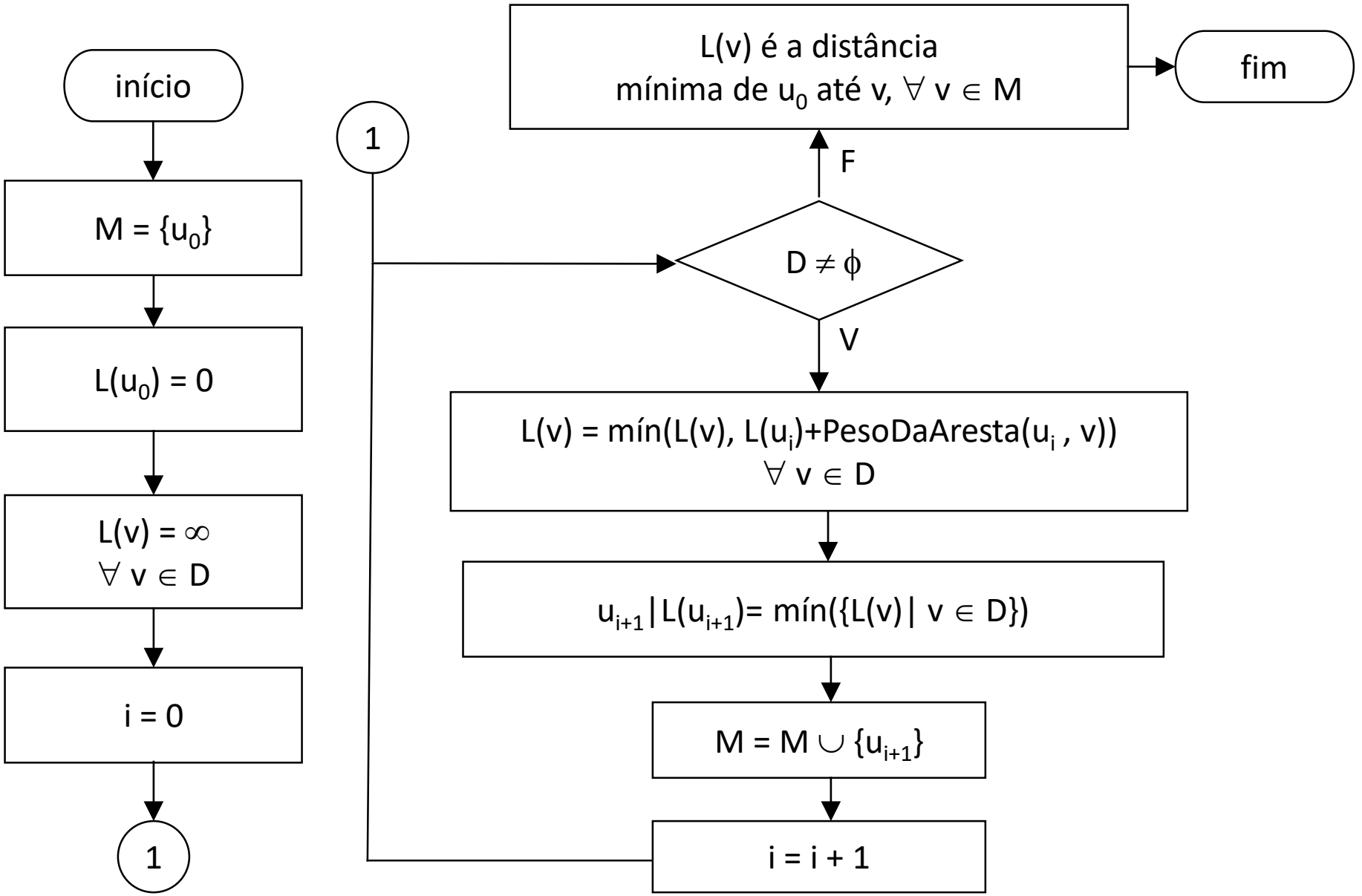
Grafos
Caminho Mínimo e Árvore Geradora Mínima

ECM404

Algoritmo Dijkstra

- Encontrar o caminho mais curto é um problema clássico na teoria dos grafos. Às arestas são atribuídos pesos que representam, por exemplo, a distância entre as cidades, tempos que separam a execução de certas tarefas, entre outras.

Algoritmo Dijkstra



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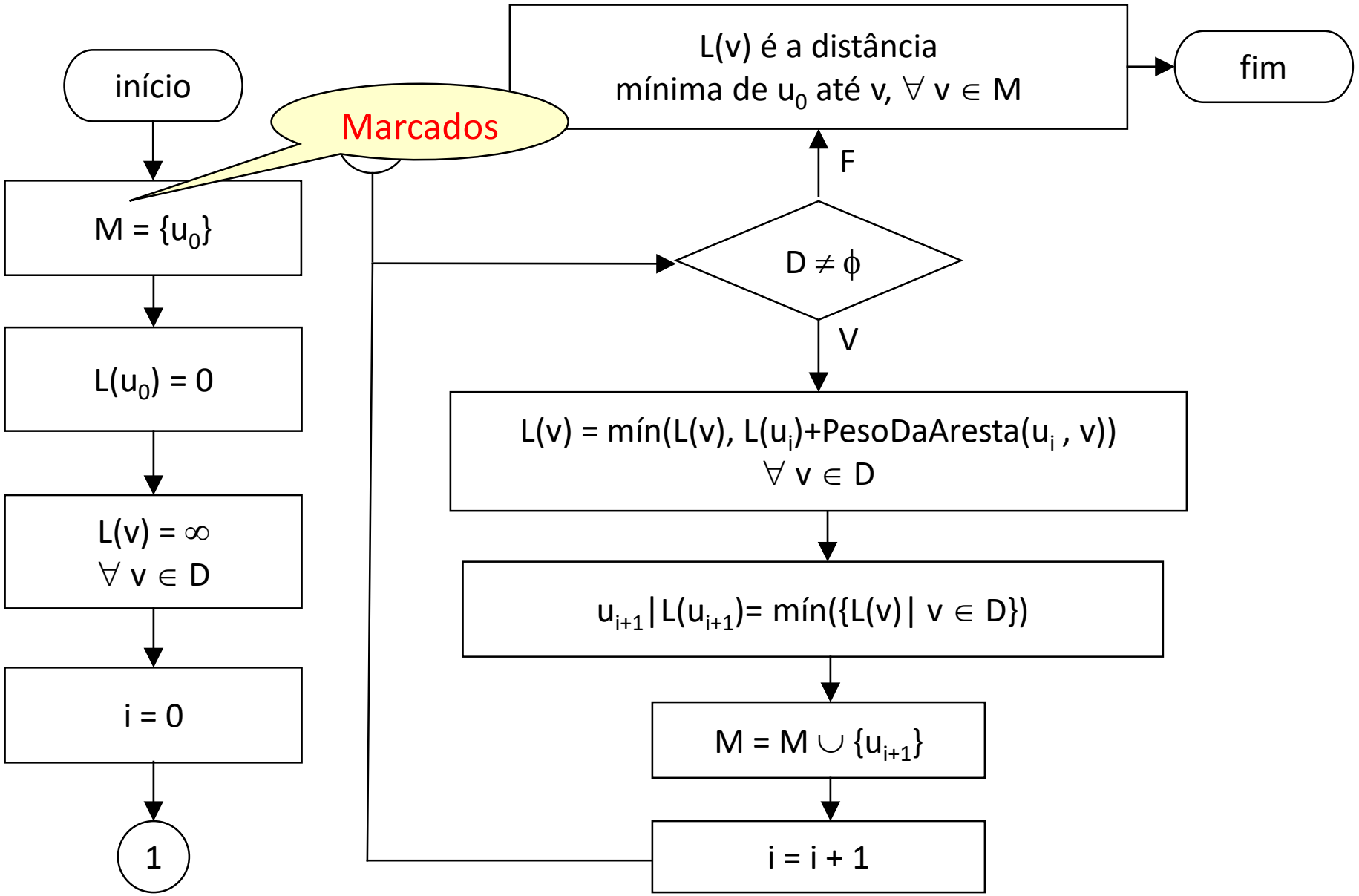

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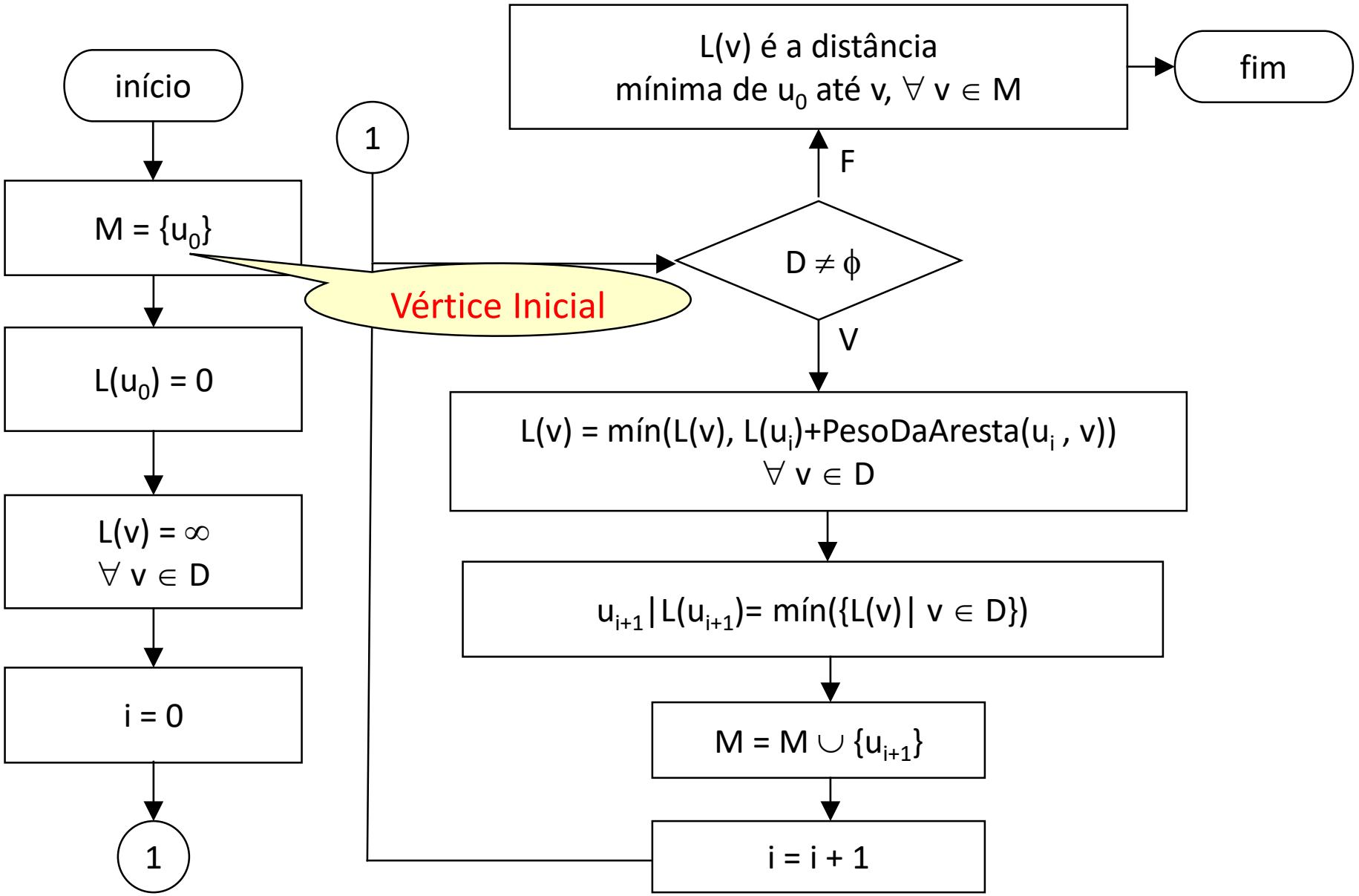

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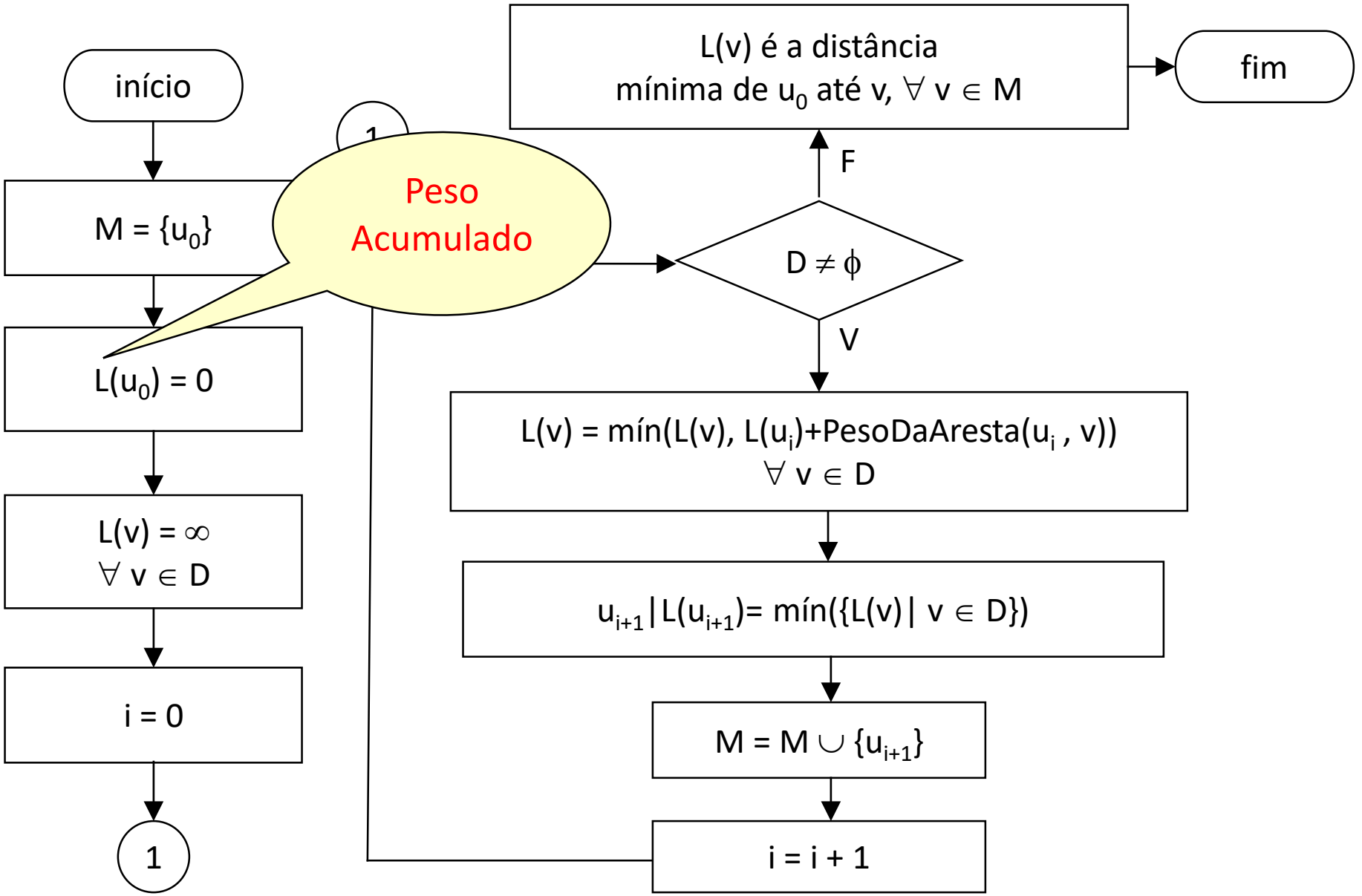

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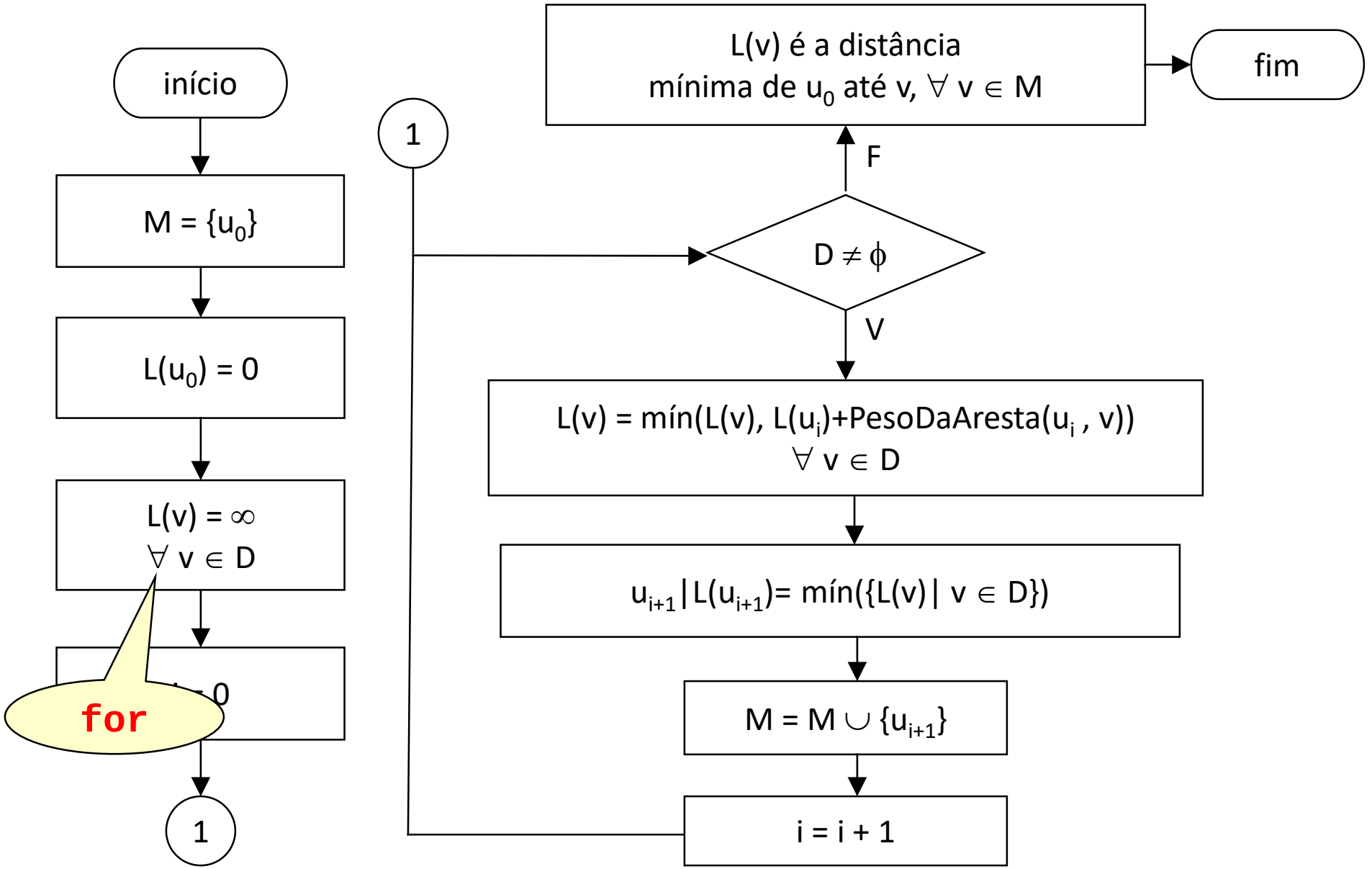

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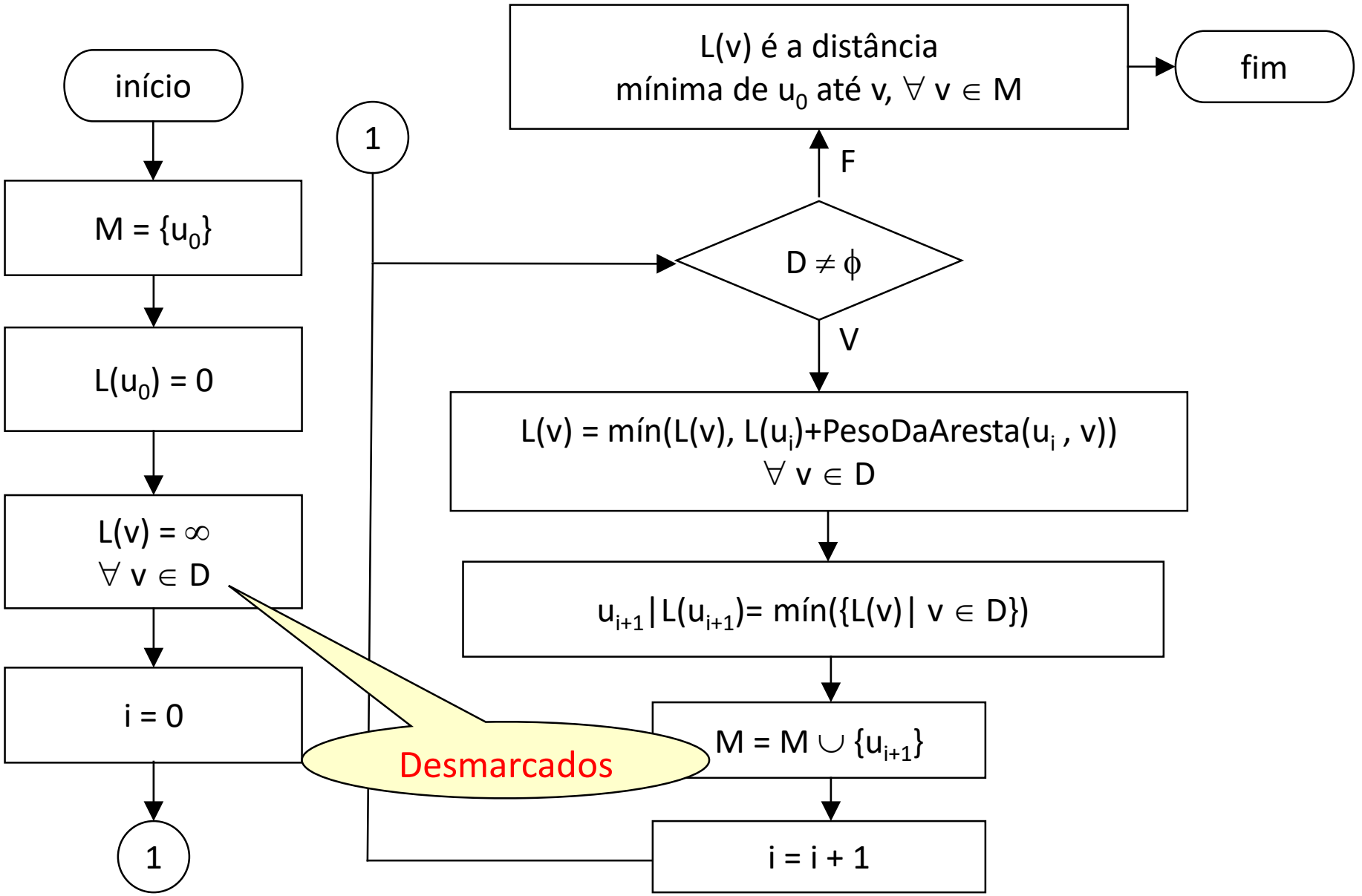

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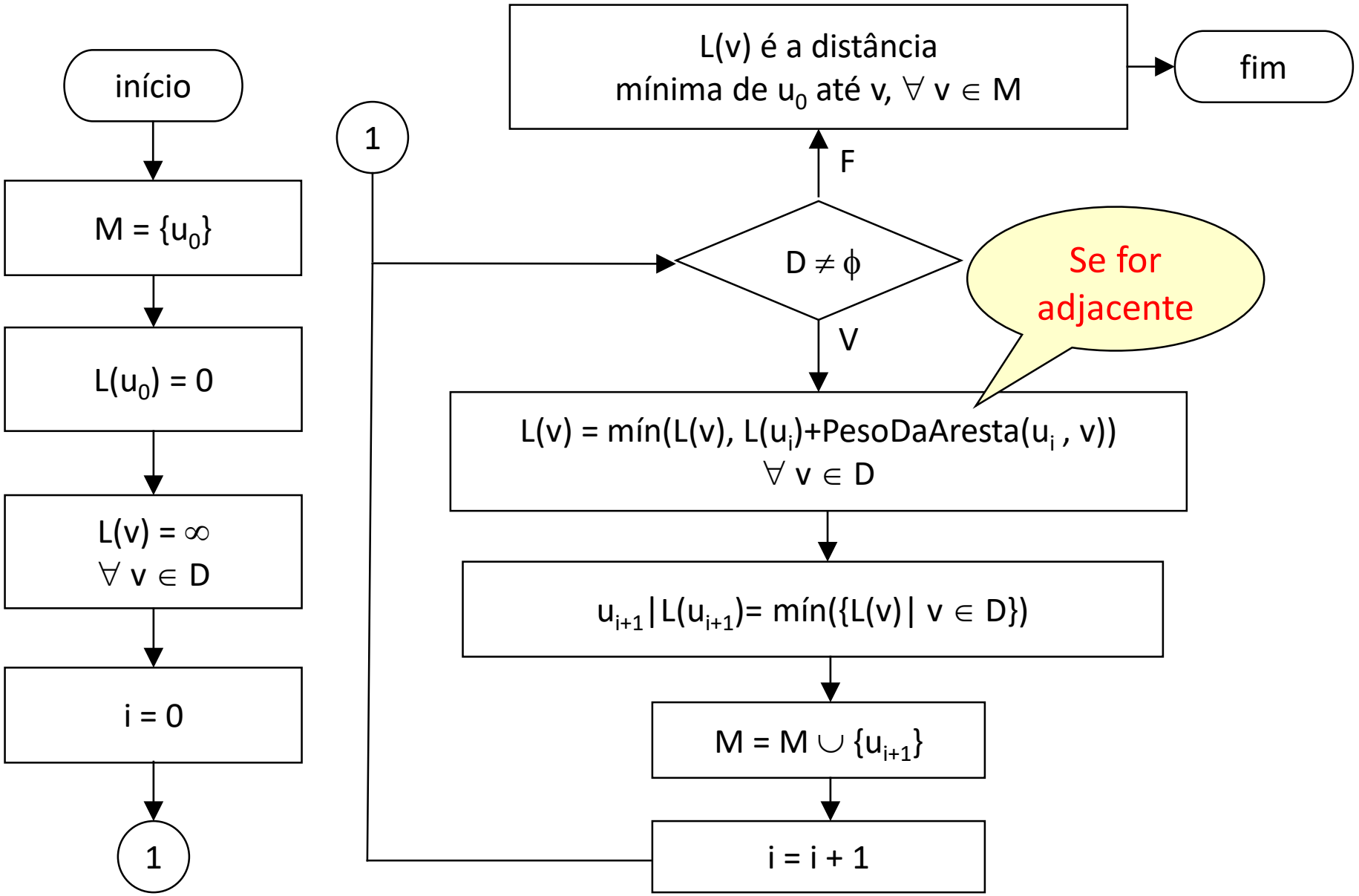
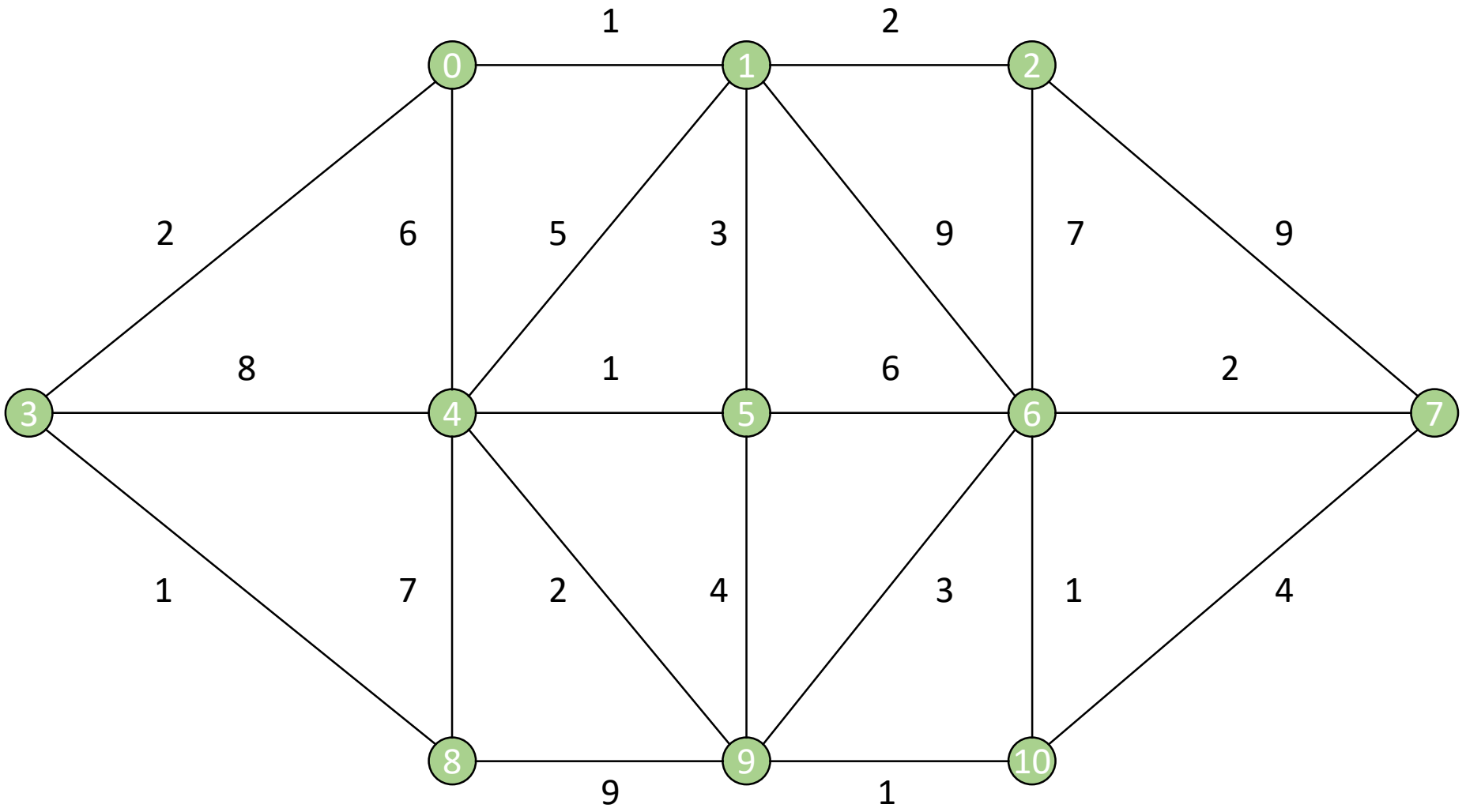


Algoritmo Dijkstra - Exemplo



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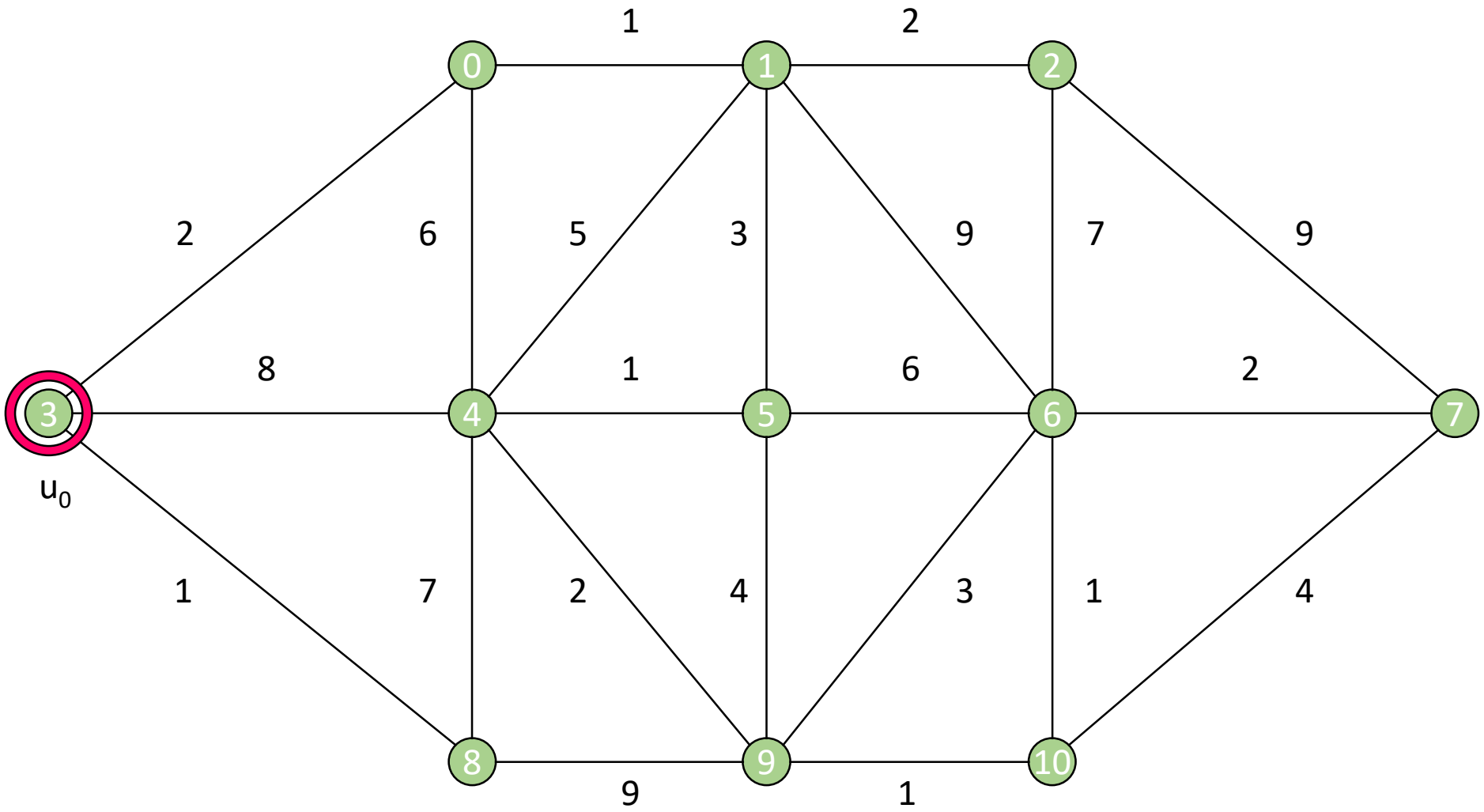

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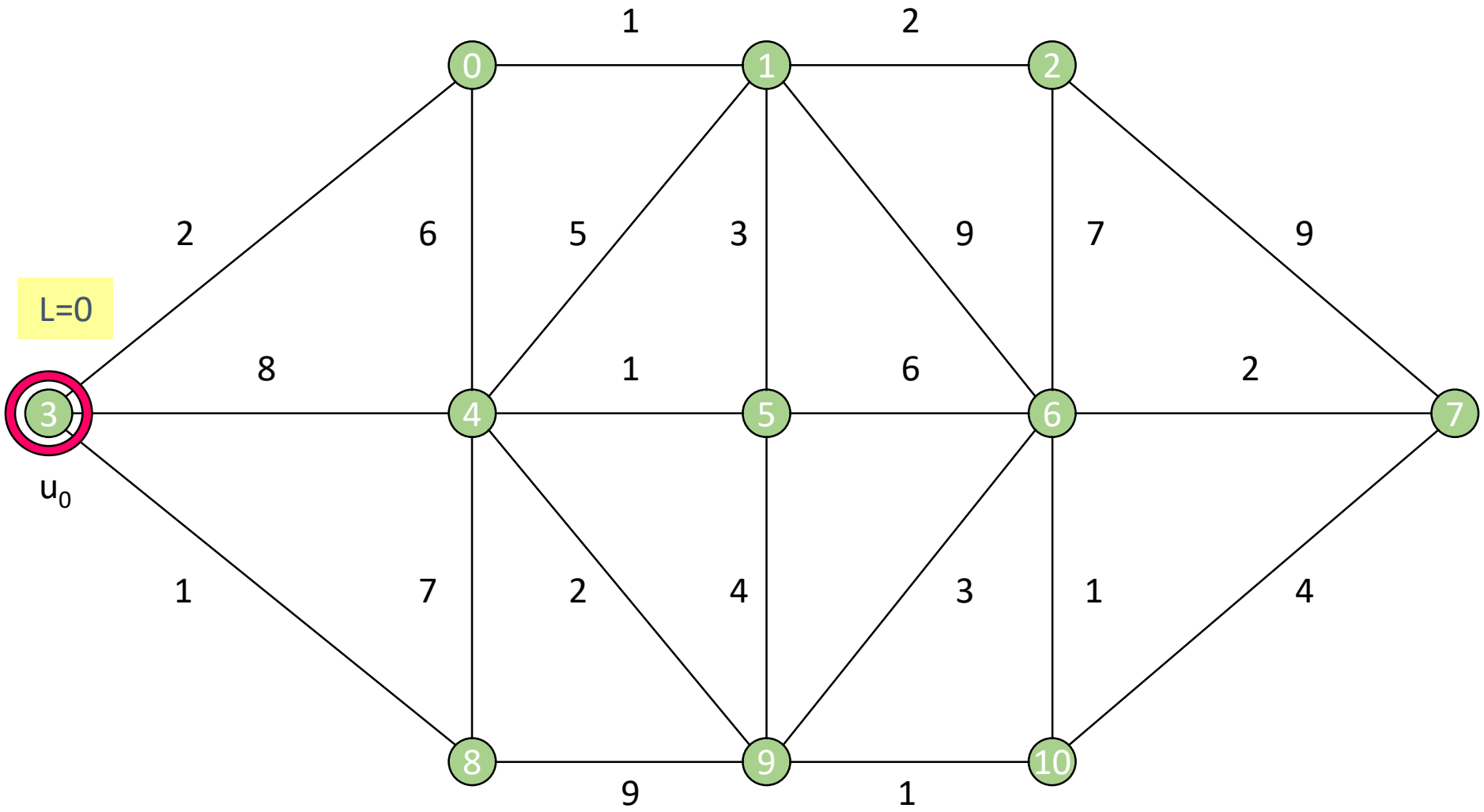

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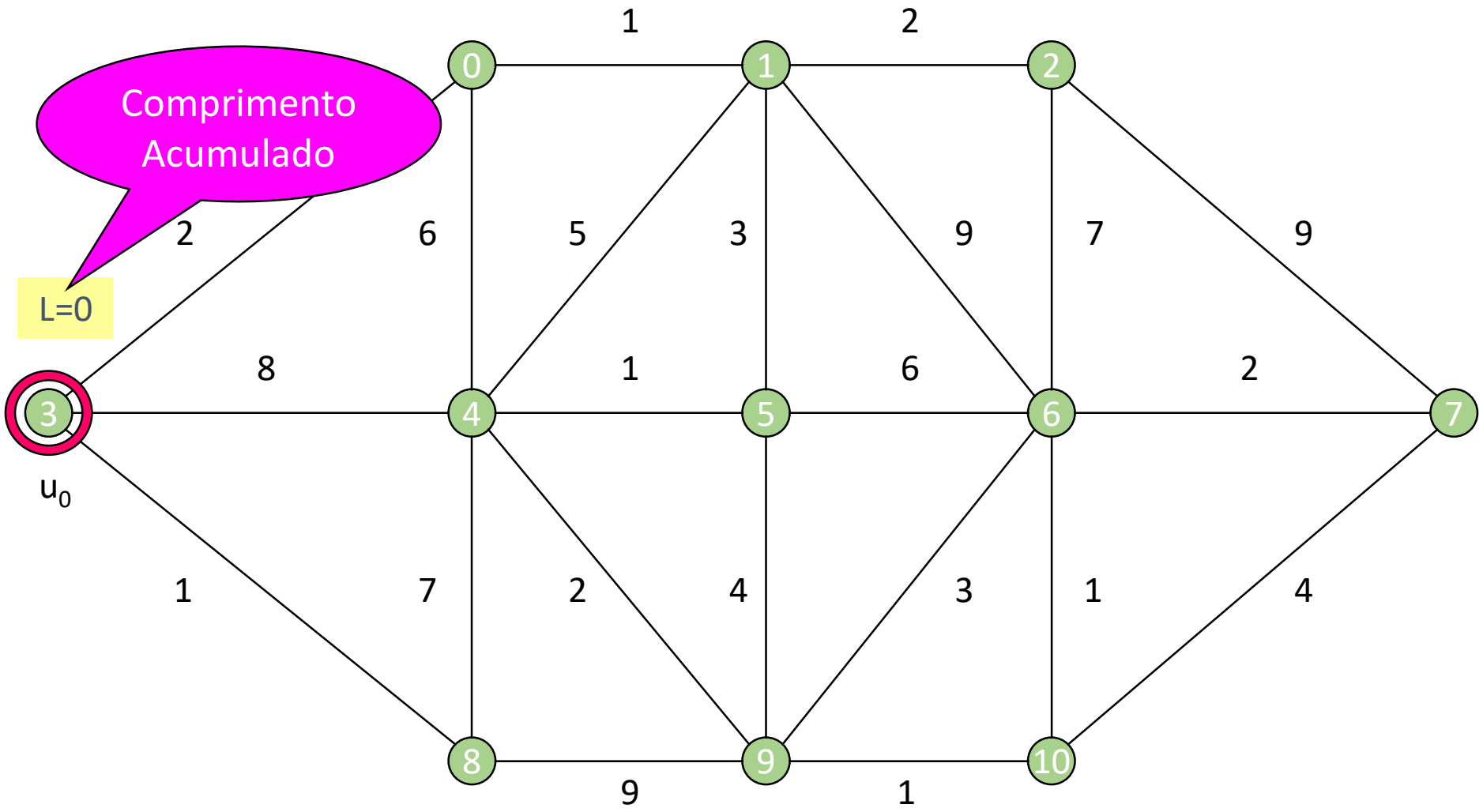

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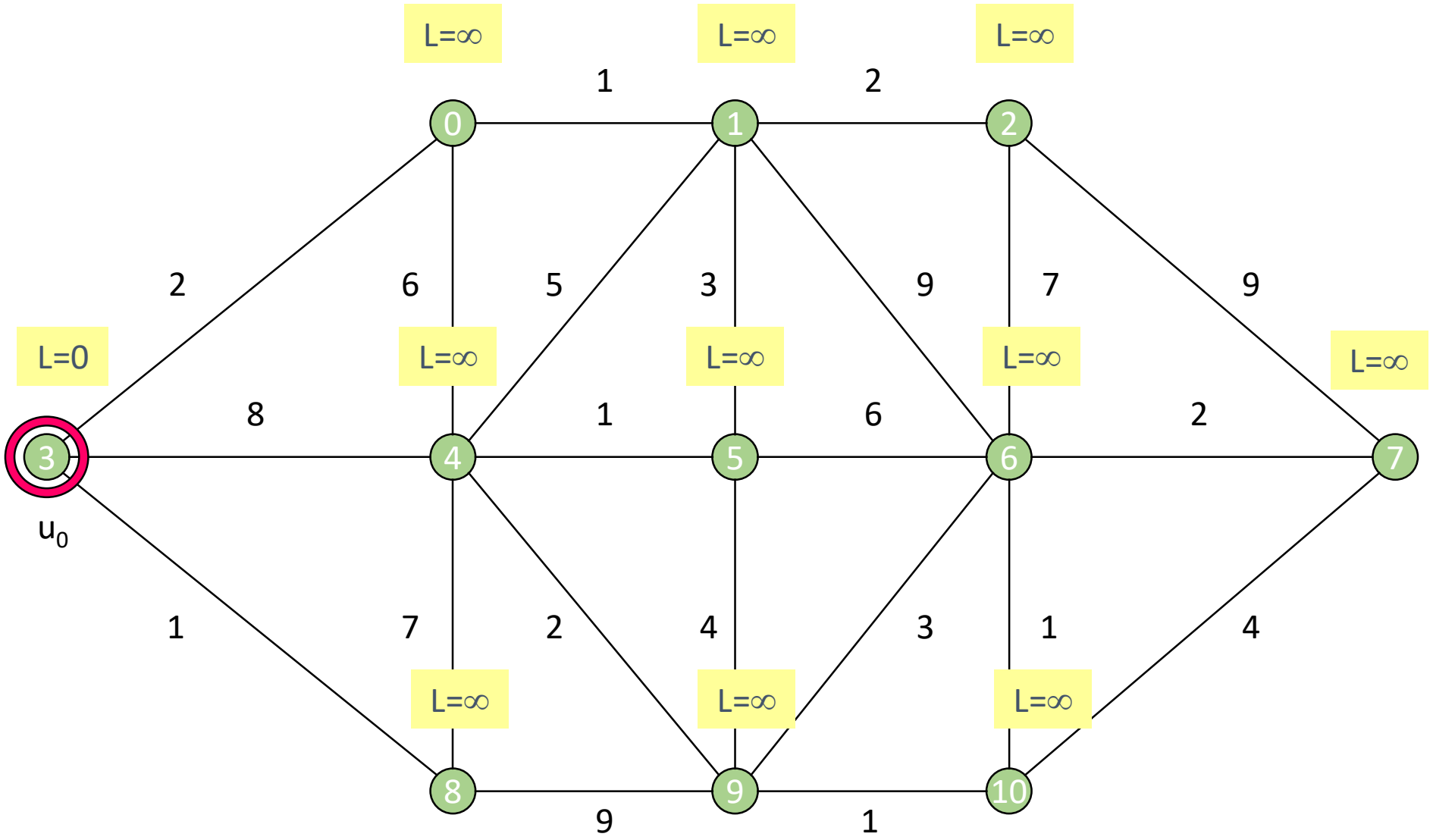

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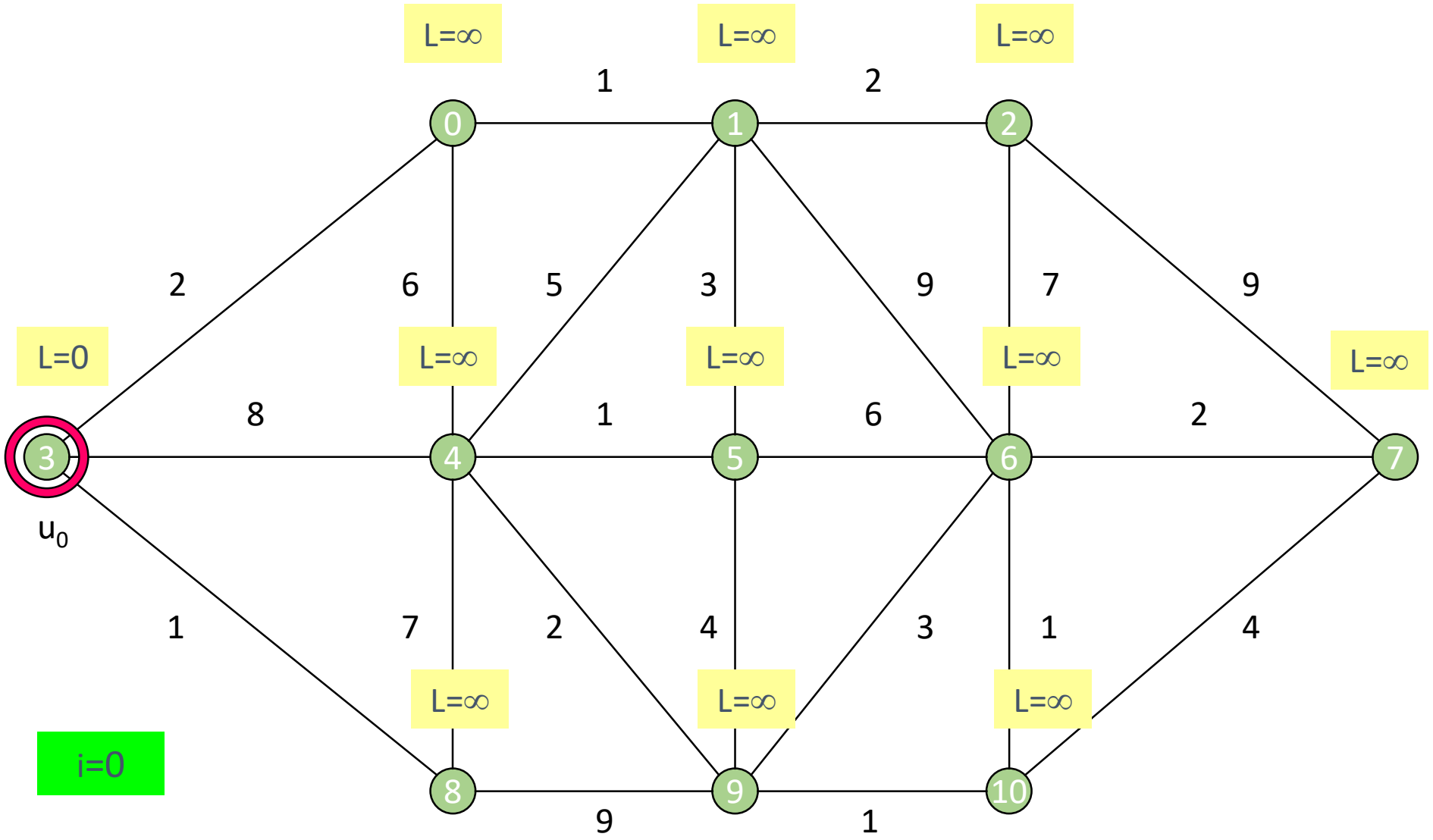

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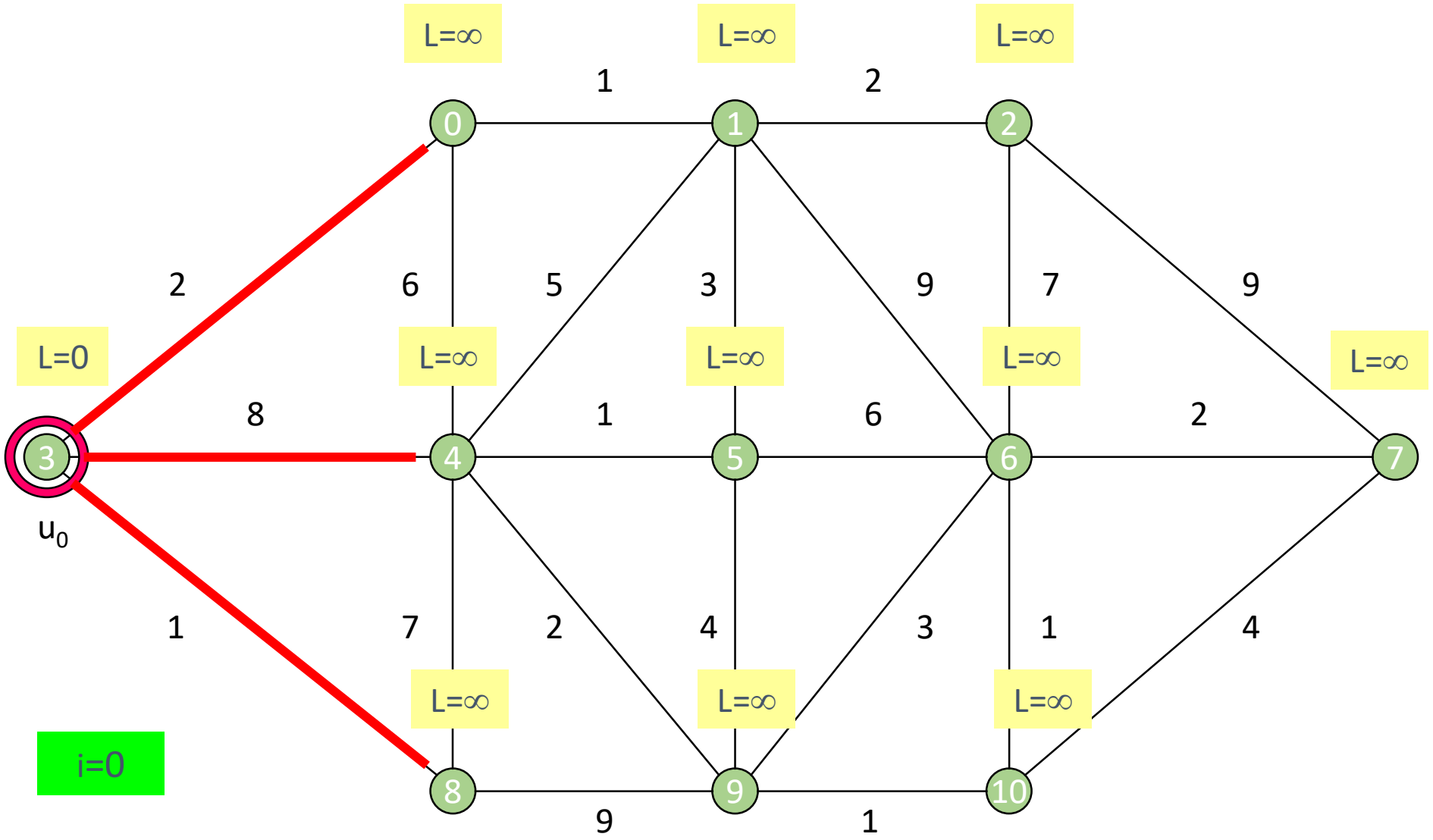

