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## 一九八一年香港中學會考

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION, 1981

Additional Mathematics II

MARKING SCHEME 产 岩 寺 寺

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<b>:</b> -				コンノコー	
	".``	Solutions		Marks	Remarks
<del></del>	(1+0	$50)^{2}d\theta = \int (1+2\cos\theta)^{2}d\theta$	0 + co-0) a0	i A	
	J		+ cos 20+1) d t		And the second
-		$= \frac{3}{2}8 + 25i$	$\frac{2}{6+\frac{\sin^2\theta}{4}+C}$	1+1+1A	-1 if omit C
				5	
2)	Area of	region = $\int_{\frac{\pi}{2}}^{\pi} \left( \sin \frac{x}{2} \right)$	- Cos 2x) dx	/M	The state of the s
		= [-200 2/2	$-\frac{\sin 2x}{2}\bigg]_{\underline{\pi}}^{\pi}$	I+I A	2 8
e.			<u> </u>		
<u>.</u>		$= 2 \cos \frac{\pi}{10} +$ $= 5 \cos \frac{\pi}{10}$	•		# 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
• •	<u>Aet</u>		( = 2.378)	5 A	ns -ve C
	· · · ]=	sin = + x = - ? coo = /	/ <sup>(</sup> 곡	14	•
		= 2007		12	
	∫ <u>₹</u>	$\cos 2x  dx = \frac{\sin 2x}{2} \Big $	7	IA	
	7		T 8-0.476.	I A	M' Carris
	An	ia = 2 cos 7/0 + 2 Sin		14	-1 for -ve answer
(3)	Putting	$u^2 = 9 - x$ , 2nd		IA	
	ď	When X	=0, u=3; ; =9, u=0.}	IA	Main ha some
	J 9	$\frac{1}{\pi} dn = \int_{3}^{0} \frac{-2u(q)}{u}$		IMHA	. 1=M for change of lim
٠	30 19-	,	•		1 A for integrand
		$= \int_{0}^{3} z(q-1)$		-	
		$= \left[ l  \mathcal{S}_{M} - \frac{2}{3} \right]$	u <sup>3</sup> ]	IA	
		= 36		IA	••
				1 4	

