$A = (-1)^{i+j} \cdot |M|$ $\qquad \begin{bmatrix} 2\cdot 1 - 1^2 & 2+1 \\ 1+1 & 2\cdot 2 - 2^2 \end{bmatrix} = \begin{bmatrix} 2-1 & 3 \\ 2 & 4-4 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & 0 \end{bmatrix}$

$(-1)^{2+10}$ $(22+10)$ 12

A:

Exercícios Propostos[1]

1. **(2,5 pt.)** Considere as matrizes $A = (a_{ij})_{2\times 2}$, tal que $a_{ij} = \begin{cases} 2j - i^2, & i = j \\ j+1, & i \neq j \end{cases}$, e $B = (b_{ij})_{2\times 2}$, onde $b_{ij} = i + j$. Determine:

   (a) **(0,5 pt.)** A e B

   (b) **(0,5 pt.)** $2A - 3B$

   (c) **(0,5 pt.)** $B^2 - BA^t$

   (d) **(0,5 pt.)** $\mathrm{tr}(A+B) - \det(B^7)$

   (e) **(0,5 pt.)** $\det(3B^tA^{-1})$

2. **(1,0 pt.)** Use o *desenvolvimento de Laplace* para calcular $\begin{vmatrix} 0 & 1 & -1 & 0 & 3 \\ 0 & 2 & 1 & 0 & 0 \\ 0 & -1 & 0 & 2 & -3 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 4 & 4 & 1 & 2 \end{vmatrix}$.

3. **(2,5 pt.)** Resolva os exercícios abaixo.

   (a) **(1,0 pt.)** Calcule x para que $B = \begin{pmatrix} 1 & x & 0 \\ 0 & 0 & -2 \\ x-1 & 15-3x & 0 \end{pmatrix}$ seja invertível.

   $-2\cdot(-5)^2 - 4\cdot(-5) + 30$
   $-2\cdot 25 + 20 + 30$
   $-50 + 50$

   (b) **(1,5 pt.)** Determine o valor de y na equação $\begin{vmatrix} -3 & 1 & -2 & 3 \\ 6 & 2 & y^3 & -1 \\ 45 & 10 & 5 & 0 \\ 3 & 1 & 2 & 0 \end{vmatrix} = -30.$

   $-2\cdot(3)^2 - 4\cdot 3 + 30$
   $-2\cdot 9 - 12 + 30$
   $-18 - 12 + 30$
   $-30 + 30$

4. **(2,0 pt.)** Verifique se $A = \begin{pmatrix} -11 & 2 & 4 \\ -4 & 0 & 2 \\ 6 & -1 & -2 \end{pmatrix}$ $\begin{matrix} -11 & 2 \\ -4 & 0 \\ 6 & -1 \end{matrix}$ é invertível usando a *regra de Sarrus* e, em caso afirmativo, calcule a sua inversa usando o método da *matriz adjunta*.

5. **(2,0 pt.)** Considere a equação matricial

$$AX + C^t = 3B - 2X$$

$\begin{vmatrix} -3 & 2 & 4 \\ 0 & 0 & 2 \\ 2 & -1 & -2 \end{vmatrix} = -2 \cdot \begin{vmatrix} -3 & 2 \\ 2 & -1 \end{vmatrix} = -2\cdot(3-4) = -2\cdot -1 = 2$

$AX + 2X = 3B - C^t$

onde todas as matrizes são *quadradas* e de mesma ordem 2×2.

   (a) **(0,5 pt.)** Determine X em função das matrizes A, B e C e comente se é necessária alguma imposição à matriz A para se resolver a equação.

   (b) **(1,5 pt.)** Sendo $A = \begin{pmatrix} -1 & 1 \\ -2 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}$ e $C = \begin{pmatrix} 1 & 3 \\ 0 & 5 \end{pmatrix}$, determine a matriz X e seu *traço*.

$AX + C^t = 2\left(\frac{3B}{2} - X\right)$ $\qquad X = 2\left(\frac{3B}{2}\right.$ $\qquad AX + C^t = X\left(\frac{3B}{X} - 2\right)$ $\qquad 4-(-3)$

$AX = 2\left(\frac{3B}{2} - X\right) - C^t$ $\qquad AX = X\left(\frac{3B}{X}\right.$ $\qquad 4+3$

[1] Coloque o nome completo nas folhas de prova e escreva o resultado final das questões à caneta. Respostas sem resolução e/ou justificativa não serão consideradas. Não é permitido o uso de quaisquer equipamentos eletrônicos. Data da Avaliação: 20/03/2024

