香港考試及評核局 HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

2 0 1 2 年 香港中 學文憑考試 HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2012

數學 延伸部分單元二(代數與微積分)

MATHEMATICS Extended Part Module 2 (Algebra and Calculus)

評 卷 參 考

MARKING SCHEME

本評卷參考乃香港考試及評核局專為今年本科考試而編寫,供閱卷員參考之用。閱卷員在完成閱卷工作後,若將本評卷參考提供其任教會考班的本科同事參閱,本局不表反對,但須切記,在任何情況下均不得容許本評卷參考落入學生手中。學生若索閱或求取此等文件,閱卷員/教師應嚴詞拒絕,因學生極可能將評卷參考視為標準答案,以致但知硬背死記,活剝生吞。這種落伍的學習態度,既不符現代教育原則,亦有違考試着重理解能力與運用技巧之旨。因此,本局籲請各閱卷員/教師通力合作,堅守上述原則。

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| | Solution | Marks | Remarks |
|----|---|-------|---|
| 1. | $f(x) = e^{2x}$ | | |
| | $f'(0) = \lim_{h \to 0} \frac{e^{2(0+h)} - e^{2(0)}}{h}$ | 1M | |
| | $h \rightarrow 0$ h | | $a^{2h}-1$ |
| | $=\lim_{h\to 0}\frac{e^{2h}-1}{2h}\cdot 2$ | 1M | Accept $\lim_{h \to 0} \frac{e^{2h} - 1}{h}$ |
| | = 2 | 1A | |
| | | (3) | |
| - | | | |
| 2. | $(1+ax)^n = 1 + nax + \frac{n(n-1)}{2}(ax)^2 + \cdots$ | 1M+1A | 1A for $\frac{n(n-1)}{2}$ |
| | Hence $na = 6$ and $\frac{n(n-1)}{2}a^2 = 16$. | 1M | 2 |
| | • | 1171 | |
| | Solving, $\frac{n(n-1)}{2} \left(\frac{6}{n}\right)^2 = 16$ | | |
| | 18(n-1) = 16n | | , |
| | n = 9 | 1A | : |
| | Therefore, $a = \frac{2}{3}$. | 1A | : |
| | | (5) | |
| | | | |
| 3. | For $n=1$, L.H.S. = $1 \times 2 = 2$ and R.H.S. = $1^2(1+1) = 2$ | | |
| | \therefore L.H.S. = R.H.S. and the statement is true for $n=1$. | 1 | |
| | Assume $1 \times 2 + 2 \times 5 + 3 \times 8 + \dots + k(3k-1) = k^2(k+1)$, where k is a positive integer. | 1 | |
| | $1 \times 2 + 2 \times 5 + 3 \times 8 + \dots + k(3k-1) + (k+1)[3(k+1)-1]$ | | |
| | $= k^{2}(k+1) + (k+1)(3k+2)$ by the assumption | 1 | |
| | $= (k+1)(k^2+3k+2)$ | 1 | |
| | $= (k+1)^{2}(k+1+1)$ Hence the statement is true for $n = k+1$. | 1 | |
| | By the principle of mathematical induction, the statement is true for all positive integers n . | 1 | Follow through |
| | | (5) | |
| | | | : |
| | | | |
| 4. | (a) $\int \frac{x+1}{x} dx = \int \left(1 + \frac{1}{x}\right) dx$ | 1M | |
| | $= x + \ln x + C$ | 1A | |
| | | | 4 |
| | (b) Let $u = x^2 - 1$. du = 2xdx | | |
| | $\int \frac{x^3}{x^2 - 1} dx = \int \frac{u + 1}{u} \cdot \frac{du}{2}$ | 1A | |
| | · · · · · | "" | |
| | $=\frac{1}{2}u + \frac{1}{2}\ln u + C$ by (a) | 1M | |
| | $= \frac{1}{2}(x^2 - 1) + \frac{1}{2}\ln x^2 - 1 + C$ | 1A | OR $\frac{1}{2}x^2 + \frac{1}{2}\ln x^2 - 1 + C$ |
| | 2 2 1 1 | (5) | |
| | | | |

| Solution | | Marks | Remarks |
|---|---|-----------|--|
| Solution $y = \frac{x^2 + x + 1}{x + 1}$ | | Marks | reina ro |
| $x+1$ $= x + \frac{1}{x+1}$ $\frac{dy}{dx} = 1 - \frac{1}{(x+1)^2}$ | | 1M 1A | OR $\frac{x^2 + 2x}{(x+1)^2}$ |
| $\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \text{when} x = -2 \text{or} 0$ | $ \begin{array}{c cccc} $ | 1M | (x+1) |
| Alternative Solution $\frac{d^2y}{dx^2} = \frac{2}{(x+1)^3}$ When $x = 0$, $\frac{d^2y}{dx^2} = 2 > 0$, when $x = -2$, $\frac{d^2y}{dx^2} = -2$ | -2 < 0 . | } 1M | |
| Hence (0,1) is a minimum point. The vertical asymptote is $x = -1$. $\lim_{x \to \pm \infty} \frac{1}{x+1} = 0$ | | 1A 1A | Accept $\lim_{x \to \infty} \frac{1}{x+1} = 0$ |
| $\therefore \text{ the oblique asymptote is } y = x$ | | 1A (6) | |
| . (a) Let the radius of the water surface be $a \text{ cm}$. By considering similar triangles, $\frac{a-3}{4-3} = \frac{h}{10}$. i.e. $a = \frac{h+30}{10}$ | | 1M | <u>4</u> → ↑ |
| $V = \frac{\pi}{3} h \left[3^2 + 3 \left(\frac{h+30}{10} \right) + \left(\frac{h+30}{10} \right)^2 \right]$ | | 1M | |
| $= \frac{\pi}{300} h[900 + 30(h+30) + (h^2 + 60h + 900)]$ $= \frac{\pi}{300} (h^3 + 90h^2 + 2700h)$ | | 1 | √ 3 |
| (b) $\frac{dV}{dt} = \frac{\pi}{300} (3h^2 + 180h + 2700) \frac{dh}{dt}$ $\therefore 7\pi = \frac{\pi}{300} [3(5)^2 + 180(5) + 2700] \frac{dh}{dt}$ | | 1M+1A | |
| $\frac{dh}{dt} = \frac{4}{7}$ i.e. the rate of increase of depth of water is $\frac{4}{7}$ cm | s ⁻¹ . | 1A | |
| | | (6) | |

