Chapter Six
Demand

Li Yan(李艳)
School of Economics
Central university of Finance and Economics

Properties of Demand Functions

- Comparative statics analysis （比较静态分析）of ordinary demand functions -- the study of how ordinary demands $x_1^*(p_1,p_2,y)$ and $x_2^*(p_1,p_2,y)$ change as prices p_1, p_2 and income y change.

Structure

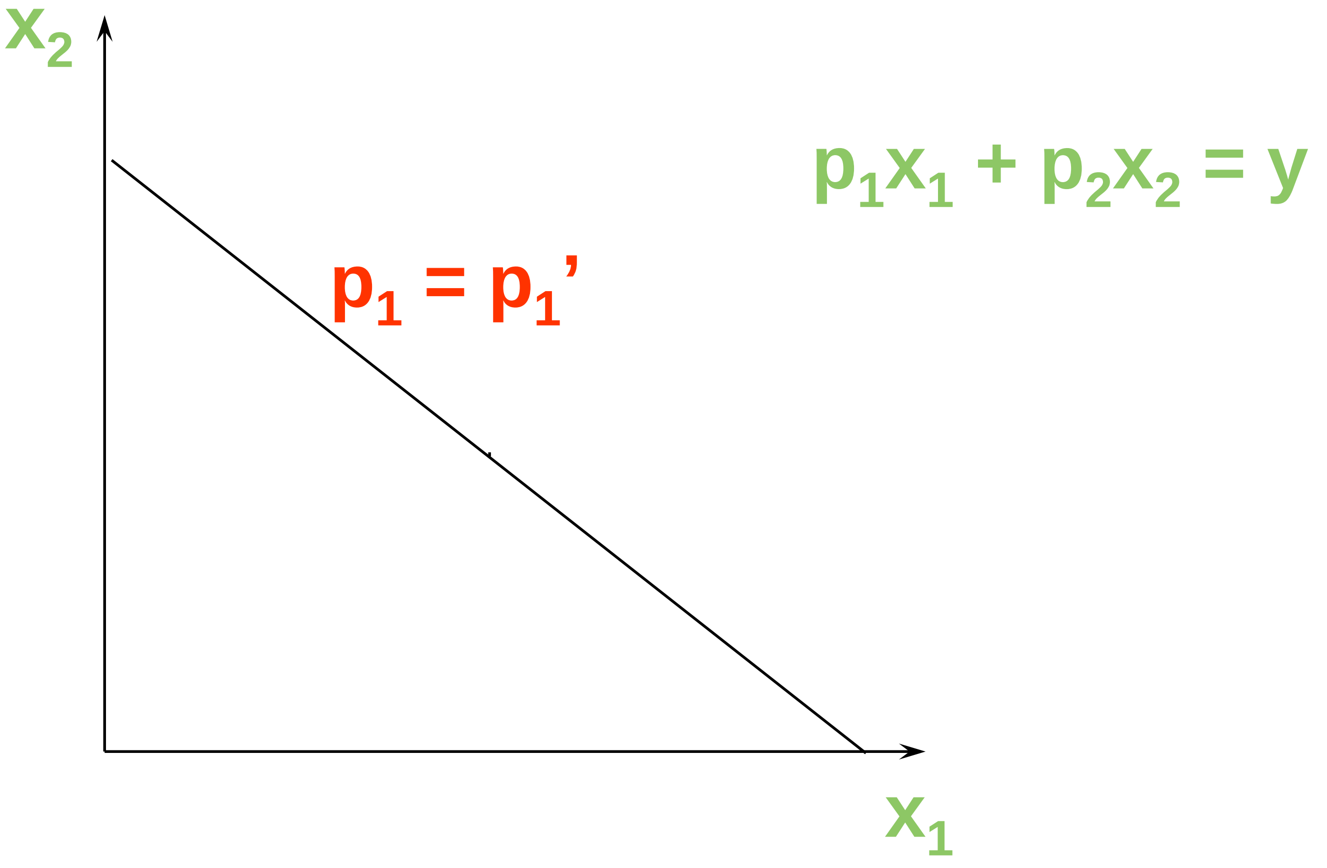
- Own-price changes
  - Price offer curve （价格提供曲线）
  - Ordinary demand curve
  - Inverse demand curve （反需求函数）
- Income changes
  - Income offer curve （收入提供曲线）
  - Engel curve （恩格尔曲线）
- Cross-price effects

Own-Price Changes

- How does $x_1^*(p_1,p_2,y)$ change as p_1 changes, holding p_2 and y constant?
- Suppose only p_1 increases, from p_1' to p_1'' and then to p_1'''.

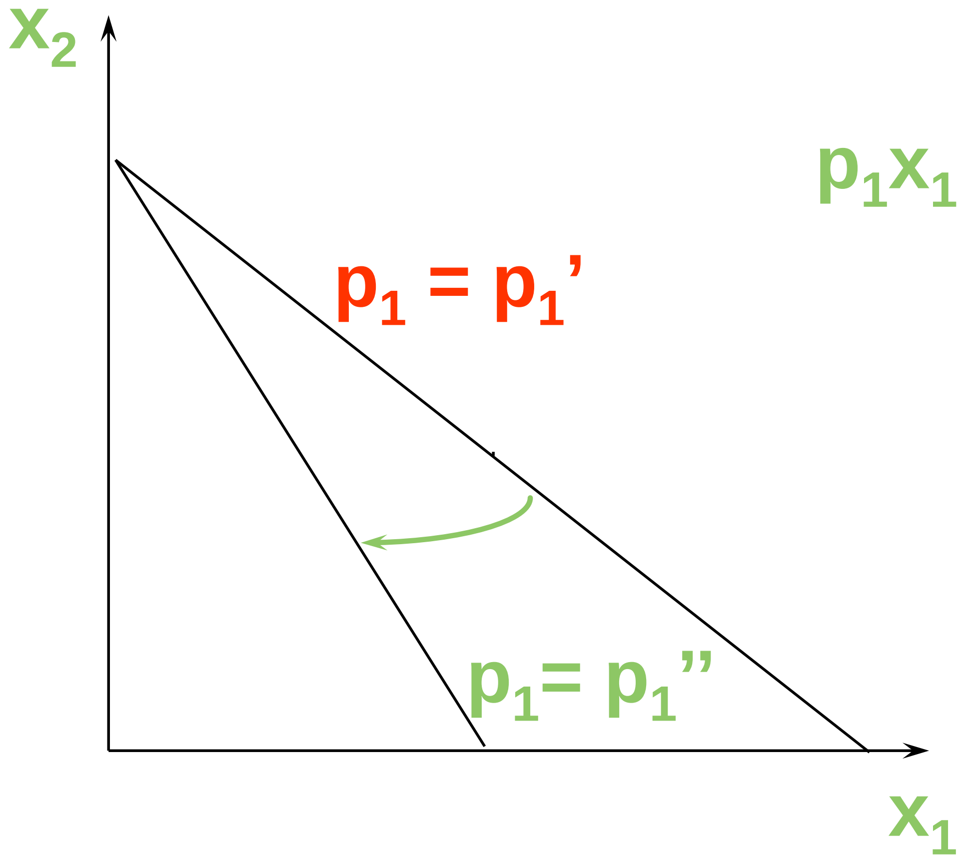
Own-Price Changes

Fixed p_2 and y.



Own-Price Changes

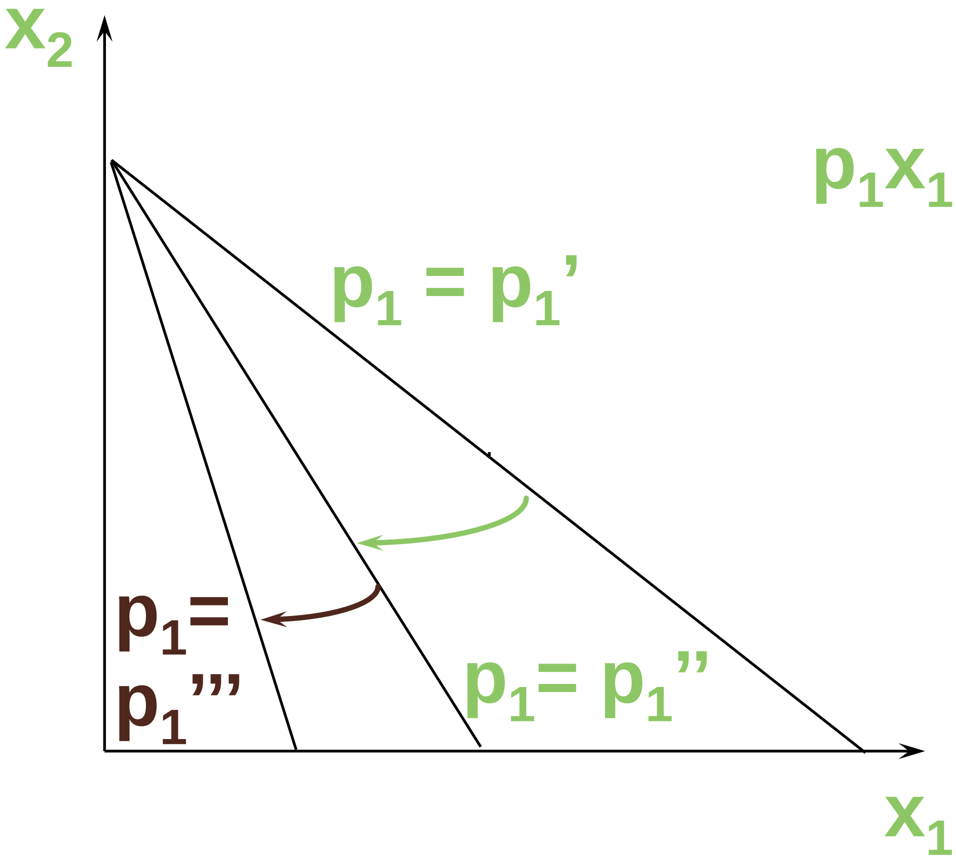
Fixed p_2 and y.



$p_1x_1 + p_2x_2 = y$

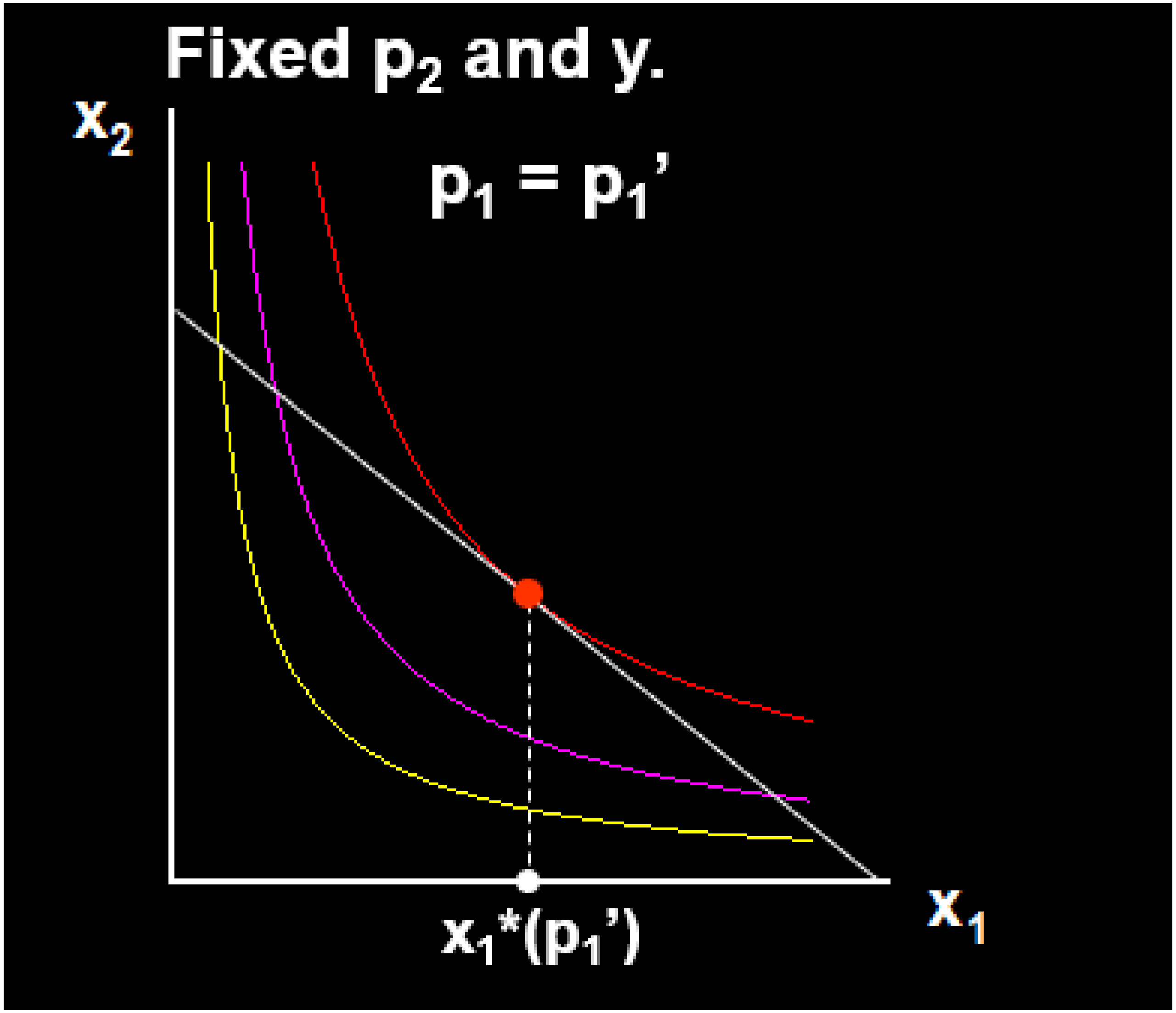
Own-Price Changes

Fixed p_2 and y.

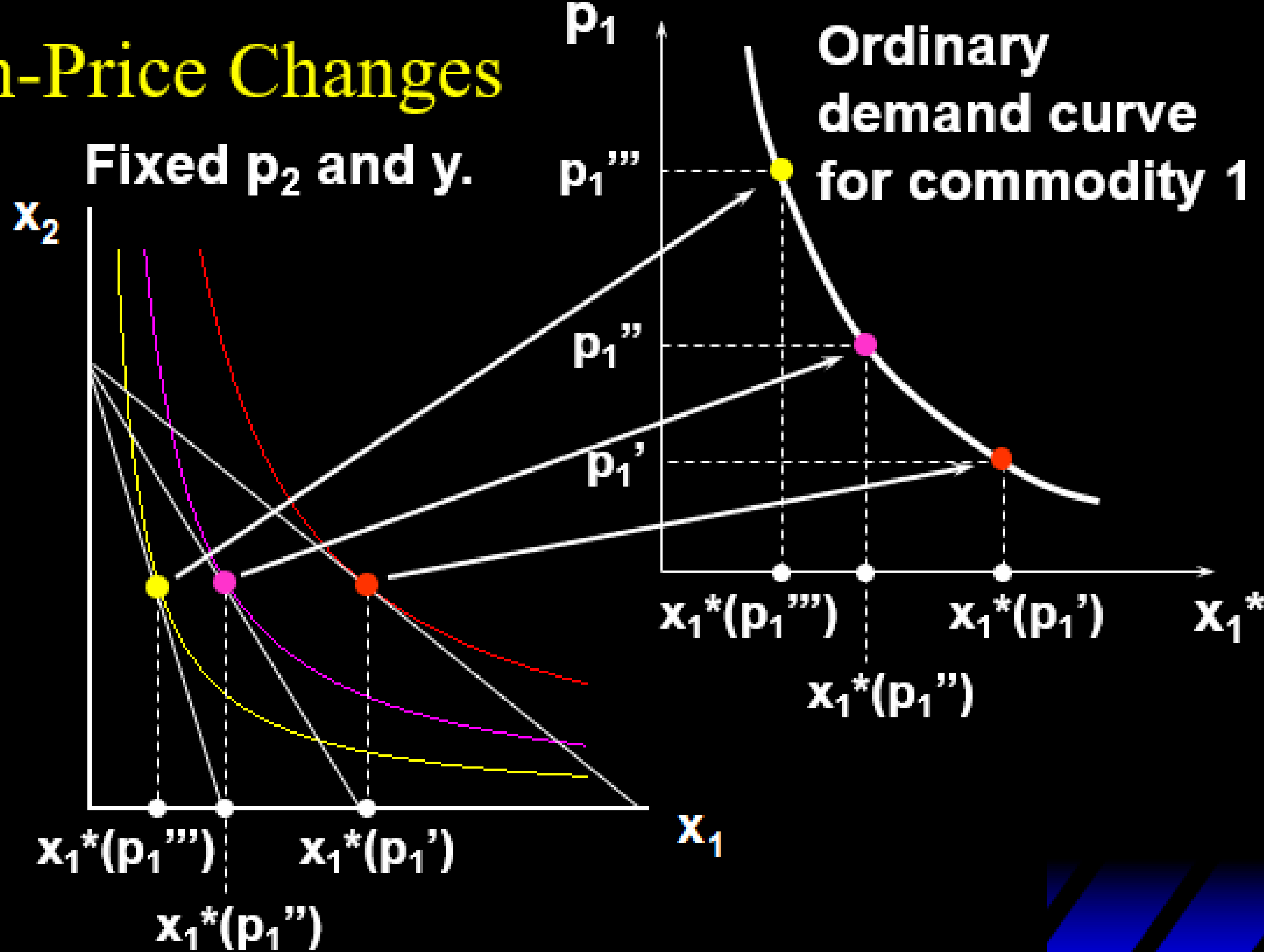


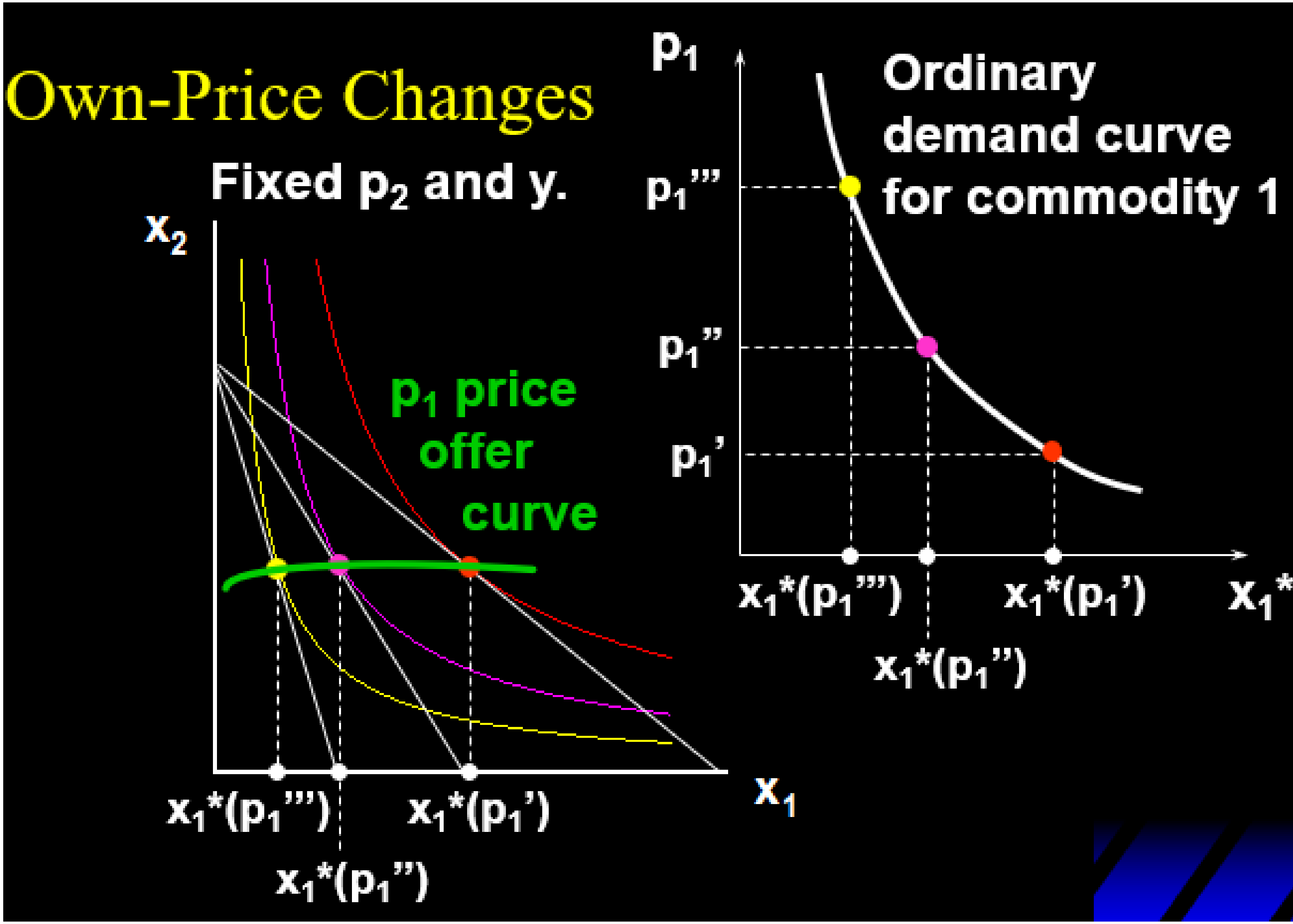
$p_1x_1 + p_2x_2 = y$

Own-Price Changes



Own-Price Changes




Own-Price Changes
Fixed p2 and y.
x2
p1 price offer curve
x1*(p1''')
x1*(p1')
x1*(p1'')
x1
p1
Ordinary demand curve for commodity 1
p1'''
p1''
p1'
x1*

Own-Price Changes

- The curve containing all the utility-maximizing bundles traced out as p_1 changes, with p_2 and y constant, is the p_1-price offer curve.
- The plot of the x_1-coordinate of the p_1-price offer curve against p_1 is the ordinary demand curve for commodity 1.

Own-Price Changes

- What does a p_1 price-offer curve look like for Cobb-Douglas preferences?

Take $U(x_1, x_2) = x_1^a x_2^b.$

 Then the ordinary demand functions for commodities 1 and 2 are

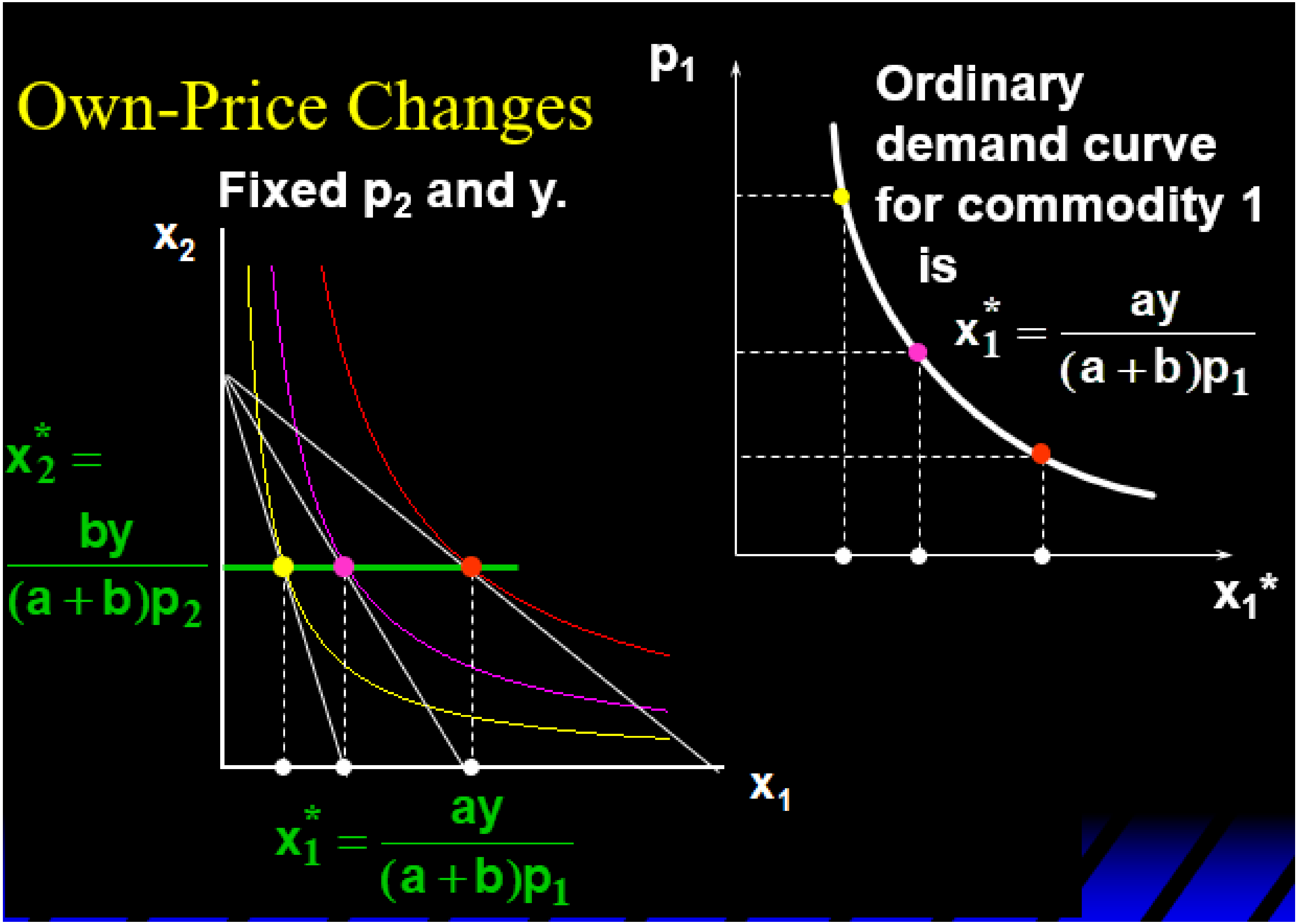
Own-Price Changes

$$x_1^*(p_1,p_2,y) = \frac{a}{a+b} \times \frac{y}{p_1}$$

and

$$x_2^*(p_1,p_2,y) = \frac{b}{a+b} \times \frac{y}{p_2}.$$

Notice that x_2* does not vary with p_1 so the p_1 price offer curve is **flat** and the ordinary demand curve for commodity 1 is a **rectangular hyperbola**.