$AX + 2X = 3B - C^t$
$X(A+2)$

$X = (3B - C^t)\cdot(A+2)^{-1}$

4, $38 \cdot 15 = 190 + 380 = 570$

$15 \cdot 6 = 90$

8,3

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① a) $A = \begin{bmatrix} 2\cdot1-1^2 & 2+1 \\ 1+1 & 2\cdot2-2^2 \end{bmatrix} = \begin{bmatrix} 2-1 & 3 \\ 2 & 4-4 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & 0 \end{bmatrix}$

C 0,5

$B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$

C ~~0,5~~

b) $2\cdot\begin{bmatrix} 1 & 3 \\ 2 & 0 \end{bmatrix} - 3\cdot\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 4 & 0 \end{bmatrix} - \begin{bmatrix} 6 & 9 \\ 9 & 12 \end{bmatrix} = \begin{bmatrix} 2-6 & 6-9 \\ 4-9 & 0-12 \end{bmatrix}$

$2A-3B = \begin{bmatrix} -4 & -3 \\ -5 & -12 \end{bmatrix}$

C 0,5

c) $B^2 = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2\cdot2+3\cdot3 & 2\cdot3+3\cdot4 \\ 3\cdot2+4\cdot3 & 3\cdot3+4\cdot4 \end{bmatrix} = \begin{bmatrix} 13 & 18 \\ 18 & 25 \end{bmatrix}$

$B\cdot A^t = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} 2\cdot1+3\cdot3 & 2\cdot2+3\cdot0 \\ 3\cdot1+4\cdot3 & 3\cdot2+4\cdot0 \end{bmatrix} = \begin{bmatrix} 8 & 4 \\ 15 & 6 \end{bmatrix}$

$B^2 - BA^t = \begin{bmatrix} 13 & 18 \\ 18 & 25 \end{bmatrix} - \begin{bmatrix} 8 & 4 \\ 15 & 6 \end{bmatrix} = \begin{bmatrix} 5 & 14 \\ 3 & 19 \end{bmatrix}$

0,4

d) $tr(A+B) - det(B^7)$ $\quad |B| = \begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix} = 2\cdot4 - 3\cdot3 = 8-9 = -1$

$= tr(A) + tr(B) - (det(B))^7$

$= (1+0) + (2+4) - (-1)^7$

$= 1 + 6 - (-1)$

$= 7+1$

$= \boxed{8}$

C 0,5

e) $\det(3B^t \cdot A^{-1})$

$|A| = \begin{vmatrix} 1 & 3 \\ 2 & 0 \end{vmatrix} = 0 - 6 = -6$

$= \det(3B^t) \cdot \det(A^{-1})$

$= \det(3B) \cdot \frac{1}{\det A}$

$|B| = \begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix} = 8 - 9 = -1$

$= 3^2 \cdot \det B \cdot \frac{1}{\det A}$

$= 9 \cdot (-1) \cdot \frac{1}{-6}$

0,5

$= -9 \cdot \frac{-1}{6} = \frac{9}{6} = \boxed{\frac{3}{2}}$

(2) $\begin{vmatrix} 0 & 1 & -1 & 0 & 3 \\ 0 & 2 & 1 & 0 & 0 \\ 0 & -1 & 0 & 2 & -3 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & 4 & 4 & 1 & 2 \end{vmatrix} = -1 \cdot \overset{-1 \cdot -1 = +1}{(-1)^5} \cdot D_{14}$

$= D_{14}$

$= \begin{vmatrix} 1 & -1 & 0 & 3 \\ 2 & 1 & 0 & 0 \\ -1 & 0 & 2 & -3 \\ 4 & 4 & 1 & 2 \end{vmatrix} = -a_{21} \cdot D_{21} + a_{22} \cdot D_{22}$

$= -2 \cdot \begin{vmatrix} -1 & 0 & 3 \\ 0 & 2 & -3 \\ 4 & 1 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 0 & 3 \\ -1 & 2 & -3 \\ 4 & 1 & 2 \end{vmatrix}$

$= -2 \cdot \left[-1 \cdot D_{11} + 3 \cdot D_{13} \right] + 1 \cdot D_{11} + 3 \cdot D_{13}$

$= -2 \cdot \left[-\begin{vmatrix} 2 & -3 \\ 1 & 2 \end{vmatrix} + 3 \cdot \begin{vmatrix} 0 & 2 \\ 4 & 1 \end{vmatrix} \right] + \begin{vmatrix} 2 & -3 \\ 1 & 2 \end{vmatrix} + 3 \cdot \begin{vmatrix} -1 & 2 \\ 4 & 1 \end{vmatrix}$

