

# DSA Homework 1 Report

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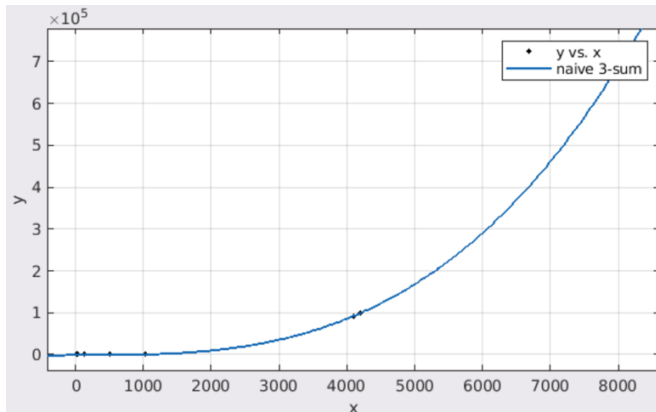
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## Question 1

Size	$O(N^3)(ms)$	$O(N^2 \lg N)(ms)$
8	0.004	0.005
32	0.049	0.071
128	3.274	0.99
512	193.615	20.405
1024	1548.38	78.406
4096	92434.9	1419.19
4192	100535	1500.41
8192	742342	5962.08

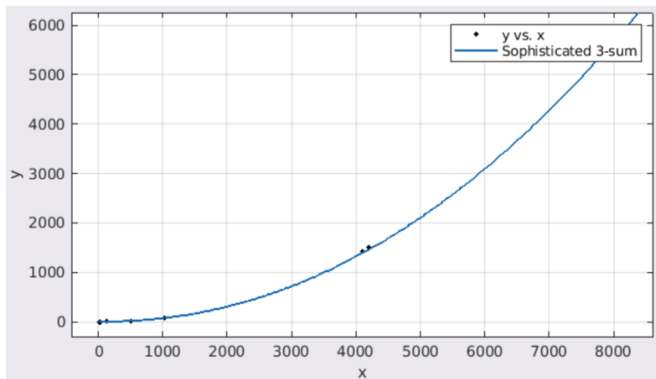
In question 1, I took advantage of the cftool in MATLAB. Detailed analysis will be shown in question 3.

### 1. Naive 3-sum



From this figure and the data, I believe the naive 3-sum algorithm is a  $O(N^3)$  algorithm.

### 2. Sophisticated 3-sum



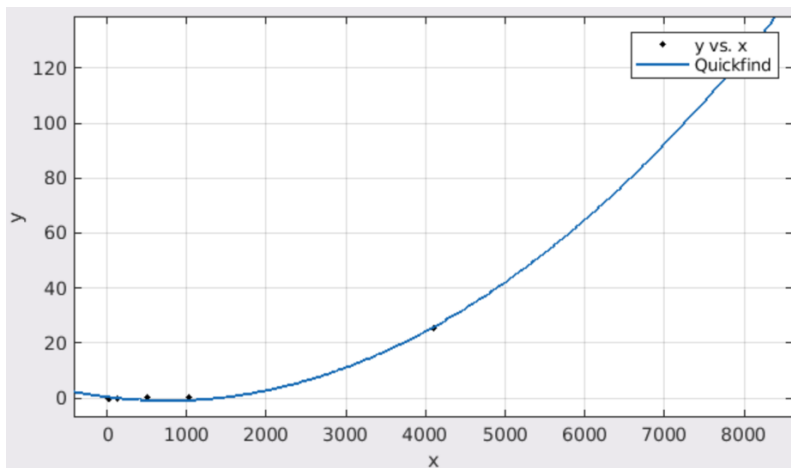
From this figure and the data, I believe the sophisticated 3-sum is a  $O(N^2 \lg N)$  algorithm.

## Question 2

Size	Quick find (ms)	Quick union (ms)	Weighted quick union (ms)
8	0.011	0.001	0.001
32	0.014	0.002	0.001
128	0.017	0.004	0.003
512	0.409	0.073	0.023
1024	0.425	0.312	0.031
4096	25.273	8.72	0.165
8192	131.815	6.055	0.378