$\therefore \ \mathcal{X} = /20^{\circ} \times n - 7.54^{\circ}  (7^{\circ}32') $ 1A		Solutions	J	Marks	Lemantes
$P_{\text{suting}} = \gamma \left( \cos 3 \times \cos 9 - \sin 3 \times \sin \theta \right)$ $P_{\text{suting}} = \gamma \cos \theta = 12$ $\gamma \sin \theta = 5$ $\gamma = \sqrt{12 + 5^{2}} = 13$ $Coo \theta = \frac{12}{13} = 0.9231$ $\therefore \theta = Coo^{\frac{1}{13}} = 2.5 \cdot b^{2}$ $= 2.5 \cdot b^{2} = 2.5 \cdot b^{2} = 13$ $12 \cos 3 \times -5 \sin 3 \times = 13$ $13 \cos \left( 3 \times +22.52^{2} \right) = 13$ $\cos \left( 3 \times -22.52^{2} \right) = 13$ $\cos \left( 3 \times -22.52^{2} \right) = 13$ $200 \left( 3 \times -22.52^{2} \right) = 13$ $100 \left( 3 \times -22.52^{2} \right) = 14$ $100 \left( 3 \times -22.$	<u>(4)</u>	$12 \cos 3x - 5 \sin 3x = \gamma \cos (3x + \theta)$			
$7 \cos \theta = 12$ $7 \sin \theta = 5$ $7 = \sqrt{12+5^{2}} = 13$ $C \cos \theta = \frac{12}{13}  (= 0.9231)$ $\therefore 9 = \cos \frac{1/3}{13}$ $= 22.62^{\circ} \qquad - \qquad  A $ $12 \cos 3x - 5 \sin 3x = 13$ $13 \cos (3x + 22.62^{\circ}) =  3 $ $3x + 22.62^{\circ} = 36 \cos n,  n = 0, \pm 1, \pm 2, \dots  A $ $\therefore x = 120^{\circ} n - 7.54^{\circ}  (7^{\circ} 32^{\circ})$ $= 120^{\circ} x - 8^{\circ},  n = 0, \pm 1, \pm 2, \dots  A $ $= 120^{\circ} x - 8^{\circ},  n = 0, \pm 1, \pm 2, \dots  A $ $= 120^{\circ} x - 8^{\circ},  n = 0, \pm 1, \pm 2, \dots  A $ $= 120^{\circ} x - 8^{\circ},  n = 0, \pm 1, \pm 2, \dots  A $ $= 120^{\circ} x - 8^{\circ},  n = 0, \pm 1, \pm 2, \dots  A $ $= 14^{\circ} \sin \theta \cos \theta = \frac{1}{2}$ $\sin \theta \cos \theta \cos \theta = \frac{1}{2}$ $\sin \theta \cos \theta = \frac{1}{2}$ $\sin \theta \cos \theta \cos \theta = \frac{1}{2}$ $\sin \theta \cos \theta \cos \theta = \frac{1}{2}$ $\sin \theta \cos$	$\odot$	= 7 (coo 3 x cos 8 - 5 in 3 x	(sino)		
$ \gamma = \sqrt{12 + 5^{2}} = 13 $ $ Cor \theta = \frac{12}{13}                                   $	-				
$Coo \theta = \frac{12}{13}                                   $				1.4	<b>-</b>
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$				"	
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$					
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$				14	Acreht 22.6°
12 $\cos 3x - 5 \sin 3x = 13$ 13 $\cos (3x + 22 \cdot 5z^2) = 13$ 20 $(3x + 22 \cdot 5z^2) = 13$ 3x + 22 \( 5z^2 = 3 \text{ for } xn \), $n = 0, \pm 1, \pm 2,$ 1A $n = 0,$ optional  2 $(20^2 xn - 7.54^2)$ ( $7^2 32^2$ )  1A $n = 0,$ optional  (corr. to the hearest degree)  6  5 $\sin \theta + \cos \theta = \frac{1}{2}$ 5 $\sin \theta + \cos \theta = \frac{1}{2}$ 6  1A  1A  Aet  1M  1A  Aet  1M  1A  1A  1A  1A  1A  1A  1A  1A  1A					•
13 Cot $(3x+22.62)=13$ Coo $(3x+22.62)=1$ $3x+22.62=360 \times n$ , $n=0,\pm 1,\pm 3$ 14 $n=0,$ optional  2 = $120^{\circ} \times n-8^{\circ}$ , $n=0,\pm 1,\pm 2,$ (corn. to the heavest degree)  6  1A $n=0,$ optional  (corn. to the heavest degree)  1A $n=0,$ optional					
$3x+2z\cdot6z=36\circ xn, n=0,\pm 1,\pm 2,$ $1A                                    $	,			JM	for sub.
$ \begin{array}{lll} \vdots & \chi =  20^{\circ} \times n - 7.54^{\circ} & (7^{\circ}32') & 1A \\ & =  20^{\circ} \times n - 8^{\circ},  n = 0, \pm 1, \pm 2, \dots & 1A \\ & (\text{corr. } + \text{o the heavest degree}) & 6 \end{array} $ $ \begin{array}{lll} \text{Sin } \theta + \cos \theta = \frac{h}{2} \\ \text{Sin } \theta \cos \theta = \frac{1}{2} \end{array} $ $ \begin{array}{lll} \text{Sin } \theta \cos \theta = \frac{1}{2} \end{array} $ $ \begin{array}{lll} \text{IA} & \underline{A2t} \\ \text{Sin } \theta \cos \theta = \frac{1}{2} \Rightarrow \sin \theta \cos \theta \end{array} $ $ \begin{array}{lll} \text{IM} & \underline{A2t} \\ \text{Sin } \theta \cos \theta = \frac{1}{2} \Rightarrow \sin \theta \cos \theta \end{array} $ $ \begin{array}{lll} \text{IM} & \underline{A2t} \\ \text{Sin } \theta \cos \theta = \frac{1}{2} \Rightarrow \sin \theta \cos \theta \end{array} $ $ \begin{array}{lll} \text{IM} & \underline{A2t} \\ \text{Sin } \theta \cos \theta = \frac{1}{2} \Rightarrow \sin \theta \cos \theta \end{array} $ $ \begin{array}{lll} \text{IM} & \underline{A2t} \\ \text{Sin } \theta \cos \theta = \frac{1}{2} \Rightarrow \sin \theta \cos \theta \end{array} $ $ \begin{array}{lll} \text{IM} & \underline{A2t} \\ \text{Sin } \theta \cos \theta = \frac{1}{2} \Rightarrow \sin \theta \cos \theta $ $ \begin{array}{lll} \text{IM} & \underline{A2t} \\ \text{Sin } \theta \cos \theta = \frac{1}{2} \Rightarrow \sin \theta \cos \theta \end{array} $ $ \begin{array}{lll} \text{Sin } \theta \cos \theta = \frac{1}{\sqrt{2}} \Rightarrow \sin \theta \cos \theta $ $ \begin{array}{lll} \text{Sin } \theta \cos \theta = \frac{1}{\sqrt{2}} \Rightarrow \cos \theta $ $ \begin{array}{lll} \text{Sin } \theta \cos \theta = \frac{1}{\sqrt{2}} \Rightarrow \cos \theta $ $ \begin{array}{lll} \text{Sin } \theta \cos \theta = \frac{1}{\sqrt{2}} \Rightarrow \cos \theta $ $ \begin{array}{lll} \text{Sin } \theta \cos \theta = \frac{1}{\sqrt{2}} \Rightarrow \cos \theta $ $ \begin{array}{lll} \text{Sin } \theta \cos \theta = \frac{1}{\sqrt{2}} \Rightarrow \cos \theta $ $ \begin{array}{lll} \text{Sin } \theta \cos \theta = \frac{1}{\sqrt{2}} \Rightarrow \cos \theta $ $ \begin{array}{lll} \text{Sin } \theta \cos \theta = \frac{1}{\sqrt{2}} \Rightarrow \cos \theta $ $ \begin{array}{lll} \text{Sin } \theta \cos \theta = \frac{1}{\sqrt{2}} \Rightarrow \cos \theta $ $ \begin{array}{lll} \text{Sin } \theta \cos \theta = \frac{1}{\sqrt{2}} \Rightarrow \cos \theta $ $ \begin{array}{lll} \text{Sin } \theta \cos \theta = \frac{1}{\sqrt{2}} \Rightarrow \cos \theta $ $ \begin{array}{lll} \text{Sin } \theta \cos \theta = \frac{1}{\sqrt{2}} \Rightarrow \cos \theta $ $ \begin{array}{lll} \text{Sin } \theta \cos \theta = \frac{1}{\sqrt{2}} \Rightarrow \cos \theta $ $ \begin{array}{lll} \text{Sin } \theta \cos \theta = \frac{1}{\sqrt{2}} \Rightarrow \cos \theta $ $ \begin{array}{lll} \text{Sin } \theta \cos \theta = \frac{1}{\sqrt{2}} \Rightarrow \cos \theta $ $ \begin{array}{lll} \text{Sin } \theta \cos \theta = \frac{1}{\sqrt{2}} \Rightarrow \cos \theta $ $ \begin{array}{lll} \text{Sin } \theta \cos \theta = \frac{1}{\sqrt{2}} \Rightarrow \cos \theta $ $ \begin{array}{lll} \text{Sin } \theta \cos \theta = \frac{1}{\sqrt{2}} \Rightarrow \cos \theta $ $ \begin{array}{lll} \text{Sin } \theta \cos \theta = \frac{1}{\sqrt{2}} \Rightarrow \cos \theta $ $ \begin{array}{lll} \text{Sin } \theta \cos \theta = \frac{1}{\sqrt{2}} \Rightarrow \cos \theta $ $ \begin{array}{lll} \text{Sin } \theta \cos \theta = \frac{1}{\sqrt{2}} \Rightarrow \cos \theta $ $ \begin{array}{lll} \text{Sin } \theta \cos \theta = \frac{1}{\sqrt{2}} \Rightarrow \cos \theta $ $ \begin{array}{lll} \text{Sin } \theta \cos \theta = \frac{1}{\sqrt{2}} \Rightarrow \cos \theta $ $ \begin{array}{lll} \text{Sin } \theta \cos \theta = \frac{1}{\sqrt{2}} \Rightarrow \cos \theta $ $ \begin{array}{lll} \text{Sin } \theta \cos \theta = \frac{1}{\sqrt{2}} \Rightarrow \cos \theta $ $ \begin{array}{lll} \text{Sin } \theta \cos \theta = \frac{1}{\sqrt{2}} \Rightarrow \cos \theta $ $ \begin{array}{lll} \text{Sin } \theta \cos \theta = \frac{1}{\sqrt{2}} \Rightarrow \cos \theta $ $ \begin{array}{lll} \text{Sin } \theta \cos \theta = \frac{1}{\sqrt{2}} \Rightarrow \cos \theta $ $ \begin{array}{lll} \text{Sin } \theta \cos \theta = \frac{1}{\sqrt{2}} \Rightarrow \cos \theta $ $ \begin{array}{lll} \text{Sin } \theta \cos \theta = \frac{1}{\sqrt{2}} \Rightarrow \cos \theta $ $ \begin{array}{lll} S$		Cro (3 x + 22.62°) = (			
