E14082181 蘇品瑄

電工 2nd 期中

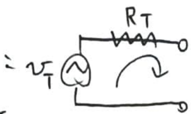
1、 $$\frac{V_{out}}{V_{in}} = \frac{\frac{R_2R_3}{R_2+R_3} + Z_L + Z_{C_1+C_2}}{R_1 + \left(\frac{R_2R_3}{R_2+R_3} + Z_L + Z_{C_1+C_2}\right)} = \frac{\left(\frac{200^2}{400} + 4j\omega + \frac{1}{10^{-4}j\omega}\right) \times 400\times10^{-4}j\omega}{\left[300 + \left(\frac{200^2}{400} + 4j\omega + \frac{1}{10^{-4}j\omega}\right)\right] \times 400\times10^{-4}j\omega}$$

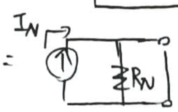
$$\left[C_1+C_2 = (20+80)\times10^{-6} = 10^{-4}\right]$$

$$= \frac{4j\omega - 0.16\omega^2 + 400}{12j\omega + 4j\omega - 0.16\omega^2 + 400} = \frac{(-0.16\omega^2+400) + j(4\omega)}{(-0.16\omega^2+400) + j(16\omega)}$$ #

2、 $$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.5\times0.1}} = 4.472 \text{ rad/s}$$ #

$$B = \frac{\omega_0}{Q} = \omega_0^2 RC = \frac{RC}{LC} = \frac{4}{0.5} = 8 \text{ rad/s}$$ #

3、 Thévenin :  $I_T = \frac{V_T}{R_T}$

Norton :  I_N

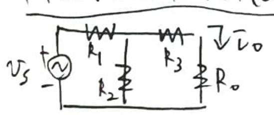
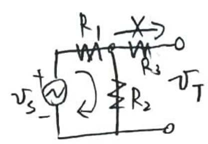
$$I_N = I_T = \frac{V_T}{R_T} \Rightarrow V_T = I_N R_T = I_N R_N \Rightarrow R_T = R_N$$

4、 ① Remove the loading

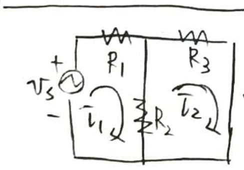
② Zero all voltage or current source

③ Compute the total resistance between load terminals with the load removed.

Thévenin Voltage :

 $\Rightarrow$  , $$v_T = v_{R_2} = v_s \times \frac{R_2}{R_1+R_2}$$ #

Norton current :



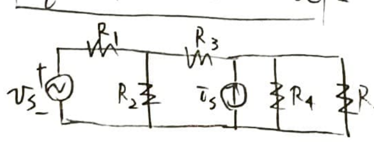
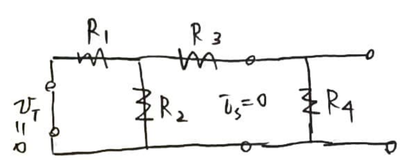
$$\begin{cases} v_s - R_1 i_1 - R_2(i_1 - i_2) = 0 & -① \\ -R_2(i_2 - i_1) - R_3 i_2 = 0 \Rightarrow i_1 = \frac{R_2+R_3}{R_2} i_2 & -② \end{cases}$$

②代入①

$$v_s = \frac{(R_1+R_2)(R_2+R_3)}{R_2} i_2 - R_2 i_2 = \left[\frac{(R_1+R_2)(R_2+R_3)}{R_2} - R_2\right] i_2$$

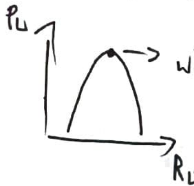
$$i_N = i_2 = \frac{v_s R_2}{(R_1+R_2)(R_2+R_3) - R_2^2} = \frac{v_s R_2}{R_1R_2 + R_1R_3 + R_2R_3}$$ #

Equivalent resistance :

 $\Rightarrow$ 

$$R_{eq} = \left[(R_1 \| R_2) + R_3\right] \| R_4$$ #

6、 $P_L = I_L^2 R_L = \dfrac{V_T^2}{(R_L+R_T)^2} R_L = V_T^2 (R_L+R_T)^{-2} R_L$



$\rightarrow$ when $\dfrac{dP_L}{dR_L} = 0$, P_L is max

$\Rightarrow \dfrac{dP_L}{dR_L} = \dfrac{d\left[V_T^2 (R_L+R_T)^{-2} R_L\right]}{dR_L} = V_T^2(-2)(R_L+R_T)^{-3} R_L + V_T^2 (R_L+R_T)^{-2}$

