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Additional Mathematics I

MARKING SCHEME



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Solution	Marks	Remarks
$= [1 + 4(2x) + 6(2x)^{2} + \cdots][1 + 7(-x) + 21(-x)^{2} + \cdots]$	2+1A	2A for tirst girch Taccor
$= 1 + x - 11 x^2 + \cdots$	/ A	i.
Coefficient of x2 is -11.	1 A 5	_
Alternatively,		
$(1+2x)^{4}(^{1}-x)^{7}$ $= [1+4(2x)+6(2x)^{2}+\cdots][1+7(-x)+21(-x)^{2}+\cdots]$	1 2+1A	
Coeff. of $x^2 = (xz) + 3x(-7) + 24x1$	1A	
=-11	/ A	
Alternatively,	5_	
Coeds of X		
$= {}_{4}C_{2}(2)^{2}\times1 + {}_{4}C_{1}\times2\times7 C_{1}\times(-1) + 1\times7 C_{2}$	(+1+1 A	
$=24\times1+8\times(-7)+1\times21$	(A	
, = -11	1 A 5	-
$\frac{2 \cdot \log_{73} 52}{209,78} = \frac{\log_{7} 52}{209,78}$	14	or my the said
104 4 + 609, 13		6 1 1 1 2 2 2 4 2 2 A
= 273 log3 2+ log3 13	IM	1
$\tau l_{aa} 7 + l_{aa} - 13$	IM	for $log(a^p) = p log a$
$= \frac{2 2033 + 2033}{\log_3 2 + \log_3 3 + \log_3 13}$ $= \frac{2a + b}{1 + a + b}$	2A	有用 award
= 1+a+b	5	
nit tiple	J	
Alternatively, $log_3 2 = a \Rightarrow 2 = 3^a$ $log_3 13 = 6 \Rightarrow 13 = 3^b$	l A	
Let $log_{78}52 = X$ $78^{x} = 52$	(M	
$(2x3 \times 13)^{x} = 2x2x13$		Put 2=3 a
$2^{x-2} \times 3^{x} \times 13^{x-1} = 1$ $(3^{4})^{x-2} \times 3^{x} \times (3^{b})^{x-1} = 3^{o}$	10	1 Put 2=3 h 13 = 3 h 4 equalifyedica
ax-2a+x+bx-b=0		4 Court Indica
$x = \frac{2a+b}{1+a+b}$	()	7
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	Howke	Remarks
Dolumon		,
s. Put $u = \frac{x-1}{x+1}$		
$du = acc^2 u$	/A	
$\frac{du}{du} = Aic^2 u$		
$\frac{du}{dx} = \frac{(x+1) - (x-1)}{(x+1)^2}$		
$=\frac{2}{(x+1)^2}$	IA	
du du cl. Pt.		