Own-Price Changes
Fixed p_2 and y.
x_2
$x_2^* = \frac{by}{(a+b)p_2}$
$x_1^* = \frac{ay}{(a+b)p_1}$
x_1
p_1
Ordinary demand curve for commodity 1 is
$x_1^* = \frac{ay}{(a+b)p_1}$
x_1^*

Own-Price Changes

- What does a p_1 price-offer curve look like for a perfect-complements utility function?

$$U(x_1, x_2) = \min\{x_1, x_2\}.$$

Then the ordinary demand functions for commodities 1 and 2 are

Own-Price Changes

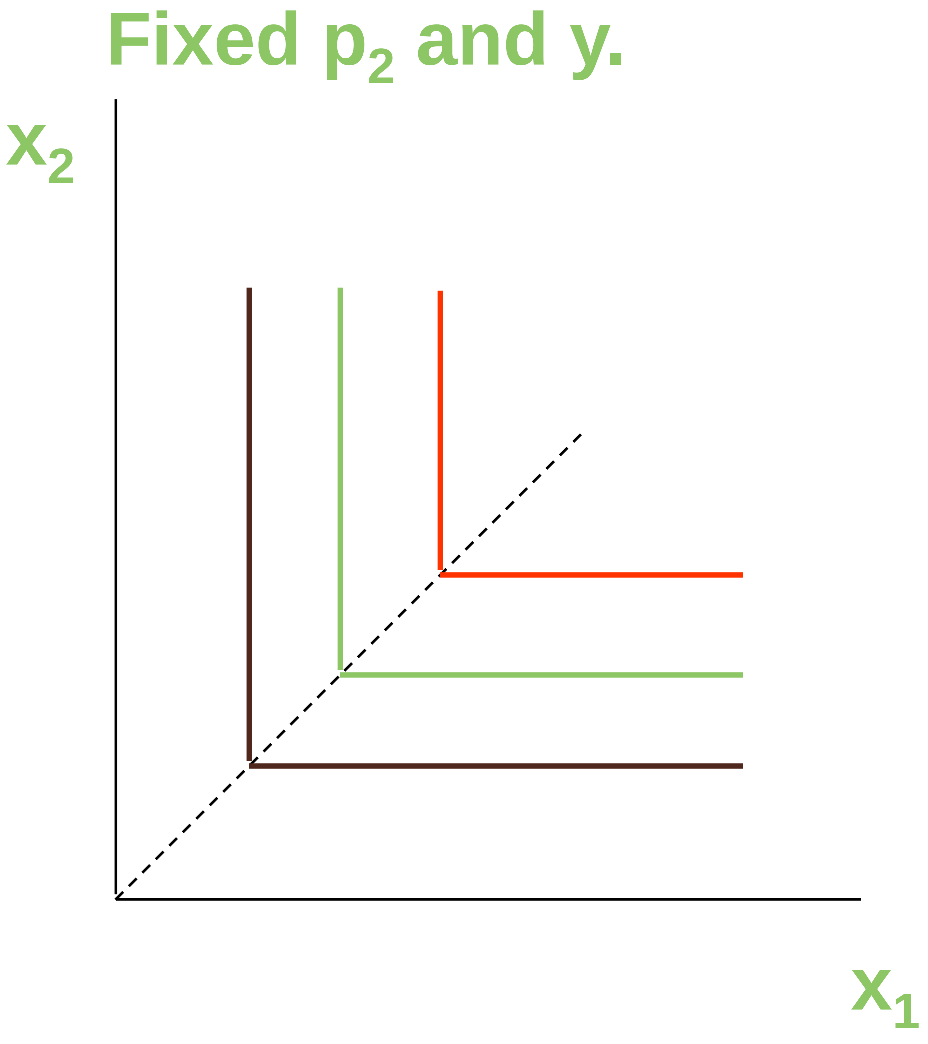
$$x_1^*(p_1,p_2,y) = x_2^*(p_1,p_2,y) = \frac{y}{p_1+p_2}.$$

With p_2 and y fixed, higher p_1 causes smaller x_1* and x_2*.

As $p_1 \to 0, \quad x_1^* = x_2^* \to \dfrac{y}{p_2}.$

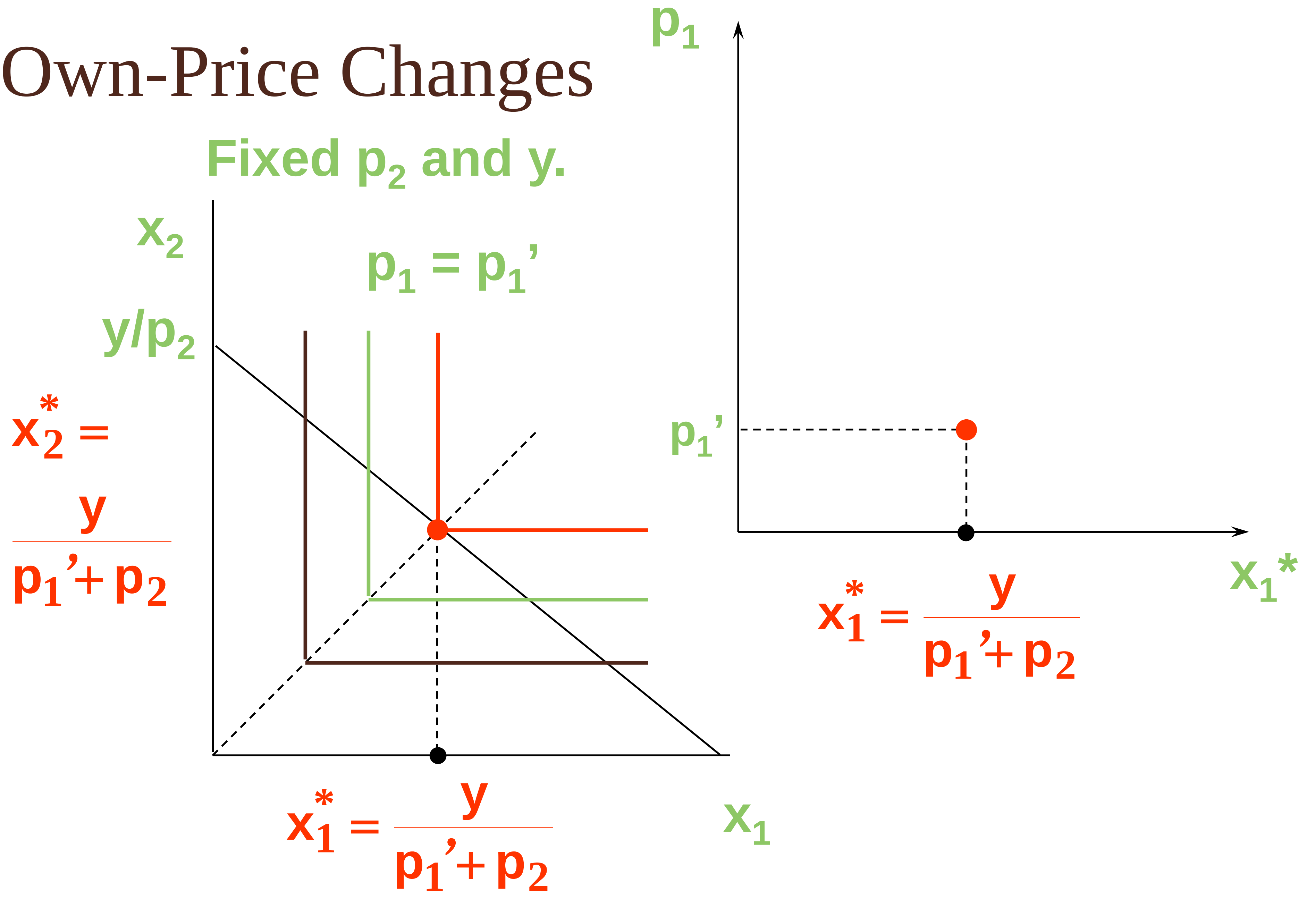
As $p_1 \to \infty, \quad x_1^* = x_2^* \to 0.$

Own-Price Changes



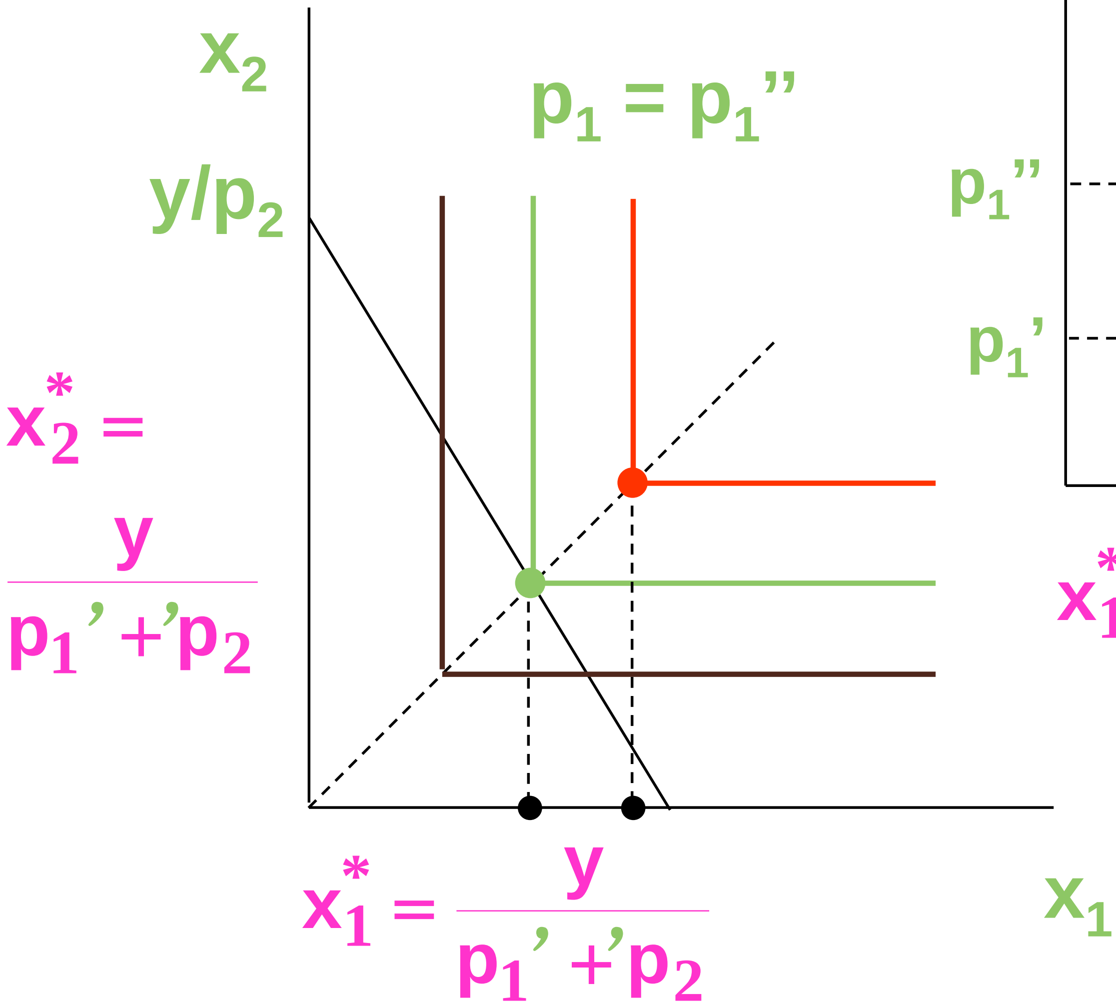
Own-Price Changes

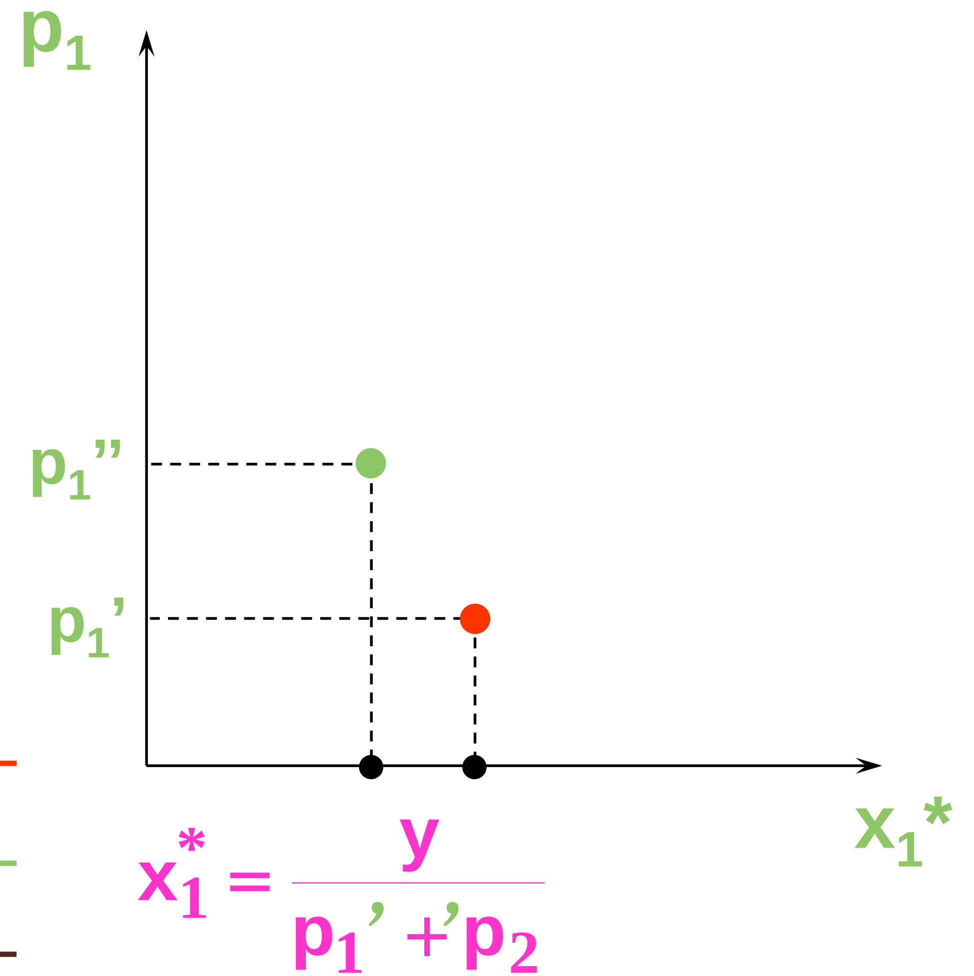
Fixed p_2 and y.



Own-Price Changes

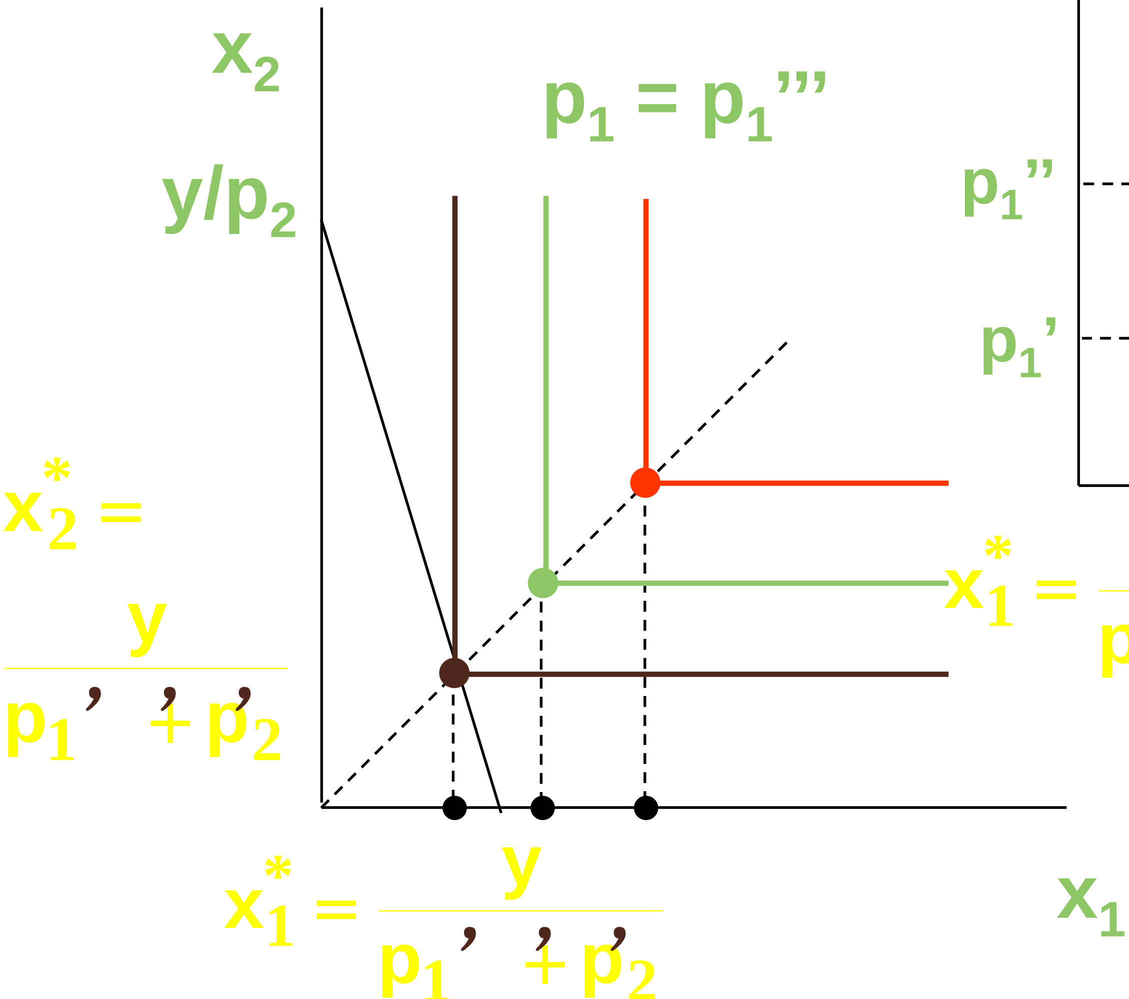
Fixed p_2 and y.

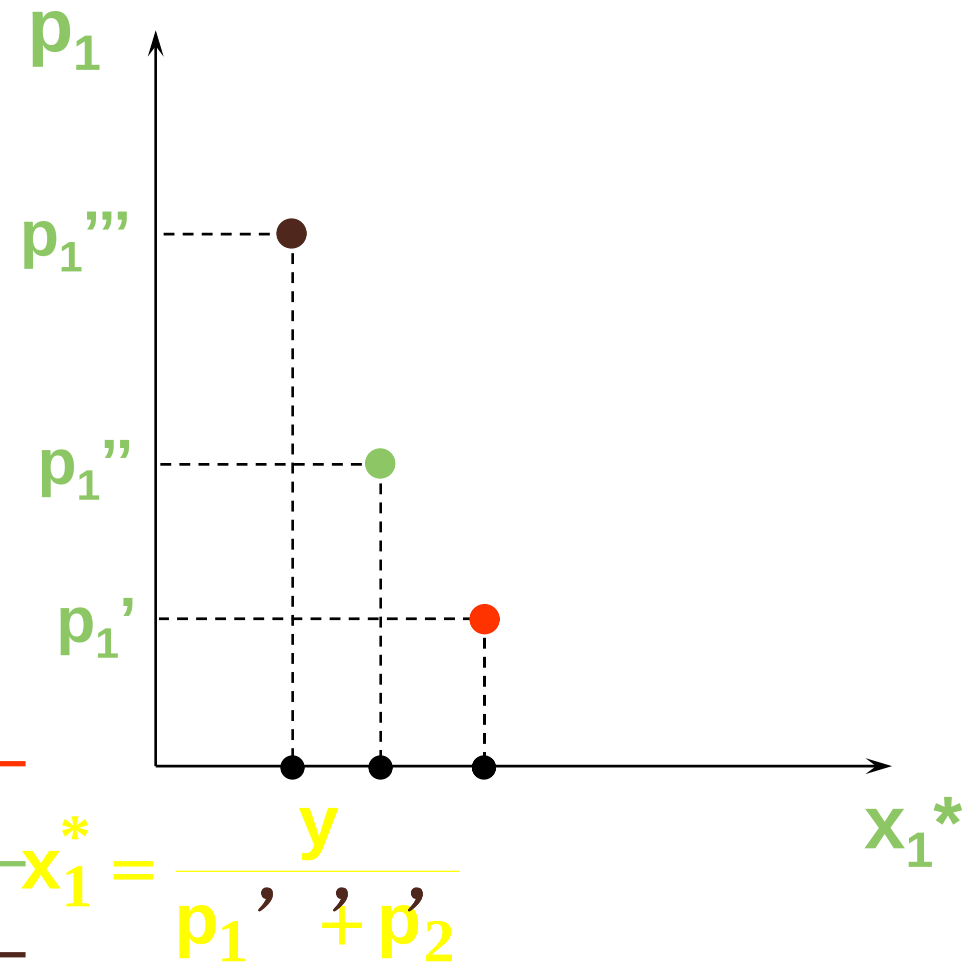




Own-Price Changes

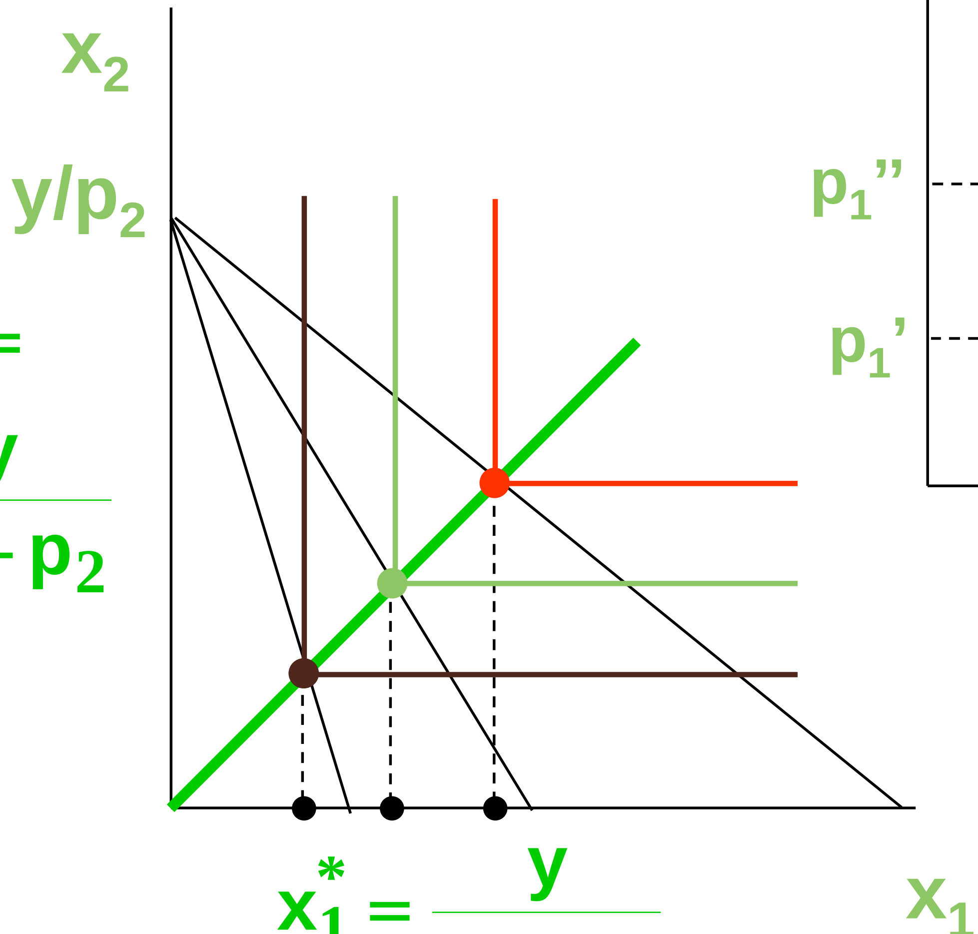
Fixed p_2 and y.





