

Chapter Seven

Revealed Preference

显示性偏好

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Revealed Preference Analysis

- Suppose we observe the demands (consumption choices) that a consumer makes for different budgets. This reveals information about the consumer's preferences. We can use this information to ...

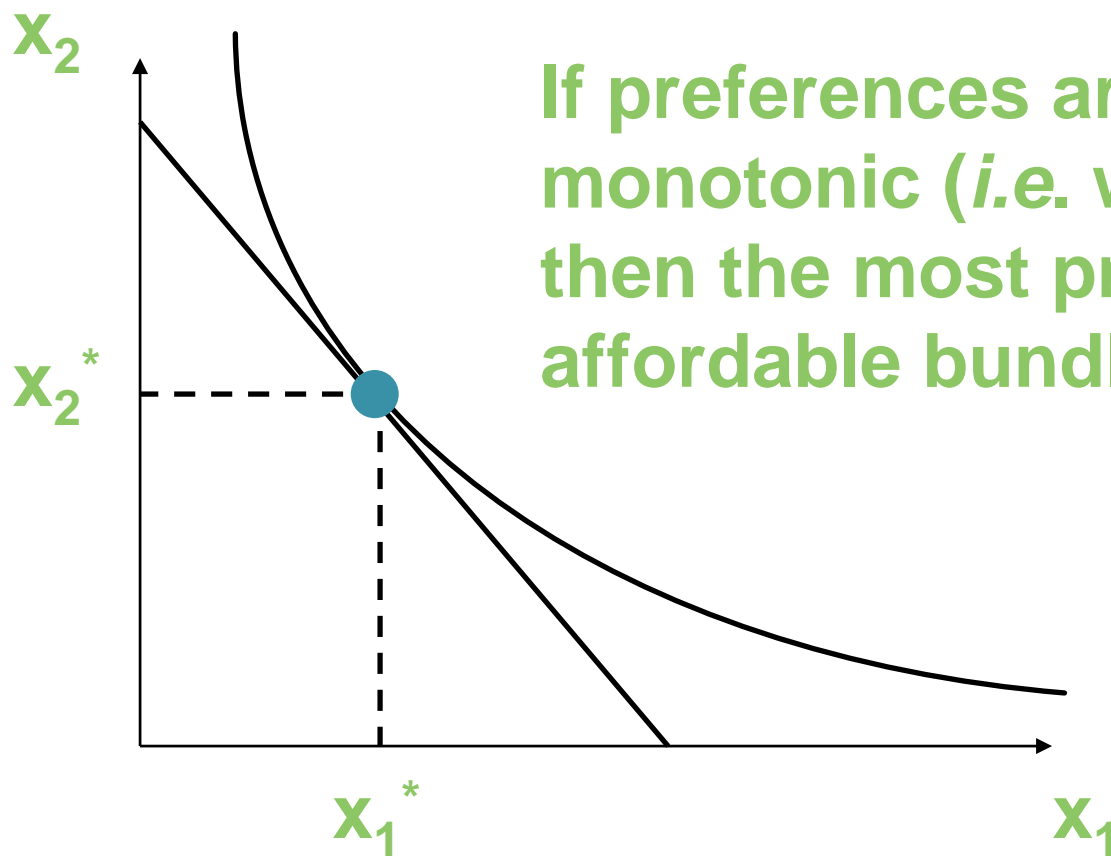
Revealed Preference Analysis

- Test the behavioral hypothesis that a consumer chooses the most preferred bundle from those available.
- Discover the consumer's preference relation.

Assumptions on Preferences

- Preferences
 - do not change while the choice data are gathered.
 - are strictly convex.
 - are monotonic.
- Together, convexity and monotonicity imply that the most preferred affordable bundle is unique.

Assumptions on Preferences

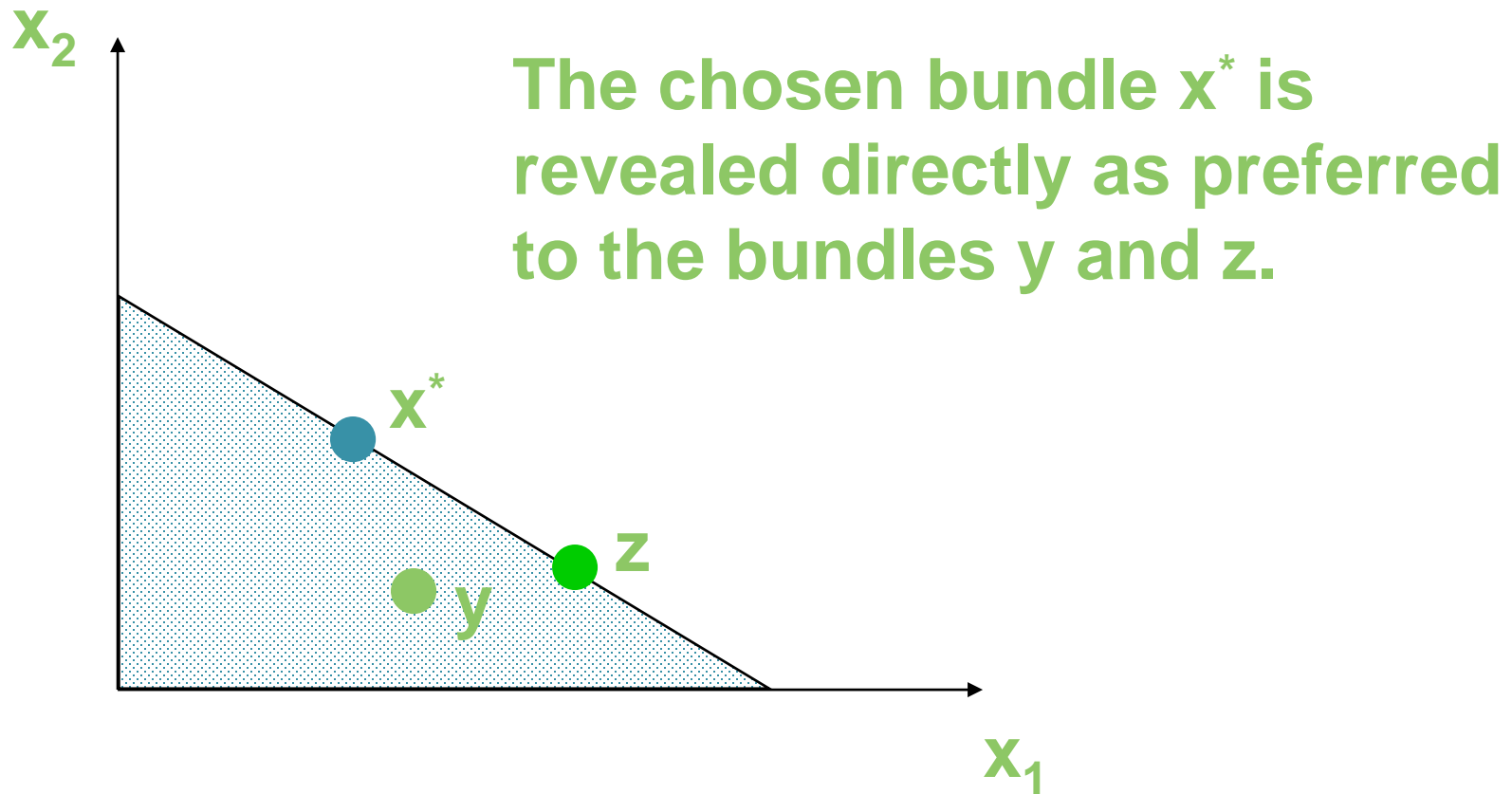


If preferences are convex and monotonic (*i.e.* well-behaved) then the most preferred affordable bundle is unique.

Direct Preference Revelation

- Suppose that the bundle x^* is chosen when the bundle y is affordable. Then x^* is **revealed directly** as preferred to y (otherwise y would have been chosen).

Direct Preference Revelation



Direct Preference Revelation

- That x is revealed directly as preferred to y will be written as

$$x \succ_D y.$$

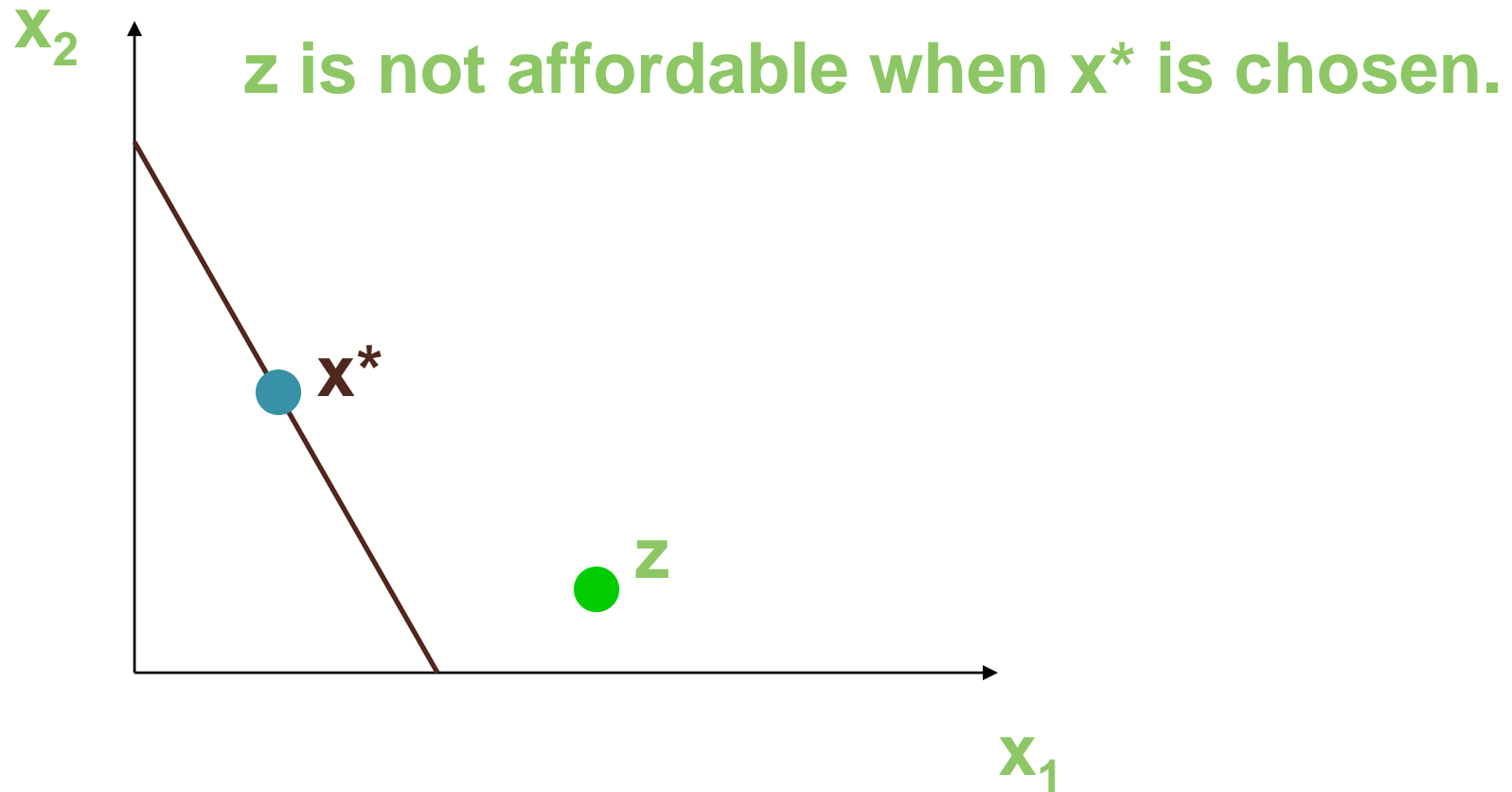
Indirect Preference Revelation

- Suppose x is revealed directly preferred to y , and y is revealed directly preferred to z . Then, by transitivity, x is **revealed indirectly** as preferred to z . Write this as

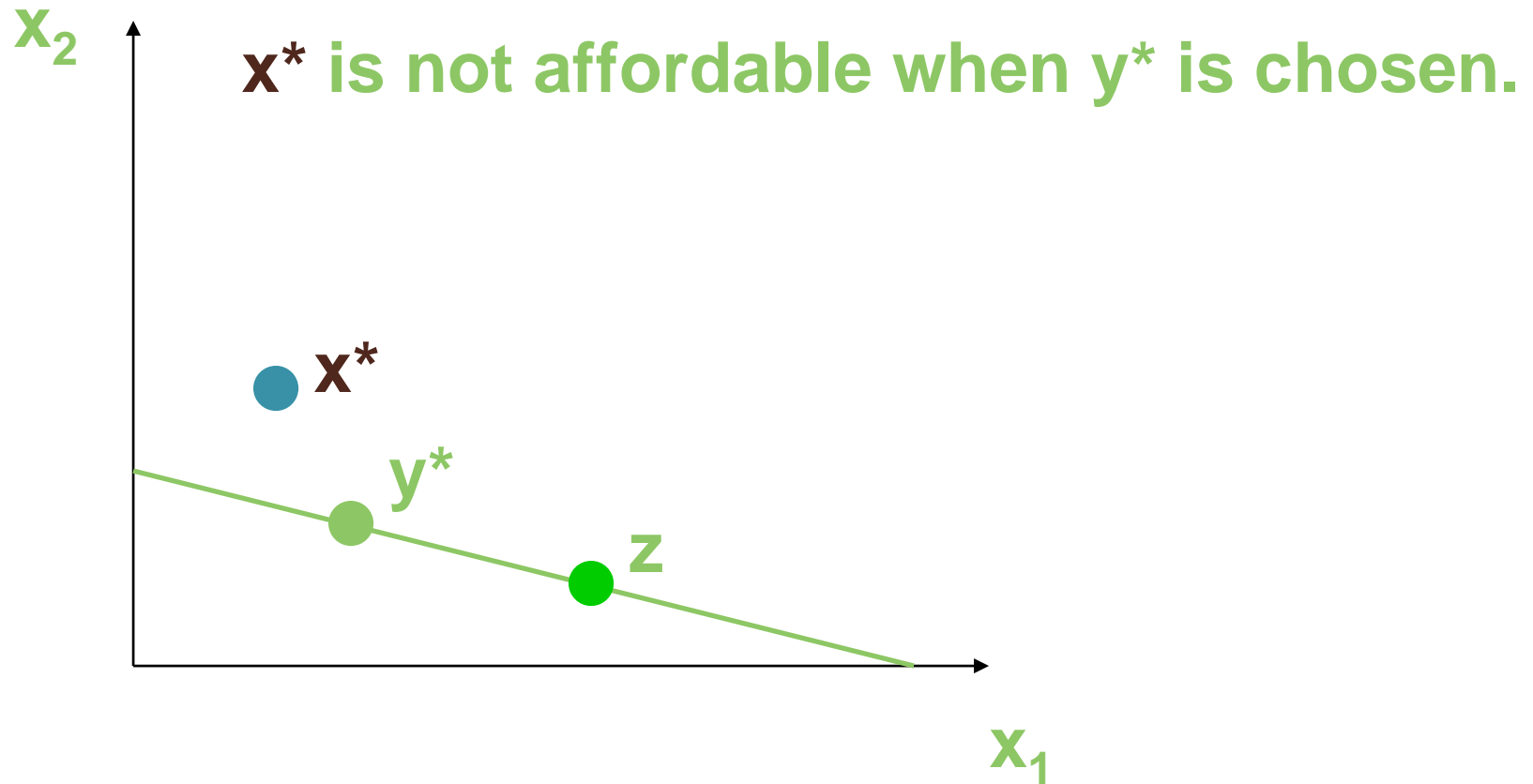
$$x \succ_I z$$

$$\text{so } x \succ_D y \text{ and } y \succ_D z \implies x \succ_I z.$$

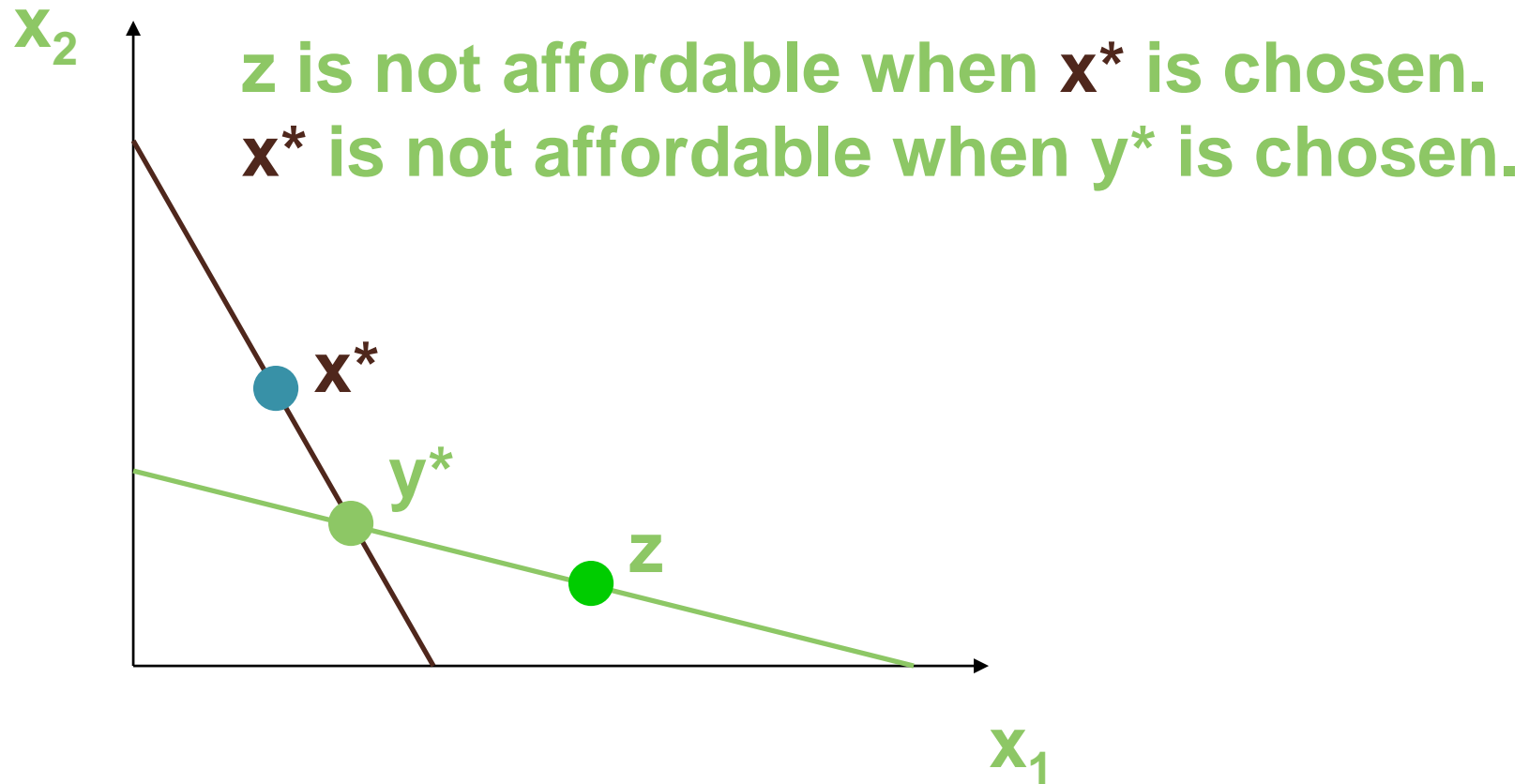
Indirect Preference Revelation



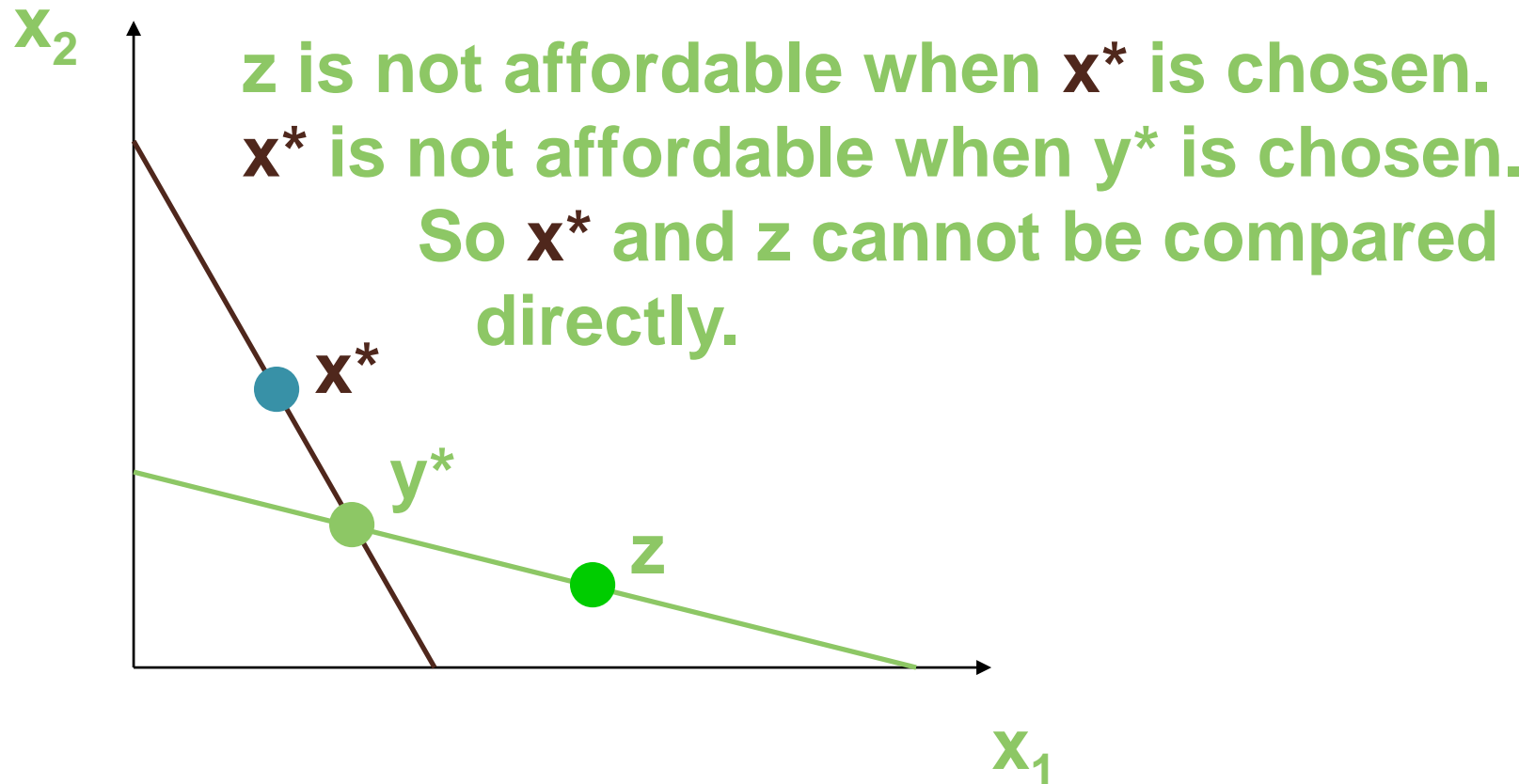
Indirect Preference Revelation



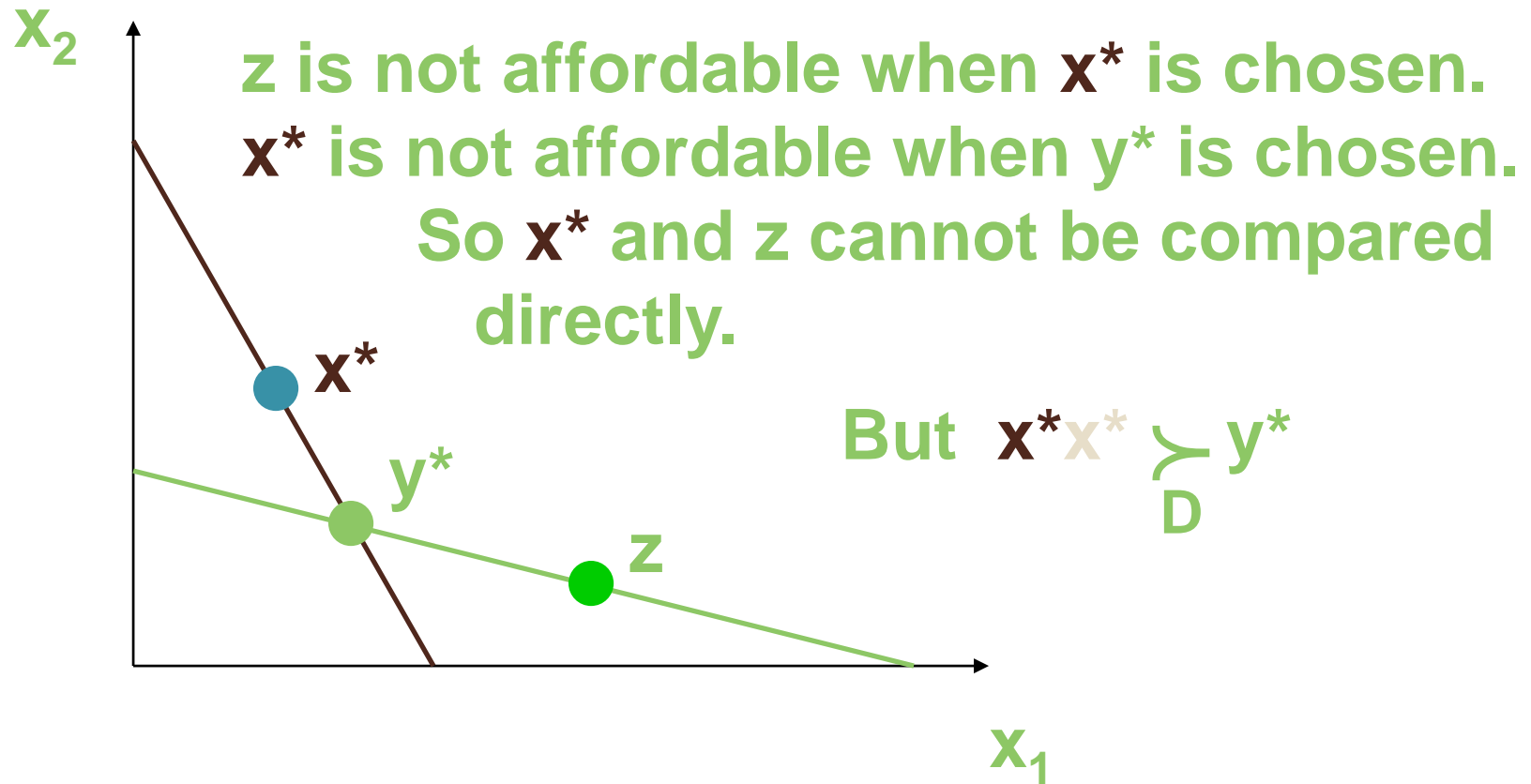
Indirect Preference Revelation



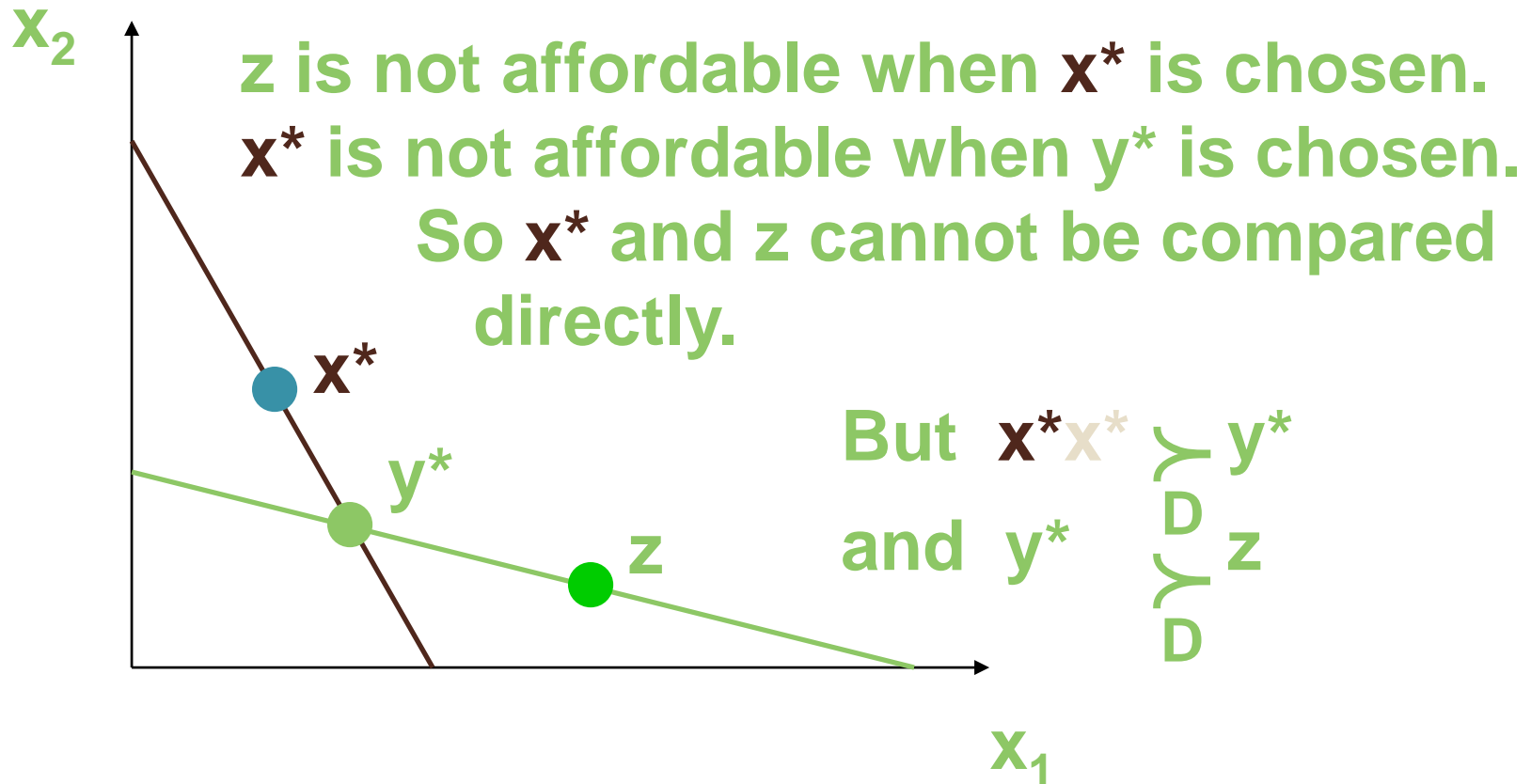
Indirect Preference Revelation



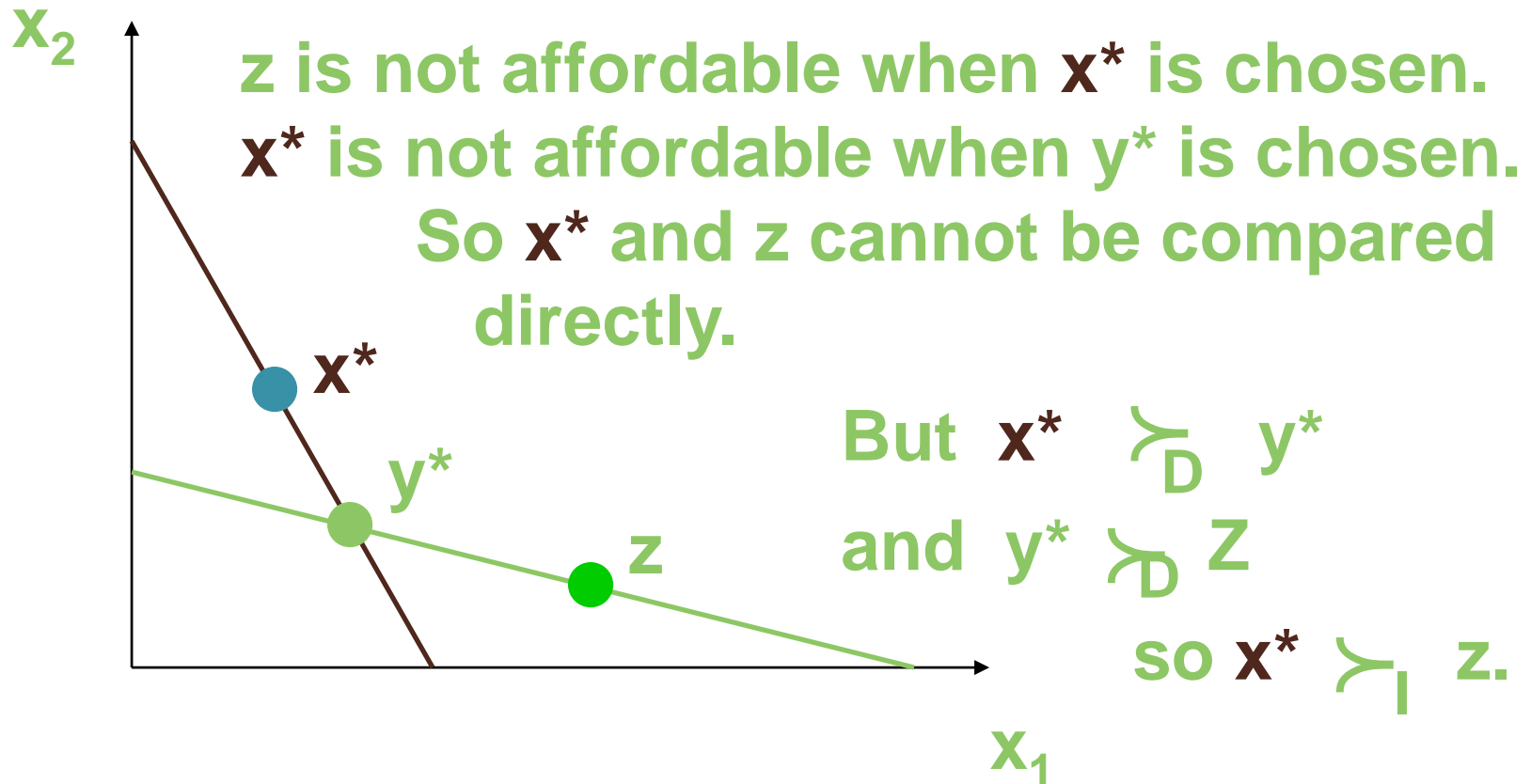
Indirect Preference Revelation



Indirect Preference Revelation



Indirect Preference Revelation



Two Axioms of Revealed Preference

- To apply revealed preference analysis, choices must satisfy two criteria -- the Weak and the Strong Axioms of Revealed Preference.

