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PURE MATHEMATICS (PAPER I)

MARKING SCHEME

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	SOLUTION DE LE COMPANY DE L'ANDRE L'AN	MARK	
(a) f(x) -	a-x 0 1 0 b-x 0 1 0 c-x		
-	$-x^{3} + (a+b+c)x^{2} - (ab+bc+ca-1)x + (abc-b)$ $-x^{3} - (ab+bc+ca-1)x + (abc-b)$ as $a+b+c = 0$	2	
`	$ \begin{pmatrix} a^{2}+1 & 0 & a+c \\ 0 & b^{2} & 0 \\ a+c & 0 & c^{2}+1 \end{pmatrix} \begin{pmatrix} a & 0 & 1 \\ 0 & b & 0 \\ 1 & 0 & c \end{pmatrix} $		
- ($ \begin{pmatrix} a^{3}+2a+c & 0 & a^{2}+ac+c^{2}+1 \\ 0 & b^{3} & 0 \\ a^{2}+ac+c^{2}+1 & 0 & a+2c+c^{3} \end{pmatrix} $	2	
	$= -A^3 - (ab+bc+ca-1)A + (abc-b)I$ $= \begin{pmatrix} -a^3-a-b-c-a^2b-a^2c & 0 & -a^2-2ac-c^2-ab-bc \\ 0 & -b^3-ab^2-b^2c & 0 \\ -a^2-2ac-c^2-ab-bc & 0 & -a-c-c^3-bc^2-ac^2-b \end{pmatrix}$	2	
	$-a^{2}(a+b+c) - (a+b+c) = 0$ $-a(a+c+b) - c(a+c+b) = 0$ $-a(a+c+b) - c(a+c+b) = 0$ $-(a+c+b) - c^{2}(c+b+a) = 0$		
	- (0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	3	
	·	10	
	$-A^3 - (ab+bc+ca-1)A + (abc-b)I = 0$ $3 = \lambda A + \mu I$,		
	$\lambda = -(ab+bc+ca-1)$ $\mu = abc-b$	1	
For A	$= \begin{pmatrix} 1 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix} , \lambda = 2 , \mu = 1 .$. 1	
= ? = ?	$\lambda_{1}^{2} + \mu_{1}^{2}$ $\lambda_{2}^{3} + 3\lambda_{1}^{2} + 3\lambda_{1}^{2} + 3\lambda_{2}^{2} + \mu_{1}^{3}$ $\lambda_{3}^{3} + 3\lambda_{1}^{2} + 3\lambda_{1}^{2} + 3\lambda_{2}^{2} + \mu_{1}^{3}$ $\lambda_{1}^{2} + \lambda_{1}^{2} + \lambda_{2}^{3} + \lambda_{1}^{3} + \lambda_{2}^{3}$ $\lambda_{1}^{2} + \lambda_{2}^{2} + \lambda_{1}^{2} + \lambda_{2}^{3} + \lambda_{2}^{3}$ $\lambda_{1}^{2} + \lambda_{2}^{2} + \lambda_{1}^{2} + \lambda_{2}^{3}$	2	may sub. A either of these steps
-	$ \begin{pmatrix} 55 & 0 & 34 \\ 0 & -1 & 0 \\ 34 & 0 & 21 \end{pmatrix} $	7	-1 for eac! - wrong entr
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SOLUTION	···	 -	P.
2. (a) For any $\lambda \in \mathbb{R}$		MARK	REMARK
2. (a) For any $d \in \mathbb{R}$, $u, v \in \mathbb{R}^3$, $g(d u) = \underline{a} \cdot (d u)$	٠.		
- 从 (a ⋅ Ū)		· .	
• d g (u) -	•		
$8(\underline{u} + \underline{v}) = \underline{a} \cdot (\underline{u} + \underline{v})$			
= 8(n) + 8(v)	••		1
∴ g is linear.			
		2+1	
(b) Since h is linear, for $u = dx + 3y + 8z$.		3 "	
$n(\underline{u}) = h(Ax + By + y)$			
$= h(\sqrt{x}) + h(\beta y) + h(\beta z)$ $= \alpha h(x) + \beta h(y) + \beta h(z)$			1
$\frac{\partial^2 f}{\partial z} = \int_{\mathbb{R}^n} \int_{\mathbb$		2	1 1
Let (e, e ₂ , e ₃) be the usual base of R ³ . For any	1	1	· · · · · · · · · · · · · · · · · · ·
$u = (u_1, u_2, u_3) \in \mathbb{R}^3, u = u_1 e_1 + u_2 e_2 + u_3 e_3$	l		· · · · · · · · · · · · · · · · · · ·
<u> </u>	- 1	2	u as a lin.
Since h is linear, $h(\underline{u}) = u_1 h(\underline{e}_1) + u_2 h(\underline{e}_2) + u_3 h(\underline{e}_3)$	- 1	-	comb. of ba
Define b = $(b(c))$ $(b(c))$ $(b(c))$ $(b(c))$ $(b(c))$ $(b(c))$) •	- 1	recepts.
Define $\underline{b} = (h(\underline{e_1}), h(\underline{e_2}), h(\underline{e_3}))$, then	- 1		
$h(\underline{u}) \sim \underline{b} \cdot \underline{u}$ for any $\underline{u} \in \mathbb{R}^3$.	1	,	
	 	3	
(c) Let $f: \mathbb{R}^3 \to \mathbb{R}$ be a linear function. By (b), there exist	 -	7	
b = (f(a) f(a) f(a) f(b) at the exist	s		
$\underline{b} = (f(\underline{e}_1), f(\underline{e}_2), f(\underline{e}_3)) \in \mathbb{R}^3 \text{s.t. } f(\underline{u}) = \underline{b} \cdot \underline{u} \forall \underline{u} \in \mathbb{R}^3$.		
Consider the set $H = \{\underline{u} \in \mathbb{R}^3 : f(\underline{u}) = 0\}$			
$= \left\{ \underline{\mathbf{u}} \in \mathbb{R}^3 : f(\underline{\mathbf{e}}_1)\mathbf{u}_1 + f(\underline{\mathbf{e}}_2)\mathbf{u}_2 + f(\underline{\mathbf{e}}_3)\mathbf{u}_3 = 0 \right\}$	l		
Since f		3	
Since f is not identically zero, $\underline{b} \neq \underline{0}$	1	1	
It is therefore a plane through the origin.			
Conversely if ::		1	
of H can be written as Au ₁ + Bu ₂ + Cu ₃ = 0, where A, B, C are not all zero.	- 1		
	1		
Define $\underline{a} = (A, B, C)$, then by (a), the non-zero function	-		
$f: \mathbb{R}^3 \to \mathbb{R}$ defined by	- 1		
$f(\underline{u}) = \underline{a} \cdot \underline{u} + \underline{u} = (u_1, u_2, u_3) \in \mathbb{R}^3$ is linear.			
Further $H = \left\{ \underline{u} \in \mathbb{R}^3 : f(\underline{u}) = 0 \right\}$			
•	2	4	•
	7	_	
		1.	
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and the state of t			
SOLUTION		MARK	Tall and
(a) When n is even,	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	IIAAA.	REMARK
$A_n = (n-3) + (n-5) + + 1$	4.	,	小型就有现
$=\frac{1}{2}(\frac{n-2}{2})(n-3+1)$	•		由而得深勢
$=(\frac{n-2}{2})^2$. ••		成八六3分!
when n is odd,		1	
$A_n = (n-3) + (n-1) + \dots + 2$			•
$=\frac{1}{2}\left(\frac{n-5}{2}+1\right)(n-3+2)$		1	ì
$=(\frac{n-1}{2})(\frac{n-3}{2})$			
· · · · · · · · · · · · · · · · · · ·		1	
(b) (1) To form a non-degenerate and	1	5	
(b) (i) To form a non-degenerate triangle with the longest s n, we have y < x < t = n, x + y > t. The number of such triangles.	1de		
The number of such triangles is equal to the number integral points in (a), i.e. A	of		
(ii) The number of possible triangles formed is	- 1	3	
$B_{21} = \sum_{1=4}^{2k} A_{1}$		2	
<u>k</u> <u>k-!</u>			
$= \sum_{i=2}^{k} \Lambda_{2i} + \sum_{i=2}^{k-1} \Lambda_{2i+1}$		1	
<u>k</u>			
$= \sum_{i=2}^{k} (i-1)^2 + \sum_{i=2}^{k-1} i(i-1)$			
$= \sum_{i=1}^{k-1} i^2 + \sum_{i=2}^{k-1} i^2 - \sum_{i=2}^{k-1} i$			
1 2 122			
$= 2 \times \frac{1}{6} (k-1)(k)(2k-1) - 1 - \frac{(k-2)(k+1)}{2}$		ai	100 m 2
$=\frac{k(k-1)(4k-5)}{6}$	2		order of F
i.	3		#** 4 · · ·
(c) $p(2k) = \frac{B_1 k}{C_1}$	2	\dashv	!
$\frac{1}{6}k(k-1)(4k-5)$			
$\frac{(2k)(2k-1)(2k-2)}{3\cdot 2}$			
$\frac{4k-5}{4(2k-1)}$	1		
$\lim_{k \to \infty} p(2k) = \lim_{k \to \infty} \frac{4k - 5}{4(2k - 1)}$	1 '		$\left\{ \sum_{i=1}^{N} \sum_{j=1}^{N} \left(\sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{j=1}^{N} \sum_{j=1}^{N} \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{j=1}^{N$
= 1 2			
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DECTDICTED THAT THE		- 1	

