Central University of Finance and Economics
School of Economics
Intermediate Microeconomics, Fall 2017
HW 1, Answer key

Question1. (6 points)

a) TRUE

$U=x_1+x_2$, good 1 and good 2 are perfect substitutes. If p_1 equal to p_2 or just below p_2, then a relatively small price increase of good 1 could make $p_1>p_2$, which would lead the consumer to stop consuming good 1

b) UNCERTAIN

Revenue is given by price multiplied by quantity: R=P*Q(P), where the amount sold is a function of price.

Take the derivative of revenues with respect to price:

$$\frac{dR}{dP} = Q + P\frac{dQ}{dP}$$

With the demand curve provided, we see that $\frac{dR}{dP} = 10 - 2P$ .This is greater than zero if P<5. So, here we are uncertain of the effect of changing price on revenues since it depends on the level of price.

(This can also be seen from the elasticity of demand: $\frac{dR}{dP} = Q(1 + \frac{P}{Q}\frac{dQ}{dP}) = Q(1-|\varepsilon|)$.

So, if demand is elastic, then revenue falls when the price is increased. The intuition for this is that with elastic demand, quantity demanded falls proportionally faster than price is increased. Hence revenue has to fall when price is increased.)

Question2. (12 points)

a) $\max_{x,y} \; u(x,y) = -\frac{1}{x} - \frac{1}{y}$

$s.t. \;\; p_x x + p_y y = m$

1) *Solution1: (use tangency condition)*

At the optimal point, we have:

①Tangency condition:

$$MRS = \frac{MU_x}{MU_y} = \frac{-\frac{1}{x^2}}{-\frac{1}{y^2}} = \frac{p_x}{p_y},$$

$$y = \sqrt{\frac{p_x}{p_y}}x$$

②Budget equation:

$$p_x x + p_y y = m$$

$$p_x x + p_y \sqrt{\frac{p_x}{p_y}} x = m$$

$$x(p_x, p_y, m) = \frac{m}{p_x + \sqrt{p_x p_y}}$$

Thus the change in quantity demanded is 22.25-11.31=10.94.

2) *Solution2: (use Lagrangian)*

$$L = -\frac{1}{x} - \frac{1}{y} - \lambda(p_x x + p_y y - m)$$

F.O.C.
$$\frac{\partial L}{\partial x} = \frac{1}{x^2} - \lambda p_x = 0$$
$$\frac{\partial L}{\partial y} = \frac{1}{y^2} - \lambda p_y = 0$$
$$\frac{\partial L}{\partial \lambda} = -(p_x x + p_y y - m) = 0$$

Solve the first two equation, we get: $y = \sqrt{\frac{p_x}{p_y}} x$.

Others are the same with the solution1.

b) $x(p_x, p_y, m) = \frac{m}{p_x + \sqrt{p_x p_y}}$

$$\varepsilon_d = \frac{p_x}{x} \frac{\partial x}{\partial p_x} = -\frac{p_x}{\frac{m}{p_x + \sqrt{p_x p_y}}} \bullet \frac{m}{(p_x + \sqrt{p_x p_y})^2} \bullet (1 + \frac{p_y}{2\sqrt{p_x p_y}})$$

Solve it, we get: $\varepsilon_d = -\frac{p_x}{p_x + \sqrt{p_x p_y}}(1 + \frac{p_y}{2\sqrt{p_x p_y}})$

c) $\varepsilon_I = \frac{m}{x} \frac{\partial x}{\partial m} = \frac{m}{\frac{m}{p_x + \sqrt{p_x p_y}}} \frac{1}{p_x + \sqrt{p_x p_y}} = 1$

d) $\frac{\partial x}{\partial m} = \frac{1}{p_x + \sqrt{p_x p_y}} > 0$, good x is a normal good.