$= 120^{\circ} \times n - 8^{\circ},  n = 0, \pm 1, \pm 2, \dots $ $(com. to the hearest degree)$ $5 \text{ in } \theta + coo \theta = \frac{h}{2}$ $5 \text{ in } \theta + coo \theta = \frac{1}{2}$ $h^{2} = 4\left(3 \sin^{2}\theta + \cos^{2}\theta + 2 \sin\theta \cos\theta\right)$ $= 4\left(1 + 1\right)$ $h = 2\sqrt{2},  since  0^{\circ} < \theta < 90^{\circ}$ $h = 2\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)$		3x+22.62°= 360×n, n=	o, ±1,±3,	/4	n=v, optional
(5) $\sin \theta + \cos \theta = \frac{h}{2}$ $\sin \theta \cos \theta = \frac{1}{2}$ $h^{2} = 4\left(\sin^{2}\theta + \cos^{2}\theta + 2\sin\theta\cos\theta\right)$ $= 4\left(1+1\right)$ $h = 2\sqrt{2}$ $\sin \theta \cos \theta = \frac{1}{2} \Rightarrow \sin^{2}\theta \cos^{2}\theta + 2\sin^{2}\theta\cos\theta$ $= 4\left(1+1\right)$ $h = 2\sqrt{2}$ $\sin \theta \cos \theta = \frac{1}{2} \Rightarrow \sin^{2}\theta \cos^{2}\theta + \sin^{2}\theta \cos^{2}\theta + \sin^{2}\theta \cos\theta$ $\sin^{2}\theta \cos^{2}\theta + \cos^{2}\theta + \cos^{2}\theta \cos\theta$ $\sin^{2}\theta \cos^{2}\theta \cos^{2}\theta + \cos^{2}\theta \cos^{2}\theta \cos\theta$ $\sin^{2}\theta \cos^{2}\theta \cos^{2}\theta \cos\theta$ $\sin^{2}\theta \cos^{2}\theta \cos^{2}\theta \cos\theta$ $\sin^{2}\theta \cos^{2}\theta \cos^{2}\theta \cos\theta$ $\sin^{2}\theta \cos\theta \cos\theta$ $\sin^{2}\theta \cos\theta$ $\sin^{2}\theta \cos\theta \cos\theta$ $\sin^{2}\theta \cos\theta$ $\sin^$				14	-
(5) $\sin \theta + \cos \theta = \frac{h}{2}$ $\sin \theta \cos \theta = \frac{1}{2}$ $h^2 = 4\left(\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta\right)$ $= 4\left(1+1\right)$ $h = 2\sqrt{2}$ , $\sin \theta \cos \theta = \frac{1}{2} \Rightarrow \sin^2\theta \cos^2\theta = \frac{1}{2}$ $\sin \theta \cos \theta = \frac{1}{2} \Rightarrow \sin^2\theta \cos^2\theta = \frac{1}{2}$ $\sin \theta \cos \theta = \frac{1}{2} \Rightarrow \sin^2\theta \cos^2\theta = \frac{1}{2}$ $\sin \theta \cos \theta = \frac{1}{2} \Rightarrow \sin^2\theta \cos^2\theta = \frac{1}{2}$ $\sin \theta \cos \theta = \frac{1}{2} \Rightarrow \sin^2\theta \cos^2\theta = \frac{1}{2}$ $\sin \theta \cos \theta = \frac{1}{2} \Rightarrow \sin^2\theta \cos^2\theta = \frac{1}{2}$ $\sin \theta \cos \theta = \frac{1}{2} \Rightarrow \sin^2\theta \cos^2\theta = \frac{1}{2} \Rightarrow \cos^2\theta \cos^$					n=0, optional
	(5)	,	iree)		
$h^{2} = 4\left(5\sin^{2}\theta + \cos^{2}\theta + 25\sin\theta\cos\theta\right)$ $= 4\left(1+1\right)$ $h = 2\sqrt{2}, \text{ since } 0^{\circ} < \theta < 90^{\circ}$ $h = 2\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)$ $h = 2\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)$	<u> </u>	<b>~</b>	ν		
$= 4(1+1)$ $h = 2\sqrt{2}, \text{ since } 0^{\circ} < \theta < 90^{\circ}$ $h = 2\sqrt{2}, \text{ since } 0^{\circ} < \theta < 90^{\circ}$ $h = 2\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)$	•		1 W 7		
$h = 2\sqrt{2}, \text{ since } 0^{\circ} < \theta < 90^{\circ}$ $h = 2\sqrt{2}, \text{ since } 0^{\circ} < \theta < 90^{\circ}$ $h = 2\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)$					
$\frac{1}{6} = 2(\frac{1}{16} + \frac{1}{6})$			c 0.15	forsal	1 (
	-	2 - 202 , since o 10 270	in the stay		1 - 0
				0	_
i l					