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SOLUTION	1.00	MARK	REMARK
(a) $z^{m} = 1$ iff $(\cos \frac{2\pi}{n} + i\sin \frac{2\pi}{n})^{m} = 2\pi $	1		
$\inf \cos \frac{2\pi m}{n} + \sin \frac{2\pi m}{n} = 1$		1	
iff n m		2	:
$\sum_{r=0}^{n-1} z^{mr} = \sum_{r=0}^{n-1} 1 = n$. 2	pi.
(ii) If $n \nmid m$, $\sum_{r=0}^{n-1} z^{mr} = \frac{1-z^{mn}}{1-z^{m}}$	•	2	
$r=0 1-z^{-}$ $= 0 as z^{mn} = 1$		1	
	•	7	•
$\sum_{r=0}^{n-1} f(z^r) z^{(n-j)r} = \sum_{r=0}^{n-1} \left[\sum_{k=0}^{n-1} a_k(z^r)^k \right] z^{(n-j)r}$	n-j)r	1	
$= \sum_{r=0}^{n-1} \sum_{k=0}^{n-1} a_k z^{(n+k-j)}$	r	1	.
$= \sum_{k=0}^{n-1} \left\{ a_k \sum_{r=0}^{n-1} z^{(n+k-j)r} \right\}$]	1	
As $0 \le k, j \le n-1$, $0 \le n+k-j \le 2$	n •		
$n \mid (n+k-1) \qquad \text{iff } k=1$. 1	
By (a), $\sum_{r=0}^{n-1} z^{(n+k-j)r} = \begin{cases} n & \text{if } k = j \\ 0 & \text{if } k \neq j \end{cases}$		1	
$\sum_{r=0}^{n-1} f(z^r) z^{(n-j)r} = na_j$		1	
: <u>.</u> •		6	
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SOLUTION	MARK	REMARK
4. (c) Let $f(x) = \sum_{j=0}^{n-1} a_j x^j$.	†	
By (b), $a_j = \frac{1}{n} \sum_{r=0}^{n-1} f(z^r) z^{(n-j)r}$		
$f(x) = \frac{1}{n} \sum_{j=0}^{n-1} \left[\sum_{r=0}^{n-1} f(z^r) z^{(n-j)r} \right] x^j$	1	
$= \frac{1}{n} \sum_{j=0}^{n-1} \left[\sum_{r=0}^{n-1} (g(z^r) - (z^{nr}-1)h(z^r))z^{(n-j)r} \right] x^j$	2	
$= \frac{1}{n} \sum_{j=0}^{n-1} \left[\sum_{r=0}^{n-1} g(z^r) z^{(n-j)r} \right] x^j \text{since } z^{nr} - 1 = 0.$	1	
5. (a) (1) For	4	
 (a) (1) For any x , y , z ∈ A , 1. f(x) = f(x) and g(x) = g(x) ∴ xRx . 		
2. If xRy , then $f(x) = f(y)$ and $g(x) = g(y)$ f(y) = f(x), g(y) = g(x) and yRx.	1	
3. If xRy and yRz, then $f(x) = f(y)$, $g(x) = g(y)$ and $f(y) = f(z)$, $g(y) = g(z)$		
f(x) = f(z) and g(x) = g(z) and xRz .		
	1	
(ii) Let $A = B = \{1, 2, 3\}$. The following example of f and g does not define an equivalence relation S:		
Here 182 and 283 but 1\$3.		
$\begin{array}{c c} 1 & 1 \\ 2 & 2 \\ 3 & 3 \end{array}$		
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SOLUTION			MARK	REMARK
(b) For any $a/R \in A/R$, define $h(a/R) = f(a)$.				
		•	2	
$a/R = a'/R \Rightarrow aRa' \Rightarrow f(a) = f(a')$. h is a well-defined mapping.			1	
Furthermore, for any $a \in A$, $h \cdot u(a)$	a) = h(a/R)	••		į
- h · u - f f(a))			i
Let h': $A/R \rightarrow B$ be any mapping surface u is surjective, any element, the image of some element, e.g. a,		f = f.	1	
∴ h(a/R) = h • u(a) = h' • u(a) = h'(a/R) for any a/R				
Hence $h = h'$, i.e. h is unique.			3	
If g is a constant mapping and if obviously h must be surjective. Furthermore, let a/R , $a'/R \in A/R$ such that $h(a/R) = h(a'/R)$.	f is surjecti	ve,	1	
Then $h \cdot u(a) = h \cdot u(a')$ $\Rightarrow f(a) = f(a')$.				
As $g(a) = g(a')$, $aRa' \Rightarrow a/R = a$ \(\therefore\) h is bijective	2'/R .	•	2	
			10	
e) (1) $\vec{d}\beta = 1 \Rightarrow \vec{d}\beta \vec{d}\beta = 1$				
=> c = 1 B				
As $ A \cdot \beta \le 1$, the above is poonly if $ A = \beta = 1$.	ssible		.	
· 22 = 1				
and $\frac{1}{\alpha}\beta = 1 \Rightarrow \frac{1}{\alpha}\beta = 1$,		
⇒ d = J3			3	,
(11) By (1), 1 - d, ≠ 0.				
$\frac{ \alpha - \beta }{ 1 - \overline{\alpha}\beta } \leqslant 1 \iff \alpha - \beta \leqslant 1 - \overline{\alpha}\beta $ $\stackrel{(-)}{\leftarrow} (\alpha - \beta) (\overline{\alpha} - \overline{\beta}) \leqslant$ $\stackrel{(-)}{\leftarrow} (\alpha + \beta) (\overline{\alpha} - \overline{\beta}) \leqslant$ $\stackrel{(-)}{\leftarrow} (1 - \alpha\overline{\alpha}) (1 - \beta\overline{\beta})$ which is true as $ \alpha \cdot \alpha \leqslant 1$	$(1 - \overline{\lambda}\beta)(1 - \overline{\lambda}\beta)(1 - \overline{\lambda}\beta) = \lambda$ $1 \ge 0$	4js) 13 - 2js		
which is true as 21. 181 < 1.			3	
If $ a < 1$, the equality holds if	f 1 - β5 = 0	. 1	1	
i.e. 1 \$1 - 1 .	17		•	
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SOLUTION	1.5.	P.7-
(b) (1) $ f(1) = f(-1) = L$	MARK	REMARK
$\Rightarrow \left \frac{1-a}{b-1} \right = 1 \text{ and } \left \frac{-1-a}{-b-1} \right = 1$	-	
$\Rightarrow a-1 - b-1 \text{ and } a+1 - b+1 $.•	
. a , b are equidistant from 1 and from -1 .		·
Hence b = a or a	3	
Alternatively:		
f(1) = f(-1) = 1		
$a + \overline{a} = b + \overline{b}$ and $a\overline{a} = b\overline{b}$		
i.e. $Re(a) = Re(b)$		
and $ a = b $ Hence $b = a$ or a		
$ f(1) = 1 \Rightarrow \left \frac{1-a}{1b-1}\right = 1$		
$\Rightarrow i-a = i+b $		
Together with the above result, this implies to	2	
Writing $f(z) = \frac{z-a}{az-1}$	2	
If $ z = 1$, $ z = 2$		•
$ f(z) ^2 = \frac{(z-a)(\overline{z}-\overline{a})}{(\overline{a}z-1)(a\overline{z}-1)}$		
$\frac{2\overline{z} + a\overline{a} - a\overline{z} - a\overline{z}}{a\overline{a}z\overline{z} - a\overline{z} - \overline{a}z + 1}$	'	
$\frac{1+a\overline{3}-a\overline{z}-\overline{a}z}{a\overline{a}-a\overline{z}-\overline{a}z+1}$		
- I ,		
(11) If a = 1,	2	
$f(z) = \frac{z - n}{z}$		
$=\frac{1}{a}\left(\frac{z-a}{z-a}\right)$		
$=\frac{1}{\overline{a}}$ (-a) which is constant.	3	
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SOLUTION	MARK		SOLUTION		P. (
f(1) = f(-1) = L = 1	- Italia	REMARK 1	7. (a) $(x_1 \ x_2 \ x_3)$ $\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$	MARK	REMARK
$\left \frac{1-a}{b-1}\right = 1$ and $\left(\frac{-1-a}{-b-1}\right) = 1$	-		``````````````````````````````````````		
a-1 - b-1 and $ a+1 - b+1 $			$= \frac{(a_{11}x_1 + a_{21}x_2 + a_{31}x_3 - a_{12}x_1 + a_{22}x_2 + a_{32}x_3 - a_{13}x_1 + a_{23}x_2 + a_{32}x_3 - a_{13}x_1 + a_{23}x_2 + a_{32}x_3 - a_{13}x_1 + a_{23}x_2 + a_{33}x_2 + a_{33}x_3 - a_{13}x_1 + a_{23}x_2 + a_{33}x_3 -$	3×3)	
a , b are equidistant from 1 and from -1 .			$= x_1 + x_2 + x_3 (: A \in \mathcal{D} \Rightarrow \sum_{j=1}^{3} a_{ij} = 1)$	3)×3	en de la companya de La companya de la co
e b-a or a	3		$\sum_{j=1}^{2} a_{ij} = 1$		
vely:			$Simtle-1 = \begin{pmatrix} a_{11} & a_{12} & a_{13} \end{pmatrix} / \gamma $	2	
= f(-1) = 1	1		Similarly $\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} a_{11}y_1 + a_{12}y_2 + a_{13}y_3 \\ a_{21}y_1 + a_{22}y_2 + a_{23}y_3 \\ a_{31}y_1 + a_{22}y_2 + a_{33}y_3 \end{pmatrix}$		
$-\frac{a}{a} + 1 = b\frac{b}{b} - b - \frac{b}{b} + 1$ $+\frac{a}{a} + 1 = b\frac{b}{b} + b + \frac{b}{b} + 1$			$\begin{array}{cccccccccccccccccccccccccccccccccccc$		· ·
p + p .			$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	^y 3	
Re(b)	-		1=1 a _{1j} = 1)	,	
or a				3	•
			(b) Let $B = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} \in \mathfrak{D}$		
$\begin{vmatrix} = 1 \Rightarrow \left \frac{1-a}{1b-1} \right = 1$ $\Rightarrow 1-a = 1+b $			\\ \\ 31 \\ \^{32} \\ \^{533} \/ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\		
er with the above result, this implies $b = \overline{a}$			(i) The ij-entry of $JB = \frac{1}{3} \sum_{i=1}^{3} b_{ij}$	•	
$g f(z) = \frac{z-a}{az-1}$	2		$=\frac{1}{3}$		
= 1 ,			The 1j-entry of BJ = $\frac{1}{2}$	1	
$\frac{(z) ^2 = \frac{(z-a)(\overline{z}-\overline{a})}{(\overline{a}z-1)(a\overline{z}-1)}$.		The ij-entry of BJ = $\frac{1}{3}\sum_{j=1}^{3}b_{ij}$.	
$= \frac{2\overline{z} + a\overline{a} - a\overline{z} - \overline{a}z}{a\overline{a}z\overline{z} - a\overline{z} - \overline{a}z + 1}$,	
$\frac{1 + a\overline{a} - a\overline{z} - \overline{a}z}{a\overline{a} - a\overline{z} - \overline{a}z + 1}$			JB = J = BJ		
		.	(11) By (1) BJ - J u B ∈ Đ		•
* I,	2		$\therefore J \in S(B) \Rightarrow S(B) \neq \emptyset$		
- 1,			(111)By (1) JB ~ J ♥ B ∈ Ŋ	.	
$ = \frac{z - a}{\bar{a}z - 1} $ $ = \frac{1}{a} \cdot z - a $			$\therefore \mathcal{S} \subset S(J)$		
$= \frac{1}{\overline{a}} \left(\frac{z - a}{z - a} \right)$ $= \frac{1}{\overline{a}} (-a) \text{ which is constant.}$			$\begin{array}{c} C & \mathcal{O} \\ \Rightarrow & S(J) = \mathcal{O} \end{array} $ (By definition) 1		.
a wilch is constant.	3				
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SOLUTION	t!	-P.9
(c) If A is invertible, $A^{-1}A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.	MARK	REMARK
	1	
Let $(x_1 x_2 x_3)$ be the first row of A^{-1} . By (a) ,		
$x_1 + x_2 + x_3 = 1 + 0 + 0 = 1$. Similarly for relative to the second state of the		
. Similarly for other rows and columns. $A^{-1} \in \mathcal{S}$ $X \in S(A) \implies AY = A$	2	,
$X \in S(A) \Rightarrow AX = J$ $\Rightarrow X = A^{-1}J$ = J By (b) (1)		
Since $J \in S(A)$ by $(b)(11)$, $S(A) = \{J\}$	1	- 17 17
[3]	1	
d) If A is since	4	
d) If A is singular, the system of equations	.	
$A\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \text{ has a non-zero solution } \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$	2	r jan
with b ₁ + b ₂ + b ₃ = 0 by (a)	1	
Let $C = \begin{pmatrix} b_1 & b_1 & -2b_1 \\ b_2 & b_2 & -2b_2 \\ b_3 & b_3 & -2b_3 \end{pmatrix}$, then $C \neq 0$ but C has zero row and column sums and $AC = 0$.		
Further $J + C \in \mathcal{D}$ and $A(J + C) = AJ + AC$	1	
$= J \Rightarrow J + C \in S(A)$ $= S(A) \neq \{J\}$	1	
	5	

