6 PURP 'ATHS	RESIRICIED 内部文件		P.1
	SOLUTIONS	MARKS	RETARKS
1. (a) (i)	If $u \in \mathcal{F}$, $\mathcal{U}^{T} = v^{-1}$	·	
		1	let -
•	$ v^{T} = v^{-1} $ $ v^{T} = v^{-1} $ $ v^{T} = v^{T} $ $ v^{T} = v^{T} $ $ v^{T} = v^{T} $	-	
. 🖡	U = 1 as U > 0.	l	4445
(11	If $U = \begin{pmatrix} a & b \\ -b & a \end{pmatrix}$ with $a^2 + b^2 = 1$,		In s
	If $U = \begin{pmatrix} a & b \\ -b & a \end{pmatrix}$ with $a^2 + b^2 = 1$,		
	$\frac{U^{-1}}{U} = \frac{1}{ U } \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$	1	<u>u</u> _'
	$= \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$ $= U^{T}$		•
	Only if' part	1	- 100 - 100
P	Suppose $U = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathcal{F}$, then $U^T = U^{-1}$		
	$\Rightarrow \begin{pmatrix} a & c \\ b & d \end{pmatrix} = \frac{1}{ U } \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \qquad \cdots$	1	
	$\Rightarrow \begin{pmatrix} a & c \\ b & d \end{pmatrix} = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \cdot as \boxed{U = 1 \text{ by (1)}}$	1	
F	$\begin{array}{cccc} \Rightarrow & a = d & \text{and} & c = -b. \\ U = \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \end{array}$		
P	and $ U = a^2 + b^2 = 1$	<u>l</u> 7	
	(2)		

	SOLUTIONS	And the second s	-86	MARKS	REMARKS
(b) (1) Let	$U = \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \in \mathcal{F}$		I 		
UBU	$\begin{pmatrix} p & 0 \\ 0 & q \end{pmatrix}$		a*• .	-	
· , ->	$B = D^{-1} \begin{pmatrix} p & 0 \\ 0 & q \end{pmatrix} \begin{pmatrix} D^T \end{pmatrix}^{-1} \dots$	**************************************	••	1	:
. (у	$\begin{pmatrix} x \\ z \end{pmatrix} = \begin{pmatrix} a & -b \\ b & a \end{pmatrix} \begin{pmatrix} b & 0 \\ 0 & q \end{pmatrix} \begin{pmatrix} a & b \\ -b & a \end{pmatrix}$		·	1	• • • • • • • • • • • • • • • • • • •
	$-\left(\begin{array}{cc} a^{2}p + b^{2}q & ab(p - q) \\ ab(p - q) & b^{2}p + a^{2}q \end{array}\right)$				
1.e.	$x = y$ $B = B^{T}$	•		1	
(ii) If E	$B = B^{T}$, let $B = \begin{pmatrix} w & x \\ x & z \end{pmatrix}$.				
For	$U = \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \in \hat{f}, a^2 + b^2 = 1$			_	• • • • • • • • • • • • • • • • • • • •
1-ע ביט	$= \begin{pmatrix} aw + bx & ax + bz \\ -bw + ax & -bx + az \end{pmatrix} \begin{pmatrix} a - bx + az \\ -bx + ax \end{pmatrix}$	b a			
	$= \begin{pmatrix} a^2w + 2abx + b^2z & (a^2-b^2)x + ab(z-w) & b^2w - b$	$ \begin{array}{ccc} 2)x + ab(z-y) \\ 2abx + a^2z \end{array} $		1	
(If x		1 0 satisfies	the		·
	condition. # 0 equating $(a^2 - b^2)x + ab$	(Z = W)	-	1	
, a 2 _		•	-	1	•
(4 +)	$(c^{2})a^{4} - (4 + c^{2})a^{2} + 1 = 0$ $(4 + c^{2}) \pm \sqrt{c^{2}(4 + c^{2})}$ $(4 + c^{2})$	•	-		• .
	$a = \left(\frac{(4+c^2)}{2(4+c^2)} \pm \sqrt{c^2(4+c^2)}\right)$) 1 ¹ / ₁ . 0 < a < 1	.		

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		- SOLUTIONS		: 86	MARKS	REMARKS
2.	(a) (i) For an	y given a, b, c, (E) has a unique	solution iff I		
]	1 k k	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$			i.	
ļ.	k(k - 1	. l			1	
	;	# 0 and k # 1			1	
]],		- 0, the system bed + y - z - a -y = b y = c	A = contay 57	lastities.	1	
[C.,	∴ <u>f</u>	or any values of a. ystem is consistent	b. c with b =		1	
]. :	(iii)For a	- b - c - 0 and i	t = 1, the system	reduces to		
	x	+ y - z = 0		1	1.	
		1) and (1, -1, 0)				
i	· soluti	ons as one is not a	scalar multiple	of the other.	$\frac{1+1}{8}$	
1 .	(b) Assuming, f	or contradiction, the	hat (E) is consid	steat, then		··.
I .	x + y	ż = a .				
ı		y + kz = b				
		y - kz = c				
l	•	x, y, z ∈ R		2	2	
ر ارد ع _{ر خ} ا	-t:-ax ₀ + by ₀ +	$cz_0 = (x+y-z)x_0 + $)x + (x ₀ -y ₀ +z ₀)y			
•	•	= 0 + 0 + 0	74 - (40)0-20/)	. ,0 0 0 0		
			· y _{0•-} z ₀) <u>satisfi</u> e	s the 2nd system	2	
	This contra	idicts the assumption				
					6	
			پروان د د د د سخی			

SOLUTIONS 9	MARKS	REMARKS		SOLUTIONS & MARK	S REMARK
SOLUTIONS			3. (a)	(1) 'If' part	
Alternatively				If $x_k (1 \le k \le n)$ is a linear combination of the other	
				vectors, let	
(b) As (x_0, y_0, z_0) satisfies the 2nd system.				$\underline{x}_{k} = \lambda_{1}\underline{x}_{1} + \lambda_{2}\underline{x}_{2} + \dots + \lambda_{k-1}\underline{x}_{k-1} + \lambda_{k+1}\underline{x}_{k+1} + \dots + \lambda_{n}\underline{x}_{n}$	
$\left(\begin{array}{cccccccccccccccccccccccccccccccccccc$			j	Then $\lambda_1 \underline{x}_1 + \lambda_2 \underline{x}_2 + \cdots - \underline{x}_k + \cdots + \lambda_n \underline{x}_n = \underline{0}$	
$\begin{cases} x_0 - y_0 + z_0 - 0 \end{cases}$				Since the coefficient of $x_i \neq 0$, the vectors	
$-x_0 + ky_0 - kz_0 = 0$	1		L	$\underline{x}_1, \underline{x}_2, \ldots, \underline{x}_n$ are linearly dependent.	
For any real x, y, z,			.	'Only if' part	•
$x(x_0 - ky_0 + k^2z_0) + y(x_0 - y_0 + z_0) + z(-x_0 + ky_0 - kz_0) = 0$	2			If \underline{x}_1 , \underline{x}_2 ,, \underline{x}_n are linearly dependent,	
$(x + y - z)x_0 + (-kx - y + kz)y_0 + (k^2x + y - kz)z_0 = 0$.	1			$\exists \lambda_1, \lambda_2, \dots, \lambda_n$, not all zero, such that	
Since $(a, b, c) \cdot (x_0, y_0, z_0) \neq 0$, the system				$\lambda_{1}\underline{x}_{1} + \lambda_{2}\underline{x}_{2} + \cdots + \lambda_{\underline{n}}\underline{x}_{\underline{n}} = \underline{0}$	
x + y - z = a			1	Let $\lambda_k \neq 0$ for some k .	
-kx - y + kz - b			•	Then	
$k^2x + y - kz = c$				$\lambda_{k^{\underline{x}_{k}}} = -[\lambda_{1}\underline{x}_{1} + \lambda_{2}\underline{x}_{2} + \dots + \lambda_{k-1}\underline{x}_{k-1} + \lambda_{k+1}\underline{x}_{k+1} + \dots + \lambda_{n}\underline{x}_{n}]$	
cannot hold simultaneously	2			/ k=k (11-1 /2-2 / k-1-k-1 / k+1-k+1 / k+1-k+1	
i.e. (E) is not solvable.	<u>-6</u>	1 · · · · · · · · · · · · · · · · · ·		$\underline{\mathbf{x}}_{k} = -\frac{1}{\lambda_{k}} \left[\lambda_{1} \underline{\mathbf{x}}_{1} + \lambda_{2} \underline{\mathbf{x}}_{2} + \dots + \lambda_{k-1} \underline{\mathbf{x}}_{k-1} + \lambda_{k+1} \underline{\mathbf{x}}_{k+1} + \dots + \lambda_{n} \underline{\mathbf{x}}_{n} \right] $	
			1	**	
	-	} {		(ii) Let \underline{x}_{i_1} , \underline{x}_{i_2} ,, \underline{x}_{i_k} be k of the vectors which are linearly dependent, where $1 \le i_1 \le i_2 \le \le i_k \le n$.	
· · · · · · · · · · · · · · · · · · ·			11		
				$\exists u_1, u_2, \dots, u_k$, not all zero, such that	
				$u_{1}\underline{x}_{1_{1}} + u_{2}\underline{x}_{1_{2}} + \dots + u_{k}\underline{x}_{1_{k}} = 0$	
				Let $\lambda_i = \begin{cases} u_j & \text{if } l = i_j \\ 0 & \text{otherwise.} \end{cases}$	
				• •	
·		· .		Then $\exists \lambda_1, \lambda_2, \dots, \lambda_n$, not all zero, such that	
				$\lambda_{1}\underline{x}_{1} + \lambda_{2}\underline{x}_{2} + \cdots + \lambda_{n}\underline{x}_{n} = \underline{0}.$	
				the n given vectors are linearly dependent.	
		1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1			
	E-74-7				- Arman