The Weak Axiom of Revealed Preference (WARP)

- If the bundle x is revealed directly as preferred to the bundle y then it is never the case that y is revealed directly as preferred to x ; *i.e.*

$$x \succ_D y \Rightarrow \text{not } (y \succ_D x).$$

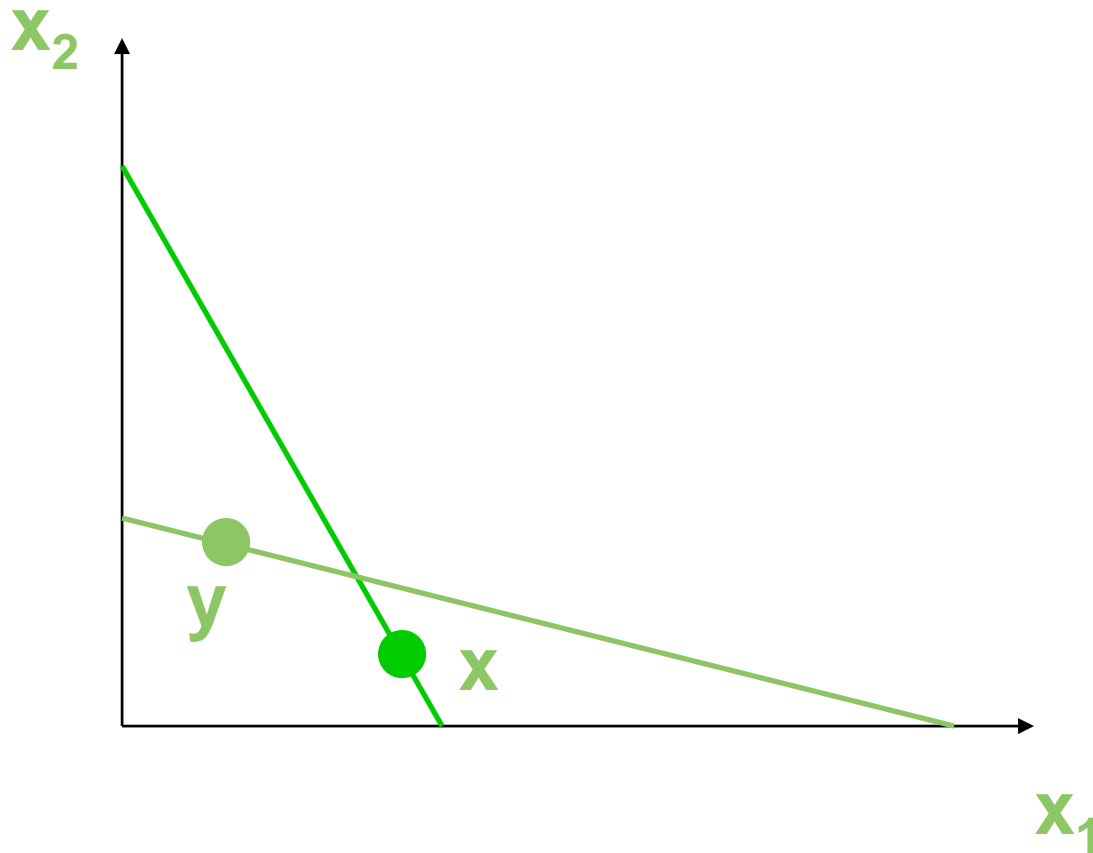
The Weak Axiom of Revealed Preference (WARP)

- Choice data which violate the WARP are inconsistent with economic rationality.
- The WARP is a **necessary** condition for applying economic rationality to explain observed choices.

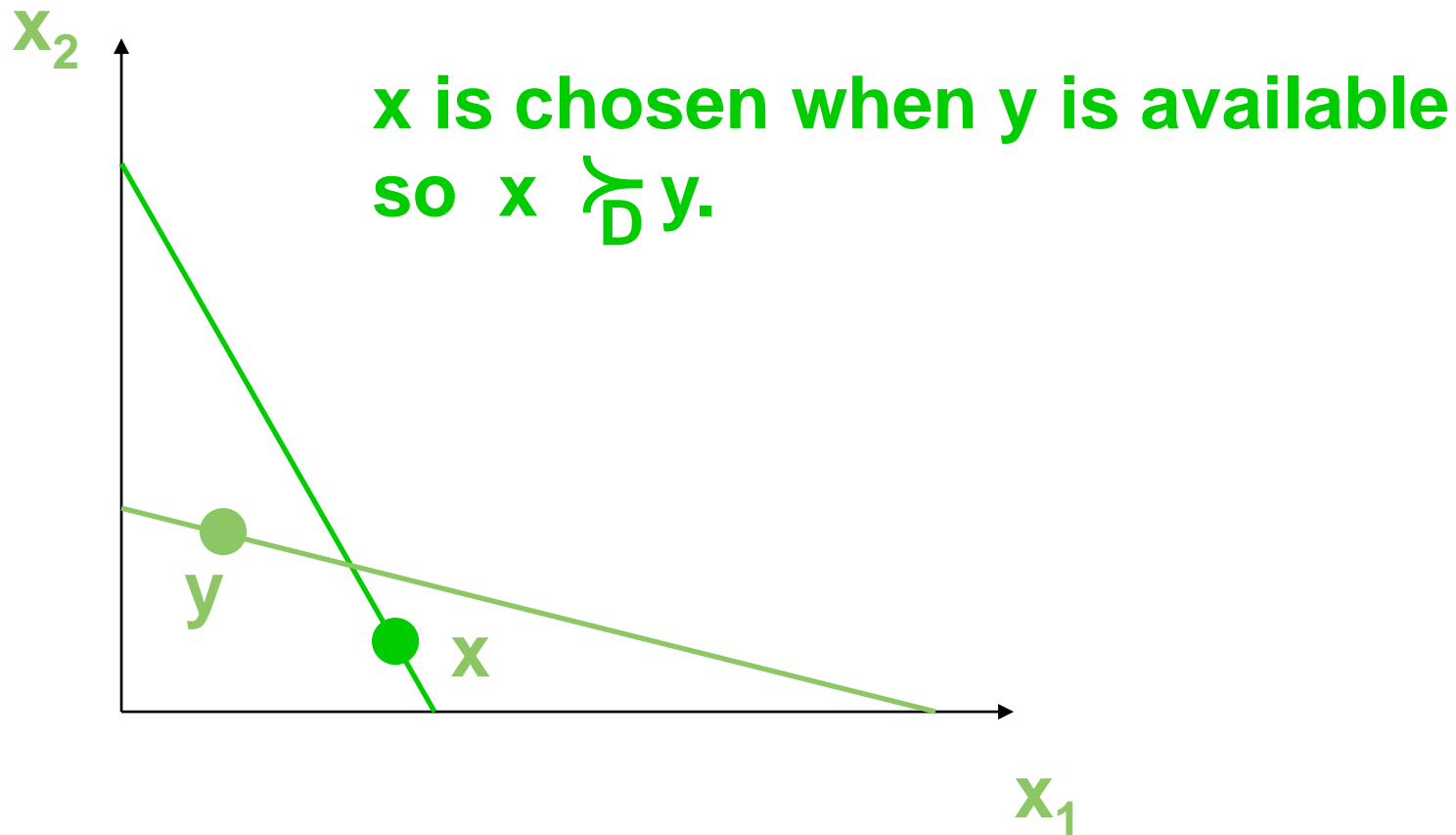
The Weak Axiom of Revealed Preference (WARP)

- What choice data violate the WARP?

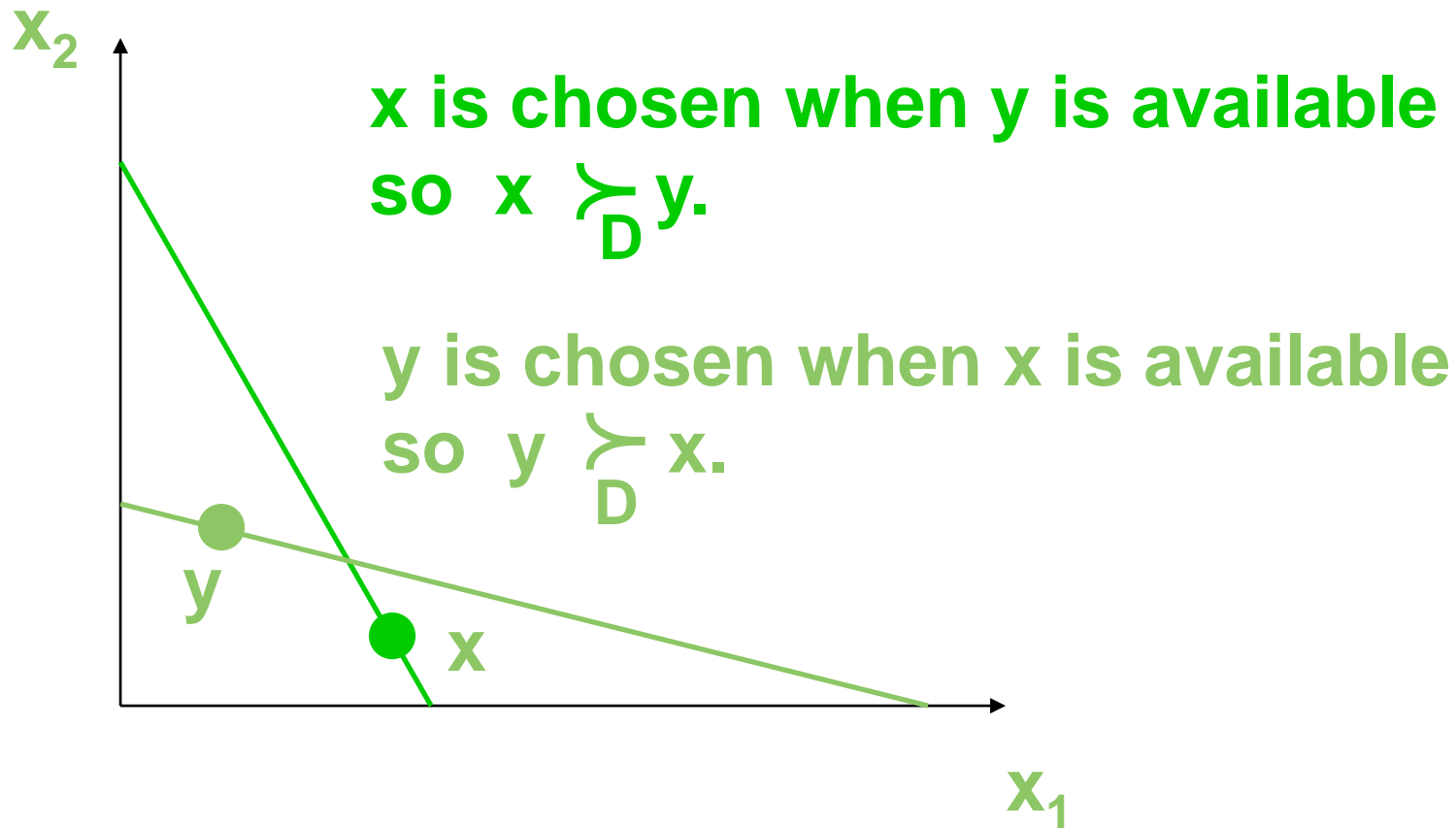
The Weak Axiom of Revealed Preference (WARP)



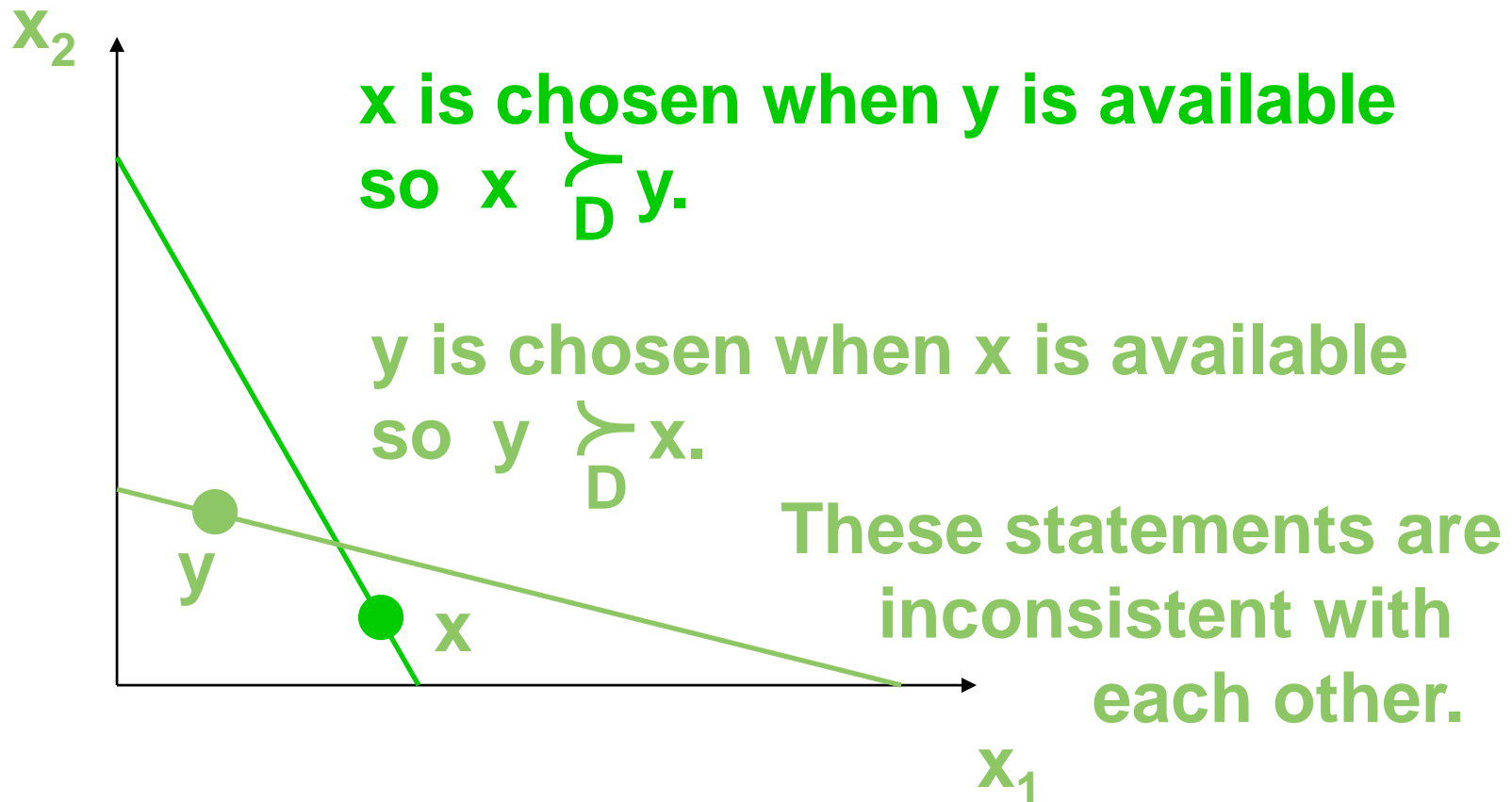
The Weak Axiom of Revealed Preference (WARP)



The Weak Axiom of Revealed Preference (WARP)



The Weak Axiom of Revealed Preference (WARP)



Checking if Data Violate the WARP

- A consumer makes the following choices:
 - At prices $(p_1, p_2) = (\$2, \$2)$ the choice was $(x_1, x_2) = (10, 1)$.
 - At $(p_1, p_2) = (\$2, \$1)$ the choice was $(x_1, x_2) = (5, 5)$.
 - At $(p_1, p_2) = (\$1, \$2)$ the choice was $(x_1, x_2) = (5, 4)$.
- Is the WARP violated by these data?

Checking if Data Violate the WARP

Choices Prices	(10, 1)	(5, 5)	(5, 4)
(2, 2)	\$22	\$20	\$18
(2, 1)	\$21	\$15	\$14
(1, 2)	\$12	\$15	\$13

Checking if Data Violate the WARP

Choices Prices	(10, 1)	(5, 5)	(5, 4)
(\$2, \$2)	\$22	\$20	\$18
(\$2, \$1)	\$21	\$15	\$14
(\$1, \$2)	\$12	\$15	\$13

Red numbers are costs of chosen bundles.

Checking if Data Violate the WARP

Choices Prices	(10, 1)	(5, 5)	(5, 4)
(\$2, \$2)	\$22	\$20	\$18
(\$2, \$1)	\$21	\$15	\$14
(\$1, \$2)	\$12	\$15	\$13

Circles surround affordable bundles that were not chosen.

Checking if Data Violate the WARP

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	(10, 1)	(5, 5)	(5, 4)
(10, 1)		D	D
(5, 5)			D
(5, 4)	D		

Checking if Data Violate the WARP

Choices Prices	(10, 1)	(5, 5)	(5, 4)
(\$2, \$2)	\$22	\$20	\$18
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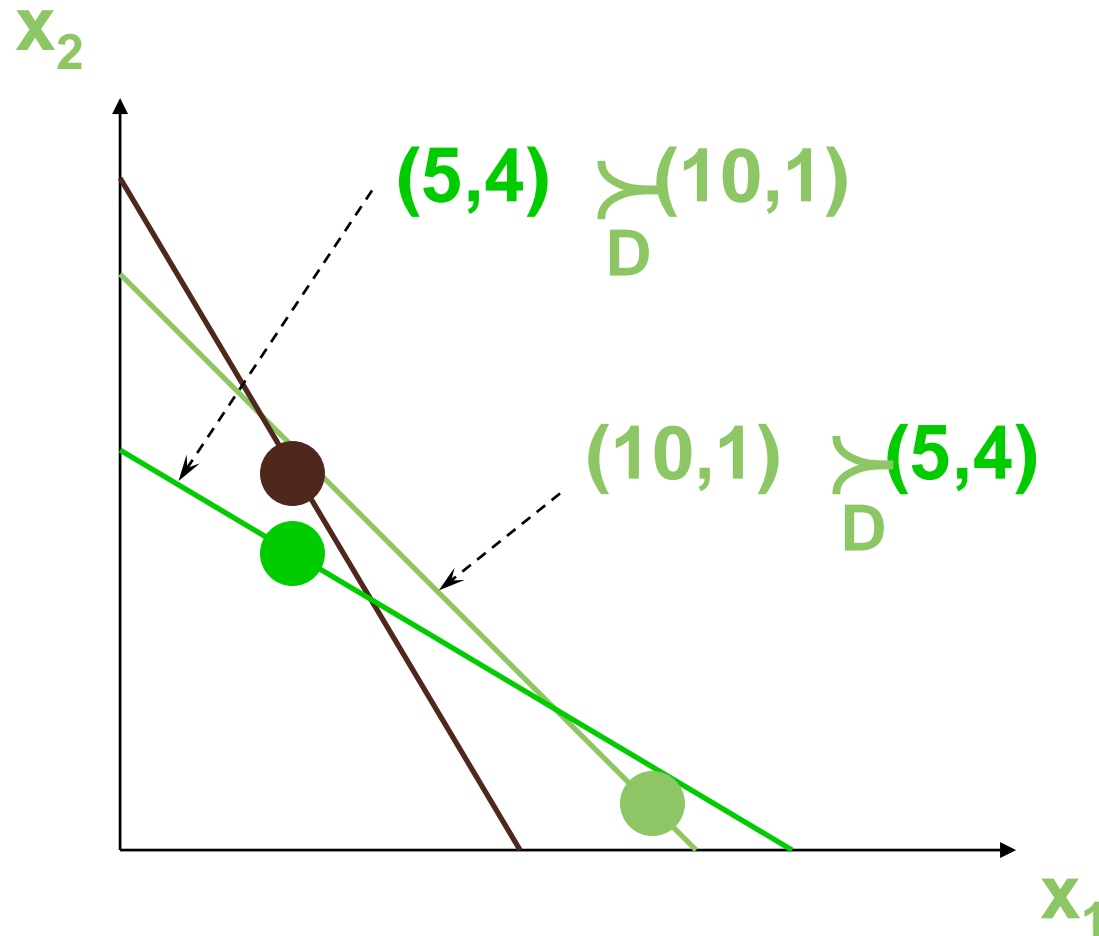
	(10, 1)	(5, 5)	(5, 4)
(10, 1)		D	D
(5, 5)			D
(5, 4)			

Checking if Data Violate the WARP

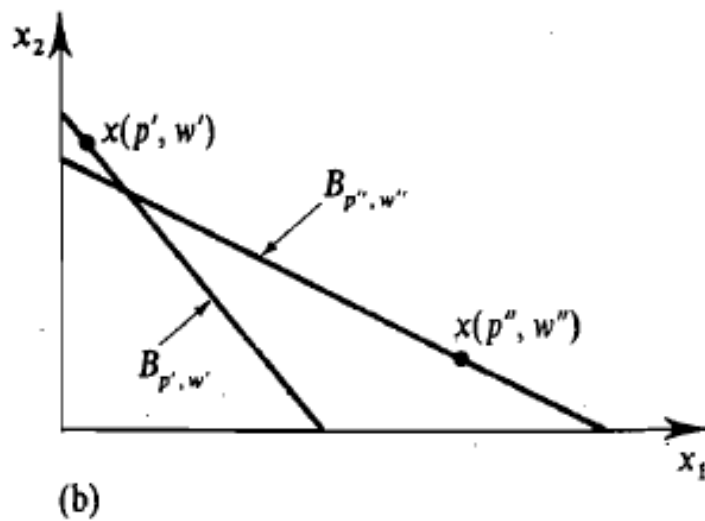
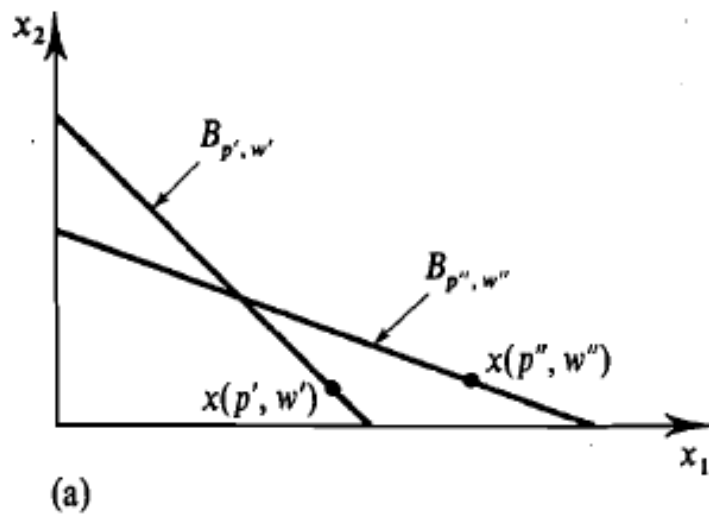
$(10,1)$ is directly revealed preferred to $(5,4)$, but $(5,4)$ is directly revealed preferred to $(10,1)$, so the WARP is violated by the data.

	$(10, 1)$	$(5, 5)$	$(5, 4)$
$(10, 1)$		D	D
$(5, 5)$			D
$(5, 4)$			

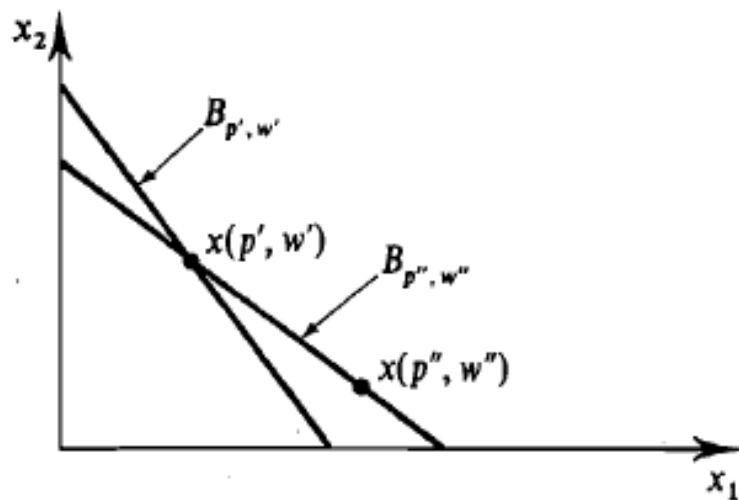
Checking if Data Violate the WARP



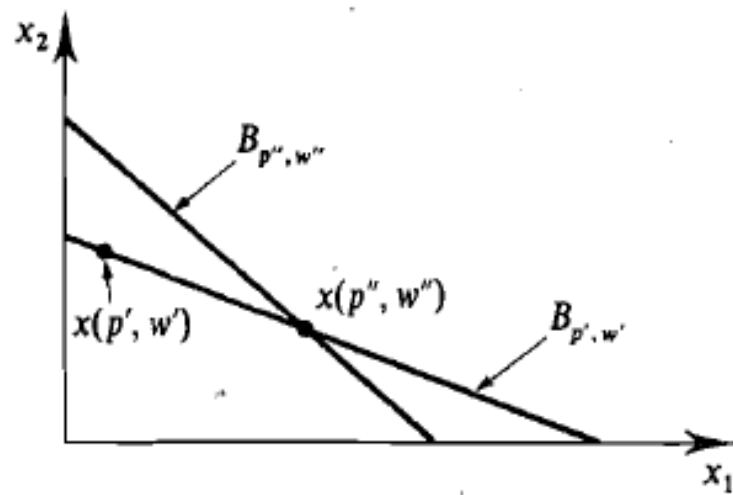
WA



WA



(c)



(d)

The Strong Axiom of Revealed Preference (SARP)

- If the bundle x is revealed (directly or indirectly) as preferred to the bundle y and $x \neq y$, then it is never the case that the y is revealed (directly or indirectly) as preferred to x ; i.e.

$$x \succ_D y \text{ or } x \succ_I y$$

$$\Rightarrow \text{not } (y \succ_D x \text{ or } y \succ_I x).$$

The Strong Axiom of Revealed Preference

- What choice data would satisfy the WARP but violate the SARP?