(i) 
$$+(ii)$$
  $2 - h \sin \theta + l = 0$  ...(ii)  $2 \cos^2 \theta - h \cos \theta + l = 0$   $2 \cos^2 \theta - h \cos \theta + l = 0$   $2 - h (\sin \theta + \cos \theta) + 2 = 0$   $2 \cos^2 \theta - h (\sin \theta + \cos \theta) + 2 = 0$   $2 \sin \theta + \cos \theta$  (i)  $-(ii)$   $2 (\sin^2 \theta - \cos^2 \theta) - h (\sin \theta - \cos \theta) = 0$  (Sin  $\theta - \cos \theta$ )  $2 (\sin \theta + \cos \theta) - h = 0$  ... Sin  $\theta - \cos \theta = 0$  or Sin  $\theta + \cos \theta = \frac{h}{2}$  141 The both cases  $\sin \theta = \cos \theta = \frac{1}{\sqrt{2}}$  ...  $h = 2\sqrt{2}$ 

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P.4.

سر		~11	1.7
	Solutions	Warks	Remarks
	$C_2 - C_1$ , $6x - 3y - 3 = 0$	IM	
_	$\gamma = \langle x - 1 \rangle$		
	5 ub. in C, $\chi^2 + (2\chi - 1)^2 + 7(2\chi - 1) + 11 = 0$	IM	
	$5\chi^{2} + 10\chi + 5 = 0$		÷.
÷.	x = -1 $y = -3$	1.4	1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1
(b)	Centre of $C_2 = (-3, -2)$ $\left[ of C_1 = (0, -\frac{7}{2}) \right]$	FA	(b) Act I  Tangent to C, at P.
		ı	-1x+-3y+= 2(y+-3)+11
	Slope of line joining Pandcentre = $-\frac{1}{2}$ Slope of tangent = 2 : common tangent at P is $y+3=2(x+1)$	2A	6 2x-y-1=0
		1	/ (1. Aut 117
	(b) AUI	1 A	(b) ALL III
	When Cound to meet, the common chord		$\frac{dy}{dx}\Big _{x=-1} = 2 \qquad (2)$
	6x-3y-3=0	-	Tangent is 2x-y-1=0
	67 2x-y-1=0	2,4	
194-1	Since they touch each other extenthe above		
	agnation is that of the common tangents	th IA	
<u>_</u> ;)	Let $P = (x, y)$ .		
	$\chi = \frac{s+t}{2}$	14	
	$\gamma = \frac{3}{2}(s-t)$	IA	
	$3 = \chi + \frac{y}{3}, t = \chi - \frac{y}{3} \left( -\text{or } \frac{S+t=2\chi}{5-t=\frac{2}{3}y} \right)$	/A	
	$ST = \int (S-t)^{2} + 9(S+t)^{2} = 2$	1A	
	$\Rightarrow \qquad (\frac{24}{3})^2 + 9(2x)^2 = 4$	IM	
	$\frac{3^2}{4} + 9x^2 = 1$	1 A	
	i.e. 81x+ y2-9=0	6	

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Solutions	Marks	Remarks
8) (a) Putting y=sinx, dy=coordx	1 A	
when $x=0$ , $y=0$ ; $y=1$ .	I A	The state of the s
$\int_{0}^{\frac{\pi}{2}} c_{10}^{3} x \sin^{2}x  dx = \int_{0}^{1} (1 - y^{2}) y^{2} dy$	ĨM+IA	Love of the second
$= \begin{bmatrix} \frac{1}{4} \frac{3}{5} - \frac{1}{4} \frac{5}{5} \end{bmatrix}_0^1$	IA	
$= \frac{2}{15}  (\rightleftharpoons 0.1333)$	1A	· · · · · · · · · · · · · · · · · · ·
(b) (i) $\frac{1}{\chi^2 + 3} - \frac{1}{(x+1)^2} = \frac{(x+1)^2 - (x^2 + 3)}{(x^2 + 3)(x+1)^2}$		
$= \frac{2(x-1)}{(x^{2}+3)(x+1)^{2}}, (x+1)$	2.A	
(ii) putting $x = \sqrt{3} \tan \theta$ , $dx = \sqrt{3} \sec^2 \theta d\theta$	/ A	
when $X = 0$ , $\theta = 0$ ; $\frac{1}{3}$ .	1.4	
$\int_{0}^{3} \frac{dx}{x^{2}+3} = \int_{0}^{\frac{\pi}{3}} \frac{\sqrt{3} \sec^{2}\theta}{3(\tan^{2}\theta+1)} d\theta$	IM+IA	
$= \frac{\int_{3}^{3}}{3} \int_{0}^{\frac{\pi}{3}} d\theta$	1A	
$= \frac{\pi\sqrt{3}}{9}$	IA	
$\int_{0}^{3} \frac{2(x-1)}{(x^{2}+3)(x+1)^{2}} dx = \int_{0}^{3} \frac{1}{(x^{2}+3)^{2}} - \frac{1}{(x+1)^{2}} dx$	2M	Using result in (b)
$= \int_0^3 \frac{dx}{x^2+3} - \int_0^3 \frac{dx}{(x+1)^2}$	I A	
$ = \frac{\sqrt{1}\sqrt{3}}{q} + \left[\frac{1}{x+1}\right]_0^3 $	0+2 A	
$= \frac{77\sqrt{3}}{9} - \frac{3}{4}  (= -0.1454)$	)   1A	
	14	

