Chapter 5
Choice

Li Yan(李艳)
School of Economics
Central university of Finance and Economics

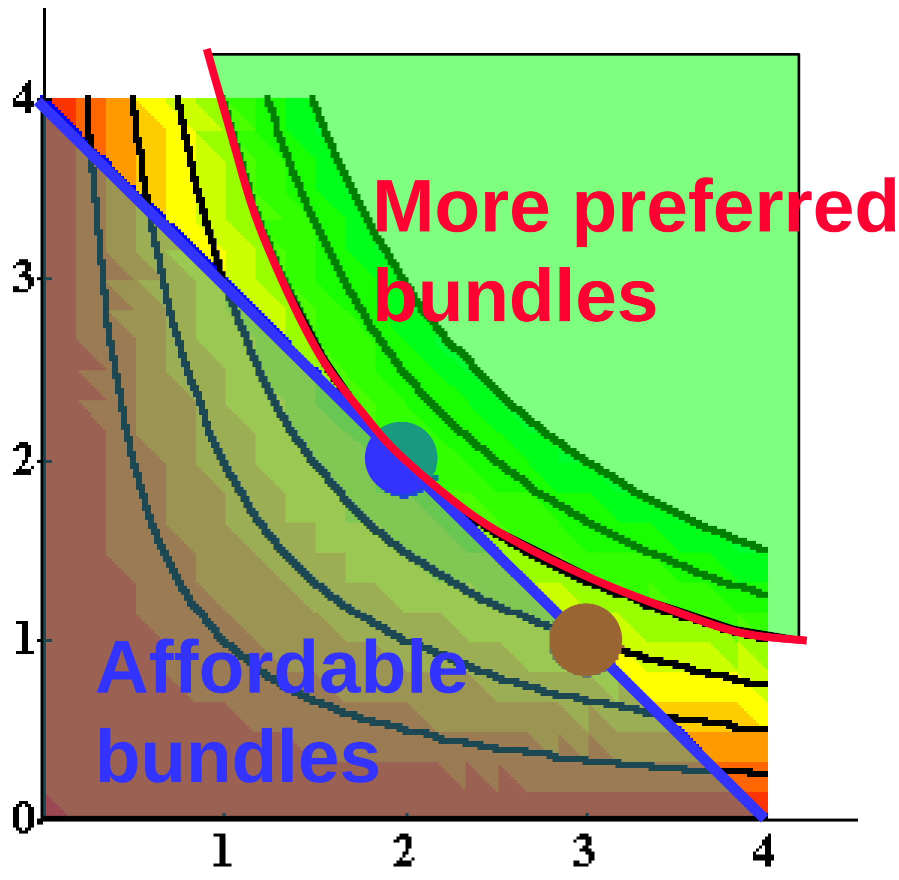
Structure

- Rational constrained choice
- Computing ordinary demands
  - Interior solution （内在解）
  - Corner solution （角点解）
  - “Kinky” solution
- Example: Choosing taxes

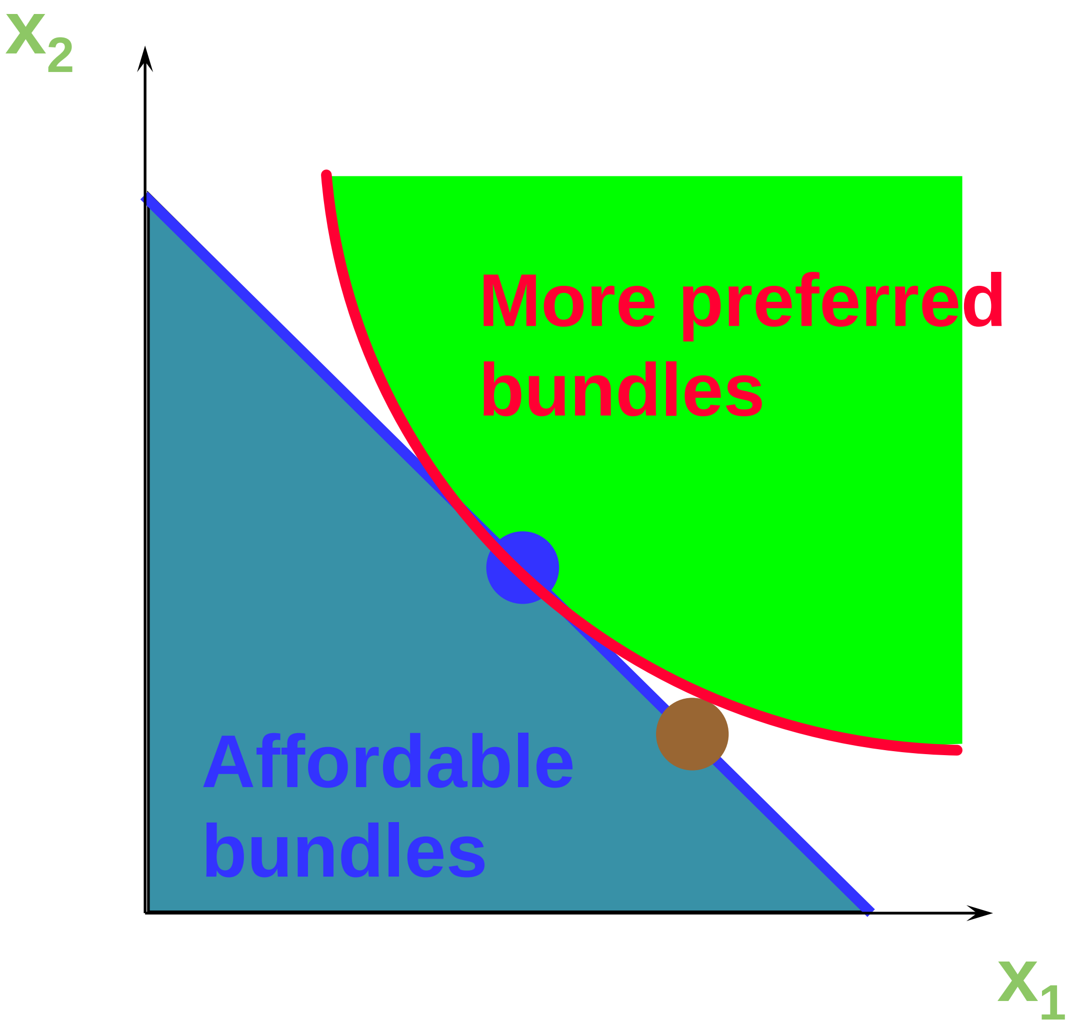
Economic Rationality

- The principal behavioral postulate is that a decision-maker chooses its most preferred alternative from those available to it.
- The available choices constitute the choice set.
- How is the most preferred bundle in the choice set located?

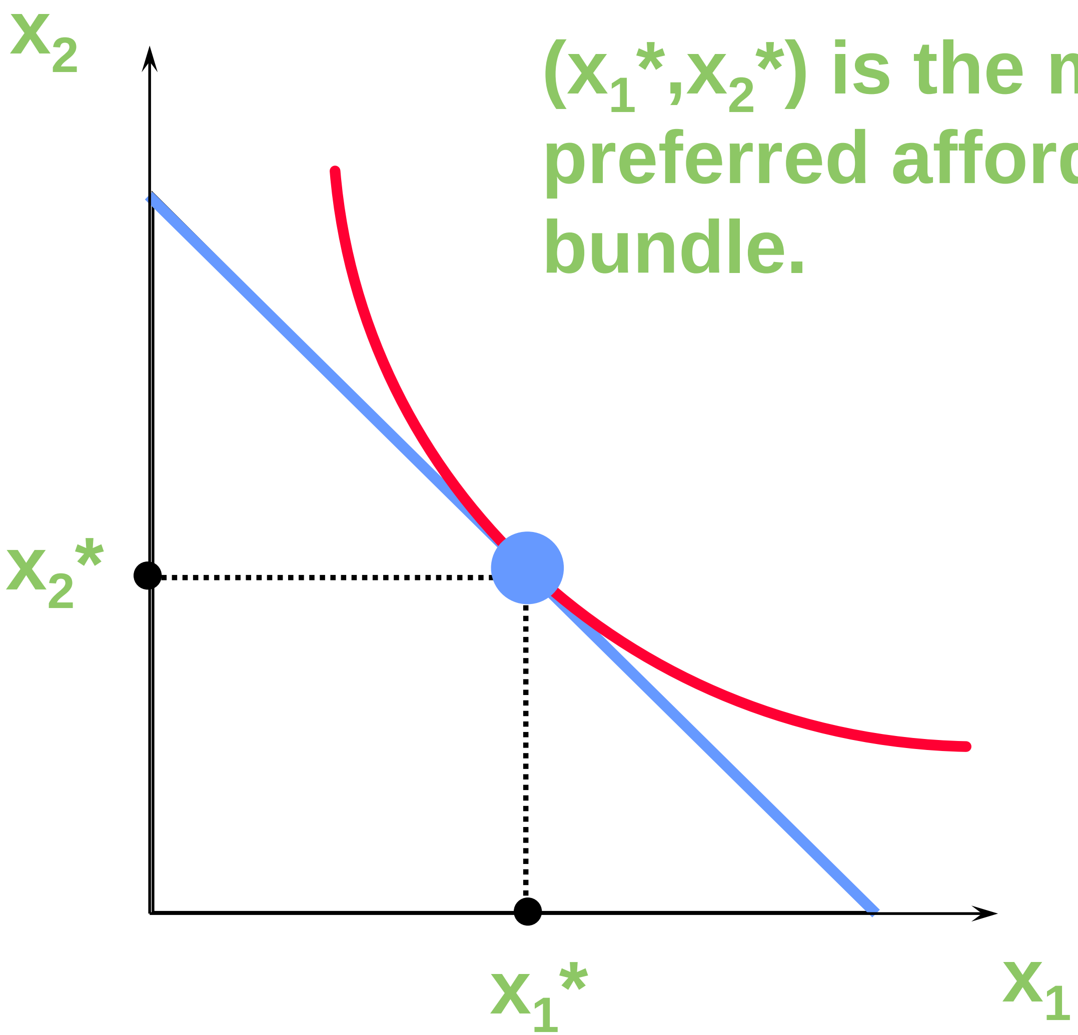
Rational Constrained Choice



Rational Constrained Choice



Rational Constrained Choice



(x_1^*, x_2^*) is the most preferred affordable bundle.

