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57º

1. Dados os vetores $\vec{u} = 2\vec{i} - 3\vec{j}$, $\vec{v} = \vec{i} - \vec{j}$ e $\vec{w} = -2\vec{i} + \vec{j}$,
determinar

a) $2\vec{u} - \vec{v}$
 $2(2\vec{i} - 3\vec{j}) - (\vec{i} - \vec{j})$
 $(4\vec{i} - 6\vec{j}) - (\vec{i} - \vec{j})$
 $3\vec{i} - 5\vec{j}$

b) $\frac{1}{2}\vec{u} - 2\vec{v} - \vec{w}$

$\frac{1}{2}(2\vec{i} - 3\vec{j}) - 2(\vec{i} - \vec{j}) -$
 $(\vec{i} - 2\vec{i} + \vec{j})$
 $(1\vec{i} - 1,5\vec{j}) - (2\vec{i} - 2\vec{j}) - (-2\vec{i} + \vec{j})$

c) $\vec{v} - \vec{u} + 2\vec{w}$

$(-1\vec{i} + 0,5\vec{j}) - (-2\vec{i} + \vec{j})$
 $(\vec{i} + -0,5\vec{j})$

$(\vec{i} - \vec{j}) - (2\vec{i} - 3\vec{j}) + 2(-2\vec{i} + \vec{j})$
 $(-1\vec{i} + 2\vec{j}) + (-4\vec{i} + 2\vec{j}) = 0(-5\vec{i} + 4\vec{j})$

d) $3\vec{u} - \frac{1}{2}\vec{v} - \frac{1}{2}\vec{w}$

$3(2\vec{i} - 3\vec{j}) - \frac{1}{2}(\vec{i} - \vec{j}) - \frac{1}{2}(-2\vec{i} + \vec{j})$

$(6\vec{i} - 9\vec{j}) - (0,5 - 0,5) - (-1 + 0,5) = 0 (5,5\vec{i} - 9,5\vec{j}) - (-1 + 0,5) = (6,5\vec{i} - 9\vec{j})$

FORONI:

② dados os vetores $\vec{u} = (3, -1)$ e $\vec{v} = (-1, 2)$ determinar o vetor \vec{x} tal que

$$2\vec{u} + \frac{1}{3}\vec{x} = 2\vec{v} - \vec{x}$$

$$\left\{ \begin{array}{l} \frac{1}{3} + 1 = \frac{4}{3} \\ -1 - \vec{x} = -\frac{15}{2} \end{array} \right.$$

$$4(\vec{u} - \vec{v}) + \frac{1}{3}\vec{x} = 2\vec{v} - \vec{x}$$

$$\left\{ \begin{array}{l} -2\vec{u} - \vec{v} = -\frac{9}{2} \\ \vec{x} + 6 = \frac{15}{2} \end{array} \right.$$

$$4\vec{u} - 4\vec{v} = \frac{1}{3}\vec{x} = 2\vec{v} - \vec{x}$$

$$\left\{ \begin{array}{l} 4\vec{u} - 4\vec{v} = 3 \\ \vec{x} + 6 = \frac{15}{2} \end{array} \right.$$

$$\frac{4}{3}\vec{x} = -2\vec{u} + 4\vec{v} \cdot \frac{3}{4}$$

$$\vec{x} = -\frac{3}{2}\vec{u} + 3\vec{v}$$

$$\vec{x} = -\frac{3}{2}(3, -1) + 3(-1, 2)$$

$$\vec{x} = \left(-\frac{9}{2} - 3, \frac{3}{2} + 6 \right) = \left(-\frac{15}{2}, \frac{15}{2} \right)$$