Own-Price Changes

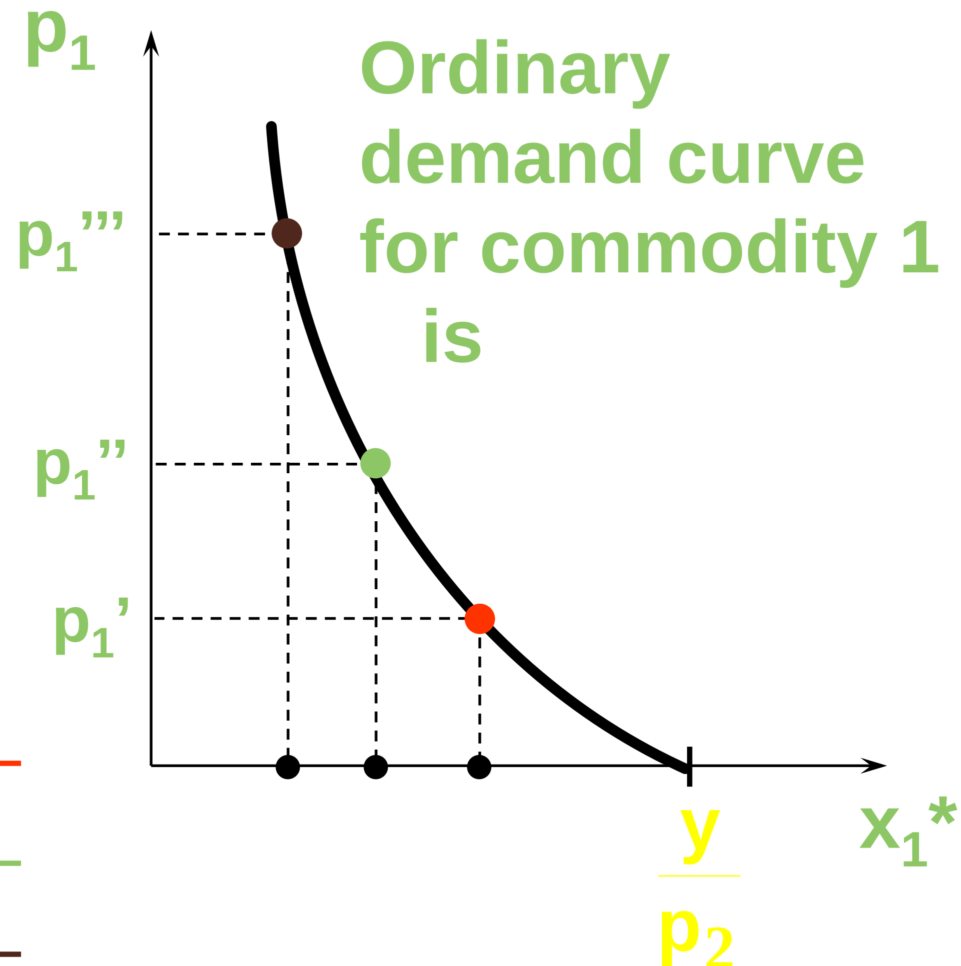
Fixed p_2 and y.



$$x_2^* = \frac{y}{p_1 + p_2}$$

$$x_1^* = \frac{y}{p_1 + p_2}$$

Ordinary demand curve for commodity 1 is



Own-Price Changes

- What does a p_1 price-offer curve look like for a perfect-substitutes utility function?

$$U(x_1, x_2) = x_1 + x_2.$$

Then the ordinary demand functions for commodities 1 and 2 are

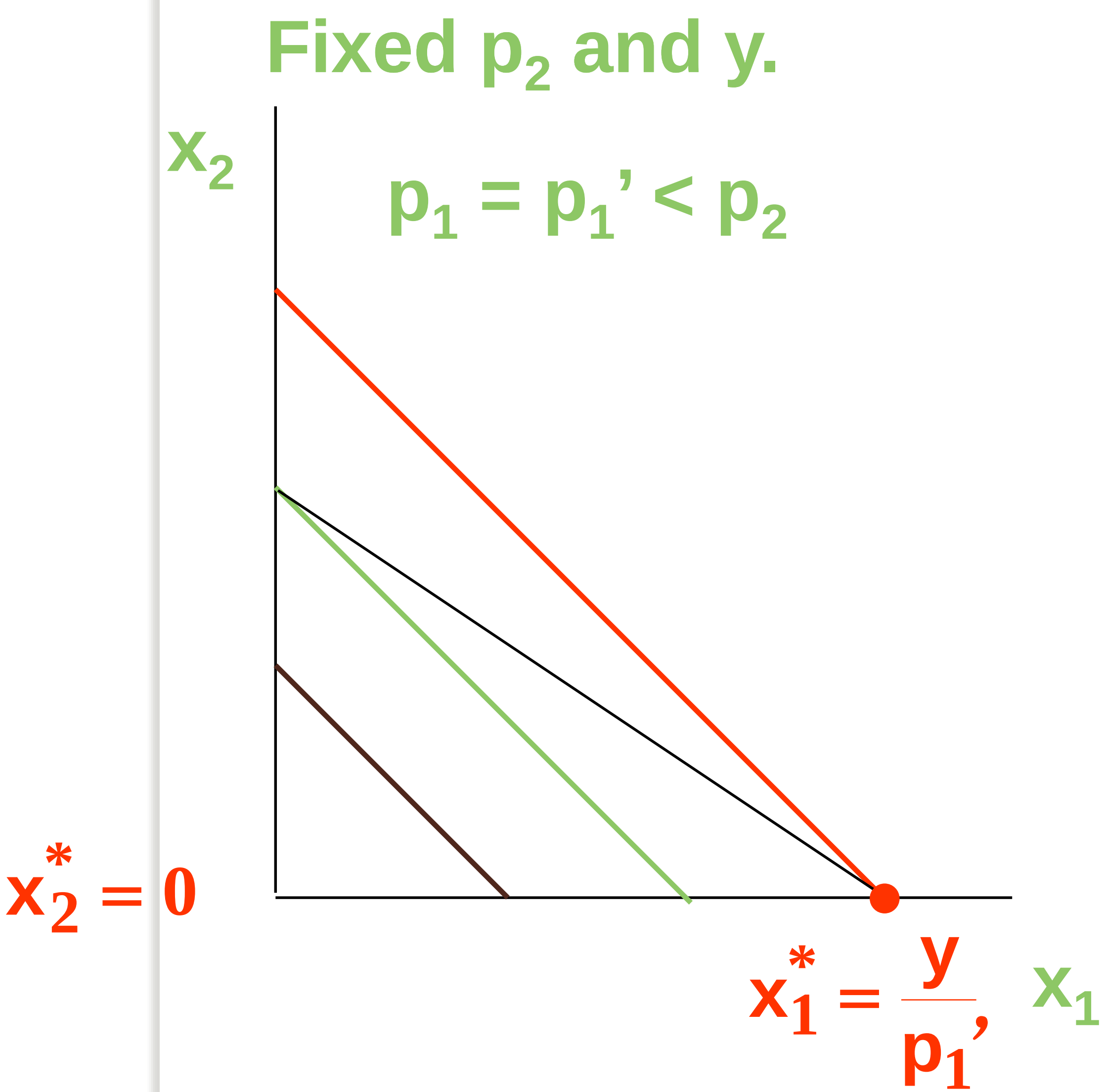
Own-Price Changes

$$x_1^*(p_1,p_2,y) = \begin{cases} 0 & \text{, if } p_1 > p_2 \\ y/p_1 & \text{, if } p_1 < p_2 \end{cases}$$

and

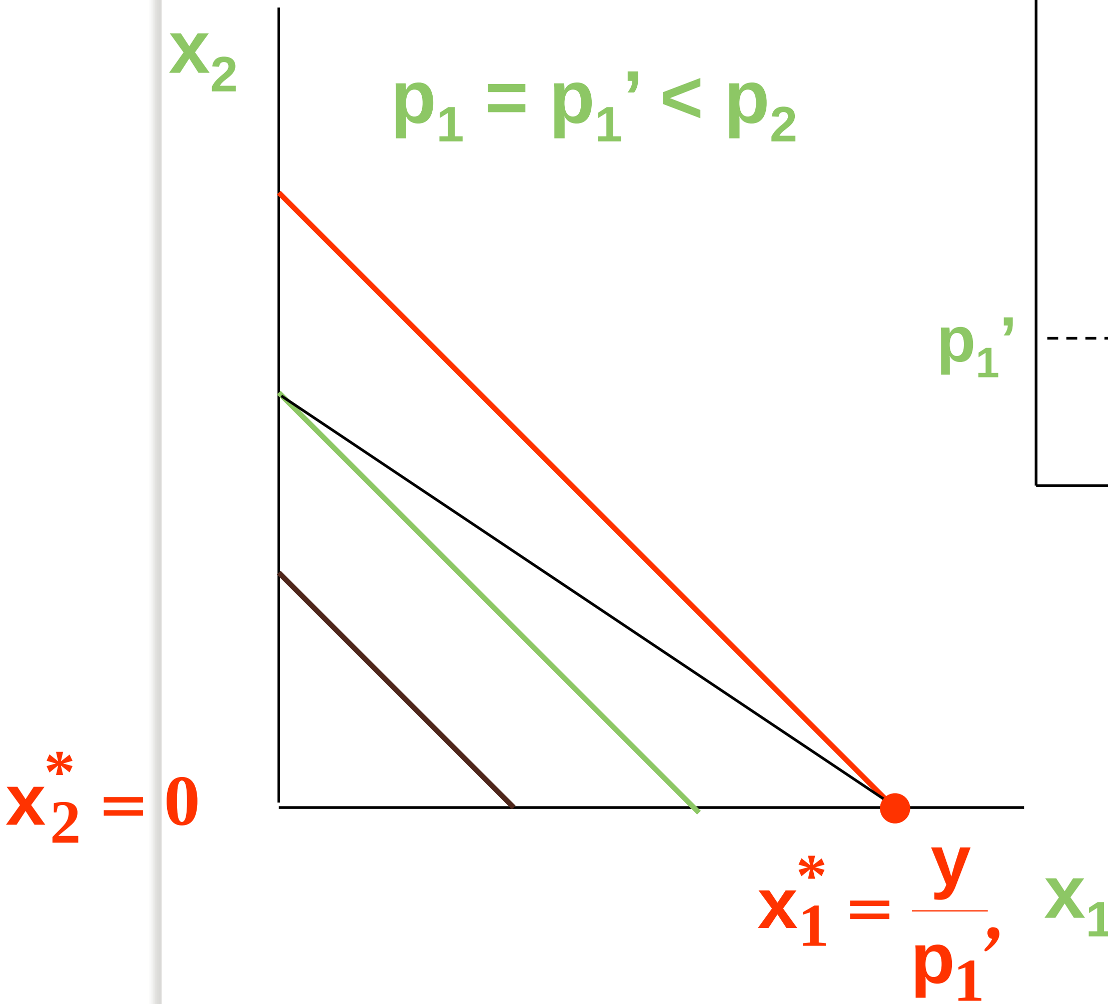
$$x_2^*(p_1,p_2,y) = \begin{cases} 0 & \text{, if } p_1 < p_2 \\ y/p_2 & \text{, if } p_1 > p_2. \end{cases}$$

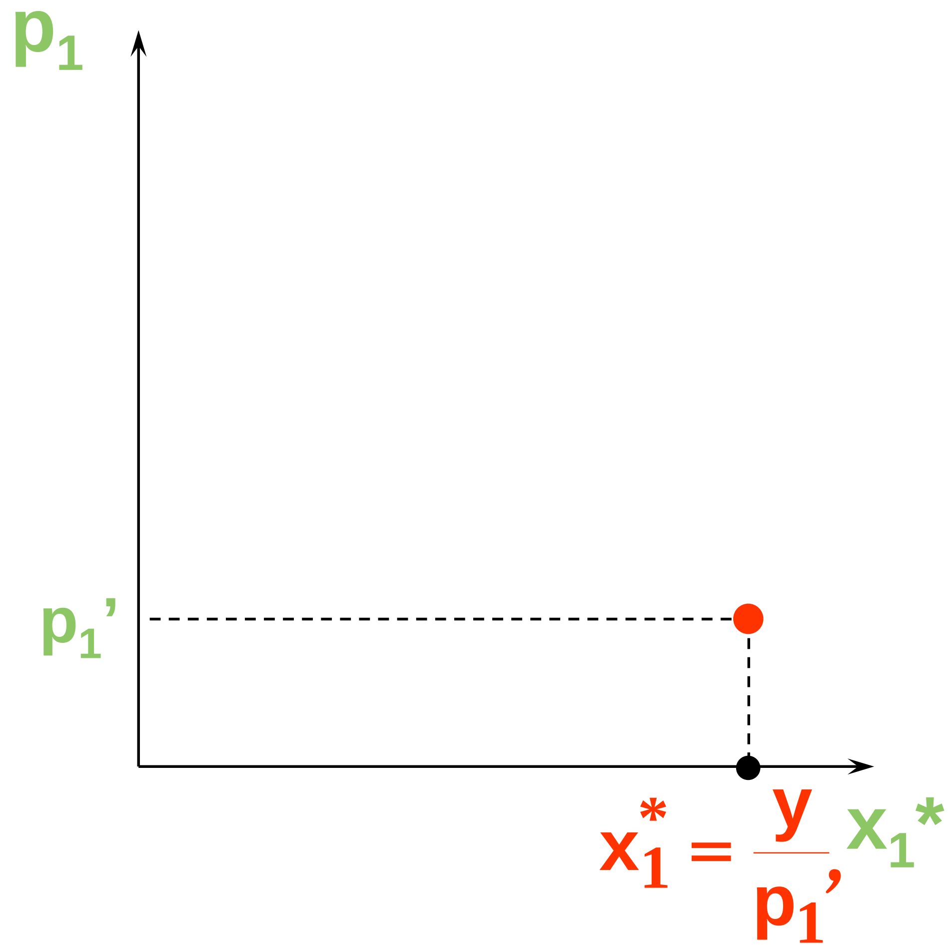
Own-Price Changes



Own-Price Changes

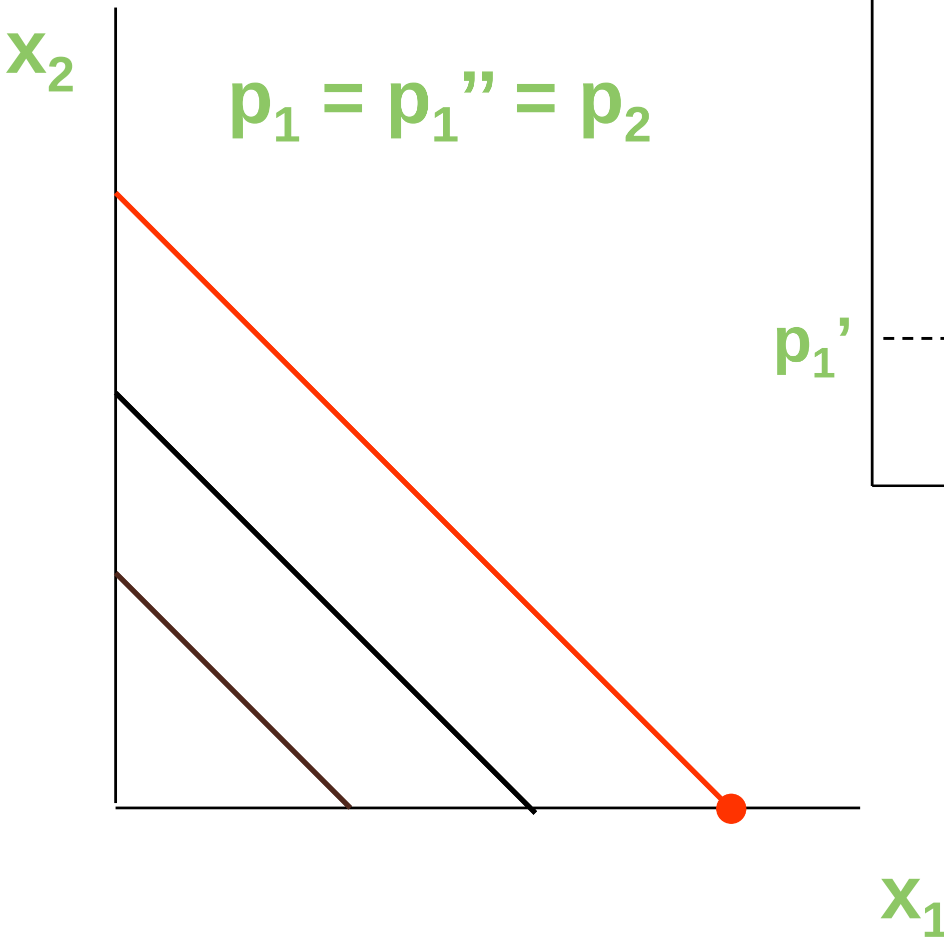
Fixed p_2 and y.

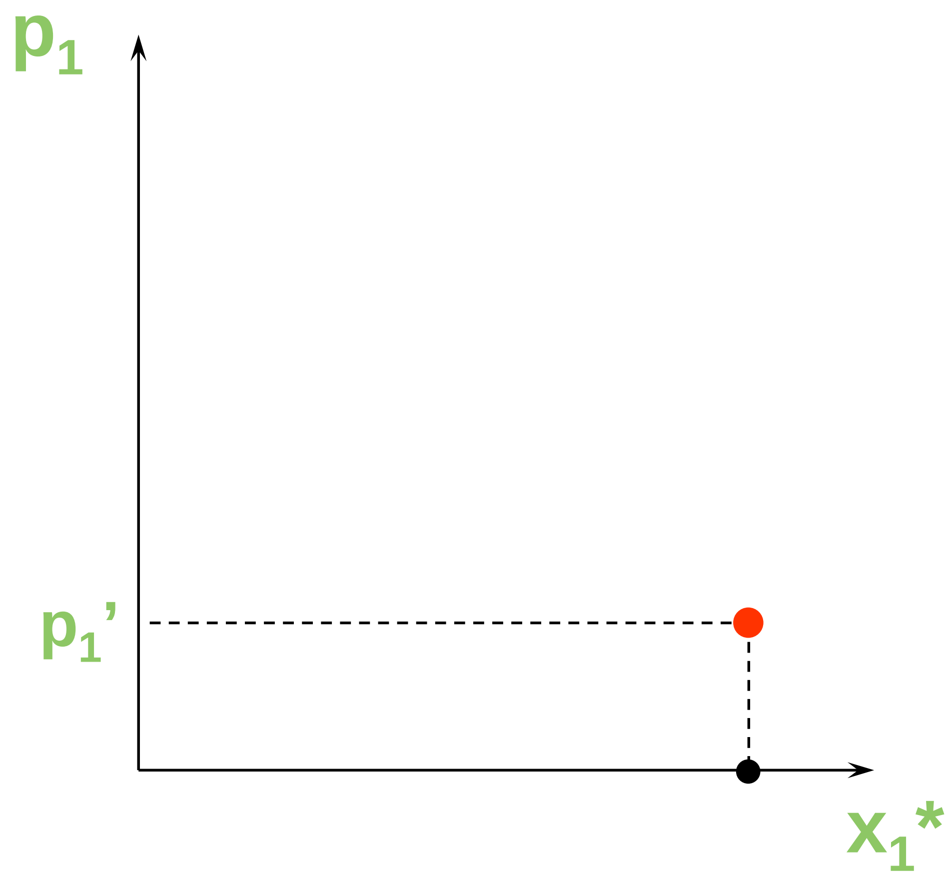




Own-Price Changes

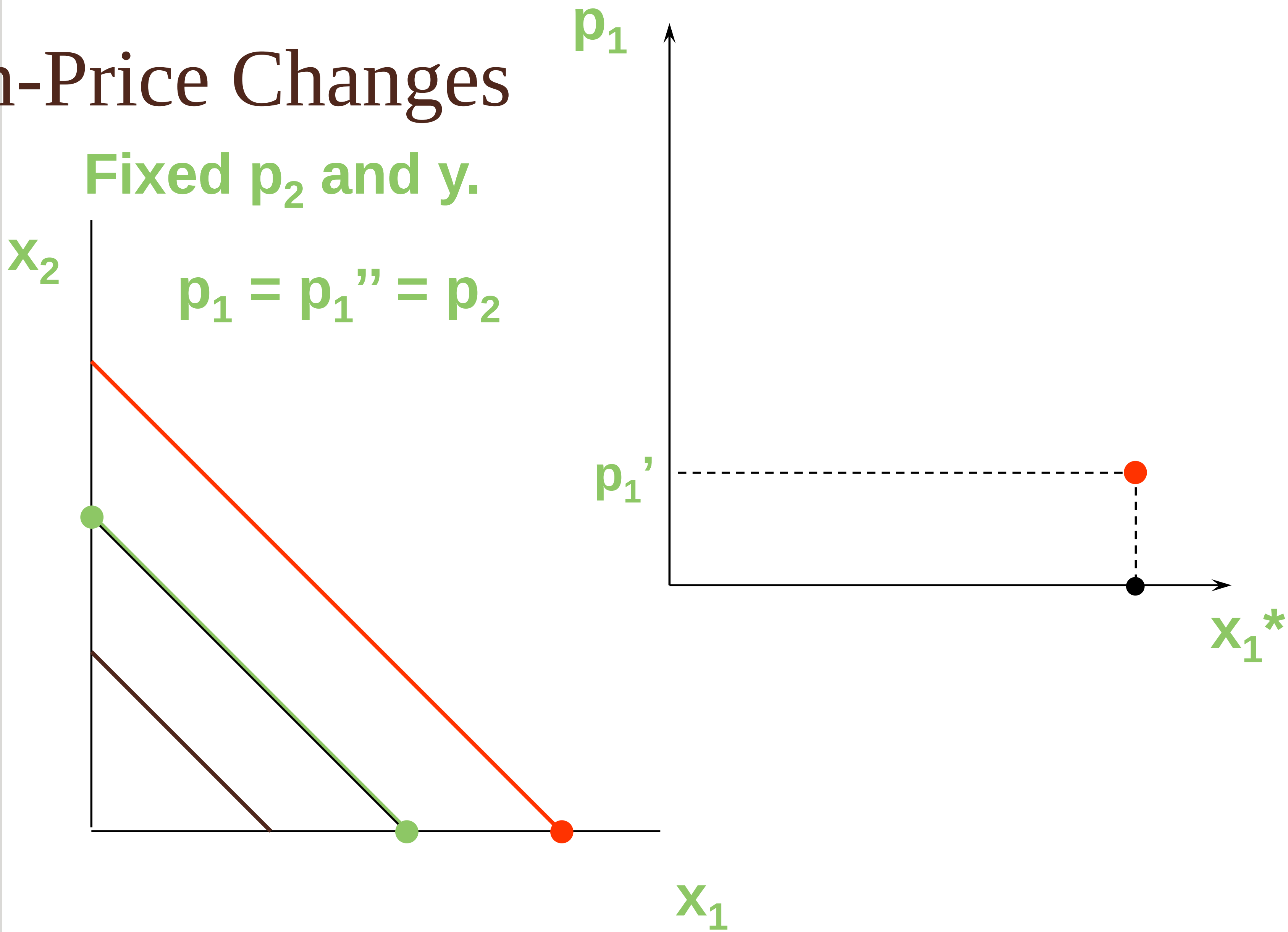
Fixed p_2 and y.



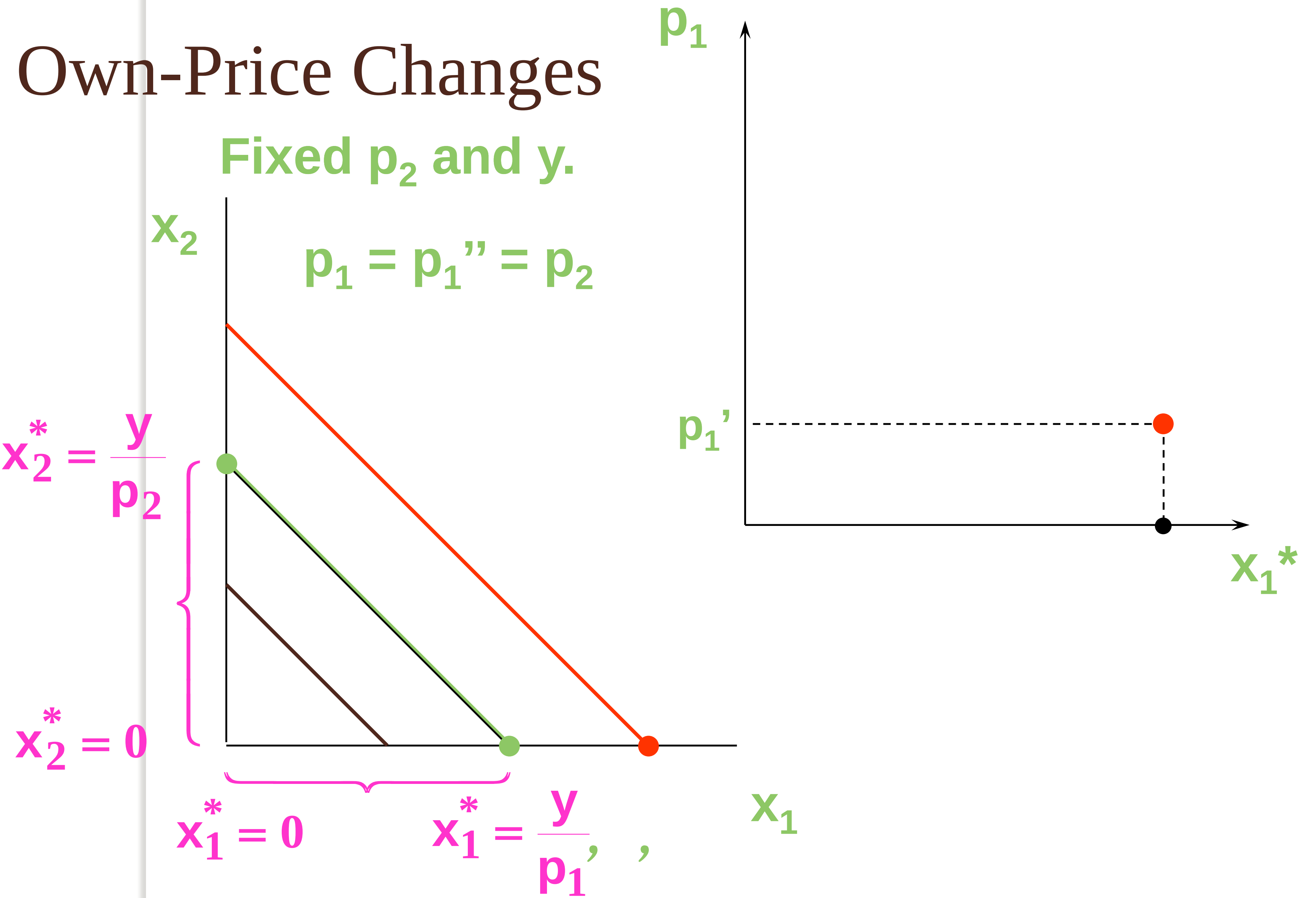


Own-Price Changes

Fixed p_2 and y.