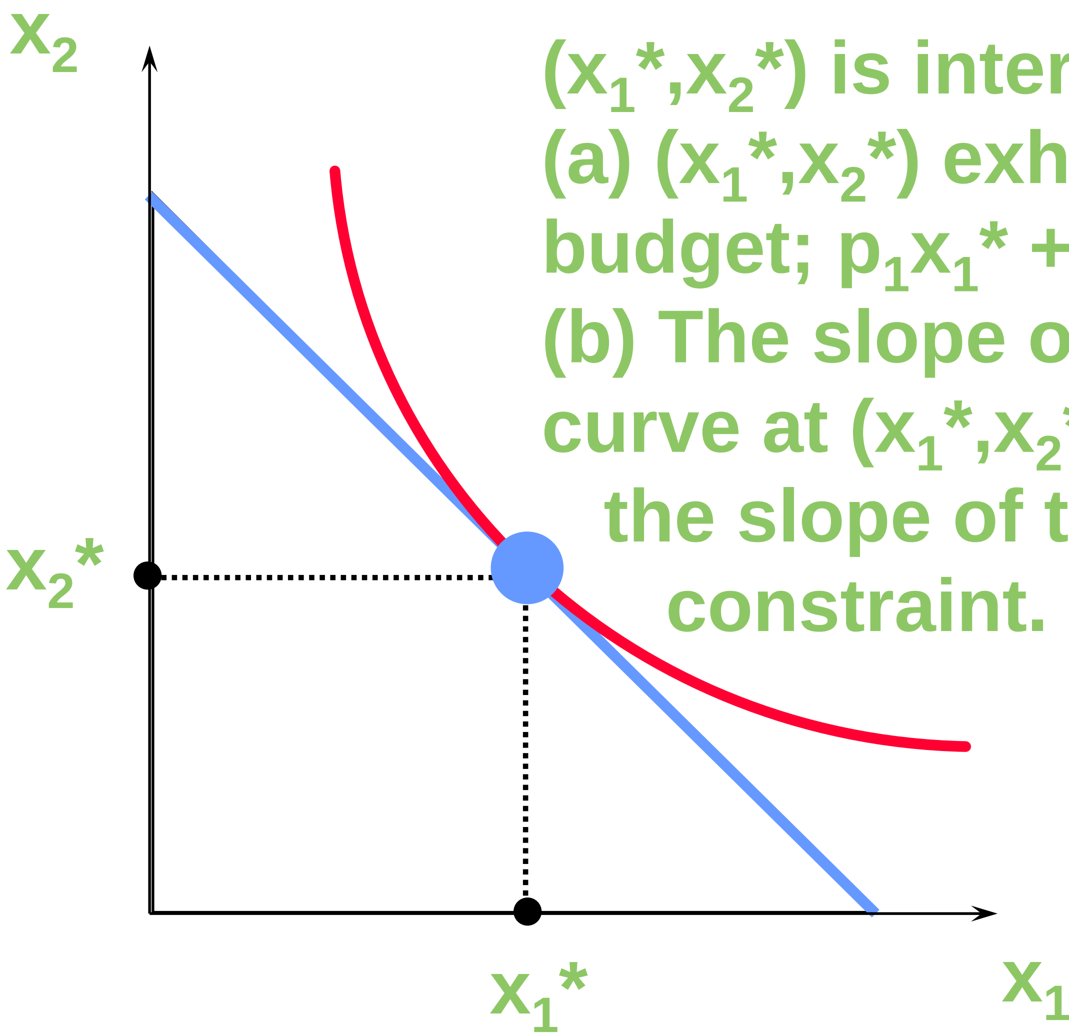
Rational Constrained Choice

- The most preferred affordable bundle is called the consumer's ORDINARY DEMAND （or DEMAND,一般需求）at the given prices and budget.
- Ordinary demands will be denoted by $x_1^*(p_1,p_2,m)$ and $x_2^*(p_1,p_2,m)$.

Rational Constrained Choice

- When $x_1^* > 0$ and $x_2^* > 0$ the demanded bundle is INTERIOR.
- If buying (x_1^*, x_2^*) costs \$m then the budget is exhausted.

Rational Constrained Choice



(x_1^*,x_2^*) is interior.
(a) (x_1^*,x_2^*) exhausts the budget; $p_1x_1^* + p_2x_2^* = m$.
(b) The slope of the indiff. curve at (x_1^*,x_2^*) equals the slope of the budget constraint.

Rational Constrained Choice

- (x_1^*, x_2^*) satisfies two conditions:
- (a) the budget is exhausted;

$$p_1 x_1^* + p_2 x_2^* = m$$

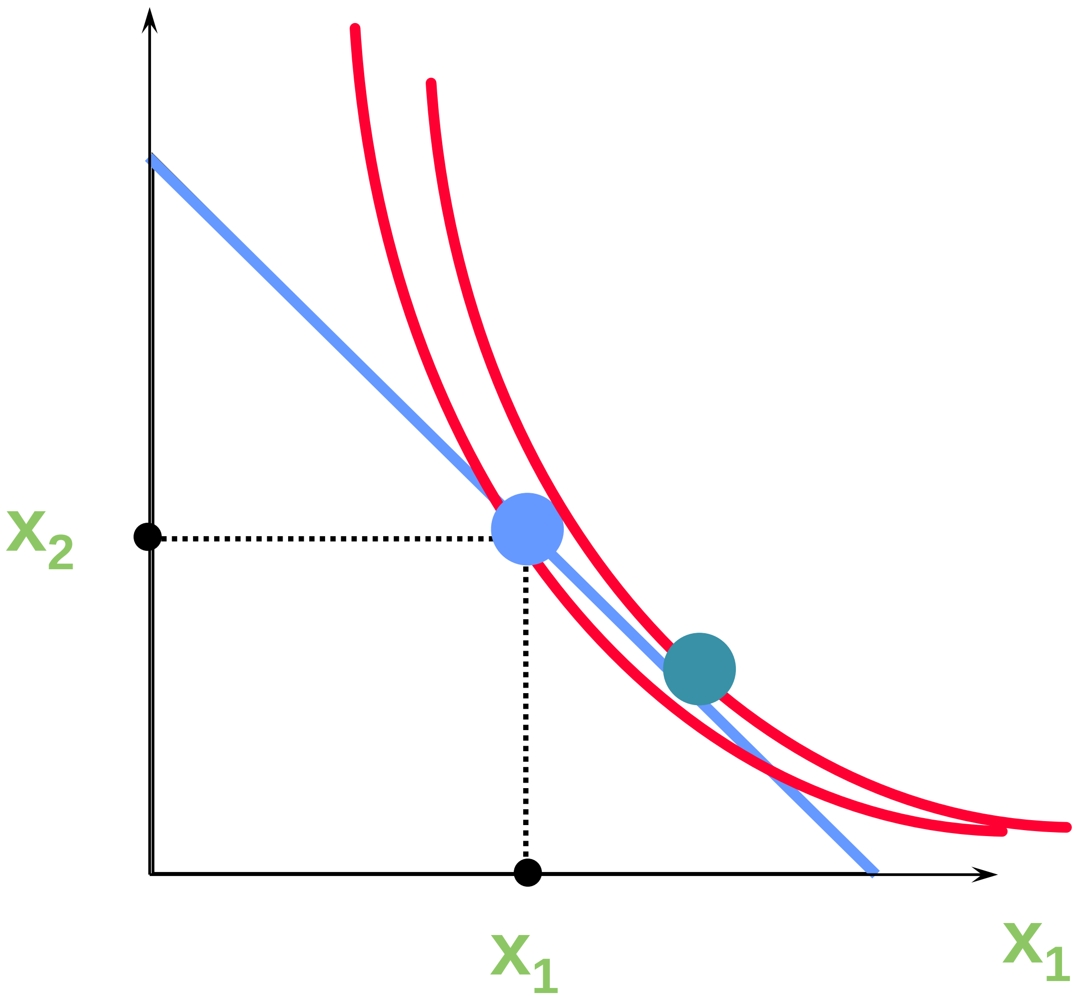
- (b) tangency: the slope of the budget constraint, $-p_1/p_2$, and the slope of the indifference curve containing (x_1^*, x_2^*) are equal at (x_1^*, x_2^*).

Meaning of the Tangency Condition

- Consumer's marginal willingness to pay equals the market exchange rate.
- Suppose at a consumption bundle (x_1, x_2),

$$MRS = 2, P_1/P_2 = 1$$

  - The consumer is willing to give up 2 unit of x_2 to exchange for an additional unit of x_1
  - The market allows her to give up only 1 unit of x_2 to obtain an additional x_1
- (x_1, x_2) is not optimal choice
- She can be better off increasing her consumption of x_1.



x_2
x_1
x_1

Computing Ordinary Demands

- Solve for 2 simultaneous equations.
  - Tangency
  - Budget constraint
- The conditions may be obtained by using the Lagrangian multiplier method （拉格朗日方程）, i.e., constrained optimization in calculus.

Computing Ordinary Demands

- How can this information be used to locate (x_1^*, x_2^*) for given p_1, p_2 and m?

Computing Ordinary Demands - a Cobb-Douglas Example.