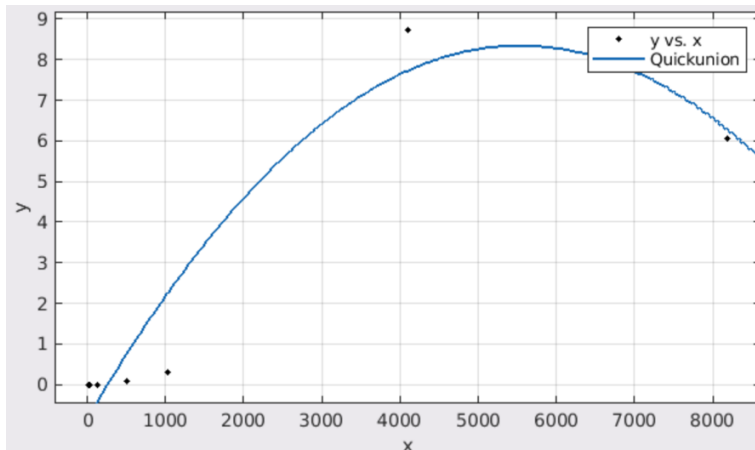
In question 2, I also used the cftool to finish the curve fitting of the data. Detailed analysis will be shown in question 3.

### 1. Quick find



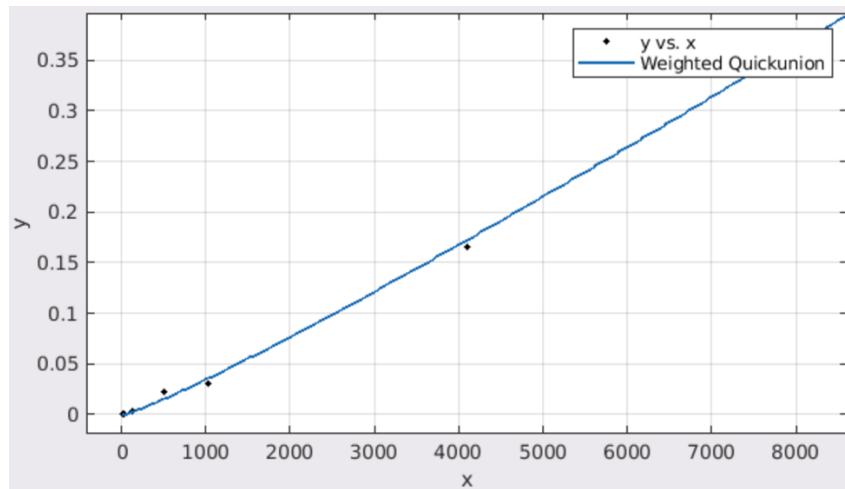
In the curve fitting, I tried  $O(N)$  and  $O(N^2)$  and they both worked well, but the SSE of  $O(N^2)$  is smaller, so I think quick find is a  $O(N^2)$  algorithm in the worst case.

### 2. Quick union



It is obvious that quick union is at least a  $O(MN)$  algorithm and it could be  $O(N^2)$  in the worst case.

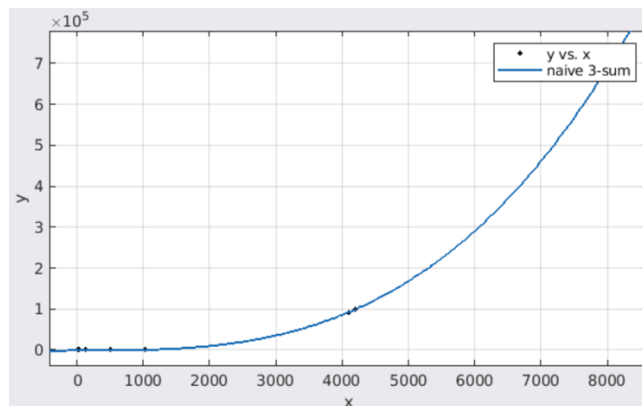
### 3. Weighted quick union



From this figure and the data, we can tell that weighted quick union is a  $O(M \cdot \lg N)$  algorithm. In the worst case, it could be  $O(N \cdot \lg N)$ .

### Question 3

#### 1. Naive 3-sum

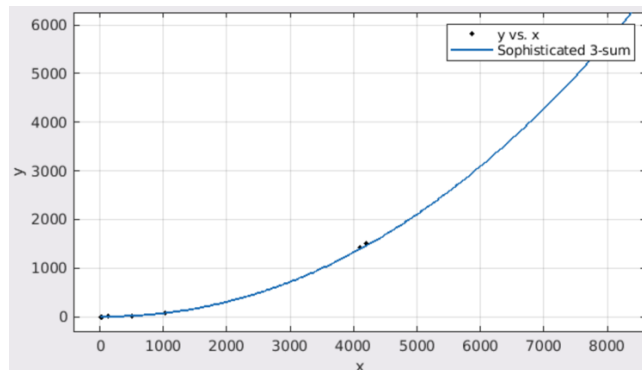


General model:  
 $f(x) = a \cdot x^3$   
 Coefficients (with 95% confidence bounds):  
 $a = 1.35e-06$  (1.349e-06, 1.352e-06)  
 Goodness of fit:  
 SSE: 1.26e+06  
 R-square: 1  
 Adjusted R-square: 1  
 RMSE: 424.3

Suppose that  $f(N) = 1.35 \cdot 10^{-6} \cdot N^3$ .