du = du du Chain Rute	2M	
$=\frac{2}{(x+1)^2}\cdot \text{ rec}^2\left(\frac{x-1}{x+1}\right)$	IA	
(211)	5	
$x = \frac{2\cos\theta + \sqrt{4\cos^2\theta - 4}}{2}$		
~		
$= \cos \theta \pm i \sin \theta$	I A	- De Moure's Theorem
$\chi^{\eta} = \cos \eta \theta + i \sin \eta \theta$	IM + IA	
: Regd egt. is		Alternatively, Sum of roots = zeono
$(x - \cos n \theta - i \sin n \theta)(x - \cos n \theta + i \sin n \theta)^{-1}$		Prod. of roots = 1
$x^2 - 2\cos n\theta \cdot x + 1 = 0$ Dedut i mark	2A	$x^{2}-2cnn\theta\cdot x+1=0$
- Jeans I miller		
5. $f(x) = (x + \frac{a}{2})^2 + (b - \frac{a^2}{4})$	1+1 A	Alternatively,
	IM	$\frac{df}{dx} = 2x + a$ 1A
	/ A	$\frac{df}{dx} = 0$
$=f(-\frac{a}{2})$		$\Rightarrow x = -\frac{\alpha}{2} \qquad 1 A$
f(x) - f(- \frac{2}{2})	IM	$\frac{d^2f}{dx^2} = 2 > 0$
$= (x^2 + ax + b) - (\frac{a^2}{4} - \frac{a^2}{2} + b)$		dx^2 ?: f is min. $at - \frac{a}{2}$ 1A
$= x^2 + ax + \frac{a^2}{4}$		Since f is quadratic,
$= \left(\chi + \frac{q}{2}\right)^2$	IA	: f(xc) > f(-2). 1M
70 for all x	/A	explain this is
$f(x) > f(-\frac{a}{2}) \text{for all } x$	I A	absolute minimum
The min value of x = J13 x +5 is		
$(\frac{\sqrt{13}}{\sqrt{13}})^2 - \sqrt{13} \cdot \frac{\sqrt{13}}{\sqrt{2}} + 5$ or $5 - 4(\sqrt{13})^2$	IM	
= 幸	1A	

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Solution wax = y lor xx	Marks	Remarks
$b^{\dot{\alpha}} = (ab)^{\dot{\beta}}$		Alternationly,
$x \log a = \frac{3}{2} \log (ab), 9 \log b = \frac{3}{2} \log (ab)$	1+1 >1	x log a = y log 6 1M
$x = \frac{\log(ab)}{\log a} \cdot 3 \qquad y = \frac{\log(ab)}{\log b} \cdot 3$	1+1 A	$ \chi \log a = 3(\log a + \log b) $
		$(x-z)\log a = z\log b$ 1M + 1A
$\frac{\chi u}{\sqrt{2}} = \frac{207(42) \cdot 3 \cdot \frac{\chi_0}{\sqrt{2}} \cdot \frac{\chi_0}{\sqrt{2}} \cdot \frac{\chi_0}{\sqrt{2}}}{\sqrt{2}} \cdot \frac{\chi_0}{\sqrt{2}} \cdot \frac{\chi_0}{\sqrt{2}} \cdot \frac{\chi_0}{\sqrt{2}} \cdot \frac{\chi_0}{\sqrt{2}}$	IM	$\therefore \frac{x}{x-3} = \frac{4}{3}$
$\frac{\chi y}{\chi + y} = \frac{\frac{\log(ab)}{\log a} \cdot 3 \cdot \frac{\log ab}{\log a} \cdot 3}{\frac{\log ab}{\log a} \cdot 3 + \frac{\log ab}{\log a} \cdot 3}$		
4 () -		$z = \frac{xy}{x+y}$ 1A
$= \frac{\log(ab) \cdot 3}{\log a + \log b}$		
= 3	IA	eliminate lugar
	6	lug b
Alternatively, $a^{x} = b^{y} \Rightarrow b = a^{x}$	l A	
$(ab)^3 = (a \cdot a^{\frac{3}{4}})^3$	IM	
$= a^{\frac{43+x3}{2}}$	(A	
$A^{T} = a^{\frac{43+23}{3}}$	IM	
	14	
$x = \frac{93 + 23}{9}$ equate index	IM	
$=\frac{xy}{x+y}$	IA	
	6	
$7 - y = z^2$		$\begin{array}{ c c c c c c c c c c c c c c c c c c c$
		x ² 9 4 0 1 4 9
5 4		For graph of y=x
4=1x-2	2A	
	弘思	2A & 1+1+1 A
-3 -2 -1 0 1 2 3		
from the graphs, $1x-21 < x^2$	100	
	IM	If equality sign in les
iff x < -2 or x > t	1+1+1/	If equality sign inch is in the answer, deducts I mark.