$$(9) 3\vec{x} - (2\vec{v} - \vec{u}) = 2(4\vec{x} - 3\vec{u})$$

$$3\vec{x} - \vec{v} = \vec{u} - 6\vec{u}$$

$$3\vec{x} - 4\vec{u} = \vec{v} - 6\vec{u}$$

$$-5\vec{x} = (-1 + 2) - (18 + 6) = 0 \quad | :4$$

$$-5\vec{x} = 1 - 24$$

$$-5\vec{x} = -23 \quad (=)$$

$$\vec{x} < 23$$

$$\vec{x} = \frac{23}{5}$$

$$\vec{x} = (0, \frac{23}{5})$$

③ dadas as pontas A (-1, 3), B (2, 5), C (3, -1) e O (0, 0)
calcular

$$a) \vec{AB} = \vec{B} - \vec{A}$$

$$\vec{A} = \vec{O} - \vec{B} + \vec{A}$$

$$(-1, 3) - (0, 0) = (2, 5) - (-1, 3)$$

$$(-1, 3) - (3, -1)$$

$$(-4, 2)$$

$$b) \vec{OC} - \vec{OB}$$

$$\vec{C} = \vec{O} + \vec{C} - \vec{B}$$

$$(3, 1) - (0, 0) = (3, -1) + (2, 5)$$

$$(3, -1) + (1, 6)$$

$$(2, 5)$$

$$c) 3\vec{BA} - 4\vec{CB}$$

$$3(-1, 3) - (2, 5) - 4(2, 5) - (3, -1)$$

$$(-3, 9) - (2, 5) - (8, 10) - (3, -1)$$

$$(-5, 3) - (5, 19)$$

$$(-10, -16)$$

④ $\vec{u} = (2, -4)$, $\vec{v} = (-5, 1)$ e $\vec{w} = (-12, 6)$, determinar a_1 e a_2 tais que $\vec{w} = a_1\vec{u} + a_2\vec{v}$.

$$\vec{w} = a_1\vec{u} + a_2\vec{v}$$

$$(-12, 6) = a_1 \cdot (2, -4) + a_2 \cdot (-5, 1)$$

$$(-12, 6) = 2a_1 - 4a_2 - 5a_2 + a_2$$

$$(-12, 6) = -2a_1 + -4a_2$$

⑤ Dadas as pontas A (3, -4) e B (-1, 1) e o vetor $\vec{v} = (-2, 3)$, calcular

$$a) (\vec{B} - \vec{A}) + 2\vec{v}$$

$$((-1+1) - (3+4)) + 2(-2, 3)$$

$$(-4, -3) + (-4, 6)$$

$$(-8, 3)$$

$$b) (\vec{A} - \vec{B}) - \vec{v}$$

$$((3, -4) - (-1, 1)) - (-2, 3)$$

$$(4, -5) - (-2, 3)$$

$$(6, -8)$$

FORON:

$$C) B + 2(B - A)$$

$$(-1, 1) + 2((-1, 1) - (-3, -4))$$

$$(-1, 1) + 2(-4, 5)$$

$$(-1, 1) + (-8, 10)$$

$$(-9, 11)$$

$$D) 3\vec{v} - 2(A - B)$$

$$3(-2, 3) - 2((3, -4) - (-1, 2))$$

$$(-6, 9) - 2(4, -5)$$

$$(-6, 9) - (8, -10)$$

$$(-14, 19)$$

6) Sejam os vetores $A(-5, 1)$ e $B(1, 3)$. Determinar o vetor $\vec{v} = (a, b)$ tal que

$$a) B = A + 2\vec{v}$$

$$(1, 3) - (-5, 1) + 2\vec{v}$$

$$2\vec{v} = (-5, 1) + (1, -3)$$

$$2\vec{v} = (-6, -2)$$

$$\vec{v} = \left(-\frac{6}{2}, -\frac{2}{2}\right) = \vec{v} = (-3, -1)$$

$$b) A = B + 3\vec{v}$$

$$(-5, 1) = (1, 3) + 3\vec{v}$$

$$3\vec{v} = (1, 3) + (5, -1)$$

$$3\vec{v} = (6, 2)$$

$$\vec{v} = \left(\frac{6}{3}, \frac{2}{3}\right) \Rightarrow \vec{v} = \left(2, \frac{2}{3}\right)$$