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	solutions	MARKS	REMARKS
	[a ₁ b ₁ c ₁]		
. (b) (1)	If $a_2 b_2 c_2 = 0$, then the system	•	
•	(a ₁ x + b ₁ y + c ₁ x = 0 = trind show (c)		
	$\begin{cases} a_1x + b_1y + c_1z = 0 \\ a_1x + b_2y + c_3z = 0 \end{cases}$	· !	
•			
	- \ \ \(a_3 \times + \bar b_3 \times + \cap z = 0 \\ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \	2	
	has a non-trivial solution $(\lambda_1, \lambda_2, \lambda_3)$.	•	
	i.e. $\exists \lambda_1, \lambda_2, \lambda_3$, not all zero, such that	1	
	$\lambda_1(a_1, a_2, a_3) + \lambda_2(b_1, b_2, b_3) + \lambda_3(c_1, c_2, c_3) = (0,0,0)$		
•	$\underline{\mathbf{x}}_1$, $\underline{\mathbf{x}}_2$, $\underline{\mathbf{x}}_3$ are linearly dependent.		43
<u> </u>	a ₁ b ₁ c ₁		CARREL CONTRACTOR
(ii)	If a ₂ b ₂ c ₂ # 0, then the system	100	
	$\frac{x_1}{a_1}, \frac{x_2}{a_2}, \frac{x_3}{a_3} \text{ are linearly dependent.}$ If $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \neq 0$, then the system $\begin{vmatrix} a_1x + b_1y + c_1z = d_1 \\ a_2x + b_2y + c_2z = d_2 \\ a_3x + b_3y + c_3z = d_3 \end{vmatrix}$	Clay	
1	$\begin{pmatrix} a_1x + b_1y + c_1z = d_1 \\ 2 & 2 \end{pmatrix}$		
*	$\begin{cases} a_2x + b_2y + c_2z = d_2 \end{cases}$	م مارسا	of rectors
	$\left(a_3x + b_3y + c_3z - d_3\right).$	3.7	·/·
	$\begin{cases} a_2x + b_2y + c_2z = d_2 \\ a_3x + b_3y + c_3z = d_3 \end{cases}$ has a unique solution $(\lambda_1, \lambda_2, \lambda_3)$	2	
	1.e. $\underline{x}_4 = \lambda_1 \underline{x}_1 + \lambda_2 \underline{x}_2 + \lambda_3 \underline{x}_3$		
	by (a)(1), \underline{x}_1 , \underline{x}_2 , \underline{x}_3 , \underline{x}_4 are linearly dependent.	1	
)	For any four vectors $\underline{x}_1 = (a_1, a_2, a_3), \underline{x}_2 = (b_1, b_2, b_3),$		
	$\underline{x}_3 = (c_1, c_2, c_3), \underline{x}_4 = (d_1, d_2, d_3)$ in \mathbb{R}^3 .		
	a ₁ b ₁ c ₁	ľ	
	either a ₂ b ₂ c ₂ = 0		
-	a ₃ b ₃ -c ₃	11-2	
	a ₁ b ₁ c ₁		
1	or a ₂ b ₂ c ₂ ≠ 0	i	
	a ₃ b ₃ c ₃		
4	In each case, the four vectors are linearly dependent	1	
/	by (b)(i) and (ii)	713	The second second

). i	solutions86	MARKS	REMARKS
4. (a)	(1)	Suppose f is injective.		10.181.5
	•	Since fof = f , for any x ∈ X		
		fof(x) = f(x)	ī	
		$\Rightarrow f(f(x)) = f(x) \qquad$	1	1. 45
		⇒ f(x) = x		J
		i.e. $f = \frac{1}{X}$		
	ì	Suppose f is surjective.		
	•	For any $y \in X$, let $-y = f(x)$ for some $x \in X$	1	1. Surjete.
		f(y) = f(f(x))		
	•	= fof(x) = f(x)	l	
- " <u> </u>		y	1	- 2
•		i.e. $\mathbf{f} = \mathbf{i}_{\mathbf{y}}$		
	(11)	If $X = \{a, b\}$, consider $f: X \to X$ defined by		
* 2	٠,٠	f(a) = f(b) = a .		V 1. F .
	,	my significant		
		Clearly fof = f but		
		f is neither injective		•
•		nor surjective.	2	
utut	- `	If X contains more than two elements; let a, b be two		
Ì		distinct elements of X . Consider f defined by		-
		$f(a) = f(b) = a$ and $f(x) = x$ for other $x \in X$.		
1		f is neither injective nor surjective but it satisfies		
<u>.</u>		fof = f and f is non-constant.	2	
		g C	9	
· (b)	ĵ (Ε)	≠ Ø .		
	For	any $A \subset E$, $hoh(A) = h(h(A))$	1	
		- h(A ∩ B) ∩ B		
] !	-	= A \cap B = h(A)	ı	
	·.	hoh = h .		
•				
	T.E.	h is injective or surjective, by (a)(1),		
-	h =	1 P(E)	1	1.0
	••	E = h(E) - E ∩ B	1	1
السعام		er = Bergin Property Property	77.1	1/2 - Carried

