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5.5 DATA COMPRESSION

- ▶ *introduction*
- ▶ *run-length encoding*
- ▶ *Huffman compression*
- ▶ *LZW compression*



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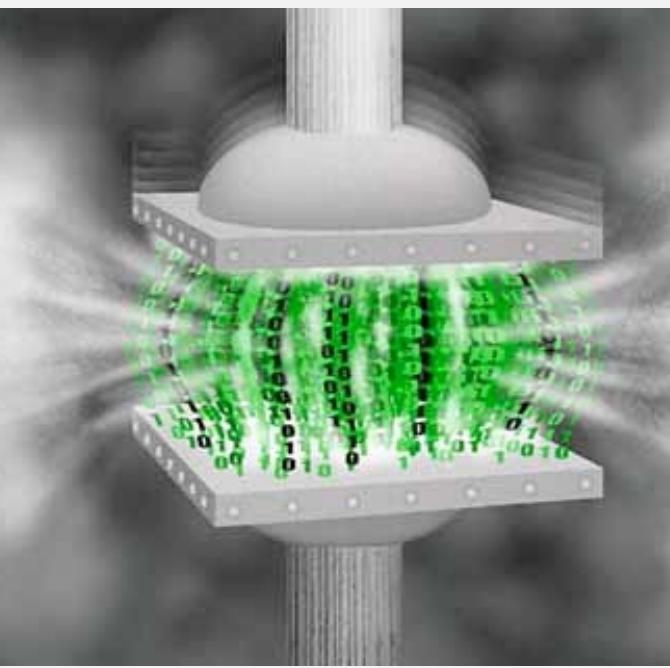
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Data compression

Compression reduces the size of a file:

- To save **space** when storing it.
- To save **time** when transmitting it.
- Most files have lots of redundancy.



Who needs compression?

- Moore's law: # transistors on a chip doubles every 18–24 months.
- Parkinson's law: data expands to fill space available.
- Text, images, sound, video, sensors, ...

Every day, we create:

- 500 million Tweets.
- 300 billion emails.
- 350 million Facebook photos.
- 500,000 hours YouTube video.

Basic concepts ancient (1950s), best technology recently developed.

Applications

Generic file compression.

- Files: Gzip, bzip2, 7z, PKZIP,
- File systems: ZFS, HFS+, ReFS, GFS, APFS,



Multimedia.

- Images: GIF, JPEG, PNG, RAW,
- Sound: MP3, AAC, Ogg Vorbis,
- Video: MPEG, HDTV, H.264, HEVC,



Communication. Fax, Skype, WeChat, Zoom,



Databases. SQL, Google, Facebook, NSA,



Smart sensors. Phone, watch, car, health,



Lossless compression and expansion

Message. Bitstream B we want to compress.

Compress. Generates a “compressed” representation $C(B)$.

Expand. Reconstructs original bitstream B .

uses fewer bits
(you hope)



Compression ratio. Bits in $C(B)$ / bits in B .

Ex. 50–75% or better compression ratio for English language.

Data representation: genomic code

Genome. String over the alphabet { A, T, C, G }.

Goal. Encode an n -character genome: A T A G A T G C A T A G ...

Standard ASCII encoding.

- 8 bits per char.
- $8n$ bits.

char	hex	binary
'A'	41	01000001
'T'	54	01010100
'C'	43	01000011
'G'	47	01000111

Two-bit encoding.

- 2 bits per char.
- $2n$ bits.

char	binary
'A'	00
'T'	01
'C'	10
'G'	11

compression ratio = 25%
(compared to ASCII)

0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
NUL	SOH	STX	ETX	EOT	ENQ	ACK	BEL	BS	HT	LF	VT	FF	CR	SO	SI
DLE	DC1	DC2	DC3	DC4	NAK	SYN	ETB	CAN	EM	SUB	ESC	FS	GS	RS	US
SP	!	"	#	\$	%	&	'	()	*	+	,	-	.	/
0	1	2	3	4	5	6	7	8	9	:	;	<	=	>	?
@	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
P	Q	R	S	T	U	V	W	X	Y	Z	[\]	^	_
`	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o
p	q	r	s	t	u	v	w	x	y	z	{		}	~	DEL

hexadecimal-to-ASCII conversion table

Fixed-length code. k -bit code supports alphabet of size 2^k .

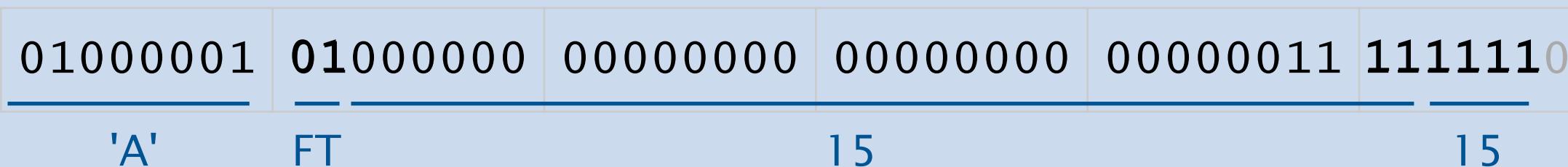
Amazing but true. Some genomic databases in 1990s used ASCII.

Writing binary data

Binary standard output. Write **bits** to standard output.

```
public class BinaryStdOut
    void write(boolean b)           write the specified bit
    void write(char c)             write the specified 8-bit char
    void write(char c, int r)      write the r least significant bits of the specified char
    [similar methods for byte (8 bits); short (16 bits); int (32 bits); long and double (64 bits)]
    void close()                  close the bitstream
```

```
BinaryStdOut.write('A');      ← write as 8-bit character
BinaryStdOut.write(false);   | ← write bits
BinaryStdOut.write(true);    | ←
BinaryStdOut.write(15);       ← write 32-bit integer
BinaryStdOut.write(15, 4);    ← write 4 least significant bits of integer
BinaryStdOut.close();
```

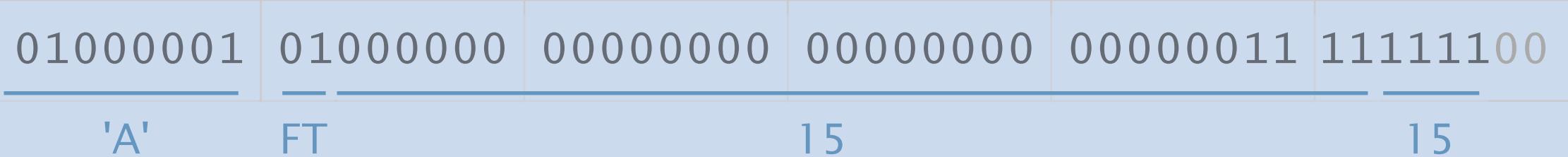


Reading binary data

Binary standard input. Read **bits** from standard input.

```
public class BinaryStdIn  
    boolean readBoolean()      read 1 bit of data and return as a boolean value  
    char readChar()           read 8 bits of data and return as a char value  
    char readChar(int r)      read r bits of data and return as a char value  
    [similar methods for byte (8 bits); short (16 bits); int (32 bits); long and double (64 bits)]  
    boolean isEmpty()         is the bitstream empty?
```

```
char c = BinaryStdIn.readChar();          ← read 8-bit character  
boolean b1 = BinaryStdIn.readBoolean();   |← read bits  
boolean b2 = BinaryStdIn.readBoolean();   |← read 32-bit integer  
int x = BinaryStdIn.readInt();           |← read 4-bit integer  
int y = BinaryStdIn.readInt(4);
```