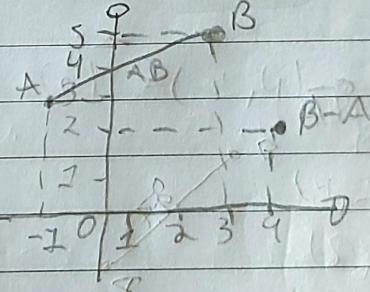
7) Representar em um gráfico o vetor \vec{AB} e os vetores resultante de vetores nulos, nos casos

$$a) A(-1, 3) \text{ e } B(3, 5)$$

$$B - A$$

$$(3, 5) - (-1, 3)$$

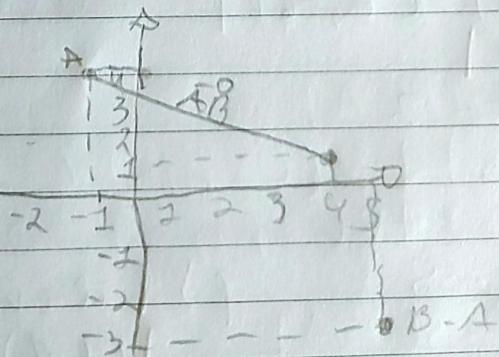
$$(4, 2)$$

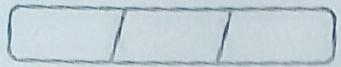


$$b) A(-1, 4), B(4, 1)$$

$$B - A = (4, 1) - (-1, 4)$$

$$= (5, -3)$$



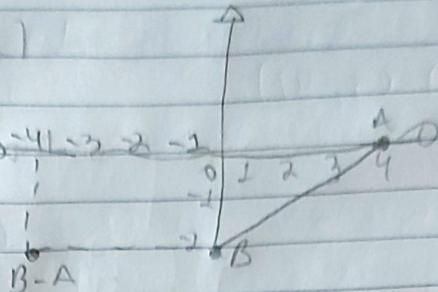


c) $A(4,0)$, $B(0,-2)$

$$B-A$$

$$(0,-2) - (4,0)$$

$$(-4, -2)$$

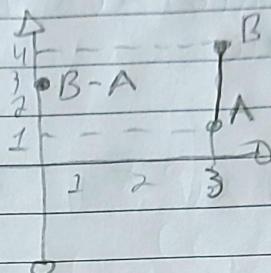


d) $A(3,1)$, $B(3,4)$

$$B-A$$

$$(3,4) - (3,1)$$

$$(0,3)$$



16) dados os vetores $\vec{u} = (1, -1)$, $\vec{v} = (-3, 4)$ e $\vec{w} = (8, -6)$, calcule

e) $|\vec{u}|$

$$|\vec{u}| = \sqrt{1^2 + 1^2}$$

$$|\vec{u}| = \sqrt{2}$$

$$e) |\vec{w}| = \sqrt{8^2 + (-6)^2}$$

$$|\vec{w}| = \sqrt{64 + 36}$$

$$|\vec{w}| = \sqrt{100}$$

$$|\vec{w}| = 10$$

f) $|2\vec{u} - \vec{w}|$

$$|2(1, -1) - (8, -6)|$$

$$|(2-2) - (8, -6)|$$

$$|-6 + 4|$$

$$|-2| = 2$$

d) $\frac{\vec{v}}{|\vec{v}|} \cdot \frac{(-3, 4)}{5} = \frac{1}{5}$

$$|\vec{v}| = \sqrt{(-3)^2 + 4^2}$$

$$|\vec{v}| = \sqrt{9 + 16}$$

$$|\vec{v}| = \sqrt{25} = 5$$

e) $|\vec{v}| = 5$

f) $|\vec{u} + \vec{v}|$

$$|(1, -1) + (-3, 4)|$$

$$|(-2 + 3)|$$

$$|1| = 1$$

g) $|\vec{w} - 3\vec{u}|$

$$|(8, -6) - 3(1, -1)|$$

$$|(8, -6) - (3, 3)|$$

$$|5 - 3|$$

$$|2| = 2$$

$$2) \begin{vmatrix} \vec{u} \\ |\vec{u}| \end{vmatrix} \quad |\vec{u}| = \sqrt{1^2 + (-1)^2} = \sqrt{2} \quad (1, -1) \Rightarrow \left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}} \right)$$

