8,8

Aluno: Lucas Carrijo Ferrari - 2024.1.08.016

1) a) $D = \begin{vmatrix} 4 & -1 \\ 3 & -2 \end{vmatrix} = -8 - (-3) = -8 + 3 = -5$ $\qquad x = \frac{-10}{-5} = 2$

$D_x = \begin{vmatrix} 3 & -1 \\ -4 & -2 \end{vmatrix} = -6 - (4) = -10$ $\qquad y = \frac{-25}{-5} = 5$

$D_y = \begin{vmatrix} 4 & 3 \\ 3 & -4 \end{vmatrix} = -16 - 9 = -25$

$4 \cdot 2 - 5 = 3$ $\qquad 3 \cdot 2 - 2 \cdot 5 = -4$

$8 - 5 = 3$ $\qquad 6 - 10 = -4$

$3 = 3$ $\qquad -4 = -4$

$$\boxed{\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}}$$

1,0

$L_1 = L_1 - L_3$

b) $\begin{cases} x + 4y + 3z = 1 \\ 0 \cdot x + 2y + 3z = 2 \\ x + 3y - 2z = -7 \end{cases}$

$D = \begin{vmatrix} 1 & 4 & 3 \\ 0 & 2 & 3 \\ 1 & 3 & -2 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 5 \\ 0 & 2 & 3 \\ 1 & 3 & -2 \end{vmatrix} = 1 \cdot (-1)^4 \cdot \begin{vmatrix} 1 & 5 \\ 2 & 3 \end{vmatrix} = 3 - 10 = -7$

$D_x = \begin{vmatrix} 1 & 4 & 3 \\ 2 & 2 & 3 \\ -7 & 3 & -2 \end{vmatrix} = \begin{vmatrix} 1 & 3 & 3 \\ 2 & 0 & 3 \\ -7 & 10 & -2 \end{vmatrix} = -3 \cdot \begin{vmatrix} 2 & 3 \\ -7 & -2 \end{vmatrix} + -10 \cdot \begin{vmatrix} 1 & 3 \\ 2 & 3 \end{vmatrix}$

$= -3 \cdot (-4 + 21) - 10 \cdot (3 - 6)$

$= -3 \cdot 17 - 10 \cdot -3$

$= -51 + 30$

$= -21$

$D_y = \begin{vmatrix} 1 & 1 & 3 \\ 0 & 2 & 3 \\ 1 & -7 & -2 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 3 \\ 0 & 2 & 3 \\ 0 & -8 & -5 \end{vmatrix} = \begin{vmatrix} 2 & 3 \\ -8 & -5 \end{vmatrix} = -10 - (-24)$

$= -10 + 24$

$= 14$

$$D_z = \begin{vmatrix} 1 & 4 & 1 \\ 0 & 2 & 2 \\ 1 & 3 & -7 \end{vmatrix} = \begin{vmatrix} 1 & 4 & 1 \\ 0 & 2 & 2 \\ 0 & -1 & -8 \end{vmatrix} = \begin{vmatrix} 2 & 2 \\ -1 & -8 \end{vmatrix} = -16-(-2) = -16+2 = -14$$

$$x = \frac{-21}{-7} = 3 \qquad y = \frac{14}{-7} = -2 \qquad z = \frac{14}{-7} = 2$$

$3+4\cdot -2+3\cdot 2 = 1$

$3-8+6 = 1$

$3-2 = 1$

$1 = 1$

$2\cdot -2+3\cdot 2\cdot 1 = 3$

$-4+7 = 3$

$3 = 3$

$3+3\cdot -2-2\cdot 2 = -7$

$3-6-4 = -7$

$-3-4 = -7$

$-7 = -7$

$$\boxed{\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 2 \end{bmatrix}}$$

1,0

2 a)