		ブイエ	P. D
	Solutions	Marks	Remarks
(a)	$y = \int k(x-\frac{1}{4}) dx$		2
;	$= k\left(\frac{x^2}{2} - \frac{x}{4}\right) + C$	I A+IA	
9	Sut. (x, y)= (-1, 4), (0,1),	14	
	$\int 4 = k(\pm + \pm) + c$		$f = f^{i}$
	l = C	IA.	A A A A A A A A A A A A A A A A A A A
	Equation of curve is $y = 2x^2 + 1$	IA.	3
_ (·b)	$y = \frac{1}{\sqrt{x}} - \frac{1}{x} + 1$	/A . 6	
	Solving $y = 2x^2 + 1$ with $y = 2x + 3$	IM	3
	$2x^2-3x-2=0$	1	
	$X = 2 \text{ or } -\frac{1}{2} \qquad \text{(12)}$	IA	Aet h
	Area of region = $\int_{0}^{2} [(2x+3)-(2x^{2}-x+1)] dx$	im+IM-	A Jagara
· · · · · ·	$= \int_0^2 \left[ -2x^2 + 3x + 2 \right] dx$	2A	$\int_{0}^{2} (2x+3) dx = x^{2} + 3x \Big _{0}^{2}$ = 10
	$= \left[ -\frac{2}{3} \chi^3 + \frac{3}{2} \chi^2 + 2 \chi \right]_0^2$		$\int_{0}^{2} (2x^{2} - \chi + 1) dx = \frac{2}{3}x^{3} + \frac{1}{2}x^{3}$
	$=\frac{14}{3} \left( =4.667\right)$	IA -	$4 \text{ reg regled} = 10 - \frac{16}{3} = \frac{14}{3} \text{ (Im)}$
;-	Volume = 1150 (2x+3)2dx - 7150 (2x2-x+1)2dx	. I M	for V= 11 5 42 4x
:	$= \pi \int_0^2 (4x^2 + 12x + 9) dx - \pi \int_0^2 (4x^4 + 5x^2 + 2x + 1) dx$	T ( M	for V <sub>1</sub> -V <sub>2</sub> Alt
	$= \pi \left[ -\frac{4}{5}x^{5} + x^{4} - \frac{x^{3}}{3} + 7x^{2} + 8x \right]^{2} - \frac{1}{16}x^{5}$	3 A	$V_1 = \pi \int_0^2 (2x+3)^2 dx \qquad (1)$
	= II [- \frac{4}{5} + 32 + 16 - \frac{2}{3} + 7 + 4 + 3 + 2] \warder \text{ward.} 3		$= \pi \int_0^2 (4x^2 + 12x + 9) dx$ $= - \left( \frac{4}{3} \right) \left( \frac{2}{3} \right) dx$
	= 31 11 = (= 31.737 = 99.69)	2.A	$ = \pi \left( \frac{4}{3} x^{3} + 6 x^{2} + 9 x \right)^{2} (1) $ $ = 52 \frac{2}{3} \pi $ (1)
If	omit T, award at most 5 marks	7	$V_2 = \pi \int_0^2 (2x^2 x + 1)^2 dx$
	<del>-</del> -		$= \pi \left[ \frac{4}{5} x^{5} - x^{4} + \frac{5}{3} x^{3} - x^{2} \right]$ $= 20 \frac{74}{15} \pi \qquad (6)$
	PECTDICTED +++r-	 	V = V1-V2 = 3/1/5-11
	RESTRICTED 內部	义行	(IM+IA)

Solutions	marks	Remarks
(10)(a) In ABFE,		M for iclea of using cos
$FE^2 = BE^2 + BF^2 - 2BF \cdot BF \cos FBE$	IMHIA	
$= k^{2} + r^{2}k^{2} - 2rk^{2}\cos 60^{\circ}$	1A	
$= \xi^2 \left( 1 - r + r^2 \right) $ 14	k/	
In DAFG, FGLAC IA	H	D
$FG^{2} = (A F \sin FAG)^{2}$ $A = \begin{bmatrix} A & A & A \\ A & A \end{bmatrix}$	E - 6	
$= \left[ (1-r)k \sin 45^{\circ} \right] $ 1A	· F	rk
$=\frac{k^2}{2}(1-2\gamma+\gamma^2)$		B /
K (1-1)2	8	
(b) In \$ EFG,	IM	N. A. C. P. C. C.
$EG = \int FE^2 - FG^2$	1 1-1	
$= \int_{1}^{2} (1-r+\gamma^{2}) - \frac{\lambda^{2}}{2} (1-2r+\gamma^{2})$	I A	
$= \mathring{R} \left(\frac{1+\gamma^2}{2}\right)^{\frac{1}{2}}$	14	
EN = AE Coo 45°	IMHIA	14 for 45°
$= \frac{\cancel{k}}{\cancel{J_2}}$	- <i>1</i> A	
Sin A - EN	IM	
= EG		
$\frac{1}{\int 1+\gamma^2}$	1A 8	
(c) Since 0≤r≤1, 1≤5in6≤1	1+1	A Accept strict
(c) Since $0 \le r \le 1$ , $\frac{1}{\sqrt{2}} \le \sin \theta \le 1$		in equalities
45° ≤ 0 ≤ 90°	1+1	• , ,
Significant to the second of t	4	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
must just de = 8 = 3 de 7 de		= 15
Company of the Property of the Company of the Compa		