Binary representation

Q. How to examine the contents of a bitstream (e.g., when debugging)?

standard character stream

```
~> more dna.txt  
ATCGCA
```

bitstream represented with hex digits

```
~> java HexDump < dna.txt  
41 54 43 47 43 41  
48 bits
```

lots of unprintable characters
(don't `System.out.print()` binary data)

	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	NUL	SOH	STX	ETX	EOT	ENQ	ACK	BEL	BS	HT	LF	VT	FF	CR	SO	SI
1	DEL	DC1	DC2	DC3	DC4	NAK	SYN	ETB	CAN	EM	SUB	ESC	FS	GS	RS	US
2	SP	!	"	#	\$	%	&	'	()	*	,	-	.	/	
3	0	1	2	3	4	5	6	7	8	9	:	;	<	=	>	?
4	@	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
5	P	Q	R	S	T	U	V	W	X	Y	Z	[\]	^	_
6	`	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o
7	p	q	r	s	t	u	v	w	x	y	z	{		}	~	DEL

hexadecimal-to-ASCII conversion table

bitstream of binary file represented with hex digits

```
~> java HexDump < us.gif  
47 49 46 38 39 61 8e 01 01 01 d5 00 00 94 18 29  
06 02 03 84 29 4a 6b 18 4a 73 29 6b 6e 5d 6e 4a  
45 4a f7 ef f7 42 00 52 1f 00 37 42 10 6b 31 00  
63 31 08 5a 7e 70 8d 36 08 6b 4a 21 7b 5b 36 86  
...  
b7 de 7b f3 dd b7 df 7f 03 1e 38 cc 41 00 00 3b  
99200 bits
```

Universal data compression

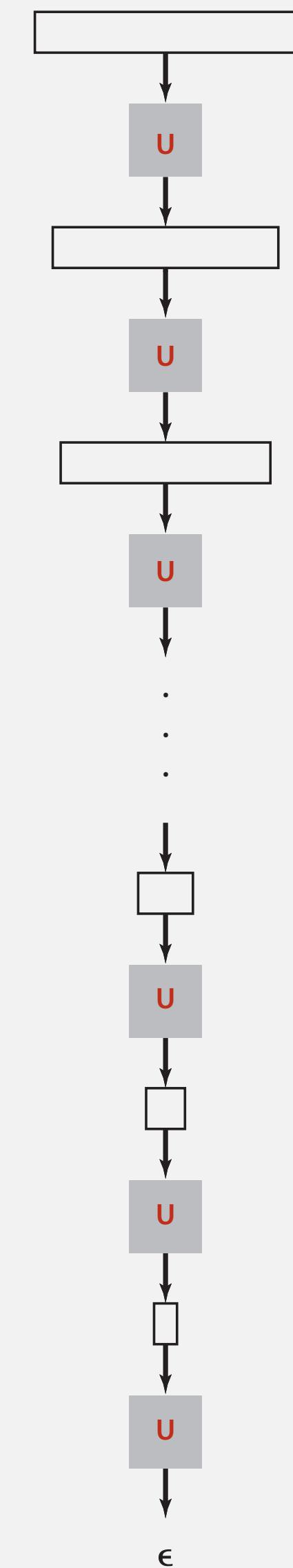
Proposition. No algorithm can compress every bitstring.

Pf 1. [by contradiction]

- Suppose you have a universal data compression algorithm U that can compress every bitstream.
 - Given bitstring B_0 , compress it to get a shorter bitstring B_1 .
 - Compress B_1 to get a shorter bitstring B_2 .
 - Continue until reaching bitstring of length 0.
 - Implication: all bitstrings can be compressed to 0 bits!

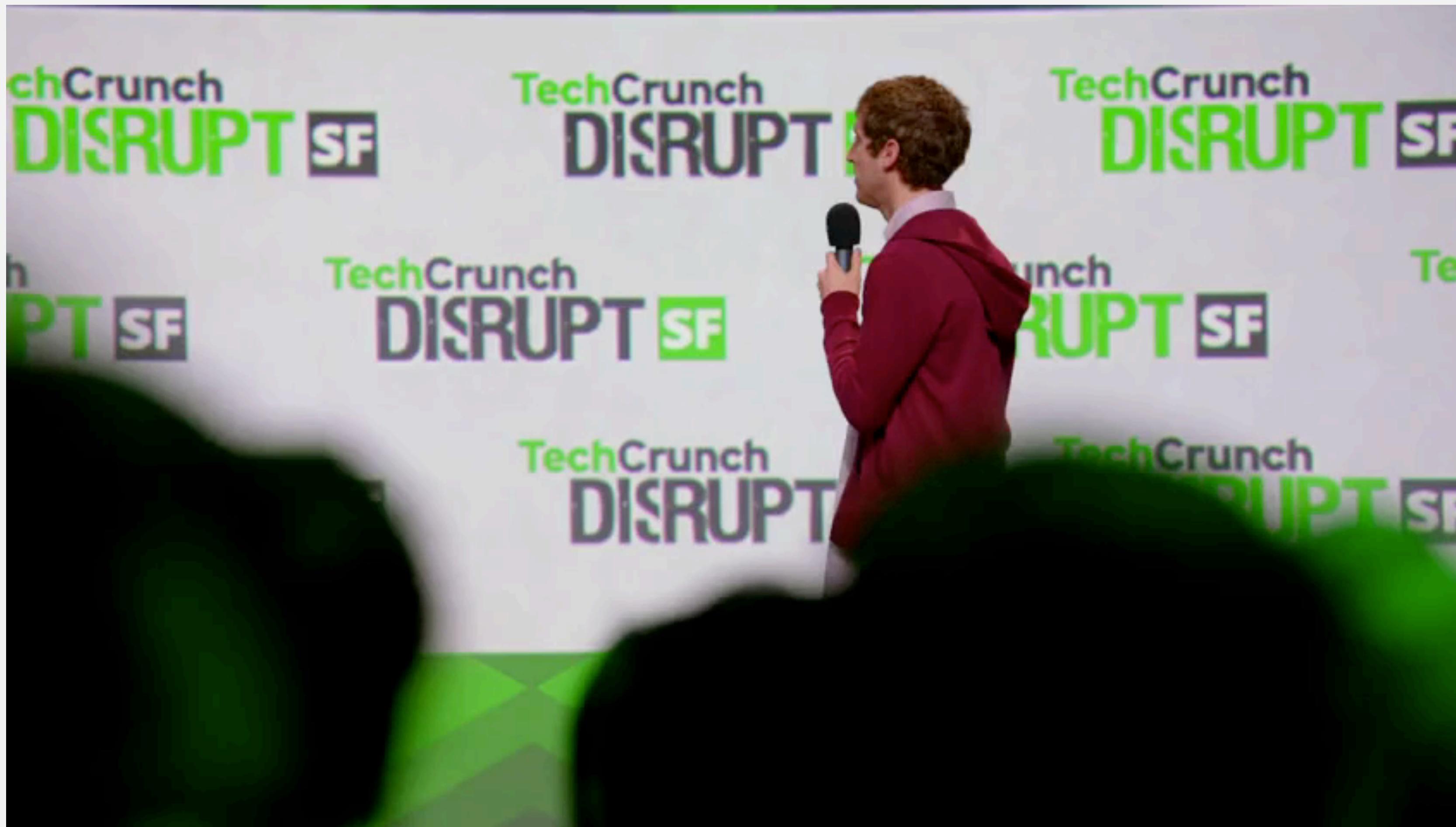
Pf 2. [by counting]