The Strong Axiom of Revealed Preference

- Consider the following data:

$$A: (p_1, p_2, p_3) = (1, 3, 10) \text{ \& } (x_1, x_2, x_3) = (3, 1, 4)$$

$$B: (p_1, p_2, p_3) = (4, 3, 6) \text{ \& } (x_1, x_2, x_3) = (2, 5, 3)$$

$$C: (p_1, p_2, p_3) = (1, 1, 5) \text{ \& } (x_1, x_2, x_3) = (4, 4, 3)$$

The Strong Axiom of Revealed Preference

A: (\$1,\$3,\$10)
(3,1,4).

B: (\$4,\$3,\$6)
(2,5,3).

C: (\$1,\$1,\$5)
(4,4,3).

Choice Prices	A	B	C
A	\$46	\$47	\$46
B	\$39	\$41	\$46
C	\$24	\$22	\$23

The Strong Axiom of Revealed Preference

Choices Prices	A	B	C
A	\$46	\$47	\$46
B	\$39	\$41	\$46
C	\$24	\$22	\$23

In situation A,
bundle A is
directly revealed
preferred to
bundle C;

$$A \succ_D C.$$

The Strong Axiom of Revealed Preference

Choices Prices	A	B	C
A	\$46	\$47	\$46
B	\$39	\$41	\$46
C	\$24	\$22	\$23

In situation B,
bundle B is
directly revealed
preferred to
bundle A;

$$B \succ_D A.$$

The Strong Axiom of Revealed Preference

Choices Prices	A	B	C
A	\$46	\$47	\$46
B	\$39	\$41	\$46
C	\$24	\$22	\$23

In situation C,
bundle C is
directly revealed
preferred to
bundle B;

$$C \succ_D B.$$

The Strong Axiom of Revealed Preference

Choices Prices	A	B	C
A	\$46	\$47	\$46
B	\$39	\$41	\$46
C	\$24	\$22	\$23

	A	B	C
A			D
B	D		
C		D	

The Strong Axiom of Revealed Preference

Choices Prices	A	B	C
A	\$46	\$47	\$46
B	\$39	\$41	\$46
C	\$24	\$22	\$23

	A	B	C
A			D
B	D		
C		D	

The data do not violate the WARP.

The Strong Axiom of Revealed Preference

We have that

$A \succsim_D C$, $B \succsim_D A$ and $C \succsim_D B$

so, by transitivity,

$A \succsim_I B$, $B \succsim_I C$ and $C \succsim_I A$.

	A	B	C
A			D
B	D		
C		D	

The data do not violate the WARP but ...

The Strong Axiom of Revealed Preference

We have that

$A \succsim_D C$, $B \succsim_D A$ and $C \succsim_D B$

so, by transitivity,

$A \succsim_I B$, $B \succsim_I C$ and $C \succsim_I A$.

	A	B	C
A		I	D
B	D		I
C	I	D	

The data do not violate the WARP but ...

The Strong Axiom of Revealed Preference

$B \succ_D A$ is inconsistent
with $A \succ_I B$.

	A	B	C
A		I	D
B	D		I
C	I	D	

The data do not violate the WARP but ...

The Strong Axiom of Revealed Preference

$A \succ_D C$ is inconsistent
 with $C \succ_I A$.

	A	B	C
A			
B			
C			

	A	B	C
A			
B			
C			

	A	B	C
A			
B			
C			

The data do not violate the WARP but ...

The Strong Axiom of Revealed Preference

$C \succ_D B$ is inconsistent
 with $B \succ_I C$.

	A	B	C
A		I	D
B	D		I
C	I	D	

The data do not violate the WARP but ...

The Strong Axiom of Revealed Preference

The data do not violate the WARP but there are 3 violations of the SARP.

	A	B	C
A		I	D
B	D		I
C	I	D	

The Strong Axiom of Revealed Preference

- That the observed choice data satisfy the SARP is a condition **necessary and sufficient** for there to be a well-behaved preference relation that “rationalizes” the data.
- So our 3 data cannot be rationalized by a well-behaved preference relation.

Recovering Indifference Curves

- Suppose we have the choice data satisfy the SARP.
- Then we can discover approximately where are the consumer's indifference curves.
- How?

Recovering Indifference Curves

- Suppose we observe:
A: $(p_1, p_2) = (\$1, \$1)$ & $(x_1, x_2) = (15, 15)$
B: $(p_1, p_2) = (\$2, \$1)$ & $(x_1, x_2) = (10, 20)$
C: $(p_1, p_2) = (\$1, \$2)$ & $(x_1, x_2) = (20, 10)$
D: $(p_1, p_2) = (\$2, \$5)$ & $(x_1, x_2) = (30, 12)$
E: $(p_1, p_2) = (\$5, \$2)$ & $(x_1, x_2) = (12, 30)$.
- Where lies the indifference curve containing the bundle $A = (15, 15)$?

Recovering Indifference Curves

- The table showing direct preference revelations is:

Recovering Indifference Curves

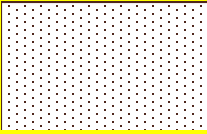
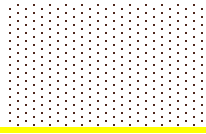
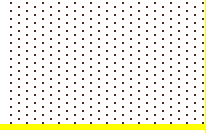
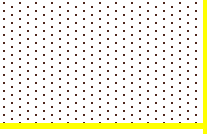
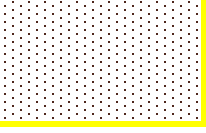
	A	B	C	D	E
A		D	D		
B					
C					
D	D	D	D		
E	D	D	D		

Direct revelations only; the WARP is not violated by the data.

Recovering Indifference Curves

- Indirect preference revelations add no extra information, so the table showing both direct and indirect preference revelations is the same as the table showing only the direct preference revelations:

Recovering Indifference Curves

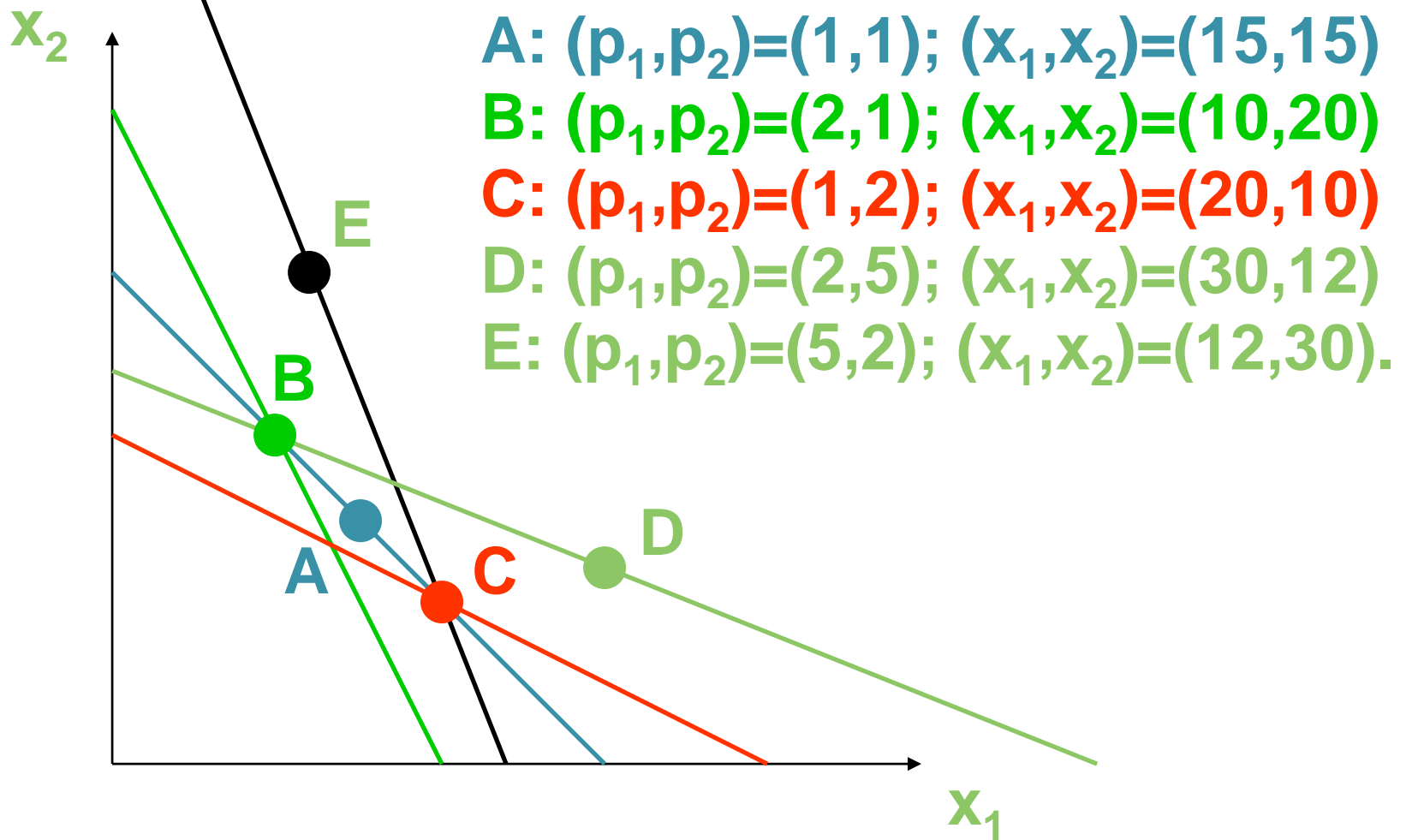
	A	B	C	D	E
A		D	D		
B					
C					
D	D	D	D		
E	D	D	D		

Both direct and indirect revelations; neither WARP nor SARP are violated by the data.

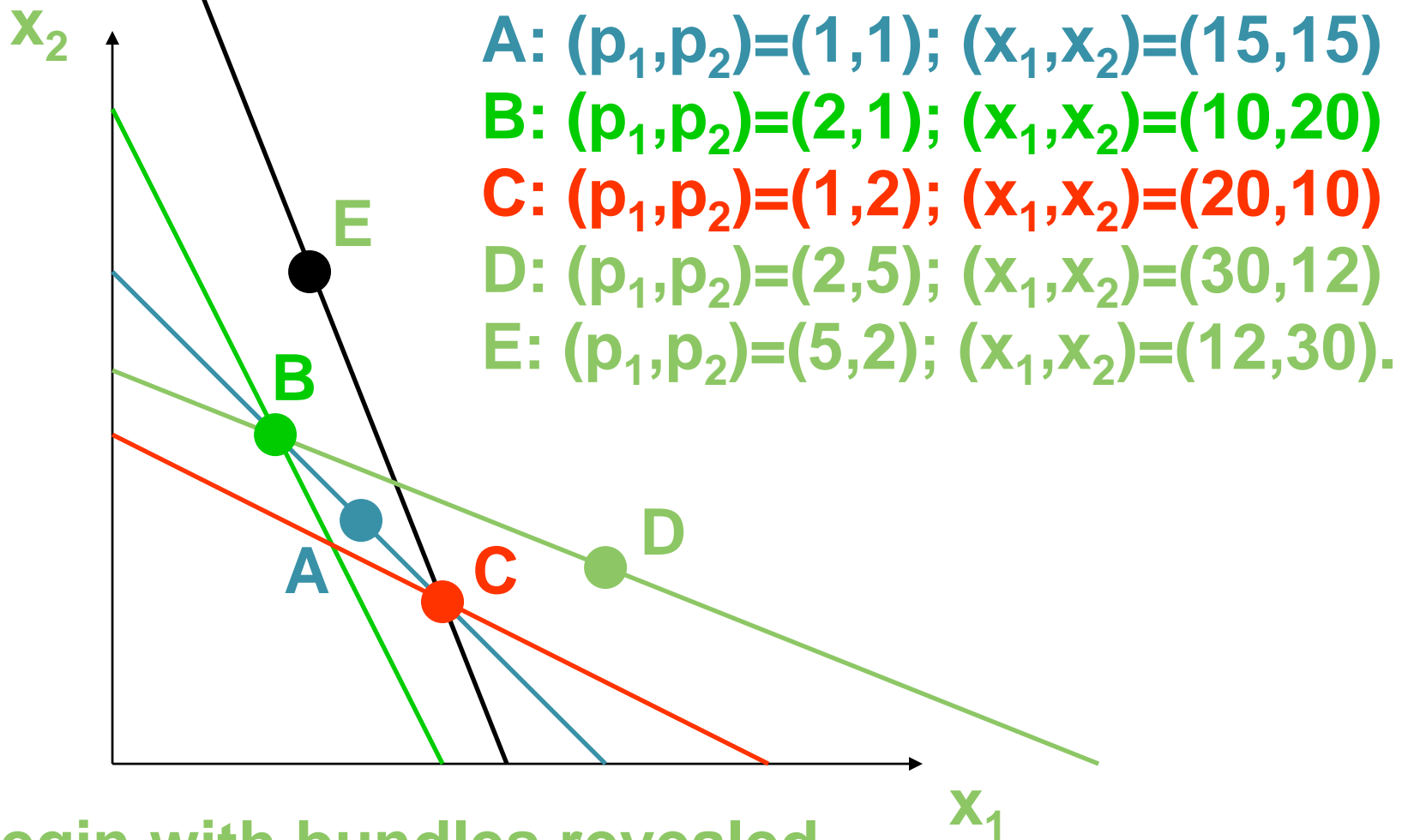
Recovering Indifference Curves

- Since the choices satisfy the SARP, there is a well-behaved preference relation that “rationalizes” the choices.

Recovering Indifference Curves



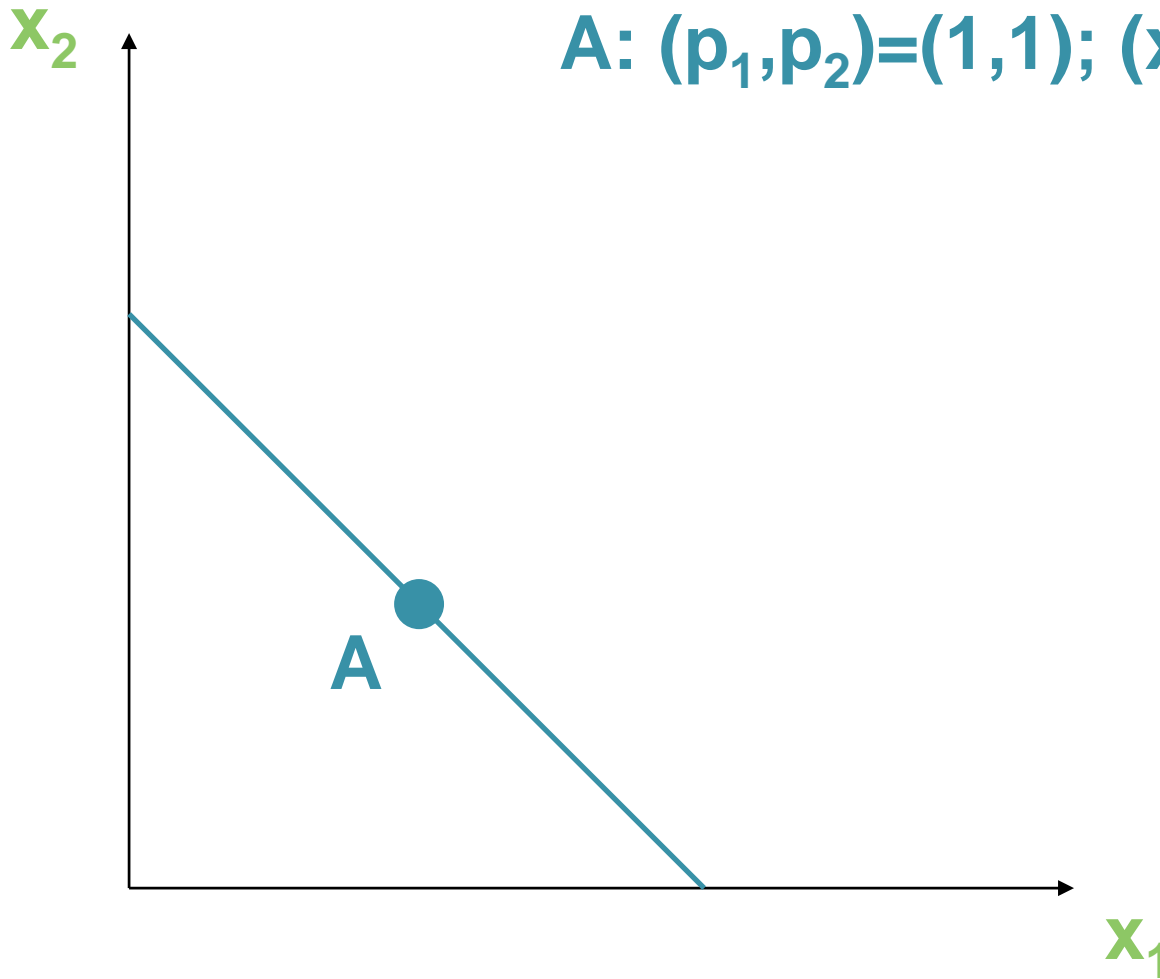
Recovering Indifference Curves



Begin with bundles revealed
to be less preferred than bundle A.

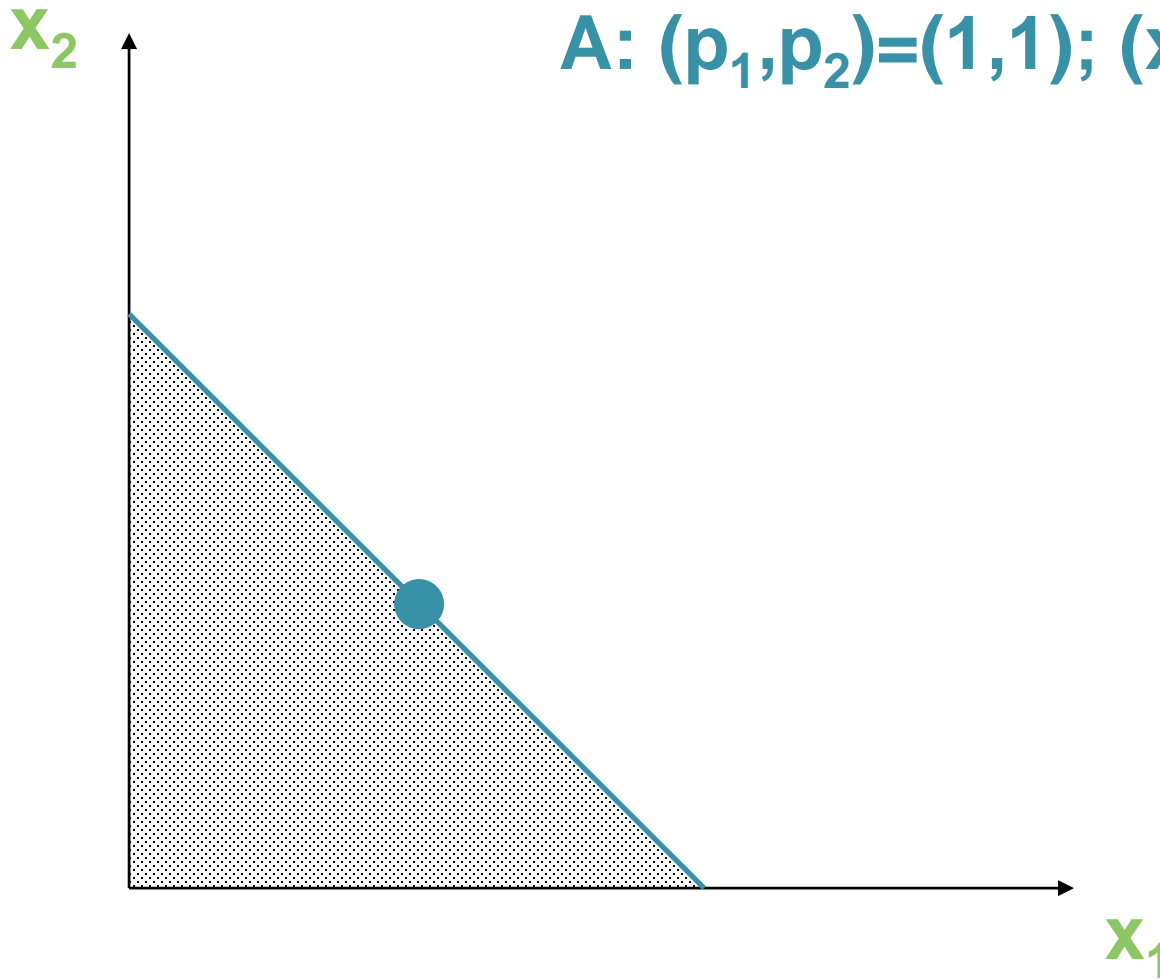
Recovering Indifference Curves

A: $(p_1, p_2) = (1, 1)$; $(x_1, x_2) = (15, 15)$.



Recovering Indifference Curves

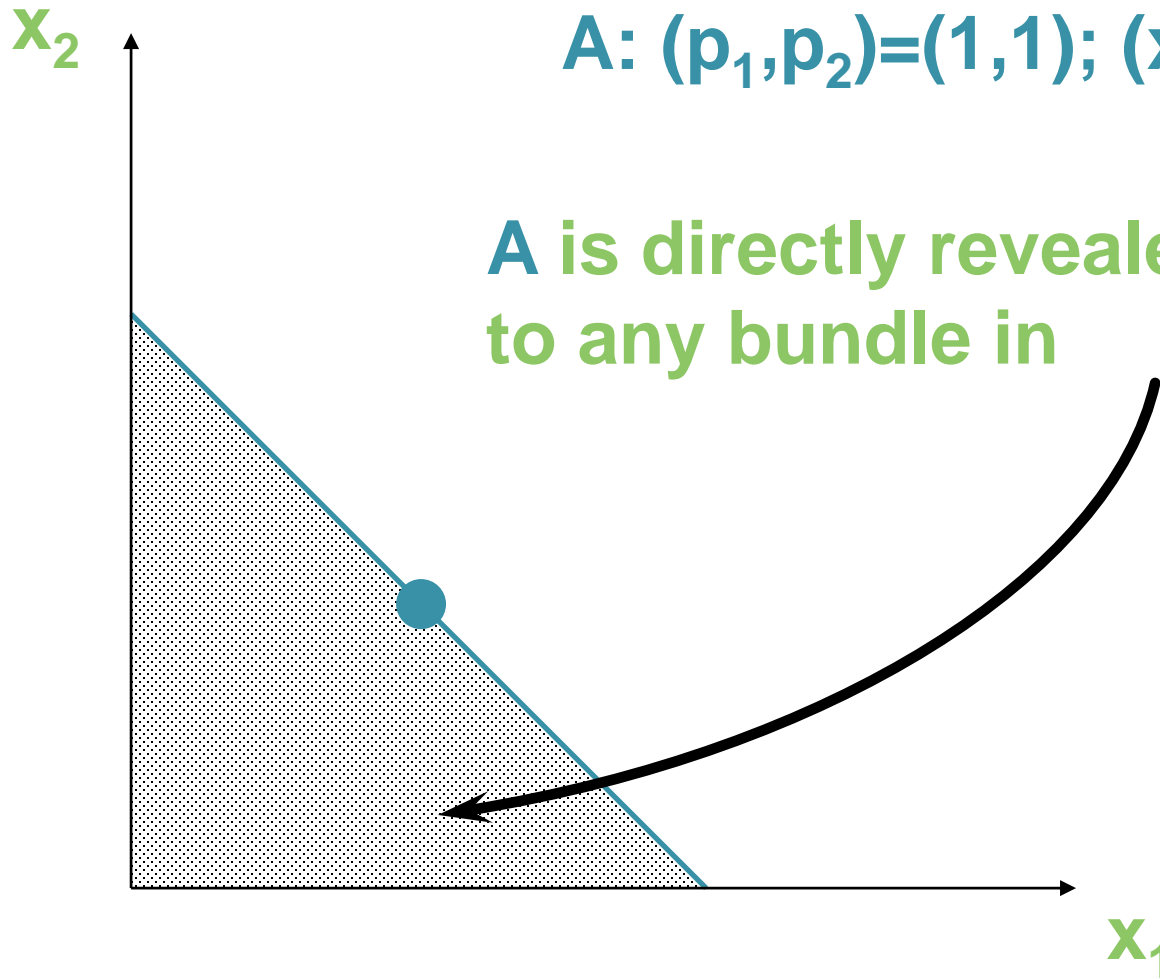
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Recovering Indifference Curves

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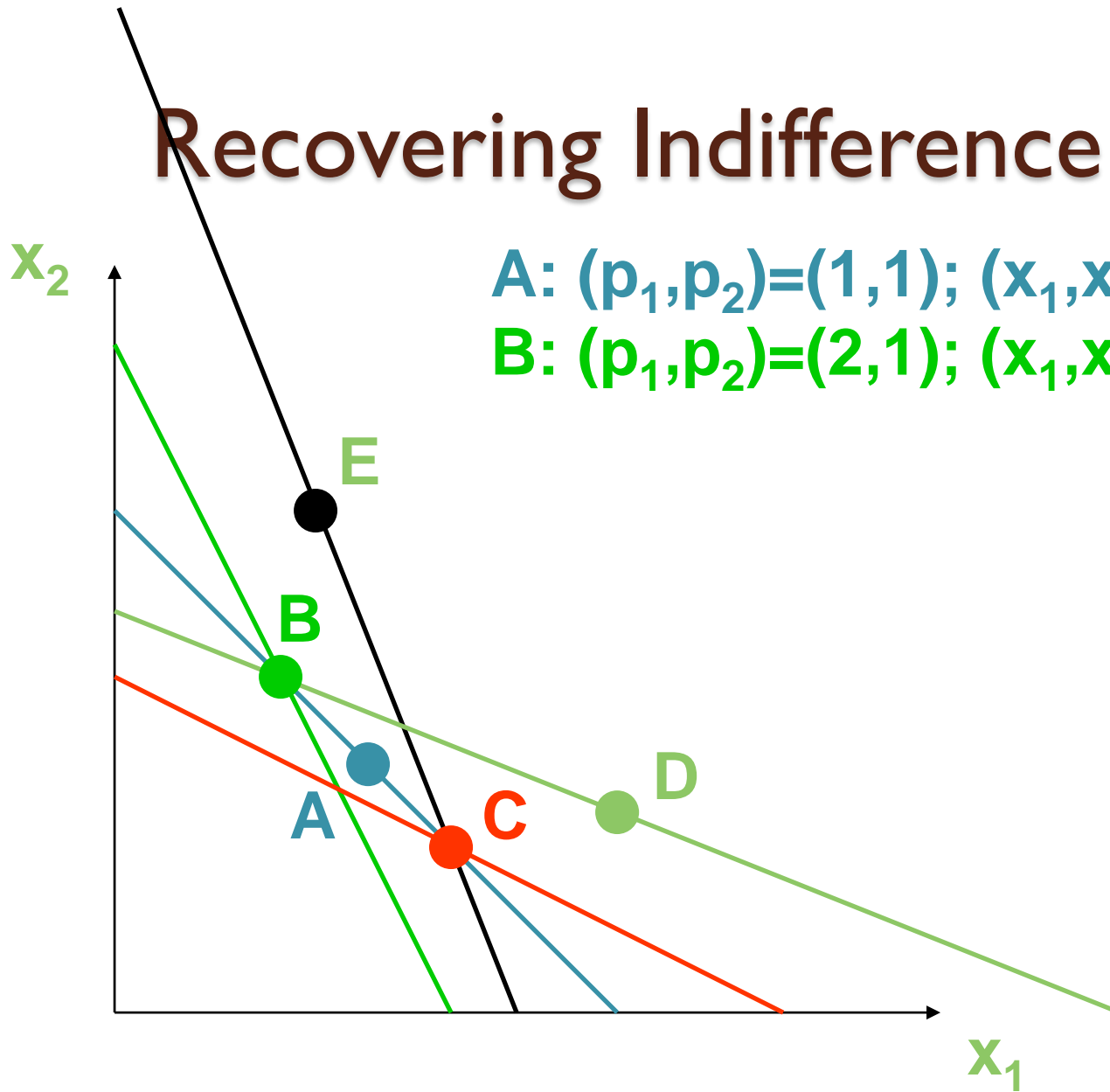
A is directly revealed preferred to any bundle in



Recovering Indifference Curves

A: $(p_1, p_2) = (1, 1)$; $(x_1, x_2) = (15, 15)$

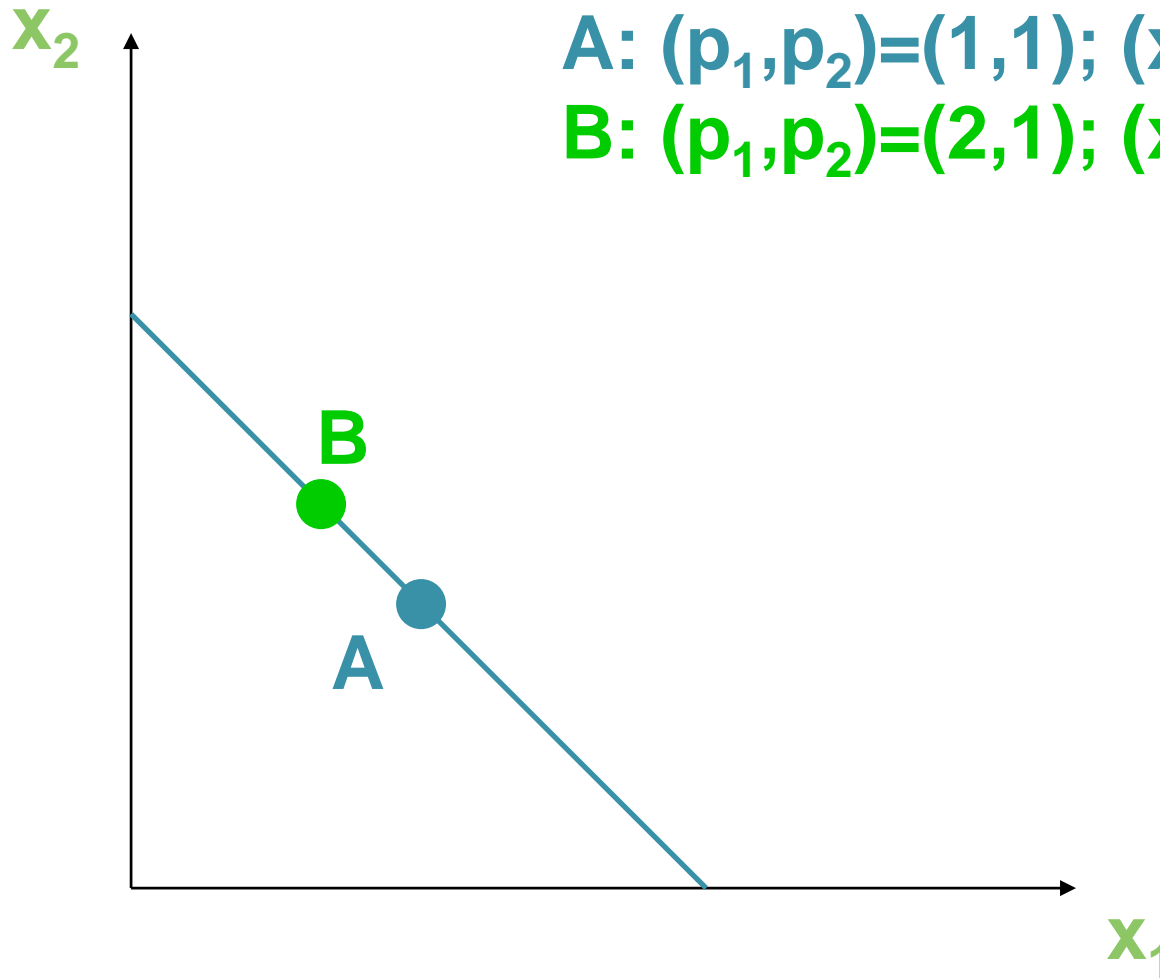
B: $(p_1, p_2) = (2, 1)$; $(x_1, x_2) = (10, 20)$.