- Suppose that the consumer has Cobb-Douglas preferences.

$$U(x_1, x_2) = x_1^{\alpha} x_2^{\beta}$$

Computing Ordinary Demands - a Cobb-Douglas Example.

- At (x_1*,x_2*), MRS = p_1/p_2 so the tangency condition (MRS = p_1/p_2) is
- $$MRS = \frac{\alpha x_2}{\beta x_1} = \frac{p_1}{p_2}$$

$$x_2 = \frac{\beta p_1}{\alpha p_2} x_1 \qquad (1)$$

Computing Ordinary Demands - a Cobb-Douglas Example.

- (x_1^*, x_2^*) also exhausts the budget so

$$p_1 x_1 + p_2 x_2 = m \qquad (2)$$

Computing Ordinary Demands - a Cobb-Douglas Example.

- The solution to the simultaneous equations (1) and (2) is:

$$x_1^* = \frac{\alpha}{\alpha+\beta}\frac{m}{p_1}$$

$$x_2^* = \frac{\beta}{\alpha+\beta}\frac{m}{p_2}$$

Lagrange Multipliers

$$Max\,U(x_1, x_2) \quad s.t.\ p_1x_1 + p_2x_2 = m$$

$$L = U(x_1, x_2) + \lambda(m - p_1x_1 - p_2x_2)$$

$$\frac{\partial L}{\partial x_1} = \frac{\partial U}{\partial x_1} - \lambda p_1 = 0$$

$$\frac{\partial L}{\partial x_2} = \frac{\partial U}{\partial x_2} - \lambda p_2 = 0$$

$$\frac{\partial L}{\partial \lambda} = m - p_1x_1 - p_2x_2 = 0$$

Equal Marginal Principle

$$\lambda = \frac{\partial U / \partial x_1}{p_1} = \frac{\partial U / \partial x_2}{p_2}$$

$$\textit{In the case of } U(x_1, x_2, \ldots, x_n),$$

$$\lambda = \frac{\partial U / \partial x_1}{p_1} = \frac{\partial U / \partial x_2}{p_2} = \ldots = \frac{\partial U / \partial x_n}{p_n}$$

Understanding lamda

$$\frac{dU}{dm} = \frac{\partial U}{\partial x_1}\frac{dx_1}{dm} + \frac{\partial U}{\partial x_2}\frac{dx_2}{dm}$$

$$Since\ dm = p_1 dx_1 + p_2 dx_2,\ \lambda = \frac{\partial U}{\partial x_1} / p_1 = \frac{\partial U}{\partial x_2} / p_2$$

$$\frac{dU}{dm} = \lambda p_1 \frac{dx_1}{dm} + \lambda p_2 \frac{dx_2}{dm} = \lambda (p_1 dx_1 + p_2 dx_2) / dm$$

$$\Rightarrow \frac{dU}{dm} = \lambda$$

$\boldsymbol{\lambda\ is\ the\ shadow\ price\ of\ income}$

How to Allocate Time Efficiently?

$$Max\, U = s_1 + \ldots + s_n = \sum_{i=1}^{n} s_i$$

$$s.t.\, (1)\ s_i = f_i(t_i),\ f_i^{'}(t_i) > 0,\ f_i^{''}(t_i) < 0$$

$$(2) \sum_{i=1}^{n} t_i \leq T$$

$$\Rightarrow \partial f_1(t_1) / \partial t_1 = \partial f_2(t_2) / \partial t_2 = \ldots. = \partial f_n(t_n) / \partial t_n = \lambda$$

$\lambda : shadow\ price\ of\ time$

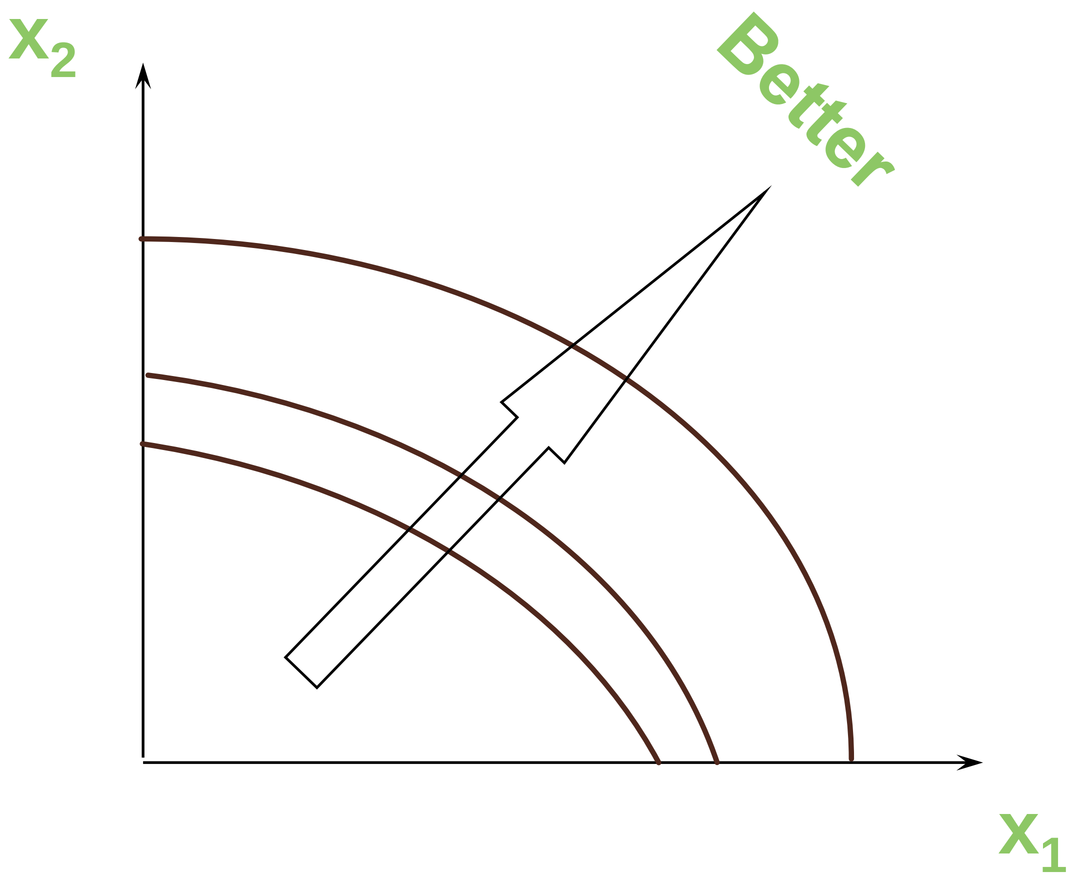
Rational Constrained Choice: Summary

- When $x_1^* > 0$ and $x_2^* > 0$
  and (x_1^*, x_2^*) exhausts the budget,
  and indifference curves have no
  'kinks', the ordinary demands are obtained by
  solving:
- (a) $p_1x_1^* + p_2x_2^* = y$
- (b) the slopes of the budget constraint, $-p_1/p_2$, and of
  the indifference curve containing (x_1^*, x_2^*) are equal
  at (x_1^*, x_2^*).

Rational Constrained Choice

- But what if $x_1^* = 0$?
- Or if $x_2^* = 0$?
- If either $x_1^* = 0$ or $x_2^* = 0$ then the ordinary demand (x_1^*, x_2^*) is at a corner solution (角点解) to the problem of maximizing utility subject to a budget constraint.

Examples of Corner Solutions -- the Non-Convex Preferences Case



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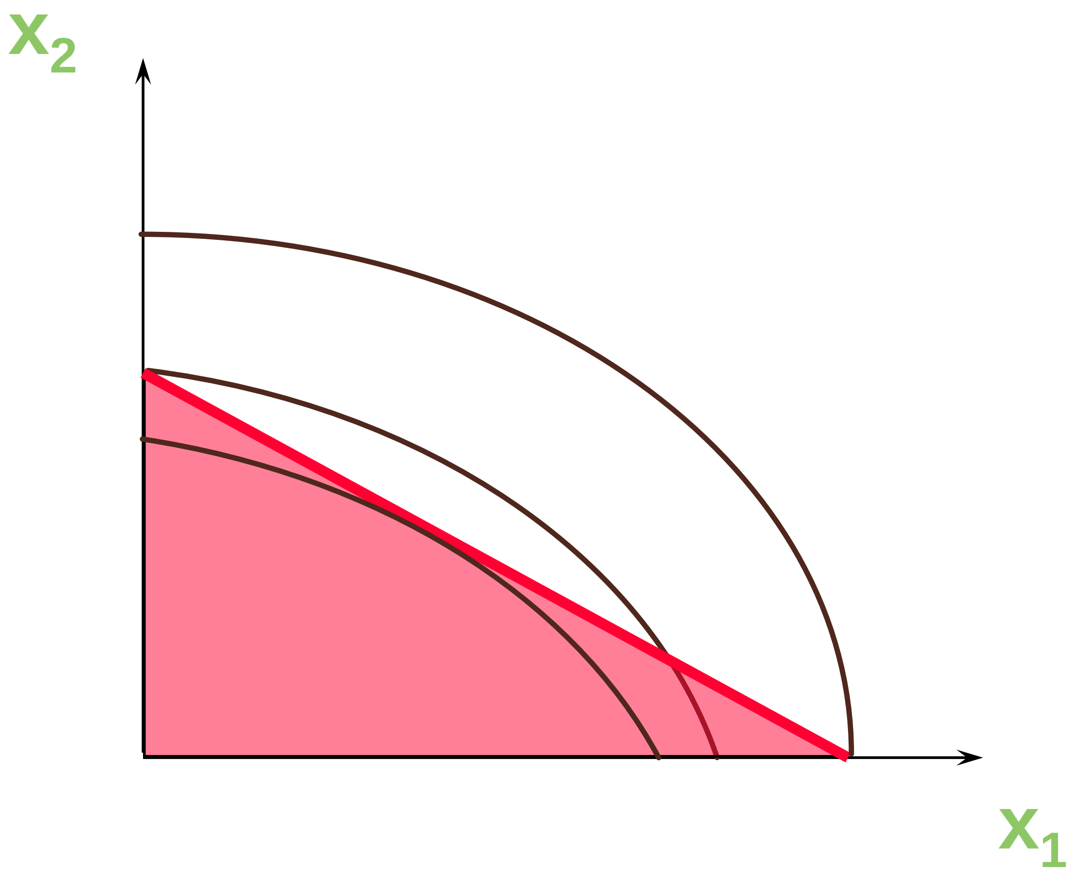