- Suppose your algorithm that can compress all 1,000-bit strings.
 - 2^{1000} possible bitstrings with 1,000 bits.
 - Only $1 + 2 + 4 + \dots + 2^{998} + 2^{999} = 2^{1000} - 1$ can be encoded with ≤ 999 bits.
 - Similarly, only 1 in 2^{499} bitstrings can be encoded with ≤ 500 bits!



Universal data compression

Pied Piper. Claims 1 : 3.8 lossless compression of arbitrary data.





Did Pied Piper achieve a lossless compression ratio of 1 : 3.8 for arbitrary files?

- A. Yes.
- B. No.

Rdenudcany in Enlgsih Inagugae

Q. How much redundancy in the English language?

A. Quite a bit.

“ ... randomising letters in the middle of words [has] little or no effect on the ability of skilled readers to understand the text. This is easy to denmtrasote. In a pubiltacion of New Scnieitst you could ramdinoze all the letetrs, keipeng the first two and last two the same, and reibadailty would hadrly be aftcfeed. My ansaylis did not come to much beucase the thoery at the time was for shape and senqeuce retigcionon. Saberi’s work sugsegs we may have some pofrweul palrlael prsooscers at work. The resaon for this is suerly that idnetiyfing coentnt by paarllel prseocsing speeds up regnicoiton. We only need the first and last two letetrs to spot chganes in meniang.” — Graham Rawlinson

The gaol of data cmperisoson is to inetdify rdenudcany and epxloit it.



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Run-length encoding (RLE)

Simple type of redundancy in a bitstream. Long runs of repeated bits.

`000000000000000011111100000001111111111` ← 40 bits

run of length 15 run of length 7 run of length 7 run of length 11

Representation. 4-bit counts to represent alternating runs of 0s and 1s:

15 0s, then 7 1s, then 7 0s, then 11 1s.

`1111011101111011` ← now only 16 bits

15 7 7 11

Q. How many bits r to store each run length?

A. Typically 8 bits (but 4 on this slide for brevity).

Q. What to do when run length exceeds max count $2^r - 1$?

A. Intersperse runs of length 0.

Applications. JPEG, TIFF, BMP, ITU-T T4 Group 3 Fax, ...



What is the best compression ratio achievable from run-length encoding when using 8-bit counts?

- A. 1 / 256
- B. 1 / 16
- C. 8 / 255
- D. 1 / 8
- E. 16 / 255

Run-length encoding: Java implementation

```
public class RunLength
{
    public static void compress()
    { /* see textbook */ }

    public static void expand()
    {
        boolean value = false;
        while (!BinaryStdIn.isEmpty())
        {
            int run = BinaryStdIn.readInt(8);
            for (int i = 0; i < run; i++)
                BinaryStdOut.write(value);
            value = !value;
        }
        BinaryStdOut.close();
    }
}
```

initial run is of 0s

read 8-bit count from standard input

write run of 0s or 1s to standard output

flip bit (for next run)

pad last byte with 0s (if needed)
and close output stream



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Variable-length codes

Key idea. Use different number of bits to encode different characters.

Ex. Morse code: • • • — — — • • •

Issue. Ambiguity.

SOS ?

VZE ?

EEJIE ?

EEWNI ?

A	• —	N	— •
B	— • • •	O	— — — —
C	— • — — •	P	• — — — •
D	— • •	Q	— — • —
E	•	R	• — •
F	• • — — •	S	• • •
G	— — — •	T	—
H	• • • •	U	• • —
I	• •	V	• • • —
J	• — — — —	W	• — —
K	— • —	X	— • • —
L	• — — • •	Y	— — • — —
M	— —	Z	— — — • •

codeword for S
is a prefix of
codeword for V

In practice. Use a short gap to separate characters.

Variable-length codes

Q. How do we avoid ambiguity?

A. Ensure that no codeword is a **prefix** of another.

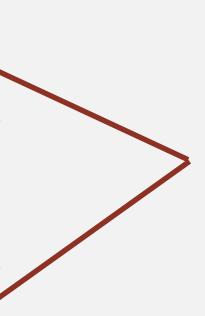
Ex 1. Fixed-length code.

Ex 2. Append special “stop” character to each codeword.

Ex 3. General prefix-free code.

