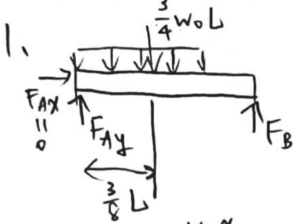
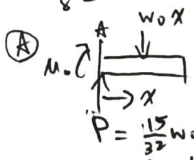
材力 期中 3rd

1.



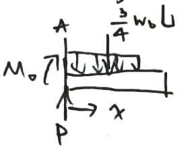
$$\begin{cases} F_{Ay}+F_B-\frac{3}{4}w_0L=0 \\ \Sigma M_A=0=\frac{3}{4}w_0L\left(\frac{3}{8}L\right)-F_BL \end{cases} \Rightarrow \begin{cases} F_{Ay}=\frac{15}{32}w_0L \\ F_B=\frac{9}{32}w_0L \end{cases}$$

Ⓐ



$$M_1=M_0+Px-\frac{w_0x^2}{2}$$

$P=\frac{15}{32}w_0L$



$$M_2=M_0+Px-\frac{3}{4}w_0L\left(x-\frac{3}{8}L\right)$$
$$=M_0+Px-\frac{3}{4}w_0Lx+\frac{9}{32}w_0L^2$$

$$\Delta_A=\frac{\partial U}{\partial P}=\int_0^{\frac{3}{4}L}\frac{M_1\frac{\partial M_1}{\partial P}}{EI}dx+\int_{\frac{3}{4}L}^{L}\frac{M_2\frac{\partial M_2}{\partial P}}{EI}dx$$

$$=\int_0^{\frac{3}{4}L}\frac{\left(M_0+Px-\frac{w_0x^2}{2}\right)(x)}{EI}dx+\int_{\frac{3}{4}L}^{L}\frac{\left(M_0+Px-\frac{3}{4}w_0Lx+\frac{9}{32}w_0L^2\right)x}{EI}dx$$

$$=\frac{1}{EI}\left(\left[\frac{\overset{0}{M_0}}{2}x^2+\frac{\overset{(\frac{15}{32}w_0L)}{P}}{3}x^3-\frac{w_0}{8}x^4\right]_0^{\frac{3}{4}L}+\left[\frac{\overset{0}{M_0}+\frac{9}{32}w_0L^2}{2}x^2+\frac{\overset{(\frac{15}{32}w_0L)}{P}-\frac{3}{4}w_0L}{3}x^3\right]_{\frac{3}{4}L}^{L}\right)$$

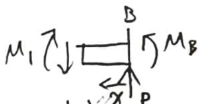
$$=\frac{1}{EI}$$

$$\theta_A=\frac{\partial U}{\partial M_0}=\int_0^{\frac{3}{4}L}\frac{\left(M_0+\underset{(\frac{15}{32}w_0L)}{P}x-\frac{w_0x^2}{2}\right)(1)}{EI}dx+\int_{\frac{3}{4}L}^{L}\frac{M_0+\frac{9}{32}w_0L+\left(\underset{(\frac{15}{32}w_0L)}{P}-\frac{3}{4}w_0L\right)x}{EI}dx$$

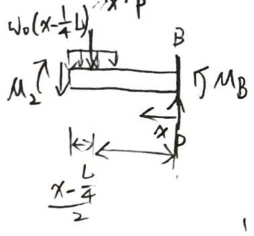
$$=\frac{1}{EI}\left\{\left[\overset{0}{M_0}x+\frac{P}{2}x^2-\frac{w_0}{6}x^3\right]_0^{\frac{3}{4}L}+\left[\left(\overset{0}{M_0}+\frac{9}{32}w_0L\right)x+\frac{P-\frac{3}{4}w_0L}{2}x^2\right]_{\frac{3}{4}L}^{L}\right\}$$

$$=$$

Ⓑ



$$M_1=M_B+Px$$



$$M_2=M_B+Px-w_0\left(x-\frac{1}{4}L\right)\left(\frac{x}{2}-\frac{L}{8}\right)$$
$$=M_B+Px-w_0\left(\frac{x^2}{2}-\frac{L}{4}x+\frac{L^2}{32}\right)$$

$$\Delta_B=\frac{\partial U}{\partial P}=\int_0^{\frac{1}{4}L}\frac{(M_B+Px)x}{EI}dx+\int_{\frac{1}{4}L}^{L}\frac{\left[M_B+Px-w_0\left(\frac{x^2}{2}-\frac{L}{4}x+\frac{L^2}{32}\right)\right]x}{EI}dx$$