SOLUTION	VAD'	THE REPORT
(a) Since a non-zero polynomial has only a finite number of zeroes	HARK	REMARK
g(n) = a f(n) = 0 and hence $a = 0$ for only a finite number of $n = 0$	2	
Assume, for contradition, that $\deg f(x) > \deg g(x)$.		
Then if n is sufficiently large, $g(n)$, a_n , $f(n) \neq 0$ and $ f(n) > g(n) $.		141
This contradicts the fact the $g(n) = a f(n)$. Hence $deg f(x) \leq deg g(x)$.	3	· <u>.</u>
••	5	•
(b) As deg f(x) ≤ deg g(x), by the Euclidean algorithm, there exist polynomials h(x) and r(x) with deg r(x) < deg f(x) such that g(x) = h(x)f(x) + r(x)		<u>:</u>
5 · · · · · · · · · · · · · · · · · · ·	3	
It is obvious that the coefficients of h(x) and r(x) are rational. Since for any $n \in \mathbb{N}$, $\exists a \in 2$ such that $g(n) = a_n f(n)$,	1	
$r(n) = a_n f(n) - h(n)f(n)$		•
= [a _n - h(n)]f(n)(*)	2	
Let M be a common multiple of the denominators of coefficients of r(x) and h(x), then the polynomials of Mr(x) and Mh(x) have integral coefficients.		
From (*), for any $n \in \mathbb{N}$, $\exists b_n = [Ma_n - Mh(n)] \in 2$		
such that $\operatorname{Hr}(n) = [\operatorname{Ha}_n - \operatorname{Mh}(n)]f(n)$		
By (a), deg $f(x) \le \deg r(x)$ which contradicts the definition of $r(x)$ unless $r(x) = 0$, i.e. $g(x) = f(x)h(x)$	2	
c) If $deg f(x) = deg g(x)$, since $g(x) = f(x)h(x)$, $h(x)$ is constant.	9	
By (a), there is an n such that $g(n) = a_n f(n)$ and a_n , $g(n)$, $f(n)$ are non-zero integers.		
Then $h(n) = \frac{g(n)}{f(n)}$	2	•
- a _n		
	3	

