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> 附加數學卷二 ADDITIONAL MATHEMATICS PAPER II

> > 評卷 参考 MARKING SCHEME

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本評卷參考並非標準答案, 故極不宜 落於學生手中, 以免引起誤會。

週有學生求取此文件時, 閱卷員應嚴 予拒絕。閱卷員如向學生披露本評卷參考 內容, 即遠背閱卷員守則。

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_ 1994 HKCE Add. Maths. II Marking Scheme

	Solution	Marks	Remarks
1.	$\int (\sin x - \cos x)^2 dx$		
	$= \int (\sin^2 x - 2\sin x \cos x + \cos^2 x) dx$	1A	
	(SIN X = ZSIN XCOSX + COS Z, CA		
	$= \int (1 - 2\sin x \cos x) dx$,
. •			
	$= x - \sin^2 x + c$ (where c is a constant)	1A+2A	A for dx = x
			2A for \2 sinxcosxdx
			$= \sin^2 x$ $(pp-1) for omitting c$
	Alternative solution		(pp-1) for omitting dx
_	$x + \cos^2 x + c$, $x + \frac{1}{2}\cos 2x + c$		
		4	
2.	$\cos(x - 7^\circ) = 2\cos(x + 7^\circ)$		
	cosxcos7° + sinxsin7° = 2cosxcos7° - 2sinxsin7°	1A+1A	1A for L.H.S., 1A for R.H.
	3sinxsin7° = cosxcos7°		
	gog 7.9		
	$\tan x = \frac{\cos 7^{\circ}}{3\sin 7^{\circ}}$	1	
	x = 180n° + 69.8°	1M+1A	lM for 180n° + α
		4	In for 100n , a
			A Part of the State of the Stat
3.	(a) $(1-2x)^3 = 1-6x+12x^2-8x^3$, ,	
, •	(a) $(1-2x)^3 = 1-6x+12x^2-8x^3$	1A	
	$(1 + \frac{1}{x})^5 = 1 + \frac{5}{x} + \frac{10}{x^2} + \frac{10}{x^3} + \frac{5}{x^4} + \frac{1}{x^5}$	1A	. •
	X		
	(b) $(1-2x)^3(1+\frac{1}{x})^5 = (1-6x+12x^2-8x^3)$		
	$\left(1 + \frac{5}{x} + \frac{10}{x^2} + \frac{10}{x^3} + \frac{5}{x^4} + \frac{1}{x^5}\right)$		
	$(\frac{1}{x}, \frac{1}{x^2}, \frac{1}{x^3}, \frac{1}{x^4}, \frac{1}{x^5})$		
		1M	For collecting terms
	(i) Constant term = $1 - 6(5) + 12(10) - 8(10)$		
	= 11	1A	
	(ii) Coefficient of $x = -6 + 12(5) - 8(10)$		
	= -26	1A	
	. ==	1A 5	
		1	