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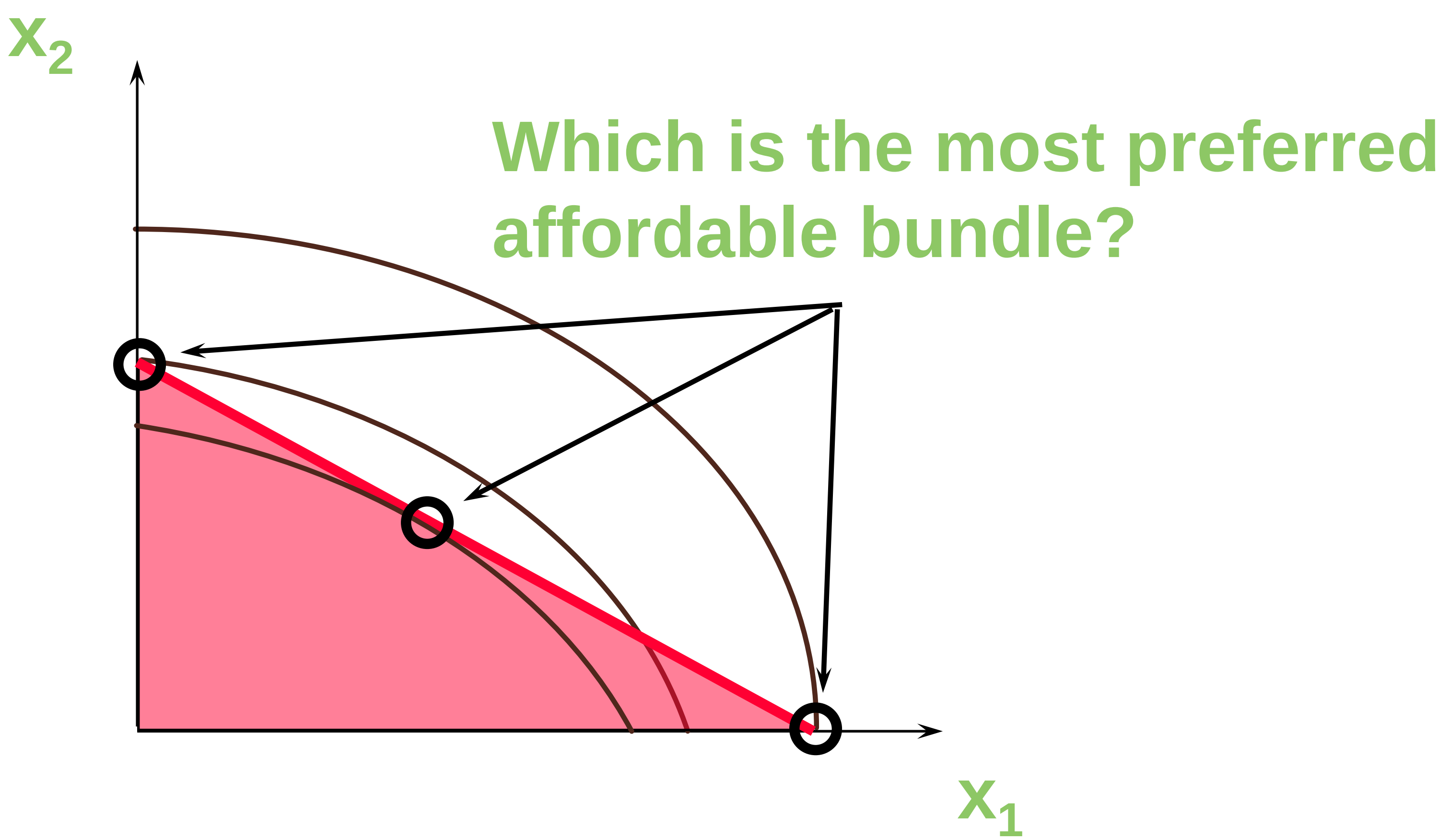

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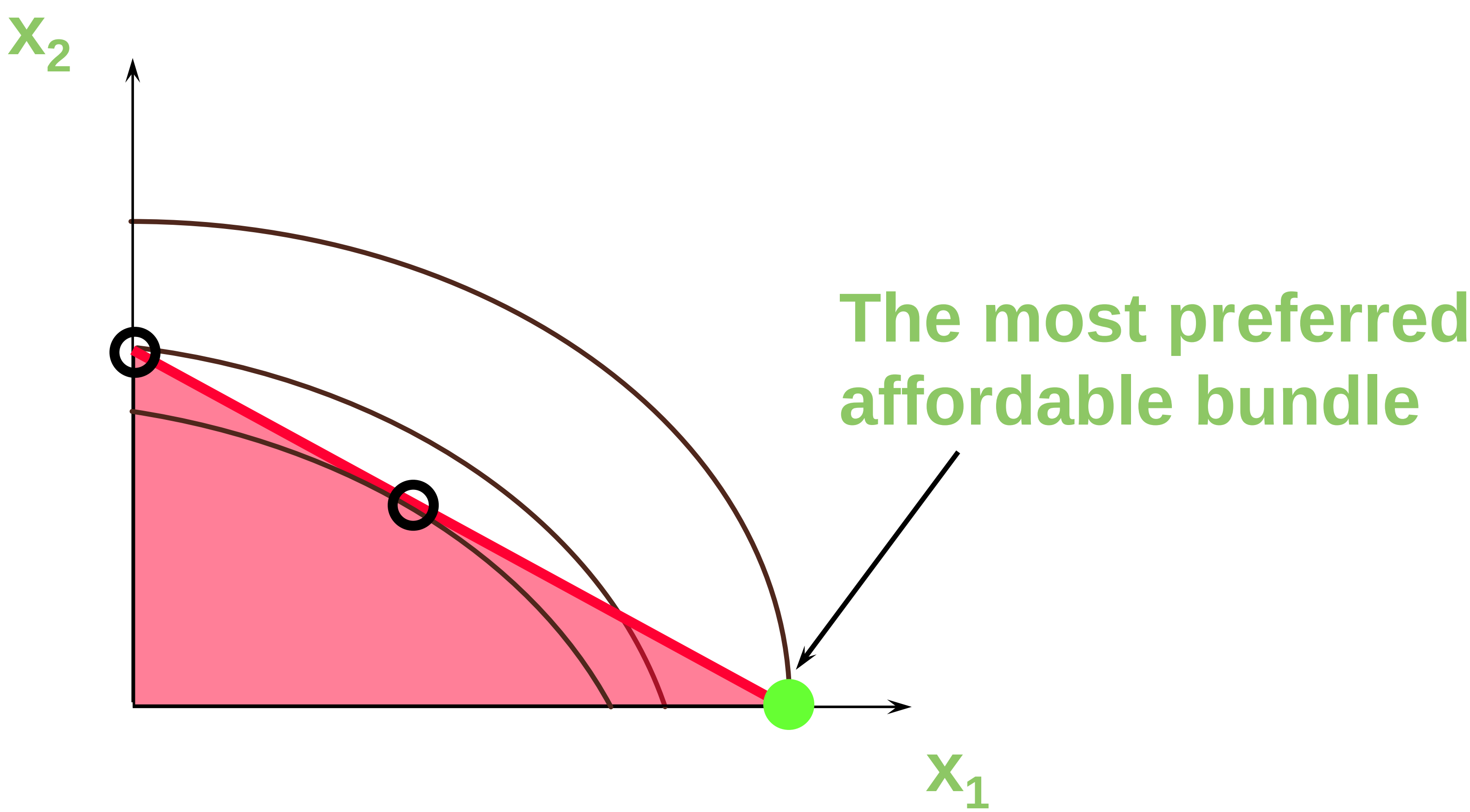

