HW6_1 solution

P6.16

From Figure P6.16, we see that the period of the signals is 10 ms. Therefore, the frequency is 100 Hz. Because the input reaches a positive peak at *t* = 2 ms, it has a phase angle of

$$\theta_{in} = -(t_d / T) \times 360^\circ = -(2/10) \times 360^\circ = -72^\circ$$

The output reaches its peak at 4 ms, and its phase angle is

$$\theta_{out} = -(t_d / T) \times 360^\circ = -(4/10) \times 360^\circ = -144^\circ$$

The transfer function is

$$H(100) = \frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = \frac{2\angle -144^\circ}{1\angle -72^\circ} = 2\angle -72^\circ$$

P6.25*

The half-power frequency of the filter is

$$f_B = \frac{1}{2\pi RC} = 500\,\text{Hz}$$

The transfer function is given by Equation 6.9 in the text:

$$H(f) = \frac{1}{1 + j(f/f_B)}$$

The given input signal is

$$v_{in}(t) = 5\cos(500\pi t) + 10\cos(1000\pi t - 30^\circ) + 15\cos(2000\pi t + 60^\circ)$$

which has components with frequencies of 250, 500, and 1000 Hz.

Evaluating the transfer function for these frequencies yields:

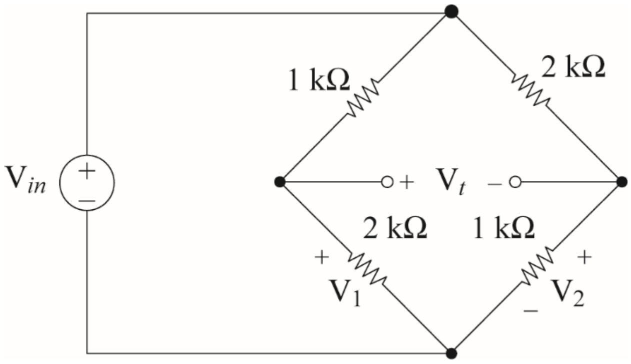
$$H(250) = \frac{1}{1 + j(250/500)} = 0.8944\angle -26.57^\circ$$
$$H(500) = 0.7071\angle -45^\circ$$
$$H(1000) = 0.4472\angle -63.43^\circ$$

Applying the appropriate value of the transfer function to each component of the input signal yields the output:

$$v_{out}(t) = 4.472\cos(500\pi t - 26.57^\circ) + 7.071\cos(1000\pi t - 75^\circ) + 6.708\cos(2000\pi t + 3.43^\circ)$$

P6.30*

The circuit seen by the capacitance is:



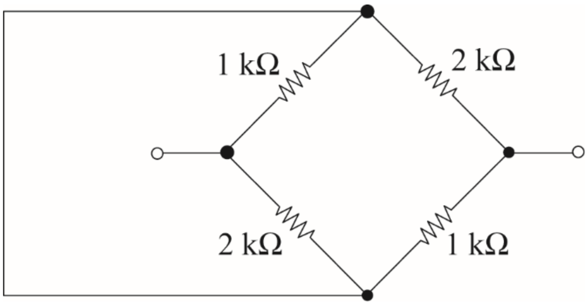
The open-circuit or Thévenin voltage is

$$\mathbf{V}_t = \mathbf{V}_1 - \mathbf{V}_2$$
$$= \mathbf{V}_{in}\frac{2000}{2000+1000} - \mathbf{V}_{in}\frac{1000}{1000+2000}$$

Thus, we obtain:

$$\mathbf{V}_t = \frac{1}{3}\mathbf{V}_{in} \qquad (1)$$

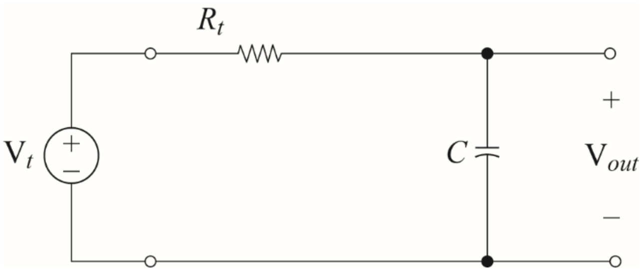
Zeroing the source, we have



The Thévenin resistance is

$$R_t = \frac{1}{1/1000 + 1/2000} + \frac{1}{1/2000 + 1/1000}$$

Thus, the equivalent circuit is:



As in the text, this circuit has the transfer function:

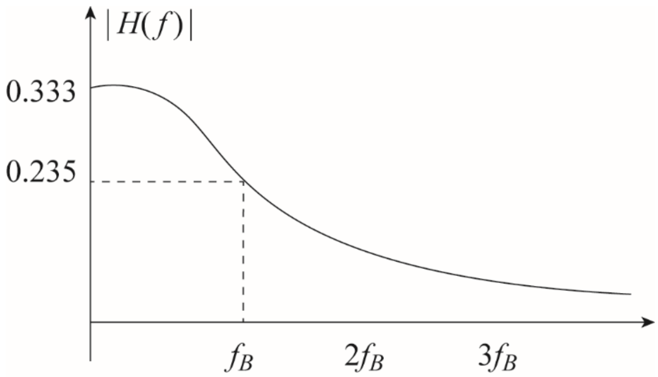
$$\frac{\mathbf{V}_{out}}{\mathbf{V}_t} = \frac{1}{1 + j(f/f_B)} \qquad (2)$$

where $f_B = \dfrac{1}{2\pi R_t C} = \dfrac{1}{2\pi 1333 \times 10^{-5} \times 2} = 5.97\text{ Hz}$