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4. (a) Area of $\triangle POR = \frac{1}{2} 4x + 0 + 2y - 6x - 8 $ $= -x + y - 4 $ $= -x + y - 4 = 4$ $-x + y - 4 = \pm 4$ $x - y = 0 \text{ or } x - y + 8 = 0$ $\therefore \text{ the equation of the curve is } x - y = 0$ $\frac{\text{Alternative solution}}{(b) (-x + y - 4)^2} = 4^2$ $x^2 - 2xy + y^2 + 8x - 8y = 0$ $\therefore \text{ the equation of the curve is } x^2 - 2xy + y^2 + 8x - 8y = 0$ $\frac{x^2 - 2xy + y^2 + 8x - 8y = 0}{5}$ 5. For $n = 1$, L.H.S. = $\frac{1}{2}$ $R.H.S. = 3 - \frac{2 + 3}{2} = \frac{1}{2}$ $\therefore \text{ the statement is true for } n = 1$ $\text{Assume } \frac{1}{2} + \frac{3}{2} + \frac{5}{2} + \dots + \frac{2k - 1}{2^k} = 3 - \frac{2k + 3}{2^k}$ $\text{(for some positive integer } k)$ $\text{Then } \frac{1}{2} + \frac{3}{2^k} + \frac{5}{2^k} + \dots + \frac{2k - 1}{2^{k-1}} + \frac{2(k + 1) - 1}{2^{k-1}}$ $= 3 - \frac{2k + 3}{2^k} + \frac{2k + 1}{2^{k-1}}$ $= 3 - \frac{2(2k + 3) - (2k + 1)}{2^{k-1}}$ $= 3 - \frac{2(2k + 3) - (2k + 1)}{2^{k-1}}$ $= 3 - \frac{2(2k + 3) - (2k + 1)}{2^{k-1}}$ (\triangle the statement is also true for $n = k + 1$ if it is true for $n = k$) (By the Principle of mathematical induction)	Solut	tion	Marks	Remarks	
(b) $ -x+y-4 =4$ $-x+y-4=\pm 4$ $x-y=0$ or $x-y+8=0$ \therefore the equation of the curve is $x-y=0$ \therefore the equation of the curve is $x^2-2xy+y^2+8x-8y=0$ \therefore the equation of the curve is $x^2-2xy+y^2+8x-8y=0$ \Rightarrow 1 8. H.s. $=3-\frac{2+3}{2}=\frac{1}{2}$ \Rightarrow the statement is true for $n=1$ Assume $\frac{1}{2}+\frac{3}{2^2}+\frac{5}{2^2}+\dots+\frac{2k-1}{2^k}=3-\frac{2k+3}{2^k}$ (for some positive integer k) Then $\frac{1}{2}+\frac{3}{2^2}+\frac{5}{2^3}+\dots+\frac{2k-1}{2^k}+\frac{2(k+1)-1}{2^{k+1}}$ $=3-\frac{2k+3}{2^k}+\frac{2k+1}{2^{k+1}}$ $=3-\frac{2(2k+3)-(2k+1)}{2^{k+1}}$ $=3-\frac{2(2k+3)-(2k+1)}{2^{k+1}}$ $=3-\frac{2k+5}{2^{k+1}}$ (= $3-\frac{2(k+1)+3}{2^{k+1}}$) (∴ the statement is also true for $n=k+1$ if it is true for $n=k$	(a)	Area of $\Delta PQR = \frac{1}{2} 4x + 0 + 2y - 6x - 8 $:	
$-x + y - 4 = \pm 4$ $x - y = 0 \text{ or } x - y + 8 = 0$ $\therefore \text{ the equation of the curve is } x - y = 0$ $\text{or } x - y + 8 = 0$ $\frac{\text{Alternative solution}}{\text{(b)} (-x + y - 4)^2 = 4^2}$ $x^2 - 2xy + y^2 + 8x - 8y = 0$ $\therefore \text{ the equation of the curve is } x^2 - 2xy + y^2 + 8x - 8y = 0$ $\frac{\text{5}}{5}$ $\text{For n = 1, L.H.S.} = \frac{1}{2}$ $\text{R.H.S.} = 3 - \frac{2 + 3}{2} = \frac{1}{2}$ $\text{the statement is true for n = 1}$ $\text{Assume } \frac{1}{2} + \frac{3}{2^3} + \frac{5}{2^3} + \dots + \frac{2k - 1}{2^k} = 3 - \frac{2k + 3}{2^k}$ $\text{(for some positive integer } k)$ $\text{Then } \frac{1}{2} + \frac{3}{2^2} + \frac{5}{2^3} + \dots + \frac{2k - 1}{2^k} + \frac{2(k + 1) - 1}{2^{k - 1}}$ $= 3 - \frac{2k + 3}{2^k} + \frac{2k + 1}{2^{k - 1}}$ $= 3 - \frac{2(2k + 3) - (2k + 1)}{2^{k - 1}}$ $= 3 - \frac{2(2k + 3) - (2k + 1)}{2^{k - 1}}$ $= 3 - \frac{2(k + 3) - (2k + 1)}{2^{k - 1}}$ $= 3 - \frac{2(k + 3) - (2k + 1)}{2^{k - 1}}$ $= 3 - \frac{2(k + 3) - (2k + 1)}{2^{k - 1}}$ $\text{($:$ the statement is also true for n = k + 1$ if it is true for n = k}$		$= \left -x + y - 4 \right $	1A+1A	1A for $(-x + y - 4)$ or $(x - y + 4)$	
$x-y=0 \text{ or } x-y+8=0$ $\therefore \text{ the equation of the curve is } x-y=0$ $\text{or } x-y+8=0$ $\frac{\text{Alternative solution}}{(b) (-x+y-4)^2=4^2}$ $x^2-2xy+y^2+8x-8y=0$ $\therefore \text{ the equation of the curve is } x^2-2xy+y^2+8x-8y=0$ $\frac{x^2-2xy+y^2+8x-8y=0}{2A}$ $\frac{5}{5}$ i. For $n=1$, L.H.S. $=\frac{1}{2}$ $R.H.S. = 3-\frac{2+3}{2}=\frac{1}{2}$ $\therefore \text{ the statement is true for } n=1$ $Assume \frac{1}{2}+\frac{3}{2^2}+\frac{5}{2^3}+\ldots+\frac{2k-1}{2^k}=3-\frac{2k+3}{2^k}$ $\text{(for some positive integer } k\text{)}$ $\text{Then } \frac{1}{2}+\frac{3}{2^2}+\frac{5}{2^3}+\ldots+\frac{2k-1}{2^k}+\frac{2(k+1)-1}{2^{k-1}}$ $=3-\frac{2k+3}{2^k}+\frac{2k+1}{2^{k+1}}$ $=3-\frac{2(2k+3)-(2k+1)}{2^{k-1}}$ $=3-\frac{2(2k+3)-(2k+1)}{2^{k-1}}$ $=3-\frac{2(2k+3)-(2k+1)}{2^{k-1}}$ ($\therefore \text{ the statement is also true for } n=k+1$ if it is true for $n=k$)	(b)	$\left -x+y-4\right =4$	1M	Accept no absolute value	
∴ the equation of the curve is $x - y = 0$ or $x - y + 8 = 0$ Alternative solution (b) $(-x + y - 4)^2 = 4^2$ $x^2 - 2xy + y^2 + 8x - 8y = 0$ ∴ the equation of the curve is $x^2 - 2xy + y^2 + 8x - 8y = 0$ ∴ the statement is true for $n = 1$ Assume $\frac{1}{2} + \frac{3}{2^2} + \frac{5}{2^3} + \dots + \frac{2k-1}{2^k} = 3 - \frac{2k+3}{2^k}$ (for some positive integer k) Then $\frac{1}{2} + \frac{3}{2^2} + \frac{5}{2^3} + \dots + \frac{2k-1}{2^k} + \frac{2(k+1)-1}{2^{k-1}}$ = $3 - \frac{2k+3}{2^k} + \frac{2k+1}{2^{k-1}}$ = $3 - \frac{2(2k+3) - (2k+1)}{2^{k-1}}$ = $3 - \frac{2k+5}{2^{k-1}}$ (= $3 - \frac{2(k+1)+3}{2^{k-1}}$) (∴ the statement is also true for $n = k + 1$ if it is true for $n = k$)		$-x + y - 4 = \pm 4$			
$x^{2} - 2xy + y^{2} + 8x - 8y = 0$ $\therefore \text{ the equation of the curve is } x^{2} - 2xy + y^{2} + 8x - 8y = 0$ $5. \text{For n = 1, L.H.S.} = \frac{1}{2}$ $R.H.S. = 3 - \frac{2+3}{2} = \frac{1}{2}$ $\therefore \text{ the statement is true for n = 1}$ $\text{Assume } \frac{1}{2} + \frac{3}{2^{2}} + \frac{5}{2^{3}} + \dots + \frac{2k-1}{2^{k}} = 3 - \frac{2k+3}{2^{k}}$ $\text{(for some positive integer } k)$ $\text{Then } \frac{1}{2} + \frac{3}{2^{2}} + \frac{5}{2^{3}} + \dots + \frac{2k-1}{2^{k}} + \frac{2(k+1)-1}{2^{k-1}}$ $= 3 - \frac{2k+3}{2^{k}} + \frac{2k+1}{2^{k-1}}$ $= 3 - \frac{2(2k+3) - (2k+1)}{2^{k}}$ $= 3 - \frac{2(2k+3) - (2k+1)}{2^{k-1}}$ $= 3 - \frac{2(2k+3) - (2k+1)}{2^{k-1}}$ (the statement is also true for n = k + 1 if it is true for n = k)		\therefore the equation of the curve is $x - y = 0$	1A+1A		
$\frac{1}{x^2 - 2xy + y^2 + 8x - 8y = 0}$ $\frac{5}{5}$ 5. For $n = 1$, L.H.S. $= \frac{1}{2}$ R.H.S. $= 3 - \frac{2+3}{2} = \frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2} + \frac{3}{2^2} + \frac{5}{2^3} + \dots + \frac{2k-1}{2^k} = 3 - \frac{2k+3}{2^k}$ $\frac{1}{2} + \frac{3}{2^2} + \frac{5}{2^3} + \dots + \frac{2k-1}{2^k} + \frac{2(k+1)-1}{2^{k+1}}$ $\frac{1}{2} + \frac{3}{2^2} + \frac{5}{2^3} + \dots + \frac{2k-1}{2^k} + \frac{2(k+1)-1}{2^{k+1}}$ $\frac{1}{2} + \frac{3}{2^k} + \frac{2k+1}{2^{k+1}}$ $\frac{1}{2} + \frac{3}{2^k} + \frac{3}{2^k} + \frac{3}{2^k} + \frac{3}{2^{k+1}}$ $\frac{1}{2} + \frac{3}{2^k} + \frac$	•		1M		
5. For $n = 1$, L.H.S. $= \frac{1}{2}$ R.H.S. $= 3 - \frac{2+3}{2} = \frac{1}{2}$ \therefore the statement is true for $n = 1$ Assume $\frac{1}{2} + \frac{3}{2^2} + \frac{5}{2^3} + \dots + \frac{2k-1}{2^k} = 3 - \frac{2k+3}{2^k}$ (for some positive integer k) Then $\frac{1}{2} + \frac{3}{2^2} + \frac{5}{2^3} + \dots + \frac{2k-1}{2^k} + \frac{2(k+1)-1}{2^{k+1}}$ $= 3 - \frac{2k+3}{2^k} + \frac{2k+1}{2^{k+1}}$ $= 3 - \frac{2(2k+3) - (2k+1)}{2^{k+1}}$ $= 3 - \frac{2(2k+3) - (2k+1)}{2^{k+1}}$ (\therefore the statement is also true for $n = k+1$ if it is true for $n = k$)		. the equation of the curve is	2A		
R.H.S. $= 3 - \frac{2+3}{2} = \frac{1}{2}$ the statement is true for $n = 1$ Assume $\frac{1}{2} + \frac{3}{2^2} + \frac{5}{2^3} + \dots + \frac{2k-1}{2^k} = 3 - \frac{2k+3}{2^k}$ (for some positive integer k) Then $\frac{1}{2} + \frac{3}{2^2} + \frac{5}{2^3} + \dots + \frac{2k-1}{2^k} + \frac{2(k+1)-1}{2^{k-1}}$ $= 3 - \frac{2k+3}{2^k} + \frac{2k+1}{2^{k-1}}$ $= 3 - \frac{2(2k+3) - (2k+1)}{2^{k-1}}$ $= 3 - \frac{2k+5}{2^{k-1}} (= 3 - \frac{2(k+1)+3}{2^{k-1}})$ (the statement is also true for $n = k+1$ if it is true for $n = k$)		·			
∴ the statement is true for n = 1 Assume $\frac{1}{2} + \frac{3}{2^2} + \frac{5}{2^3} + \dots + \frac{2k-1}{2^k} = 3 - \frac{2k+3}{2^k}$ (for some positive integer k) Then $\frac{1}{2} + \frac{3}{2^2} + \frac{5}{2^3} + \dots + \frac{2k-1}{2^k} + \frac{2(k+1)-1}{2^{k-1}}$ $= 3 - \frac{2k+3}{2^k} + \frac{2k+1}{2^{k-1}}$ $= 3 - \frac{2(2k+3) - (2k+1)}{2^{k+1}}$ $= 3 - \frac{2(2k+3) - (2k+1)}{2^{k+1}}$ 1 (∴ the statement is also true for n = k + 1 if it is true for n = k)	For n	$n = 1$, L.H.S. $= \frac{1}{2}$			
Assume $\frac{1}{2} + \frac{3}{2^2} + \frac{5}{2^3} + \dots + \frac{2k-1}{2^k} = 3 - \frac{2k+3}{2^k}$ [1] (for some positive integer k) Then $\frac{1}{2} + \frac{3}{2^2} + \frac{5}{2^3} + \dots + \frac{2k-1}{2^k} + \frac{2(k+1)-1}{2^{k-1}}$ $= 3 - \frac{2k+3}{2^k} + \frac{2k+1}{2^{k+1}}$ $= 3 - \frac{2(2k+3) - (2k+1)}{2^{k+1}}$ $= 3 - \frac{2k+5}{2^{k+1}} (= 3 - \frac{2(k+1)+3}{2^{k+1}})$ (In the statement is also true for $n = k+1$ if it is true for $n = k$)		R.H.S. = $3 - \frac{2+3}{2} = \frac{1}{2}$	1		
Then $\frac{1}{2} + \frac{3}{2^2} + \frac{5}{2^3} + \dots + \frac{2k-1}{2^k} + \frac{2(k+1)-1}{2^{k-1}}$ $= 3 - \frac{2k+3}{2^k} + \frac{2k+1}{2^{k+1}}$ $= 3 - \frac{2(2k+3) - (2k+1)}{2^{k+1}}$ $= 3 - \frac{2k+5}{2^{k+1}} (= 3 - \frac{2(k+1)+3}{2^{k+1}})$ (: the statement is also true for $n = k+1$ if it is true for $n = k$)			1		
$= 3 - \frac{2(2k+3) - (2k+1)}{2^{k+1}}$ $= 3 - \frac{2k+5}{2^{k+1}} (= 3 - \frac{2(k+1)+3}{2^{k+1}})$ (the statement is also true for $n = k+1$ if it is true for $n = k$)					
$= 3 - \frac{2k+5}{2^{k+1}} (= 3 - \frac{2(k+1)+3}{2^{k+1}})$ (the statement is also true for $n = k+1$ if it is true for $n = k$)	= 3	$3 - \frac{2k+3}{2^k} + \frac{2k+1}{2^{k+1}}$	1		
(: the statement is also true for n = k + 1 if it is true for n = k)	= 3 ·	$3 - \frac{2(2k+3) - (2k+1)}{2^{k+1}}$,	
if it is true for n = k)	= 3	$3 - \frac{2k+5}{2^{k+1}} \ (= 3 - \frac{2(k+1)+3}{2^{k+1}})$	1		
(By the Principle of mathematical induction)			-		
\therefore the statement is true for all +ve integers n. $\frac{1}{5}$			15		

