

Central University of Finance and Economics
School of Economics
Intermediate Microeconomics, Fall 2017
HW 1
(Date: Friday, November 17, 2017)

Question 1. True/False/Uncertain (6 points)

Take the non-italicized statements as given. Indicate whether the italicized statements are TRUE, FALSE, or UNCERTAIN and explain why.

- a) If utility is given by $U = x_1 + x_2$, *the consumer may stop consuming one good altogether in response to a slight change in price.*
- b) Suppose demand is given by $Q = 10 - P$. Suppose there is only one firm serving the market. Its revenues are given by $P * Q(P)$. *Increasing price increases revenue.*

Question 2. (12 points)

Suppose a consumer's preferences are represented by the utility function

$$U(x, y) = -\frac{1}{x} - \frac{1}{y}.$$

The price for good x is p_x , the price for good y is p_y , and the consumer has m to spend.

- a) Derive the consumer's demand for good x as a function of p_x , p_y , and m . (Note: The solution is interior.)
- b) Calculate the price elasticity of demand for good x.
- c) Calculate the income elasticity for good x.
- d) Is the good x a normal good or an inferior good?

Question 3. (12 points)

Suppose a consumer has a demand function of the form:

$$x(p_x, p_y, p_z, m) = 1 + m \left(\frac{1}{4p_x} + \frac{1}{20p_z} + \frac{m}{400p_x^2} \right).$$

Suppose that while the price of good x decreases from 4 to 2, income and other prices remain constant at $m = 100$, $p_y = 3$ and $p_z = 2$. Find the change in quantity demanded. Find the magnitudes of the Slutsky substitution effect and income effect.

Question 4. (15 points)

Dave is deciding how much to work in the coming year. He derives utility from consumption, C, but he also really likes taking leisure time L. He must divide his

available hours between work and leisure for every hour of leisure he takes he must work one fewer hours. The function that describes his preferences is given by:

$$U(C, L) = C^{1/4} L^{3/4}$$

He can earn a wage of w , and suppose the price of consumption is given by $p=1$.

Finally, Dave receives some non-wage income, m , which does not depend on the number of hours he works.

- What is Dave's full income if he can work 2000 hours in a year? Write down his budget constraint.
- Solve Dave's utility maximization problem and write down his optimal consumption of C and L . How does Dave's consumption depend on w and m ? Is leisure a normal good?
- Suppose Dave's wages are initially given by $w = 10$ and his non-labor income by $m=100$. Find his optimal levels of leisure and consumption. If Dave's wage increases to $w = 20$, what are his new levels of leisure and consumption?
- Graphically depict the income and substitution effects on leisure hours associated with the change in wage from 10 to 20. Do the income and substitution effects have the same sign? What is the intuition for this?

Question 5. (30 points)

Peter spends his income on books (x_1) and CD (x_2).

- Suppose the price of a book is $p_1=10$, the price of a CD is $p_2=5$, and Peter's daily budget is $m = 40$. Show graphically Peter's budget constraint, marking his real incomes in terms of books and CDs. On the same graph, show how his budget set is affected by a gift of 2 CDs (assume that he can always dispose the gift).
- Peter's preferences are given by the following utility function

$$U(x_1, x_2) = x_1 + 2\ln x_2$$

- Find Peter's marginal rate of substitution (MRS) for any bundle (x_1, x_2) (give the formula for MRS).
 - What is the value of MRS at consumption bundle $(5, 8)$ (give a number)?
 - Suppose Peter "consumes" 5 books and 8 CDs and one takes away 0.0001 of a book. What is compensation in terms of CDs is sufficient to make Peter indifferent?
 - Depict the indifference curve map in a commodity space. Mark the slope of the indifference curve at bundle $(5, 8)$.
- Given $U(x_1, x_2) = x_1 + 2\ln x_2$, write down the necessary conditions for the optimal choice (assume interior solution). Provide economic intuition behind the two conditions.

- d) Using the two conditions from c) find the optimal consumption levels of both types of commodities (x_1, x_2) for:

- $p_1 = 10, p_2 = 5, \text{ and } m = 40$ (give two numbers)

And after the price of a book increases:

- $p_1 = 30, p_2 = 5, \text{ and } m = 40$ (give two numbers)

Is each of the solutions interior? Illustrate the change on the graph.

- e) In d), is the marginal utility of the last dollar invested in books and CDs equal? (Find two numbers for the parameters before and after the change.) In case they are not, explain why not equalizing the marginal utility of the last dollar is consistent with optimum.

Question 6. (10 points)

Consider a model with two periods, each with one composite consumption good. Prices are the stable and normalized to 1. Suppose a consumer is endowed with income 20 in the first period and 60 in the second period. The consumer receives a 10% interest rate on savings and is subject to a 20% interest rate on borrowing. The banks prohibit the consumer from borrowing more than 20.

- Find this consumer's intertemporal budget constraint.
- Carefully graph the budget set. Label all intercepts.

Question 7. (15 points)

The Stone-Geary utility function:

$$U(x_1, x_2) = (x_1 - \gamma_1)^{\beta_1} (x_2 - \gamma_2)^{\beta_2}$$

It is an important generalization of the Cobb-Douglas Utility Function. Consider the special case where $\beta_1 = 1, \beta_2 = 1, \gamma_1 > 0$ and $\gamma_2 > 0$. The price for good x_1 is p_1 , the price for good x_2 is p_2 , and the consumer has m to spend.

- Graph the indifference curves associated with this preference.
- Derive the consumer's demand for good x_1 as a function of p_1 , p_2 , and m .
- Are good x_1 and x_2 gross substitutes, gross complements, or unrelated goods?