| | Solution | Marks | Remarks |
|-------|---|--------|---|
| 7. (a | | Widths | Romarks |
| 7. (6 | $= (6\mathbf{i} + 2\mathbf{j} - \mathbf{k}) \times (2\mathbf{i} + \mathbf{j}) $ | 1M | |
| | , | 11V1 | $\frac{E}{}$ |
| | $= \mathbf{i} - 2\mathbf{j} + 2\mathbf{k} $ | | C/ $F/$ |
| | $=\sqrt{1^2+2^2+2^2}$ | | 1 |
| | =3 | 1A | |
| | | | //B $/D$ |
| (t | The volume of the parallelepiped OADBECFG | | O A |
| | $= (6\mathbf{i} + 2\mathbf{j} - \mathbf{k}) \times (2\mathbf{i} + \mathbf{j}) \cdot (5\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ | 1M | |
| | $= (\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}) \cdot (5\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ | | |
| | =1.5+(-2)(-1)+2.2 | | |
| | =11 | | |
| | | | volume |
| | Hence, the distance between point C and plane $OADB$ is $\frac{11}{3}$. | 1M+1A | 1M for height = $\frac{\text{volume}}{\text{base}}$ |
| | | (5) | base |
| | <i>k</i> | (3) | |
| | | | |
| | | | |
| 0 (- | $\begin{array}{cccccccccccccccccccccccccccccccccccc$ | | ; |
| 8. (a | The augmented matrix is $\begin{pmatrix} 1 & 1 & 1 & 0 \\ 2 & -1 & 5 & 6 \end{pmatrix}$ | | |
| | | | |
| | $ \sim \begin{pmatrix} $ | 1M | |
| | $\begin{pmatrix} 0 & -3 & 3 & & 6 \end{pmatrix}$ | | |
| | $\sim \begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & -2 \end{pmatrix}$ | | |
| | $\begin{pmatrix} 0 & 1 & -1 & & -2 \end{pmatrix}$ | | |
| | Let $z = t$, where t is a real number. Then $y = t - 2$ and $x = 2 - 2t$. | 1A | OR Solution Set = |
| | | | $\{(2-2t, t-2, t): t \in \mathbf{R}\}$ |
| | | | |
| (t | Substitute $(x, y, z) = (2 - 2t, t - 2, t)$ into the last equation: | 1M | |
| | $(2-2t)-(t-2)+\lambda(t)=4$ | , | |
| | $(\lambda - 3)t = 0$ | | |
| | When $\lambda \neq 3$, $t = 0$. | | |
| | | 1.4 | |
| | (x, y, z) = (2, -2, 0) | 1A | |
| | When $\lambda = 3$, t can be any real number. | | |
| | $\therefore (x, y, z) = (2-2t, t-2, t)$ | 1A | |
| | Alternative Collection | | |
| | Alternative Solution | | |
| | | | 1 |
| | The augmented matrix is $ \begin{pmatrix} 1 & 1 & 1 & 0 \\ 2 & -1 & 5 & 6 \\ 1 & -1 & \lambda & 4 \end{pmatrix} $ | | |
| | $\begin{pmatrix} 1 & -1 & \lambda & & 4 \end{pmatrix}$ | | • |
| | (1 1 1 0) | | |
| | $ \sim \begin{pmatrix} $ | 1M | |
| | | 1111 | |
| | | | |
| | $\begin{pmatrix} 1 & 1 & 1 & & 0 \end{pmatrix}$ | | |
| | ~ 0 1 -1 -2 | | |
| | $ \sim $ | | |
| | When $\lambda \neq 3$, $z = 0$. | | |
| | $\therefore (x, y, z) = (2, -2, 0)$ | 1A | |
| | | IA | |
| | When $\lambda = 3$, z can be any real number. | 1.4 | |
| | \therefore $(x, y, z) = (2-2t, t-2, t)$, where t is a real number. | 1A | |
| | | (5) | * · · · · · · · · · · · · · · · · · · · |
| | | (3) | |

| | | Solution | Marks | Remarks |
|----|-----|---|------------|---|
| 9. | (a) | $\int x \sin x dx = x(-\cos x) - \int (-\cos x) dx$ | 1M | |
| | | $= -x\cos x + \sin x + C$ | 1 A | W - |
| | (b) | The volume $= \pi \int_0^{\pi} x \sin x dx$ | 1M | $ \begin{array}{c c} 1M & \text{for } V = \pi \int y^2 dx \\ y_0 & & & \\ \end{array} $ |
| | | $= \pi[-x\cos x + \sin x]_0^{\pi}$ | | $y = \sqrt{x \sin x}$ |
| | | $= \pi [-x \cos x + \sin x]_0$ $= \pi^2$ | 1 A | |
| | | | (4) | ο π |
| 0. | (a) | Let F be a point on AB such that $OF \perp AB$. Let OA be x . | | |
| | | $\therefore AF = \frac{1}{2}$ and $\angle AOF = 2\theta$ (properties of isos. \triangle) | | 020 |
| | | In $\triangle OAF$, $\sin 2\theta = \frac{1}{2} \frac{1}{x}$ | 1M | A y Y F |
| | | Alternative Solution | | |
| | | In $\triangle OAB$, $\frac{1}{\sin 4\theta} = \frac{x}{\sin(90^\circ - 2\theta)}$ | 1M | OR $x^2 = x^2 + 1^2 - 2x\cos(90^\circ - 26^\circ)$ |
| | | $x = \frac{1}{2\sin 2\theta} $ (1) | | |
| | | In $\triangle OAY$, $\frac{y}{\sin \theta} = \frac{x}{\sin(90^\circ + \theta)}$ (2) | 1M | |
| | | Substitute (1) into (2): $\frac{y}{\sin \theta} = \frac{1}{2\sin 2\theta} \cdot \frac{1}{\cos \theta}$ | 1M | |
| | | Alternative Solution | | |
| | | In $\triangle OAY$, $\frac{y}{\sin \theta} = \frac{OY}{\sin \angle OAY}$ | 1M | 0 |
| | | In $\triangle OBY$, $\frac{1-y}{\sin 3\theta} = \frac{OY}{\sin \angle OBY}$ | 1M | $\theta/3\theta$ |
| | | $ sin 3\theta sin \angle OBY \therefore \angle OAY = \angle OBY (base \angle s, isos. \Delta s) $ | | A y Y 1-y |
| | | $\therefore \frac{y}{\sin \theta} = \frac{1 - y}{3 \sin \theta - 4 \sin^3 \theta}$ | 1M | |
| | | $\sin \theta = 3\sin \theta - 4\sin^3 \theta$ $3y - 4y\sin^2 \theta = 1 - y$ | | |
| | | $4y(1-\sin^2\theta)=1$ | | |
| | • | $y = \frac{1}{4}\sec^2\theta$ | 1 | · . |
| | | | | |
| | (b) | $0^{\circ} < 4\theta < 180^{\circ}$ | 1M | |
| | | $\begin{array}{cccccccccccccccccccccccccccccccccccc$ | | |
| | | $\frac{1}{4}\sec^2 0^\circ < y < \frac{1}{4}\sec^2 45^\circ \text{ since } \sec^2 \theta \text{ is an increasing function for } 0^\circ < \theta < 45^\circ $ | | Accept using "≤" sign |
| | | i.e. $\frac{1}{4} < y < \frac{1}{2}$ | 1A | |
| | | · · · · · · · · · · · · · · · · · · · | (6) | 17 |