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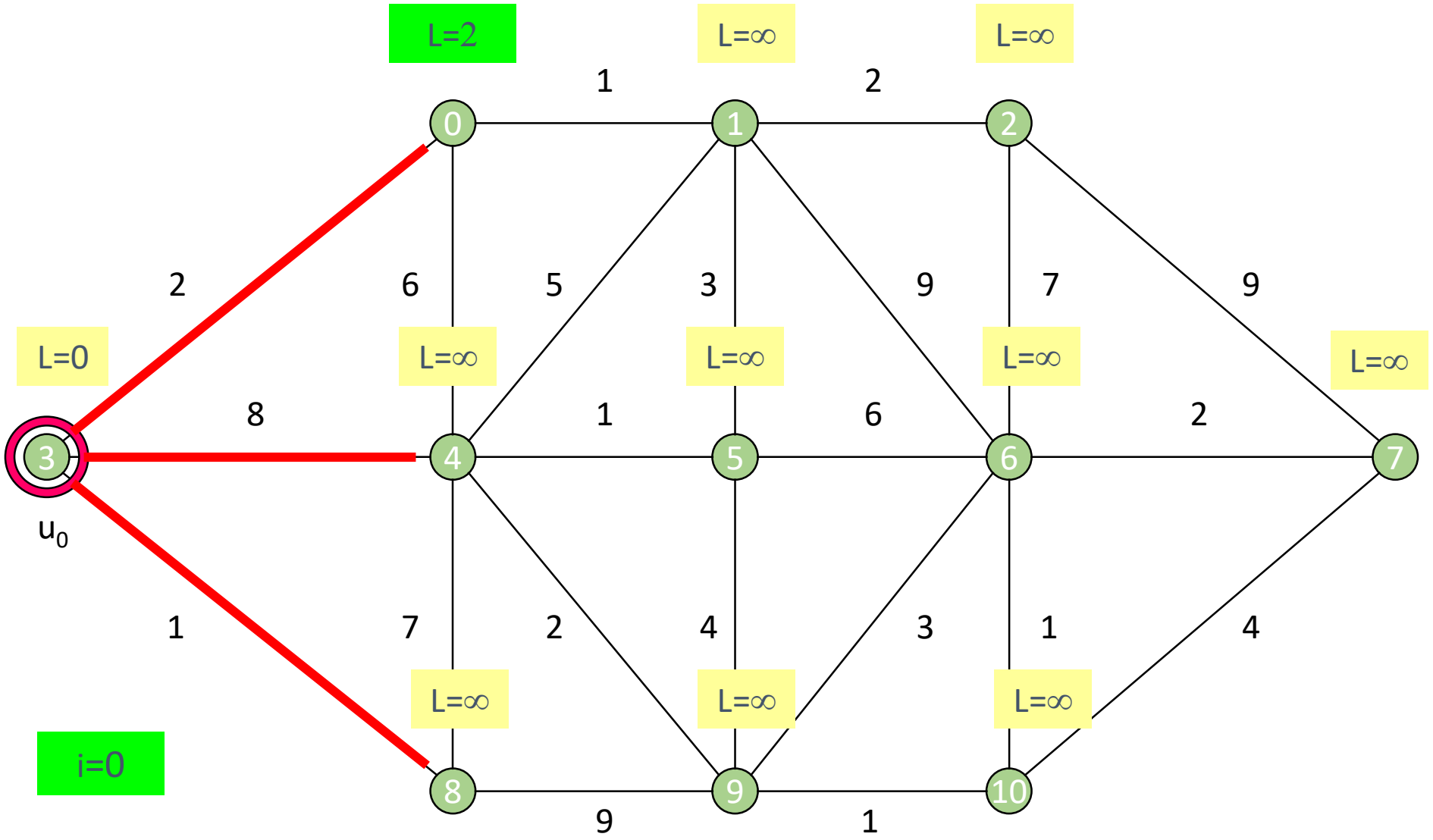

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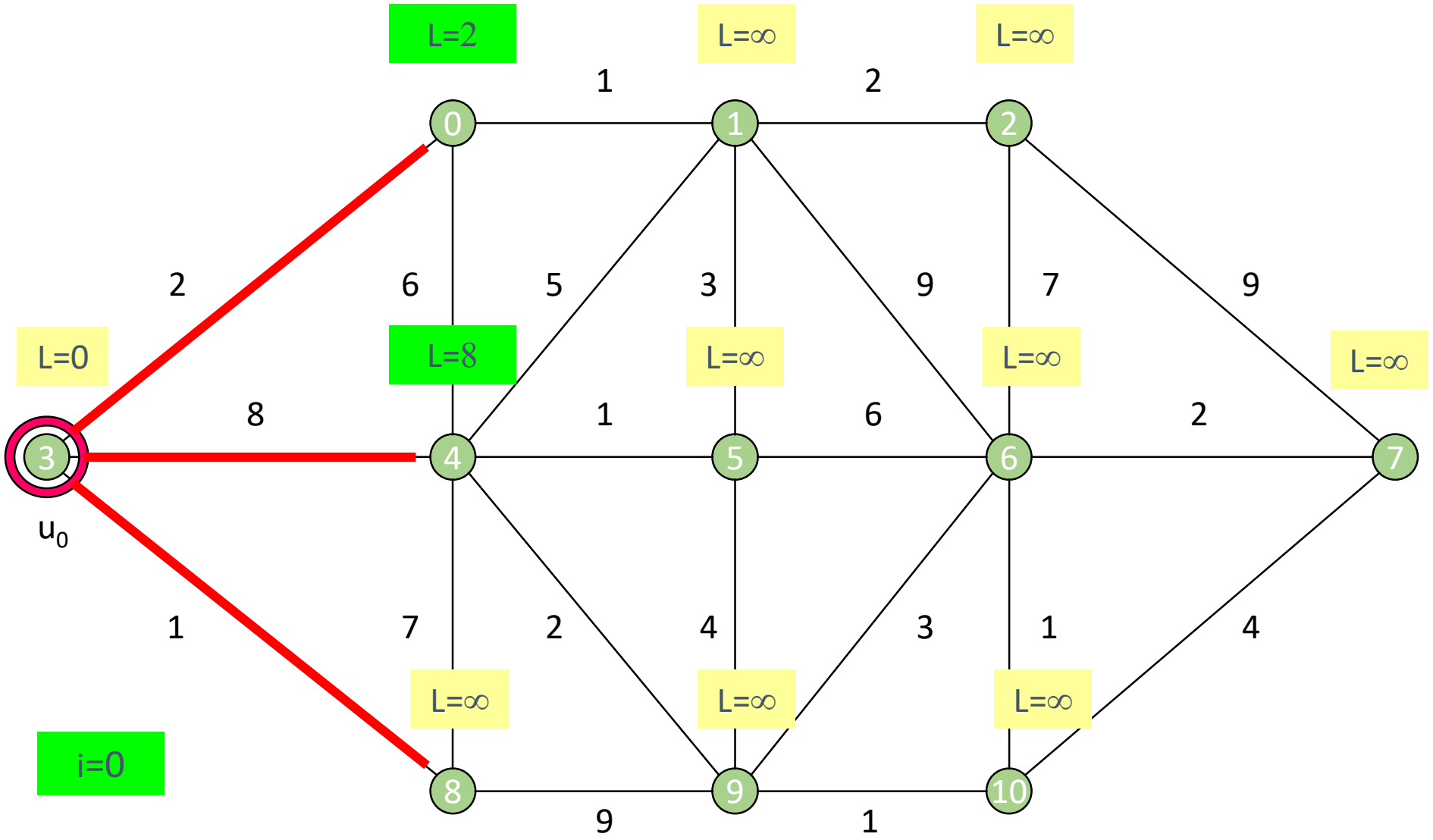

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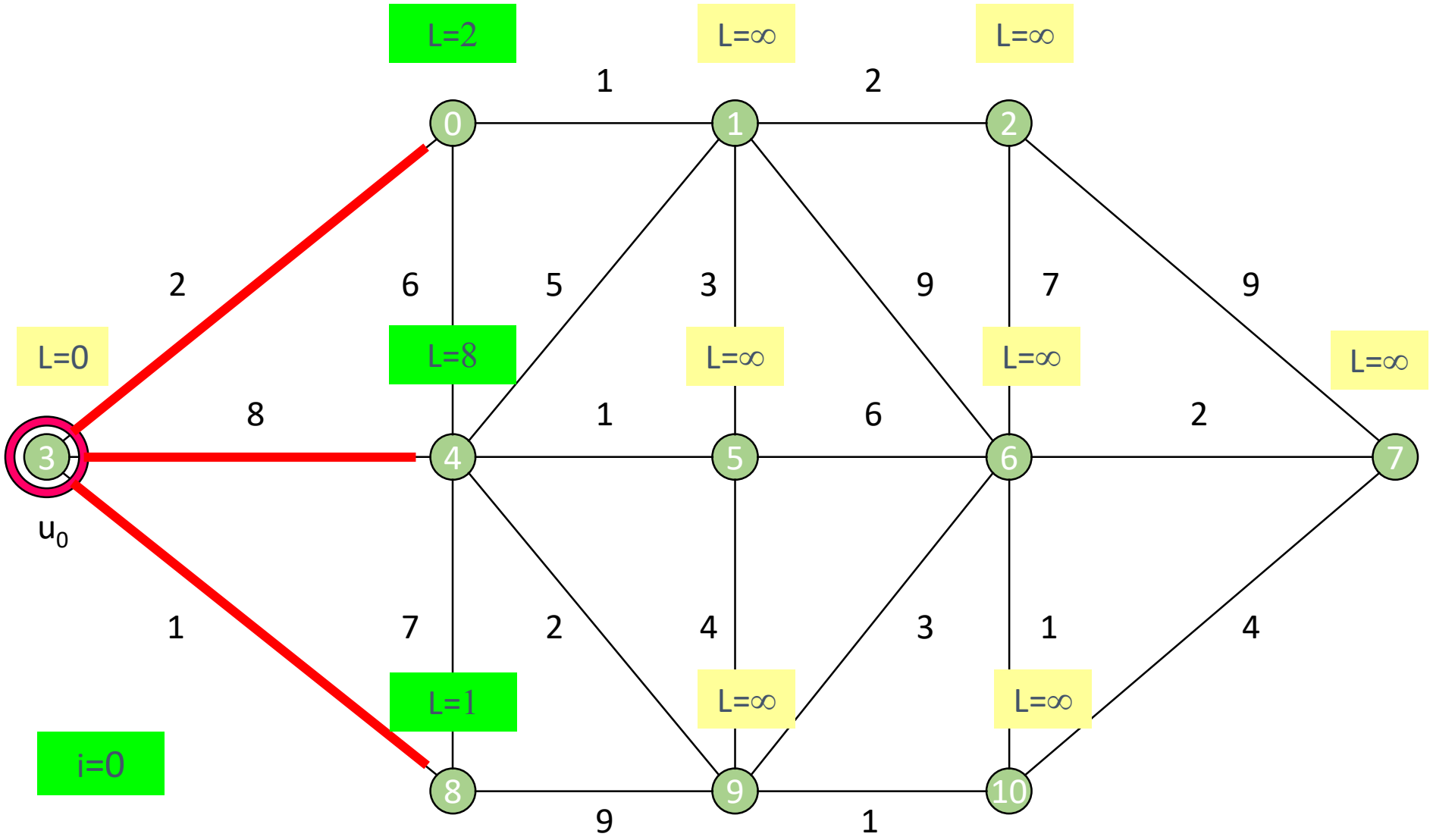

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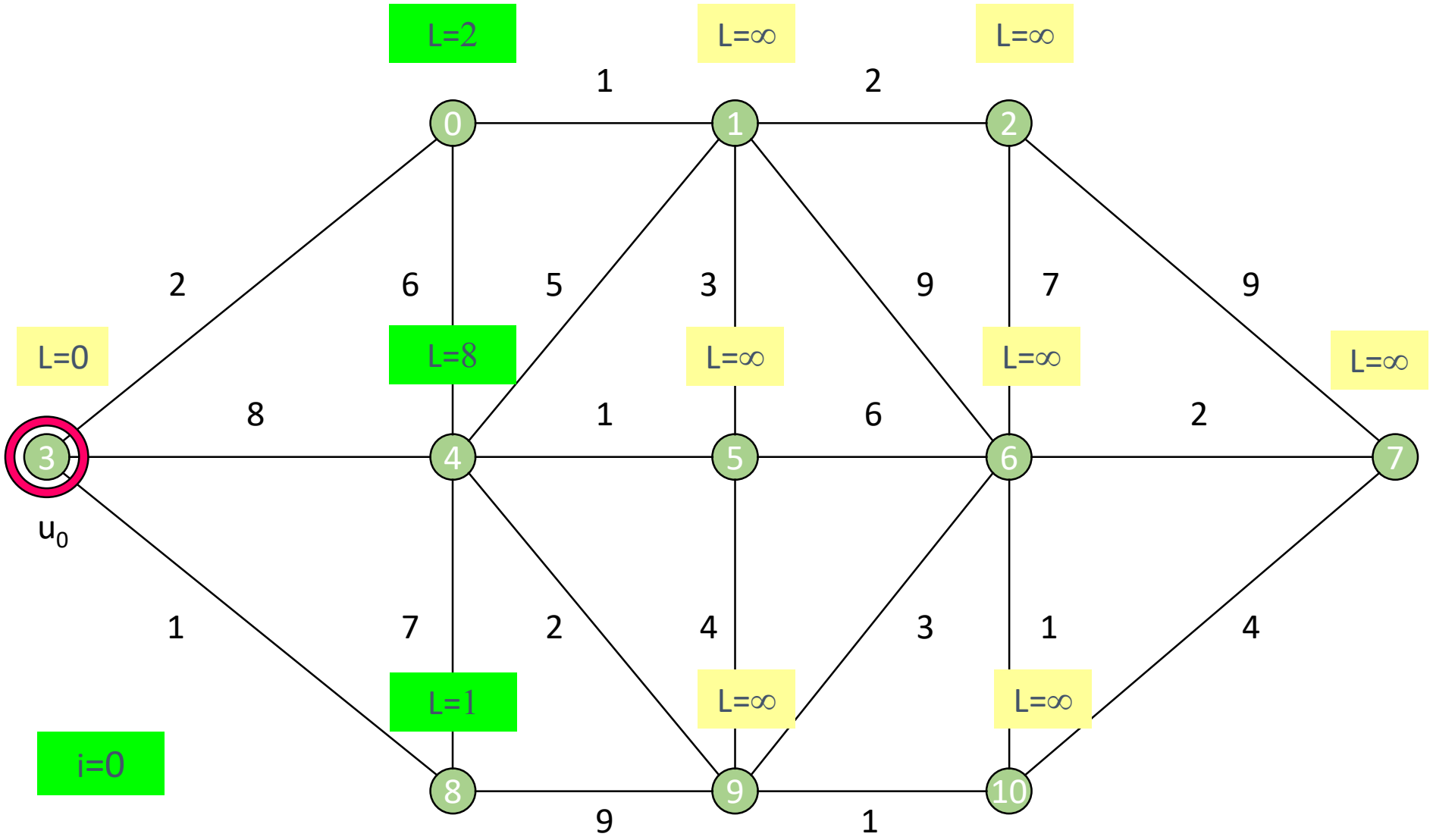

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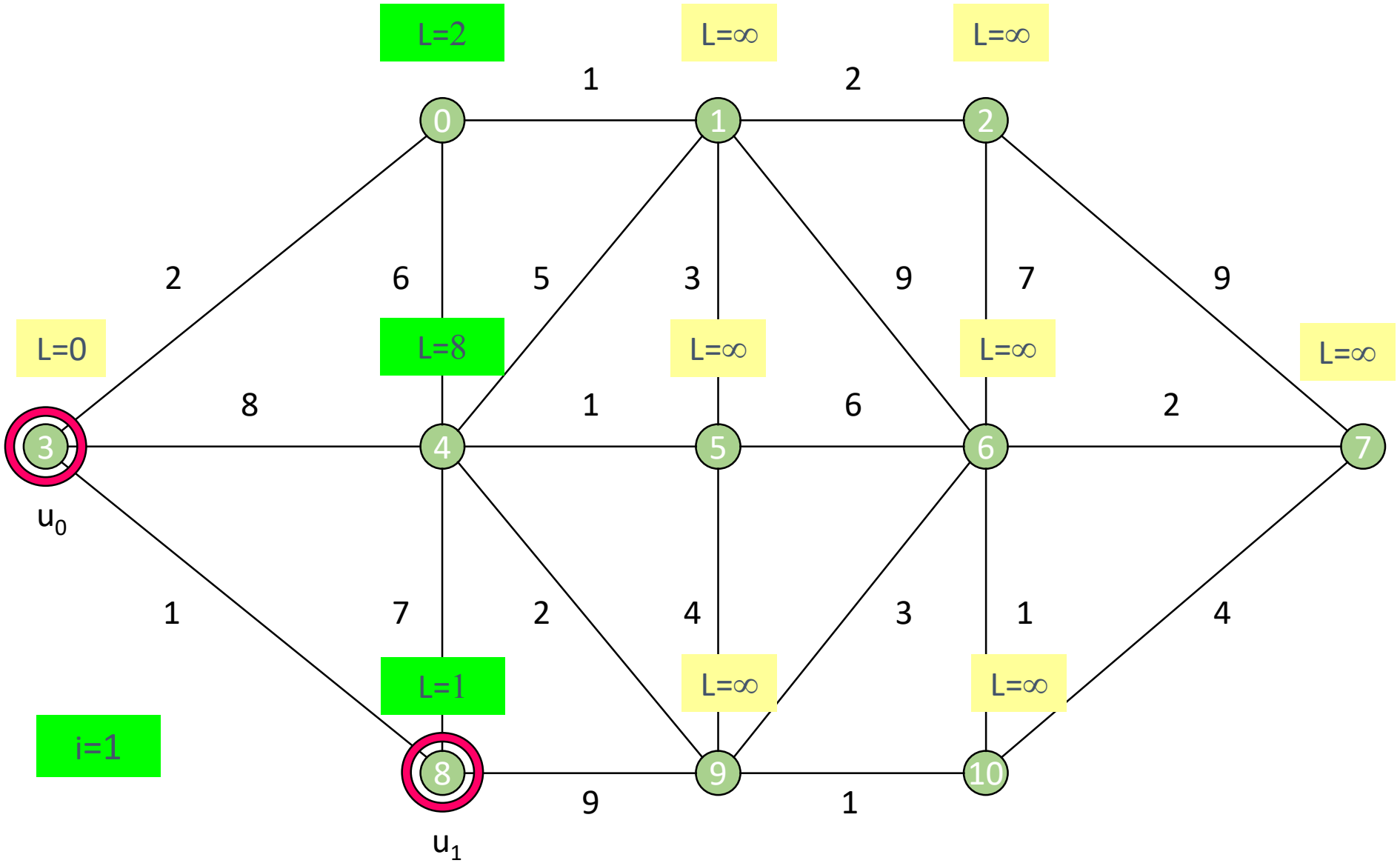

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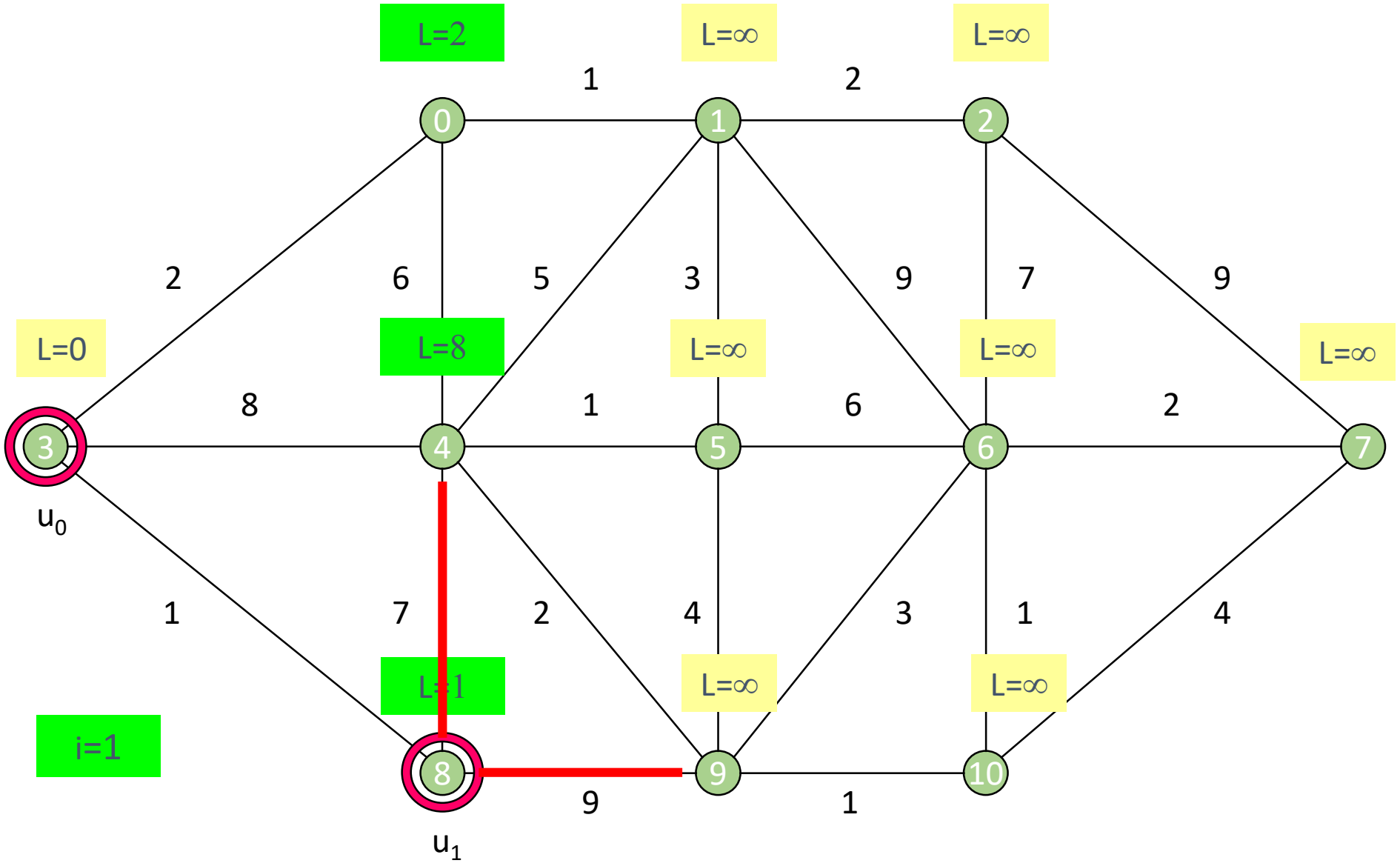

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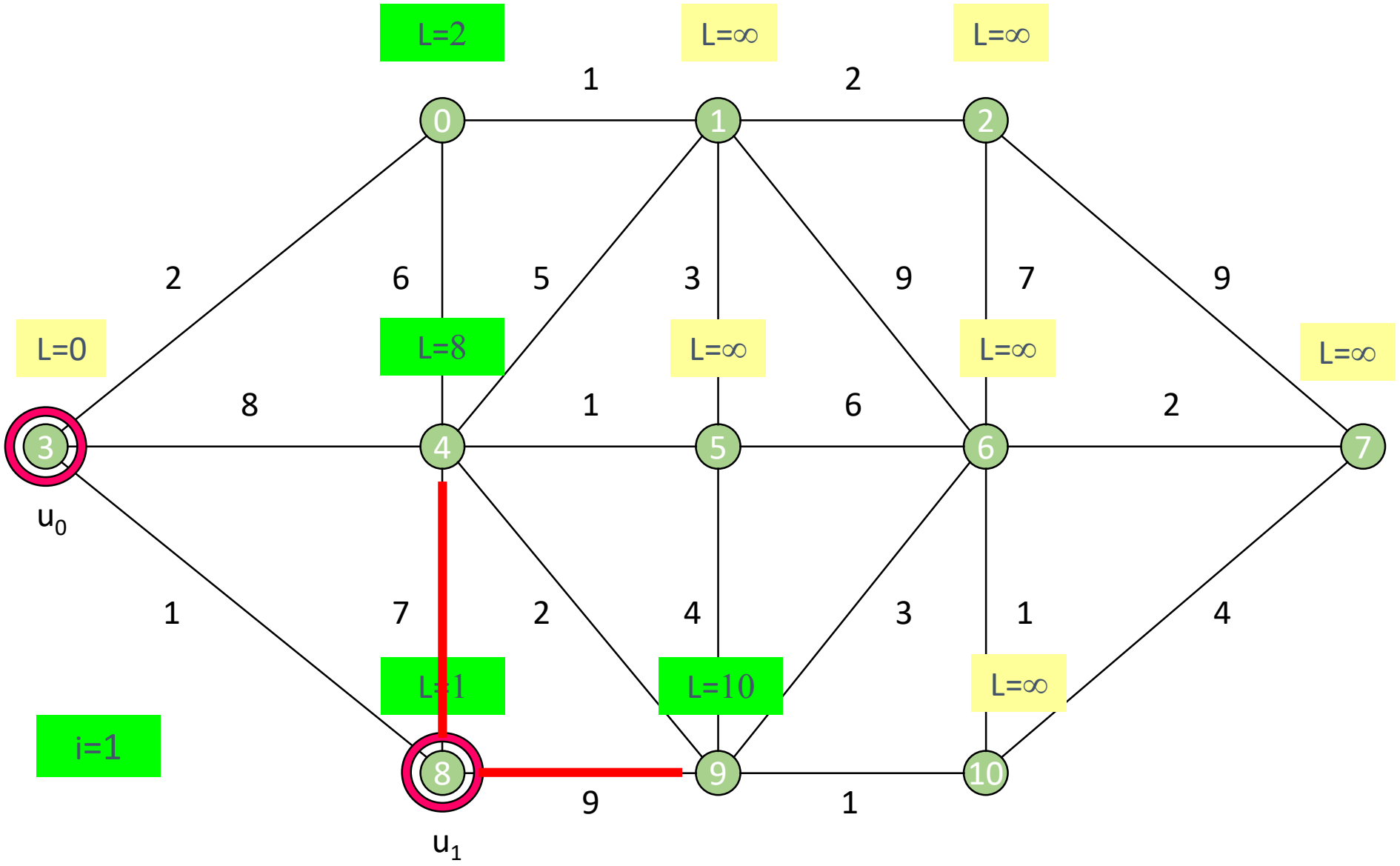

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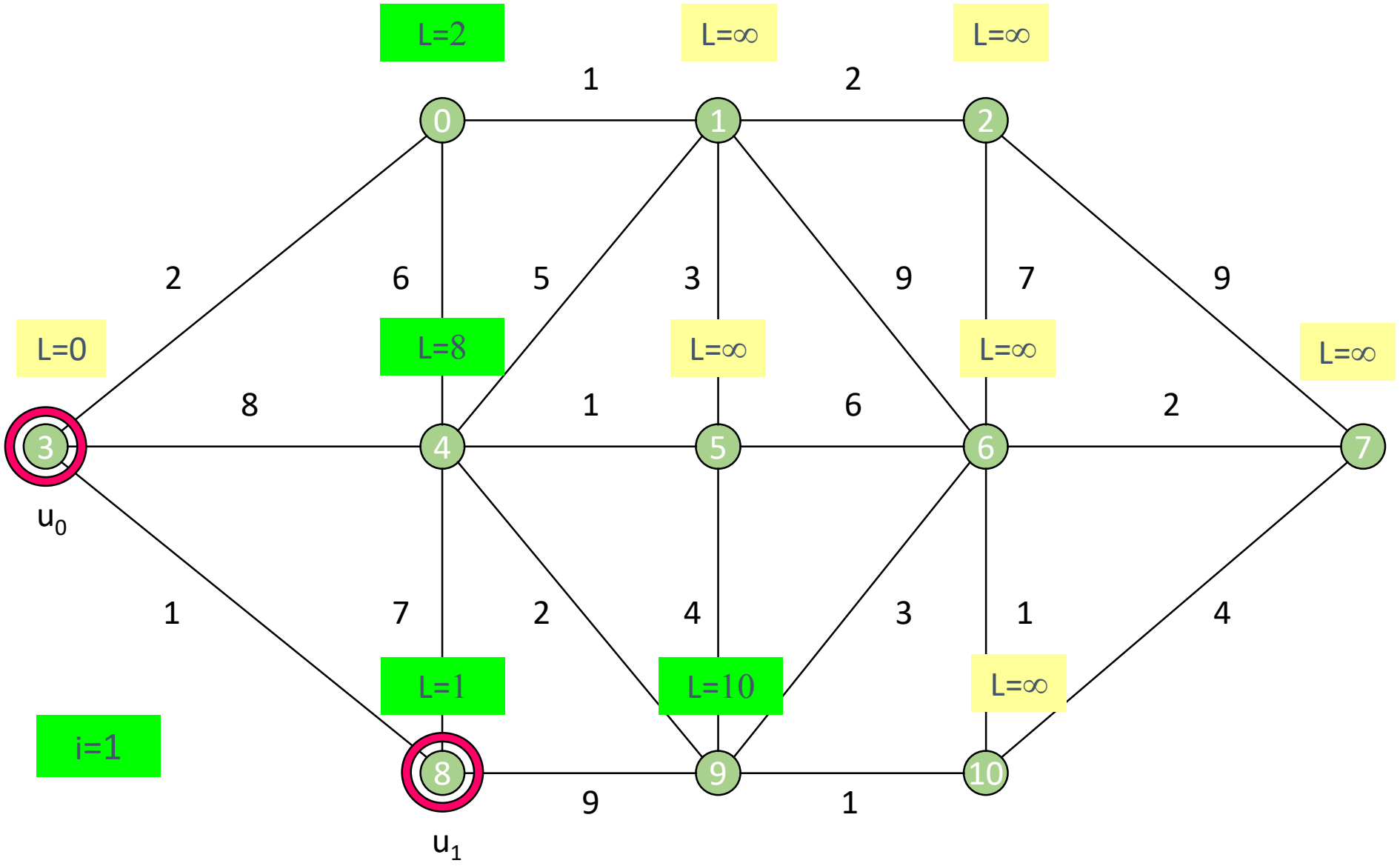

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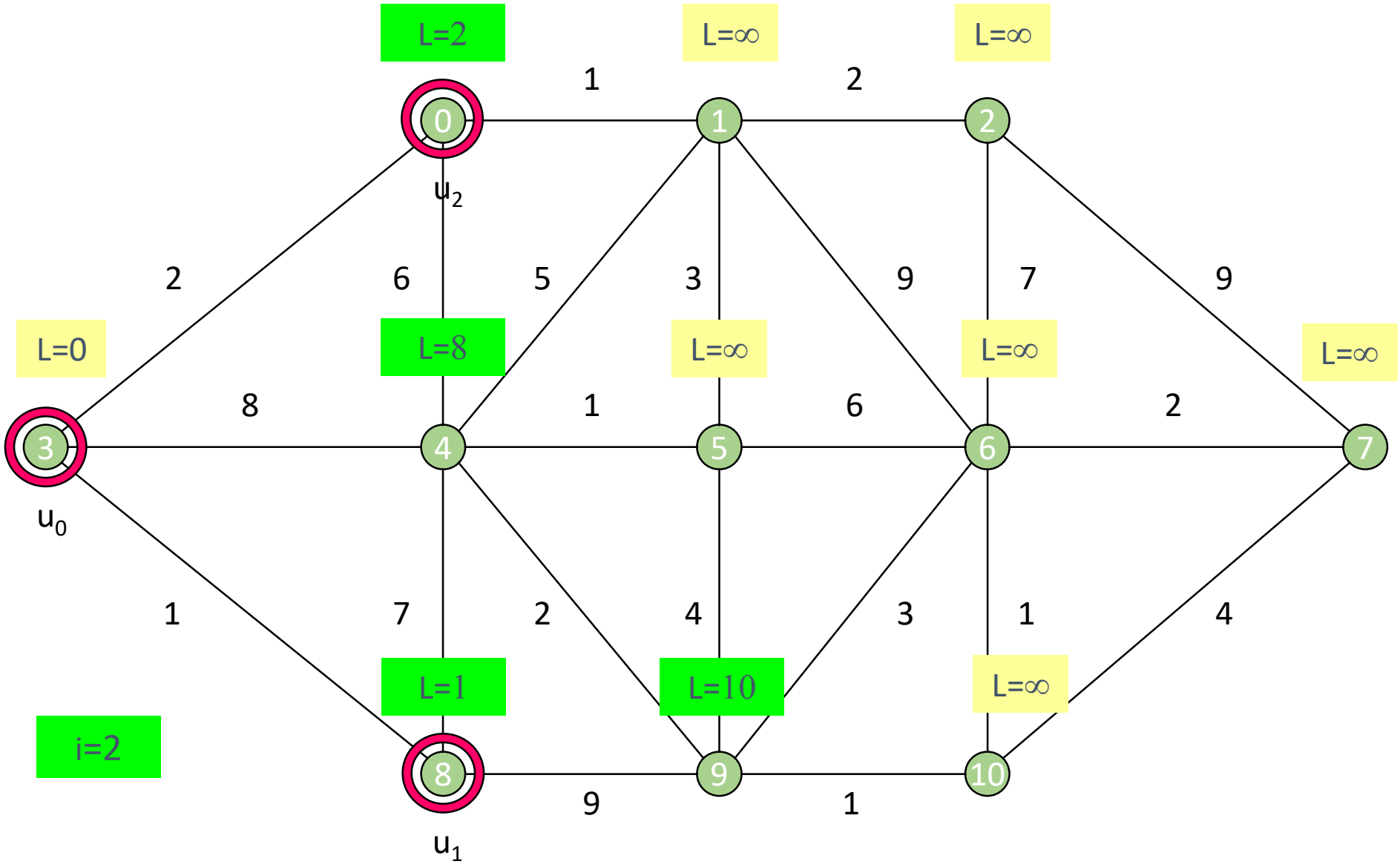

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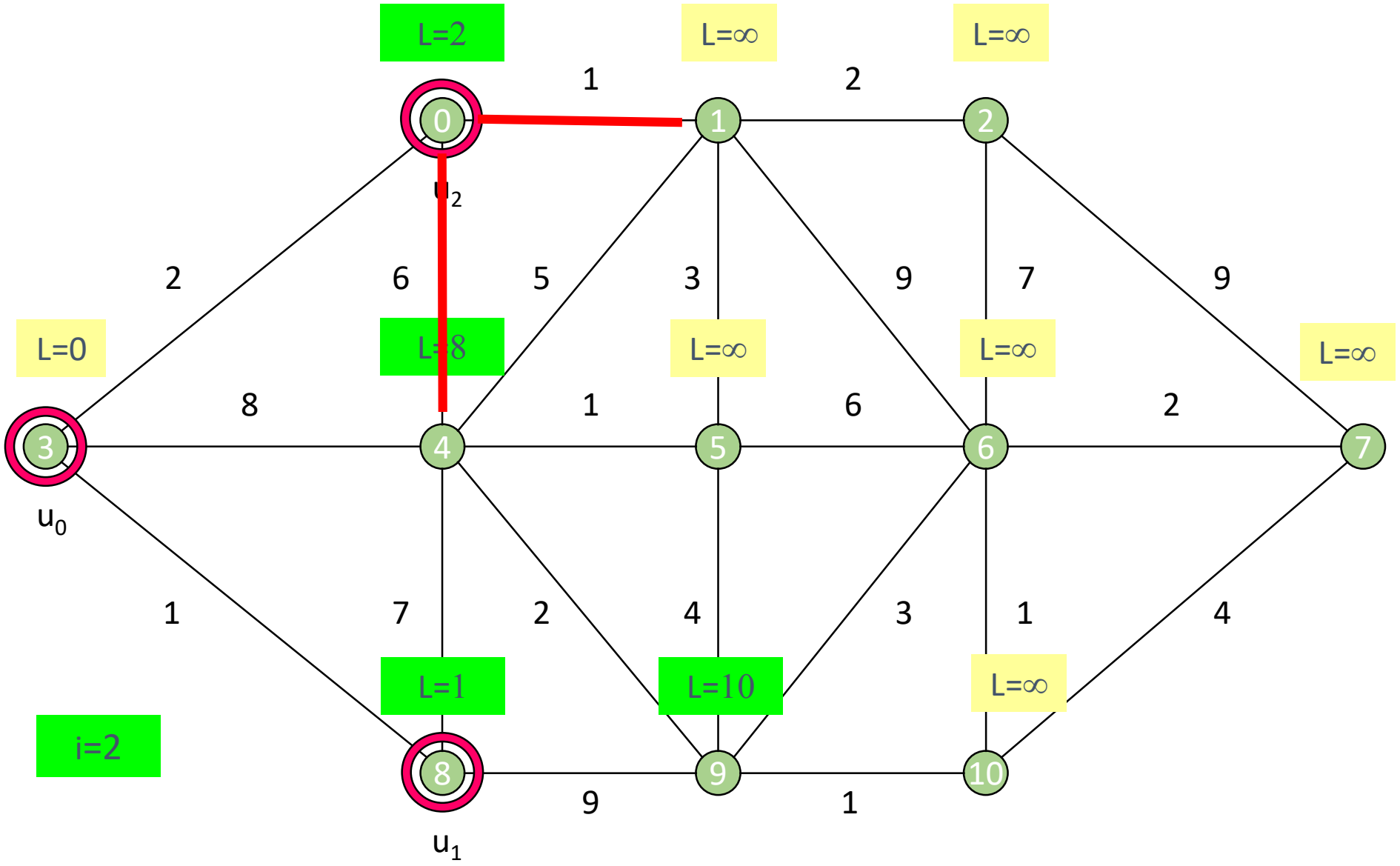

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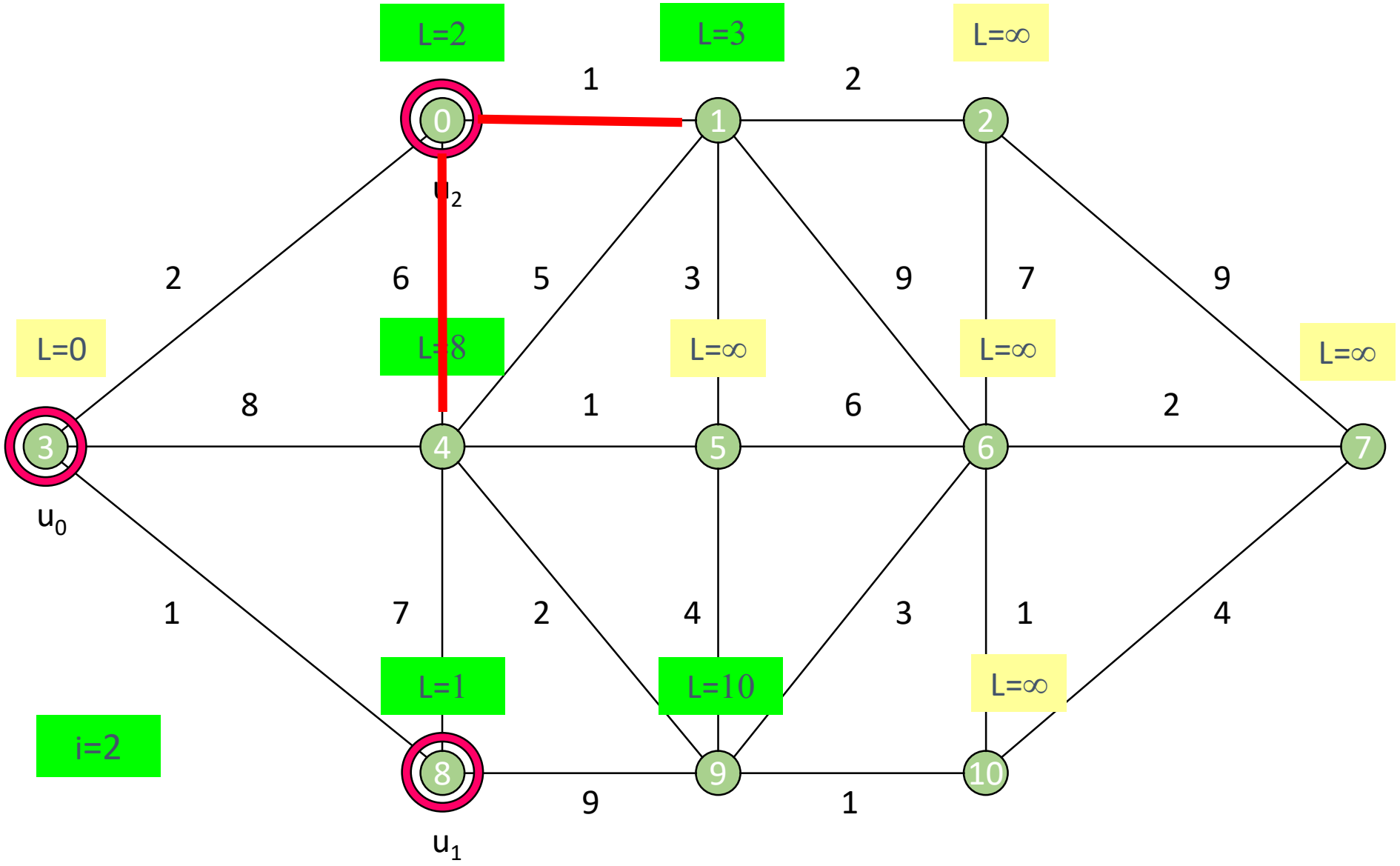

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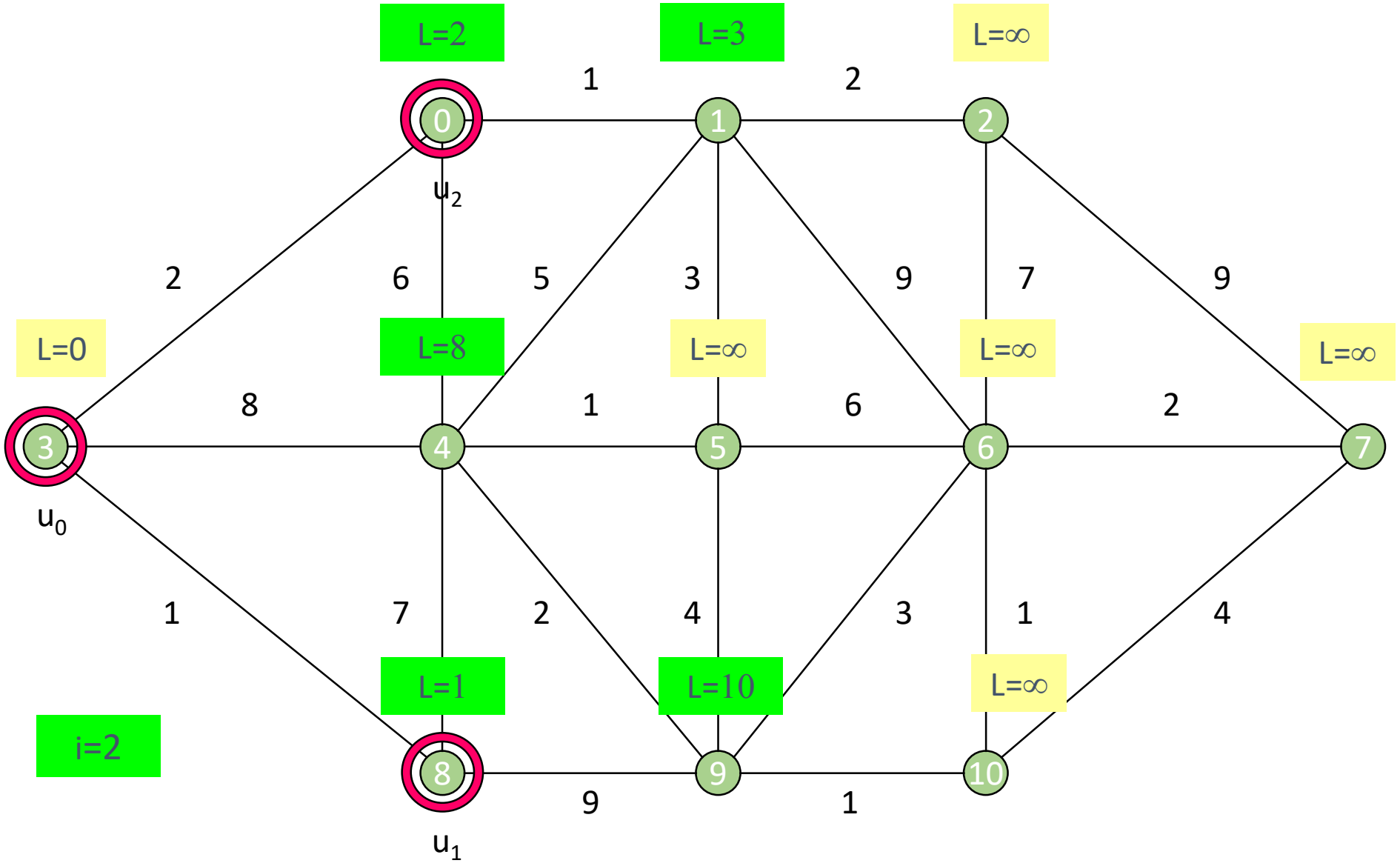

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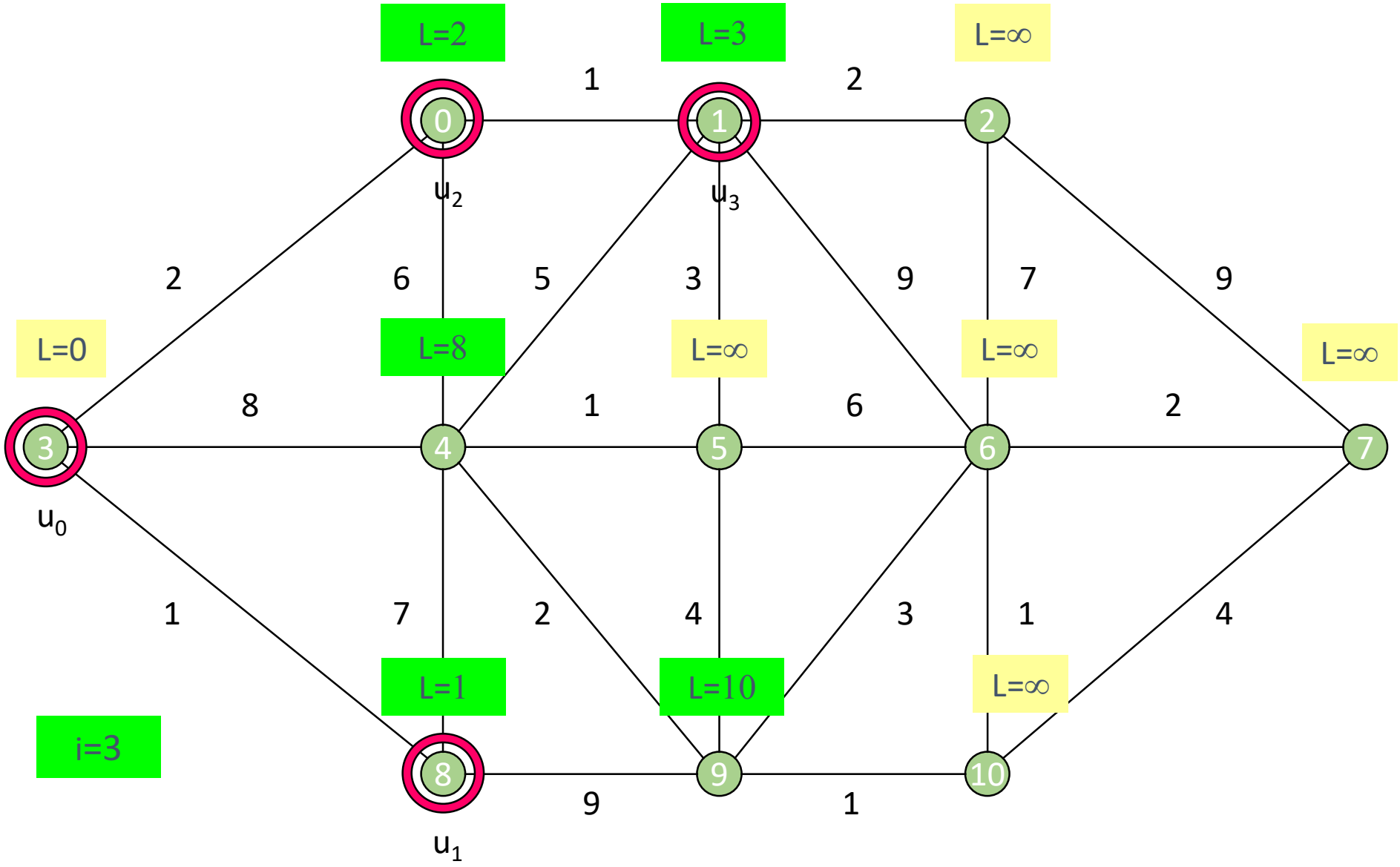

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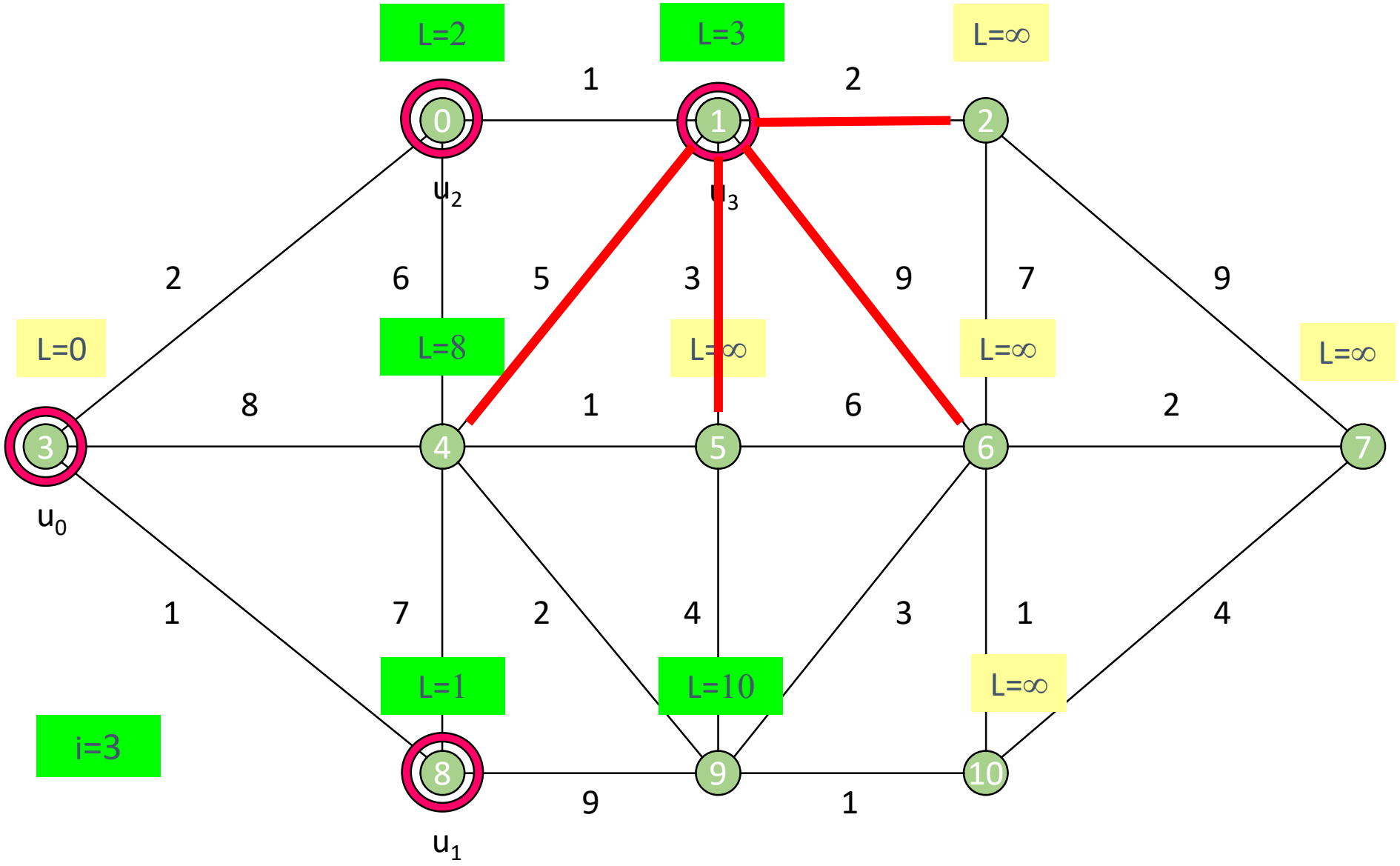

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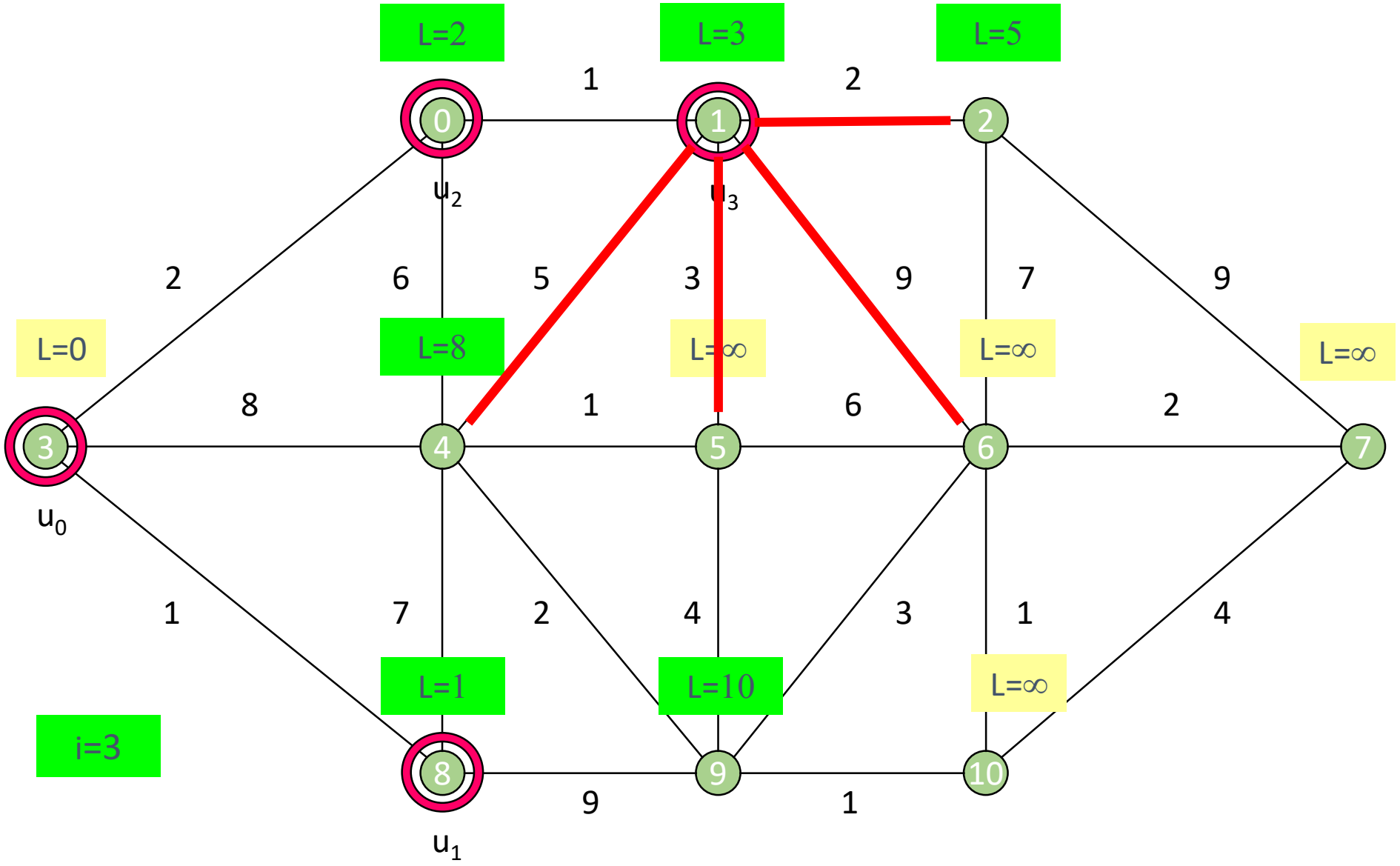

