School of Engineering
Department of Electrical and Computer Engineering

**332:494:01/599:02 – Smart Grid – spring 2021**
**Homework Assignment – Set 1**

**General guidelines for homework assignments:** Homework should be submitted online (via Canvas under the 'assignment' Homework 1)

## Question 1:

For the **single-phase** system in Figure 1, it is given that the load $Z_{Load}$ is consuming $16kVA$ at a $pf = 0.6\ leading$. The voltage across the load $\boldsymbol{V}_{Load} = 120\angle 0^{\circ} V(rms)$. The line impedance is $Z_{Line} = 1 + j2\Omega$

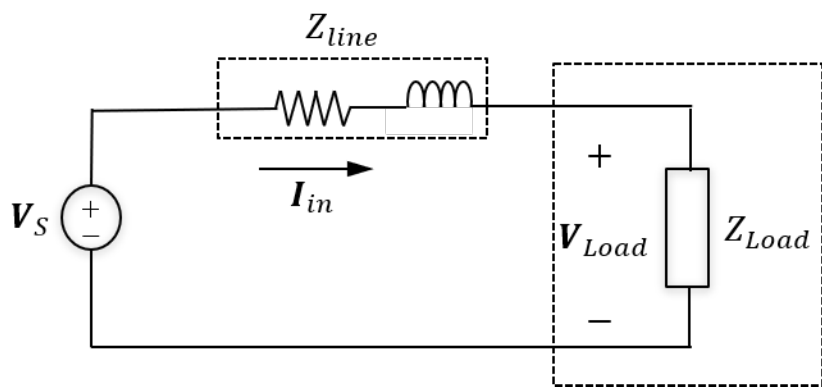



Figure 1

(a) For the load impedance, $Z_{Load}$, draw the power triangle and find (**include units** for each of the first three values): $P_{Load}$; $Q_{Load}$; $|S_{Load}|$; $S_{Load}$ and the power factor angle $\theta_{pf}$

$$|\boldsymbol{S_{Load}}| = \boldsymbol{16\ kVA}$$
$$pf = 0.6\ lead \rightarrow \theta_{pf} = -cos^{-1}(0.6) = \boldsymbol{-53.13}^{\circ}$$
$$\rightarrow P_{Load} = |S_{Zload}| \cdot pf = \boldsymbol{9.6kW};\ \boldsymbol{P_{Load} = 9.6kW}$$
$$Q_{Load} = |S_{Zload}| \cdot sin(-53.13^{\circ}) = -12.8\ \boldsymbol{kVAr}$$
$$\boldsymbol{Q_{Load} = -12.8\ kVAr}$$

$$S_{Load} = 9.6kW - j12.8kVAr = 16kVA\angle{-53.13^{\circ}}$$

(b) Find the load impedance: $Z_{load}$

$$Z_{load} = \frac{|V_{Load}|^2}{S_{Load}^*} = \frac{|120|^2}{16kVA\angle 53.13°} = 0.9\angle{-53.13°}\Omega$$

$$Z_{load} = 0.54 - j0.72\ \Omega$$

(c) Find the power losses on the line impedance $S_{line}(Z_{line})$

$$Z_{Line} = 1 + j2\Omega$$

$$I_{Line} = I_{Load} = \frac{S_{Load}^*}{V_{Load}^*} = \frac{16kVA\angle 53.13°}{120\angle 0^\circ} = 133.34\angle{-53.13°}A$$

$$S_{line} = |I_{Line}|^2 \cdot Z_{Line} = (133.34)^2(1 + j2) = 17.778kW + j35.556kVAr$$

## Question 2:

Given the single-phase system in Figure 2, given that:

- $V_{Load} = 13{,}800\angle 0^\circ V_{rms}$
- Load 1: 60kW at 0.8 power factor **lead;**
- Load 2: 160 kVA at 0.5 power factor **lag**;

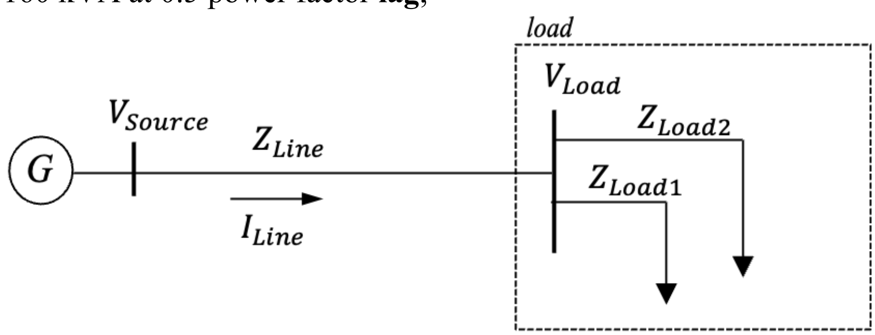



Figure 2

(a) Find and draw the power triangle for load $Z_{Load1}$: what is the real power $P_{Z1}$ and reactive power $Q_{Z1}$ consumed by load $Z_{Load1}$