	RESINIC				H8.
	Solutions		Vhanks	Remarks	
1) (a) Solving L	$-1$ , and $L_2$ , $3y-6=0$		1M	L <sub>2</sub>	Li
<b>V</b>	y = 2 X = 4	}	1.1	A	
A = (4)	· -			. /49	(1)
	=(X,71), then AG: GI	D= 2 = 1 DG= -3 = 1)	(optime)	100.	-7).
:- {t-6	$= \frac{2x_1+4}{2+1}$ $= \frac{2y_1+2}{2+1}$	abantood		B G(01-12) / H	
- ×1=	$= \frac{3 + 4}{2}$		-1A		
$\mathcal{Y}_{i}$	$= \frac{3 t - 20}{2}$		1A 5	441	\ \ \ \
(b) Slope o	$7AH = \frac{2-70}{4} = 3$			Alt Egnof AH o	
	$AG = \frac{2 - (t - 6)}{4 - t} = 3$		IM	$\gamma_{+10} = \frac{2 \div 10}{4}$	_
A CO	$\therefore t = 2$	· · · · · · · · · · · · · · · · · · ·	IA	or 3x-y-10 Sub. G(t,t-6)	$0 = 0$ ( $I_{M_j}$
	$D = (1, -7)$ $D = 13^{2} + 9^{2} - 7 \int_{0}^{7} 0$		1A	t = 2	<u>(1,4</u>
A.	$D = \int 3^{2} + 9^{2}$ $= 3\sqrt{10} \ (= 9.487)$		1A		*
tand	$BAD = \frac{3-1}{1+3\times 1}$	<u> </u>	<u>1M</u>		
	= ½		1A		
(c) sin B	$AD = \frac{1}{\sqrt{1^2 + 2^2}} = \frac{1}{\sqrt{5}}$ 0.	. 1	I A		÷ .
Area of	$ABD = \frac{1}{2}AB \times AD \sin \beta AD$		1 M		•
	= 1 x14/2 x 3 /10 x 1/5	(6)		<u>-</u>	
	= 42 a 9 ABC = 84 mits		1 A	•	
· an	A M DAB C = 04 llmis		4		

,			
	Solutions	Marks	Remark
(1)(d)	Let $P = (x, y)$ .		
	Area of $\triangle APD = \pm \frac{1}{2} \left[ (2x - 4y) + (-28 - 2) + (y + 7x) \right]$	   +1 A	for + and -
<del>-</del>	- = 42	>*	signs optiona
	:. Locus of Pis 9x-3y-30=±84		
	i.e. 3x-y+18=0	14	
	3x-y-38=0	<u>h</u>	;
<b>-</b> ,		4	
	(d) Let $P=(x,y)$ , $h=height of SADC with AD as base.$	•	·
<del>.</del>	$\frac{1}{2} \times h \times 3\sqrt{10} = 42$		•
	$h = \frac{28}{\sqrt{10}}$	I A	
	$\frac{3 \times -4 - 10}{\pm \sqrt{10}} = \frac{28}{\sqrt{10}}$	/ A	
	Lower of P is 3x-y+18=0	l A	
vi.	3 x-y-38=0	14	
. ·			
		·	
			·
. '			
			-

Solutions	Marks	Remarks
L: 7 = mx+2		4+ 1/2 = - 2m
$C: x^{\frac{1}{2}}y^{\frac{1}{2}} = 1$		m +/
		$\mathcal{X}_{1}\mathcal{X}_{2}=\frac{3}{m^{2}+1}$
Sub. L in C,		
$\chi^2 + (2nx+2)^2 = 1$	IM	$y_1 + y_2 = \frac{1}{m^2 + 1}$ $y_1 y_2 = \frac{1}{m^2 + 1}$ Alt $m^2 + 1$
$(m^2+1)\chi^2 + 4m\chi + 3 = 0$	/ / /	$\int_{0}^{\infty} \frac{1}{y^2} \frac{1}{y^2} = \frac{2 - i h^2}{i h^2}$
(11041) K 1 1 MK 1 3 = 0	/A	Alt
$\chi = \frac{-4  \text{m}  \text{f}  \sqrt{16  \text{m}^2 - 12  (\text{m}^2 + 1)}}{2  (\text{m}^2 + 1)} - \frac{12  (\text{m}^2 + 1)}{12  (\text{m}^2 + 1)}$		$(x_1 - x_2) = (X_1 + X_2) - 4x$
		$=\frac{16m^2}{(m^2+1)^2}-\frac{12}{(m^2+1)}$
$= \frac{-2 m \pm \sqrt{m^2 - 3}}{m^2 + 1}$	IA	$(m+1)^{2}$ $(m+1)^{2}$
<i>M</i> -71		$= \frac{4 (\ln^2 - 3)}{(\ln^2 + 1)^2} $ (1)
y = mx + 2		$(y_1 - y_2) = y_1^2(x_1 - x_2)$
$= 2 \pm m \sqrt{m^2 - 3}$	1 A	
$m^2+1$	'''	$=\frac{4m^2(m^2-3)}{(m^2+1)^2}$
		(m T)
•		
$B = \left(\frac{-2m - \sqrt{m^2 - 3}}{m^2 + 1}, \frac{2 - m\sqrt{m^2 - 3}}{m^2 + 1}\right)$		
m+1 $m+1$		
$AB = \int \left(\frac{2\sqrt{m^2-3}}{m+1}\right)^2 + \left(\frac{2m\sqrt{n^2-3}}{m+1}\right)^2$	-  - IM	Triber 1
$\frac{1}{m+1} = \frac{1}{m+1}$		
$= \sqrt{\frac{4(m^{4}-2m^{2}-3)}{m+1}}$		
$\int m + 1$		4
$=2\sqrt{\frac{m^2-3}{m^2+1}}$	14	
$-\lambda \int \frac{m^2+1}{m^2+1}$	177	<u> </u>
	5	
	~·	
	対域など	