| | | Solution | Marks | Remarks |
|--------|---------------------------------------|--|----------|--|
| 1. (a) | $\begin{vmatrix} 1-\\ 2\end{vmatrix}$ | $\begin{vmatrix} -x & 4 \\ 2 & 3-x \end{vmatrix} = 0$ | | |
| | | $\frac{3-x}{(3-x)(3-x)-2\cdot 4} = 0$ | 1M | |
| | | -4x - 5 = 0 -1 or 5 | 1A | |
| | ~ – | | | |
| | | | (2) | |
| | | | | |
| (b) |) (i) | $ \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = -1 \cdot \begin{pmatrix} a \\ b \end{pmatrix} $ | | |
| | | $\begin{cases} a+4b=-a\\ 2a+3b=-b \end{cases}$ | 1M € | |
| | | a+2b=0 | (1) 1A < | |
| | | $ \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} c \\ 1 \end{pmatrix} = 5 \begin{pmatrix} c \\ 1 \end{pmatrix} $ | | Either one |
| | | | | Either one |
| | | $\begin{cases} c+4=5c \\ 2c+3=5 \end{cases}$ | 2) | |
| | | $ \begin{vmatrix} c & = 1 \\ a & c \\ b & 1 \end{vmatrix} = 1 $ | 2) = | |
| | | $\begin{vmatrix} b & 1 \end{vmatrix}^{-1}$ By (2), $a-b=1$ | (3) 1M | For $a-bc=1$ |
| | | Solving (1) and (3), we have $a = \frac{2}{3}$ and $b = \frac{-1}{3}$. | | 101 4 - 50 - 1 |
| | | , (,) | | |
| | | $\therefore P = \begin{pmatrix} \frac{2}{3} & 1 \\ \frac{-1}{2} & 1 \end{pmatrix}$ | 1A | |
| | | $\left(\overline{3}^{-1}\right)$ | | |
| | | | | |
| | (ii) | $P^{-1} = \frac{1}{\frac{2}{3} + \frac{1}{3}} \begin{pmatrix} 1 & \frac{1}{3} \\ -1 & \frac{2}{3} \end{pmatrix}^{t}$ | | |
| | () | $\frac{2}{3} + \frac{1}{3} \left(-1 \frac{2}{3} \right)$ | | |
| | | $= \begin{pmatrix} 1 & -1 \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix}$ | 1M | |
| | | | | |
| | | $P^{-1} \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix} P = \begin{pmatrix} \frac{1}{3} & -\frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix} \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} \frac{2}{3} & 1 \\ -\frac{1}{3} & 1 \end{pmatrix}$ | | |
| | | | | OR $ \begin{pmatrix} 1 & -1 \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix} \begin{pmatrix} \frac{-2}{3} & 5 \\ \frac{1}{3} & 5 \end{pmatrix} $ |
| | | $\begin{pmatrix} -1 & 1 \\ 5 & 10 \end{pmatrix} \begin{pmatrix} \frac{2}{3} & 1 \end{pmatrix}$ | | $\begin{bmatrix} 1 & -1 \\ 1 & 2 \end{bmatrix} \begin{pmatrix} -2 \\ 3 & 5 \end{bmatrix}$ |
| | | $= \begin{pmatrix} -1 & 1 \\ \frac{5}{3} & \frac{10}{3} \end{pmatrix} \begin{pmatrix} \frac{2}{3} & 1 \\ -\frac{1}{3} & 1 \end{pmatrix}$ $= \begin{pmatrix} -1 & 0 \\ 0 & 5 \end{pmatrix}$ | IM | $ \left \begin{array}{c c} OR & \left(\frac{1}{3} & \frac{2}{3} \right) & \frac{1}{2} & 5 \end{array} \right $ |
| | | $\begin{pmatrix} 1 & 0 \end{pmatrix}$ | 1 A | (3) |
| | | -(0 5) | IA IA | |
| | | | | |
| | | | | |

| Solution | Marks | Remarks |
|--|-------|---|
| (iii) $P^{-1} \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix} P = \begin{pmatrix} -1 & 0 \\ 0 & 5 \end{pmatrix}$ | | |
| $ \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix} = P \begin{pmatrix} -1 & 0 \\ 0 & 5 \end{pmatrix} P^{-1} $ | 1M | |
| $ \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}^{12} = \underbrace{P \begin{pmatrix} -1 & 0 \\ 0 & 5 \end{pmatrix} P^{-1} P \begin{pmatrix} -1 & 0 \\ 0 & 5 \end{pmatrix} P^{-1} \cdots P \begin{pmatrix} -1 & 0 \\ 0 & 5 \end{pmatrix} P^{-1}}_{12 \text{ times}} $ | | |
| $=P\begin{pmatrix} -1 & 0\\ 0 & 5 \end{pmatrix}^{12}P^{-1}$ | 1M | |
| $= \begin{pmatrix} \frac{2}{3} & 1\\ -\frac{1}{3} & 1 \end{pmatrix} \begin{pmatrix} 1 & 0\\ 0 & 5^{12} \end{pmatrix} \begin{pmatrix} \frac{1}{3} & \frac{-1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix}$ | 1M | For $\begin{pmatrix} 1 & 0 \\ 0 & 5^{12} \end{pmatrix}$ |
| $= \begin{pmatrix} \frac{2}{3} & 5^{12} \\ \frac{-1}{3} & 5^{12} \end{pmatrix} \begin{pmatrix} \frac{1}{3} & \frac{2}{3} \end{pmatrix}$ | | For $\begin{pmatrix} 1 & 0 \\ 0 & 5^{12} \end{pmatrix}$ $OR \begin{pmatrix} \frac{2}{3} & 1 \\ \frac{-1}{3} & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{5^{12}} & \frac{-1}{3} \\ \frac{5^{12}}{3} & \frac{2 \cdot 5^{12}}{3} \end{pmatrix}$ |
| $= \begin{pmatrix} \frac{5^{12} + 2}{3} & \frac{2 \cdot 5^{12} - 2}{3} \\ \frac{5^{12} - 1}{3} & \frac{2 \cdot 5^{12} + 1}{3} \end{pmatrix}$ | 1A | OR (81380209 162760416 81380208 162760417 |
| | (11) | |
| Let $\overrightarrow{AG} = \frac{\overrightarrow{AC} + \lambda \overrightarrow{AD}}{1 + \lambda}$. Since \overrightarrow{AG} lies on a median, $\overrightarrow{AG} = k \frac{\overrightarrow{AC} + \overrightarrow{AB}}{2} = k \frac{\overrightarrow{AC} + 2\overrightarrow{AD}}{2}$ for some k . | | |
| Comparing the two expressions of \overrightarrow{AG} , we get $\lambda = 2$. | | C A |
| $\overrightarrow{AG} = \frac{\overrightarrow{AC} + 2\overrightarrow{AD}}{3}$ | 1A | F |
| $=\frac{\overrightarrow{AC} + \overrightarrow{AB}}{3}$ | | O O |
| $=\frac{3}{(\mathbf{c}-\mathbf{a})+(\mathbf{b}-\mathbf{a})}$ | 1M | A E D For tip-to-tail method |
| Alternative Solution 1 Let M be the mid-point of BC . | | |
| $\overrightarrow{AG} = \frac{2}{3} \overrightarrow{AM}$ | 1A | |
| $=\frac{2}{3}\cdot\frac{\overrightarrow{AC}+\overrightarrow{AB}}{2}$ | | , |
| $=\frac{3}{(\mathbf{c}-\mathbf{a})+(\mathbf{b}-\mathbf{a})}$ | 1M | For tip-to-tail method |
| | | • |
| $\overrightarrow{AG} = \overrightarrow{OG} - \overrightarrow{OA}$ | 1M | |
| $=\frac{\mathbf{a}+\mathbf{b}+\mathbf{c}}{3}-\mathbf{a}$ | 1A | For $\frac{a+b+c}{3}$ |
| $=\frac{\mathbf{b}+\mathbf{c}-2\mathbf{a}}{3}$ | 1.4 | 3 |
| =3 | 1A | |

