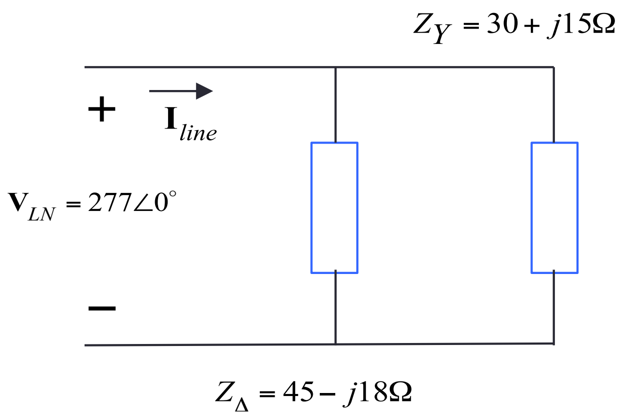
School of Engineering
Department of Electrical and Computer Engineering

332:494:01/599:02 – Smart Grid – spring 2021
Homework Assignment – Set 2 SOLUTION

Question 1

A balanced Y-connected voltage source with a phase voltage $V_{an} = 277\angle 0° V$ delivers to a balanced delta connected load with impedance $Z_{delta} = 45 - j18\Omega$ that is in parallel with a Y-connected load with impedance $Z_Y = 30 + j15\Omega$.

1. Calculate the total current deliver to the loads



Load 1: $Z_{L1} = \frac{Z_\Delta}{3} = (15 - j6)\Omega$

Load 2: $Z_{L2} = Z_Y = (30 + j15)\Omega$

$$I_{Line} = \frac{V_{load.an}}{Z_{L1} \parallel Z_{L2}} = \frac{277\angle 0°}{(15 - j6) \parallel (30 + j15)} = 23.459\angle 6.546° A$$

2. Calculate the total real and reactive power for the loads

$$S_{gen.1\emptyset} = V_{load.an} \cdot I_{Line}^{*} = 277\angle 0° \cdot 23.459\angle - 6.546° delivered$$

$$S_{gen.1\emptyset} = 6.498\angle - 6.546°\ kVA\ delivered$$

$$S_{gen.3\emptyset} = 3S_{gen.1\emptyset} = 19.494\angle - 6.546°\ kVA\ \ delivered$$

$$P_{gen.3\emptyset} = 19494 \cdot \cos(-6.546°) = 19.367 kW\ \ \ delivered$$

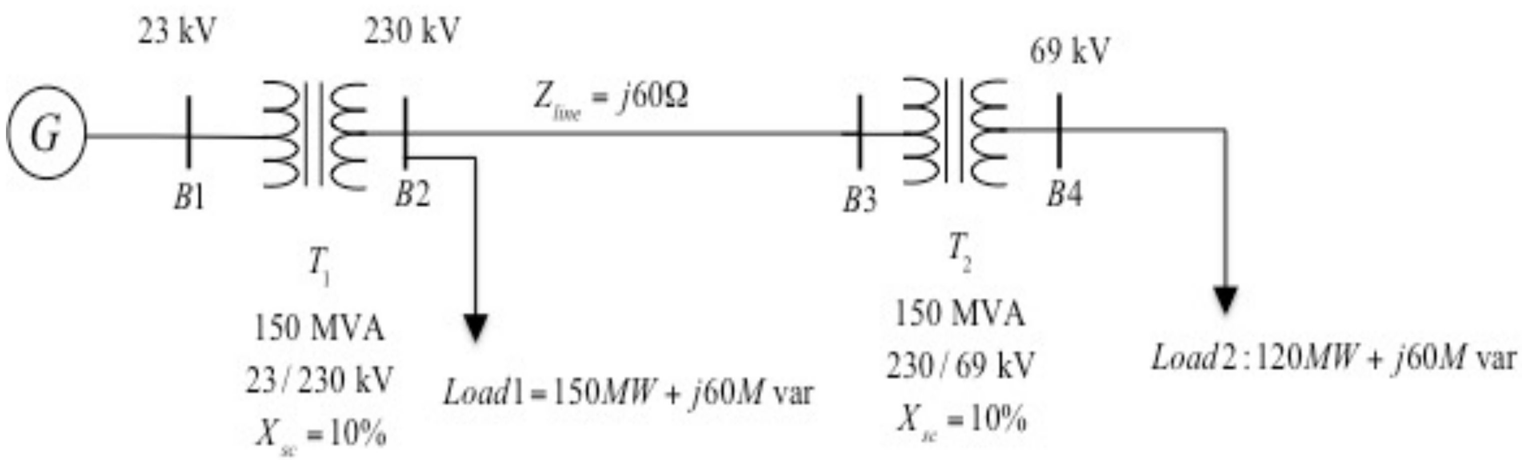
$$Q_{gen.3\emptyset} = 19494 \cdot \sin(-6.546°) = -2.22 kVAr\ \ \ delivered$$

Question 2

Consider the three-phase system in Figure 1.

From the generator bus (marked as B1) power is delivered to a load of $150MW$ and $60M$var at bus 3 and a load of $120MW$ and $60MVAr$ at bus 4 (all values in 3-Phase).

The line voltage magnitude at bus 4 (B4) for load 2 needs is kept at 69 kV.



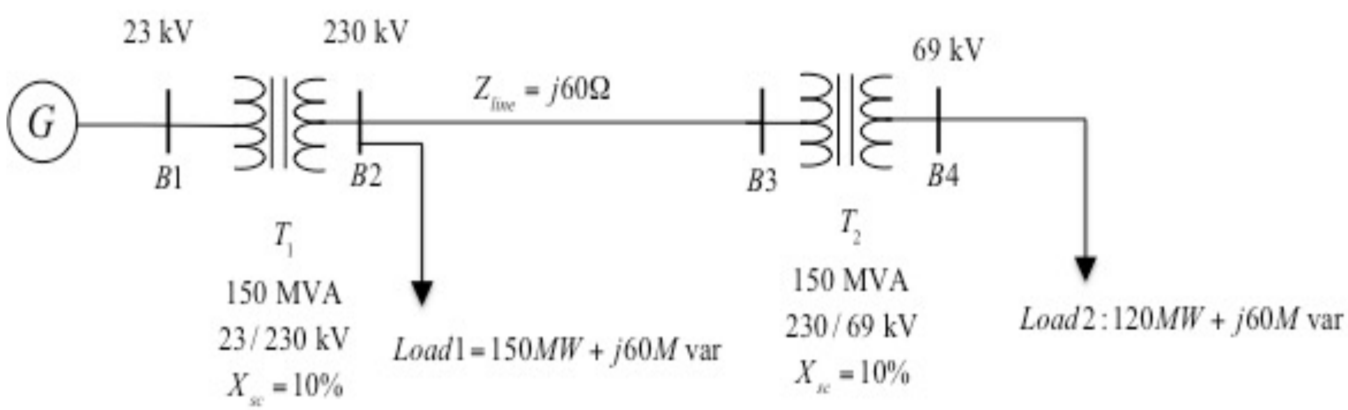
Question 2

(a) Find the per-unit representation of the system with a power base value of:

- **Option one:** single-phase value of $100 MVA$ for the power and phase voltage base value of $23/\sqrt{3}\ kV$ at bus B1
- **Option two**: three-phase value of $300 MVA$ for the power and line voltage base value of $23\ kV$ at bus B1

Per the option you chose:

- Find the base values for the systems in figure 1
- Find all per-unit line impedances: transformers T1 and T2 impedances
- Find all per-unit values for all given powers, impedances, and voltages



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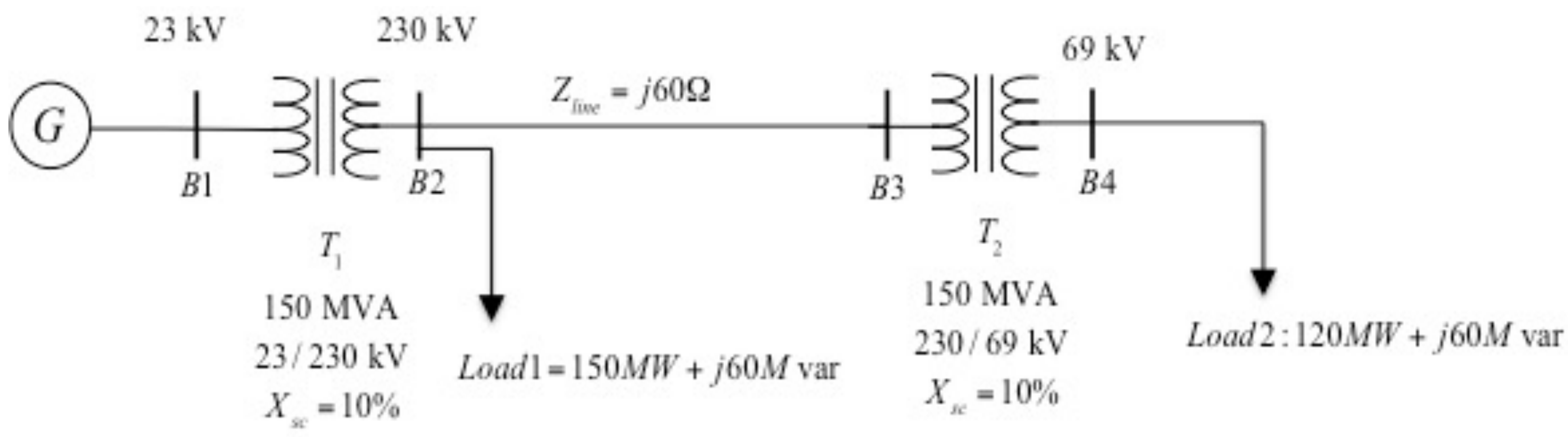
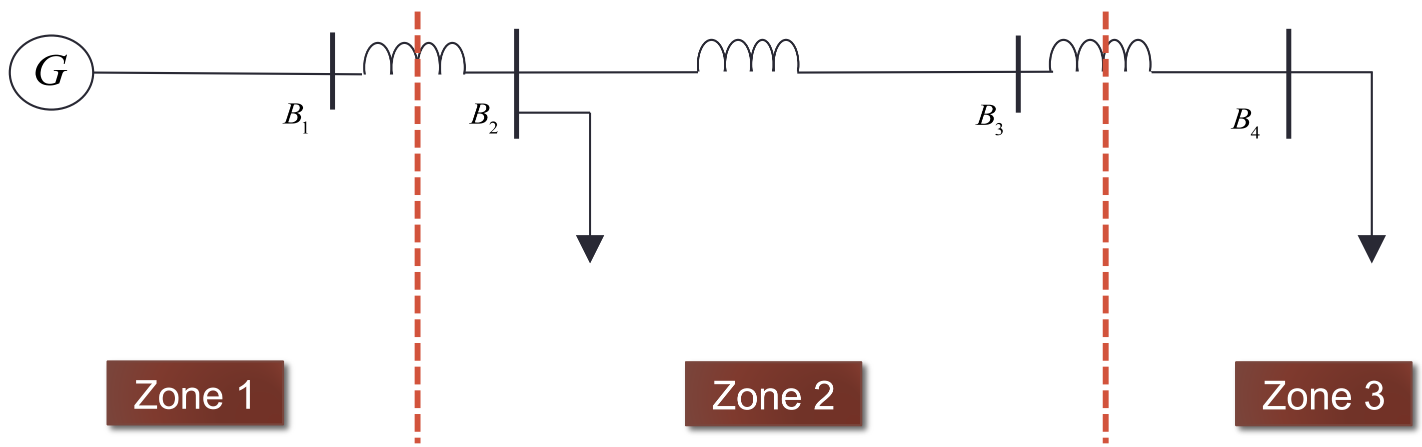
Question 2


23 kV
230 kV
69 kV
G
$Z_{line} = j60\Omega$
B1
B2
B3
B4
T_1
150 MVA
23 / 230 kV
$X_{sc} = 10\%$
$Load1 = 150MW + j60M$ var
T_2
150 MVA
230 / 69 kV
$X_{sc} = 10\%$
$Load2 : 120MW + j60M$ var


G
B_1
B_2
B_3
B_4
Zone 1
Zone 2
Zone 3

Question 2

(a) Find the per-unit representation of the system with a power base value of:

Per the option you chose … single-phase

- Find the base values for the systems in figure 1

$$S_{b.1\emptyset} = 100MVA; P_{b.1\emptyset} = 100MW; Q_{b.1\emptyset} = 100MVAr;$$

$$V_{b1.LN} = \frac{23}{\sqrt{3}}kV$$

$$V_{b1.LN} = \frac{23}{\sqrt{3}}kV \rightarrow \quad V_{b2.LN} = V_{b1.LN}\frac{V_{T1.ZN2}}{V_{T1.ZN1}} = \frac{230}{\sqrt{3}}kV$$

$$V_{b2..LN} = \frac{230}{\sqrt{3}}kV \rightarrow. \quad V_{b3.LN} = V_{b2.LN}\frac{V_{T2.ZN3}}{V_{T2.ZN2}} = \frac{69}{\sqrt{3}}kV$$

$$I_{b1} = \frac{S_{b.1\emptyset}}{V_{b1.LN}} = 7.53kA$$

$$I_{b2} = \frac{S_{b.1\emptyset}}{V_{2.LN}} = 0.753kA$$

$$I_{b3} = \frac{S_{b.1\emptyset}}{V_{3.LN}} = 2.51kA$$

$$Z_{b1} = \frac{V_{b1.LN}^2}{S_{b.1\emptyset}} = 1.7634\Omega$$

$$Z_{b2} = \frac{V_{b2.LN}^2}{S_{b.1\emptyset}} = 176.34\Omega$$

$$Z_{b3} = \frac{V_{b3.LN}^2}{S_{b.1\emptyset}} = 15.89\Omega$$

Question 2

(a) Find the per-unit representation of the system with a power base value of:

Per the option you chose … THREE-phase

- Find the base values for the systems in figure 1

$$S_{b.3\emptyset} = 300MVA; P_{b.3\emptyset} = 300MW; Q_{b.3\emptyset} = 300MVAr;$$

$$V_{b1.LL} = 23kV$$

$$V_{b1.LL} = 23kV \rightarrow \quad V_{b2.LL} = V_{b1.LL}\frac{V_{T1.ZN2}}{V_{T1.ZN1}} = 230kV$$

$$V_{b2.LL} = 230kV \rightarrow . \quad V_{b3.LL} = V_{b2.LL}\frac{V_{T2.ZN3}}{V_{T2.ZN2}} = 69kV$$

$$I_{b1} = \frac{S_{b.3\emptyset}}{\sqrt{3}\cdot V_{b1.LL}} = 7.53kA$$

$$I_{b2} = \frac{S_{b.3\emptyset}}{\sqrt{3}\cdot V_{2.LL}} = 0.753kA$$

$$I_{b3} = \frac{S_{b.3\emptyset}}{\sqrt{3}\cdot V_{3.LL}} = 2.51kA$$

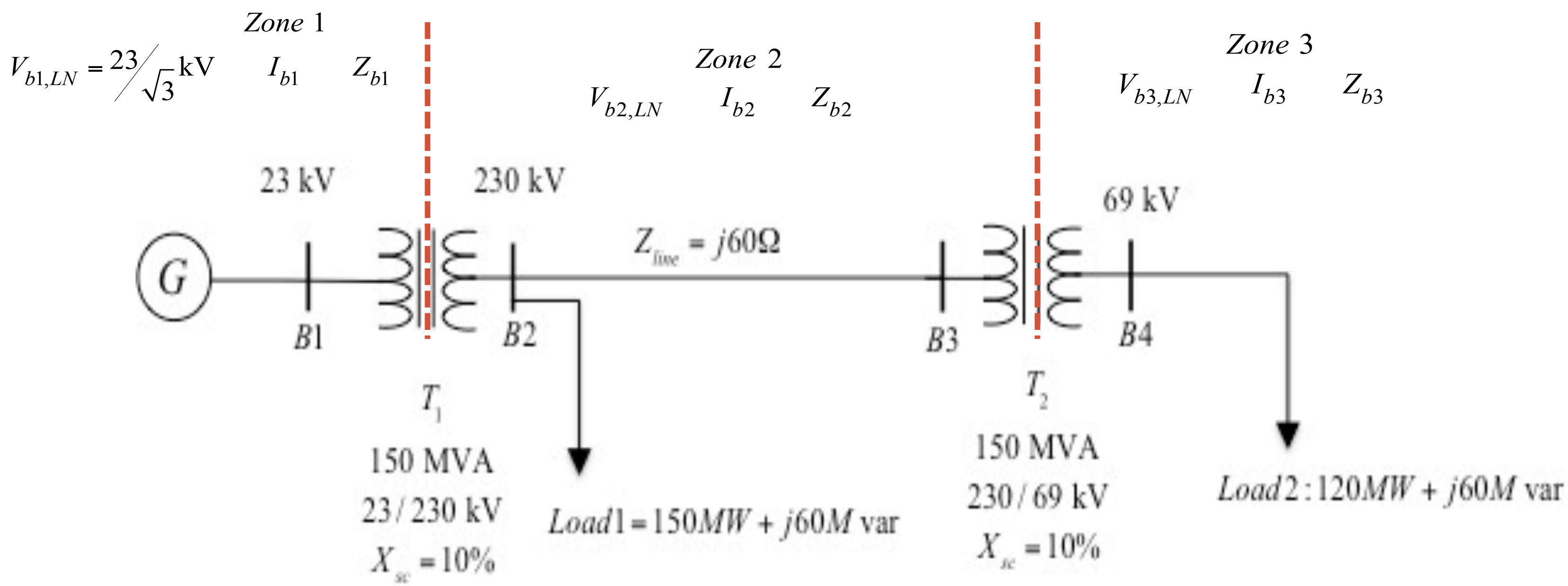
$$Z_{b1} = \frac{V_{b1.LL}^2}{S_{b.3\emptyset}} = 1.7634\Omega$$

$$Z_{b2} = \frac{V_{b2.LL}^2}{S_{b.3\emptyset}} = 176.34\Omega$$

$$Z_{b3} = \frac{V_{b3.LL}^2}{S_{b.3\emptyset}} = 15.89\Omega$$

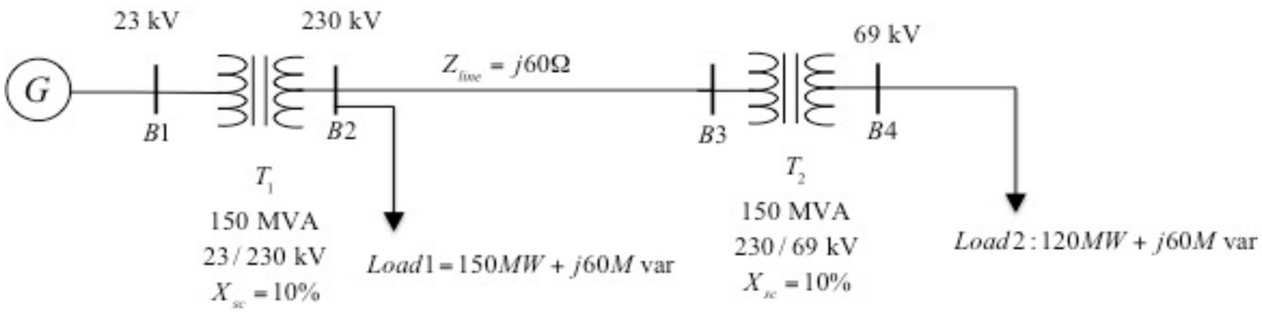
Question 2

(a.1) Find the base values for the systems in figure 1: with a power base value of $100MVA$ per-phase (single-phase) and phase voltage base value of $23/\sqrt{3}$ kV.