Question3. (12 points)

We have the original consumption of x:

$$x(4,3,2,100) = 1 + 100(\frac{1}{16} + \frac{1}{40} + \frac{100}{400 \bullet 4^2}) \approx 11.31$$

And the final consumption of x:

$$x'(2,3,2,100) = 1 + 100(\frac{1}{8} + \frac{1}{40} + \frac{100}{400 \bullet 4}) = 22.25$$

Finding the income associated with the pivoted budget line:

$$\Delta m = x \bullet \Delta p_x = 11.31 \bullet 2 = 22.62$$

$$m' = 100 - 22.62 = 77.38$$

Therefore, $x^s(2,3,2,77.38) = 1 + 77.38(\frac{1}{8} + \frac{1}{40} + \frac{77.38}{400 \bullet 4}) \approx 16.35$

The substitution effect is 16.35-11.31=5.04

The income effect is 22.25-16.35=5.90

Question4. (20 points)

a) Dave's full income is 2000w+m

His budget constraint is C+wL=m+2000w

(Dave spends his income consuming C at the price of p_c=1.)

b) $\max_{C,L} \; U(C,L) = C^{\frac{1}{4}} L^{\frac{3}{4}}$

$s.t. \;\; C + wL = m + 2000w$

1) <u>Solution1: (use tangency condition)</u>

At the optimal point, we have:

①Tangency condition:

$$MRS = \frac{MU_L}{MU_C} = \frac{\frac{3}{4} C^{\frac{1}{4}} L^{-\frac{1}{4}}}{\frac{1}{4} C^{-\frac{3}{4}} L^{\frac{3}{4}}} = \frac{3C}{L} = \frac{w}{1},$$

3C=wL

②Budget equation:

$C + wL = m + 2000w,$

Substituting 3C=wL into it, we have:

$C + 3C = m + 2000w$

$C = 500w + \frac{m}{4}$

Here we can conclude that if w goes up, C goes up. If m goes up, C goes up, too.

To find L: $L = \frac{3C}{w} = \frac{1500w + \frac{3}{4}m}{w} = 1500 + \frac{3m}{4w}$

$\frac{\partial L}{\partial m} = \frac{3}{4w} > 0$, as m goes up, L goes up. Therefore, leisure is a normal good.

2) *Solution2: (use Lagrangian)*

$$L = C^{\frac{1}{4}} L^{\frac{3}{4}} - \lambda(C + wL - m - 2000w)$$

F.O.C. $\frac{\partial L}{\partial C} = \frac{1}{4} C^{-\frac{3}{4}} L^{\frac{3}{4}} - \lambda = 0$

$$\frac{\partial L}{\partial C} = \frac{3}{4} C^{\frac{1}{4}} L^{-\frac{1}{4}} - w\lambda = 0$$

$$\frac{\partial L}{\partial \lambda} = -(C + wL - m - 2000w) = 0$$

Solve the first two equation, we get: 3C=wL

Others are the same with the solution1.

3) *Solution3: (use the properties of Cobb-Douglas preference)*

$$C = \frac{1}{4}\frac{2000w + m}{1} = 500w + \frac{m}{4}$$

$$L = \frac{3}{4}\frac{2000w + m}{w} = 1500 + \frac{3m}{4w}$$

Others are the same with the solution1.

c) When w=10, m=100

C=500*10+100/4=5025

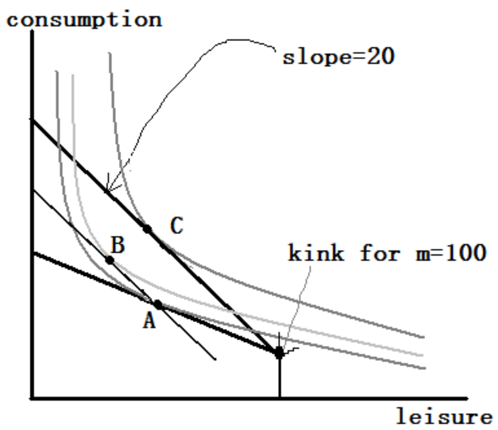
L=1500+3*100/(4*10)=1507.5

When w=20, m=100

C=500*20+100/4=10025

L=1500+3*100/(4*20)=1503.75

d) Income effect and substitution effect will have the opposite signs.



(Here we can calculate the quantity of leisure in B:

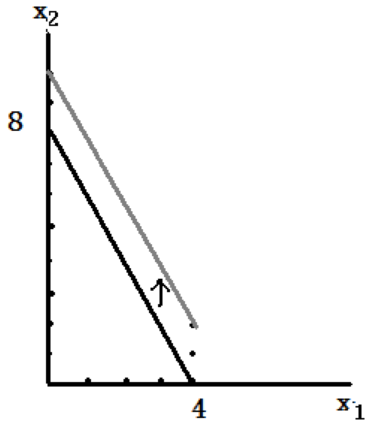
$$L_B = \frac{3}{4}\frac{5025 + 20 \times 1507.5}{20} = 1319.0625$$, which is less than the quantity of leisure in C.