	也以去」	-

•		
Solution	Marles	Remarks
$\frac{1}{1}(\dot{a}) + (-x) = \frac{2(-x)}{(-x)^2 + 1} \qquad (\ddot{x}) \wedge (-x) \longrightarrow$	IM	a must
$= \frac{-2x}{x^2 + 1}$ $= -f(x) \text{for all } x$	1A 2	
(b) $\frac{du}{dx} = \frac{2(x^2+1)-2x\cdot 2x}{(x^2+1)^2}$	IM 4	For Quotient Rule
$=\frac{2-2x^{2}}{(x^{2}+1)^{2}}$	1 A	
$\frac{d^2y}{dx^2} = \frac{-4x(x^2+1)^2 - (z-2x^2) \cdot 2(x^2+1) \cdot 2x}{(x^2+1)^4}$	- IM+	- For Quolient Rule
$=\frac{4x(x^2+1)[-(x^2+1)-(2-2x^2)]}{(x^2+1)^4}$		
$= \frac{4x(x^2-3)}{(x^2+1)^3}$	2A	Accept $\frac{d^2y}{dx^2} = \frac{4(x^5-2x^3-x^3-x^2+1)^4}{(x^2+1)^4}$
(C) Put $\frac{dy}{dx} = 0$ Put $\frac{dy}{dx} = 0$ $x = \pm 1$	1 M 1+1 A	
At $x=1$, $\frac{d^2y}{dx}<0$, if is max.	i	Test max or nun.
Awarded Max. point = (1,1) FEAPA	T A	and realise
of the At $x=-1$, $\frac{d^2y}{dx^2} > 0$, if is mim. A very \longrightarrow ! Min. point = $(-1,-1)$	1 A	that max, a min. may be detirmined from 14"
(d) 7 †	(A	1
y = f(x-1) $y = f(x)$	IA IA	For the 2 turning pt For 2 tails not cutting x-axis
-1 0 1/2	→X IM	shifting horizontally
	IM.	shifting I wit towards
(-1,-1)	(A	for graph of y=fix-1
	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	+

(a) $V = \frac{4}{3}\pi r^{3} + \pi r^{2}R$ $\therefore h = \frac{V}{4r} - \frac{4}{3}r$ (b) (i) Surface area of cylindes = $2\pi r h$ $= 2\pi r \left(\frac{V}{4r^{2}} - \frac{4}{3}r\right)$ Surface area of cylindes = $2\pi r h$ $= 2\pi r \left(\frac{V}{4r^{2}} - \frac{4}{3}r\right)$ $\therefore C = 2\pi r \left(\frac{V}{4r^{2}} - \frac{4}{3}r\right) h + 4\pi r^{2} 22h$ $= \frac{2k}{4} \frac{V}{r^{2}} + \frac{13}{3}k\pi r^{2}$ $= \frac{2k}{4} \frac{V}{r^{2}} + \frac{32}{3}k\pi r$ $\frac{dC}{dr} = 0 \Rightarrow (32k\pi r^{3} - 6kV) = 0$ $r = 3\frac{7V}{16\pi}$ $\frac{d^{2}C}{dr^{2}} = \frac{4kV}{r^{2}} + \frac{32}{3}k\pi$ $= \frac{7}{4} \frac{4c}{r^{2}}$ $\frac{d^{2}C}{dr^{2}} = \frac{4kV}{r^{2}} + \frac{32}{3}k\pi$ $= \frac{7}{4} \frac{4c}{r^{2}} + \frac{4c}{r^{2}} + \frac{4c}{r^{2}}$ $= \frac{7}{4} \frac{7}{16\pi} + \frac{4c}{r^{2}} + \frac{7}{4} \frac{4c}{r^{2}}$ $\therefore C \text{ is suin. 2would anly if the whole is count}$ (iii) $\frac{7}{R} = \frac{r}{\sqrt{16\pi}} - \frac{4c}{4r} + \frac{4c}{r^{2}} + $	Solutions	Marks	R.emarks
