Ejercicio 5

a)

Para que $f(x) = \begin{cases} k\,x^2 & 0 < x \leq 10 \\ 0 & para\ todo\ otro\ x \end{cases}$

sea función de densidad de probabilidad se debe verificar que:

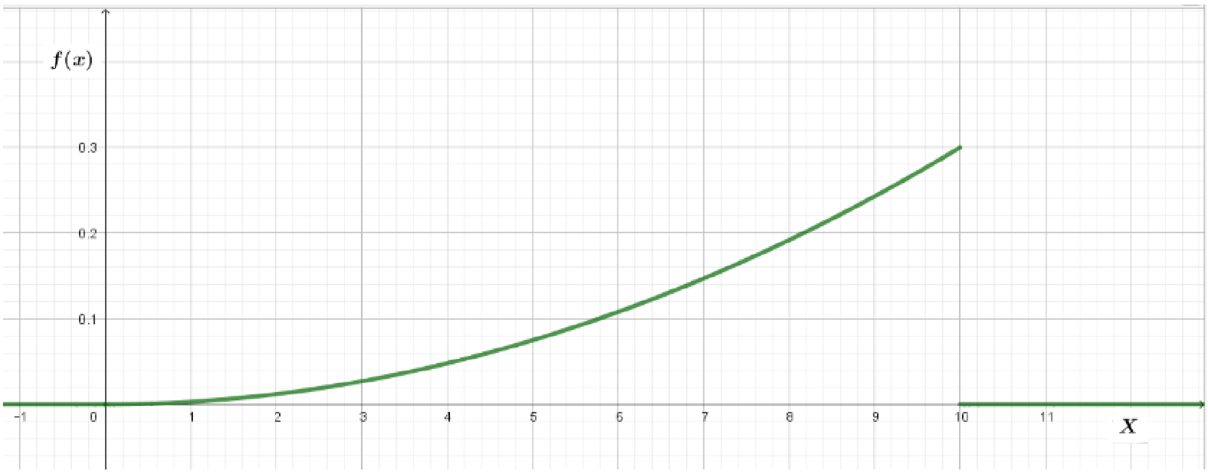
i) $f(x) \geq 0 \quad \forall x \in R_X$ (*no negatividad)*

ii) y que $\int_0^{10} f(x)dx = 1$ *(ley de cierre)*

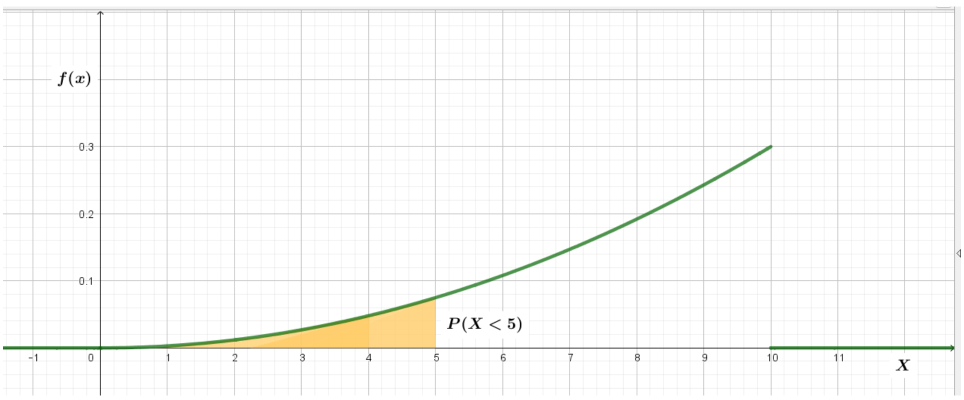
i) como $0 < x \leq 10$ deberá ser $\boldsymbol{k \geq 0}$

ii) $\int_0^{10} kx^2 \cdot dx = 1$ entonces $\frac{1}{3}kx^3\Big|_0^{10} = 1 \quad \rightarrow \quad \frac{1}{3}k10^3 = 1 \quad \rightarrow k = \frac{3}{1.000} \quad \rightarrow \boldsymbol{k = 0{,}003}$

