

Exercícios sobre a reta

(1) $\vec{v} = \vec{BA} = A - B = \cancel{(2, -3, 4)} - (1, -1, 2) = \underbrace{(1, 4, 2)}_{\vec{v}}$

$(x, y, z) = (1, -1, 2) + t(1, 4, 2)$ eq. vetorial da reta r .

$$\begin{cases} x = 1 + t \Rightarrow \frac{5}{2} = 1 + t \\ y = -1 + 4t \Rightarrow -4 = -1 + 4t \\ z = 2 + 2t \Rightarrow 5 = 2 + 2t \end{cases} \quad \begin{array}{l} \frac{5}{2} - 1 = t \Rightarrow t = \frac{3}{2} \\ -4 + 1 = t \Rightarrow t = -\frac{3}{4} \\ \frac{5 - 2}{2} = t \Rightarrow t = \frac{3}{2} \end{array}$$

$C \notin r$

$$\begin{array}{ll} -1 = 1 + t \Rightarrow t = -1 - 1 = -2 & (x, y, z) = (-3, 1, 1) \text{ representam} \\ 3 = -1 + 4t \Rightarrow t = \frac{3+4}{4} = \frac{7}{4} = 1 & \text{o ponto } C \\ 4 = 2 + 2t \Rightarrow t = \frac{4-2}{2} = \frac{2}{2} = 1 & \end{array}$$

(2) $r: (x, y, z) = (-1, 2, 3) + t(2, -3, 0)$

$$\begin{cases} x = -1 + 2t \\ y = 2 - 3t \\ z = 3 + 0t \end{cases}$$

(3) escrever os eq. paramétricos da reta que passa por $A(-1, 2, 3)$ e é paralela à reta $r: (x, y, z) = (1, 4, 3) + t(0, 0, 1)$

$r: (x, y, z) = (1, 4, 3) + t(0, 0, 1)$

$\vec{v} = \vec{u}$

ou que

não paralelas

no entanto

temo que

mo vetor

(4) $r: \begin{cases} x = 2 + t \\ y = 3 - t \\ z = -4 + 2t \end{cases}$ ou $\begin{cases} x = 2 + 3 = x = 5 \\ t = 3 - t = 0 \Rightarrow t = 3 - 3 = 0 \\ z = -4 + 2 \cdot 0 = -4 + 2 = 2 \end{cases}$

FORON:

$z = -4 + 2 \cdot 2 = 0$

$P(5, 2, 2)$

$$\text{def } X = 6 \quad \begin{cases} 6 = 2 + t = 0 \quad t = 4 \\ 4 = 3 - t = 0 \quad 3 - 4 = -1 \\ Z = -4 + 2t = 0 \quad -4 + 8 = 4 \end{cases}$$

$P(6, -1, 4)$

$$C) Z = 24 \quad \begin{cases} X = 24 + 10 = X = 34 \\ 4 = 3 - 10 = 4 = -7 \\ 24 = -4 + 2t = 0 \quad t = \frac{24 + 4}{2} = 10 \end{cases}$$

$$P(34, -7, 24)$$

$$\textcircled{5} \quad A(4, -3, -2)$$

$$D: \begin{cases} X = 1 + 3t \\ Y = 2 - 4t \\ Z = 3 - t \end{cases} \quad \text{se } P(m, n, s) \in \pi$$

$$(X, Y, Z) = (1, 2, 3) + t(3, -4, -1) = \text{or} \\ (m, n, s) = (4, -3, -2) + t(3, -4, -1) = \text{or}$$

$$\begin{cases} m = 4 + 3t = 0 \quad m = 4 + 3 \cdot -3 = -5 \\ n = -3 + 4t \quad n = -3 + 4 \cdot -3 = -15 \\ -5 = -2 - t = 0 \quad t = -5 + 2 = -3 \end{cases}$$