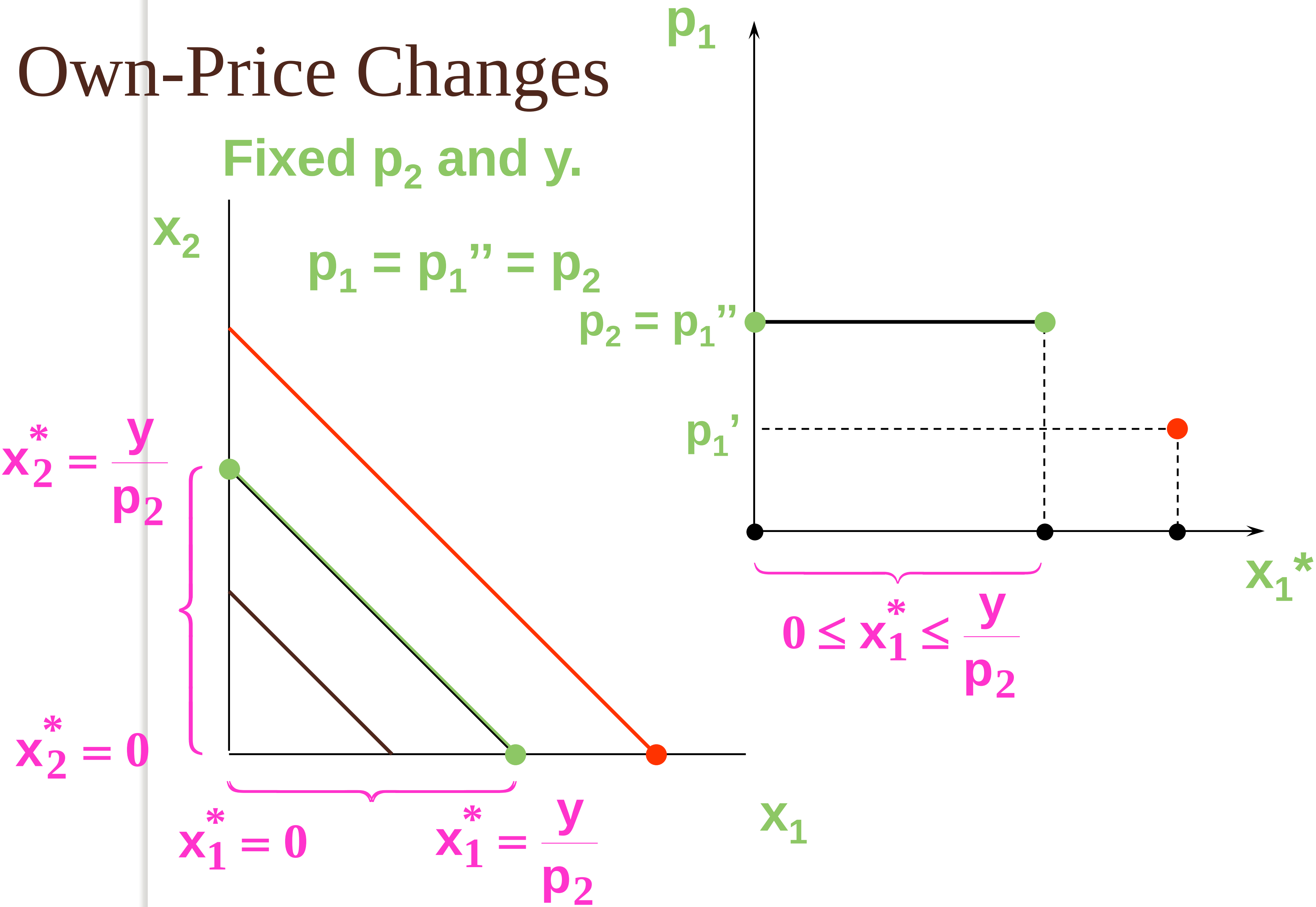


Own-Price Changes



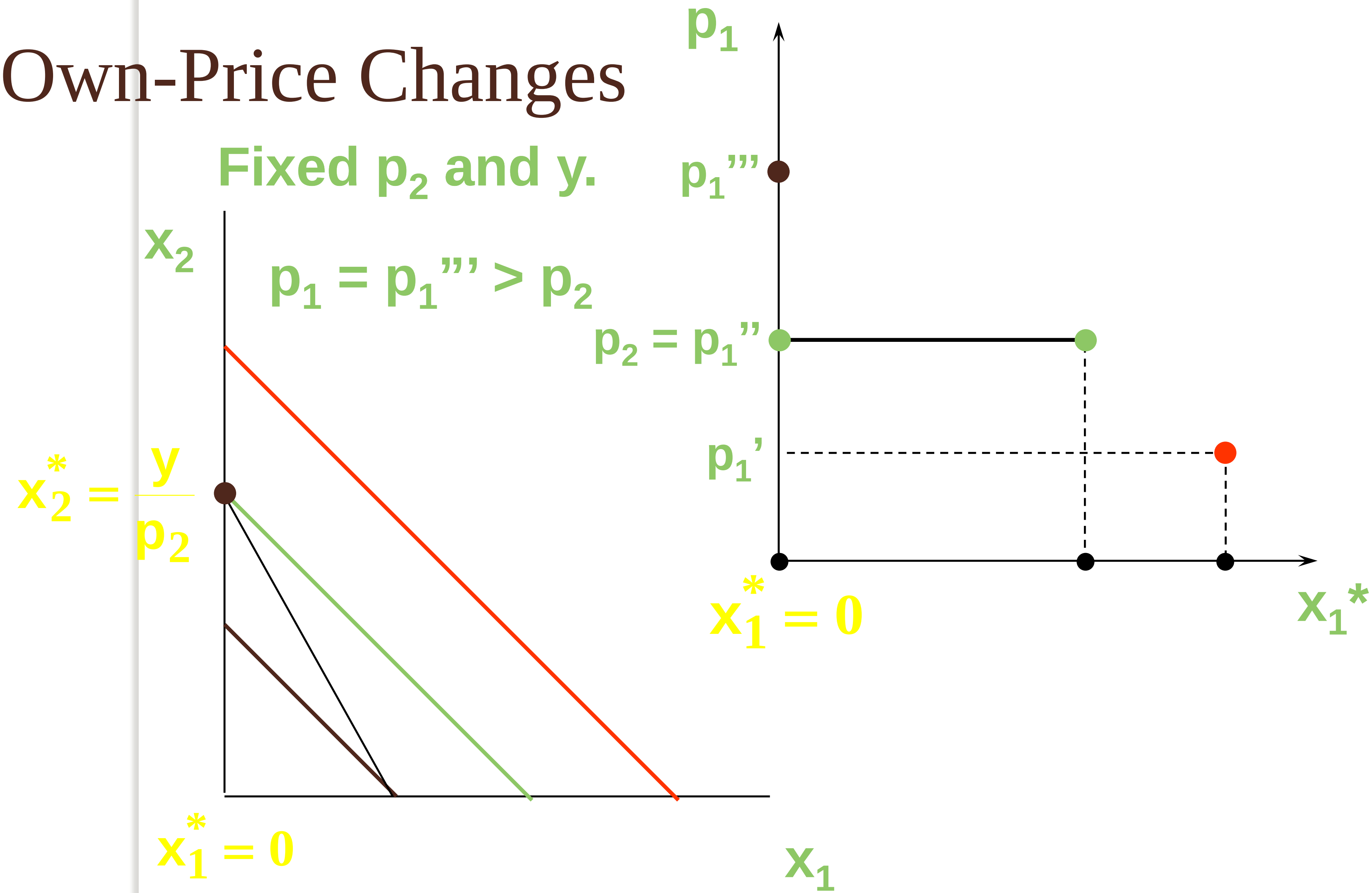
Own-Price Changes

Fixed p_2 and y.

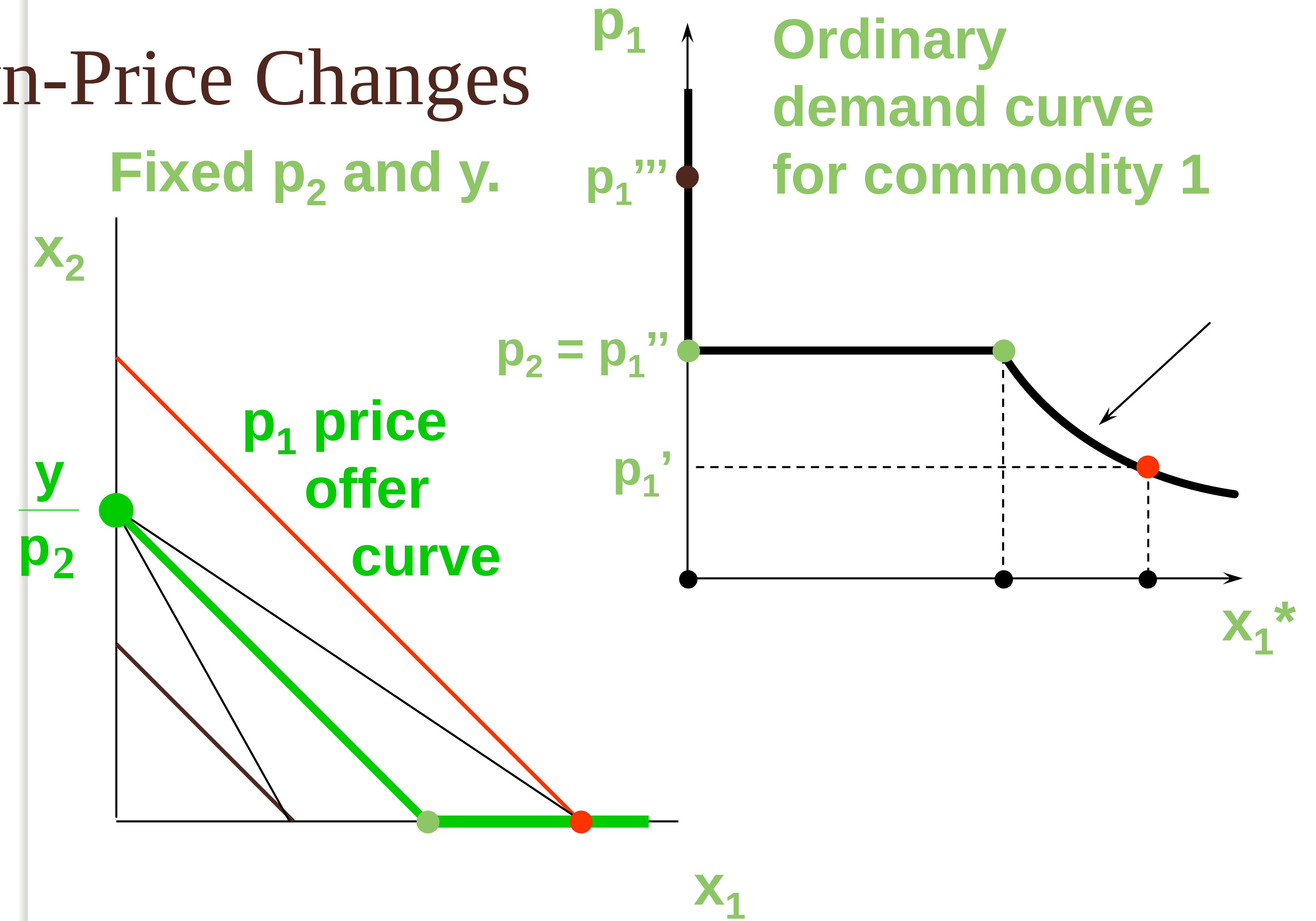


Own-Price Changes

Fixed p_2 and y.



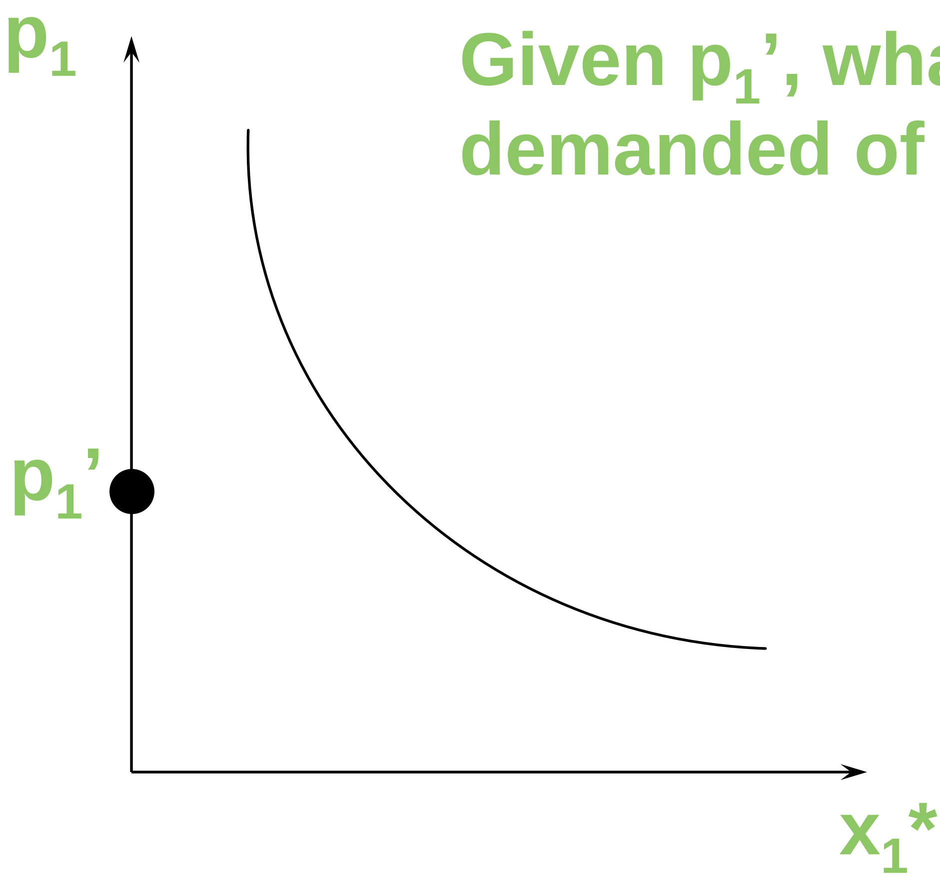
Own-Price Changes



Own-Price Changes

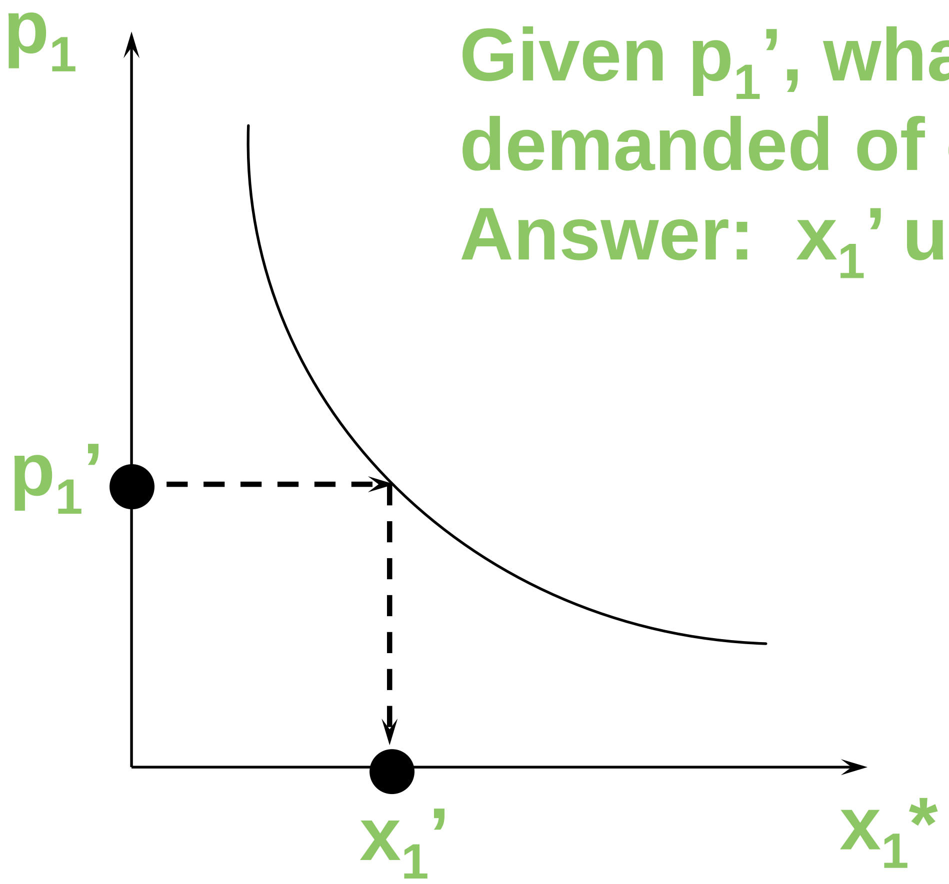
- Usually we ask "Given the price for commodity 1 what is the quantity demanded of commodity 1?"
- But we could also ask the inverse question "At what price for commodity 1 would a given quantity of commodity 1 be demanded?"

Own-Price Changes



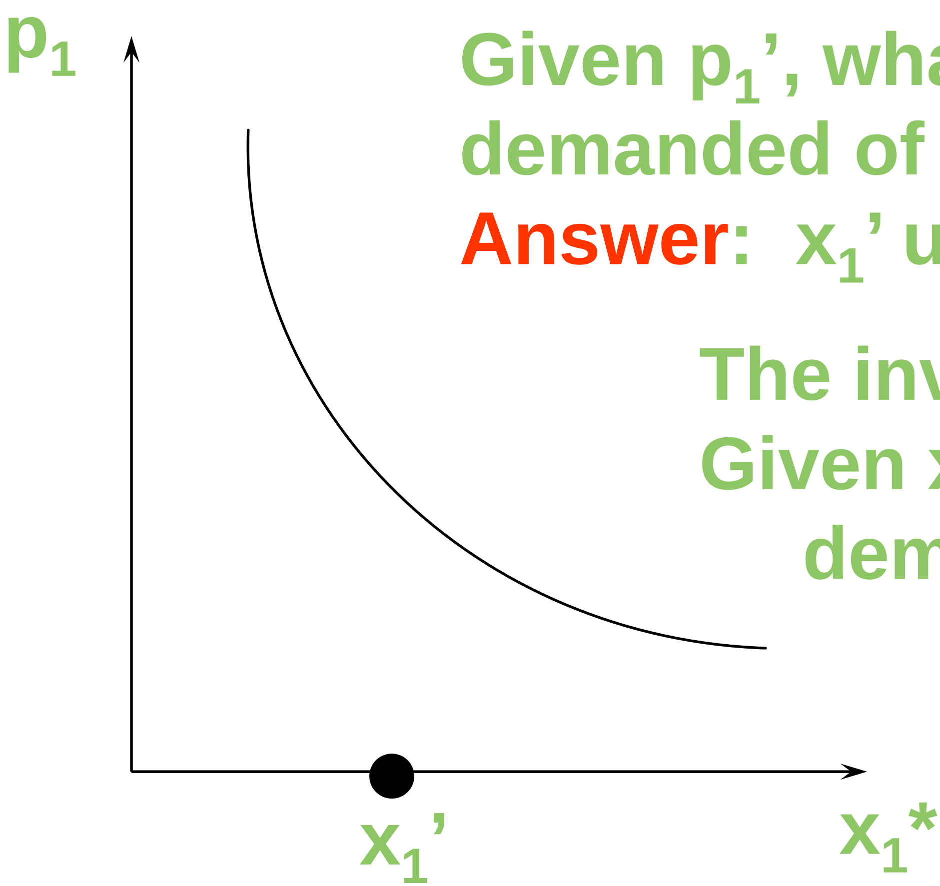
Given p_1', what quantity is demanded of commodity 1?

Own-Price Changes



Given p_1', what quantity is demanded of commodity 1?
Answer: x_1' units.

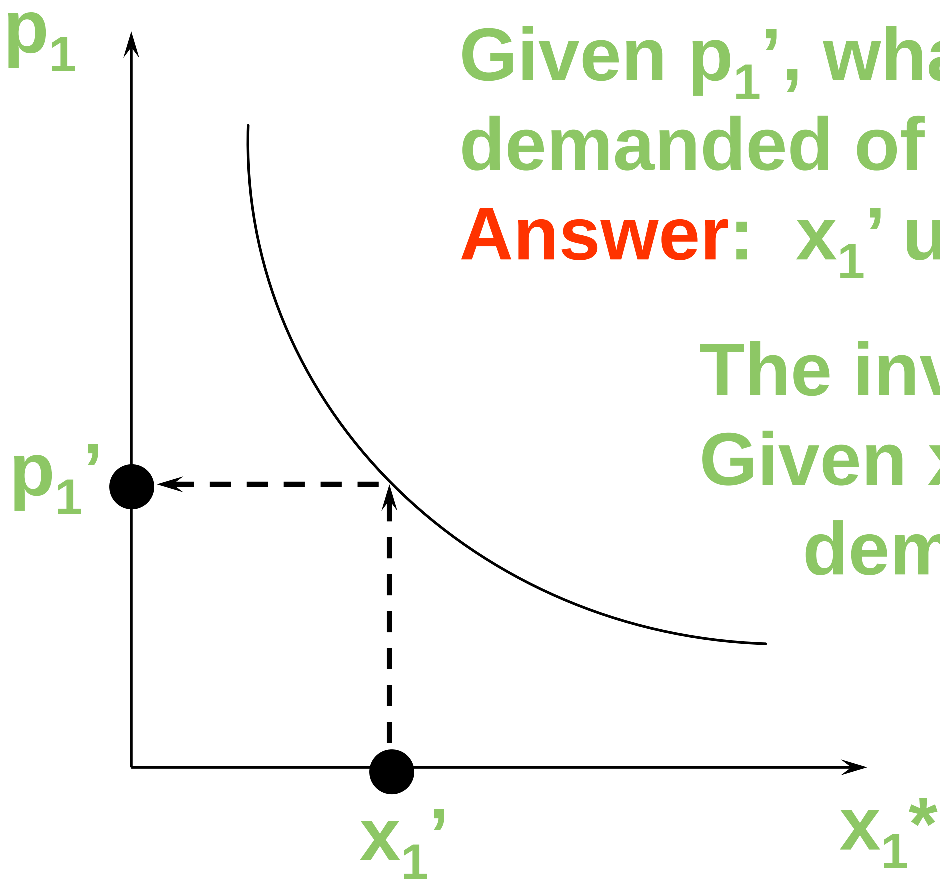
Own-Price Changes



Given p_1', what quantity is demanded of commodity 1?
Answer: x_1' units.

The inverse question is: Given x_1' units are demanded, what is the price of commodity 1?

Own-Price Changes



Given p_1', what quantity is demanded of commodity 1?
Answer: x_1' units.

The inverse question is: Given x_1' units are demanded, what is the price of commodity 1?
Answer: p_1'

Own-Price Changes

- Taking quantity demanded as given and then asking what must be price describes the inverse demand function of a commodity.

Own-Price Changes

A Cobb-Douglas example:

$$x_1^* = \frac{ay}{(a+b)p_1}$$

is the ordinary demand function and

$$p_1 = \frac{ay}{(a+b)x_1^*}$$

is the inverse demand function.

Own-Price Changes

A perfect-complements example:

$$x_1^* = \frac{y}{p_1 + p_2}$$

is the ordinary demand function and

$$p_1 = \frac{y}{x_1^*} - p_2$$

is the inverse demand function.

Meaning of the Inverse Demand Function

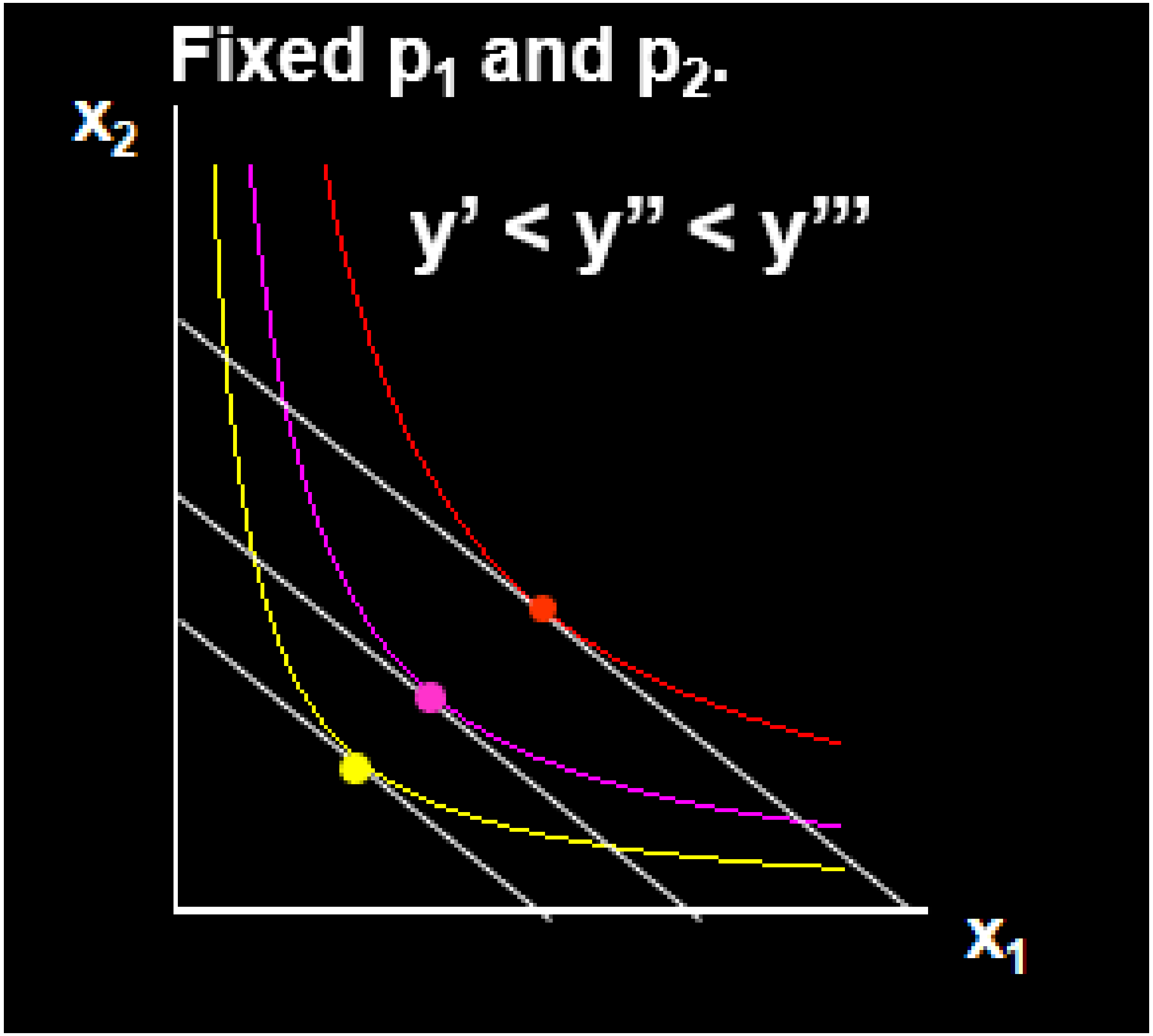
- At optimal choice
- $|MRS| = P_1/P_2$ or $p_1 = p_2|MRS|$

- If taking good 2 as money on other goods, then p_2=1 and p_1=MRS.
- This is the marginal willingness to pay.

