Homework 3

Convex Optimization

Due March 5

1. Given symmetric matrices $F_0, F_1, ..., F_n$,cast the following optimization problem as an SDP

$$\begin{array}{ll} \min & (Ax+b)^T F(x)^{-1}(Ax+b) \\ \text{s.t.} & F(x) \succ 0 \end{array}$$

where $F(x) = F_0 + x_1 F_1 + \cdots + x_n F_n$

2. Given symmetric matrices W0 and W1, find the dual of the optimization min

$$\begin{array}{ll} \min & x^T W_0 X \\ \text{s.t.} & x^T W_1 x \leq 1. \end{array}$$

3. Use the duality idea to prove that the set $\{x \mid x \in \mathbb{R}^n, Ax \leq b\}$ is empty if the set

$$\left\{\lambda \mid \lambda \in \mathbb{R}^m, \lambda \geq 0, A^T\lambda = 0, b^T\lambda < 0\right\}$$

is nonempty (where $A \in \mathbb{R}^{m\times n}$).

4. *Separating hyperplane between two polyhedra*: formulate the following problem as an LP (feasibility) problem. Find a separating hyperplane that strictly separates two polyhedra

$$P_1 = \{x \mid Ax \preceq b\}, \quad P_2 = \{x \mid Cx \preceq d\}$$

then find a vector $a \in \mathbb{R}^n$ and a scalar γ such that

$$a^T x > \gamma \quad \forall x \in P_1, \quad a^T x < \gamma \quad \forall x \in P_2$$

Hence $\inf_{x\in P_2} a^T x > \gamma > \sup_{x\in P_2} a^T x$ Use LP duality to simplify the infimum and supremum in these conditions.