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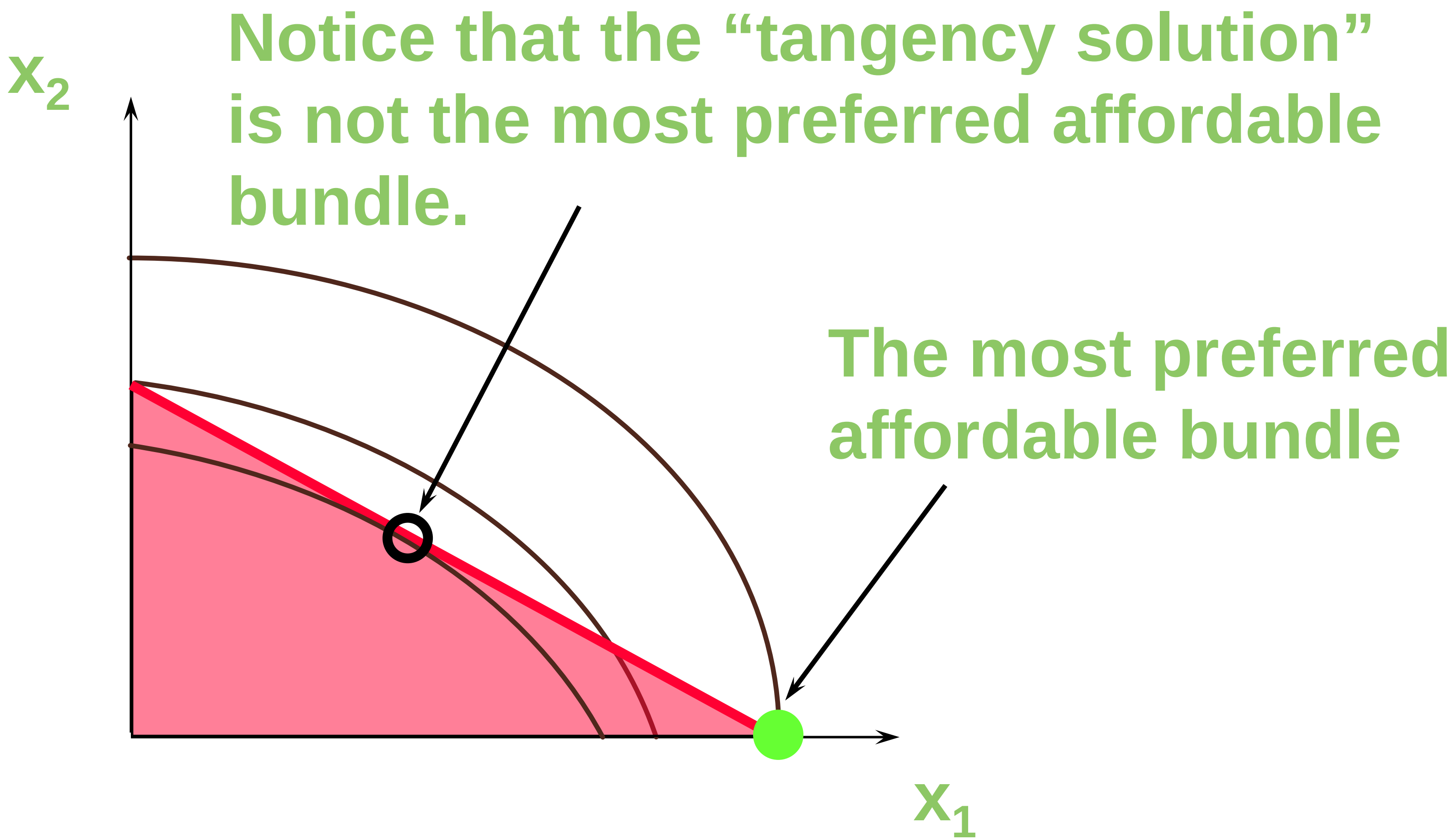
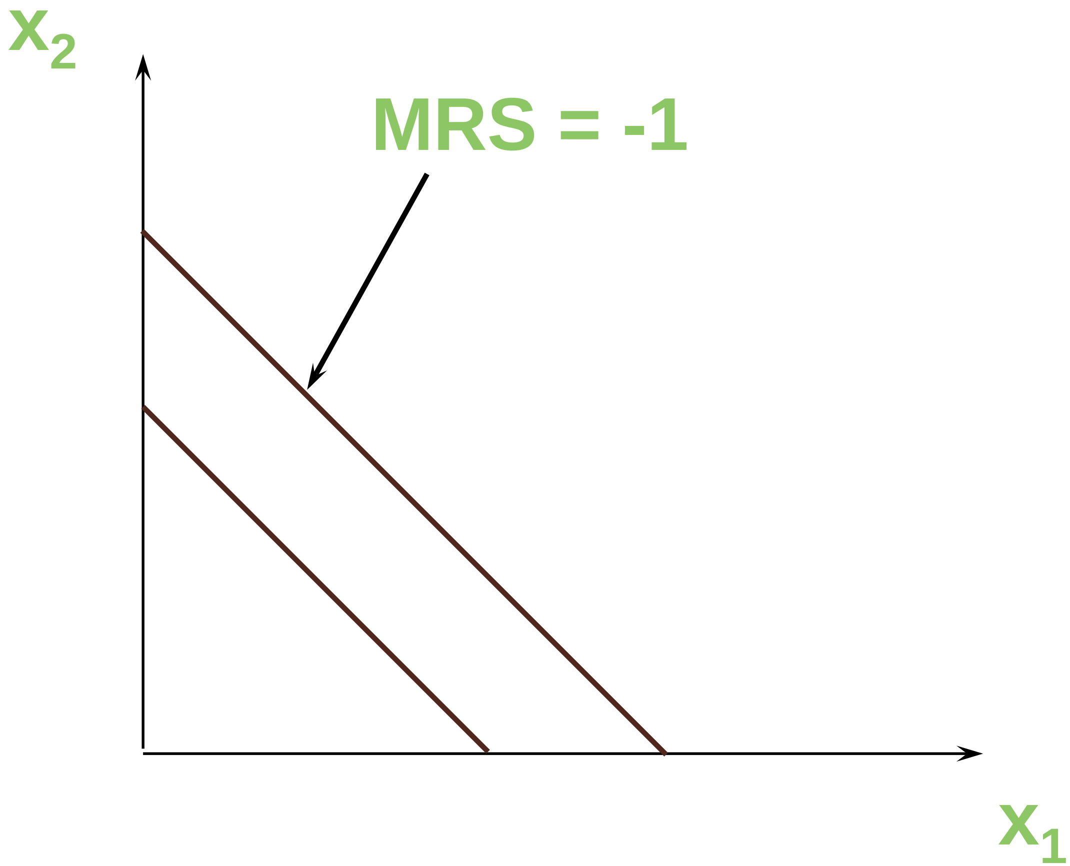
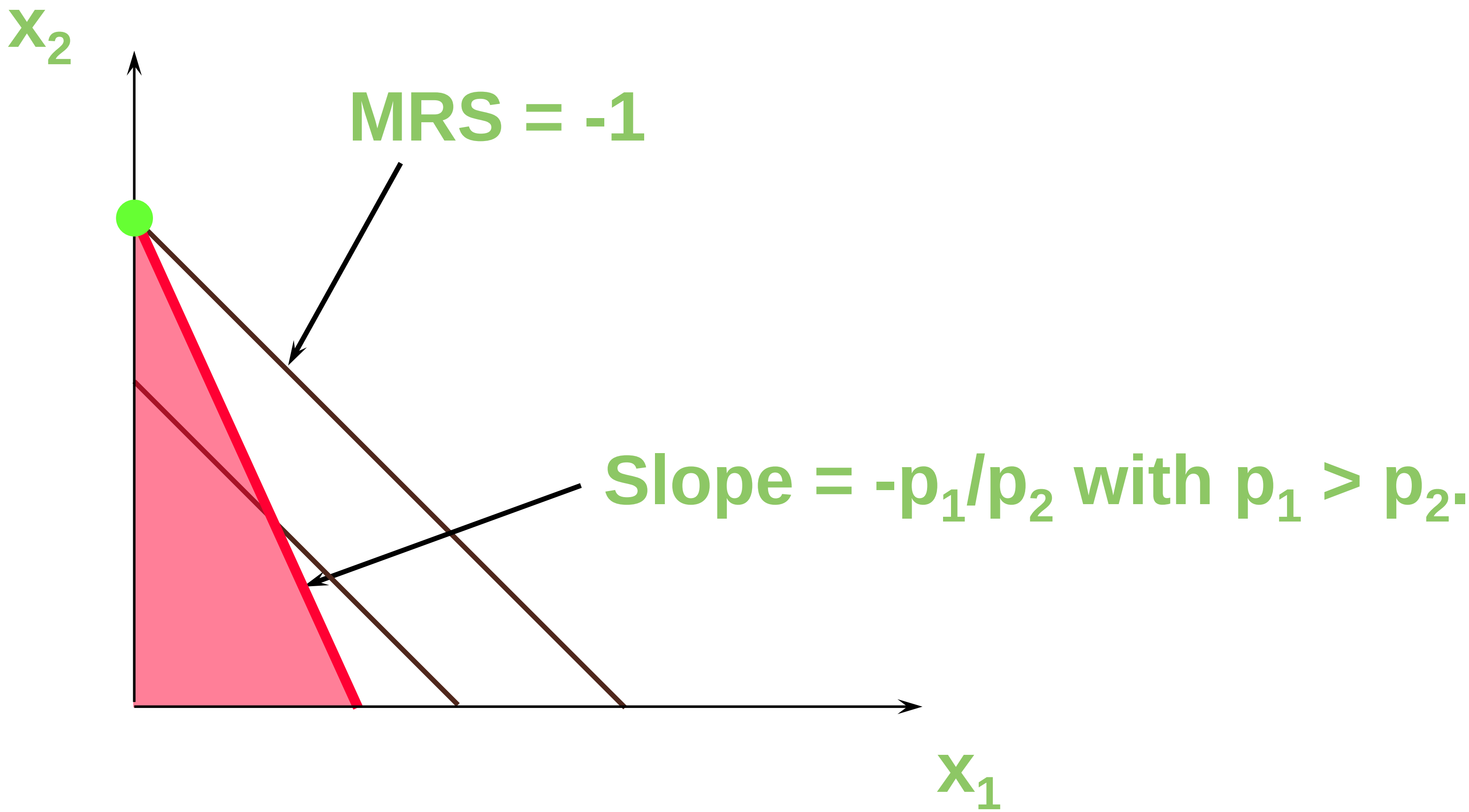