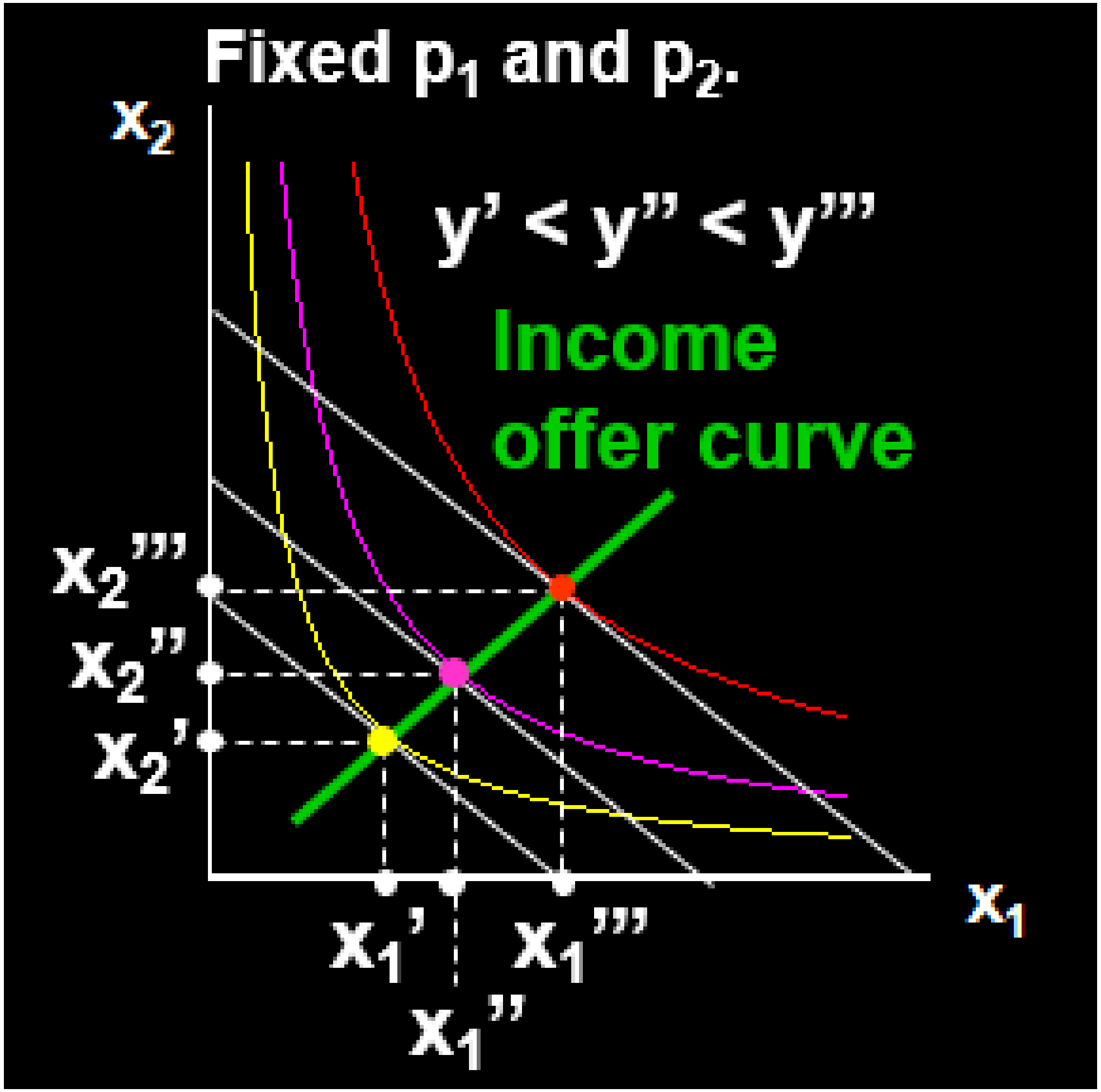
Income Changes

- How does the value of $x_1^*(p_1,p_2,y)$ change as y changes, holding both p_1 and p_2 constant?

Income Changes



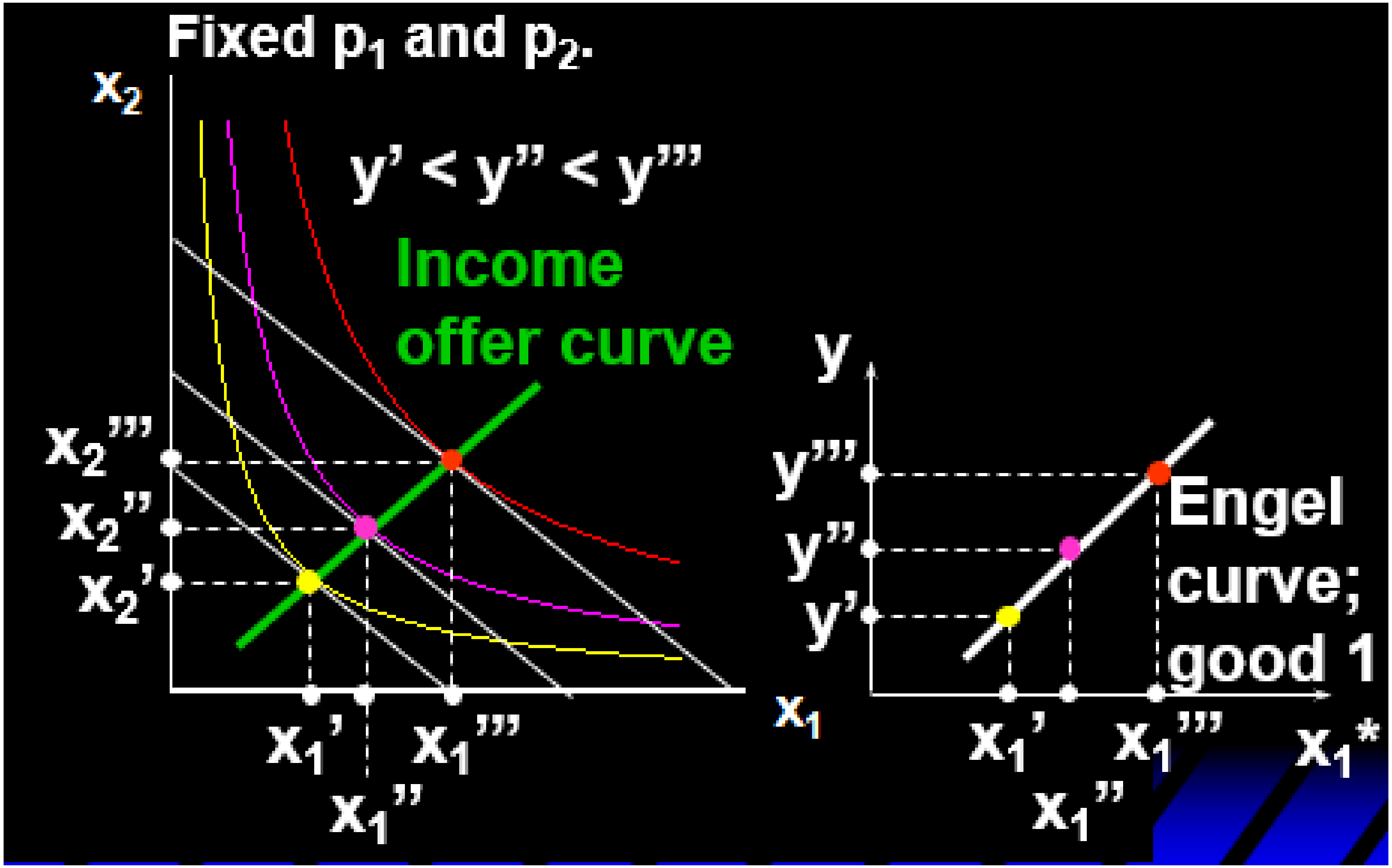
Income Changes



Income Changes

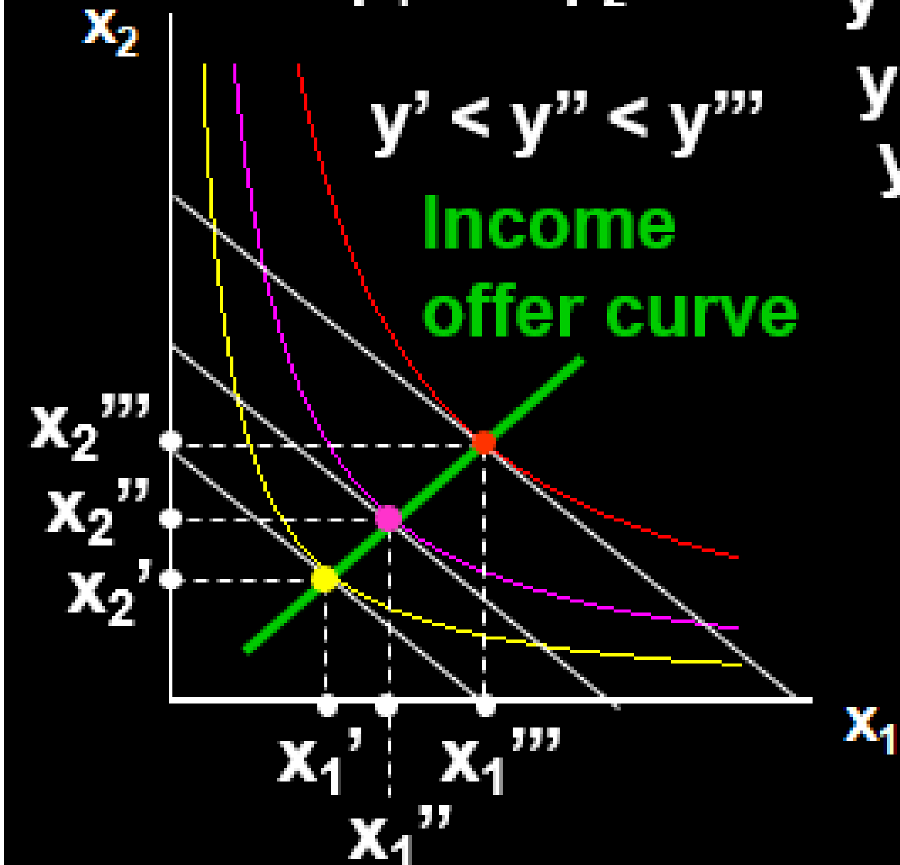
- A plot of quantity demanded against income is called an Engel curve.

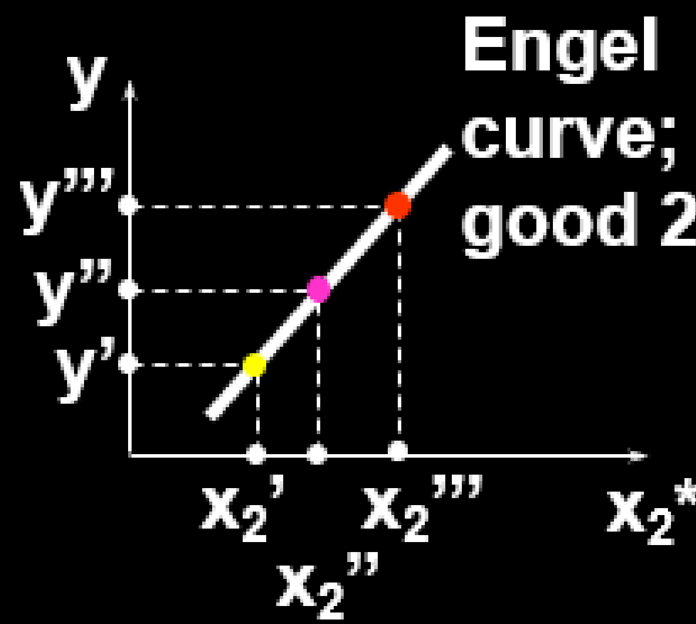
Income Changes



Income Changes

Fixed p_1 and p_2.





Income Changes and Cobb-Douglas Preferences

- An example of computing the equations of Engel curves; the Cobb-Douglas case.

$$U(x_1, x_2) = x_1^a x_2^b.$$

- The ordinary demand equations are

$$x_1^* = \frac{ay}{(a+b)p_1}; \quad x_2^* = \frac{by}{(a+b)p_2}.$$

Income Changes and Cobb-Douglas Preferences

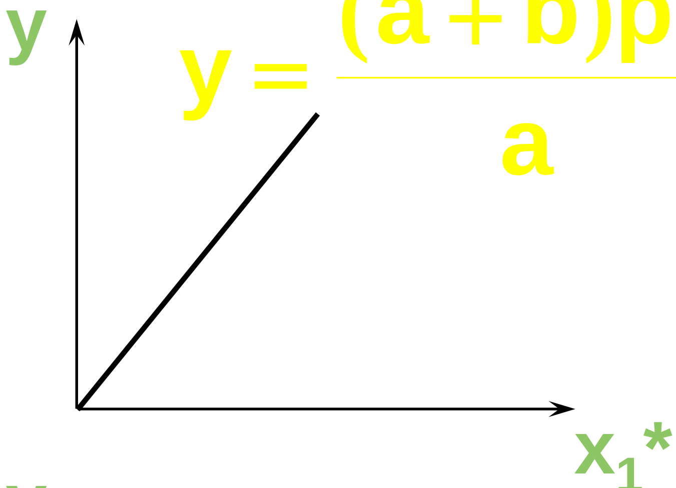
$$x_1^* = \frac{ay}{(a+b)p_1}; \quad x_2^* = \frac{by}{(a+b)p_2}.$$

Rearranged to isolate y, these are:

$$y = \frac{(a+b)p_1}{a} x_1^*$$ Engel curve for good 1

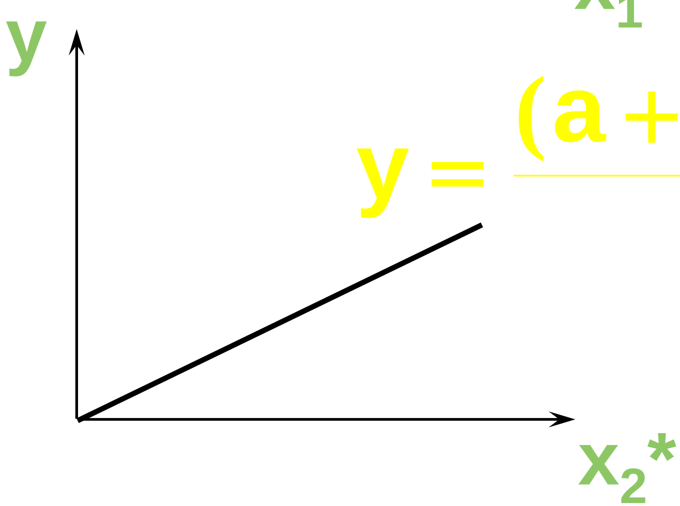
$$y = \frac{(a+b)p_2}{b} x_2^*$$ Engel curve for good 2

Income Changes and Cobb-Douglas Preferences



$$y = \frac{(a+b)p_1}{a} x_1^*$$

Engel curve for good 1



$$y = \frac{(a+b)p_2}{b} x_2^*$$

Engel curve for good 2

Income Changes and Perfectly-Complementary Preferences

- Another example of computing the equations of Engel curves; the perfectly-complementary case.

$$U(x_1, x_2) = \min\{x_1, x_2\}.$$

- The ordinary demand equations are

$$x_1^* = x_2^* = \frac{y}{p_1 + p_2}.$$

Income Changes and Perfectly-Complementary Preferences

$$x_1^* = x_2^* = \frac{y}{p_1 + p_2}.$$

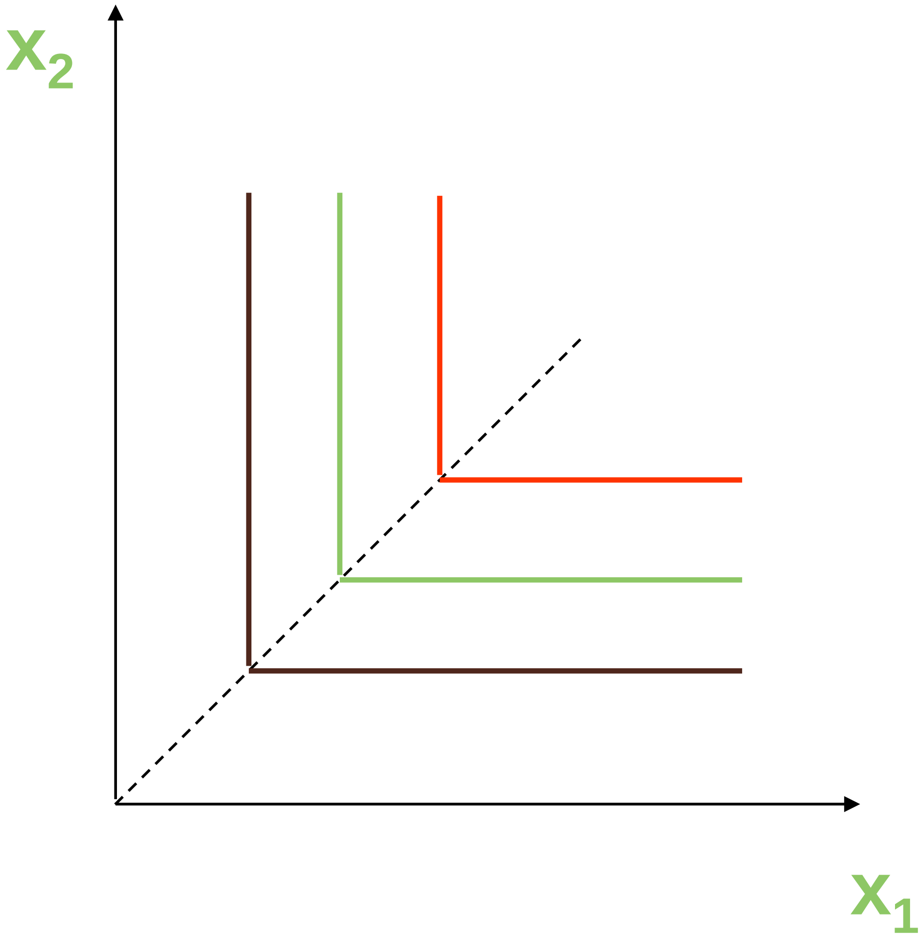
Rearranged to isolate y, these are:

$$y = (p_1 + p_2)x_1^*$$ Engel curve for good 1

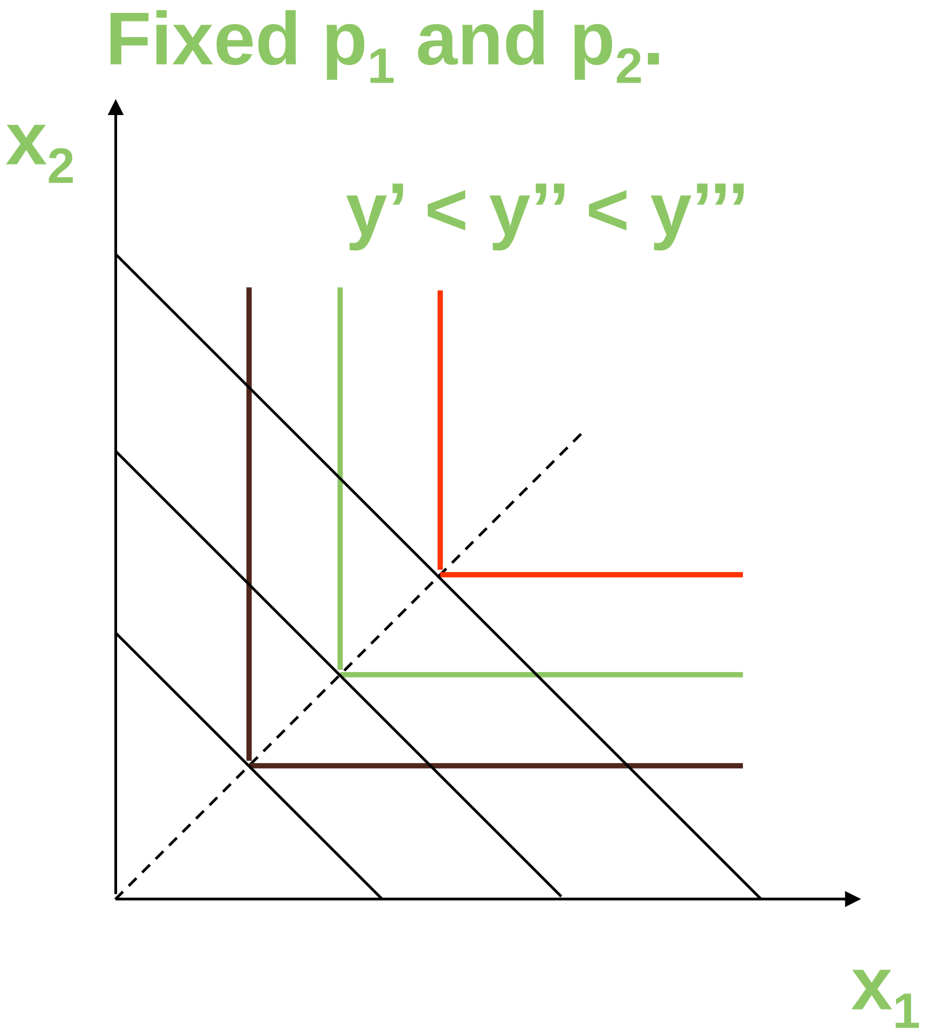
$$y = (p_1 + p_2)x_2^*$$ Engel curve for good 2

Income Changes

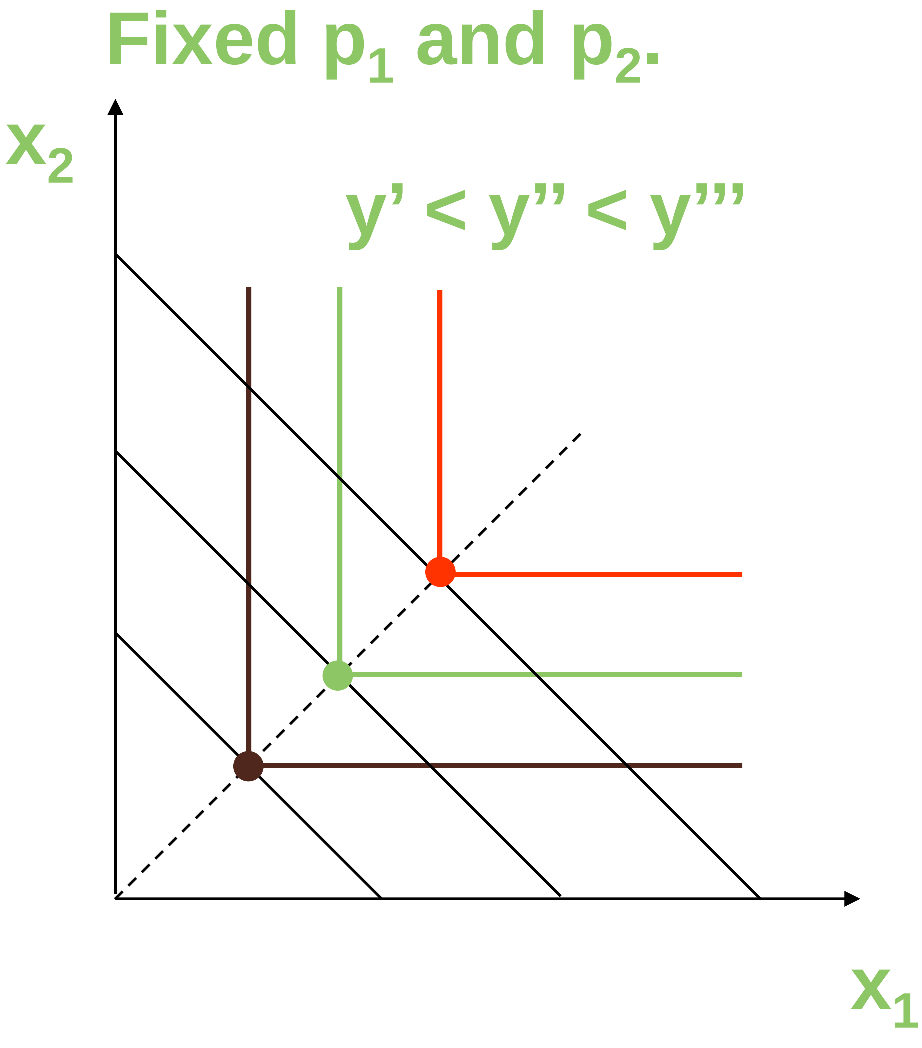
Fixed p_1 and p_2.