Thus the sign of income effect is positive and different of the sign of substitution effect.)

Question5. (25 points)

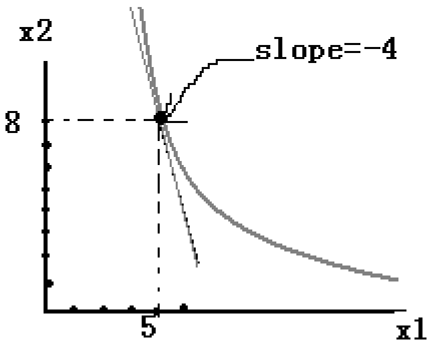
a) His budget line without the CD gifts is :$10x_1+5x_2=40$,

After receiving a gift of 2 CDs, his new budget line is $10x_1+5x_2=40+2*5=50$, and x_1 is NO more than 40/10=4.



b) $$MRS = \frac{MU_{x_1}}{MU_{x_2}} = \frac{1}{\frac{2}{x_2}} = \frac{x_2}{2}$$

- At the consumption bundle (5,8), the value of MRS is 4.
- Peter has to be compensated 0.0001*4=0.0004 to make him indifferent.



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c) There are two necessary conditions for the *optimal* choice:

- The Tangency condition: $MRS = \frac{p_1}{p_2}$.

It means that the rate of exchange at which the consumer is willing to stay put (MRS) must be equal to the price ratio.

- The budget constraint: $p_1x_1 + p_2x_2 = m$

It means that all the money should be exhausted by consuming both of the goods.

d) $MRS = \dfrac{MU_{x_1}}{MU_{x_2}} = \dfrac{1}{\frac{2}{x_2}} = \dfrac{x_2}{2}$

- when $p_1=10$, $p_2=5$, $m=40$

$MRS=x_2/2=10/5=2$, $x_2=4$.

$x_1=(40-4*5)/10=2$.

Hence: $x_1=2$ and $x_2=2$.

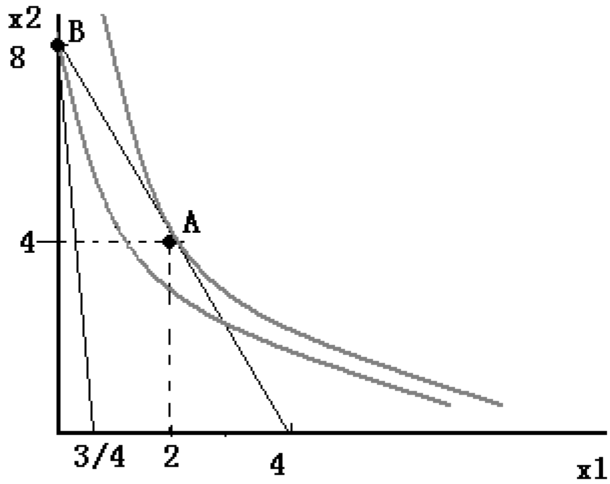
- when $p_1=30$, $p_2=5$, $m=40$

$MRS=x_2/2=30/5=6$, $x_2=12$.

$x_1=(40-12*5)/30= -2/3<0$

Hence: $x_1=0$ and $x_2=m/p_2=40/5=8$

While the first bundle $(x_1,x_2)=(2,4)$ is interior, the second bundle $(x_1,x_2)=(0,8)$ is at the corner (boundary).



e) $\dfrac{MU_1}{p_1} = \dfrac{1}{p_1}$, $\dfrac{MU_2}{p_2} = \dfrac{2}{x_2 p_2}$

f) when $p_1=10$, $p_2=5$, $m=40$, $(x_1,x_2)=(2,4)$

$$\frac{MU_1}{p_1} = \frac{1}{p_1} = \frac{1}{10}, \quad \frac{MU_2}{p_2} = \frac{2}{x_2 p_2} = \frac{2}{4\times 5} = \frac{1}{10}$$

It's equal.

g) when $p_1=30$, $p_2=5$, $m=40$, $(x_1,x_2)=(0,8)$

$$\frac{MU_1}{p_1} = \frac{1}{p_1} = \frac{1}{30}, \quad \frac{MU_2}{p_2} = \frac{2}{x_2 p_2} = \frac{2}{8\times 5} = \frac{1}{20}$$

It's not equal.

