$x^2y'' - (2+3x)y = 0$

$\begin{cases} xp(x) = 0 \Rightarrow p_0 = 0 \\ x^2Q(x) = -(2+3x) \Rightarrow q_0 = -2 \end{cases}$

$r(r-1) + \overset{(0)}{p_0} r + \overset{(-2)}{q_0} = r^2 - r - 2 = (r-2)(r+1) \Rightarrow \begin{matrix} r_1 = 2 \\ r_2 = -1 \end{matrix}$ $\Rightarrow$ case 2 #

sub $y = \sum_{n=0}^{\infty} a_n x^{n+r}$

$\sum_{n=0}^{\infty}(n+r)(n+r-1)a_n x^{n+r} - 3\sum_{n=0}^{\infty} a_n x^{n+r+1} - 2\sum_{n=0}^{\infty} a_n x^{n+r}$

$\Rightarrow \begin{cases} x^r : [r(r-1) - 2]a_0 = 0 \Rightarrow r_1 = 2,\ r_2 = -1 \\ x^{n+r} : \sum_{n=1}^{\infty} \{[(n+r)(n+r-1) - 2]a_n - 3a_{n-1}\} x^{n+r} = 0 \end{cases}$

$\Rightarrow a_n = \dfrac{3}{(n+r-2)(n+r+1)} a_{n-1}$

Set $r = r_2 = -1$

$a_n = \dfrac{3}{n(n-3)} a_{n-1} \quad (n = 1, 2, 3, \dots)$ $\nearrow$ $a_1 = -\frac{3}{2}a_0,\ a_2 = -\frac{3}{2}(-\frac{3}{2}a_0) = \frac{9}{4}a_0$

$n=3 \Rightarrow \underbrace{n(n-3)}_{0} a_3 = 3a_2 = 3 \times \frac{9}{4}a_0 \Rightarrow a_0 = 0$ $\Rightarrow$ 矛盾!

$\therefore y_2 = \sum_{n=0}^{\infty} a_n x^{n-1}$ fails to result a sol'n.

set $r = r_1 = 2$

$a_n = \dfrac{3}{n(n+3)} a_{n-1} \quad (n = 1, 2, 3, \dots) \rightarrow \begin{matrix} a_1 = \frac{3}{4}a_0,\ a_2 = \frac{3}{10}(\frac{3}{4}a_0) = \frac{9}{40}a_0 \\ a_3 = \frac{3}{18}(\frac{9}{40}a_0) = \frac{31}{80}a_0 \end{matrix}$

$\therefore y_1 = a_0 x^2 (1 + \frac{3}{4}x + \frac{9}{40}x^2 + \frac{3}{80}x^3 + \dots)$

$y_2 = y_1 \int \dfrac{e^{-\int p(x)dx}}{y_1^2} dx,\ p(x) = 0 \Rightarrow \int p(x)dx = 0$

$= y_1 \int \frac{1}{y_1^2} dx = y_1 \int \dfrac{1}{x^4(1 + \frac{3}{2}x + \frac{81}{80}x^2 + \dots)} dx = y_1 \int (\frac{1}{x^4} - \frac{3}{2x^3} + \frac{99}{80x^2} + \dots) dx$

$= y_1 (-\frac{1}{3x^3} + \frac{3}{4x^2} - \frac{99}{80x} \dots)$

$y = c_1 y_1 + c_2 y_2 = c_1 x^2 (1 + \frac{3}{4}x + \frac{9}{40}x^2 + \dots) + c_2 y_1 (-\frac{1}{3x^2} + \frac{3}{4x^2} - \frac{99}{80x} + \dots)$ #