

CÁLCULO MULTIVARIABLE

DICCIONARIO ESPAÑOL-INGLÉS DE VOCABULARIO ACADÉMICO



CÁLCULO MULTIVARIABLE

A



Área

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Medida de una superficie.

$$\begin{aligned} &= -2\pi \int_{9a^2}^0 (9a^2 - t) \sqrt{t} dt \quad t = 9a^2 - \rho^2 \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$



$$\begin{aligned} &= 2 \left[9a^2 \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_0^{9a^2} \\ &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{54}{5} \pi a^5. \end{aligned}$$

Area
é-rea



Measure of a surface.

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Aproximación



$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2 - \rho^2} d\rho$$

Valor cercano a uno real.

$$\begin{aligned} &= -2\pi \int_0^0 (9a^2 - t) \sqrt{t} dt \quad t = 9a^2 - \rho^2 \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$



Approximation
ap-roksi-méishon

$$\begin{aligned} &= 2\pi \left[9a^2 \frac{2}{3} t^{3/2} - \frac{2}{5} t^{5/2} \right]_0^{9a^2} \\ &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{24}{5} \pi a^5 \end{aligned}$$



Close value to a real one.

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Altura

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Distancia vertical.

$$t = 9a^2 - \rho^2$$

$$\begin{aligned} &= -2\pi \int_{9a^2}^0 (9a^2 - t) \sqrt{t} dt \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$



$$\begin{aligned} &= 2\pi \left[\frac{9a^2 \cdot 2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_0^{9a^2} \\ &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{64}{5} \pi a^5. \end{aligned}$$

Height



Vertical distance.

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Aceleración



$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Cambio de velocidad.

$$t = 9a^2 - \rho^2$$

$$\begin{aligned} &= -2\pi \int_{9a^2}^0 (9a^2 - t) \sqrt{t} dt \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$



Acceleration
ak-se-le-rei-shon

$$\begin{aligned} &= 2\pi \left[9a^2 \cdot \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_{9a^2}^0 \\ &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{648\pi}{5} a^5. \end{aligned}$$



Change of speed.

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Análisis

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Estudio detallado de algo.

$$\begin{aligned} &= 2\pi \int_0^{3a} \int_0^0 (9a^2-t)\sqrt{t} dt \quad t = 9a^2 - \rho^2 \\ &= 2\pi \int_0^{9a^2} (9a^2\sqrt{t} - t\sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2\sqrt{t} dt - \int_0^{9a^2} t\sqrt{t} dt \right] \end{aligned}$$



Analysis

$$\begin{aligned} &= 2\pi \left[\frac{2}{3} 9a^2 t^{3/2} - \frac{2}{5} t^{5/2} \right]_{t=0}^{t=9a^2} \\ &= 2 \cdot \frac{2}{3} \pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{64}{15} \pi a^5 \end{aligned}$$

a-ná-li-sis



Detailed study of something.

CÁLCULO MULTIVARIABLE

B



Base

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Fundamento o soporte de algo.



$$\begin{aligned} &= -2\pi \int_{9a^2}^0 (9a^2 - t) \sqrt{t} dt \quad t = 9a^2 - \rho^2 \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$

Base

$$\begin{aligned} &= 2 \cdot \left[9a^2 \cdot \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_0^{9a^2} \\ &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{648\pi}{5} a^5. \end{aligned}$$

beis



Foundation or support of something.

CÁLCULO MULTIVARIABLE

B

Bivariable



$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Con dos variables.

$$\begin{aligned}
 &= -2\pi \int_0^0 (9a^2-t)\sqrt{t} dt & t = 9a^2 - \rho^2 \\
 &= 2\pi \int_0^{9a^2} (9a^2\sqrt{t} - t\sqrt{t}) dt \\
 &= 2\pi \left[\int_0^{9a^2} 9a^2\sqrt{t} dt - \int_0^{9a^2} t\sqrt{t} dt \right]
 \end{aligned}$$



Bivariate

$$\begin{aligned}
 &= 2\pi \left[9a^2 \frac{2}{3} t^{3/2} - \frac{2}{5} t^{5/2} \right]_0^{9a^2} \\
 &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\
 &= \frac{648\pi}{5} a^5.
 \end{aligned}$$

bai-vé-reit



With two variables.

CÁLCULO MULTIVARIABLE

B



Borde

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Límite de una región.

$$t = 9a^2 - \rho^2$$

$$\begin{aligned} &= 2\pi \int_0^{9a^2} (9a^2 - t) \sqrt{t} dt \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$



$$\begin{aligned} &= 2 \cdot \left[9a^2 \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_0^{9a^2} \\ &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{648\pi}{5} a^5. \end{aligned}$$

Edge edch



Boundary of a region.

CÁLCULO MULTIVARIABLE

B



Brújula

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Indica dirección.

$$t = 9a^2 - \rho^2$$

$$\begin{aligned} &= -2\pi \int_0^0 (9a^2 - t) \sqrt{t} dt \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$



$$\begin{aligned} &= 2\pi \left[9a^2 \frac{2}{3} t^{3/2} - \frac{2}{5} t^{5/2} \right]_0^{9a^2} \\ &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{54\pi}{5} a^5 \end{aligned}$$

Compass
kómpas



Indicates direction.

CÁLCULO MULTIVARIABLE

B



Bloque

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Porción sólida de materia.

$$t = 9a^2 - \rho^2$$

$$= 2\pi \int_{9a^2}^0 (9a^2 - t) \sqrt{t} dt$$

$$= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt$$

$$= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right]$$

$$= 2\pi \left[9a^2 \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_0^{9a^2}$$

$$= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right)$$

$$= \frac{648\pi}{5} a^5$$



Block
blok



Solid piece of matter.

CÁLCULO MULTIVARIABLE

C



Campo

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Región donde actúa una magnitud.



$$\begin{aligned} &= -2\pi \int_{9a^2}^0 (9a^2 - t) \sqrt{t} dt \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$

Field

$$\begin{aligned} &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{64}{5} \pi a^5. \end{aligned}$$

field



Region where a magnitude acts.

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C



Curva

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2 - \rho^2} d\rho$$

Línea sin ángulos.

$$t = 9a^2 - \rho^2$$

$$\begin{aligned} &= -2\pi \int_{9a^2}^0 (9a^2 - t) \sqrt{t} dt \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$



Curve

$$\begin{aligned} &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{648\pi}{5} a^5. \end{aligned}$$

kerv



Line without angles.

CÁLCULO MULTIVARIABLE

C



Cambio

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Variación de una cantidad.

$$\begin{aligned} &= -2\pi \int_{9a^2}^0 (9a^2-t)\sqrt{t} dt \quad t = 9a^2 - \rho^2 \\ &= 2\pi \int_0^{9a^2} (9a^2\sqrt{t} - t\sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2\sqrt{t} dt - \int_0^{9a^2} t\sqrt{t} dt \right] \end{aligned}$$



$$\begin{aligned} &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{648\pi}{5} a^5. \end{aligned}$$

Change
cheinch



Variation of a quantity.

CÁLCULO MULTIVARIABLE

C

Continui-
dad



$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2 - \rho^2} d\rho$$

Sin interrupciones.

$$t = 9a^2 - \rho^2$$

$$\begin{aligned} &= -2\pi \int_{9a^2}^0 (9a^2 - t) \sqrt{t} dt \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$



Continuity

kon-ti-nuí-ti

$$\begin{aligned} &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{648\pi}{5} a^5. \end{aligned}$$



Without interruptions.

CÁLCULO MULTIVARIABLE

C

Coordenada



$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Número que indica posición.

$$\begin{aligned} &= -2\pi \int_0^0 (9a^2 - t) \sqrt{t} dt \quad t = 9a^2 - \rho^2 \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$



Coordinate
koor-di-neit

$$\begin{aligned} &= 2\pi \left[9a^2 \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_0^{9a^2} \\ &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{648\pi}{5} a^5. \end{aligned}$$



Number indicating position.