香港考益局

HONG KONG EXAMINATIONS AUTHORITY

一九八四年香港高級程度合考 HONG KONG ADVANCED LEVEL EXAMINATION, 1984

PURE MATHEMATICS (PAPER II)
MARKING SCHEME

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SOLUTION MARK REMARK: (a) $u_{k+2} = \int_0^{\pi} \frac{\sin(k+2)x}{\sin x} dx$ $= \int_0^{\pi} \frac{\sin(k+2)x}{\cos x} \frac{\sin x}{\sin x} dx$ $= \int_0^{\pi} \frac{\sin kx}{\sin x} (1 - 2\sin^2 x) + 2\cos kx \sin x \cos x} dx$ $= \int_0^{\pi} \frac{\sin kx}{\sin x} dx + \int_0^{\pi} (-2\sin kx \sin x + 2\cos kx \cos x) dx$ $= u_k + 2 \int_0^{\pi} \cos(k+1)x dx$ $= u_k + \frac{2}{k+1} \left[\sin(k+1)x \right]_0^{\pi}$ $= u_k$ $\therefore u_k = u_{k-2}$ $= \text{etc}$ $= \int_0^{\pi} \frac{\sin kx}{\sin x} dx + \int_0^{\pi} (-2\sin kx \sin x + 2\cos kx \cos x) dx$ $= u_k + \frac{2}{k+1} \left[\sin(k+1)x \right]_0^{\pi}$ $= u_k$ $= u_k - 2$ $= \text{etc}$ $= \int_0^{\pi} \frac{\sin kx}{\sin x} dx + \int_0^{\pi} \frac{\sin kx}{\sin x} dx + 2\cos kx \cos x dx$ $= u_k + 2 \int_0^{\pi} \cos(k+1)x dx + 2\cos kx \cos x dx$ $= u_k + 2 \int_0^{\pi} \cos(k+1$, , , , , , , , , , , , , , , , , , , ,			and the same of th
$= \int_{0}^{\pi} \frac{\sinh x \cos 2x + \cos kx \sin 2x}{\sin x} dx$ $= \int_{0}^{\pi} \frac{\sinh x (1 - 2\sin^{2}x) + 2\cos kx \sin x \cos x}{\sin x} dx$ $= \int_{0}^{\pi} \frac{\sinh x}{\sin x} dx + \int_{0}^{\pi} (-2\sinh x \sin x + 2\cos kx \cos x) dx$ $= u_{k} + 2 \int_{0}^{\pi} \cos(k + 1)x dx$ $= u_{k} + \frac{2}{k + 1} \left[\sin(k + 1)x \right]_{0}^{\pi}$ $= u_{k}$ $\therefore u_{k} = u_{k-2}$ $= \text{etc}$ $= \int_{0}^{\pi} \frac{\sinh x \cos^{2}x + \cos^{2}x \sin x \cos x}{\sin x + 2\cos kx \cos x} dx$ $= u_{k} + 2 \int_{0}^{\pi} \cos(k + 1)x dx$ $= u_{k} + \frac{2}{k + 1} \left[\sin(k + 1)x \right]_{0}^{\pi}$ $= u_{k}$ $\therefore u_{k} = u_{k-2}$ $= \text{etc}$ $= \int_{0}^{\pi} \frac{\sinh x \cos^{2}x + \cos^{2}x \sin x \cos x}{\sinh x + 2\cos kx \cos x} dx$ $= u_{k} + 2 \int_{0}^{\pi} \cos(k + 1)x dx$ $= u_{k} +$			MARK	REMARK
$-\int_{0}^{\pi} \frac{\sin kx}{\cos kx} \frac{(1 - 2\sin^{2}x) + 2\cos kx \sin x \cos x}{\sin x} dx$ $-\int_{0}^{\pi} \frac{\sin kx}{\sin x} dx + \int_{0}^{\pi} (-2\sin kx \sin x + 2\cos kx \cos x) dx$ $= u_{k} + 2\int_{0}^{\pi} \cos (k + 1)x dx$ $= u_{k} + \frac{2}{k + 1} \left[\sin(k + 1)x \right]_{0}^{\pi}$ $= u_{k}$ $\therefore u_{k} = u_{k-2}$ $= \cot c$ $-\int_{0}^{\pi} \frac{\sin kx}{\sin kx} \frac{(1 - 2\sin^{2}x) + 2\cos^{2}kx \cos x}{\sin kx} + 2\cos^{2}kx \cos x} dx$ $= u_{k} + 2\int_{0}^{\pi} \cos(k + 1)x dx$ $= u_{k} $	(a) $u_{k+2} = \int_0^{\pi} \frac{\sin(k+2)x}{\sin x} dx$	e e e e e e e e e e e e e e e e e e e		
$-\int_{0}^{\sqrt{\sin kx}} dx + \int_{0}^{\pi} (-2\sin kx \sin x + 2\cos kx \cos x) dx$ $= u_{k} + 2\int_{0}^{\sqrt{\cos kx}} \cos k + 1)x dx$ $= u_{k} + \frac{2}{k+1} \left[\sin(k+1)x \right]_{0}^{\pi}$ $= u_{k}$ $\therefore u_{k} = u_{k-2}$ $= \text{etc}$ $-\begin{cases} u_{0} & \text{if } k \text{ is even} \\ u_{1} & \text{if } k \text{ is odd.} \end{cases}$ $= \begin{cases} 0 & \text{if } k \text{ is even} \\ \frac{\pi}{4} & \text{if } k \text{ is odd.} \end{cases}$	$= \int_0^{\pi} \frac{\sin kx \cos 2x + \cos kx \sin 2x}{\sin x} dx$	•.		
	$= \int_0^{\pi} \frac{\sin kx \ (1 - 2\sin^2 x) + 2\cos kx \sin x \cos x}{\sin x} dx$	* •		
	$-\int_0^{\tau} \frac{\sin kx}{\sin x} dx + \int_0^{\tau} (-2\sin kx \sin x + 2\cos kx \cos x)$	x) dx		
$ \begin{array}{c} -u_k \\ \vdots \\ u_k = u_{k-2} \\ = \text{etc} \\ -\left(\begin{array}{c} u_0 & \text{if } k \text{ is even} \\ u_1 & \text{if } k \text{ is odd.} \end{array} \right) \\ = \begin{cases} 0 & \text{if } k \text{ is even} \\ \pi & \text{if } k \text{ is odd.} \end{cases} $	$= u_k + 2 \int_0^{\pi} \cos(k+1)x dx \cdots$			
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$u_k + \frac{2}{k+1} [\sin(k+1)x]_0^{\pi}$			
