## Ejercicio 4

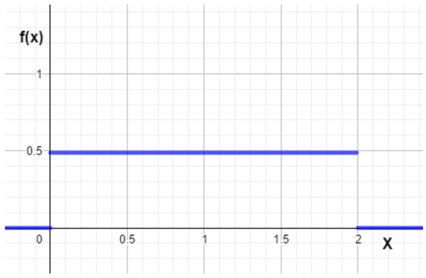
Sea X una variable aleatoria (v.a) continua que se distribuye según:

$$f(x)=\begin{cases} 1/2 & 0\le x\le 2 \\ 0 & \forall_{otro}\ x \end{cases}$$

a) Verifique que f(x) es función de densidad. Grafique f(x).

Debo verificar que: i) $f(x)\ge 0 \quad \forall x\ \epsilon\ R_X$ *(no negatividad)*

ii) y que $\int_0^2 f(x)dx = 1$ *(ley de cierre)*



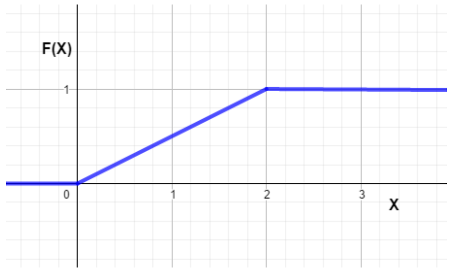

i) $\frac{1}{2}\ge 0\ \forall x\ \epsilon\ [0,2]$

ii) $\int_0^2 f(x)dx = \int_0^2 \frac{1}{2}dx = \frac{1}{2}x\Big|_0^2 = 1$

b) Halle la función de distribución acumulada. Grafíquela.

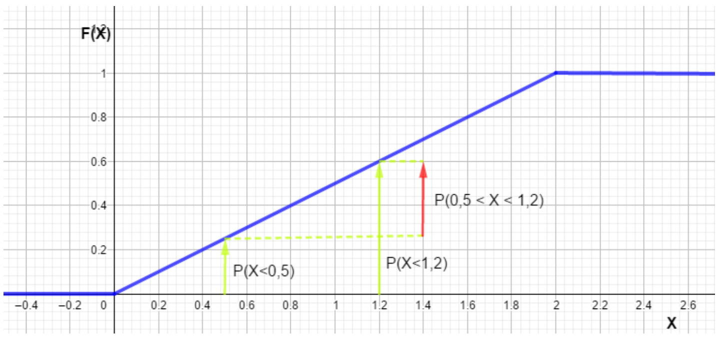
$$F(x_0)=P(X\le x_0)=\int_0^{x_0} f(s)ds=\int_0^{x_0}\frac{1}{2}ds=\frac{1}{2}s\Big|_0^{x_0}=\frac{1}{2}x_0$$

$$F(X)=\begin{cases} 0 & si\ x<0 \\ \frac{1}{2}x & si\ 0\le x\le 2 \\ 1 & si\ x>2 \end{cases}$$




c) Calcule y represente gráficamente las siguientes probabilidades:

i) $\boldsymbol{P(0,5<X<1,2)} = F(1{,}2)-F(0{,}5) = 0{,}5\cdot 1{,}2 - 0{,}5\cdot 0{,}5 = \mathbf{0{,}35}$

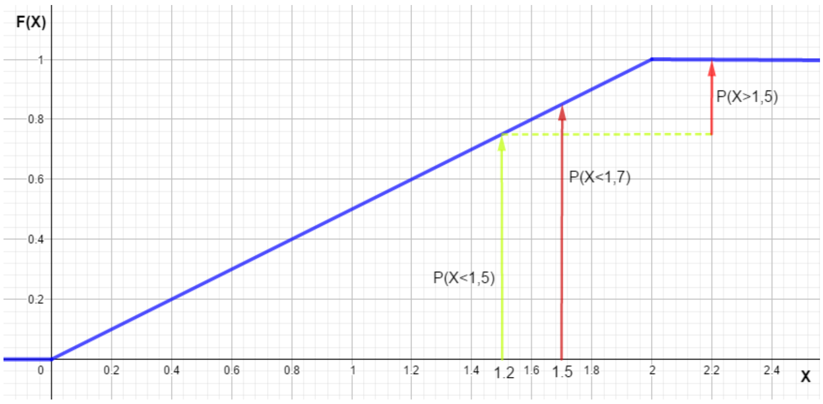




ii) $\boldsymbol{P(X>1,5)} = 1-F(1{,}5) =$

$= 1-0{,}5\cdot 1{,}5 = \mathbf{0{,}25}$

iii) $\boldsymbol{P(X\le 1,7)} = F(1{,}7) = 0{,}5\cdot 1{,}7 =$

$= \mathbf{0{,}85}$




iv) $\boldsymbol{P(X=1)=0}$

v) $\boldsymbol{F(-1)=P(X\le -1)=0}$

vi) $\boldsymbol{F(1,7)} = P(X\le 1{,}7) = \mathbf{0{,}85}$

vii) $\boldsymbol{F(4)} = P(X\le 4) = \mathbf{1}$

d) Calcule E(x) y σ(x).

$$\boldsymbol{\mu = E(X)} = \int_0^2 x\cdot f(x)\cdot dx = \int_0^2 x\cdot\frac{1}{2}\cdot dx = \frac{1}{4}x^2\Big|_0^2 = \mathbf{1}$$

$$\boldsymbol{\sigma(X)} = \sqrt{\int_0^2 x^2\cdot f(x)\cdot dx - \mu^2} = \sqrt{\int_0^2 x^2\cdot\frac{1}{2}\cdot dx - \mu^2} = \sqrt{\frac{1}{6}x^3\Big|_0^2 - 1} = \frac{4}{3}$$