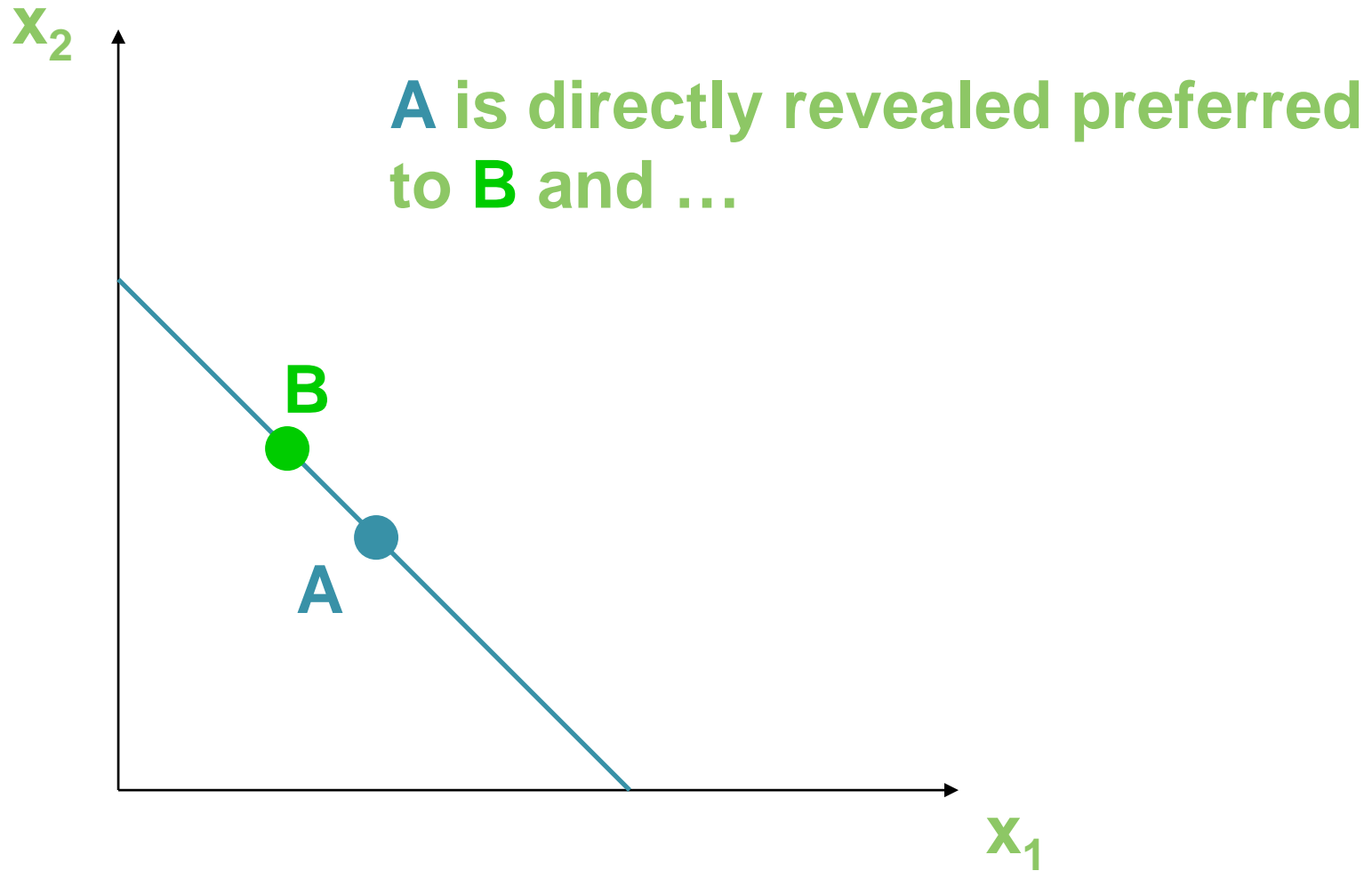
Recovering Indifference Curves

A: $(p_1, p_2) = (1, 1)$; $(x_1, x_2) = (15, 15)$

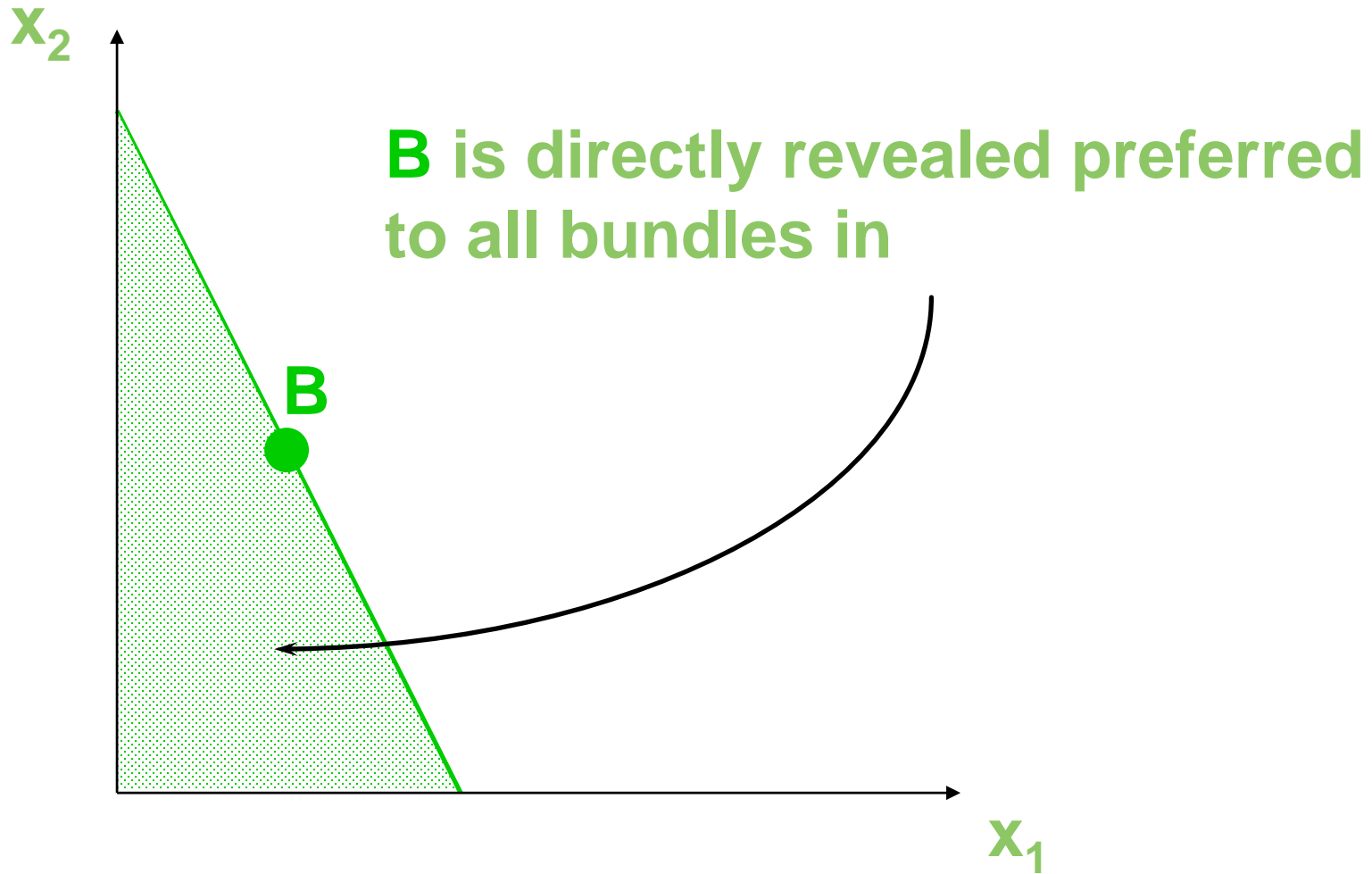
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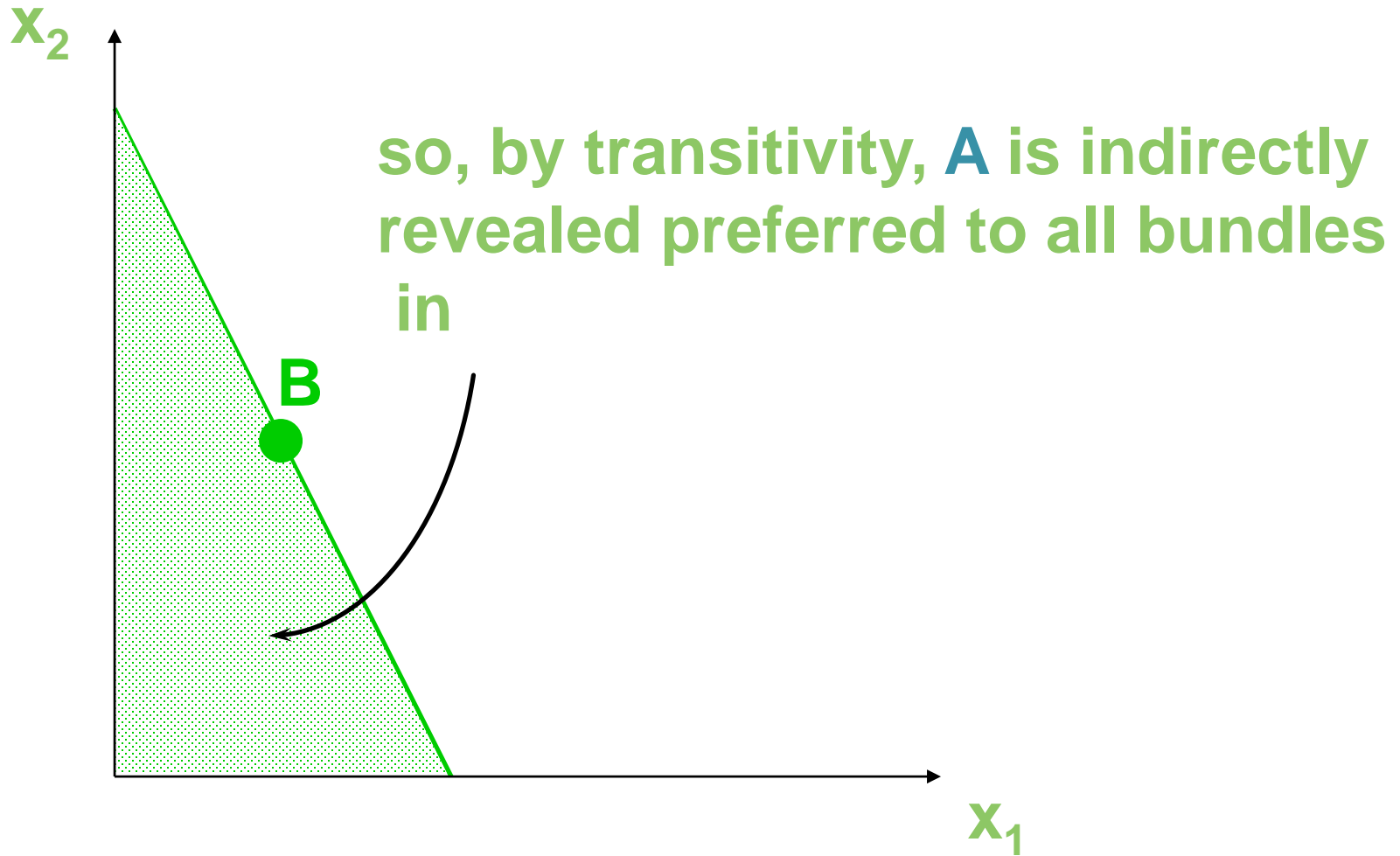
Recovering Indifference Curves



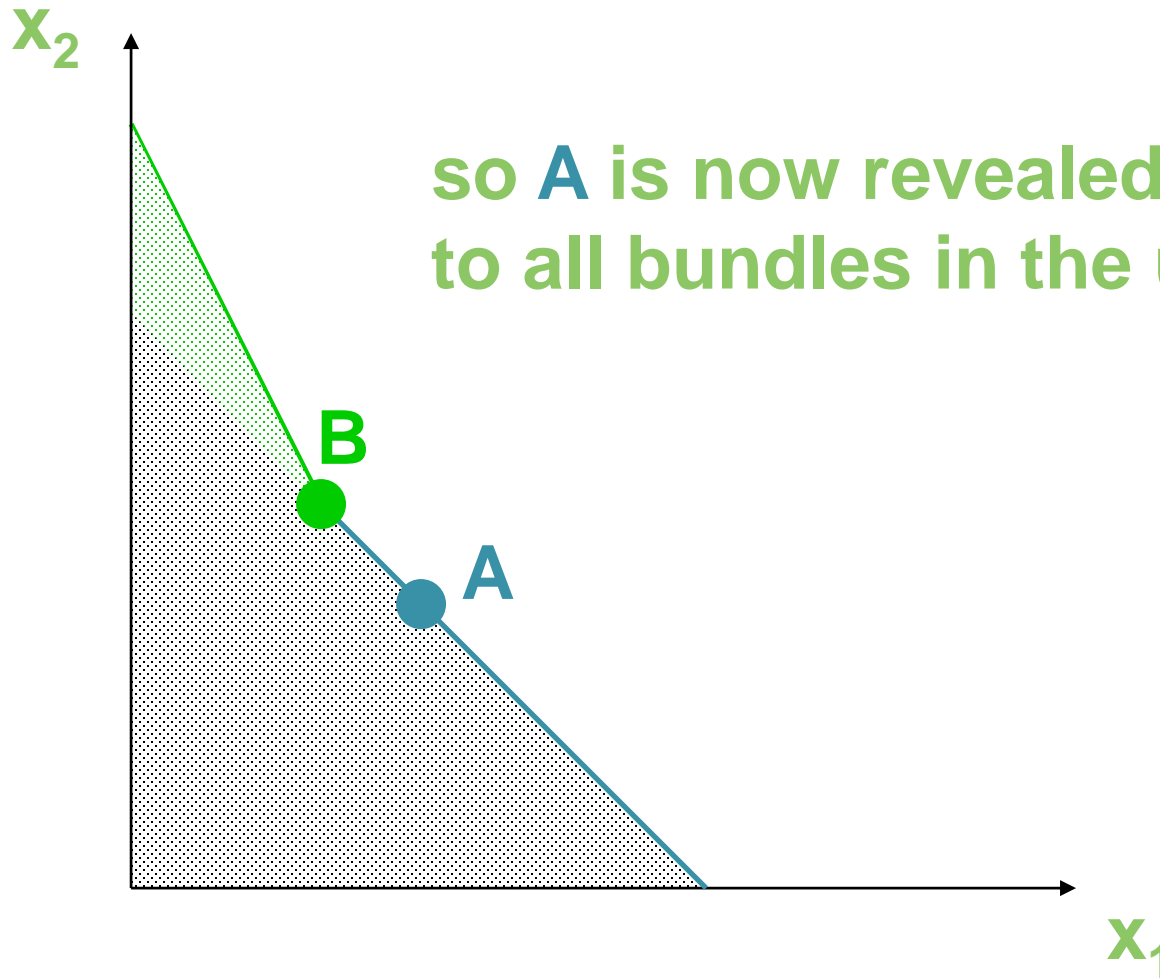
Recovering Indifference Curves



Recovering Indifference Curves



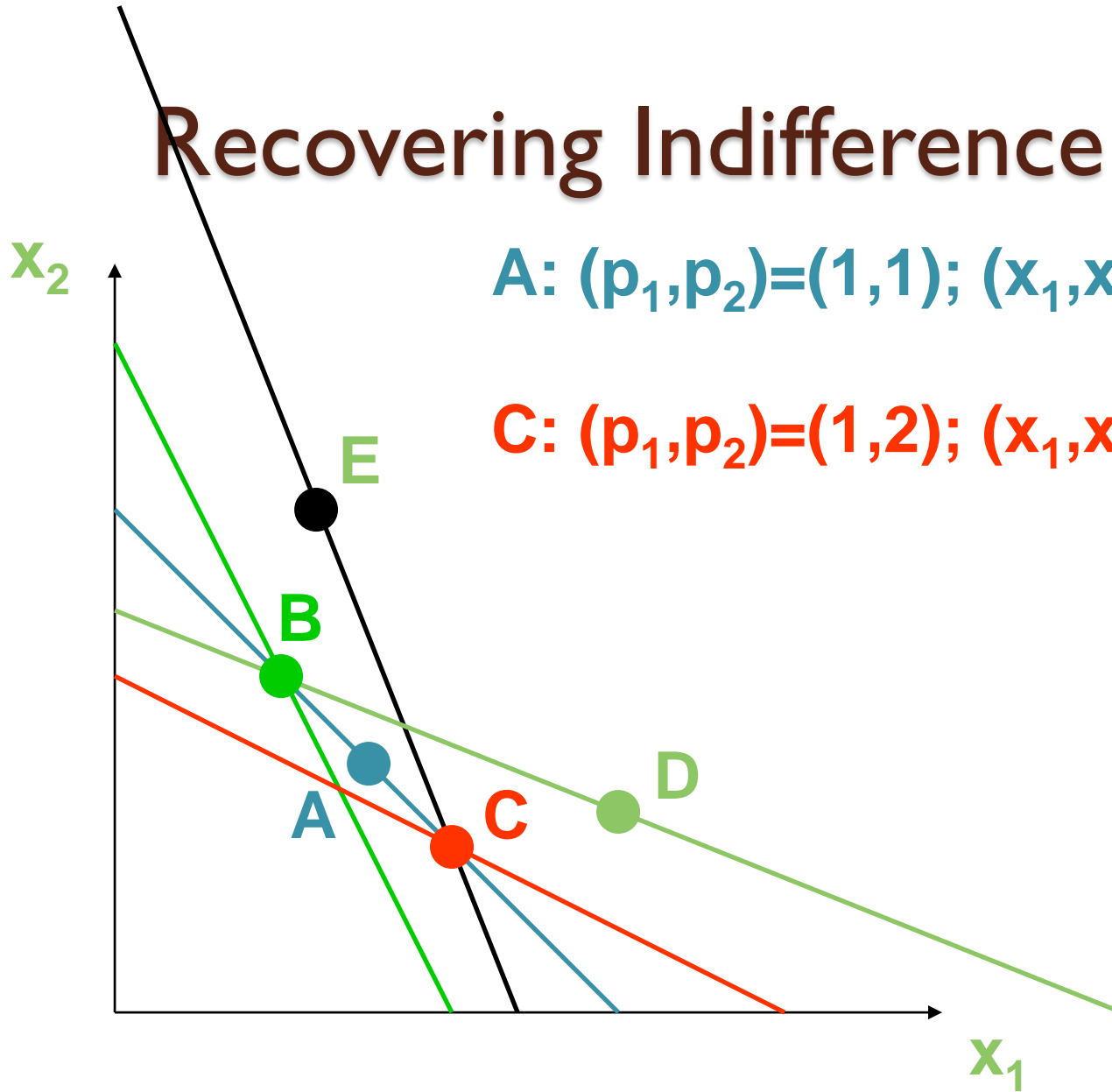
Recovering Indifference Curves



Recovering Indifference Curves

A: $(p_1, p_2) = (1, 1)$; $(x_1, x_2) = (15, 15)$

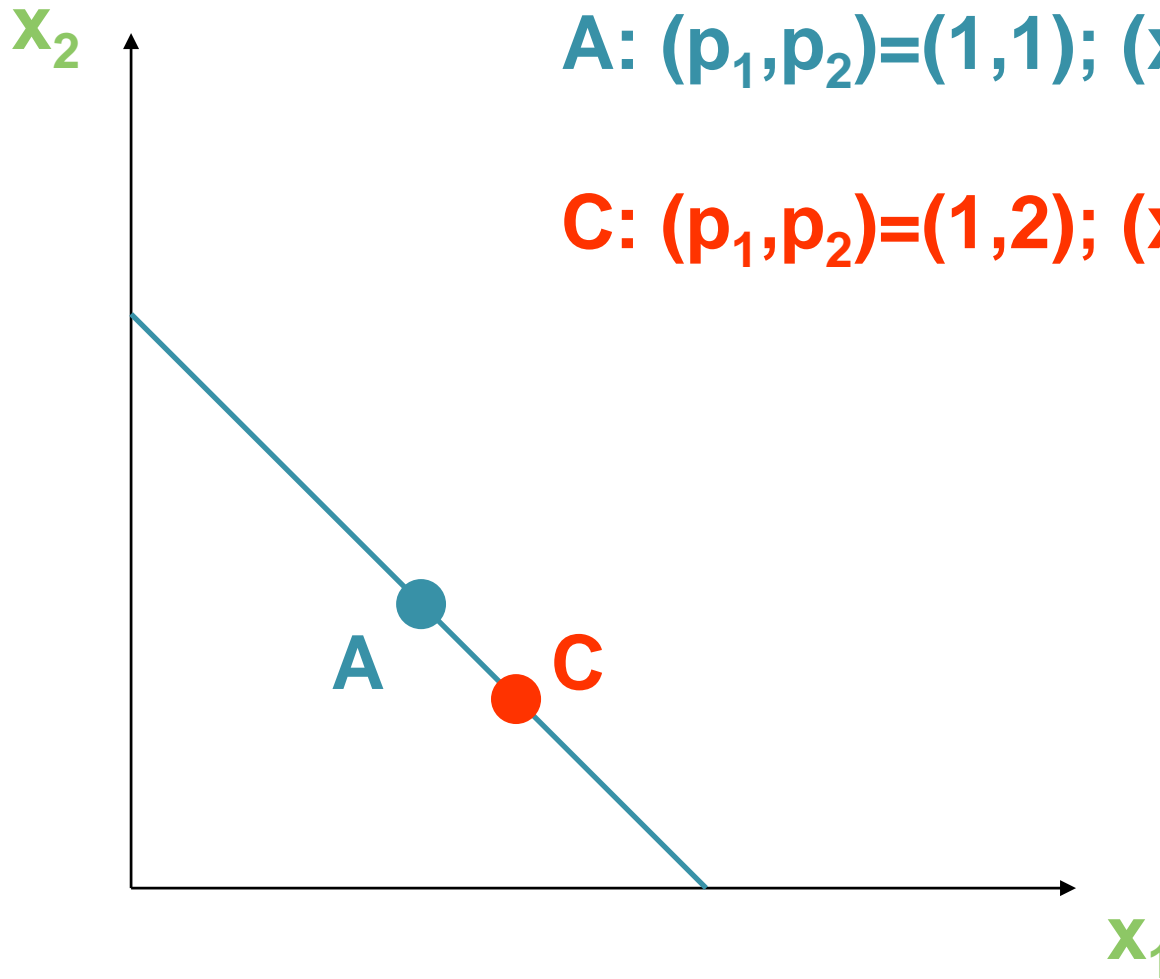
C: $(p_1, p_2) = (1, 2)$; $(x_1, x_2) = (20, 10)$.



