HW6_2 solution

P6.72*

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = 1.125 \text{ MHz}$$

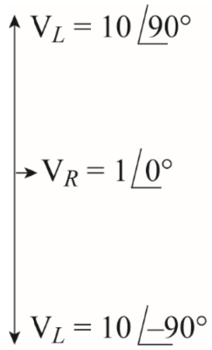
$$Q_s = \frac{2\pi f_0 L}{R} = 5$$

$$B = \frac{f_0}{Q_s} = 255 \text{ kHz}$$

$$f_H \cong f_0 + \frac{B}{2} = 1.2375 \text{ MHz}$$

$$f_L \cong f_0 - \frac{B}{2} = 1.0125 \text{ MHz}$$

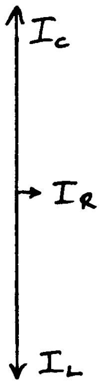
At the resonant frequency:



$\mathbf{V}_R = 1\angle 0^\circ$ $\quad$ $\mathbf{V}_L = 5\angle 90^\circ$ $\quad$ $\mathbf{V}_C = 5\angle -90^\circ$

P6.81

$$Q_p = \frac{f_0}{B} = 20$$



$$C = \frac{Q_p}{2\pi f_0 R} = 318.3 \text{ pF}$$

$$L = \frac{R}{2\pi f_0 Q_p} = 0.7958\ \mu\text{H}$$

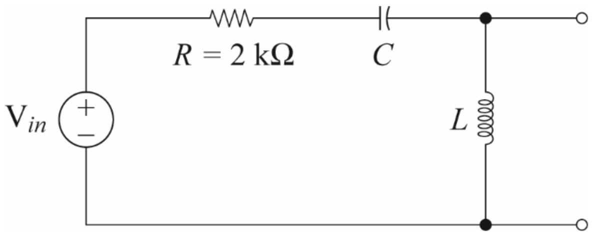
$$\mathbf{I} = \mathbf{I}_R = 1\angle 0^\circ \text{ mA}$$

$$\mathbf{I}_L = \frac{\mathbf{V}}{j2\pi f_0 L} = \frac{R\mathbf{I}}{j2\pi f_0 L} = 20\angle -90^\circ \text{ mA}$$

$$\mathbf{I}_C = \frac{\mathbf{V}}{1/(j2\pi f_o C)} = \frac{R\mathbf{I}}{1/(j2\pi f_0 C)} = 20\angle +90^\circ \text{ mA}$$

P6.88*

The circuit diagram of a second-order highpass filter is:



$$L = \frac{RQ_s}{2\pi f_0} = 3.184\,\text{mH} \qquad C = \frac{1}{Q_s R(2\pi f_0)} = 796\,\text{pF}$$

P6.98

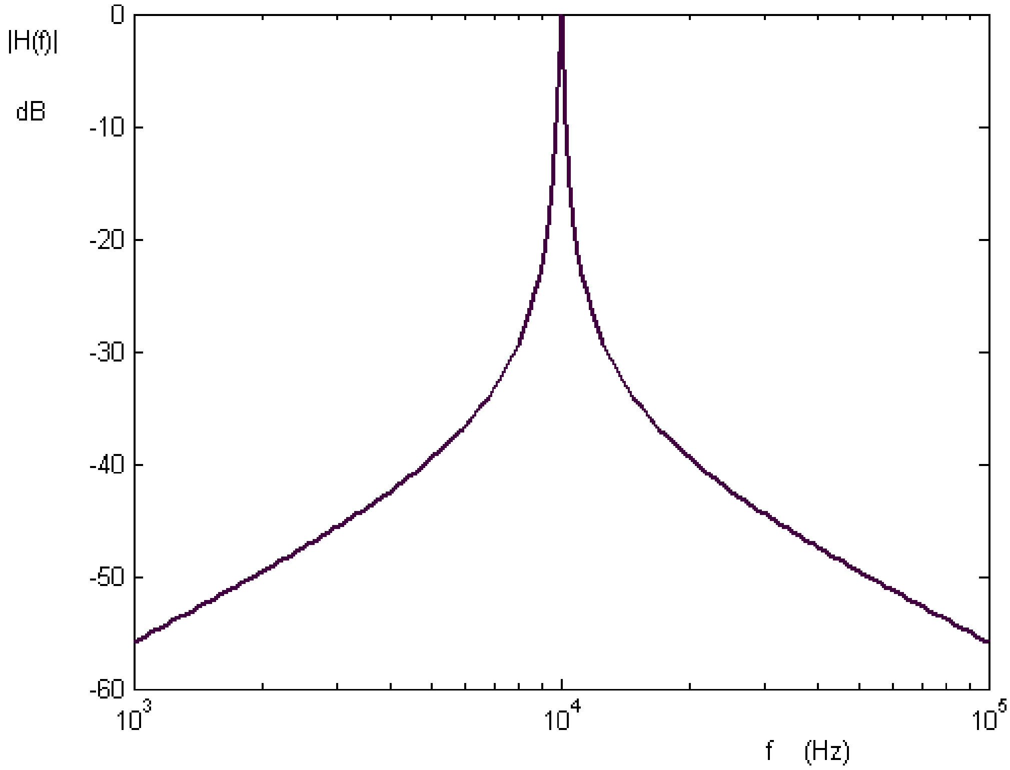
(a) Applying the voltage-division principle, we have