Using Equation (1) to substitute for $\mathbf{V}_t$ in Equation (2) and rearranging, we have

$$\frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = \frac{1/3}{1 + j(f/f_B)}$$

A sketch of the transfer-function magnitude is:



P6.46*

(a) The overall transfer function is the product of the transfer

functions of the filters in cascade:

$$H(f) = H_1(f) \times H_2(f) = \frac{1}{[1 + j(f/f_B)]^2}$$

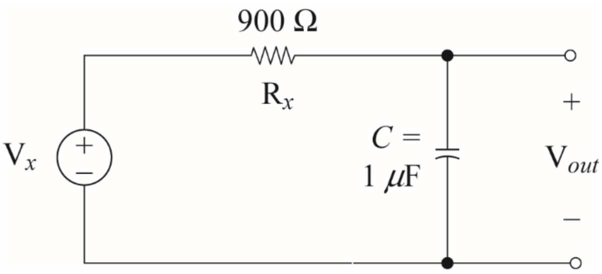
(b)
$$|H(f)| = \frac{1}{1 + (f/f_B)^2}$$
$$|H(f_{3dB})| = \frac{1}{\sqrt{2}} = \frac{1}{1 + (f_{3dB}/f_B)^2}$$
$$(f_{3dB}/f_B)^2 = \sqrt{2} - 1$$
$$f_{3dB} = f_B\sqrt{\sqrt{2} - 1} = 0.6436 f_B$$

P6.57

First, we find the Thévenin equivalent for the source and the resistances. The Thévenin resistance is

$$R_t = \frac{1}{1/R_1 + 1/R_2} = 900\ \Omega$$

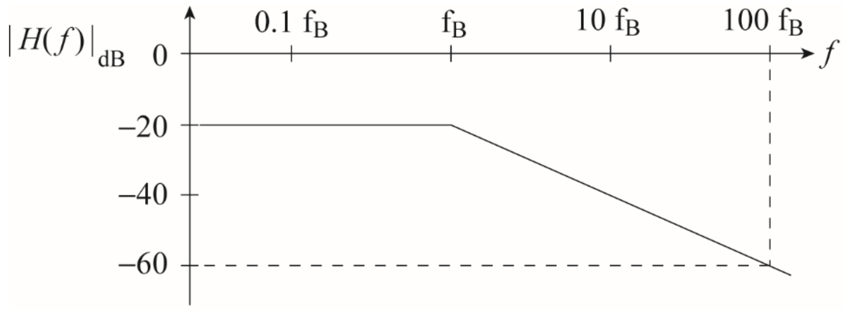
and the Thévenin voltage is
$$\mathbf{V}_t = \frac{R_2}{R_1 + R_2}\mathbf{V}_{in} = 0.1\mathbf{V}_{in}$$
Thus, an equivalent for the original circuit is:

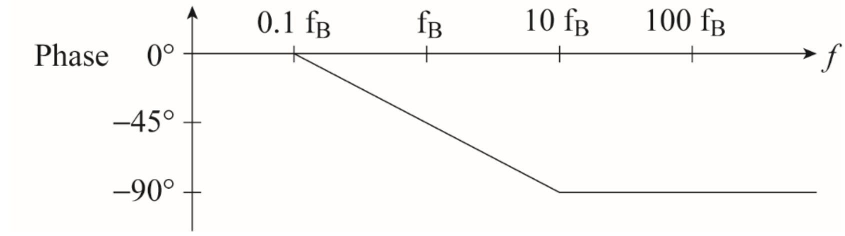


This is a lowpass filter having a transfer function given by Equation 6.8 (with changes in notation):
$$\frac{\mathbf{V}_{out}}{\mathbf{V}_t} = \frac{1}{1 + j(f/f_B)}$$
where $f_B = 1/(2\pi R_t C) = 8.84\ \text{Hz}$.

Using the fact that $\mathbf{V}_t = 0.1\mathbf{V}_{in}$, we have $H(f) = \frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = \frac{0.1}{1 + j(f/f_B)}$

The Bode plots are:





P6.65*

The input signal is given by

$$v_{in}(t) = 10 + 15\cos(2000\pi t)$$

This signal has components at $f = 0$ Hz and $f = 1000$ Hz. The transfer-function values at these frequencies are:

$$H(0) = \frac{j0}{1 + j0} = 0$$

$$H(1000) = \frac{j1}{1 + j1} = 0.7071\angle 45^\circ$$

Applying these transfer-function values to the respective components yields:

$$v_{out}(t) = 10.60\cos(2000\pi t + 45^\circ)$$