(b) (i) Surface area of cylindes = $2\pi r h$ $= 2\pi r \left(\frac{V}{4r^2} - \frac{L}{3}r\right)$ Surface area of ends = $4\pi r^2$ $\therefore C = 2\pi r \left(\frac{V}{4r^2} - \frac{L}{3}r\right) R + 4\pi r^2 x 2 R$ $= \frac{2RV}{7} + \frac{16}{3} R \pi r^2$ $(ii) \frac{dC}{dr} = -\frac{2RV}{7^2} + \frac{32}{3} R \pi r$ $\frac{dC}{dr} = 0 \Rightarrow \left(32R\pi r^3 - 6RV\right) = 0$ $r = 3 \frac{3V}{16\pi}$ $\frac{d^2C}{dr^2} = \frac{4LV}{7^3} + \frac{32}{3} R \pi$ $2R$ $\frac{d^2C}{dr^2} = \frac{4LV}{7^3} + \frac{32}{3} R \pi$ $\frac{d^2C}{dr^2} = \frac{4LV}{4r^3} + \frac{4LV}$	$(9) (a) V = \frac{4}{3} \pi r^3 + \pi r^2 h$	1+14	
Surface area of ends = $4\pi r^2$ Surface area of ends = $4\pi r^2$ $C = 2\pi r \left(\frac{V}{\pi r^2} - \frac{4}{3}r\right) R + 4\pi r^2 \times 2R$ $= \frac{2RV}{\pi r^4} + \frac{16}{3}R\pi r^2$ $= \frac{2RV}{dr} + \frac{16}{3}R\pi r^2$ $= \frac{2RV}{dr} + \frac{32}{3}R\pi r$ $= \frac{3}{10\pi} \frac{1}{16\pi} \frac{1}{4\pi}$ Altimatively, Checking sign of $\frac{d^2r}{dr}$ If $r = \frac{3}{12V}$, $\frac{d^2r}{dr} > 0$ Find $\frac{d^2r}{dr} = \frac{d^2r}{dr} = d^$	$\therefore h = \frac{V}{\pi r^2} - \frac{4}{3} \gamma$		
Surface and of ends = $4\pi r^2$ $C = 2\pi r \left(\frac{V}{\pi r^2} - \frac{4}{3}r\right) R + 4\pi r^2 \times 2R$ $= \frac{2RV}{r} + \frac{16}{3}R\pi r^2$ $= \frac{2RV}{r^2} + \frac{32}{3}R\pi r$ $= \frac{dC}{dr} = 0 \Rightarrow \left(32R\pi r^3 - 6RV\right) = 0$ $= \frac{3}{16\pi}$ $\frac{d^2C}{dr^2} = \frac{4RV}{r^3} + \frac{32}{3}R\pi$ $= \frac{1}{16\pi}$ $\frac{d^2C}{dr^2} = \frac{4RV}{r^3} + \frac{32}{3}R\pi$ $= \frac{1}{16\pi}$	(b) (i) Surface area of cylindes = 2 Trh		Sub. 2
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$			
(ii) $\frac{d^{2}c}{dv} = -\frac{2kv}{r^{2}} + \frac{32}{3}k\pi \tau$ $\frac{d^{2}c}{dr} = 0 \Rightarrow (32k\pi r^{3} - 6kv) = 0$ $r = 3\sqrt{\frac{3V}{16\pi}}$ $\frac{d^{2}c}{dr^{2}} = \frac{4kv}{r^{3}} + \frac{32}{3}k\pi$ $If r = 3\sqrt{\frac{3V}{16\pi}}, \frac{d^{2}c}{dr^{2}} > 0 \frac{d^{2}c}{dr^{2}} = \frac{4kv}{r^{3}} + \frac{32}{3}k\pi \frac{d^{2}c}{dr^{2}} = \frac{4kv}{r^{3}} + \frac{4c}{3}k\pi \frac{d^{2}c}{dr^{2}} = 0 \frac{d^{2}c}{dr^{2}} = \frac{4kv}{r^{3}} + \frac{4c}{3}k\pi \frac{d^{2}c}{dr^{2}} =$	$\therefore C = 2\pi r \left(\frac{V}{\pi r^2} - \frac{U}{3} r \right) k + 4\pi r^2 \times 2k$	1+114	
