Homework 2

Convex Optimization

Due February 19

1. Given $A \in \mathbb{R}^{m \times n}$ and $b \in \mathbb{R}^m$, cast each of the following problems as LP

- min $\quad \|Ax - b\|_1 \quad$ s.t. $\quad \|x\|_\infty \leq 1$
- min $\quad \|x\|_1 \quad$ s.t. $\quad \|Ax - b\|_\infty \leq 1$
- min $\quad \|Ax - b\|_1 + \|x\|_\infty$

2. Consider the L4-norm approximation problem:

$$\min \quad \|Ax - b\|_4 = \left[\sum_{i=1}^{m} (a_i^t x - b_i)^4 \right]^{1/4}$$

where $A \in \mathbb{R}^{m \times n}$ and $b \in \mathbb{R}^m$. Formulate this problem as a QCQP.

3. Consider the LP problem:

$$\begin{aligned} \min \quad & e^T x + f \\ \text{s.t.} \quad & a_i^T x + b_i = 0, \quad i = 1, \cdots, m \\ & c_i^T x + d_i \leq 0, \quad i = 1, cdots, k \end{aligned}$$

Find $A_0, \cdots, A_n$ to formulate this problem as a SDP:

$$\begin{aligned} \min \quad & e^T x + f \\ \text{s.t.} \quad & A_0 + A_1 x_1 + \cdots + A_n x_n \preceq 0 \end{aligned}$$

4. Consider the optimization problem

$$\min \quad f(x) \quad \text{s.t.} \quad x \geq 0$$

where f is convex. Let x^* be a point such that

$$x^* \geq 0, \quad \nabla f(x^*) \geq 0, \quad [\nabla f(x^*)]_i \, x_i^* = 0, \quad i = 1, \cdots, n$$

Prove that x^* is a solution of the optimization problem.