Since $c_{\mathbf{r}}^{2n+1} = \frac{2n+1}{2n+1-r}$ $c_{\mathbf{r}}^{2n+1} = \frac{2n+1}{2r-2} c_{\mathbf{r}}^{2n+1}$ when A and B toss the $c_{\mathbf{r}}^{2n+1} = \frac{2n+1}{2n+1-r} = \frac{2n+1}{2n+1-r}$ $c_{\mathbf{r}}^{2n+1} = \frac{1}{2} \cdot 2^{2n+1} = 2^{2n}$ $c_{\mathbf{r}}^{2n+1} = 2^{2n}$ $c_{\mathbf{r}}^{2n+$			ن تا داد د	Pappara	messer, and the	- ρ. 8	PIRF ATHS I	
5. (a) (1) Putting $x = 1$ in $(1 + x)^{2n+1} = \frac{2n+1}{x^{2n}} c^{2n+1} x^{\frac{1}{x^{2n}}}$ obtain r heads $(r = 0, \frac{n}{r} \frac{1}{2^{r}} \cdot \frac{1}{2^{n-r}} - \frac{n}{2^{r}} \cdot \frac{1}{2^{n-r}} - \frac{n}{2^{n}} \cdot \frac{1}{2^{n}} \cdot \frac{1}{2^{n}$			COLUTIONS		9.1			SOLUTIONS
the coefficient of $x^{k} = \frac{1}{x^{k}}$ in $(1 + x)^{2n+1} = \frac{\sum_{r=0}^{n+1} c^{2n+1}}{\sum_{r=0}^{r} c^{2n+1}} x^{r}$. 1 obtain r heads $(r = 0, c^{n} \frac{1}{2} - \frac{1}{2n-r} - \frac{C^{n}}{2n}$. 1 $c^{n} \frac{1}{2} - \frac{1}{2n-r} - \frac{C^{n}}{2n}$. 1 $c^{n} \frac{1}{2} - \frac{1}{2n-r} - \frac{C^{n}}{2n}$. 1 When A and B coss the A will obtain $k \in L \leq k$. 1 a will obtain $k \in L \leq k$. 1 a is a is a in a is a is a in a is a in a is a in a is a in a in a is a in					- 86 H	ARKS REG	5. (b) (1) W	nen B tosses n coins
). (a)	(1) Putting x = 1	in $(1 + x)^{2n+1} =$	$\sum_{i=1}^{2n+1} c^{2n+1} x^{r^{\frac{1}{2}}}$	- 1			
Since $ C^{2n+1} = C^{2n+1} $ when A and B toss the $\sum_{r=1}^{n+1} C^{2n+1}_{n+r} = \frac{1}{2} \cdot 2^{2n+1} = \frac{1}{2^2} \cdot 2^{2n+1} = \frac{1}{2} \cdot 2^{2n+1} = \frac{1}{2} \cdot 2^{2n+1} = \frac{1}{2^2} \cdot$				1-0				
Since $C_{\mathbf{r}}^{-} = C_{2n+1-\mathbf{r}}^{-1}$. A will obtain $k \in 1 \le k$ $\sum_{r=1}^{n+1} C_{n+r}^{2n+1} = \frac{1}{2} \cdot 2^{2n+1} = 2^{2n}$ (ii) $(1+x)^m (1+\frac{1}{x})^n = \frac{1}{x^n} \sum_{r=0}^{m+n} C_r^{m+n} x^r$ For $m-n \le k \le m$, the coefficient of x^k is C_{n+k}^{m+n} . On the other hand, $(1+x)^m (1+\frac{1}{x})^n = (\sum_{s=0}^{m} C_s^m x^s) (\sum_{r=0}^{m} C_r^{n+\frac{1}{x^r}})$ The coefficient of $x^k = \sum_{r=0}^{m-k} C_{n+r}^m C_r$.			$2^{2n+1} = \frac{2}{5}$	c^{2n+1}	-			
$\sum_{r=1}^{n+1} \frac{c^{2n+1}}{c^{n+r}} = \frac{1}{2} \cdot 2^{2n+1} = 2^{2n}$ $(1i) (1+x)^m (1+\frac{1}{x})^n = \frac{1}{x^n} (1+x)^{m+n}$ $= \frac{1}{x^n} \sum_{r=0}^{m+n} c^{m+n}_{r} x^r$ $(1i) \text{The probability that A}$ For $m-n \le k \le m$, the coefficient of x^k is C^{m+n}_{n+k} . $(1+x)^m (1+\frac{1}{x})^n = (\sum_{s=0}^m c^m_{s} x^s) (\sum_{r=0}^n c^n_{r} \frac{1}{x^r})$ $(1+x)^m (1+\frac{1}{x})^n = (\sum_{s=0}^m c^m_{s} x^s) (\sum_{r=0}^n c^n_{r} \frac{1}{x^r})$ 1 The coefficient of $x^k = \sum_{r=0}^{m-k} c^m_{r} c^n_{r}$.		Since $c_r^{2n+1} = c_2^2$	n+1	r=0 r		1	1 -	•
(ii) $(1+x)^m(1+\frac{1}{x})^n = \frac{1}{x^n} (1+x)^{m+n}$ $= \frac{1}{x^n} \sum_{r=0}^{m+n} c_r^{m+n} x^r$ $= \frac{1}{x^n} \sum_{r=0}^{m+n} c_r^{m+n} x^r$ On the other hand, $(1+x)^m(1+\frac{1}{x})^n = (\sum_{s=0}^m c_s^m x^s)(\sum_{r=0}^m c_r^n \frac{1}{x^r})$ $= \frac{1}{2^{2n+1}} \cdot 2^{2n}$ The coefficient of $x^k = \sum_{r=0}^{m-k} c_r^m \frac{n}{x^r}$;	· _					A	•
(ii) $(1+x)^{m}(1+\frac{1}{x})^{n} = \frac{1}{x^{n}}(1+x)^{m+n}$ $= \frac{1}{x^{n}}\sum_{r=0}^{m+n}c_{r}^{m+n}x^{r}$ 1 (ii) The probability that A For $m-n \leq k \leq m$, the coefficient of x^{k} is c_{n+k}^{m+n} . On the other hand, $(1+x)^{m}(1+\frac{1}{x})^{n} = (\sum_{s=0}^{m}c_{s}^{m}x^{s})(\sum_{r=0}^{n}c_{r}^{n}\frac{1}{x^{r}})$ 1 $= \frac{1}{2^{2n+1}} \cdot 2^{2n}$ The coefficient of $x^{k} = \sum_{r=0}^{m-k}c_{r}^{n}c_{r}^{n}$.	•			*************	. !		В	is $\sum_{n+1-k}^{n+1-k} \cdot \frac{c}{n+1} \cdot \frac{c}{n}$
$=\frac{1}{x^n}\sum_{r=0}^{m+n}c_r^{m+n}x^r$ 1 For $m-n \le k \le m$, the coefficient of x^k is c_n^{m+n} . On the other hand, $(1+x)^m(1+\frac{1}{x})^n = (\sum_{s=0}^m c_s^m x^s)(\sum_{r=0}^n c_r^n \frac{1}{x^r})$ $=\frac{1}{2^{2n+1}}\cdot 2^{2n}$ The coefficient of $x^k = \sum_{r=0}^{m-k}c_r^m c_r^n$.	,		C	`.				
For $m-n \le k \le m$, the coefficient of x^k is C_{n+k}^{m+n} . In than B is $\sum_{k=1}^{n+1} \frac{1}{2^{2n+1}} C_n^2$ on the other hand, $ (1+x)^m (1+\frac{1}{x})^n = (\sum_{s=0}^m C_s^m x^s) (\sum_{r=0}^n C_r^n \frac{1}{x^r}) $ $ = \frac{1}{2^{2n+1}} \cdot 2^{2n} $ $ = \frac{1}{2} \dots$ The coefficient of $x^k = \sum_{r=0}^{m-k} C_{k+r}^m C_r^n$.	1	$(11) (1 + x)^{-}(1 + \frac{x}{1})^{-}$	$\frac{1}{x^n} (1+x)^{m+n}$			·	YE NHI-	火氣:出生
For $m-n \le k \le m$, the coefficient of x^k is C_{n+k}^{m+n} . In than B is $\sum_{k=1}^{n+1} \frac{1}{2^{2n+1}} C_n^2$ on the other hand, $ (1+x)^m (1+\frac{1}{x})^n = (\sum_{s=0}^m C_s^m x^s) (\sum_{r=0}^n C_r^n \frac{1}{x^r}) $ $ = \frac{1}{2^{2n+1}} \cdot 2^{2n} $ $ = \frac{1}{2} \dots$ The coefficient of $x^k = \sum_{r=0}^{m-k} C_{k+r}^m C_r^n$.	-		1 N+n m+n r					••
On the other hand, $ (1+x)^m (1+\frac{1}{x})^n = (\sum_{s=0}^m c_s^m x^s) (\sum_{r=0}^n c_r^n \frac{1}{x^r}) $ $ = \frac{1}{2^{2n+1}} \cdot 2^{2n} $ $ = \frac{1}{2^{2n+1}} \cdot 2^{2n} $ The coefficient of $x^k = \sum_{r=0}^{m-k} c_r^m c_r^n$.				2 ·	1		(ii) Th	me probability that A
On the other hand, $ (1+x)^m (1+\frac{1}{x})^n = (\sum_{s=0}^m c_s^m x^s) (\sum_{r=0}^n c_r^n \frac{1}{x^r}) $ $ = \frac{1}{2^{2n+1}} \cdot 2^{2n} $ $ = \frac{1}{2^{2n+1}} \cdot 2^{2n} $ The coefficient of $x^k = \sum_{r=0}^{m-k} c_r^m c_r^n$.		For $m - n \le k \le m$,	the coefficient of	of xk is cm+n				$\frac{n+1}{2}$ $\frac{1}{2}$ c^2
The coefficient of $x^k = \sum_{r=0}^{m-k} \sum_{k+r}^{m} c_k^n$.	•	On the other hand,		n+k	!			k-1 2 ²ⁿ⁺¹ n
The coefficient of $x^k = \sum_{r=0}^{m-k} \sum_{k+r=r}^{m} c_k^n$.	•	(I) + => == . I	. п. <u>Та</u> — е П		- [•		l2n
The coefficient of $x^k = \sum_{r=0}^{m-k} \sum_{k+r=r}^{m} c_k^n$.	•	$(1+x)(1+\frac{x}{2})$	$C_{s=0}^{m} = \left(\sum_{s=0}^{\infty} C_{s}^{m} \times S\right) \left(\sum_{s=0}^{\infty} C_{s}^$	$C_r^n \frac{1}{x^r}$	1	•		$=\frac{1}{2^{2n+1}}$
					!	:		$-\frac{1}{2}$
			. n-k -	•				
		the coefficient of	$x^k = \sum_{r=0}^{\infty} C_{k+r} C_r$	•	-	1		
Hence the result.						j	, r	
		Hence the result.			$-\frac{1}{7}$			
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SOLUTIONS 86	MARKS	REMARK
5. (b) (i) When B tosses n coins, the probability that he will		1 47 % 1 4 4 5 1 4
obtain r heads (r = 0, 1,, n) is	-	
$c_r^n \frac{1}{2^r} \cdot \frac{1}{2^{n-r}} = \frac{c_r}{2^n}$.	1	
When A and B toss their coins, the probability that	1	
A will obtain $k (l \le k \le n + l)$ more heads than		
n+1-k' Cn+1		
B is $\sum_{r=0}^{n+1-k} \frac{c_{k+r}^{n+1}}{2^{n+1}} \cdot \frac{c_r^n}{2^n} = \frac{1}{2^{2n+1}} \cdot \sum_{r=0}^{n+1-k} c_{k+r}^{n+1} c_r^n \dots$	2	:
		:
$Y \leq N + 1 - k$ $\frac{1}{2^{2n+1}} c_{n+k}^{2n+1}$	1	1
2 ^{2a+1} a+k	!	
(ii) The probability that A will obtain more heads		
(AL) the productate, that is made obtained and and and and and and and and and an		<u> </u>
than B is $\sum_{k=1}^{n+1} \frac{1}{2^{2n+1}} c_{n+k}^{2n+1} \dots hy.(Ai)$	2	
than b is $\frac{L}{k-1}$ $\frac{1}{2^{2n+1}}$ $\frac{L}{n+k}$	-	
•		
$=\frac{1}{2^{2n+1}}\cdot 2^{2n}$	1	
•		1
$=\frac{1}{2}$	1	1.0
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Harrison	SOLUTIONS			86	HARKS .	REMA
6(4)	Tet f(x) = 2 ^{p-1} (1 + x ^p) - (1 +	x) ^p - (1 - x) ¹	P	Ţ		
	$f'(x) = p[(2x)^{p-1} - (1+x)^{p-1} +$					
**	For 0 ≤ x ≤ 1, p ≥ 2,			**		
	$\left(\frac{1-x}{1+x}\right)^{p-1} \leqslant \frac{1-x}{1+x}$ and $\left(\frac{2}{1+x}\right)^{p-1}$	$(x)^{p-1} \le \frac{2}{1}$	<u>. </u>		21.5	
	$\left(\frac{1-x}{1+x}\right)^{p-1} + \left(\frac{2x}{1+x}\right)^{p-1} \le \frac{1}{1}$	-	x 	. ••		
	1+x' 1+x' 1	+x 1+x			1	-
•	1.e. $(1-x)^{p-1} + (2x)^{p-1} \le (1+x)^{p-1}$	x) ^{p-1}				
-1	∴ f'(x) ≤ 0 and f is decre	asing in [0, 1	i.	. 2	I	
	f(x) > f(l)					
1.3	- 0		•			
/	i.e. $(1 + x)^p + (1 - x)^p \in 2^{p-1}$	1 + x ^p)			1	
	h'(6) = prsin9[(1 + r ² - 2rcos9)	2-120	<u>.</u>			·
	· · · · · · · · · · · · · · · · · · ·				1	
	For r > 0, (9) \ 0	_	-			
	and h'(0) > 0	-				
•	$h(\theta)$ is decreasing in [0,		asing in $(\frac{\pi}{2})$,π].		
	$h(\theta) \leq h(0) + h(\pi)$ if $\theta \in [0,$	-			I .	
	Noting that $h(\theta + n\eta) = h(\theta)$			į		
	$h(\theta) \leq h(0)$ for an	y 9 € R.	·		1 4	
(c)	The inequality is trivial if eit	her z, or z	z, equals ze	ero.		
	Without loss of generality, supp	\sim -				
	We shall prove $\left 1 + \frac{z_1}{z_1}\right ^p + \left 1\right ^p$	-	-		1	
	Let $\frac{z_1}{z_1} = r \operatorname{cis} \theta$, $0 < r \le 1$.	z,	[21]		1	
	$\left 1 + \frac{z_1}{z_1}\right ^2 = \left 1 + rcis\theta\right ^2$			-		
	$= (1 + r\cos\theta)^2 + (r\sin\theta)^2$	θ)²			ļ	
	= 1 + r ² + 2rcos0	·"			ı	
		r ² +2rcosθ) ^τ +	(1+r²-2rcos	2)2		
ē	- z ₁ z ₁	+ r) ^p + (1 - r	:) ^p by (b)		1	
		$\frac{1}{2}(1+r^p)$	by (a)			
		$\frac{1}{1}\left(1+\left \frac{z_{\nu}}{z_{1}}\right ^{p}\right)$	i.			
A L					<u> </u>	