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D

Derivada



$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Tasa de cambio instantánea.

$$\begin{aligned} &= -2\pi \int_{9a^2}^0 (9a^2 - t)\sqrt{t} dt \quad t = 9a^2 - \rho^2 \\ &= 2\pi \int_0^{9a^2} (9a^2\sqrt{t} - t\sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2\sqrt{t} dt - \int_0^{9a^2} t\sqrt{t} dt \right] \end{aligned}$$



Derivative
de-rí-va-tiv

$$\begin{aligned} &= 2 \cdot \left[9a^2 \cdot \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_{0}^{9a^2} \\ &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{648\pi}{5} a^5. \end{aligned}$$



Instantaneous rate of change.

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D



Diferencia

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Distinción entre valores.

$$\begin{aligned} &= -2\pi \int_0^0 (9a^2 - t) \sqrt{t} dt \quad t = 9a^2 - \rho^2 \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$



Difference
di-fé-rens

$$\begin{aligned} &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{648\pi}{5} a^5. \end{aligned}$$



Distinction between values.

CÁLCULO MULTIVARIABLE

D



Dimensión

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Medida en una dirección.

$$\begin{aligned} &= -2\pi \int_{9a^2}^0 (9a^2 - t) \sqrt{t} dt \quad t = 9a^2 - \rho^2 \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$



Dimension
di-men-shon



Measure in one direction.

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D

Divergente



$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Que se separa.

$$t = 9a^2 - \rho^2$$

$$\begin{aligned} &= -2\pi \int_{9a^2}^0 (9a^2 - t) \sqrt{t} dt \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$



Divergent
dai-vér-yent



That separates.

CÁLCULO MULTIVARIABLE

D



Dirección

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Orientación de una línea.

$$\begin{aligned} &= -2\pi \int_0^0 (9a^2 - t)\sqrt{t} dt \quad t = 9a^2 - \rho^2 \\ &= 2\pi \int_0^{9a^2} (9a^2\sqrt{t} - t\sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2\sqrt{t} dt - \int_0^{9a^2} t\sqrt{t} dt \right] \end{aligned}$$



**Direction
dai-rek-shon**

$$\begin{aligned} &= 2\pi \left[9a^2 \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_0^{9a^2} \\ &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{648\pi}{5} a^5. \end{aligned}$$



Orientation of a line.

CÁLCULO MULTIVARIABLE



E

Ecuación

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2 - \rho^2} d\rho$$

Relación entre variables. $t = 9a^2 - \rho^2$

$$\begin{aligned} &= -2\pi \int_0^0 (9a^2 - t) \sqrt{t} dt \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$



Equation
e-kuei-shon



Relationship between variables.

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E

Escalar

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2 - \rho^2} d\rho$$

Cantidad sin dirección. $t = 9a^2 - \rho^2$

$$\begin{aligned} &= 2\pi \int_0^0 (9a^2 - t) \sqrt{t} dt \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$



Scalar

$$\begin{aligned} &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{648\pi}{5} a^5 \end{aligned}$$

ské-lar



Quantity without direction.

CÁLCULO MULTIVARIABLE



E

Eje

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Línea de referencia.

$$t = 9a^2 - \rho^2$$

$$= 2\pi \int_{9a^2}^0 (9a^2 - t) \sqrt{t} dt$$

$$= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt$$

$$= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right]$$

$$= 2\pi \left[9a^2 \cdot \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_0^{9a^2}$$

$$= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right)$$

$$= \frac{64\pi}{5} a^5.$$

Axis

axis



Reference line.

CÁLCULO MULTIVARIABLE



E

Extremo

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Punto máximo o mínimo.

$$\begin{aligned} &= -2\pi \int_0^0 (9a^2 - t) \sqrt{t} dt \quad t = 9a^2 - \rho^2 \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$



Extreme eks-trím

$$\begin{aligned} &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{648\pi}{5} a^5 \end{aligned}$$



Maximum or minimum
point.

CÁLCULO MULTIVARIABLE



E

Elipse

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Curva cerrada ovalada.

$$\begin{aligned} &= -2\pi \int_0^0 (9a^2 - t) \sqrt{t} dt \quad t = 9a^2 - \rho^2 \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$



Elipse

$$\begin{aligned} &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{648\pi}{5} a^5 \end{aligned}$$

e-lips



Oval closed curve.

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F



Función

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2 - \rho^2} d\rho$$

Relación entre variables. $t = 9a^2 - \rho^2$

$$\begin{aligned} &= -2\pi \int_0^0 (9a^2 - t) \sqrt{t} dt \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$



Function
fánk-shon



Relation between variables.

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F



Flujo

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2 - \rho^2} d\rho$$

Movimiento de un fluido.

$$t = 9a^2 - \rho^2$$

$$\begin{aligned} &= -2\pi \int_{9a^2}^0 (9a^2 - t) \sqrt{t} dt \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$



$$\begin{aligned} &= 2 \cdot \left[9a^2 \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_0^{9a^2} \\ &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{648\pi}{5} a^5. \end{aligned}$$

Flow
flou



Movement of a fluid.

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F



Frontera

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Límite de una región.

$$t = 9a^2 - \rho^2$$

$$\begin{aligned} &= 2\pi \int_0^{9a^2} (9a^2 - t) \sqrt{t} dt \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$



Boundary
báun-da-ri

$$\begin{aligned} &= 2\pi \left[9a^2 \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_{0}^{9a^2} \\ &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{24\pi}{5} a^5. \end{aligned}$$



Limit of a region.

CÁLCULO MULTIVARIABLE

F



Factor

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2 - \rho^2} d\rho$$

Elemento multiplicador.

$$t = 9a^2 - \rho^2$$

$$\begin{aligned} &= -2\pi \int_{9a^2}^0 (9a^2 - t) \sqrt{t} dt \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$



Factor

$$\begin{aligned} &= 2\pi \left[9a^2 \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_0^{9a^2} \\ &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{24\pi}{5} a^5. \end{aligned}$$

fák-tor



Multiplying element.

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F



Frecuencia

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2 - \rho^2} d\rho$$

$$\begin{aligned} &= 2\pi \int_0^0 (9a^2 - t)\sqrt{t} dt & t = 9a^2 - \rho^2 \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t\sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t\sqrt{t} dt \right] \end{aligned}$$

Número de repeticiones.



Frequency

$$\begin{aligned} &= 2\pi \left[9a^2 \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_0^{9a^2} \\ &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{64\pi}{5} a^5. \end{aligned}$$

frí-kuen-si



Number of repetitions.

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Gradiente

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Vector de derivadas parciales.

$$= -2\pi \int_0^0 (9a^2 - t) \sqrt{t} dt \quad t = 9a^2 - \rho^2$$

$$= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt$$

$$= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right]$$



Gradient
gréi-dient

$$= 2\pi \left[9a^2 \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_0^{9a^2}$$

$$= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right)$$

$$= \frac{54\pi}{5} a^5$$



Vector of partial derivatives.

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Gráfica

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Representación visual.

$$t = 9a^2 - \rho^2$$

$$\begin{aligned} &= -2\pi \int_{9a^2}^0 (9a^2 - t) \sqrt{t} dt \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$



Graph

$$\begin{aligned} &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{6}{5} \pi a^5. \end{aligned}$$

graf



Visual representation.

CÁLCULO MULTIVARIABLE

G



Gauss

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Científico del teorema de flujo.

$$\begin{aligned} &= 2\pi \int_0^{3a} (9a^2 - t) \sqrt{t} dt \quad t = 9a^2 - \rho^2 \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$



$$\begin{aligned} &= 2\pi \left[9a^2 \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_0^{9a^2} \\ &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{5}{5} a^5. \end{aligned}$$

Gauss
gauss



Scientist of the flux theorem.

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Geometría

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Estudio de figuras y espacios.

$$\begin{aligned} &= 2\pi \int_0^{3a} (9a^2 - t) \sqrt{t} dt \quad t = 9a^2 - \rho^2 \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$



Geometry
yó-me-tri

$$\begin{aligned} &= 2\pi \left[9a^2 \cdot \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_{0}^{9a^2} \\ &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{648\pi}{5} a^5. \end{aligned}$$



Study of shapes and spaces.