| | Solution | Marks | Remarks |
|-------|---|----------------|--|
|) (i) | Since O is the circumcentre of the $\triangle ABC$, $OD \perp AB$. $\therefore OD//CE$ $\angle DOG = \angle CFG$ (alt. $\angle s$, $OD//CF$) | 1 | C C |
| | $\angle ODG = \angle FCG$ (alt. \angle s, $OD//CF$) | | F |
| | $\angle OGD = \angle FGC$ (vert. opp. $\angle s$) | | |
| | $\therefore \Delta DOG \sim \Delta CFG (A.A.A.)$ $FG: GO = CG: GD (corr. sides, \sim \Delta s)$ | 1 | |
| | = 2:1 | 1A | $egin{array}{cccccccccccccccccccccccccccccccccccc$ |
| (ii) | $\overrightarrow{AG} = \frac{\overrightarrow{AF} + 2\overrightarrow{AO}}{3}$ | 1M | For using (b)(i) |
| | $\overrightarrow{AF} = 3\overrightarrow{AG} - 2\overrightarrow{AO}$ | | |
| | | 134 | Farancia (a) |
| | $=3\cdot\frac{\mathbf{b}+\mathbf{c}-2\mathbf{a}}{3}-2(-\mathbf{a})$ | 1M | For using (a) |
| | $\overrightarrow{AH} = \overrightarrow{AG} + \overrightarrow{GF}$ | | 1 1 |
| | $= \overrightarrow{AG} + \overrightarrow{OG}$ $= \overrightarrow{AG} + 2\overrightarrow{OG}$ | 1 _M | For using (b)(i) |
| | $= \frac{\mathbf{b} + \mathbf{c} - 2\mathbf{a}}{3} + 2 \cdot \frac{\mathbf{a} + \mathbf{b} + \mathbf{c}}{3}$ | | |
| | = | 1M | For using (a) |
| | Alternative Solution 2 | | : |
| | $\overrightarrow{AF} = \overrightarrow{AC} + \overrightarrow{CF}$ | 111 | |
| | $= \overrightarrow{AC} + 2\overrightarrow{OD}$ $= \overrightarrow{AC} + \overrightarrow{OA} + \overrightarrow{OB}$ | 1M | |
| | = AC + OA + OB $= (c - a) + a + b$ | 1M | |
| | Alternative Solution 3 | | |
| | $\overrightarrow{AF} = \overrightarrow{OF} - \overrightarrow{OA}$ | | ' |
| | $=3\overrightarrow{OG}-\overrightarrow{OA}$ | 1M | For using (b)(i) |
| | $=3\cdot\frac{\mathbf{a}+\mathbf{b}+\mathbf{c}}{3}-\mathbf{a}$ | 1M | |
| | $= \mathbf{b} + \mathbf{c}$ | 1 | |
| | $\overrightarrow{AF} \cdot \overrightarrow{BC} = (\mathbf{b} + \mathbf{c}) \cdot (\mathbf{c} - \mathbf{b})$ | 1M | |
| | $=\left \mathbf{c}\right ^{2}-\left \mathbf{b}\right ^{2}$ | | |
| | = 0 (: O is the circumcentre) | 1A | |
| | $\therefore AF \perp BC$ | | |
| | \therefore AF is another altitude of $\triangle ABC$. | | |
| | Alternative Solution | 124 | |
| | $\overrightarrow{BF} \cdot \overrightarrow{AC} = (\overrightarrow{BA} + \overrightarrow{AF}) \cdot \overrightarrow{AC}$ $= (\mathbf{a} - \mathbf{b} + \mathbf{b} + \mathbf{c}) \cdot (\mathbf{c} - \mathbf{a})$ | 1M | |
| | | | |
| | $= \mathbf{c} ^2 - \mathbf{a} ^2$ $= (\mathbf{c} + \mathbf{c}) $ is the circumcentre. | 1A | |
| | $= 0 \qquad (\because O \text{ is the circumcentre})$ $\therefore BF \perp AC$ | IA | |
| | $\therefore BF \perp AC$ $\therefore BF \text{ is another altitude of } \Delta ABC .$ | | |
| | \therefore F is the orthocentre of $\triangle ABC$. | 1 | |
| | | (9) | - |
| | | | |

| | Solution | Marks | Remarks |
|-------------|--|-----------|---------------|
| 13. (a) (i) | $\tan u = \frac{-1 + \cos\frac{2\pi}{5}}{\sin\frac{2\pi}{5}}$ | | |
| | $= \frac{-1 + 1 - 2\sin^2\frac{\pi}{5}}{2\sin\frac{\pi}{5}\cos\frac{\pi}{5}}$ | 1M | |
| | $= -\tan \frac{\pi}{5}$ $= \tan \frac{-\pi}{5}$ $\therefore u = \frac{-\pi}{5} \text{for } \frac{-\pi}{2} < u < \frac{\pi}{2}$ | 1 | |
| (ii) | $\tan v = \frac{1 + \cos \frac{2\pi}{5}}{\sin \frac{2\pi}{5}}$ | | |
| | $ \frac{\sin\frac{2\pi}{5}}{2\sin\frac{\pi}{5}\cos\frac{\pi}{5}} = \frac{1+2\cos^2\frac{\pi}{5}-1}{2\sin\frac{\pi}{5}\cos\frac{\pi}{5}} $ | 1M | |
| | $= \cot \frac{\pi}{5}$ $= \tan \left(\frac{\pi}{2} - \frac{\pi}{5}\right)$ | | |
| | $\therefore v = \frac{3\pi}{10} \text{for } \frac{-\pi}{2} < v < \frac{\pi}{2}$ | 1A (4) | |
| (b) (i) | $x^2 + 2x\cos\frac{2\pi}{5} + 1$ | | · |
| | $x^{2} + 2x\cos\frac{2\pi}{5} + \cos^{2}\frac{2\pi}{5} + \sin^{2}\frac{2\pi}{5}$ $= (x + \cos\frac{2\pi}{5})^{2} + \sin^{2}\frac{2\pi}{5}$ | 1A | |
| (ii) | $\int_{-1}^{1} \frac{\sin\frac{2\pi}{5}}{x^2 + 2x\cos\frac{2\pi}{5} + 1} dx = \int_{-1}^{1} \frac{\sin\frac{2\pi}{5}}{(x + \cos\frac{2\pi}{5})^2 + \sin^2\frac{2\pi}{5}} dx$ | | |
| | Let $x + \cos \frac{2\pi}{5} = \sin \frac{2\pi}{5} \tan \theta$ | 1M | · · |
| | $\therefore dx = \sin \frac{2\pi}{5} \sec^2 \theta d\theta$ | 1A | |
| | When $x = -1$, $\tan \theta = \frac{-1 + \cos \frac{2\pi}{5}}{\sin \frac{2\pi}{5}}$ which gives $\theta = \frac{-\pi}{5}$ (by (a)(i)) | 1M | For using (a) |
| | When $x = 1$, $\tan \theta = \frac{1 + \cos \frac{2\pi}{5}}{\sin \frac{2\pi}{5}}$ which gives $\theta = \frac{3\pi}{10}$ (by (a)(ii)) | | |

