

Développements limités en 0 des fonctions usuelles

$$\begin{aligned}
 e^x &= \sum_{k=0}^n \frac{x^k}{k!} + o(x^n) \\
 &= 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \cdots + \frac{x^n}{n!} + o(x^n) \\
 \cos x &= \sum_{k=0}^n (-1)^k \frac{x^{2k}}{(2k)!} + o(x^{2n+1}) \\
 &= 1 - \frac{x^2}{2} + \frac{x^4}{24} + \cdots + (-1)^n \frac{x^{2n}}{(2n)!} + o(x^{2n+1}) \\
 \sin x &= \sum_{k=0}^n (-1)^k \frac{x^{2k+1}}{(2k+1)!} + o(x^{2n+2}) \\
 &= x - \frac{x^3}{6} + \cdots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+2}) \\
 \operatorname{ch} x &= \sum_{k=0}^n \frac{x^{2k}}{(2k)!} + o(x^{2n+1}) \\
 &= 1 + \frac{x^2}{2} + \frac{x^4}{24} + \cdots + \frac{x^{2n}}{(2n)!} + o(x^{2n+1}) \\
 \operatorname{sh} x &= \sum_{k=0}^n \frac{x^{2k+1}}{(2k+1)!} + o(x^{2n+2}) \\
 &= x + \frac{x^3}{6} + \cdots + \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+2}) \\
 (1+x)^\alpha &= 1 + \sum_{k=1}^n \alpha(\alpha-1)\dots(\alpha-k+1) \frac{x^k}{k!} + o(x^n) \\
 &= 1 + \alpha x + \alpha(\alpha-1) \frac{x^2}{2} + \cdots + \alpha(\alpha-1)\dots(\alpha-n+1) \frac{x^n}{n!} + o(x^n) \\
 \frac{1}{1+x} &= \sum_{k=0}^n (-1)^k x^k + o(x^n) \\
 &= 1 - x + x^2 + \cdots + (-1)^n x^n + o(x^n) \\
 \frac{1}{1-x} &= \sum_{k=0}^n x^k + o(x^n) \\
 &= 1 + x + x^2 + \cdots + x^n + o(x^n) \\
 \ln(1+x) &= \sum_{k=1}^n (-1)^{k-1} \frac{x^k}{k} + o(x^n) \\
 &= x - \frac{x^2}{2} + \frac{x^3}{3} + \cdots + (-1)^{n-1} \frac{x^n}{n} + o(x^n) \\
 \ln(1-x) &= - \sum_{k=1}^n \frac{x^k}{k} + o(x^n) \\
 &= -x - \frac{x^2}{2} - \frac{x^3}{3} + \cdots - \frac{x^n}{n} + o(x^n) \\
 \operatorname{Arctan} x &= \sum_{k=0}^n (-1)^k \frac{x^{2k+1}}{2k+1} + o(x^{2n+2}) \\
 &= x - \frac{x^3}{3} + \cdots + (-1)^n \frac{x^{2n+1}}{2n+1} + o(x^{2n+2}) \\
 \tan x &= x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + o(x^6)
 \end{aligned}$$