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G



Global

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Referido al todo.

$$t = 9a^2 - \rho^2$$

$$\begin{aligned} &= -2\pi \int_{9a^2}^0 (9a^2 - t) \sqrt{t} dt \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$



Global

$$\begin{aligned} &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{108}{5} \pi a^5 \end{aligned}$$

gló-bal



Referring to the whole.

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H



Hessiana

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Matriz de segundas derivadas parciales.



$$\begin{aligned} &= 2\pi \int_0^{3a} \left(3a^2 \sqrt{t} - t\sqrt{t} \right) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t\sqrt{t} dt \right] \end{aligned}$$

Hessian

$$\begin{aligned} &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{548\pi}{5} a^5. \end{aligned}$$



Matrix of second-order partial derivatives.

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H



Hiperplano

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Subespacio de dimensión n-1.

$$= -2\pi \int_0^{9a^2} (9a^2-t)\sqrt{t} dt$$

$$t = 9a^2 - \rho^2$$

$$= 2\pi \int_0^{9a^2} (9a^2\sqrt{t} - t\sqrt{t}) dt$$

$$= 2\pi \left[\int_0^{9a^2} 9a^2\sqrt{t} dt - \int_0^{9a^2} t\sqrt{t} dt \right]$$



Hyperplane
jái-per-plein

$$= 2\pi \left[0 \cdot \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_{0}^{9a^2}$$

$$= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right)$$

$$= \frac{36\pi}{5} a^5$$



Subspace of dimension n-1.

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H

Hipersu- perficie



$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Superficie en dimensión superior.

$$\begin{aligned} &= -2\pi \int_0^0 (9a^2 - t) \sqrt{t} dt \quad t = 9a^2 - \rho^2 \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$



Hypersurface
jái-per-sér-fes

$$\begin{aligned} &= 2\pi \left[9a^2 \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_0^{9a^2} \\ &= 2 \cdot 27\pi a^5 \left(\frac{6}{5} - \frac{18}{5} \right) \\ &= \frac{54\pi}{5} a^5. \end{aligned}$$



Surface in higher dimension.

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H



Hipersfera

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2 - \rho^2} d\rho$$

Esfera en n dimensiones.

$$\begin{aligned} &= -2\pi \int_{9a^2}^0 (9a^2 - t) \sqrt{t} dt \quad t = 9a^2 - \rho^2 \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$



Hypersphere
jái-per-sfír

$$\begin{aligned} &= 2\pi \left[9a^2 \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_0^{9a^2} \\ &= \frac{2 \cdot 27\pi a^5}{6} \left(6 - \frac{18}{5} \right) \\ &= \frac{6}{5} \pi a^5 \end{aligned}$$



Sphere in n dimensions.

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H



Helicoide

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Superficie generada por una hélice.

$$\begin{aligned} &= -2\pi \int_0^0 (9a^2 - t) \sqrt{t} dt \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$



Helicoid

$$\begin{aligned} &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{648\pi}{5} a^5 \end{aligned}$$



Surface generated by a helix.

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Integral doble



$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2 - \rho^2} d\rho$$

Integral sobre \mathbb{R}^2 .

$$t = 9a^2 - \rho^2$$

$$\begin{aligned} &= -2\pi \int_{9a^2}^0 (9a^2 - t) \sqrt{t} dt \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$



Double integral
dá-bol in-te-gral

$$\begin{aligned} &= 2\pi \left[9a^2 \left(\frac{2}{3} t^{3/2} - \frac{2}{5} t^{5/2} \right) \right]_{9a^2}^0 \\ &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{648\pi}{5} a^5. \end{aligned}$$



Integral over \mathbb{R}^2 .

CÁLCULO MULTIVARIABLE



Integral triple



$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2 - \rho^2} d\rho$$

Integral sobre \mathbb{R}^3 .

$$t = 9a^2 - \rho^2$$

$$\begin{aligned} &= -2\pi \int_{9a^2}^0 (9a^2 - t) \sqrt{t} dt \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$



Triple integral
trí-pol in-te-gral

$$\begin{aligned} &= 2\pi \left[9a^2 \left(\frac{2}{3} t^{3/2} - \frac{2}{5} t^{5/2} \right) \right]_{9a^2}^0 \\ &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{648\pi}{5} a^5. \end{aligned}$$



Integral over \mathbb{R}^3 .

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Integral múltiple



$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2 - \rho^2} d\rho$$

Integral en varias variables.

$$\begin{aligned} &= -2\pi \int_0^0 (9a^2 - t)\sqrt{t} dt \quad t = 9a^2 - \rho^2 \\ &= 2\pi \int_0^{9a^2} (9a^2\sqrt{t} - t\sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2\sqrt{t} dt - \int_0^{9a^2} t\sqrt{t} dt \right] \end{aligned}$$



Multiple integral
mól-ti-pol ín-te-gral

$$\begin{aligned} &= 2\pi \left[9a^2 \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_{0^{9a^2}}^{9a^2} \\ &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{9408\pi}{5} a^5. \end{aligned}$$



Integral in several variables.

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Integrando

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Función que se integra.

$$\begin{aligned} &= -2\pi \int_0^0 (9a^2 - t) \sqrt{t} dt \quad t = 9a^2 - \rho^2 \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$



Integrand
ín-te-gránd

$$\begin{aligned} &= 2\pi \left[9a^2 \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_0^{9a^2} \\ &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{9}{5} 27\pi a^5 \end{aligned}$$



Function being integrated.

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Isosuperficie



$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Conjunto donde la función es constante (3D).



Isosurface

ái-so-sér-fes



Set where the function is constant (3D).

CÁLCULO MULTIVARIABLE

J

Jacobiano



$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Determinante de derivadas parciales.



$$\begin{aligned} &= 2\pi \int_0^{3a} \left(9a^2 \sqrt{t} - t\sqrt{t} \right) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t\sqrt{t} dt \right] \end{aligned}$$

$$\begin{aligned} &= 2\pi \left[9a^2 \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_{t=0}^{t=9a^2} \\ &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{648\pi}{5} a^5. \end{aligned}$$

Jacobian ya-kó-bian



Determinant of partial derivatives.

CÁLCULO MULTIVARIABLE

J



Junto

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

En el mismo lugar o posición.



$$\begin{aligned} &= -2\pi \int_0^0 (9a^2 - t) \sqrt{t} dt \quad t = 9a^2 - \rho^2 \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$

Together

tu-gé-der



In the same place or position.

CÁLCULO MULTIVARIABLE

J

Justifica-
ción



$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2 - \rho^2} d\rho$$

Explicación razonada.

$$\begin{aligned} &= -2\pi \int_0^0 (9a^2 - t)\sqrt{t} dt & t = 9a^2 - \rho^2 \\ &= 2\pi \int_0^{9a^2} (9a^2\sqrt{t} - t\sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2\sqrt{t} dt - \int_0^{9a^2} t\sqrt{t} dt \right] \end{aligned}$$



Justification
yus-ti-fi-kei-shon

$$\begin{aligned} &= 2\pi \left[\frac{2}{3} 9a^2 t^{3/2} - \frac{2}{5} t^{5/2} \right]_0^{9a^2} \\ &= \frac{2 \cdot 27\pi a^5}{5} \left(6 - \frac{18}{5} \right) \\ &= \frac{648\pi}{5} a^5. \end{aligned}$$



Reasoned explanation.

CÁLCULO MULTIVARIABLE

J



Juicio

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Capacidad para evaluar.

$$t = 9a^2 - \rho^2$$

$$\begin{aligned} &= -2\pi \int_{a^2}^0 (9a^2 - t) \sqrt{t} dt \\ &= 2\pi \int_0^{a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$



Judgment

yudj-ment



Ability to evaluate.

CÁLCULO MULTIVARIABLE



J

Jirón

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Parte pequeña o fragmento.

$$\begin{aligned} &= -2\pi \int_{9a^2}^0 (9a^2-t)\sqrt{t} dt \quad t = 9a^2 - \rho^2 \\ &= 2\pi \int_0^{9a^2} (9a^2\sqrt{t} - t\sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2\sqrt{t} dt - \int_0^{9a^2} t\sqrt{t} dt \right] \end{aligned}$$



$$\begin{aligned} &= 2\pi \left[9a^2 \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_0^{9a^2} \\ &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{648\pi}{5} a^5. \end{aligned}$$

Piece
pis



Small part or fragment.