Question 2

(a.2) Find all per-unit line impedances: transformers T1 and T2 impedances



Single phase base:

$$Z_{T1} = jX_{T1.sc}\frac{V_{T1.ZN1.LN}^2}{S_{T1.1\emptyset}}\frac{S_{b.1\emptyset}}{V_{b1.LN}^2} = j0.1\frac{\left(23/\sqrt{3}\cdot 10^3\right)^2}{150/3\cdot 10^6}\cdot\frac{100\cdot 10^6}{\left(23/\sqrt{3}\cdot 10^3\right)^2} = j0.2\ pu$$

$$Z_{T2} = jX_{T2.sc}\frac{V_{T2.ZN2.LN}^2}{S_{T2.1\emptyset}}\frac{S_{b.1\emptyset}}{V_{b2.LN}^2} = j0.1\frac{\left(230/\sqrt{3}\cdot 10^3\right)^2}{150/3\cdot 10^6}\cdot\frac{100\cdot 10^6}{\left(230/\sqrt{3}\cdot 10^3\right)^2} = j0.2\ pu$$

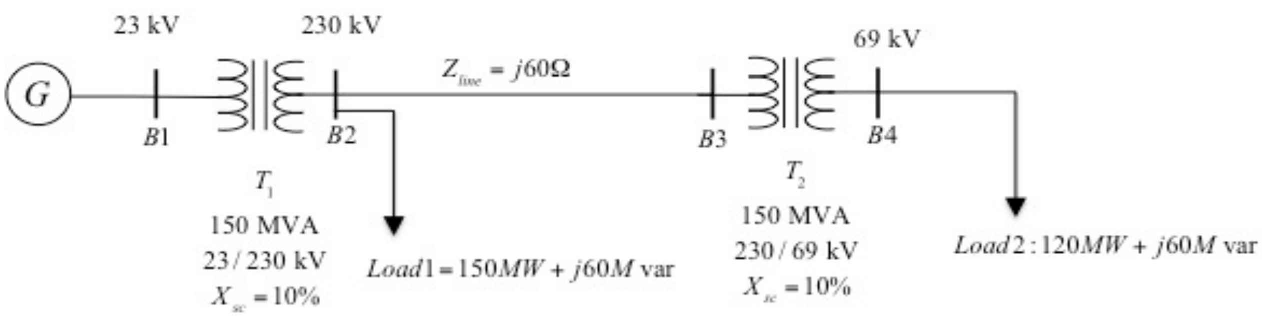
Three phase base:

$$Z_{T1} = jX_{T1.sc}\frac{V_{T1.ZN1.LL}^2}{S_{T1.3\emptyset}}\frac{S_{b.3\emptyset}}{V_{b1.LL}^2} = j0.1\frac{(23\cdot 10^3)^2}{150\cdot 10^6}\cdot\frac{300\cdot 10^6}{(23\cdot 10^3)^2} = j0.2\ pu$$

$$Z_{T2} = jX_{T2.sc}\frac{V_{T2.ZN2.LL}^2}{S_{T2.3\emptyset}}\frac{S_{b.3\emptyset}}{V_{b2.LL}^2} = j0.1\frac{(230\cdot 10^3)^2}{150\cdot 10^6}\cdot\frac{300\cdot 10^6}{(230\cdot 10^3)^2} = j0.2\ pu$$

Question 2

(a.3) Find all per-unit values for all given **powers**, impedances, and voltages



Single phase base:

$$S_{Load1.pu} = \frac{S_{Load1.1\emptyset}}{S_{b.1\emptyset}} = \frac{(150 + j60) \cdot 10^6/3}{100 \cdot 10^3} = 0.5 + j0.2 \ pu$$

$$S_{Load2.pu} = \frac{S_{Load2.1\emptyset}}{S_{b.1\emptyset}} = \frac{(120 + j60) \cdot 10^6/3}{100 \cdot 10^3} = 0.4 + j0.2 \ pu$$

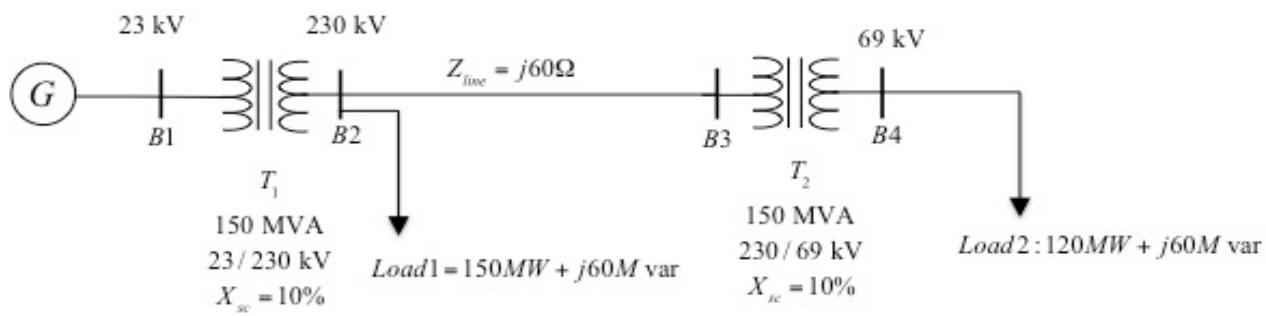
Three phase base:

$$S_{Load1.pu} = \frac{S_{Load1.3\emptyset}}{S_{b.3\emptyset}} = \frac{(150 + j60) \cdot 10^6}{300 \cdot 10^3} = 0.5 + j0.2 \ pu$$

$$S_{Load2.pu} = \frac{S_{Load2.3\emptyset}}{S_{b.3\emptyset}} = \frac{(120 + j60) \cdot 10^6}{300 \cdot 10^3} = 0.4 + j0.2 \ pu$$