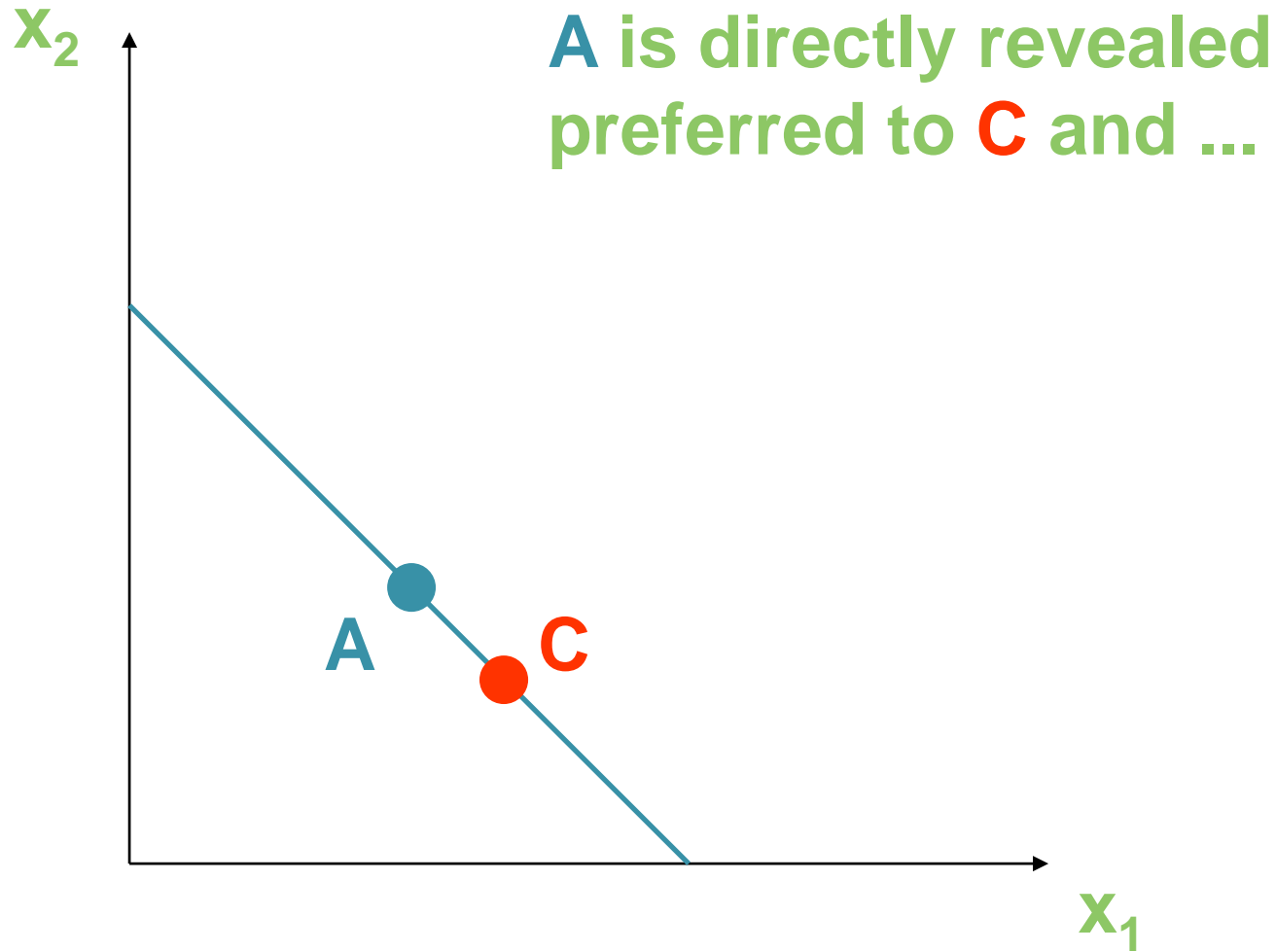
Recovering Indifference Curves

A: $(p_1, p_2) = (1, 1)$; $(x_1, x_2) = (15, 15)$

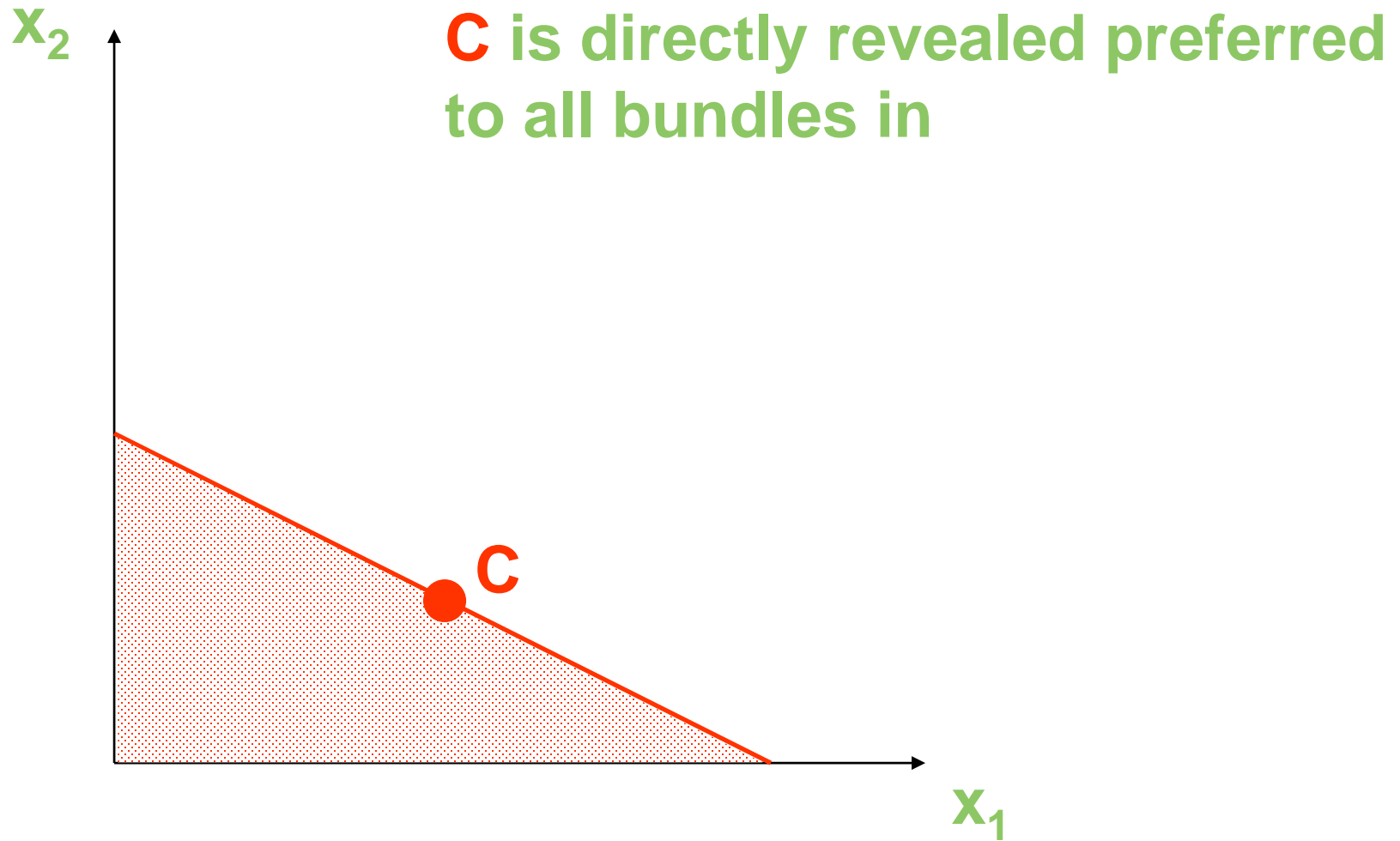
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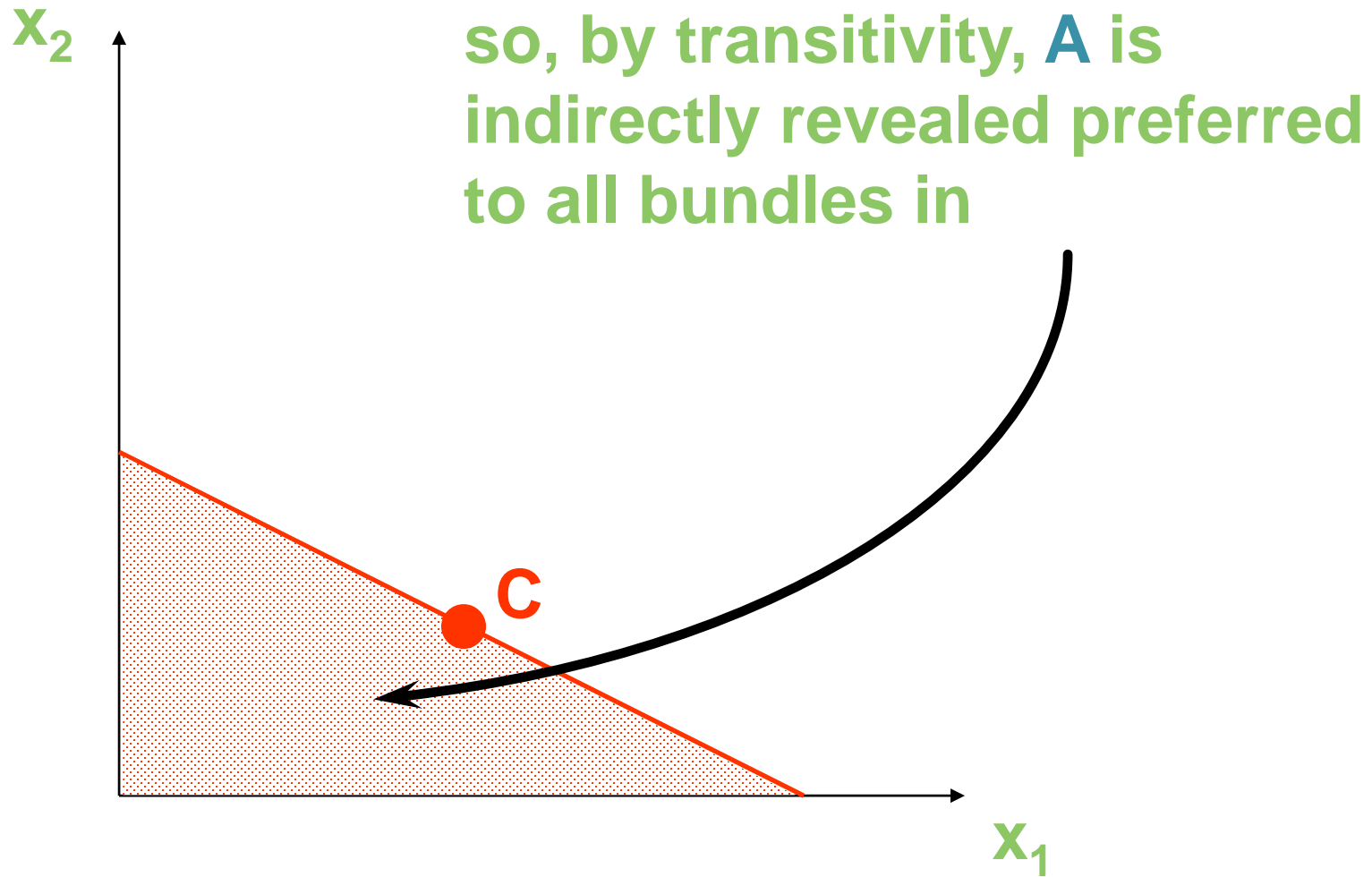
Recovering Indifference Curves



Recovering Indifference Curves

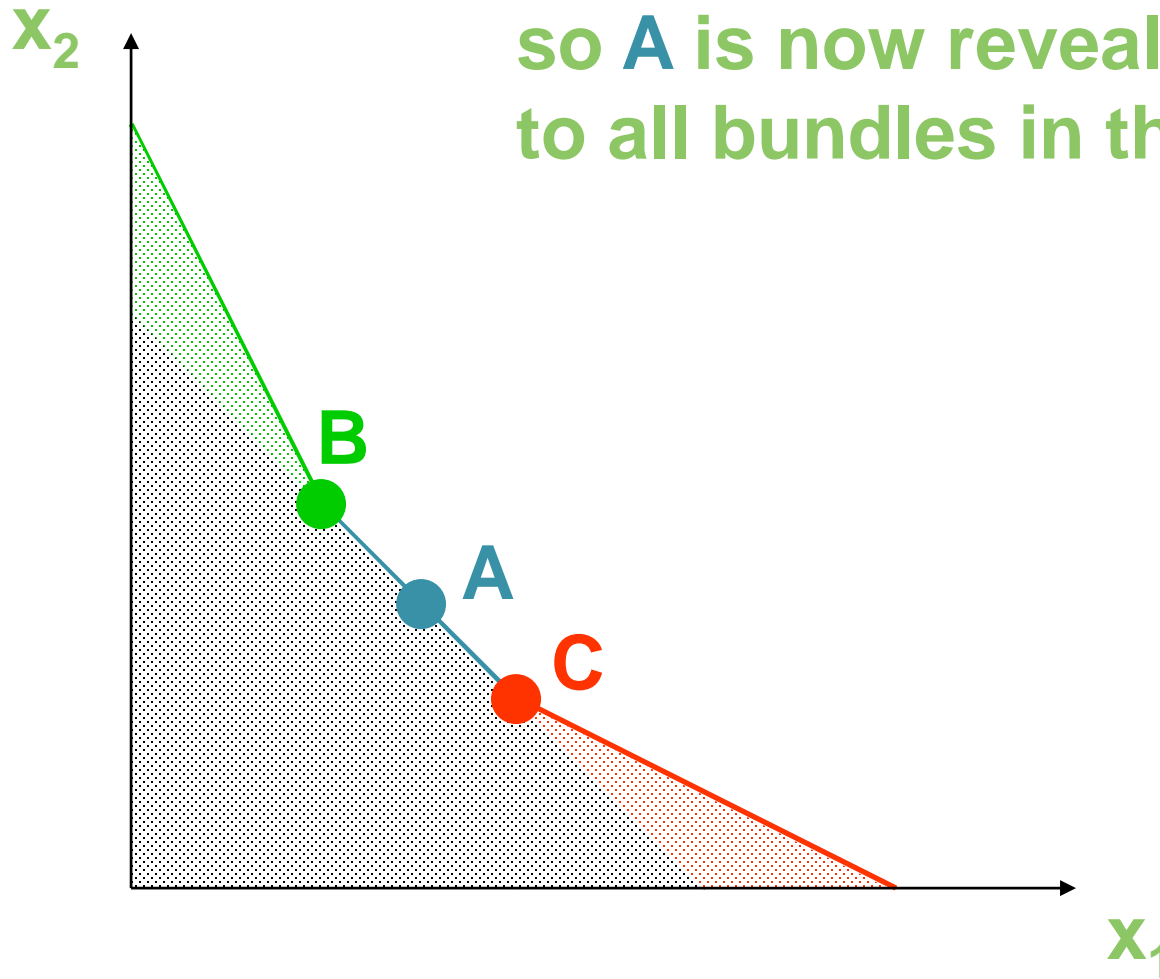


Recovering Indifference Curves

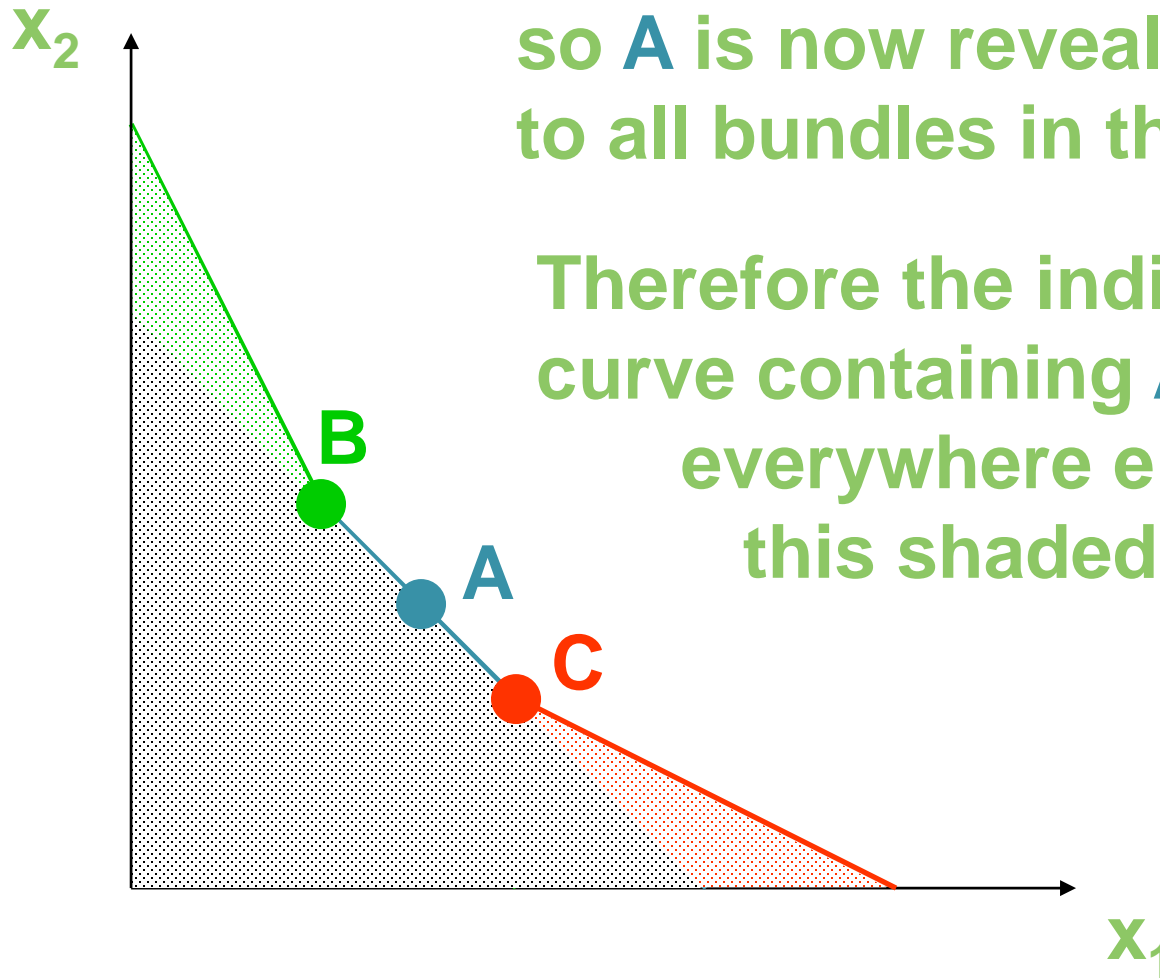


Recovering Indifference Curves

so **A** is now revealed preferred to all bundles in the union.



Recovering Indifference Curves



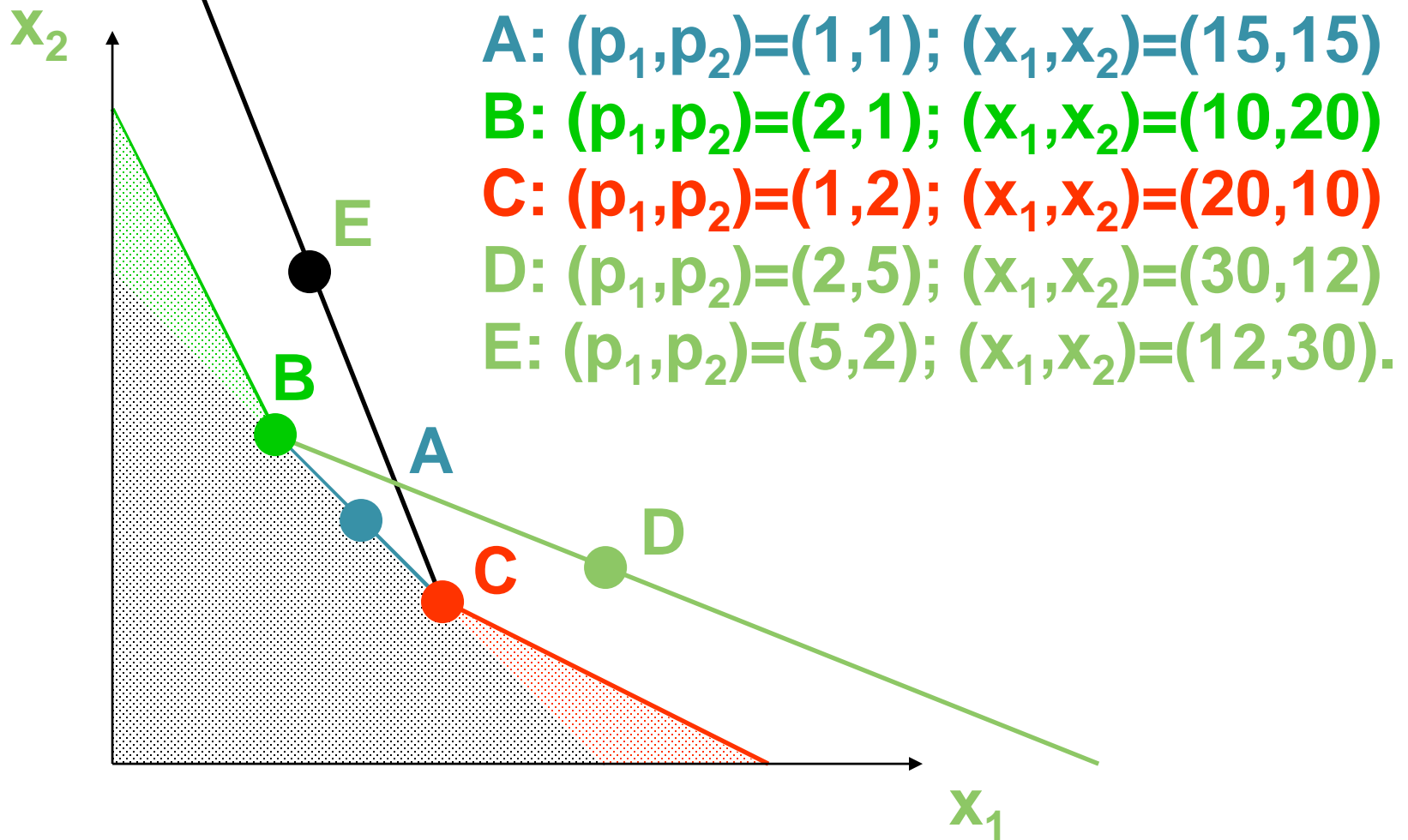
so **A** is now revealed preferred to all bundles in the union.

Therefore the indifference curve containing **A** must lie everywhere else above this shaded set.

Recovering Indifference Curves

- Now, what about the bundles revealed as more preferred than **A**?

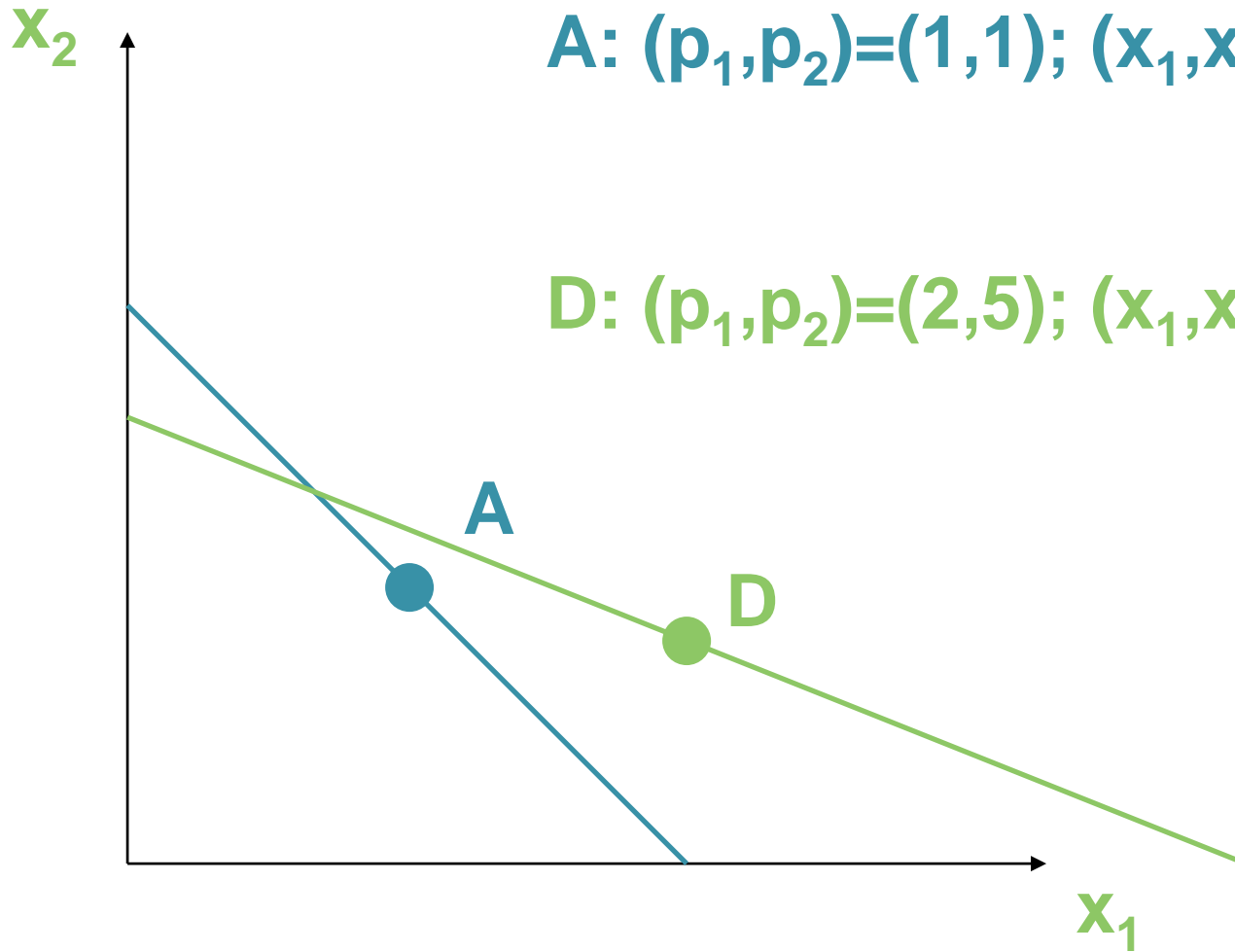
Recovering Indifference Curves



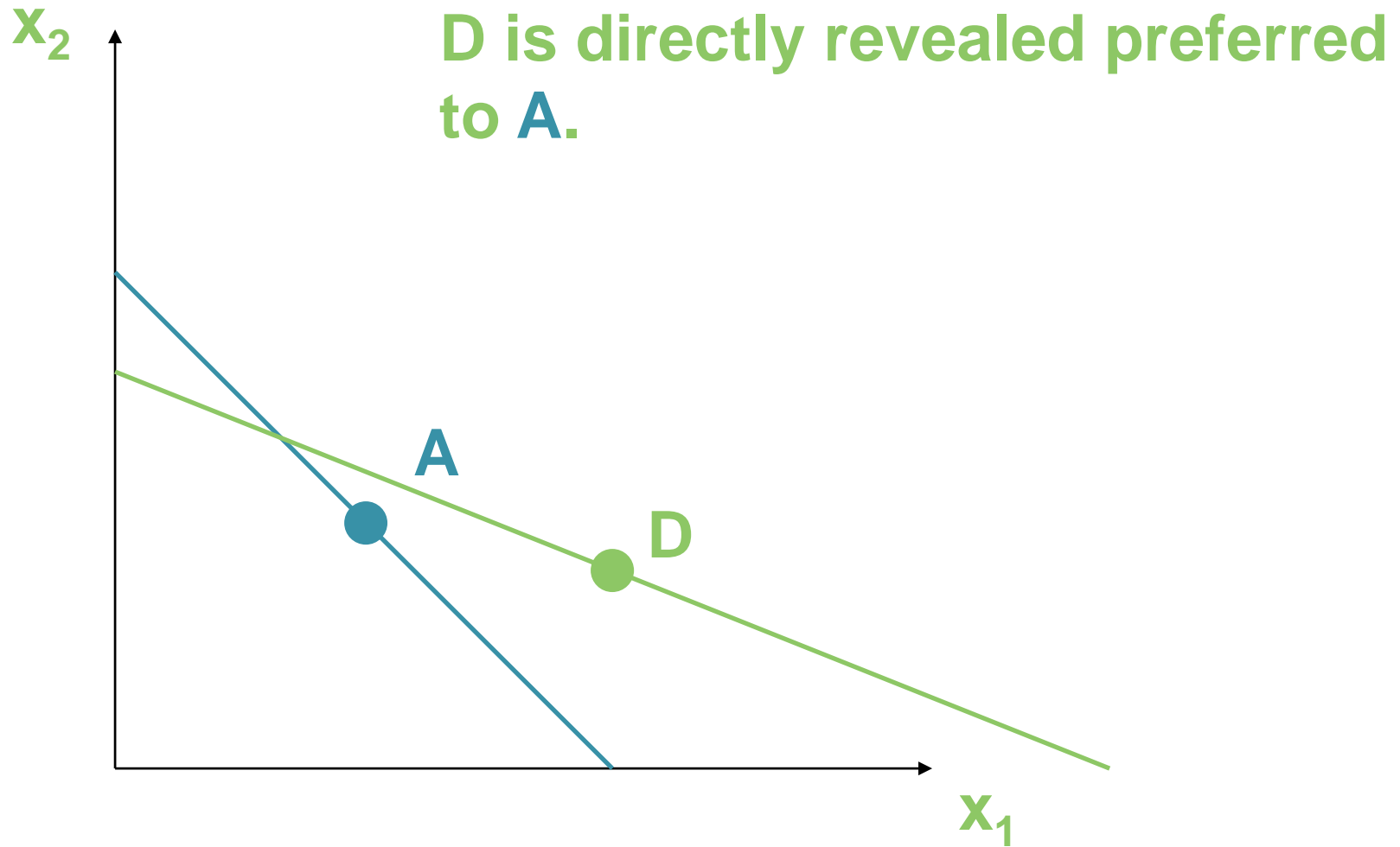
Recovering Indifference Curves

A: $(p_1, p_2) = (1, 1)$; $(x_1, x_2) = (15, 15)$

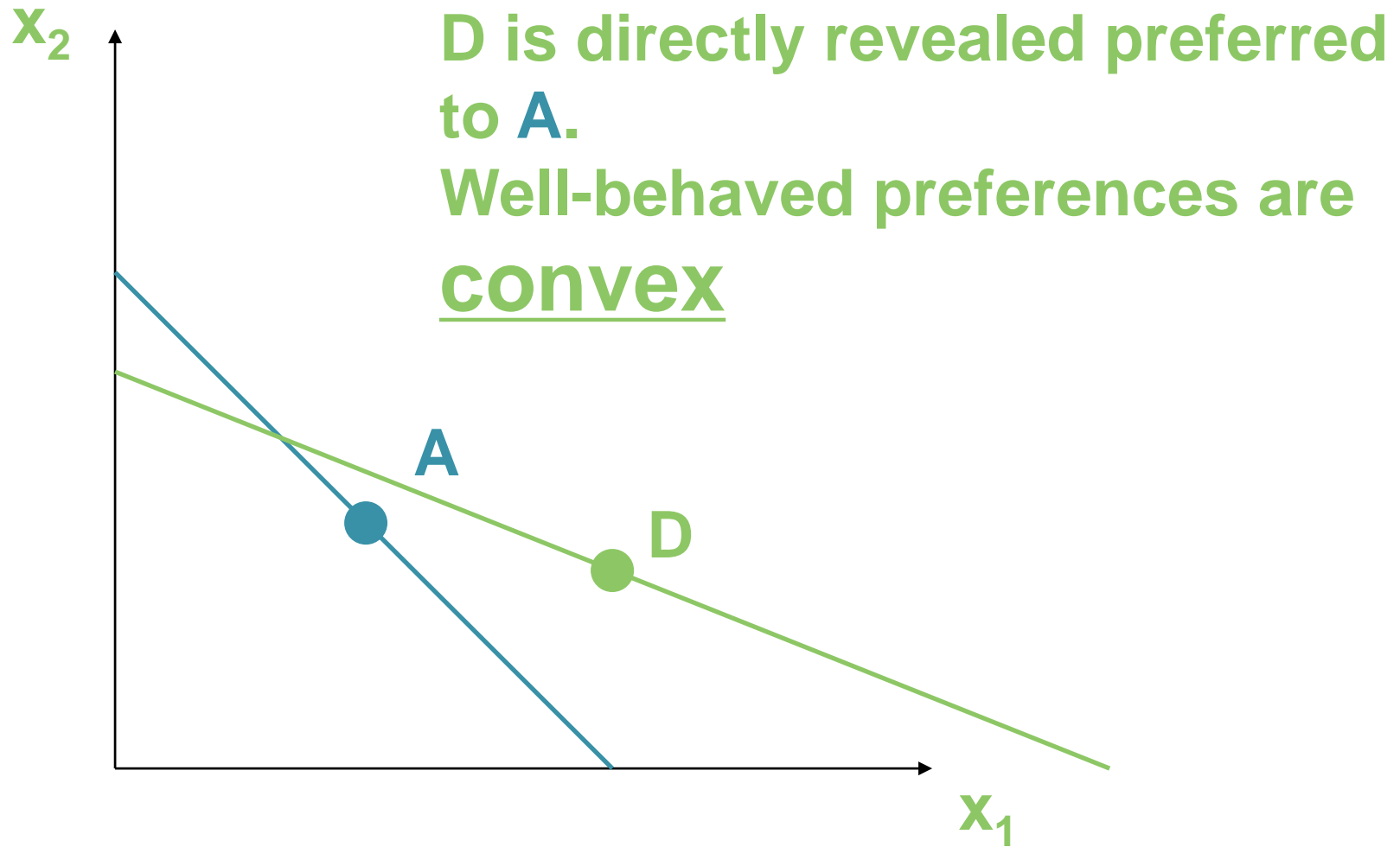
D: $(p_1, p_2) = (2, 5)$; $(x_1, x_2) = (30, 12)$.