<i>char</i>	<i>codeword</i>
!	101
A	11
B	00
C	010
D	100
R	011

no codeword
is a prefix of another



Prefix-free codes: compression

Q. How to represent the prefix-free code for compression?

A. A symbol table or array.

<i>char</i>	<i>codeword</i>
!	101
A	11
B	00
C	010
D	100
R	011

↑ ↑
key/index value

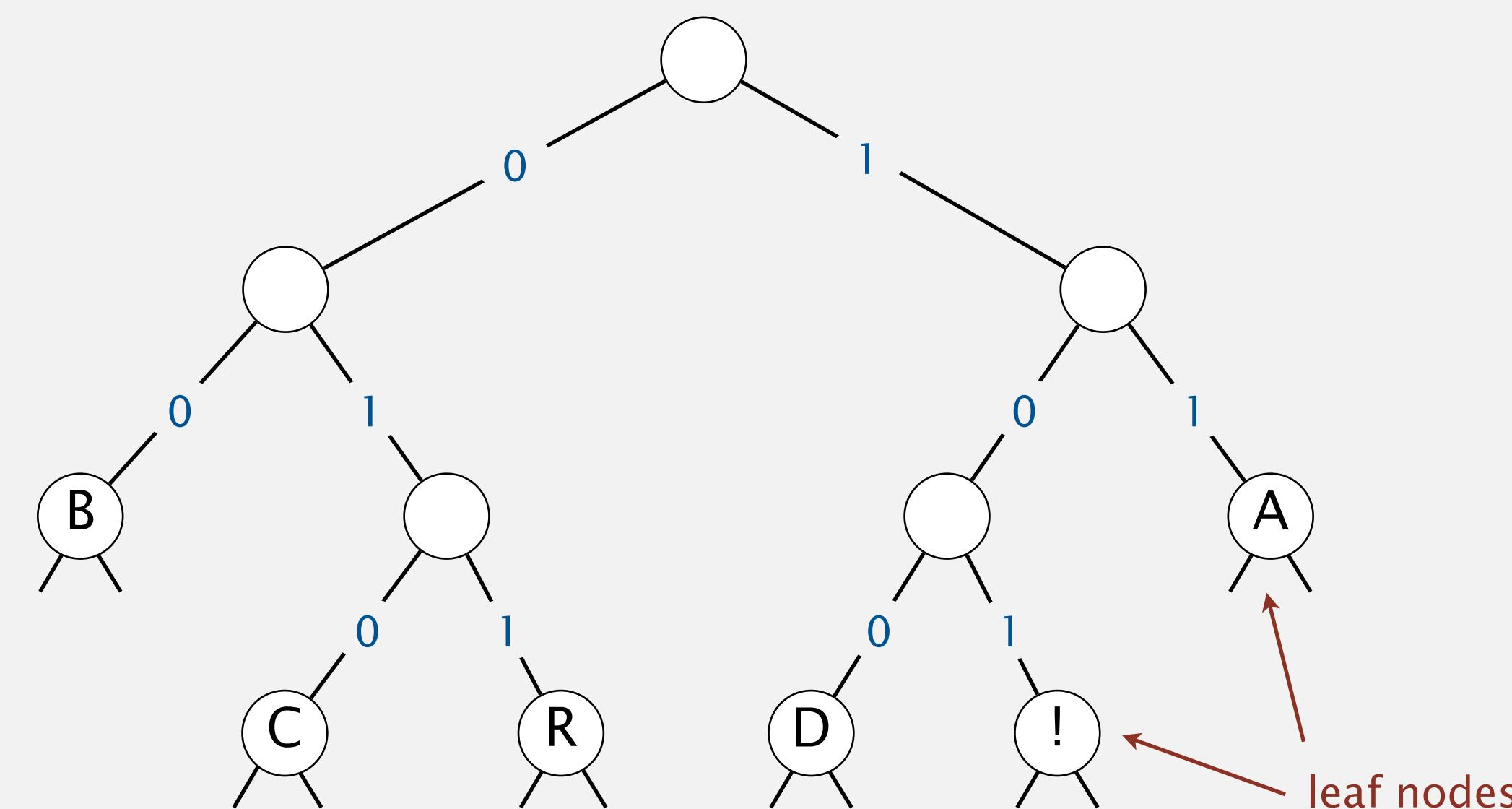
Prefix-free codes: trie representation

Q. How to represent the prefix-free code for expansion?

A. A **binary trie**.

- Characters in leaves.
- Codeword is path from root to leaf.

char	codeword
!	101
A	11
B	00
C	010
D	100
R	011



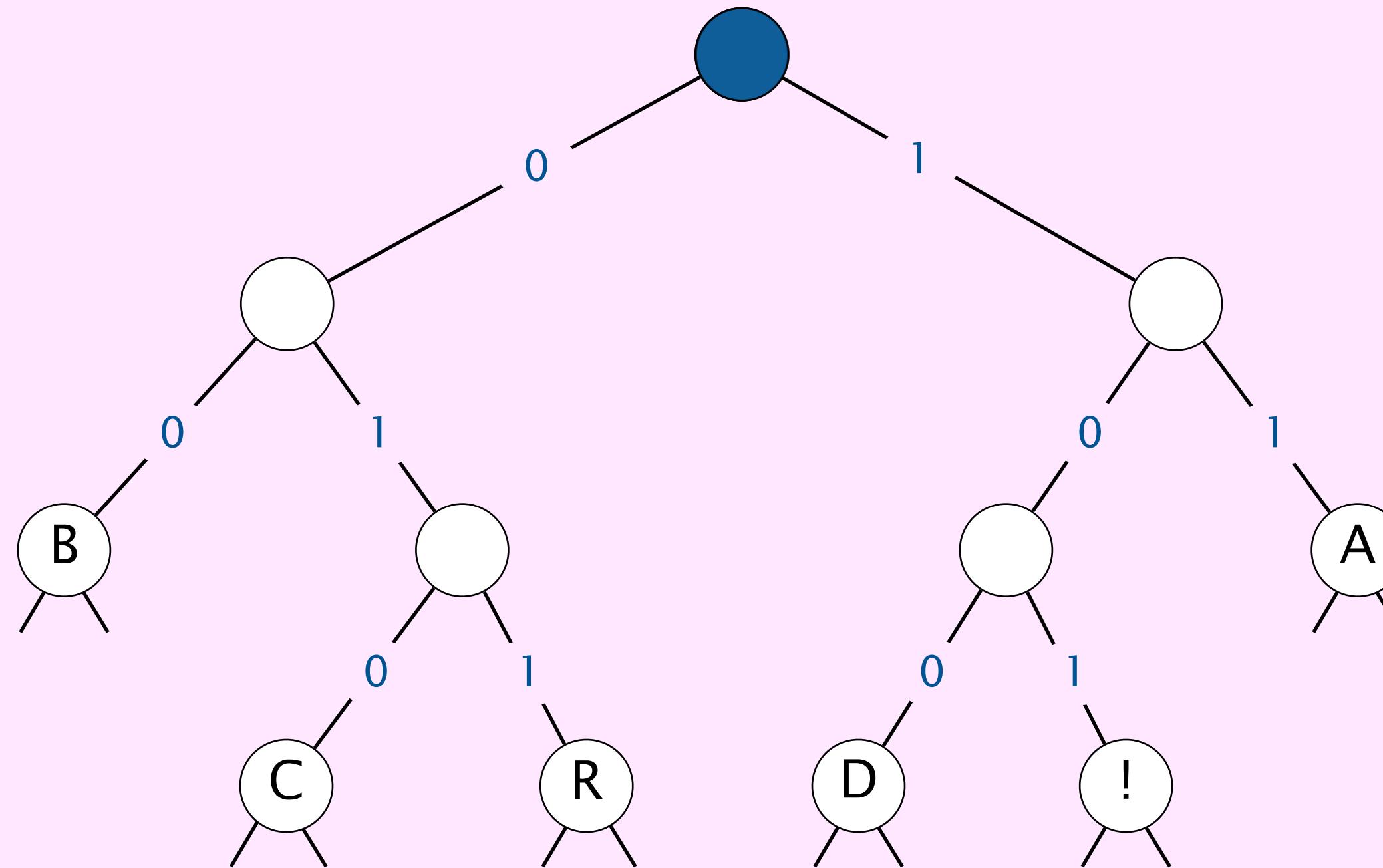
Prefix-free codes: expansion



Expansion.

- Start at root.
- Go left if bit is 0; go right if 1.
- If leaf node, write character and restart at root node.

 1 1 0 0 0 1 1 1 1 0 1 1 0 1 0 1 1 1 0 0 1 1 0 0 0 1 1 1 1 0 1
A B R A C A D A B R A !