1,0

$= -2 \cdot \left[-7 + 3 \cdot -8 \right] + 7 + 3 \cdot -9$

$= -2 \cdot \left[-7 - 24 \right] + 7 - 27$

$= -2 \cdot -31 - 20$

$= 62 - 20$

$\boxed{\det = 42}$

$= 42$

2

3 a) $\det(B) \neq 0$

$$|B| = \begin{vmatrix} 1 & x & 0 \\ 0 & 0 & -2 \\ x-1 & 15-3x & 0 \end{vmatrix} = a_{23} \cdot A_{23}$$

$$= -2 \cdot (-1)^5 \cdot \begin{vmatrix} 1 & x \\ x-1 & 15-3x \end{vmatrix}$$

$$= 2 \cdot \left[15-3x - (x \cdot (x-1)) \right]$$

$$= 2 \cdot \left[15-3x - x^2 + x \right]$$

$$= 2 \cdot \left[-x^2 - 2x + 15 \right]$$

$$= -2x^2 - 4x + 30$$

$$-2x^2 - 4x + 30 \neq 0$$

$$-x^2 - 2x + 15 \neq 0$$

$$S = \frac{-b}{a} = \frac{2}{-1} = -2 \begin{cases} -5 \\ 3 \end{cases}$$

$$P = \frac{c}{a} = \frac{15}{-1} = -15 \begin{cases} -5 \\ 3 \end{cases}$$

$$\boxed{\mathbb{R} - \{-5, 3\}}$$

1,0

b)
$$\begin{vmatrix} -3 & 1 & -2 & 3 \\ 6 & 2 & y^3 & -1 \\ 45 & 10 & 5 & 0 \\ 3 & 1 & 2 & 0 \end{vmatrix} \begin{matrix} \\ \\ :5 \\ \\ \end{matrix} = \begin{vmatrix} -3 & 1 & -2 & 3 \\ 6 & 2 & y^3 & -1 \\ 9 & 2 & 1 & 0 \\ 3 & 1 & 2 & 0 \end{vmatrix} \cdot 5 = \begin{vmatrix} -1 & 1 & -2 & 3 \\ 2 & 2 & y^3 & -1 \\ 3 & 2 & 1 & 0 \\ 1 & 1 & 2 & 0 \end{vmatrix} \cdot 15$$

$:3$

$L_1 + L_2 \cdot 3 \to L_1$

$$= \begin{vmatrix} 5 & 7 & -2+3y^3 & 0 \\ 2 & 2 & y^3 & -1 \\ 3 & 2 & 1 & 0 \\ 1 & 1 & 2 & 0 \end{vmatrix} \cdot 15 = -1 \cdot (-1)^6 \cdot \begin{vmatrix} 5 & 7 & -2+3y^3 \\ 3 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix}$$

$$= -15 \cdot \begin{vmatrix} 5 & 7 & -2+3y^3 \\ 2 & 0 & -3 \\ 1 & 1 & 2 \end{vmatrix}$$

1,2

$$= -15 \cdot \left[-2 \cdot \begin{vmatrix} 7 & -2+3y^3 \\ 1 & 2 \end{vmatrix} + 3 \cdot \begin{vmatrix} 5 & 7 \\ 1 & 1 \end{vmatrix} \right]$$

~~0,7~~

$$= -15 \cdot \left[-2 \cdot (14 + 2 - 3y^3) + 3 \cdot (5-7) \right]$$

$$= -15 \cdot \left[-2 \cdot (16 - 3y^3) + 3 \cdot -2 \right]$$

$$= -15 \cdot \left[-32 + 6y^3 - 6 \right]$$

$$= -15 \cdot (-38 + 6y^3)$$

$$= 570 - 90y^3$$

$$-30 = 570 - 90y^3$$

$$90y^3 = 570 + 30$$

$$9\cancel{0}y^3 = 60\cancel{0}$$

$$y^3 = \frac{60}{9} = \frac{20}{3}$$

$$y = \frac{\sqrt[3]{20}}{\sqrt[3]{3}}$$

$$\boxed{y = \frac{\sqrt[3]{20}}{\sqrt[3]{3}}}$$

(4) $|A| = \begin{vmatrix} -11 & 2 & 4 \\ -4 & 0 & 2 \\ 6 & -1 & -2 \end{vmatrix} \begin{matrix} -11 & 2 \\ -4 & 0 \\ 6 & -1 \end{matrix}$