= etc \[\begin{aligned} u_0 & \text{if k is even} & \text{if k is odd.} & \text{if k is odd.} & \text{if k is odd.} \end{aligned} \] \[\begin{aligned} 0 & \text{if k is even} & \text{if k is odd.} & \text{if k is odd.} & \text{if k is odd.} \end{aligned} \] \[\begin{aligned} \begin{aligned} 0 & \text{if k is odd.} \end{aligned} \]	- u _k		3	
To if k is even I f k is odd.				
= {0 if k is even π if k is odd. 6	u ₀ if k is even			
	•			
	(π if k is odd.	-		
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ATHS II		P.2			
- SOLUTION	MARK	rinkk	RESTRICTED 汽部文件	947-37	
b) (1) Put $u = \cos^{m-1}\theta$, $dv = \cos\theta \sin^{n}\theta d\theta$, then $du = -(n-1)\cos^{m-2}\theta$. $\sin\theta d\theta$, $v = \frac{1}{n+1}\sin^{n+1}\theta$	-		TI TOTAL DE LES PERSONATE	· Participa	ម្រាប់ ្នាន់
$I(m, n) = \left[\frac{1}{n+1}\cos^{m-1}\theta \sin^{n+1}\theta\right]_0^{\frac{\pi}{2}} + \frac{m-1}{n+1} \int_0^{\frac{\pi}{2}} \cos^{m-2}\theta \sin^{n+2}\theta d\theta$		1	SCLUTION MM.	ANSE NE	EE.
$= (\frac{m-1}{n+1}) \text{ I } (m-2, n+2) , m \ge 2$			Let $y = \frac{(b-c)^{a-1}}{(a-c)^{a-1}}$, $y = e^{(a)}/eyae$		
(ii) For $n \ge 0$, $I(1, n) = \begin{cases} \frac{\pi}{2} \\ 0 \\ \cos \theta \\ \sin^n \theta \\ d\theta \end{cases}$	3		$\frac{d}{dx} = \frac{-(4 - 1)(11 - 2)^{4-2}}{(2^{3} + 2^{3})^{2}} = \frac{(7^{-1})^{3}}{(2^{3} + 2^{3})^{2}} = \frac{1}{2}$		1
$=\frac{1}{n+1}\sin^{n+1}\theta \Big _{0}^{\frac{\pi}{2}}$			$R_{a} = \frac{1}{(a-1)!} \left\{ (Ca + a)^{a-1} e^{(a-1)!} e^{(a)} e^{(a)} + \frac{1}{2} e^{(a-1)!} \frac{1}{2} e^{(a-1)!} e^{(a)} e^{(a)} \right\}$	e .	-
$=\frac{1}{n+1}$	2		· · · · · · · · · · · · · · · · · · ·		}
(iii) Put $u = \sin^{n-1}\theta$, $dv = \sin\theta d\theta$,			$\mathcal{R}_{1} = \begin{cases} 0 & \text{if } (a) \text{ is} \\ 0 & \text{of } (a) \text{ is} \end{cases}$:	
then $du = (n-1)\sin^{n-2}\theta \cos\theta d\theta$, $v = -\cos\theta$.		•	= E(D)_= E(O)		
For $n \ge 2$, $I(0, n) = \begin{cases} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{cases}$			$R_{2} = R_{1} + 66\%0$	<i>;</i>	
$= \left[-\sin^{n-1}\theta \cos\theta\right]_{0}^{\frac{\pi}{4}} + (n-1) \int_{0}^{\frac{\pi}{4}} \sin^{n-2}\theta \cos^{2}\theta d\theta$		'	1	;	
$= (n-1) \left\{ \int_{0}^{\frac{\pi}{4}} \sin^{n-2}\theta \ d\theta - \int_{0}^{\frac{\pi}{4}} \sin^{n}\theta \ d\theta \right\}$			$R_{n} = R_{n-1} - \frac{2^{n-1}}{(n-1)^{n}} = \frac{2^{n-1}}{(n-1)^{n}}$:	
$i \cdot I(0, n) = (\frac{n-1}{n}) I(0, n-2)$.	3		$= R_{n+2} - \frac{n^{(n)}}{(n-2)!} f^{(n-2)}(0) - \frac{n^{(n)}}{(n-1)!} f^{(n-1)}(0)$		
(iv) $I(6, 4) = \frac{5}{5}I(4, 6)$ by (1)	-1		n-2 $(n-2)$: $n-2$ $(n-2)$: $n-2$ $(n-2)$: $n-2$		
$= \frac{5 \cdot 3 \cdot 1}{5 \cdot 7 \cdot 9} \text{ I(0, 10)}$			$= R_1 - h \varepsilon^*(0) - \frac{h^2}{2!} \varepsilon^{**}(0) - \dots - \frac{h^{n-1}}{(n-1)!} \varepsilon^{(n-1)}(0)$		
$\frac{5 \cdot 3 \cdot 1}{5 \cdot 7 \cdot 9} - \frac{9}{10} I(0, 8) \qquad \text{by (111)}$	1	<u> </u>	Subst. $R_1 = f(h) - f(0)$, we have		
$-\frac{5 \cdot 3 \cdot 1}{5 \cdot 7 \cdot 3} \qquad \frac{9 \cdot 7 \cdot 5 \cdot 3 \cdot 1}{10 \cdot 8 \cdot 6 \cdot 4 \cdot 2} \text{ t(0, 0)}$			$\vec{z}(h) = \vec{z}(0) + h\vec{z}'(0) + \frac{h^2}{2!} \vec{z}''(0) + \dots + \frac{h^{(n-1)}}{(n-1)!} \vec{z}'^{(n-1)}(0) + \vec{z}_n $!
- 3π 512	1		$(n-1)! \sim \frac{(n-1)!}{5}$		
	11				
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	Tagasta is a special of	A State of the Sta	Provide	d by de	l li

-	SOLUTION			
(c) Putting	E/W		Twine	1
. on (~1	1), which is infinit	ely differentiable	MARK	RE
f'(x) =	$\frac{1}{1+x}$		1	
f ⁽ⁿ⁾ (x)	$\frac{(-1)^{n-1}(n-1)!}{(1+x)^n}.$			
.		· · · · · · · · · · · · · · · · · · ·	1	
ву (b), f	r any $0 < h < 1$, $\ln(1 + h) = \ln 1 + h$	h ² h ³ (4	- 1	
Since R ₅	$=\frac{1}{4!}\int_{0}^{h} (h-t)^{4} \cdot \frac{(-1)^{4}(4!)}{(1+t)^{3}} dt$		1	
	>0 as (h - r)4 (+t5	F07 - 1 (10		
: ln(1	$2 - 3 + 4 - R_5 > 0$	201 EE(0, h) .	1	
Similarly,	$R_6 = \frac{1}{5!} \int_0^h \frac{(h-t)^5 (-1)^5 (5!)}{(1+t)^6} dt$,	
	< o			
- In(1 + p	$h - h + \frac{h^2}{2} - \frac{h^3}{3} + \frac{h}{4} = \frac{h^5}{5}$			
	× h ⁵ / ₅			

