FIT5047 Second Theory Assignment

Due date: 23rd April 2021 (23:59).

Evaluation: 100 marks = 5%.

Submission: Moodle.

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Propositional Logic - Syntax and Semantics

[5+5+5+5 = 20]

Decide whether the following propositional logic sentences are either valid, satisfiable (but not valid), or unsatisfiable. Justify your answer.

(a) $\neg B \to (A \land \neg B)$ [5]

(b) $((\neg A \lor B) \land (\neg C \to \neg B)) \to (A \to C)$ [5]

Convert the following propositional logic sentences into Conjunctive Normal Form (CNF):

(c) $(C \to D) \to \neg(D \to B)$ [5]

(d) $(A \leftrightarrow B) \to (\neg A \land C)$ [5]

(a) $\neg(\neg B) \lor (A \land \neg B) \equiv B \lor (A \land \neg B) \equiv (B \lor A) \land (B \lor \neg B)$

A	B	$\neg B$	$(B \lor A)$	$(B \lor \neg B)$	$(B \lor A) \land (B \lor \neg B)$
T	T	F	T	T	T
T	F	T	T	T	T
F	T	F	T	T	T
F	F	T	F	T	F

Because $\neg B \to (A \land \neg B)$ is equal to $(B \lor A) \land (B \lor \neg B)$. According to the table, $(B \lor A) \land (B \lor \neg B)$ has 3 true models and 1 false model, so $\neg B \to (A \land \neg B)$ is satisfiable (but not valid).

(b) $((\neg A \lor B) \land (\neg C \to \neg B)) \to (A \to C) \equiv \neg ((\neg A \lor B) \land (\neg(\neg C) \land \neg B)) \land (\neg A \land C) \equiv ((A \land \neg B) \lor (\neg C \lor B)) \land (\neg A \land C)$

A	B	C	$\neg A$	$\neg$B	$\neg$C	$(A \land \neg B)$	$(\neg C \lor B)$	$((A \land \neg B) \lor (\neg C \lor B))$	$(\neg A \land C)$	(a) $((A \land \neg B) \lor (\neg C \lor B)) \land (\neg A \land C)$
T	T	T	F	F	F	F	T	T	F	F
T	T	F	F	F	T	F	T	T	F	F
T	F	T	F	T	F	T	F	T	F	F
T	F	F	F	T	T	T	T	T	F	F
F	T	T	T	F	F	F	T	T	T	T
F	F	T	T	T	F	F	F	F	T	F
F	T	F	T	F	T	F	T	T	F	F
F	F	F	T	T	T	F	T	T	F	F

Because $((\neg A \lor B) \land (\neg C \to \neg B)) \to (A \to C)$ is equal to $((A \land \neg B) \lor (\neg C \lor B)) \land (\neg A \land C)$. According to the table, has 1 true model and 7 false models, so $((\neg A \lor B) \land (\neg C \to \neg B)) \to (A \to C)$ is satisfiable (but not valid).

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(c) $(C \to D) \to \neg(D \to B) \equiv (\neg\, (\neg C \vee D)) \vee (\neg(\neg D \vee B)) \equiv (C \wedge \neg D) \vee (D \wedge \neg B)$

C	D	$\neg B$	$\neg D$	$(C \wedge \neg D)$	$(D \wedge \neg B)$	$(C \wedge \neg D) \vee (D \wedge \neg B)$
T	T	T	F	F	T	T
T	T	F	F	F	F	F
T	F	T	T	T	F	T
F	T	T	F	F	T	T
F	F	T	T	F	F	F
F	T	F	F	F	F	F
F	F	F	T	F	F	F

According to the table, has 3 true models and 4 false models, so $(C \to D) \to \neg(D \to B)$ is satisfiable (but not valid).

(d) $(A \leftrightarrow B) \to (\neg A \wedge C) \equiv \neg\, ((\neg A \vee B) \wedge (\neg B \vee A)) \vee (\neg A \wedge C) \equiv (A \wedge \neg B) \vee (B \wedge \neg A) \vee (\neg A \wedge C)$

A	B	C	$\neg A$	$\neg B$	$(A \wedge \neg B)$	$(B \wedge \neg A)$	$(\neg A \wedge C)$	$(A \wedge \neg B) \vee (B \wedge \neg A) \vee (\neg A \wedge C)$
T	T	T	F	F	F	F	F	F
T	T	F	F	F	F	F	F	F
T	F	T	F	T	T	F	F	T
F	T	T	T	F	F	T	T	T
F	F	T	T	T	F	F	T	T
F	T	F	T	F	F	T	F	T
F	F	F	T	T	F	F	F	F

According to the table, has 4true models and 3 false models, so $(A \leftrightarrow B) \to (\neg A \wedge C)$ is satisfiable (but not valid).