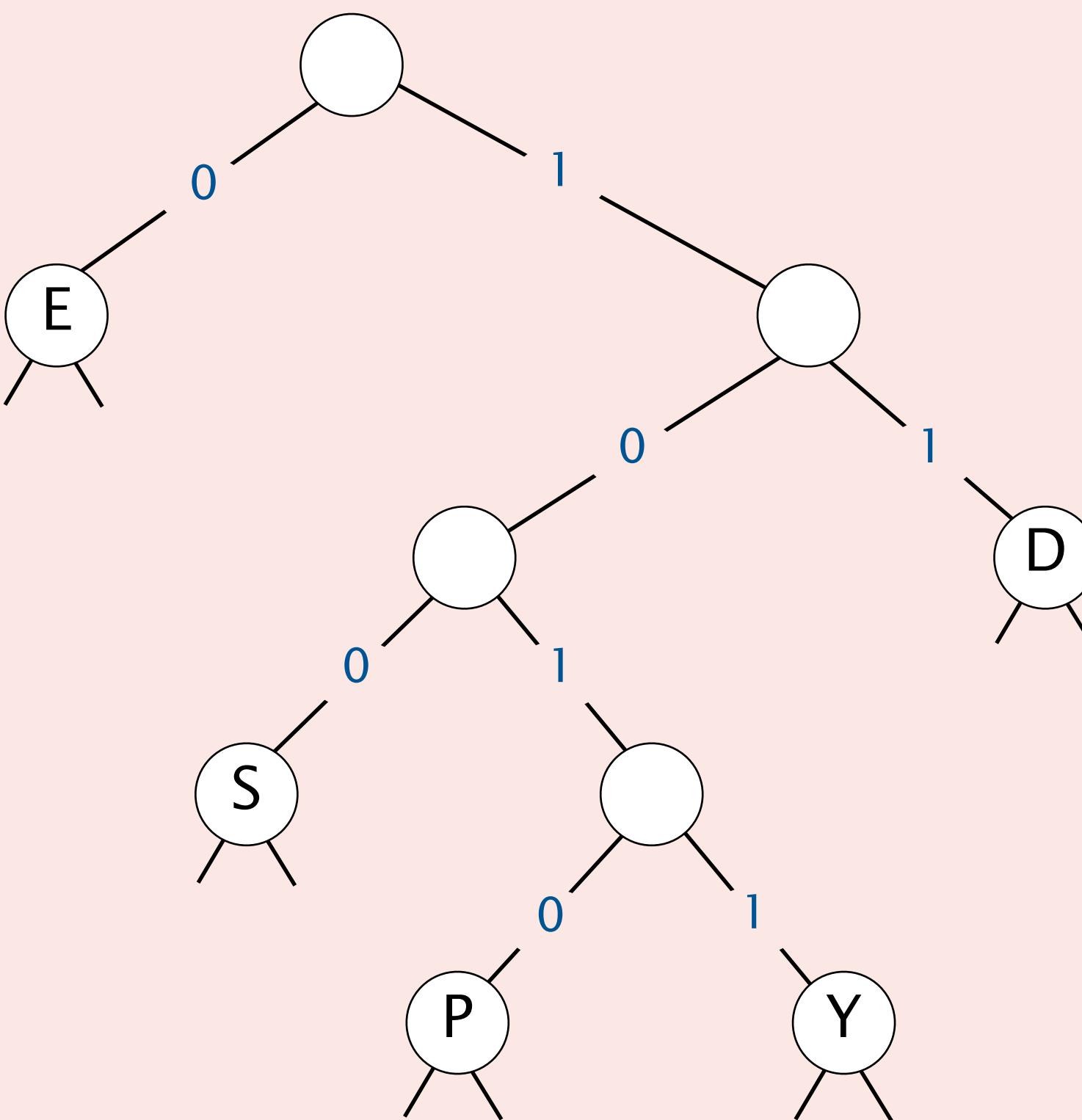




Consider the following trie representation of a prefix-free code.

Expand the compressed bitstring 1 0 0 1 0 1 0 0 0 1 1 1 0 1 1 ?

- A. PEED
- B. PESDEY
- C. SPED
- D. SPEEDY



Prefix-free codes: expansion

```
public void expand()
{
    Node root = readTrie();
    int n = BinaryStdIn.readInt();

    for (int i = 0; i < n; i++)
    {
        Node x = root;
        while (!x.isLeaf())
        {
            if (!BinaryStdIn.readBoolean())
                x = x.left;
            else
                x = x.right;
        }
        BinaryStdOut.write(x.ch, 8);
    }
    BinaryStdOut.close(); ← don't forget this!
}
```

The diagram illustrates the flow of data for the `expand()` method. A vertical blue bar on the left separates the code from its annotations. Arrows point from the code to the right, labeled with descriptions of the I/O operations:

- `Node root = readTrie();` → read encoding trie
- `int n = BinaryStdIn.readInt();` → read number of encoded characters (32 bits)
- `for (int i = 0; i < n; i++)` → for each encoded character i
- `Node x = root;` → follow path from root to leaf to determine character
- `if (!BinaryStdIn.readBoolean())` → read 0 or 1 (1 bit)
- `BinaryStdOut.write(x.ch, 8);` → write character (8 bits)
- `BinaryStdOut.close();` ← don't forget this!

Running time. Linear in input size (number of bits).

Huffman compression overview

Static model. Use the same prefix-free code for all messages.

Dynamic model. Use a custom prefix-free code for each message.

