

$$A = (-1)^{1+1} \cdot 1 \cdot 1$$

$$\begin{bmatrix} 2 \cdot 1 & 1^2 & 2 \cdot 1 \\ 1 \cdot 1 & 2 \cdot 2 & 2^2 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 4 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$$

A:

Exercícios Propostos¹

(22.10)
12

1. (2,5 pt.) Considere as matrizes $A = (a_{ij})_{2 \times 2}$, tal que $a_{ij} = \begin{cases} 2j - i^2, & i = j \\ j + 1, & i \neq j \end{cases}$, e $B = (b_{ij})_{2 \times 2}$, onde $b_{ij} = i + j$. Determine:

(a) (0,5 pt.) A e B

(d) (0,5 pt.) $\text{tr}(A + B) - \det(B^T)$

(b) (0,5 pt.) $2A - 3B$

(e) (0,5 pt.) $\det(3B^t A^{-1})$

(c) (0,5 pt.) $B^2 - BA^t$

2. (1,0 pt.) Use o desenvolvimento de Laplace para calcular

$$\begin{vmatrix} 0 & 1 & -1 & 0 & 3 \\ 0 & 2 & 1 & 0 & 0 \\ 0 & -1 & 0 & 2 & -3 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 4 & 4 & 1 & 2 \end{vmatrix}$$

3. (2,5 pt.) Resolva os exercícios abaixo.

- (a) (1,0 pt.) Calcule x para que $B = \begin{pmatrix} 1 & x & 0 \\ 0 & 0 & -2 \\ x-1 & 15-3x & 0 \end{pmatrix}$ seja invertível.

$$\begin{aligned} & -2 \cdot (-5)^2 - 4 \cdot (-5) + 30 \\ & -2 \cdot 25 + 20 + 30 \\ & -50 + 50 \end{aligned}$$

- (b) (1,5 pt.) Determine o valor de y na equação

$$\begin{vmatrix} -3 & 1 & -2 & 3 \\ 6 & 2 & y^3 & -1 \\ 45 & 10 & 5 & 0 \\ 3 & 1 & 2 & 0 \end{vmatrix} = -30$$

$$\begin{aligned} & -2 \cdot (3)^2 - 4 \cdot 3 + 30 \\ & -2 \cdot 9 - 12 + 30 \\ & -18 - 12 + 30 \\ & -30 + 30 \end{aligned}$$

4. (2,0 pt.) Verifique se $A = \begin{pmatrix} -11 & 2 & 4 \\ -4 & 0 & 2 \\ 6 & -1 & -2 \end{pmatrix}$ é invertível usando a regra de Sarrus e, em caso afirmativo, calcule a sua inversa usando o método da matriz adjunta.

5. (2,0 pt.) Considere a equação matricial

$$AX + C^t = 3B - 2X$$

$$\begin{vmatrix} -3 & 2 & 4 \\ 0 & 0 & 2 \\ 2 & -1 & -2 \end{vmatrix} = -2 \cdot \begin{vmatrix} -3 & 2 \\ 2 & -1 \end{vmatrix} = -2 \cdot (3 - 4) = -2 \cdot (-1) = 2$$

onde todas as matrizes são quadradas e de mesma ordem 2×2 .

- (a) (0,5 pt.) Determine X em função das matrizes A , B e C e comente se é necessária alguma imposição à matriz A para se resolver a equação.

- (b) (1,5 pt.) Sendo $A = \begin{pmatrix} -1 & 1 \\ -2 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}$ e $C = \begin{pmatrix} 1 & 3 \\ 0 & 5 \end{pmatrix}$, determine a matriz X e seu traço.

$$\begin{aligned} AX + C^t &= 2 \left(\frac{3B}{2} - X \right) \\ AX &= 2 \left(\frac{3B}{2} - X \right) - C^t \end{aligned}$$

$$\begin{aligned} AX + C^t &= X \left(\frac{3B}{X} - 2 \right) \\ AX &= X \left(\frac{3B}{X} \right) \end{aligned}$$