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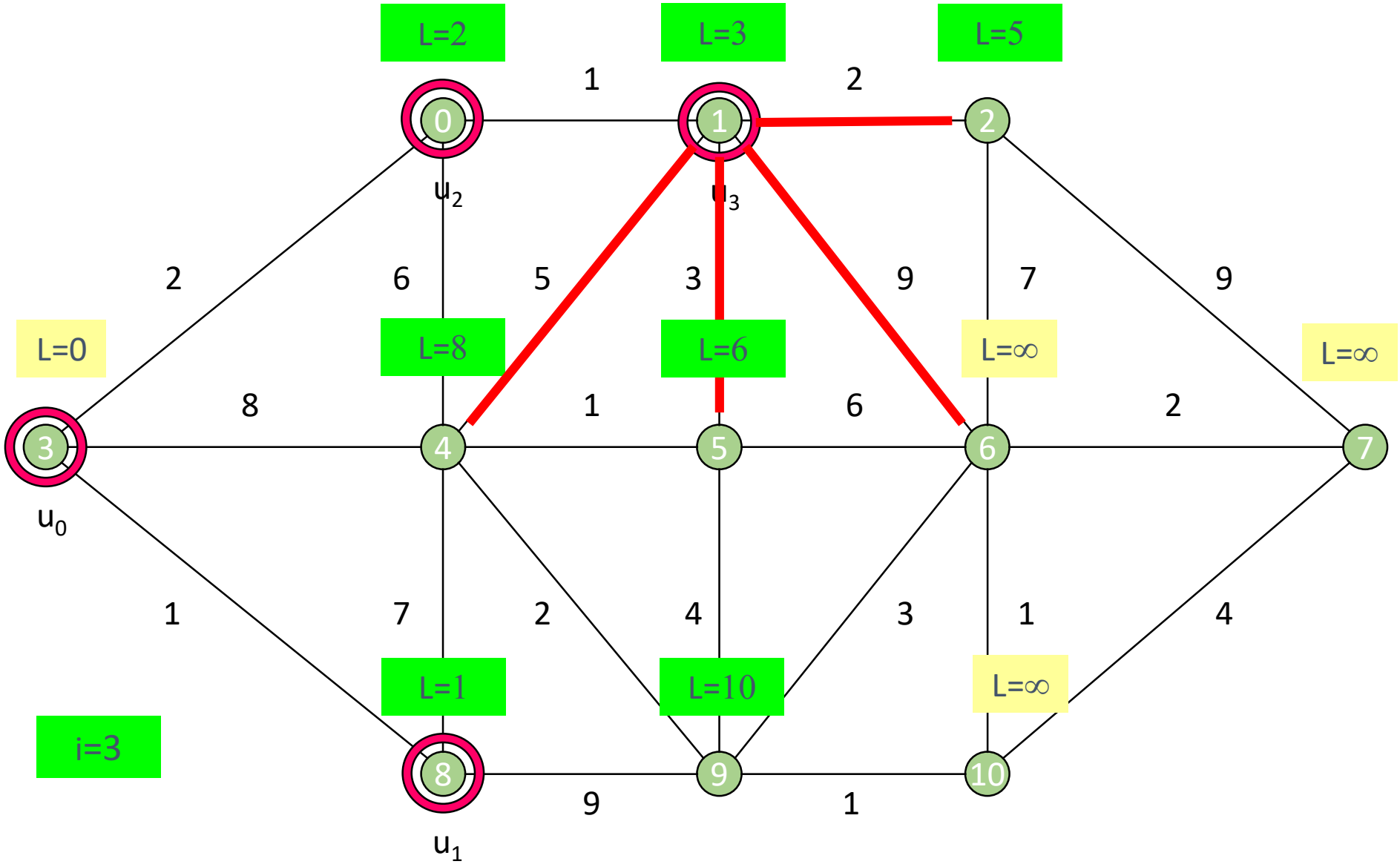

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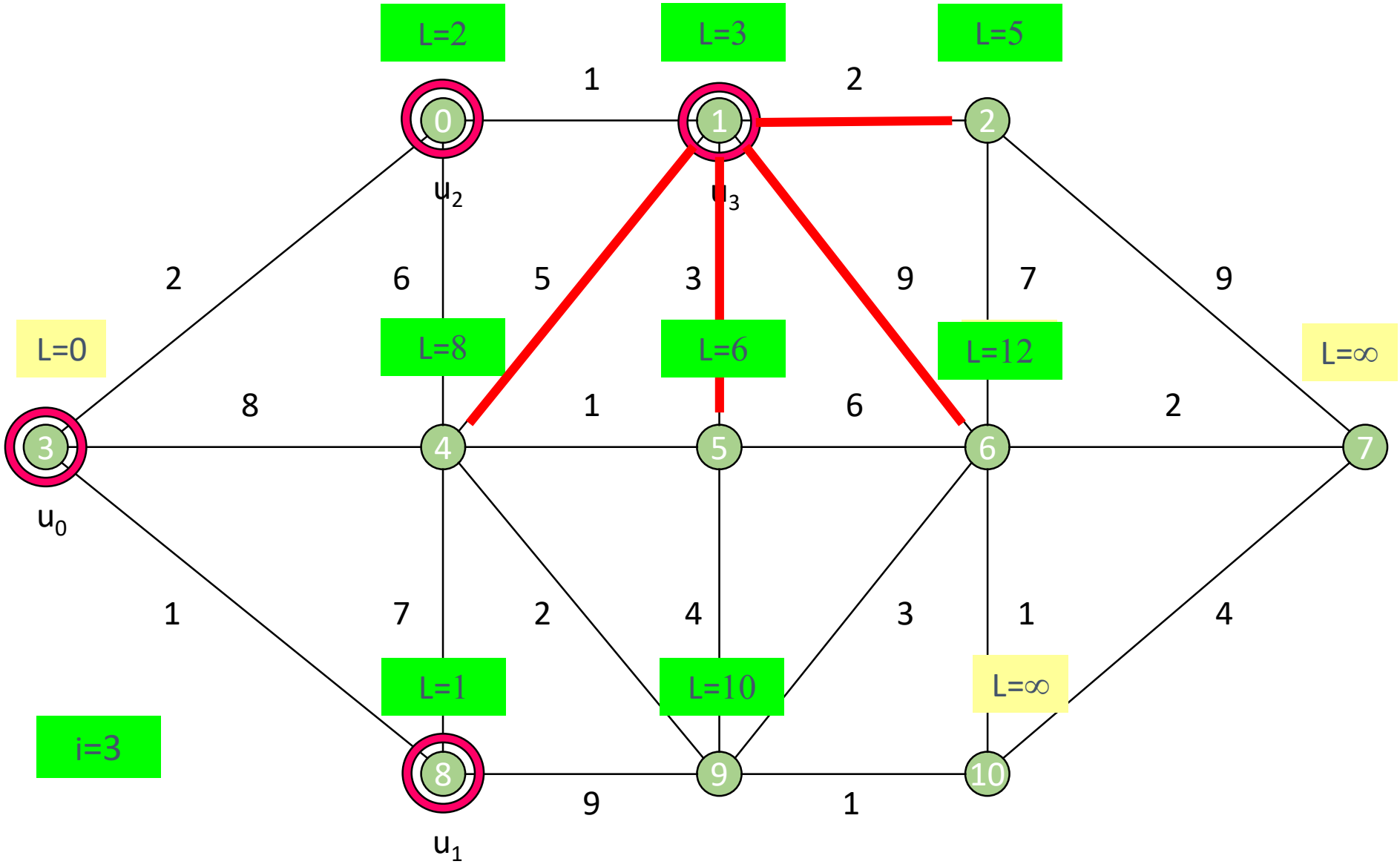

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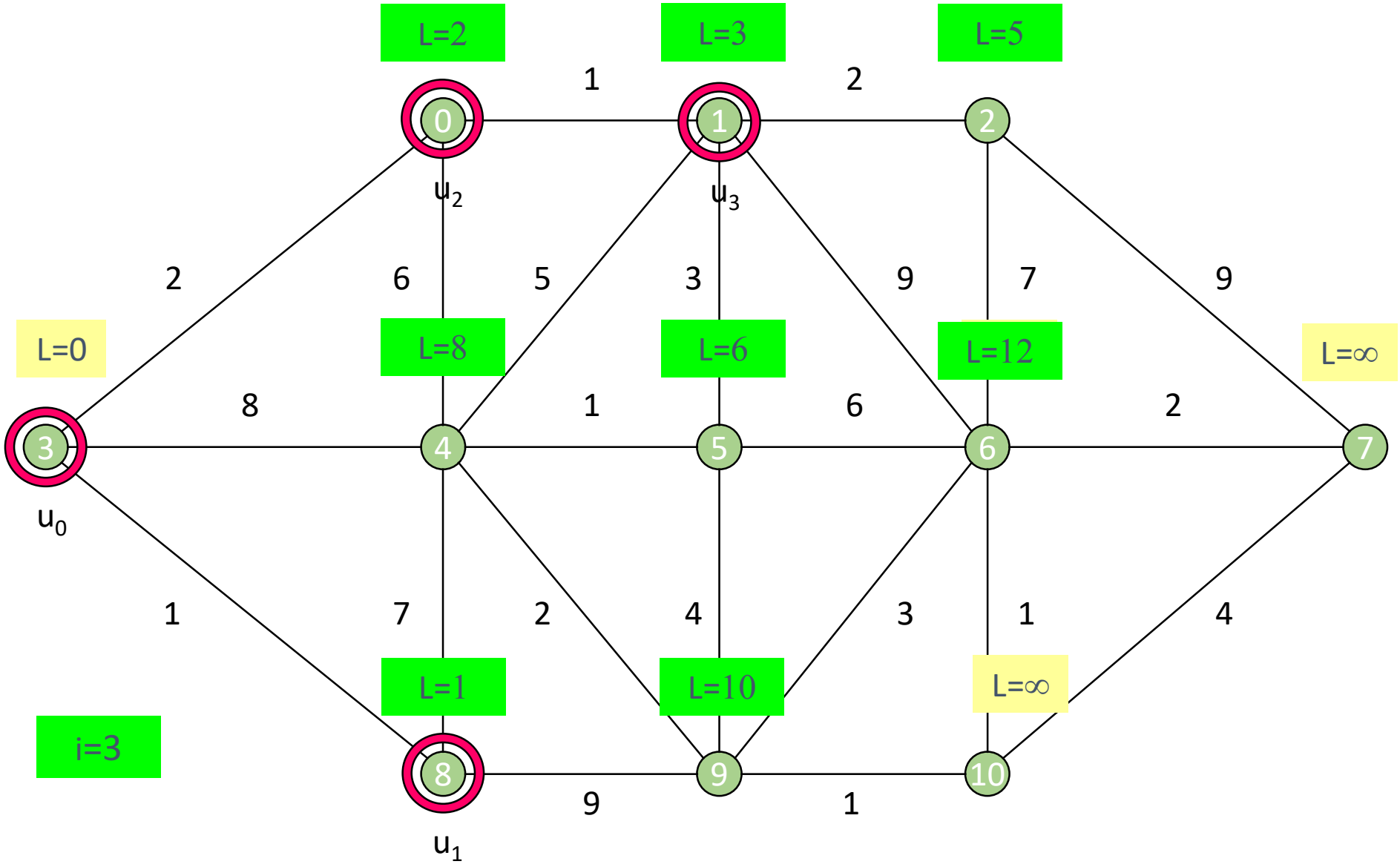

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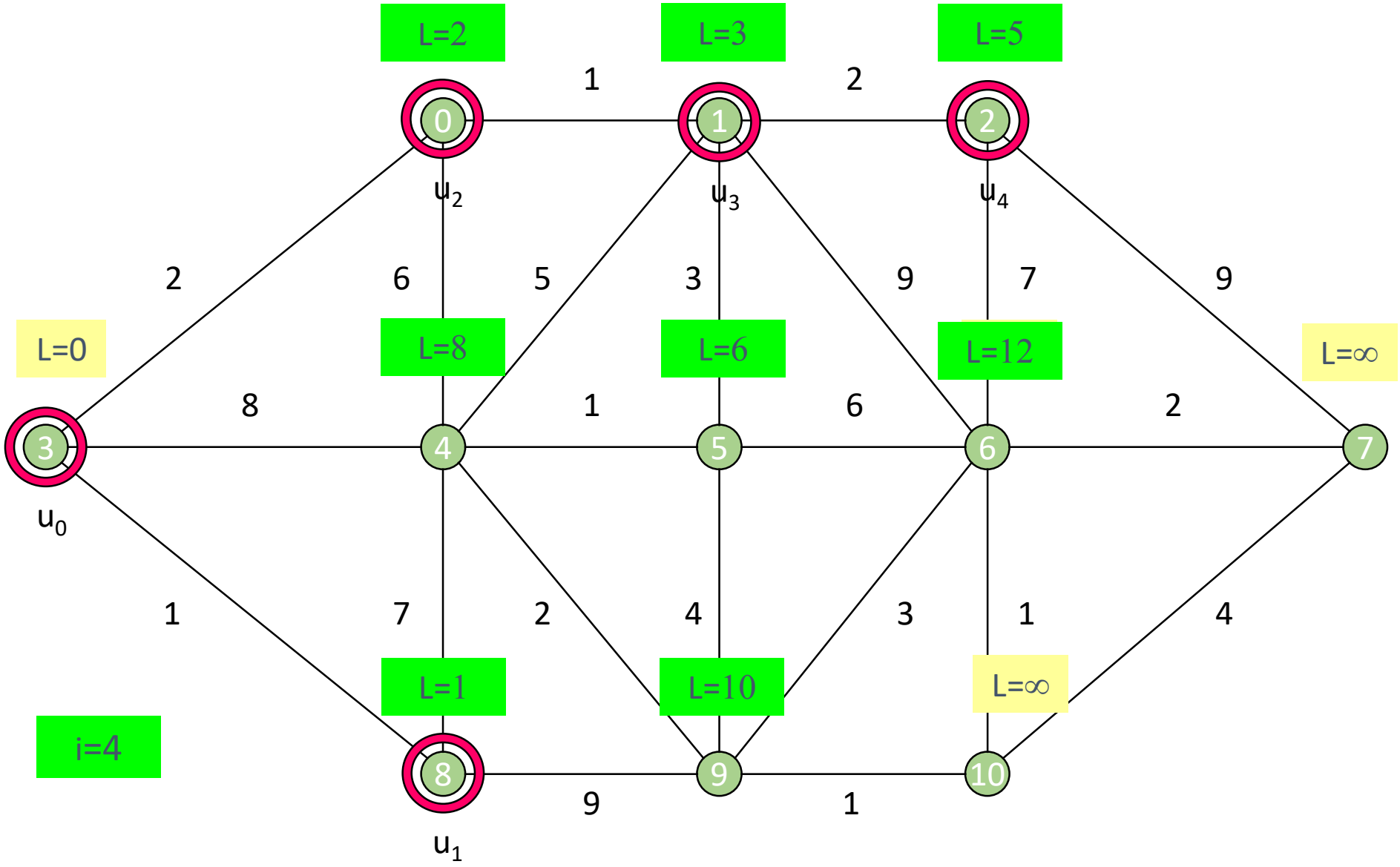

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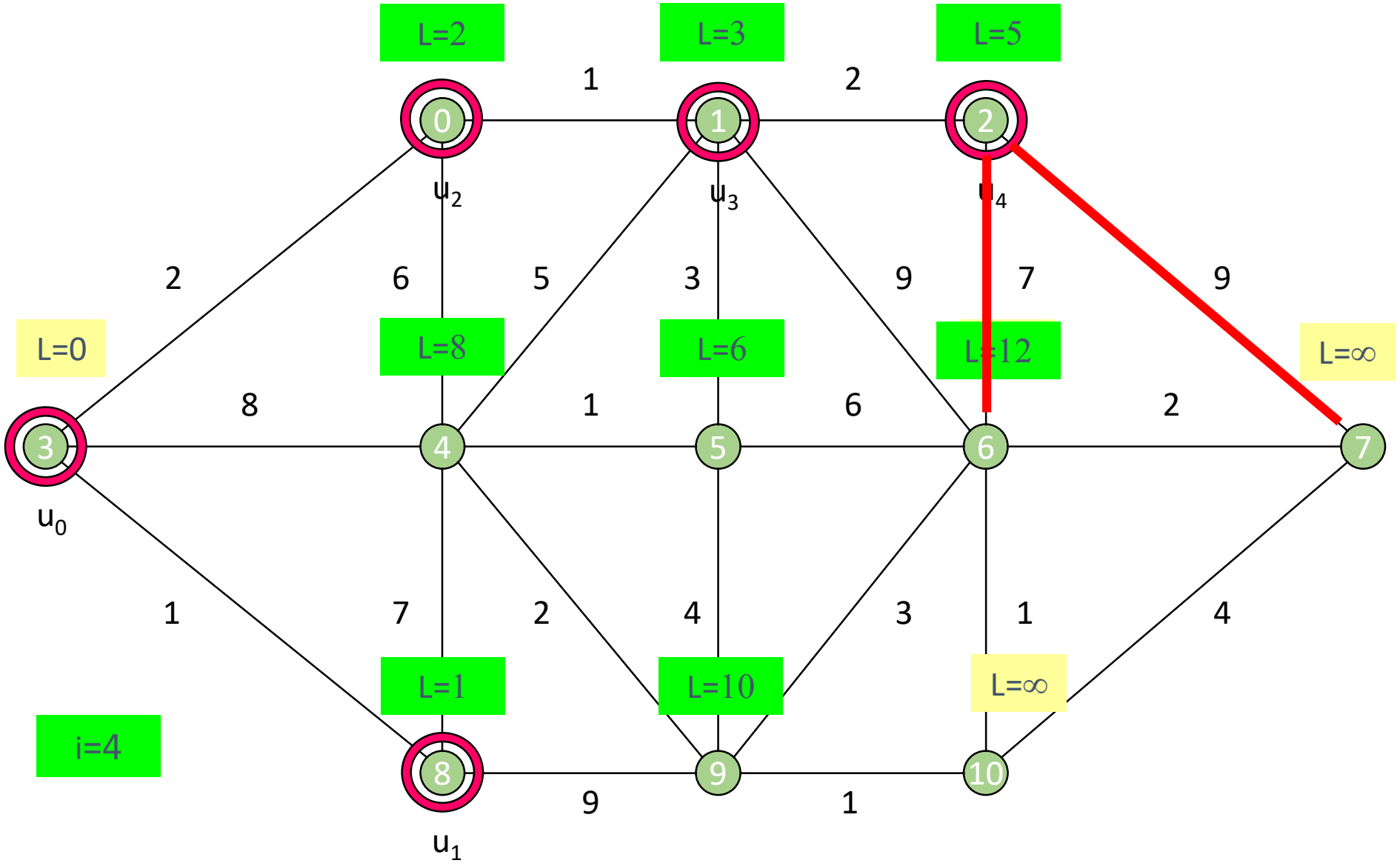

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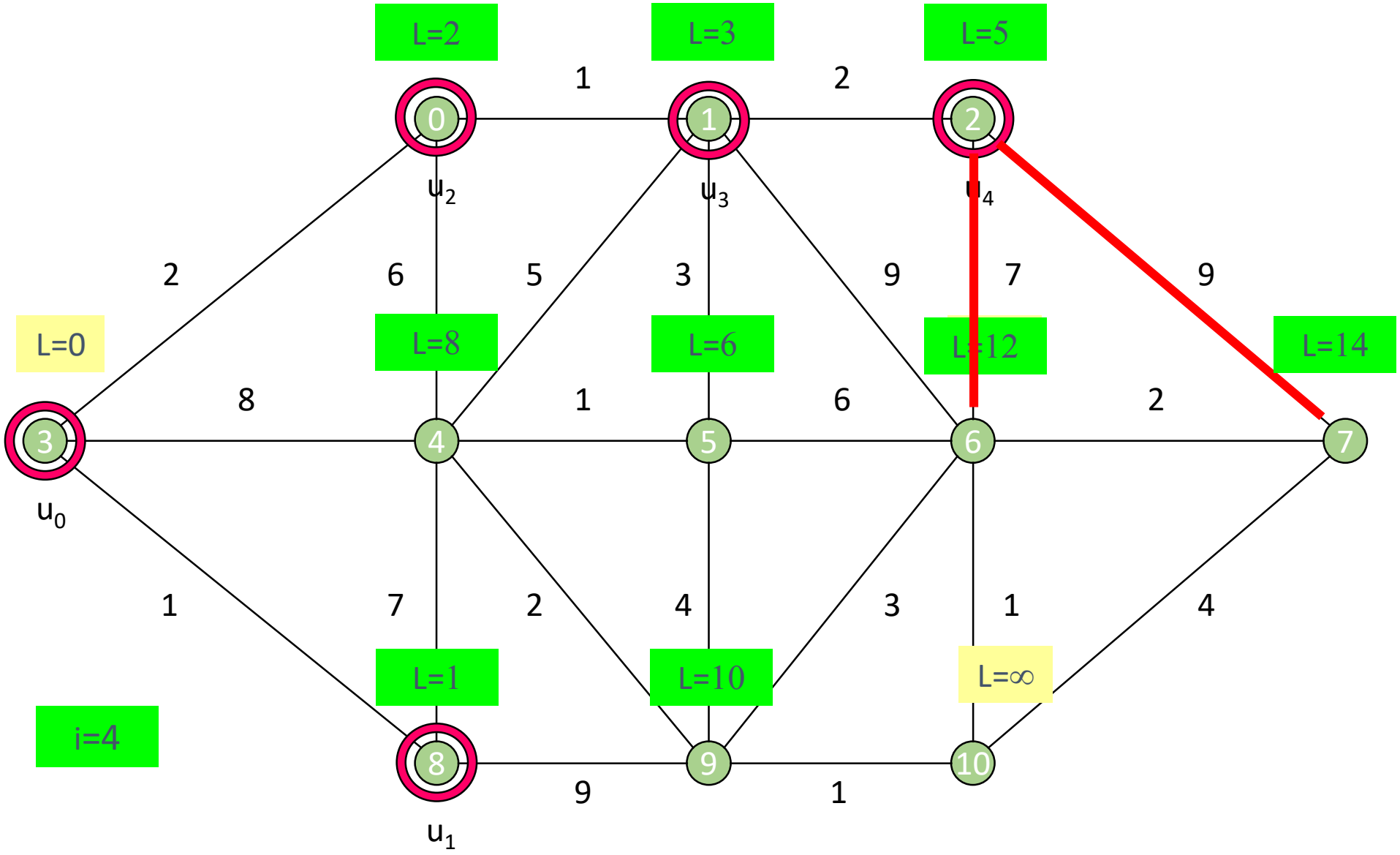

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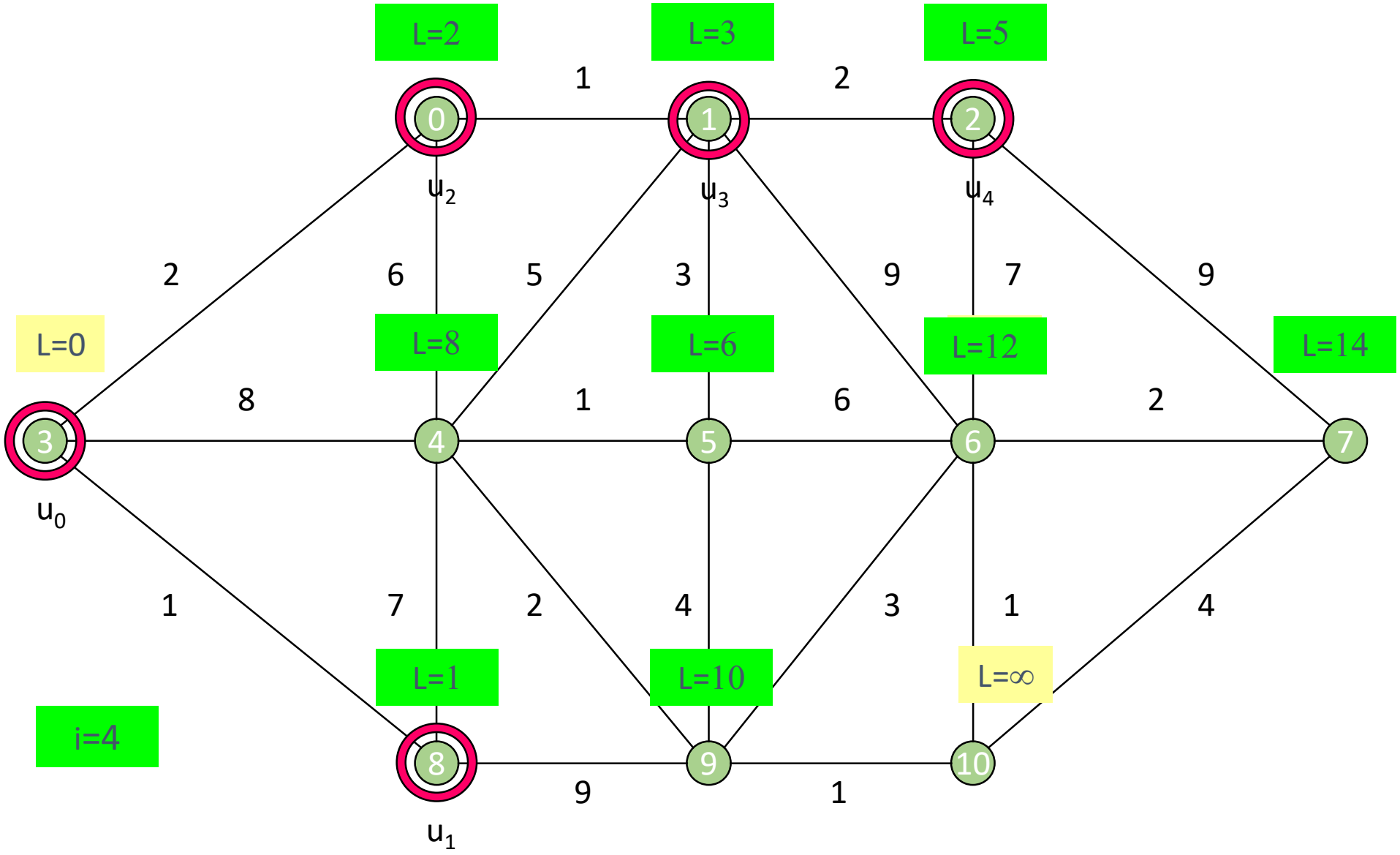

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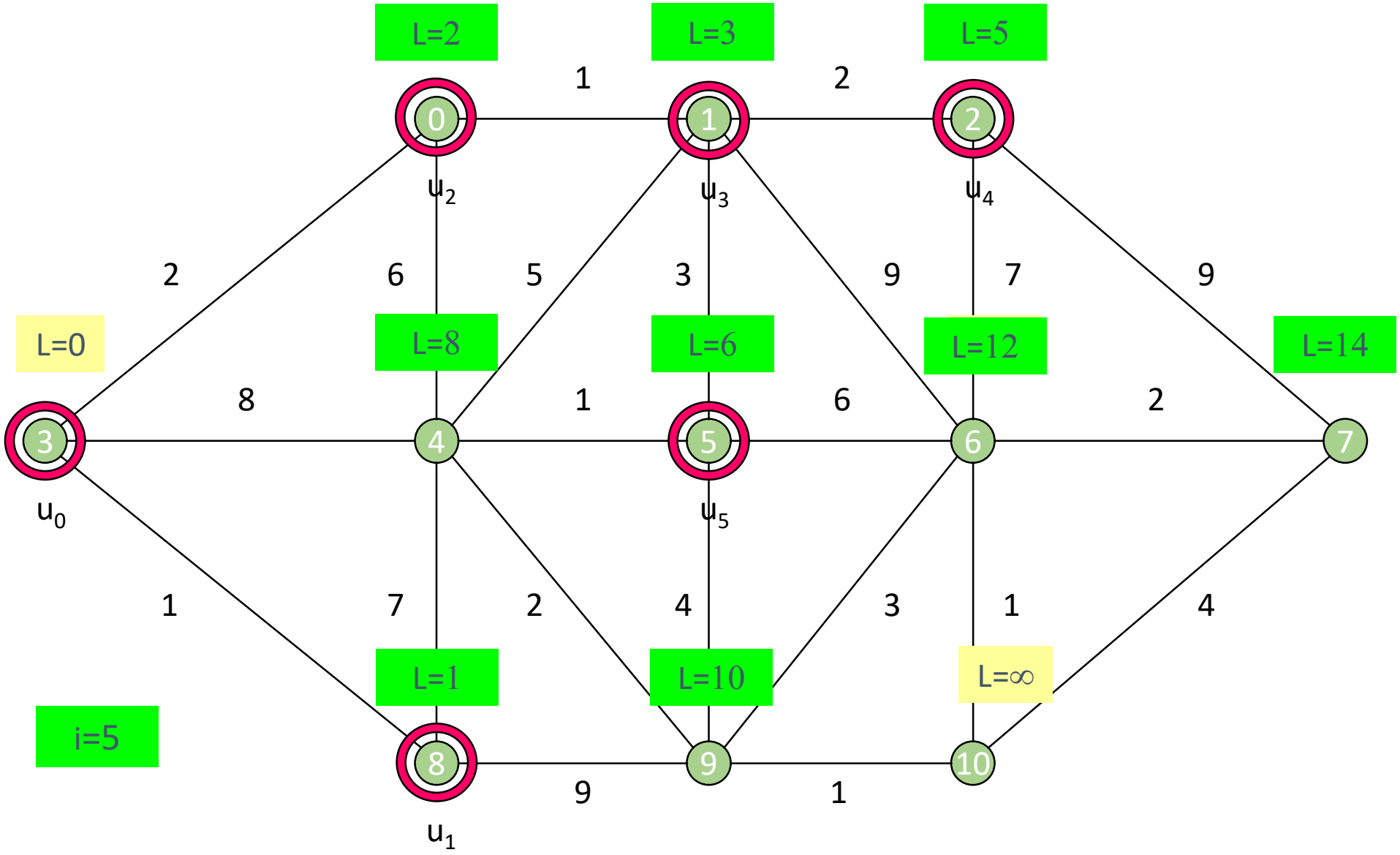

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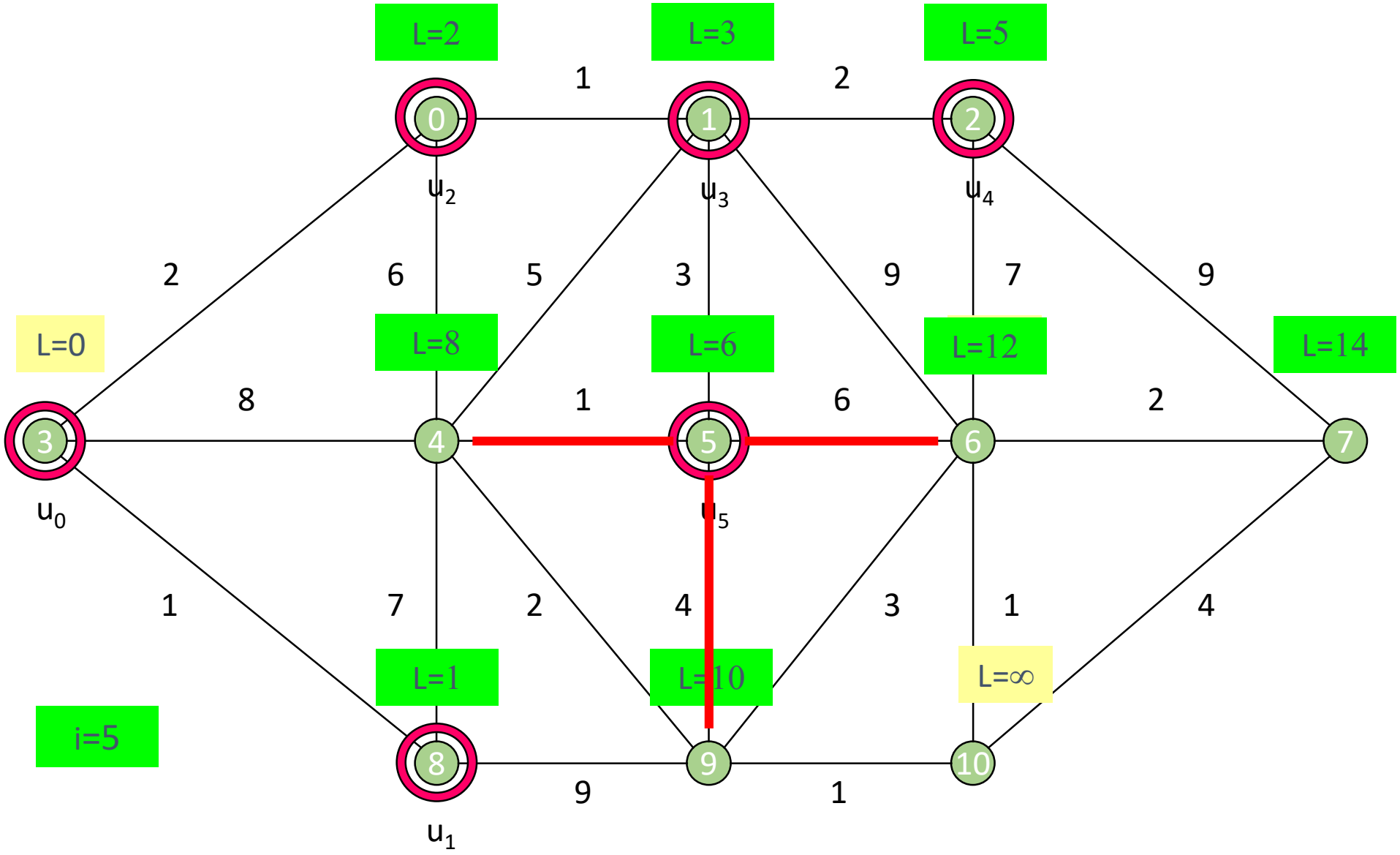

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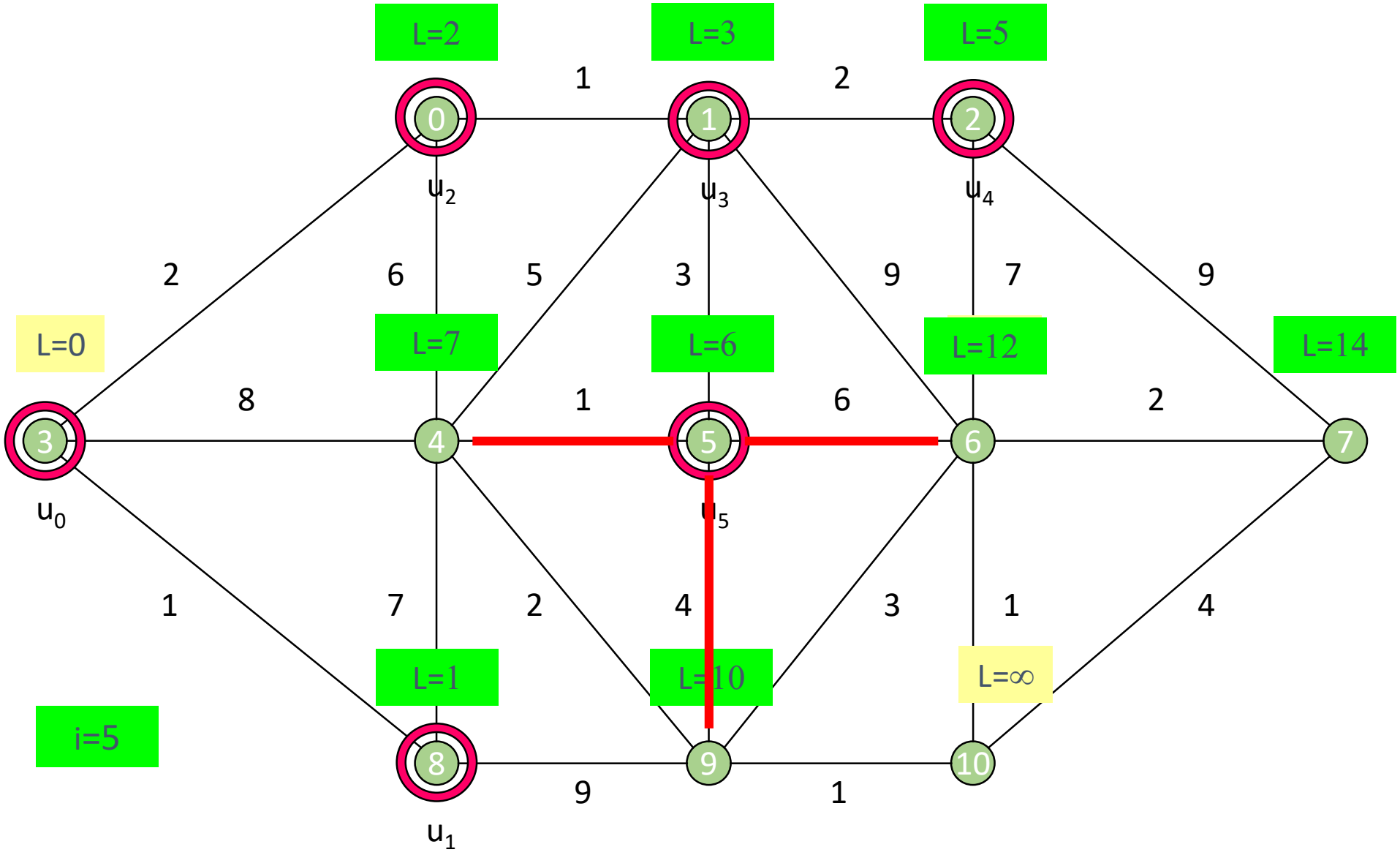

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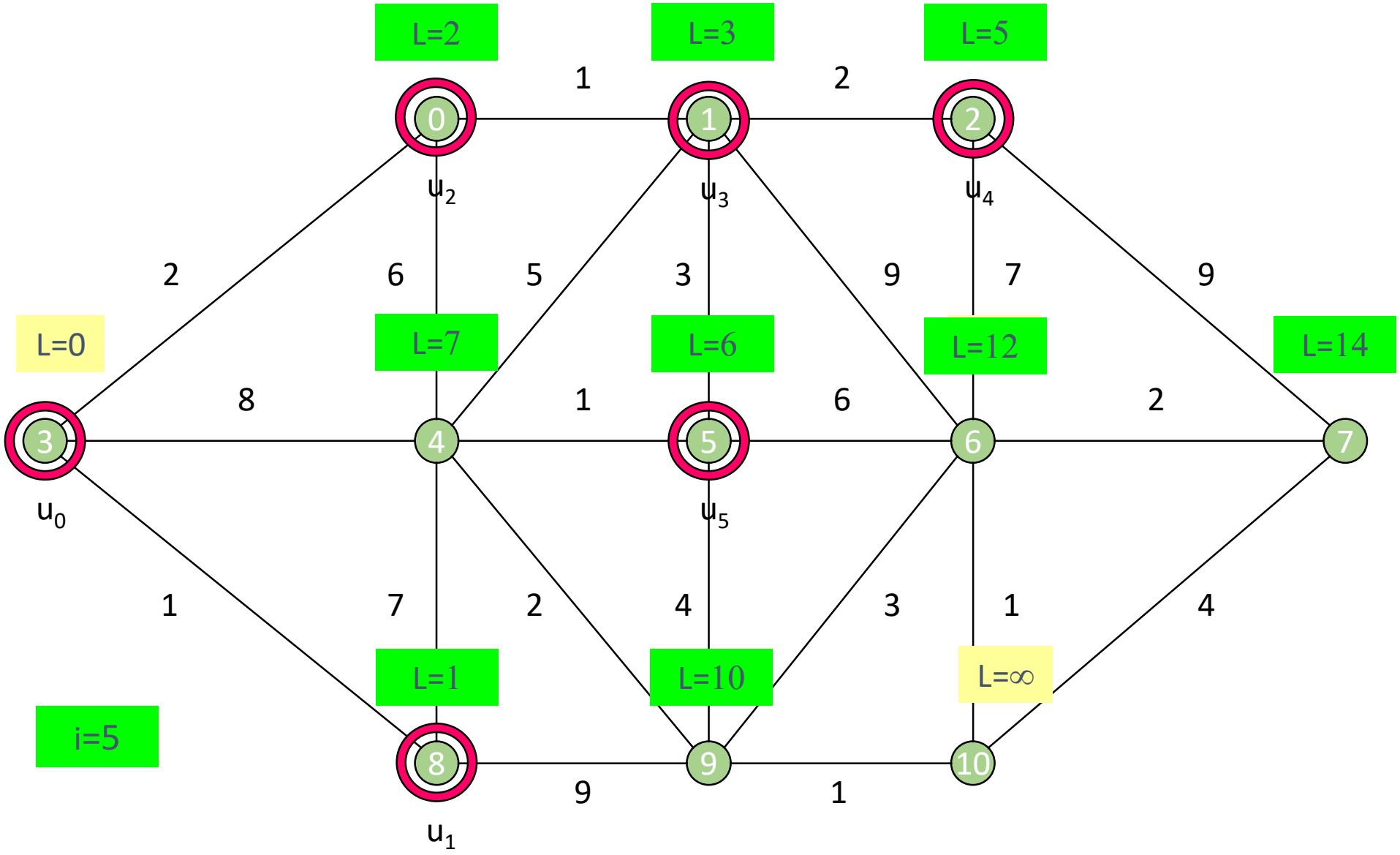

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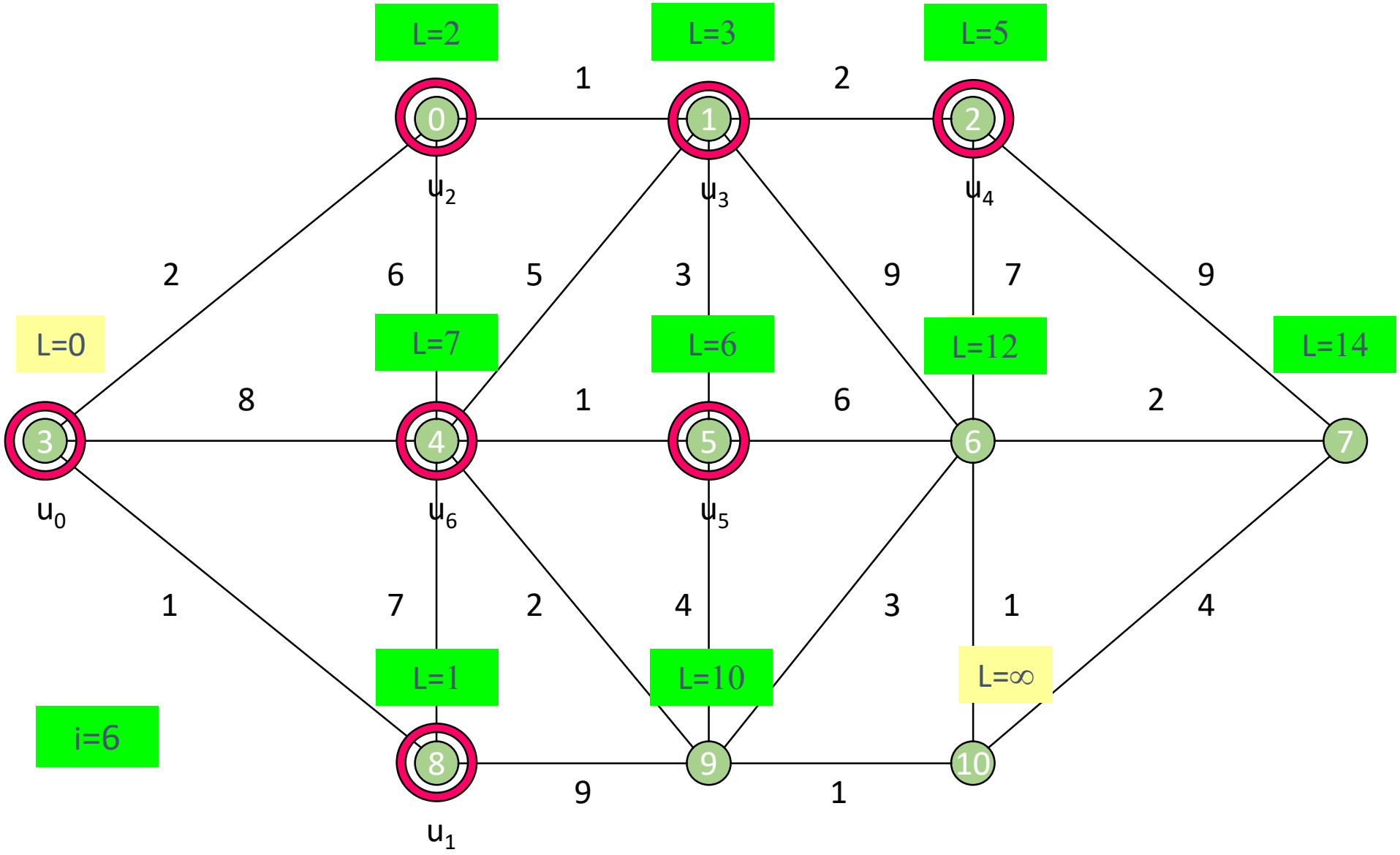

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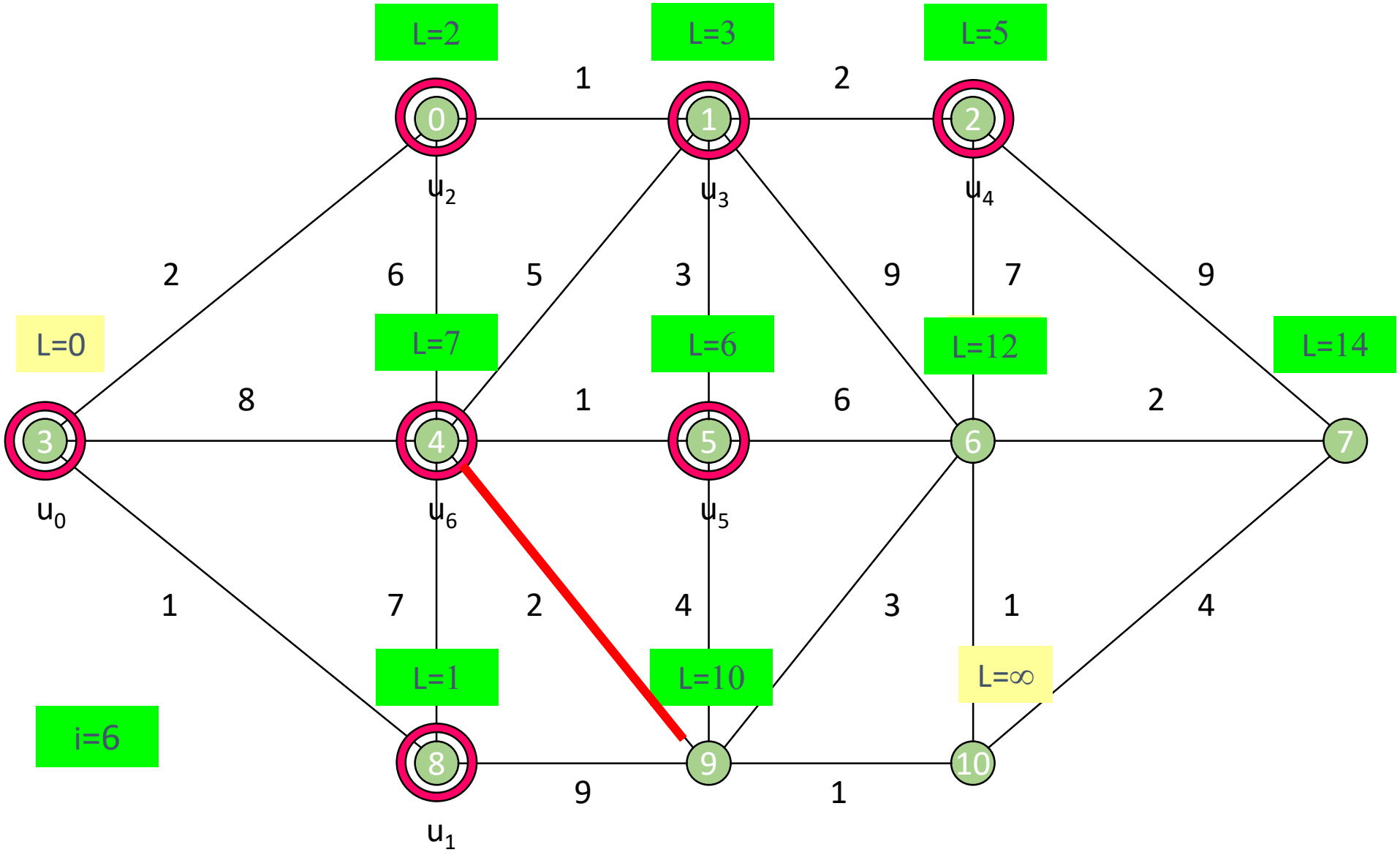

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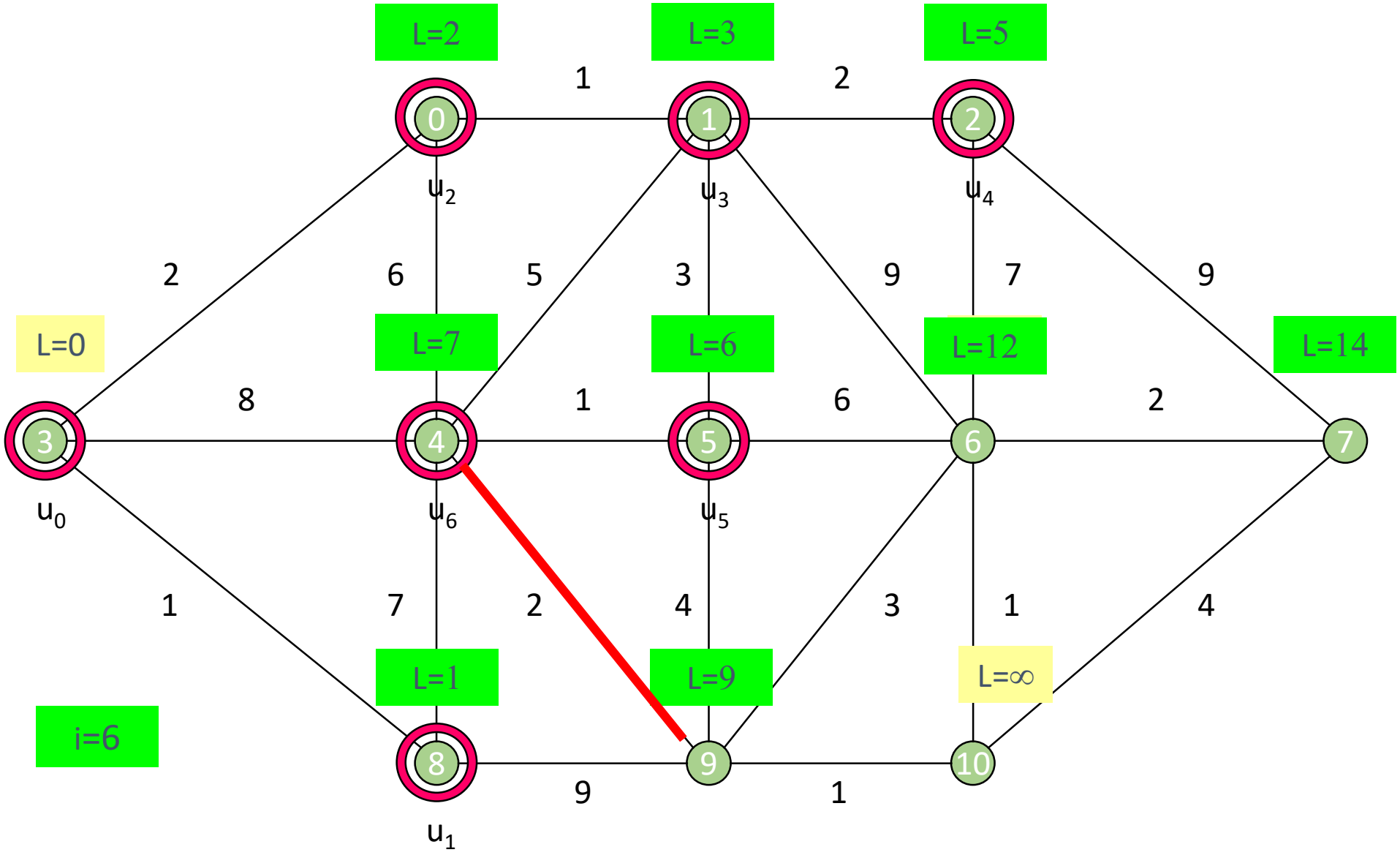

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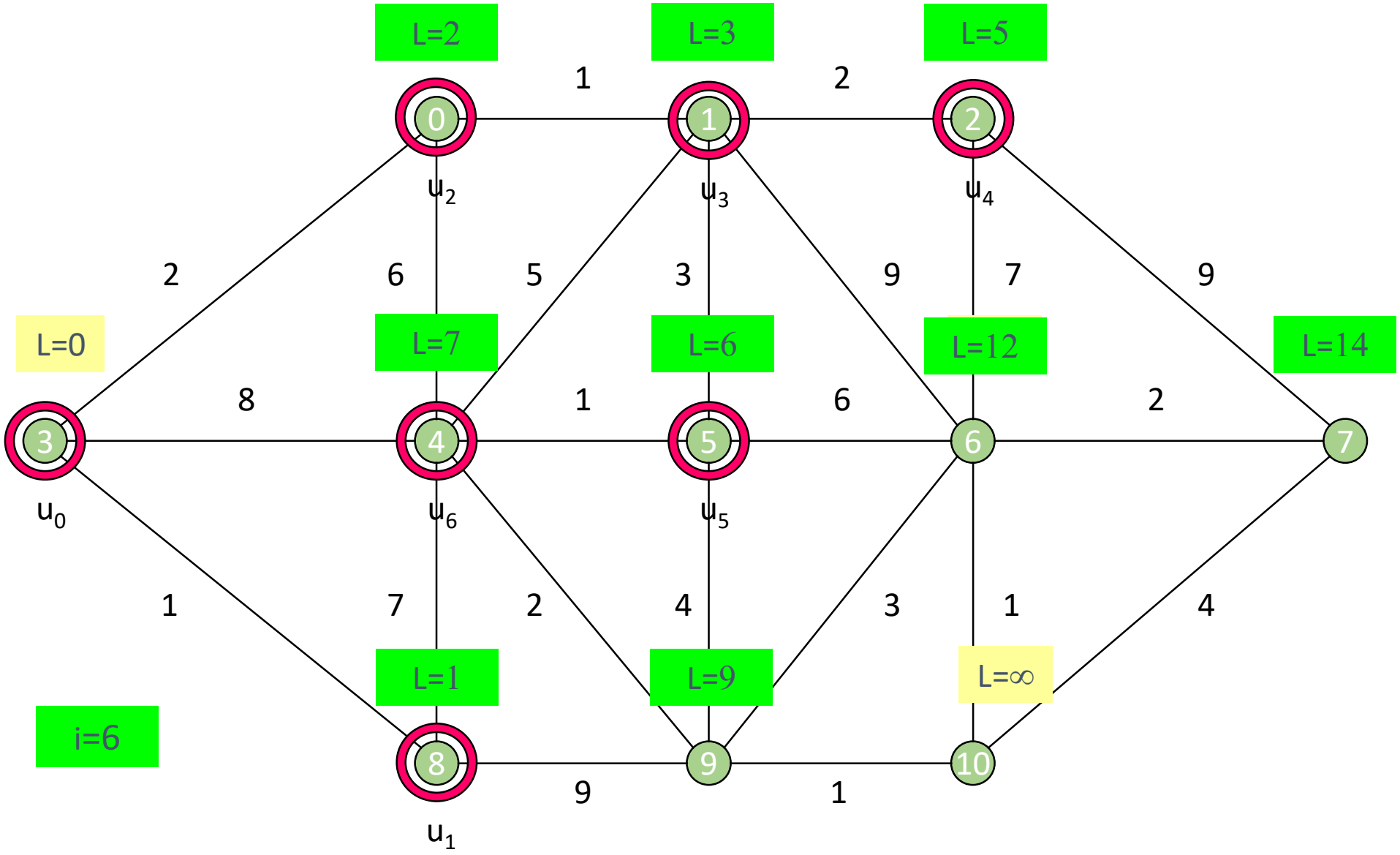