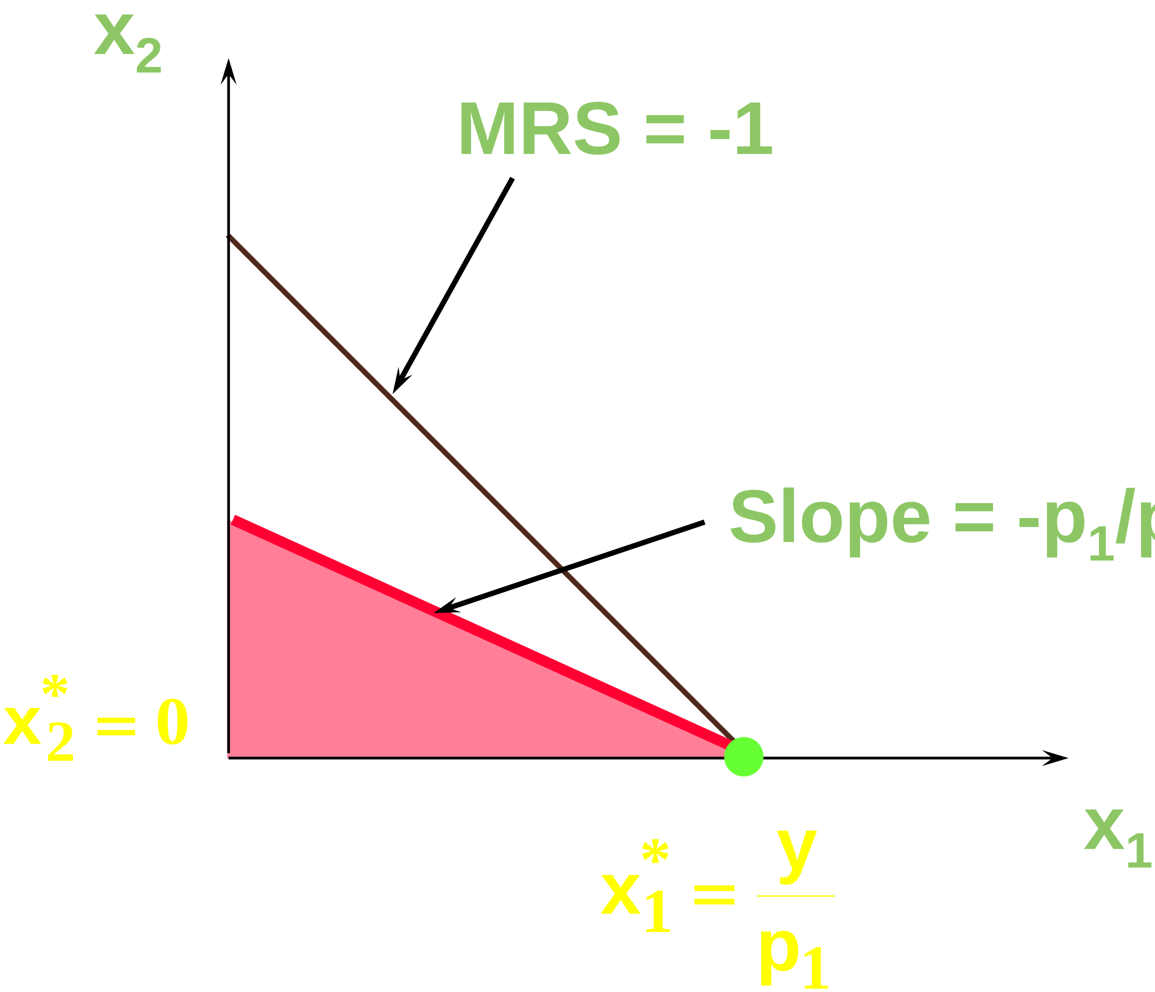
Examples of Corner Solutions -- the Perfect Substitutes Case



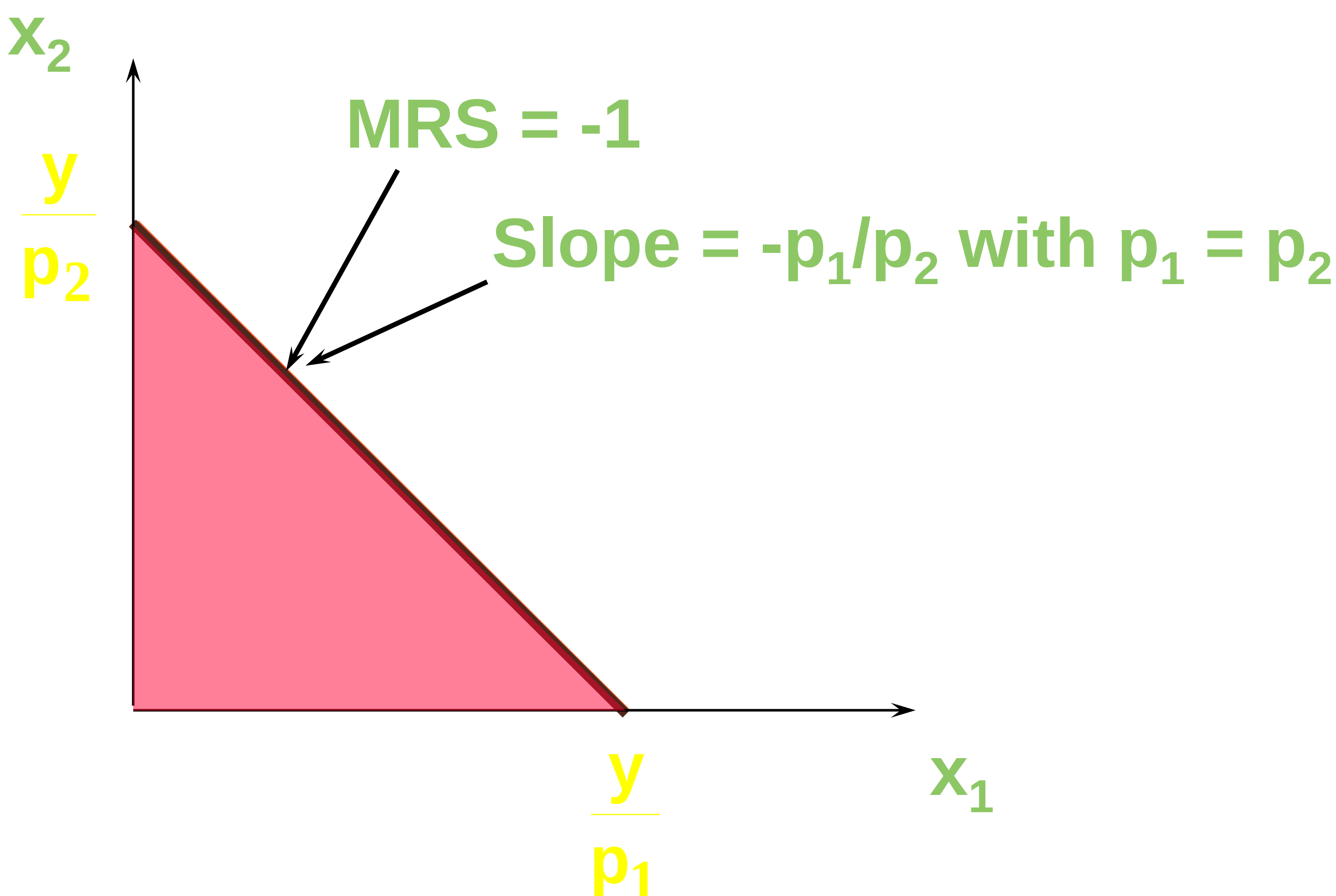
Examples of Corner Solutions -- the Perfect Substitutes Case



Examples of Corner Solutions -- the Perfect Substitutes Case



Examples of Corner Solutions -- the Perfect Substitutes Case



Demand curve

$$x_1 = \begin{cases} m/p_1 & \text{when } p_1 < p_2; \\ \text{any number between 0 and } m/p_1 & \text{when } p_1 = p_2; \\ 0 & \text{when } p_1 > p_2. \end{cases}$$

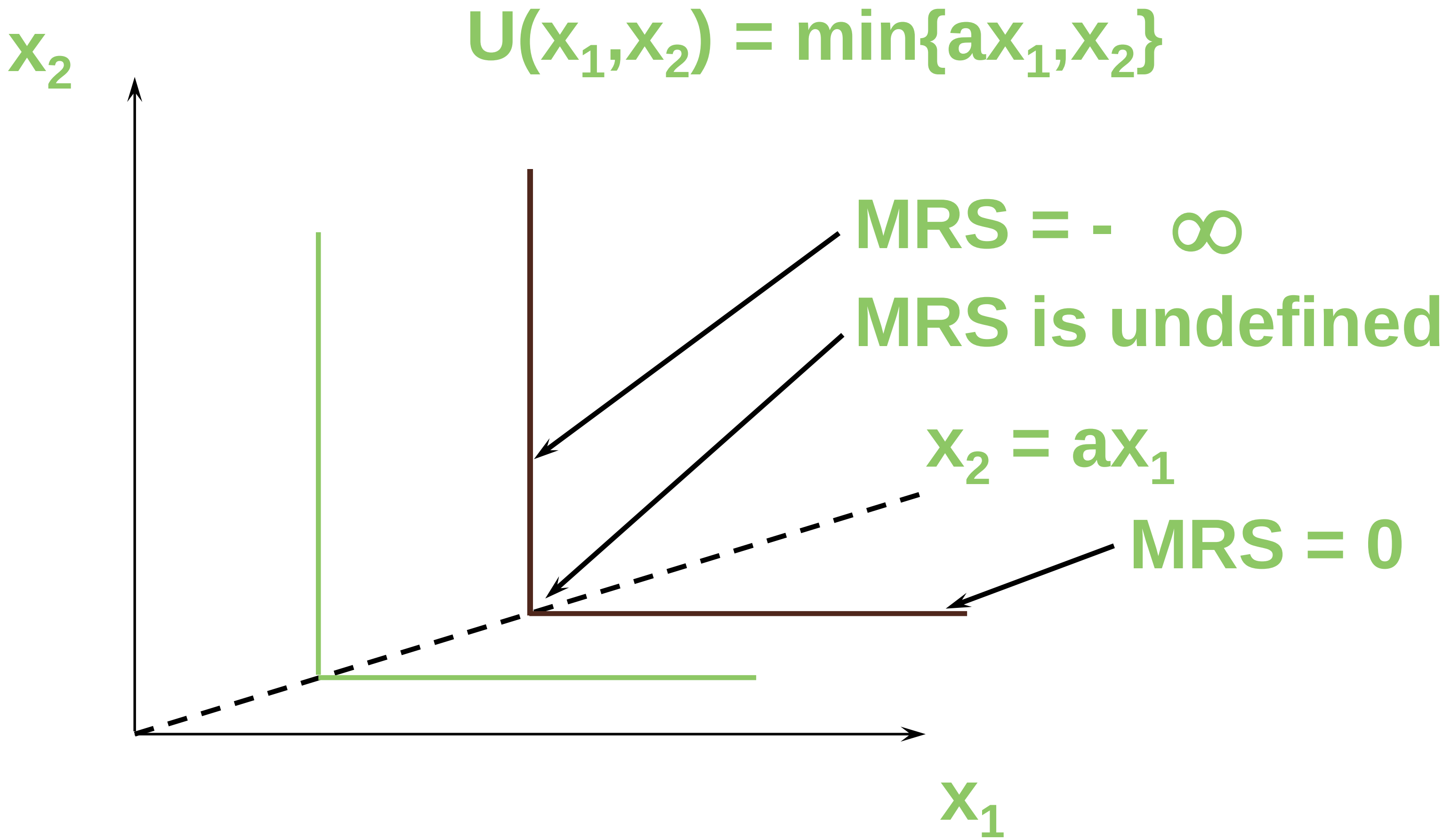
Is Tangency Condition Sufficient?