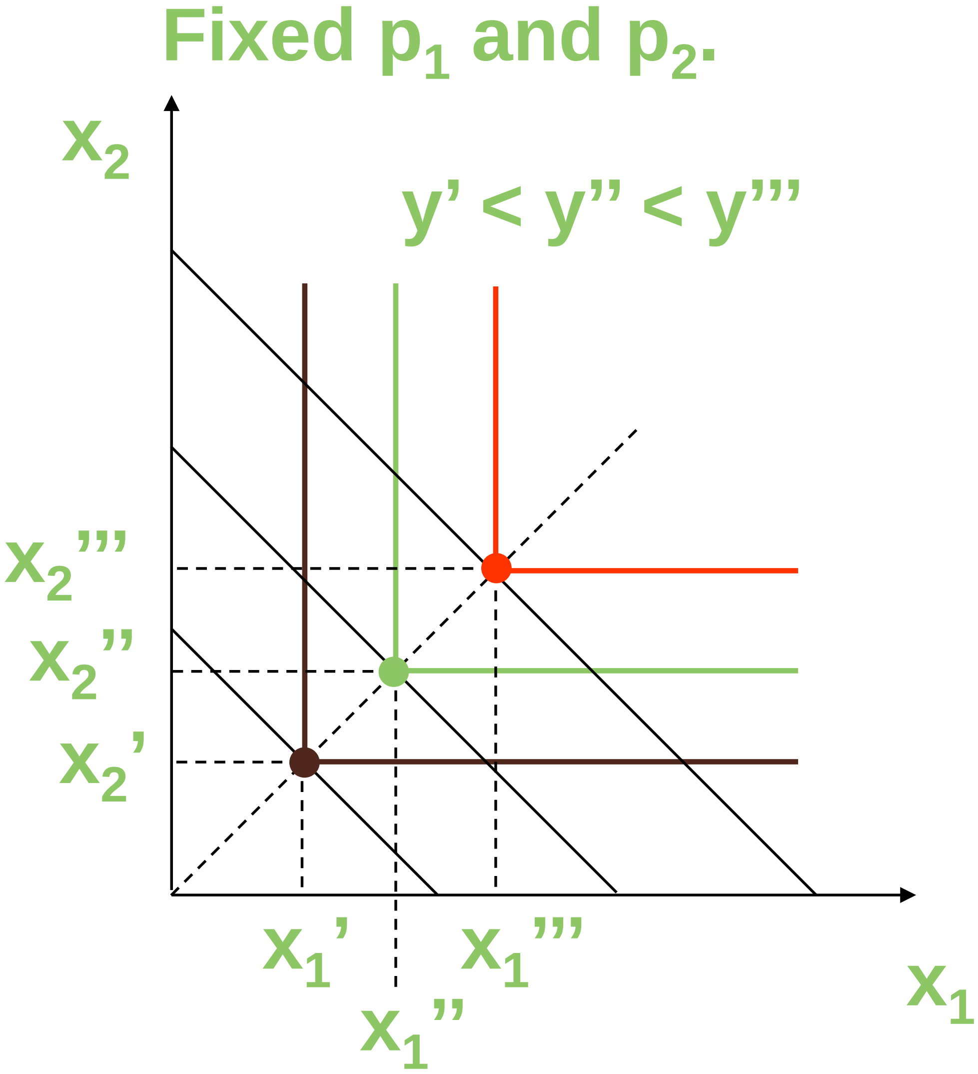
Income Changes



Income Changes

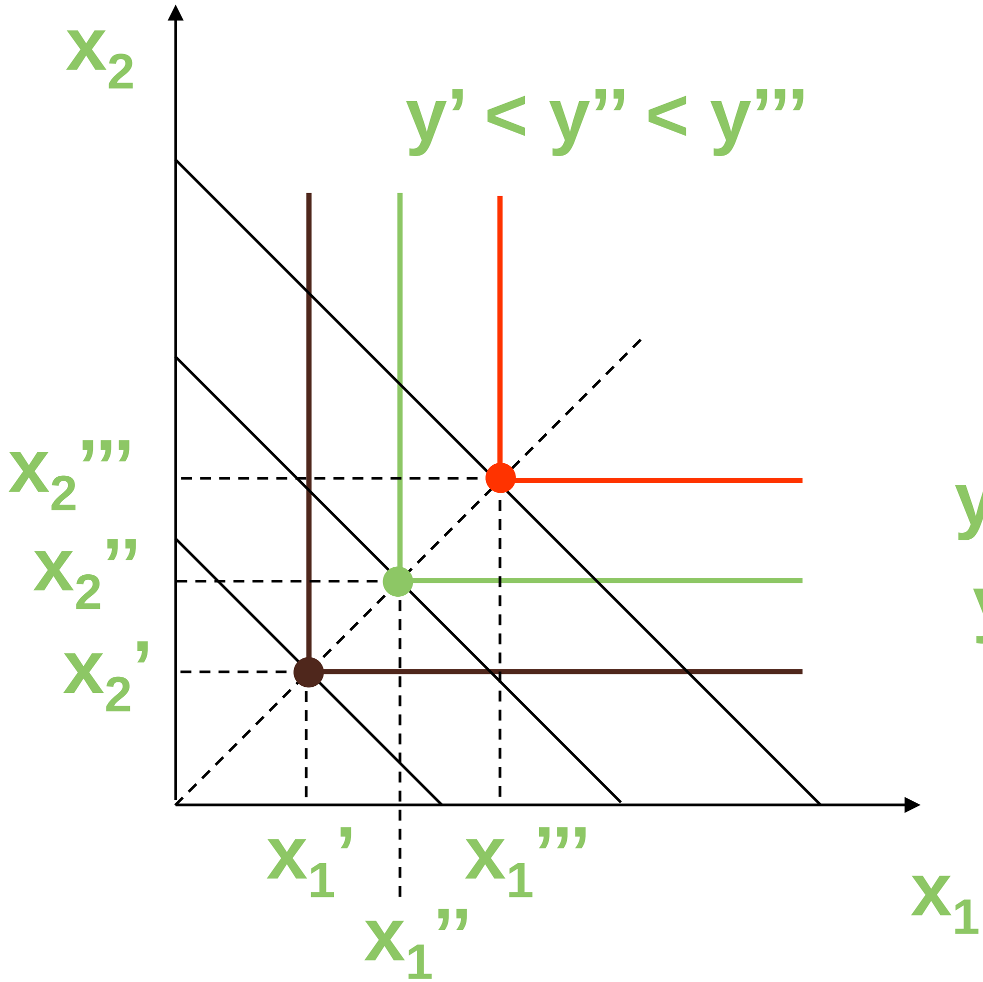


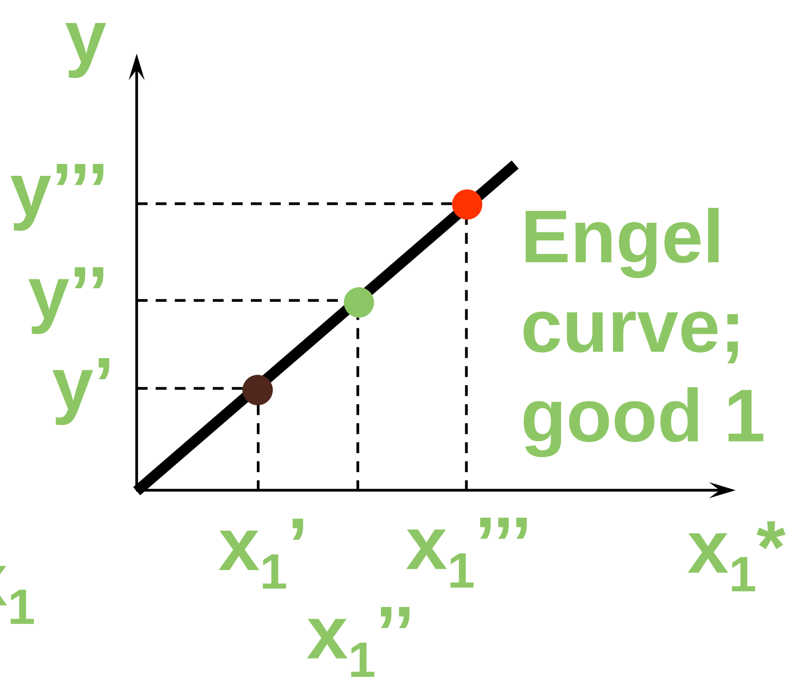
Income Changes



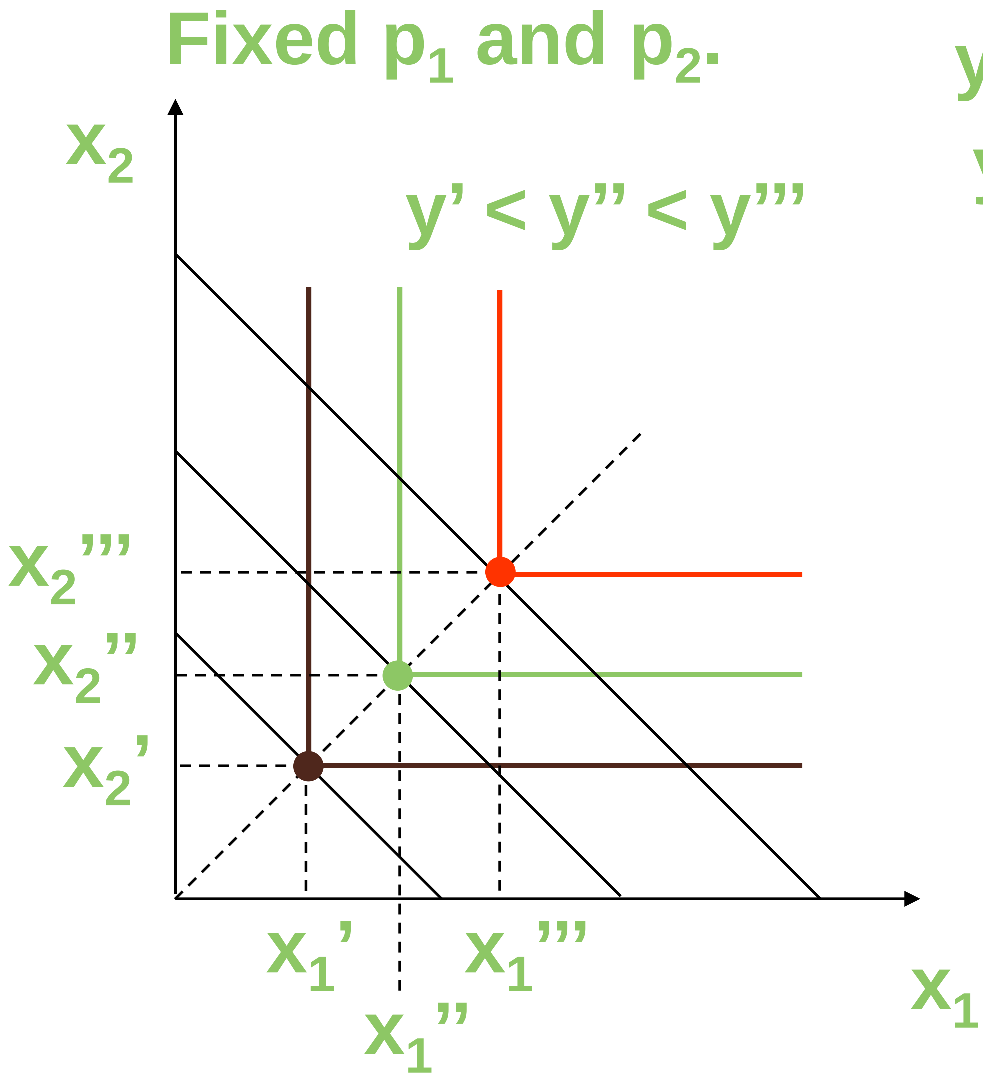
Income Changes

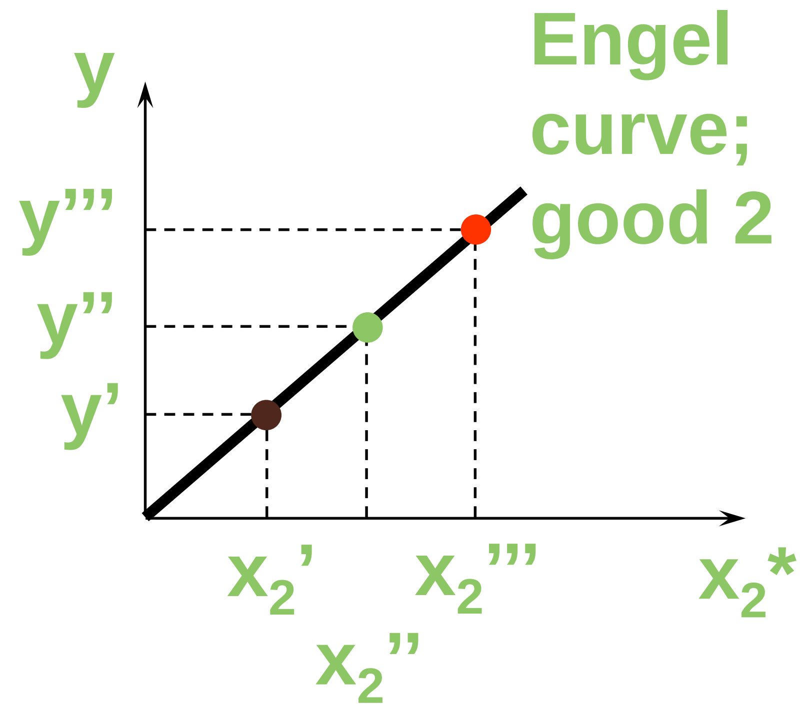
Fixed p_1 and p_2.



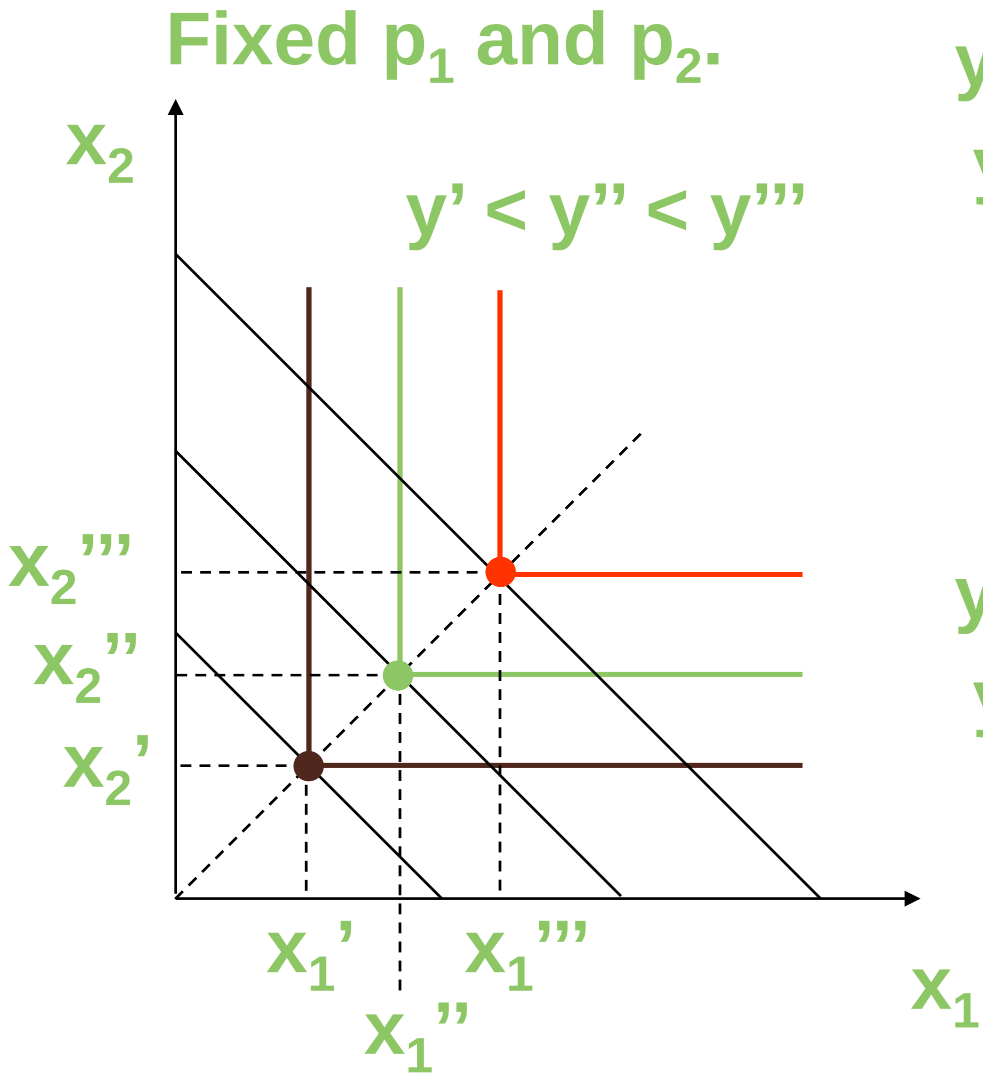


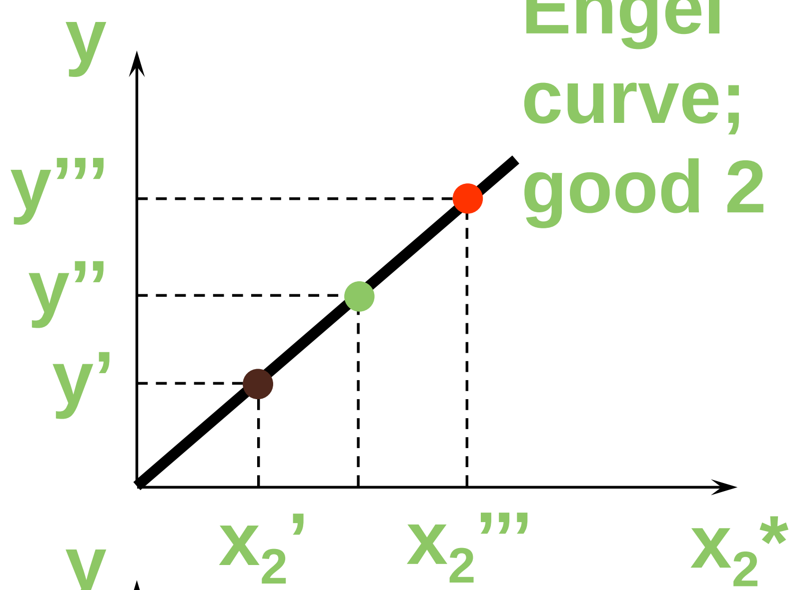
Income Changes

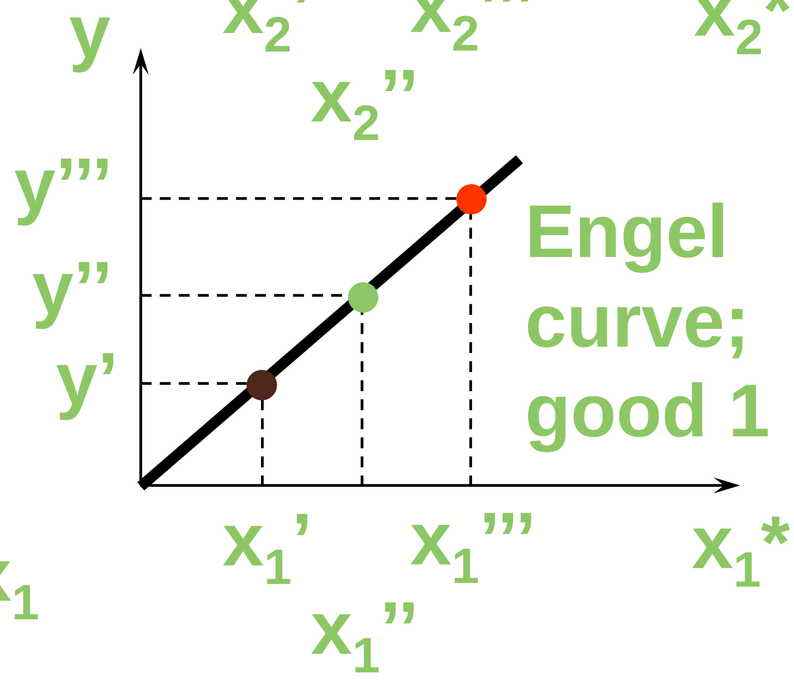




Income Changes





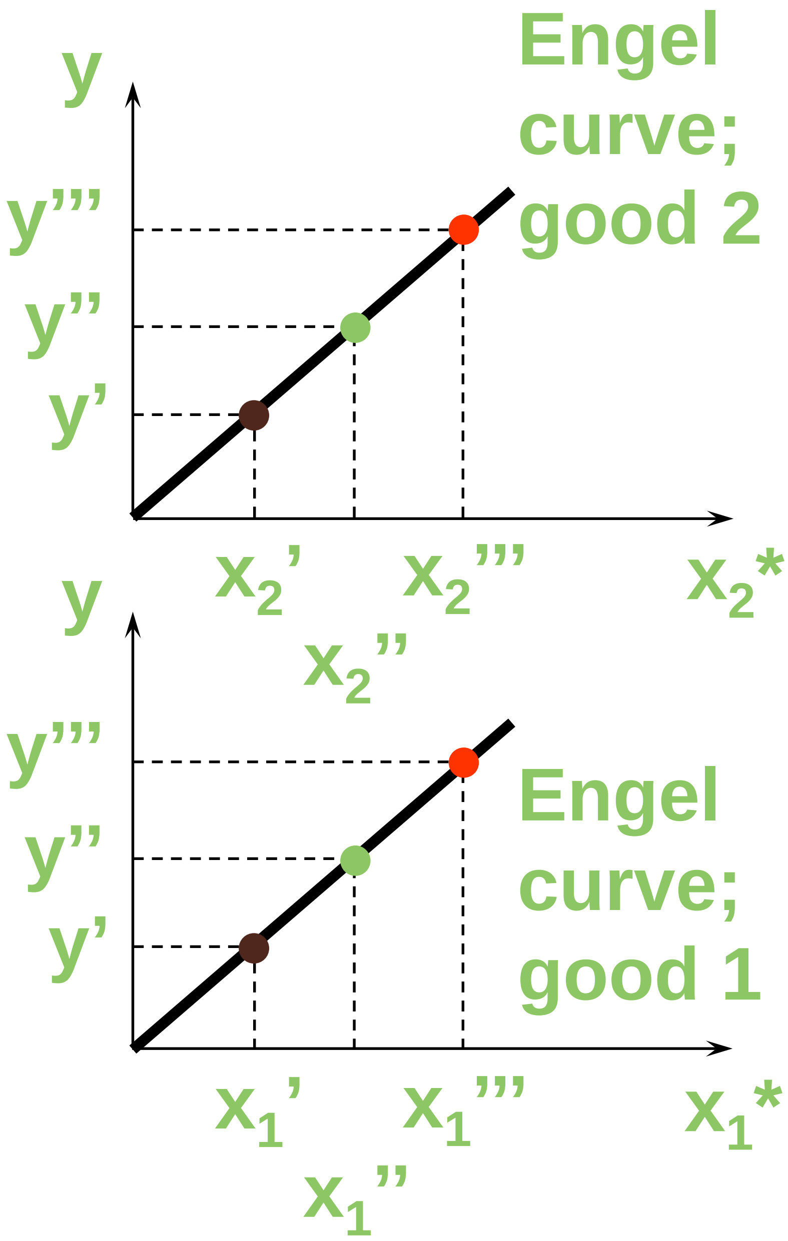


Income Changes

Fixed p_1 and p_2.

$$y = (p_1 + p_2)x_2^*$$

$$y = (p_1 + p_2)x_1^*$$



Income Changes and Perfectly-Substitutable Preferences

- Another example of computing the equations of Engel curves; the perfectly-substitution case.

$$U(x_1, x_2) = x_1 + x_2.$$

- The ordinary demand equations are

Income Changes and Perfectly-Substitutable Preferences

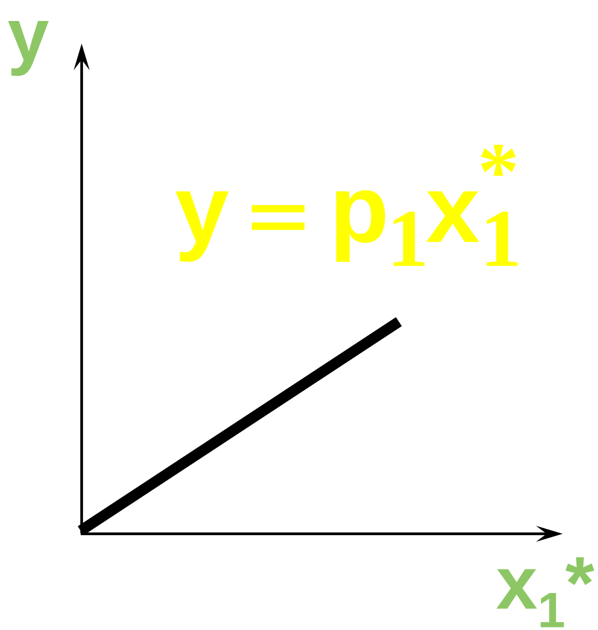
$$x_1^*(p_1,p_2,y) = \begin{cases} 0 & , \text{if } p_1 > p_2 \\ y/p_1 & , \text{if } p_1 < p_2 \end{cases}$$

$$x_2^*(p_1,p_2,y) = \begin{cases} 0 & , \text{if } p_1 < p_2 \\ y/p_2 & , \text{if } p_1 > p_2. \end{cases}$$

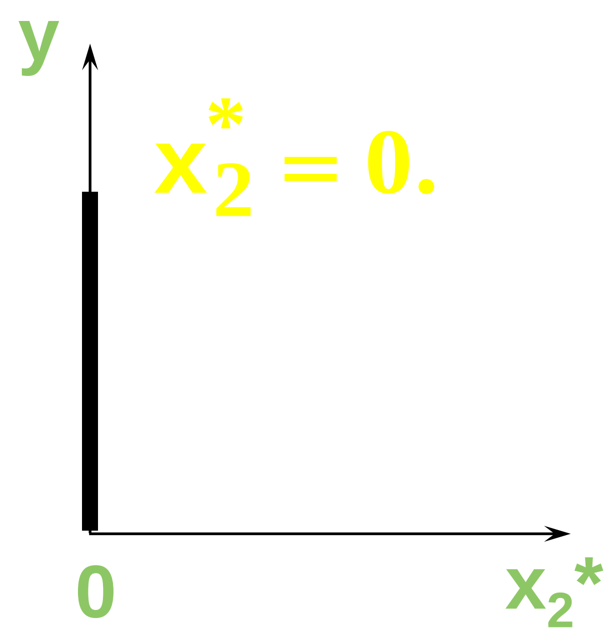
Suppose $p_1 < p_2$. Then $x_1^* = \dfrac{y}{p_1}$ and $x_2^* = 0$

$\Rightarrow$ $y = p_1 x_1^*$ and $x_2^* = 0.$

Income Changes and Perfectly-Substitutable Preferences



Engel curve for good 1



Engel curve for good 2

Income Changes

- In every example so far the Engel curves have all been straight lines?
  Q: Is this true in general?
- A: No. Engel curves are straight lines if the consumer's preferences are homothetic.

Homotheticity （位似偏好）

- A consumer's preferences are homothetic if and only if

$$(x_1,x_2) \prec (y_1,y_2) \Leftrightarrow (kx_1,kx_2) \prec (ky_1,ky_2)$$

for every $k > 0$.

- That is, the consumer's MRS is the same anywhere on a straight line drawn from the origin.