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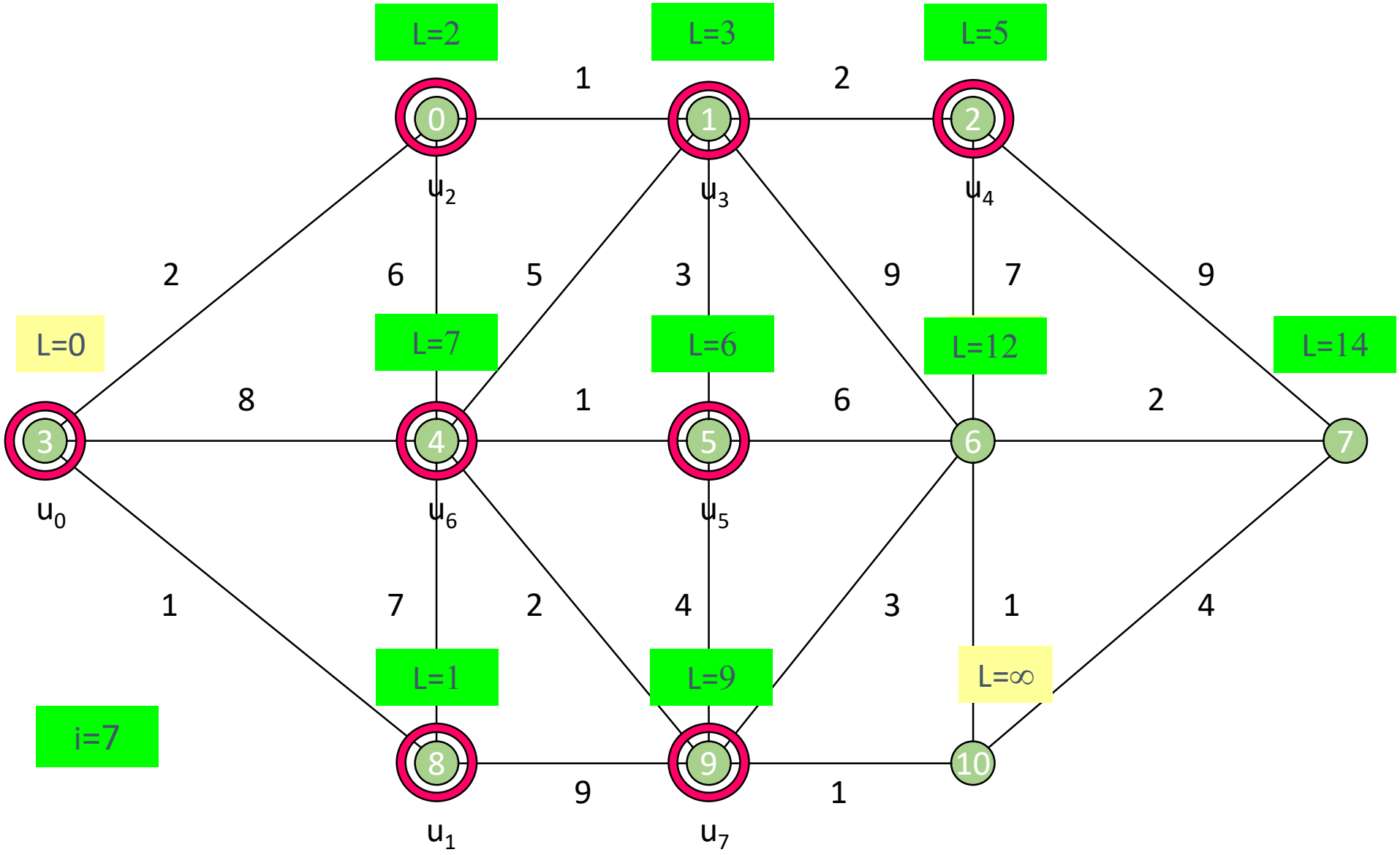

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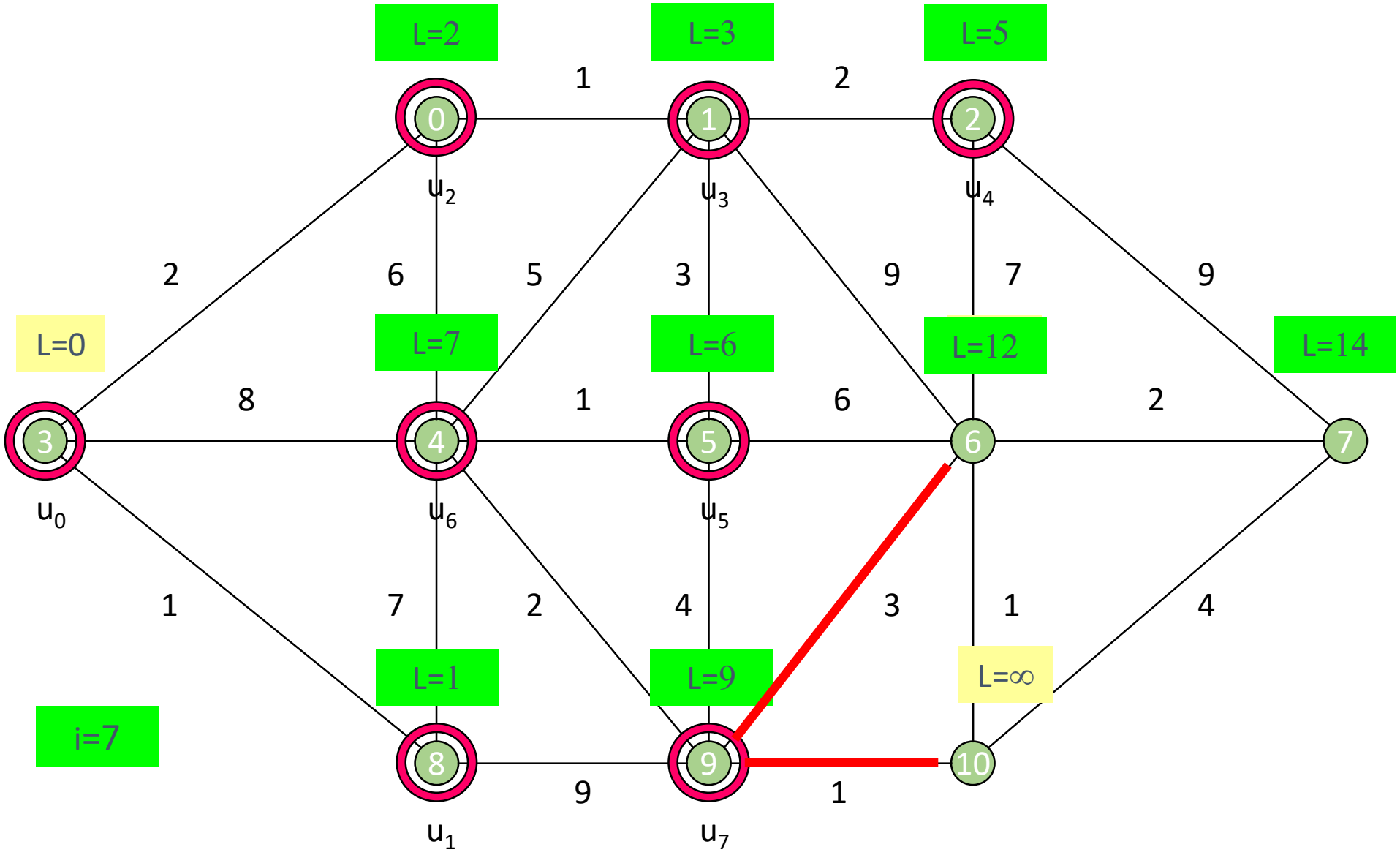

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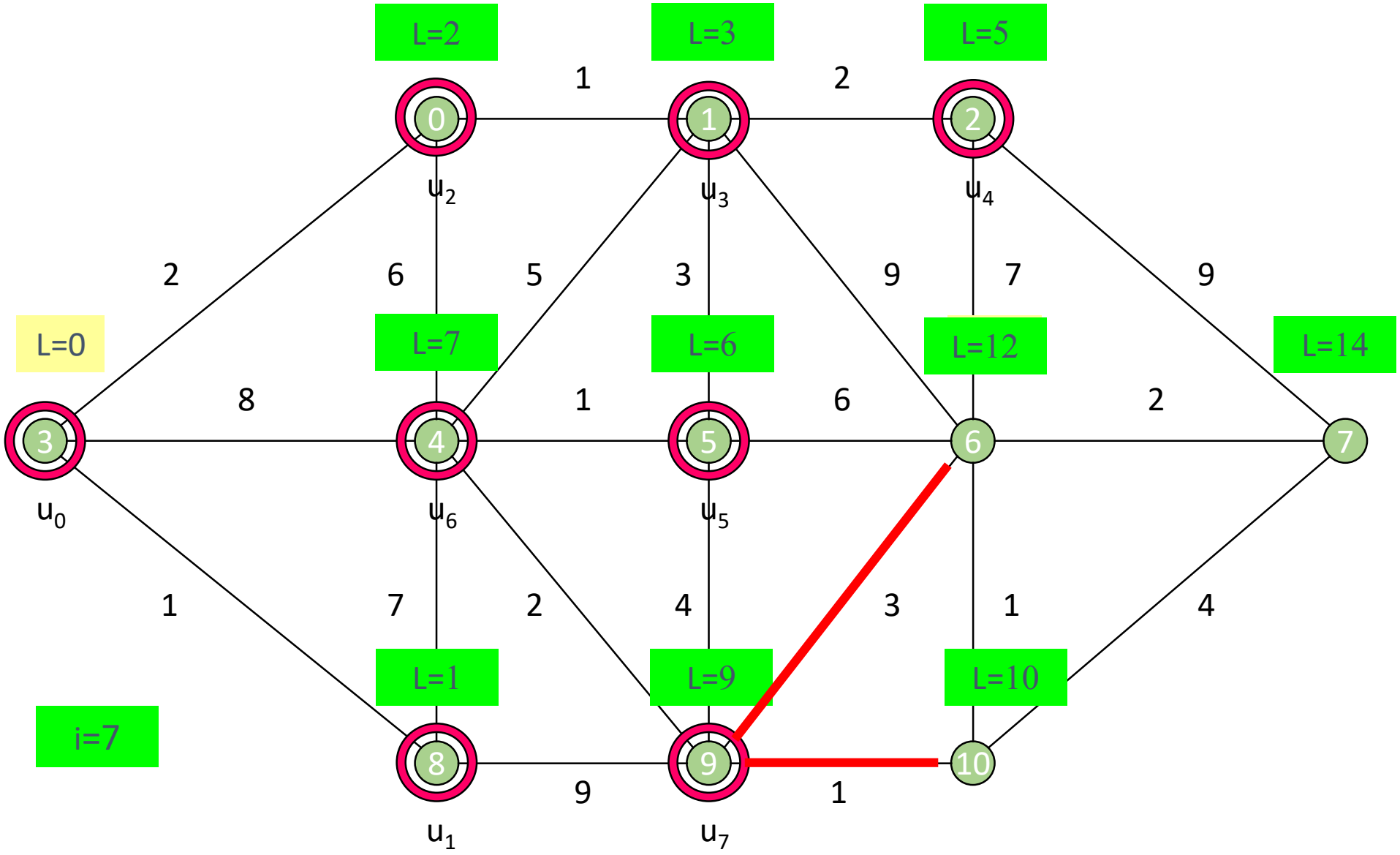

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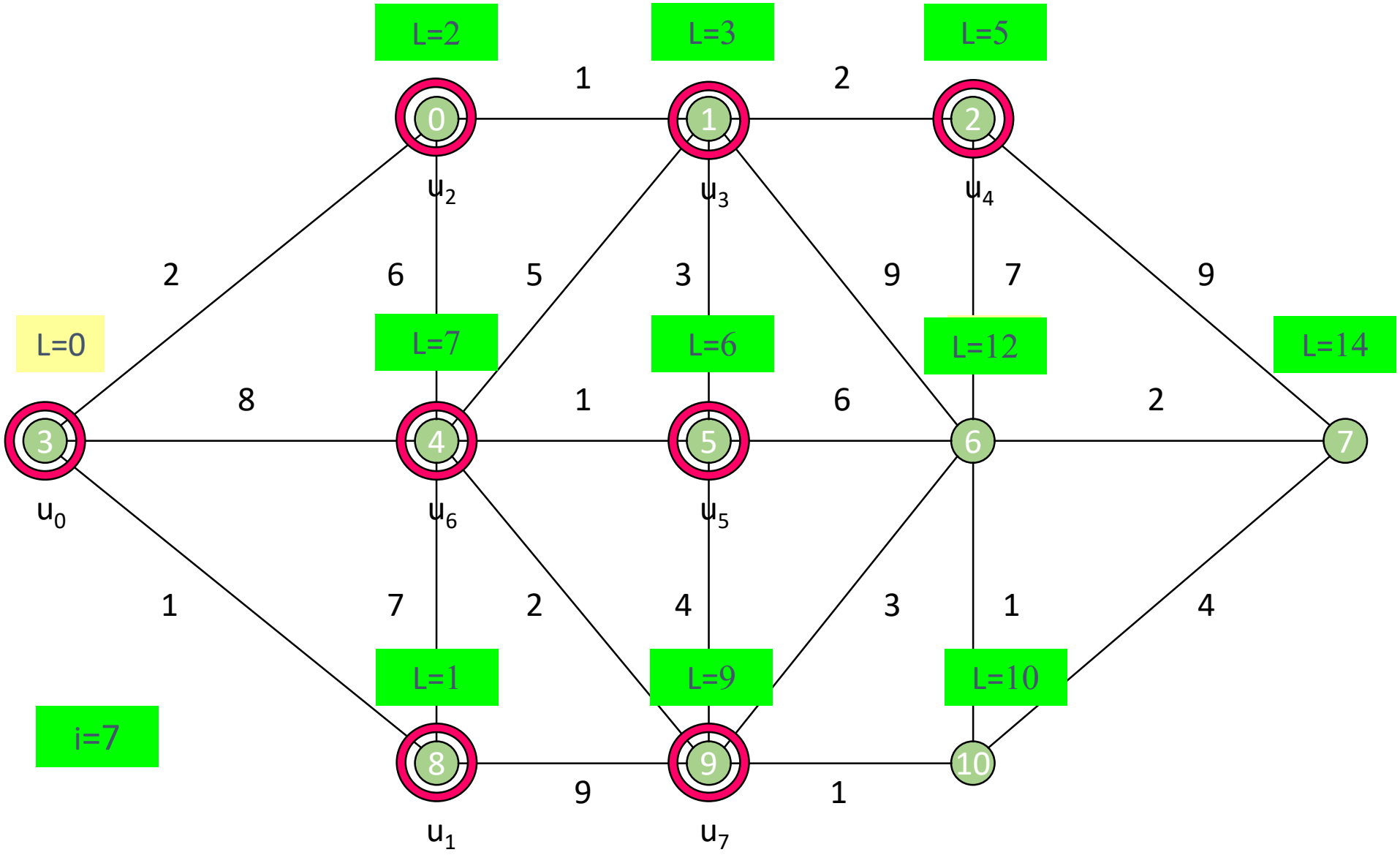
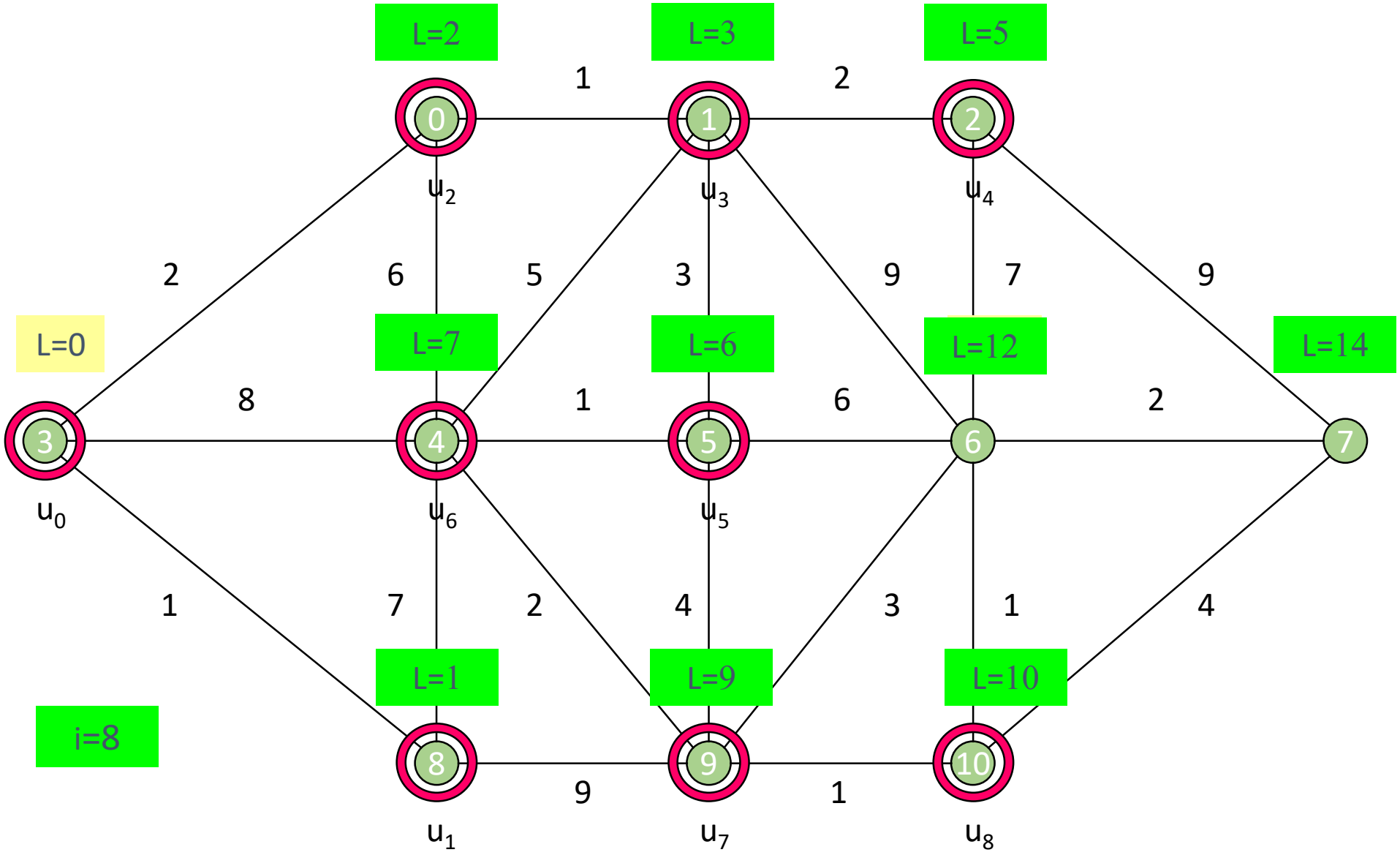
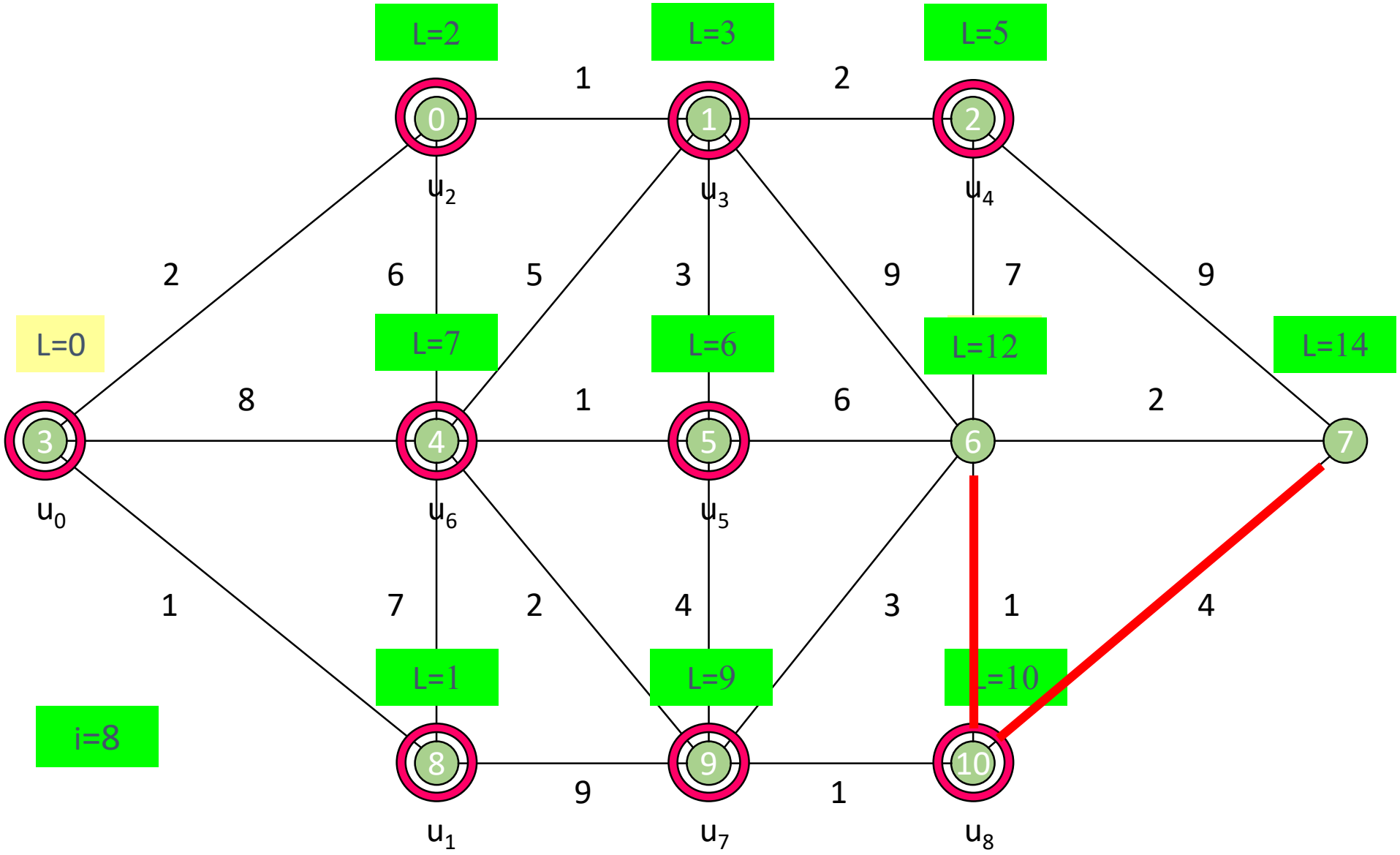


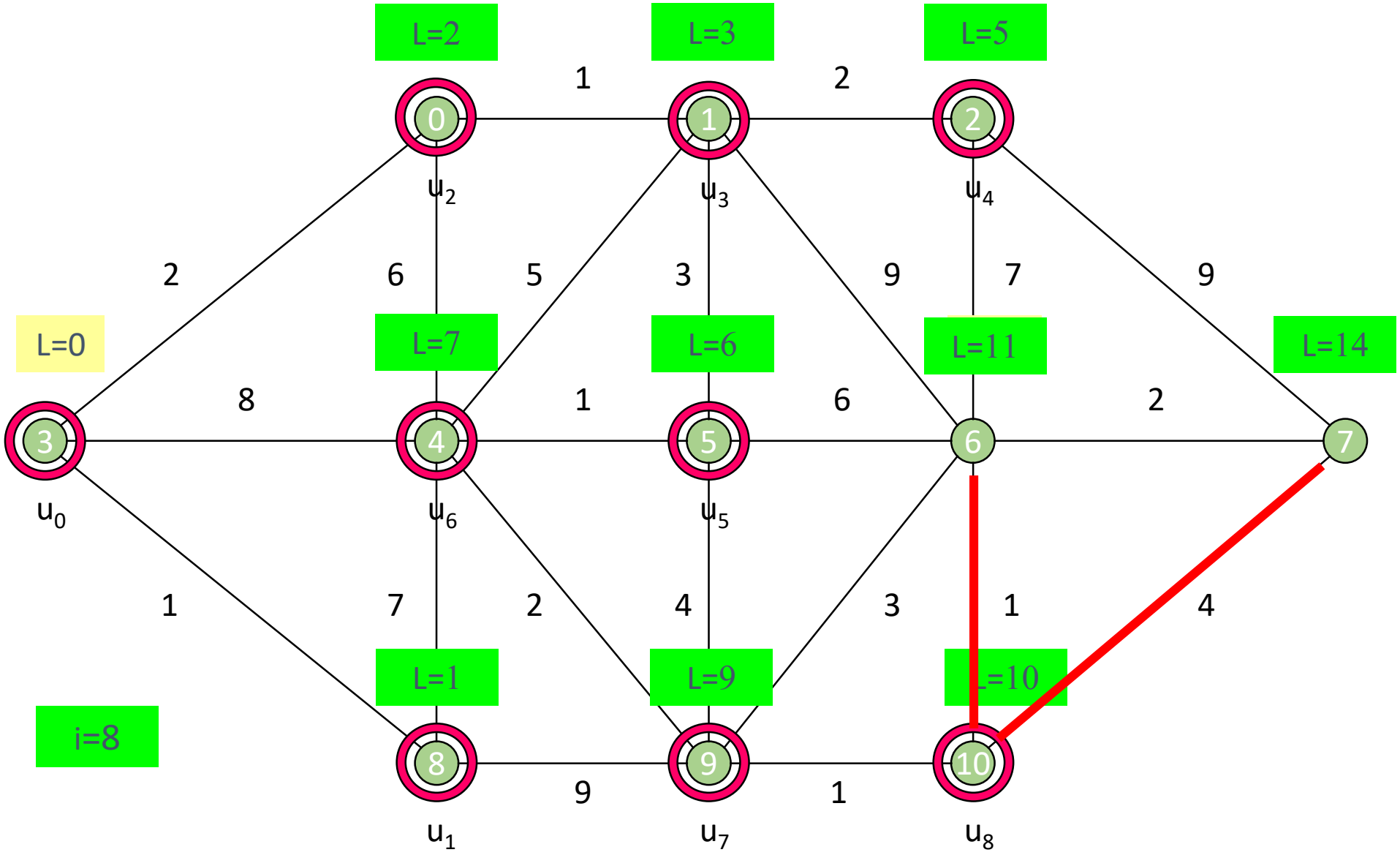
Algoritmo Dijkstra - Exemplo



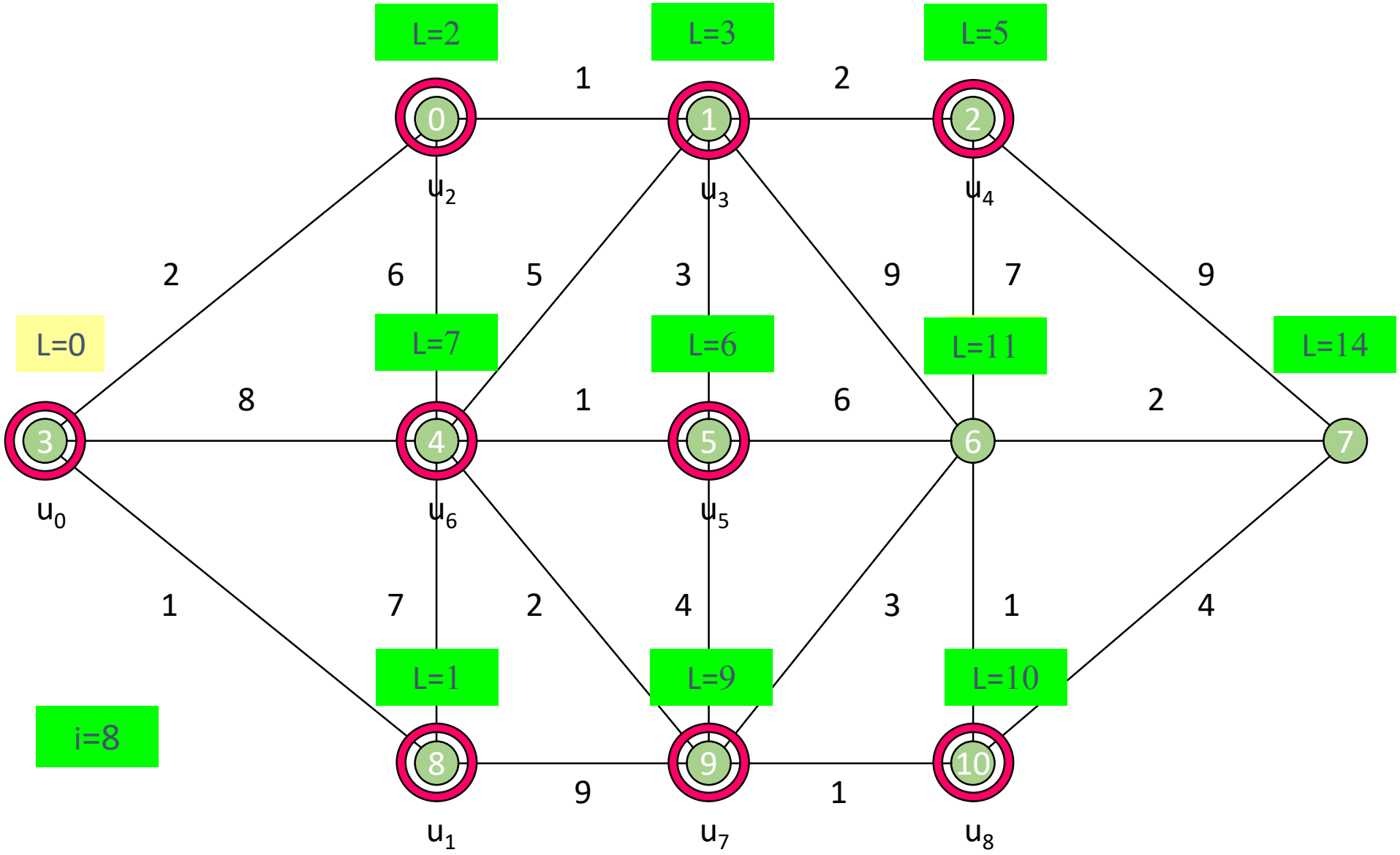
Algoritmo Dijkstra - Exemplo



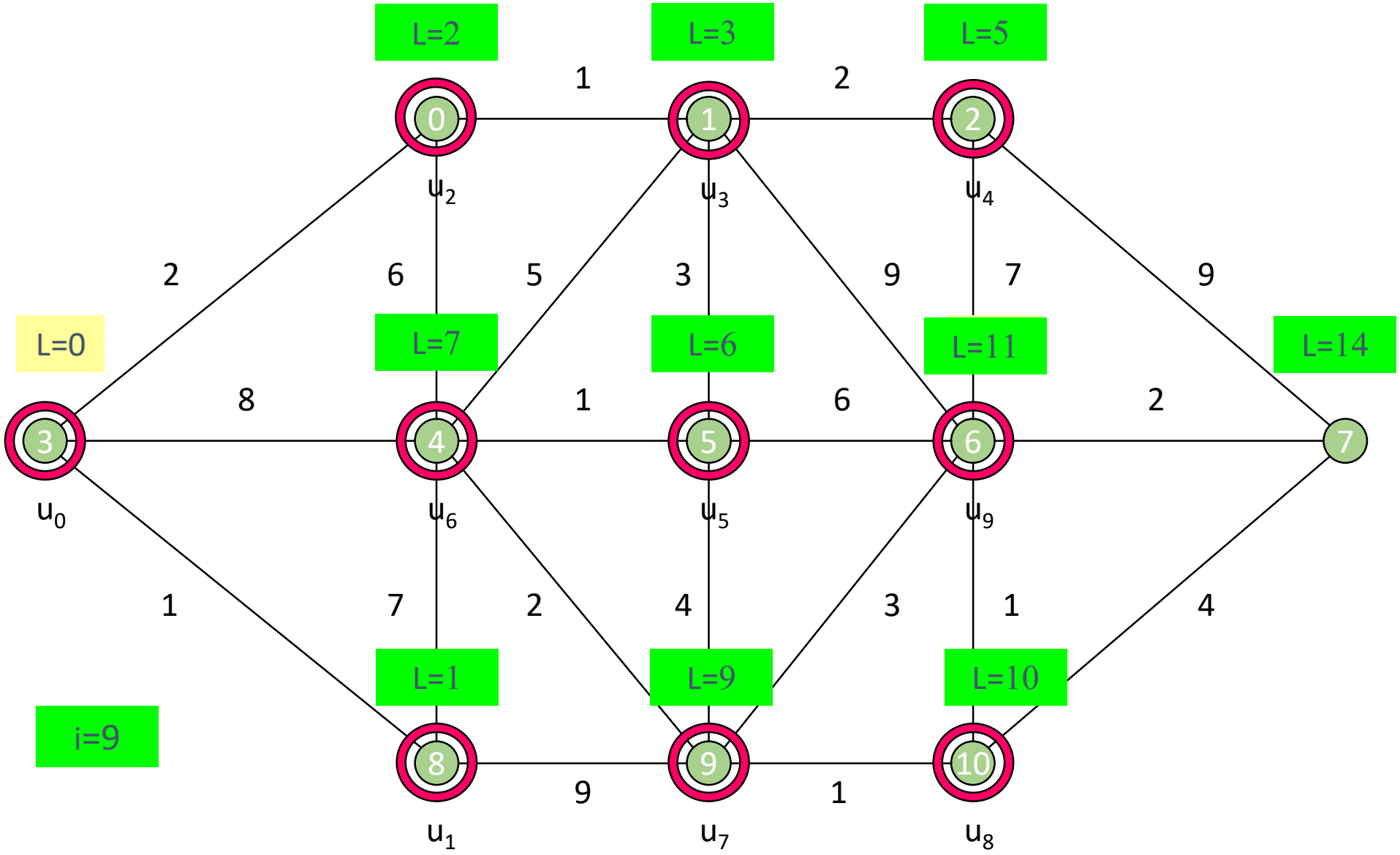
Algoritmo Dijkstra - Exemplo



Algoritmo Dijkstra - Exemplo



Algoritmo Dijkstra - Exemplo



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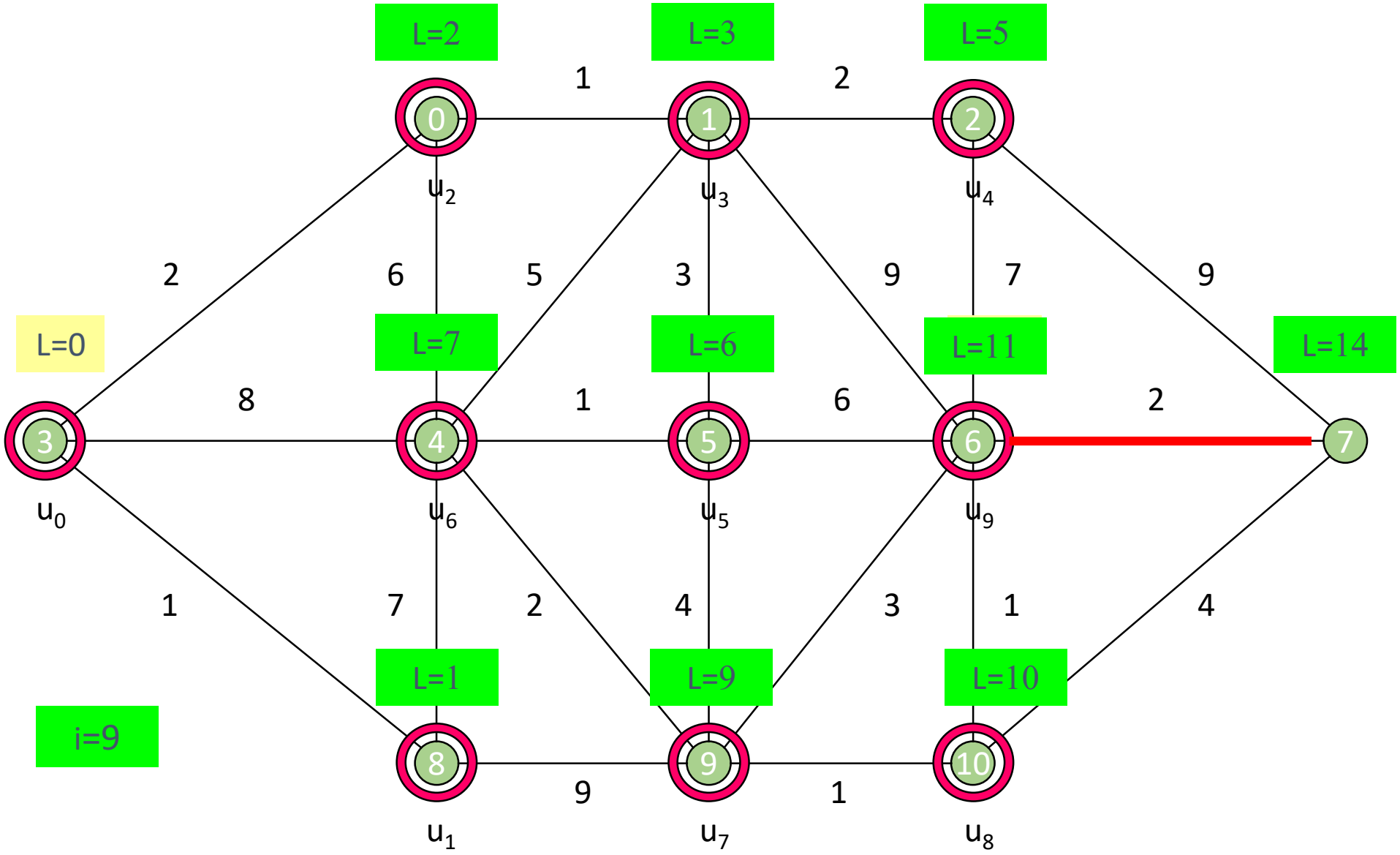
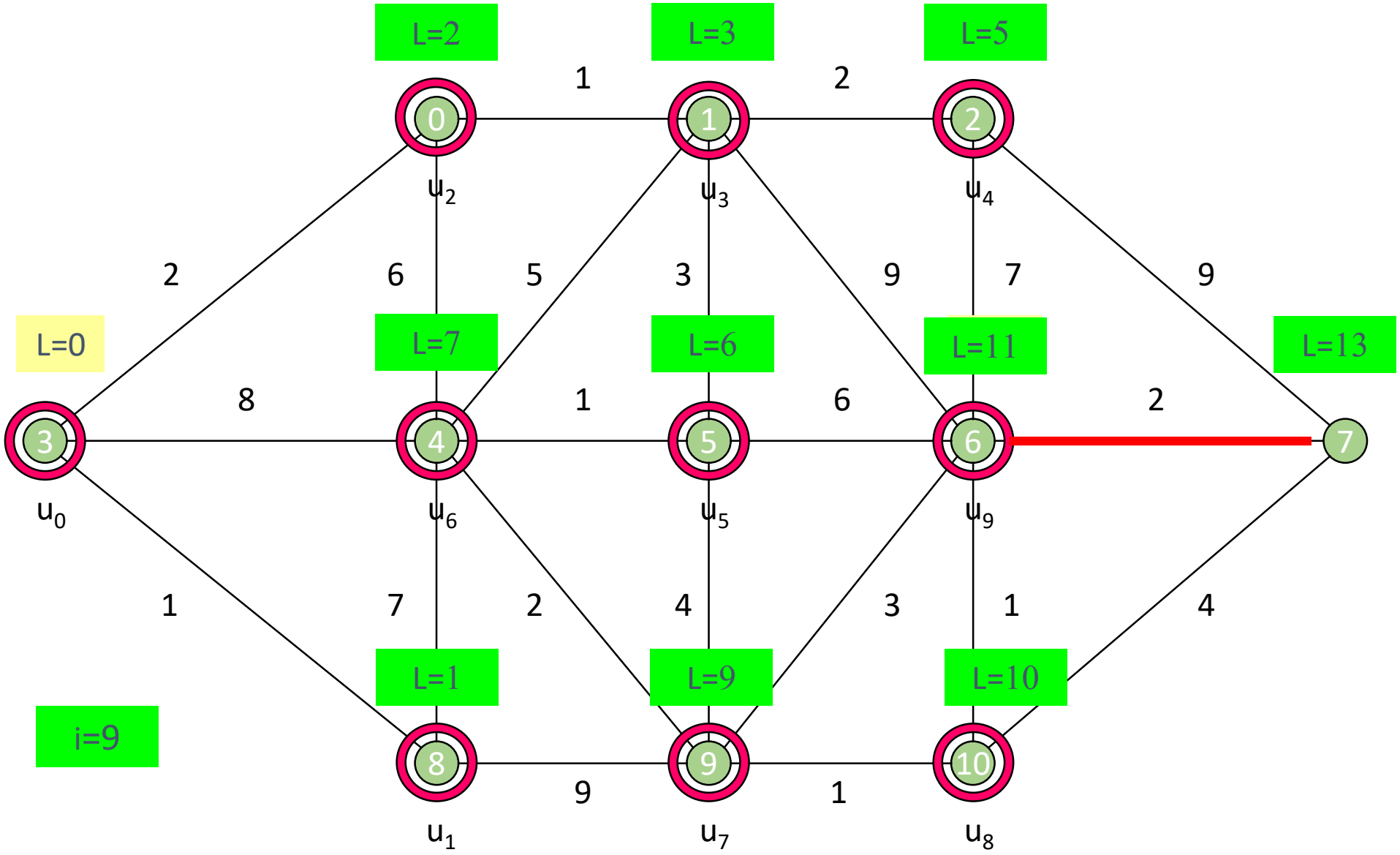
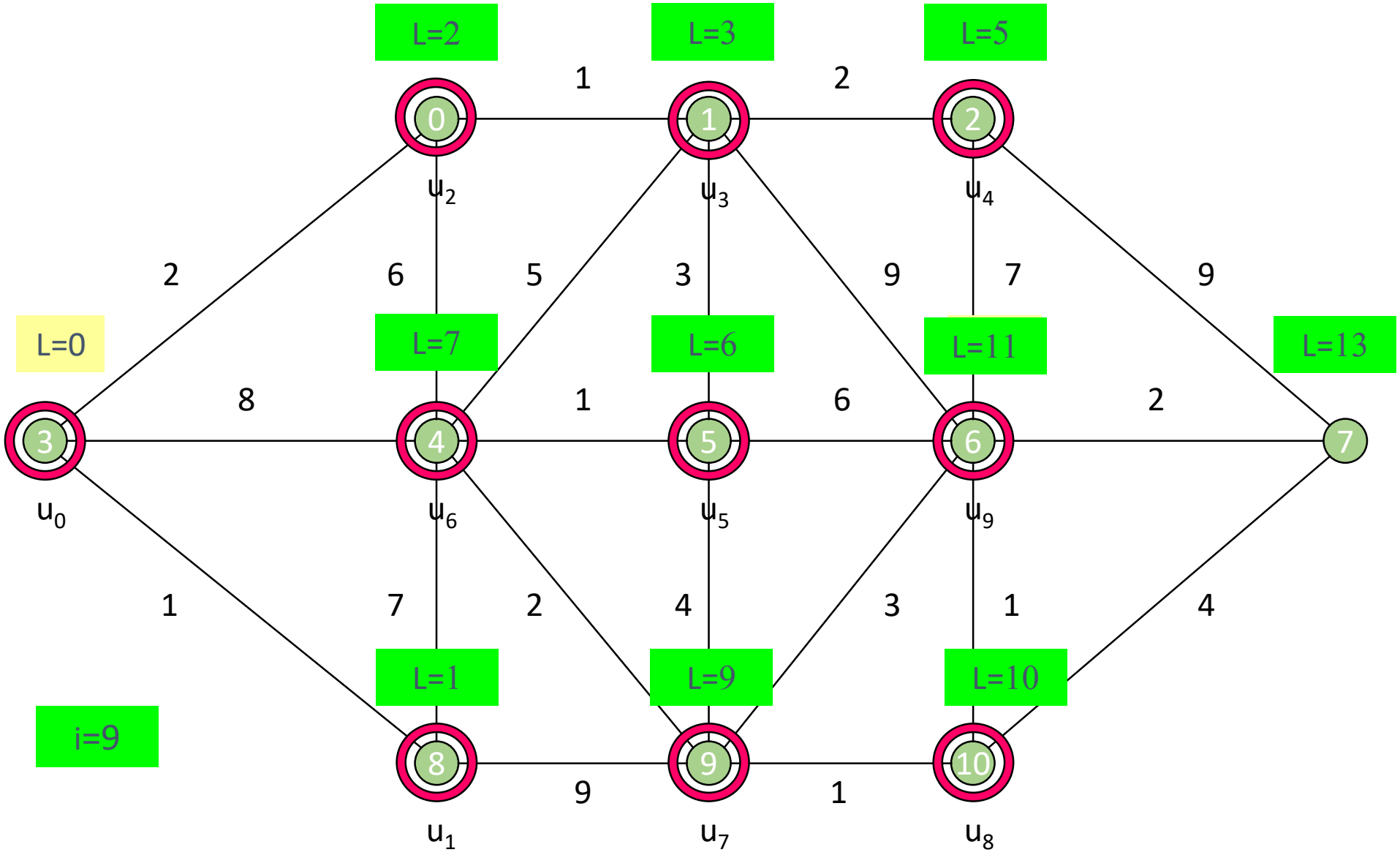


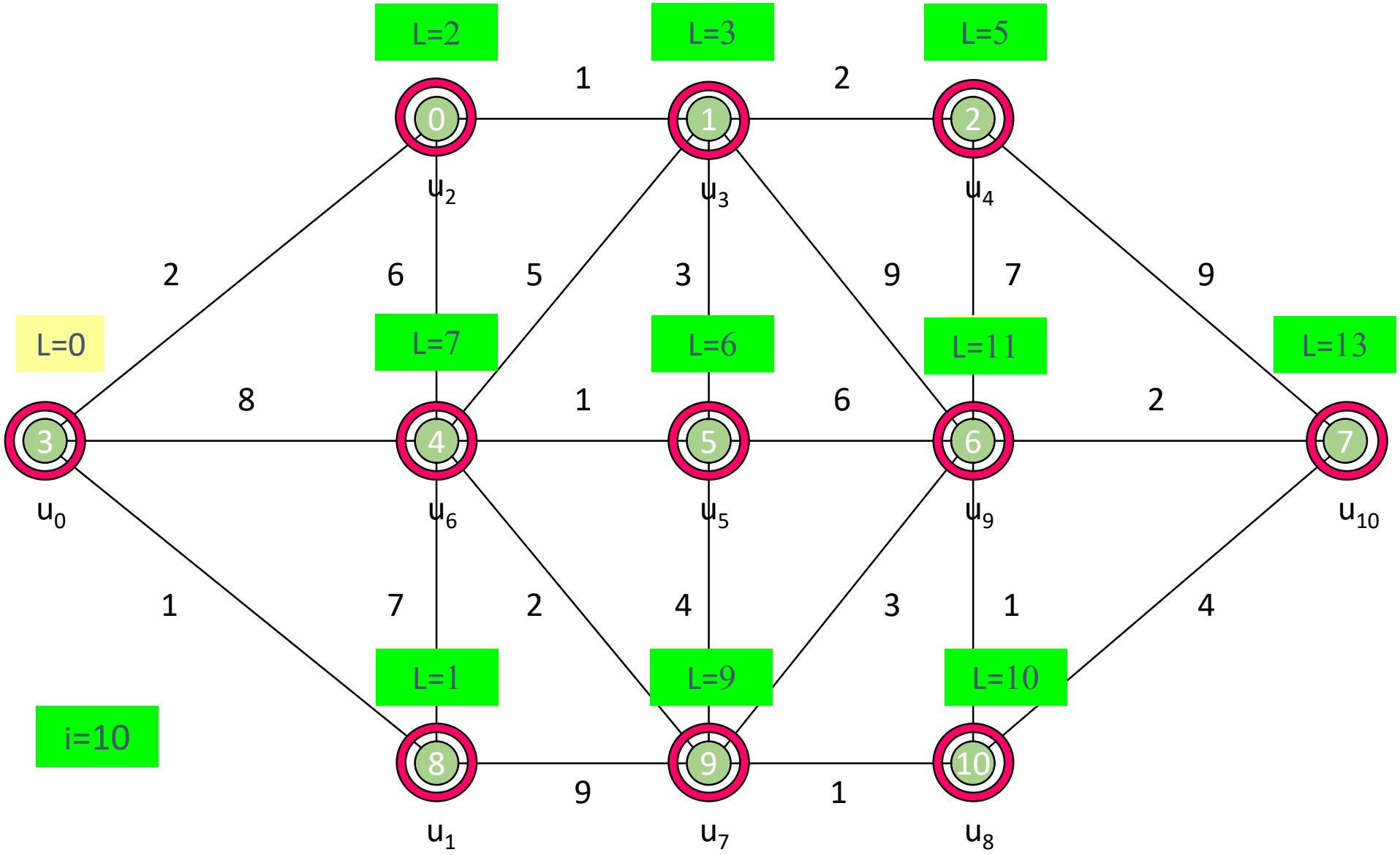
Algoritmo Dijkstra - Exemplo



Algoritmo Dijkstra - Exemplo



Algoritmo Dijkstra - Exemplo



Algoritmo Dijkstra – Perguntas

L=

0	2
1	3
2	5
u_0= 3	0
4	7
5	6
6	11
7	13
8	1
9	9
10	10

- Qual a distância mínima entre os vértices:
  - 3 e 0 => 2
  - 3 e 1 => 3
  - 3 e 10 = > 10
  - 3 e 9 => 9
  - 3 e 7 => 13

Algoritmo Dijkstra – Perguntas

L=

0	2
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  - 3 e 0 => 2
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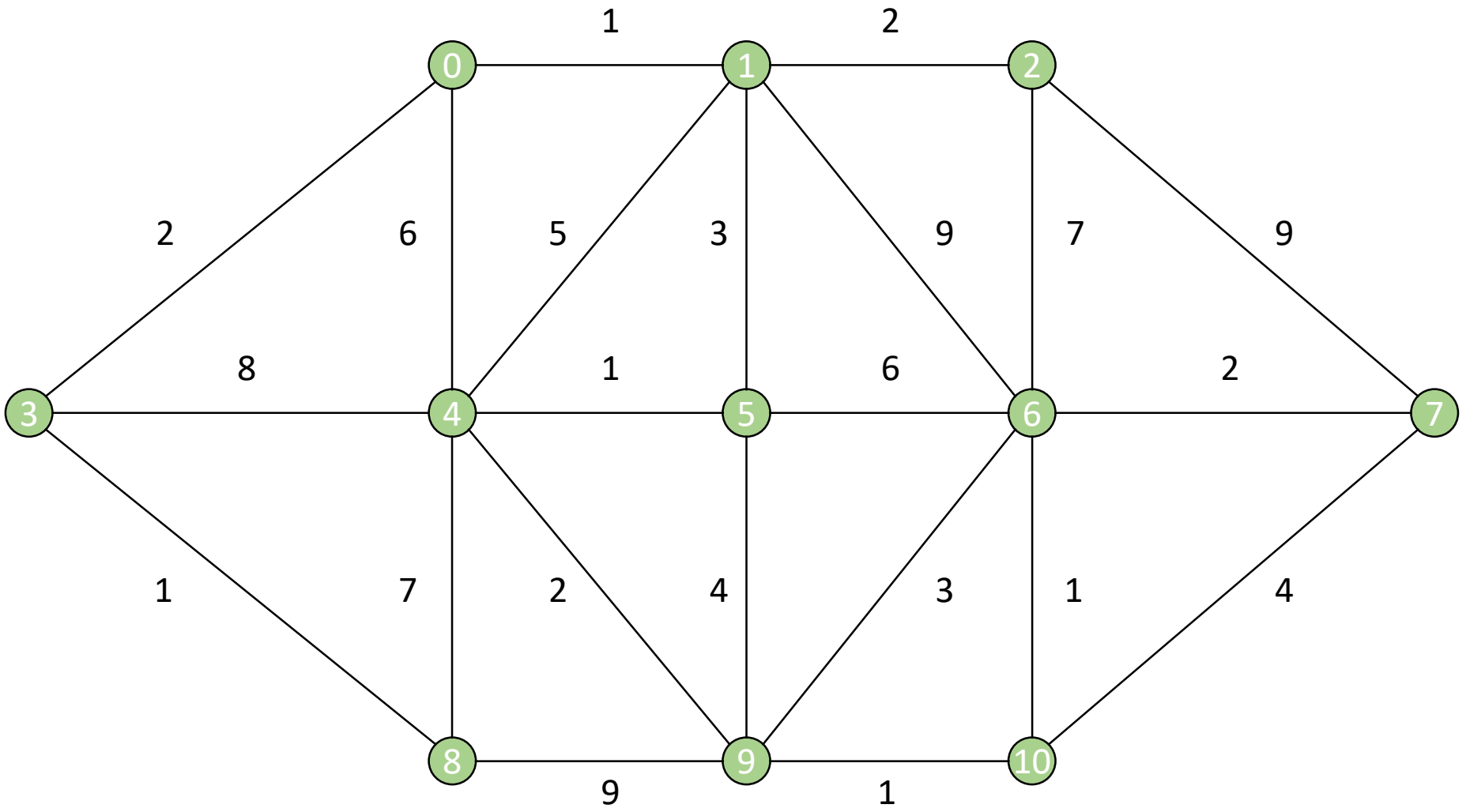
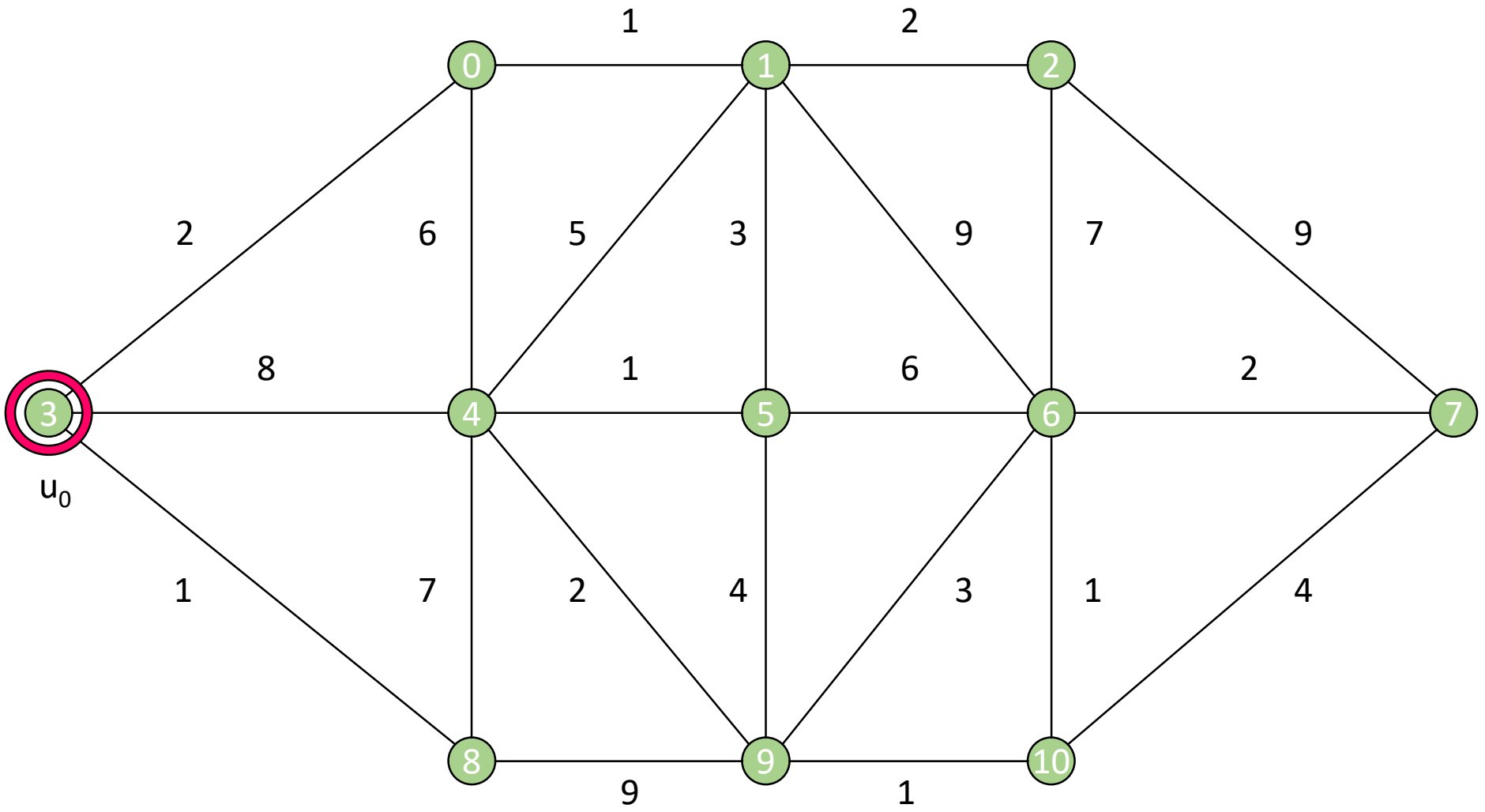
  - 3 e 10 = > 10
  - 3 e 9 => 9
  - 3 e 7 => 13

Qual **caminho** possui distância mínima entre os vértices 3 e 7 ?