CÁLCULO MULTIVARIABLE

K

Kilómetro



$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Mil metros de longitud.

$$t = 9a^2 - \rho^2$$

$$\begin{aligned} &= -2\pi \int_{9a^2}^0 (9a^2 - t) \sqrt{t} dt \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$



Kilometer
ki-ló-me-ter



One thousand meters in length.

CÁLCULO MULTIVARIABLE

K



Kappa

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Símbolo usado en vectores y tenso-
res.



$$\begin{aligned} &= 2\pi \int_0^{3a} \left(9a^2 \sqrt{t} - t\sqrt{t} \right) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t\sqrt{t} dt \right] \end{aligned}$$

$$\begin{aligned} &= 2\pi \left[9a^2 \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_0^{9a^2} \\ &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{24}{5} \pi a^5 \end{aligned}$$

Kappa

ká-pa



Symbol used in vectors and
tensors.

CÁLCULO MULTIVARIABLE

K



Kernel

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2 - \rho^2} d\rho$$

Función núcleo en integrales.

$$\begin{aligned} &= 2\pi \int_0^{3a} \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} (9a^2 - t) \sqrt{t} dt \quad t = 9a^2 - \rho^2 \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$



Kernel

$$\begin{aligned} &= 2\pi \left[0 \rho^2 \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_0^{9a^2} \\ &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{54\pi}{5} a^5. \end{aligned}$$

kér-nel



Core function in integrals.

CÁLCULO MULTIVARIABLE

K



Kepleriano

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Relacionado con las leyes de Kepler.

$$= -2\pi \int_{9a^2}^0 (9a^2-t)\sqrt{t} dt \quad t=9a^2-\rho^2$$

$$= 2\pi \int_0^{9a^2} (9a^2\sqrt{t}-t\sqrt{t}) dt$$

$$= 2\pi \left[\int_0^{9a^2} 9a^2\sqrt{t} dt - \int_0^{9a^2} t\sqrt{t} dt \right]$$



Keplerian

keplé-rian

$$= 2\pi \left[9a^2 \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_{t=0}^{t=9a^2}$$

$$= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right)$$

$$= \frac{948\pi}{5} a^5$$



Related to Kepler's laws.

CÁLCULO MULTIVARIABLE

K



Kinemática

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Parte de la mecánica que estudia el movimiento.



$$= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right]$$

Kinematics
ki-ne-má-tiks



Branch of mechanics
studying motion.

CÁLCULO MULTIVARIABLE

L



Límite

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Valor al que $t = 9a^2 - \rho^2$ tiende una función.

$$\begin{aligned} &= -2\pi \int_{9a^2}^0 (9a^2 - t) \sqrt{t} dt \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$



Limit

$$\begin{aligned} &= 2\pi \left[9a^2 \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_0^{9a^2} \\ &= \frac{2}{5} 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{18\pi}{5} a^5. \end{aligned}$$

límit



Value a function approaches.

CÁLCULO MULTIVARIABLE

L



Laplaciano

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Operador diferencial de segundo orden.



$$\begin{aligned} &= 2\pi \int_0^{3a} \left(9a^2 \sqrt{t} - t\sqrt{t} \right) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t\sqrt{t} dt \right] \end{aligned}$$

Laplacian
la-plá-shan



Second-order differential operator.

CÁLCULO MULTIVARIABLE

L



Lineal

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Que tiene forma de línea.

$$\begin{aligned} &= -2\pi \int_0^0 (9a^2-t)\sqrt{t} dt \quad t = 9a^2 - \rho^2 \\ &= 2\pi \int_0^{9a^2} (9a^2\sqrt{t} - t\sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2\sqrt{t} dt - \int_0^{9a^2} t\sqrt{t} dt \right] \end{aligned}$$



$$\begin{aligned} &= 2\pi \left[9a^2 \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_0^{9a^2} \\ &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{648\pi}{5} a^5 \end{aligned}$$

Linear
lí-niar



Having the form of a line.

CÁLCULO MULTIVARIABLE

L



Longitud

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Medida de una distancia.

$$\begin{aligned} &= -2\pi \int_0^0 (9a^2 - t)\sqrt{t} dt \quad t = 9a^2 - \rho^2 \\ &= 2\pi \int_0^{9a^2} (9a^2\sqrt{t} - t\sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2\sqrt{t} dt - \int_0^{9a^2} t\sqrt{t} dt \right] \end{aligned}$$



$$\begin{aligned} &= 2\pi \left[0 \cdot \frac{2}{3} t^{\frac{3}{2}} \right]_0^{9a^2} \\ &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{648\pi}{5} a^5. \end{aligned}$$

Length
lenz



Measurement of distance.

CÁLCULO MULTIVARIABLE

L



Logaritmo

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Inversa de la función exponencial.

$$= -2\pi \int_0^0 (9a^2 - t) \sqrt{t} dt \quad t = 9a^2 - \rho^2$$

$$= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt$$

$$= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right]$$

Logarithm

ló-ga-rizm



Inverse of the exponential function.

CÁLCULO MULTIVARIABLE

M

Magnitud



$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Propiedad que puede medirse.

$$\begin{aligned} &= -2\pi \int_{9a^2}^0 (9a^2 - t) \sqrt{t} dt \quad t = 9a^2 - \rho^2 \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$



$$\begin{aligned} &= 2\pi \left[9a^2 \frac{2}{3} t^{3/2} - \frac{2}{5} t^{5/2} \right]_0^{9a^2} \\ &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{8\pi}{5} a^5. \end{aligned}$$

Magnitud

mag-ni-tud



Property that can be measured.

CÁLCULO MULTIVARIABLE

M



Matriz

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Arreglo rectangular de números.

$$\begin{aligned} &= -2\pi \int_{9a^2}^0 (9a^2 - t) \sqrt{t} dt \quad t = 9a^2 - \rho^2 \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$



Matrix

méi-triks



Rectangular array of numbers.

CÁLCULO MULTIVARIABLE

M



Máximo

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Valor más alto.

$$t = 9a^2 - \rho^2$$

$$\begin{aligned} &= -2\pi \int_0^0 (9a^2 - t)\sqrt{t} dt \\ &= 2\pi \int_0^{9a^2} (9a^2\sqrt{t} - t\sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2\sqrt{t} dt - \int_0^{9a^2} t\sqrt{t} dt \right] \end{aligned}$$



$$\begin{aligned} &= 2\pi \left[9a^2 \frac{2}{3} t^{3/2} - \frac{2}{5} t^{5/2} \right]_0^{9a^2} \\ &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{64}{5} \pi a^5 \end{aligned}$$

Maximum

má-ksi-mom



Highest value.

CÁLCULO MULTIVARIABLE

M



Mínimo

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Valor más bajo.

$$t = 9a^2 - \rho^2$$

$$\begin{aligned} &= -2\pi \int_0^0 (9a^2 - t) \sqrt{t} dt \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$



$$\begin{aligned} &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{648\pi}{5} a^5. \end{aligned}$$

Minimum

mí-ni-mom



Lowest value.

CÁLCULO MULTIVARIABLE

M



Modelo

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Representación de un sistema.



$$\begin{aligned} &= -2\pi \int_0^0 (9a^2 - t)\sqrt{t} dt \quad t = 9a^2 - \rho^2 \\ &= 2\pi \int_0^{9a^2} (9a^2\sqrt{t} - t\sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2\sqrt{t} dt - \int_0^{9a^2} t\sqrt{t} dt \right] \end{aligned}$$

$$= 2\pi \left[9a^2 \cdot \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_0^{9a^2}$$

$$= 2 \cdot 2\pi a^5 \left(6 - \frac{18}{5} \right)$$

$$= \frac{64\pi}{5} a^5$$

Modelo mó-del



Representation of a system.

