Central University of Finance and Economics
School of Economics
Intermediate Microeconomics, Spring 2017
Homework 2, Answer key

Question 1. (15 points)

a) To find the marginal revenue, we first have to "invert" the demand function to the inverse demand function as $P = 100 - Q$. The total revenue is then $TR(Q) = P \times Q = 100Q - Q^2$.

MR: $MR(Q) = 100 - 2Q$

Marginal cost is the derivative of TC with respect to Q

MC: $MC(Q) = Q + 10$

AC: $AC(Q) = \frac{Q}{2} + \frac{20}{Q} + 10$

b) For perfect competition, we have $P(Q) = MC(Q)$. We get Q = 45 and P = 55.

Profit is then defined as $\pi(Q) = TR(Q) - TC(Q) = 100Q - Q^2 - \frac{Q^2}{2} - 10Q - 20 = 992.5$

c) For monopolist, we have $MR(Q) = MC(Q)$. We get Q = 30 and P = 70.

Profit is then $\pi(Q) = 100Q - Q^2 - \frac{Q^2}{2} - 10Q - 20 = 1330$

Question 2. (15 points)

a)

$$\min_{K,L} \quad 4K + 36L$$

$$s.t. \quad Y = K^{\frac{1}{2}} L^{\frac{1}{2}}$$

In order to calculate the cost minimizing input, we need

$$TRS = -\frac{MP_K}{MP_L} = \frac{r}{w}$$

We get K = 9L.

Now we input "K = 9L" into the production function $Y = K^{\frac{1}{2}} L^{\frac{1}{2}}$

$$Y = 3L^{\frac{1}{2}} L^{\frac{1}{2}} = 3L$$

If Y = 300, we get $\begin{cases} K^* = 900 \\ L^* = 100 \end{cases}$.

The cost minimizing bundle is $(K^*, L^*) = (900, 100)$.

The total cost of production is : $C = 4 \times 900 + 36 \times 100 = 7200$.

The average cost of production is: $AC = \frac{C}{Y} = \frac{7200}{300} = 24$

b) The short run production function is:

$$Y = (900)^{\frac{1}{2}} L^{\frac{1}{2}} \Rightarrow Y = 30L^{\frac{1}{2}}$$

This exhibits decreasing returns to scale.

c) The amount of labor needed to produce at minimum cost will be solved using the equation:

$$Y = 30L^{\frac{1}{2}} \rightarrow 450 = 30L^{\frac{1}{2}} \Rightarrow L^* = 225$$

The variable costs of production is VC = 225 × 36 = 8100.

The total cost can be calculated by adding the variable cost with the fixed cost:

FC = 900 × 4 = 3600   VC = 8100.

TC = FC + VC = 8100 + 3600=11700

The average cost is: $\frac{TC}{Y} = \frac{11700}{450} = 26$

Question 3. (20 points)

a) Total supply is $S(p) = S_C(p) + S_{US}(p) = p$

b) From supply equal to demand ($D(p) = S(p)$) we find: $12 - 2p = p$. So $\begin{cases} p^* = 4 \\ q^* = 4 \end{cases}$.

c)

$$CS = \frac{(6-4)\times 4}{2} = 4$$

Total $PS = \frac{4\times 4}{2} = 8$. $PS_C = \frac{4\times 2}{2} = 4$ and $PS_{US} = \frac{4\times 2}{2} = 4$.

$$TS = CS + PS_C + PS_{US} = 12$$

No deadweight loss.

d) From supply equal to demand we find: $12 - 2p = \frac{1}{2}p$. So $\begin{cases} p' = 4.8 \\ q' = 2.4 \end{cases}$.

e)

$$CS = \frac{(6-4.8)\times 2.4}{2} = 1.44$$

Total $PS = PS_{US} = \frac{4.8\times 2.4}{2} = 5.76$ $PS_C = 0$

$$TS = CS + PS_C + PS_{US} = 7.2$$

Deadweight loss = $12 - 7.2 = 4.8$

The change in US surplus is $\Delta TS_{US} = 7.2 - 8 = -0.8$.