$L_2 = L_2 - 3 \cdot L_1$

$$\begin{bmatrix} 1 & 1 & 1 & \sqrt{3} \\ 3 & -15 & -3m^2 & -4 \\ 1 & -1 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & \sqrt{3} \\ 3 & -15 & -3m^2 & -4 \\ 2 & 0 & 0 & 3+\sqrt{3} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & \sqrt{3} \\ 0 & -18 & -3m^2-3 & -4-3\sqrt{3} \\ 2 & 0 & 0 & 3+\sqrt{3} \end{bmatrix}$$

$$\frac{2\sqrt{3}-3-\sqrt{3}}{2} = \frac{-3+\sqrt{3}}{2}$$

$$= \begin{bmatrix} 0 & 1 & 1 & \sqrt{3} - \frac{3+\sqrt{3}}{2} \\ 0 & -18 & -3m^2-3 & -4-3\sqrt{3} \\ 1 & 0 & 0 & \frac{3+\sqrt{3}}{2} \end{bmatrix} = \begin{bmatrix} 0 & 18 & 18 & 18 \cdot \frac{-3+\sqrt{3}}{2} \\ 0 & -18 & -3m^2-3 & -4-3\sqrt{3} \\ 1 & 0 & 0 & \frac{3+\sqrt{3}}{2} \end{bmatrix}$$

$$= \begin{bmatrix} \sim\sim\sim \\ 0 \quad 0 \quad -3m^2-3+18 \quad -4-3\sqrt{3}+\cancel{18}^{\,9} \cdot \frac{-3+\sqrt{3}}{\cancel{2}} \\ \sim\sim\sim \end{bmatrix}$$

1,0

$z \cdot (-3m^2+15) = -4-3\sqrt{3} + 9 \cdot (-3+\sqrt{3})$

$z = \dfrac{\sim\sim\sim}{-3m^2+15}$

$-3m^2+15 \neq 0$

$-m^2+5 \neq 0$

$-m^2 \neq -5$

$m^2 \neq 5$

$m \neq \pm\sqrt{5}$

$$\boxed{\mathbb{R} - \{\pm\sqrt{5}\}}$$

$$\begin{cases} 4x - 2y - 2z = 3x \\ 2xn \quad \vdots \end{cases}$$

b) $\begin{bmatrix} 4 & -n & 2 \\ 2n & 2 & 1 \\ 3 & -n & 3 \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3x \\ 3y \\ 3z \end{bmatrix}$

$$\begin{bmatrix} 1 & -n & -2 & 0 \\ 2n & -1 & 1 & 0 \\ 3 & -n & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -n & 2 & 0 \\ 2n & -1 & 1 & 0 \\ 2 & 0 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -n & 2 & 0 \\ 2n & -1 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -n & 1 & 0 \\ 2n & -1 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

$L_3 = L_3 - L_1$

$$\begin{bmatrix} 0 & -n & 1 \\ 2n-1 & -1 & 0 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -n & 1 \\ 1 & \frac{-1}{2n-1} & 0 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -n & 1 \\ 0 & \frac{-1}{2n-1} & -1 \\ 1 & 0 & 1 \end{bmatrix}$$

(continua no outro almaço)

3 a)

$$\begin{array}{cccc} 2 & -4 & 2 & -4 \end{array}$$

$$\begin{bmatrix} 1 & -2 & 1 & -2 \\ -2 & 3 & -3 & 3 \\ 3 & -1 & -2 & 9 \end{bmatrix} = \begin{bmatrix} 1 & -2 & 1 & -2 \\ 0 & -1 & -1 & -1 \\ 5 & -5 & 0 & 5 \end{bmatrix} = \begin{bmatrix} 1 & -2 & 1 & -2 \\ 0 & 1 & 1 & 1 \\ 1 & -1 & 0 & 5 \end{bmatrix}$$

operações??

$$= \begin{bmatrix} 0 & -1 & 1 & -7 \\ 0 & 1 & 1 & 1 \\ 1 & -1 & 0 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & -3 \\ 0 & 1 & 1 & 1 \\ 1 & -1 & 0 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & -3 \\ 0 & 1 & 0 & 4 \\ 1 & -1 & 0 & 5 \end{bmatrix}$$

?

$$= \begin{bmatrix} 0 & 0 & 1 & -3 \\ 0 & 1 & 0 & 4 \\ 1 & 0 & 0 & 9 \end{bmatrix} \qquad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 4 \\ -3 \end{bmatrix}$$

grau de liberdade = 0

$$\begin{bmatrix} 1 & 0 & 0 & 9 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -3 \end{bmatrix}$$