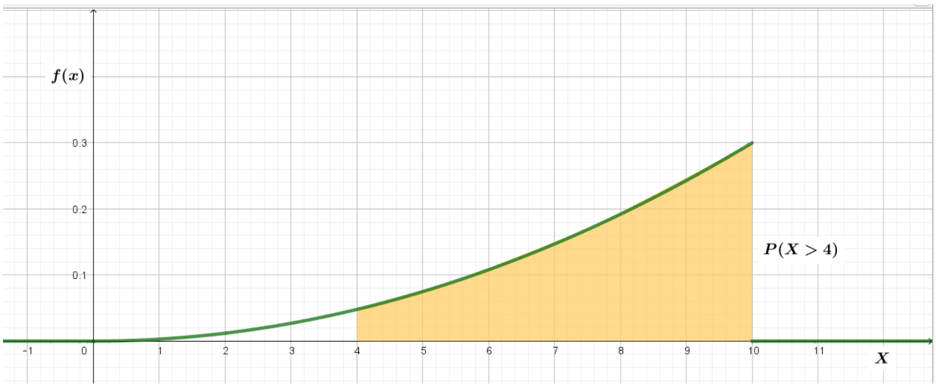
luego $f(x) = \begin{cases} 0{,}003x^2 & si\ 0 < x \leq 10 \\ 0 & \forall o\ x \end{cases}$



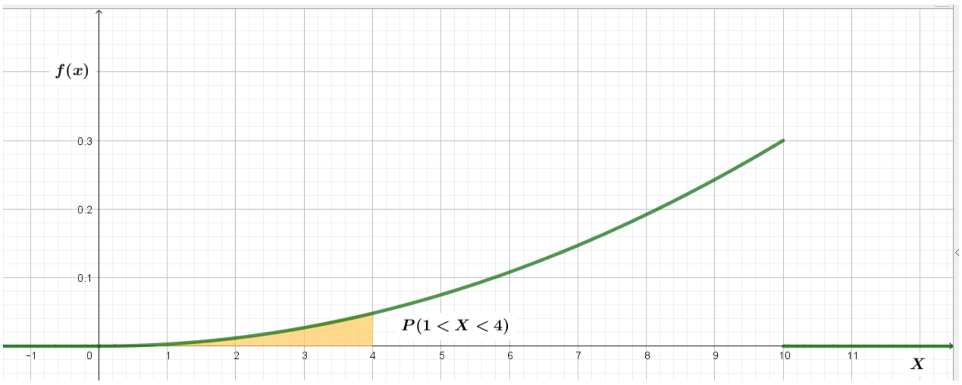
b) y c)



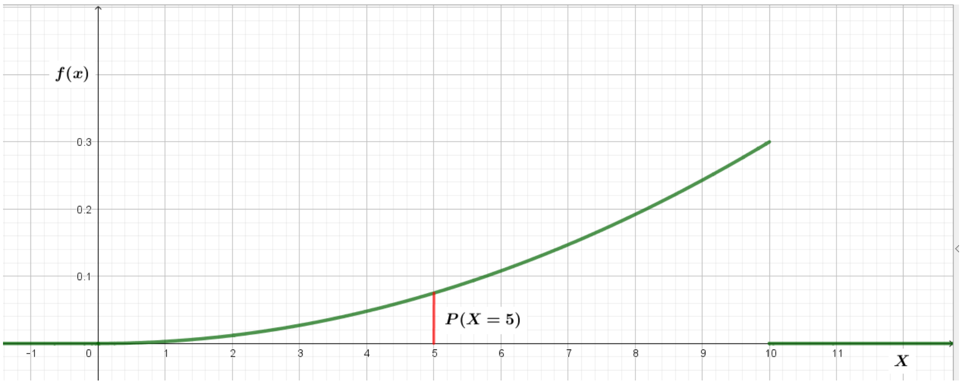
$$P(X < 5) = \int_0^5 0{,}003x^2dx = 0{,}001x^3\Big|_0^5 = 0{,}001 \cdot 5^3 = 0{,}125$$



$$P(X > 4) = 1 - P(X < 4) = 1 - \int_0^4 0{,}003x^2dx = 0{,}001x^3\Big|_0^4 = 1 - 0{,}001 \cdot 4^3 = 0{,}936$$



$$\mathrm{P(1 < X < 4) = P(X < 4) - P(X < 1)} = \int_0^4 0{,}003\mathrm{x}^2\mathrm{dx} - \int_0^1 0{,}003\mathrm{x}^2\mathrm{dx} = 0{,}001\mathrm{x}^3\Big|_0^4 - 0{,}001\mathrm{x}^3\Big|_0^1 = 0{,}063$$