Question 2

(a.3) Find all per-unit values for all given powers, **impedances, and voltages.**



Single phase base:

$$Z_{Line.pu} = \frac{Z_{line}}{Z_{b2}} = \frac{j60}{176.34} = j0.34 \ pu$$

$$V_{B4.pu} = \frac{V_{B4.LN}}{V_{b3.LN}} = \frac{69/\sqrt{3} \cdot 10^3 \angle 0°}{69/\sqrt{3} \cdot 10^3} = 1\angle 0° \ pu$$

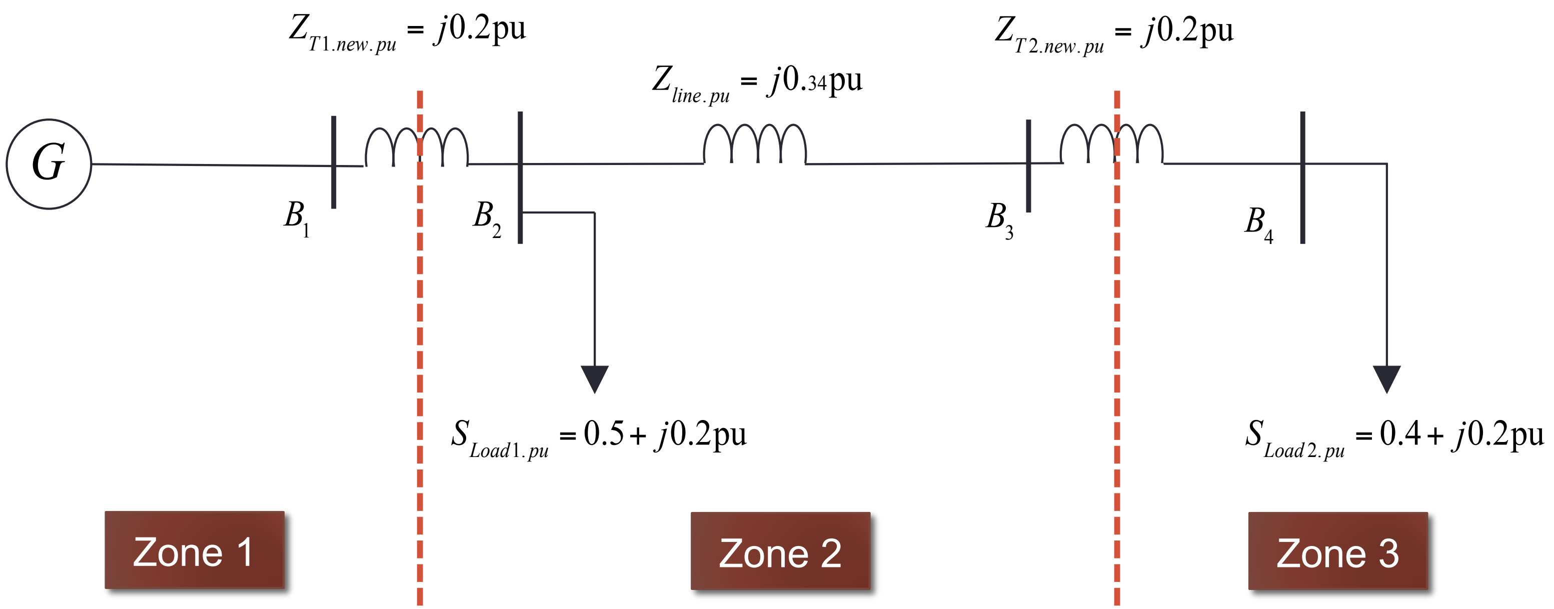
Three phase base:

$$Z_{Line.pu} = \frac{Z_{line}}{Z_{b2}} = \frac{j60}{176.34} = j0.34 \ pu$$

$$V_{B4.pu} = \frac{V_{B4.LL}}{V_{b3.LL}} = \frac{69 \cdot 10^3 \angle 0°}{69 \cdot 10^3} = 1\angle 0° \ pu$$

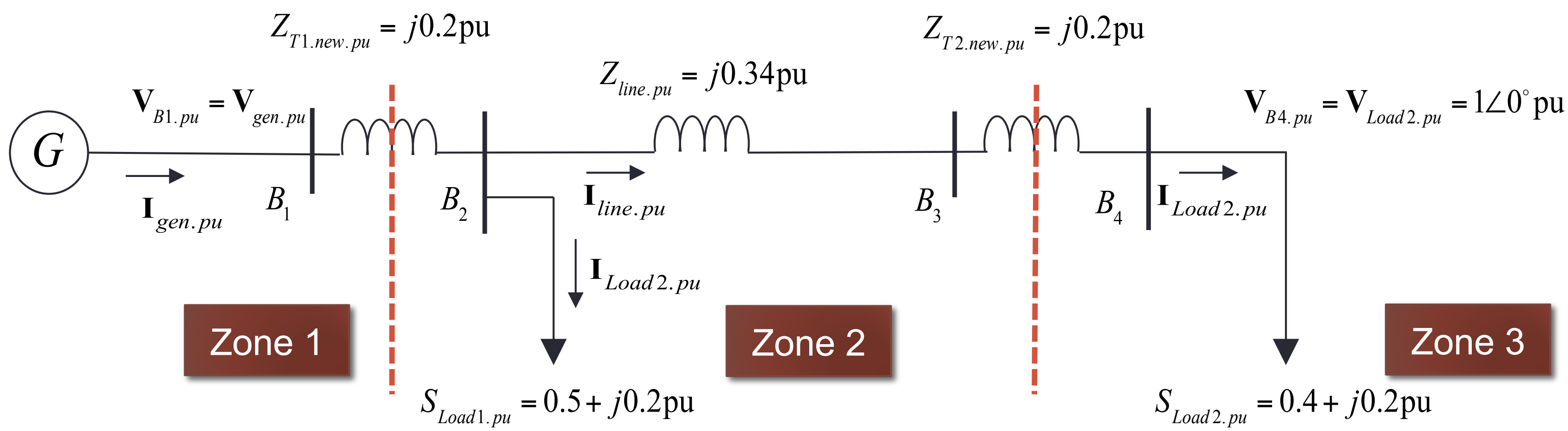
Question 2

PU one line diagram:



Question 2

(b) What is the line voltage magnitude at the generating source at bus 1 (magnitude of the line voltage at B1)?