CÁLCULO MULTIVARIABLE

N



Normal

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Perpendicular a una superficie.

$$= -2\pi \int_0^0 (9a^2 - t) \sqrt{t} dt \quad t = 9a^2 - \rho^2$$

$$= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt$$

$$= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right]$$

$$= 2\pi \left[0 \cdot \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_0^{9a^2}$$

$$= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right)$$

$$= \frac{54\pi}{5} a^5$$

ENG



Normal

$$= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right)$$

$$= \frac{54\pi}{5} a^5$$



Perpendicular to a surface.

CÁLCULO MULTIVARIABLE

N



Nivel

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Altura o posición relativa.

$$\begin{aligned} &= -2\pi \int_{9a^2}^0 (9a^2 - t) \sqrt{t} dt \quad t = 9a^2 - \rho^2 \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t\sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t\sqrt{t} dt \right] \end{aligned}$$



Level

$$\begin{aligned} &= 2\pi \left[9a^2 \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_0^{9a^2} \\ &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{9432\pi}{5} a^5. \end{aligned}$$

lé-vel



Height or relative position.

N



Trabajo de fuerza.

$$= -2\pi \int_0^0 (9a^2 - t)\sqrt{t} dt$$

$$= 2\pi \int_0^{9a^2} (9a^2\sqrt{t} - t\sqrt{t}) dt$$

$$= 2\pi \left[\int_0^{9a^2} 9a^2\sqrt{t} dt - \int_0^{9a^2} t\sqrt{t} dt \right]$$



Newton

$$= \frac{648\pi}{5} a^5.$$



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CÁLCULO MULTIVARIABLE

N



Nabla

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Símbolo del operador gradiente.

$$\begin{aligned} &= -2\pi \int_0^0 (9a^2-t)\sqrt{t} dt \quad t = 9a^2 - \rho^2 \\ &= 2\pi \int_0^{9a^2} (9a^2\sqrt{t} - t\sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2\sqrt{t} dt - \int_0^{9a^2} t\sqrt{t} dt \right] \end{aligned}$$



Nabla

$$\begin{aligned} &= 2\pi \left[9a^2 \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_0^{9a^2} \\ &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{648\pi}{5} a^5. \end{aligned}$$

ná-bla



Symbol for gradient operator.

CÁLCULO MULTIVARIABLE

N

Número real



$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2 - \rho^2} d\rho$$

Valor continuo sin imaginarios.

$$\begin{aligned} &= -2\pi \int_0^0 (9a^2 - t)\sqrt{t} dt \quad t = 9a^2 - \rho^2 \\ &= 2\pi \int_0^{9a^2} (9a^2\sqrt{t} - t\sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2\sqrt{t} dt - \int_0^{9a^2} t\sqrt{t} dt \right] \end{aligned}$$



Real number
rí-al nám-ber



Continuous value without
imaginaries.

CÁLCULO MULTIVARIABLE

O

Operador



$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Símbolo que realiza una operación.

$$\begin{aligned} &= -2\pi \int_0^0 (9a^2 - t) \sqrt{t} dt \quad t=9a^2-\rho^2 \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$



Operator
ó-pe-rei-tor



Symbol performing an operation.

CÁLCULO MULTIVARIABLE

O



Ortogonal

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

En ángulo recto.

$$t = 9a^2 - \rho^2$$

$$\begin{aligned} &= -2\pi \int_{9a^2}^0 (9a^2 - t) \sqrt{t} dt \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$



Orthogonal

or-thó-go-nal

$$\begin{aligned} &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{648\pi}{5} a^5 \end{aligned}$$



At right angles.

CÁLCULO MULTIVARIABLE

O



Orden

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2 - \rho^2} d\rho$$

Secuencia o disposición.

$$\begin{aligned} &= -2\pi \int_{9a^2}^0 (9a^2 - t) \sqrt{t} dt \quad t = 9a^2 - \rho^2 \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t\sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t\sqrt{t} dt \right] \end{aligned}$$



Order

$$\begin{aligned} &= \left[9a^2 \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_0^{9a^2} \\ &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{18\pi}{5} a^5. \end{aligned}$$

órden



Sequence or arrangement.

CÁLCULO MULTIVARIABLE

O



Origen

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Punto (0,0,0) de referencia. $t = 9a^2 - \rho^2$

$$\begin{aligned} &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$



$$\begin{aligned} &= 2\pi \left[9a^2 \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_0^{9a^2} \\ &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{64\pi}{5} a^5 \end{aligned}$$

Origen

ó-ri-yin



Reference point (0,0,0).

CÁLCULO MULTIVARIABLE

O

Oscilación



$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Movimiento repetido o alternante.

$$\begin{aligned} &= -2\pi \int_0^0 (9a^2 - t) \sqrt{t} dt \quad t = 9a^2 - \rho^2 \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$



Oscillation

ó-si-lei-shon

$$\begin{aligned} &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{648\pi}{5} a^5. \end{aligned}$$



Repeated or alternating motion.

CÁLCULO MULTIVARIABLE

P



Plano

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Superficie plana que se extiende infinitamente.



$$\begin{aligned} &= 2\pi \int_0^{3a} \left(9a^2 \sqrt{t} - t\sqrt{t} \right) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t\sqrt{t} dt \right] \\ &= 2\pi \left[9a^2 \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_0^{9a^2} \\ &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{48\pi}{5} a^5. \end{aligned}$$

Plane
plein



Flat surface extending infinitely.

CÁLCULO MULTIVARIABLE

P



Parábola

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Curva simétrica generada por una ecuación cuadrática.



$$= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right]$$

$$= 2\pi \left[9a^2 \cdot \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_{t=0}^{t=9a^2}$$

$$= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right)$$

$$= \frac{648\pi}{5} a^5$$

Parabola
pa-rá-bo-la



Symmetrical curve from a quadratic equation.

CÁLCULO MULTIVARIABLE

P



Parcial

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Derivada respecto a una variable.

$$= -2\pi \int_0^0 (9a^2 - t) \sqrt{t} dt$$

$$= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt$$

$$= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right]$$



Partial

pár-shal

$$= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) = \frac{54\pi}{5} a^5$$



Derivative with respect to one variable.

CÁLCULO MULTIVARIABLE

P



Pendiente

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Grado de inclinación de una recta.



$$\begin{aligned} &= 2\pi \int_0^{9a^2} (9a^2 - t) \sqrt{t} dt \quad t = 9a^2 - \rho^2 \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$

$$\begin{aligned} &= 2\pi \left[9a^2 \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_0^{9a^2} \\ &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{648\pi}{5} a^5. \end{aligned}$$

Slope
sloup



Degree of tilt of a line.

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P



Proyección

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Sombra o representación sobre un plano.



$$\begin{aligned} &= 2\pi \int_0^{3a} \left(9a^2 \sqrt{t} - t\sqrt{t} \right) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t\sqrt{t} dt \right] \end{aligned}$$

Projection
pro-yék-shon



Shadow or representation on a plane.

CÁLCULO MULTIVARIABLE



Queso

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Figura usada en ejemplos volumétricos.



$$\begin{aligned} &= 2\pi \int_0^{3a} \left(9a^2 \sqrt{t} - t\sqrt{t} \right) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t\sqrt{t} dt \right] \end{aligned}$$

$$\left[9a^2 \cdot \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_0^{9a^2}$$

$$= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right)$$

$$= \frac{648\pi}{5} a^5.$$

Cheese
chís



Shape used in volume examples.

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Q



Quadrante

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Una de las cuatro partes del plano cartesiano.



$$= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right]$$

Quadrant

kuó-drant



One of four parts of the Cartesian plane.

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Q



Quociente

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Resultado de una división.

$$\begin{aligned} &= -2\pi \int_{9a^2}^0 (9a^2-t)\sqrt{t} dt \quad t = 9a^2 - \rho^2 \\ &= 2\pi \int_0^{9a^2} (9a^2\sqrt{t} - t\sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2\sqrt{t} dt - \int_0^{9a^2} t\sqrt{t} dt \right] \end{aligned}$$



Quotient

kuó-shent

$$\begin{aligned} &= 2\pi \left[9a^2 \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_0^{9a^2} \\ &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{648\pi}{5} a^5. \end{aligned}$$



Result of a division.