$$\boldsymbol{P_{Z1} = 60\ kW};\ \theta_{pf1} = -cos^{-1}(0.8) = \boldsymbol{-36.87}°$$

$$\rightarrow |S_{Z1}| = \frac{P_{Z1}}{pf1} = 75\ \boldsymbol{kVA};\ \boldsymbol{|S_{Z1}| = 75\ kVA}$$

$$Q_{Z1} = P_{Z1} \cdot tan(-36.87°) = |S_{Z1}| \cdot sin(-36.87°) = \boldsymbol{-45\ kVAr}$$

$$\boldsymbol{Q_{Z1} = -45\ kVAr}$$

(b) Find and draw the power triangle for load $Z_{Load2}$: what is the real power $P_{Z2}$ and reactive power $Q_{Z2}$ consumed by load $Z_{Load2}$

$$|\boldsymbol{S_{Z2}}| = \mathbf{160\ kVA};\ \theta_{pf2} = cos^{-1}(0.5) = \mathbf{60^\circ}$$
$$\rightarrow P_{Z2} = |S_{Z2}| \cdot cos(60^\circ) = \mathbf{80kW};\ \boldsymbol{P_{Z2}} = \mathbf{80kW}$$
$$Q_{Z2} = |S_{Z2}| \cdot sin(60^\circ) = \mathbf{138.564\ kVAr}$$
$$\boldsymbol{Q_{Z2}} = \mathbf{138.564\ kVAr}$$

(c) Find the power factor (*pf*) for the total load $Z_{Load\_eq} = Z_{Load1}||Z_{Load2}$ and the total apparent power $|S_{Load\_eq}|$ in Figure 3

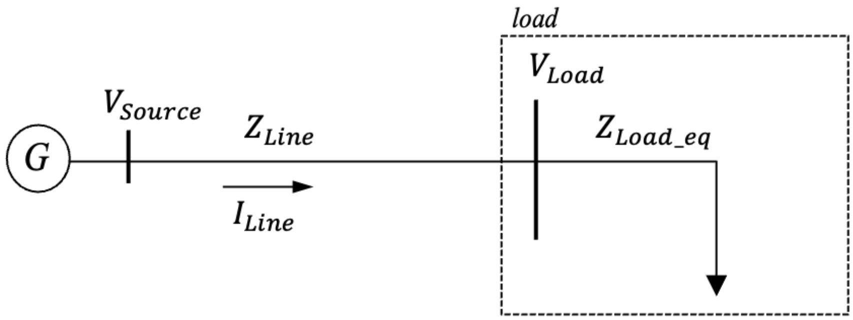



Figure 3

$$\rightarrow P_{Zeq} = P_{Z1} + P_{Z2} = \mathbf{140kW};$$
$$\rightarrow Q_{Zeq} = Q_{Z1} + Q_{Z2} = \mathbf{93.564\ kVAr}$$

$$\rightarrow |S_{Zeq}| = \sqrt[2]{P_{Zeq}^2 + Q_{Zeq}^2} = \mathbf{168.387kVA}$$
$$|\boldsymbol{S_{Z2}}| = \mathbf{168.387kVA}$$
$$\theta_{pf(eq)} = tan^{-1}\left(\frac{Q_{Zeq}}{P_{Zeq}}\right) = 33.755^\circ$$
$$pf_{eq} = \cos(33.755^\circ) = \mathbf{0.83}$$
$$\boldsymbol{pf_{eq}} = \mathbf{0.83}$$

(d) Find the line current phasor $\mathbf{I}_{line}$

$$I_{Line} = I_{Load} = \frac{S_{Zeq}^*}{V_{Load}^*} = \frac{168.387 \cdot 10^3 \angle -33.755^\circ}{13{,}800\angle 0^\circ} = 12.2\angle -33.755^\circ A$$

(e) A capacitor is added in parallel to $Z_{Load\_eq}$ (see figure 4). If the power grid frequency is *f*=60Hz, calculate the size of the capacitor $C_{comp}$ required in order to correct the power factor for the load to 0.95 lag