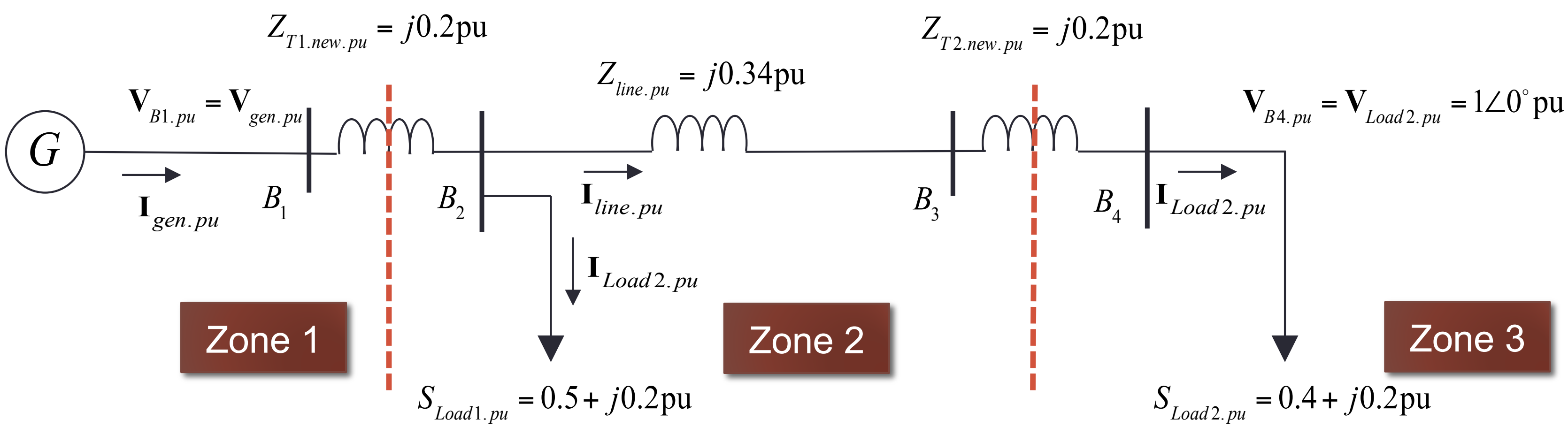
The voltage at bus 1 can be calculated as:

$$I_{Line.pu} = I_{Load2.pu} = \frac{S^*_{load2.pu}}{V^*_{B4.pu}} = \frac{0.4 - j0.2}{1\angle 0^\circ} = 0.4 - j0.2 = 0.4472\angle -26.565^\circ$$

$$V_{B2.pu} = V_{B1.pu} + I_{Line.pu}\left(Z_{Line.pu} + Z_{T2.pu}\right)$$

$$V_{B2.pu} = 1\angle 0^\circ + 0.4472\angle -26.565^\circ \cdot (j0.34 + j0.2) = 1.108 + j0.216 = 1.129\angle 11.031^\circ$$

Question 2



$$V_{B2.pu} = 1\angle 0^{\circ} + 0.4472\angle -26.565^{\circ} \cdot (j0.34 + j0.2) = 1.108 + j0.216 = 1.129\angle 11.031^{\circ}\ pu$$

$$I_{Load1.pu} = \frac{S^*_{load1.pu}}{V^*_{B2.pu}} = \frac{0.5 - j0.2}{1.129\angle -11.031^{\circ}} = 0.4686 - j0.089 = 0.477\angle -10.77^{\circ}\ pu$$

$$V_{B1.pu} = V_{B2.pu} + I_{gen.pu}\left(Z_{T1.pu}\right)$$

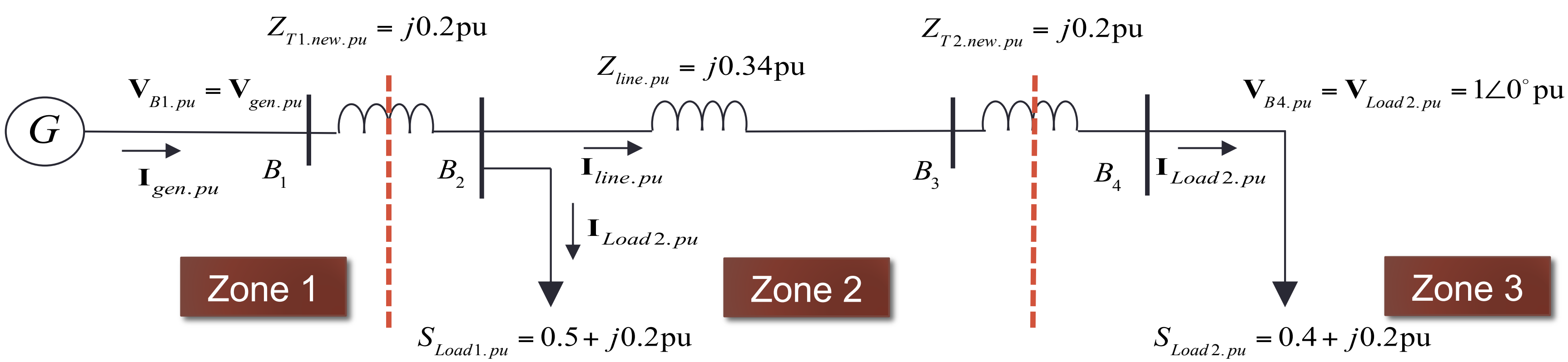
$$I_{gen.pu} = I_{Load1.pu} + I_{Line.pu} = 0.4686 - j0.089 + 0.4 - j0.2 = 0.8686 - j0.289 = 0.9155\angle -18.41^{\circ}\ pu$$

$$V_{B1.pu} = 1.129\angle 11.031^{\circ} + 0.9155\angle -18.41^{\circ} \cdot (j0.2) = 1.166 + j0.3897 = 1.229\angle 18.4844^{\circ}$$

$$\left|V_{gen.LL}\right| = \left|V_{B1.LL}\right| = \left|V_{B1.pu}\right| \cdot V_{b1.LL} = 1.229 \cdot 23k = 28.267kV$$

Question 2

(c) Find the real and reactive power for the generator (remember, it's a three-phase system).