CÁLCULO MULTIVARIABLE

Q



Quiralidad

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Propiedad de no ser superponible
con su imagen especular.



$$= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right]$$

Chirality

kai-rá-li-ti



Property of not being superimposable on its mirror image.

CÁLCULO MULTIVARIABLE

Q



Química

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Ciencia que estudia la materia y sus cambios.



$$\begin{aligned} &= 2\pi \int_0^{3a} \left(9a^2 \sqrt{t} - t\sqrt{t} \right) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t\sqrt{t} dt \right] \end{aligned}$$

Chemistry
ké-mis-tri



Science studying matter and its changes.

CÁLCULO MULTIVARIABLE

R



Radio

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Distancia del centro al borde de un círculo.



$$= 2\pi \int_0^{3a} \left(9a^2 \sqrt{t} - t\sqrt{t} \right) dt$$

$$= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t\sqrt{t} dt \right]$$

Radius

$$= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right)$$

$$= \frac{540\pi}{5} a^5$$



Distance from center to edge of a circle.

CÁLCULO MULTIVARIABLE

R



Rotación

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Giro alrededor de un punto o eje.

$$= -2\pi \int_0^0 (9a^2 - t) \sqrt{t} dt \quad t = 9a^2 - \rho^2$$

$$= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt$$

$$= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right]$$



Rotation
ro-téi-shon

$$= 2\pi \left[9a^2 \left(\frac{2}{3} t^{3/2} \right) - \left(\frac{2}{5} t^{5/2} \right) \right]_{t=0}^{t=9a^2}$$

$$= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right)$$

$$= \frac{24\pi}{5} a^5$$



Turn around a point or axis.

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R



Región

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2 - \rho^2} d\rho$$

Zona o área del espacio. $t = 9a^2 - \rho^2$

$$\begin{aligned} &= 2\pi \int_0^{3a} \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} (9a^2\sqrt{t} - t\sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2\sqrt{t} dt - \int_0^{9a^2} t\sqrt{t} dt \right] \end{aligned}$$



$$\begin{aligned} &= 2\pi \left[9a^2 \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_0^{9a^2} \\ &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{648\pi a^5}{5} \end{aligned}$$

Region
rí-yon



Area of space.

CÁLCULO MULTIVARIABLE

R



Rango

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Conjunto de valores posibles de una función.



$$= 2\pi \int_0^{3a} \left(9a^2 \sqrt{t} - t\sqrt{t} \right) dt$$

$$= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t\sqrt{t} dt \right]$$

Range

$$= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right)$$

$$= \frac{648\pi}{5} a^5$$

réinchn



Set of possible function values.

CÁLCULO MULTIVARIABLE

R



Resultado

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2 - \rho^2} d\rho$$

Valor obtenido tras una operación.



$$\begin{aligned} &= -2\pi \int_0^0 (9a^2 - t) \sqrt{t} dt \quad t = 9a^2 - \rho^2 \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$

$$= 2\pi \left[9a^2 \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_0^{9a^2}$$

$$= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right)$$

$$= \frac{648\pi}{5} a^5$$

Result
ri-sólt



Value obtained after an operation.

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S



Superficie

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Área que delimita un cuerpo.

$$\begin{aligned} &= 2\pi \int_0^{3a} \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} (9a^2 - t) \sqrt{t} dt \quad t = 9a^2 - \rho^2 \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$



Surface

$$\begin{aligned} &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{648\pi}{5} a^5. \end{aligned}$$

sér-fes



Area bounding a body.

CÁLCULO MULTIVARIABLE

S



Suma

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2 - \rho^2} d\rho$$

Resultado de agregar valores.

$$\begin{aligned} &= -2\pi \int_0^0 (9a^2 - t)\sqrt{t} dt \quad t = 9a^2 - \rho^2 \\ &= 2\pi \int_0^{9a^2} (9a^2\sqrt{t} - t\sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2\sqrt{t} dt - \int_0^{9a^2} t\sqrt{t} dt \right] \end{aligned}$$



Sum som

$$\begin{aligned} &= 2\pi \left[\frac{3}{2} \cdot 2 \cdot t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_0^{9a^2} \\ &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{648\pi}{5} a^5. \end{aligned}$$



Result of adding values.

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S



Simetría

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Equilibrio de forma o posición.

$$\begin{aligned} &= 2\pi \int_0^{3a} \int_0^0 (9a^2-t)\sqrt{t} dt \quad t = 9a^2 - \rho^2 \\ &= 2\pi \int_0^{9a^2} (9a^2\sqrt{t} - t\sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2\sqrt{t} dt - \int_0^{9a^2} t\sqrt{t} dt \right] \end{aligned}$$



Symmetry

$$\begin{aligned} &= 2\pi \left[9a^2 \cdot \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_{t=0}^{t=9a^2} \\ &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{648\pi}{5} a^5. \end{aligned}$$

sí-me-trí



Balance of form or position.

CÁLCULO MULTIVARIABLE

S

Sistema



$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Conjunto de elementos relaciona-
dos.



$$\begin{aligned} &= 2\pi \int_0^{3a} \left(9a^2 \sqrt{t} - t\sqrt{t} \right) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t\sqrt{t} dt \right] \end{aligned}$$

System

$$\begin{aligned} &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{540\pi}{5} a^5. \end{aligned}$$

sistema



Set of related elements.

CÁLCULO MULTIVARIABLE

S



Scalar

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Cantidad con magnitud pero sin dirección.



$$\begin{aligned} &= 2\pi \int_0^{3a} \left(9a^2 \sqrt{t} - t\sqrt{t} \right) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t\sqrt{t} dt \right] \end{aligned}$$

$$\begin{aligned} &= 2\pi \left[9a^2 \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_0^{9a^2} \\ &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{648\pi}{5} a^5. \end{aligned}$$

Scalar ské-lar



Quantity with magnitude but
no direction.

CÁLCULO MULTIVARIABLE



Tangente

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Recta que toca una curva en un punto.



$$\begin{aligned} &= 2\pi \int_0^{3a} \left(9a^2 \sqrt{t} - t \sqrt{t} \right) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$

Tangent
tán-yent



Line that touches a curve at one point.

CÁLCULO MULTIVARIABLE



Teorema

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Proposición demostrada matemáticamente.



$$= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right]$$

Theorem

$$= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) = \frac{548\pi}{5} a^5$$



Proposition proven mathematically.

CÁLCULO MULTIVARIABLE

T



Triple

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Que tiene tres partes o componentes.



$$\begin{aligned} &= 2\pi \int_0^{3a} \left(9a^2 \sqrt{t} - t\sqrt{t} \right) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t\sqrt{t} dt \right] \end{aligned}$$

$$\begin{aligned} &= 2\pi \left[6a^2 t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_0^{9a^2} \\ &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{18\pi}{5} a^5 \end{aligned}$$

Triple
trí-pol



Having three parts or components.

CÁLCULO MULTIVARIABLE

T

Trayecto-
ria



$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Camino seguido por un punto o cuerpo.



$$\begin{aligned} &= 2\pi \int_0^{3a} \left(9a^2 \sqrt{t} - t\sqrt{t} \right) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t\sqrt{t} dt \right] \end{aligned}$$

Trajectory
tra-yék-to-ri



Path followed by a point or
body.

CÁLCULO MULTIVARIABLE



Transformación



$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Cambio de forma o posición.

$$\begin{aligned} &= 2\pi \int_0^{3a} (9a^2 - t) \sqrt{t} dt \quad t = 9a^2 - \rho^2 \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$



Transformation
trans-for-méi-shon

$$\begin{aligned} &= 2 \cdot \left[9a^2 \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_{t=0}^{t=9a^2} \\ &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{64\pi}{5} a^5. \end{aligned}$$



Change of shape or position.

