system still converges. From the bode diagram in figure 6 we note that the loop gain has the desired phase margin and cross-over frequency.

Are the output signals coupled?
As can be seen in figure 5, the output signals are coupled. Both outputs react to both inputs. Worth noting is that the effects are quite a bit worse in the non-minimum phase case.