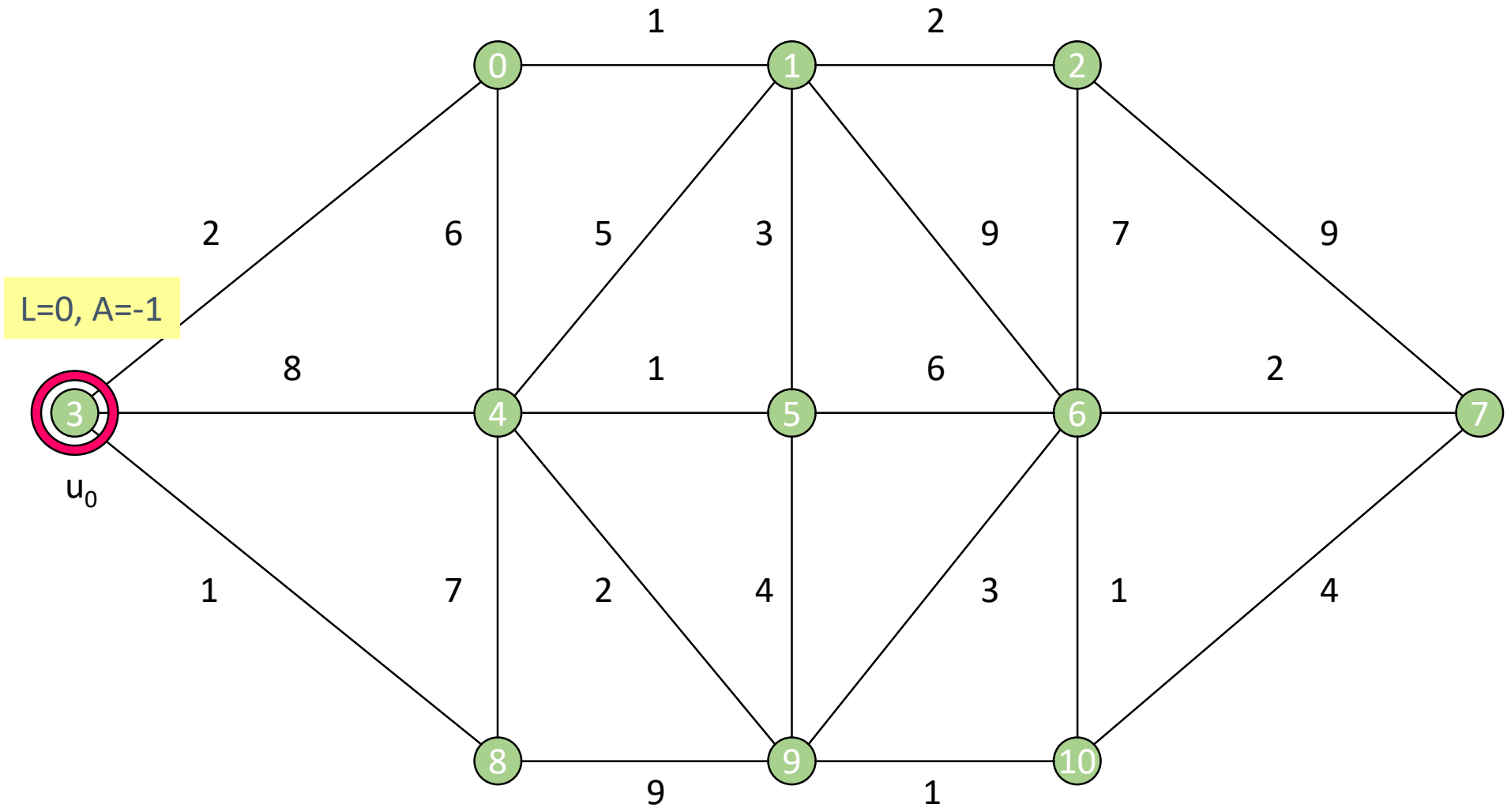
Caminho do vértice 3 até o vértice 7



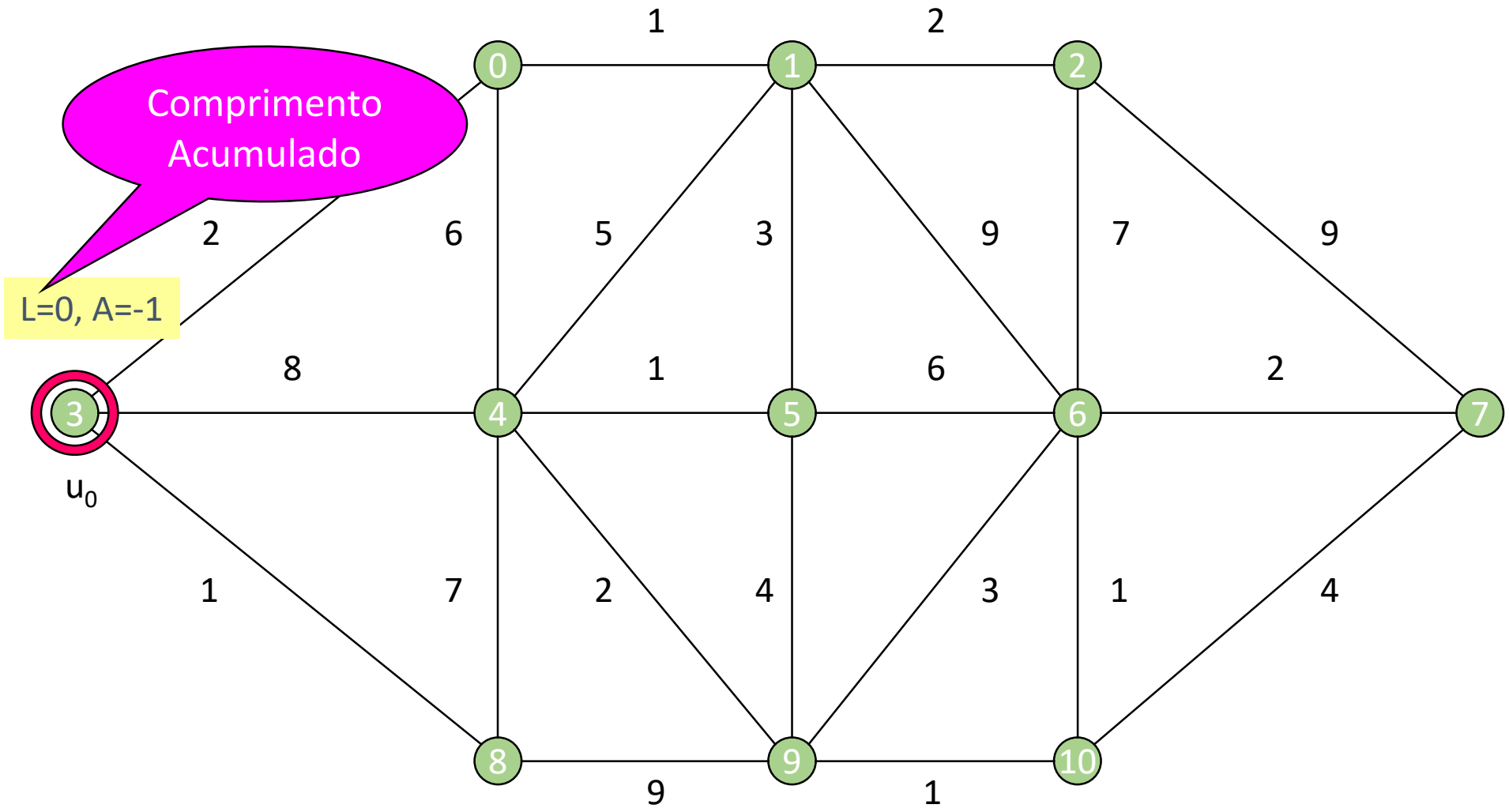
Caminho do vértice 3 até o vértice 7



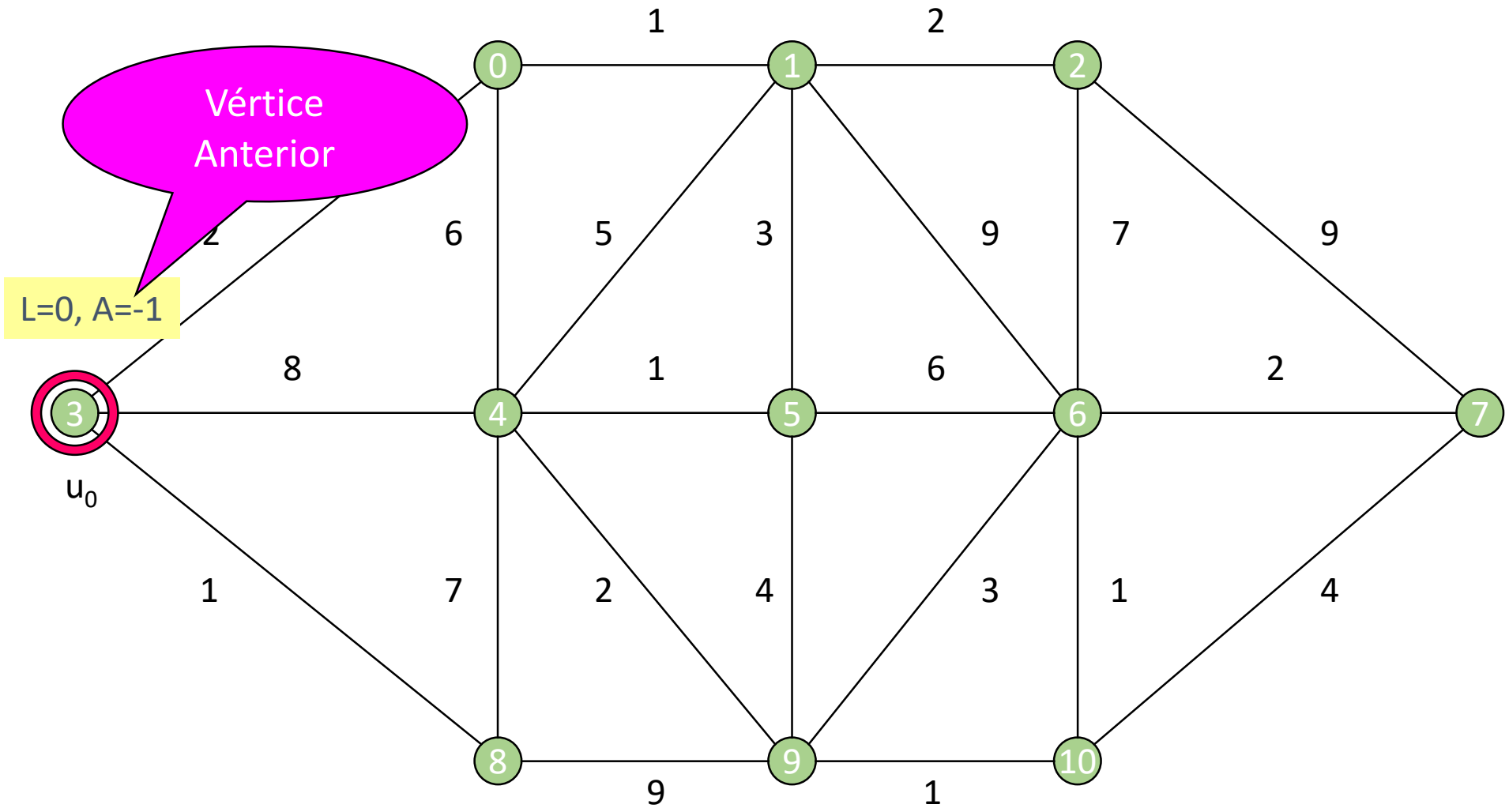
Caminho do vértice 3 até o vértice 7



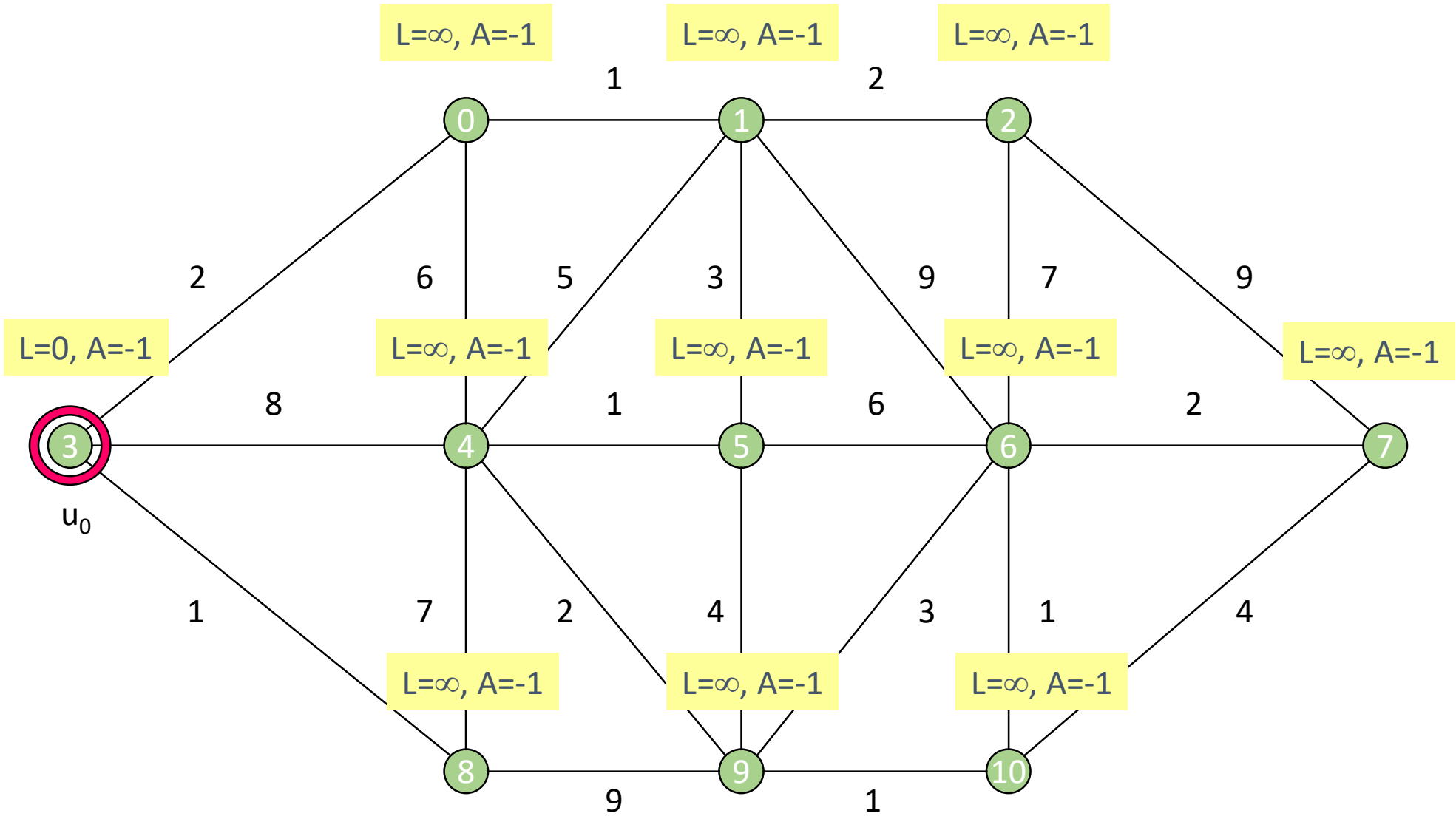
Caminho do vértice 3 até o vértice 7



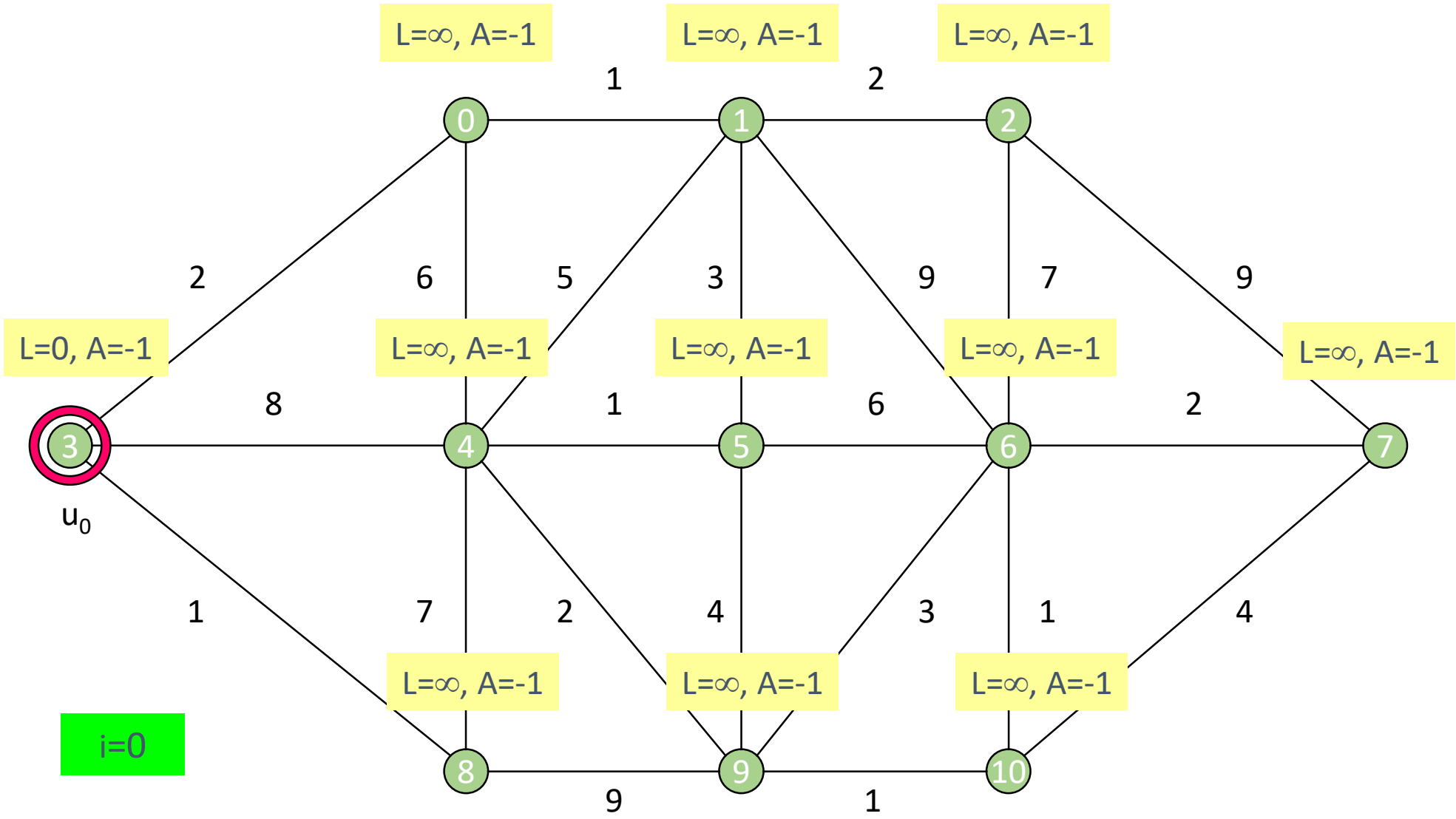
Caminho do vértice 3 até o vértice 7



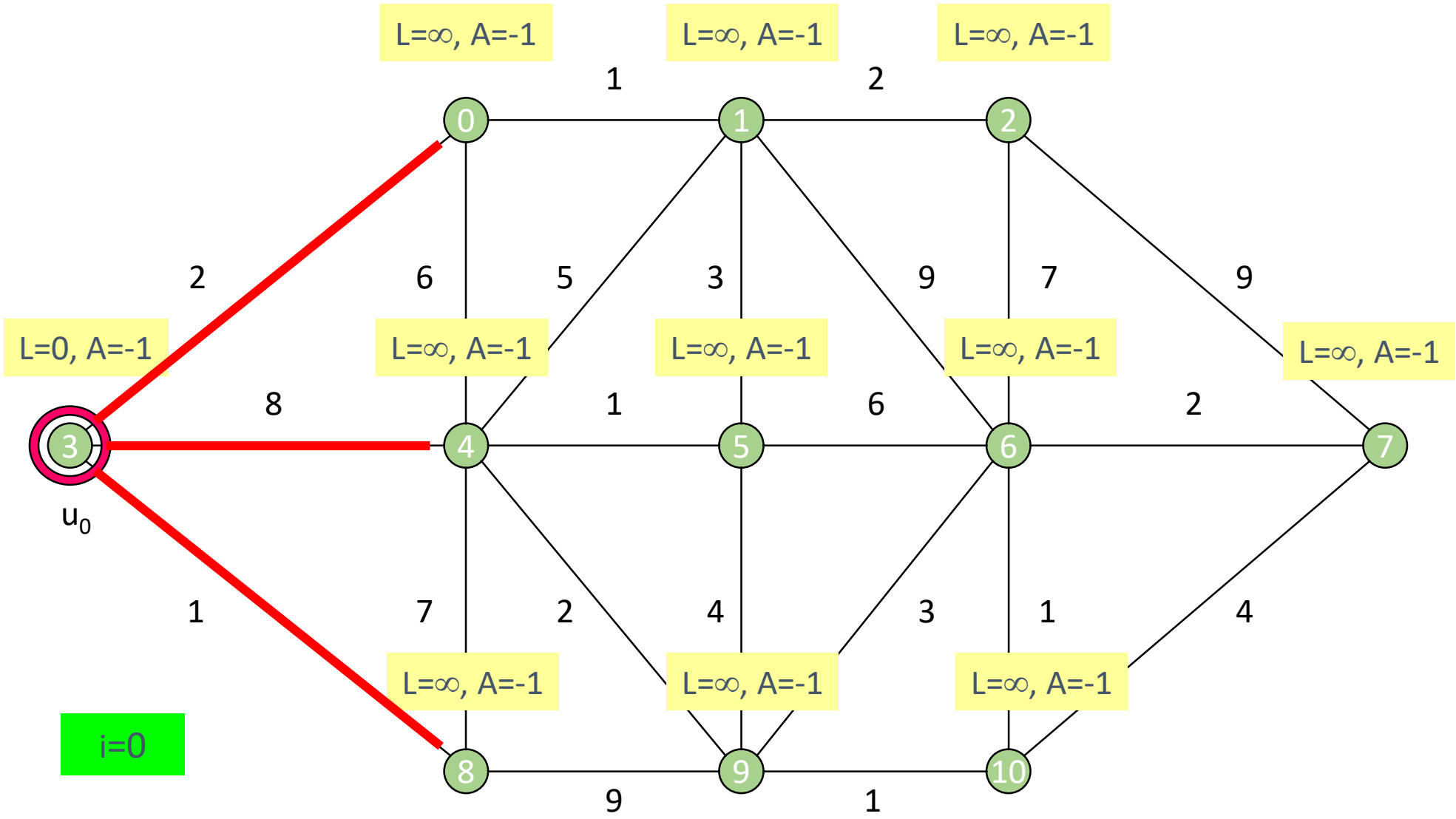
Caminho do vértice 3 até o vértice 7



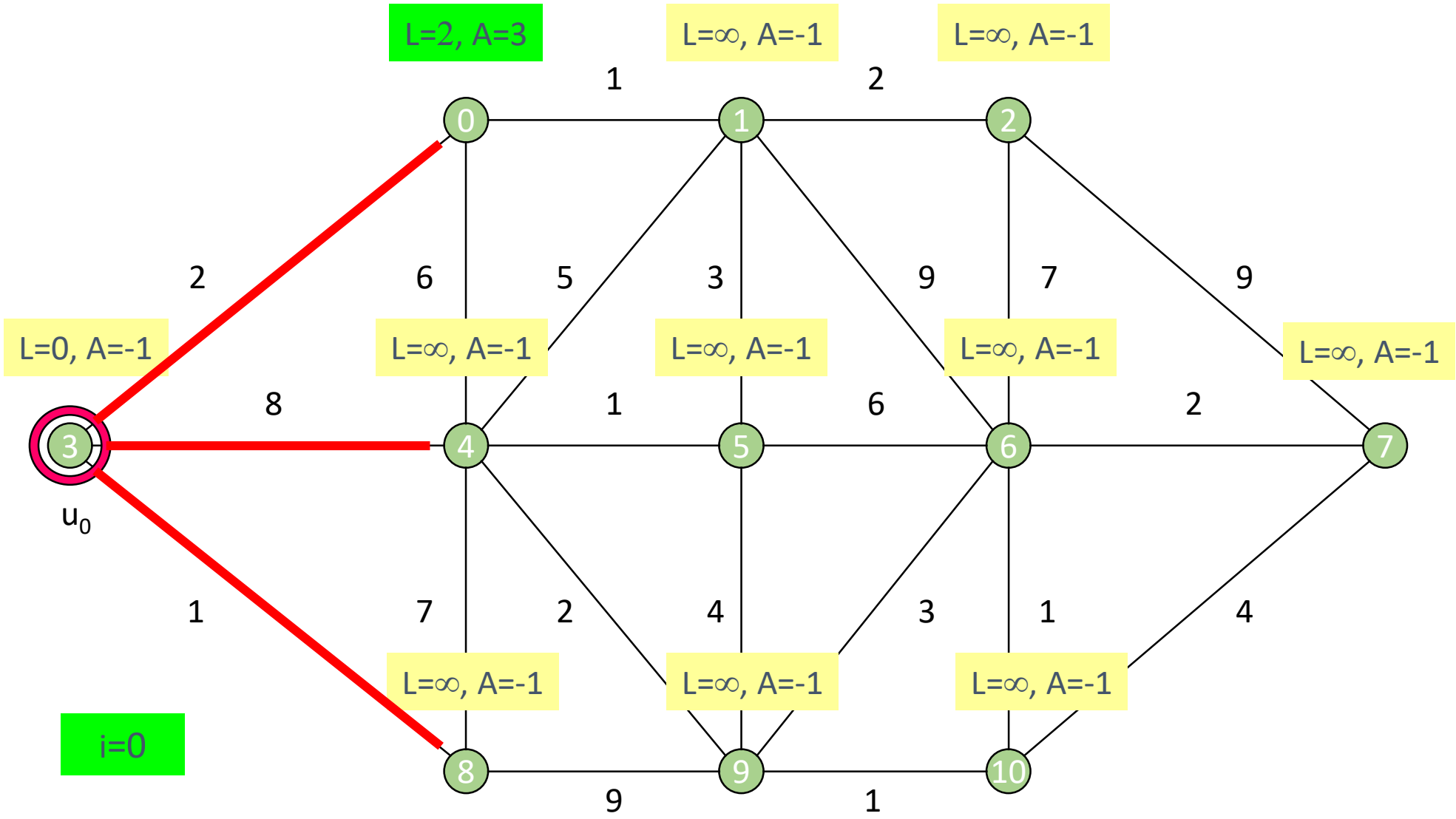
Caminho do vértice 3 até o vértice 7



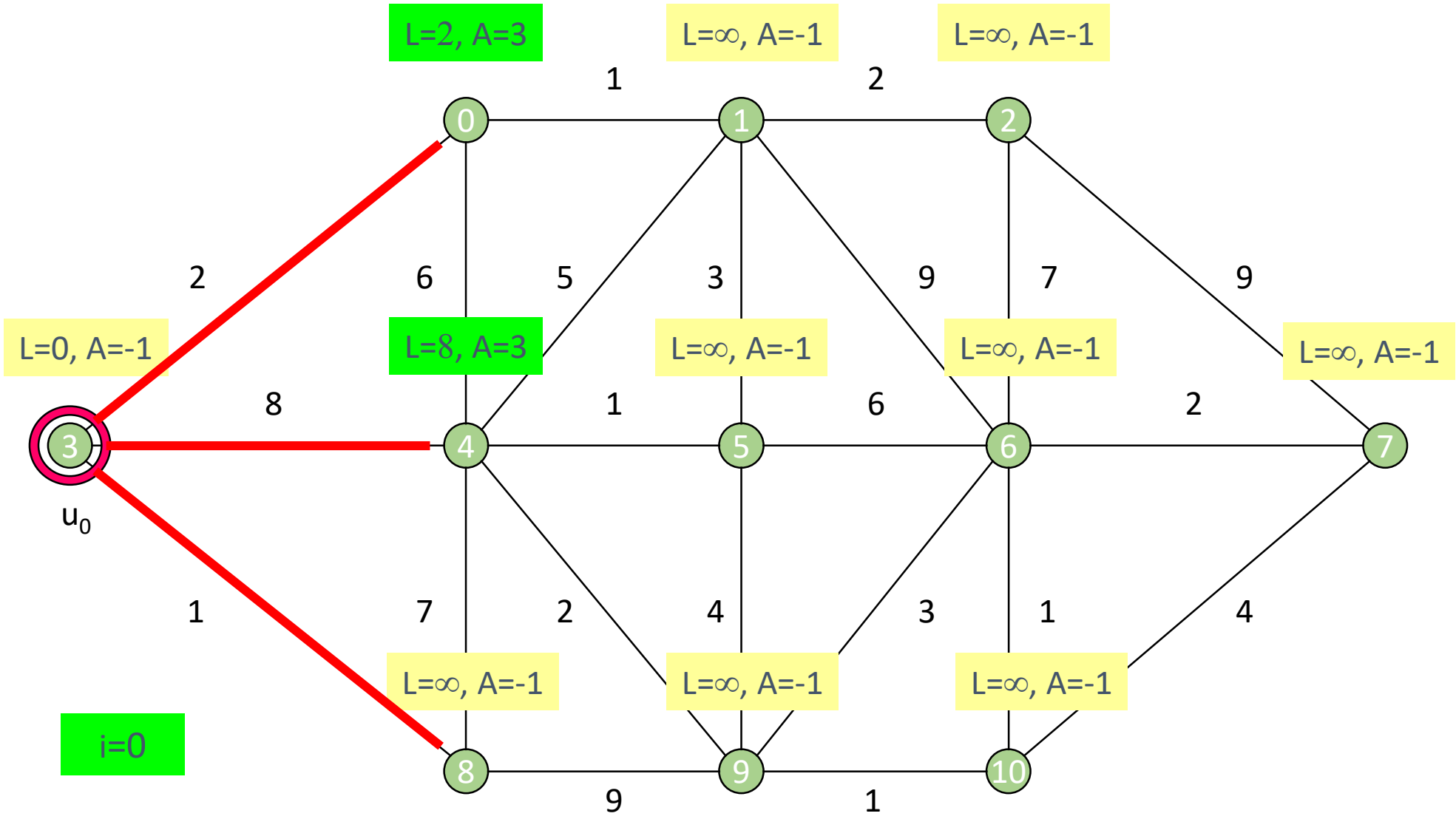
Caminho do vértice 3 até o vértice 7



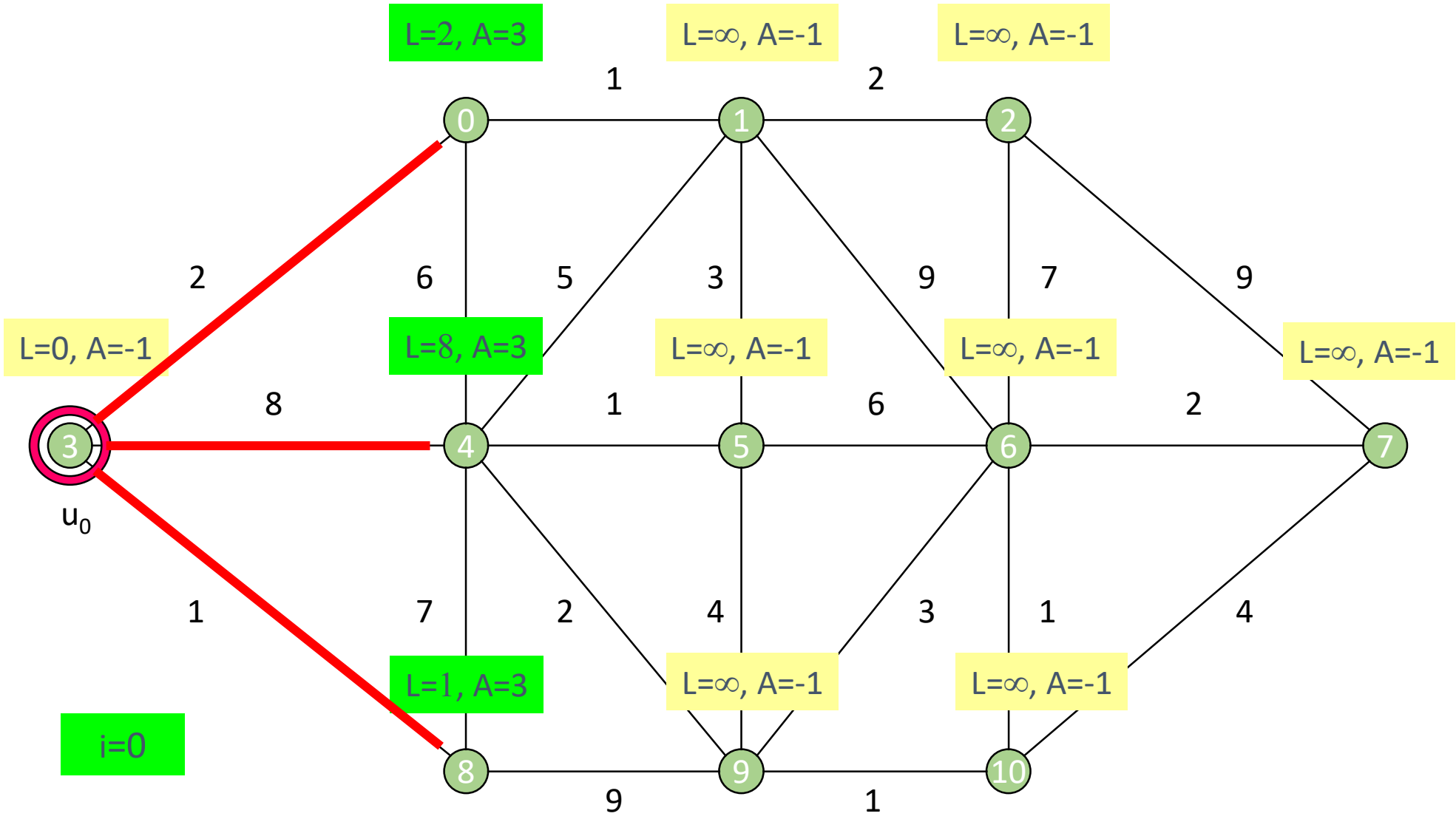
Caminho do vértice 3 até o vértice 7



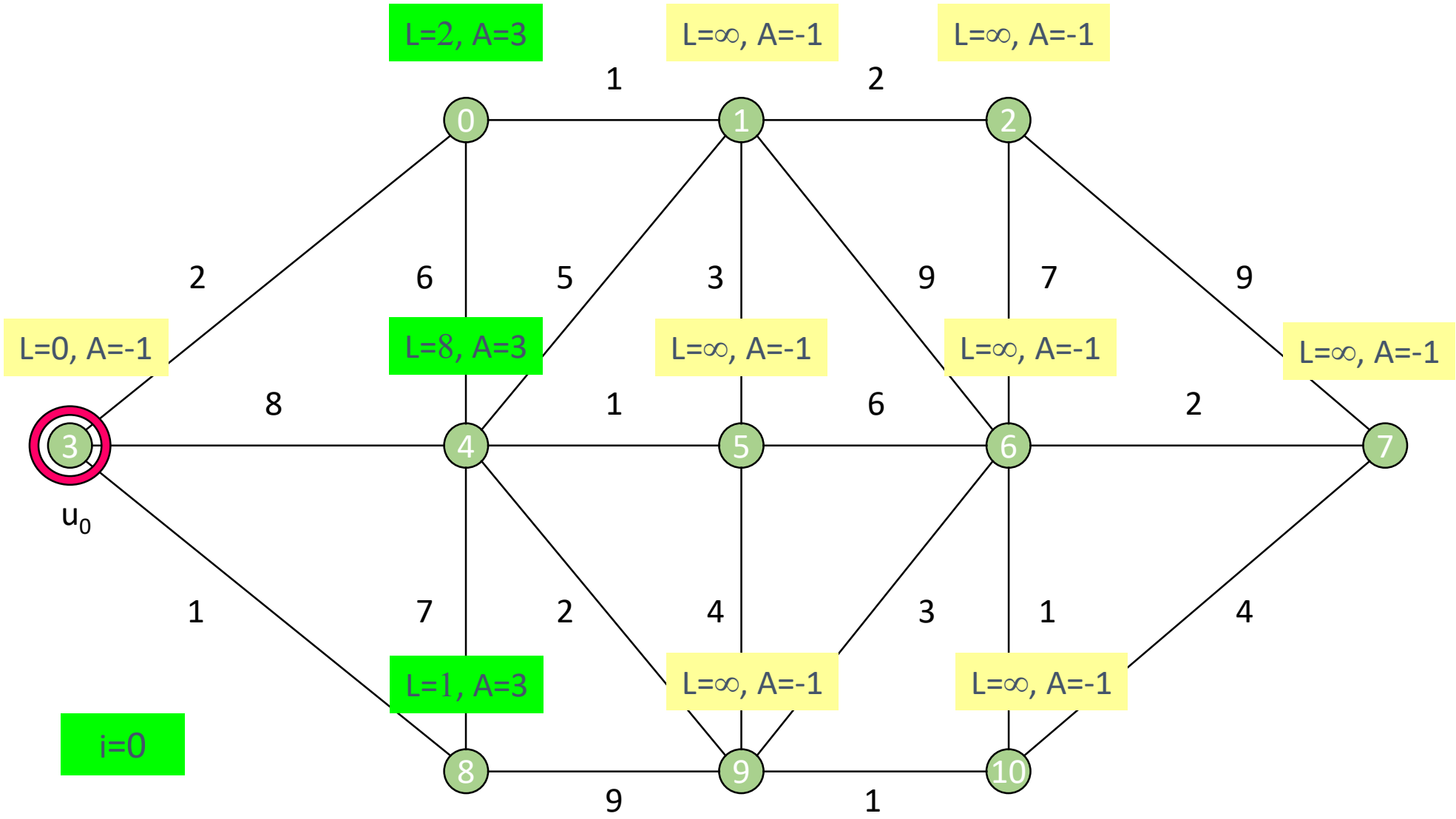
Caminho do vértice 3 até o vértice 7



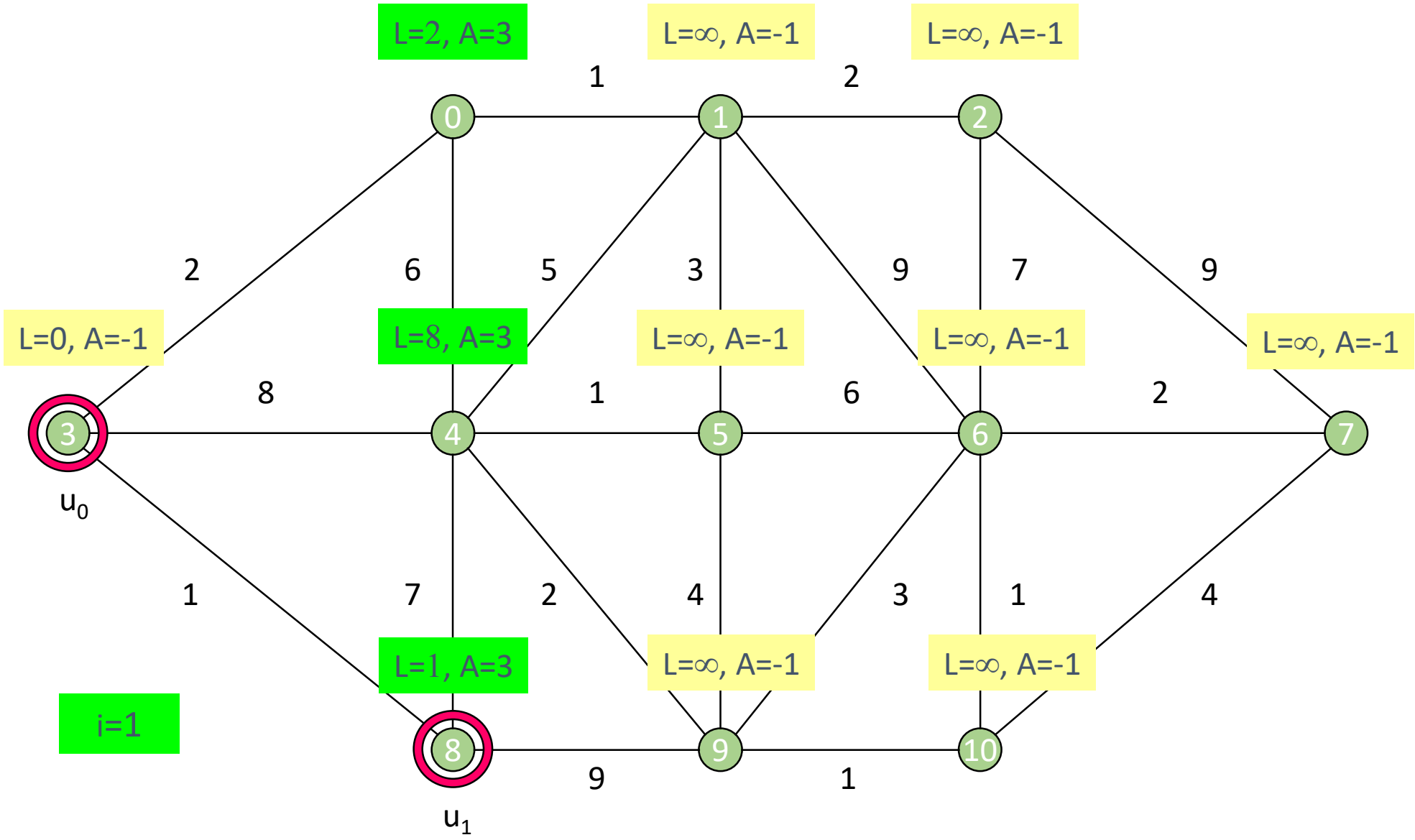
Caminho do vértice 3 até o vértice 7



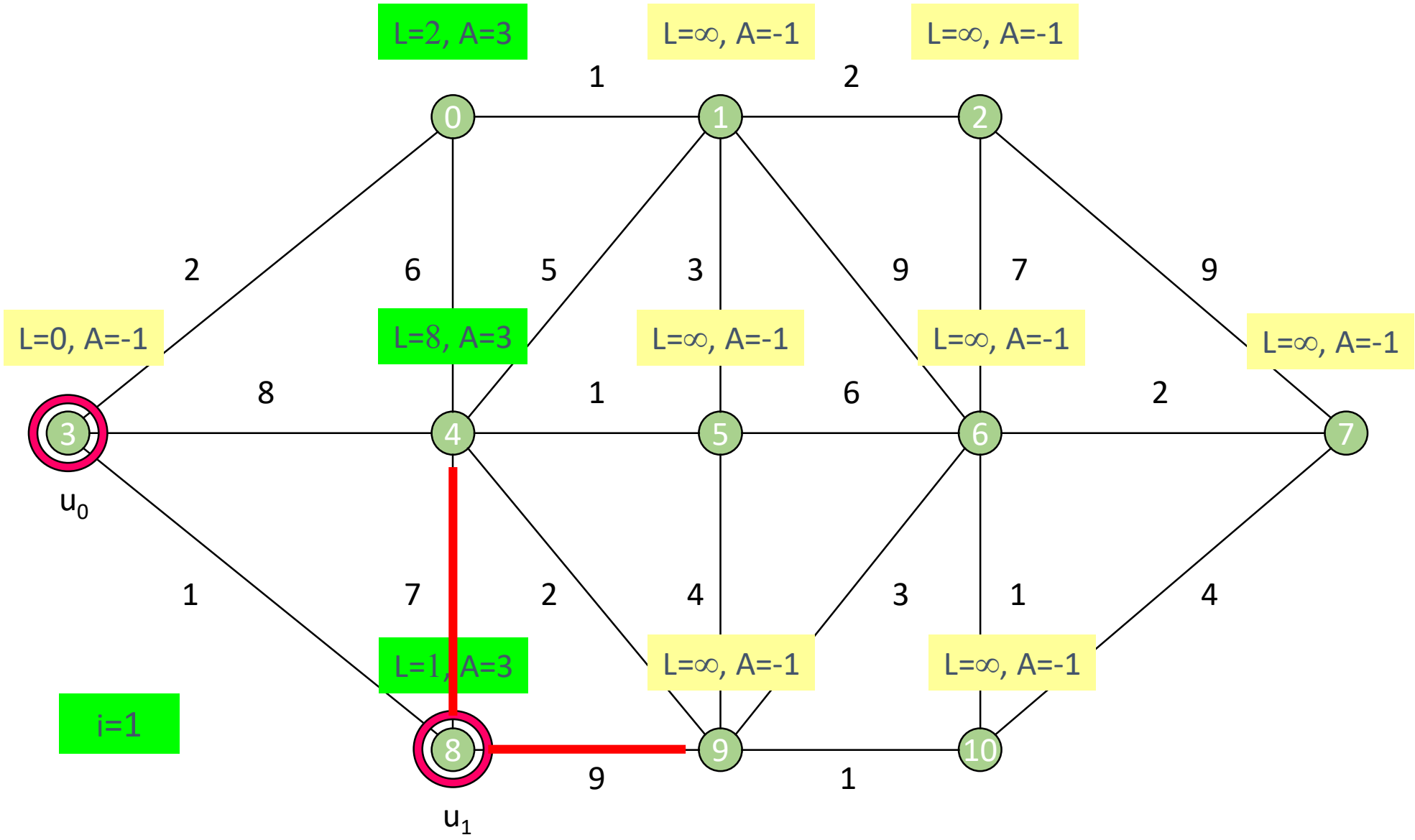
Caminho do vértice 3 até o vértice 7