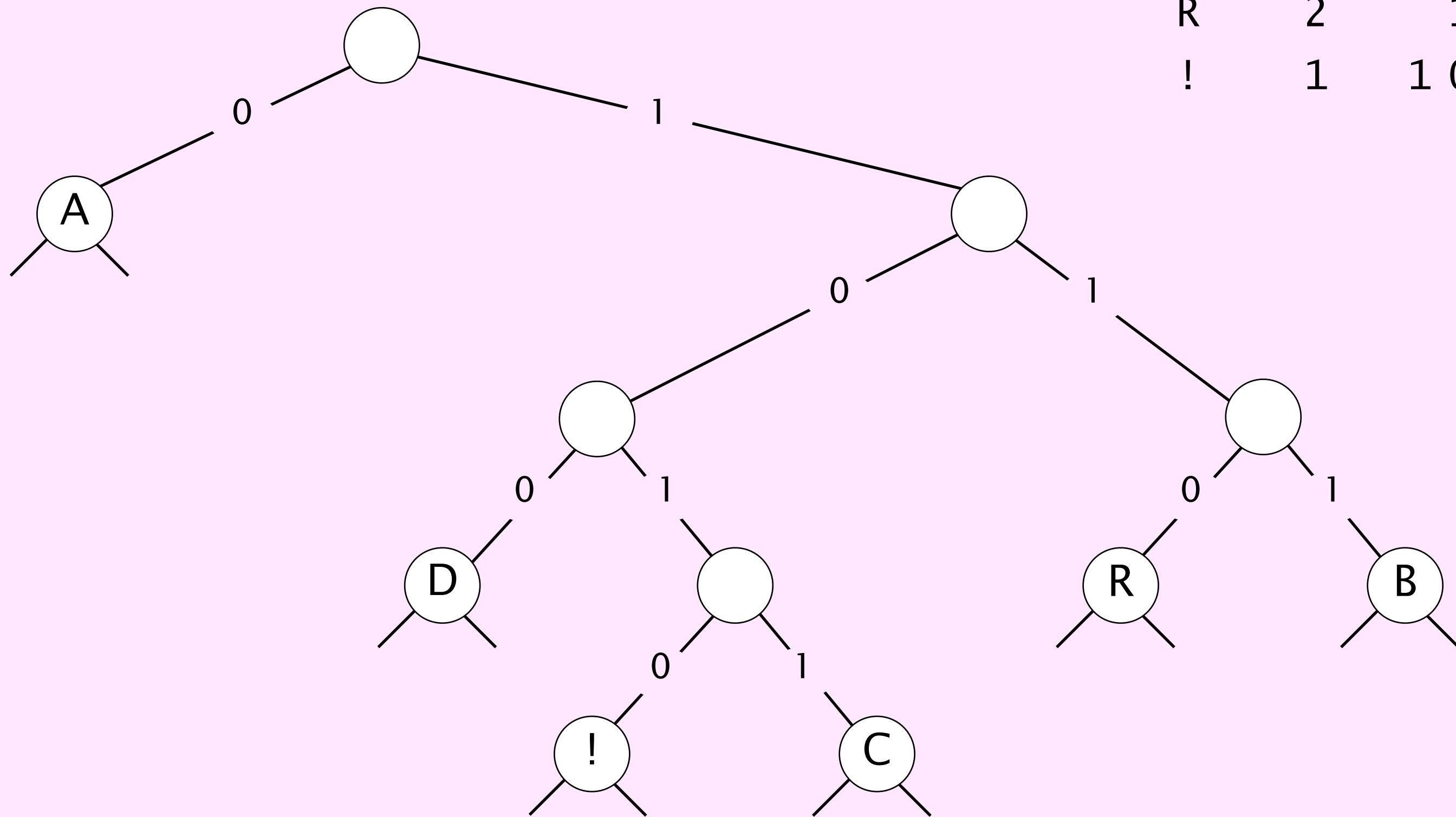
Compression.

- Read message.
- Build **best prefix-free code** for message using Huffman's algorithm. [next]
- Write prefix-free code.
- Compress message using prefix-free code.

Expansion.

- Read prefix-free code.
- Read compressed message and expand using prefix-free code.

Huffman's algorithm demo



char	freq	encoding
A	5	0
B	2	1 1 1
C	1	1 0 1 1
D	1	1 0 0
R	2	1 1 0
!	1	1 0 1 0

Huffman's algorithm

Huffman's algorithm:

- Count frequency $\text{freq}[c]$ of each character c in input.
- Start with one node corresponding to each character c (with weight $\text{freq}[c]$).
- Repeat until single trie formed:
 - select two tries with min weight $\text{freq}[i]$ and $\text{freq}[j]$
 - merge into single trie with weight $\text{freq}[i] + \text{freq}[j]$

Proposition. Huffman's algorithm computes an **optimal** prefix-free code for a given message.

Pf. See textbook.

no prefix-free code
uses fewer bits

Applications:



Constructing a Huffman trie: Java implementation

```
private static Node buildTrie(int[] freq)
{
    MinPQ<Node> pq = new MinPQ<Node>();
    for (char c = 0; c < R; c++)
        if (freq[c] > 0)
            pq.insert(new Node(c, freq[c], null, null));
}

while (pq.size() > 1)
{
    Node x = pq.delMin();
    Node y = pq.delMin();
    Node parent = new Node('\0', x.freq + y.freq, x, y);
    pq.insert(parent);
}

return pq.delMin();
}
```