¹Coloque o nome completo nas folhas de prova e escreva o resultado final das questões à caneta. Respostas sem resolução e/ou justificativa não serão consideradas. Não é permitido o uso de quaisquer equipamentos eletrônicos. Data da Avaliação: 20/03/2024

$$\begin{aligned} AX - 2X &= 3B - C^t \\ X(A - 2) \end{aligned}$$

$$X = (3B - C^t) \cdot (A - 2)^{-1}$$

$$\begin{array}{r} 3 \\ 4 \\ 38 \\ 15 \\ 190 \\ 38 \\ 570 \end{array}$$

8,3

$$1) a) A = \begin{bmatrix} 2 \cdot 1 - 1^2 & 2+1 \\ 1+1 & 2 \cdot 2 - 2^2 \end{bmatrix} = \begin{bmatrix} 2-1 & 3 \\ 2 & 4-4 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$$

$$b) 2 \cdot \begin{bmatrix} 1 & 3 \\ 2 & 0 \end{bmatrix} - 3 \cdot \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 4 & 0 \end{bmatrix} - \begin{bmatrix} 6 & 9 \\ 9 & 12 \end{bmatrix} = \begin{bmatrix} 2-6 & 6-9 \\ 4-9 & 0-12 \end{bmatrix}$$

$$2A - 3B = \begin{bmatrix} -4 & -3 \\ -5 & -12 \end{bmatrix}$$

$$c) B^2 = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2 \cdot 2 + 3 \cdot 3 & 2 \cdot 3 + 3 \cdot 4 \\ 3 \cdot 2 + 4 \cdot 3 & 3 \cdot 3 + 4 \cdot 4 \end{bmatrix} = \begin{bmatrix} 13 & 18 \\ 18 & 25 \end{bmatrix}$$

$$B \cdot A^t = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} 2 \cdot 1 + 3 \cdot 3 & 2 \cdot 2 + 3 \cdot 0 \\ 3 \cdot 1 + 4 \cdot 3 & 3 \cdot 2 + 4 \cdot 0 \end{bmatrix} = \begin{bmatrix} 8 & 4 \\ 15 & 6 \end{bmatrix}$$

$$B^2 - BA^t = \begin{bmatrix} 13 & 18 \\ 18 & 25 \end{bmatrix} - \begin{bmatrix} 8 & 4 \\ 15 & 6 \end{bmatrix} = \begin{bmatrix} 5 & 14 \\ 3 & 19 \end{bmatrix}$$

$$d) \text{tr}(A+B) - \det(B^2)$$

$$= \text{tr}(A) + \text{tr}(B) - (\det(B))^2$$

$$= (1+0) + (2+4) - (-1)^2$$

$$= 1 + 6 - (-1)$$

$$= 7 + 1$$

$$= 8$$

$$|B| = \begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix} = 2 \cdot 4 - 3 \cdot 3 = 8 - 9 = -1$$

$$e) \det(3B^t \cdot A^{-1})$$

$$= \det(3B^t) \cdot \det(A^{-1})$$

$$= \det(3B) \cdot \frac{1}{\det A}$$

$$= 3^2 \cdot \det B \cdot \frac{1}{\det A}$$

$$= 9 \cdot (-1) \cdot \frac{1}{-6}$$

$$= \frac{-9 \cdot -1}{6} = \frac{9}{6} = \boxed{\frac{3}{2}}$$

$$|A| = \begin{vmatrix} 1 & 3 \\ 2 & 0 \end{vmatrix} = 0 - 6 = -6$$

$$|B| = \begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix} = 8 - 9 = -1$$

0,5

$$\begin{aligned} \textcircled{2} \begin{vmatrix} 0 & 1 & -1 & 0 & 3 \\ 0 & 2 & 1 & 0 & 0 \\ 0 & -1 & 0 & 2 & -3 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & 4 & 4 & 1 & 2 \end{vmatrix} &= -1 \cdot (-1)^{5+1} \cdot D_{14} \\ &= D_{14} \\ &= \begin{vmatrix} 1 & -1 & 0 & 3 \\ 2 & 1 & 0 & 0 \\ -1 & 0 & 2 & -3 \\ 4 & 4 & 1 & 2 \end{vmatrix} = -a_{21} \cdot D_{21} + a_{22} \cdot D_{22} \\ &= -2 \cdot \begin{vmatrix} -1 & 0 & 3 \\ 0 & 2 & -3 \\ 4 & 1 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 0 & 3 \\ -1 & 2 & -3 \\ 4 & 1 & 2 \end{vmatrix} \end{aligned}$$

$$= -2 \cdot [-1 \cdot D_{11} + 3 \cdot D_{13}] + 1 \cdot D_{11} + 3 \cdot D_{13}$$