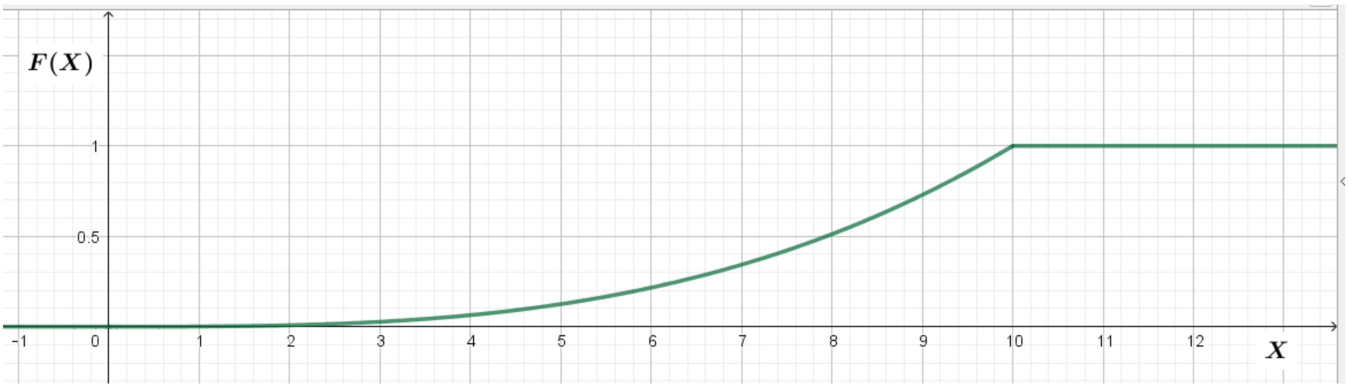
$$P(X = 5) = \int_5^5 0{,}003x^2dx = 0$$

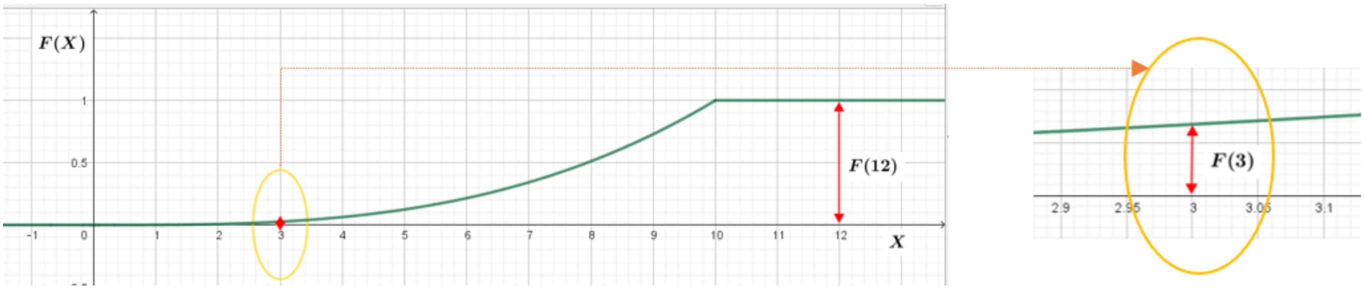
d)

Función de distribución de probabilidad acumulada

$$F(x_0) = P(X \leq x_0) = \int_0^{x_0} f(s)ds = \int_0^{x_0} 0{,}003s^2ds = 0{,}001s^3\Big|_0^{x_0} = 0{,}001{x_0}^3$$

$$F(X) = \begin{cases} 0 & si \quad x < 0 \\ 0{,}001x^3 & si \quad 0 \leq x \leq 10 \\ 1 & si \quad x > 10 \end{cases}$$





$F(X = 3) = 0{,}001 \cdot 3^3 = 0{,}027 \qquad F(X = 12) = 1$

e)

$$\mu = E(X) = \int_{R_X} x \cdot f(x) \cdot dx = \int_0^{10} x \cdot 0{,}003 \cdot x^2 \cdot dx = 0{,}00075 \cdot x^4|_0^{10} = 7{,}5$$

$$V(X) = \int_{R_X} x^2 \cdot f(x) \cdot dx - \mu^2 = \int_0^{10} x^2 \cdot 0{,}003 \cdot x^2 \cdot dx - 7{,}5^2 = 0{,}0006 \cdot x^5|_0^{10} - 7{,}5^2 = 3{,}75$$