initialize PQ with singleton tries

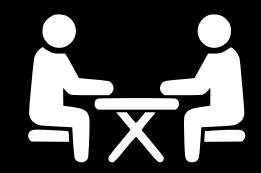
merge two smallest tries

not used for internal nodes

total frequency

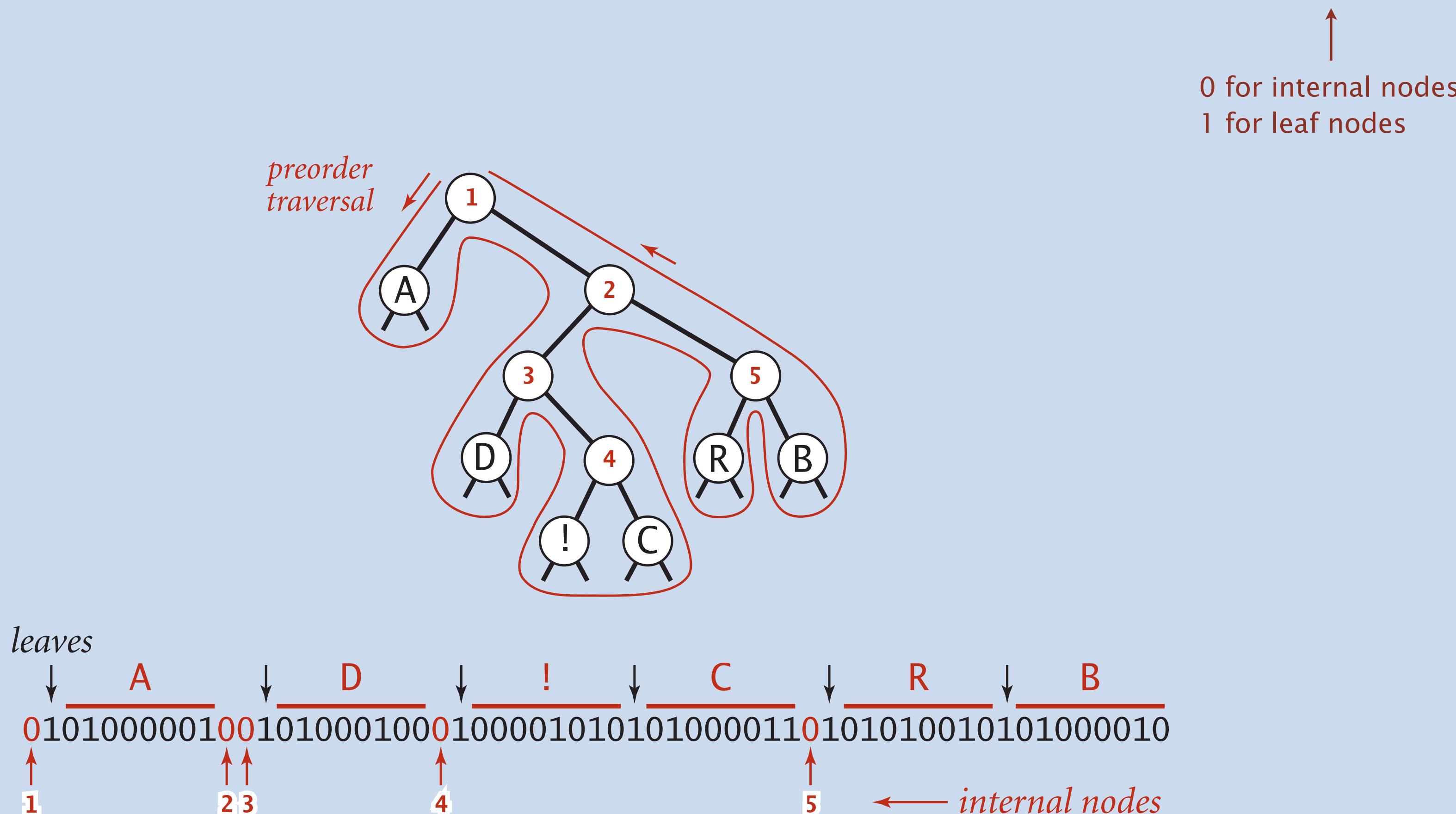
two subtrees

ENCODING THE BINARY TRIE



Q. How to transmit the binary trie?

A. Write preorder traversal; mark leaf nodes and internal nodes with a bit.





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- ▶ *Huffman compression*
- ▶ ***LZW compression***

Statistical methods

Static model. Same model for all messages.

- Fast but not optimal: different messages have different statistical properties.
- Ex: ASCII, Morse code.

Dynamic model. Generate model based on message.

- Preliminary pass needed to generate model; must transmit the model.
- Ex: Huffman code.

Adaptive model. Progressively learn and update model as you read message.

- More refined modeling can produce better compression.
- Ex: LZW.

LZW compression demo (for 7-bit chars and 8-bit codewords)

<i>input</i>	A	B	R	A	C	A	D	A	B	R	A	B	R	A	B	R	A
<i>matches</i>	A	B	R	A	C	A	D	A	B	R	A	B	R	A	B	R	A
<i>value</i>	41	42	52	41	43	41	44	81		83		82		88		41	80

LZW compression for A B R A C A D A B R A B R A B R A

key	value
:	:
A	41
B	42
C	43
D	44
:	:

key	value
AB	81
BR	82
RA	83
AC	84
CA	85
AD	86

key	value
DA	87
ABR	88
RAB	89
BRA	8A
ABRA	8B

codeword table

Input.

- 7-bit ASCII chars.
- ASCII 'A' is 41_{16} .

Codeword table.

- 8-bit codewords.
- Codewords for single chars are ASCII values.
- Use codewords 81_{16} to FF_{16} for multiple chars.
- Stop symbol = 80_{16} .

LZW expansion demo (for 7-bit chars and 8-bit codewords)

<i>value</i>	41	42	52	41	43	41	44	81	83	82	88	41	80
<i>output</i>	A	B	R	A	C	A	D	A B	R A	B R	A B R	A	

LZW expansion for 41 42 52 41 43 41 44 81 83 82 88 41 80

key	value
:	:
41	A
42	B
43	C
44	D
:	:

key	value
81	AB
82	BR
83	RA
84	AC
85	CA
86	AD

key	value
87	DA
88	ABR
89	RAB
8A	BRA
8B	ABRA

codeword table

Input.

- 7-bit ASCII chars.
- ASCII 'A' is 41_{16} .

Codeword table.

- 8-bit codewords.
- Codewords for single chars are ASCII values.
- Use codewords 81_{16} to FF_{16} for multiple chars.
- Stop symbol = 80_{16} .



Data compression: quiz 4

Which is the LZW compression for ABABABA ?

- A. 41 42 41 42 41 42 80
- B. 41 42 41 81 81 80
- C. 41 42 81 81 41 80
- D. 41 42 81 83 80

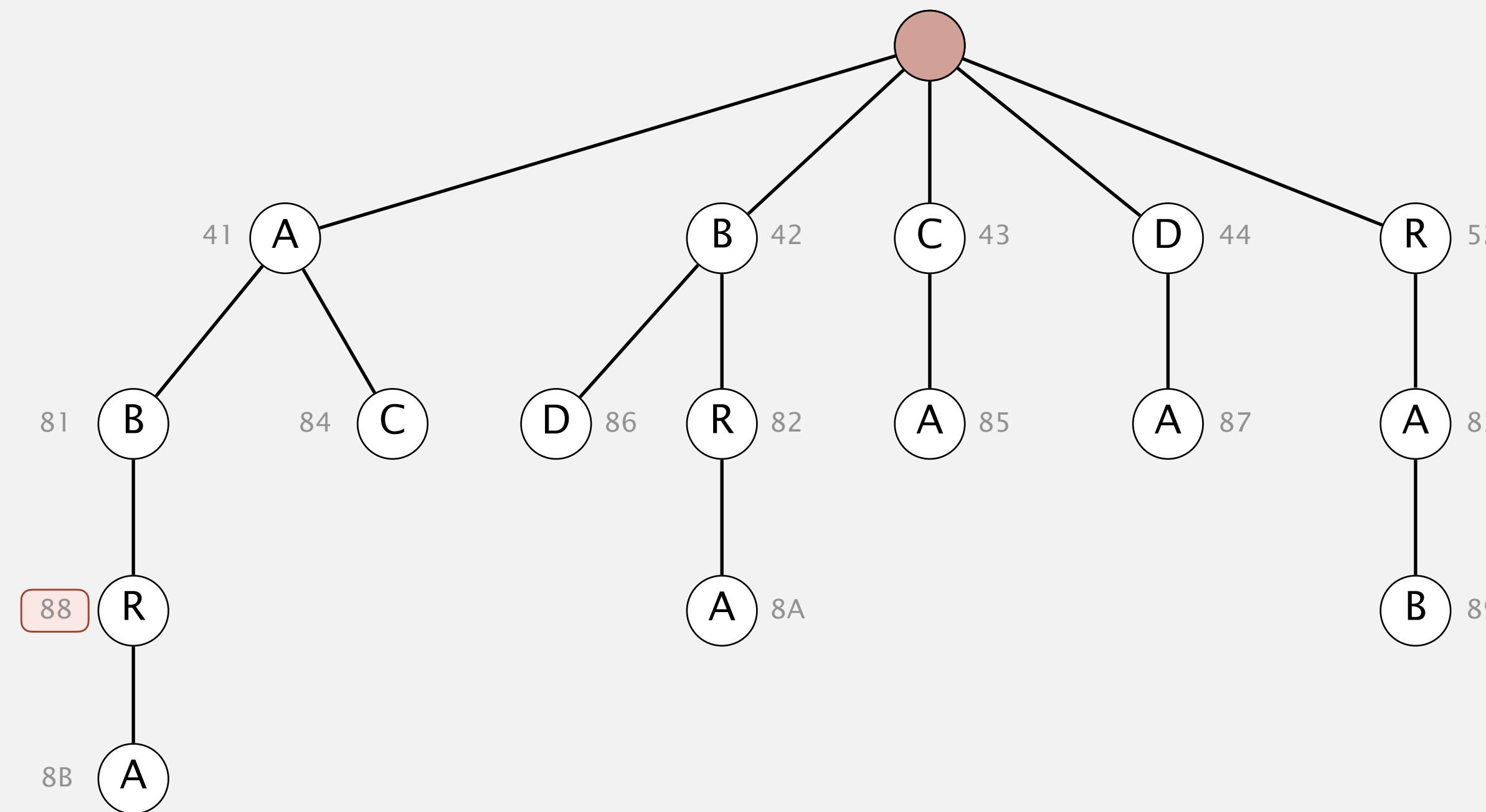
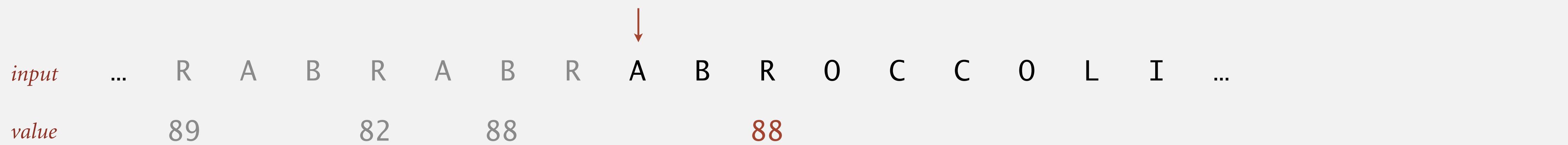