19) $\vec{v} = (a, -2)$ tenha módulo 4

$$\begin{aligned} |\vec{v}| &= \sqrt{a^2 + (-2)^2} = 4 \quad (\sqrt{a^2 + 4})^2 = 4^2 \\ 16 &= a^2 + 4 \\ a^2 &= 12 \Rightarrow a = \pm \sqrt{12} \end{aligned}$$

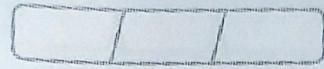
20) calcule os valores de a para que o vetor $\vec{u} = (a, \frac{1}{2})$ seja unitário

$$\begin{aligned} |\vec{u}| &= 1 \quad |\vec{u}| = \sqrt{a^2 + \left(\frac{1}{2}\right)^2} \\ 1 &= \sqrt{a^2 + \frac{1}{4}} \\ 1^2 &= \left(\sqrt{a^2 + \frac{1}{4}}\right)^2 \\ 1 &= a^2 + \frac{1}{4} \\ a^2 + \frac{1}{4} &= 1 \\ a^2 &= 1 - \frac{1}{4} \\ a^2 &= \frac{4-1}{4} \\ a^2 &= \frac{3}{4} \\ a &= \pm \sqrt{\frac{3}{4}} \\ a &= \pm \frac{\sqrt{3}}{2} \end{aligned}$$

21) Dado o vetor $\vec{v} = (1, -3)$, determinar o vetor para ele a \vec{w} que temos:

a) sentido contrário ao de \vec{v} e duas vezes o módulo de \vec{v}

$$\begin{aligned} -2 \cdot \vec{v} &= -2(1, -3) \\ -2\vec{v} &= (-2, 6) \end{aligned}$$



2) o mesmo sentido de \vec{v} e módulo 2;

$$\frac{\vec{v}}{|\vec{v}|} = \frac{(1, -3)}{\sqrt{10}} = \left(\frac{1}{\sqrt{10}}, \frac{-3}{\sqrt{10}} \right) \Rightarrow \text{o vetor}$$

$$2 \cdot \left(\frac{1}{\sqrt{10}}, \frac{-3}{\sqrt{10}} \right) = 0 \left(\frac{2}{\sqrt{10}}, \frac{-6}{\sqrt{10}} \right)$$

(c) sentido contrário ao de \vec{v} e módulo 4.

$$-4 = \left(\frac{1}{\sqrt{10}}, \frac{-3}{\sqrt{10}} \right) = 0 \left(\frac{4}{\sqrt{10}}, \frac{-12}{\sqrt{10}} \right)$$

31) dado os pontos $A(2, -2, 3)$ e $B(1, 1, 5)$ e o vetor $\vec{v} = (1, 3, -4)$, calcular:

a) $A + 3\vec{v}$

$$(2, -2, 3) + 3(1, 3, -4) = 0(2, -2, 3) + (3, 9, -12) = (5, 7, -9)$$

b) $(A - B) - \vec{v}$

$$((2, -2, 3) - (1, 1, 5)) - (1, 3, -4)$$

$$(1, -3, -2) - (1, 3, -4) = 0(0, -6, 2)$$

c) $B + 2(B - A)$

$$(1, 1, 5) + 2((1, 1, 5) - (2, -2, 3))$$

$$(1, 1, 5) + 2(-1, 3, 2) = 0(1, 1, 5) + (-2, 6, 4) = (-1, 7, 9)$$

d) $2\vec{v} - 3(B - A)$

$$2(1, 3, -4) - 3((1, 1, 5) - (2, -2, 3))$$

$$(2, 6, -8) - 3(-1, 3, 2)$$

$$(2, 6, -8) - (-3, 9, 6) = 0 = (5, -3, -14)$$

FORON:

(32) dadas as pontos A(3, -4, -2) e B(-2, 1, 0), determine o ponto N interno ao segmento AB tal que $\vec{AN} = \frac{2}{5}\vec{AB}$

$$\vec{AB} = \frac{2}{5}(B-A)$$

$$\vec{AB} = \frac{2}{5}((-2, 1, 0) - (3, -4, -2)) = 0(-5, 5, 2)$$

$$\vec{AN} = \frac{2}{5}(-5, 5, 2) = 0\left(\frac{-10}{5}, \frac{10}{5}, \frac{4}{5}\right) = 0(-2, 2, 4/5)$$

$$\vec{AN} = (-2, 2, 4/5)$$

$$N - A = (x, y, z) - (3, -4, -2)$$

$$x - 3 = -2 \Rightarrow x = -2 + 3 = 1 \quad N = (1, -2, 4/5)$$

$$y + 4 = 2 \quad y = 2 - 4 = -2$$

$$z + 2 = 4/5 \quad z = 4/5 - 2 = -\frac{6}{5}$$

(33) sabendo que $3\vec{u} - 4\vec{v} - 2\vec{w} = 0$, determinar a, b, c sejam do $\vec{u} = (2, -1, 1)$, $\vec{v} = (a, b - 2, 3)$ e $\vec{w} = (4, -1, 0)$

$$3(2, -1, 1) - 4(a, b - 2, 3) - 2(4, -1, 0) = 0$$

$$(6, -3, 3) - (4a, 4b - 8, 12) = (8, -2, 0)$$

$$6 - 4a = 8 \quad a = -\frac{1}{2}$$

$$-3 - 4b + 8 = -2 \quad b = \frac{7}{4}$$

$$3 - 12 = 0 \quad c = 4$$

$$6 - 4a = 8$$

$$-4a = 8 - 6$$

$$-4a = 2$$

$$a = \frac{2}{-4} = -\frac{1}{2}$$

$$3c - 12 = 0$$

$$3c = 12$$

$$c = \frac{12}{3}$$

$$c = 4$$

$$-3 - 4b + 8 = -2$$

$$-3 - 4b + 2 = -2 - 8 + 3$$

$$-4b = -7 - (-1)$$

$$b = \frac{7}{4}$$

(35) dados os vetores $\vec{u} = (2, 3, -1)$, $\vec{v} = (1, 1, 1)$, $\vec{w} = (-3, 4, 0)$

a) determinar o vetor \vec{x} de modo que $3\vec{u} - \vec{v} + \vec{x} = 4\vec{x} + 2\vec{w}$

$$4\vec{x} - \vec{x} = 3\vec{u} - \vec{v} - 2\vec{w}$$

$$3\vec{x} = 3(2, 3, -1) - (1, 1, 1) - 2(-3, 4, 0)$$

$$3\vec{x} = (6, 9, -3) - (1, 1, 1) - (-6, 8, 0)$$

$$3\vec{x} = (6, 9, -3) - (7, 7, 1)$$

$$3\vec{x} = (-1, 16, -4)$$

$$3(x, y, z) = (-1, 16, -4) \Rightarrow \left(\frac{-1}{3}, \frac{16}{3}, \frac{-4}{3} \right) = 0 \times$$

$$\begin{cases} 3x = -1 \\ 3y = 16 \\ 3z = -4 \end{cases} \quad \begin{cases} 3x = -1 \\ x = -\frac{1}{3} \end{cases} \quad \begin{cases} 3y = 16 \\ y = \frac{16}{3} \end{cases} \quad \begin{cases} 3z = -4 \\ z = -\frac{4}{3} \end{cases}$$