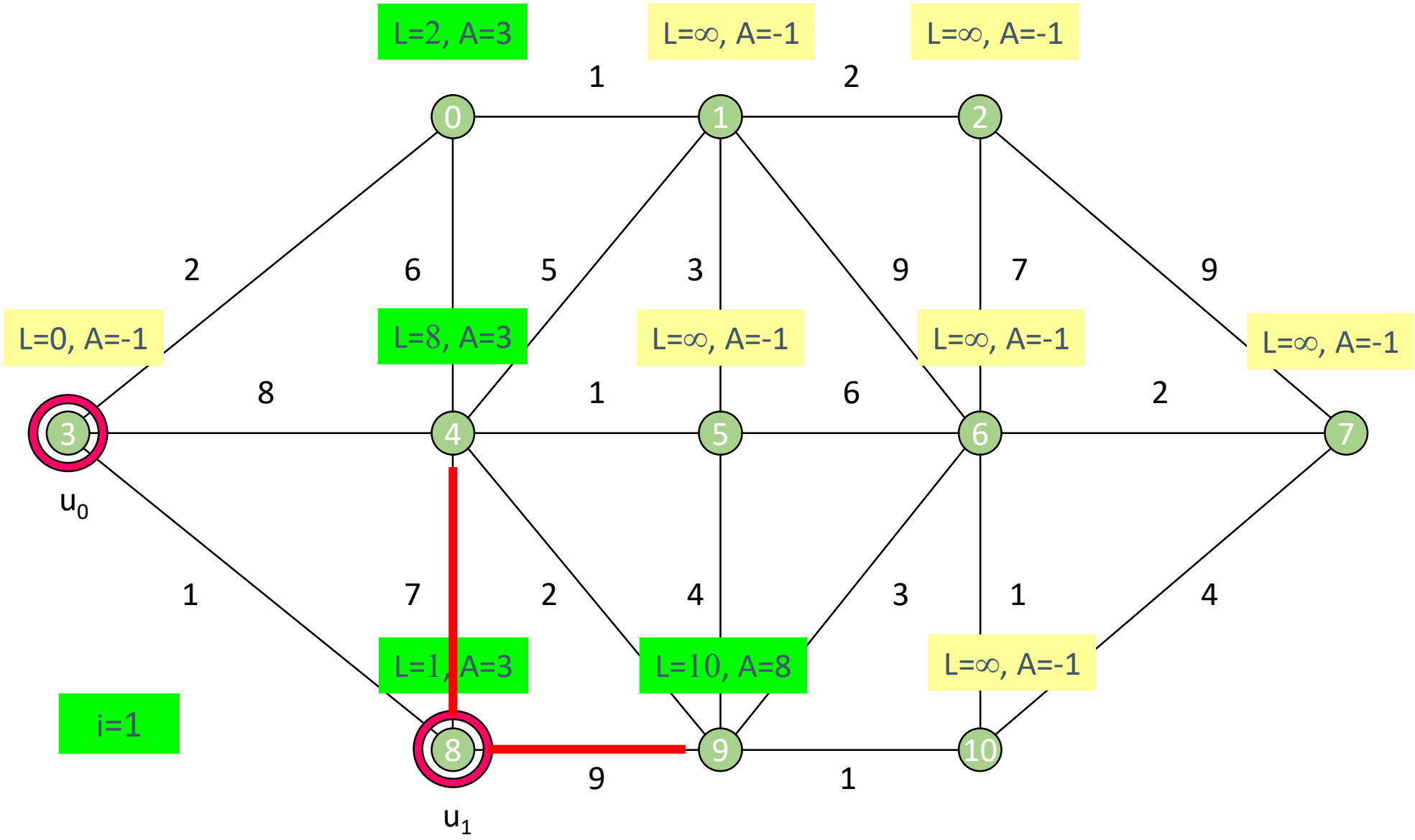
Caminho do vértice 3 até o vértice 7



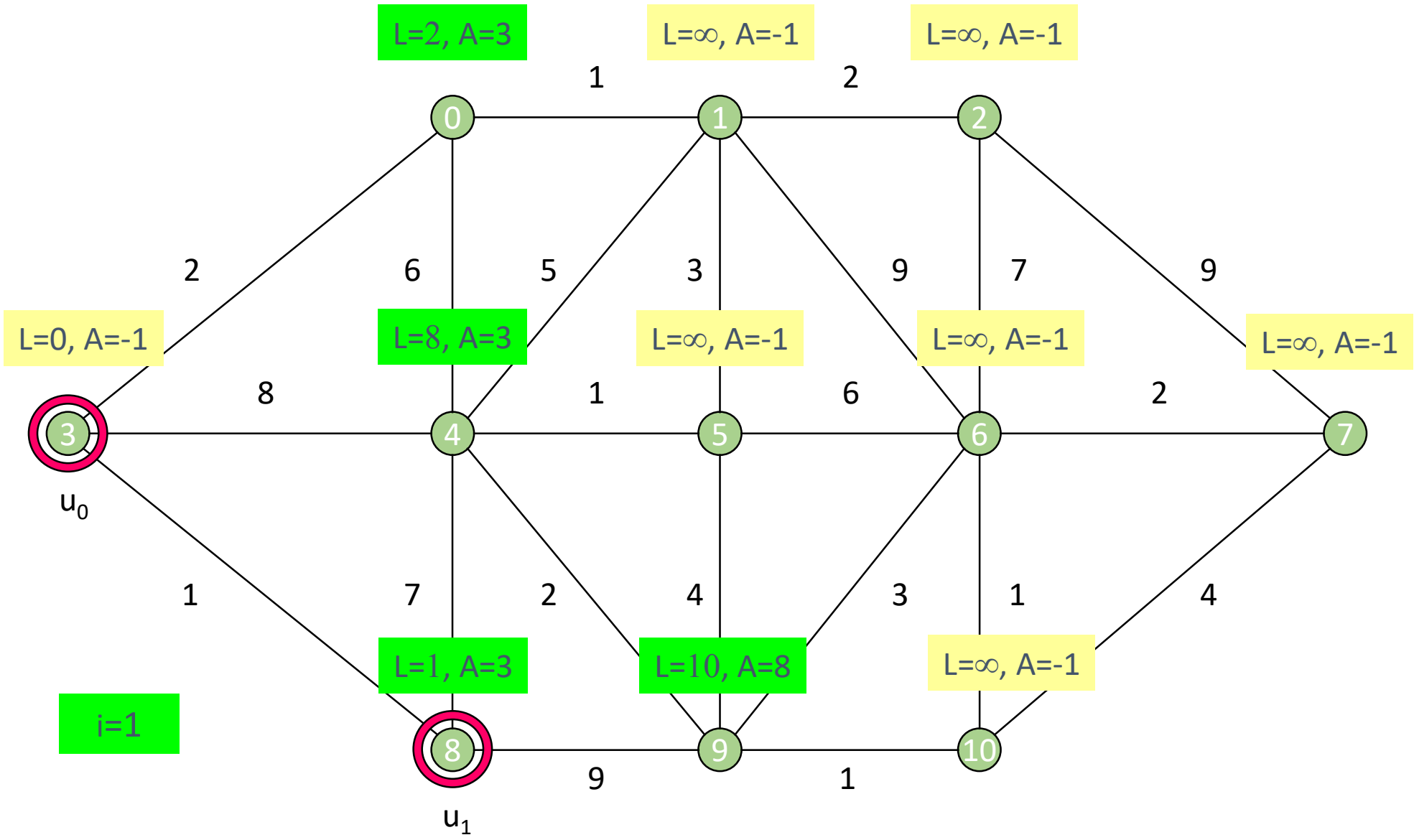
Caminho do vértice 3 até o vértice 7



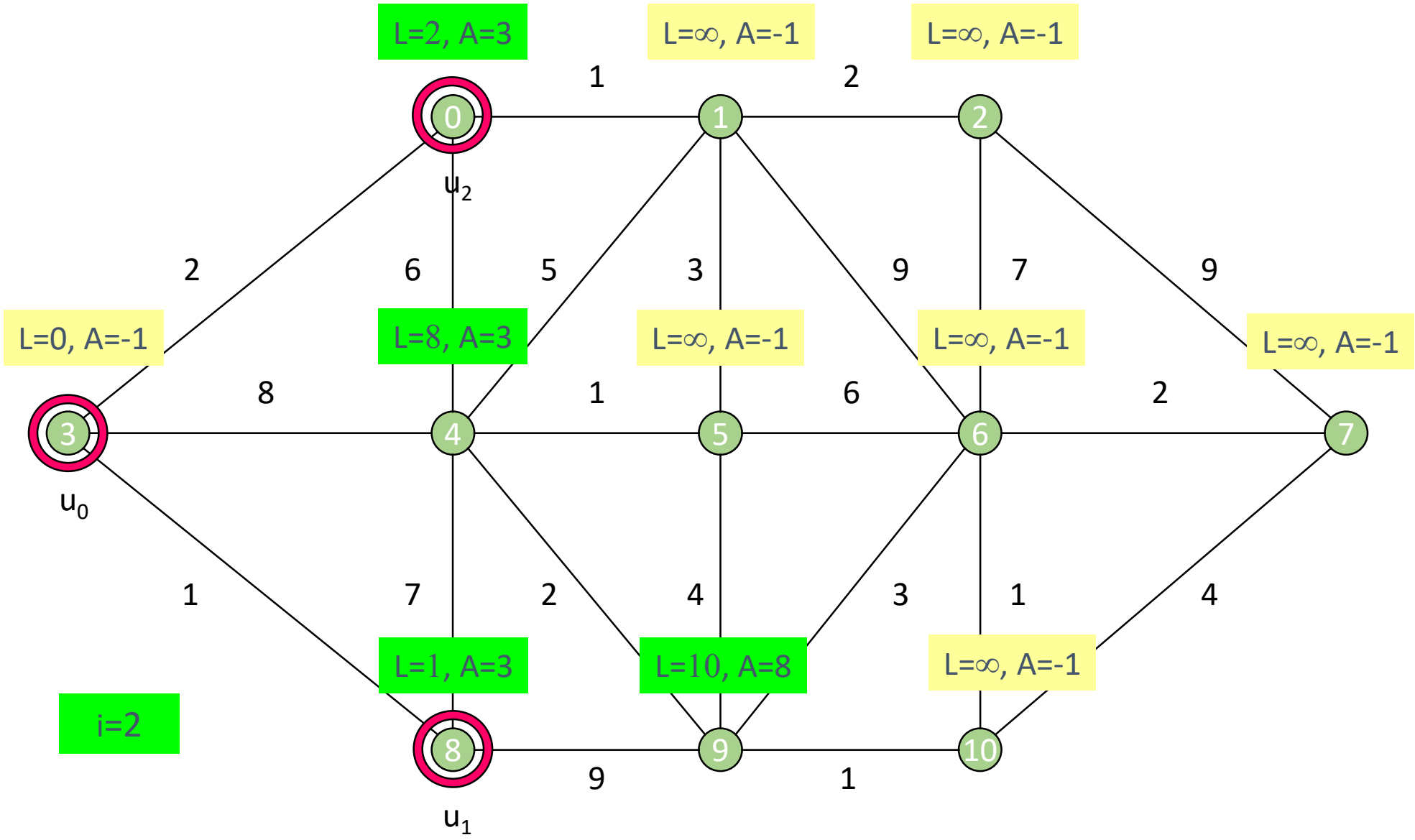
Caminho do vértice 3 até o vértice 7



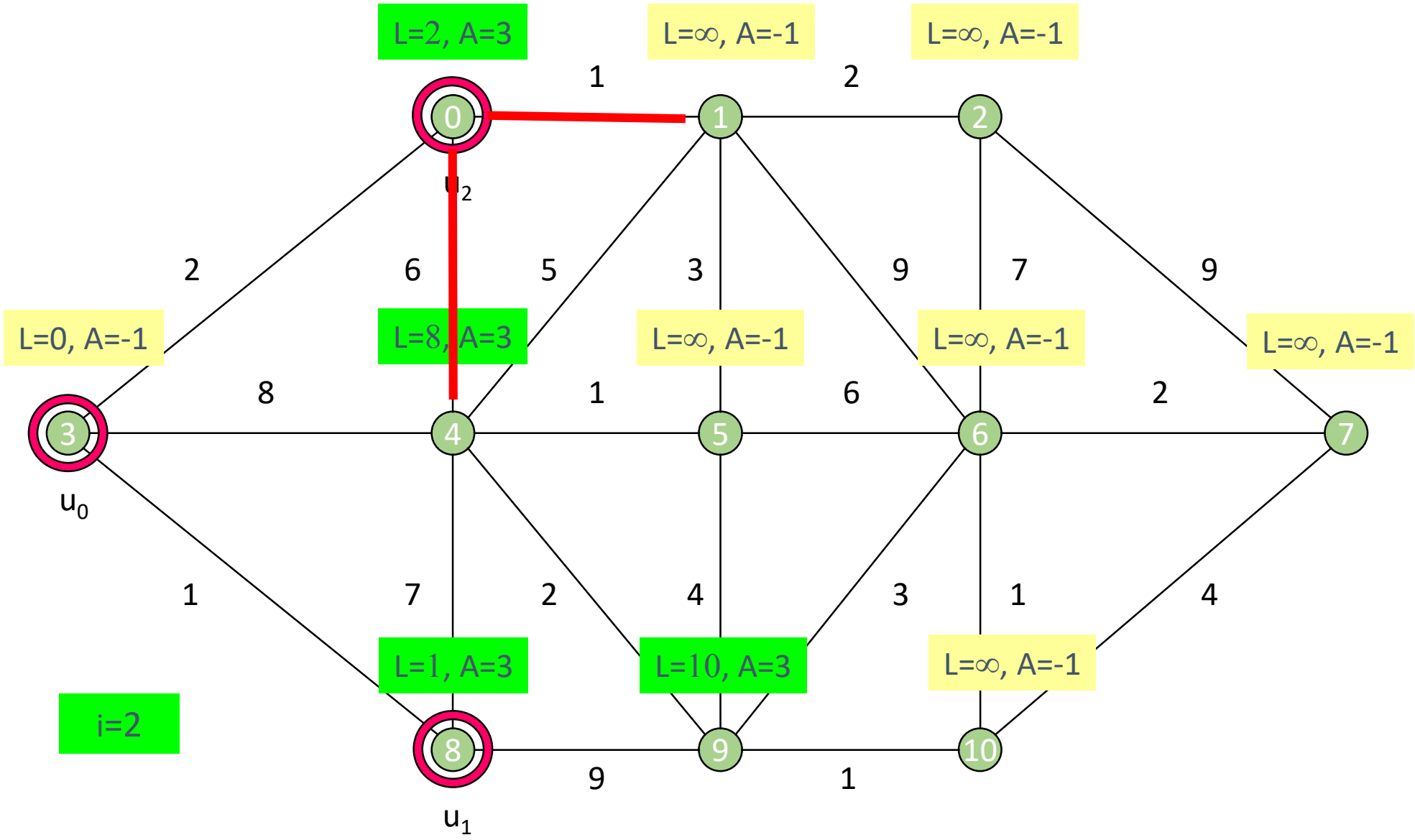
Caminho do vértice 3 até o vértice 7



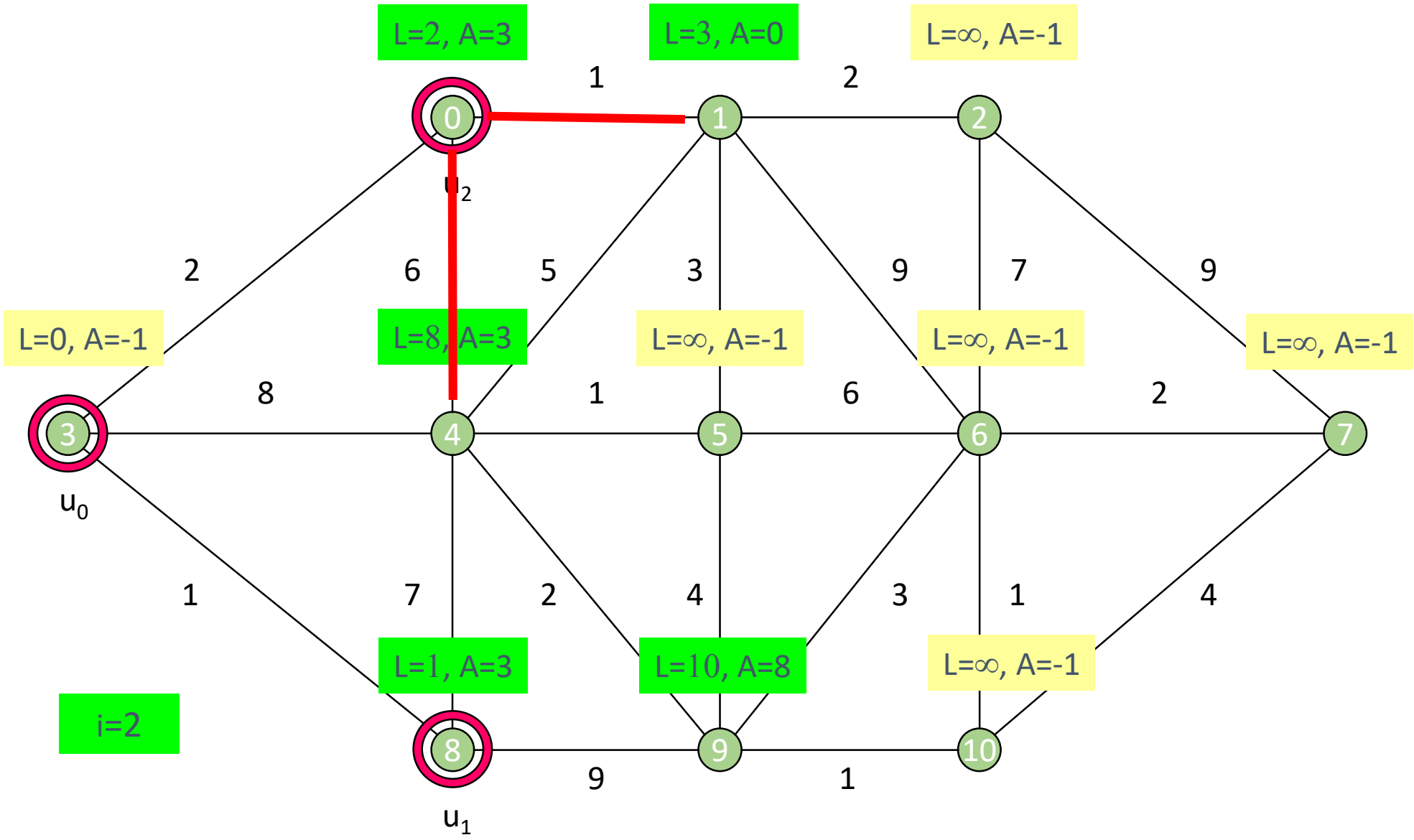
Caminho do vértice 3 até o vértice 7



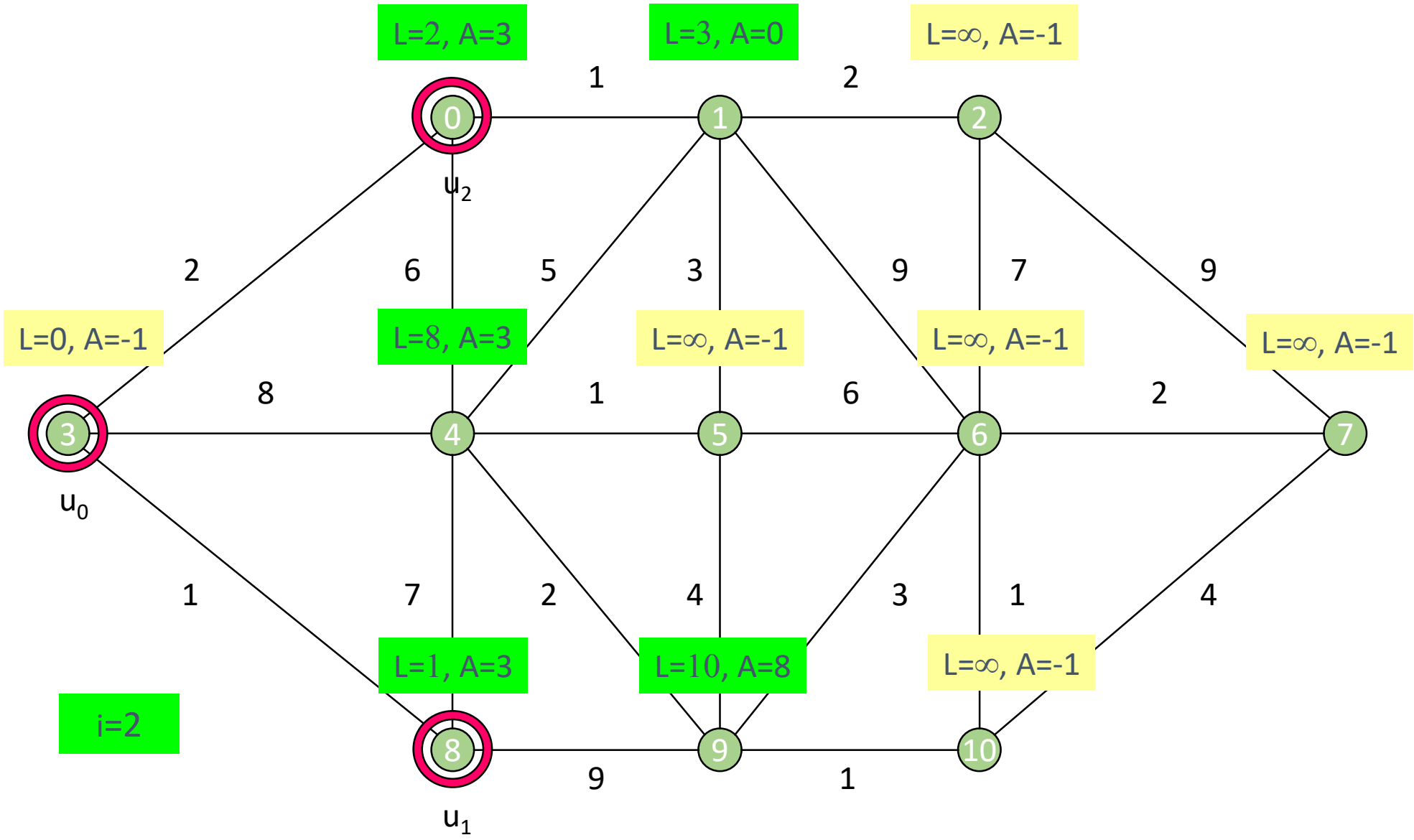
Caminho do vértice 3 até o vértice 7



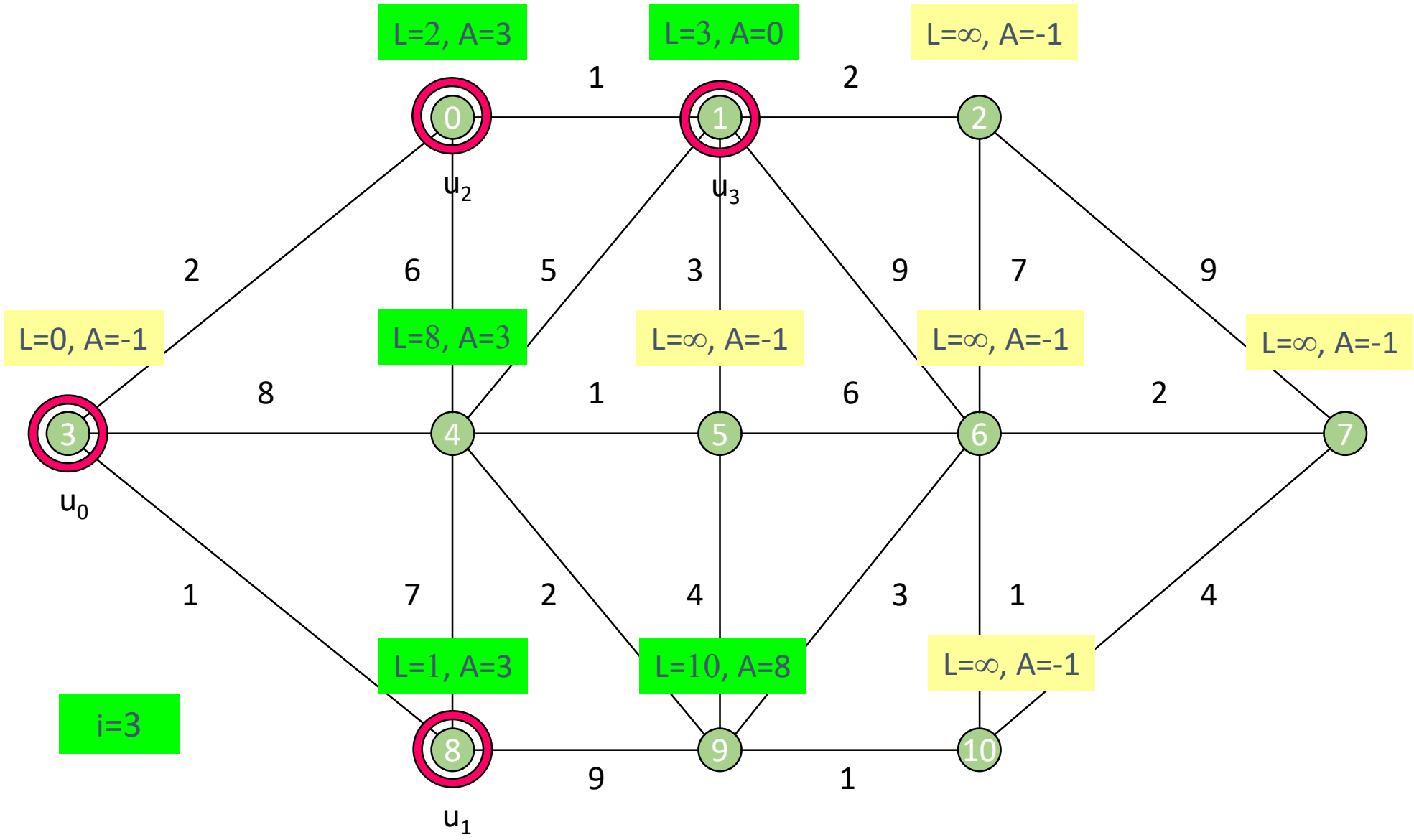
Caminho do vértice 3 até o vértice 7



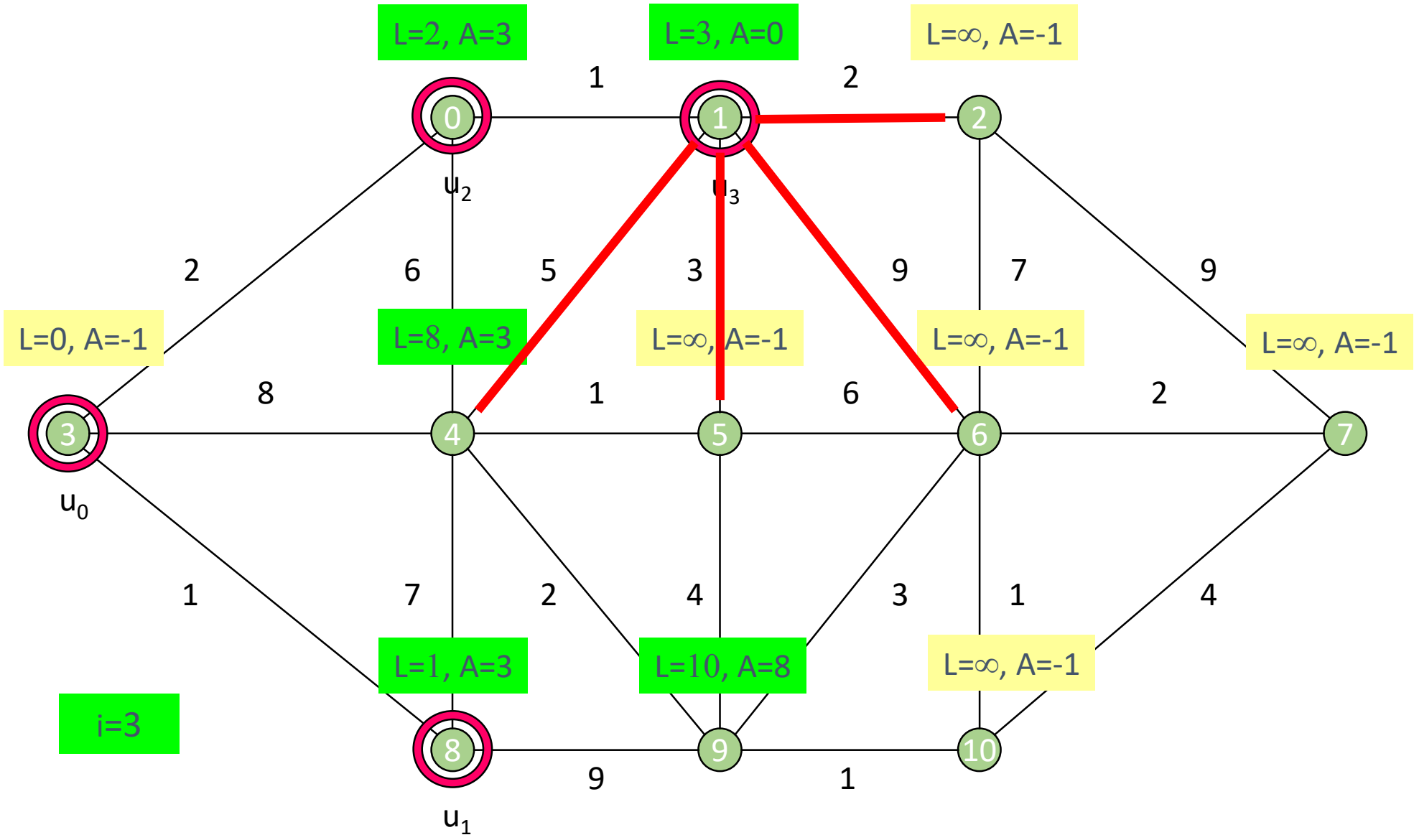
Caminho do vértice 3 até o vértice 7



Caminho do vértice 3 até o vértice 7



Caminho do vértice 3 até o vértice 7



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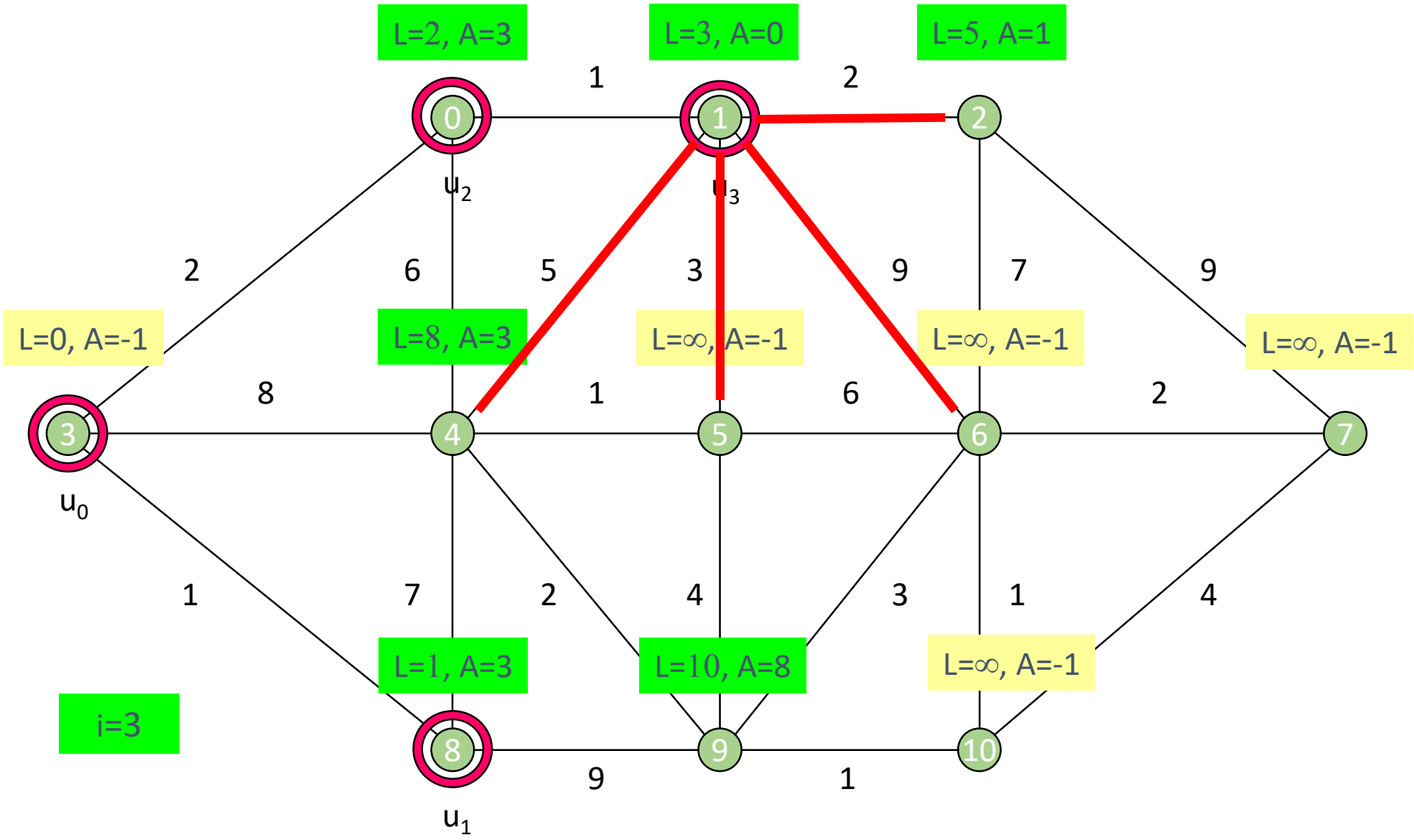

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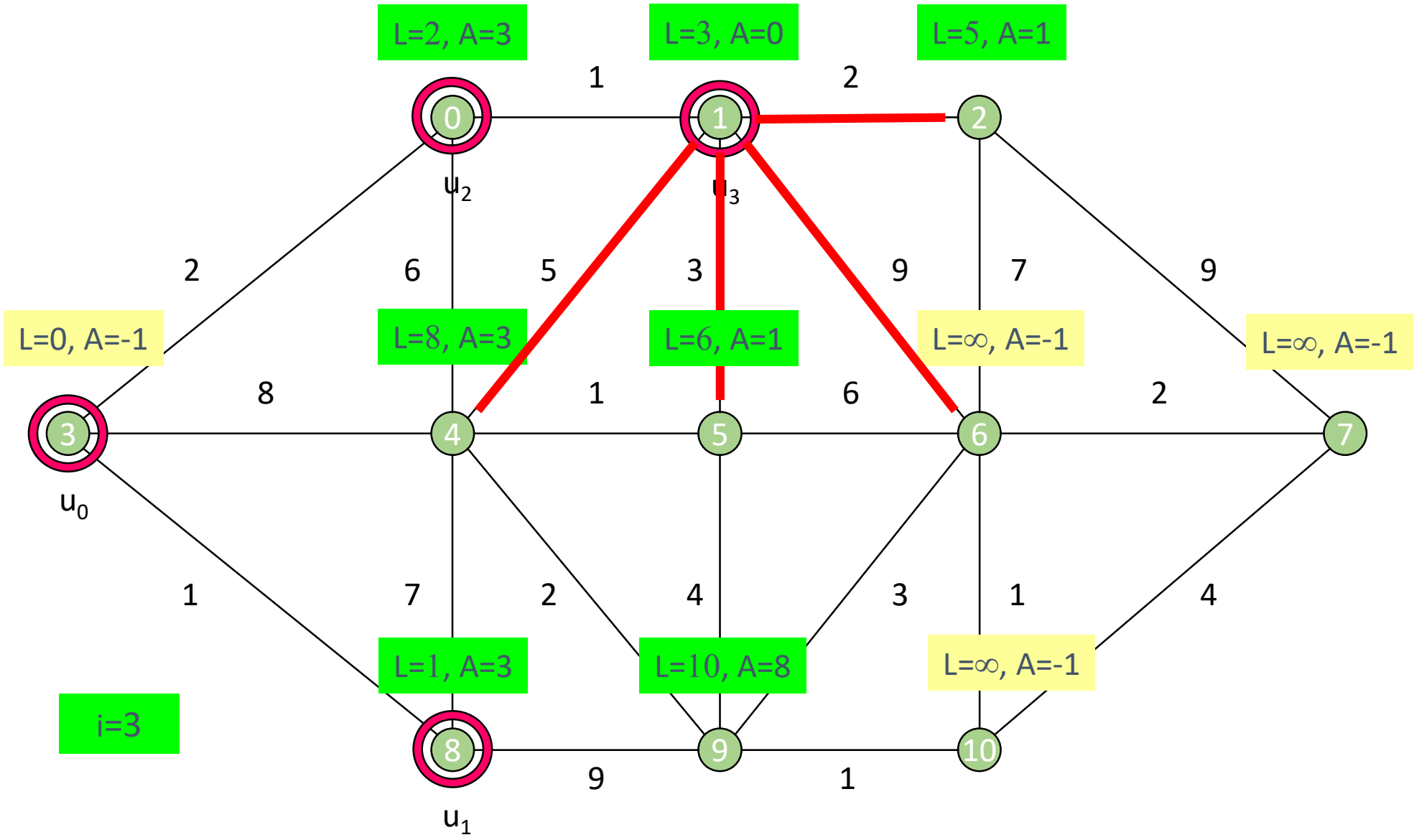
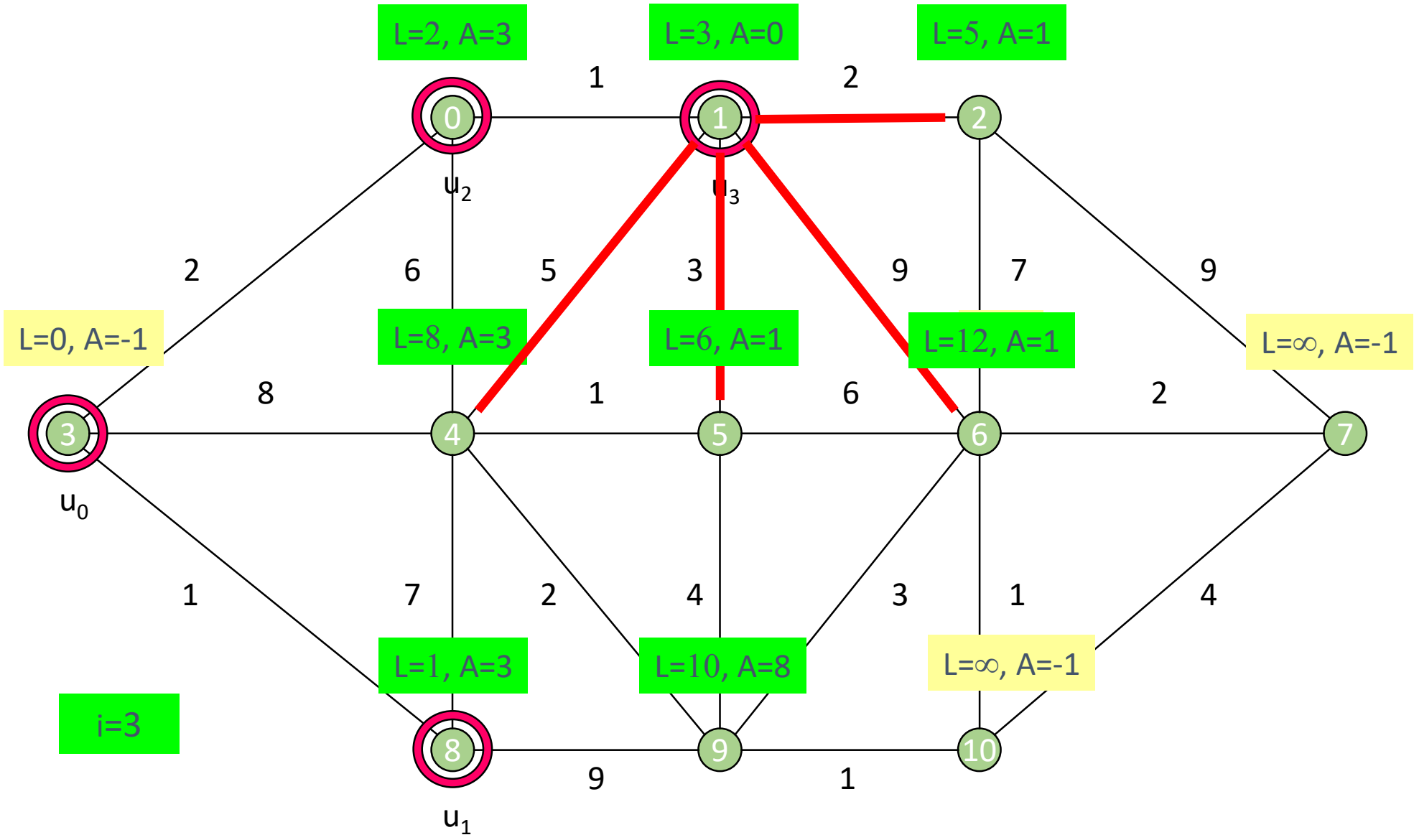
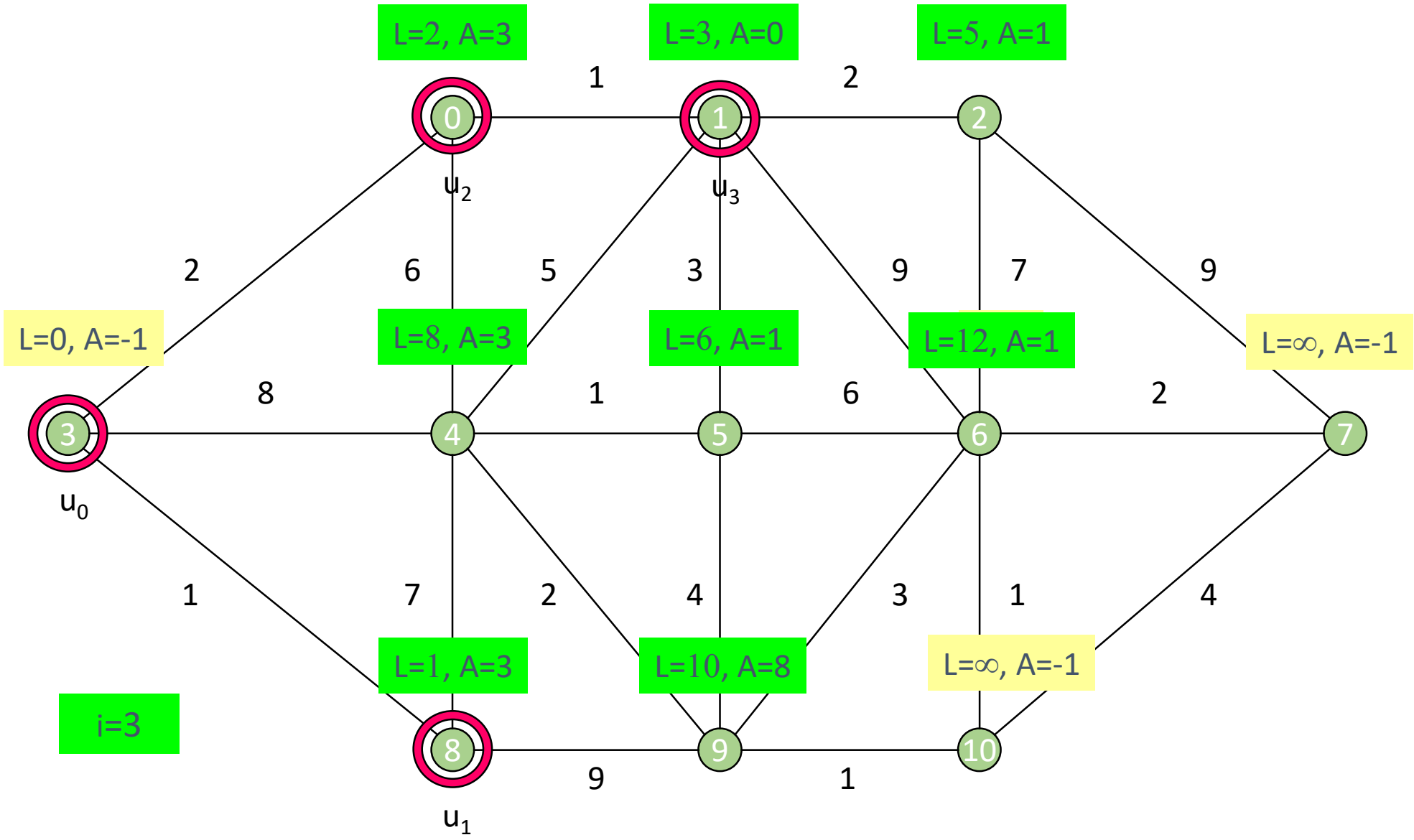