$$=\frac{1}{EI}\left\{\left[\frac{\overset{0}{M_B}}{2}x^2+\frac{\overset{(\frac{9}{32}w_0L)}{P}}{3}x^3\right]_0^{\frac{1}{4}L}+\left[\frac{\overset{0}{M_B}-\frac{w_0L^2}{32}}{2}x^2+\frac{\overset{(\frac{9}{32}w_0L)}{P}+\frac{w_0L}{4}}{3}x^3-\frac{w_0}{8}x^4\right]_{\frac{1}{4}L}^{L}\right.$$

$$=$$

$$\theta_B=\frac{\partial U}{\partial M_B}=\int_0^{\frac{1}{4}L}\frac{(M_B+\underset{(\frac{9}{32}w_0L)}{P}x)(1)}{EI}dx+\int_{\frac{1}{4}L}^{L}\frac{M_B+\underset{(\frac{9}{32}w_0L)}{P}x-w_0\left(\frac{x^2}{2}-\frac{L}{4}x+\frac{L^2}{32}\right)}{EI}dx$$

$$=\frac{1}{EI}\left\{\left[\overset{0}{M_B}x+\frac{P}{2}x^2\right]_0^{\frac{1}{4}L}+\left[\left(\overset{0}{M_B}-\frac{w_0L^2}{32}\right)x+\frac{P+\frac{w_0L}{4}}{2}x^2-\frac{w_0}{6}x^3\right]_{\frac{1}{4}L}^{L}\right.$$

$$=$$

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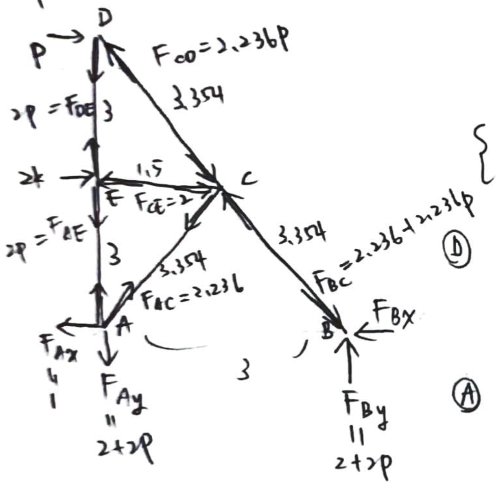
2、



$\sqrt{3^2+1.5^2}=3.354$, $E=200\,GPa\ (10^6)$, $A=400\,mm^2=4\times10^{-4}\,m^2$

$\left\{\begin{array}{l} F_{Ay}=F_{By} \\ \Sigma M_B=0=2\times3+P\times6-F_{Ay}\times3 \end{array}\right. \Rightarrow F_{Ay}=F_{By}=2+2P$

(D) $\frac{1.5}{3.354}F_{CD}=P \Rightarrow F_{CD}=2.236P$

$F_{DE}=\frac{3}{3.354}F_{CD}=2P$

(A) $\frac{3}{3.354}F_{AC}+2P=2+2P \Rightarrow F_{AC}=2.236$

$F_{Ax}=\frac{1.5}{3.354}F_{AC}=1$

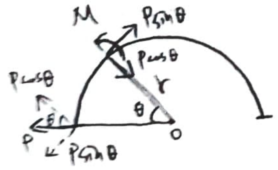
(B) $\frac{3}{3.354}F_{BC}=2+2P \Rightarrow F_{BC}=2.236+2.236P$

$$\Delta_D=\frac{\partial U}{\partial P}=\Sigma\frac{F\frac{\partial F}{\partial P}L}{AE}=\frac{1}{AE}\left[2^2P\times3\times2+2.236^2P\times3.354+(2.236+2.236P)\times2.236\times3.354\right]\times1000$$

$A=4\times10^{-4}$, $E=200\times10^6$, $P=4$

$=3.0865\,mm$ #

4、

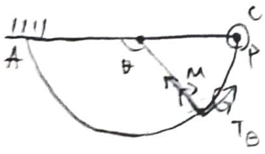


$\Sigma M_O=0=M-P(\sin\theta)r \Rightarrow M=Pr\sin\theta$

$$\Delta_A=\frac{\partial U}{\partial P}=\int_0^{\pi}\frac{M\frac{\partial M}{\partial P}}{EI}r\,d\theta=\int_0^{\pi}\frac{P(r\sin\theta)^2}{EI}r\,d\theta=\frac{Pr^3}{EI}\int_0^{\pi}\frac{1-\cos2\theta}{2}d\theta$$

$$=\frac{Pr^3}{EI}\left[\frac{\theta}{2}-\frac{\sin2\theta}{4}\right]_0^{\pi}=\frac{P r^3\pi}{2EI}\ \#$$

