Formulaire de Trigonométrie

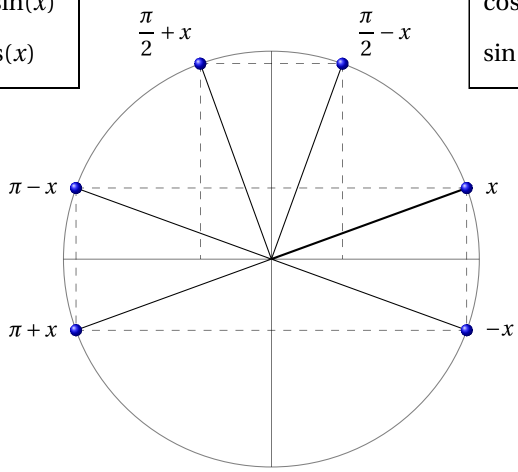
Formules élémentaires

Une lecture efficace du cercle trigonométrique permet de retrouver les relations suivantes :
Soit $x \in \mathbb{R}$,

$$\cos\left(\frac{\pi}{2}+x\right) = -\sin(x) \qquad \sin\left(\frac{\pi}{2}+x\right) = \cos(x)$$

$$\cos\left(\frac{\pi}{2}-x\right) = \sin(x) \qquad \sin\left(\frac{\pi}{2}-x\right) = \cos(x)$$

$$\cos(\pi - x) = -\cos(x) \qquad \sin(\pi - x) = \sin(x)$$



$$\cos(\pi + x) = -\cos(x) \qquad \sin(\pi + x) = -\sin(x)$$

$$\cos(-x) = \cos(x) \qquad \sin(-x) = -\sin(x)$$

Valeurs remarquables

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π
$\cos(\theta)$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1
$\sin(\theta)$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0
$\tan(\theta)$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$		0

Relations entre cos, sin et tan

Soit $x \in \mathbb{R}$,

$$\cos^2(x) + \sin^2(x) = 1$$

Si $x \not\equiv \frac{\pi}{2} [\pi]$

$$1 + \tan^2(x) = \frac{1}{\cos^2(x)}$$

Formules d'addition

Soient $a, b \in \mathbb{R}$,

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b) \qquad \cos(a-b) = \cos(a)\cos(b) + \sin(a)\sin(b)$$

$$\sin(a+b) = \sin(a)\cos(b) + \sin(b)\cos(a) \qquad \sin(a-b) = \sin(a)\cos(b) - \sin(b)\cos(a)$$

Si $(a \not\equiv \frac{\pi}{2}[\pi])$, $(b \not\equiv \frac{\pi}{2}[\pi])$ et $(a+b \not\equiv \frac{\pi}{2}[\pi])$, alors

$$\tan(a+b) = \frac{\tan(a)+\tan(b)}{1-\tan(a)\tan(b)}$$

Si $(a \not\equiv \frac{\pi}{2}[\pi])$, $(b \not\equiv \frac{\pi}{2}[\pi])$ et $(a-b \not\equiv \frac{\pi}{2}[\pi])$, alors

$$\tan(a-b) = \frac{\tan(a)-\tan(b)}{1+\tan(a)\tan(b)}$$

Formules de duplication

Soit $a \in \mathbb{R}$,

$$\cos(2a) = \cos^2(a) - \sin^2(a) = 2\cos^2(a) - 1 = 1 - 2\sin^2(a)$$

$$\sin(2a) = 2\sin(a)\cos(a)$$

Si $(a \not\equiv \frac{\pi}{2}[\pi])$ et $(a \not\equiv \frac{\pi}{4}\left[\frac{\pi}{2}\right])$, alors

$$\tan(2a) = \frac{2\tan(a)}{1-\tan^2(a)}$$

Transformation de produits en sommes

Soient $a, b \in \mathbb{R}$,

$$\sin(a)\cos(b) = \frac{1}{2}\left[\sin(a+b) + \sin(a-b)\right]$$
$$\cos(a)\cos(b) = \frac{1}{2}\left[\cos(a+b) + \cos(a-b)\right]$$
$$\sin(a)\sin(b) = -\frac{1}{2}\left[\cos(a+b) - \cos(a-b)\right]$$

Equations trigonométriques

Soient $a, b \in \mathbb{R}$,

$$\cos(a) = \cos(b) \iff \left\{\begin{array}{l} a \equiv b[2\pi] \\ ou \\ a \equiv -b[2\pi] \end{array}\right. \qquad \sin(a) = \sin(b) \iff \left\{\begin{array}{l} a \equiv b[2\pi] \\ ou \\ a \equiv \pi - b[2\pi] \end{array}\right. \qquad \tan(a) = \tan(b) \iff a \equiv b[\pi]$$