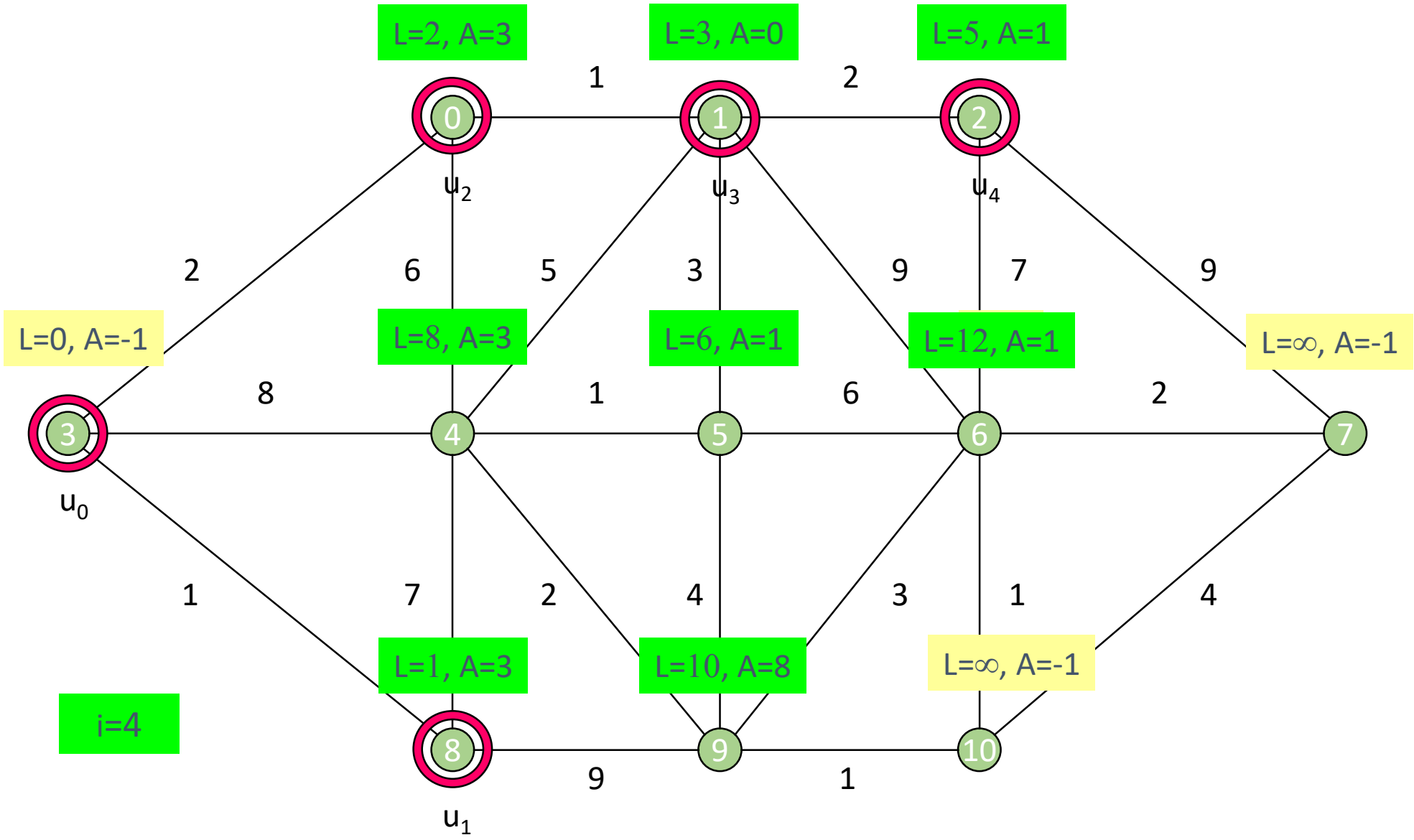
Caminho do vértice 3 até o vértice 7



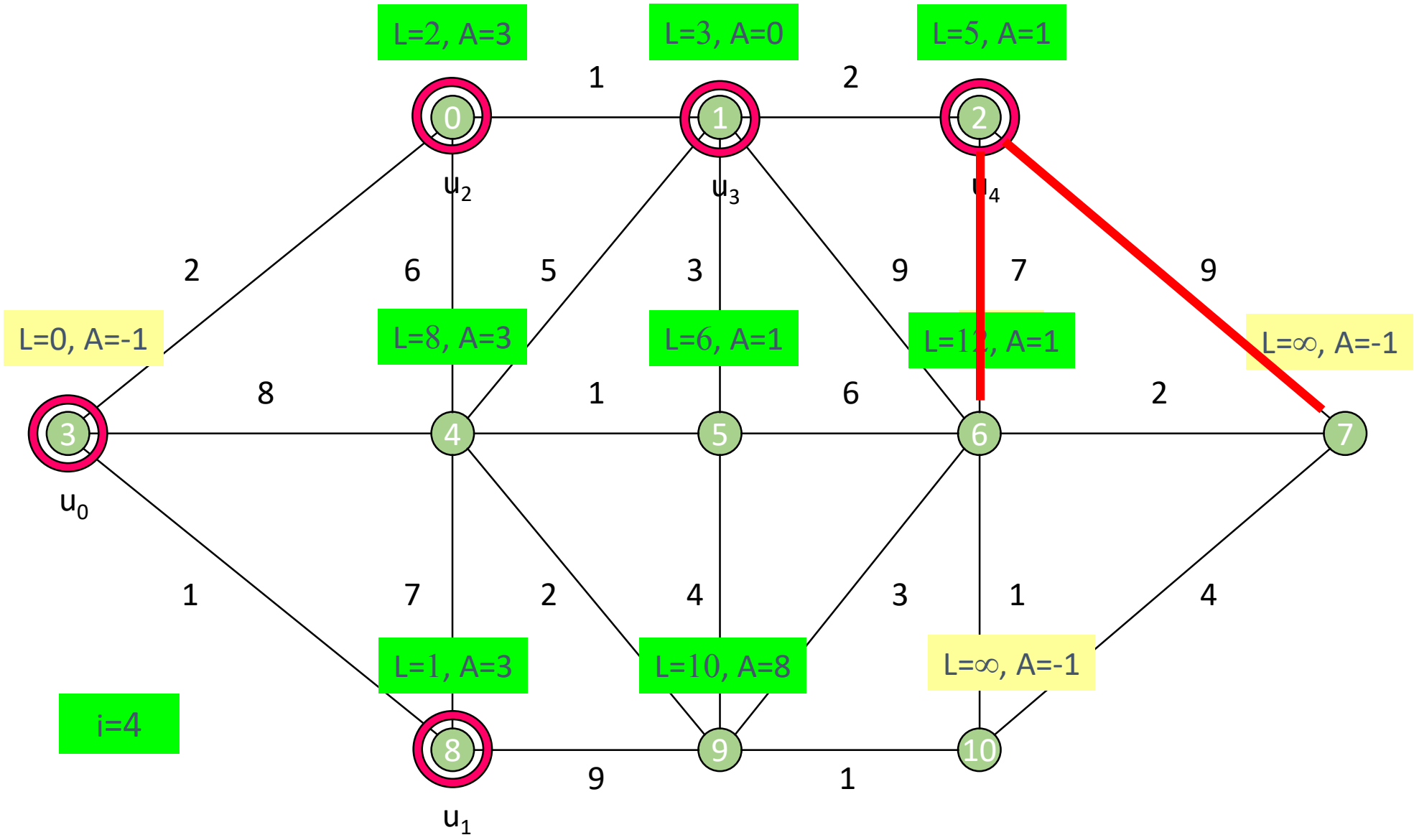
Caminho do vértice 3 até o vértice 7



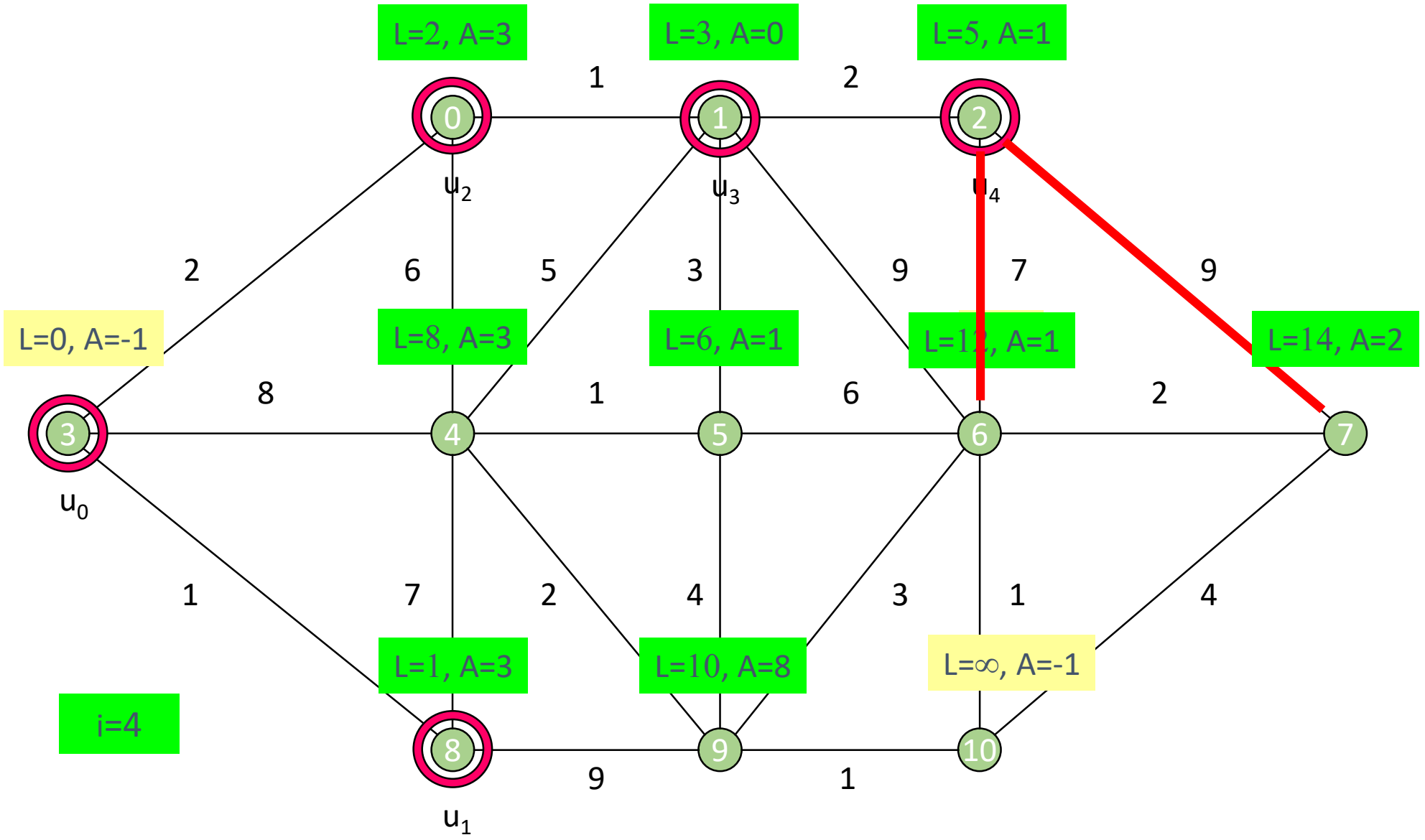
Caminho do vértice 3 até o vértice 7



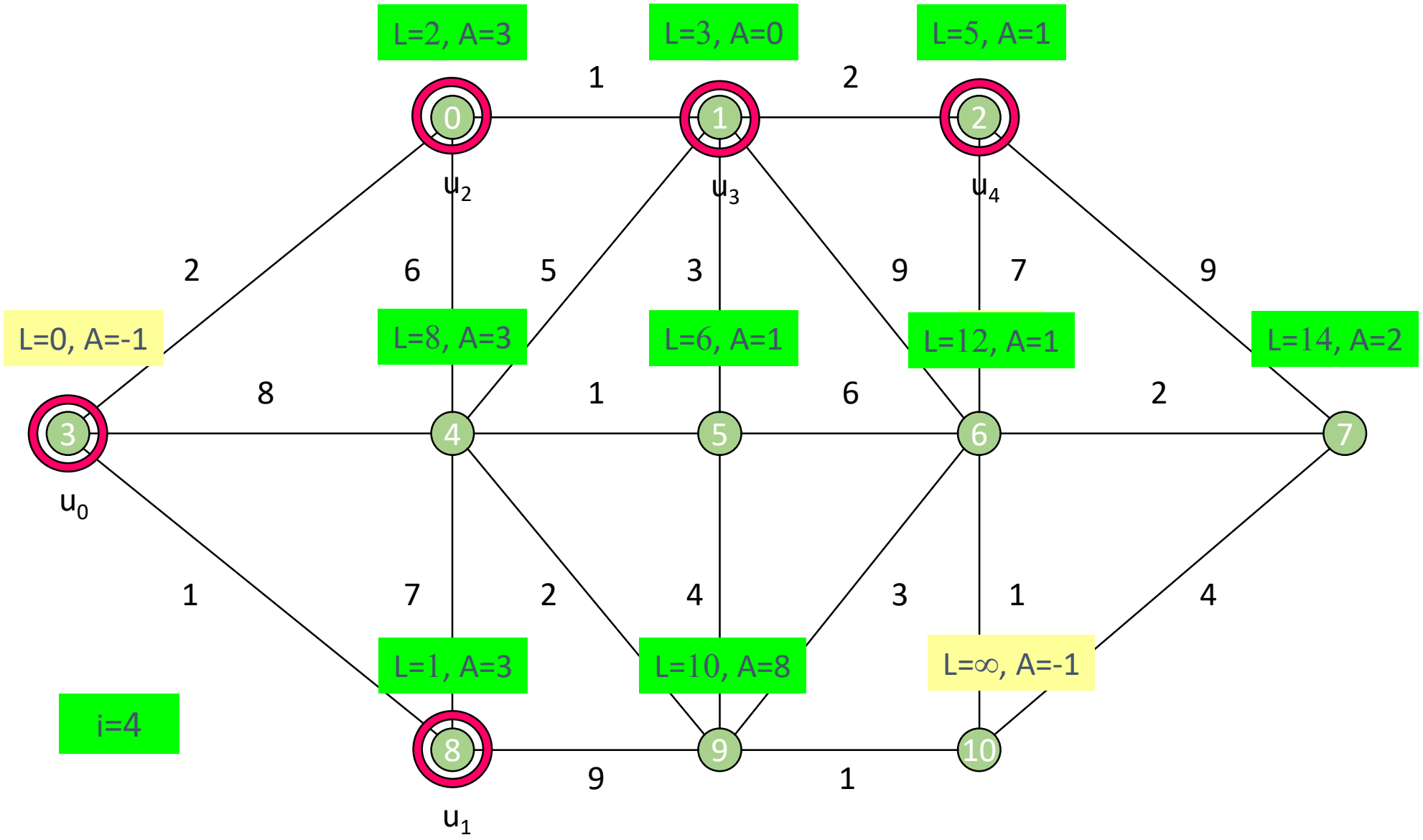
Caminho do vértice 3 até o vértice 7



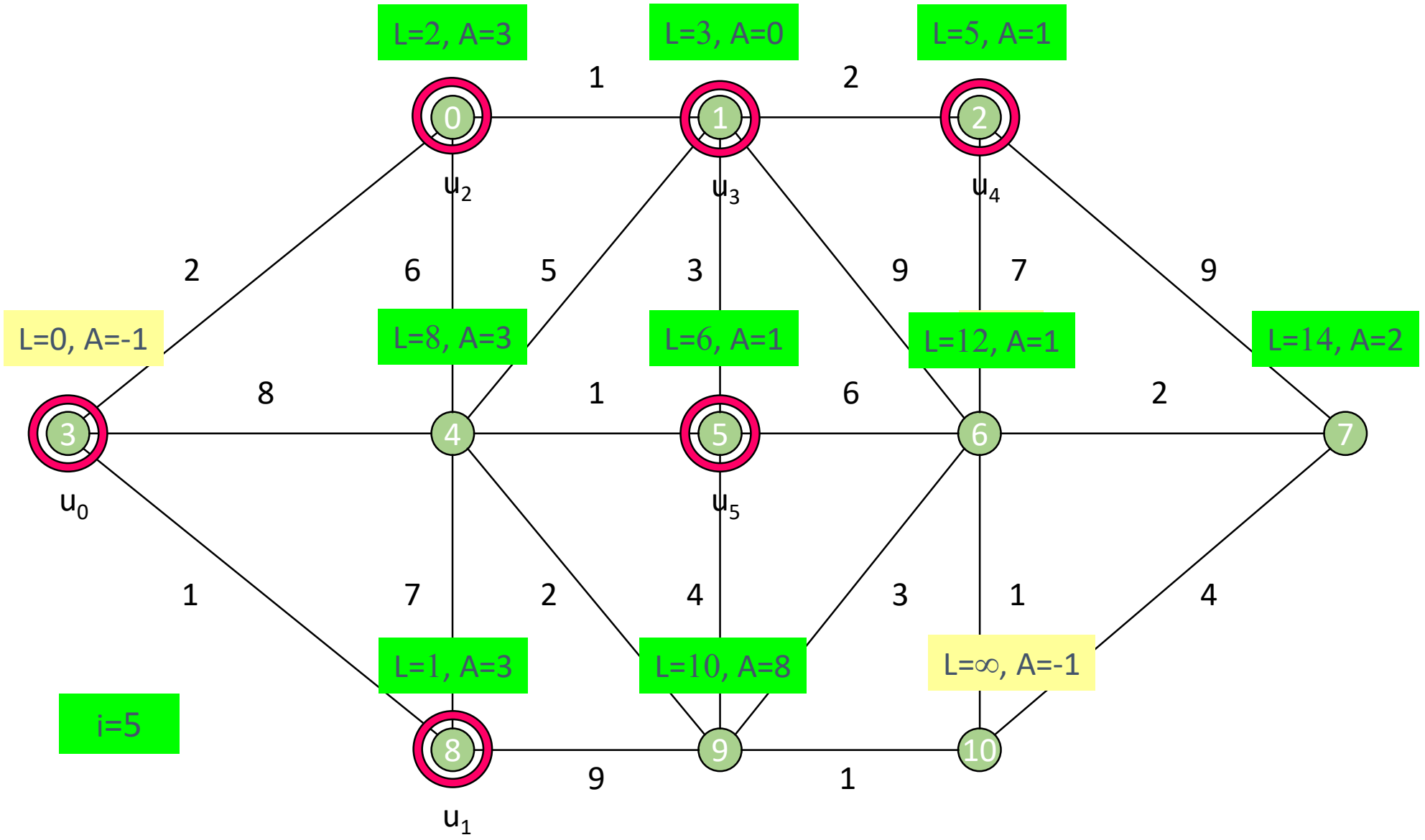
Caminho do vértice 3 até o vértice 7



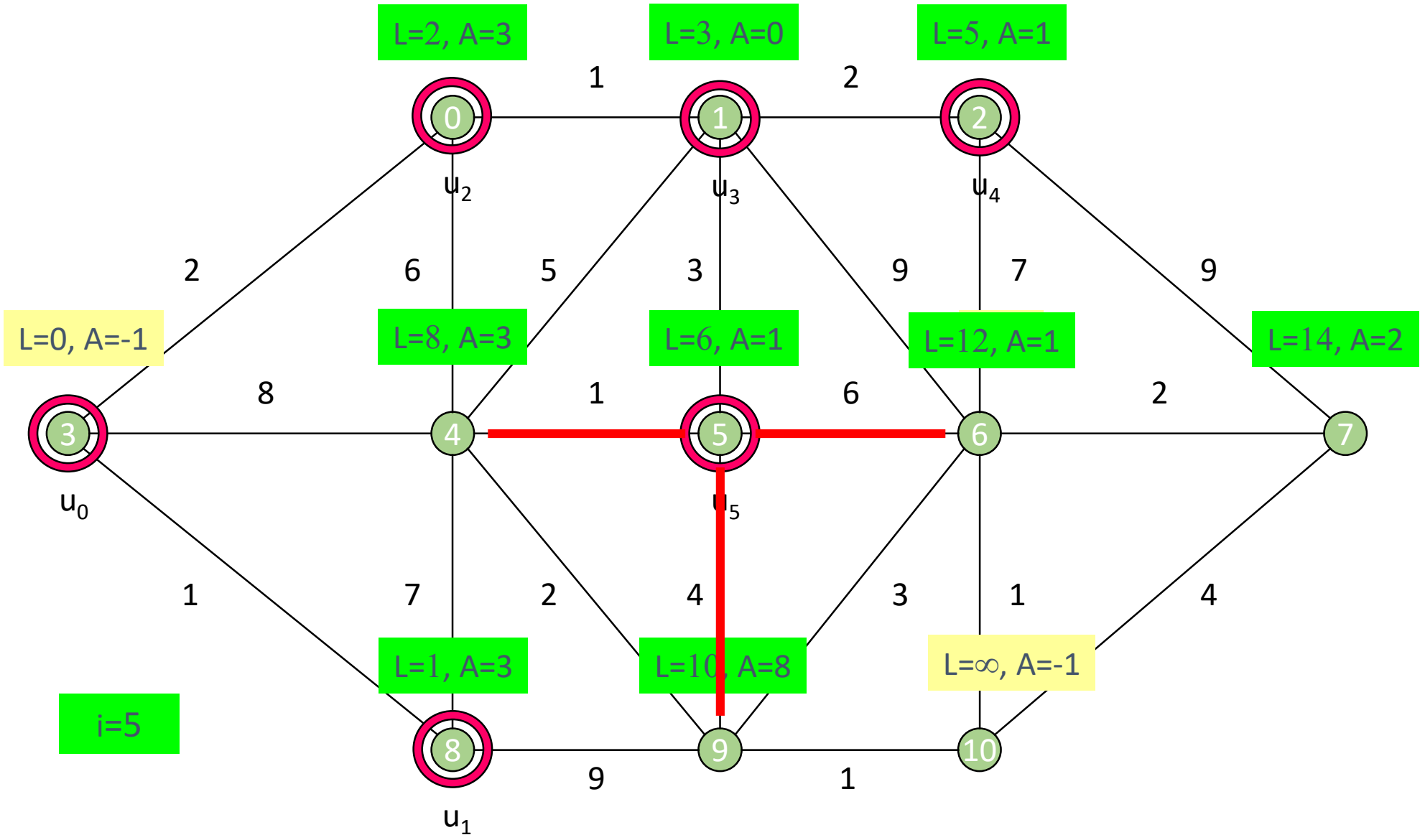
Caminho do vértice 3 até o vértice 7



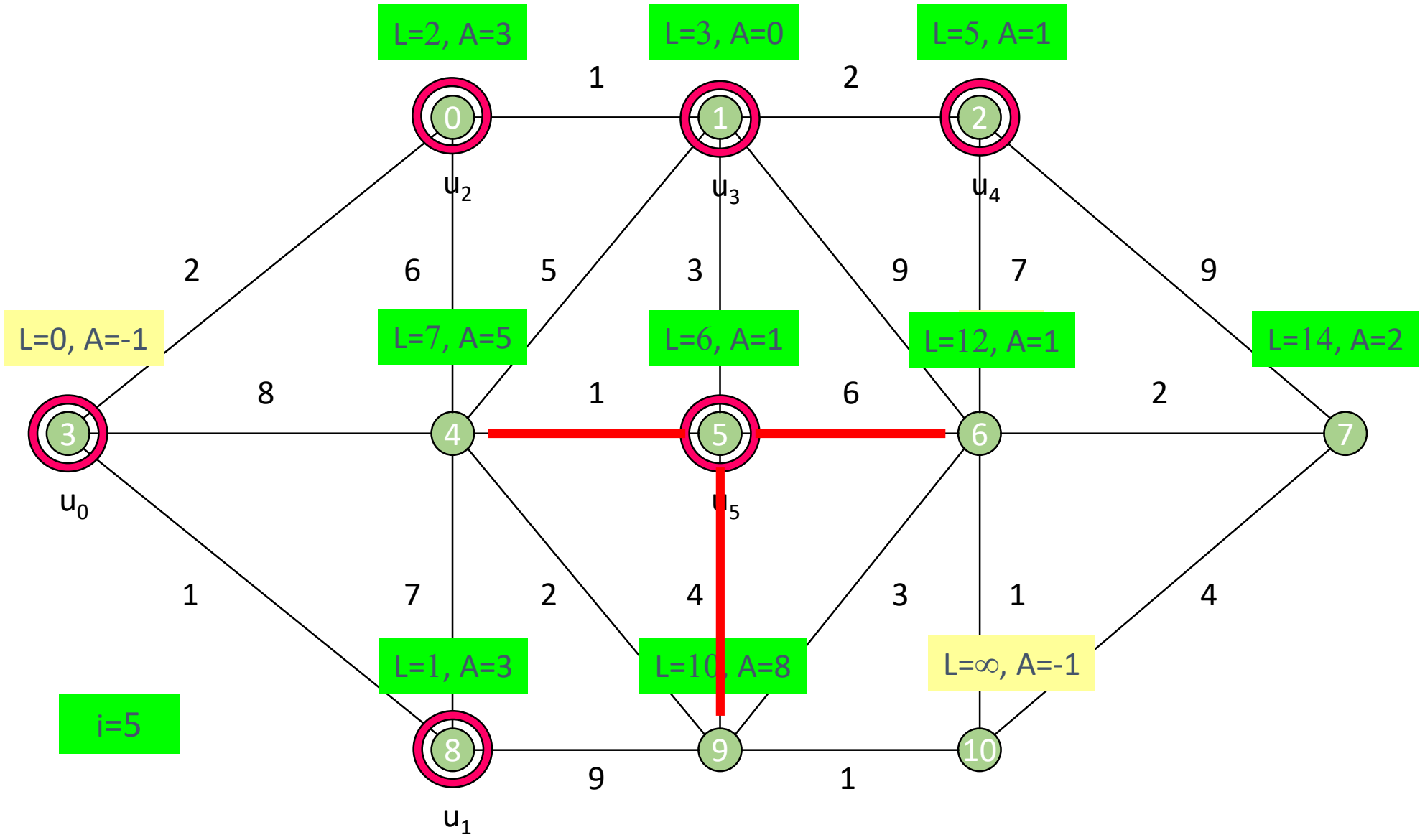
Caminho do vértice 3 até o vértice 7



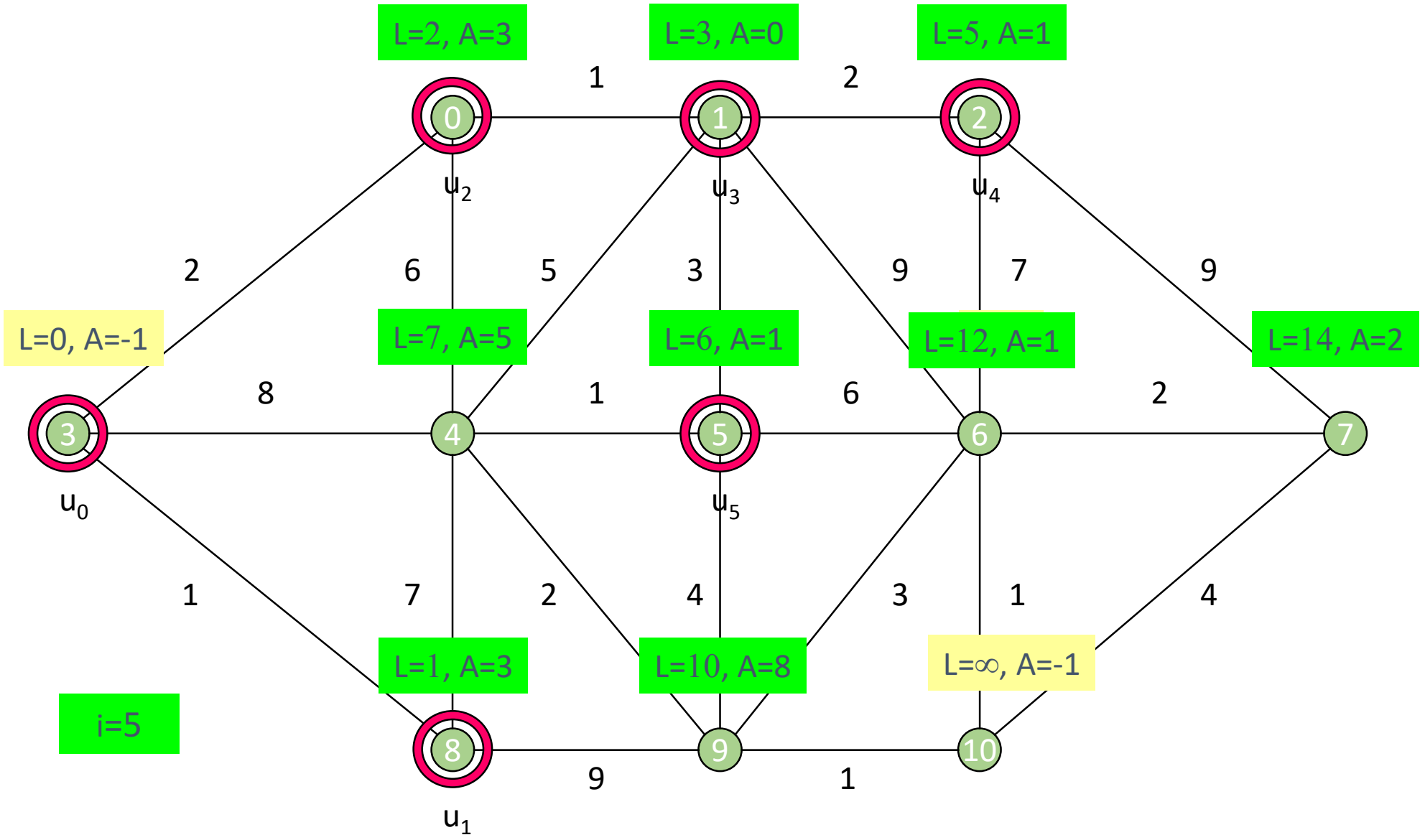
Caminho do vértice 3 até o vértice 7



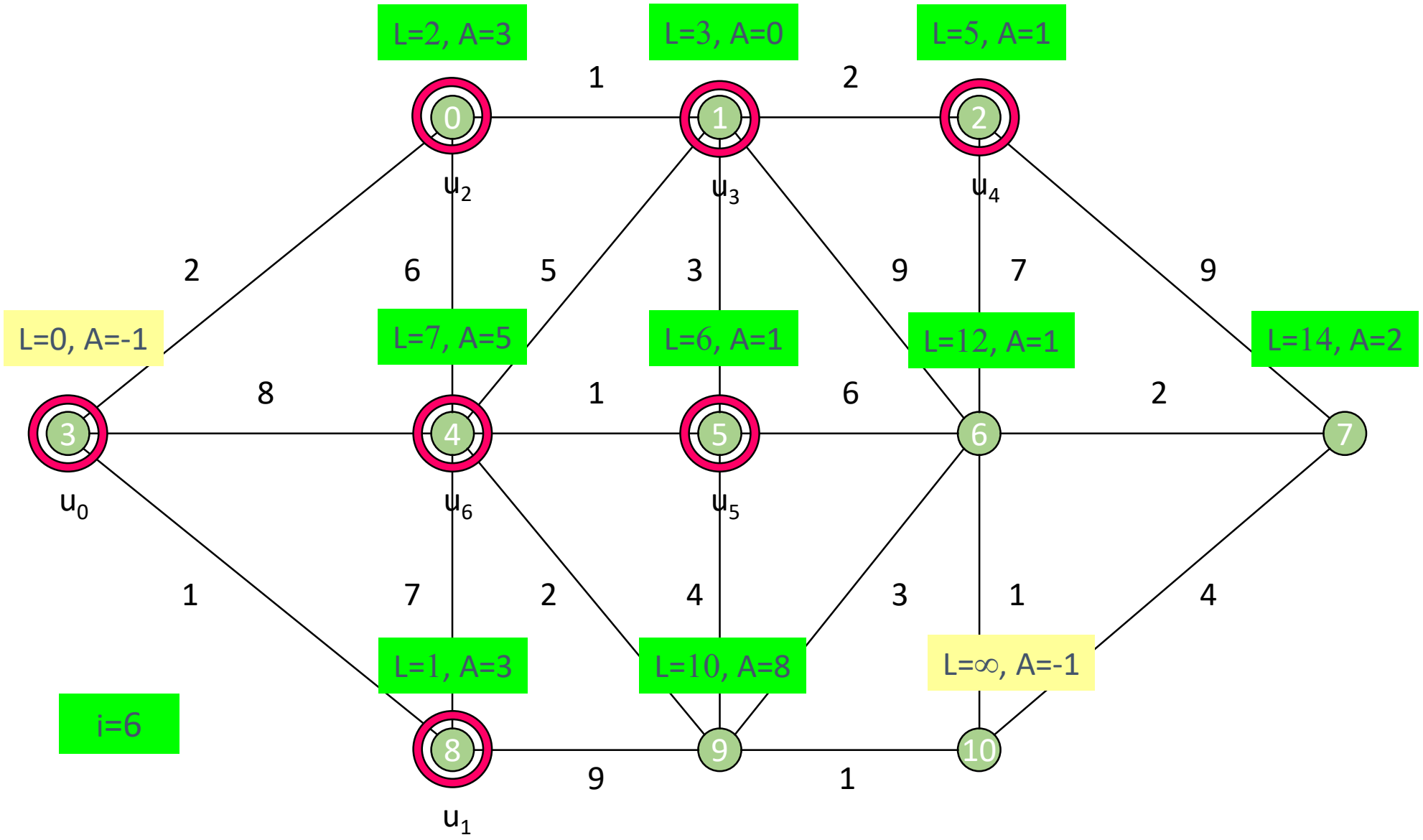
Caminho do vértice 3 até o vértice 7



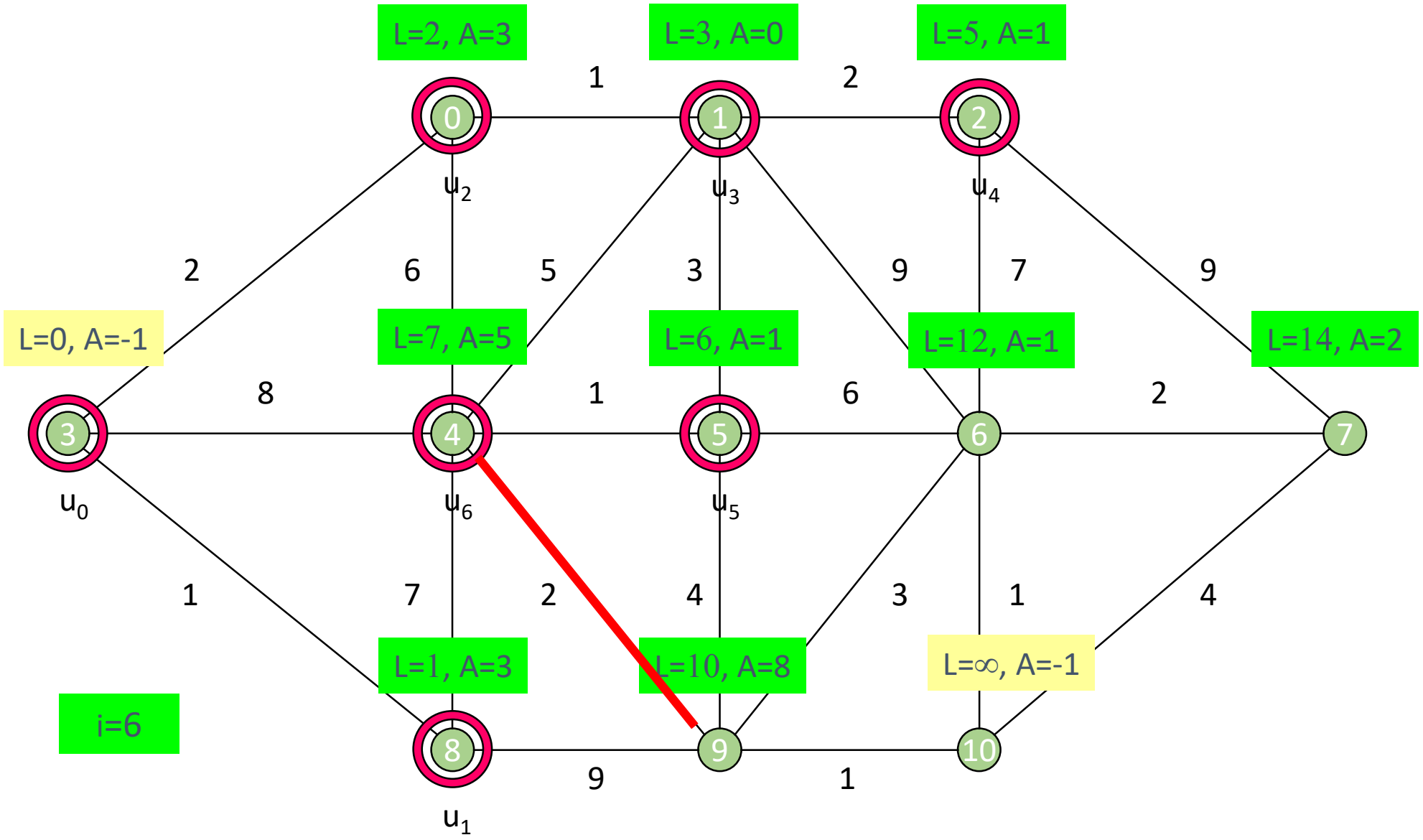
Caminho do vértice 3 até o vértice 7



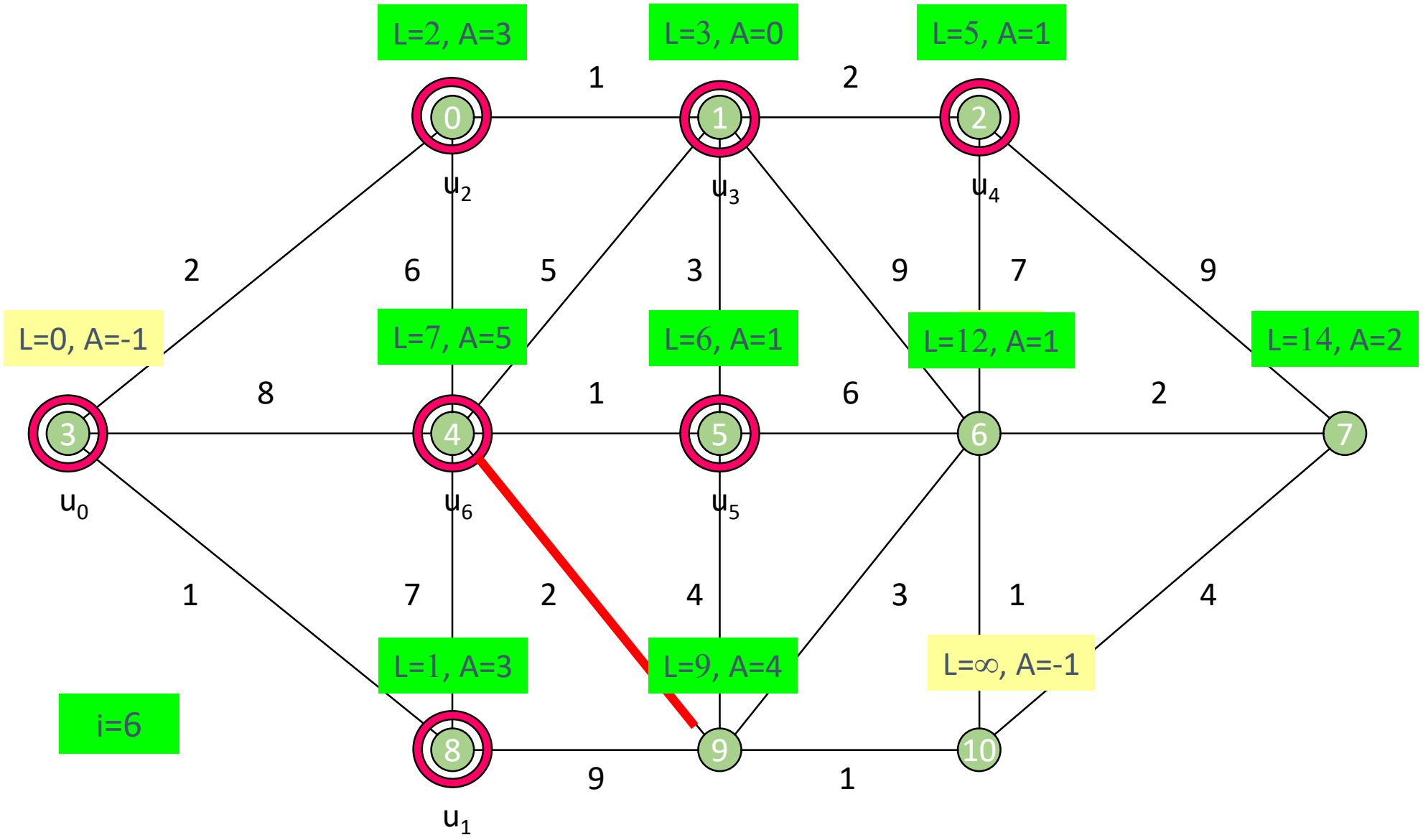
Caminho do vértice 3 até o vértice 7



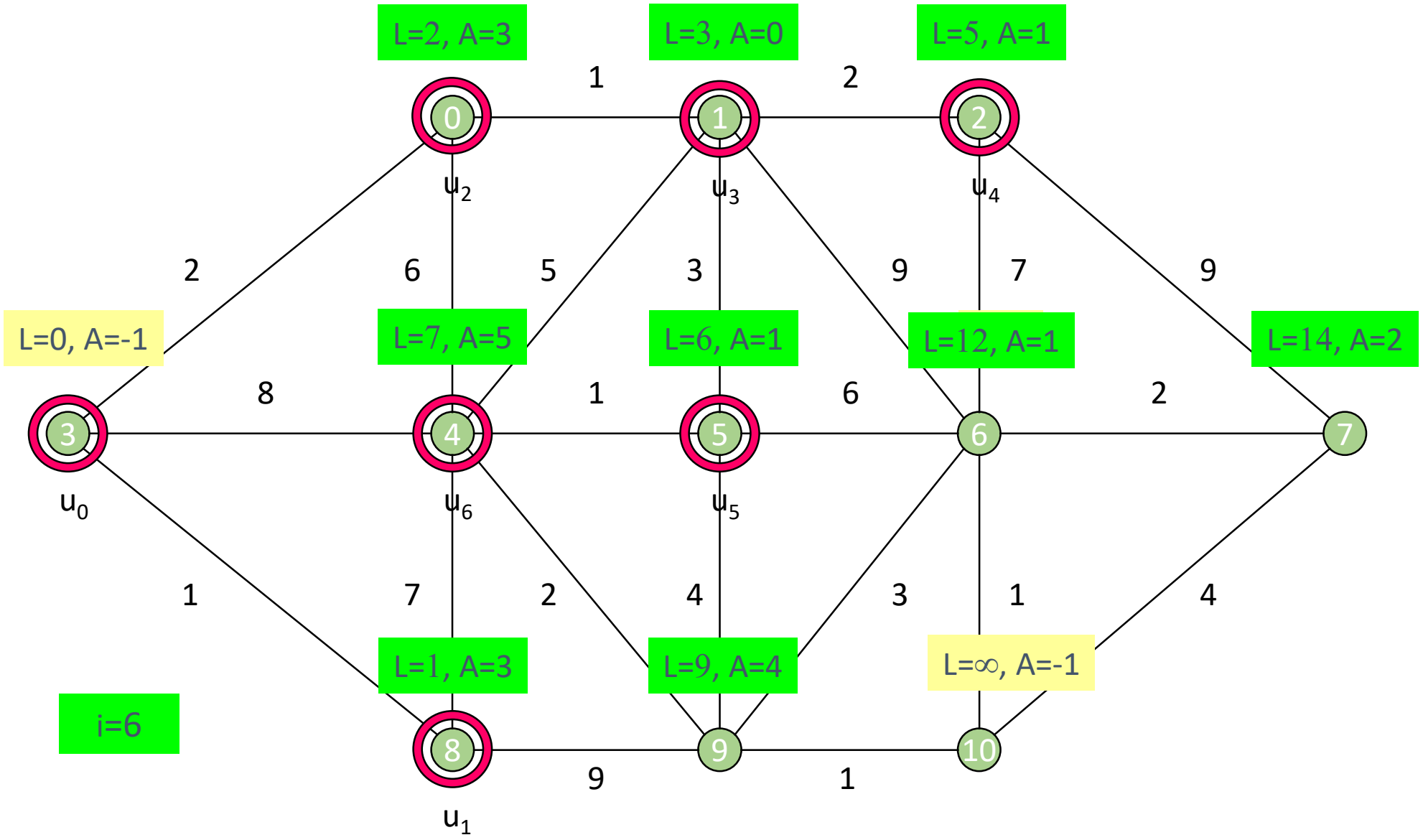
Caminho do vértice 3 até o vértice 7



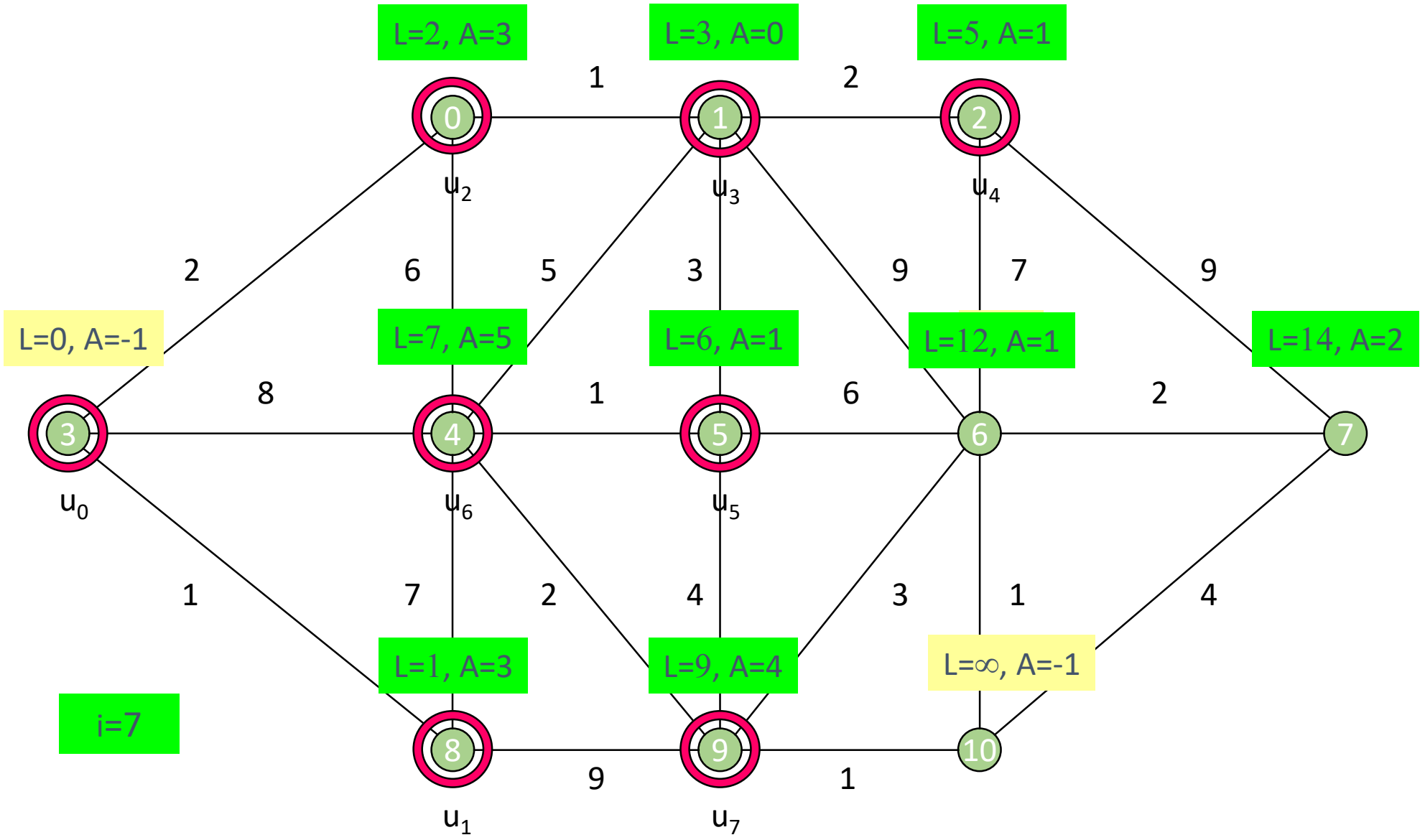
Caminho do vértice 3 até o vértice 7



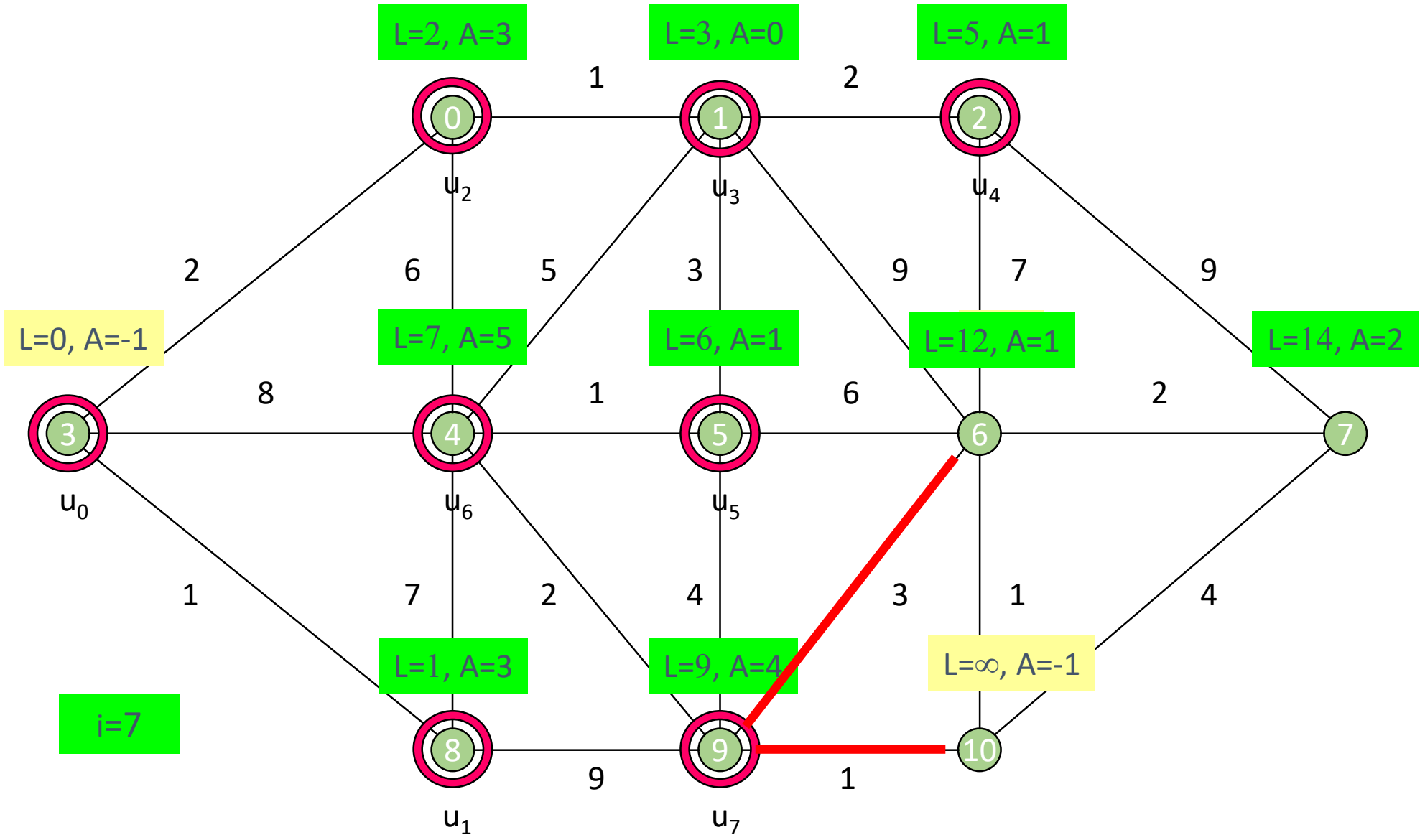
Caminho do vértice 3 até o vértice 7



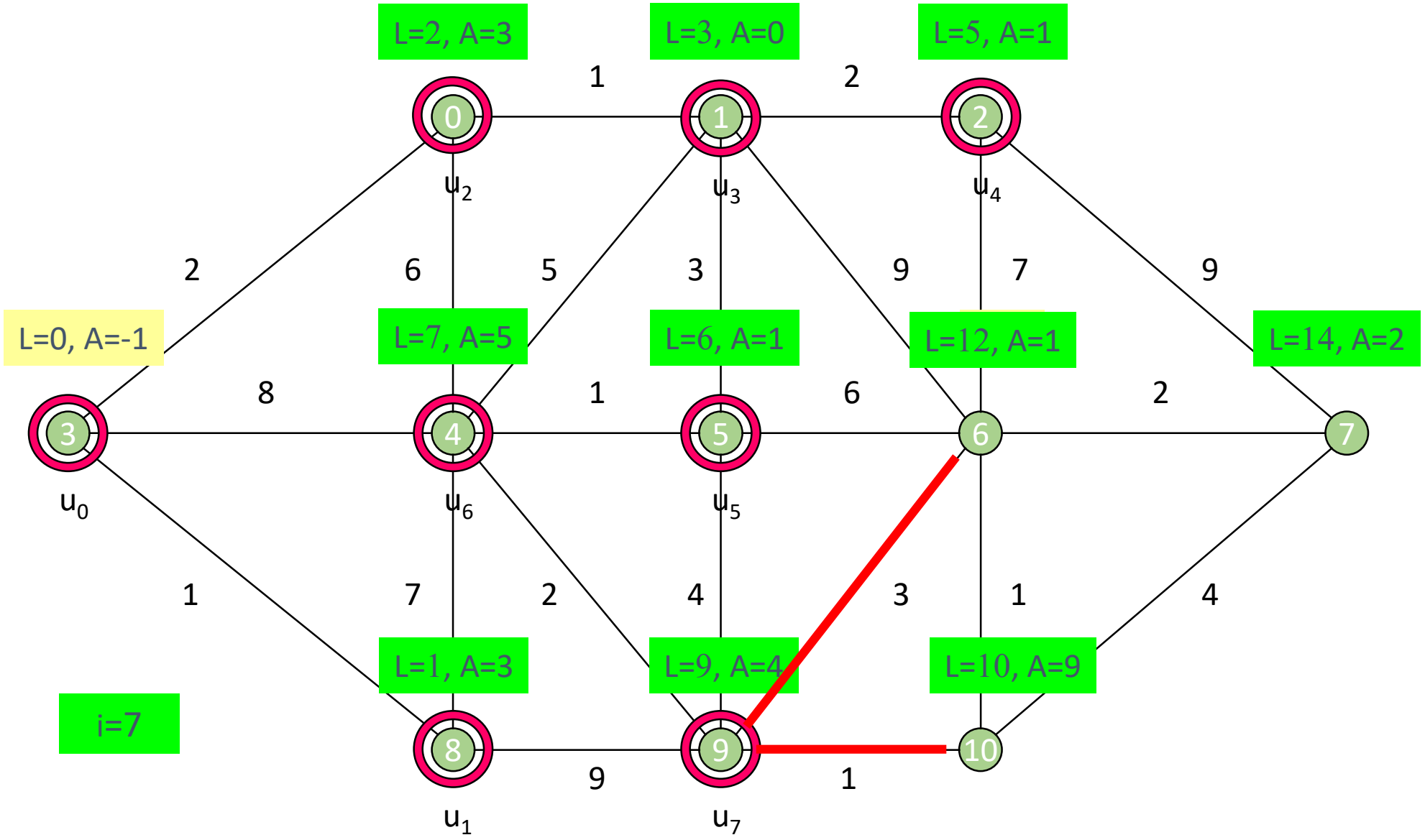
Caminho do vértice 3 até o vértice 7



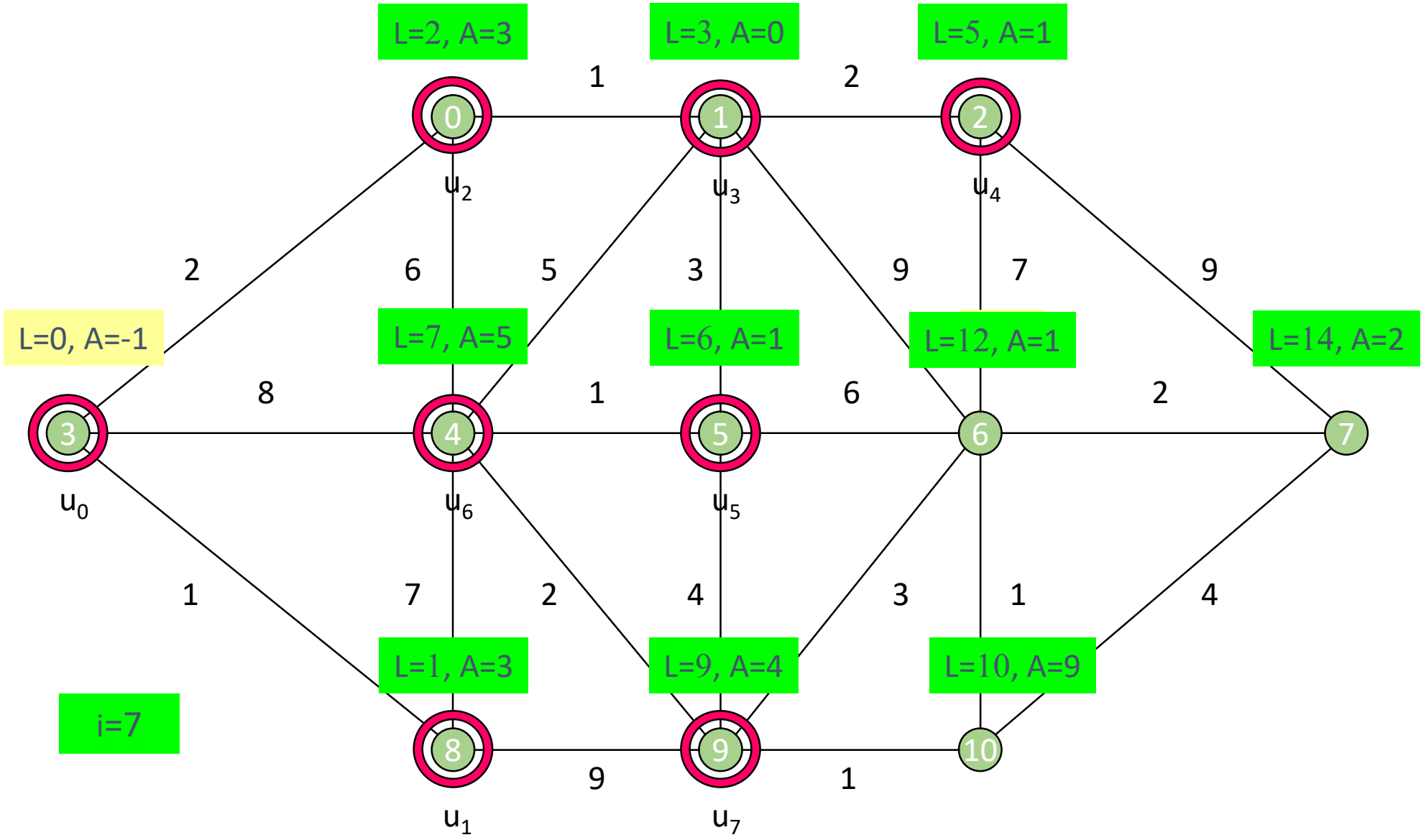
Caminho do vértice 3 até o vértice 7



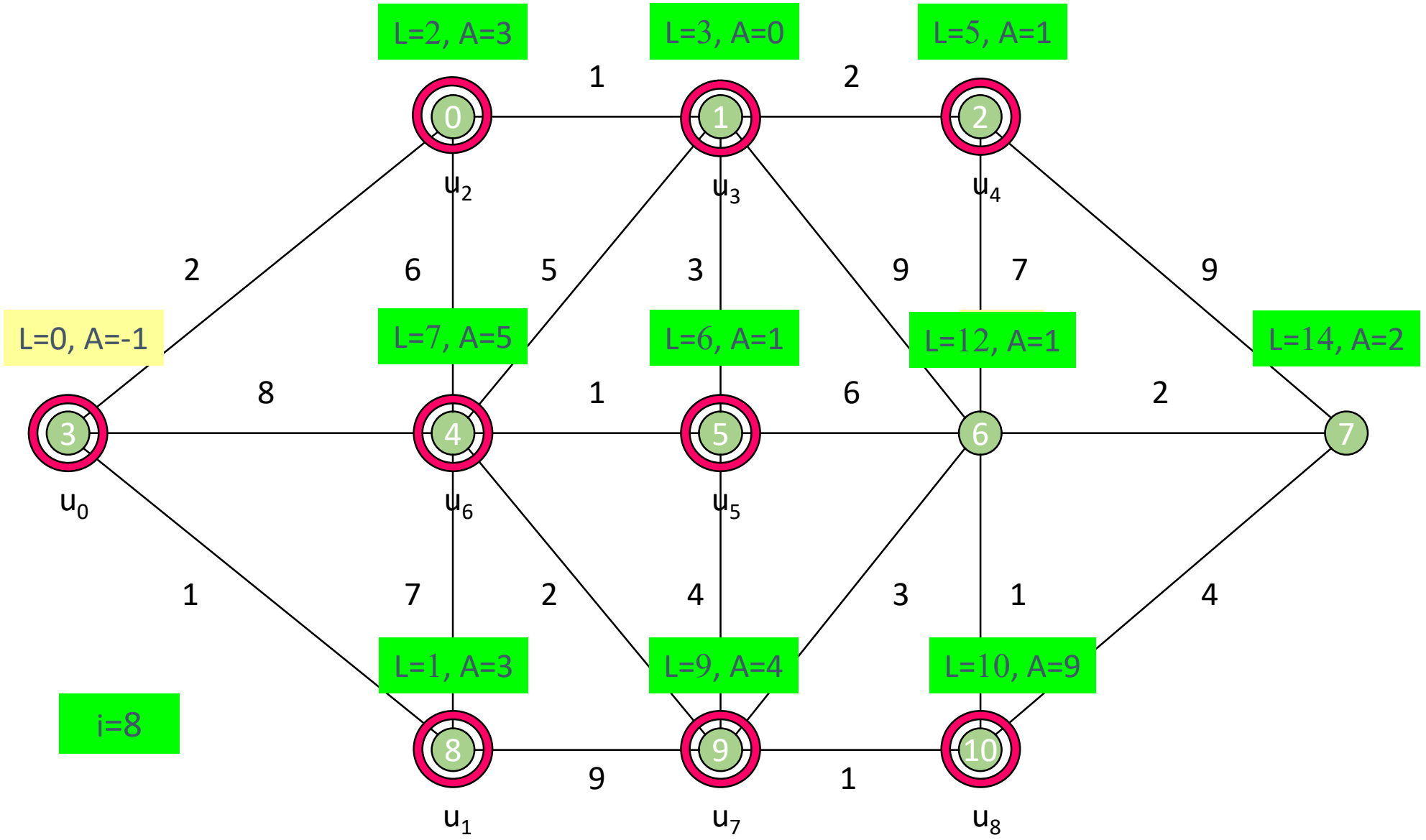
Caminho do vértice 3 até o vértice 7



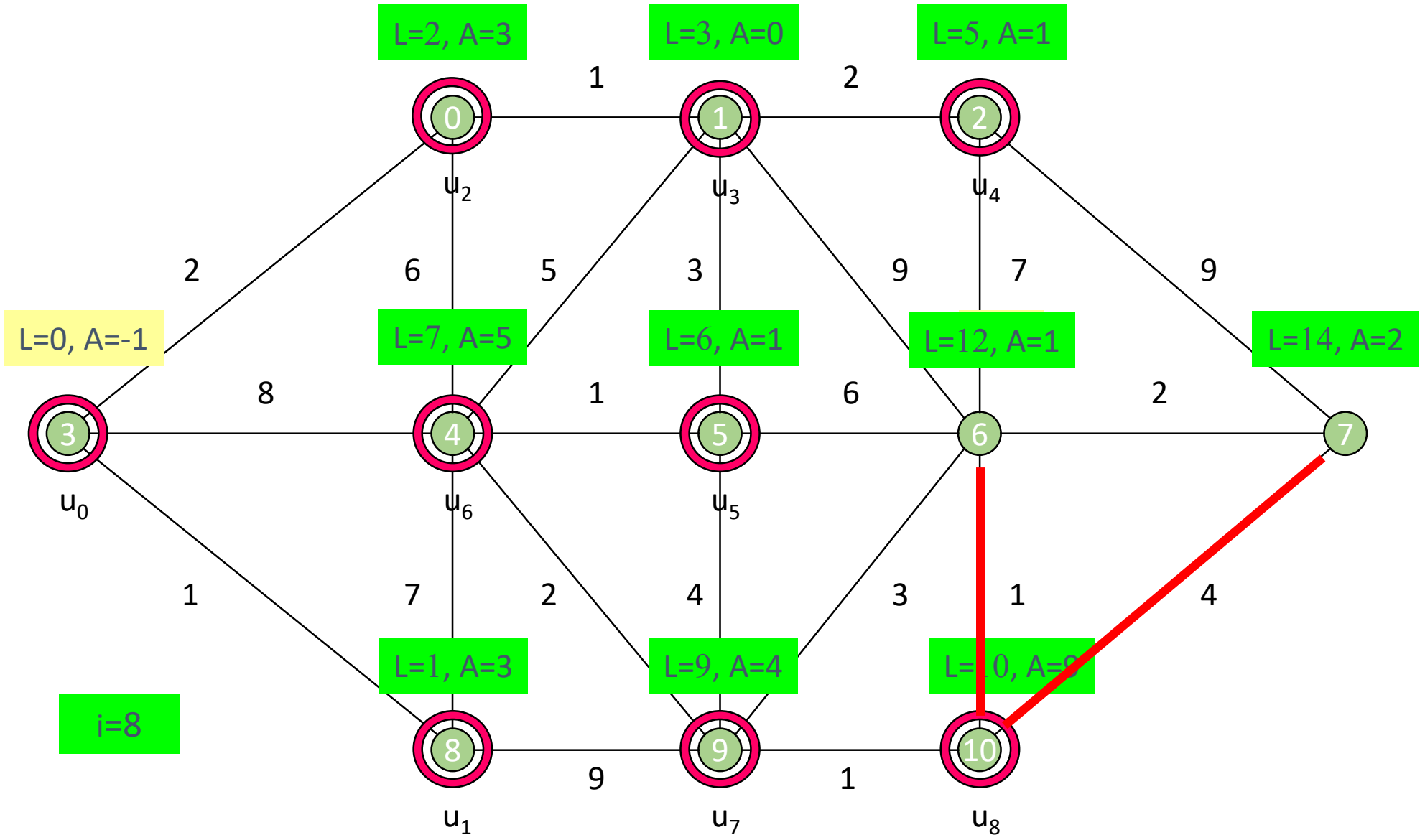
Caminho do vértice 3 até o vértice 7



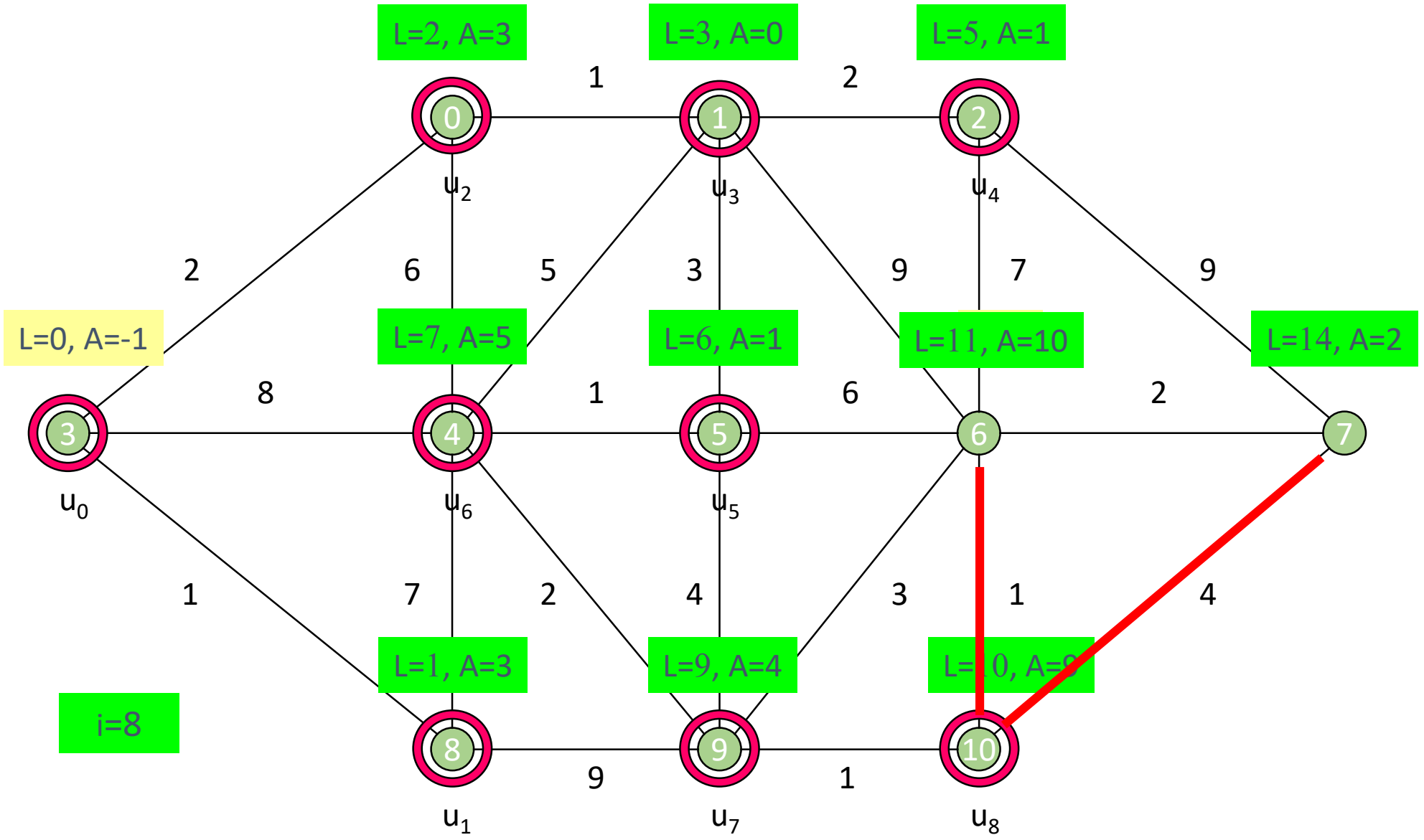
Caminho do vértice 3 até o vértice 7



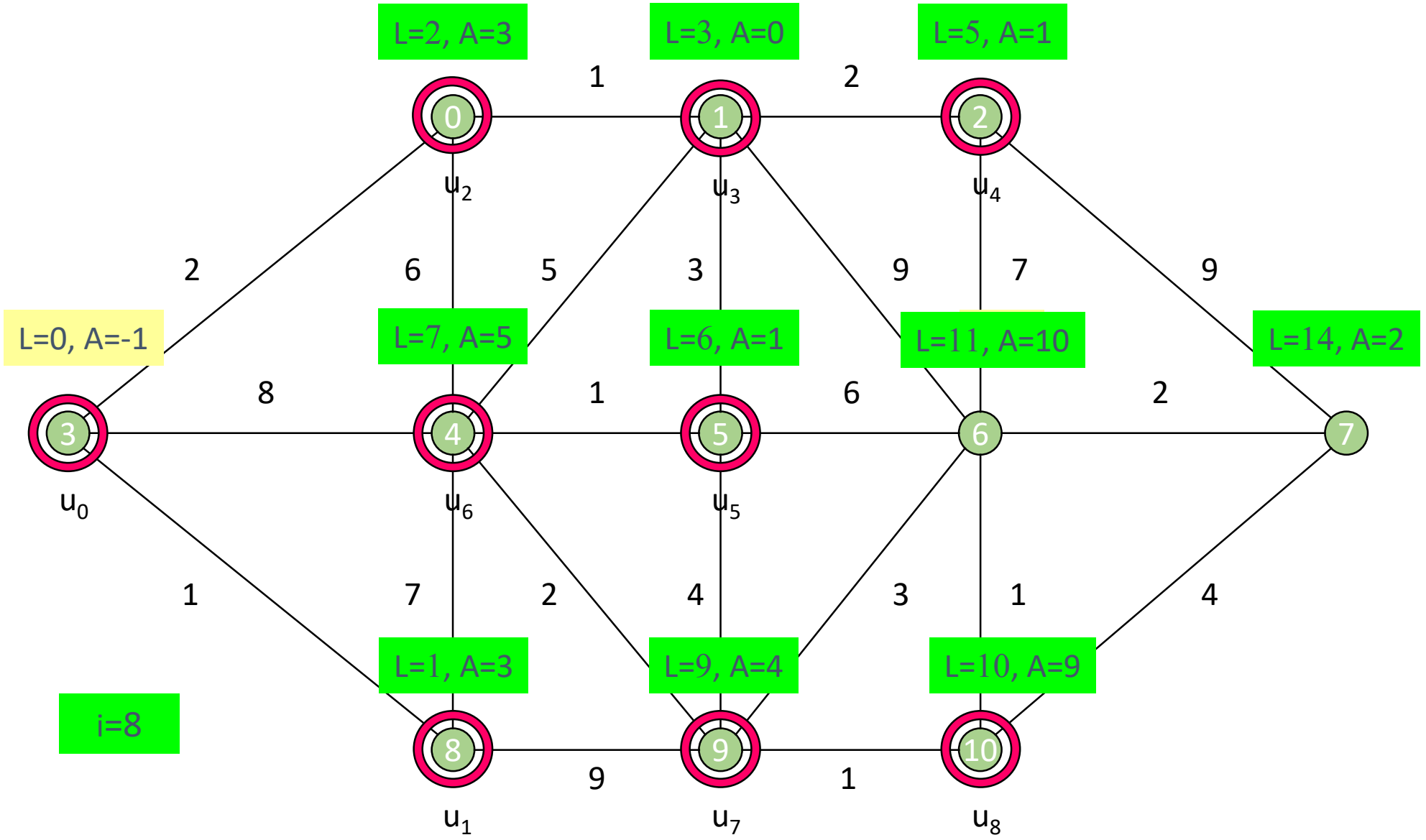
Caminho do vértice 3 até o vértice 7



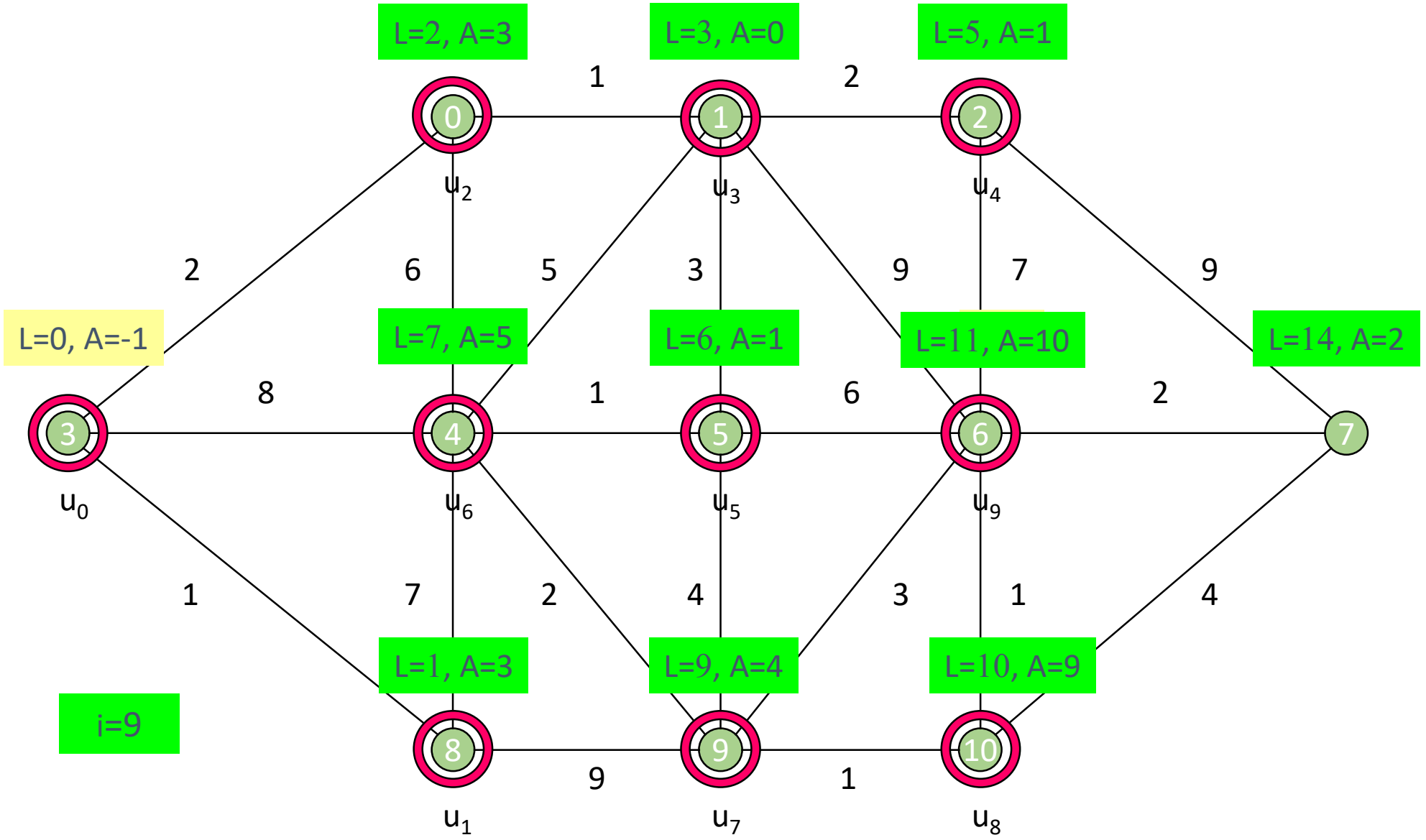
Caminho do vértice 3 até o vértice 7



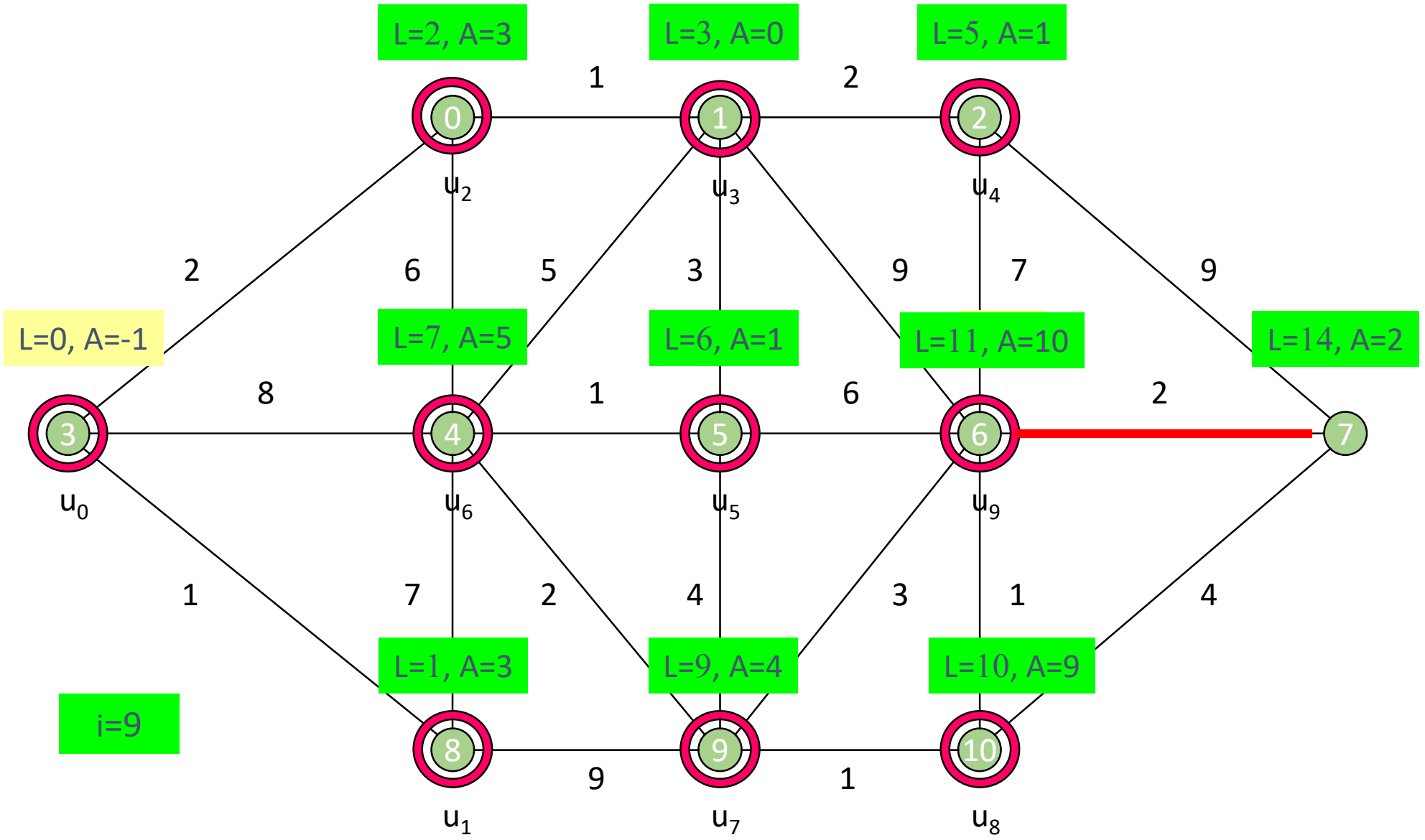
Caminho do vértice 3 até o vértice 7



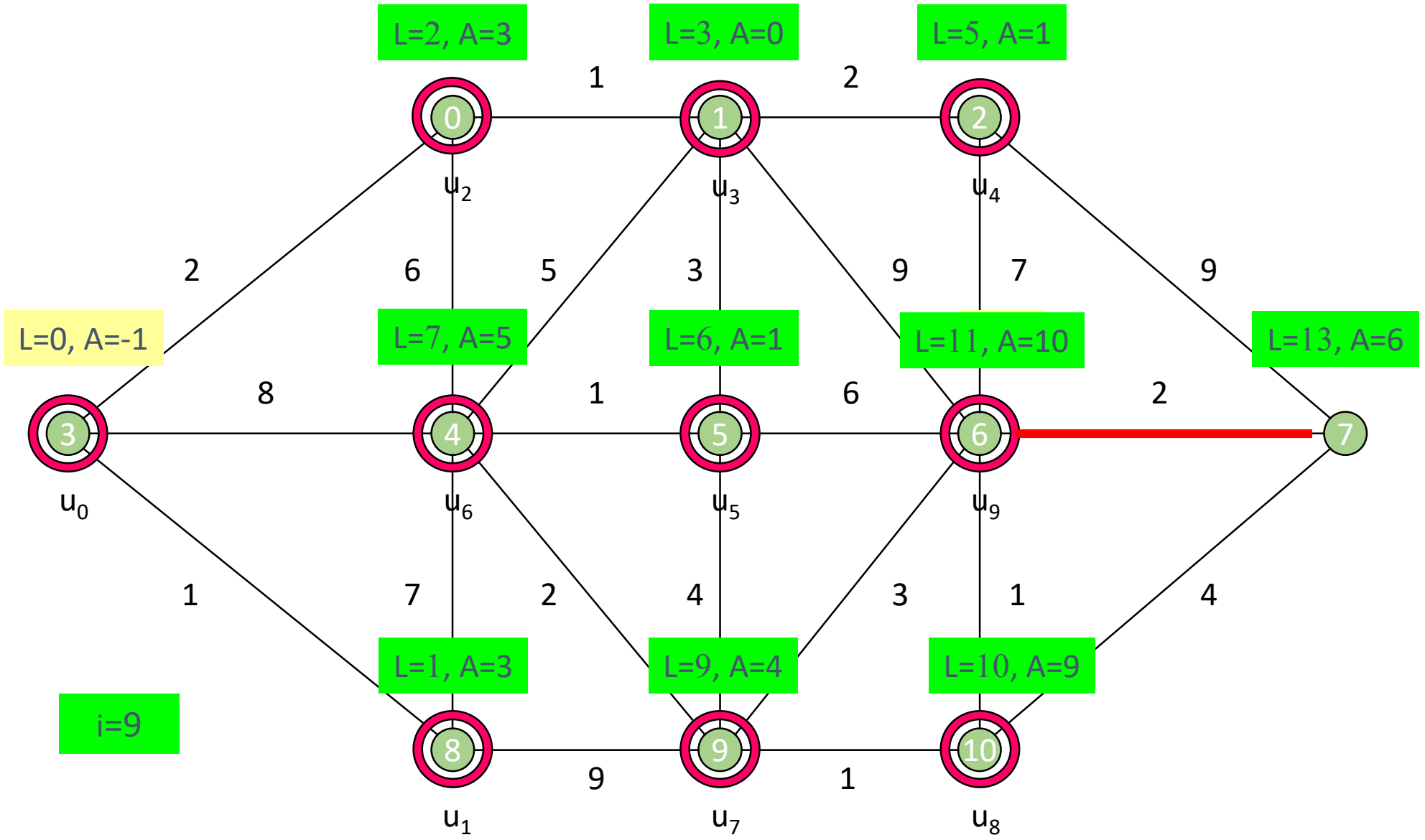
Caminho do vértice 3 até o vértice 7



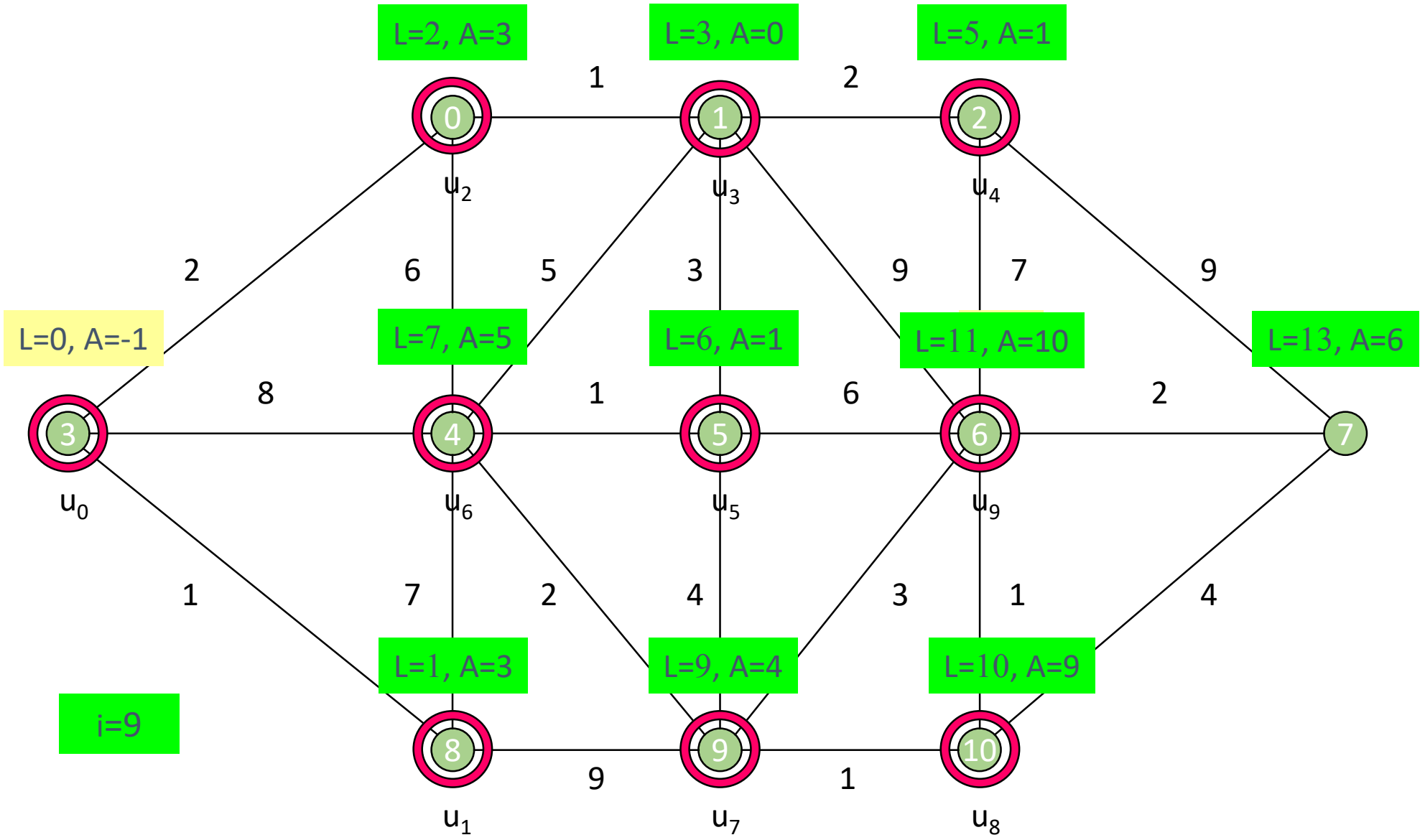
