2SC5710 – Theory and algorithmics for wave control

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Department: DÉPARTEMENT MATHÉMATIQUES
Language of instruction: FRANCAIS
Campus: CAMPUS DE PARIS - SACLAY
Workload (HEE): 60
On-site hours (HPE): 34,50

Description

Acoustic pollution is today considered a major annoyance by the population, while electromagnetic pollution raises many questions and questions related to health problems. Practical achievements to control this pollution represent a major technical and industrial challenge, but do not yet benefit from the new developments that mathematical analysis, numerical simulation and computer science can bring to it.

This course focuses on the design of innovative products (acoustic or electromagnetic coating) intended to control acoustic / electromagnetic pollution.
Having introduced the physical context in the "Context and challenges" module, it aims to give students the concepts in mathematics and computer science associated with the design and manufacture of such products. This course offers two training paths:

1) Theoretical analysis path, directed by Ms. Rozanova-Pierrat: a path based on mathematical theory (EDPs, functional analysis, derived from Fréchet) to master the control of waves to define the form of an acoustic or electromagnetic coating. In particular, in this way are treated the EDPs on the irregular edges (fractals) and is developed the method of optimization of the forms and the concept of the derivative compared to the boundary.

2) Digital analysis path, numerical and algorithmic methods, directed by M. Magoulès: this path aims to deepen the numerical methods, it is oriented towards scientific calculation. In particular, in this way are treated the numerical methods and the implementation of these for the propagation of the waves and the method of optimization of the forms.

The two tracks prepare for the three EIs proposed subsequently. More precisely, they study the methods (theoretical or numerical) which are useful for the themes of the three EIs: for controlling external acoustic waves (near motorways, airports, construction sites) or internal (sound insulation in offices by perforated panels or acoustic liners in aircraft reactors for

example), as well as in the control of electromagnetic pollution (anechoic chambers).

Quarter number
ST5

Prerequisites (in terms of CS courses)

Knowledge of SG courses of the 1st year: Algorithmics and complexity (for the Scientific Computing track) and EDPs (for the Mathematical Methods track)
SPI course "Wave Physics" is recommended for the EI "Electromagnetic pollution control"

Syllabus
The control problems described by partial differential equations (observability, controllability, controllability) are developed in the course of the "Automatique et Contrôle" of the ST, which is further explored in the context of the control of the dissipation of the wave energy. The observability in this case depends on the geometry of the absorbing edge. To illustrate the reasons for requiring geometric irregularity, we introduce the notions of fractal geometry with known results in physics (for two tracks) and mathematics (for the track "Theoretical analysis"). In particular, we present the phenomena of localization and absorption of waves (acoustic or electromagnetic) which are connected by the spectral analysis of the model. To better understand the environmental and sociological issues of acoustic barrier development, we present some psychoacoustic aspects, which show the importance of the dissipation of certain frequencies. The course studies in particular, for a fixed frequency, obtaining an optimal form for a frequency model on the Helmholtz equations in order to allow its use in the IE on two types of controls: geometric and topological. We then consider the main difficulty of having an "almost optimal" shape over a wide band of frequencies, important from a psychoacoustic point of view and industrial interests. In this context are also presented the most suitable numerical methods (thoroughly for the track "Scientific computing") in order to quickly and robustly determine the optimal shape or the "almost optimal" shape of the geometry on a frequency band by numerical simulation.

Course outline per session:
1) Course/TD on the common theme of two tracks with adapted levels of presentation: EDPs introduction: Delta operators, nabla, div, edge of a domain, an external normal, integration by parts. Wave propagation models.
2) Course/TD

a) Theoretical Analysis track: Traces, extensions, compact sets, compact operators. Poisson equation.
b) Numerical Analysis track: Poisson equation, finite element method, finite difference method, numerical implementation.
3) Course
a) Theoretical Analysis track: Fractal boundary. Analysis of the Poisson problem with mixed boundary conditions and the associated spectral problem.
b) Numerical Analysis track: Pre-fractal boundary. Spectral problem and associated numerical methods, implementation of the different boundary conditions, numerical error.
4) Course/TD
a) Theoretical Analysis track: Helmholtz model with an absorbing edge, its theoretical resolution and dependence on the acoustic energy of the frequencies
b) Digital Analysis track: Advanced numerical methods in the context of waves
5) Course/TD on the common theme in two ways: Numerical resolution of the Helmholtz problem with edge dissipation and the associated spectral problem
6) Machine and TP tutorials: Launch of the eigenmode localization project for two tracks
7) Course
a) Theoretical Analysis track: Parametric optimization (existence of an optimal form)
b) Digital Analysis track: Introduction of parametric, algorithmic and associated digital optimization
8) Course
a) Theoretical Analysis track: Fréchet derivative and the derivative with respect to a parameter. Lagragian method.
b) Numerical Analysis track: Concept of the Fréchet derivative. Numerical implementation of parametric optimization
9) Course/TD
a) Theoretical Analysis track: Optimization of forms. Shape derivative.
b) Digital Analysis track: Introduction to optimizing shapes. Numerical implementation of shape optimization
10) Course with an adaptation to the two paths: Numerical algorithm of the optimization of the forms and the optimality on a range of frequencies. Wave control, link with th commun "Automatic" course.

