1SL1500 – PDE - Partial Differential Equations

Instructors: John Cagnol
Department: DÉPARTEMENT MATHÉMATIQUES
Language of instruction: FRANCAIS, ANGLAIS
Campus: CAMPUS DE PARIS - SACLAY
Workload (HEE): 50
On-site hours (HPE): 33,00

Description

Partial differential equations (or PDEs for short) are equations whose solutions are functions. They appear naturally in modeling in physics, mechanics, biology, economics, finance, and more generally in all engineering fields.

In this course, you will learn the basics of PDEs. We will start with recalling the situation of the ordinary differential equations (ODE) for which we will examine the well-posedness of the questions. Then, you will understand the different classes of PDEs, including elliptic, parabolic and hyperbolic. You will see how one can prove the existence and uniqueness of solutions of some elliptic equations.

You will see how to numerically approximate the solutions of elliptic and parabolic partial differential equations using two standard techniques : the Finite Element Method and the Finite Difference Method. Both of these techniques lead to a huge linear system, so we will see the basics of numerical linear algebra to tackle this problem. You will also learn about FEniCSx.

In this course, you will also see the theory of distributions which generalizes the concept of functions. You will learn how to successfully use distributions and apply them. You will also learn about Sobolev spaces which are useful in the context of PDEs.

Quarter number
ST2 and SG3

Prerequisites (in terms of CS courses)
Convergence, Integration and Probabability
Modeling (co-requirement)
Information Systems and Programming

Syllabus
Chapter I - Ordinary Differential Equations
Chapter II - Classification of PDEs and Modeling
Chapter III - Distributions
Chapter IV - The Variational Formulation
Chapter V - The Finite Element Method
Chapter VI - The Finite Difference Method
Chapter VII - Numerical Linear Algebra
Chapter VIII - Parabolic PDEs

Chapters I and V are conducted over two sessions each.

Class components (lecture, labs, etc.)

The course is available in :

French

Face-to-face (sections 3 to 6) or blended learning (section 1)

English

blended leaning (section 2)

Blended learning means that the lectures take place in the form of video capsules and the TD in person.

Students who need additional help are enrolled in MR (support sessions). They benefit from additional sessions led by students enrolled in the 2nd year "Teaching Assistant" elective class under the responsibility of the teaching staff. The support sessions are compatible with face-to-face French and distance or mixed English.

A special section is available for a few students with an exceptional mathematical skills who wish to study partial differential equations in greater depth. Admission is subject to approval by Pauline Lafitte.

The section and lab group of CIP determines the section and lab group for PDE (not applicable for the special section or in case of compelling reasons).

Grading
The assessment consists of :

- For all students:
    - A **continuous assessment CC**, under the responsibility of each course instructor.
    - A **final exam EF**, under the responsibility of John Cagnol.

- For all students enrolled in support sessions
    - A **support session score MR** grading the student's involvement, placed under the responsibility of relevant instructors.
- Optional for students who are not in the reinforced mode
    a. **Un companion project PR**, under the responsibility of Lionel Gabet

The overall grade is

For students without a companion project or support sessions :
Max (EF, 70% EF + 30% CC)

For students with a companion project:
Max (85% EF + 15% PR, 68% EF + 20% CC + 12% PR)

For students with support sessions :
Max (85% EF + 15% MR, 68% EF + 20% CC + 12% MR)

Course support, bibliography

Erick Herbin & Pauline Lafitte
CIP and PDE Lecture Notes

Haïm Brézis
Functional Analysis, Sobolev Spaces and Partial Differential Equations.
Springer, 2011.

Grégoire Allaire
Numerical Analysis and Optimization:
An Introduction to Mathematical Modelling and Numerical Simulation
Oxford University Press, USA, 2007

Hans Petter Langtangen & Anders Logg
Solving PDEs in Python. The FEniCS Tutorial I
Springer Open, 2007

Resources

This course is composed of ten lectures and ten lab sessions.

Courses are given in one of the following sections :
Section 1 - FR - Blended learning - Ph. Bouafia & guests
Section 2 - EN - Blended learning - John Cagnol
Section 3 - FR - Face-to-Face - Ludovic Goudenège
Section 4 - FR - Face-to-Face - Philippe Bouafia

Section 5 - FR - Face-to-Face - Vincent Lescaret
Section 6 - FR - Face-to-Face - Aymeric Vié
as well as a special section taught by Pauline Lafitte, for which students must to apply, that will be explained by e-mail.

Each section is subdivisez in lab groups.
A support session is assigned to certain students.

In addition, students who have to work on a modeling problem leading to a PDE in the context of a project, a ST, an associative activity or a personal interest, can ask to benefit from a companion project. This is the completion of additional work. This work is optional and subject to acceptation. It is under the responsibility of Lionel Gabet.

Description of the skills acquired at the end of the course
For all : C1, C2, C6
For students involved in a companion project: additionally C4, C7