1,0

b) $\begin{bmatrix} 3 & -4 & -2 & -13 \\ 1 & -3 & 1 & -1 \\ -1 & -2 & 4 & 11 \end{bmatrix} = \begin{bmatrix} 5 & 0 & -10 & -35 \\ 1 & -3 & 1 & -1 \\ -1 & -2 & 4 & 11 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -2 & -7 \\ 0 & -5 & 5 & 10 \\ -1 & -2 & 4 & 11 \end{bmatrix}$

$L_1 = L_1 - 2L_3$ $\quad L_2 = L_2 + L_3$

$L_3 = L_3 + L_1$ $\quad = \begin{bmatrix} 1 & 0 & -2 & -7 \\ 0 & -1 & 1 & 2 \\ 0 & -2 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -2 & -7 \\ 0 & -1 & 1 & 2 \\ 0 & -1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -2 & -7 \\ 0 & -1 & 1 & 2 \end{bmatrix}$

$x - 2z = -7 \qquad x = -7 + 2z$

$-y + z = 2 \qquad -y = 2 - z \longrightarrow y = z - 2$

$\begin{bmatrix} 1 & 0 & -2 & -7 \\ 0 & -1 & 1 & 2 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -7 + 2\lambda \\ \lambda - 2 \\ \lambda \end{bmatrix}$ grau de liberdade = 1

1,5

(4) $\begin{bmatrix} 1 & 1 & -1 & 1 \\ 3 & 2 & -1 & 2 \\ 2 & 3 & (12-a^2) & 2a-5 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 & 1 \\ & & & \\ 0 & 1 & 12-a^2+2 & 2a-5-2 \end{bmatrix}$

-7

$L_2 = L_2 - 3L_1 \qquad L_3 = L_3 - 2L_1$

$= \begin{bmatrix} 1 & 1 & -1 & 1 \\ 0 & -1 & 2 & -1 \\ 0 & 1 & 14-a^2 & 2a-7 \end{bmatrix} = \begin{bmatrix} & & & \\ & & & \\ 0 & 0 & 16-a^2 & 2a-8 \end{bmatrix}$

1,5

$z(16 - a^2) = 2a - 8$

$z = \dfrac{2a-8}{16-a^2}$

$a = -4: \dfrac{2 \cdot (-4) - 8}{16 - (-4)^2} = \dfrac{-8-8}{16-16} = \dfrac{-16}{0}$

$a = 4: \dfrac{2 \cdot 4 - 8}{16 - 4^2} = \dfrac{8-8}{16-16} = \dfrac{0}{0}$

sistema sem solução : $a = -4$

" com solução única : $a \to \mathbb{R} - \{\pm 4\}$

" com soluções infinitas : $a = 4$

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(5) $\left[\begin{array}{ccc|ccc} 3 & 1 & 6 & 1 & 0 & 0 \\ 2 & -1 & 3 & 0 & 1 & 0 \\ 1 & 0 & 2 & 0 & 0 & 1 \end{array}\right] = \left[\begin{array}{ccc|ccc} 3 & 1 & 6 & 1 & 0 & 0 \\ 5 & 0 & 9 & 1 & 1 & 0 \\ 1 & 0 & 2 & 0 & 0 & 1 \end{array}\right]$

$L_1 = L_1 - 3L_3$ $\left[\begin{array}{ccc|ccc} 0 & 1 & 0 & 1 & 0 & -3 \\ 5 & 0 & 9 & 1 & 1 & 0 \\ 1 & 0 & 2 & 0 & 0 & 1 \end{array}\right] = \left[\begin{array}{ccc|ccc} 0 & 1 & 0 & 1 & 0 & -3 \\ 0 & 0 & -1 & 1 & 1 & -5 \\ 1 & 0 & 2 & 0 & 0 & 1 \end{array}\right] = \left[\begin{array}{ccc|ccc} 0 & 1 & 0 & 1 & 0 & -3 \\ 0 & 0 & -1 & 1 & 1 & -5 \\ 1 & 0 & 0 & 2 & 2 & -9 \end{array}\right]$