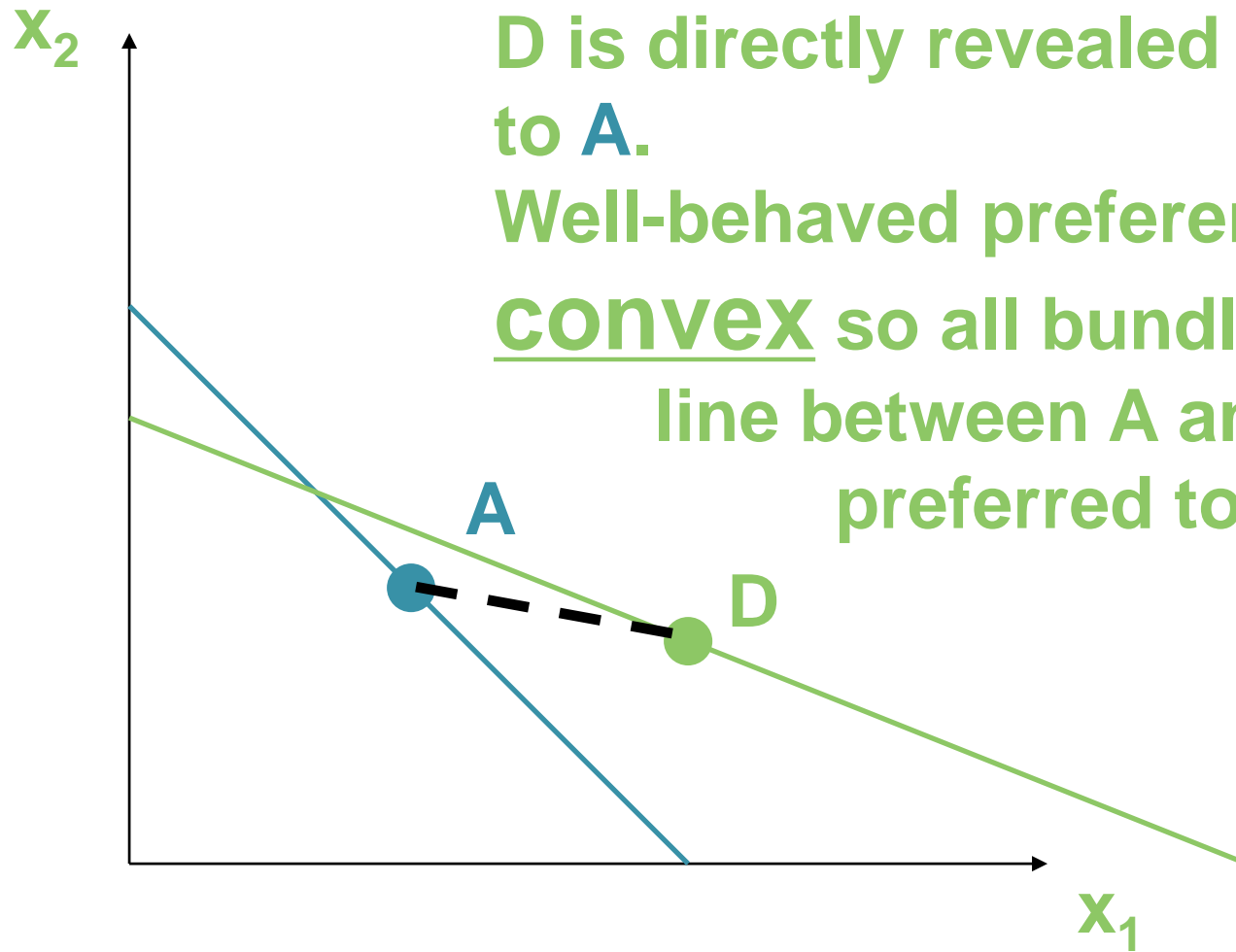
Recovering Indifference Curves



Recovering Indifference Curves



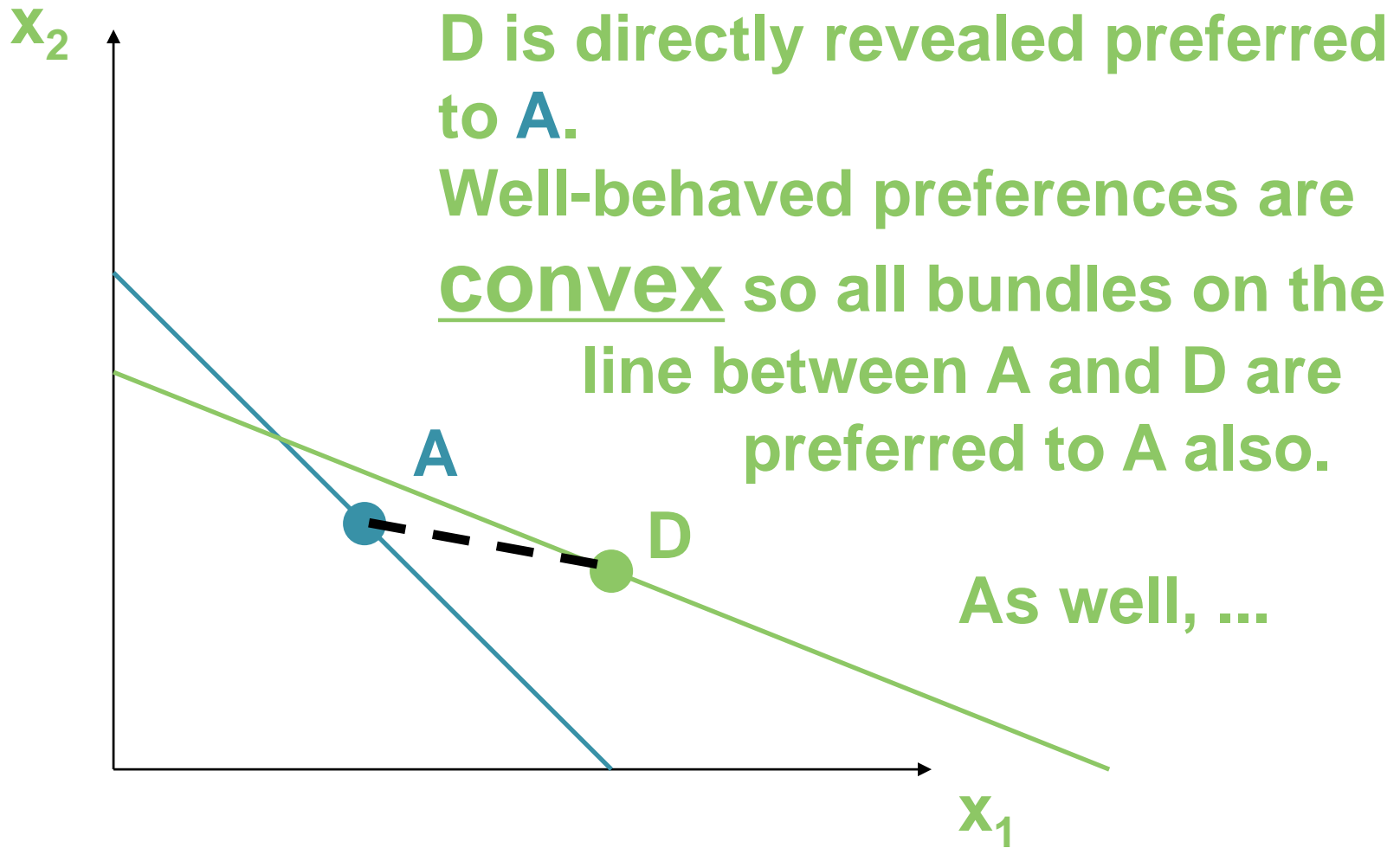
Recovering Indifference Curves



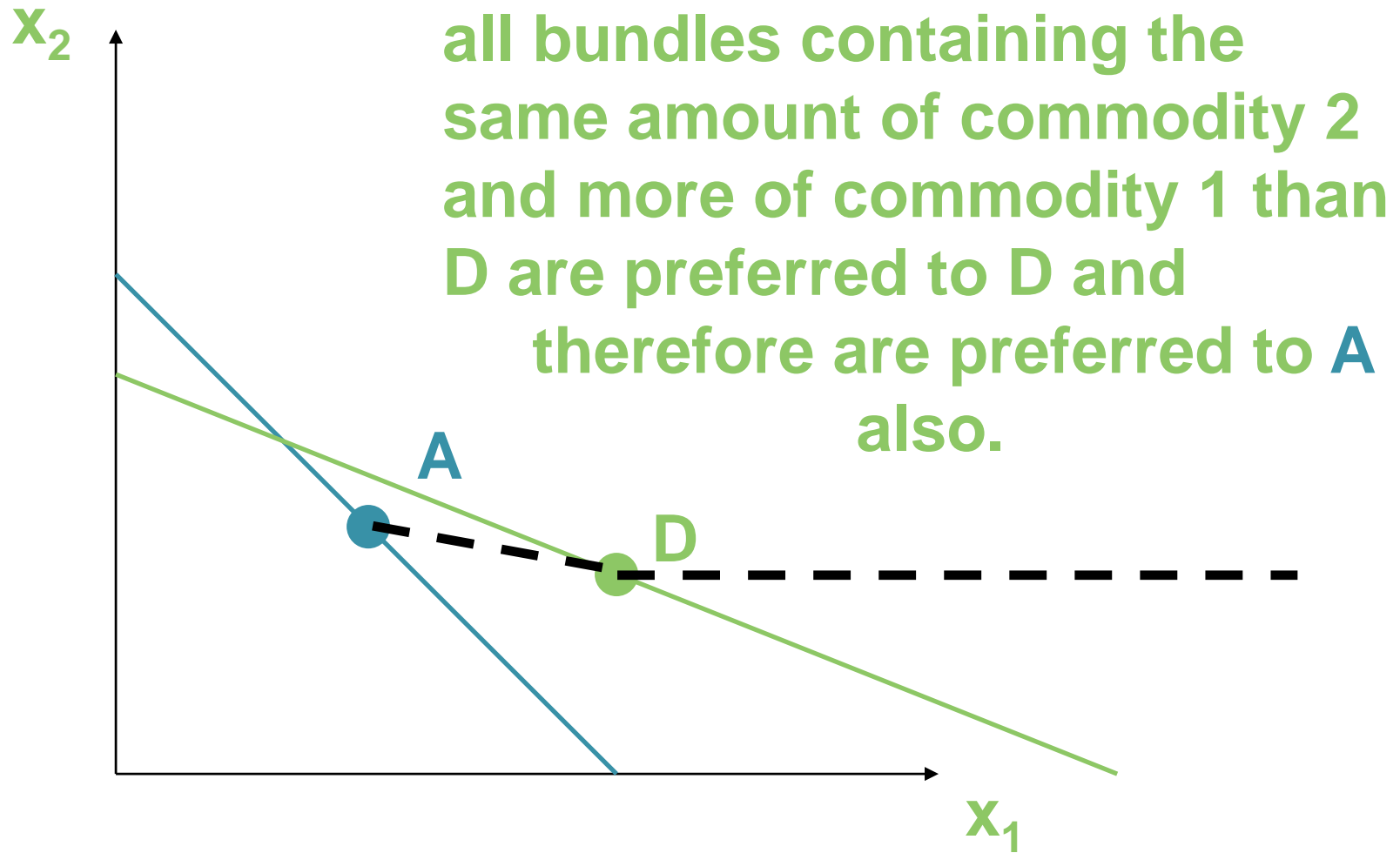
D is directly revealed preferred to A.

Well-behaved preferences are convex so all bundles on the line between A and D are preferred to A also.

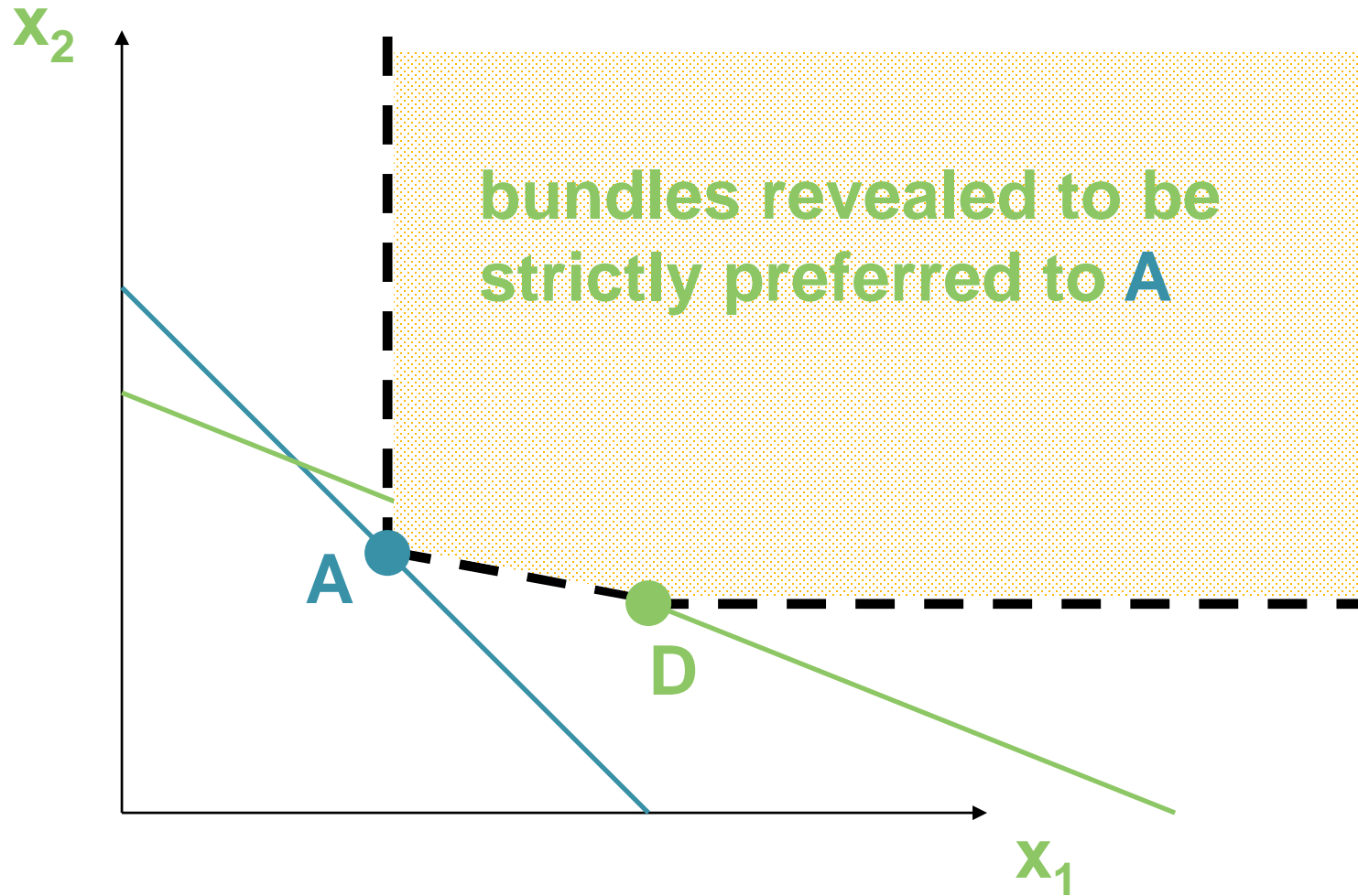
Recovering Indifference Curves



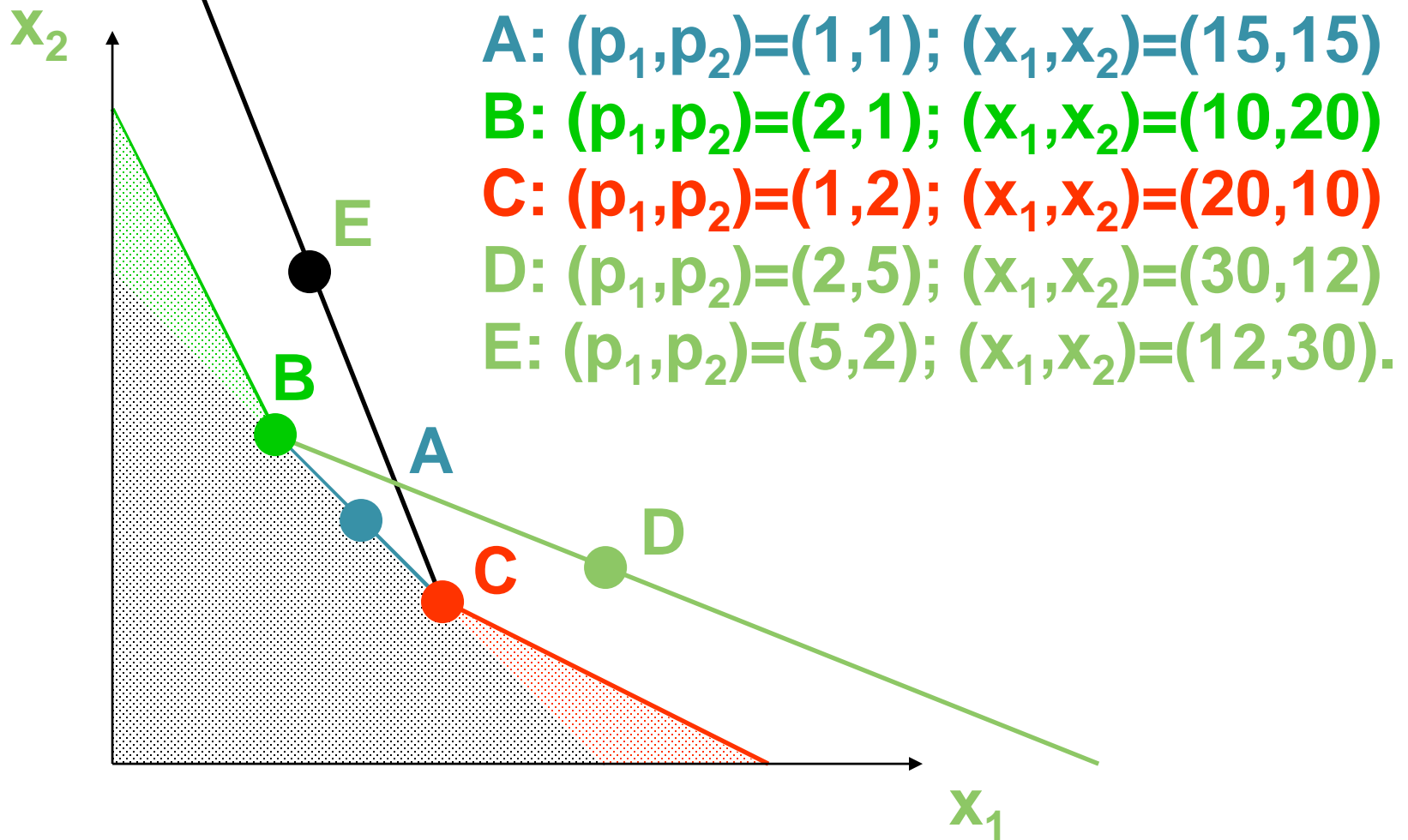
Recovering Indifference Curves



Recovering Indifference Curves



Recovering Indifference Curves



Recovering Indifference Curves

x_2

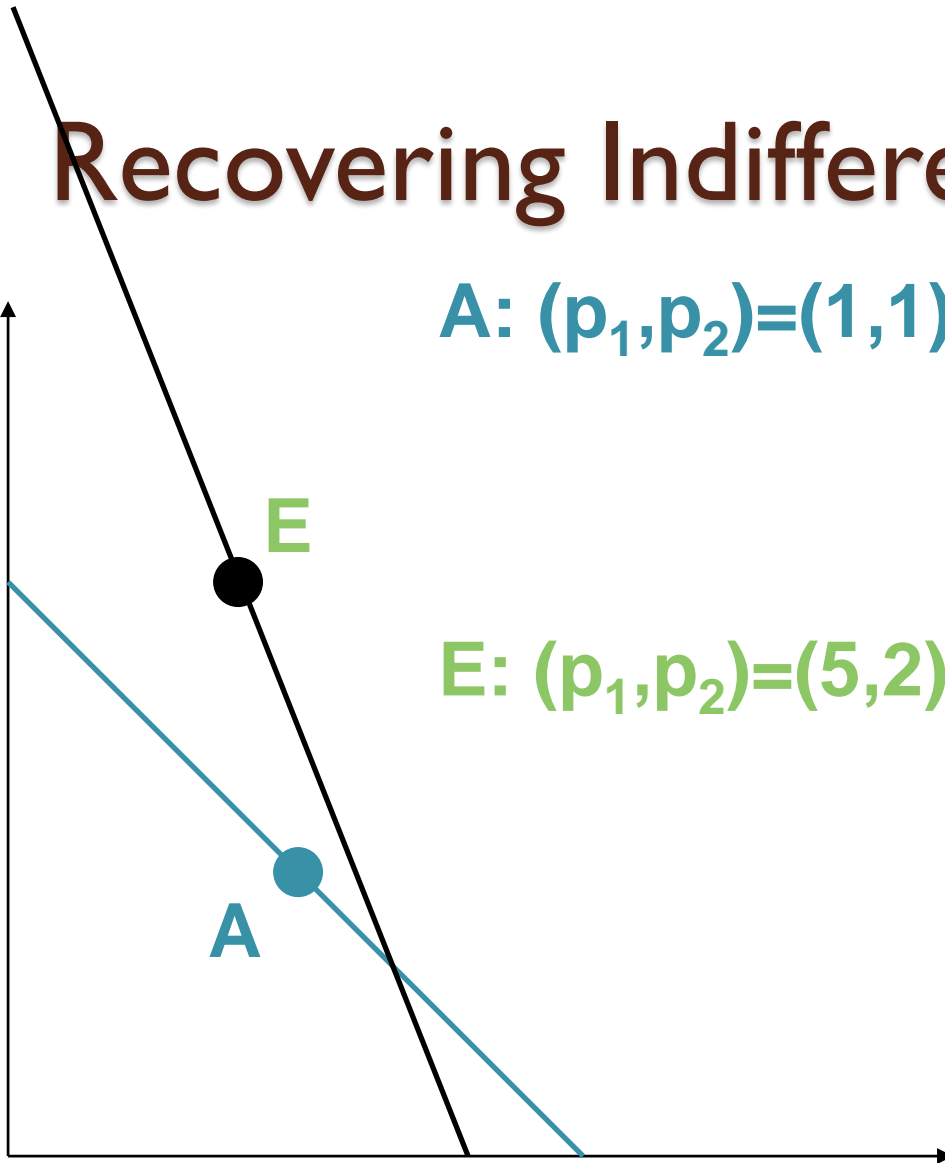
A: $(p_1, p_2) = (1, 1)$; $(x_1, x_2) = (15, 15)$

E

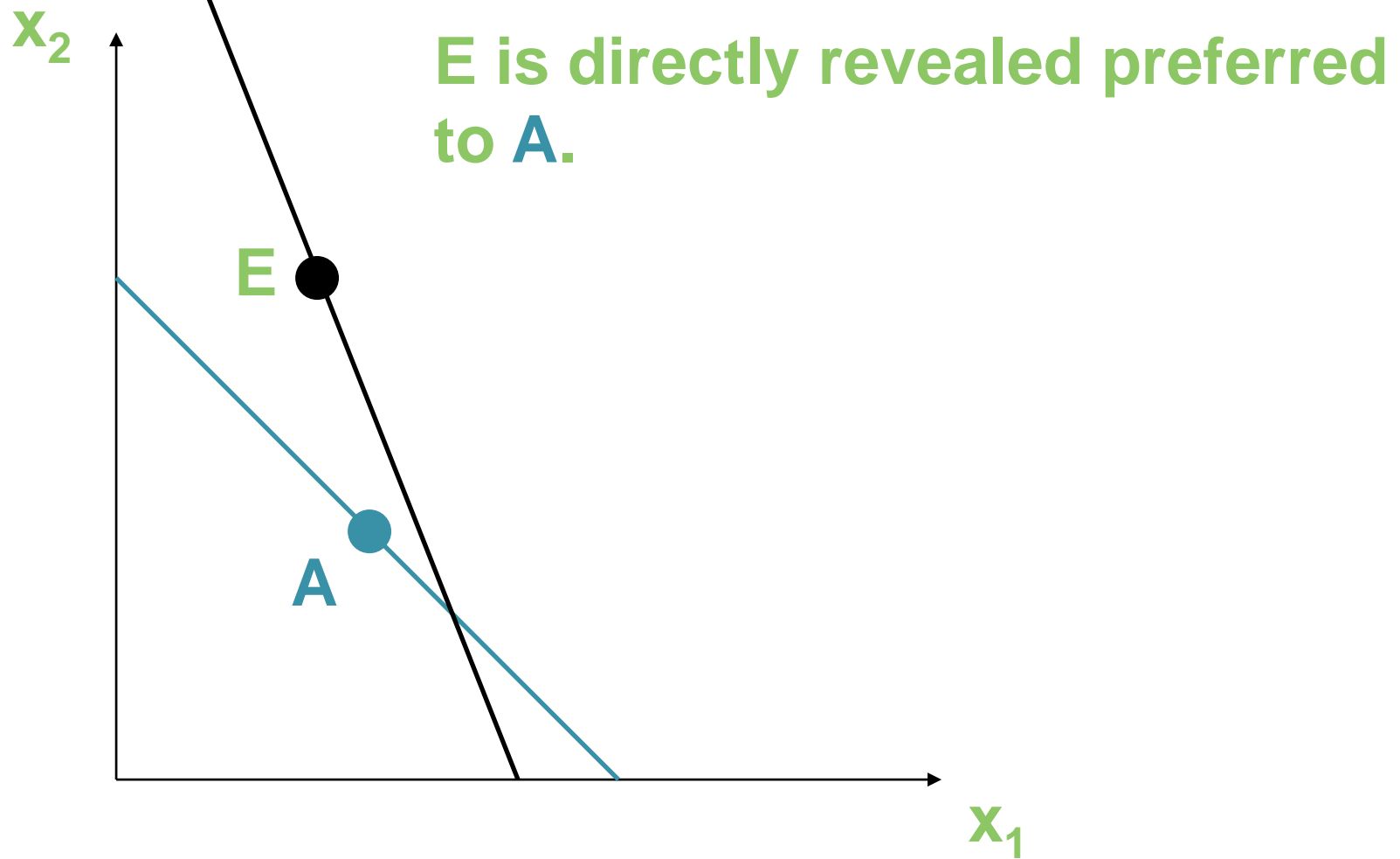
E: $(p_1, p_2) = (5, 2)$; $(x_1, x_2) = (12, 30)$.