SOLUTION	MARK	DELLAR
(a) Since a0 - b0,	+	REMARK
if P = (x, y) ,		
$x = (a - b)\cos\theta + b\sin\left(\frac{\pi}{3} - \theta - \left(\frac{\pi}{2} - \theta\right)\right)$	2	
= $(a - b)\cos\theta + b\sin\left(\frac{\pi}{2} - (\emptyset - \theta)\right)$		
· = (a - b)cosθ : bcos(∅ - θ)	1	
$= (a - b)\cos\theta + b\cos(\frac{a-b}{b})\theta$	2	
$y = (a-b)\sin \theta - b\cos(\frac{\pi}{2} - (\theta - \theta))$		
$= (a-b)\sin\theta - b\sin(\frac{a-b}{b})\theta.$	2	
(h) rc . Q	7	
(b) If $b = \frac{a}{4}$, $x = \frac{3}{4} a \cos \theta + \frac{a}{4} \cos 3\theta$		
$=\frac{3}{4} a\cos\theta + \frac{a}{4} \left[4 \cos^3\theta - 3\cos\theta \right]$		•
= acos ³ θ	2	
$y = \frac{3}{4} \operatorname{asin9} - \frac{a}{4} \operatorname{sin30}$		
$=\frac{3}{4} \operatorname{asin} \theta - \frac{a}{4} \left[3 \sin \theta - 4 \sin^3 \theta \right]$		
= asin ³ 0	2	
$x^{\frac{1}{3}} + y^{\frac{2}{3}} = a^{\frac{1}{2}}\cos^2\theta + a^{\frac{2}{3}}\sin^2\theta$		
$= e^{\frac{\lambda}{3}}$	1	
	5	
(c) Length of hypocycloid		
$= \int_0^{2\sqrt{1}} \int \left(\frac{dx}{d\theta} \right)^2 + \left(\frac{dy}{d\theta} \right)^2 d\theta$		
(25)		
. 	2	٠
= 4		
- 6a [sin²0]		
- 6a .	,	
	5	

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b x + a + a + m + = 1				
$\therefore \ell^2 = m^2 = \frac{1}{a^2 + b^2}$				
Equations of common tangents to	(E)	and	(F) are	
$x \pm y = \pm \sqrt{a^2 + b^2}$				
	•			

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MATHS II

	SOLITION					P
· (c) Equation of	tangent at S(s		Salah ing Salah	i de-	MARK	REMARK
81 X S.	8cur ar 2(\$	1 , s ₂) is			-	REPURK
$\frac{s_1 \times a^2}{a^2} + \frac{s_2}{b^2}$	1/2 = 1.	• • • •	•			6 1 46
If R(h, k)	lies on the tar		** ** ***	.	-	
$\frac{s_1 h}{a^2} + \frac{s_2}{b^2}$	<u>k</u> _ ,	rgent,				
a* b²	1	• •		••	2	. •
Similarly, fo	r the tangent a	t T(t ₁ , t ₂),		.	į	•
$\frac{t_1h}{a^2} + \frac{t_2}{b^2}$	<u>k</u> _ 1	1(1, 52)	we have			
		• • •		ľ		
: the straight	line $\frac{xh}{2} + \frac{yk}{2}$	= 1 passes thro	÷ .		1	1 1 197
	a- 62	r passes thro	ough S and	T .	3	
(d) By (n) the -t	o-1 o-			- [5	. The same
(d) By (a), the ch	ord ST is tan	gent to (F) if	£	Ī		-10.4 kg
$\frac{h^2b^2}{a^2} + \frac{k^2a^2}{b^2} = 1$	l, where (L	, k) lies outsi			- 1	E CONTRACTOR
- U		, k/ lles outsi	de (E)].	2	
. the locus of	R consists o	of the parts of t	ha alla.			
	W.	- 120 OL (me ellipse			
(G):	y² • * —— = 1	which lie out				
$(\frac{a^2}{b})^2$	$\left(\frac{b^2}{a}\right)^2$	which lie out	side (E) .		1	
		, .				
	1	R				
)(G)	——→ x	2	-	
	1	(E)	•	- 5	\dashv	
		(F) -		-	-	•
	7					
(0)	ł	•				
(G) , locus of	R					
•				1		
•						•
	·		1	1	1.	
. R				1 .	1 .	