$$H(f) = \frac{R}{R + j2\pi f L - j/(2\pi f C)}$$

(b) A MATLAB program to produce the desired plot is

```
R = 10;
L = 0.01;
C = 2.533e-8;
f = logspace(3,5,2000);
w = 2*pi*f;
H = R./(R+j*w*L + 1./(j*w*C ));
semilogx(f,20*log10(abs(H)))
```

The resulting plot is



(c) At very low frequencies, with the capacitance considered to be an open circuit, no current flows and $H(f)$ becomes very small in magnitude as shown in the plot.

(d) At very high frequencies with the inductance considered as an open circuit, no current flows and $H(f)$ becomes very small in magnitude as shown in the plot.

P6.100

(a) Writing a current equation at the node joining the inductance and resistance, we have

$$\frac{1}{L}\int_0^t [y(t) - x(t)]dt + i_L(0) + \frac{y(t)}{R} = 0$$

Taking the derivative with respect to time we have

$$\frac{1}{L}y(t) - \frac{1}{L}x(t) + \frac{1}{R}\frac{dy(t)}{dt} = 0$$

Multiplying each term by L and using the fact that the time constant is $\tau = L/R$, we obtain

$$y(t) - x(t) + \tau \frac{dy(t)}{dt} = 0$$

Then we approximate the derivative and write the following approximation to the differential equation.

$$y(n) - x(n) + \tau \frac{y(n) - y(n-1)}{T} = 0$$

Solving for $y(n)$, we obtain the equation for the digital filter:

$$y(n) = \frac{\tau / T}{1 + \tau / T} y(n-1) + \frac{1}{1 + \tau / T} x(n)$$

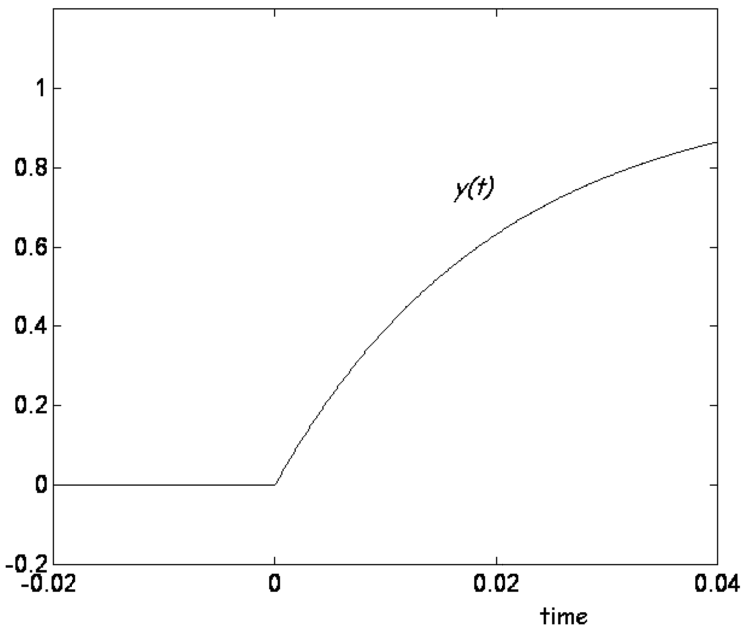
(b) For the values given the time constant is $\tau = L/R = 20 \text{ ms}$. The step input is

$$\begin{aligned} x(t) &= 0 \quad \text{for } t < 0 \\ &= 1 \quad \text{for } t \geq 0 \end{aligned}$$

Using the methods of Chapter 4, we have

$$\begin{aligned} y(t) &= 0 \quad \text{for } t < 0 \\ &= 1 - \exp(t/\tau) \quad \text{for } t \geq 0 \end{aligned}$$

A plot of $y(t)$ versus t is:



(c) The sampling interval is T = 1/500 = 2 ms, and we have τ/T = 10. Thus the defining equation for the digital filter is:

$$y(n) = \frac{10}{11} y(n-1) + \frac{1}{11} x(n)$$

The step input to the digital filter is defined as

$$x(n) = 0 \quad \text{for } n < 0$$
$$= 1 \quad \text{for } n \geq 0$$

A list of MATLAB commands to compute and plot x and y is:

```
t = -20e-3:2e-3:60e-3;
x = ones(size(t));
for n = 1:10
x(n) = 0;
end
y =zeros(size(t))
for n = 2:31
y(n) = (10/11)*y(n-1) +(1/11)*x(n);
end
plot(t,y,'wo')
hold
plot(t,x, 'wx')
axis([-20e-3 40e-3 -0.2 1.2])
```