

w | v

Produto escalar

① Dados os vetores $\vec{u} = (2, -3, -1)$ e $\vec{v} = (1, -1, 4)$, calcular

$$a) 2\vec{u} \cdot (-\vec{v}) \quad 2(2, -3, -1) \cdot (-\vec{v})$$

$$(4, -6, -2) \cdot (-1, +1, -4)$$

$$-4 - 6 + 8$$

$$-2$$

$$b) (\vec{u} + 3\vec{v}) \cdot (\vec{v} - 2\vec{u})$$

$$(2, -3, -1) + 3(1, -1, 4) \quad (1, -1, 4) - 2(2, -3, -1)$$

$$(5, -6, 11) \cdot (5, -7, 2)$$

$$(25, -42, 22)$$

$$c) (\vec{u} + \vec{v}) \cdot (\vec{u} - \vec{v})$$

$$(2, -3, -1) + (1, -1, 4) \cdot (2, -3, -1) - (1, -1, 4)$$

$$(3, -4, 3) \cdot (1, -2, -5) = 0 (3 + 8 - 15 = 11 - 15) \\ -4$$

$$d) (\vec{u} + \vec{v}) \cdot (\vec{v} - \vec{u})$$

$$(3, -4, 3) \cdot (1, -1, 4) - (2, -3, -1)$$

$$(3, -4, 3) \cdot (-1, 2, 5) = 0 - 3 - 8 + 15 \\ -11 + 25$$

FORON:



② Dados os vetores $\vec{u} = (2, a, -1)$, $\vec{v} = (3, 1, -2)$ e $\vec{w} = (2, -1, -2, 4)$
determinar a de modo que $\vec{u} \cdot \vec{v} = (\vec{u} + \vec{v}) \cdot (\vec{v} + \vec{w})$

$$\vec{u} \cdot \vec{v} = (2, a, -1) \cdot (3, 1, -2)$$

$$(2, a, -1) + (3, 1, -2) = (3, 1, -2) + (2, -1, -2, 4)$$

$$\vec{u} \cdot \vec{v} = 23 + a + (-1)^4(-2)$$

$$(5, a+1, -3) = (2a+1, 1, 2)$$

$$\vec{u} \cdot \vec{v} = 6 + a + 2$$

$$5(2a+1) + (a+1)(-2) + (-3)(2)$$

$$\vec{u} \cdot \vec{v} = a + 8$$

$$10 + 5 + (-a-2) - 6$$

$$(a+1)(4+6) = a - 2$$

$$a+8 = 9a-2$$

$$8+2 = 9a-a$$

$$10 = 8a$$

$$a = \frac{10}{8} = \frac{5}{4}$$

$$③ \text{ se } B-A = (2, -2, 2) - (4, 0, -1) = (-2, -2, 3)$$

$$A(4, 0, -1)$$

$$3x + 2v = x + (AB \cdot u)v$$

$$B(2, -2, 3)$$

$$(3-1)x = (AB \cdot u)v - 2v$$

$$C(1, 3, 2)$$

$$2x = (-2, -2, 3) - (2, 1, 2) - 2(-1, -2, 3)$$

$$\vec{u} = (2, 1, 1)$$

$$2x = (-6, -2, 2)$$

$$V = (-1, -2, 3)$$

$$x = (-3, -1, 1)$$

$$\text{se } (BC \cdot V)x = (u \cdot V)V - 3x$$

$$u \cdot V = (2)(-2) + (1)(-2) + (3)(3) = -7$$

$$(C-B) \cdot V = (-2, 5, 1) - (-2, -2, 3) = 8$$

$$-7$$

$$Rx = (u \cdot V)V - 3x$$

$$8x - (-1)(-2, 3) - 3(-3, -1, 1)$$

$$Rx = (1, 2, -3) + (9, 3, -3)$$

$$Rx = (10, 5, -6)$$

$$x = (5/4, 5/8, -3/8)$$

Q) $\vec{u} = (2, -2, 3)$ $\vec{v} \cdot \vec{u} = -42$ o vector \vec{v} se é paralelo.