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	Sol	ution	Marks	Remarks	
6.	(a)	Slope of $L_1 = 2$, slope of $L_2 = 3$	1A		
		$\tan\theta = \frac{3-2}{1+3(2)}$	1M	Accept $\frac{2-3}{1+2(3)}$	
		$=\frac{1}{7}$	1A	1M for using inclination	
	(p)	Let m be the slope of the line			
		$\frac{2-m}{1+2m}=\frac{1}{7}$	1M	Accept $\frac{m-2}{1+2m}=\frac{1}{7}$	
		$m = \frac{13}{9}$	1A		
		∴ the equation of the line is $y = \frac{13}{9}x$	<u>1A</u>	or 13x - 9y = 0	
_		•	_6		
7.	(a)	$x^3 = x^3 - 6x^2 + 12x$	1A		
		$6x^2 - 12x = 0$			
		x = 0 or 2			
		The coordinates of A are (2, 8)	1A		
	(b)	Area = $\int_0^2 [(x^3 - 6x^2 + 12x) - x^3] dx$	1M+1A	$\int_{\alpha}^{\beta} (y_2 - y_1) dx$	
		$= \int_0^2 (-6 x^2 + 12 x) dx$			
		$= \left[-2 \mathbf{x}^3 + 6 \mathbf{x}^2\right]_0^2$	1A	For primitive function only	
		= 8	1A 6		
	(a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 8 - 10x$			
		$y = 8x - 5x^2 + k$	1A	·	
	•	Put $x = 1$, $y = 13$, $k = 10$	1M	For substituting (1, 13) and	
		\therefore The equation of C is $y = 8x - 5x^2 + 10$	1A	finding k	
	(b)	At $x = 0$,			
		$\frac{dy}{dx} = 8$	1A		
		dx			
		∴slope of normal = $-\frac{1}{8}$	1M		
		y = 10	IM		
		The equation of the normal is			
		$\frac{y-10}{x-0}=-\frac{1}{8}$		•	
		$y = -\frac{1}{8}x + 10$	1 <u>A</u> 7	or $x + 8y - 80 = 0$	
			. —		