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(a) Putting $z = x+iy$, $q = q_0+iq_1$, $r = r_0+ir_1$, q_0 , q_1 , r_0 , $r_1 \in \mathbb{R}$. (*) becomes $x^2+y^2+(q_0+iq_1)(x+iy)+(r_0+ir_1)(x-iy)+s = 0 + Oc$. i.e. $\begin{cases} x^2+y^2+(q_0+r_0)x+(r_1-q_1)y+s=0 & \dots & (1) \\ (q_1+r_1)x+(q_0-r_0)y=0 & \dots & (1i) \\ (q_1+r_1)x+(q_0-r_0)y=0 & \dots & (1i) \\ \end{cases}$ Further the origin lies inside (i) $\begin{cases} x^2+y^2+(q_0+r_0)x+(r_1-q_1)y+s=0 & \dots & (1i) \\ (q_1+r_1)x+(q_0-r_0)y=0 & \dots & (1i) \\ \end{cases}$ If $q_1+r_1=q_0=r_0=0$, (if and (ii) and therefore (*)) has infinitely many solutions. Otherwise (ii) is a straight line through the origin intersecting (i) at two points (*) has 2 solutions. $\begin{cases} x^2+y^2+(q_0+r_0)x+(r_1-q_1)y+(r_1-2+r_1)(r_1-2+r_1)+(r_1-2+r_2)(r_2+2-r_2) \\ \vdots & \vdots & \vdots \\ \end{cases}$ (b) $\begin{cases} tu_1\bar{u}_2+(1-t)v_1\bar{v}_2+(1-t)v_1\bar{v}_2+(1-t)v_1(r_1-2-r_1)+(r_1-2+r_2)+(r_1-2+r_2) \\ \vdots & \vdots & \vdots \\ \end{cases}$ Expanding and using $ u_1 ^2+ u_2 ^2- v_1 ^2+ v_2 ^2-1$, we obtain $t(v_1\bar{v}_2-u_1\bar{u}_2)+ z ^2+pz+q\bar{z}+t-1=0$ where $\begin{cases} x^2+y^2+(1-t)v_1\bar{v}_2+(1-t)(r_1-2-r_1)+(r_1-2+r_2) \\ \vdots & \vdots \\ 0 \text{ as } 0 < t < 1. \end{cases}$ By (a), this equation has at least two solutions. $\begin{cases} x^2+y^2+(q_0+r_0)x+(r_1-q_1)y+(r_$		SOLUTIONS	86 MARKS	REMARKS
(*) becomes $x^2+y^2+(q_0+iq_1)(x+iy)+(r_0+ir_1)(x-iy)+s=0+0c$. 1.e. $\begin{cases} x^2+y^2+(q_0+r_0)x+(r_1-q_1)y+s=0&\dots(1)\\ (q_1+r_1)x+(q_0-r_0)y=0&\dots(1) \end{cases}$ 1. If $q_1+r_1=q_0-r_0=0$, (if) and (if) $\begin{cases} x^2+y^2+(q_0+r_0)x+(r_1-q_1)y+s=0&\dots(1)\\ (x^2+y^2+q_0+r_0)y=0&\dots(1) \end{cases}$ 1. Further the origin lies inside (i) $\begin{cases} x^2+y^2+(q_0+r_0)x+(q_0+r_0)y+s=0\\ (x^2+q_0+r_0)y=0&\dots(1) \end{cases}$ 1. If $q_1+r_1=q_0-r_0=0$, (if) and (if) $\begin{cases} x^2+y^2+(q_0+r_0)x+(q_0+r_0)x+(q_0+r_0)\\ (x^2+q_0+r_0)y=0&\dots(1) \end{cases}$ 1. If $q_1+r_1=q_0-r_0=0$, (if) and (if) $\begin{cases} x^2+y^2+(q_0+r_0)x+(q_$	(a) Putting	$z = x+iy$, $q = q_0+iq$, $r = r_0+ir$, q_0 , q_0 , r_0 , r_0	I . ∈ ℝ,	
1.e. $\begin{cases} x^2 + y^2 + (q_0 + r_0)x + (r_1 - q_1)y + s = 0 & \dots(1) \\ (q_1 + r_1)x + (q_0 - r_0)y = 0 & \dots(11) & 1 \\ \end{cases}$ As 3 < 0. (1) is a circle with positrine radius. Further the origin lies inside (1) $ \begin{cases} 1 & \text{for } r_0 \\ \text{for } r_0 \\ \text{for } r_0 \end{cases} = 0, \text{ (i) and (ii) (and therefore (*))} $ has infinitely many solutions. Otherwise (ii) is a straight line through the origin intersecting (1) at two points. (*) has 2 solutions. (b) $ [tu_1 \bar{u}_2 + (1 - t)v_1 \bar{v}_2][(u_1 + v_1)(\bar{u}_1 + \bar{v}_1) + (u_2 + \bar{v}_2)(\bar{u}_2 + \bar{v}_2)] $ $ = (u_1 + v_1)(\bar{u}_2 + \bar{v}_2) $ $ \begin{cases} 1 \\ \text{Expanding and using } u_1 ^2 + u_2 ^2 - v_1 ^2 + v_2 ^2 = 1, \text{ we obtain } 1 \end{cases} $ $ t(v_1 \bar{v}_2 - u_1 \bar{u}_2) z ^2 + Pz + Q\bar{z} + (1 - t)(u_1 \bar{u}_2 - v_1 \bar{v}_2) = 0, P, Q \in \mathbb{R}. $ Since $ u_1 \bar{u}_2 \neq v_1 \bar{v}_2, t \geq 0, \text{ this can be written as } $ $ z ^2 + pz + q\bar{z} + \frac{t - 1}{t} = 0 $ $ \text{where } \begin{cases} -1 \\ 1 \\ 0 \text{ as } 0 < t < 1. \end{cases} $ By (a), this equation has at least two solutions. $ \begin{vmatrix} 1 \\ 6 \\ 1 \end{vmatrix} $				
(q ₁ +r ₁)x + (q ₀ -r ₀)y = 0		22	1	
Further the origin lies inside (1) If $q_1 + r_1 = q_0 - r_0 = 0$, (i) and (ii) (and therefore (*)) has infinitely many solutions. Otherwise (ii) is a straight line through the origin intersecting (i) at two points. (*) has 2 solutions. [*] [*] [*] (b) $[tu_1\bar{u}_2+(1-t)v_1\bar{v}_2][(u_1+zv_1)(\bar{u}_1+zv_1)+(u_2+zv_2)(\bar{u}_2+z\bar{v}_2)]$ $= (u_1+zv_1)(\bar{u}_2+z\bar{v}_2)$ [*] **Expanding and using $ u_1 ^2+ u_2 ^2= v_1 ^2+ v_2 ^2=1$, we obtain $t(v_1\bar{v}_2-u_1\bar{u}_2) z ^2+pz+q\bar{z}+(1-t)(u_1\bar{u}_2-v_1\bar{v}_2)=0$, $p,q\in\mathbb{R}$. Since $u_1\bar{u}_2+v_1\bar{v}_2$, $t\geq 0$, this can be written as $ z ^2+pz+q\bar{z}+\frac{t-t}{t}=0$ where $(z)^2+pz+q\bar{z}+\frac{t-t}{t}=0$ 1 By (a), this equation has at least two solutions. 1 1 1 1 1 1 1 1 1 1 1 1 1	1.e.	$q_1 + r_1 \times + (q_0 - r_0) y = 0$ (11)	1	
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has infinitely many solutions. Otherwise (ii) is a straight line through the origin intersecting (i) at two points (*) has 2 solutions. [1] 8 (b) [tu₁u₂+(1-t)v₁v₂][(u₁+zv₁)(u₁+zv₁)+(u₂+zv₂)(u₂+zv₂)] - (u₁+zv₁)(u₂+zv₂) [2] 1 Expanding and using u₁ ²+ u₂ ²- v₁ ²+ v₂ ²-1, we obtain t(v₁v₂-u₁u₂) z ²+Pz+Qz+(1-t)(u₁u₂-v₁v₂)-0, P,Q∈R. Since u₁u₂+v₁v₂, t>0, this can be written as z ²+pz+qz+t-1/t = 0 where (-1/t) 0 as 0 < t < 1. By (a), this equation has at least two solutions. 1/t)	Further t	the origin lies inside (1)	(<u>1-8</u>)	
Otherwise (ii) is a straight line through the origin intersecting (i) at two points (*) has 2 solutions. $\frac{1}{8}$ (b) $\left[tu_1 \overline{u}_2 + (1-t)v_1 \overline{v}_2 \right] \left[(u_1 + zv_1)(\overline{u}_1 + z\overline{v}_1) + (u_2 + zv_2)(\overline{u}_2 + z\overline{v}_2) \right]$ $= (u_1 + zv_1)(\overline{u}_2 + z\overline{v}_2)$ $\frac{1}{8}$ Expanding and using $ u_1 ^2 + u_2 ^2 = v_1 ^2 + v_2 ^2 = 1$, we obtain $t(v_1 \overline{v}_2 - u_1 \overline{u}_2) z ^2 + Pz + Q\overline{z} + (1-t)(u_1 \overline{u}_2 - v_1 \overline{v}_2) = 0$, $P, Q \in \mathbb{R}$. Since $u_1 \overline{u}_2 \neq v_1 \overline{v}_2$, $t \geq 0$, this can be written as $ z ^2 + pz + q\overline{z} + \frac{t-1}{t} = 0$ where $\left(\frac{t-1}{t} \right) = 0$ $2 = \frac{t-1}{t} = 0$ $2 = \frac{t-1}{t} = 0$ $3 = \frac{t-1}{t} = 0$ $4 = \frac{t-1}{t} = 0$ $3 = \frac{t-1}{t} = 0$ $4 = \frac{t-1}{t} = 0$	If q _I +	$r_1 = q_0 - r_0 = 0$, (i) and (ii) (and therefore (*))		
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(b) $\left[tu_1 \overline{u}_2 + (1-t)v_1 \overline{v}_2 \right] \left[(u_1 + zv_1) (\overline{u}_1 + z\overline{v}_1) + (u_2 + zv_2) (\overline{u}_2 + z\overline{v}_2) \right]$ $= (u_1 + zv_1) (\overline{u}_2 + z\overline{v}_2)$ $= (u_1 + zv_1) (\overline{u}_2 + z\overline{v}_2)$ $t(v_1 \overline{v}_2 - u_1 \overline{u}_2) z ^2 + v_2 ^2 = v_1 ^2 + v_2 ^2 = 1, \text{ we obtain } 1$ $t(v_1 \overline{v}_2 - u_1 \overline{u}_2) z ^2 + v_2 ^2 + (1-t) (u_1 \overline{u}_2 - v_1 \overline{v}_2) = 0, \text{ p.q } \in \mathbb{R}.$ $ z ^2 + v_2 ^2 + v_1 \overline{v}_2 + v_2 ^2 = 0, \text{ this can be written as } z ^2 + v_2 ^2 + v_1 ^2 = 0$ $ z ^2 + v_2 ^2 + v_1 ^2 = 0$ $ z ^2 + v_2 ^2 + v_1 ^2 = 0$ $ z ^2 + v_2 ^2 + v_1 ^2 = 0$ $ z ^2 + v_1 ^2 + v_1 ^2 = 0$ $ z ^2 + v_1 ^2 + v_1 ^2 = 0$ $ z ^2 + v_1 ^2 + v_1 ^2 = 0$ $ z ^2 + v_1 ^2 + v_1 ^2 = 0$ $ z ^2 + v_1 ^2 + v_1 ^2 = 0$ $ z ^2 + v_1 ^2 + v_1 ^2 + v_2 ^2 = 1, \text{ we obtain } 1$ $ z ^2 + v_1 ^2 + v_1 ^2 + v_2 ^2 = 1, \text{ we obtain } 1$ $ z ^2 + v_1 ^2 + v_1 ^2 + v_2 ^2 = 1, \text{ we obtain } 1$ $ z ^2 + v_1 ^2 + v_1 ^2 + v_2 ^2 = 1, \text{ we obtain } 1$ $ z ^2 + v_1 ^2 + v_2 ^2 + v_1 ^2 + v_2 ^2 = 1, \text{ we obtain } 1$ $ z ^2 + v_1 ^2 + v_2 ^2 + v_1 ^2 + v_2 ^2 = 1, \text{ we obtain } 1$ $ z ^2 + v_1 ^2 + v_2 ^2 + v_2 ^2 + v_1 ^2 + v_2 ^2 + v_1 ^2 + v_2 ^2 + v_1 ^2 + v_2 ^2 + v_2 ^2 + v_1 ^2 + v_2 ^2 + v_1 ^2 + v_2 ^2 + v_1 ^2 + v_2 ^2 + v_2 ^2 + v_1 ^2 + v_2 ^2 + v_1 ^2 + v_2 ^2 + v_1 ^2 + v_2 ^2 + v_2 ^2 + v_2 ^2 + v_1 ^2 + $	Otherwise	(ii) is a straight line through the origin	1	
(b) $\left[tu_1 \overline{u}_2 + (1-t)v_1 \overline{v}_2 \right] \left[(u_1 + zv_1)(\overline{u}_1 + z\overline{v}_1) + (u_2 + zv_2)(\overline{u}_2 + z\overline{v}_2) \right]$ $= (u_1 + zv_1)(\overline{u}_2 + z\overline{v}_2)$ $= (u_1 $	intersect	ing (i) at two points (*) has 2 solutions.	1	-
$= (u_1 + zv_1)(\overline{u_2} + \overline{zv_2})$ $\text{Expanding and using } u_1 ^2 + u_2 ^2 = v_1 ^2 + v_2 ^2 = 1, \text{ we obtain } 1$ $t(v_1\overline{v_2} - u_1\overline{u_2}) z ^2 + Pz + Q\overline{z} + (1-t)(u_1\overline{u_2} - v_1\overline{v_2}) = 0, P, Q \in \mathbb{R}.$ $ z ^2 + pz + q\overline{z} + \frac{t-1}{t} = 0$ $\text{where } (t) = (1-t)(0) \text{ as } 0 < t < 1.$ $\text{By (a), this equation has at least two solutions.}$	(b) Ftu. ū. +(1)		8	<u>-</u>
Expanding and using $ u_1 ^2 + u_2 ^2 = v_1 ^2 + v_2 ^2 = 1$, we obtain $t(v_1\overline{v}_2 - u_1\overline{u}_2) z ^2 + Pz + Q\overline{z} + (1-t)(u_1\overline{u}_2 - v_1\overline{v}_2) = 0$, P,Q $\in \mathbb{R}$. It Since $u_1\overline{u}_2 \neq v_1\overline{v}_2$, $t \geq 0$, this can be written as $ z ^2 + pz + q\overline{z} + \frac{t-1}{t} = 0$ where (-1) (0) as $0 < t < 1$. By (a), this equation has at least two solutions. $ z ^2 + z + z + z + z + z + z + z + z + z + $			···	
$t(v_1\overline{v}_2 - u_1\overline{u}_2) z ^2 + Pz + Q\overline{z} + (1-t)(u_1\overline{u}_2 - v_1\overline{v}_2) = 0, P, Q \in \mathbb{R}.$ Since $u_1\overline{u}_2 \neq v_1\overline{v}_2$, $t \geq 0$, this can be written as $ z ^2 + pz + q\overline{z} + \frac{t-1}{t} = 0$ where $\underbrace{(-1)}_{t} = 0$ as $0 < t < 1$. By (a), this equation has at least two solutions. $\frac{1}{6}$		2 2	(n 1	
Since $u_1 \overline{u}_2 \neq v_1 \overline{v}_2$, $t \geq 0$, this can be written as $ z ^2 + pz + q\overline{z} + \frac{t-1}{t} = 0$ where (-1)	$t(v_1\overline{v}_1 - v_2)$	$ z_1 ^2 + Pz + Qz + (1-t)(u, \overline{u}) - v \overline{v} = 0$	2	**
$ z ^2 + pz + q\overline{z} + \frac{t-1}{t} = 0$ where (-1) 0 as $0 < t < 1$. By (a), this equation has at least two solutions. $\frac{1}{6}$			1	
where (-1) 0 as 0 < t < 1. By (a), this equation has at least two solutions. 1			. ,	J. 30
By (a), this equation has at least two solutions. 1 6 1 7 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1				
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	SOLUTIONS 80	MARKS	REMARKS						SOL	UTIONS			.		· · · · · · · · · · · · · · · · · · ·	MARKS.	REMARKS
(a)	If $d_1(x)$ and $d_2(x)$ divide each other, let $d_1(x) = h_1(x) d_2(x)$	ŀ			(c)			_	- :	+ n ₁ (x)g(x) 1	+	્. જ	f f(x)	and g(x)	1	
	and $d_2(x) = h_2(x) d_1(x)$ for some polynomials $h_1(x)$, $h_2(x)$.	٠,		F					•					d(x) =	kd ₁ (x)	:	-
	Then $\deg d_1(x) = \deg h_1(x) + \deg d_2(x)$	1										_		2	zero k·		
•	= deg $h_1(x) + [deg h_2(x) + deg d_1(x)]$			F					- :		-				= ₁ (x).		
	deg $h_1(x) = \deg h_2(x) = 0$.	•				i			-0 (-> - c	_,	0,,,,,,	.,,		(x) = 1		1 4	
	$h_1(x) = k \neq 0$ (as $d_1(x)$ and $d_2(x) \neq 0$)	1			÷	•			•				0	 	-1 ()	- 2	
	$d_1(x) = k d_2(x)$.	3		P											,		
(b)	(i) If s(x) divides f(x) and g(x),			•	· · · · · · · · · · · · · · · · · · ·						٠.						
	let $f(x) = h_1(x)s(x)$, $g(x) = h_2(x)s(x)$ for some	1			$\sum_{i,j} \sum_{i=1}^{n} \sum_{j=1}^{n} (i - \sum_{j=1}^{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{j=1}^{n} \sum_{j=1}^{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{$. •							•			!
	polynomials h ₁ (x), h ₂ (x).			. •							•			•	. •		
	p(x) = n(x)f(x) + n(x)g(x)					٠	•			•							
	= $[\pi(x)h_1(x) + \pi(x)h_2(x)]s(x)$			F													
	i.2. s(x) divides p(x)	ı															
	1	By 7	uclidean					•									-
	Let $f(x) = q(x)p(x) + r(x)$, where $\gamma(x) \neq 0$ and deg $r(x) \leq \deg p(x)$. Then $r(x) = f(x) - q(x)p(x)$		Mgorithm	F											-		
	= $f(x) - q(x)[\pi(x)f(x) + \pi(x)g(x)]$	1		•													
	= $[1 - q(x) = (x)]f(x) + [-n(x)q(x)]g(x)$												•				
	$r(x) \in A$. $\leftarrow N c n - 3ero$ non-zero	1													•		
•	(iii) Let $d_1(x) = a_1(x)f(x) + n_1(x)g(x)$ and suppose $d_1(x) \nmid f(x)$.																
	Let $f(x) = h(x)d_1(x) + r(x)$, where $r(x) \neq 0$ and deg $r(x) \leq \deg d_1(x)$.			F						-				• .			
	Then by (ii), $r(x) \in A$.	1			:							•					
	But deg r(x) < deg d ₁ (x) contradicts the definition			F	i	_			-				~ -				
	of d ₁ (x).	1			. !								-				
	d ₁ (x) divides f(x).				i		*								•		
- •	Similarly d ₁ (x) divides g(x).														* '.		
	$d_1(x)$ is a common divisor of $f(x)$ and $g(x)$.	1	ينجه ينجه		. 1								~. ·				
	By (1) $d_1(x)$ is a G.C.D. of $f(x)$ and $g(x)$.	1										ilar kiri.					