CÁLCULO MULTIVARIABLE

U



Unidad

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Cantidad adoptada como referencia.



$$\begin{aligned} &= -2\pi \int_{9a^2}^0 (9a^2 - t) \sqrt{t} dt \quad t = 9a^2 - \rho^2 \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$

Unit

iú-nit

$$\begin{aligned} &= \frac{2 \cdot 27\pi a^5}{5} \left(6 - \frac{18}{5} \right) \\ &= \frac{920\pi}{5} a^5. \end{aligned}$$



Quantity taken as reference.

CÁLCULO MULTIVARIABLE

U

Universal



$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Que aplica a todo.

$$= -2\pi \int_0^0 (9a^2 - t) \sqrt{t} dt$$

$$t = 9a^2 - \rho^2$$

$$= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt$$

$$= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right]$$



Universal yu-ni-vér-sal

$$= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) = \frac{648\pi}{5} a^5$$



That applies to everything.

CÁLCULO MULTIVARIABLE

U



Ubicación

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2 - \rho^2} d\rho$$

Lugar donde algo se encuentra.

$$= -2\pi \int_0^0 (9a^2 - t) \sqrt{t} dt \quad t = 9a^2 - \rho^2$$

$$= 2\pi \int_0^0 (9a^2 \sqrt{t} - t \sqrt{t}) dt$$



$$= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right]$$

Location lo-kéi-shon

$$= 2\pi \left[9a^2 \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_{0^{9a^2}}$$

$$= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right)$$

$$= \frac{24\pi}{5} a^5$$



Place where something is found.

CÁLCULO MULTIVARIABLE

U



Uniforme

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Que no cambia en el espacio.

$$\begin{aligned} &= -2\pi \int_0^0 (9a^2 - t) \sqrt{t} dt \quad t = 9a^2 - \rho^2 \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$



Uniform yú-ni-form

$$\begin{aligned} &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{648\pi}{5} a^5. \end{aligned}$$



That does not change in space.

CÁLCULO MULTIVARIABLE

U



Urbano

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Relacionado con la ciudad.

$$\begin{aligned} &= -2\pi \int_0^0 (9a^2-t)\sqrt{t} dt \quad t = 9a^2 - \rho^2 \\ &= 2\pi \int_0^{9a^2} (9a^2\sqrt{t} - t\sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2\sqrt{t} dt - \int_0^{9a^2} t\sqrt{t} dt \right] \end{aligned}$$



$$\begin{aligned} &= 2\pi \left[9a^2 \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_0^{9a^2} \\ &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{548\pi}{5} a^5. \end{aligned}$$

Urban úr-ban



Related to the city.

CÁLCULO MULTIVARIABLE

V



Vector

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2 - \rho^2} d\rho$$

Magnitud con dirección y sentido.

$$\begin{aligned} &= -2\pi \int_0^0 (9a^2 - t)\sqrt{t} dt \quad t = 9a^2 - \rho^2 \\ &= 2\pi \int_0^{9a^2} (9a^2\sqrt{t} - t\sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2\sqrt{t} dt - \int_0^{9a^2} t\sqrt{t} dt \right] \end{aligned}$$



Vector

$$\begin{aligned} &= 2\pi \left[9a^2 \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_0^{9a^2} \\ &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{648\pi}{5} a^5. \end{aligned}$$

vék-tor



Quantity with direction and sense.

CÁLCULO MULTIVARIABLE

V



Variable

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Elemento que puede cambiar de valor.



$$= 2\pi \int_0^{3a} \left(9a^2 \sqrt{t} - t \sqrt{t} \right) dt$$

$$= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right]$$

Variable

$$= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right)$$

$$= \frac{648\pi}{5} a^5$$



Element that can change in value.

CÁLCULO MULTIVARIABLE

V



Volúmen

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Espacio ocupado por un cuerpo.

$$\begin{aligned} &= -2\pi \int_0^0 (9a^2 - t) \sqrt{t} dt \quad t = 9a^2 - \rho^2 \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$



$$\begin{aligned} &= 2\pi \left[9a^2 \left[\frac{2}{3} t^{\frac{3}{2}} \right]_0^{9a^2} - \left[\frac{2}{5} t^{\frac{5}{2}} \right]_0^{9a^2} \right] \\ &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{648\pi}{5} a^5. \end{aligned}$$

Volume

vó-lum



Space occupied by a body.

CÁLCULO MULTIVARIABLE

V



Velocidad

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Cambio de posición respecto al tiempo.



$$\begin{aligned} &= 2\pi \int_0^{3a} \left(9a^2 \sqrt{t} - t\sqrt{t} \right) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t\sqrt{t} dt \right] \end{aligned}$$

Velocity

$$\begin{aligned} &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{64\pi}{5} a^5. \end{aligned}$$

ve-ló-si-ti



Change of position over time.

CÁLCULO MULTIVARIABLE

V



Valor

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Cantidad o número asignado.

$$\begin{aligned} &= -2\pi \int_{9a^2}^0 (9a^2-t)\sqrt{t} dt \quad t = 9a^2 - \rho^2 \\ &= 2\pi \int_0^{9a^2} (9a^2\sqrt{t} - t\sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2\sqrt{t} dt - \int_0^{9a^2} t\sqrt{t} dt \right] \end{aligned}$$



$$\begin{aligned} &= 2\pi \left[9a^2 \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_0^{9a^2} \\ &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{54\pi}{5} a^5. \end{aligned}$$

Value
vã-liu



Assigned quantity or number.

CÁLCULO MULTIVARIABLE

W



Watt

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2 - \rho^2} d\rho$$

Unidad de potencia.

$$t = 9a^2 - \rho^2$$

$$\begin{aligned} &= 2\pi \int_0^{9a^2} (9a^2 - t) \sqrt{t} dt \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$



$$\begin{aligned} &= 2 \cdot \left[9a^2 \cdot \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_0^{9a^2} \\ &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{64\pi}{5} a^5. \end{aligned}$$

Watt
uat



Unit of power.

CÁLCULO MULTIVARIABLE

W



Work

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Trabajo realizado por una fuerza.

$$= -2\pi \int_{9a^2}^0 (9a^2 - t) \sqrt{t} dt \quad t = 9a^2 - \rho^2$$

$$= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt$$

$$= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right]$$

$$= 2\pi \left[9a^2 \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_0^{9a^2}$$

$$= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right)$$

$$= \frac{648\pi}{5} a^5$$



Work



Work done by a force.

CÁLCULO MULTIVARIABLE

W



Wave

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Onda o vibración periódica.



$$\begin{aligned} &= 2\pi \int_0^{3a} \int_0^{9a^2-\rho^2} (9a^2-t)\sqrt{t} dt \quad t = 9a^2 - \rho^2 \\ &= 2\pi \int_0^{9a^2} (9a^2\sqrt{t} - t\sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2\sqrt{t} dt - \int_0^{9a^2} t\sqrt{t} dt \right] \\ &= 2\pi \left[9a^2 \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_0^{9a^2} \\ &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{6}{5} \pi a^5. \end{aligned}$$

Wave



Periodic oscillation or vibration.

CÁLCULO MULTIVARIABLE

W



Weight

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Medida del efecto de la gravedad.



$$\begin{aligned} &= -2\pi \int_0^{9a^2} (9a^2 - t) \sqrt{t} dt \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$

$$\begin{aligned} &= 2\pi \left[6a^2 \frac{2}{3} t^{3/2} - \frac{2}{5} t^{5/2} \right]_0^{9a^2} \\ &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{6}{5} \pi a^5 \end{aligned}$$

Weight



Measure of gravitational effect.

CÁLCULO MULTIVARIABLE

W

Wireframe



$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Modelo de líneas que representa formas 3D.



Wireframe

uai-er-freim



Line model representing 3D shapes.

CÁLCULO MULTIVARIABLE



X

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Letra usada como variable descono-
cida.



$$\begin{aligned} &= 2\pi \int_0^{3a} \left(3a^2 \sqrt{t} - t\sqrt{t} \right) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t\sqrt{t} dt \right] \\ &= 2\pi \left[9a^2 \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_0^{9a^2} \\ &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{648\pi}{5} a^5. \end{aligned}$$

X
éks



Letter used as unknown va-
riable.