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	Sol	ution	Marks	Remarks
9.	(a)	A, B are equidistant from the centre $(h - 5)^2 + (k - 5)^2 = (h - 7)^2 + (k - 1)^2$ $h^2 - 10h + 25 + k^2 - 10k + 25 =$ $h^2 - 14h + 49 + k^2 - 2k + 1$	2A	
		h = 2k	1A	
		Alternative solution Mid point of AB = (6, 3), slope of AB = - 2 Equation of perpendicular bisection of AB	1A	
		$\frac{y-3}{x-6}=\frac{1}{2}$	1M	
		x = 2y Since (h, k) lies on the perpendicular bisector $\therefore h = 2k$, 1A	
		Equation of C is $(x - h)^2 + (y - k)^2 = (h - 7)^2 + (k - 1)^2$ $(x - 2k)^2 + (y - k)^2 = (2k - 7)^2 + (k - 1)^2$	1M	or = $(h - 5)^2 + (k - 5)^2$
	(b)	$x^2 + y^2 - 4kx - 2ky + 30k - 50 = 0$ Slope of line joining centre (2k, k) and B(7, 1)	<u>1</u> _5	
		$=\frac{k-1}{h-7}$	1M	Accept $\frac{k-1}{h-k}$
		Slope of tangent at $B = \frac{7-2k}{k-1}$	1M	or $\frac{7-h}{k-1}$
		Since slope of tangent at $B = \frac{1}{2}$		
		$\frac{7-2k}{k-1} = \frac{1}{2}$ $k = 3$		In one unknown
		∴ Equation of C is $x^2 + y^2 - 12x - 6y + 40 = 0$		or $(x - 6)^2 + (y - 3)^2 = 5$
			•	