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Propositional Logic - Resolution

[10+10 = 20]

Use resolution-refutation to decide whether or not the following goals can be proved, from the Knowledge Bases given below.

(a) Goal: C [10]

$1 : (A \to B) \to B$

$2 : (A \to A) \to C$

$3 : (C \to D) \to \neg(D \to B)$

(b) Goal: $\neg(D \to A)$ [10]

$1 : A \lor B \lor \neg C$

$2 : A \lor B \lor C$

$3 : D \lor \neg B$

$4 : \neg A \lor \neg B$

(a) Goal: C

1: $(A \to B) \to B \equiv (\text{A} \land \neg \text{B}) \lor \text{B}$

2: $(A \to A) \to C \equiv (\text{A} \land \neg \text{A}) \lor \text{C} \equiv \text{C}$

3: $(C \to D) \to \neg\, (D \to B) \equiv (\text{C} \land \neg D) \lor (\text{D} \land \neg \text{B})$

Negated goal: 4: $\neg C$

4 and 2: 4: ¬C 2: C

resolvent: NIL

This case can proved

(b)Goal: $\neg(D \to A) \equiv \text{D} \land \neg \text{A}$ [10]

$1 : A \lor B \lor \neg C$

2: $A \lor B \lor C$

3: $D \lor \neg B$

4: $\neg A \lor \neg B$

Negated goal: 5: ¬ D ∨ A

5 and 4: 5 .¬ D ∨ A 4. $\neg A \lor \neg B$

resolvent:6: ¬ D ∨$\neg B$

6 and 3: 6. ¬ D ∨$\neg B$ 3. $D \lor \neg B$

resolvent:7. ¬B∨¬B

7 and 1: 7. ¬B∨¬B 1. $A \vee B \vee \neg C$

resolvent: 8: A ∨¬B ∨ ¬C

8 and 2 : 8: A ∨¬B ∨ ¬C *2:* $A \vee B \vee C$

resolvent: 9 : A

This case can't proved

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Propositional Logic - Forward Chaining

[5+5+5+5 = 20]

This question is about Horn clauses and Forward chaining.

R1: $L \rightarrow P$

R2: $(A \wedge B) \rightarrow L$

R3: $P \rightarrow Q$

R4: $L \rightarrow A$

A

B

Apply forward reasoning to the Horn clauses above in order to prove Q. After each rule application, show the values to be inserted in the columns named:

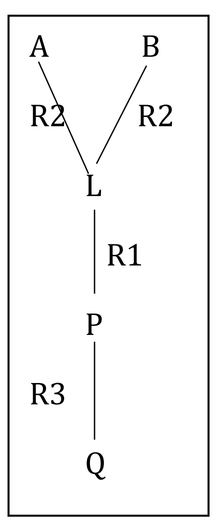
(a) AGENDA [5] (b) COUNT [5](c)INFERRED [5]

in the table provided below, in the way seen in the lectures. Also,

(d)show the search graph resulting from the application of the rules. [5]

R1: $\neg L \vee P$ R2: $\neg A \vee \neg B \vee L$ R3: $\neg P \vee Q$ R4: $\neg L \vee A$

AGENDA	COUNT				INFERRED
	R1	R2	R3	R4	
A,B	1	2	1	1	
B	1	1	1	1	A
L	1	0	1	1	A,B
P	0	0	1	1	A,B,L
Q	0	0	0	1	A,B,L,P



First Order Logic - Substitution and Unification

[5+5+5+5 = 20]

(a) Calculate the composition of substitutions s_1s_2 and s_2s_1, given the two substitutions s_1 = $\{x|f(y,A),y|B,z|g(B,w)\}$ and s_2 = $\{x|A,y|h(B,z),z|C,w|D\}$ [5]

(b) Apply the composed substitutions s_1s_2 and s_2s_1 of part (a) to $P(x,f(y),z,g(w))$, that is, calculate predicates $P(x,f(y),z,g(w))s_1s_2$ and $P(x,f(y),z,g(w))s_2s_1$. [5]

The following questions are about unification. Are the following literals unifiable? If so, provide the most general unifier (mgu). If not unifiable, explain why not.