$L_2 = L_2 - 5L_3$ $L_3 = L_3 + 2L_2$

$= \left[\begin{array}{ccc|ccc} 0 & 1 & 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & -1 & -1 & 5 \\ 1 & 0 & 0 & 2 & 2 & -9 \end{array}\right] = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 2 & -9 \\ 0 & 1 & 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & -1 & -1 & 5 \end{array}\right]$

$$\boxed{A^{-1} = \begin{bmatrix} 2 & 2 & -9 \\ 1 & 0 & -3 \\ -1 & -1 & 5 \end{bmatrix}}$$

1,5

(2) b) $\begin{bmatrix} 0 & -n & 1 \\ 0 & \frac{-1}{2n-1} - n & 0 \\ & & \end{bmatrix}$

$y\left(\frac{-1}{2n-1} - n\right) = 0$

$y = \dfrac{0}{\frac{-1}{2n-1} - n}$

$\frac{-1}{2n-1} - n = 0$

$-1 - n \cdot (2n-1) = 0$

$-1 - 2n^2 + n = 0$

$-2n^2 + n - 1 = 0$

$\Delta = 1^2 - 4 \cdot -2 \cdot -1$

$\Delta = 1 - 9 = -8$

0,3

Exercícios Propostos[1]

1. **(2,0 pt.)** Resolva os sistemas lineares usando a regra de Cramer.

   (1,0 pt.) $\begin{cases} 4x - y = 3 \\ 3x - 2y = -4 \end{cases}$

   (1,0 pt.) $\begin{cases} x + 4y + 3z = 1 \\ 2y + 3z + 1 = 3 \\ x + 3y - 2z = -7 \end{cases}$

2. **(2,0 pt.)** Considere os exercícios abaixo.

   (1,0 pt.) O sistema linear $\begin{cases} x + y + z = \sqrt{3} \\ 3x - 15y - 3m^2 z = -4 \\ x - y - z = 3 \end{cases}$ *admite uma única solução* para quais valores de m? Justifique.

   (1,0 pt.) Sendo $A = \begin{pmatrix} 4 & -n & -2 \\ 2n & 2 & 1 \\ 3 & -n & 3 \end{pmatrix}$ e X uma matriz coluna de ordem 3×1, encontre o(s) valor(es) de n para que o sistema $AX = 3X$ *admita infinitas soluções* para a incógnita X. Justifique sua resposta.

3. **(3,0 pt.)** Para cada um dos sistemas abaixo, obtenha a *forma escalonada reduzida* da matriz ampliada, determine o conjunto solução e o grau de liberdade associado.

   (1,5 pt.) $\begin{cases} x - 2y + z = -2 \\ -2x + 3y - 3z = 3 \\ 3x - y - 2z = 9 \end{cases}$

   $9 - 2 \cdot 4 - 3 = -2$ $\quad 6 - 8 = -2$
   $9 - 8 - 3 = -2$ $\quad -2 = -2$

   (1,5 pt.) $\begin{cases} 3x - 4y - 2z = -13 \\ x - 3y - z = -1 \\ -x - 2y + 4z = 11 \end{cases}$

   $(-7 + 2z) - 3 \cdot (3 - 2) - z = -1$

4. **(1,5 pt.)** Considere o seguinte sistema linear: $\begin{cases} x + y - z = 1 \\ 3x + 2y - z = 2 \\ 2x - 3y - (12 - a^2)z = 2a - 5 \end{cases}$

   Encontre todos os valores de a para os quais o sistema *não tem solução*, *tem solução única* ou *tem infinitas soluções*.

5. **(1,5 pt.)** Use a matriz identidade e as *operações elementares entre linhas* para encontrar a matriz inversa de $A = \begin{pmatrix} 3 & 1 & 6 \\ 2 & -1 & 3 \\ 1 & 0 & 2 \end{pmatrix}$.

   $6 + 1 - 6 \quad 6 + 0 - 6 \quad -27 - 3 + 30$

   $-2 + 1 + 1 - 1 = 0$

   $-2 + 0$

13 + 22 = 35

2 + 17 + 03 = 51