$\frac{d^{2}C}{dr^{2}} = \frac{32k\pi r^{3} - 6kV}{16\pi}$ $r = \frac{3}{3}\frac{3V}{16\pi}$ $\frac{d^{2}C}{dr^{2}} = \frac{4kV}{r^{3}} + \frac{32}{3}R\pi$ $\frac{d^{2}C}{dr^{2}} = \frac{4kV}{r^{3}} + \frac{32}{3}R\pi$ $\frac{d^{2}C}{dr^{2}} = \frac{4kV}{r^{3}} + \frac{32}{3}R\pi$ $\frac{d^{2}C}{dr^{2}} = \frac{1M+1A}{r^{3}}$ $$	$= \frac{2kV}{\gamma} + \frac{16}{3}k\pi r^2$	1 A	,
$\frac{d^{2}C}{dr^{2}} = 0 \Rightarrow (32k\pi r^{3} - 6kV) = 0$ $r = 3\sqrt{\frac{3V}{16\pi}}$ $\frac{d^{2}C}{dr^{2}} = \frac{4kV}{r^{3}} + \frac{32}{3}2\pi$ $If r = 3\sqrt{\frac{3V}{16\pi}}, \frac{d^{2}C}{dr^{2}} > 0$ $Sign declaring sign of d$	(ii) $\frac{dc}{dr} = -\frac{2bV}{r^2} + \frac{32}{3}k\pi r$, lA	
$\frac{d^{2}C}{dr^{2}} = \frac{4kV}{r^{3}} + \frac{32}{3} \frac{2}{8}\pi$ $If r = \frac{3}{3} \frac{3V}{16\pi}, \frac{d^{2}C}{dr^{2}} > 0$ $I = \frac{3}{16\pi} \frac{3V}{16\pi}, \frac{d^{2}C}{dr^{2}} > 0$ $I = \frac{3}{16\pi} \frac{3V}{16\pi} = \frac{3}{16\pi} \frac{3V}{16\pi}$ $I = \frac{3\pi r^{3}}{3V - 4\pi r^{3}}$ $= \frac{3\pi r^{3}}{3V - 4\pi r^{3}}$ $= \frac{3\pi r^{3}}{3V - 4\pi r^{3}}$ $= \frac{1}{4}$ IM IM IM IM IM	$\frac{dC}{d\gamma} = 0 \Rightarrow (32k\pi r^3 - 6kV) = 0$	1M	25 de = 0
If $\tau = \frac{3}{3V}$, $\frac{d^2C}{dr^2} > 0$ Find $\frac{d^2C}{dr^2}$ [M+1A] $C \text{ is min. awarded only if the above is correct}$ $(iii) \frac{\gamma}{R} = \frac{\gamma}{\frac{V}{\pi r^2} - \frac{V}{3}r}$ $= \frac{3\pi r^3}{3V - 4\pi r^3}$ $= \frac{3\pi \cdot \frac{3V}{16\pi}}{3V - 4\pi \cdot \frac{3V}{16\pi}}$ $= \frac{1}{4}$ IM $2A$	$\gamma = 3 \frac{3 V}{16 \pi}$	/A	Alternatively,
$ \begin{array}{lll} & = & \sqrt{16\pi}, & \sqrt{4r^2} > 0 & \text{sign} d d d d d d d d d d d d d d d d d d d$			
$(iii) \frac{\gamma}{R} = \frac{\gamma}{\frac{V}{\pi r^{2}} - \frac{\psi}{3} r}$ $= \frac{3\pi r^{3}}{3V - 4\pi r^{3}}$ $= \frac{3\pi \cdot \frac{3V}{16\pi}}{3V - 4\pi \cdot \frac{3V}{16\pi}}$ $= \frac{1}{4}$ IM	If $\gamma = \frac{3\sqrt{3V}}{16\pi}$, $\frac{d^2C}{dr^2} > 0$ Find $\frac{d^2C}{dr^2}$	IM+IA	Correct working Di