- Tangency condition is sufficient and necessary if

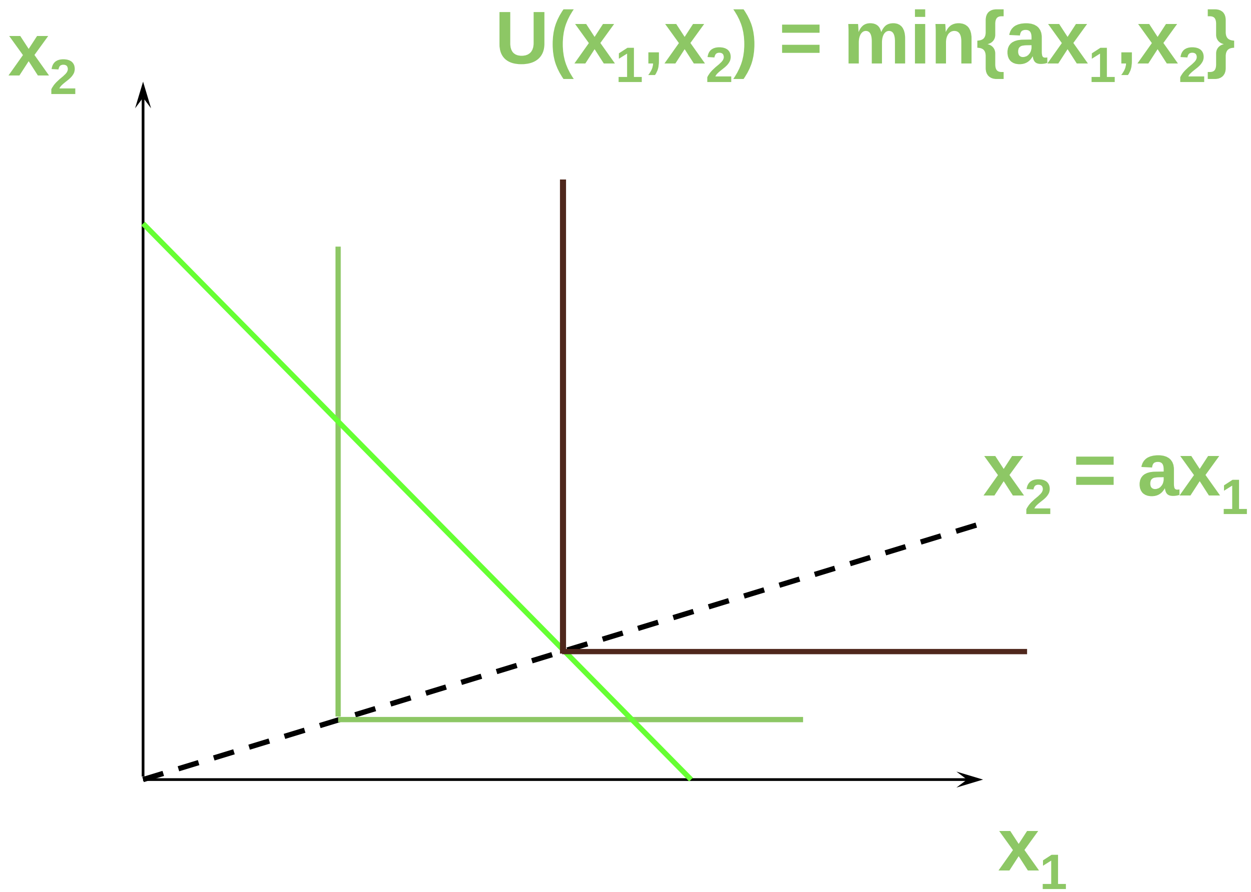
(1) Preferences are convex

(2) Solutions are interior

Examples of ‘Kinky’ Solutions -- the Perfect Complements Case



Examples of 'Kinky' Solutions -- the Perfect Complements Case



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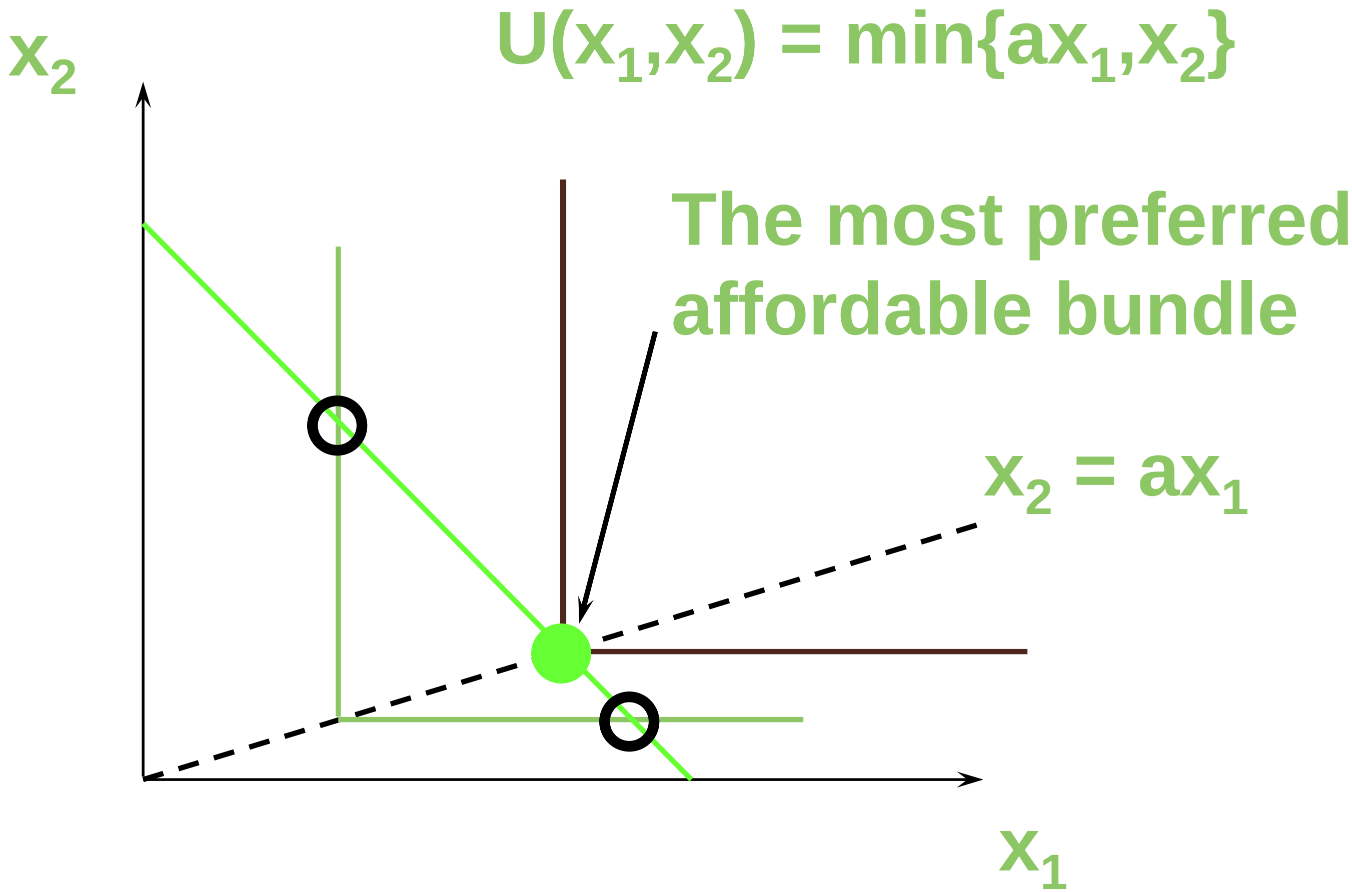
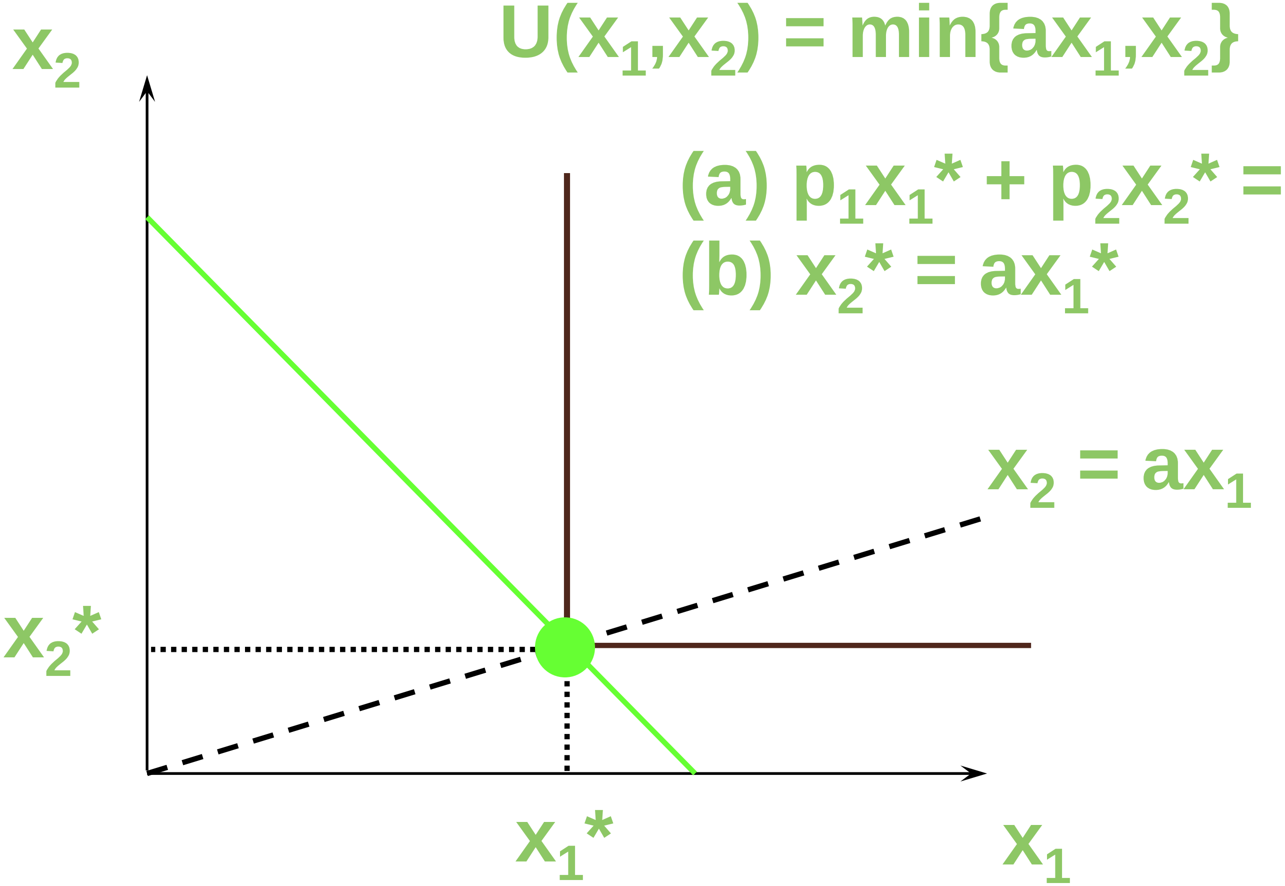