$$= -2 \cdot \left[- \begin{vmatrix} 2 & -3 \\ 1 & 2 \end{vmatrix} + 3 \cdot \begin{vmatrix} 0 & 2 \\ 4 & 1 \end{vmatrix} \right] + \begin{vmatrix} 2 & -3 \\ 1 & 2 \end{vmatrix} + 3 \cdot \begin{vmatrix} -1 & 2 \\ 4 & 1 \end{vmatrix}$$

$$= -2 \cdot [-7 + 3 \cdot -8] + 7 + 3 \cdot -9$$

$$= -2 \cdot [-7 - 24] + 7 - 27$$

$$= -2 \cdot -31 - 20$$

$$= 62 - 20$$

$$= 42$$

$$\boxed{\det = 42}$$

1,0

③ a) $\det(B) \neq 0$

$$|B| = \begin{vmatrix} 1 & x & 0 \\ 0 & 0 & -2 \\ x-1 & 15-3x & 0 \end{vmatrix} = a_{23} \cdot A_{23} = -2 \cdot (-1)^{2+3} \cdot \begin{vmatrix} 1 & x \\ x-1 & 15-3x \end{vmatrix}$$

$$= 2 \cdot [15 - 3x - (x \cdot (x-1))] = 2 \cdot [15 - 3x - x^2 + x] = 2 \cdot [-x^2 - 2x + 15]$$

$$= -2x^2 - 4x + 30$$

$$-2x^2 - 4x + 30 \neq 0$$

$$-x^2 - 2x + 15 \neq 0$$

$$S = \frac{-b}{a} = \frac{2}{-1} = -2 \quad \begin{matrix} -5 \\ 3 \end{matrix}$$

$$P = \frac{c}{a} = \frac{15}{-1} = -15 \quad \begin{matrix} -5 \\ 3 \end{matrix}$$

$$\mathbb{R} - \{-5, 3\}$$

1,0

C

b)

$$\begin{vmatrix} -3 & 1 & -2 & 3 \\ 6 & 2 & y^3 & -1 \\ 45 & 10 & 5 & 0 \\ 3 & 1 & 2 & 0 \end{vmatrix} \stackrel{:3}{=} \begin{vmatrix} -3 & 1 & -2 & 3 \\ 6 & 2 & y^3 & -1 \\ 9 & 2 & 1 & 0 \\ 3 & 1 & 2 & 0 \end{vmatrix} \stackrel{:3}{=} \begin{vmatrix} -1 & 1 & -2 & 3 \\ 2 & 2 & y^3 & -1 \\ 3 & 2 & 1 & 0 \\ 1 & 1 & 2 & 0 \end{vmatrix} \cdot 15$$

$$= \begin{vmatrix} 5 & 7 & -2+3y^3 & 0 \\ 2 & 2 & y^3 & -1 \\ 3 & 2 & 1 & 0 \\ 1 & 1 & 2 & 0 \end{vmatrix} \cdot 15 = -1 \cdot (-1)^{1+4} \cdot \begin{vmatrix} 5 & 7 & -2+3y^3 \\ 3 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix}$$

$$= -15 \cdot \begin{vmatrix} -2 & 7 & -2+3y^3 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix} = -15 \cdot [-2 \cdot (14 + 2 - 3y^3) + 3 \cdot (5 - 7)]$$

$$= -15 \cdot [-2 \cdot (16 - 3y^3) + 3 \cdot (-2)]$$

$$= -15 \cdot [-32 + 6y^3 - 6]$$

$$= -15 \cdot (-38 + 6y^3)$$

$$= 570 - 90y^3$$

$$-30 = 570 - 90y^3$$

$$90y^3 = 570 + 30$$

$$90y^3 = 600$$

$$y^3 = \frac{600}{90} = \frac{20}{3}$$

$$y = \sqrt[3]{\frac{20}{3}}$$

$$y = \frac{\sqrt[3]{20}}{\sqrt[3]{3}}$$

1,2

0,7

3

4) $|A| = \begin{vmatrix} -11 & 2 & 4 \\ -4 & 0 & 2 \\ 6 & -1 & -2 \end{vmatrix} = \begin{vmatrix} -11 & 2 \\ -4 & 0 \\ 6 & -1 \end{vmatrix}$
 $0 + 22 + 16 = 38$ $0 + 24 + 16 = 40$