$= \frac{3\pi r^{3}}{3V - 4\pi r^{3}}$ $= \frac{3\pi \cdot \frac{3V}{16\pi}}{3V - 4\pi \cdot \frac{3V}{16\pi}}$ $= \frac{1}{4}$ $= 2A$			met
$= \frac{3\pi \cdot \frac{3V}{16\pi}}{3V - 4\pi \cdot \frac{3V}{16\pi}}$ $= \frac{1}{4}$ $= 2A$	$\frac{\tilde{n}}{\tilde{n}} = \frac{\gamma}{\frac{1}{\sqrt{1 + \frac{1}{3}r}}}$ $\tilde{n} = \frac{\gamma}{\sqrt{1 + \frac{1}{3}r}}$ $\tilde{n} = \frac{\gamma}{\sqrt{1 + \frac{1}{3}r}}$.) IM	
$= \frac{3\pi \cdot \frac{3V}{16\pi}}{3V - 4\pi \cdot \frac{3V}{16\pi}}$ $= \frac{1}{4}$ $2A$	$= \frac{3\pi r^3}{3V - 4\pi r^3}$	-	
$= \frac{3V - 4\pi \cdot \frac{3V}{16\pi}}{4}$ $= \frac{4}{4}$ $= 2A$	$3\pi \cdot \frac{3V}{16\pi}$	IM	•
: $\gamma: k = 1 - 4$			
	:. 7: h= 1-4	17	

Solutions	linarks	Remarks
(10) (a) $\frac{df}{dx} = 3x^2 + 2ax + b$	IM+IA	
f has stationary values at a, B		
=> α , β are roots of $3x^2+2ax+b=0$	2M -	if the following
	IA	the correct
-	6	Alte active
(b) Discriminant of $f'(x) = 0$ is $(2a)^{2} - 4(3b)$ $= 4a^{2} - 12b$ $= 5a$	I.M	Alternatively, (L-B)2>0 IM
Since $d \neq \beta$ (d, β real)		$\lambda + \beta^{2} > 2\lambda \beta$ $(\lambda + \beta)^{2} > 4\lambda \beta$ IA
- $(4a^2-12b70)$ $4nD>0$	114	$\frac{4a^{2}}{9} > 4 \cdot \frac{h}{3}$ i.e. $a^{2} > 3b$ IA
1.4. 0/>28	1A 3	
(c) $\frac{f(\alpha) - f(\beta)}{\alpha - \beta} = \frac{(\alpha^3 + a\alpha^2 + b\alpha + c) - (\beta^3 + a\beta^2 + b\beta + c)}{\alpha - \beta}$	14	$\begin{array}{c} \cdot \alpha = -\frac{3}{2} (\alpha + \beta) \\ b = 3\alpha\beta \end{array}$ $\begin{array}{c} 1 \\ 2 \\ \end{array}$
$= \frac{(\alpha^3 - \beta^3) + a(\alpha^2 - \beta^2) + b(\alpha - \beta)}{\alpha - \beta}$	(2, 2, β	$a^{2}-3b = \frac{4}{4}(x+\beta) - \frac{9}{4}\alpha\beta$ $= \frac{9}{4}(\alpha - \beta)^{2} \qquad 1A$
_ '	2A	>0 1A
$= \frac{\alpha^{2} + \alpha \beta + \beta^{2} + \alpha (\alpha + \beta) + b}{(\alpha + \beta)^{2} - \alpha \beta + \alpha (\alpha + \beta) + b}$	IM	For $\lambda^2 + \beta^2 = (\alpha + \beta)^2 - 2\kappa \beta$
$- \mathcal{K} \xrightarrow{\alpha\beta} \longrightarrow = \frac{\alpha^2}{4} - \frac{b}{3} - \frac{2a^2}{3} + b$		For all p = costps = 2xp