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	SOLUTIONS	86	MARKS	REMARKS:
a) (1)	For $0 \le \theta \le \frac{\pi}{4}$, $0 \le \tan \theta \le 1$.	I	I	
	$0 \leq \tan^{n+1}\theta \leq \tan^n\theta$			
	$0 \le I_{n+1} \le I_n \text{for } n \ge 0$		1	
;	For $n \ge 2$, $I_n = \int_0^{\pi} 4 \tan^{n-2}\theta (\sec^2\theta - 1) d\theta$		•	
(11)	_		1	
,	$= \int_{0}^{\frac{\pi}{4}} \tan^{n-2}\theta \ d \ \tan\theta - I_{n-2}$		1	
	$=\frac{\tan^{n-1}\theta}{n-1}\bigg _{0}^{\frac{\pi}{4}}-I_{n-2}$			
	$=\frac{1}{n-1}-I_{n-2}$			
	$I_n + I_{n-2} - \frac{1}{n-1}$		1	
(111)	For $n \ge 2$, $\frac{1}{n-1} = I_n + I_{n-2}$ $\ge 2I_n$ by (1)			
	$\therefore \frac{1}{2(n-1)} \geqslant I_n \qquad \dots$	-	i	. 545 T
	Further $\frac{1}{n+1} = I_{n+2} + I_n$ $\leq 2I_n$		٠	
	$\frac{1}{2(n+1)} \le I_n \qquad \dots$		1 7	
			-	