(c) *PREDICATE*$(x,f(7,3),g(z),z)$ and *PREDICATE*$(f(3,7),y,g(w),5)$ [5]

(d) *PREDICATE*$(f(x,y))$ and *PREDICATE*$(g(3,7))$ [5]

(a) s1s2 =$\{x|f(y,A),y|B,z|g(B,w)\}$ $\{x|A,y|h(B,z),z|C,w|D\}$ = $\{ x \mid f(B,A) ,y \mid B, z \mid g(B, D), w \mid D \}$
s2s1 = $\{x|A,y|h(B,z),z|C,w|D\}\{x|f(y,A),y|B,z|g(B,w)\}$ = $\{x \mid A, y \mid h(B,C), z \mid C, w \mid D\}$

(b) $P(x, f(y),z ,g(w))\ s_1s_2 =\ P(f(B,A), f(B), g(B, D) ,g(D))\ \{ x \mid f(B,A) ,y \mid B, z \mid g(B, D), w \mid D \}$
$P(x, f(y),z ,g(w))s_2s_1 =\ P(A, f(h(B,C)), C, g(D))\ \{x \mid A, y \mid h(B,C), z \mid C, w \mid D\}$

(c) Unify (PREDICATE (x, f (7,3), g (z), z), PREDICATE (f (3,7), y, g (w), 5))
Z1<- Unify(P,P)
P and P are identical -> return NIL
Z2 <- Unify ((x, f (7,3), g (z), z), (f (3,7), y, g (w), 5)))
Z1<- Unify(x,f(3,7))
Return {x| f(3,7)}
G1 <- (f (7,3), g (z), z) {x| f(3,7)} ->(f (7,3), g (z), z)
G2<- (y, g (w), 5)) { x| f(3,7)} ->(y, g (w), 5))
Z2 <- Unify ((f (7,3), g (z), z), (y, g (w), 5)))
Z1<- Unify(f (7,3),y)
Return {y| f(7,3)}
G1 <- (g (z), z) {y| f(7,3)} ->(g (z), z)
G2<- (g (w), 5)) { y| f(7,3)} ->(g (w), 5))
Z2 <- Unify ((g (z), z), (g (w), 5)))
Z1<- Unify((g (z), g (w))
Z1<- Unify(g, g)
g and g are identical return{}
Z2 <- Unify ((z), (w))
Z1<- Unify(z, w)
Return {z | w}
G1 <- (z) {z| w} ->(w)
G2<- (5)) { z| w} ->(5))
Z1 <- Unify (w, 5)
Return {w|5}
Return {x| f(3,7)} {y| f(7,3)} {z | w}{w|5} = {x| f(3,7),y| f(7,3),z | 5,w|5}

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(d) Unify (*PREDICATE*(*f*(*x*, *y*)), *PREDICATE*(*g*(3,7)))

Z1<- Unify(P,P)

P and P are identical -> return NIL

Z2 <- Unify (*f*(*x*, *y*), *g*(3,7))

Z1<- Unify(f, g)

f is a symbol and g is a symbol, but they aren’t identical

return fail

So UNIFICATION FAILS.

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First Order Logic - Resolution

[10+10 = 20]

Use resolution refutation to show the goal "G" below and show:

(a) how you construct all necessary clauses, and [10]

(b) which most general unifiers are used. [10]

1: $P(A) \rightarrow Q(B)$

2: $\forall x[P(x)]$

3: $\forall y\ [\exists z\ [R(y,z)] \rightarrow \neg Q(y)]$

G: $\forall z\ [\neg R(B,z)]$

(a) 1: $P(A) \rightarrow Q(B) \equiv \neg P(A) \vee Q(B)$

2: $\forall x[P(x)] \equiv P(x)$

3: $\forall y\ [\exists z\ [R\ (y, z)] \rightarrow \neg Q(y)] \equiv \forall y\ [\neg\ (\exists z\ [R\ (y, z)]) \vee \neg Q(y)] \equiv \forall y\ [\forall z\ [\neg R\ (y, z)] \vee \neg Q(y)]$
$\equiv \neg R\ (y, z) \vee \neg Q(y)$

G: $\forall z\ [\neg R\ (B, z)] \equiv \neg R\ (B, z)$

(b) Negated goal: 4: $R\ (B, z)$
4 and 3: 4: $\underline{R\ (B, z)}$ 3: $\underline{\neg R(y, z)} \vee \neg Q(y)$
mgu : {y | B}
resolvent: 5. $\neg Q(B)$
5 and 1: 5. $\neg Q(B)$ 1. $\neg P(A) \vee Q(B)$
resolvent: 6. $\neg P(A)$
6 and 2: 6. $\neg P(A)$ 2. $P(x)$
mgu:{x|A}
resolvent: NIL

so mgu = {x|A,y|B}