•	IM+IA	
$= \frac{2}{9} \left(3b - a^2 \right)$	1A 7	
(d) Since $a^2 > 3b$, $\frac{2}{9}(3b - a^2) < 0$	IM	
$\frac{f(\lambda)-f(\beta)}{\lambda-\beta}<0$	IM	
$f(\alpha) - f(\beta) < 0$ $if \alpha - \beta > 0$	2A	
$f(\alpha) < f(\beta)$		
if $\alpha > \beta$		
	4	_

Solutions		<i>thavis</i>	Remarks
(i) (a) Let $P(n): 1^2 + 2^2 + \dots + n^2 = \frac{1}{6}n(n)$			• • • • • • • • • • • • • • • • • • •
$P(1) \text{ is true since } R.S. = \frac{1}{6} \times 1 \times 1$	2 x 3 = 1 = 2.5.	14	
Assume p(n) is true for n=k;		14	assume a = & is true is
12. $1^2 + 2^2 + \dots + k^2 = \frac{1}{6} k (k)$	+1)(2h+1)	-	
$1^{2}+2^{2}++k^{2}+(k+1)^{2}=\frac{1}{6}\lambda(k+1)$	(2h+1) + (k+1)2	1A	In (k+1) to both the
$=\frac{(2k+1)}{6}\left((2k+1) \right)$	(+k)+6&+y]		
•	(2h+3)	l A	
= $\frac{1}{6}(k+1)(1$ By M.I. $P(n)$ is true $\forall n$	k+1)+1/2(k+1)+1)	IA	
By M.I. P(n) is true Yn	EN - of	1M	July awarded if above conest
(b) (i) Length of each brick = x		14	•
•	- = ~-	1/24	: - -
No. of the v-th layer = 726	(-) ³	$\int_{\mathcal{A}} d\mathbf{A}$	
There are altogether n layer	\$	1A	V= V, +V2 + 1 . V2
: vol. of the solid = $\left(\frac{X}{n}\right)^3$	$\begin{pmatrix} 2 & 2 \\ +2 & +\cdots +n^2 \end{pmatrix}$	IM+/A	
(1) Height of Agramid = nx	$\frac{r}{c} = \chi$	14	
· · · · · · · · · · · · · · · · · ·	$\frac{\chi^3}{3}$	IA	
Vol. of solid - vol. of pyn	unid		
$= \left(\frac{\gamma}{n}\right)^{3} \left(1^{\frac{2}{7}} 2^{\frac{2}{7}} \cdots + n^{2}\right) - \frac{\chi^{3}}{3}$		IM	Volume FO ist
$= \left(\frac{x}{h}\right)^{3} \cdot \frac{1}{6} n (n+1)(2n+1) - \frac{x^{3}}{3}$		lm:	it (2) mut 1
$= \frac{\chi^3}{3} \left[\frac{1}{2n^2} (2n+1)(2n+1) - 1 \right]$	•		
$= \frac{13}{3} \left(\frac{3n+1}{2n^2} \right) \qquad \left(\pi \right)$	3(3,1)	24	2. A 10 10 10 10 10 10 10 10 10 10 10 10 10
>`a ` ` ` ` ` ` ` ` ` ` ` ` ` ` ` ` ` `		IA:	aute independent
vol. of solid is always gren pymind. If nis very large to	ter Than vol. of	re to zero /A	quite independent
	! 	· · · · · · · · · · · · · · · · · · ·	

C 0]	Thanks	Remarks
J stutions		De Moure's Thm
2) (a) $con + \omega = \left[con \frac{2nk\pi}{5} + i sin \frac{2nk\pi}{5}\right] + \left[cos \frac{2nk\pi}{5} + i sin \frac{2nk\pi}{5}\right]$		