$0 + 22 + 16 = 38 \qquad 0 + 24 + 16 = 40$

$|A| = 40 - 38 = 2$

$\boxed{|A| \neq 0 \text{, dessa forma, existe } A^{-1}}$

$A^{-1} = \dfrac{Adj\,A}{|A|}$

$Cof\,A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} \begin{vmatrix} 0 & 2 \\ -1 & -2 \end{vmatrix} & -\begin{vmatrix} -4 & 2 \\ 6 & -2 \end{vmatrix} & \begin{vmatrix} -4 & 0 \\ -6 & -1 \end{vmatrix} \\ -\begin{vmatrix} 2 & 4 \\ -1 & -2 \end{vmatrix} & \begin{vmatrix} -11 & 4 \\ 6 & -2 \end{vmatrix} & -\begin{vmatrix} -11 & 2 \\ 6 & -1 \end{vmatrix} \\ \begin{vmatrix} 2 & 4 \\ 0 & 2 \end{vmatrix} & -\begin{vmatrix} -11 & 4 \\ -4 & 2 \end{vmatrix} & \begin{vmatrix} -11 & 2 \\ -4 & 0 \end{vmatrix} \end{bmatrix}$

$Adj\,A = \begin{bmatrix} 2 & 0 & 4 \\ 4 & -2 & 6 \\ 4 & 1 & 8 \end{bmatrix}$

$= \begin{bmatrix} -2 & -(8-12) & 4 \\ -(-4--4) & 22-24 & -(11-12) \\ -4 & -(-22--16) & 8 \end{bmatrix}$

$= \begin{bmatrix} 2 & 4 & 4 \\ 0 & -2 & 1 \\ 4 & 6 & 8 \end{bmatrix}$

2,0

$\boxed{A^{-1} = \begin{bmatrix} \frac{2}{2} & 0 & \frac{4}{2} \\ \frac{4}{2} & -\frac{2}{2} & \frac{6}{2} \\ \frac{4}{2} & \frac{1}{2} & \frac{8}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 2 & \frac{1}{2} & 4 \end{bmatrix}}$

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(5) a) $Ax + C^t = 3B - 2x$

$Ax = 3B - 2x - C^t$

$Ax + 2x = 3B - C^t$

$x(A+2) = 3B - C^t$

$\cancel{x = A^{-1} \cdot (3B}$

$x = (3B - C^t) \cdot (A+2)^{-1}$

$\boxed{x = (3B - C^t) \cdot (A+2)^{-1}}$

0,2

A deve ser inversível, seu determinante não pode ser 0

b) $|A| = -3 - (-2) = -3 + 2 = -1$

$Adj\,A = \begin{bmatrix} 3 & -1 \\ 2 & -1 \end{bmatrix}$ $3B = \begin{bmatrix} 9 & -6 \\ 6 & -3 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} -3 & 1 \\ -2 & 1 \end{bmatrix}$ C $C^t = \begin{bmatrix} 1 & 3 \\ 0 & 5 \end{bmatrix}$

0,5

$x = \begin{bmatrix} 9-1 & -6-3 \\ 6-0 & -3-5 \end{bmatrix} \cdot (A+2)^{-1}$

$= \begin{bmatrix} 8 & -9 \\ 6 & -8 \end{bmatrix} \cdot (A+2)^{-1}$

$= \begin{bmatrix} 8 & -9 \\ 6 & -8 \end{bmatrix} \times A^{-1} + \begin{bmatrix} 8 & -9 \\ 6 & -8 \end{bmatrix} \times \frac{1}{2}$

$= \begin{bmatrix} 8 & -9 \\ 6 & -8 \end{bmatrix} \cdot \begin{bmatrix} -3 & 1 \\ -2 & 1 \end{bmatrix} + \begin{bmatrix} 4 & -\frac{9}{2} \\ 3 & -4 \end{bmatrix}$

$= \begin{bmatrix} -24+18 & 8-9 \\ -18+16 & 6-8 \end{bmatrix} + \begin{bmatrix} 4 & -\frac{9}{2} \\ 3 & -4 \end{bmatrix}$

$= \begin{bmatrix} -6 & -1 \\ -2 & -2 \end{bmatrix} + \begin{bmatrix} 4 & -\frac{9}{2} \\ 3 & -4 \end{bmatrix}$

$= \begin{bmatrix} -2 & -\frac{2}{2} + -\frac{9}{2} \\ 1 & -6 \end{bmatrix} = \begin{bmatrix} -2 & -\frac{11}{2} \\ 1 & -6 \end{bmatrix}$

$\boxed{x = \begin{bmatrix} -2 & -\frac{11}{2} \\ 1 & -6 \end{bmatrix}}$