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ne	1	·
lution	Marks	Remarks
Alternative solution :	<u> </u>	1
Method (2):		
$x^2 + y^2 - 4ky - 2ky + 30k - 50 = 0$		
$2x + 2y\frac{dy}{dx} - 4k - 2k\frac{dy}{dx} = 0$	1M	
$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4k - 2x}{2y - 2k}$		
At B (7, 1), $\frac{dy}{dx} = \frac{4k-14}{2-2k} = \frac{1}{2}$	1M+1M	1M for substituting (7, 1)
k = 3	1A	1M for equating $\frac{1}{2}$
equation of C is $x^2 + y^2 - 12x - 6y + 40 = 0$	1A	
Method (3):		
The equation of tangent at B is		
· ·	1,,	
$y-1=\frac{1}{2}(x-7)$	lA	
x - 2y - 5 = 0		
Distance from centre $(2k, k)$ of circle to the line		
$= \left \frac{2k - 2k - 5}{\sqrt{5}} \right $	1M	
= √ 5		
$\sqrt{5} = \sqrt{5k^2 - 30k + 50}$	1M	
$k^2 - 6k + 9 = 0$		
k = 3	1A	
: equation of C is $x^2 + y^2 - 12x - 6y + 40 = 0$	1A	
Method (4):		
The equation of tangent at B is		
$y-1=\frac{1}{2}(x-7)$	1A	
$y = \frac{1}{2}(x - 5)$ (or $x = 2y + 5$)		
Substitute into C		
$x^{2} + \frac{1}{4}(x-5)^{2} - 4kx - 2k\frac{1}{2}(x-5) + 30k - 50 = 0$	1M	or substitute $x = 2y + 5$
$5x^2 - 10(2k + 1)x + 140k - 175 = 0$		$ 5y^2 + (20 - 10k)y + (10k - 25) = 0$
Dis. = $100(2k + 1)^2 - 20(140k - 175) = 0$	1M	$(20 - 10k)^2 - 20(10k - 25) = 0$
$k^2-6k+9=0$		
k = 3	1A	
: equation of C is $x^2 + y^2 - 12x - 6y + 40 = 0$	1	