b) a_1, a_2, a_3 tais que $a_1\vec{u} + a_2\vec{v} + a_3\vec{w} = (-2, 13, -5)$

$$a_1 \cdot (2, 3, -1) + a_2 \cdot (1, 1, 1) + a_3 \cdot (-3, 4, 0) = (-2, 13, -5)$$

$$(2a_1, 3a_1, -a_1) + (a_2, a_2, a_2) + (-3a_3, 4a_3, 0) = (-2, 13, -5)$$

$$2a_1 + a_2 + (-3a_3) = -2$$

$$3a_1 + a_2 + 4a_3 = 13$$

$$-a_1 + a_2 + 0 = -5$$

$$\begin{vmatrix} 2 & 1 & -3 & -2 \\ 3 & -1 & 4 & 13 \\ -1 & 1 & 0 & -5 \end{vmatrix}$$

$$\begin{vmatrix} 2 & 1 & -3 & -2 \\ 3 & -1 & 4 & 13 \\ -1 & 1 & 0 & -5 \end{vmatrix}$$

$$\begin{vmatrix} 2 & 1 & -3 & -2 \\ 3 & -1 & 4 & 13 \\ -1 & 1 & 0 & -5 \end{vmatrix}$$

$$-a_1 + a_2 = -5$$

$$a_1 = a_2 + 5$$

$$a = -3 + 5$$

$$a = 2$$

$$c = -3 + 4$$

$$c = 1$$

$$3 \times 4$$

$$a_1 = 2, a_2 = -3, a_3 = 1$$

$$2(a_2 + 5) + a_2 + (-3a_3) = -2$$

$$2a_2 + 10 + a_2 - 3a_3 = -2$$

$$3a_2 + 10 - 3a_3 = -2$$

$$3a_2 = -2 - 10 + 3a_3$$

$$3a_2 = -12 + 3a_3 \quad (\div 3)$$

$$3(a_2 + 5) + a_2 + 4(a_2 + 4) = 13$$

$$3a_2 + 15 - a_2 + 4a_2 + 16 = 13$$

$$6a_2 + 31 = 13$$

$$6a_2 = 13 - 31$$

$$6a_2 = -18$$

$$a_2 = \frac{-18}{6}$$

$$a_2 = -3$$

$$\text{FORON: } a_2 = a_3 - 4$$

$$a_3 = a_2 + 4$$

(49) verificar se são unitários os seguintes vetores:

$$\vec{u} = (1, 1, 1) \text{ e } \vec{v} = \left(\frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}} \right)$$

$$|\vec{u}| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$$

\vec{u} é unitário

$$\begin{aligned} |\vec{v}| &= \sqrt{\left(\frac{1}{\sqrt{6}}\right)^2 + \left(-\frac{2}{\sqrt{6}}\right)^2 + \left(\frac{1}{\sqrt{6}}\right)^2} = \sqrt{\frac{1^2}{6} + \frac{4}{6} + \frac{1}{6}} = \sqrt{\frac{6}{6}} = 1 \\ &= \sqrt{\frac{1+4+1}{6}} = \sqrt{\frac{6}{6}} = 1 \end{aligned}$$

\vec{v} é unitário

(50) determinar o valor de N para que o vetor $\vec{v} = (m, -\frac{1}{2}, \frac{3}{4})$ seja unitário

$$|\vec{v}| = \sqrt{m^2 + \left(\frac{-1}{2}\right)^2 + \left(\frac{3}{4}\right)^2} = \sqrt{m^2 + \frac{1}{4} + \frac{9}{16}}$$

$$\sqrt{m^2 + \frac{13}{16}} = \sqrt{m^2 + \frac{13}{16}} = \sqrt{3 + \frac{13}{16}}$$

$$\sqrt{\frac{16}{16}} = 1$$