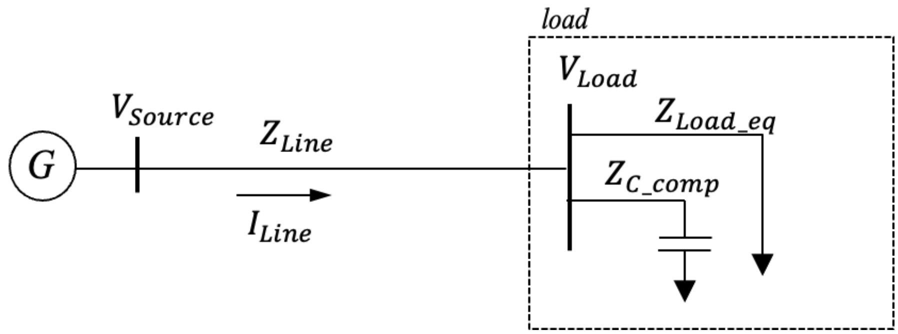


Figure 4

$$\theta_{pf(new)} = cos^{-1}(0.95) = 18.195°$$

$$\frac{Q_{Znew}}{P_{Zeq}} = tan(\theta_{pf(new)}) = 0.3287 \rightarrow Q_{Znew} = 0.3287 \cdot 140kW = 46.016kVAr$$

$$Q_{CAP} + Q_{Zeq} = Q_{Znew}$$

$$\rightarrow Q_{CAP} = Q_{Znew} - Q_{Zeq} = 46.016k - 93.564\,k = -47.548kVAr$$

$$S_{cap} = jQ_{CAP} = -j47.548kVAr$$

$$Z_{CAP} = \frac{|V_{Load}|^2}{S_{cap}^*} = \frac{13{,}800^2}{(-j47.548 \cdot 10^3)^*} = -j4005.216$$

$$Z_{CAP} = -j\frac{1}{\omega \cdot C_{cap}} \rightarrow C_{cap} = \frac{1}{\omega \cdot |Z_{CAP}|} = \frac{1}{2\pi 60 \cdot 4005.216} = 0.6623\ \mu F$$

## Question 3:

A small manufacturing plant is located 3km down a transmission line, which has a series reactance of j0.4 Ω/km. The line resistance is negligible. The plant is a three-phase load with a line voltage of 690V (Assume a positive sequence and a phase voltage $V_{an}$ that serves as reference with angle ∠0°). It consumes 200 kW at 0.85 power factor lagging.

(a) Determine the line voltage at the source

$$V_{Load.LN} = \frac{|V_{Load.LL}|}{\sqrt{3}} \angle 0° = \frac{690}{\sqrt{3}} \angle 0° = 398.37\angle 0° V$$

(LN→ line-to-neutral voltage or phase voltage; LL → line-to-line voltage or line voltage)

$$Z_{Line} = d \cdot Z_{Line-per-km} = 3 \cdot j0.4 = j1.2\Omega$$

Given the info for the load:

$$\boldsymbol{P_{load.3\emptyset} = 200\ kW};\ \theta_{pfload} = cos^{-1}(0.85) = \mathbf{31.788}°$$

$$\rightarrow |S_{load.3\emptyset}| = \frac{P_{load}}{\theta_{pfload}} = 235.29\ \boldsymbol{kVA};\ \ |\boldsymbol{S}_{load.3\emptyset}| = \mathbf{235.29}\ \boldsymbol{kVA}$$

$$Q_{load.3\emptyset} = P_{load.3\emptyset} \cdot tan(31.788°) = |S_{load.3\emptyset}| \cdot sin(31.788°)$$
$$= \mathbf{123.95\ kVAr}$$
$$\boldsymbol{Q_{load.3\emptyset} = 123.95\ kVAr}$$

$$S_{source.1\emptyset} = \frac{S_{source.3\emptyset}}{3} = 66.67kW + j41.3167kVAr = 78.43\angle 31.788° kVA$$

We can use the load power and voltage to calculate the load current (which in this case is the same as the line current):

$$I_{Line} = I_{Load} = \frac{S^*_{load.1\emptyset}}{V^*_{Load.LN}} = \frac{(78.43 \cdot 10^3 \angle -31.788°)}{398.37\angle 0°} = 196.879\angle -31.788° A$$