The generated power at bus 1 can be calculated as:

$$S_{gen.pu} = V_{gen.pu} I^*_{gen.pu} = V_{B1.pu} I^*_{gen.pu} = 1.229\angle 18.4844° \cdot (0.9155\angle 18.41°) = 1.125\angle 36.89°$$

$$S_{gen} = S_{gen.pu} \cdot S_{b1.3\emptyset} = 1.125\angle 36.89° \cdot 300M = 337.5\ \angle 36.89°\ MVA = 270MW + j202.55MVAr$$

$$P_{gen} = 270MW$$
$$Q_{gen} = 202.55MVAr$$

Question 3

The three-phase power and line-line voltage ratings are given for the electric power system in figure 2. For the per-unit analysis, a base value of 100MVA is chosen for the three-phase power and a base value of 20kV is chosen for the line voltage on generator 1 end.

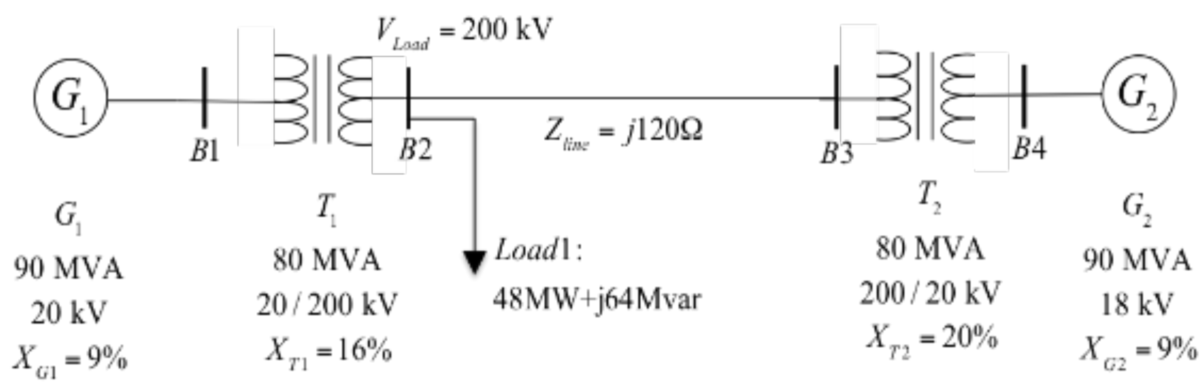
(a) Find the base values for the systems in figure 2

(a) Find all per-unit line impedances: transformers T1 and T2 impedances; generators G1 and G2 impedances;

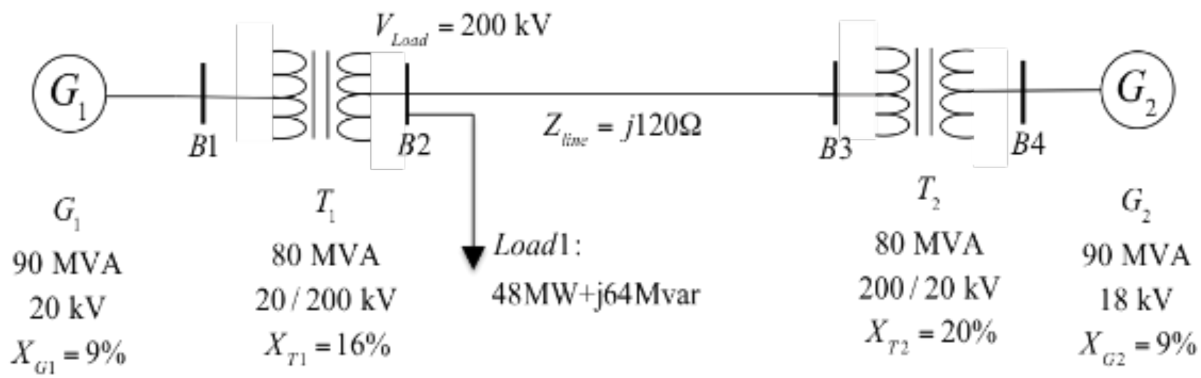
(b) Find all per-unit values for all given powers, and voltages

(c) Find load1 impedance

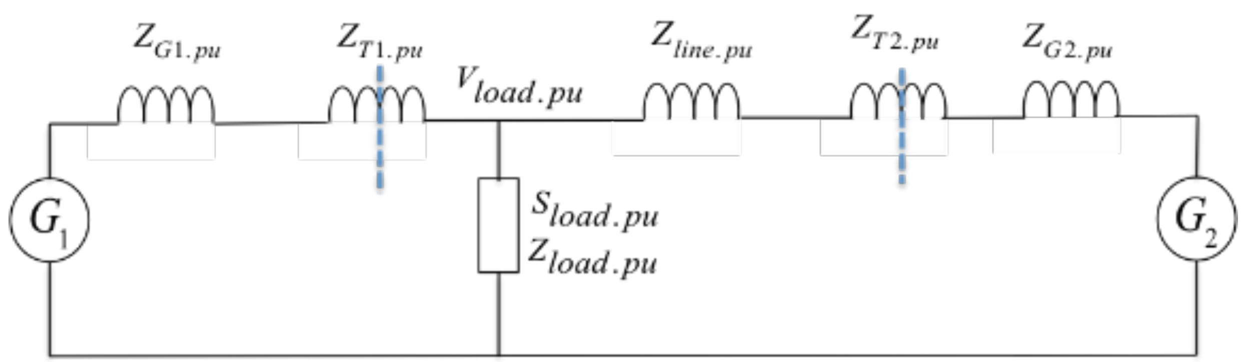
(d) Find the current flow through the load



Question 3



$V_{Load} = 200$ kV
G_1
$B1$
$B2$
$Z_{line} = j120\Omega$
$B3$
$B4$
G_2
G_1
90 MVA
20 kV
$X_{G1} = 9\%$
T_1
80 MVA
20 / 200 kV
$X_{T1} = 16\%$
$Load1$:
48MW+j64Mvar
T_2
80 MVA
200 / 20 kV
$X_{T2} = 20\%$
G_2
90 MVA
18 kV
$X_{G2} = 9\%$



