509 Homework 2 Haowong Wang mw814

Q1. (a) Equivalent to the LP: minimize $1^T y$

subject to $-y \le Ax-b \le y$ $(x \in R^n$ and $y \in R^m)$

$-1 \le x \le 1$

(b) Equivalent to the LP: minimize $1^T y$

subject to $-y \le x \le y$ $(x \in R^n$ and $y \in R^n)$

$-1 \le Ax-b \le 1$

(c) Equivalent to the LP: minimize $1^T y + t$

subject to $-y \le Ax-b \le y$ $(x \in R^n,\ y \in R^m,\ t \in R)$

$-t1 \le x \le t1$

Q2. Adding slack variables t_i for $i=1,\cdots,m$, the problem can be converted to

$$\min_{x,t} \sum_{i=1}^{m} t_i^2$$

$$\text{s.t. } (a_i^T x - b_i)^2 \le t_i,\ i=1,\cdots,m.$$

which is a convex QCQP

Q3. $a_i^T x + b_i = 0$ $i=1,2,\cdots,m$ $\Rightarrow$ $a_i^T x + b_i \le 0$ & $a_i^T x + b_i \ge 0$ $i=1,\cdots,m$

$c_i^T x + d_i \le 0,$ $i=1,2,\cdots,k$ $\qquad$ $c_i^T x + d_i \le 0$ $i=1,\cdots,k$

To transform it into SDP, $[A_0\ A_1\ \cdots\ A_n]\begin{bmatrix} 1 \\ x_1 \\ \vdots \\ x_n \end{bmatrix} \le 0$

We have A_t $\begin{cases} \begin{bmatrix} b_i \\ d_i \\ -b_i \end{bmatrix} & (t=0) \\ \begin{bmatrix} a_i^T \\ c_i^T \\ -a_i^T \end{bmatrix} & (t=1,2,\cdots,n) \end{cases}$

Q4. Since $f(x)$ is convex, consider FOC: $f(x) \ge f(x^*) + \nabla f(x^*)^T (x-x^*)$

$\Rightarrow f(x) \ge f(x^*) + \nabla f(x^*)^T x - \nabla f(x^*)^T x^*$

$\because [\nabla f(x^*)]_i x_i^* = 0$ $\quad\therefore f(x) \ge f(x^*) + \nabla f(x^*)^T x$

$\because x \ge 0,\ \nabla f(x^*) \ge 0$ $\quad\therefore \nabla f(x^*)^T x \ge 0$

$\therefore f(x^*) + \nabla f(x^*)^T x \ge f(x^*)$ $\quad\therefore f(x) \ge f(x^*)$ $\quad\therefore x^*$ is optimal solution