$= V_T^2 (R_L+R_T)^{-2} \left[\underbrace{-2(R_L+R_T)^{-1} R_L + 1}_{=0}\right] = 0$

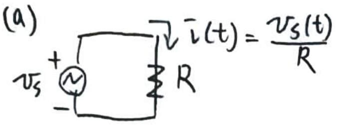
$\Rightarrow 1 = 2\dfrac{R_L}{R_L+R_T}$

$\Rightarrow R_L + R_T = 2R_L$

$\Rightarrow R_T = R_L$

$\therefore \dfrac{dP_L}{dR_L} = 0$ when $R_L = R_T = R_N$ #

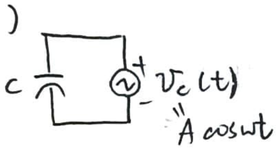
9、(a)



$v_S(t) = A\cos\omega t$

$\begin{cases} V_S(j\omega) = Ae^{j0^\circ} \\ I_S(j\omega) = \frac{A}{R} e^{j0^\circ} \end{cases} \Rightarrow Z_R(j\omega) = \dfrac{V_S(j\omega)}{I_S(j\omega)} = R$ #

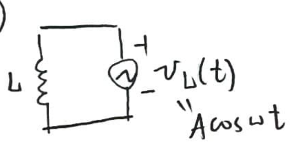
(b)



$i_C(t) = C\dfrac{dv_C(t)}{dt} = C\left[-A\omega\sin(\omega t)\right] = \omega CA\cos(\omega t + 90^\circ)$

$Z_C(j\omega) = \dfrac{V_C(j\omega)}{I_C(j\omega)} = \dfrac{Ae^{j0^\circ}}{\omega CAe^{j90^\circ}} = \dfrac{1}{\omega C} e^{j(-90^\circ)} = -j\dfrac{1}{\omega C} = \dfrac{1}{j\omega C}$ #

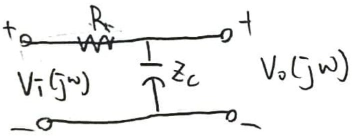
(c)



$i_L(t) = \frac{1}{L}\int A\cos\omega t\,dt = \dfrac{A}{L}\dfrac{\sin\omega t}{\omega} = \dfrac{A}{\omega L}\cos(\omega t - 90^\circ)$

$Z_C(j\omega) = \dfrac{V_L(j\omega)}{I_L(j\omega)} = \dfrac{Ae^{j0^\circ}}{\frac{A}{\omega L}e^{j(-90^\circ)}} = \omega Le^{j90^\circ} = j\omega L$ #

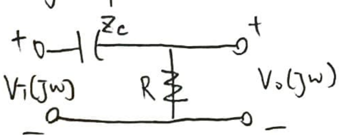
7、① law-pass filter



law $\omega \Rightarrow$ C 斷路 $\Rightarrow$ 過

high $\omega \Rightarrow$ C 短路 $\Rightarrow$ 擋

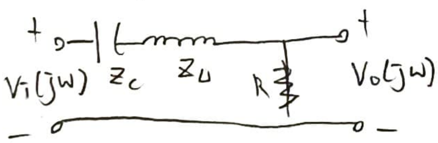
② high-pass filter



law $\omega \Rightarrow$ C 斷路 $\Rightarrow$ 擋

high $\omega \Rightarrow$ C 短路 $\Rightarrow$ 過

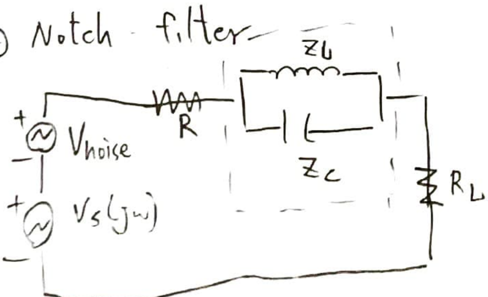
③ Band-pass filter



law $\omega \Rightarrow \begin{cases} \text{C 斷路} \\ \text{L 短路} \end{cases} \Rightarrow$ 擋

high $\omega \Rightarrow \begin{cases} \text{C 短路} \\ \text{L 斷路} \end{cases} \Rightarrow$ 擋