For an interior solution, the marginal utilities from a dollar for each of the two goods MUST be the same.

Question6. (10 points)

a) If the consumer saves in period 1:

$$C_2 = m_2 + (1+r_s)(m_1 - C_1)$$

C_2=60+1.1(20-C_1), if $C_1 \leqslant 20$

If the consumer borrows in period 1:

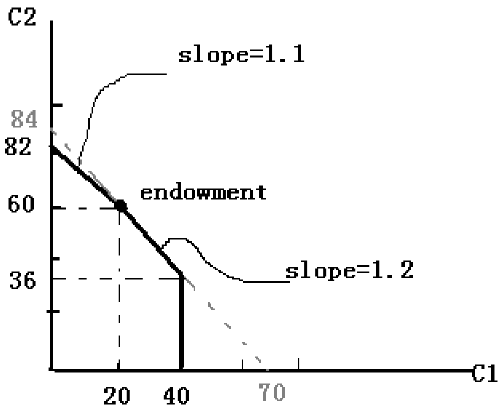
$$C_2 = m_2 - (1+r_b)(C_1 - m_1)$$

C_2=60-1.2(C_1-20), if $20 \leqslant C_1 \leqslant 40$

Thus the consumers' budget constraint is:

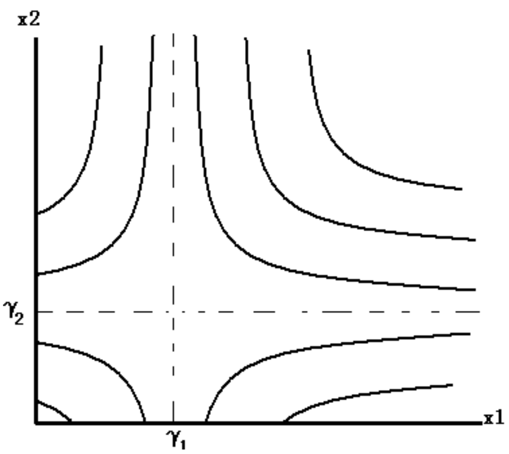
$$C_2 = \begin{cases} 60 + 1.1(20 - C_1), & if \quad C_1 \le 20 \\ 60 + 1.2(20 - C_1) & if \quad 20 \le C_1 \le 40 \end{cases}$$

b)



Question7. (15 points)

a)



b) $U(x_1,x_2)=(x_1-\gamma_1)(x_2-\gamma_2)$

A. <u>Solution1: (use tangency condition)</u>

From $$MRS = \frac{MU_{x_1}}{MU_{x_2}} = \frac{x_2 - \gamma_2}{x_1 - \gamma_1} = \frac{p_1}{p_2}$$

We can get: $p_2 x_2 - p_2\gamma_2 = p_1 x_1 - \gamma_1 p_1$.

Substituting it into budget equation $p_1 x_1 + p_2 x_2 = m$:

$p_1x_1 + (p_1 x_1 - \gamma_1 p_1 + p_2\gamma_2) = m$

$$x_1(p_1, p_2, m) = \frac{m + \gamma_1 p_1 - \gamma_2 p_2}{2p_1}$$

B. <u>Solution2: (use Lagrangian)</u>

$$L = (x_1 - \gamma_1)(x_2 - \gamma_2) - \lambda(p_1 x_1 + p_2 x_2 - m)$$

F.O.C:

$$\begin{cases} \dfrac{\partial L}{\partial x_1} = (x_2 - \gamma_2) - \lambda p_1 = 0 \\ \dfrac{\partial L}{\partial x_2} = (x_1 - \gamma_1) - \lambda p_2 = 0 \\ \dfrac{\partial L}{\partial \lambda} = -(p_1 x_1 + p_2 x_2 - m) = 0 \end{cases}$$

From the first two equation we can get:

$p_2 x_2 - p_2\gamma_2 = p_1 x_1 - \gamma_1 p_1$.

Others are the same with solution 1.

c) Good x_1 is a gross complement to good x_2 because $\dfrac{\partial x_1}{\partial p_2} = -\dfrac{\gamma_2}{2p_1} < 0$.