Assume that  $g(N) = N^3$  and  $c = 1$ . To get  $f(N) < c \cdot g(N)$  ( $N > N_c$ ), we can set  $N_c = 1$ .

#### 2. Sophisticated 3-sum

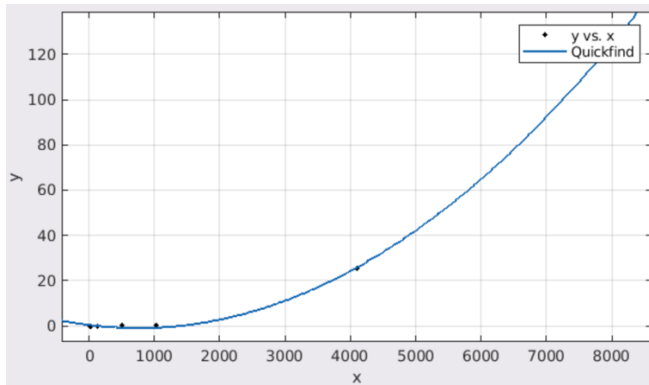


General model:  
 $f(x) = a \cdot x^2 \cdot \log_2(x) + b$   
 Coefficients (with 95% confidence bounds):  
 $a = 6.838e-06$  (6.764e-06, 6.912e-06)  
 $b = 13.06$  (-10.97, 37.08)  
 Goodness of fit:  
 SSE: 3477  
 R-square: 0.9999  
 Adjusted R-square: 0.9999  
 RMSE: 24.07

Suppose that  $f(N) = 6.838 \cdot 10^{-6} \cdot N^2 \cdot \log(N) + 13.06$ .

Assume that  $g(N) = N^2 \cdot \log(N)$  and  $c = 7$ . To get  $f(N) < c \cdot g(N)$  ( $N > N_c$ ), we can set  $N_c = 10$ .

### 3. Quick find



Linear model Poly2:

$$f(x) = p1 \cdot x^2 + p2 \cdot x + p3$$

Coefficients (with 95% confidence bounds):

$p1 = 2.401e-06$  (2.236e-06, 2.566e-06)

$p2 = -0.003679$  (-0.005013, -0.002345)

$p3 = 0.6929$  (-0.5789, 1.965)

Goodness of fit:

SSE: 3.127

R-square: 0.9998

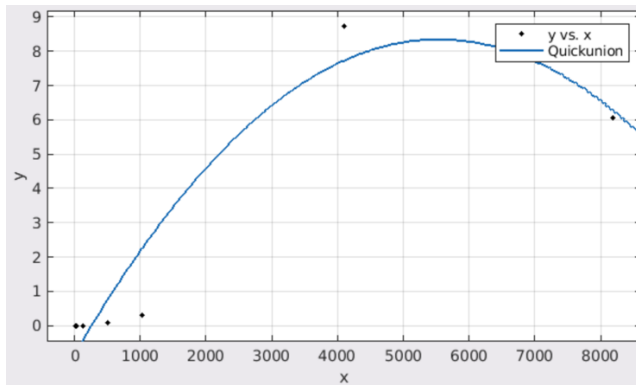
Adjusted R-square: 0.9997

RMSE: 0.8841

Suppose that  $f(N) = 2.401 \cdot 10^{-6} \cdot N^2 - 0.0036 \cdot N + 0.6929$ .

Assume that  $g(N) = N^2$  and  $c = 3$ . We can get  $N_c = 1$ .