| Solution | Marks | Remarks |
|---|--------|---------------|
| $\therefore \int_{-1}^{1} \frac{\sin\frac{2\pi}{5}}{x^2 + 2x\cos\frac{2\pi}{5} + 1} dx = \int_{-\frac{\pi}{5}}^{\frac{3\pi}{10}} \frac{\sin^2\frac{2\pi}{5}\sec^2\theta}{\sin^2\frac{2\pi}{5}(\tan^2\theta + 1)} d\theta$ $= [\theta]_{-\frac{\pi}{2}}^{\frac{3\pi}{10}}$ | 1A | For integrand |
| $= [0] \frac{\pi}{5}$ $= \frac{\pi}{2}$ | 1A (6) | |
| (c) $\int_{-1}^{1} \frac{\sin \frac{7\pi}{5}}{x^2 + 2x \cos \frac{7\pi}{5} + 1} dx = \int_{-1}^{1} \frac{-\sin \frac{2\pi}{5}}{x^2 - 2x \cos \frac{2\pi}{5} + 1} dx$ | 1A | |
| Let $y = -x$. dy = -dx When $x = -1$, $y = 1$; when $x = 1$, $y = -1$. | 1M | |
| $\therefore \int_{-1}^{1} \frac{\sin \frac{7\pi}{5}}{x^2 + 2x \cos \frac{7\pi}{5} + 1} dx = \int_{1}^{-1} \frac{-\sin \frac{2\pi}{5}}{y^2 + 2y \cos \frac{2\pi}{5} + 1} dx$ | | |
| Alternative Solution $\int_{-1}^{1} \frac{\sin \frac{7\pi}{5}}{x^2 + 2x \cos \frac{7\pi}{5} + 1} dx = \int_{-1}^{1} \frac{\sin \frac{7\pi}{5}}{(x + \cos \frac{7\pi}{5})^2 + \sin^2 \frac{7\pi}{5}} dx$ | | |
| Let $x + \cos \frac{7\pi}{5} = \sin \frac{7\pi}{5} \tan \theta$ $\therefore dx = \sin \frac{7\pi}{5} \sec^2 \theta d\theta$ | 1M | |
| When $x = -1$, $\theta = \frac{3\pi}{10}$; when $x = 1$, $\theta = \frac{-\pi}{5}$. $\therefore \int_{-1}^{1} \frac{\sin \frac{7\pi}{5}}{x^2 + 2x \cos \frac{7\pi}{5} + 1} dx = \int_{\frac{3\pi}{10}}^{\frac{-\pi}{5}} \frac{\sin^2 \frac{7\pi}{5} \sec^2 \theta}{\sin^2 \frac{7\pi}{5} (\tan^2 \theta + 1)} d\theta$ | 1A | |
| $=\frac{-\pi}{2}$ by (b)(ii) | 1A (3) | |
| 14. (a) $y = kx^p$ $\frac{dy}{dx} = kpx^{p-1}$ | 1A | |
| The slope of the tangent to Γ at A is kpa^{p-1} . | | |
| $\therefore \frac{ka^{p} - 0}{a - (-a)} = kpa^{p-1}$ $p = \frac{1}{2}$ | 1M | |
| 2 | (3) | |