12) (1)(i) L meets C at two district points  1 if $2\sqrt{\frac{m^2-3}{m^2+1}} > 0$ 1.2. $m < -\sqrt{3}$ or $m > \sqrt{3}$ (ii) L is a tangent to C iff $AB = 0$ 1.2. $m = \pm \sqrt{3}$ (iii) L does not meet C iff $AB < 0$ 1.2. $-\sqrt{3} < m < \sqrt{3}$ (c) Since $m = \pm \sqrt{3}$ , sub. in $A_{0}B \ne (a)$ $P = \left(-\frac{2m+\sqrt{m^2-3}}{m^2+1}, \frac{2+m\sqrt{m^2-3}}{m^2+1}\right)$ $= \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ $Q = \left(-\frac{2m-\sqrt{m^2-3}}{m^2+1}, \frac{2-m\sqrt{m^2-3}}{m^2+1}\right)$ $= \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ 1.2. $a_{0}a_{0} \ne p \ Q$ is $a_{0}a_{0} = \frac{1}{2}$ (d) Equ. region is $a_{0}a_{0} = \frac{1}{2}$	ZIT	
1ff $2\sqrt{\frac{m^2-3}{m^2+1}} > 0$ 1if. $m < -\sqrt{3}$ or $m > \sqrt{3}$ (ii) L is a tangent to C iff $AB = 0$ 1if. $m = \pm \sqrt{3}$ (iii) L does not excet C iff $AB < 0$ 1if. $-\sqrt{3} < m < \sqrt{3}$ (c) Since $m = \pm \sqrt{3}$ , sub. in $A_{0} = 3$ of (a) $P = \left(\frac{-2m + \sqrt{m^2-3}}{m^2+1}, \frac{2+m\sqrt{m^2-3}}{m^2+1}\right)$ $= \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ $Q = \left(\frac{-3m - \sqrt{m^2-3}}{m^2+1}, \frac{2-m\sqrt{m^2-3}}{m^2+1}\right)$ $= \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ 12 an. of $PQ = \frac{1}{2}$ $Q = \frac{1}{2}$	nanhs	Remarks
(ii) _L is a stangent to C iff $AB = 0$ 1.8. $m = \pm \sqrt{3}$ (iii) L does not meet C iff $AB < 0$ 1.8. $-\sqrt{3} < m < \sqrt{3}$ (c) Since $m = \pm \sqrt{3}$ , sub. in $A \ne B$ of (a) $P = \left(\frac{-2m + \sqrt{m^2 - 3}}{m^2 + 1}, \frac{2 + m \sqrt{m^2 - 3}}{m^2 + 1}\right)$ $= \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ $= \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ $\therefore 2qn. of PQ is y = \frac{1}{2} (a 2y - 1 = 0)  (d) Equ. meg'ed is y^2 + y^2 + \frac{1}{2}(2y + 1) = 0$	2M	In (i) (ii) (iii), 24 for 1st correct idea about length of AB.
(c) Since $m = \pm 13$ , sub. in $A_{07}B_{07}(a)$ $P = \left(\frac{-2m + \sqrt{m^{2}-3}}{m^{2}+1}, \frac{2+m\sqrt{m^{2}-3}}{m^{2}+1}\right)$ $= \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ $Q = \left(\frac{-2m - \sqrt{m^{2}-3}}{m^{2}+1}, \frac{2-m\sqrt{m^{2}-3}}{m^{2}+1}\right)$ $= \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ $\therefore 2q_{11} \cdot of PQ_{12} \cdot q_{12} = \frac{1}{2} \left(\pi 2q_{-1}=0\right)$ (d) Equ. reglad is $y^{2} + q^{2} + 2f(2q_{11}) = 0$	IA IA	
$P = \left(\frac{-2m + \sqrt{m^{2}-3}}{m^{2}+1}, \frac{2+m\sqrt{m^{2}-3}}{m^{2}+1}\right)$ $= \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ $= \left(\frac{-2m - \sqrt{m^{2}-3}}{m^{2}+1}, \frac{2-m\sqrt{m^{2}-3}}{m^{2}+1}\right)$ $= \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ $\therefore lgn. of PQ = y = \frac{1}{2} (m2y-1=0)$ $(a) Equ. meg'=4 is y^{2}+y^{2$	1A 5	In this case , A, 8-a identical
$Q = \left(\frac{-2m - \sqrt{m^{2}-3}}{m^{2}+1}, \frac{2-m\sqrt{m^{2}-3}}{m^{2}+1}\right), \sqrt{m^{2}-3}$ $= \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ $\therefore 2qn. \text{ of } PQ = \frac{1}{2}  (424-1=0)$ $(4)  Equ. \text{ reg'ad is } \qquad \chi^{2}+4^{2}+\frac{1}{2}(24-1=0)$	FA HIA	
(a) Equ. reg'zu is $y^2 + y^2 + b(2y-1=0)$	M. 1+1A	
	1A 5 2M+	
	2A	