n - 3	SOLUTIONS			86 HARKS	REMARKS
(b) For	n > 1.		!	I	
-	•				1.
-2n-	$-1 = \frac{1}{2n} - I_{2n-1}$			1	
	$= \frac{1}{2n} - \frac{1}{2n-2} + I_{2n-3}$	+(1-11	nta		- '
•	/ = •••			- 1:0	
	$= \frac{1}{2n} - \frac{1}{2n-2} + \frac{1}{2n-4}$	$- \dots + (-1)^{n-1} \frac{1}{2}$	-+ (-I) ⁿ I,	1	- 7i
† •	$= \frac{(-1)^{n-1}}{2} (a_n - 2I_1)$		•	. 1	
• • • • • • • • • • • • • • • • • • •					
Now	$I_1 = \int_0^{\frac{\pi}{4}} \tan \theta \ d\theta$	•	•	i	,
		• • • • • • • • • • • • • • • • • • •		i	:
	$= -\int_{0}^{\frac{\pi}{4}} \frac{1}{\cos \theta} d \cos \theta$	1 <u>2</u> 2	ر در در این در در این در این در این در این در این در	- 1	
					•
	.= [-ln cos0] \(\frac{7}{4}	e y a estab.			
	,				:
	$=\frac{1}{2} \ln 2$			1	
	$I_{2n+1} = \frac{(-1)^{n-1}}{2} (a_n - \ln 2)$) -		1	
From	(a)(iii) $\frac{1}{2(2n+2)} \le I_{2n}$	$\frac{1}{2(2n)}$) [•
•	both $\lim_{n \to \infty} \frac{1}{2(2n+2)}$ and		l zero)		
		h>00 2(2a)	•	į .	·
\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\					
(\i.e.	lim a = ln 2	•••••	••••••	$-\frac{1}{1-7}$	•
V	$\frac{1}{2} \Omega_{M} = 2 \int_{M+1}^{M+1}$	+1~	•		٠.
	$a_{N} = \frac{2 \ln t}{(-1)^{N-1}}$	-	_		
	in an = lime [Mant + 0	\exists		
	1300 MODE	(-1)14 1 m	۔ ک		
	•	•			
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SOLUTIONS	MARKS	REMARKS
For C + 0, let the equations of the two lines be		- 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
$y - m_1 x - C_1 = 0$ and $y = m_2 x - C_2 = 0$.	-	
Then the given equation can be written as		
* $(y - m_1 x - C_1)(y - m_2 x - C_2) = 0$	1	- 19 - 1944 - 197 - 1944
$m_1 m_2 x^2 - (m_1 + m_2) xy + y^2 + (m_1 c_2 + m_2 c_1) x - (c_1 + c_2) y + c_1 c_2 = 0$, -	
Comparing coefficients, we have		
$\left({}^{m_{1}m_{2}} - \frac{A}{C}, {}^{m_{1}} + {}^{m_{2}} - \frac{B}{C} \right)$	1.	
(1) If $A + C = 0$, $m_1 m_2 = -1$. A = C		
2	1	
(ii) If $A + C \neq 0$, $tan^2 < -(\frac{m_1 - m_2}{1 + m_1 m_2})^2$	-1	
$\frac{(m_1 + m_2)^2 - 4m_1m_2}{(1 + m_m)^2}$ $(-\frac{B}{C})^2 - \frac{4A}{C}$	i	**
$ \underbrace{\begin{array}{c} \cdot C \\ \cdot C \end{array}}_{=2} \underbrace{\begin{array}{c} \cdot C \\ \cdot C \end{array}}_{=2} $;iii ==	
$\frac{B^2 - 4AC}{(A+C)^2}$	1	ŀ
*Note: Candidates may also consider $Ax^2 + Bxy + Cy^2 = 0$. For $C = 0$,		•
-Case 1 Ar' (C 7 : * (Ar + 6) = 0		
If B # 0, then the pair of straight lines are given by		
$(y - m_1 x - c_1)(x - c_2) = 0$. 241
$-m_1x^2 + xy - (c_1 - m_1c_2)x - c_2y + c_1c_2 = 0$		
∴ m ₁ = - A/B		1
$x - C_2 = 0$ is a vertical line,		
If $A + C = 0$, then $A = 0$ and therefore $m_1 = 0$.		İ
i.e. the two lines intersect at right angles	1	
If A + C + 0, then A + 0.		
$\tan^2 \alpha = \tan^2 (\frac{\pi}{2} - \beta)$, (where $\tan \beta = \pi_1$)		
$= \cot^2 \beta = \frac{1}{m_1^2} = \frac{B^2}{A^2} = \frac{B^2 - 4AC}{(A+C)^2} (as \ C = 0)$	1	
Case 2	. -	
If B = 0 (in which case A \neq 0 and A + C \neq 0), then both	- TT	
straight lines are vertical.	and the same	
$\therefore \approx -0.0 \Rightarrow \tan^2 \approx 4 = 0 = \frac{B^2 - 4AC}{(A+C)^2}$ (as B = C = 0)		