$= 2 \cos \frac{2h R\pi}{5}$] A	
(b) $\omega^5 = c\omega 5 = \frac{2k\pi}{5} + i \sin 5 = \frac{2k\pi}{5}$	1 A	,
$= CO 2 k\pi + \lambda Sin 2 k\pi$	LA	ALL
$\int 1 + \omega + \omega^2 + \omega^3 + \omega'' = \frac{1 - \omega^5}{1 - \omega}$	347	$\omega^{5} = 0 \qquad (1A)$ $(\omega - 1) (1 + \omega + \omega^{2} + \omega^{3} + \omega^{3} + \omega^{3})$
		$\omega \neq 1, 1 + \omega + \omega^{2} + \omega^{4} = 0$
= 0 (C.w.) X	6	@
7 2 4 75 2 3 4		
$= \int_{0}^{\infty} 1 + \omega + \omega^{2} + \omega^{3} + \omega^{4} = \omega^{4} + \omega + \omega^{2} + \omega^{3} + \omega^{4}$	IA.	从一点分
$= \omega(\omega^{4} + 1 + \omega + \omega^{2} + \omega^{3})$	1A	
$\therefore (1-\omega)(1+\omega+\omega^2+\omega^3+\omega^4)=0$	IA	
$\frac{1+\omega+\omega^2+\omega^2+\omega^4=0}{(1+\omega+1)^2+\omega^4=0}$	<u>/.≱</u>	13.20
$(0) (\omega + \omega^{-1})^{-} + (\omega^{2} + \omega^{-2})^{-} = \omega^{2} + 2 + \omega^{2} + \omega^{4} + 2 + \omega^{7}$	14	$(\omega+\omega^{-1})^{2}+(\omega^{-2}+\omega^{-2})^{-1}$
$\frac{1}{\omega_{1}^{2}+1}=\frac{2}{\omega_{1}^{2}+\omega_{2}^{2}+\omega_{3}^{2}+\omega_{4}^{2}+\omega_{5}^{2}}=\frac{2}{\omega_{1}^{2}+2+\omega_{3}^{2}+\omega_{4}^{2}+2+\omega_{5}^{2}}$	2194	= W+2+ du+ W+2+ du (1)
$= 3 + (1 + \omega + \omega^2 + \omega^3 + \omega^4)$	14	$=\frac{\omega^2+2\omega^2+\omega^2+\omega^2+2\omega^2+1}{\omega^2}.$
= 3 	14	= a1 + 2w + w + w + 2w + 2w + 2 pma
\star 2M for $\omega^{-n} = \omega^{-n+5}$	6	$= \frac{(1+\omega+\omega^2+\omega^3+\omega^4+3\omega^4)}{(1+\omega+\omega^2+\omega^3+\omega^4)}$
	-	= 3
(d) From (a) $\cos \frac{2k\pi}{5} = \frac{\omega + \omega}{2}$, $\sin \frac{\psi k\pi}{5} = \frac{\omega^2 + \omega^2}{2}$	1+1	2mfor articles.
$\therefore \left(\cos \frac{2 \cancel{k} \tau}{5} \right)^2 + \left(\cos \frac{4 \cancel{k} \tau}{5} \right)^2 = \left(\frac{\omega + \omega^{-1}}{2} \right)^2 + \left(\frac{\omega^2 + \omega^{-2}}{2} \right)^2$		$\omega^n = \omega^{n-r}$
	_ 1	1 Sule
$\frac{1}{\omega} = \frac{1}{\omega} \left((\omega + \omega^{2}) + (\omega^{2} + \omega^{2}) \right)$	1 1A	
= 3 (rom(c)	11	
	5	
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