| (ii) The shaded area = area of $\triangle PQA$ + area on the left of Γ from O to A - area of sector PAS $= \frac{1}{2} \cdot 1 \cdot \sqrt{4 - 1} + \int_{0}^{2\sqrt{3}} \frac{y}{3} \left(\frac{y}{2\sqrt{3}} \right)^{2} dy - \frac{1}{2} (2)^{2} \frac{\pi}{6}$ $IM = \frac{1}{2} \cdot 1 \cdot \sqrt{4 - 1} + \int_{0}^{3} \frac{3}{3} \left(\frac{y}{2\sqrt{3}} \right)^{2} dy - \frac{1}{2} (2)^{2} \frac{\pi}{6}$ $IM = \frac{1}{2} \cdot 1 \cdot \sqrt{4 - 1} + \frac{3}{2} \cdot \frac{3}{2} \cdot \frac{3}{2} \cdot \frac{3}{2}$ | | Solution | Marks | Remarks |
|---|---------|---|------------|--|
| $k-t = -\sqrt{3} \text{for } \sqrt{3} \text{frejected}$ Slope of $AP = \frac{k-t}{1-0}$ Slope of $AB = \frac{k}{2} \text{(by (a))}$ $\therefore (k-t)\frac{k}{2} = -1 (2) 1A$ Substitute (1) into (2): $(-\sqrt{3})\frac{k}{2} = -1$ Alternative Solution According to the figure, $\angle ROB = \angle RAP = \frac{\pi}{2}$. $\angle ORB = \angle ARP \text{(vert. opp. } \angle S)$ $\therefore \angle RBO = \angle RPA \text{and let the angles be } \theta$. Since $PA = 2 \text{and } QA = 1$, $\theta = \frac{\pi}{6}$ Slope of $AB = \frac{k}{2} \text{(by (a))}$ $\therefore \tan \frac{\pi}{6} = \frac{k}{2}$ $k = \frac{2\sqrt{3}}{3}$ 1 (ii) The shaded area = area of $\triangle PQA$ + area on the left of Γ from O to A - area of sector PAS $= \frac{1}{2} \cdot 1 \cdot \sqrt{4-1} + \int_{0}^{2\sqrt{3}} \left(\frac{y}{2\sqrt{3}}\right)^{\frac{2}{3}} dy - \frac{1}{2}(2)^{2} \frac{\pi}{6}$ $= \frac{\sqrt{3}}{2} + \frac{3}{4} \left[\frac{y^{3}}{3}\right]^{\frac{3}{3}} - \frac{\pi}{3}$ $1M$ $= \frac{1}{2} \cdot 1 \cdot \sqrt{4-1} + \frac{2\sqrt{3}}{3} \left(\frac{y}{3}\right)^{\frac{2\sqrt{3}}{3}} - \frac{\pi}{3}$ $1M$ $= \frac{\sqrt{3}}{2} + \frac{3}{4} \left[\frac{y^{3}}{3}\right]^{\frac{3}{3}} - \frac{\pi}{3}$ $1M$ | (b) (i) | | | |
| Slope of $AP = \frac{k-t}{1-0}$ Slope of $AB = \frac{k}{2}$ (by (a)) $\therefore (k-t)\frac{k}{2} = -1$ Substitute (1) into (2): $(-\sqrt{3})\frac{k}{2} = -1$ Alternative Solution According to the figure, $\angle ROB = \angle RAP = \frac{\pi}{2}$. $\angle ORB = \angle ARP$ (vert. opp. $\angle s$) $\therefore \angle RBO = \angle RPA$ and let the angles be θ . Since $PA = 2$ and $QA = 1$, $\theta = \frac{\pi}{6}$ Slope of $AB = \frac{k}{2}$ (by (a)) $\therefore \tan \frac{\pi}{6} = \frac{k}{2}$ $k = \frac{2\sqrt{3}}{3}$ 1 (ii) The shaded area = area of $\triangle PQA$ + area on the left of Γ from O to A - area of sector PAS $= \frac{1}{2} \cdot 1 \cdot \sqrt{A-1} + \int_{0}^{2\sqrt{3}} \frac{\sqrt{y}}{2\sqrt{3}} dy - \frac{1}{2}(2)^{2} \frac{\pi}{6}$ $= \frac{\sqrt{3}}{2} + \frac{3}{4} \left[\frac{\sqrt{3}}{3} \right]_{0}^{2\sqrt{3}} - \frac{\pi}{3}$ IM $B(-1, 0)$ B | | | 1M | OR $t = k + \sqrt{4 - 1}$ |
| Slope of $AP = \frac{k-1}{1-0}$ Slope of $AB = \frac{k}{2}$ (by (a)) $\therefore (k-t)\frac{k}{2} = -1$ Substitute (1) into (2): $(-\sqrt{3})\frac{k}{2} = -1$ Alternative Solution According to the figure, $\angle ROB = \angle RAP = \frac{\pi}{2}$. $\angle ORB = \angle ARP$ (vert. opp. $\angle S$) $\therefore \angle RBO = \angle RPA$ and let the angles be θ . Since $PA = 2$ and $QA = 1$, $\theta = \frac{\pi}{6}$ Slope of $AB = \frac{k}{2}$ (by (a)) $\therefore \tan \frac{\pi}{6} = \frac{k}{2}$ $k = \frac{2\sqrt{3}}{3}$ 1 (ii) The shaded area = area of $\triangle PQA$ + area on the left of Γ from O to A - area of sector PAS $= \frac{1}{2} \cdot 1 \cdot \sqrt{A-1} + \int_{0}^{2\sqrt{3}} \frac{\sqrt{y}}{2\sqrt{3}}^2 dy - \frac{1}{2}(2)^2 \frac{\pi}{6}$ $= \frac{\sqrt{3}}{2} + \frac{3}{4} \left[\frac{y^3}{3} \right]_{0}^{2\sqrt{3}} - \frac{\pi}{3}$ $ M $ $ M $ $ A $ $ $ | | 1 | | $\bigwedge^{\mathcal{N}}_{C}$ |
| Substitute (1) into (2): $(-\sqrt{3})\frac{k}{2} = -1$ Alternative Solution According to the figure, $\angle ROB = \angle RAP = \frac{\pi}{2}$. $\angle ORB = \angle ARP$ (vert. opp. $\angle S$) $\therefore \angle RBO = \angle RPA$ and let the angles be θ . Since $PA = 2$ and $QA = 1$, $\theta = \frac{\pi}{6}$ Slope of $AB = \frac{k}{2}$ (by (a)) $\therefore \tan \frac{\pi}{6} = \frac{k}{2}$ (ii) The shaded area = area of ΔPQA + area on the left of Γ from O to A - area of sector PAS $= \frac{1}{2} \cdot 1 \cdot \sqrt{4 - 1} + \int_{0}^{2\sqrt{3}} \frac{y}{3} \left(\frac{y}{2\sqrt{3}} \right)^{2} dy - \frac{1}{2} (2)^{2} \frac{\pi}{6}$ $= \frac{\sqrt{3}}{2} + \frac{3}{4} \left[\frac{y^{3}}{3} \right]_{0}^{2\sqrt{3}} - \frac{\pi}{3}$ 1M $= \frac{1}{2} \cdot 1 \cdot \sqrt{4 - 1} + \int_{0}^{2\sqrt{3}} \frac{y}{3} \left(\frac{y}{2\sqrt{3}} \right)^{2} dy - \frac{1}{2} (2)^{2} \frac{\pi}{6}$ $= \frac{\sqrt{3}}{2} + \frac{3}{4} \left[\frac{y^{3}}{3} \right]_{0}^{2\sqrt{3}} - \frac{\pi}{3}$ 1M $= \frac{\sqrt{3}}{2} + \frac{3}{4} \left[\frac{y^{3}}{3} \right]_{0}^{2\sqrt{3}} - \frac{\pi}{3}$ | | Slope of $AP = \frac{\kappa - t}{1 - 0}$ | | |
| Substitute (1) into (2): $(-\sqrt{3})\frac{k}{2} = -1$ Alternative Solution According to the figure, $\angle ROB = \angle RAP = \frac{\pi}{2}$. $\angle ORB = \angle ARP$ (vert. opp. $\angle S$) $\therefore \angle RBO = \angle RPA$ and let the angles be θ . Since $PA = 2$ and $QA = 1$, $\theta = \frac{\pi}{6}$ Slope of $AB = \frac{k}{2}$ (by (a)) $\therefore \tan \frac{\pi}{6} = \frac{k}{2}$ (ii) The shaded area $= \arcsin \Delta PQA + \arcsin \theta$ left of Γ from O to $A - \arcsin \theta$ sector PAS $= \frac{1}{2} \cdot 1 \cdot \sqrt{4 - 1} + \int_{0}^{2\sqrt{3}} \frac{y}{2\sqrt{3}} dy - \frac{1}{2}(2)^{2} \frac{\pi}{6}$ $= \frac{\sqrt{3}}{2} + \frac{3}{4} \left[\frac{y^{3}}{3} \right]^{2\sqrt{3}} - \frac{\pi}{3}$ IM $= \frac{1}{2} \cdot 1 \cdot \sqrt{4 - 1} + \int_{0}^{2\sqrt{3}} \frac{y}{2\sqrt{3}} dy - \frac{1}{2}(2)^{2} \frac{\pi}{6}$ $= \frac{\sqrt{3}}{2} + \frac{3}{4} \left[\frac{y^{3}}{3} \right]^{2\sqrt{3}} - \frac{\pi}{3}$ IM $= \frac{\sqrt{3}}{2} + \frac{3}{4} \left[\frac{y^{3}}{3} \right]^{2\sqrt{3}} - \frac{\pi}{3}$ IM $= \frac{\sqrt{3}}{2} + \frac{3}{4} \left[\frac{y^{3}}{3} \right]^{2\sqrt{3}} - \frac{\pi}{3}$ IM $= \frac{\sqrt{3}}{2} + \frac{3}{4} \left[\frac{y^{3}}{3} \right]^{2\sqrt{3}} - \frac{\pi}{3}$ | | Slope of $AB = \frac{k}{2}$ (by (a)) | 1M | P(0,t) |
| Substitute (1) into (2): $(-\sqrt{3})\frac{2}{2} = -1$ Alternative Solution According to the figure, $\angle ROB = \angle RAP = \frac{\pi}{2}$. $\angle ORB = \angle ARP$ (vert. opp. $\angle S$) $\therefore \angle RBO = \angle RPA$ and let the angles be θ . Since $PA = 2$ and $QA = 1$, $\theta = \frac{\pi}{6}$ Slope of $AB = \frac{k}{2}$ (by (a)) $\therefore \tan \frac{\pi}{6} = \frac{k}{2}$ $k = \frac{2\sqrt{3}}{3}$ 1 (ii) The shaded area = area of $\triangle PQA$ + area on the left of Γ from O to A - area of sector PAS $= \frac{1}{2} \cdot 1 \cdot \sqrt{4 - 1} + \int_{0}^{2\sqrt{3}} \left(\frac{y}{2\sqrt{3}}\right)^{2} dy - \frac{1}{2}(2)^{2} \frac{\pi}{6}$ $= \frac{\sqrt{3}}{2} + \frac{3}{4} \left[\frac{y^{3}}{3}\right]_{0}^{3} - \frac{\pi}{3}$ 1M $= \frac{\sqrt{3}}{2} + \frac{3}{4} \left[\frac{y^{3}}{3}\right]_{0}^{3} - \frac{\pi}{3}$ | | $\therefore (k-t)\frac{k}{2} = -1 \qquad (2)$ | 1A | |