Examples of 'Kinky' Solutions -- the Perfect Complements Case

$U(x_1,x_2) = \min\{ax_1,x_2\}$

(a) $p_1x_1^* + p_2x_2^* = m$
(b) $x_2^* = ax_1^*$



Examples of 'Kinky' Solutions -- the Perfect Complements Case

(a) $p_1x_1^* + p_2x_2^* = m$; (b) $x_2^* = ax_1^*$.

Substitution from (b) for x_2^* in (a) gives $p_1x_1^* + p_2ax_1^* = m$ which gives

$$x_1^* = \frac{m}{p_1 + ap_2}; \; x_2^* = \frac{am}{p_1 + ap_2}.$$

Choosing Taxes: Various Taxes

- Quantity tax: on x: $(p+t)x$
- Value tax: on px: $(1+t)px$
  - Also called ad valorem tax
- Lump sum tax: T
- Income tax:
  - Can be proportional or lump sum

Income Tax vs. Quantity Tax

- Original budget: $p_1x_1 + p_2x_2 = m$
- After quantity tax:

$$(p_1+t)x_1 + p_2x_2 = m$$

- At optimal choice (x_1^*, x_2^*)
  - $(p_1+t)x_1^* + p_2x_2^* = m$ (5.2)
  - Tax revenue: $R^*=tx_1^*$
- With an income tax, budget is:

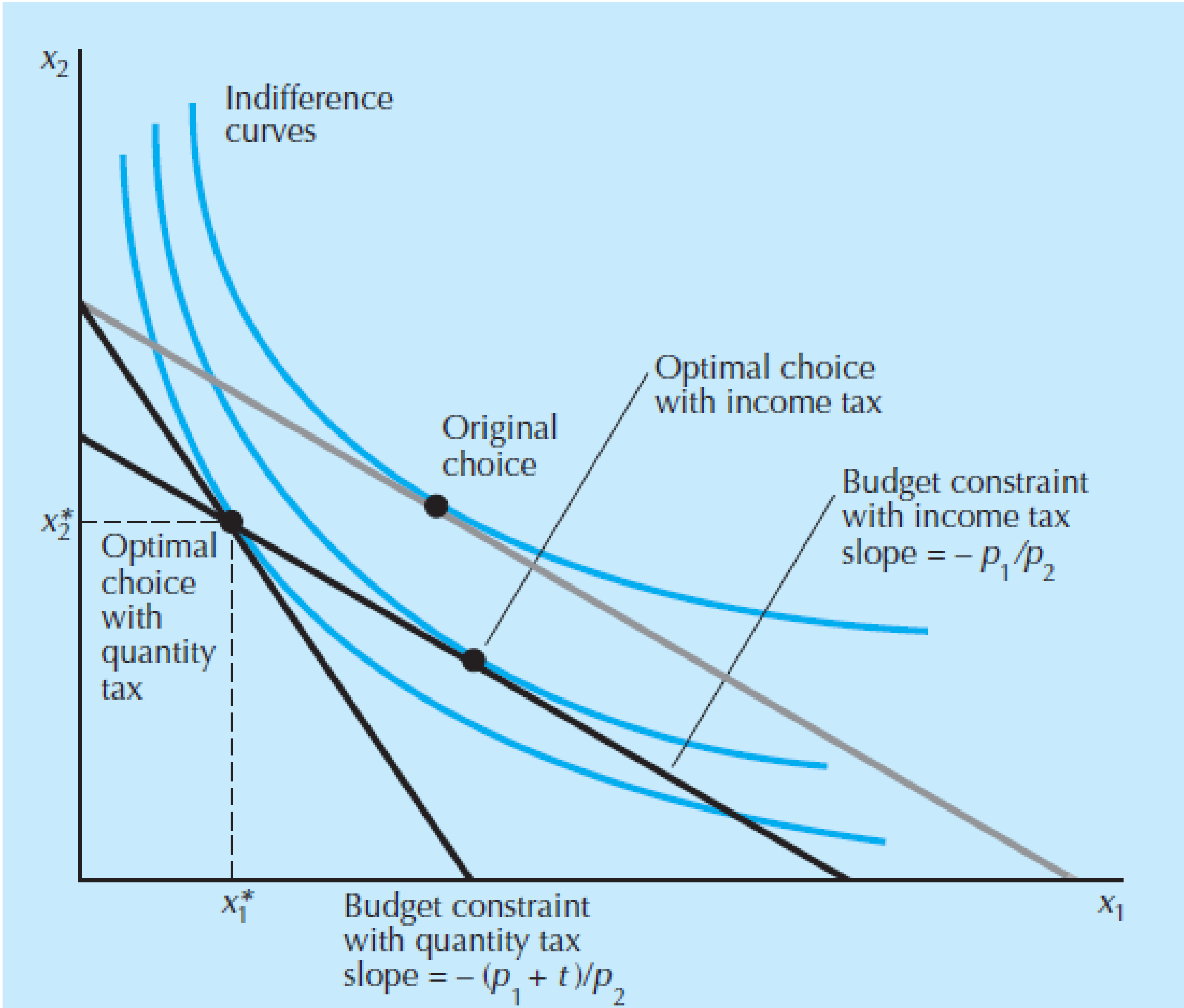
$$p_1x_1 + p_2x_2 = m- tx_1^*$$

Income vs. Quantity Tax

- Proposition: (x_1^*, x_2^*) is affordable under income tax
- Equivalent to: prove that (x_1^*, x_2^*) satisfies budget constraint under income tax.
- Or, budget constraint holds at point (x_1^*, x_2^*).

$$p_1x_1^* + p_2x_2^* = m - tx_1^*$$

- Which is true according to (5.2).
- It is not an optimal choice because prices are different.
- Conclusion: The optimal choice must be more preferred to (x_1^*, x_2^*)



x_2
Indifference curves
Optimal choice with income tax
Original choice
Budget constraint with income tax slope = $-p_1/p_2$
x_2^*
Optimal choice with quantity tax
x_1^*
Budget constraint with quantity tax slope = $-(p_1 + t)/p_2$
x_1

Estimating utility function - Choice based Method

Year	p_1	p_2	m	x_1	x_2	s_1	s_2	Utility
1	1	1	100	25	75	.25	.75	57.0
2	1	2	100	24	38	.24	.76	33.9
3	2	1	100	13	74	.26	.74	47.9
4	1	2	200	48	76	.24	.76	67.8
5	2	1	200	25	150	.25	.75	95.8
6	1	4	400	100	75	.25	.75	80.6
7	4	1	400	24	304	.24	.76	161.1

- $U(x_1, x_2) = x_1^{1/4}\ x_2^{3/4}$