$Z_{G1.pu}$
$Z_{T1.pu}$
$V_{load.pu}$
$Z_{line.pu}$
$Z_{T2.pu}$
$Z_{G2.pu}$
G_1
$S_{load.pu}$
$Z_{load.pu}$
G_2

Question 3

(a) Find the per-unit representation: **Per the option you chose … THREE-phase**

- Find the base values for the systems in figure 1

$$S_{b.3\emptyset} = 100MVA; P_{b.3\emptyset} = 100MW; Q_{b.3\emptyset} = 100MVAr;$$

$$V_{b1.LL} = 20kV$$

$$V_{b1.LL} = 20kV \rightarrow \qquad V_{b2.LL} = V_{b1.LL}\frac{V_{T1.ZN2}}{V_{T1.ZN1}} = 200kV$$

$$V_{b2..LL} = 200kV \rightarrow. \qquad V_{b3.LL} = V_{b2.LL}\frac{V_{T2.ZN3}}{V_{T2.ZN2}} = 20kV$$

$$I_{b1} = \frac{S_{b.3\emptyset}}{\sqrt{3}\cdot V_{b1.LL}} = 2.886kA$$

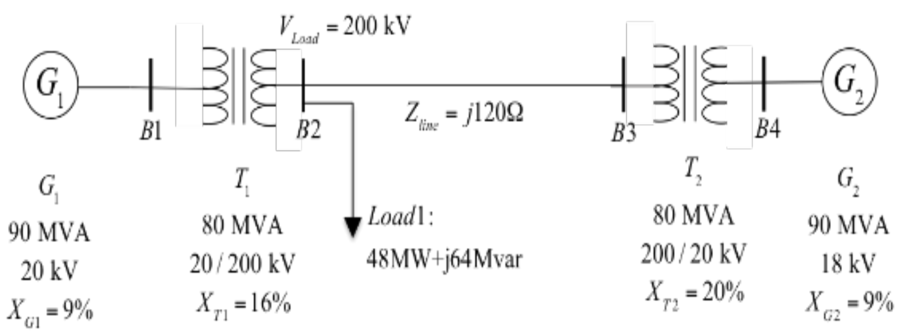
$$I_{b2} = \frac{S_{b.3\emptyset}}{\sqrt{3}\cdot V_{2.LL}} = 0.2886kA$$

$$I_{b3} = \frac{S_{b.3\emptyset}}{\sqrt{3}\cdot V_{3.LL}} = 2.886kA$$

$$Z_{b1} = \frac{V_{b1.LL}^2}{S_{b.3\emptyset}} = 4\Omega$$

$$Z_{b2} = \frac{V_{b2.LL}^2}{S_{b.3\emptyset}} = 400\Omega$$

$$Z_{b3} = \frac{V_{b3.LL}^2}{S_{b.3\emptyset}} = 4\Omega$$



Question 3

Part (a):

$S_{b,3\phi} = 100$ MVA			
	zone 1	zone 2	zone 3
$V_{bi,LL}$	20 kV	200 kV	20 kV
I_{bi}	2.886 kA	0.2886 kA	2.886 kA
Z_{bi}	4Ω	400Ω	4Ω

Question 3

(a) Find all per-unit line impedances: transformers T1 and T2 impedances; generators G1 and G2 impedances.

$$Z_{T1} = jX_{T1.sc}\frac{V_{T1.ZN1.LL}^2}{S_{T1.3\emptyset}}\frac{S_{b.3\emptyset}}{V_{b1.LL}^2} = j0.16\frac{(20\cdot 10^3)^2}{80\cdot 10^6}\cdot\frac{100\cdot 10^6}{(20\cdot 10^3)^2} = j0.2\ \ pu$$

$$Z_{T2} = jX_{T2.sc}\frac{V_{T2.ZN2.LL}^2}{S_{T2.3\emptyset}}\frac{S_{b.3\emptyset}}{V_{b2.LL}^2} = j0.2\frac{(200\cdot 10^3)^2}{80\cdot 10^6}\cdot\frac{100\cdot 10^6}{(200\cdot 10^3)^2} = j0.25\ \ pu$$

$$Z_{G1} = jX_{G1.sc}\frac{V_{G1..LL}^2}{S_{G1.3\emptyset}}\frac{S_{b.3\emptyset}}{V_{b1.LL}^2} = j0.09\frac{(20\cdot 10^3)^2}{90\cdot 10^6}\cdot\frac{100\cdot 10^6}{(20\cdot 10^3)^2} = j0.1\ \ pu$$

$$Z_{G2} = jX_{G2.sc}\frac{V_{G2.LL}^2}{S_{G2.3\emptyset}}\frac{S_{b.3\emptyset}}{V_{b3.LL}^2} = j0.09\frac{(18\cdot 10^3)^2}{90\cdot 10^6}\cdot\frac{100\cdot 10^6}{(20\cdot 10^3)^2} = j0.081\ \ pu$$

Question 3

(b) Find all per-unit values for all given powers, and voltages

$$S_{Load1.pu} = \frac{S_{Load1.3\emptyset}}{S_{b.3\emptyset}} = \frac{(48 + j64) \cdot 10^6}{100 \cdot 10^6} = 0.48 + j0.64 \;\; pu$$

$$V_{Load1.pu} = \frac{V_{Load1.LL}}{V_{b2.LL}} = \frac{200 \cdot 10^3}{200 \cdot 10^3} = 1\angle 0° \;\; pu$$

$$Z_{Line.pu} = \frac{Z_{Line}}{Z_{b2}} = \frac{j120}{400} = j0.3 \;\; pu$$

(c) Find load1 impedance:

$$Z_{Load1.pu} = \frac{\left|V_{Load1.pu}\right|^2}{S^*_{Load1.pu}} = \frac{1}{0.48 - j0.64} = 1.25\angle - 53.13° \;\; pu$$

$$Z_{Load1} = Z_{Load1.pu} \cdot Z_{b2} = 1.25\angle - 53.13° \cdot 400 = 300 + \mathrm{j}400\Omega$$

Question 3

(d) Find the current flow through the load

$$I_{Load1.pu} = \frac{S^*_{Load1.pu}}{V^*_{Load1.pu}} = \frac{0.48 - j0.64}{1\angle 0°} = 0.48 - j0.64 \;\; pu$$

$$I_{Load1} = I_{Load1.pu} \cdot I_{b2} = (0.48 - j0.64) \cdot 288.6 = 138.53 - \text{j}184.7 \text{ A} = 230.88\angle - 53.13° A$$