| According to the figure, $\angle ROB = \angle RAP = \frac{\pi}{2}$. $\angle ORB = \angle ARP$ (vert. opp. $\angle s$) $\therefore \angle RBO = \angle RPA$ and let the angles be θ . Since $PA = 2$ and $QA = 1$, $\theta = \frac{\pi}{6}$ Slope of $AB = \frac{k}{2}$ (by (a)) $\therefore \tan \frac{\pi}{6} = \frac{k}{2}$ $k = \frac{2\sqrt{3}}{3}$ 1 (ii) The shaded area = area of $\triangle PQA$ + area on the left of Γ from O to A - area of sector PAS $= \frac{1}{2} \cdot 1 \cdot \sqrt{4 - 1} + \int_{0}^{2\sqrt{3}} \left(\frac{y}{2\sqrt{3}}\right)^{2} dy - \frac{1}{2}(2)^{2} \frac{\pi}{6}$ $= \frac{\sqrt{3}}{2} + \frac{3}{4} \left[\frac{y^{3}}{3}\right]_{0}^{2\sqrt{3}} - \frac{\pi}{3}$ 1M | | Substitute (1) into (2): $(-\sqrt{3})\frac{k}{2} = -1$ | | |
| $\angle ORB = \angle ARP (\text{vert. opp. } \angle s)$ $\therefore \angle RBO = \angle RPA \text{ and let the angles be } \theta .$ Since $PA = 2$ and $QA = 1$, $\theta = \frac{\pi}{6}$ $\text{Slope of } AB = \frac{k}{2} (\text{by (a)})$ $\therefore \tan \frac{\pi}{6} = \frac{k}{2}$ $k = \frac{2\sqrt{3}}{3}$ 1 (ii) The shaded area = area of $\triangle PQA$ + area on the left of Γ from O to A - area of sector PAS $= \frac{1}{2} \cdot 1 \cdot \sqrt{4 - 1} + \int_{0}^{2\sqrt{3}} \frac{y}{2\frac{\sqrt{3}}{3}} \frac{y}{3} dy - \frac{1}{2}(2)^{2} \frac{\pi}{6}$ $= \frac{\sqrt{3}}{2} + \frac{3}{4} \left[\frac{y^{3}}{3} \right]_{0}^{2\sqrt{3}} - \frac{\pi}{3}$ $1M$ $B(-1, 0)$ D | | Alternative Solution | | |
| Since $PA = 2$ and let the angles be θ . Since $PA = 2$ and $QA = 1$, $\theta = \frac{\pi}{6}$ Slope of $AB = \frac{k}{2}$ (by (a)) $\therefore \tan \frac{\pi}{6} = \frac{k}{2}$ (ii) The shaded area = area of ΔPQA + area on the left of Γ from O to A - area of sector PAS $= \frac{1}{2} \cdot 1 \cdot \sqrt{4 - 1} + \int_{0}^{2\sqrt{3}} \frac{y}{2\sqrt{3}} \frac{y}{3} dy - \frac{1}{2}(2)^{2} \frac{\pi}{6}$ $= \frac{\sqrt{3}}{2} + \frac{3}{4} \left[\frac{y^{3}}{3} \right]_{0}^{2\sqrt{3}} - \frac{\pi}{3}$ IM $B(-1, 0) \xrightarrow{R}$ $A(1, k)$ $B(-1, 0) \xrightarrow{R}$ $A(1, k)$ $A(1, k)$ $A(1, k)$ $A(1, k)$ $A(1, k)$ | | According to the figure, $\angle ROB = \angle RAP = \frac{\pi}{2}$. | | N ^y |
| Slope of $AB = \frac{k}{2}$ (by (a)) $\therefore \tan \frac{\pi}{6} = \frac{k}{2}$ $k = \frac{2\sqrt{3}}{3}$ 1 (ii) The shaded area = area of $\triangle PQA$ + area on the left of Γ from O to A - area of sector PAS $= \frac{1}{2} \cdot 1 \cdot \sqrt{4 - 1} + \int_{0}^{2\sqrt{3}} \frac{y}{3} \left(\frac{y}{2\sqrt{3}} \right)^{2} dy - \frac{1}{2} (2)^{2} \frac{\pi}{6}$ $= \frac{\sqrt{3}}{2} + \frac{3}{4} \left(\frac{y^{3}}{3} \right)^{\frac{2\sqrt{3}}{3}} - \frac{\pi}{3}$ 1M $B(-1, 0)$ $B(-1, 0)$ $B(-1, 0)$ $B(-1, 0)$ $B(-1, 0)$ | | ` ** <i>'</i> | 1M | C |
| $k = \frac{2\sqrt{3}}{3}$ (ii) The shaded area = area of $\triangle PQA$ + area on the left of Γ from O to A - area of sector PAS $= \frac{1}{2} \cdot 1 \cdot \sqrt{4 - 1} + \int_{0}^{2\sqrt{3}} \left(\frac{y}{2\sqrt{3}}\right)^{2} dy - \frac{1}{2}(2)^{2} \frac{\pi}{6}$ $= \frac{\sqrt{3}}{2} + \frac{3}{4} \left[\frac{y^{3}}{3}\right]_{0}^{2\sqrt{3}} - \frac{\pi}{3}$ 1M $B(-1, 0)$ $B(-1,$ | | Since $PA = 2$ and $QA = 1$, $\theta = \frac{\pi}{6}$ | 1 A | P |
| (ii) The shaded area = area of $\triangle PQA$ + area on the left of Γ from O to A - area of sector PAS $= \frac{1}{2} \cdot 1 \cdot \sqrt{4 - 1} + \int_{0}^{2\sqrt{3}} \left(\frac{y}{2\sqrt{3}}\right)^{2} dy - \frac{1}{2}(2)^{2} \frac{\pi}{6}$ $= \frac{\sqrt{3}}{2} + \frac{3}{4} \left[\frac{y^{3}}{3}\right]_{0}^{2\sqrt{3}} - \frac{\pi}{3}$ | | Slope of $AB = \frac{k}{2}$ (by (a)) | 1M | Ø\2 |
| $k = \frac{2\sqrt{3}}{3}$ (ii) The shaded area $= \text{area of } \Delta PQA + \text{area on the left of } \Gamma \text{ from } O \text{ to } A - \text{area of sector } PAS$ $= \frac{1}{2} \cdot 1 \cdot \sqrt{4 - 1} + \int_0^{2\sqrt{3}} \left(\frac{y}{2\sqrt{3}}\right)^2 dy - \frac{1}{2}(2)^2 \frac{\pi}{6}$ $= \frac{\sqrt{3}}{2} + \frac{3}{4} \left[\frac{y^3}{3}\right]_0^{2\sqrt{3}} - \frac{\pi}{3}$ $1M$ $= \frac{1}{2} \cdot 1 \cdot \sqrt{4 - 1} + \int_0^{2\sqrt{3}} \left(\frac{y}{2\sqrt{3}}\right)^2 dy - \frac{1}{2}(2)^2 \frac{\pi}{6}$ $= \frac{\sqrt{3}}{2} + \frac{3}{4} \left[\frac{y^3}{3}\right]_0^{2\sqrt{3}} - \frac{\pi}{3}$ $1M$ $= \frac{1}{2} \cdot 1 \cdot \sqrt{4 - 1} + \frac{1}{2} \cdot \frac{\sqrt{3}}{3} - \frac{\pi}{3}$ $1M$ $= \frac{1}{2} \cdot 1 \cdot \sqrt{4 - 1} + \frac{1}{2} \cdot \frac{\sqrt{3}}{3} - \frac{\pi}{3}$ $1M \cdot 1M \cdot 1A$ | | $\therefore \tan \frac{\pi}{6} = \frac{k}{2}$ | | B(-1,0) |
| (ii) The shaded area $= \text{area of } \Delta PQA + \text{area on the left of } \Gamma \text{ from } O \text{ to } A - \text{area of sector } PAS$ $= \frac{1}{2} \cdot 1 \cdot \sqrt{4 - 1} + \int_0^{2\sqrt{3}} \left(\frac{y}{2\sqrt{3}}\right)^2 dy - \frac{1}{2} (2)^2 \frac{\pi}{6}$ $= \frac{\sqrt{3}}{2} + \frac{3}{4} \left[\frac{y^3}{3}\right]_0^{2\sqrt{3}} - \frac{\pi}{3}$ | | $k = \frac{2\sqrt{3}}{3}$ | 1 | $\begin{vmatrix} B(-1,0) & Y \\ O \end{vmatrix}$ |
| = area of ΔPQA + area on the left of Γ from O to A – area of sector PAS $= \frac{1}{2} \cdot 1 \cdot \sqrt{4 - 1} + \int_0^{2\sqrt{3}} \left(\frac{y}{2\sqrt{3}}\right)^2 dy - \frac{1}{2}(2)^2 \frac{\pi}{6}$ $= \frac{\sqrt{3}}{2} + \frac{3}{4} \left[\frac{y^3}{3}\right]_0^{2\sqrt{3}} - \frac{\pi}{3}$ $1M$ $B(-1, 0)$ $B(-1, 0)$ | | 3 | | |
| = area of ΔPQA + area on the left of Γ from O to A – area of sector PAS $= \frac{1}{2} \cdot 1 \cdot \sqrt{4 - 1} + \int_0^{2\sqrt{3}} \left(\frac{y}{2\sqrt{3}}\right)^2 dy - \frac{1}{2}(2)^2 \frac{\pi}{6}$ $= \frac{\sqrt{3}}{2} + \frac{3}{4} \left[\frac{y^3}{3}\right]_0^{2\sqrt{3}} - \frac{\pi}{3}$ $1M$ $B(-1, 0)$ $B(-1, 0)$ | (;;) | The shaded area | | A ^y |
| $= \frac{\sqrt{3}}{2} + \frac{3}{4} \left[\frac{y^3}{3} \right]_0^{\frac{2\sqrt{3}}{3}} - \frac{\pi}{3}$ $1M$ $B(-1, 0)$ $B(-1, 0)$ | (π) | = area of $\triangle PQA$ + area on the left of Γ from O to A - area of sector PAS | 1M | |
| $= \frac{\sqrt{3}}{2} + \frac{3}{4} \left[\frac{y^3}{3} \right]_0^{2\sqrt{3}} - \frac{\pi}{3}$ $1M$ $B(-1, 0)$ $B(-1, 0)$ | | $= \frac{1}{2} \cdot 1 \cdot \sqrt{4 - 1} + \int_0^{2\sqrt{3}} \left(\frac{y}{\frac{2\sqrt{3}}{2}} \right)^2 dy - \frac{1}{2} (2)^2 \frac{\pi}{6}$ | 1M+1A | |
| | | | · | |
| Alternative Solution 1 | | $= \frac{\sqrt{3}}{2} + \frac{3}{4} \left[\frac{y^3}{3} \right]_0^3 - \frac{\pi}{3}$ | 1M | B(-1,0) |
| | | Alternative Solution 1 | | / 0 |
| $t = k + \sqrt{3}$ | | | | A ^V |
| $=\frac{5\sqrt{3}}{3}$ | | $=\frac{5\sqrt{3}}{3}$ | | C |
| The shaded area = area of trapezium $OFAP$ – area of sector PAS – area under Γ from O to A 1M $P(0, t)$ | | | 1M | P(0,t) |
| $= \frac{1}{2} \left(\frac{2\sqrt{3}}{3} + \frac{5\sqrt{3}}{3} \right) (1) - \frac{1}{2} (2)^2 \frac{\pi}{6} - \int_0^1 \frac{2\sqrt{3}}{3} x^{\frac{1}{2}} dx$ $1M+1A$ | | · . | | \\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\ |
| | | | 1M | B(-1,0) $A(1,k)$ |
| $\begin{bmatrix} \frac{1}{6} - \frac{1}{3} - \frac{1}{9} \end{bmatrix}_0$ | | 6 3 9 [] 0 | 1141 | |