CÁLCULO MULTIVARIABLE



X-axis

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Eje horizontal de un plano. $t = 9a^2 - \rho^2$

$$= -2\pi \int_0^0 (9a^2 - t) \sqrt{t} dt$$

$$= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt$$

$$= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right]$$



$$= 2\pi \left[9a^2 \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_0^{9a^2}$$

$$= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right)$$

$$= \frac{360\pi}{5} a^5$$

X-axis
éks-aksis



Horizontal axis of a plane.

CÁLCULO MULTIVARIABLE



X-



coordinate

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Valor en el eje X.

$$t = 9a^2 - \rho^2$$

$$= -2\pi \int_{9a^2}^0 (9a^2 - t) \sqrt{t} dt$$

$$= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt$$

$$= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right]$$



X-coordinate

éks-kor-di-neit

$$= 2\pi \left[0 \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} \right]^{9a^2}$$

$$= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right)$$

$$= \frac{9407}{5} a^5.$$



Value on the X-axis.

CÁLCULO MULTIVARIABLE



X-



component

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2 - \rho^2} d\rho$$

Parte de un vector en el eje X.

$$\begin{aligned} &= -2\pi \int_0^0 (9a^2 - t)\sqrt{t} dt \quad t = 9a^2 - \rho^2 \\ &= 2\pi \int_0^{9a^2} (9a^2\sqrt{t} - t\sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2\sqrt{t} dt - \int_0^{9a^2} t\sqrt{t} dt \right] \end{aligned}$$



X-component

éks-kom-po-nent

$$\begin{aligned} &= 2\pi \left[9a^2 \frac{2}{3} t^{3/2} - \frac{2}{5} t^{5/2} \right]_0^{9a^2} \\ &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{36\pi}{5} a^5 \end{aligned}$$



Part of a vector along the X-axis.

CÁLCULO MULTIVARIABLE



X-



intercept

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Punto donde la curva cruza el eje X.

$$= 2\pi \int_0^{9a^2} (9a^2 - t) \sqrt{t} dt$$

$$= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right]$$



X-intercept

éks-in-ter-sept

$$= 2\pi \left[9a^2 \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_{0}^{9a^2}$$

$$= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right)$$

$$= \frac{548\pi}{5} a^5$$



Point where the curve crosses the X-axis.

CÁLCULO MULTIVARIABLE

Y



Y

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Letra usada como segunda variable.



$$\begin{aligned} &= -2\pi \int_{9a^2}^0 (9a^2 - t) \sqrt{t} dt \quad t = 9a^2 - \rho^2 \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t\sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t\sqrt{t} dt \right] \\ &= 2\pi \left[9a^2 \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_0^{9a^2} \\ &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{648\pi}{5} a^5. \end{aligned}$$



Letter used as second variable.

CÁLCULO MULTIVARIABLE

4



Y-axis

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Eje vertical de un plano. $t = 9a^2 - \rho^2$

$$\begin{aligned} &= -2\pi \int_0^0 (9a^2 - t) \sqrt{t} dt \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$



$$\begin{aligned} &= 2\pi \left[9a^2 \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_0^{9a^2} \\ &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{948\pi}{5} a^5. \end{aligned}$$

Y-axis
uai-aksis



Vertical axis of a plane.

CÁLCULO MULTIVARIABLE

Y

Y-

coordinate



$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Valor en el eje Y.

$$t = 9a^2 - \rho^2$$

$$= -2\pi \int_{9a^2}^0 (9a^2 - t) \sqrt{t} dt$$

$$= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt$$

$$= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right]$$

Y-coordinate

uai-kor-di-neit

$$= 2\pi \left[0 \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} \right] 9a^2$$

$$= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right)$$

$$= \frac{540\pi}{5} a^5.$$



Value on the Y-axis.

CÁLCULO MULTIVARIABLE

Y

Y-

component



$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Parte de un vector en el eje Y.

$$= -2\pi \int_0^0 (9a^2-t)\sqrt{t} dt$$

$$= 2\pi \int_0^{9a^2} (9a^2\sqrt{t}-t\sqrt{t}) dt$$

$$= 2\pi \left[\int_0^{9a^2} 9a^2\sqrt{t} dt - \int_0^{9a^2} t\sqrt{t} dt \right]$$



Y-component

uai-kom-po-nent

$$= 2\pi \left[9a^2 \frac{2}{3} t^{3/2} - \frac{2}{5} t^{5/2} \right]_{0^{9a^2}}$$

$$= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right)$$

$$= \frac{64\pi}{5} a^5$$



Part of a vector along the Y-axis.

CÁLCULO MULTIVARIABLE

Y



Yield

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Resultado o producción de algo.



$$\begin{aligned} &= -2\pi \int_0^0 (9a^2 - t)\sqrt{t} dt \quad t = 9a^2 - \rho^2 \\ &= 2\pi \int_0^{9a^2} (9a^2\sqrt{t} - t\sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2\sqrt{t} dt - \int_0^{9a^2} t\sqrt{t} dt \right] \end{aligned}$$

$$= 2\pi \left[9a^2 \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_0^{9a^2}$$

$$= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right)$$

$$= \frac{64}{5}\pi a^5$$

Yield

yild



Output or production of something.

CÁLCULO MULTIVARIABLE

Z



Z

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Letra usada como tercera variable.

$$= -2\pi \int_0^0 (9a^2 - t) \sqrt{t} dt$$

$$= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt$$

$$= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right]$$

$$= 2\pi \left[9a^2 \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_0^{9a^2}$$

$$= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right)$$

$$= \frac{64\pi}{5} a^5$$



Z

zed



Letter used as third variable.

CÁLCULO MULTIVARIABLE

Z



Zona

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2 - \rho^2} d\rho$$

Espacio o área determinada.

$$\begin{aligned} &= 2\pi \int_0^{3a} (9a^2 - t) \sqrt{t} dt \quad t = 9a^2 - \rho^2 \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$



Zone zón

$$\begin{aligned} &= 2\pi \left[9a^2 \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_0^{9a^2} \\ &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{64\pi}{5} a^5. \end{aligned}$$



Defined space or area.

CÁLCULO MULTIVARIABLE

Z



Z-axis

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Eje perpendicular al plano XY.

$$\begin{aligned} &= 2\pi \int_0^{3a} (9a^2 - t) \sqrt{t} dt \quad t = 9a^2 - \rho^2 \\ &= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt \\ &= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right] \end{aligned}$$



Z-axis

$$\begin{aligned} &= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right) \\ &= \frac{948\pi}{5} a^5 \end{aligned}$$

zi-aksis



Axis perpendicular to the XY plane.

CÁLCULO MULTIVARIABLE

Z

Z-

coordinate



$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Valor en el eje Z.

$$t = 9a^2 - \rho^2$$

$$= -2\pi \int_{9a^2}^0 (9a^2 - t) \sqrt{t} dt$$

$$= 2\pi \int_0^{9a^2} (9a^2 \sqrt{t} - t \sqrt{t}) dt$$

$$= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right]$$

Z-coordinate

zi-kor-di-neit

$$= 2\pi \left[0 \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} \right] 9a^2$$

$$= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right)$$

$$= \frac{945\pi}{5} a^5$$



Value on the Z-axis.

CÁLCULO MULTIVARIABLE

Z



Zeta

$$\int_0^{2\pi} d\phi \int_0^{3a} \rho^3 d\rho \int_{-\sqrt{9a^2-\rho^2}}^{\sqrt{9a^2-\rho^2}} dz = 2\pi \int_0^{3a} 2\rho^3 \sqrt{9a^2-\rho^2} d\rho$$

Letra griega usada en fórmulas matemáticas.



$$= 2\pi \left[\int_0^{9a^2} 9a^2 \sqrt{t} dt - \int_0^{9a^2} t \sqrt{t} dt \right]$$

$$= 2 \cdot \left[9a^2 \cdot \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{5} t^{\frac{5}{2}} \right]_0^{9a^2}$$

$$= 2 \cdot 27\pi a^5 \left(6 - \frac{18}{5} \right)$$

$$= \frac{64\pi}{5} a^5.$$

Zeta
zé-ta



Greek letter used in formulas.