SOLUTIONS	86 4	ARKS	REMARKS
2. (b) Let y = mx + c be the equation of a line through P.	I		7.77 m 7
Substituting in (E)		ı	
$\frac{x^2}{a^2} + \frac{(nx + c)^3}{b^2} = 1$			· · ·
$(b^2 + a^2m^2)x^2 + 2a^2mcx + a^2(c^2 - b^2) = 0.$		1 -	
For tangency, $4a^4m^2c^2 = 4a^2(c^2 - b^2)(b^2 + a^2m^2)$			
$c^2 - b^2 - a^2 \pi^2 = 0$		41	
Since y = mx + c passes through (h, k)	-		
c - k - mh			
$(k - mh)^2 - b^2 - a^2m^2 = 0.$			
$(h^2 - a^2)n^2 - 2hkm + (k^2 - b^2) = 0$.	i .	
If the tangents are perpendicular	_	-	
$\frac{k^2 - b^2}{h^2 - a^2} = -1$.	1	
$h^2 + k^2 = a^2 + b^2$		_	
. P lies on the circle $x^2 + y^2 = a^2 + b^2$	-	1 6	•
	-		
Alternatively		=	
(b) The pair of tangents through P(h, k) is			
$\left(\frac{h^2}{a^2} + \frac{k^2}{b^2} - 1\right) \left(\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1\right) = \left(\frac{hx}{a^2} + \frac{ky}{b^2} - 1\right)^2 \dots \left[\frac{1}{a^2} \left(\frac{h^2}{a^2} + \frac{k^2}{b^2} - 1\right) - \frac{h^2}{a^4}\right] x^2 + \left[\frac{1}{b^2} \left(\frac{h^2}{a^2} + \frac{k^2}{b^2} - 1\right) - \frac{k^2}{b^4}\right] x^2 + \left[\frac{1}{b^2} \left(\frac{h^2}{a^2} + \frac{k^2}{b^2} - 1\right) - \frac{k^2}{b^4}\right] x^2 + \left[\frac{1}{b^2} \left(\frac{h^2}{a^2} + \frac{k^2}{b^2} - 1\right) - \frac{k^2}{b^4}\right] x^2 + \left[\frac{1}{b^2} \left(\frac{h^2}{a^2} + \frac{k^2}{b^2} - 1\right) - \frac{k^2}{b^4}\right] x^2 + \left[\frac{1}{b^2} \left(\frac{h^2}{a^2} + \frac{k^2}{b^2} - 1\right) - \frac{k^2}{b^4}\right] x^2 + \left[\frac{1}{b^2} \left(\frac{h^2}{a^2} + \frac{k^2}{b^2} - 1\right) - \frac{k^2}{b^4}\right] x^2 + \left[\frac{1}{b^2} \left(\frac{h^2}{a^2} + \frac{k^2}{b^2} - 1\right) - \frac{k^2}{b^4}\right] x^2 + \left[\frac{1}{b^2} \left(\frac{h^2}{a^2} + \frac{k^2}{b^2} - 1\right) - \frac{k^2}{b^4}\right] x^2 + \left[\frac{1}{b^2} \left(\frac{h^2}{a^2} + \frac{k^2}{b^2} - 1\right) - \frac{k^2}{b^4}\right] x^2 + \left[\frac{1}{b^2} \left(\frac{h^2}{a^2} + \frac{k^2}{b^2} - 1\right) - \frac{k^2}{b^4}\right] x^2 + \left[\frac{1}{b^2} \left(\frac{h^2}{a^2} + \frac{k^2}{b^2} - 1\right) - \frac{k^2}{b^4}\right] x^2 + \left[\frac{1}{b^2} \left(\frac{h^2}{a^2} + \frac{k^2}{b^2} - 1\right) - \frac{k^2}{b^4}\right] x^2 + \left[\frac{1}{b^2} \left(\frac{h^2}{a^2} + \frac{k^2}{b^2} - 1\right) - \frac{k^2}{b^4}\right] x^2 + \left[\frac{1}{b^2} \left(\frac{h^2}{a^2} + \frac{k^2}{b^2} - 1\right) - \frac{k^2}{b^4}\right] x^2 + \left[\frac{1}{b^2} \left(\frac{h^2}{a^2} + \frac{k^2}{b^2} - 1\right) - \frac{k^2}{b^4}\right] x^2 + \left[\frac{1}{b^2} \left(\frac{h^2}{a^2} + \frac{k^2}{b^2} - 1\right) - \frac{k^2}{b^4}\right] x^2 + \left[\frac{h^2}{b^2} \left(\frac{h^2}{a^2} + \frac{k^2}{b^2} - 1\right) - \frac{k^2}{b^4}\right] x^2 + \left[\frac{h^2}{b^2} \left(\frac{h^2}{a^2} + \frac{k^2}{b^2} - 1\right) - \frac{k^2}{b^2}\right] x^2 + \left[\frac{h^2}{b^2} \left(\frac{h^2}{a^2} + \frac{k^2}{b^2} - 1\right) - \frac{k^2}{b^2}\right] x^2 + \left[\frac{h^2}{b^2} \left(\frac{h^2}{a^2} + \frac{k^2}{b^2} - 1\right) - \frac{k^2}{b^2}\right] x^2 + \left[\frac{h^2}{b^2} \left(\frac{h^2}{a^2} + \frac{k^2}{b^2} - 1\right) - \frac{k^2}{b^2}\right] x^2 + \left[\frac{h^2}{b^2} \left(\frac{h^2}{a^2} + \frac{k^2}{b^2} - 1\right) - \frac{k^2}{b^2}\right] x^2 + \left[\frac{h^2}{b^2} \left(\frac{h^2}{a^2} + \frac{k^2}{b^2} - 1\right) - \frac{k^2}{b^2}\right] x^2 + \left[\frac{h^2}{b^2} \left(\frac{h^2}{a^2} + \frac{k^2}{b^2} - 1\right) - \frac{k^2}{b^2}\right] x^2 + \left[\frac{h^2}{b^2} \left(\frac{h^2}{a^2} + \frac{k^2}{b^2} - 1\right) - \frac{k^2}{b^2}\right] x^2 + \left[\frac{h^2}{b^2} \left(\frac{h^2}{a^2} + \frac{k^2}{b^2} - 1\right) - \frac{k^2}{b^2}\right] x^2 + \left[\frac{h^2}{b^2} \left(\frac{h^2}{a^2} + \frac{h^2}{b^2} - 1\right) - \frac{k^2}{b^2}\right] x^2 + \left[\frac{h^2}{b^2$	·	2	
$-\frac{2hk}{a^2b^2}xy + \frac{2h}{a^2}x + \frac{2k}{b^2}y - (\frac{h^2}{a^2} + \frac{k^2}{b^2}) = 0$		1	
If the tangents are perpendicular, by (a) (1) and (11) h^2 1			
$\left(\frac{k^2}{a^2b^2} - \frac{1}{a^2}\right) + \left(\frac{h^2}{a^2b^2} - \frac{1}{b^2}\right) = 0$		2	
$k^2 + h^2 = a^2 + b^2$			
$\therefore P(h, k) \text{ lies on the circle } x^2 + y^2 = a^2 + b^2.$	-	6	
	-		
:Note: There are a number of alternative methods used in common texth	ooks		
	.÷.,		
			E