### 4. Quick union



Linear model Poly2:

$$f(x) = p1 \cdot x^2 + p2 \cdot x + p3$$

Coefficients (with 95% confidence bounds):

$p1 = -2.986e-07$  (-5.388e-07, -5.842e-08)

$p2 = 0.003311$  (0.001367, 0.005255)

$p3 = -0.8164$  (-2.669, 1.037)

Goodness of fit:

SSE: 6.637

R-square: 0.917

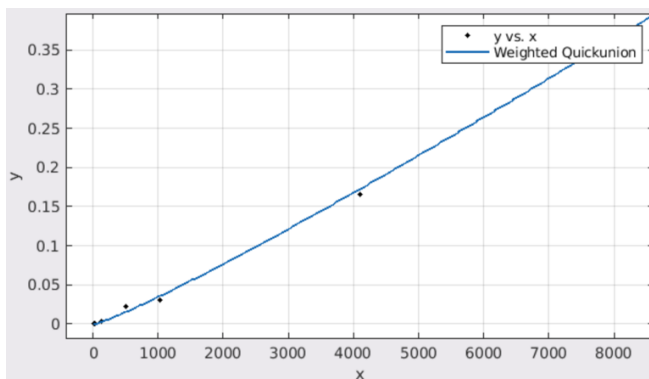
Adjusted R-square: 0.8755

RMSE: 1.288

Suppose that  $f(N) = -2.986 \cdot 10^{-7} \cdot N^2 + 0.003311 \cdot N - 0.8164$ .

Assume that  $g(N) = N^2$  and  $c = -1$ . We can get  $N_c = 1$ .

### 5. Weighted quick union



General model:

$$f(x) = a \cdot x \cdot \log_2(x) + b$$

Coefficients (with 95% confidence bounds):

$a = 3.517e-06$  (3.375e-06, 3.658e-06)

$b = -0.0001949$  (-0.000648, 0.0006091)

Goodness of fit:

SSE: 0.0001459

R-square: 0.9988

Adjusted R-square: 0.9985

RMSE: 0.005401

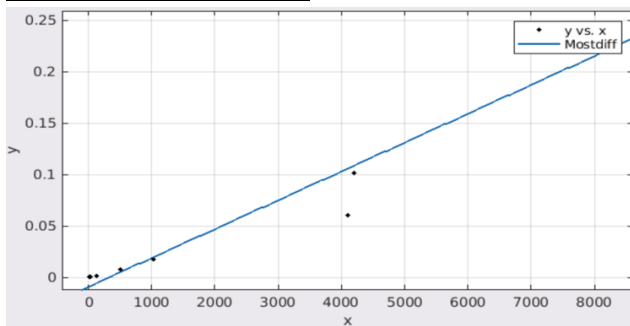
Suppose that  $f(N) = 3.517 \cdot 10^{-6} \cdot N \cdot \log(N) - 0.0001949$ .

Assume that  $g(N) = N \cdot \log(N)$  and  $c = 4$ . We can get  $N_c = 2$ .

#### Question 4

The data I used in this question is as same as question 1.

Size	Time(ms)
8	0.001
32	0.001
128	0.002
512	0.008
1024	0.018
4096	0.061
4192	0.102
8192	0.247



Linear model Poly1:

$$f(x) = p1 \cdot x + p2$$

Coefficients (with 95% confidence bounds):

p1 = 2.805e-05 (2.108e-05, 3.501e-05)

p2 = -0.008749 (-0.03372, 0.01622)

Goodness of fit:

SSE: 0.002989

R-square: 0.9418

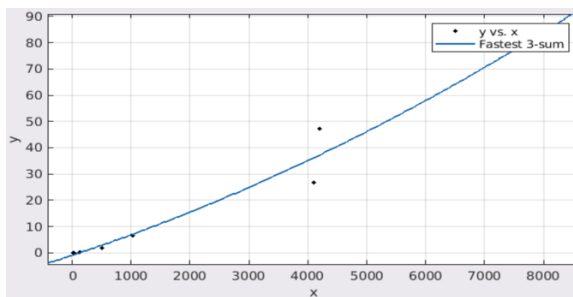
Adjusted R-square: 0.9321

RMSE: 0.02232

It is obvious that this is a linear algorithm.

#### Question 5

Size	Time(ms)
8	0.002
32	0.009
128	0.137
512	1.751
1024	6.645
4096	26.679
4192	47.309
8192	86.579



Linear model Poly2:

$$f(x) = p1 \cdot x^2 + p2 \cdot x + p3$$

Coefficients (with 95% confidence bounds):

p1 = 4.021e-07 (-4.748e-07, 1.279e-06)

p2 = 0.007409 (0.000537, 0.01428)

p3 = -0.8685 (-9.066, 7.329)

Goodness of fit:

SSE: 195.2

R-square: 0.9718

Adjusted R-square: 0.9605

RMSE: 6.248

Apparently, this is  $O(N^2)$  algorithm, which is faster than the two previous algorithms.