Income Effects -- A Nonhomothetic Example

- Quasilinear preferences are not homothetic.

$$U(x_1, x_2) = v(x_1) + x_2.$$

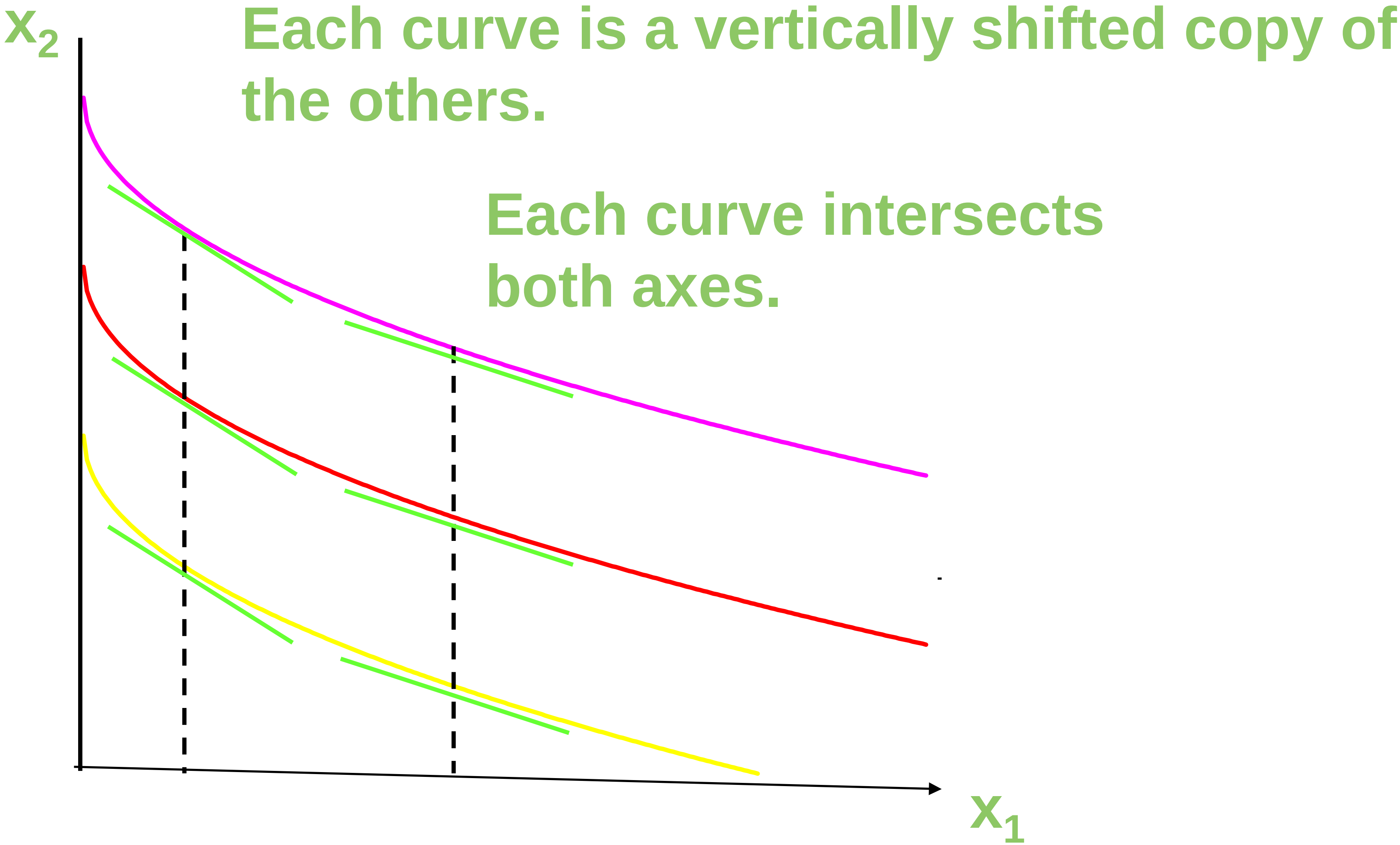
  For example,

$$\mathbf{U(x_1, x_2) = \sqrt{x_1} + x_2.}$$

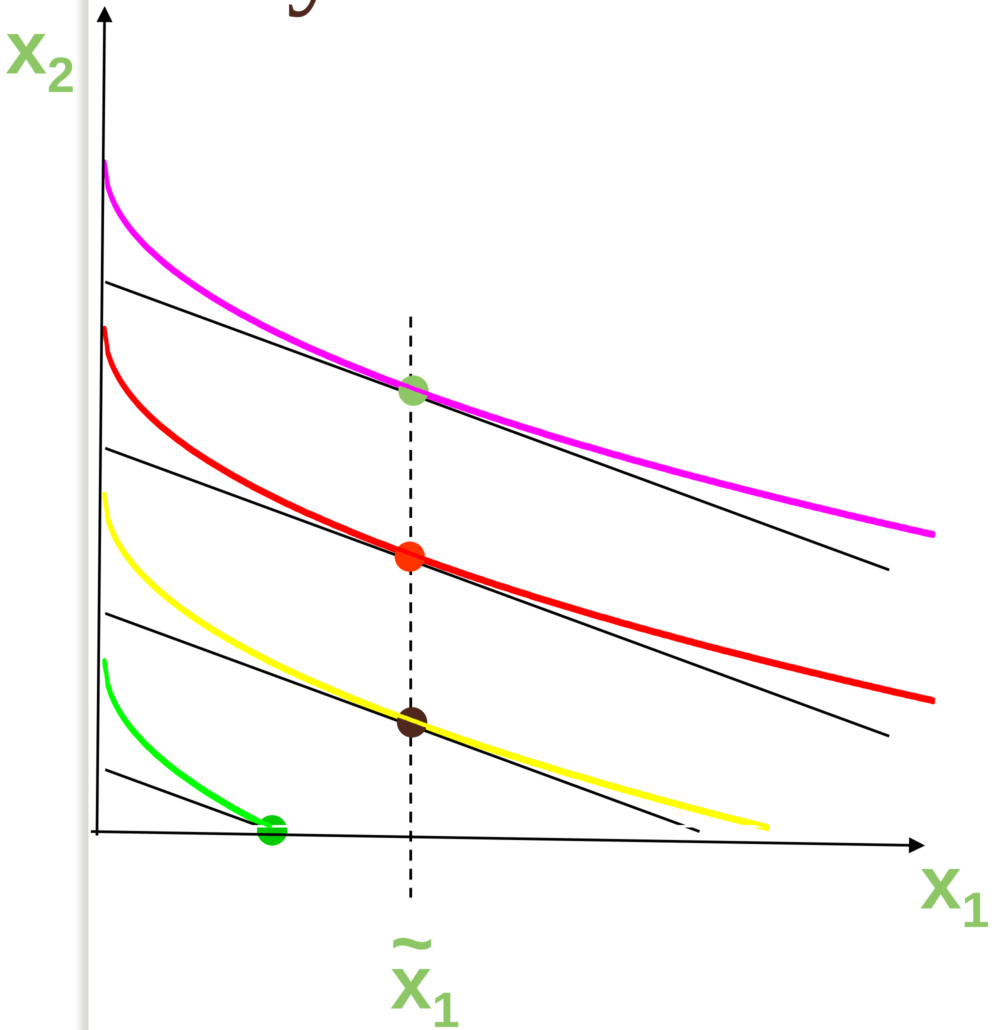
- Optimal interior consumption:

$$v'(x_1^*) = \frac{p_1}{p_2}.$$

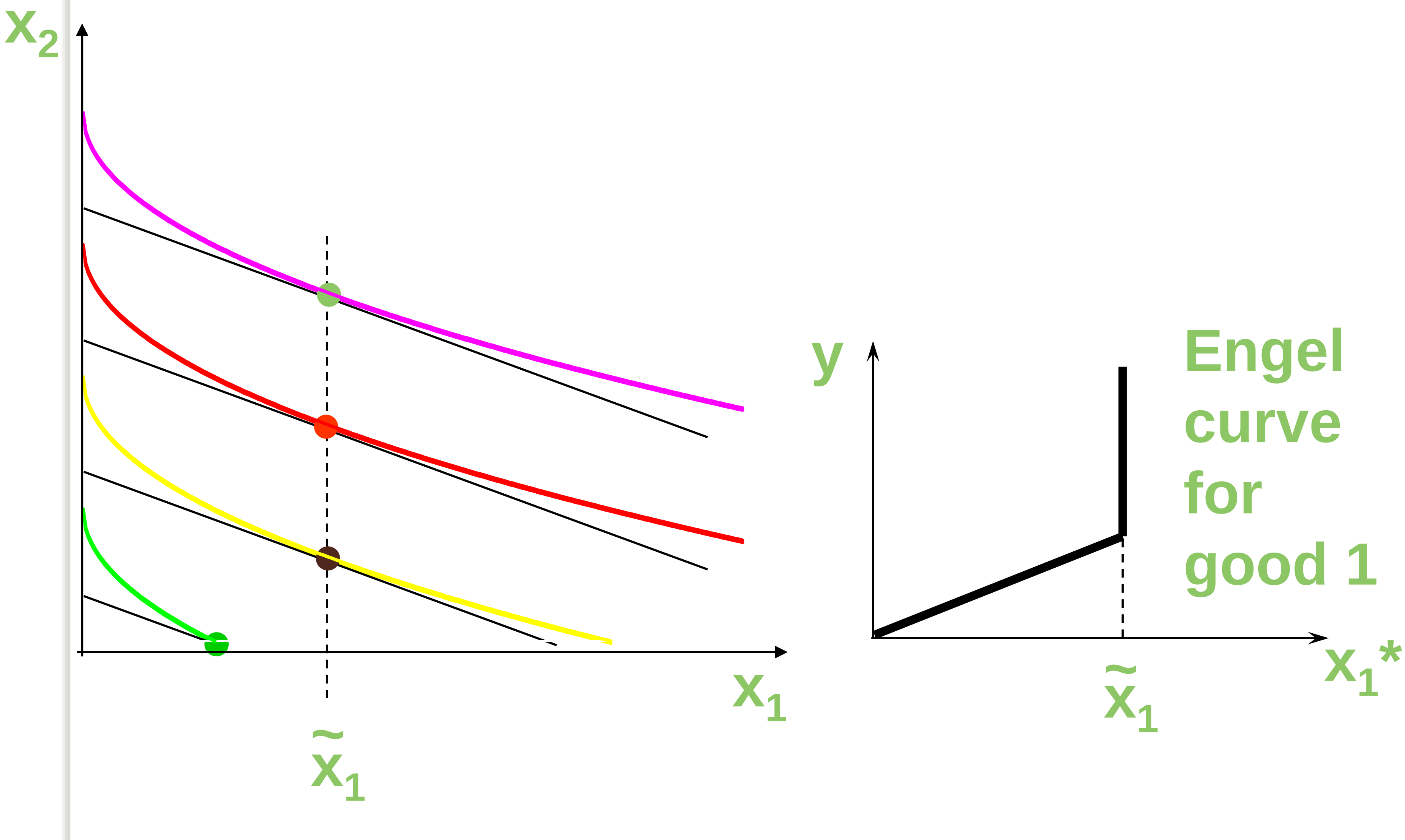
Quasi-linear Indifference Curves



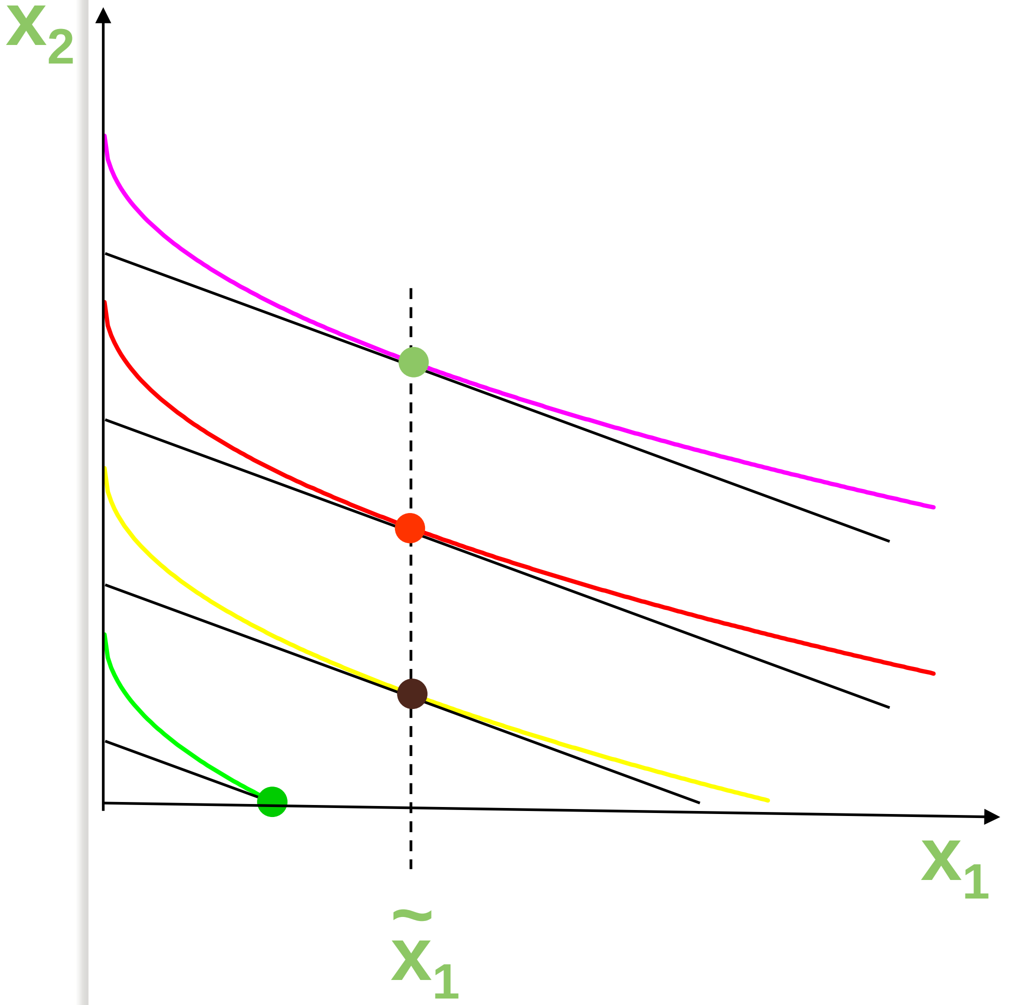
Income Changes; Quasilinear Utility

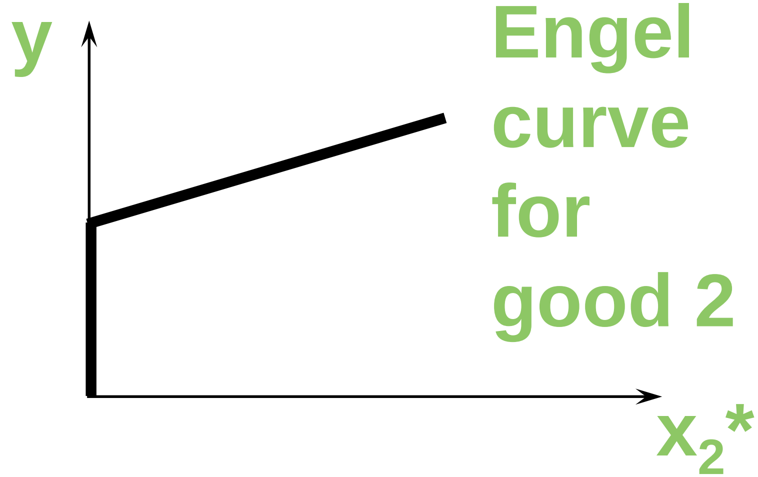


Income Changes; Quasilinear Utility

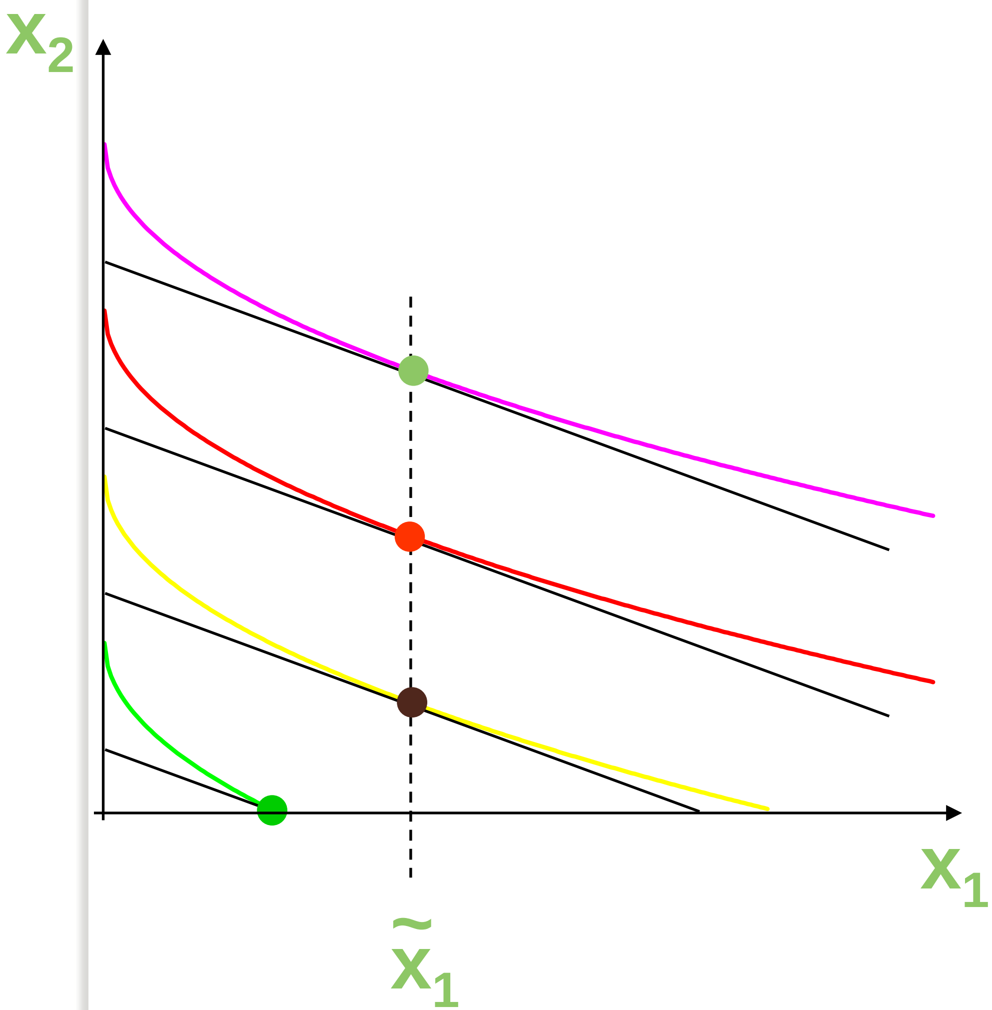


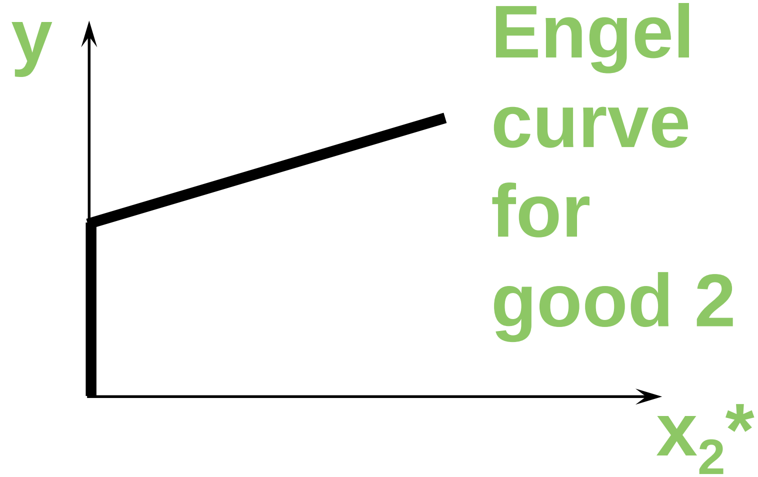
Income Changes; Quasilinear Utility

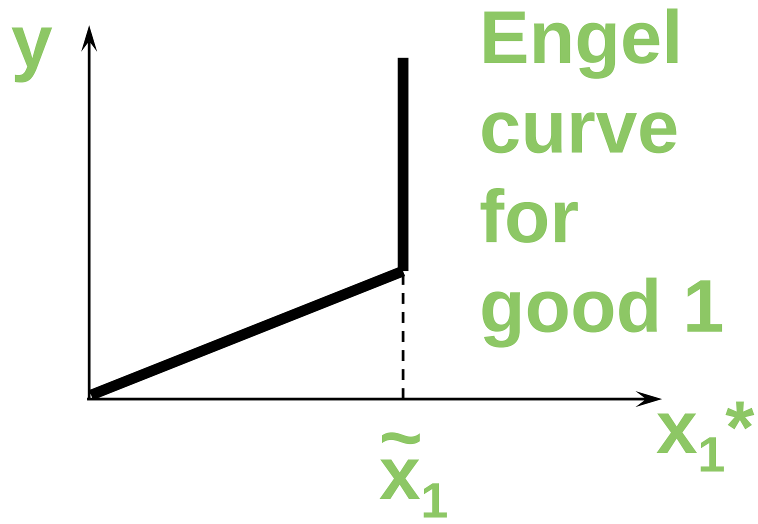




Income Changes; Quasilinear Utility







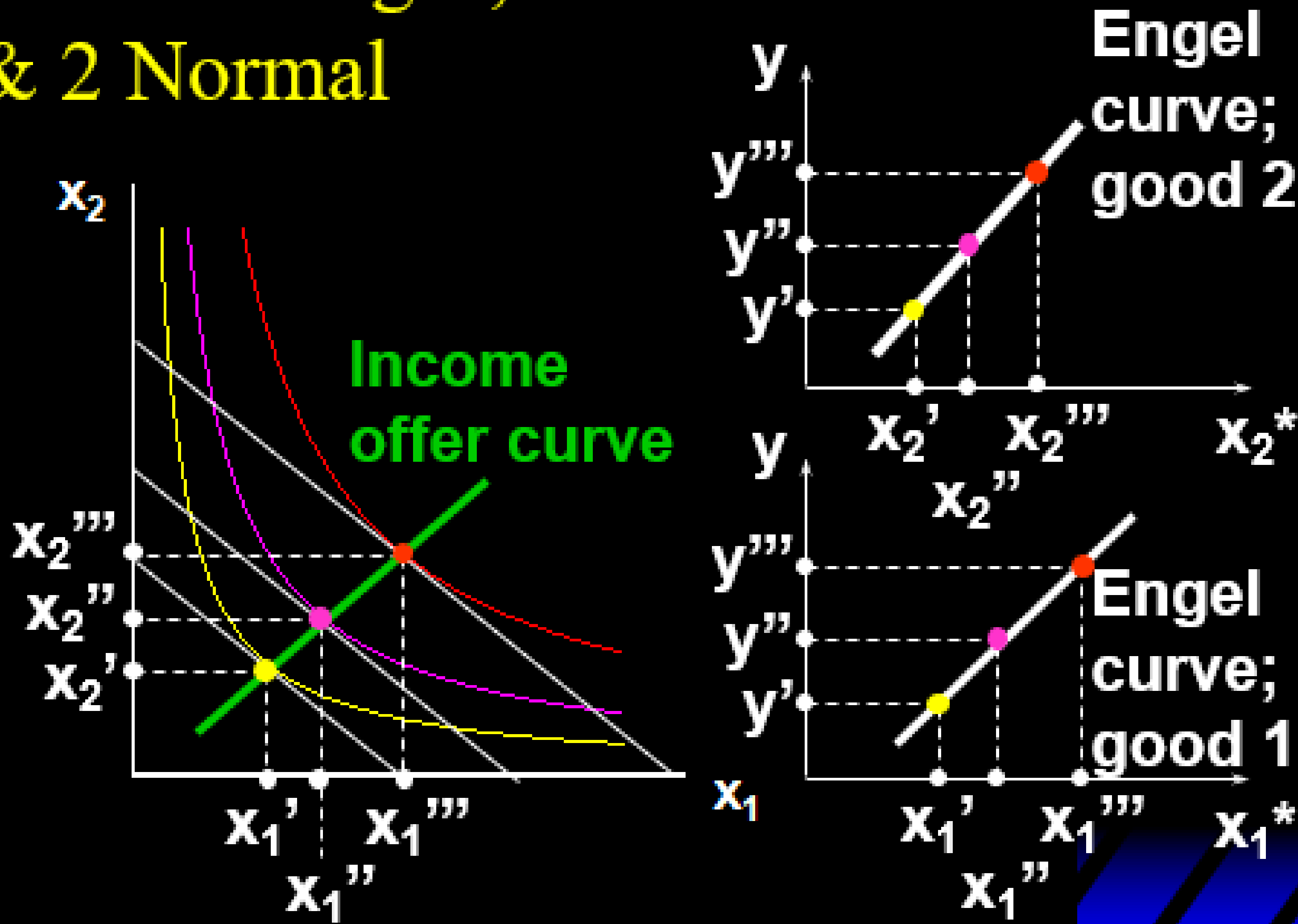
Income Effects

- A good for which quantity demanded rises with income is called normal （正常品）.
- Therefore a normal good’s Engel curve is positively sloped.

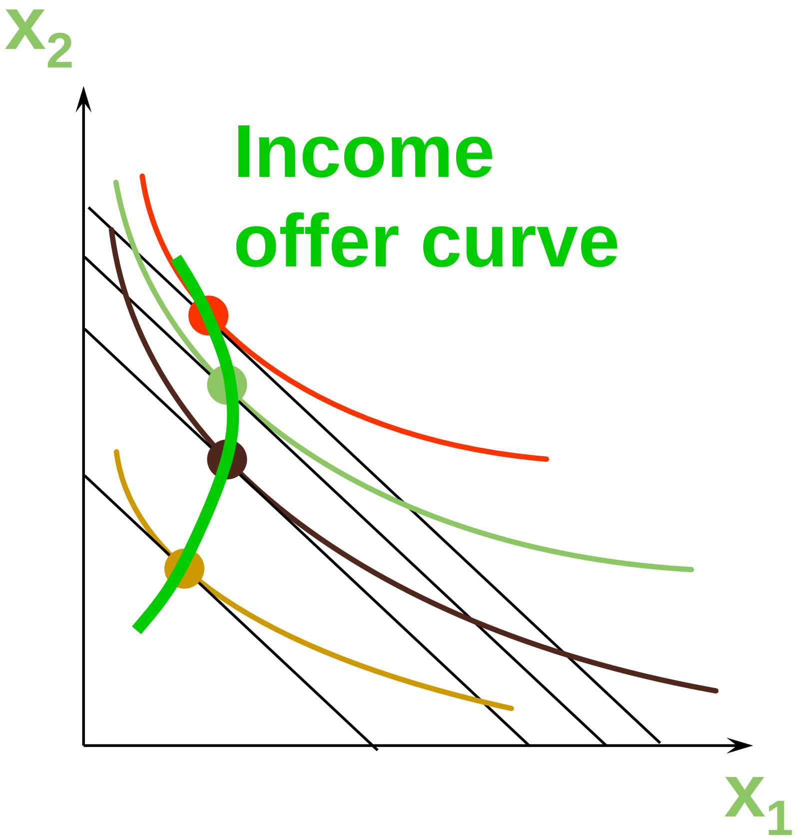
Income Effects

- A good for which quantity demanded falls as income increases is called income inferior （劣质品）.
- Therefore an income inferior good’s Engel curve is negatively sloped.