The voltage on the source side can be found using node voltages across the line impedance:

$$\frac{V_{Source.LN} - V_{Load.LN}}{Z_{Line}} = I_{Line}$$

$$\rightarrow V_{Source.LN} = V_{Load.LN} + I_{Line} \cdot Z_{Line} = 398.37\angle 0° + 196.879\angle -31.788° \cdot 1.2\angle 90°$$

$$V_{Source.LN} = 522.82 + 200.82 = 560.06\angle 21.012° V$$

$$|V_{Source.LL}| = \sqrt{3} \cdot |V_{Source.LL}| = 970.06V$$

(b) Determine the 3-phase complex power **generated** by the source

$$S_{source.3\emptyset} = 3 \cdot S_{source.1\emptyset} = 3 \cdot V_{Source.LN} \cdot I^*_{source}$$

$$I_{Source} = I_{Line}$$

$$\rightarrow S_{source.3\emptyset} = 3 \cdot 560.06\angle 21.012° \cdot 196.879\angle 31.788°$$

$$= 330.79kVA\angle 52.8°\ generated$$

$$S_{source.3\emptyset} = 200kW + j263.48kVAr\ generated$$

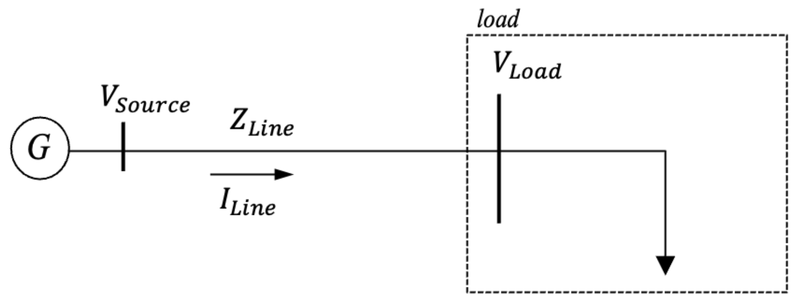
We can either mark that this real and reactive power is generated or assign the right sign:

$$S_{source.3\emptyset} = 3 \cdot S_{source.1\emptyset} = 3 \cdot (-V_{Source.LN} \cdot I^*_{source})$$

$$S_{source.3\emptyset} = -200kW - j263.48kVAr$$

***Another way to solve would be:***

$$S_{source.3\emptyset} = S_{line.3\emptyset} + S_{load.3\emptyset} = 3|I_{Line}|^2 \cdot Z_{Line} + S_{load.3\emptyset}$$




## Question4:

The following three-phase loads are connected in parallel across a 3,800 V (line-line; (Assume a positive sequence and a phase voltage $V_{an}$ that serves as reference with angle $\angle 0°$) balanced three-phase power network:

Load 1: 120 kVA at 0.9 power factor lag;

Load 2: 180 kW at 0.55 power factor lead;

Load 3: 30 kW at unity power factor.

(a) Find the total complex power of the three loads

Given:

$S_{L1.3\phi} = 120\ kVA; pf_{L1} = 0.9 lag \Rightarrow \theta_{L1} = 25.84°$

$P_{L1.3\phi} = S_{L1.3\phi}\cos(\theta_{L1}) = 108\text{ kW}; Q_{L1.3\phi} = S_{L1.3\phi}\sin(\theta_{L1}) = 52.3\ kVAr$

$P_{L2.3\phi} = 180\ kW; pf_{L2} = 0.55 lead \Rightarrow \theta_{L2} = -56.63°$

$$Q_{L2.3\phi} = S_{L2.3\phi}\sin(\theta_{L2}) = \frac{P_{L2.3\phi}}{\cos(\theta\theta_{L2})}\sin(\theta_{L2}) = -273.33\ kVAr$$

$P_{L3.3\phi} = 30\ kW; pf_{L3} = 1 \Rightarrow \theta_{L3} = 0°$

$Q_{L3.3\phi} = 0\ kVAr$

$$\boldsymbol{S_{loas.3\phi}} = P_{L1.3\phi} + P_{L2.3\phi} + P_{L3.3\phi} + Q_{L1.3\phi} + Q_{L2.3\phi} + Q_{L3.3\phi}$$

$$\boldsymbol{S_{loas.3\phi} = 318kW - j221.03\ kVAr = 387.27\angle -34.8°\ kVA}$$

(b) Find the overall power factor

$pf_{Load} = \boldsymbol{cos(\angle -34.8°) = 0.821\ lead}$

(c) Find line current in the supply line

$$|V_{Load.LL}| = 3{,}800\ V$$

$$V_{Load.LN} = \frac{|V_{Load.LL}|}{\sqrt{3}}\angle 0° = \frac{3{,}800}{\sqrt{3}}\angle 0° = 2193.93\angle 0° V$$

$$S_{Load.1\phi} = \frac{S_{3\phi_{total}}}{3} = 129.09 \cdot 10^3 \angle - 34.8°$$

$$\Rightarrow I_{line} = I_{load} = \left(\frac{S_{1\phi\_total}}{V_{Load.LN}}\right)^* = \left(\frac{129.09 \cdot 10^3 \angle - 34.8°}{2193.93\angle 0°}\right)^* = 58.84\angle 34.8° \;\; A$$