$|A| = 40 - 38 = 2$

$|A| \neq 0$, dessa forma,
 existe A^{-1}

$A^{-1} = \frac{\text{Adj } A}{|A|}$

$\text{Cof } A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} \begin{vmatrix} 0 & 2 \\ -1 & -2 \end{vmatrix} & - \begin{vmatrix} -4 & 2 \\ 6 & -2 \end{vmatrix} & \begin{vmatrix} -4 & 0 \\ -6 & -1 \end{vmatrix} \\ - \begin{vmatrix} 2 & 4 \\ -1 & -2 \end{vmatrix} & \begin{vmatrix} -11 & 4 \\ 6 & -2 \end{vmatrix} & - \begin{vmatrix} -11 & 2 \\ 6 & -1 \end{vmatrix} \\ \begin{vmatrix} 2 & 4 \\ 0 & 2 \end{vmatrix} & - \begin{vmatrix} -11 & 4 \\ -4 & 2 \end{vmatrix} & \begin{vmatrix} -11 & 2 \\ -4 & 0 \end{vmatrix} \end{bmatrix}$

$\text{Adj } A = \begin{bmatrix} 2 & 0 & 4 \\ 4 & -2 & 6 \\ 4 & 1 & 8 \end{bmatrix}$

$= \begin{bmatrix} -2 & -(8-12) & 4 \\ -(-4-4) & 22-24 & -(11-12) \\ -4 & -(-22-16) & 8 \end{bmatrix}$

$= \begin{bmatrix} 2 & 4 & 4 \\ 0 & -2 & 1 \\ 4 & 6 & 8 \end{bmatrix}$

2,0

$A^{-1} = \begin{bmatrix} \frac{2}{2} & 0 & \frac{4}{2} \\ \frac{4}{2} & -\frac{2}{2} & \frac{6}{2} \\ \frac{4}{2} & \frac{1}{2} & \frac{8}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 2 & \frac{1}{2} & 4 \end{bmatrix}$

5) a) $Ax + C^t = 3B - 2x$

$Ax = 3B - 2x - C^t$

$Ax + 2x = 3B - C^t$

$x(A+2) = 3B - C^t$

~~$x = A^{-1} \cdot (3B - C^t)$~~

$x = (3B - C^t) \cdot (A+2)^{-1}$

$x = (3B - C^t) \cdot (A+2)^{-1}$

0,2

X

A deve ser inversível, seu determinante não pode ser 0

b) $|A| = -3 - (-2) = -3 + 2 = -1$

$\text{Adj } A = \begin{bmatrix} 3 & -1 \\ 2 & -1 \end{bmatrix}$

$3B = \begin{bmatrix} 9 & -6 \\ 6 & -3 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} -3 & 1 \\ -2 & 1 \end{bmatrix}$

$C^t = \begin{bmatrix} 1 & 3 \\ 0 & 5 \end{bmatrix}$

0,5

$x = \begin{bmatrix} 9-1 & -6-3 \\ 6-0 & -3-5 \end{bmatrix} \cdot (A+2)^{-1}$

$= \begin{bmatrix} -24+18 & 8-9 \\ -18+16 & 6-8 \end{bmatrix} + \begin{bmatrix} 4 & -\frac{9}{2} \\ 3 & -4 \end{bmatrix}$

$= \begin{bmatrix} 8 & -9 \\ 6 & -8 \end{bmatrix} \cdot (A+2)^{-1}$

$= \begin{bmatrix} -6 & -1 \\ -2 & -2 \end{bmatrix} + \begin{bmatrix} 4 & -\frac{9}{2} \\ 3 & -4 \end{bmatrix}$

$= \begin{bmatrix} 8 & -9 \\ 6 & -8 \end{bmatrix} \times A^{-1} + \begin{bmatrix} 8 & -9 \\ 6 & -8 \end{bmatrix} \times \frac{1}{2}$

$= \begin{bmatrix} -2 & -\frac{3}{2} + \frac{9}{2} \\ 1 & -6 \end{bmatrix} = \begin{bmatrix} -2 & -\frac{11}{2} \\ 1 & -6 \end{bmatrix}$

$= \begin{bmatrix} 8 & -9 \\ 6 & -8 \end{bmatrix} \cdot \begin{bmatrix} -3 & 1 \\ -2 & 1 \end{bmatrix} + \begin{bmatrix} 4 & -\frac{9}{2} \\ 3 & -4 \end{bmatrix}$

$x = \begin{bmatrix} -2 & -\frac{11}{2} \\ 1 & -6 \end{bmatrix}$