A

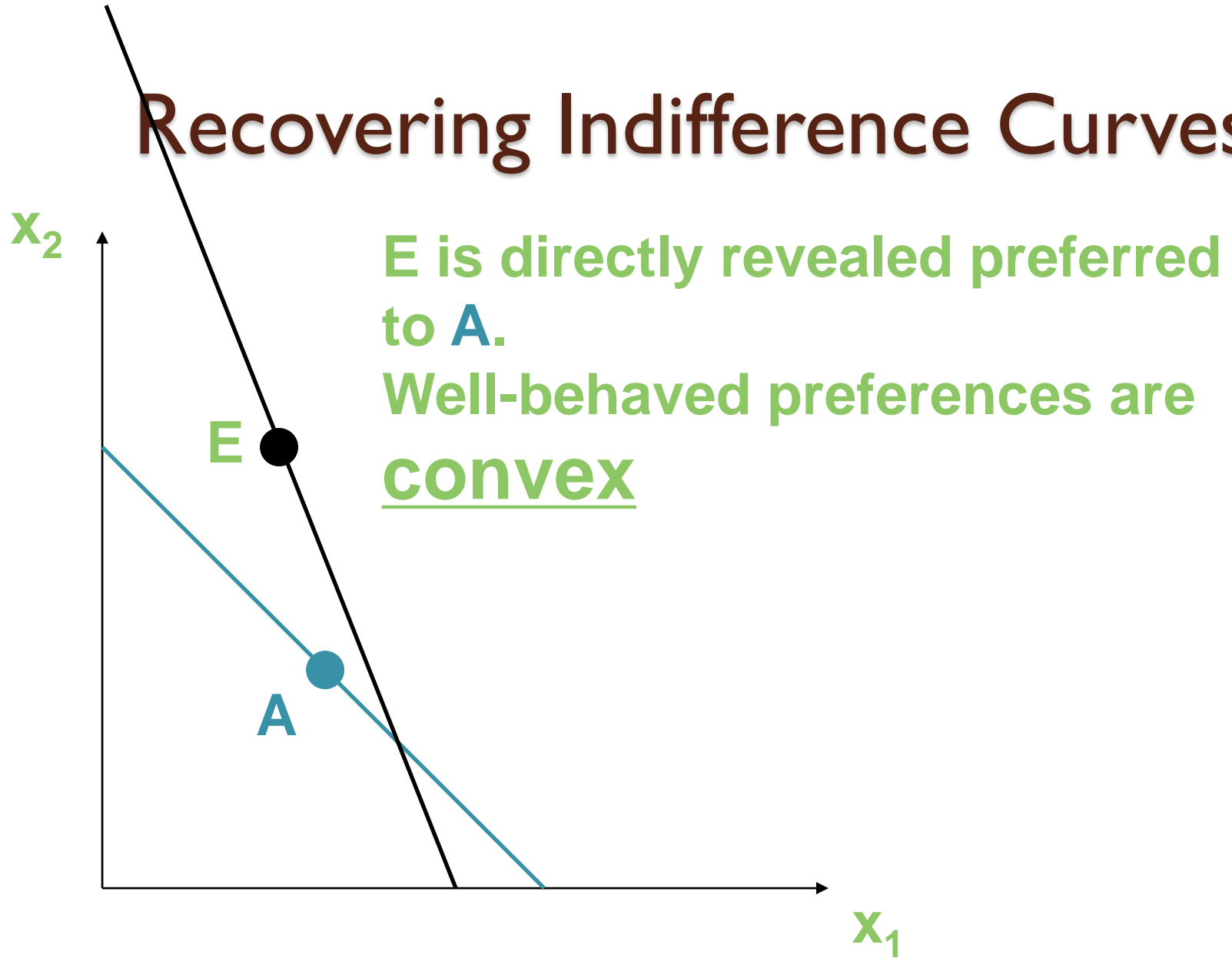
x_1



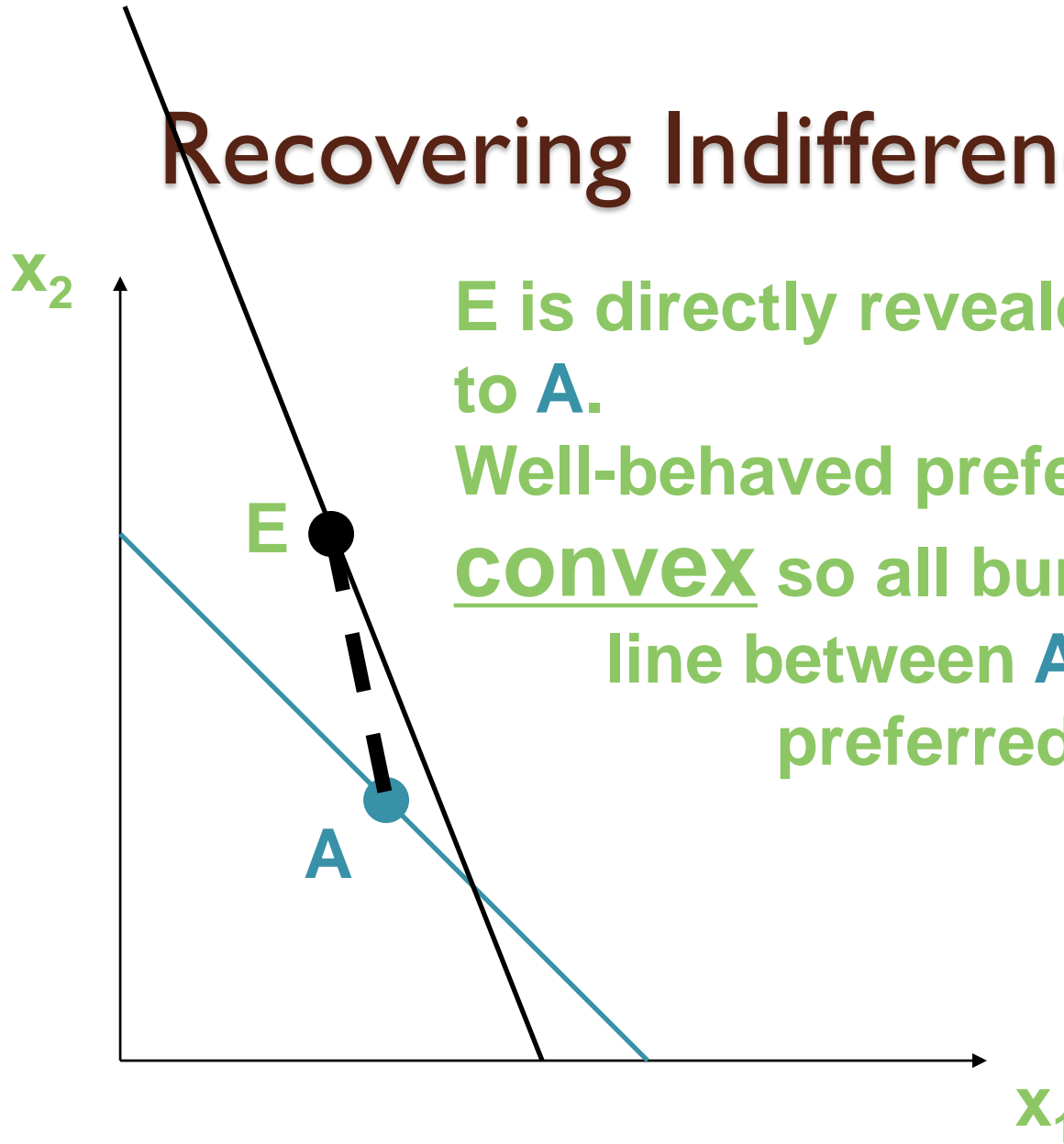
Recovering Indifference Curves



Recovering Indifference Curves



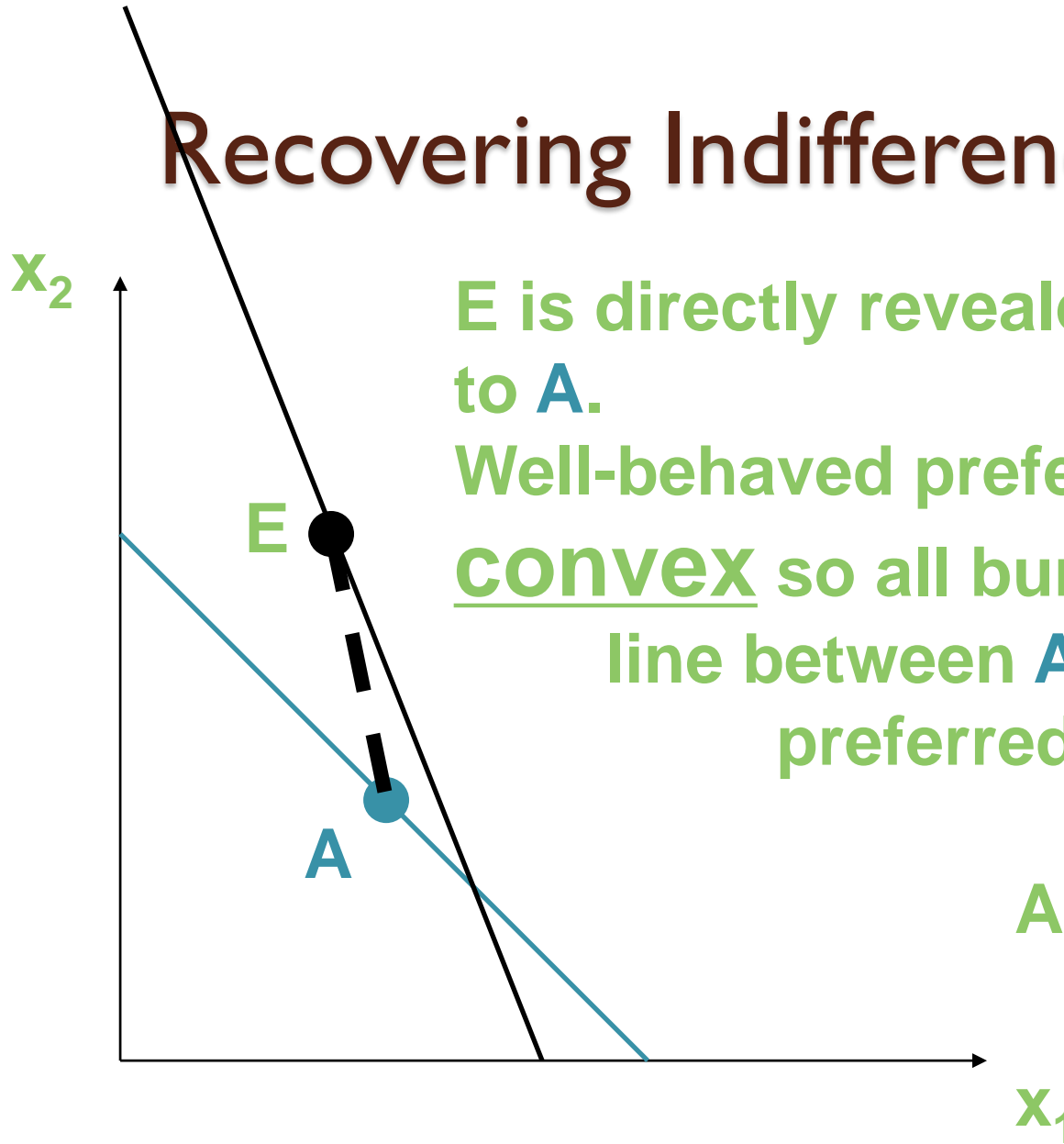
Recovering Indifference Curves



E is directly revealed preferred to A.

Well-behaved preferences are convex so all bundles on the line between A and E are preferred to A also.

Recovering Indifference Curves

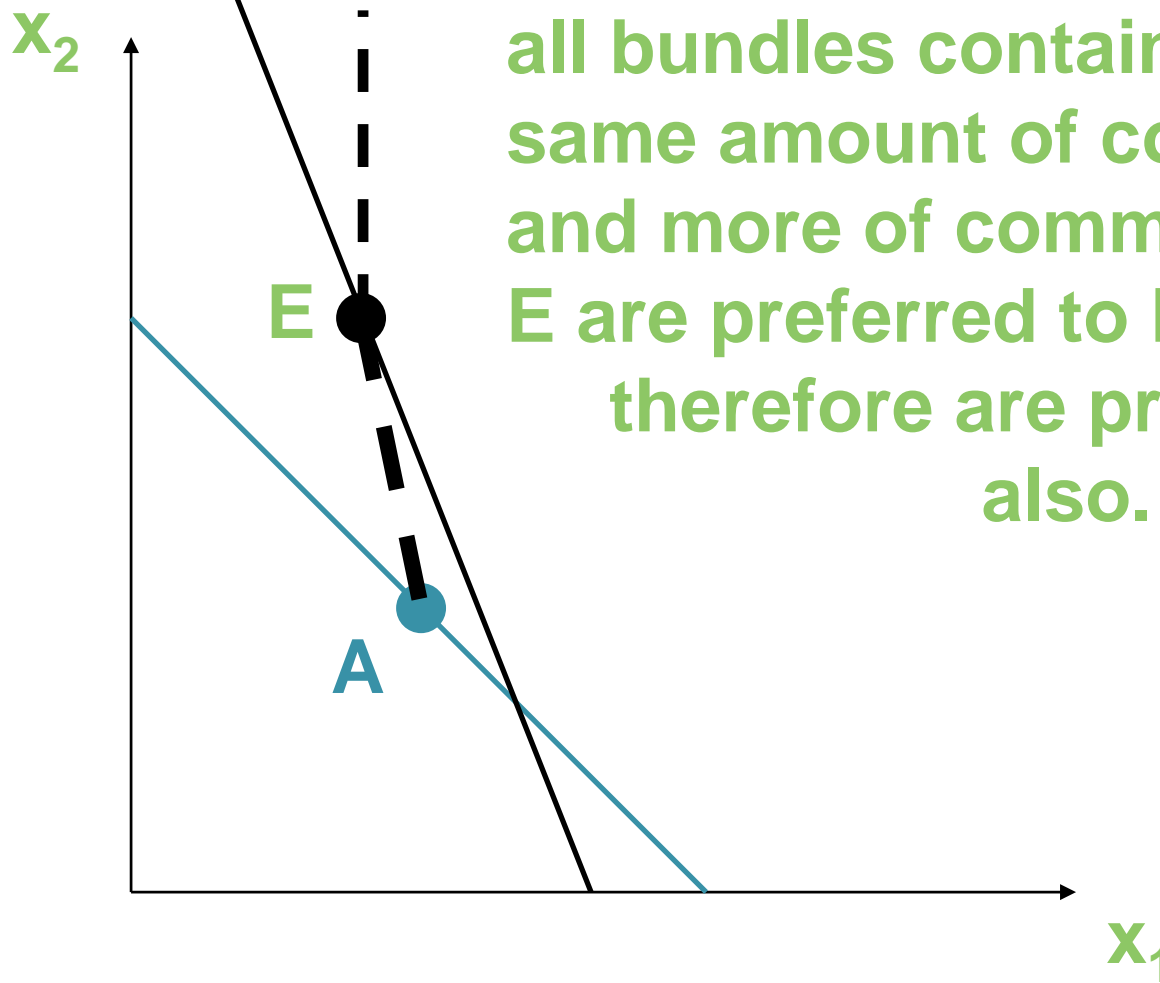


E is directly revealed preferred to A.

Well-behaved preferences are convex so all bundles on the line between A and E are preferred to A also.

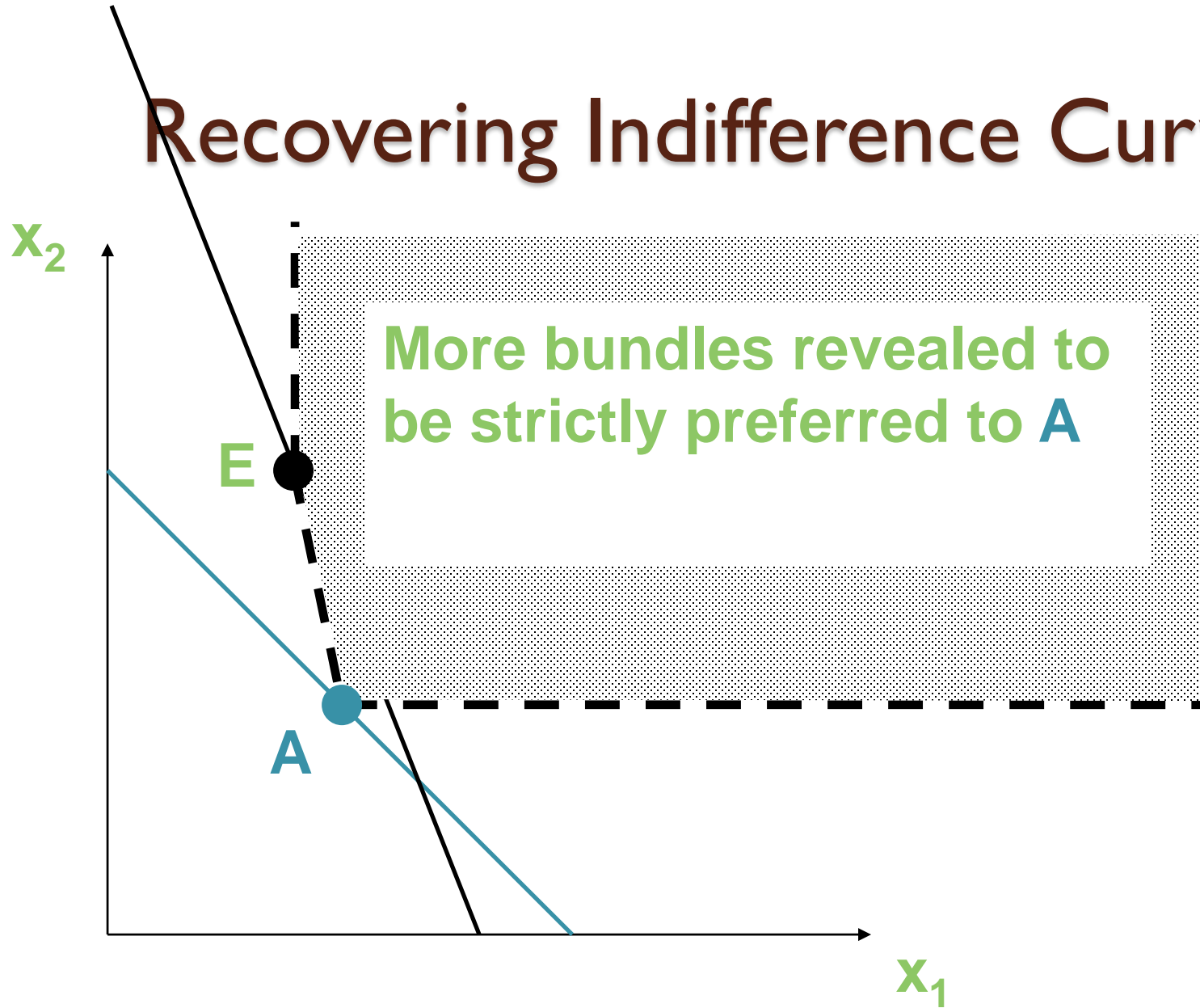
As well, ...

Recovering Indifference Curves

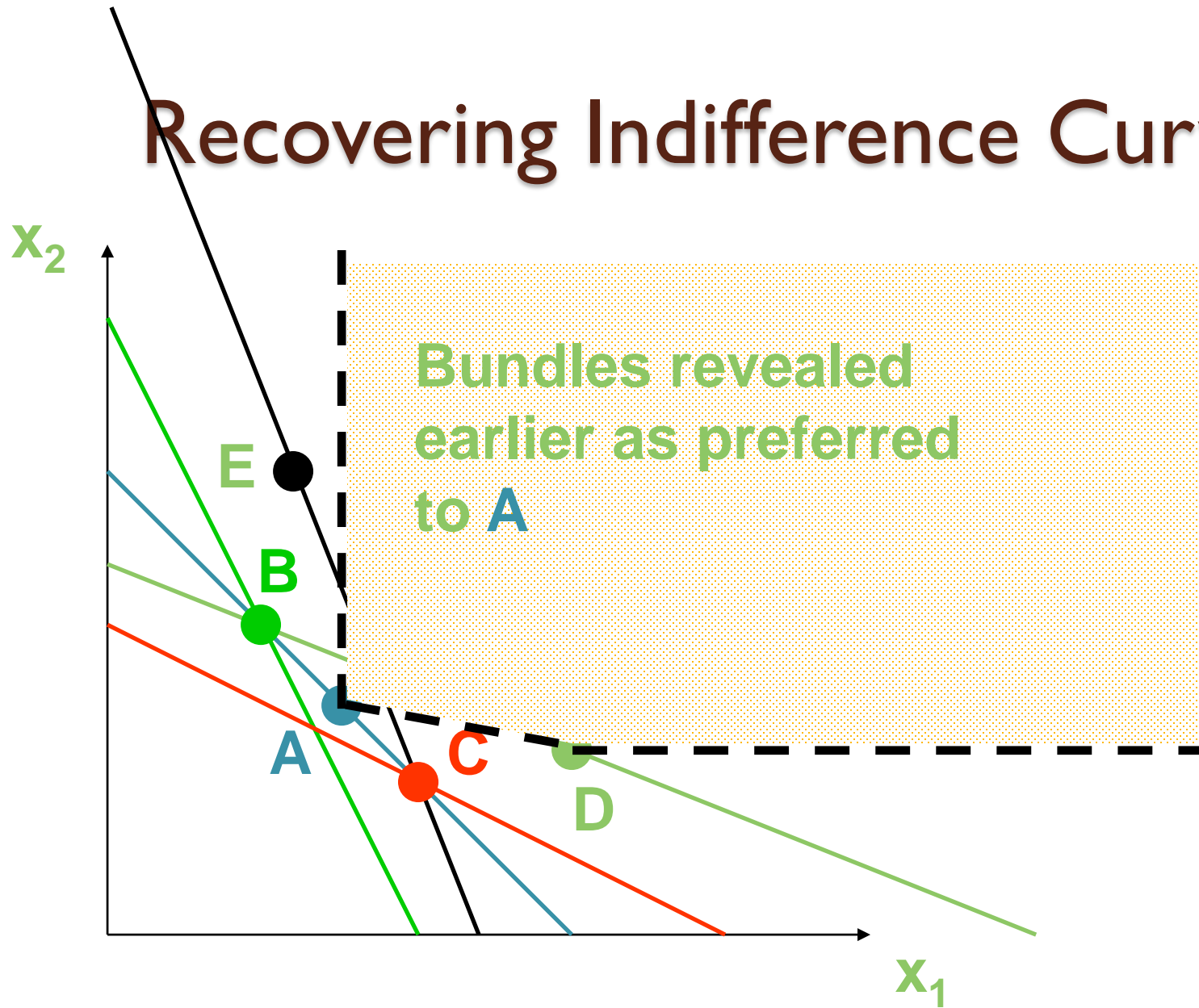


all bundles containing the same amount of commodity 1 and more of commodity 2 than E are preferred to E and therefore are preferred to A also.

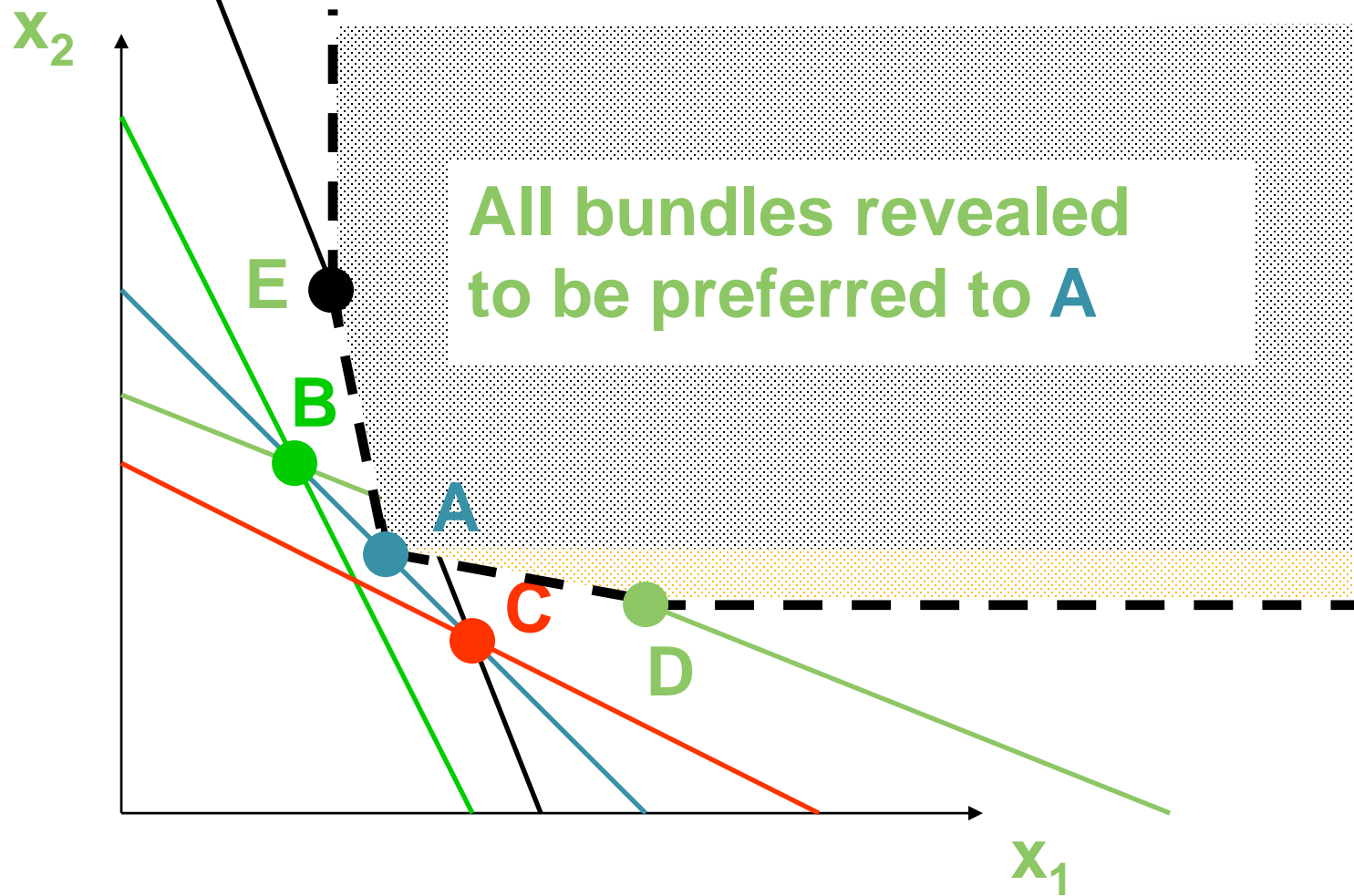
Recovering Indifference Curves



Recovering Indifference Curves



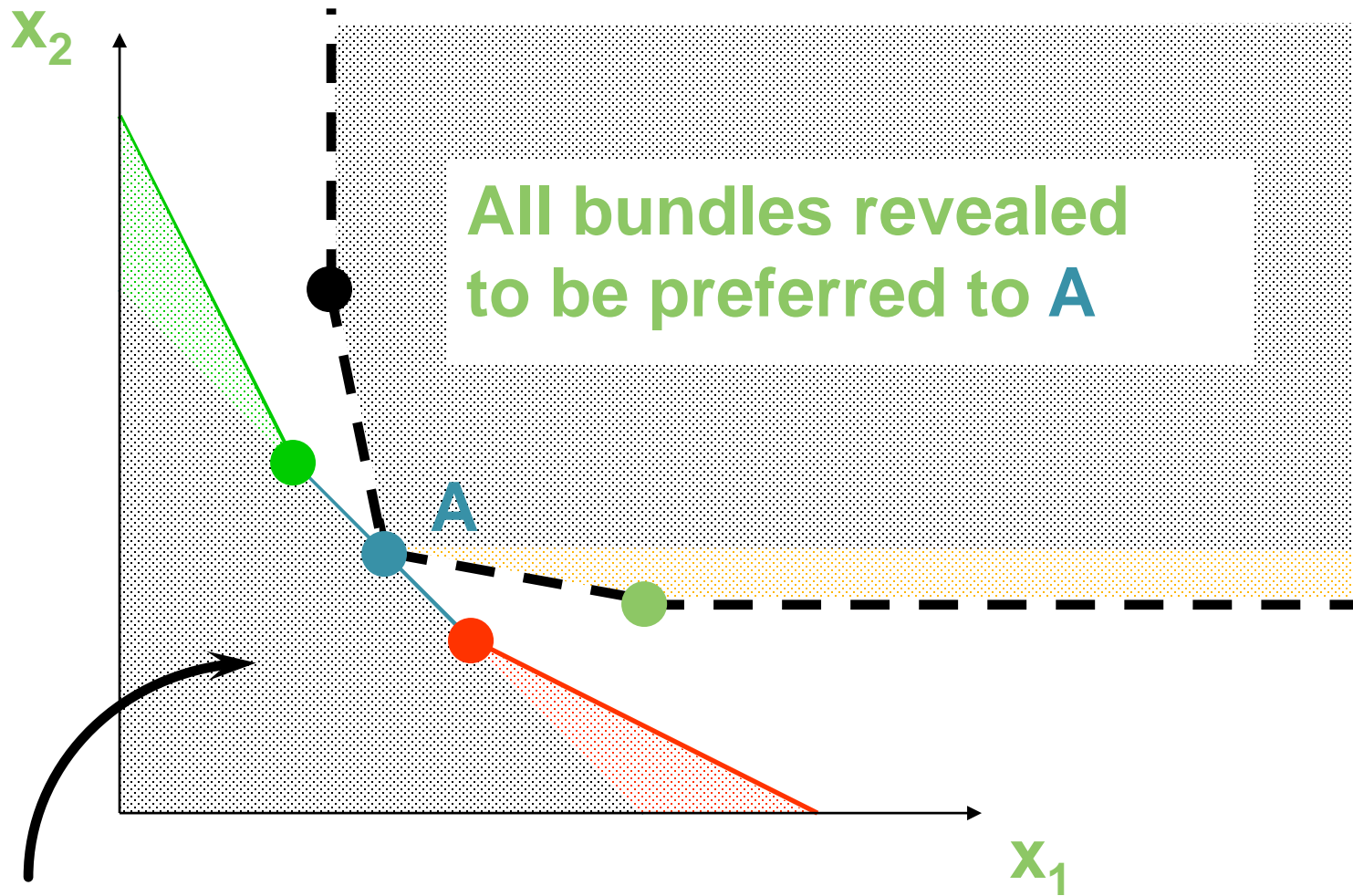
Recovering Indifference Curves



Recovering Indifference Curves

- Now we have upper and lower bounds on where the indifference curve containing bundle **A** may lie.

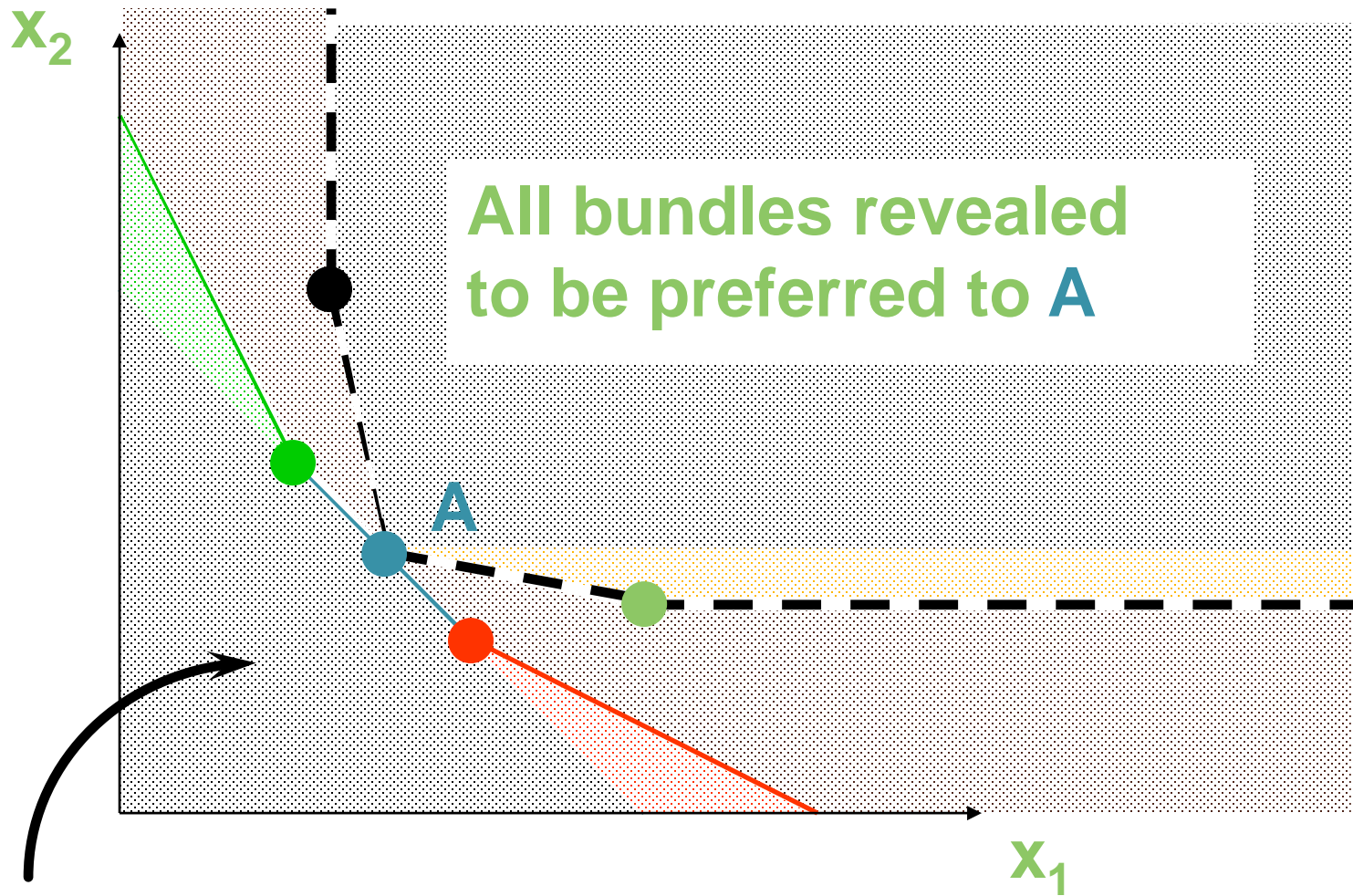
Recovering Indifference Curves



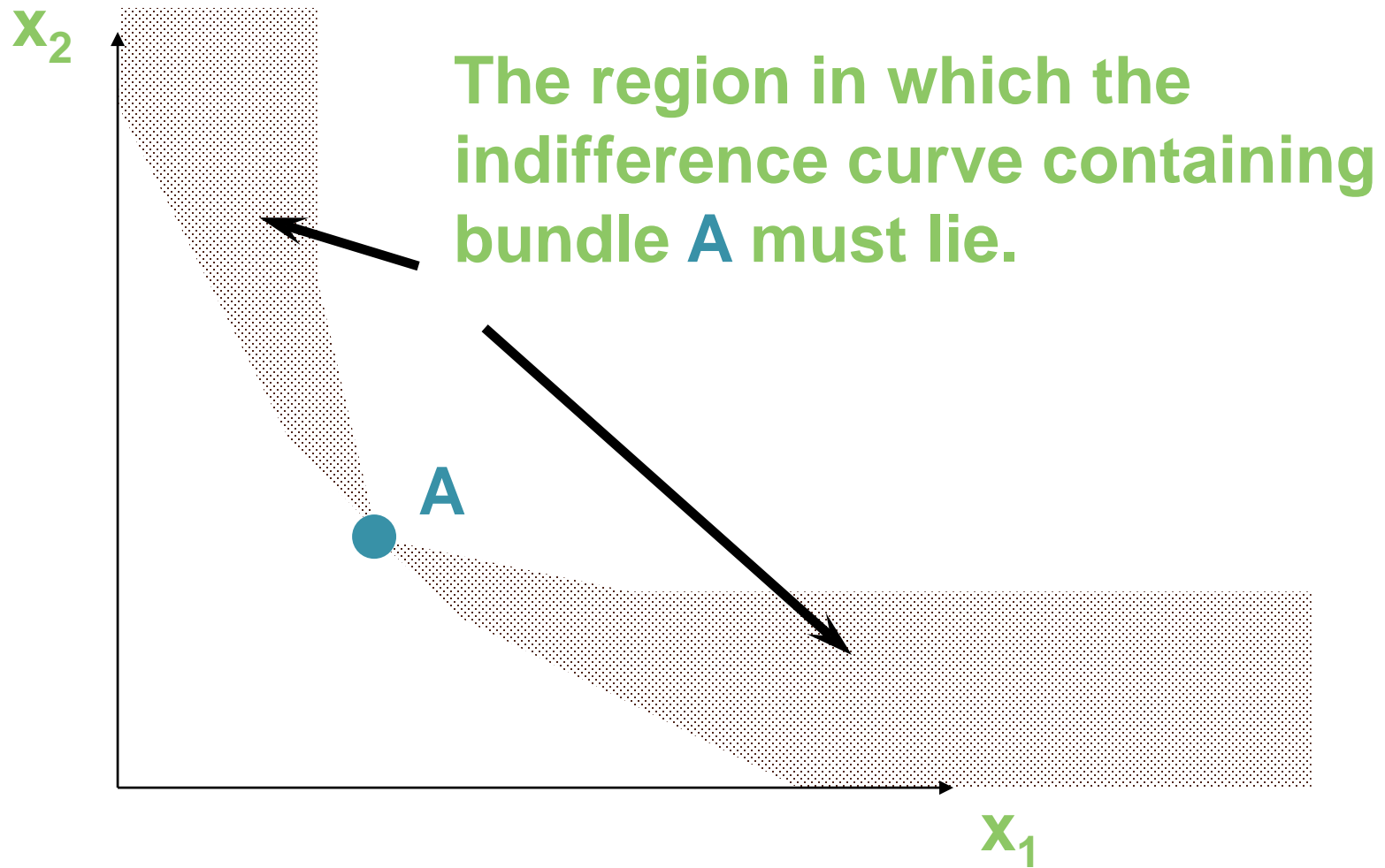
All bundles revealed
to be preferred to A

All bundles revealed to be less preferred to A

Recovering Indifference Curves



Recovering Indifference Curves



Index Numbers

- Over time, many prices change. Are consumers better or worse off “overall” as a consequence?
- Index numbers give approximate answers to such questions.