$$m = -5 \quad P(-5, -15, -5)$$

$$n = -15$$

$$(21) - \overset{+}{(1)} - \overset{-}{(2)}$$

$$\textcircled{6} \quad \text{a) } A(1, -1, 2) \text{ e } B(2, 1, 0)$$

$$AB = B - A = (1, 2, 2)$$

$$(X, Y, Z) = (1, -1, 2) + t(1, 2, 2)$$

$$\begin{cases} X = 1 + t \\ Y = -1 + 2t \\ Z = 2 + 2t \end{cases}$$

1 1

3-3

$$d) A(3, 1, 4) \text{ } e B(3, -2, 2)$$

$$\vec{AB} = (0, -3, -2)$$

$$(x, y, z) = (3, 1, 4) + t(0, -3, -2)$$

$$\begin{cases} x = 3 \\ y = 1 - 3t \\ z = 4 - 2t \end{cases}$$

$$c) A(1, 2, 3) \text{ } e B(1, 3, 2)$$

$$\vec{AB} = (0, 1, -1)$$

$$(x, y, z) = (1, 2, 3) + t(0, 1, -1)$$

$$\begin{cases} x = 1 \\ y = 2 + t \\ z = 3 - t \end{cases}$$

$$d) A(0, 0, 0) \text{ } e b(0, 1, 0)$$

$$\vec{AB} = (0, 1, 0)$$

$$(x, y, z) = (0, 0, 0) + t(0, 1, 0)$$

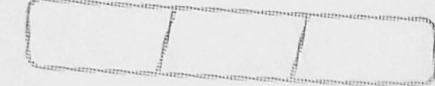
$$\begin{cases} x = 0 + 0 = 0 \\ y = 0 + t = 0 \quad t \\ z = 0 + 0 = 0 \end{cases}$$

12) $P_1(5, -5, 6) \text{ } P_2(4, -1, 12)$ pertenecen a recta?

$$r: x - 3 = \frac{y + 1}{2} = \frac{z - 2}{-2}$$

$$\frac{5 - 3}{-1} = \frac{-5 + 1}{2} = \frac{6 - 2}{-2} \Rightarrow \frac{2}{-1} = \frac{-4}{2} = \frac{4}{-2} \quad P_1 \in r$$

$$\frac{4 - 3}{-1} = \frac{-1 + 1}{2} = \frac{12 - 2}{-2} = 0 \quad \frac{1}{-1} = \frac{0}{2} = \frac{10}{2} \quad P_2 \notin r$$



(13) n: $\frac{x-1}{2} = \frac{y+3}{-1} = \frac{z}{4}$

a) $\frac{5-1}{52} = \frac{y+3}{-1} = \frac{z}{4}$

$$-2 \frac{(5-1)}{2} = -2 \frac{(y+3)}{2} = -2(z/4)$$

$$-10 + 2 = -2(y+3) = -2/12$$

$$-9 = -2y - 6 = \frac{-2}{2}$$

$$-9 + 6 = -3 \quad (5, 3/2, -3/2)$$

$$-3 = -2y = -1/12$$

$$\frac{3}{-2} = y = -\frac{3}{2}$$

b) $\frac{x-1}{2} = -5 = z/4$

$$4x - 2 = -20 = 2 \quad (-9/2, 2, -18/4)$$

$$4x = -18 = z \quad (\div 4)$$

$$x = -9/2 = z/4$$

(14) $n : \frac{3x+1}{3} = \frac{3y-2}{2} = 3+4$

com o $x=1$

$$\frac{3+1}{3} = \frac{3y-2}{2} = 3+4 \quad \frac{3y-2}{2} = 1 \Rightarrow 3y-2 = 2$$

$$\begin{aligned} 3+4 &= 1 \\ 3 &= -3 \end{aligned}$$

$$\begin{aligned} 3y &= 4 \\ y &= \frac{4}{3} \end{aligned}$$

$P\left(1, \frac{4}{3}, -3\right)$

preto diretor

$$\vec{v}^0 = \left(\frac{3}{2}, \frac{2}{3}, 1\right)$$

$$\vec{u}^0 = 3\vec{v}^0 = \left(\frac{9}{2}, 2, 3\right)$$

$$n = \frac{x+1}{3/2} = 4 - \frac{2}{2/3} = 3+4$$

(15) c)