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Marking Scheme	-	į
Solution	Marks	Remarks
(c) Radius of $C = \sqrt{(h-7)^2 + (k-1)^2}$	1M	or $\sqrt{(h-5)^2+(k-5)^2}$
$= \sqrt{5k^2 - 30k + 50}$ Distance from $(2k, k)$ to the line $y = 3x$ $= \left \frac{3(2k) - k}{\sqrt{10}} \right = \left \frac{5k}{\sqrt{10}} \right $	1:M	Accept missing absoulte signs
If the circle touches the line, $\left \frac{5k}{\sqrt{10}}\right = \sqrt{5k^2 - 30k + 50}$ $k^2 - 12k + 20 = 0$	1 1	For equating and expressing in one unknown
k = 2 or 10 ∴ The equations of the circles are $x^2 + y^2 - 8x - 4y + 10 = 0$ and $x^2 + y^2 - 40x - 20y + 250 = 0$		or $(x - 4)^2 + (y - 2)^2 = 10$ or $(x - 20)^2 + (y - 10)^2 = 250$
Alternative solution Substitute $y = 3x$ into C $x^2 + (3x)^2 - 4kx - 2k(3x) + 30k - 50 = 0$ $10x^2 - 10kx + 30k - 50 = 0$ Discriminant = $100k^2 - 40(30k - 50) = 0$ $k^2 - 12k + 20 = 0$ $k = 2$ or 10 \therefore The equations of the circles are $x^2 + y^2 - 8x - 4y + 10 = 0$ and $x^2 + y^2 - 40x - 20y + 250 = 0$	1M 1M+1A 1A 1A	

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Sol	Solution			Remarks
. (a)	$\int_0^1 \frac{\mathrm{d}x}{1+x^2} = \int_0^{\frac{\pi}{4}} \frac{\sec^2\theta \mathrm{d}\theta}{1+\tan^2\theta}$	1A+1A	lA for ir	
	$=\int_{0}^{\frac{\pi}{4}}d\theta$	1A		
	$=\frac{\pi}{4}$	1A 4		
(p)	3 + 2sinx + cosx	4		
	$=3+2(\frac{2t}{1+t^2})+\frac{1-t^2}{1+t^2}$	1A+1A	1A for si	
	$=\frac{2(2+2t+t^2)}{1+t^2}$	1		
	$t = \tan \frac{x}{2}$			
	$dt = \frac{1}{2} \sec^2 \frac{x}{2} dx$		melley place of the control of the c	
	$dx = \frac{2dt}{1+t^2}$			
	$\int \frac{dx}{3 + 2\sin x + \cos x} = \int \frac{1 + t^2}{2(2 + 2t + t^2)} \frac{2dt}{1 + t^2}$	1A	With the state of	
	$=\int \frac{\mathrm{d}t}{2+2t+t^2}$			
	$=\int \frac{\mathrm{d}t}{1+(1+t)^2}$	<u>1</u>		
(c)	Put $t = \tan \frac{x}{2}$			
	$\int_{-\frac{\pi}{2}}^{0} \frac{dx}{3 + 2\sin x + \cos x} = \int_{-1}^{0} \frac{dt}{1 + (1 + t)^{2}}$	1À		
	Put $u = 1 + t$	1A		
	$\int_{-1}^{0} \frac{dt}{1 + (1 + t)^{2}} = \int_{0}^{1} \frac{du}{1 + u^{2}}$	1A		
	$=\frac{\pi}{4}$ (using the result of (a))	1 <u>A</u>		