5、



$M=Pr\sin(\pi-\theta)$

$T=-Pr\left[1-\cos(\pi-\theta)\right]$

$1-2\cos(\pi-\theta)+\cos^2(\pi-\theta)$

$$\Delta_B=\frac{\partial U}{\partial P}=\int_0^{\pi}\left(\frac{M\frac{\partial M}{\partial P}}{EI}+\frac{T\frac{\partial T}{\partial P}}{GJ}\right)r\,d\theta=\int_0^{\pi}\left(\frac{Pr^2\sin^2(\pi-\theta)}{EI}+\frac{Pr^2\left[1-\cos(\pi-\theta)\right]^2}{GJ}\right)r\,d\theta$$

$$=Pr^3\int_0^{\pi}\left(\frac{1-\cos2(\pi-\theta)}{2EI}+\frac{1-2\sin(\pi-\theta)+\frac{1+\cos2(\pi-\theta)}{2}}{GJ}\right)d\theta$$

$$=Pr^3\left\{\frac{1}{EI}\left[\frac{\theta}{2}+\frac{\sin2(\pi-\theta)}{4}\right]+\frac{1}{GJ}\left[\theta-2\cos(\pi-\theta)+\frac{\theta}{2}-\frac{\sin2(\pi-\theta)}{4}\right]\right\}_0^{\pi}$$

$G=\frac{E}{2(1+\nu)}$, $J=2I$

$$=Pr^3\left[\frac{1}{EI}\times\frac{\pi}{2}+\frac{(1+\nu)}{EI}\left(\pi-4+\frac{\pi}{2}\right)\right]$$

$$=\frac{Pr^3}{2EI}\left[\pi+2(1+\nu)\left(\frac{3}{2}\pi-4\right)\right]\ \#$$

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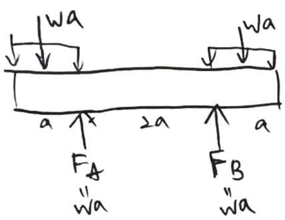
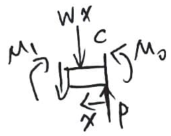
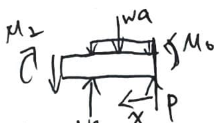
3、



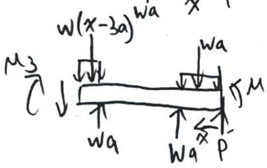
$$\begin{cases} F_A + F_B = 2wa \\ \Sigma M_A = 0 = F_B(2a) + wa\left(\frac{a}{2}\right) - wa\left(\frac{5}{2}a\right) \end{cases} \Rightarrow F_A = F_B = wa$$



$$M_1 = M_0 - \frac{wx^2}{2} + Px$$



$$M_2 = M_0 + Px + wa(x-a) - wa\left(x - \frac{a}{2}\right)$$
$$= M_0 + Px - \frac{wa^2}{2}$$



$$M_3 = M_0 + Px + wa(x-a) + wa(x-3a) - wa\left(x - \frac{a}{2}\right) - w(x-3a)\left(\frac{x}{2} - \frac{3}{2}a\right)$$
$$= M_0 + Px + w\left(a^2 - 5ax + \frac{x^2}{2}\right)$$

$$\Delta_C = \frac{\partial U}{\partial P} = \frac{1}{EI}\left\{\int_0^a \left(M_0 - \frac{wx^2}{2} + Px\right)x\,dx + \int_a^{3a}\left(M_0 + Px - \frac{wa^2}{2}\right)x\,dx + \int_{3a}^{4a}\left[M_0 + Px + w\left(a^2 - 5ax + \frac{x^2}{2}\right)\right]x\,dx\right\}$$

$$= \frac{1}{EI}\left\{\left[-\frac{w}{8}x^4\right]_0^a + \left[-\frac{wa^2}{4}x^2\right]_a^{3a} + \left[\frac{wa^2}{2}x^2 - \frac{w5a}{3}x^3 + \frac{w}{8}x^4\right]_{3a}^{4a}\right\}$$

$=$

$$\theta_C = \frac{\partial U}{\partial M_0} = \frac{1}{EI}\left\{\int_0^a \left(M_0 - \frac{wx^2}{2} + Px\right)dx + \int_a^{3a}\left(M_0 + Px - \frac{wa^2}{2}\right)dx + \int_{3a}^{4a}\left[M_0 + Px + w\left(a^2 - 5ax + \frac{x^2}{2}\right)\right]dx\right\}$$

$$= \frac{1}{EI}\left\{\left[-\frac{w}{6}x^3\right]_0^a + \left[-\frac{wa^2}{2}x\right]_a^{3a} + \left[wa^2x - \frac{5aw}{2}x^2 + \frac{w}{6}x^3\right]_{3a}^{4a}\right.$$

$=$