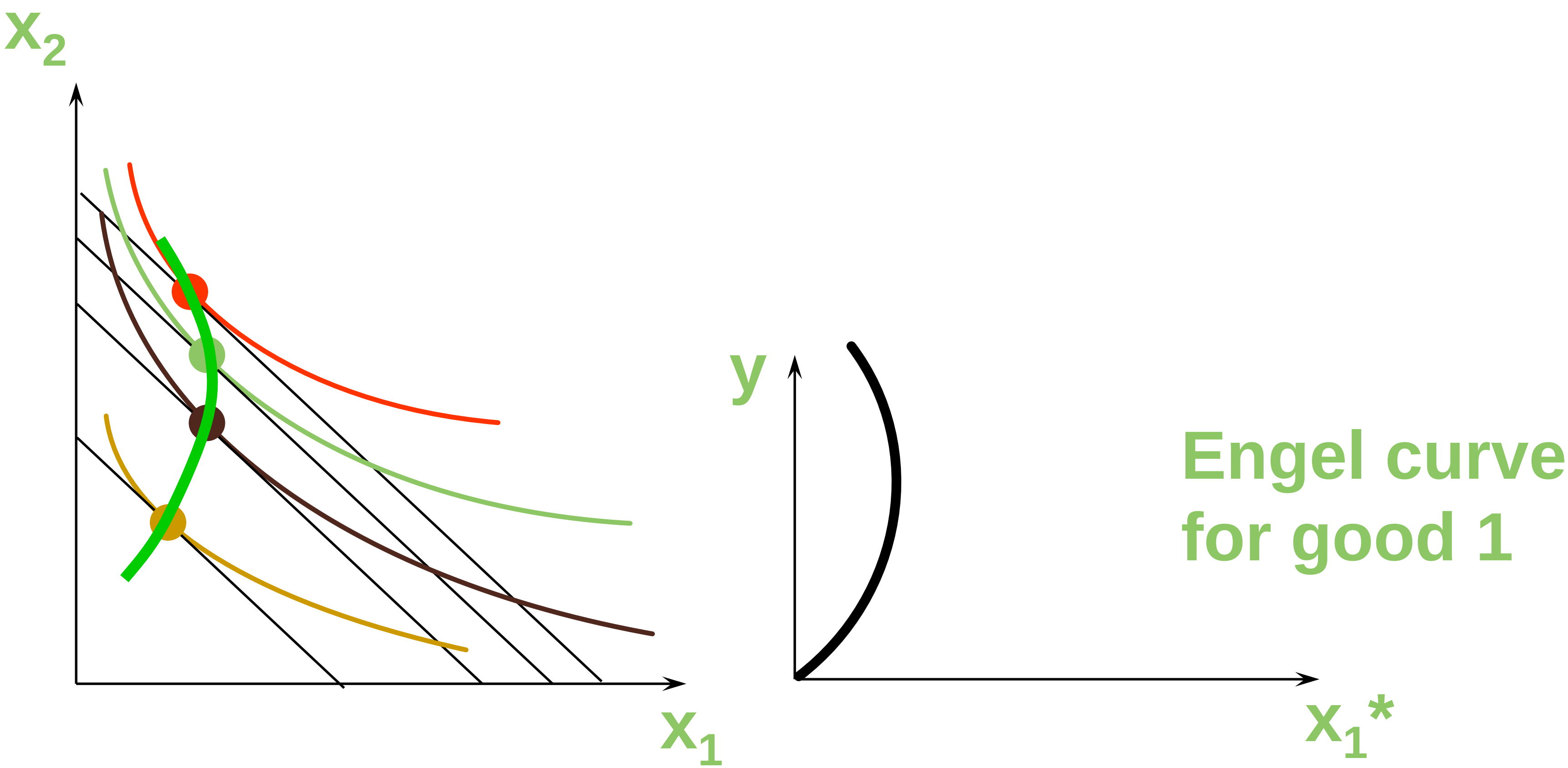
Income Changes; Goods 1 & 2 Normal



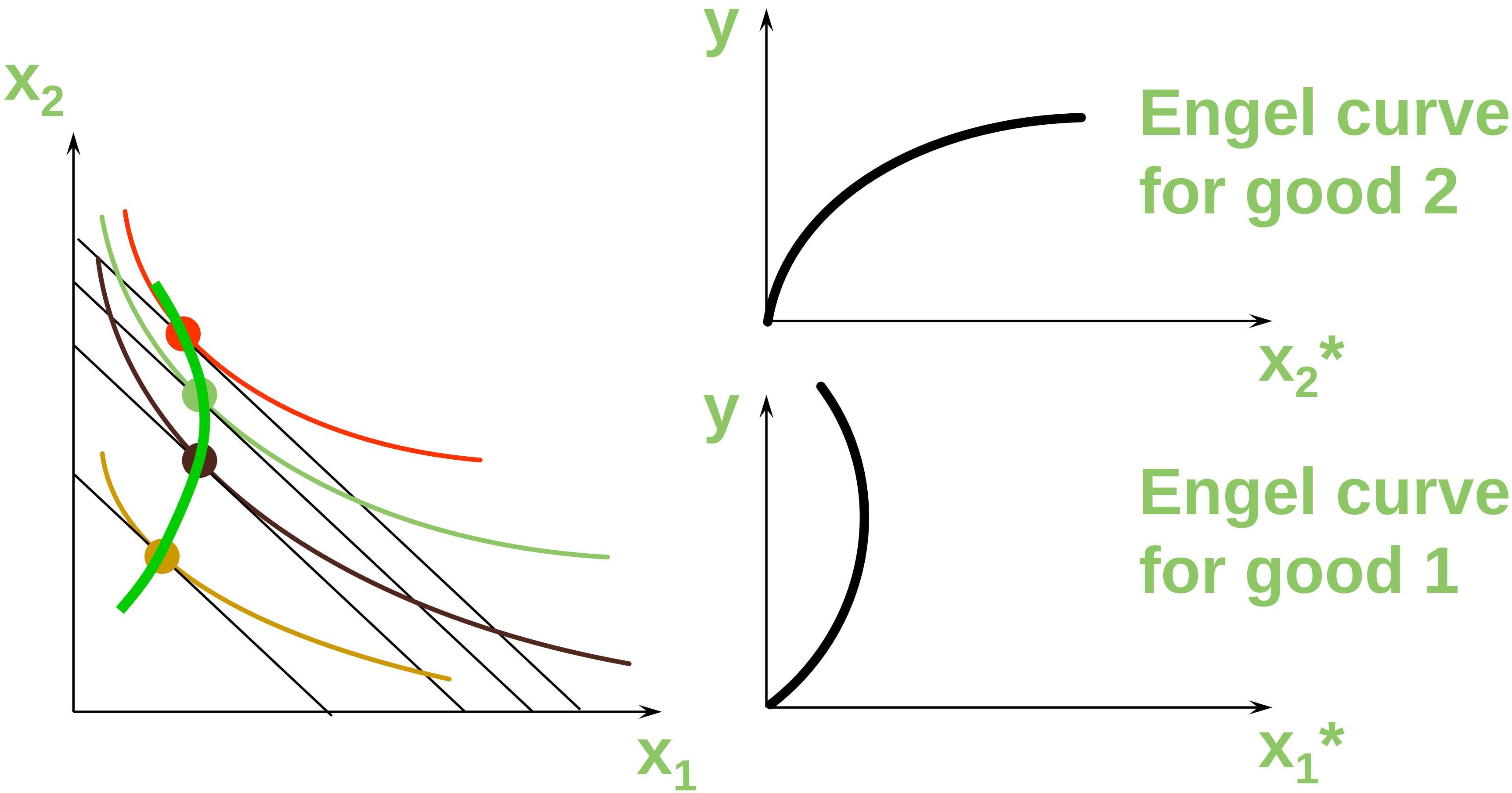
Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior



Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior



Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior

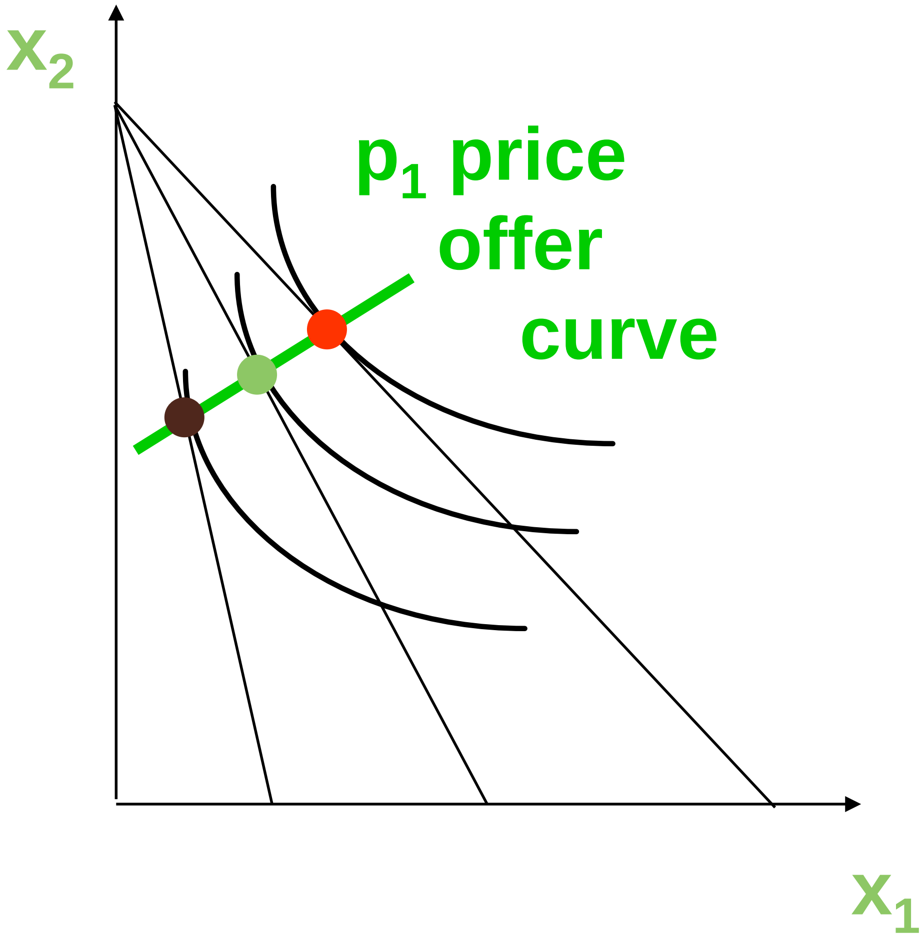


Ordinary Goods （一般商品）

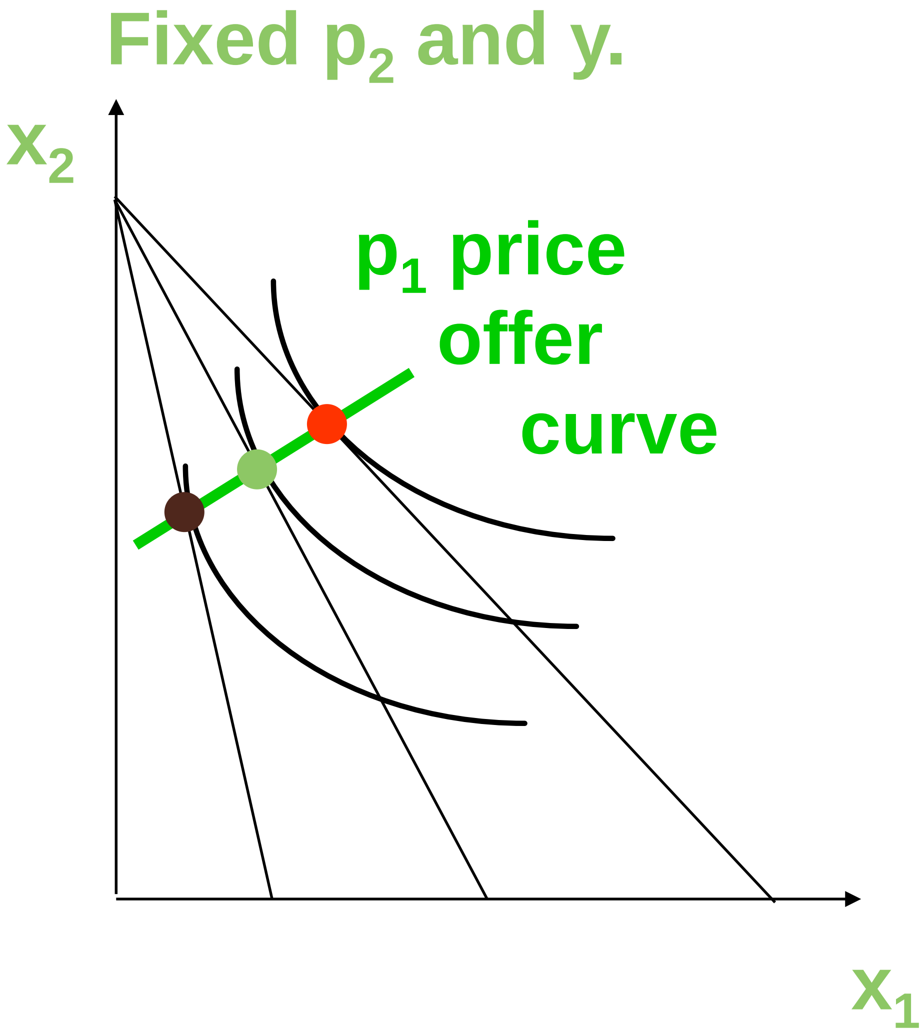
- A good is called ordinary if the quantity demanded of it always increases as its own price decreases.

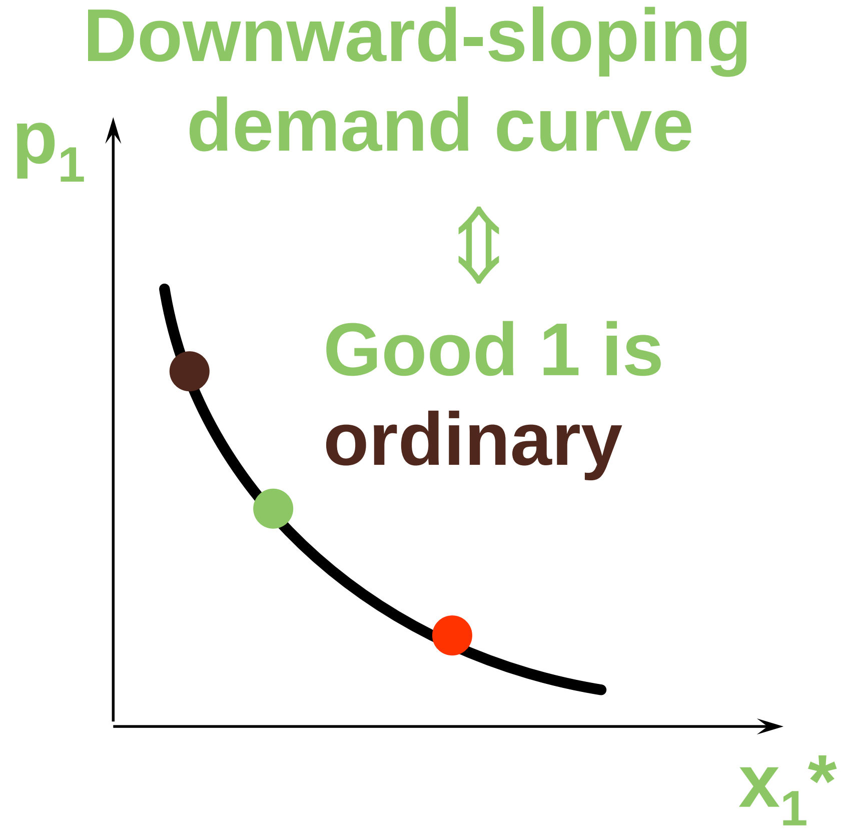
Ordinary Goods

Fixed p_2 and y.



Ordinary Goods





Giffen Goods （吉芬商品）

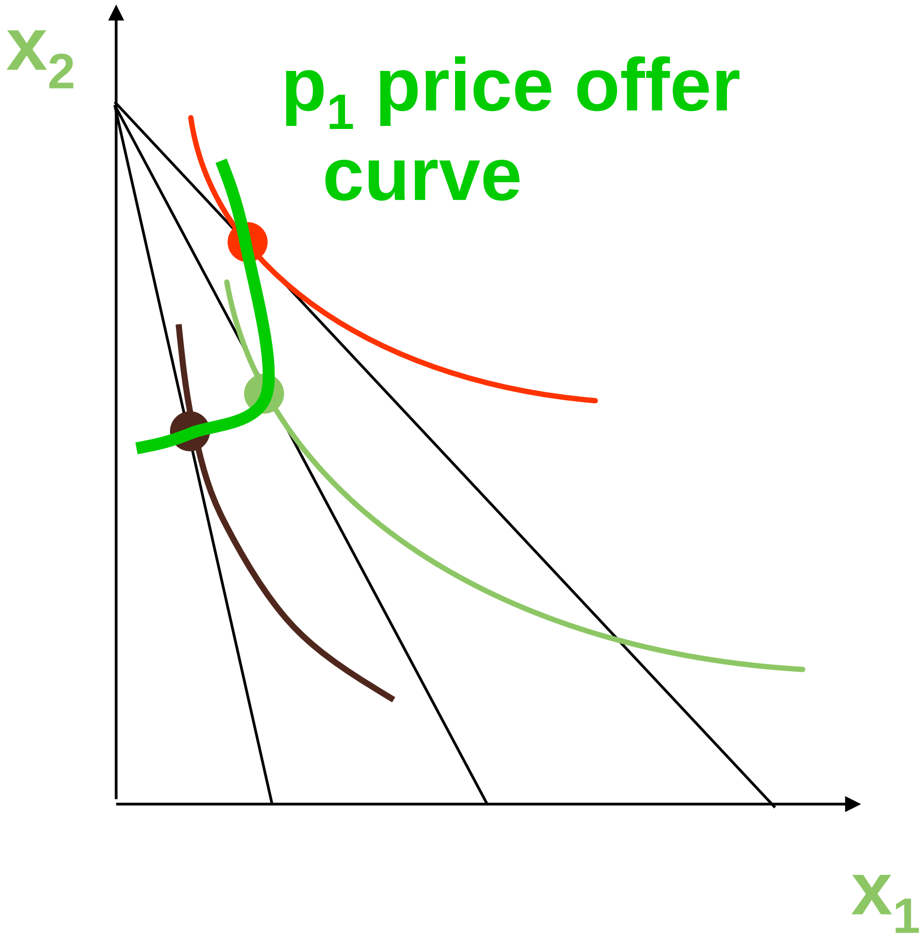
- If, for some values of its own price, the quantity demanded of a good rises as its own-price increases then the good is called Giffen.

Ordinary Goods

Fixed p_2 and y.

Ordinary Goods

Fixed p_2 and y.



Demand curve has a positively sloped part

⇕

Good 1 is Giffen

Cross-Price Effects

- If an increase in p_2
  - increases demand for commodity 1 then commodity 1 is a gross substitute for commodity 2.
  - reduces demand for commodity 1 then commodity 1 is a gross complement for commodity 2.

Cross-Price Effects

A perfect-complements example:

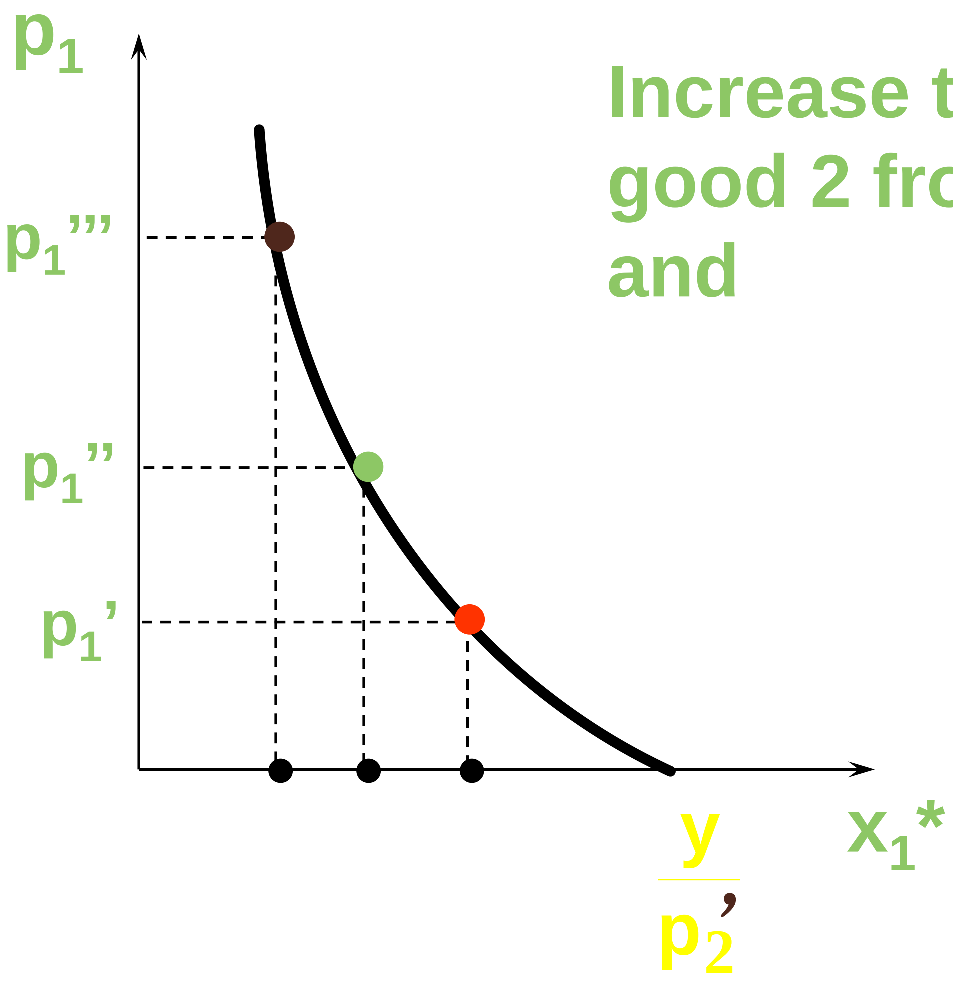
$$x_1^* = \frac{y}{p_1 + p_2}$$

so

$$\frac{\partial x_1^*}{\partial p_2} = -\frac{y}{(p_1 + p_2)^2} < 0.$$

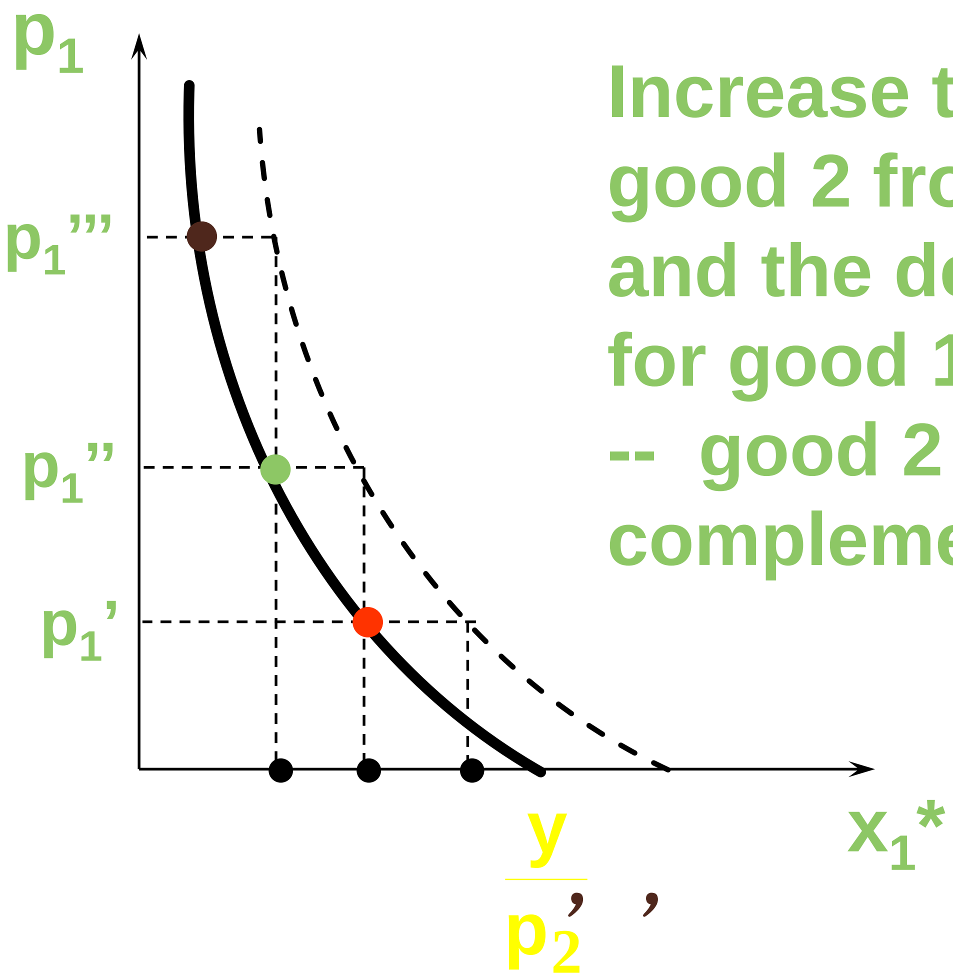
Therefore commodity 2 is a gross complement for commodity 1.

Cross-Price Effects



Increase the price of good 2 from p_2' to p_2'' and

Cross-Price Effects



Increase the price of good 2 from p_2' to p_2'' and the demand curve for good 1 shifts inwards -- good 2 is a complement for good 1.

Cross-Price Effects

A Cobb- Douglas example:

$$x_2^* = \frac{by}{(a+b)p_2}$$

so

$$\frac{\partial x_2^*}{\partial p_1} = 0.$$

Therefore commodity 1 is neither a gross complement nor a gross substitute for commodity 2.