電磁 H.W p.7　　E14086622 探三晴

Problem 7

A half-wave dipole antenna radiates 10 W at a frequency of 5 GHz. A short dipole antenna situated at a distance of 100 m is used as a receiving antenna. If both antennas are symmetrically placed in the *xy* plane and the medium is free space, (a) determine the effective area of each antenna and (b) the power absorbed by the receiving antenna. (c) If the minimal received power of 1 nW, what is the maximal distance allowed?

Answers: $(a)\ldots\ and\ A_{e\_short\ dipole} = 4.3\,cm^2, (b)\ldots, and\ (c)?49\ m.$

$P_t = 10W \qquad \lambda = c/f = 0.06$

$f = 5G$

$r = 100m$

(a)

$A_{er} = \frac{\lambda^2}{4\pi} D_r = \frac{0.06^2}{4\pi} \cdot 1.5 = 4.297 \cdot 10^{-4}\ m^2$ #

$A_{et} = \frac{\lambda^2}{4\pi} D_t = \frac{0.06^2}{4\pi} \cdot 1.69 = 4.841 \cdot 10^{-4}\ m^2$ #

(b)

$\frac{P_r}{P_t} = \frac{A_{et}A_{er}}{\lambda^2 r^2} \Rightarrow P_r = 5.78 \cdot 10^{-8}\ (W)$

(c)

$P_{rmin} = 1nW$

$\frac{P_r}{P_t} = \frac{A_{et}A_{er}}{\lambda^2 r^2}$

$\frac{1n}{10} = \frac{4.3 \cdot 10^{-4} \cdot 4.8 \cdot 10^{-4}}{0.06^2 r^2}, \quad r = 749\ m$ #

Half-wave:

$\vec{A} = \vec{a_z} \frac{\mu_0 I}{2\pi r} \frac{e^{-j\beta r}}{\sin^2\theta} \cos(\frac{\pi}{2}\cos\theta)$

$\vec{H} = \frac{1}{\mu_0} \nabla \times \vec{A} = \vec{a_\phi} \frac{jIe^{-j\beta r}}{2\pi r \sin\theta} \cos(\frac{\pi}{2}\cos\theta)$

$\vec{P_{avg}} = \frac{1}{2}\eta H_\phi^2 \vec{a_r}$

$= \frac{15 I^2}{r^2 \sin^2\theta\, \pi} \cos^2(\frac{\pi}{2}\cos\theta) \approx \frac{15}{\pi}\frac{I^2}{r^2}\sin^3\theta$

$U(\theta,\phi) = \vec{P_{avg}} \cdot r^2 = \frac{15}{\pi} I^2 \sin^3\theta$

$\Rightarrow U_{max} = \frac{15}{\pi} I^2 \big|_{\theta=90}$

$P_{rad} = \int_0^{2\pi}\int_0^{\pi} \frac{15 I^2}{\pi} \sin^3\theta \sin\theta\, d\theta\, d\phi$

$= 30 I^2 \cdot \frac{3}{8}\pi = \frac{45}{4}\pi I^2$

$D = \frac{U_{max}}{U_{avg}} = \frac{U_{max}}{P_{rad}/4\pi} = 1.698 = D_t$

short dipole:

$\vec{A} = \vec{a_z} \frac{\mu_0 I dl}{4\pi r} e^{-j\beta r}$

$\vec{H} = \mu_0 \nabla \times \vec{A} \approx \vec{a_\phi} \frac{I dl}{4\pi} \sin\theta \frac{j\beta}{r} e^{-j\beta r}$

$\vec{P_{avg}} = \frac{1}{2}\eta H_\phi^2 \vec{a_r} = \frac{\eta\beta^2 I^2 dl^2}{32\pi^2 r^2} \sin^2\theta$

$U(\theta,\phi) = \vec{P_{avg}} \cdot r^2 = \frac{15}{4\pi}\beta^2 I^2 dl^2 \sin^2\theta$

$P_{rad} = \int_0^{2\pi}\int_0^{\pi} U(\theta,\phi)\sin\theta\, d\theta\, d\phi$

$= \frac{\eta\pi I^2}{3}[\frac{dl}{\lambda}]^2$

$D = \frac{U_{max}}{U_{avg}} = \frac{U_{max}}{P_{rad}/4\pi} = \frac{3}{2}\sin^2\theta \big|_{max}$

$= 1.5 \quad \theta = 90°$

$= D_r$

电磁 H.W p.8 E14086622 楊三晴.

Problem 8

The Arecibo Observatory in Puerto Rico has a gigantic dish antenna of diameter of 1000 ft (304.8 m). It transmits power of 2.5 MW at a frequency of 430 MHz. (a) Assuming a 60 percent effective area, what is its gain in dB? (b) What is *its half-power beam width* in degrees? (c) If used as a radar and the minimum detectable received power is −130 dBW, what is its maximum range for detecting a target of radar cross-section of 1 m^2?

Answers: $(a)\ldots$, $(b)\,0.1?°$, *and* $(c)?408$ *km.*

$d = 304.8\,m = 2r$, $r = 152.4\,m$

$P_t = 2.5\,MW$

$f = 430\,MHz$

(a)

$A_e = \pi r^2 \cdot 0.6 = 43779.5$

$= \frac{\lambda^2}{4\pi} D$

$\lambda = c/f = \frac{30}{43}$

$\therefore D = 1.13\,M = 10\log(1.13)\ dB$

$= 60.53\ dB$ #.

(b)

$HPBW = 70\lambda/d$

$= 0.16\,m \approx 0.16°$ #

(c)

$P_r = -130\,dB = 10^{-13}\,W$

$\sigma = 1\,m^2$

by Radar transmission eq,

$$\frac{P_r}{P_t} = \frac{(\lambda G)^2 \sigma}{(4\pi)^3 r^4}$$

$$\frac{10^{-13}}{2.5M} = \frac{(\frac{30}{43} \cdot 1.13M)^2 \cdot 1}{(4\pi)^3 \cdot r^4}$$

$r = 9.4\,Mm$ #.