Therefore the ban increases the surplus of US producer at expense of both US consumers and Canadian producers. In addition note that the total US surplus is reduced because of the ban. US consumers loose more than US producers win with the ban. Inefficient, even if you don’t consider the welfare of Canadians.

Question 4. (20 points)

a) The monopolist problem is maximizing profit, which is total revenue minus total cost.

$$\max_q \quad aq - bq^2 - cq$$

The first order condition is $a - 2bq - c = 0$

$$q = \frac{a-c}{2b}$$

b) The problem is now

$$\max_q \quad (1-t_p)[aq - bq^2 - cq]$$

because $(1-t_p)$ doesn't depend on q at all. The fist order condition is then

$$(1-t_p)(a-2bq-c) = 0$$

because $(1-t_p) \neq 0$, which means $(a-2bq-c)=0$, and therefore the output level is the same as a)

c) The problem is now

$$\max_q \quad [aq - bq^2] - (c+t_q)q$$

The first order condition is

$$a - 2bq - c - t_q = 0$$

$$q = \frac{a-c-t_q}{2b}$$

d) The problem is now

$$\max_q \quad (1-t_r)[aq - bq^2] - cq$$

The first order condition is

$$(1-t_r)(a-2bq) - c = 0$$

$$q = \frac{1}{2b}\left[a - \frac{c}{1-t_r}\right]$$

e) The equivalence condition that we are seeking requires

$$\frac{a-c-t_q}{2b} = \frac{1}{2b}\left[a - \frac{c}{1-t_r}\right]$$

$$a - c - t_q = a - \frac{c}{1-t_r}$$

$$c + t_q = \frac{c}{1 - t_r}$$

Question 5. (30 points)

a) α_L is the amount of input L required for each unit of output. α_K is the amount of input K required to for each unit of output.

b)

$$f(\lambda L, \lambda K) = \min\left\{\frac{\lambda L}{\alpha_L}, \frac{\lambda K}{\alpha_K}\right\} = \lambda \min\left\{\frac{L}{\alpha_L}, \frac{K}{\alpha_K}\right\} = \lambda f(L, K)$$

Therefore, this production function exhibits Constant Returns to Scale (CRS). Intuitively, if for each unit of output we need α_L units of L and α_K units of K, then if we double the inputs, the resulting output exactly doubles.

c) Long-run cost curves

Since every unit of output requires α_L units of L and α_K units of K, the cost of every unit is $w\alpha_L + r\alpha_K$ and the cost of y units is

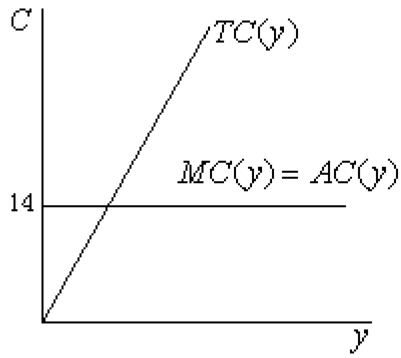
$$TC(y) = y(w\alpha_L + r\alpha_K) = y(\alpha_L + 5\alpha_K)$$

$$\Rightarrow AC(y) = \frac{TC(y)}{y} = \alpha_L + 5\alpha_K$$

$$MC(y) = TC'(y) = \alpha_L + 5\alpha_K$$

It is a general feature of the constant returns to scale technology that the average and the marginal long-run cost curves are the same.

d) $TC(y) = \ 14y \quad AC(y) = \ MC(y) = \ 14$



e) The firm cannot produce more than $\frac{K}{\alpha_K} = \frac{10}{2} = 5$. The total cost composed of the fixed cost (for the capital) and the variable cost (for the labor). Thus, the short-run curves are:

$$TC(y) = rK + ywL = 5 \cdot 10 + 1 \cdot y \cdot 4 = 50 + 4y \quad (0 \le y \le 5)$$

$$AC(y) = \frac{50+4y}{y} = \frac{50}{y} + 4 \quad (0 \leq y \leq \ 5$$

$$MC(y) = TC'(y) = 4 \quad (0 \leq y \leq 5)$$

f)