Marking Scheme

Solu	tion	Marks	Remarks
(d)	$\int_{-\frac{\pi}{2}}^{9} \frac{(2\sin x + \cos x) dx}{3 + 2\sin x + \cos x}$		
	$= \int_{-\frac{\pi}{2}}^{0} (1 - \frac{3}{3 + 2 \sin x + \cos x}) dx$	lM+lA	$1M \text{ for } k_1 + \frac{k_2}{3 + 2\sin x + \cos x}$
	$= \int_{-\frac{\pi}{2}}^{0} dx - 3 \int_{-\frac{\pi}{2}}^{0} \frac{dx}{3 + 2\sin x + \cos x}$		
	$=\frac{\pi}{2}-3(\frac{\pi}{4})$		
	$=-\frac{\pi}{4}$	1 <u>A</u>	
	At the control of the	_3	

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Markin	g Schem	e		Р.
	Sol	ution	Marks	Remarks
11.	(a)	Substitute $y = m_1 x + c_1$ into $x^2 = 8y$		
		$x^2 = 8(m_1x + c_1)$	1A	or $(\frac{y-c_1}{m_1})^2 = 8y$
		$x^2 - 8m_1x - 8c_1 = 0$		
. •		Discriminant = $64m_1^2 + 32c_1 = 0$	1M	·
		$C_1 = -2m_1^2$	1A 3	
	(b)	Equation of L_2 is $y = m_2 x - 2m_2^2$	1A	
		$y = m_1 x - 2m_1^2$		
•		$y = m_2 x - 2m_2^2$	1M	For solving the 2 eqns.
_		$0 = (m_1 - m_2) \times - 2(m_1^2 - m_2^2)$		
		$x = 2(m_1 + m_2)$	1	
		$y = m_1 x - 2m_1^2$		
		$= m_1[2(m_1 + m_2)] - 2m_1^2$		
		$= 2m_1m_2$	1_4_	
	(c)	Let (x, y) be a point on the locus.		
		$\begin{cases} x = 2(m_1 + m_2) \\ y = 2m_1m_2 \end{cases}$		
		$\left \frac{m_1-m_2}{1+m_1m_2}\right =\tan\frac{\pi}{4}$	1A	Accept no absolute sign
_		$\left(\frac{m_1 - m_2}{1 + m_1 m_2}\right)^2 = 1$	1M	For squaring both sides
,		$(m_1 + m_2)^2 - 4m_1m_2 = (1 + m_1m_2)^2 \qquad .$	1A	
		$(\frac{x}{2})^2 - 2y = (1 + \frac{y}{2})^2$	1M	For substituting $m_1 + m_2 = \frac{x}{2}$, $m_1 m_2 = \frac{y}{2}$
		$x^2 - y^2 - 12y - 4 = 0$	1A	2 2
	(d)	Let (x, y) be a point on the locus	_5	
	(-/	$(x = 2(m_1 + m_2))$		
		$y = 2m_1m_2$		
		Since $L_1 \perp L_2$, $m_1 m_2 = -1$	1A	
		$y = 2m_1m_2 = -2$		•
		∴ the equation of the locus is $y = -2$	1A	
		OR Y	1A	For a line below and parallel the x-axis
		-2 7=-2		For labelling the axes and the line
				· •