$$(15) \text{ a) } A(4,0,-3) \quad \vec{v} = (2,4,5)$$

$$(x, y, z) = (4, 0, -3) + t(2, 4, 5)$$

$$x = 4 + 2t \Rightarrow t = \frac{x-4}{2}$$

$$y = 0 + 4t \Rightarrow t = \frac{y}{4}$$

$$z = -3 + 5t \Rightarrow t = \frac{z+3}{5}$$

$$\frac{x-4}{2} = \frac{y}{4} = \frac{z+3}{5}$$

$$-x - \frac{t-4}{2} \cdot (-1) \quad x = \frac{t+4}{2}$$

$$(r: x = \frac{t+4}{2}, y = \frac{t}{4})$$

$$-y = \frac{t}{4} \cdot (-1)$$

$$y = \frac{t}{4}$$

$$z = \frac{t+3}{5}$$

$$-z = \frac{t-3}{5} \cdot (-1) \quad z = \frac{t+3}{5}$$

$$(b) A(1, -2, 3) \quad B(3, -1, -1)$$

$$\vec{AB} = (2, 1, -4)$$

$$(x, y, z) = (1, -2, 3) + t(2, 1, -4)$$

$$x = 1 + 2t \quad t = \frac{x-1}{2}$$

$$y = -2 + t \quad t = \frac{y+2}{1} = 0$$

$$z = 3 - 4t \quad t = \frac{z+3}{4}$$

$$\frac{x-1}{2} = y+2 = \frac{z-3}{4}$$

$$(x = \frac{t+1}{2}, y = t+2, z = \frac{t-3}{4})$$

$$z = \frac{t-3}{4}$$

(15) c)

$$A(-1, 2, 3) \text{ e } B(2, -1, 3)$$

$$v = \vec{AB} = B - A = (2, -1, 3) - (-1, 2, 3) = (3, -3, 0)$$

Eq. vectorial

$$(x, y, z) = (-1, 2, 3) + t(3, -3, 0)$$

Eq. paramétricas

$$\begin{cases} x = -1 + 3t \\ y = 2 - 3t \\ z = 3 \end{cases} \Rightarrow t = \frac{x+1}{3}$$



Eq. simétricas

$$\frac{x+1}{3} = \frac{y-2}{3} \quad z = 3$$

Eq. reducidas

$$x-1 = y-2$$

$$y = -x - 1 + 2 = 0 \quad y = 1 - x$$

$$\begin{cases} y = 1 - x \\ z = 3 \end{cases}$$

⑯ $A\beta = (3, -4, -2)$

$$(x, y, z) = (-1, 6, 3) + t(3, -4, 2)$$

$$x = -1 + 3t$$

$$y = 6 - 4t$$

$$z = 3 - 2t$$

$$x = x + 1/3$$

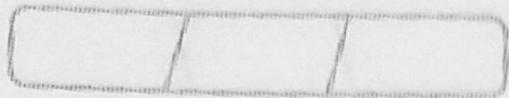
$$y = y - 6/4$$

$$z = z - 3/2$$

$$\frac{x+1}{3} = \frac{y-6}{-4} = \frac{z-3}{-2}$$

$$z = \frac{3}{2}$$

FORON:



$$\begin{array}{r} 22 \\ \times 2 \\ \hline 44 \end{array}$$

17 a) n: $\begin{cases} y = 2x + 3 \\ z = x - 1 \end{cases}$

ordenado igual a 9

$$y = 2x + 3 \quad -2x = 9 + 3 = \frac{12}{-2} = -6$$

$$z = x - 1$$

$$z = (-6) - 1 \quad (1, -6, -7)$$

$$z = -7$$

b) abscisa = 54

$$y = 2 \cdot 54 + 3 = 111 \quad (1, 111, 53)$$

$$z = 54 - 1 = 53$$

c) ordenada = 27

$$y = 2 \cdot -26 + 3 = -49 \quad (-26, -49, 27)$$

$$27 = x - 1$$

$$-x = 27 - 1$$

$$-x = 26 \quad (-1)$$

$$x = -26$$

$$x-1 = y-2$$

$$y = -x - 1 + 2 \Rightarrow y = 1 - x$$

$$\begin{cases} y = 1 - x \\ z = 3 \end{cases}$$

(21)

c) $\pi_1:$ $\begin{cases} x = 1 + \sqrt{2} t \\ y = t \\ z = 5 - 3t \end{cases}$

$\pi_2:$ $\begin{cases} x = 3 \\ y = 2 \\ z = t \end{cases}$

$$\vec{u}^{\circ} = (\sqrt{2}, 1, -3)$$

$$\vec{v}^{\circ} = (0, 0, 1)$$

$$\cos \theta = \frac{|\vec{u}^{\circ} \cdot \vec{v}^{\circ}|}{|\vec{u}^{\circ}| |\vec{v}^{\circ}|} = \frac{3}{2\sqrt{3} \cdot 1} = \frac{\sqrt{3}}{2} = 0 \quad \theta = 30^\circ$$

$$\vec{u}^{\circ} \cdot \vec{v}^{\circ} = -3$$

$$|\vec{u}^{\circ}| = \sqrt{2+1+9} = \sqrt{12} = 2\sqrt{3}$$

$$|\vec{v}^{\circ}| = 1$$