#ATHS II SOLUTION 5. (a) $\frac{1}{1}k+1^{a} \ge 0$, let $\frac{1}{N}$ be an integgr such that $\frac{1}{N} > a$. $\frac{1}{(N+1)1} - \frac{a}{N+1} \cdot \frac{a}{N!} < \frac{N}{N+1} \cdot \frac{a}{N!}$ $\frac{N+2}{(N+2)1} - \frac{a}{(N+2)} \cdot \frac{a}{(N+1)} \cdot \frac{a}{N!} < \left(\frac{N}{N+1}\right)^2 \cdot \frac{a^N}{N!}$, etc. $0 < \frac{a^N}{n!} < \left(\frac{N}{N+1}\right) \cdot \frac{a^N}{N!}$ for every $n > N$. 1 im $\frac{a^n}{n} = 0$ as $0 < \frac{N}{N+1} < 1$. If $a < 0$, the same result follows from the inequalities $\frac{1}{n!} \le \frac{a^N}{n!} \le \frac{ a ^n}{n!}$ (b) (i) $f(1-x) = (1-x)^n (1-(1-x))^n = (1-x)^n x^n (1-x)$ $f'(x) = f'(1-x)$ $f''(x) = f''(1-x)$ $f'''(x) = f''(1-x)$ $f'''(x) = f''(1-x)$ $f'''(x) = f''(1-x)$
$\frac{1}{N+1} \stackrel{a}{=} \stackrel{b}{=} 0, \text{ let } N_N \text{ be an integer such that } N > a.$ $(N+1)_1 = \frac{a}{N+1} \cdot \frac{a}{N!} < \frac{N}{N+1} \cdot \frac{a}{N!}$ $\frac{a^{N+2}}{(N+2)_1} = \frac{a}{(N+2)} \cdot \frac{a}{(N+1)} \cdot \frac{a^{N}}{N!} < \left(\frac{N}{N+1}\right)^2 \frac{a^{N}}{N!} , \text{ etc.}$ $0 < \frac{a^{n}}{n!} < \left(\frac{N}{N+1}\right) \cdot \frac{a^{n}}{n!} \text{ for every } n > N.$ $\frac{1}{n} \stackrel{a^{n}}{=} 0 \text{ as } 0 < \frac{N}{N+1} < 1.$ If $a < 0$, the same result follows from the inequalities $\frac{1}{n!} \leq \frac{a^{n}}{n!} \leq \frac{ a ^{n}}{n!}$ $(b) (1) f(1-x) = (1-x)^{n}(1-(1-x))^{n}$ $= (1-x)^{n}x^{n}$ $f'(x) = -f'(1-x)$ $f''(x) = -f'(1-x)$ $f''(x) = -f'(1-x)$
$(N+2)! = \frac{a}{(N+2)} \cdot \frac{a}{(N+1)} \cdot \frac{a}{N!} < \left(\frac{N}{N+1}\right)^2 \frac{a^N}{N!} , \text{ etc.}$ $0 < \frac{a}{n!} < \left(\frac{N}{N+1}\right) \cdot \frac{a}{N!} \text{ for every } n > N.$ $1 \cdot \frac{1}{n+\infty} \frac{a}{n!} = 0 \text{as} 0 < \frac{N}{N+1} < 1.$ If $a < 0$, the same result follows from the inequalities $-\frac{ a ^n}{n!} < \frac{a}{n!} < \frac{ a ^n}{n!}$ $(b) (1) f(1-x) = \frac{(1-x)^n(1-(1-x))^n}{n!}$ $= \frac{f(x)}{f''(x)} = \frac{-f'(1-x)}{f'''(x)} \forall x \in \mathbb{R}$
$(N+2)! = \frac{a}{(N+2)} \cdot \frac{a}{(N+1)} \cdot \frac{a}{N!} < \left(\frac{N}{N+1}\right)^2 \frac{a^N}{N!} , \text{ etc.}$ $0 < \frac{a}{n!} < \left(\frac{N}{N+1}\right) \cdot \frac{a}{N!} \text{ for every } n > N.$ $1 \cdot \frac{1}{n+\infty} \frac{a}{n!} = 0 \text{as} 0 < \frac{N}{N+1} < 1.$ If $a < 0$, the same result follows from the inequalities $-\frac{ a ^n}{n!} < \frac{a}{n!} < \frac{ a ^n}{n!}$ $(b) (1) f(1-x) = \frac{(1-x)^n(1-(1-x))^n}{n!}$ $= \frac{f(x)}{f''(x)} = \frac{-f'(1-x)}{f'''(x)} \forall x \in \mathbb{R}$
$(N+2)! = \frac{a}{(N+2)} \cdot \frac{a}{(N+1)} \cdot \frac{a}{N!} < \left(\frac{N}{N+1}\right)^2 \frac{a^N}{N!} , \text{ etc.}$ $0 < \frac{a}{n!} < \left(\frac{N}{N+1}\right) \cdot \frac{a}{N!} \text{ for every } n > N.$ $1 \cdot \frac{1}{n+\infty} \frac{a}{n!} = 0 \text{as} 0 < \frac{N}{N+1} < 1.$ If $a < 0$, the same result follows from the inequalities $-\frac{ a ^n}{n!} < \frac{a}{n!} < \frac{ a ^n}{n!}$ $(b) (1) f(1-x) = \frac{(1-x)^n(1-(1-x))^n}{n!}$ $= \frac{f(x)}{f''(x)} = \frac{-f'(1-x)}{f'''(x)} \forall x \in \mathbb{R}$
$0 < \frac{1}{n!} < \frac{n}{N+1} $ $\frac{a}{N!}$ for every $n > N$. $\frac{1 \text{im } \frac{a}{n!}}{n!} = 0 \text{as } 0 < \frac{N}{N+1} < 1.$ If $a < 0$, the same result follows from the inequalities $-\frac{ a ^n}{n!} < \frac{a}{n!} < \frac{ a ^n}{n!}$ (b) (i) $f(1-x) = (1-x)^n (1-(1-x))^n$ $= (1-x)^n x^n$ $f'(x) = -f'(1-x)$ $f''(x) = f''(1-x)$
$0 < \frac{1}{n!} < \frac{n}{N+1} $ $\frac{a}{N!}$ for every $n > N$. $\frac{1 \text{im } \frac{a}{n!}}{n!} = 0 \text{as } 0 < \frac{N}{N+1} < 1.$ If $a < 0$, the same result follows from the inequalities $-\frac{ a ^n}{n!} < \frac{a}{n!} < \frac{ a ^n}{n!}$ (b) (i) $f(1-x) = (1-x)^n (1-(1-x))^n$ $= (1-x)^n x^n$ $f'(x) = -f'(1-x)$ $f''(x) = f''(1-x)$
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(b) (1) $f(1-x) = (1-x)^n (1-(1-x))^n = (1-x)^n x^n (1-(1-x))^n = f(x) \forall x \in IR$ $f''(x) = -f'(1-x) \forall x \in IR$ $f'''(x) = f''(1-x) \exists f''(x) = f''(1-x) = f''(1-$
f'(x) = -f'(1-x) $f''(x) = f''(1-x)$
f'(x) = -f'(1-x) $f''(x) = f''(1-x)$
^ \X/ = +"/! \
$f^{(k)}(x) = (-1)^k f^{(k)}(1-x)$
(11) $f(x) = x^n \sum_{i=0}^n c_i^n (-1)^i x^{i}$
= C ⁿ _n _n n+1
$= c_0^n x^n - c_1^n x^{n+1} + \dots + c_n^n (-1)^n x^{2n}$
For $k > 2n$, $f^{(k)}(x) \equiv 0$
For $0 \le k < n$, $f^{(k)}(0) = 0$
For $n \le k \le 2n$, $f^{(k)}(x) = \sum_{i=0}^{n} c_i^n(-1)^i \frac{d^k}{dx^k} (x^{n+i})$
$\frac{1}{1=0} \frac{c_1(-1)^2}{dx^k} \frac{dx^{k-1}}{(x^{n+1})}$
At $x = 0$, $f^{(k)}(0) = C_{k-n}^{n}(-1)^{k-n} \frac{d^k}{dx^k} (x^k)$
GA 1 1
$-(-1)^{k-n}c_{k-n}^{n}$ $k!$
= (-1) ^{K-0} 00
$\frac{k!n!}{(k-n)!(2n-k)!} - k!C_{k-n}^{n} \in \mathbb{R}$ and $C_{k-n}^{k} = k!$
and $C_{2n-k}^{k} = \frac{k!}{(2n-k)!(2k-2n)!}$
4n-k $(2n-k)!(2k-2n)!$
$(2n-k) \frac{1}{2(k-k)} \in \mathcal{A}$
te follows that k!
(0) is divisible by pl
That $f^{(k)}(1)$ is also divisible by ni follows from the
fact that $f^{(k)}(1-x) = (-1)^k f^{(k)}(x)$.
(-1) t (x).
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10 11 11 11 11 11 11 11 11 11 11 11 11 1

SOLUTION (a) That $e^{-x^2} > 0$ H		MARK	REMARK
. (a) That e ^{-x²} > 0			1
		1	
$\frac{1}{e^{x^2}} \leqslant 1$			
$f(x) \leq f(x) \leq 1 \qquad \forall x \in \mathbb{R}.$		1+2	
ALTERNATIVELY,			
Since $\frac{d}{dx}e^{-x^2}$ $-2xe^{-x^2}$			
$ \begin{cases} 0 & \text{if } x \leq 0 \\ 0 & \text{if } x > 0 \end{cases} $	-		
f(x) has an absolute maximum at $x = 0$ and $f(0) = 1$.			
Since $0 < \{f(x)\}^n \le 1$,			*.
$I_n = \left[\int_{-1}^{1} \{f(x)\}^n dx \right]^{\frac{1}{n}}$			
$\leq \left[\int_{-1}^{1} dx \right]^{\frac{1}{n}}$.			
= 2 (1)			
- 2		2	
(b) $r \leqslant e^{-x^2}$		5	
⇔ In r ≤ -x²			
$\iff \ln \frac{1}{r} \geqslant x^2$			
$\Leftrightarrow -d \leq x \leq d$, where $d = \sqrt{\ln(\frac{1}{r})}$			
For $\frac{1}{e} < r < 1$, $0 < \alpha < 1$			
$0 < r \leq f(x)$	-		
$0 < r^n \leq [f(x)]^n$			•
$\int_{-\infty}^{\infty} r^{n} dx \leq \int_{-\infty}^{\infty} [f(x)]^{n} dx$			
$\leq \int_{-1}^{1} [\bar{\epsilon}(x)]^{\pi} dx$	3		
Hence $2r^n_{\infty} \leq \int_{-1}^{1} \left[f(x)\right]^n dx$			*
$\Rightarrow r\left[2\sqrt{\ln(\frac{1}{r})}\right]^{\frac{1}{n}} \leqslant I_{p}$			

