Homework 4 Haocong Wang mw814

Q1. (a) $y_{bus} = \begin{bmatrix} y_{12}+y_{13} & -y_{12} & -y_{13} \\ -y_{21} & y_{21} & 0 \\ -y_{31} & 0 & y_{31} \end{bmatrix} = \begin{bmatrix} 20-j50 & -10+j30 & -10+j20 \\ -10+j30 & 10-j30 & 0 \\ -10+j20 & 0 & 10-j20 \end{bmatrix}$

$V_1^0 = 1\angle 0^\circ$ pu, $V_2^0 = 1\angle 0^\circ$ pu, $V_3^0 = 1\angle 0^\circ$ pu

$V_2^{(k+1)} = \frac{1}{y_{22}}\left[\frac{P_2 - jQ_2}{[V_2^{(k)}]^*} - y_{21}V_1^{(k+1)} - y_{23}V_3^{(k)}\right]$, $P_2 = -P_{D2} = -3$ pu, $Q_2 = -Q_{D2} = -1.7$ pu

$\Rightarrow V_2^{(1)} = \frac{1}{10-j30}\left[\frac{-3+j1.7}{1\angle 0^\circ} - (-10+j30)(1\angle 0^\circ)\right] = 0.9154 - j0.073$

$V_3^{(k+1)} = \frac{1}{y_{33}}\left[\frac{P_3 - jQ_3}{[V_3^{(k)}]^*} - y_{31}V_1^{(k+1)} - y_{32}V_2^{(k+1)}\right]$, $P_3 = -P_{D3} = -4$pu, $Q_3 = -Q_{D3} = -2.2$ pu

$\Rightarrow V_3^{(1)} = \frac{1}{10-j20}\left[\frac{-4+j2.2}{1\angle 0^\circ} - (-10+j20)(1\angle 0^\circ)\right] = 0.832 - j0.116$

(b) (1) $S_{12} = V_1 \cdot I_{12}^* = V_1[(V_1 - V_2)y_{12}]^* = 3 + j1.7$ pu $= 300 + j170$ MVA

(2) $S_{13} = V_1 \cdot I_{13}^* = V_1[(V_1 - V_3)y_{13}]^* = 4 + j2.2 = 400 + j220$ MVA

(3) $S_1 = V_1 \cdot I_1^* = V_1(y_{11}V_1 + y_{12}V_2 + y_{13}V_3)^* = 7 + j3.9 = 700 + j390$ MVA

(4) $S_{21} = V_2 \cdot I_{21}^* = V_2[(V_2 - V_1)y_{21}]^* = -2.881 - j1.343$ pu $= -288 - j134$ MVA

$S_{loss.12} = S_{12} + S_{21} = 12 + j36$ MVA

(5) $S_{31} = V_3 \cdot I_{31}^* = V_3[(V_3 - V_1)y_{31}]^* = -3.58 - j1.37 = -358 - j137$ MVA

$S_{loss.13} = S_{13} + S_{31} = 42 + j83$ MVA

Q2. (a) $y_{bus} = \begin{bmatrix} y_{12} & -y_{12} \\ -y_{21} & y_{21} \end{bmatrix} = \begin{bmatrix} 10-j20 & -10+j20 \\ -10+j20 & 10-j20 \end{bmatrix}$

$V_2^{(k+1)} = \frac{1}{y_{22}}\left[\frac{P_2 - jQ_2}{[V_2^{(k)}]^*} - y_{21}V_1^{(k+1)}\right]$, $P_2 = -P_{D2} = -2.8$ pu, $Q_2 = -Q_{D2} = -0.6$ pu

$\Rightarrow V_2^{(k+1)} = \frac{1}{10-j20}\left(\frac{-2.8+j0.6}{[V_2^{(k)}]^*} + 10 - j20\right) \Rightarrow V_2^{(1)} = 0.92 - j0.1$ pu

(b) $S_1 = V_1 \cdot I_1^* = V_1(y_{11}V_1 + y_{12}V_2)^* = 3 + j$ pu $= 300 + j100$ MVA

(c) $S_{12} = V_1 \cdot I_{12}^* = V_1[(V_1 - V_2)y_{12}]^* = 3 + j$ pu $= 300 + j100$ MVA

$S_{21} = V_2 \cdot I_{21}^* = V_2[(V_2 - V_1)y_{21}]^* = -2.8 - j0.6$ pu $= -280 - 60$ MVA

$S_{loss.12} = S_{12} + S_{21} = 20 + j40$ MVA

Q3. (a) $y_{bus} = \begin{bmatrix} y_{13}+y_{14} & 0 & -y_{13} & -y_{14} \\ 0 & y_{23}+y_2+y_{24} & -y_{23} & -y_{24} \\ -y_{13} & -y_{32} & y_3+y_{23}+y_{13} & 0 \\ -y_{14} & -y_{42} & 0 & y_4+y_{14}+y_{24} \end{bmatrix} = \begin{bmatrix} -j130 & 0 & j80 & j50 \\ 0 & -j242 & j40 & j200 \\ j80 & j40 & -j122 & 0 \\ j50 & j200 & 0 & -j251 \end{bmatrix}$

(b) $V_2^{(k+1)} = \frac{1}{y_{22}}\left[\frac{P_2 - jQ_2}{V_2^{(k)}} - y_{21}V_1^{(k+1)} - y_{23}V_3^{(k)} - y_{24}V_4^{(k)}\right]$, $P_2 = -P_{D2} = -1.3$ pu, $Q_2 = -Q_{D2} = -0.25$ pu

$\Rightarrow V_2^{(k+1)} = \frac{1}{-j242}\left[\frac{-1.3+j0.25}{V_2^{(k)}} - j40V_3^{(k)} - j200V_4^{(k)}\right]$

$V_3^{(k+1)} = \frac{1}{y_{33}}\left(\frac{P_3 - jQ_3}{V_3^{(k)}} - y_{21}V_1^{(k+1)} - y_{32}V_2^{(k+1)} - y_{34}V_4^{(k)}\right)$, $P_3 = -2.1$ pu, $Q_3 = -0.8$ pu

$\Rightarrow V_3^{(k+1)} = \frac{1}{-j122}\left(\frac{-2.1+j0.8}{V_3^{(k)}} - j80 - j40V_2^{(k+1)}\right)$

$V_4^{(k+1)} = \frac{1}{y_{44}}\left(\frac{P_4 - jQ_4}{V_4^{(k)}} - y_{41}V_1^{(k+1)} - y_{42}V_2^{(k+1)} - y_{43}V_3^{(k+1)}\right)$, $P_4 = -1.75$ pu, $Q_4 = -0.55$ pu

$\Rightarrow V_4^{(k+1)} = \frac{1}{-j251}\left(\frac{-1.75+j0.55}{V_4^{(k)}} - j50 - j200V_2^{(k+1)}\right)$

$V_2^{(0)} = V_3^{(0)} = V_4^{(0)} = 1\angle 0^\circ$ pu, $V_1 = 1\angle 0^\circ$ pu

$\Rightarrow V_2^{(1)} = 0.99 - j0.0054$ pu, $V_3^{(1)} = 0.99 - j0.019$ Pu, $V_4^{(1)} = 0.97 - j0.022$ pu