$$(x, y, z) \cdot (2, -2, 3) = -42$$

$$|\vec{u}| = \sqrt{2^2 + (-2)^2 + 3^2} = \sqrt{4 + 1 + 9} = \sqrt{14}$$

$$\vec{v} \cdot \vec{u} = |\vec{v}| \cdot \sqrt{14} \cdot \cos \theta^1$$

$$-42 = |\vec{v}| \cdot \sqrt{14}$$

$$|\vec{v}| = -42 / \sqrt{14} \approx -19 \sqrt{2} (-1) \quad 19\sqrt{2}$$

$$\vec{v} = \left(\frac{-42}{|\vec{u}|} \right) \cdot \vec{u} = \left(\frac{-42}{\sqrt{14}} \right) \cdot (2, -2, 3) = (-12, 12, -18)$$

$$\textcircled{8} \quad |\vec{U}| = 2, |\vec{V}| = 3 \Rightarrow \vec{U} \cdot \vec{V} = -1$$

$$u|(\vec{u} - 3\vec{v}) \cdot \vec{u}$$

$$u \cdot u = 3u \cdot v$$

$$= 1|\vec{U}|^2 \cdot 3(u \cdot v)$$

$$= 2^2 \cdot 3(-1)$$

$$= -3$$

$$6) \quad 0(2v-u) \cdot (2v)$$

$$2v \cdot 2v - u \cdot 2v$$

$$4|v|^2 - 2(u \cdot v)$$

$$4(3)^2 - 2(-1)$$

$$= 5$$

$$c) (u+v) \cdot (v-u)$$

$$u \cdot v = 4u \cdot u + v \cdot v - 4v \cdot u$$

$$= 1 - 4(2^2) + 3^2 - 4(3)$$

$$= -24$$

$$d) (3u+4v) \cdot (-2u-5v)$$

$$-6u \cdot u = 15u \cdot u - 8u \cdot v -$$

$$20v \cdot v = -6(2^2) - 15(-1) -$$

$$8(-1) - 20(3)^2$$

$$= -192$$

$$\textcircled{12} \quad |\vec{u}| = 4, |\vec{v}| = 3 \text{ e o ângulo entre } \vec{u} \text{ e } \vec{v} \text{ é de } 60^\circ$$

$$u \cdot v = |u||v| \cos 60^\circ$$

$$= 4 \cdot 3 \cdot \cos 60^\circ$$

$$\cos(60^\circ) = -1/6 = 0.130^\circ$$

$$|u+v|^2 = |u|^2 + |v|^2 + 2|u||v|\cos\theta$$

$$= 4^2 + 3^2 + 2 \cdot 4 \cdot 3 \cdot (-1/2)$$

$$= 16 + 9 - 24$$

$$= 1$$

$$(u-v) \cdot (u-v) = u \cdot u - 2u \cdot v + v \cdot v$$

$$= 4^2 + 3^2 - 2(4)(3)(-1/2)$$

$$= 25$$

$$= \sqrt{25}$$

$$|\vec{u} - \vec{v}| = \sqrt{25} = 5$$

$$|u-v| = 5$$

$$|u+v| = 7$$

$$|u-v| = (6, 5\sqrt{3}, -3)$$



$$\textcircled{13} \quad a) |(2u-v) \cdot (u-2v)|$$

$$(2u-v) \cdot (u-2v) = 2u^2 - vu - 4uv + v^2$$

$$2 \cdot 2 - (-\frac{3\sqrt{2}}{2}) - 4 \cdot \frac{3\sqrt{2}}{2} + 9 \approx 4 + 3\sqrt{2} - 6\sqrt{2} + 9$$

$$|73 - 3\sqrt{2}| = |(2u-v) \cdot (u-2v)|$$

$$b) |u-2v| = \sqrt{|u|^2 + |2v|^2 - 2(u \cdot 2v)}$$

$$= \sqrt{2 + 4 \cdot 9 - 2(u \cdot 2v)}$$

$$u \cdot 2v = -2(u \cdot v)$$

$$= -2 \cdot |u| \cdot |v| \cdot \cos \frac{3\pi}{4}$$

$$= 3\sqrt{2}$$

$$\textcircled{15} \quad \vec{a} = \alpha \vec{i} + 2\vec{j} - 4\vec{k} \quad B = 2\vec{i} + (1-2\alpha)\vec{j} + 3\vec{k}$$

$$(\alpha \vec{i} + 2\vec{j} - 4\vec{k}) \cdot (2\vec{i} + (1-2\alpha)\vec{j} + 3\vec{k}) = 0$$

$$2\alpha - 2\alpha^2 - 8 = 0$$

$$-2\alpha^2 + 2\alpha - 8 = 0 \quad -\frac{1}{2}$$

$$\alpha^2 - \alpha + 4 = 0 \quad \alpha = \frac{1 \pm \sqrt{1-4 \cdot 1 \cdot 4}}{2}$$

