2EL5130 – Chaos, Fractals and complexity

Instructors: Damien Rontani
Department: CAMPUS DE METZ
Language of instruction: ANGLAIS
Campus: CAMPUS DE METZ
Workload (HEE): 60
On-site hours (HPE): 35,00
Elective Category : Fundamental Sciences
Advanced level : Yes

Description

Neural networks, electronic or optical oscillators, or even chemical reaction are various examples of dynamical systems, where the state variable describing their spatiotemporal evolution evolve in a nonlinear fashion. From the intrinsic nonlinearity present in these systems stems rich dynamical behaviors, which allows for the observation of novel phenomena of interest for scientists and engineers. We can cite chaotic dynamics for example, explaining the impossibility to provide accurate long-term weather forecasting or collective phenomena, such as synchronisation, with multiple applications in neurosciences.

This lecture will provide to the student with the fundamental tools and framework of science of complexity. It will be illustrated by multiple example from recent research articles with application in the field of engineering. The student will use analytical and numerical techniques form the resolution of problems.

Quarter number

SG8

Prerequisites (in terms of CS courses)

Basics in Linear Algebra and real analysis (level L2)
Classical Physics (Electricity, mechanics, Fluid dynamics...) (level L2)
Modeling (1CC3000 : Theory and analysis of linear dynamical systems, ordinary differential equations, and partial dérivative equations).

Syllabus

Context and Introduction (1.5h)
Discovery of Chaos theory: from H. Poincaré to E. Lorenz

B. Mandelbrot and discovery of Fractals
Examples of complex phenomena in physics, chemistry, and biology

General introduction to nonlinear systems and chaos theory (9h)
Introduction to the mathematical framework (Map, ODE, stability analysis)
Notions of attractors: fixed points, limit cycles, Torus, strange attractors
Bifurcations
Route to Chaos and strange attractors. Bifurcation diagrams.

Discrete-time systems (maps). Notions of Lyapunov Exponent. Analysis of complexity: dimension, entropy. Introduction to Cellular Automata.

The special case of nonlinear time-delay systems. Mathematical description with delay differential equations (DDE). Significance in Biology and Physics. Application of time-delay systems in photonics .

Introductions to Fractals (1.5h)
Introduction to the theory of fractals. Self-similarity and fractal dimension (Hausdorff). Cantor, Mandelbrot, and Julia sets.

Complex phenomena – Introduction to network physics (10,5h)
Definition of complex physical networks. Examples in biology (metabolism, genetic, neurosciences) and in engineering (transportation and power grids)
Collective and emergent phenomena. Notions of synchronization.
Examples of synchronization in Biology and Physics
Presentation of the Kuramoto Model.

Spreading phenomena on networks (epidemics compartmental modeling and statistical approaches on networks). Contacts networks.

Small classes and Labs (12h):

Numerical simulations of fractals (3h)

Analysis of complexity of a nonlinear map (3h)

Analysis and simulation of a network of phase oscillators and observation of synchronization (3h)

Numerical simulation and analysis of nonlinear dynamical system (3h)

Class components (lecture, labs, etc.)
Lectures with emphasized interactions with numerical and experimental demonstration. Priority given to physical interpretation and examples from

current research. The presentation of mathematical tools is ilimted to essential notions necessary for the understanding of concepts seen in class.

Small class: (x2) will be organized for the assimilation of key notions
Labs: (x3) will be organized for experimenting with concepts seen in class and will focus on examples from current research topics.

Hourly volume:
Lectures: 22,5h
Small Class / Labs: 12h

Grading

Evaluation (Modalities and weight of each quiz/evaluation in the final grade) :

- Lab Reports are 100% of the total grade (they represent individual assignments). Unjustified absences during Lab session and/or missing Lab report: the grade of 0 is given to the corresponding Lab session.

Course support, bibliography

1. S. H. Strogatz, « Nonlinear Dynamics and Chaos : with Applications in Physics, Biology, Chemistry, and Engineering », Westview Press (2001), ISBN 978-0738204536
2. A. Pikovsky, M. Rosenblum, J. Kurths, « Synchronization: a Universal Concept in Nonlinear Sciences», Cambridge University Press, 2003, ISBN 978-0521533522
3. A.-L. Barabasi, "Network Science", Cambridge University Press, 2016, ISBN 978-1107076266

Resources

Teaching staff /faculty : Damien Rontani, Marc Sciamanna

Computing Ressources with Matlab and/or Python for laboratories and small classes

Learning outcomes covered on the course

To know the scientific context and multi-disciplinary aspects of nonlinear sciences and network theory.

To identify / recognize situations where is formalism can be applied to solve a problem.

To know and master a few basic methods for the analysis of nonlinear dynamical systems and nonlinear networks.

To perform numerical simulations of dynamical systems and dynamical networks

Description of the skills acquired at the end of the course

C1 : Analyze, design, and build complex systems with scientific, technological, human, and economic components

C2 : Develop in-depth skills in an engineering field and a family of professions

C6 : Be operational, responsive, and innovative in the digital world