Which is the key data structure to implement LZW compression efficiently?

- A. Array.
- B. Red-black BST.
- C. Hash table.
- D. Trie.

Implementing LZW compression: longest prefix match

Find longest key in symbol table that is a prefix of query string.



LZW tricky case: expansion

value	41	42	81	83	80
output	A	B	A B	A B A	
<hr/> <i>codeword for 83</i>					

LZW expansion for 41 42 81 83 80

need to know code for 83
before it is in codeword table!

we can deduce that
the code for 83 is ABx
for some character x

now, we have deduced x!

key	value
:	:
41	A
42	B
43	C
44	D
:	:

key	value
81	AB
82	BA
83	ABA

codeword table

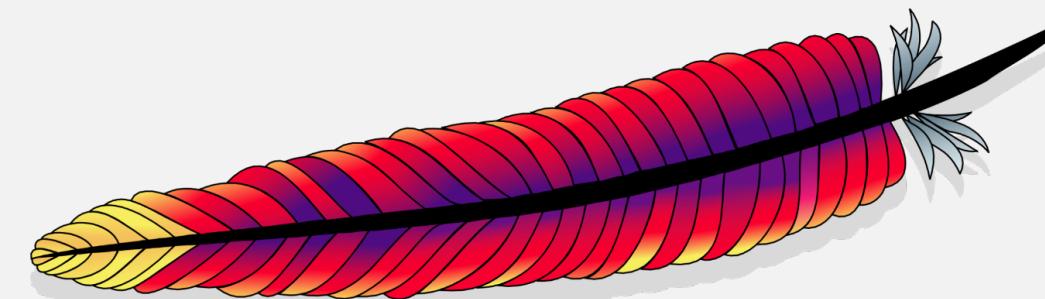
LZW in the real world

Lempel–Ziv and friends.

- LZ77.
- LZ78.
- LZW.
- Deflate / zlib = LZ77 variant + Huffman.

Unix compress, GIF, TIFF, V.42bis modem: LZW. ← previously under patent

zip, 7zip, gzip, jar, png, pdf: deflate / zlib.
iPhone, Wii, Apache HTTP server: deflate / zlib. | ← not patented
(widely used in open source)



Apache Web Server

Lossless data compression benchmarks

year	scheme	bits / char
1967	ASCII	7
1950	Huffman	4.7
1977	LZ77	3.94
1984	LZMW	3.32
1987	LZH	3.3
1987	move-to-front	3.24
1987	LZB	3.18
1987	gzip	2.71
1988	PPMC	2.48
1994	SAKDC	2.47
1994	PPM	2.34
1995	Burrows-Wheeler	2.29
1997	BOA	1.99
1999	RK	1.89

← next programming assignment

Data compression summary

Lossless compression.

- Represent fixed-length symbols with variable-length codes. [Huffman]
- Represent variable-length symbols with fixed-length codes. [LZW]

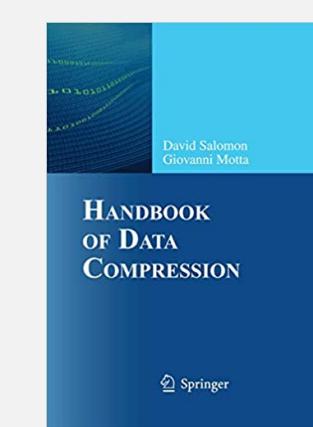
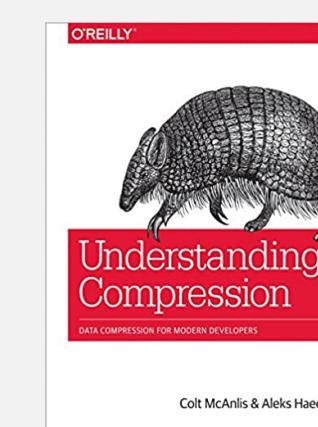
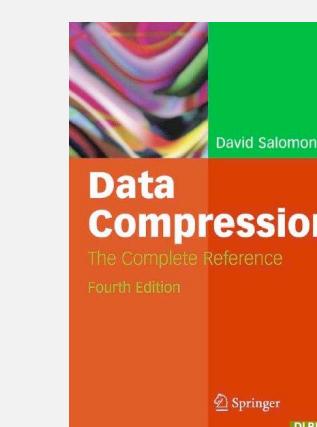
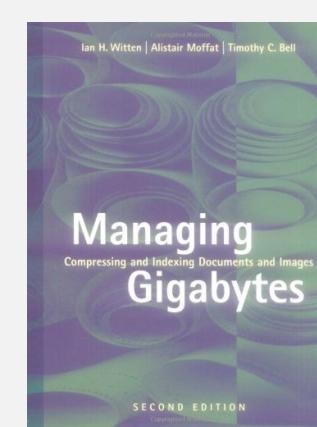
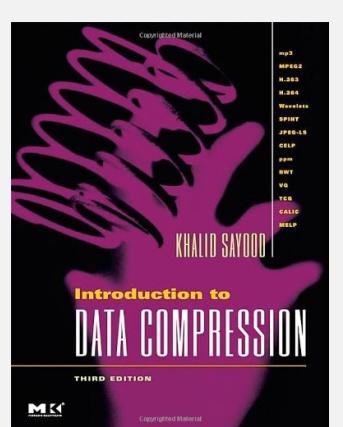
Lossy compression. [not covered in this course]

- JPEG, MPEG, MP3, ...
- FFT/DCT, wavelets, fractals, ...

$$X_k = \sum_{i=0}^{n-1} x_i \cos \left[\frac{\pi}{n} \left(i + \frac{1}{2} \right) k \right]$$

Theoretical limits on compression. Shannon entropy: $H(X) = - \sum_i^n p(x_i) \log_2 p(x_i)$

Practical compression. Exploit extra knowledge whenever possible.



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