Caminho do vértice 3 até o vértice 7



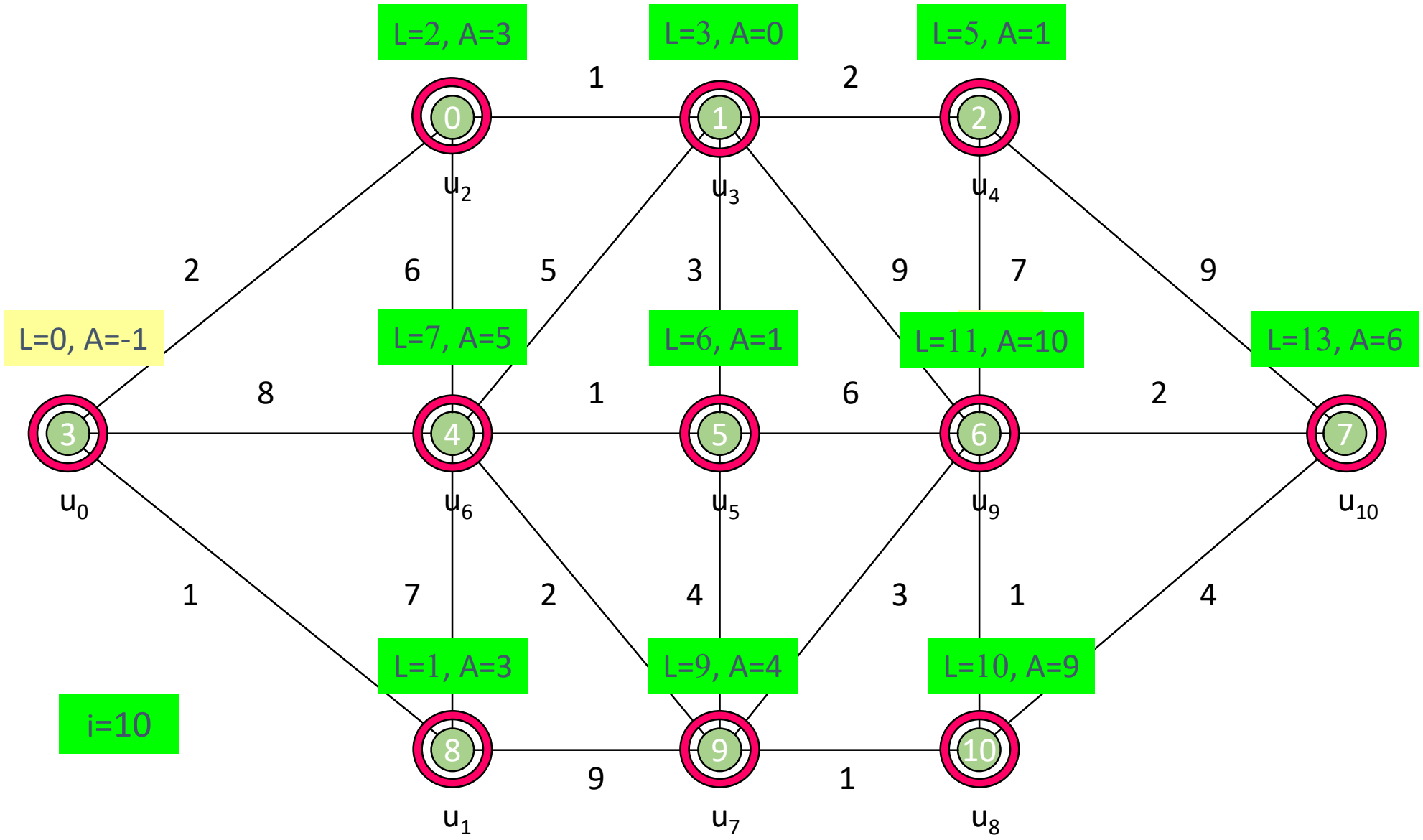
Caminho do vértice 3 até o vértice 7



Caminho do vértice 3 até o vértice 7



Caminho do vértice 3 até o vértice 7



Caminho Mínimo – Perguntas

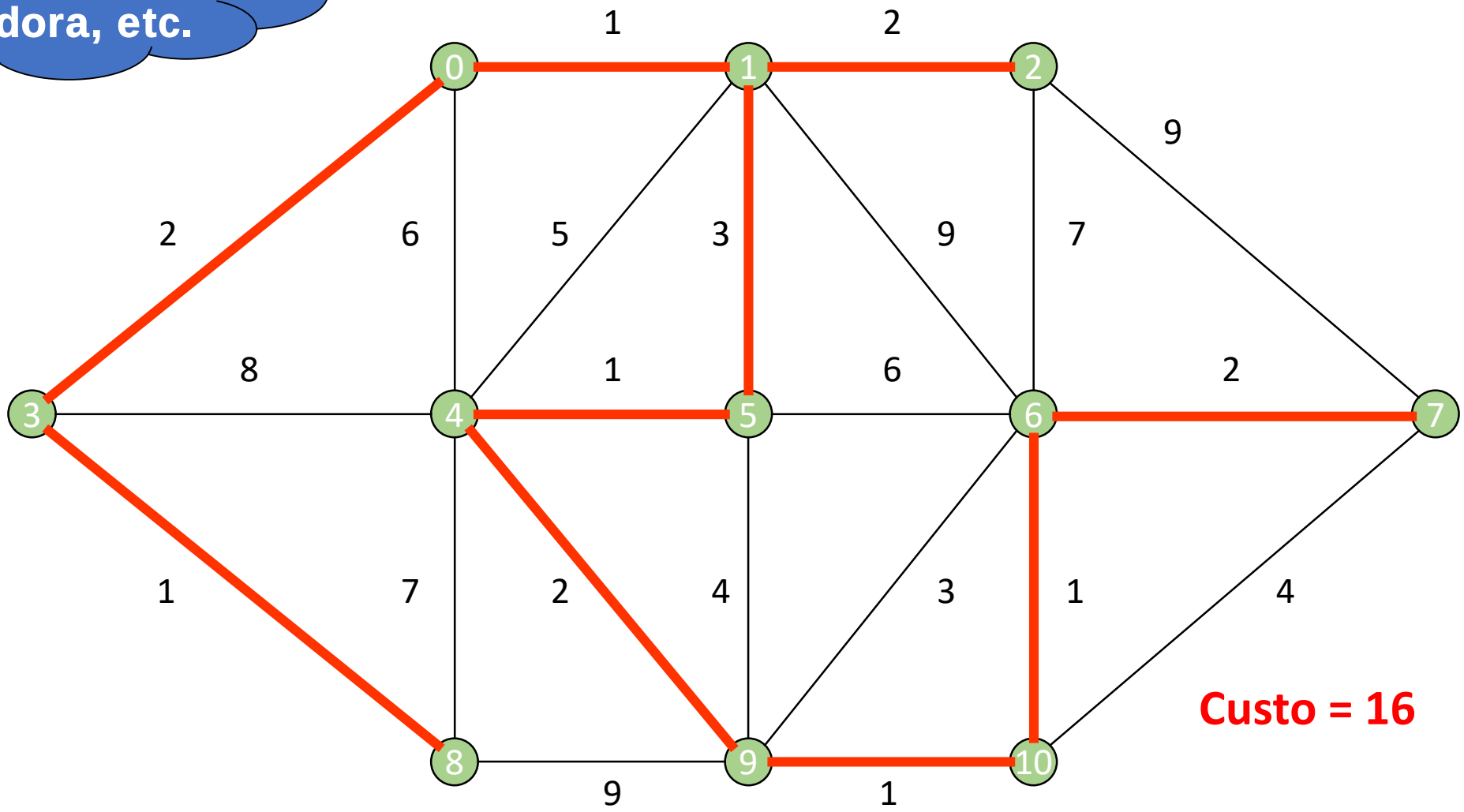
A=

0	3
1	0
2	1
u_0= 3	-1
4	5
5	1
6	10
7	6
8	3
9	4
10	9

- Qual caminho possui a menor distância entre os vértices:
  - 3 e 7 => 7 ← 6 ← 10 ← 9 ← 4 ← 5 ← 1 ← 0 ← 3
  - 3 e 1 => 1 ← 0 ← 3
  - 3 e 10 = > 10 ← 9 ← 4 ← 5 ← 1 ← 0 ← 3
  - 3 e 9 => 9 ← 4 ← 5 ← 1 ← 0 ← 3

Árvore de Cobertura Mínima

Espalhadora, Geradora, etc.

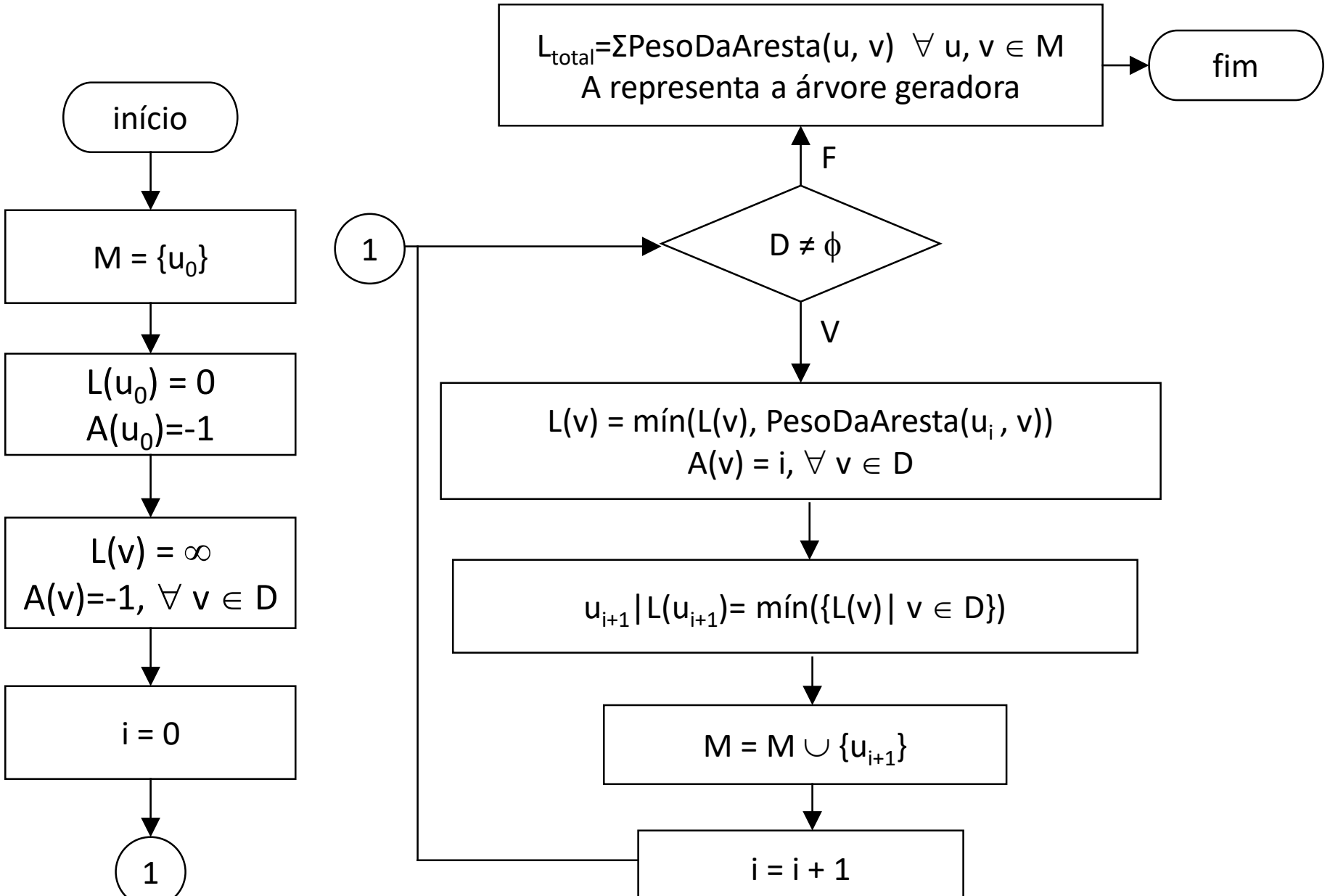


A=

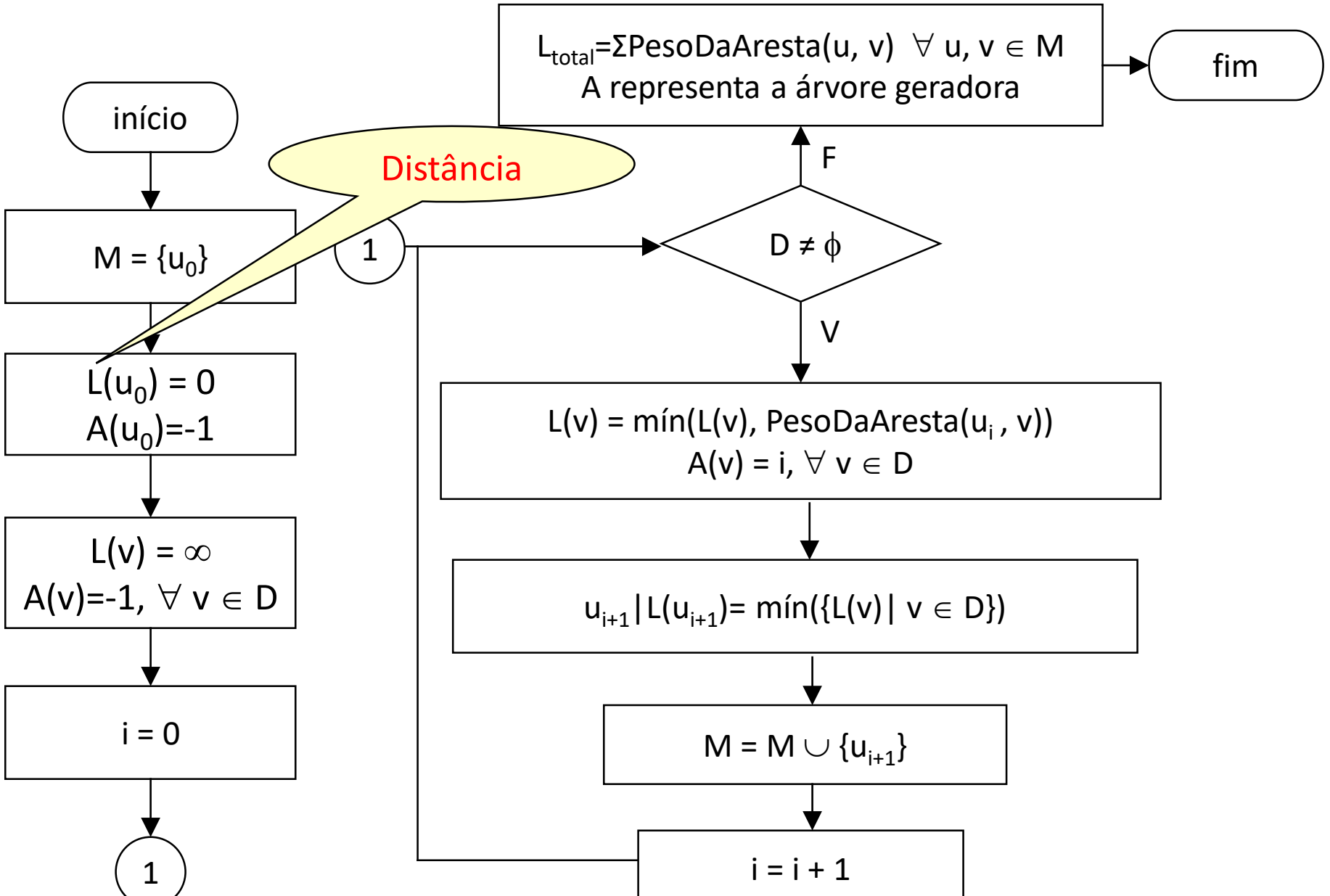
0	3
1	0
2	1
u_0= 3	-1
4	5
5	1
6	10
7	6
8	3
9	4
10	9

Custo = 16

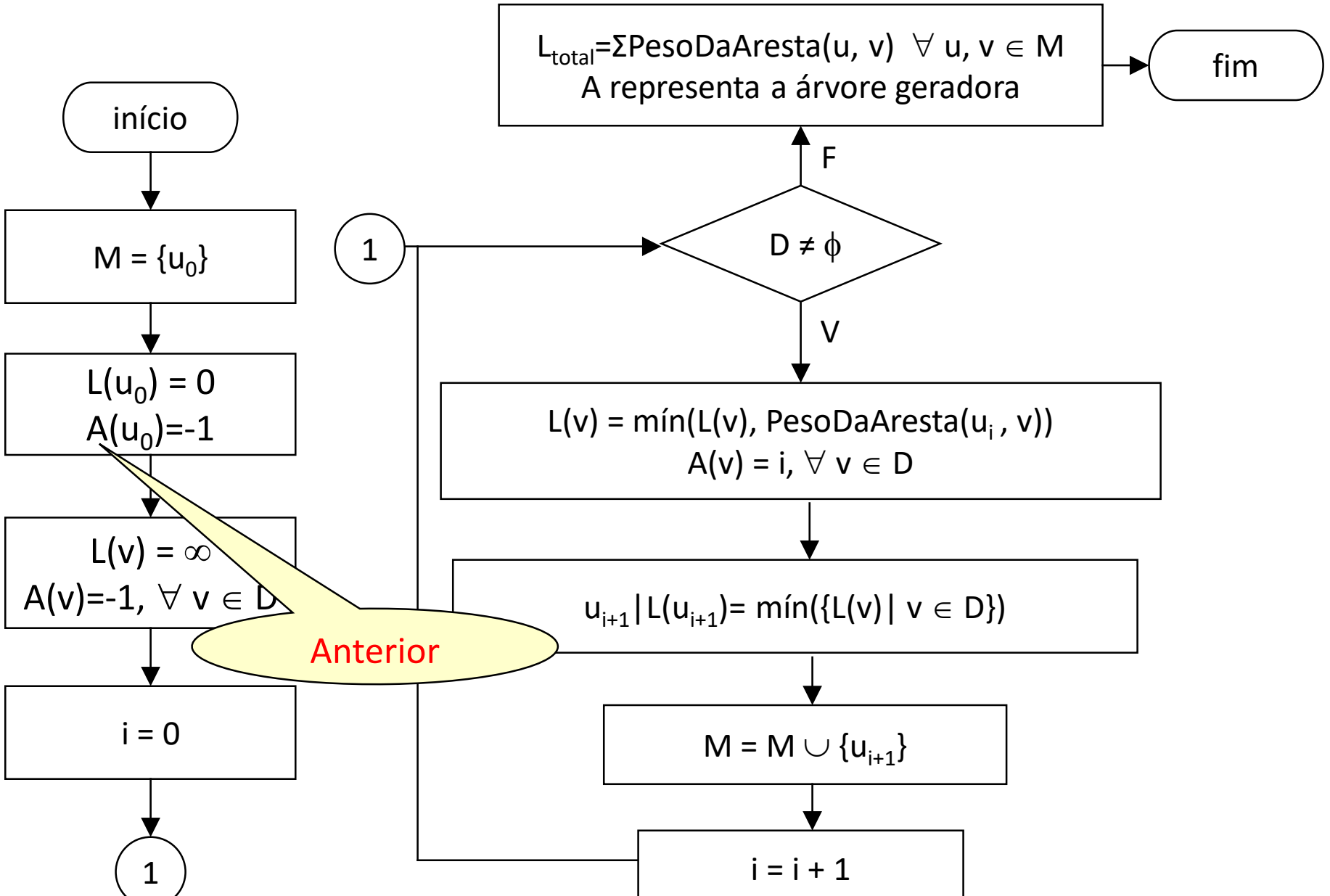
Algoritmo Prim



Algoritmo Prim



Algoritmo Prim



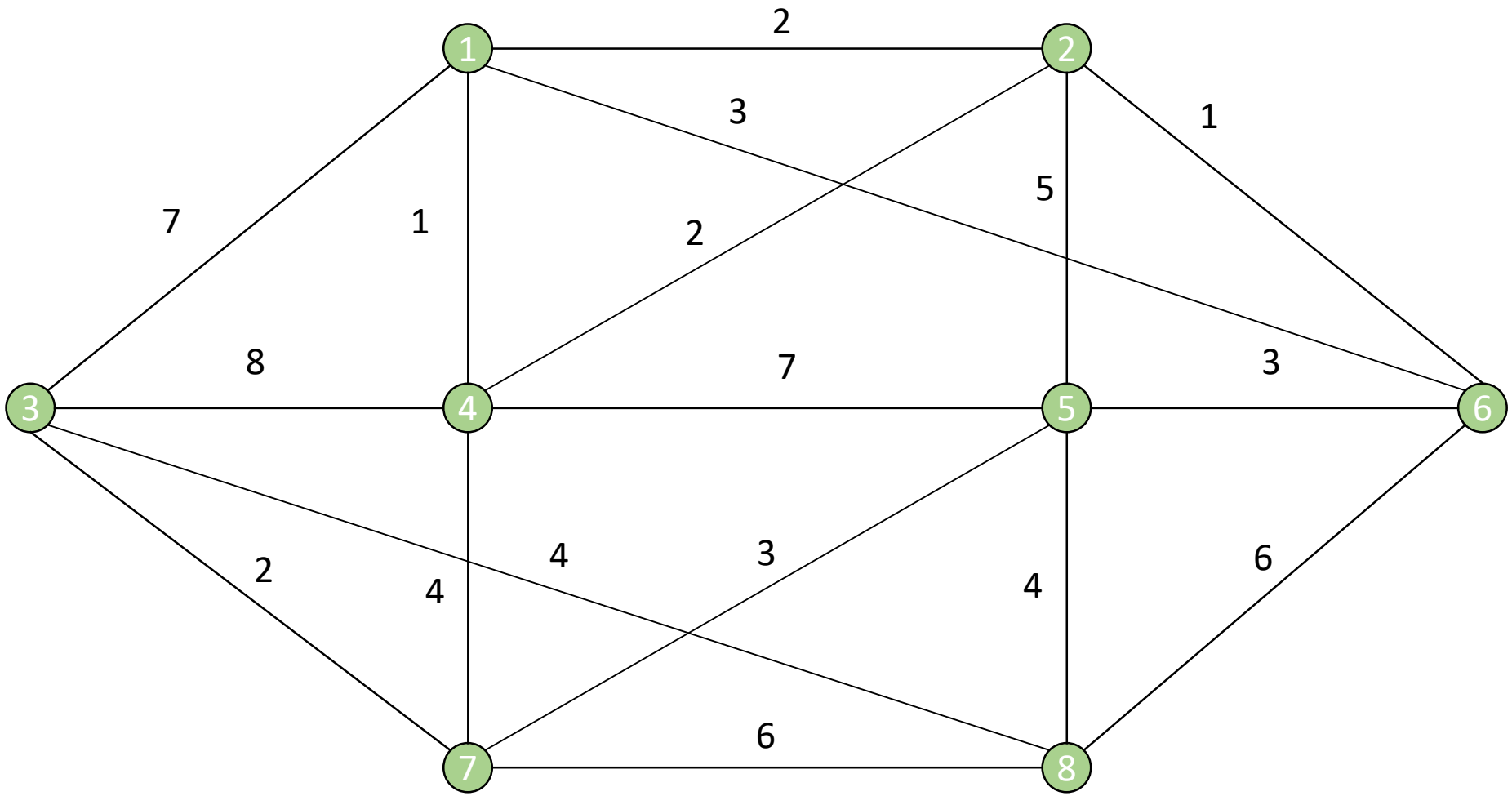
Diferença entre algoritmos

- Dijkstra:
  - Determina o custo mínimo, a partir do vértice de origem, para cada um dos demais vértices;
  - determina a árvore de custos mínimos, a partir do vértice de origem;
  - para cada vértice de origem, os custos para alcançar os demais deverão variar.

- Prim:
  - Determina uma árvore (grafo simples que interliga todos os vértices, sem formar laços) com o menor CUSTO GLOBAL possível;
  - o custo mínimo independe do vértice de origem;
  - a árvore pode ser diferente em função do vértice de origem
  - considera o problema como um TODO.

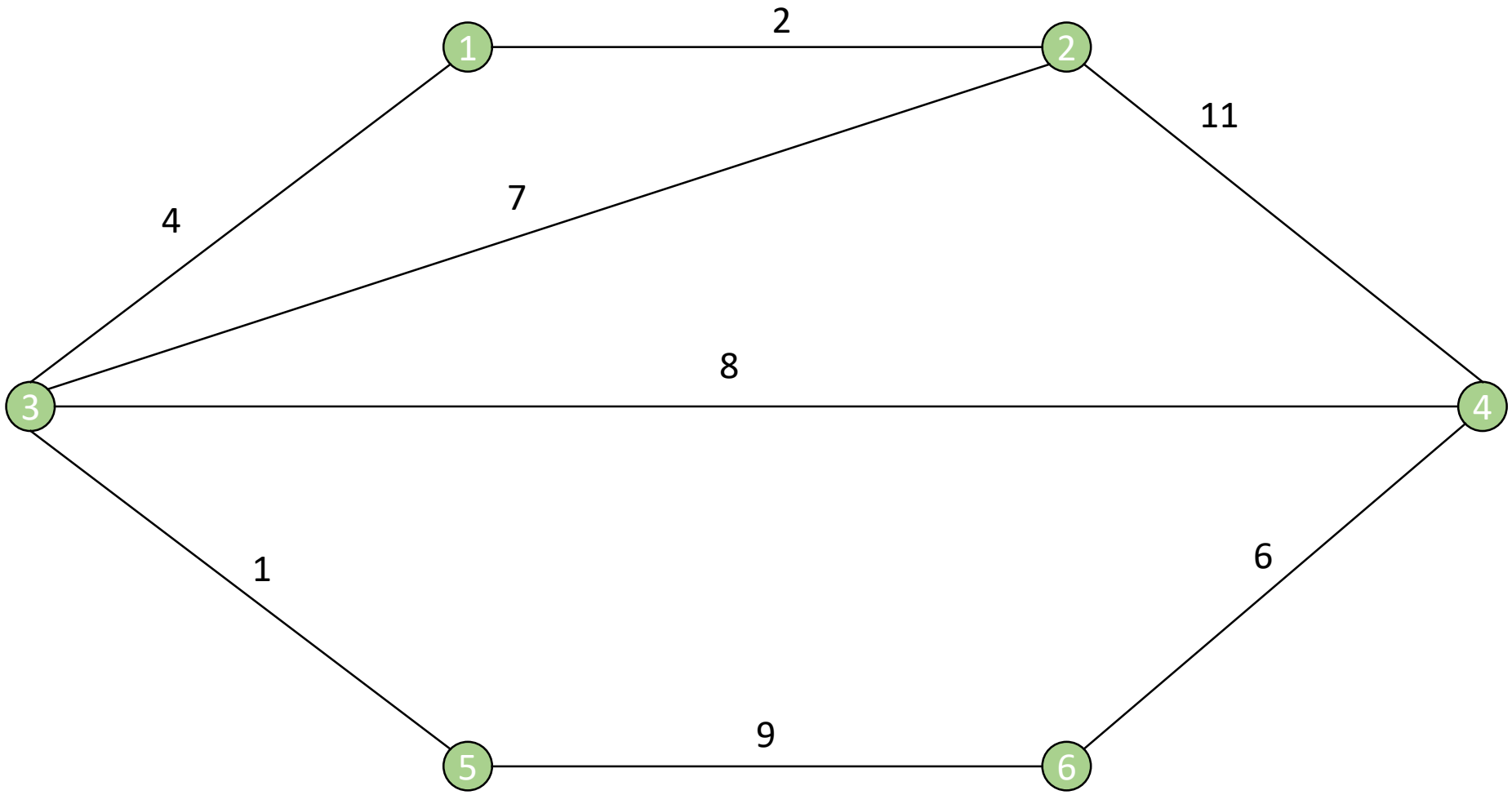
Exercícios

- Determine a árvore geradora mínima para o grafo a seguir



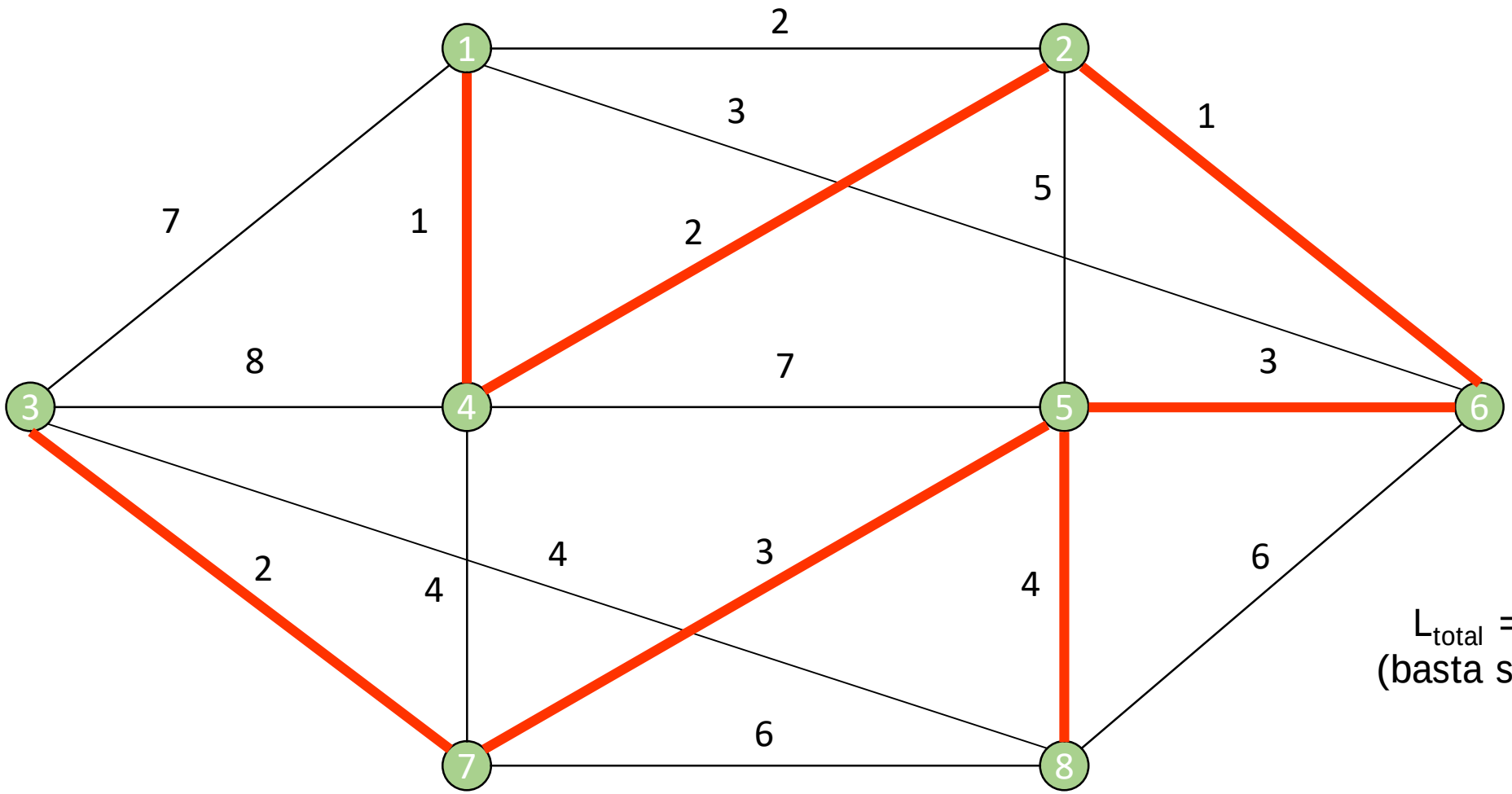
Exercícios

- Determine a árvore geradora mínima para o grafo a seguir



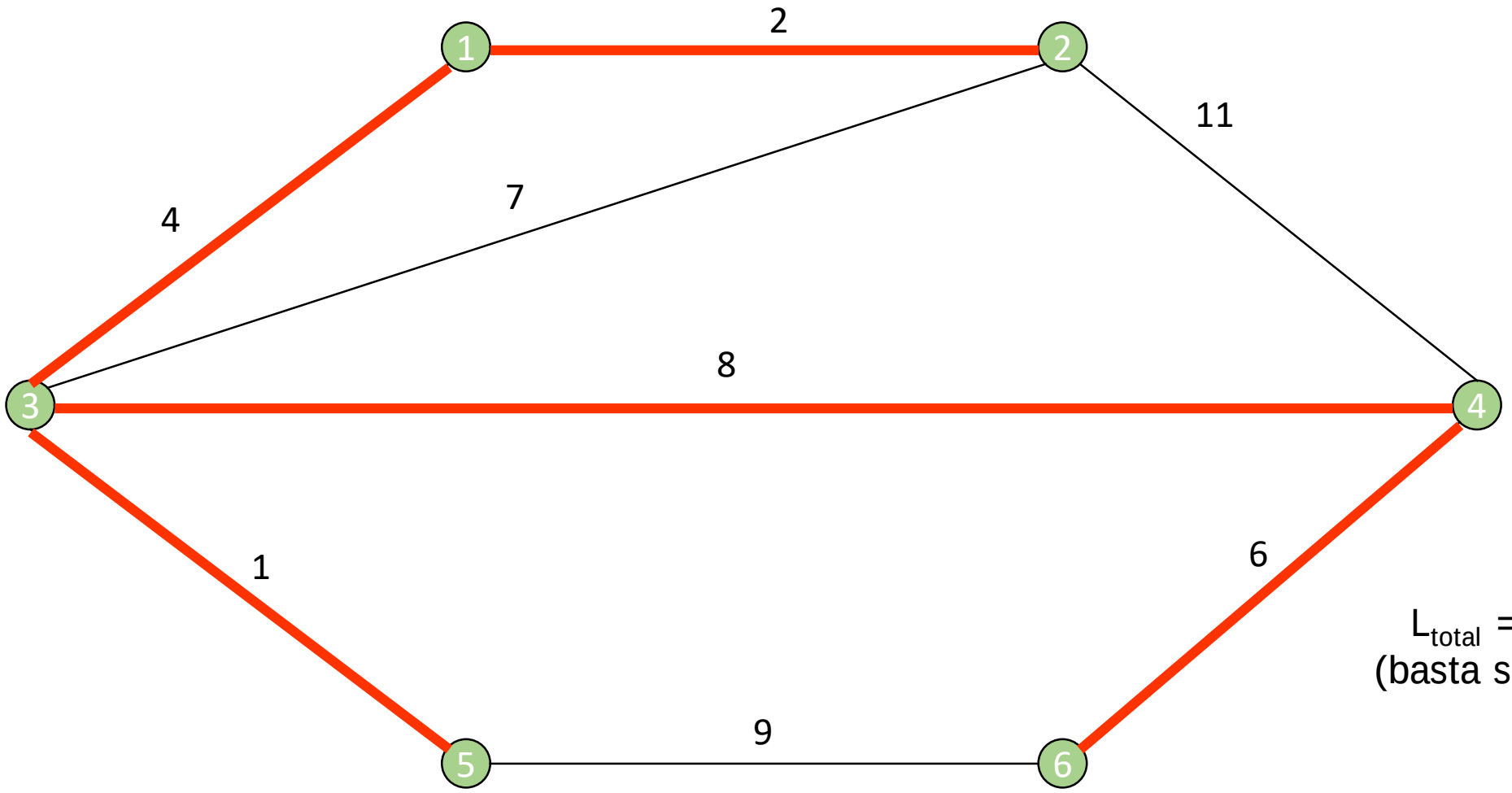
Exercícios

- Determine a árvore geradora mínima para o grafo a seguir



Exercícios

- Determine a árvore geradora mínima para o grafo a seguir



L_{total} = 21
(basta somar)