Index Numbers

- Two basic types of indices
 - price indices, and
 - quantity indices
- Each index compares expenditures in a base period and in a current period by taking the ratio of expenditures.

Quantity Index Numbers

- A quantity index is a price-weighted average of quantities demanded; *i.e.*

$$I_q = \frac{p_1 x_1^t + p_2 x_2^t}{p_1 x_1^b + p_2 x_2^b}$$

- (p_1, p_2) can be base period prices (p_1^b, p_2^b) or current period prices (p_1^t, p_2^t) .

Quantity Index Numbers

- If $(p_1, p_2) = (p_1^b, p_2^b)$ then we have the Laspeyres quantity index;

$$L_q = \frac{p_1^b x_1^t + p_2^b x_2^t}{p_1^b x_1^b + p_2^b x_2^b}$$

Quantity Index Numbers

- If $(p_1, p_2) = (p_1^t, p_2^t)$ then we have the Paasche quantity index;

$$P_q = \frac{p_1^t x_1^t + p_2^t x_2^t}{p_1^t x_1^b + p_2^t x_2^b}$$

Quantity Index Numbers

- How can quantity indices be used to make statements about changes in welfare?

Quantity Index Numbers

• If $L_q = \frac{p_1^b x_1^t + p_2^b x_2^t}{p_1^b x_1^b + p_2^b x_2^b} < 1$ then

$$p_1^b x_1^t + p_2^b x_2^t < p_1^b x_1^b + p_2^b x_2^b$$

so consumers overall were better off in the base period than they are now in the current period.

Quantity Index Numbers

- If $P_q = \frac{p_1^t x_1^t + p_2^t x_2^t}{p_1^t x_1^b + p_2^t x_2^b} > 1$ then

$$p_1^t x_1^t + p_2^t x_2^t > p_1^t x_1^b + p_2^t x_2^b$$

so consumers overall are better off in the current period than in the base period.

Price Index Numbers

- A price index is a quantity-weighted average of prices; *i.e.*

$$I_p = \frac{p_1^t x_1 + p_2^t x_2}{p_1^b x_1 + p_2^b x_2}$$

- (x_1, x_2) can be the base period bundle (x_1^b, x_2^b) or else the current period bundle (x_1^t, x_2^t) .

Price Index Numbers

- If $(x_1, x_2) = (x_1^b, x_2^b)$ then we have the Laspeyres price index;

$$L_p = \frac{p_1^t x_1^b + p_2^t x_2^b}{p_1^b x_1^b + p_2^b x_2^b}$$

Price Index Numbers

- If $(x_1, x_2) = (x_1^t, x_2^t)$ then we have the Paasche price index;

$$P_p = \frac{p_1^t x_1^t + p_2^t x_2^t}{p_1^b x_1^t + p_2^b x_2^t}$$

Price Index Numbers

- How can price indices be used to make statements about changes in welfare?
- Define the expenditure ratio

$$M = \frac{p_1^t x_1^t + p_2^t x_2^t}{p_1^b x_1^b + p_2^b x_2^b}$$

Price Index Numbers

- If

$$L_p = \frac{p_1^t x_1^b + p_2^t x_2^b}{p_1^b x_1^b + p_2^b x_2^b} < \frac{p_1^t x_1^t + p_2^t x_2^t}{p_1^b x_1^b + p_2^b x_2^b} = M$$

then

$$p_1^t x_1^b + p_2^t x_2^b < p_1^t x_1^t + p_2^t x_2^t$$

so consumers overall are better off in the current period.

Price Index Numbers

- But, if

$$P_p = \frac{p_1^t x_1^t + p_2^t x_2^t}{p_1^b x_1^t + p_2^b x_2^t} > \frac{p_1^t x_1^t + p_2^t x_2^t}{p_1^b x_1^b + p_2^b x_2^b} = M$$

then

$$p_1^b x_1^t + p_2^b x_2^t < p_1^b x_1^b + p_2^b x_2^b$$

so consumers overall were better off in the base period.

Full Indexation?

- Changes in price indices are sometimes used to adjust wage rates or transfer payments. This is called “indexation”.
- “Full indexation” occurs when the wages or payments are increased at the same rate as the price index being used to measure the aggregate inflation rate.

Full Indexation?

- Since prices do not all increase at the same rate, relative prices change along with the “general price level”.
- A common proposal is to index fully Social Security payments, with the intention of preserving for the elderly the “purchasing power” of these payments.

Full Indexation?

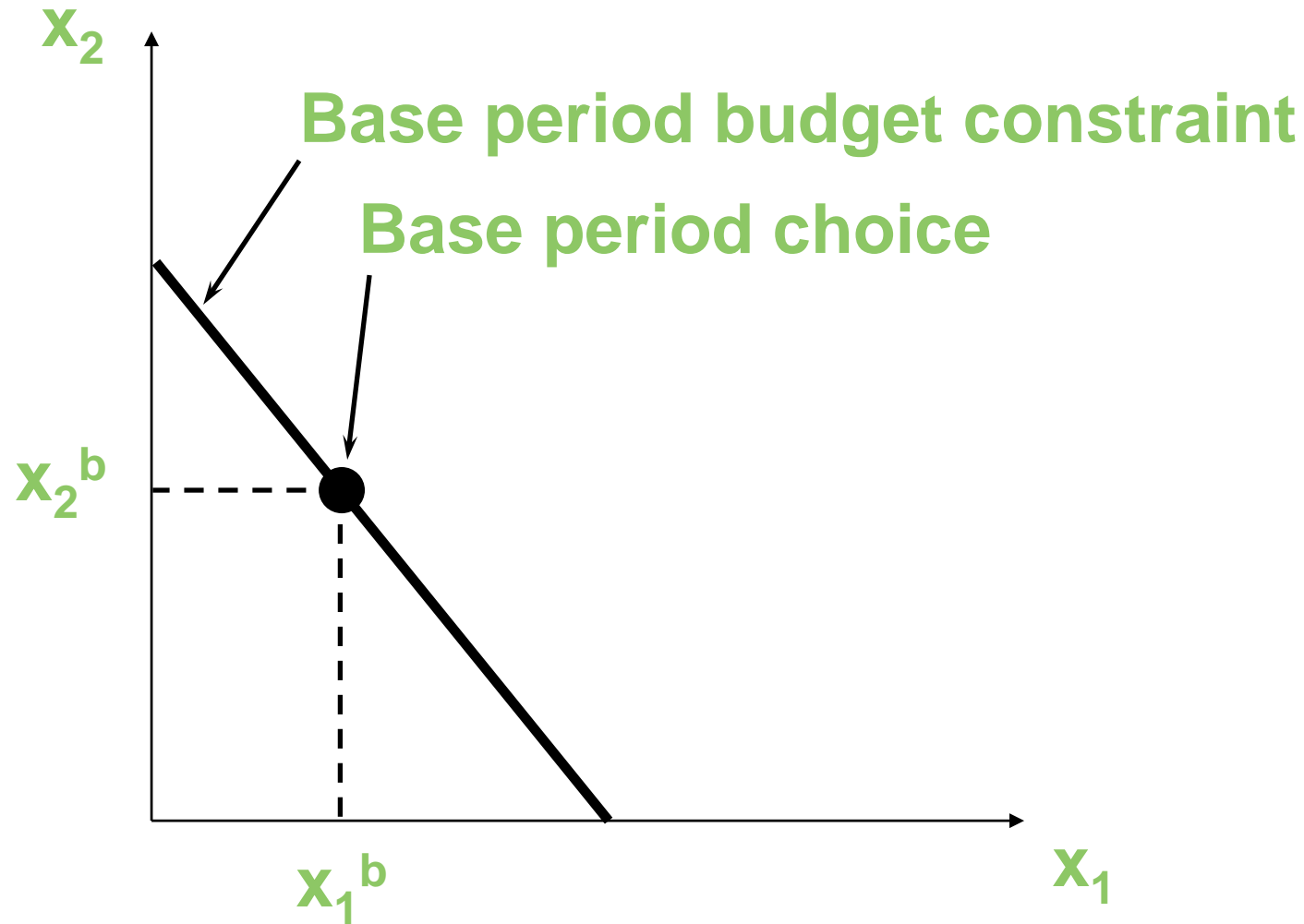
- The usual price index proposed for indexation is the Paasche quantity index (the Consumers' Price Index).
- What will be the consequence?

Full Indexation?

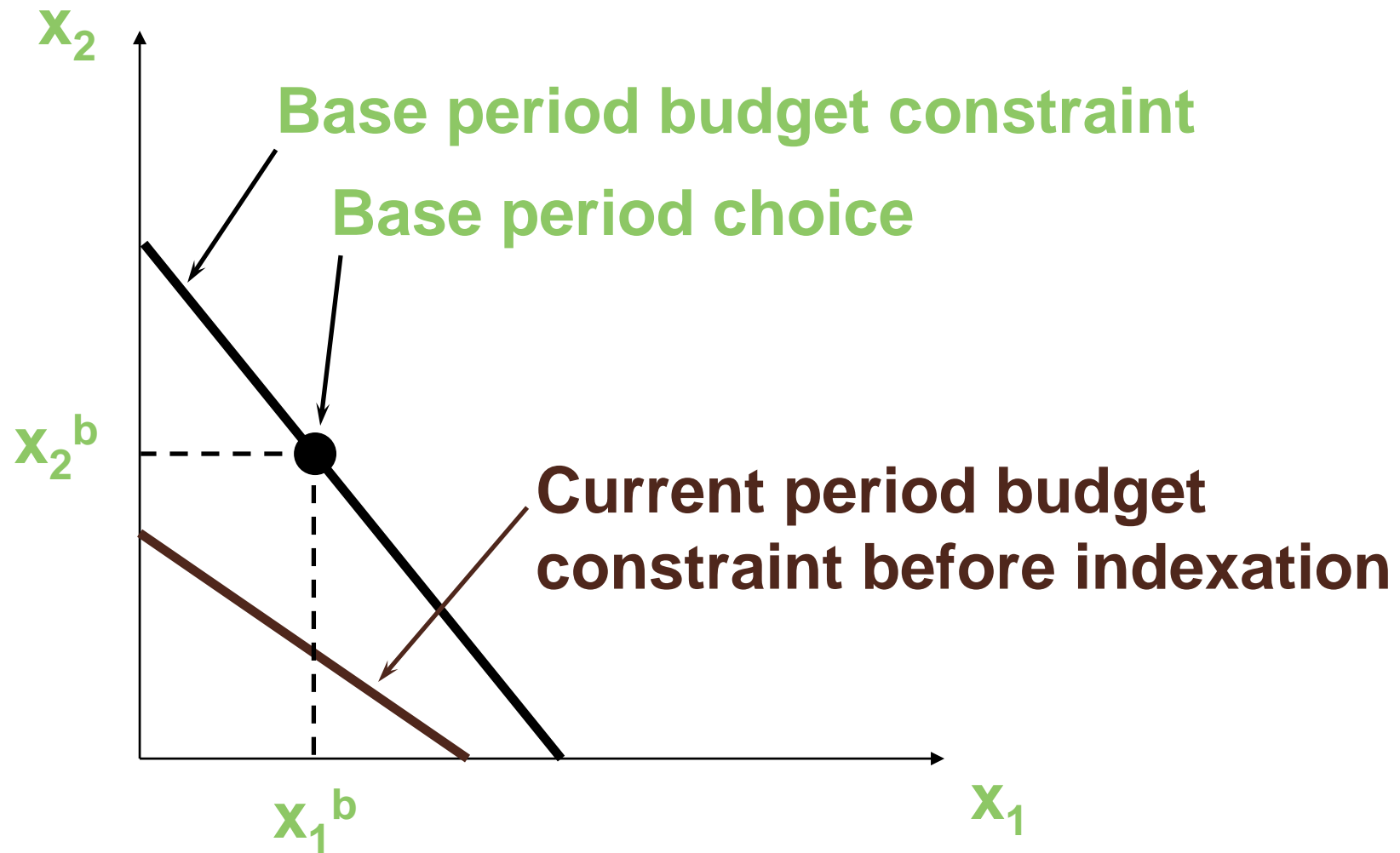
$$P_q = \frac{p_1^t x_1^t + p_2^t x_2^t}{p_1^t x_1^b + p_2^t x_2^b}$$

Notice that this index uses **current** period prices to weight both base and current period consumptions.

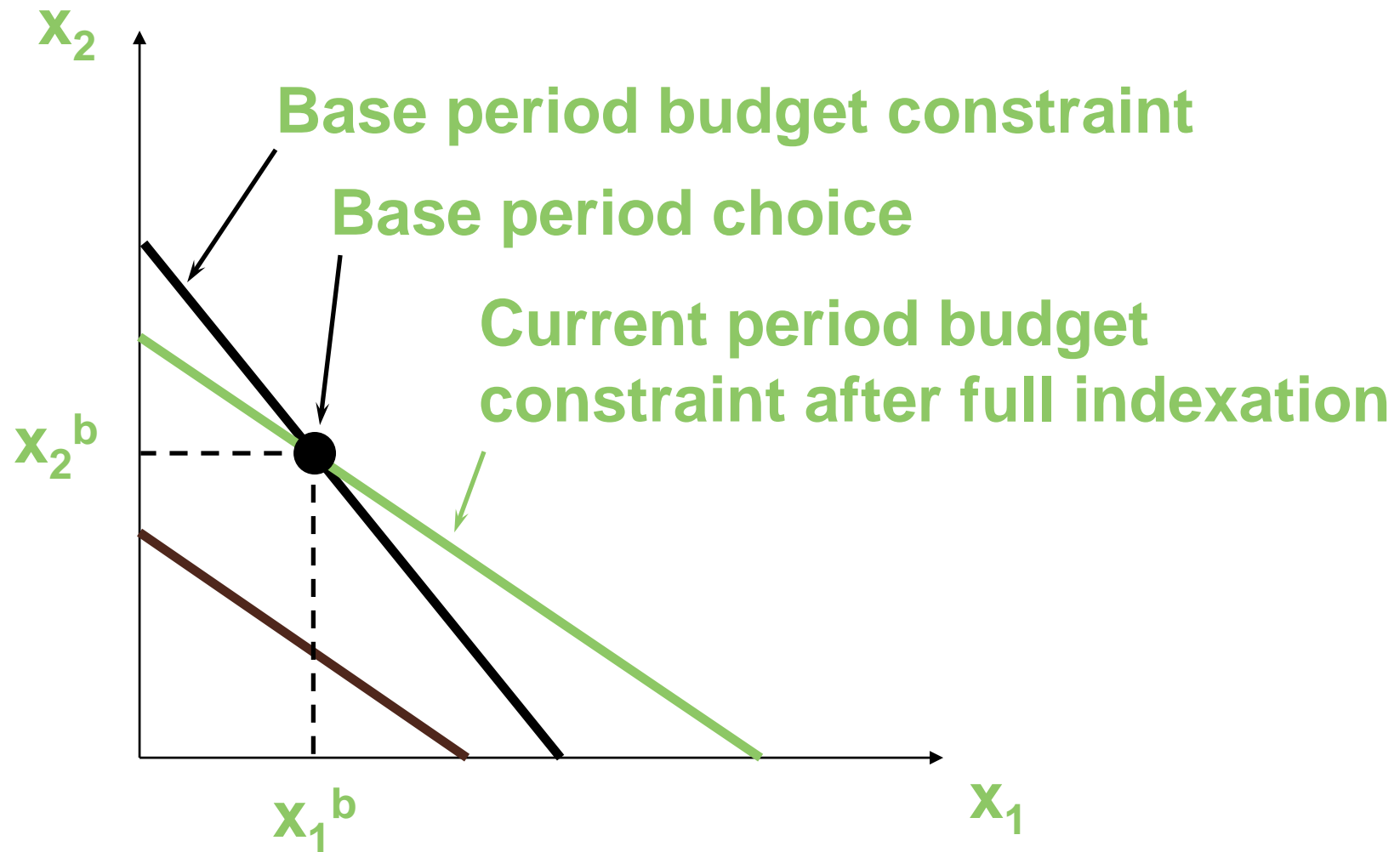
Full Indexation?



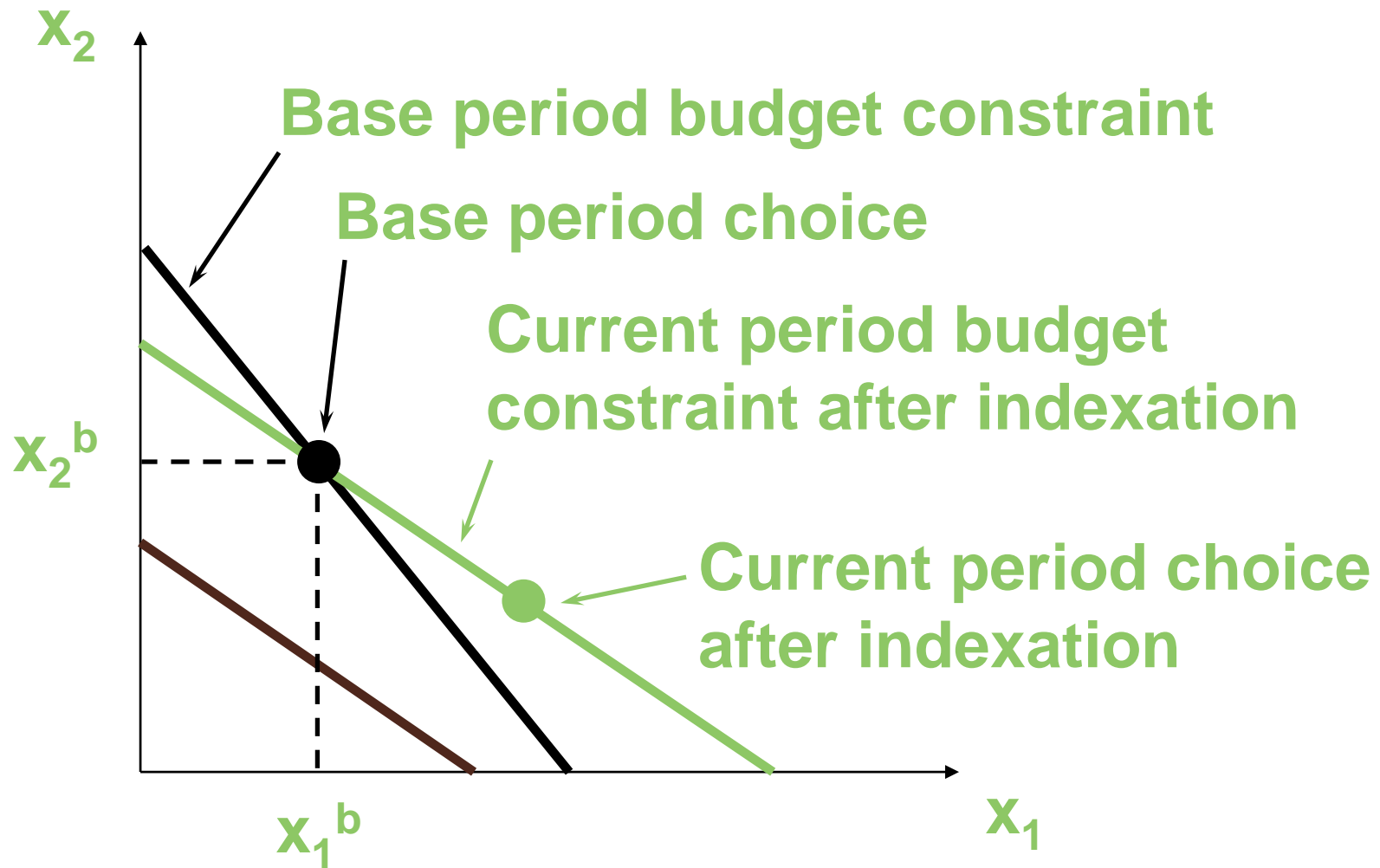
Full Indexation?



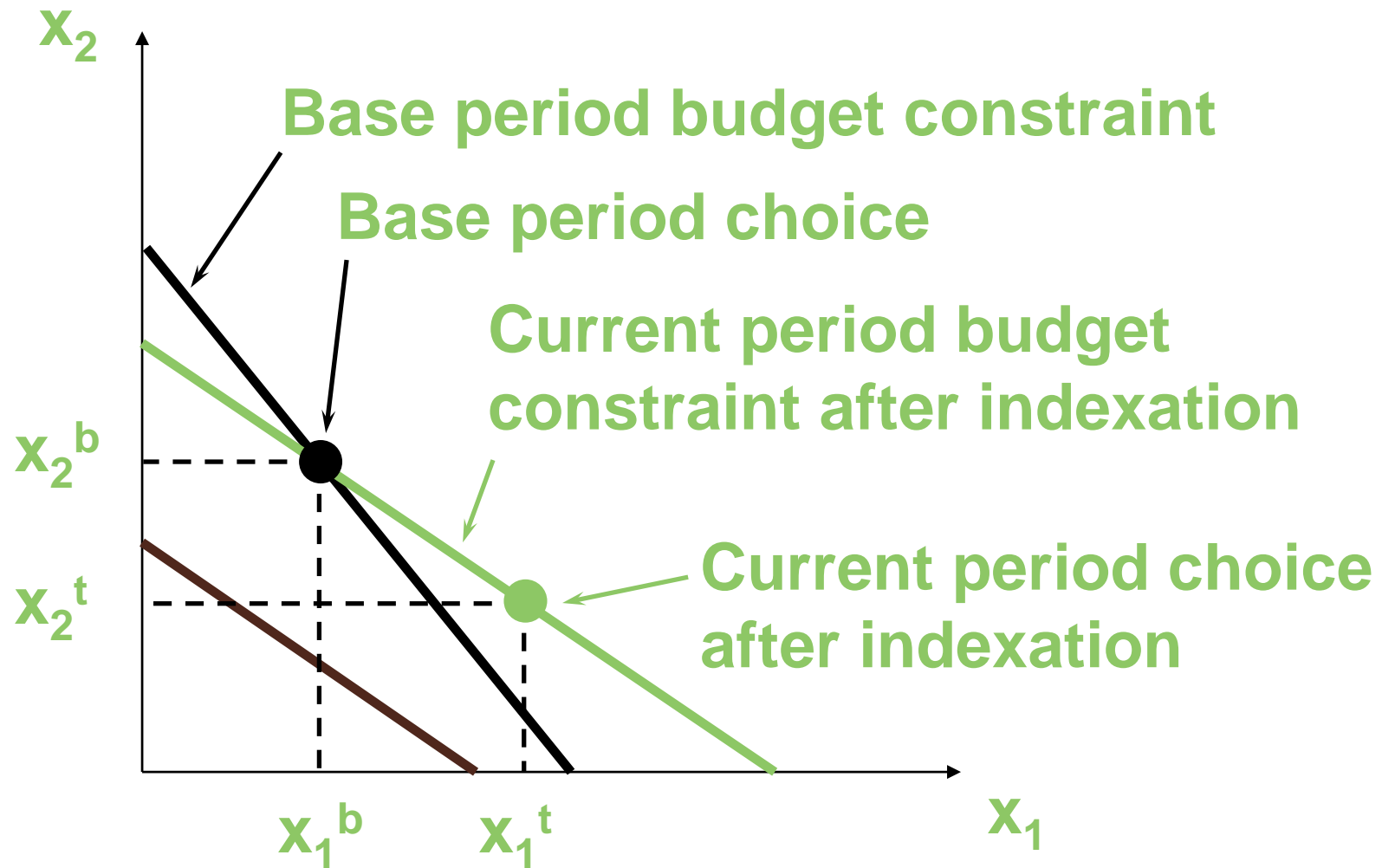
Full Indexation?



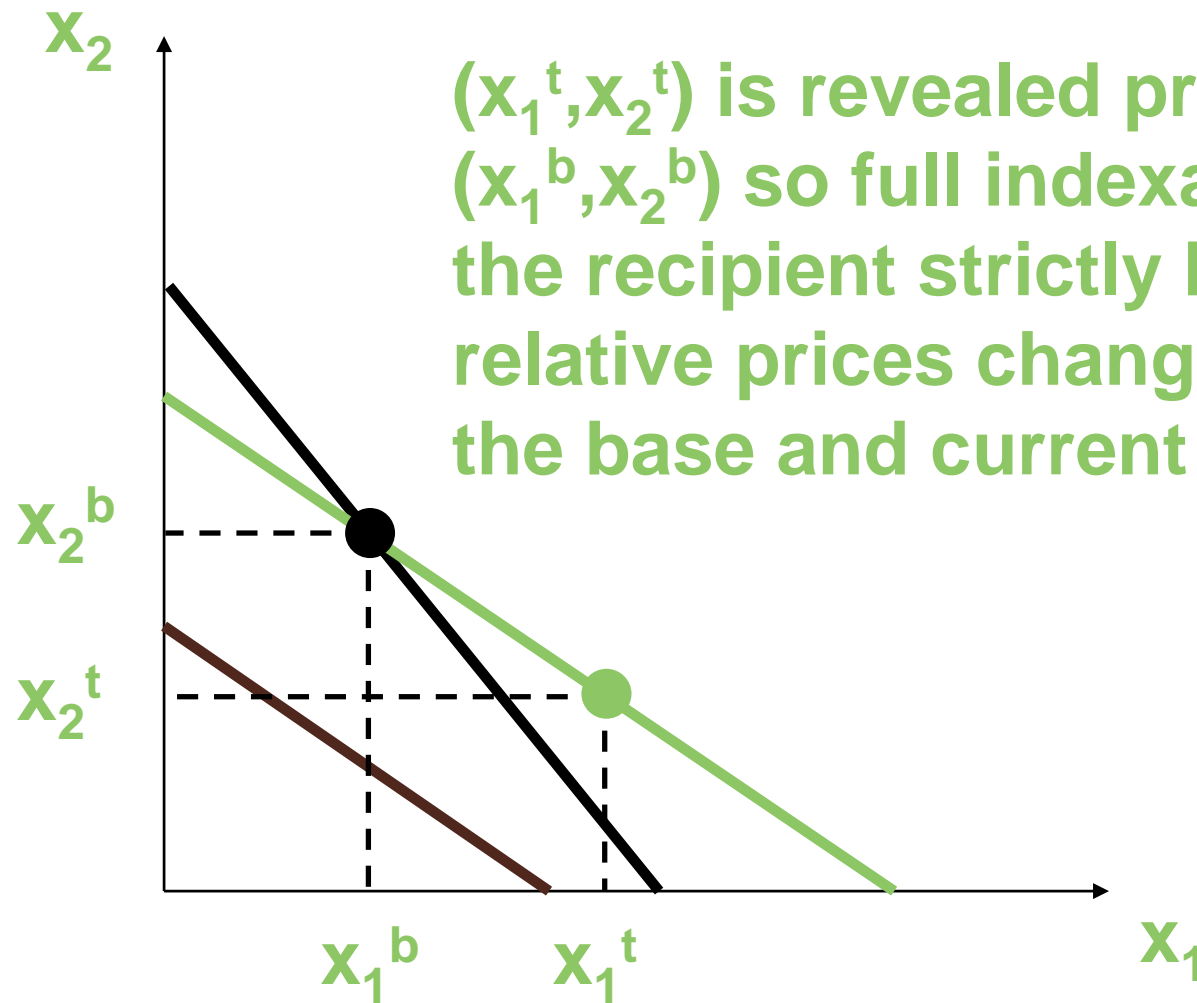
Full Indexation?



Full Indexation?



Full Indexation?



(x_1^t, x_2^t) is revealed preferred to (x_1^b, x_2^b) so full indexation makes the recipient strictly better off if relative prices change between the base and current periods.

Full Indexation?

- So how large is this “bias” in the US CPI?
- A table of recent estimates of the bias is given in the *Journal of Economic Perspectives*, Volume 10, No. 4, p. 160 (1996). Some of this list of point and interval estimates are as follows:

Full Indexation?

Author	Point Est.	Int. Est.
Adv. Commission to Study the CPI (1995)	1.0%	0.7 - 2.0%
Congressional Budget Office (1995)		0.2 - 0.8%
Alan Greenspan (1995)		0.5 - 1.5%
Shapiro & Wilcox (1996)	1.0%	0.6 - 1.5%

Full Indexation?

- So suppose a social security recipient gained by 1% per year for 20 years.
- Q: How large would the bias have become at the end of the period?

Full Indexation?

- So suppose a social security recipient gained by 1% per year for 20 years.
- Q: How large would the bias have become at the end of the period?
- A: $(1 + 0.01)^{20} = 1.01^{20} = 1.22$ so after 20 years social security payments would be about 22% “too large”.