Class components (lecture, labs, etc.)
Cours 15*1h30, TD 5*1h30, TP 3*1h (TDs on a computer, computations on a cluster), project (not included in TDs), handbook of the course, computations on a cluster, solutions of exercises

Grading

Final exam 1h30 evaluated as 70% in the final note and the project evaluated as 30% in the final note for the Theoretical Analysis Track.

Numerical project evalueted as 70% in the final note and the project evaluated as 30% in the final note for the Numerical Analysis Track.

Course support, bibliography

1. F. Magoulès, P.T.K. Ngyuen, P. Omnes, A. Rozanova-Pierrat, Optimal absorbtion of acoustic waves by a boundary} SIAM J. Control Optim. Vol. 59, No. 1, (2021), pp. 561-583.
2. Kevin Arfi, Anna Rozanova-Pierrat. Dirichlet-to-Neumann or Poincaré-Steklov operator on fractals described by d-sets. Discrete and Continuous Dynamical Systems - Series S, American Institute of Mathematical Sciences, 2019, 12 (1), pp.1-26.
3. G. Allaire Conception optimale de structures, Springer.
4. A. Henrot, M. Pierre Variation et optimisation de formes. Une analyse géométrique. Springer.
5. M. Filoche and S. Mayboroda, Universal mechanism for Anderson and weak localization, Proceedings of the National Academy of Sciences of the USA 109, 14761 (2012).
6. M. Filoche and S. Mayboroda, The landscape of Anderson localization in a disordered medium, Contemporary Mathematics, 601 (2013), 113-121

Resources

Students are divided into two groups (by the choice of students) corresponding to two mentioned tracks before the start of the specific course. Each group has courses intended to introduce concepts used in TDs and in the longer term in IE (the three IEs offered). There will be some courses on numerical methods in the mathematical way and there will be some basic theoretical courses (such as multidimensional integration) in the numerical way. It is planned to have a session of TPs (TDs on the computer) of 3h common for 2 channels at the end of which there is a digital project to render (the influence of the geometry of the wall on the localization of the eigen modes and dissipation of wave energy). Students will perform modeling, simulation, visualization and rendering of the phenomenon. To validate the Numerical Analysis track there is a numerical project and to validate the Mathematical Analysis track there is an written exam of 1.5 hours. The Mathematical Analysis track is based on the course handout, which will be made available to everyone. Students of two tracks also have

at their disposal all the subjects of the TDs with the corrections. Numerical calculations will be performed on a CentraleSupélec calculation cluster by connecting to Jupyter.

Learning outcomes covered on the course

Theoretical Analysis track:

-understand the theoretical and numerical techniques of acoustic / electromagnetic wave control

-Validate theoretical techniques of acoustic / electromagnetic wave control (shape optimization)

Numerical Analysis track, numerical and algorithmic methods :

- numerical techniques of acoustic / electromagnetic wave control

-Implement numerical methods to simulate acoustic wave propagation phenomena of large dimensions (external / internal problems and problems for a wide band of frequencies)

-Validate numerical techniques of acoustic / electromagnetic wave control

Description of the skills acquired at the end of the course

C1, C2

Theoretical Analysis track (functional analysis, shape optimization):

To be able to deal with the control problems described by the PDEs.

Know how to deal with edge irregularity including fractal to show the well-posed nature of a problem described by PDEs.

Knowing how to apply the method of optimizing shapes and deriving an energy functional from the edge of the domain.

To be able to deduce from the application objectives the constraints of control and the fact of an existence / non-existence of an optimal form.

Target the geometric scales of interest in relation to the wavelengths to be dissipated.

Being able to deal with digital aspects.

Numerical Analysis track (Scientific Computing, numerical and algorithmic methods):

To be able to deal with the control problems described by the PDEs.

To be able to deduce from the application objectives the constraints of control and the importance of an optimal form.

Target the geometric scales of interest in relation to the wavelengths to be dissipated.

Master the finite element method and finite differences and their implementation.

Knowledge of resolution methods related to the simulation of wave propagation.

Mastery of numerical difficulties related to simulation.