$$\alpha = \frac{1 \pm \sqrt{-15}}{2}$$

mão mão ortogonais

$$\textcircled{16} \quad \frac{(a+B)}{(C-a)} = 0 \quad \frac{(2+\alpha+\alpha+2, -4, \alpha)}{(2\alpha-2, 7, -\alpha-1)} = 0$$

$$4\alpha^2 + 11\alpha - 18 = 0 \quad \frac{9}{4} \quad \alpha = -0.5 \text{ ou } 2.25$$

(25) a) $u = (2, -1, -1)$ e $v = (-1, -2, 2)$

$$u \cdot v = 2 \cdot (-1) + (-1) \cdot (-1) + (-1) \cdot 2 = -5$$

$$\|u\| = \sqrt{2^2 + (-1)^2 + (-1)^2} = \sqrt{6}$$

$$\|v\| = \sqrt{(-1)^2 + (-2)^2 + 2^2} = \sqrt{6}$$

$$\Rightarrow \theta = \frac{-5}{\sqrt{6} \cdot \sqrt{6}} = -\frac{5}{6} \quad \theta = \cos^{-1} \left(-\frac{5}{6} \right) \approx 133,82^\circ$$

b) $u = (1, -2, 1)$ e $v = (-1, 1, 0)$

$$u \cdot v = 1 \cdot (-1) + (-2) \cdot 1 + 1 \cdot 0 = -3$$

$$\|u\| = \sqrt{1^2 + (-2)^2 + 1^2} = \sqrt{6}$$

$$\|v\| = \sqrt{(-1)^2 + 1^2 + 0^2} = \sqrt{2}$$

$$\Rightarrow \theta = \frac{-3}{\sqrt{6} \cdot \sqrt{2}} = -\frac{3\sqrt{2}}{4}$$

$$\theta = \cos^{-1} \left(-\frac{3\sqrt{2}}{4} \right) \approx 137,63^\circ$$

29) calcular n para que o ângulo entre os vetores
 $\vec{v} = (-3, 1, n)$ e \vec{k} seja de 30°

$$\vec{v} \cdot \vec{k} = |\vec{v}| \cdot |\vec{k}| \cdot \cos 30^\circ$$

$$(-3, 1, n) \cdot (0, 0, 1) = \\ -3n = |\vec{v}| \cdot |\vec{k}| \cdot \frac{\sqrt{3}}{2}$$

$$4n^2 = (10 + n^2) \cdot 3$$

$$4n^2 = 30 + 3n^2$$

$$4n^2 - 3n^2 = 30$$

$$n^2 = 30$$

$$n = \pm \sqrt{30}$$

① se $\vec{u} = 3\vec{i} - \vec{j} - 2\vec{k}$, $\vec{v} = 2\vec{i} + 4\vec{j} - \vec{k}$ e $\vec{w} = -\vec{i} + \vec{k}$, determine

~~a)~~ a) $(\vec{u} \times \vec{v})$

$$\begin{array}{|ccc|c|} \hline & \vec{i} & \vec{j} & \vec{k} \\ \hline \vec{u} & 3 & -1 & -2 \\ \vec{v} & 2 & 4 & -1 \\ \hline \end{array} \quad \vec{u} \times \vec{v} = (2)(-1) - (-6)(-3) - (-6)(-2) - (2)(-3) = 2 - 18 - 12 + 6 = -22$$

b) $(2\vec{v}) \times (3\vec{v})$

2. $(2, 4, -1) \times 3(2, 4, -1)$

$(4, 8, -2) \times (6, 12, -3)$

$(2\vec{v})(3\vec{v}) = (0, 0, 0)$

Produto vetorial

$$W(-1, 0, 2) \times (3, -1, -2)$$

~~$$\begin{array}{|ccc|c|} \hline & \vec{i} & \vec{j} & \vec{k} \\ \hline W & -1 & 0 & 2 \\ \vec{v} & 3 & -1 & -2 \\ \hline \end{array} \quad \vec{w} \times \vec{v} = (-1)(-2) - (0)(-3) - (2)(-1) = 2 - 0 + 2 = 4$$~~

$$e) (\vec{u} - \vec{v}) \times \vec{w}$$

$$\begin{pmatrix} (3, -1, -2) - (2, 4, -1) \end{pmatrix} \times (-1, 0, 1)$$
$$(1, -5, -1) \times (-1, 0, 1)$$