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Solution			Remarks
12. (a)	By Sine law,		
	$\frac{AC}{\sin\beta} = \frac{100}{\sin(\pi - \alpha - \beta)}$	1M+1A	$1M \text{ for } \frac{AC}{\sin B} = \frac{AB}{\sin C}$
	$AC = \frac{100\sin\beta}{\sin(\alpha + \beta)} (km)$	1	
	$PC = AC \tan \theta$	1A	·
	$= \frac{100 \sin\beta \tan\theta}{\sin(\alpha + \beta)} (km)$	<u>1</u> 5	·
(p)	(i) $AC = \frac{100 \sin 30^{\circ}}{\sin (45^{\circ} + 30^{\circ})}$		
_	= 51.76 (km)	1A	
	$AC' = \frac{100 \sin 43^{\circ}}{\sin (37^{\circ} + 43^{\circ})}$		
	= 69.25 (km)	1A	
	(ii) $\angle CAC' = 45^{\circ} - 37^{\circ} = 8^{\circ}$	1A	
	By Cosine law,		
	$CC^{2} = AC^{2} + AC^{2} - 2(AC)(AC')\cos\angle CAC'$		
	= $(51.76)^2 + (69.25)^2 - 2(51.76) (69.25)\cos 8^\circ$	1M	
	CC' = 19.38 (km)	1A	
	(iii) Increase in height		
	= P'C' - PC		
_	$= \frac{100 \sin 43^{\circ} \tan 17^{\circ}}{\sin (43^{\circ} + 37^{\circ})} - \frac{100 \sin 30^{\circ} \tan 20^{\circ}}{\sin (30^{\circ} + 45^{\circ})}$	1M+1A	or AC' tan17° - ACtan20°-1M
	= 2.33 (km)	1A	
	(iv) Let the angle of elevation be γ		
	$tan\gamma = \frac{P'C' - PC}{CC'}$	2м	
	$\gamma = 6.86^{\circ}$	1 <u>A</u> 11	

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Marking Scheme			P.11
Solution		Marks	Remarks
13. (a) (i) V =	$\int_{-a}^{-a+h} \pi x^2 dy (Accept \int_{a-h}^{a} \pi x^2 dy)$	1M+1A	1M for $\pi \int_{\alpha}^{\beta} x^2 dy$
=	$\int_{-a}^{-a+h} \pi (a^2 - y^2) dy$	1M	For integrand only
	$\pi [a^2 y - \frac{1}{3} y^3]_{-a}^{-a-h}$	1 A	For primitive function only
• =	$\pi[a^2(h-a)-\frac{1}{3}(h-a)^3+a^3-\frac{1}{3}a^3]$.		
=	$\pi(a^2h - a^3 - \frac{1}{3}(h^3 - 3h^2a + 3ha^2 - a^3) + a^3 - \frac{1}{3}a^3$	3	
	$=\pi h^2(a-\frac{1}{3}h)$	1	
(ii)	Put $y = -a + h$ into $x^2 + y^2 = a^2$		
	$x^2 = a^2 - (-a + h)^2$	1M	For finding the radius
	$= 2ah - h^2$		
	$\therefore A = \pi h (2a - h)$	<u>1</u>	
(b) (i)	$\frac{dv}{d\varepsilon} = \frac{dv}{dh} \cdot \frac{dh}{d\varepsilon}$	1M	For chain rule
	$\frac{\mathrm{d}V}{\mathrm{d}h} = \pi h (2a - h) (= A)$	1A	Accept $\frac{\mathrm{d}V}{\mathrm{d}h} = A$
	$-kA = A \frac{dh}{dt}$		•
	$\frac{\mathrm{d}h}{\mathrm{d}z} = -k$	1A	•
	$\frac{dh}{dt}$ is a constant.		
	Alternative solution		
	$\frac{\mathrm{d}V}{\mathrm{d}t} = -kA$		
	$\frac{\mathrm{d}V}{\mathrm{d}h}\frac{\mathrm{d}h}{\mathrm{d}\varepsilon} = -kA$	lm	
	$\pi (2ah - h^2) \frac{\mathrm{d}h}{\mathrm{d}t} = -k\pi h (2a - h)$	1A I	For finding $\frac{\mathrm{d}V}{\mathrm{d}h}$
	$\frac{\mathrm{d}h}{\mathrm{d}t} = -k$	1A	
	$\frac{dh}{dt}$ is a constant.		
	• • • • • • • • • • • • • • • • • • • •		•
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Solution	Marks Remarks
(ii) (1) $\frac{\mathrm{d}h}{\mathrm{d}t} = -k$	
h = -kt + c (where c is a	constant) $1M$ Accept $h = -kt$
At $t = 0$, $h = \frac{3}{4}a$ $\therefore c = \frac{3}{4}$	a 1M
At $t = 30$, $h = 0$ $\therefore k = \frac{1}{40}$	a 1M
$h = \frac{3}{4}a - \frac{1}{40}at = \frac{a}{40} $ (30)) - t) 1
(2) At t = 10	
$h = \frac{1}{40}a(30 - 10) = \frac{1}{2}a$	1A
$V = \pi h^2 (a - \frac{h}{3})$	
$= \pi (\frac{a}{2})^2 (a - \frac{1}{6}a)$	
$=\frac{5}{24}\pi a^3$	1A