(c) By (a) and (i)		P.1(=
	MARK	REMARK
$r \left[2 \sqrt{\ln\left(\frac{1}{r}\right)} \right]^{\frac{1}{n}} \le I_{n} \le 2^{\frac{1}{n}} \cdots \frac{1}{e} < r < 1$ $\text{Now } \lim_{n \to \infty} 2^{\frac{1}{n}} = 1 \text{ and } \lim_{n \to \infty} \left[\sqrt{\ln\left(\frac{1}{r}\right)} \right]^{\frac{1}{n}} = 1$ $\text{As } \lim_{n \to \infty} I_{n} \text{ exists}$	1 <u></u>	
Since r is an arbitrary number between $\frac{1}{e}$ and 1,	1	
(a) (1) $f(-x) = (-x)^{\frac{2}{3}} - ((-x)^2 - 1)^{\frac{1}{3}} = f(x)$.	1 5	
f (x) = $x^{\frac{1}{3}} - (x^2 - 1)^{\frac{1}{3}} > 0$ $x^{\frac{2}{3}} > (x^2 - 1)^{\frac{1}{3}}$		- 18 - 11 (
$\Rightarrow x^{2} > x^{2} - 1, \text{ which is true for all } x.$ $\therefore f(x) > 0 \text{ for all } x.$ $(11) \text{ Put } x^{2} = \frac{1}{y}, \text{ then } x^{\frac{2}{3}} - (x^{2} - 1)^{\frac{1}{3}} = (\frac{1}{y})^{\frac{1}{3}} - (\frac{1}{y} - 1)^{\frac{1}{3}}$	2	
$= \frac{1 - (1 - y)^{\frac{1}{3}}}{y!}$ $\lim_{x \to \infty} f(x) = \lim_{y \to 0^{+}} \frac{1 - (1 - y)^{\frac{1}{3}}}{y!}$		
$-\lim_{y \to 0^{+}} \frac{-\frac{1}{3}(1-y)^{\frac{3}{2}}(-1)}{\frac{1}{3}y^{\frac{3}{2}}} $ (by L'Hopital's rule) $-\lim_{y \to 0^{+}} (\frac{1}{(\frac{1}{y}-1)^{\frac{3}{2}}})$ $= 0$		
(1) For $x \neq 0$, $1 \text{ or } -1$, $f'(x) = \frac{2}{3} x^{-\frac{1}{3}} - \frac{1}{3} (x^2 - 1)^{\frac{-1}{3}} (2x)$ $= \frac{2}{3} \cdot \frac{(x^2 - 1)^{\frac{2}{3}} - x^{\frac{4}{3}}}{x^{\frac{1}{3}} (x^3 - 1)^{\frac{1}{3}}}$	·	
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É MATHS II		P.11
SOLUTION .	MARK	REMARK
(b) (ii) For $x > 0$, $x \neq \pm 1$, the denominator $x^{\frac{1}{3}}(x^2 - 1)^{\frac{2}{3}} > 0$. $(x^2 - 1)^{\frac{1}{3}} - x^{\frac{1}{3}} \stackrel{>}{>} 0 \text{ according as } (x^2 - 1)^2 \stackrel{>}{<} x^4$		
$\begin{array}{cccccccccccccccccccccccccccccccccccc$		
1		
the required sets are		•
$\left\{x: x-\frac{1}{J^2}\right\}$. 2	· ·
$\left\{x: x > \frac{1}{\sqrt{2}}\right\}$	1	
$\left\{x: 0 < x < \frac{1}{JZ}\right\}$	-1	
$x = 0$ $0 < x < \frac{1}{J2}$ $x = \frac{1}{J2}$ $\frac{1}{J2} < x < 1$ $x = 1$ $1 < x$		
f does not exist + 0 - does not exist		
f min. max.		
At $x = 0$, 1 and -1 $f(x) = 1$		THE PROPERTY OF THE PROPERTY O
At $x = \pm \frac{1}{12}$, $f(x) = 4^{\frac{1}{3}}$ (= 1.5876)		The state of the s
(= ± 0.7071) (0, 1) is a minimum		- contains and
	1	the grown
$(-\frac{1}{\sqrt{2}}, 4^{\frac{1}{3}}), (\frac{1}{\sqrt{2}}, 4^{\frac{1}{3}})$ are maxima.	1+1	
	9	
f(x)		,
	-	· · · · · · · · · · · · · · · · · · ·
-1 $-\frac{1}{\overline{J2}}$ 0 $\frac{1}{\overline{J2}}$ 1	3	
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2(a) = 8 (c) - b - f(c)	
Since f is strictly increasing	MARK REMARK
1 - 1 - 2 - 2 - 2 - 2 - 2 - 2 - 2 - 2 -	
B'(t) \geq 0 according as t \geq f^1(b). As g is continuously differently the second in the second	
As 8 is continuously differentiable on [0	
8 attains its greatest value at f-1(b).	······
(1) Integration	
f-1(b)	
$\int_{0}^{f^{-1}(b)} \int_{0}^{f^{-1}(b)} xf'(x)dx = xf(x) \Big _{0}^{f^{-1}(b)} - \int_{0}^{f^{-1}(b)} f^{-1}(b)$	
10	2 2
8(£ ⁻¹ (b))	
(ii) Putting $y = f(x)$, $dy = f'(x)dx$.	
as f is co-t	
$\int_{0}^{f^{-1}(b)} f(x) dx = \int_{0}^{b} f^{-1}(y) dy$	
) 0 ¹ (7) dy	
$\int_0^b f^{-1}(x) dx$	
/1	31
$\frac{By''(b)}{0}$, $\int_{0}^{b} \frac{f^{-1}(x)dx}{f} = \int_{0}^{f^{-1}(b)} xf'(x)dx$	-5
•	
* g(f ⁻¹ (b))	
$\geq bt - \int_0^t f(x) dx \forall t \in [0]$ Lience $\int_0^b f^{-1}(x) dx \geq t$	
	by (a) 1
i.e. $\int_0^a f(x)dx + \int_0^b f^{-1}(x)dx \ge ab$ Seometrically,	
the same of the sa	1
f(x)dx - area of region OAP	c /
£=1(x)dx = (*,b)	i/a
f-1(x)dx = area of region OBQ	P
area of OACB	
e equality holds when	(a, o) >x
e equality holds when $f(a) = b$.)	A Company of the Comp
PERFECTOR	
TATE OF THE PARTY	
	_

	SOLUTIO			i i	MARK	REMARK
which sat	isfics the giv	$f(x) = x^{p-1}$ ($x \in Y$	[0, ∞], p>	2)	1 -	
Since :	$\frac{1}{q} = 1$					"
	$(x) = x^{\frac{1}{p-1}}$.==-				
	(x) = x ^r =	Alexandro Markovici, Illinois Totalista	7 * · · · · · · · · · · · · · · · · · ·			
-	$=x^{q-1}$	-	+ 1 4	. J.	.	• · · · · · · · · · · · · · · · · · · ·
and the same of th		-441	***	-	1	t. jáljast m
Ву (с),	a xp-1 dx +	$\int_0^b x^{q-1} dx \ge ab$				
		$\int_0^{\infty} 0 x dx \ge ab$				
$\frac{1}{p} a^p +$	1,9				1	
P .	q = 20 ·			-	,	
en en age s, en gris en				-	3	
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