~~$$\begin{array}{r|rr} & -5 & 1 \\ \hline 1 & (+1) + (-5) + (0) - (-1) - (5) - (0) \\ & -10 \end{array}$$~~

$$f) (\vec{u} \times \vec{v}) \times \vec{w}$$

~~$$\begin{array}{r|rr} & -4 & 2 \\ \hline 2 & (2(-4) + (2)(-2) - (-2) - (-1) - (3)) = -12 + 2 + 2 - 1 + 3 \\ & (9, -11, 24) \times (-1, 0, 1) \end{array}$$~~

~~$$\begin{array}{r|rr} & -11 & -24 \\ \hline 9 & (+(-11) + (-24) + (0) - (-2) - (9) - (-1)) = -34 - 23 \\ & -37 \end{array}$$~~

$$i) \vec{u} \times \vec{v} + \vec{v} \times \vec{w}$$

~~$$\begin{array}{r|rr} & +1 & -4 & +12 & +8 & +2 & +3 \\ \hline 1 & (+1) + (-4) + (12) - (-8) - (-2) - (-3) \end{array}$$~~

~~$$(26 - 4) = 22$$~~

③ $A(2, 1, -1), B(3, 0, 1), C(2, -1, -3)$ determinieren D

$$\vec{AB} = \vec{B} - \vec{A} \Rightarrow C - B \times C - A$$

$$D - A$$

$$(x, y, z) - (2, 1, -1) = (2, -1, -3) - (3, 0, 1) \times (2, -1, -3) - (2, 1, 1)$$
$$(-2x, -y, -z) = (-1, -1, -4) \times (0, -2, -2)$$

$$= \begin{vmatrix} 2 & -1 & -3 \\ -1 & -1 & -4 \\ 0 & -2 & -2 \end{vmatrix} = 2\vec{i} + 0\vec{j} + 2\vec{k} - 8\vec{i} - 2\vec{j} - 2\vec{k}$$
$$-6\vec{i} - 2\vec{j} + 2\vec{k}$$

$$(-2x, -y, -z) = (-6, -2, 2)$$

FORON:

$$\begin{cases} -2x = -6 \Rightarrow x = \frac{-6}{-2} = +3 \\ -y = -2(-1) \Rightarrow y = 2 \\ -z = 2(-1) = 0 \Rightarrow z = -2 \end{cases}$$
$$D = (3, 2, -2)$$

⑨ ~~2. Determinant~~ $\begin{cases} x \cdot (u+2v)=0 \\ x \cdot (v-u)=0 \end{cases}$

$$(x, 6-3, 3, -4) = 0 \quad \text{c}(x, (0, 1, 2) = 0)$$

$$\begin{cases} -3x + 3y - 4z = 0 \\ x - 2y + 2z = 0 \end{cases} \quad (1, 3/5, 4/5)$$

⑩ $AB = B \cdot A = (1, -1, 1) - (2, 3, 1) = (-1, -4, 0)$

$$AC = C \cdot A = (4, 1, -2) - (2, 3, 1) = (2, -2, -3)$$

$$\therefore = (-1, -4, 0) \times (2, -2, -3)$$

$$\vec{n} = (-2, -6, 6)$$

FOR

Prodotto mixto

① a) (\bar{u}, \bar{v}, w)

$$V \times W = (22 - (-3)2, -(22 - 32), 10 - 20) = (4, 4, 0)$$

$$u \cdot (V \times W) = (3, -2, 1) \cdot (4, 4, 0) = 3 \cdot 4 + (-2)4 + 1 \cdot 0 = 2$$

$$\text{cos} \varphi$$

b) (w, u, v)

$$u \times v = (-22 - 12, 12 - 31, 3 \cdot 2 - (-27) \cdot 1) = (-6, 1, 3)$$

$$w \cdot (u \times v) = (2, 0, -3) \cdot (-6, 1, 3) = 2 \cdot (-6) + 0 \cdot (1) + (-3) \cdot 3 = -36$$

mentre, $(w, u, v) = -36$

②

$\bar{i} \cdot \bar{j} = 0$	$\bar{k} \cdot \bar{j} \cdot \bar{k} = 0$
$\bar{i} \cdot \bar{k} = 0$	
$\bar{j} \cdot \bar{k} = 0$	