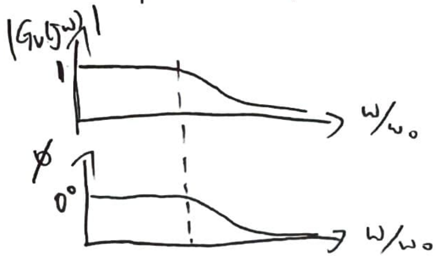
④ Notch filter



law $\omega \Rightarrow \begin{cases} \text{C 斷路} \\ \text{L 短路} \end{cases} \Rightarrow$ 過

high $\omega \Rightarrow \begin{cases} \text{C 短路} \\ \text{L 斷路} \end{cases} \Rightarrow$ 過

8. ① law- pass filter

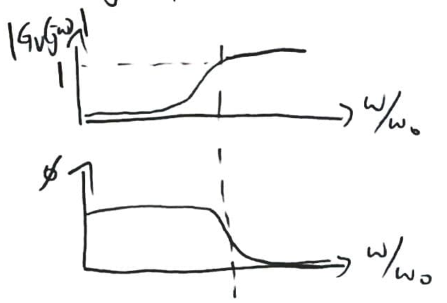


$|G_v|$ must be 1

low $\omega \Rightarrow \phi$ must be 0°, so that there won't have noise in $|G_v(j\omega)|$

high $\omega \Rightarrow \phi$ can be any number, because $|G_v(j\omega)|$ would be zero.

② high- pass filter

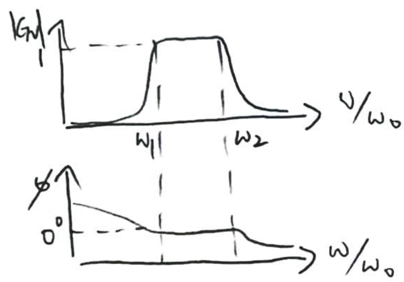


low $\omega \Rightarrow \phi$ can be any number, because $|G_v(j\omega)|$ would be zero.

high $\omega \Rightarrow \phi$ must be 0°, so that there won't have noise in $|G_v(j\omega)|$.

$|G_v|$ must be 1

③ Band - pass filter

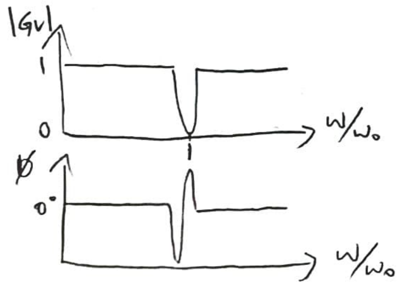


$|G_v|$ must be 1

$\omega_1 < \omega < \omega_2 \Rightarrow \phi$ must be 0°, so that there won't have noise in $|G_v|$.

$\omega < \omega_1$
$\omega > \omega_2$ $\Rightarrow \phi$ can be any number, because $|G_v(j\omega)|$ would be zero.

④ Notch filter

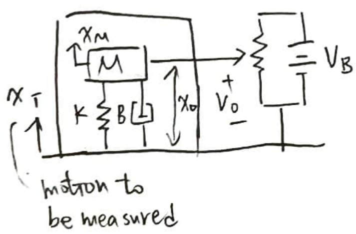


$\omega = \omega_0 \Rightarrow \phi$ can be any number, because $|G_v|$ would be zero.

$\omega \neq \omega_0 \Rightarrow \phi$ must be 0°, so that there won't have noise in $|G_v|$.

$|G_v|$ must be 1

10.



$$K x_o + B\frac{dx_o}{dt} = M\frac{d^2 x_M}{dt^2} \xrightarrow{x_M = x_i - x_o} M\left(\frac{d^2 x_i}{dt^2} - \frac{d^2 x_o}{dt^2}\right)$$

$$\begin{cases} X_i(j\omega) = |X_i| e^{j\phi_i} \\ X_o(j\omega) = |X_o| e^{j\phi_o} \end{cases}$$

$$M(j\omega)^2 X_o + B(j\omega) X_o + K X_o = M(j\omega)^2 X_i$$

$$\Rightarrow (-\omega^2 M + j\omega B + K) X_o = -\omega^2 M X_i$$

$$\Rightarrow \frac{X_o(j\omega)}{X_i(j\omega)} = \frac{-\omega^2 M}{-\omega^2 M + j\omega B + K}$$