| Solution | Marks |
|---|-------|
| Alternative Solution 2 $t = k + \sqrt{3}$ $= \frac{5\sqrt{3}}{3}$ The solution 2 | - |
| The shaded area = area of $\triangle OAP$ + area of $\triangle OAF$ - area of sector PAS - area under Γ from O to A | 1 1M |
| $= \frac{1}{2} \left(\frac{5\sqrt{3}}{3} \right) (1) + \frac{1}{2} (1) \left(\frac{2\sqrt{3}}{3} \right) - \frac{1}{2} (2)^2 \frac{\pi}{6} - \int_0^1 \frac{2\sqrt{3}}{3} x^{\frac{1}{2}} dx$ | 1M+1A |
| $=\frac{5\sqrt{3}}{6}+\frac{\sqrt{3}}{3}-\frac{\pi}{3}-\frac{4\sqrt{3}}{9}\left[x^{\frac{3}{2}}\right]_{0}^{1}$ | 1M |
| Alternative Solution 3 $t = k + \sqrt{3}$ $= \frac{5\sqrt{3}}{2}$ | |
| The equation of C is $x^2 + \left(y - \frac{5\sqrt{3}}{3}\right)^2 = 4$. | 1A |
| Hence, the equation of \widehat{AS} is $y = \frac{5\sqrt{3}}{3} - \sqrt{4 - x^2}$. | |
| The shaded area = $\int_0^1 \left(\frac{5\sqrt{3}}{3} - \sqrt{4 - x^2} - \frac{2\sqrt{3}}{3} x^{\frac{1}{2}} \right) dx$ | 1M |
| For $\int_0^1 \sqrt{4 - x^2} dx$, let $x = 2 \sin \phi$. | |
| $\therefore dx = 2\cos\phi d\phi$ | |
| When $x=1$, $\phi = \frac{\pi}{6}$; when $x=0$, $\phi = 0$. | |
| $\int_0^1 \sqrt{4 - x^2} \mathrm{d}x = \int_0^{\frac{\pi}{6}} \sqrt{4 - 4\sin^2 \phi} 2\cos \phi \mathrm{d}\phi$ | 1M |

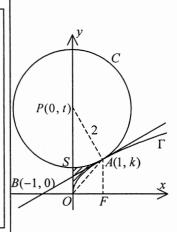
 $=\int_0^{\frac{\pi}{6}} 2(1+\cos 2\phi) \,\mathrm{d}\phi$

 $= \left[2\phi + \sin 2\phi\right]^{\frac{\pi}{6}}$

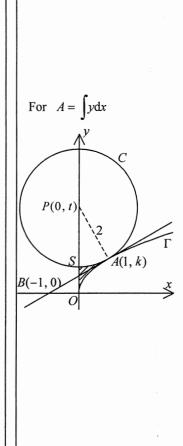
 $= \frac{\pi}{3} + \frac{\sqrt{3}}{2}$ Hence, the shaded area

 $=\frac{13\sqrt{3}}{18} - \frac{\pi}{3}$

 $= \left[\frac{5\sqrt{3}}{3} x - \frac{4\sqrt{3}}{9} x^{\frac{3}{2}} \right]_{0}^{1} - \left(\frac{\pi}{3} + \frac{\sqrt{3}}{2} \right)$



Remarks



1M