		-15.P.6
	Alternatively	(MARKS
	3. (b) $\int_{a}^{b} \left \sum_{j=1}^{2n} c_{j} x^{j} \right dx$	i
	$-\int_{a}^{0} \left \sum_{j=1}^{2n} c_{j} x^{j} \right dx + \int_{0}^{b} \left \sum_{j=1}^{2n} c_{j} x^{j} \right dx$	1
15	$ \begin{cases} 0 & 2n \\ \sum_{a} c_j x^j dx + \int_{0}^{b} \sum_{j=1}^{2n} c_j x^j dx \end{cases} $ $ = \begin{bmatrix} 0 & 2n \\ 2n & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & $	1
	$-\int_{\epsilon}^{0} \frac{\sum_{j=1}^{2n} c_{j} ^{(-1)^{j}} x^{j} dx}{\sum_{j=1}^{2n} c_{j} ^{j}} dx + \int_{0}^{b} \frac{\sum_{j=1}^{2n} c_{j} ^{j}}{\sum_{j=1}^{2n} c_{j} ^{j}} dx$ $-\sum_{j=1}^{2n} \left[\int_{0}^{\infty} (-1)^{j} \frac{x^{j+1}}{j+1} \right]_{0}^{0} + \sum_{j=1}^{2n} \left[\int_{0}^{\infty} c_{j} ^{j} \frac{x^{j+1}}{j+1} \right]_{0}^{0}$	1
	$= \sum_{j=1}^{2n} c_j (-1)^{j+1} \frac{(a)^{j+1}}{j+1} + \sum_{j=1}^{2n} c_j \frac{b^{j+1}}{j+1}$.1
	$= \sum_{j=1}^{2n} c_j \frac{ a ^{j+1}}{j+1} + \sum_{j=1}^{2n} c_j \frac{b^{j+1}}{j+1}$ $= \sum_{j=1}^{2n} c_j \frac{ a ^{j+1}}{j+1} + \sum_{j=1}^{2n} c_j \frac{b^{j+1}}{j+1}$	
	$\sum_{i=1}^{2n} c_j \frac{b^{i+1} + a ^{-j+1}}{j+1}$	<u>1</u> 5
	$3. (c) \int_{-1}^{1} \left \sum_{j=1}^{2n} \frac{(-1)^{j} (j+1)x^{j}}{2^{j}} \right dx$ $= 2 \int_{0}^{1} \left \sum_{j=1}^{2n} \frac{(-1)^{j} (j+1)x^{j}}{2^{j}} \right dx$	-
	$-2 \int_{0}^{1} \sum_{j=1}^{2n} \frac{(j+1)x^{j}}{2^{j}} dx$	
	$-2\sum_{j=1}^{2n} \left[\frac{x^{j+1}}{2^{j}}\right]_{0}^{1}$	
	$-2\sum_{j=1}^{2n}\frac{1}{2^{j}}$	
	$= 2 \left(\frac{\frac{1}{2} \left(1 - \frac{1}{2^{4n}} \right)}{1 - \frac{1}{2}} \right)$	

SOLUTIONS	MARKS	REMARKS
(a) $f'(0) = \lim_{\Delta x \to 0} \frac{(\Delta x + 0)^{n} \Delta x + 0 - 0}{\Delta x}$	1	# 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
$= \lim_{\Delta x \to 0} (\Delta x)^{\Pi} \Delta x$ $= 0$	1	
$\int \frac{d}{dx} x^{n+1} \cdot x \qquad x > 0$		
$f'(x) = \begin{cases} 0 & x = 0 \\ \frac{d}{dx} x^{n+1} & (-x) & x < 0 \end{cases}$	1	
(n+2) x ⁿ • x > 0		آئمون دراد
- 0 x - 0		·
	,	
	1	
$\int x^{n} \cdot \left x \right dx = \frac{1}{n+2} x^{n+1} \cdot \left x \right + c$	<u>1</u> <u>5</u>	
(b) $\int_{a}^{b} \left \sum_{j=1}^{2n} c_{j} x^{j} \right dx \le \int_{a}^{b} \left \sum_{j=1}^{2n} c_{j} x^{j} \right dx $	ī	
$= \sum_{j=1}^{2n} \int_{a}^{b} c_{j} x^{j} dx$		
$-\sum_{j=1}^{n} \left\{ c_{2j-1} \int_{a}^{b} x^{2j-1} dx + c_{2j} \int_{a}^{b} x^{2j} dx \right\} \qquad \dots$	1	
$- \int_{j=1}^{n} \left\{ \left c_{2j-1} \right \int_{a}^{b} x^{2j-2} \left x \right dx + \left c_{2j} \right \int_{a}^{b} x^{2j} dx \right\}$	1	
$-\sum_{j=1}^{n} \left\{ \left \left c_{2j-1} \right \frac{1}{2j} x^{2j-1} \right x \right + \left c_{2j} \right \frac{1}{2j+1} x^{2j+1} \right\}_{a}^{b} \right\} \dots$	1	
$=\sum_{j=1}^{n}\left\{\left c_{2j-1}\right \frac{1}{2j}\left[b^{2j-1}\right b\right -a^{2j-1}\left a\right \right\}+\left c_{2j}\right \frac{1}{2j+1}\left[b^{2j+1}-a^{2j+1}\right]\right\}$		
$= \sum_{j=1}^{n} \left\{ \left c_{2j-1} \right \frac{1}{2j} \left[b^{2j} + \left a \right ^{2j} \right] + \left c_{2j} \right \frac{1}{2j+1} \left[b^{2j+1} + \left a \right ^{2j+1} \right] \right\}$	-	
$-\frac{2n}{5-1} c_{j} \left(\frac{b^{j+1} + a ^{j+1}}{j+1} \right) - \cdots$	1	
	<u> </u>	

SOLUTIONS SOLUTIONS	- ρ.	
4. (a) 1(0+0)	MARKS"	
-[£(0)]²		
$\Rightarrow f(0) = 1 \text{ as } f(0) \neq 0$		
(11) We shall prove by induction:		
$\forall x \ge 0, f(x) \ge f(0) = 1$		
$rac{1}{2} \frac{1}{2} $	1 /	
$\Gamma(kx) = \Gamma_{F}(-1) \times \Gamma_{F}(-1$		
$- \mathbf{r}(\mathbf{k}\mathbf{x}) \mathbf{f}(\mathbf{x})$		<=
$= [f(x)]^{k+1}$		
$f(kx) = [f(x)]^k \forall \text{ integers } k \ge 0$ (iii) Let $x = f(x)$	1	
$= \{(1) \in \mathcal{E}(0) : \{($		
For any non-negative integer n a may be ≤ 1 , $f(n) = f(n \cdot 1)$ by (11) by (11)		
$[f(1)]^n$ by (11)		

(b) (1) For any x > 0, let n be the non-negative integer	6	
- × × \ L +		
Then $a^n \leq a^x \leq a^{n+1}$ and $\frac{1}{a^{n+1}} \leq \frac{1}{a^x} \leq \frac{1}{a^n}$ (1)	.	
13 increasing f(-)		
$a^n \leq f(x) \leq a^{n+1}$	1	
Since all quantities (and		
(11) give $\frac{a^n}{a^{n+1}} \le \frac{f(x)}{a^n} \le \frac{a^{n+1}}{a^n}$		
1.e. $\frac{1}{a} \le \frac{f(x)}{a^x} \le a$		
(11) As (*) is independent of x , replacing x by kx , we have $\frac{1}{a} < \frac{f(kx)}{kx} < a$		
$\frac{1}{a} \le \frac{f(kx)}{a^{kx}} \le a$ $\forall k > 0$		
$\frac{1}{a} \leq \left(\frac{f(x)}{a^{\lambda}}\right) \leq a$		
$\frac{1}{a^{k}} \left\langle \frac{f(x)}{a^{2}} \right\rangle \left\langle a^{k} \right\rangle$		
As both lim a and lim t equal 1		
$\lim_{k \to \infty} \frac{f(x)}{a^k} = 1$		<u> </u>
1.e. $f(x) = a^x$		137

4.5.12.N				TIONS				* ∕\$/	MARKS	
(a)	Let	х, у	be the cartesi	an coordi	nates of P	, tan	Ø be	I		REMAF
	the	gradien	t of the curve	at P		·• · · · · · · · · · · · · · · · · · ·				1 m
		X = 7 (cos9 sínô				. 45		•	
•	<u>dx</u> =		<u>r</u> - r sin0		•		••			
•	<u>dy</u>	sin9 dr	+ r cos9							
÷		tan Ø =			• • • • • • • • • • • • • • • • • • • •	*****	••••••	•	ı 	
7.			$\sin\theta \frac{dr}{d\theta} + r - \epsilon$:os9					5	
•		-	$\cos\theta \frac{dr}{d\theta} - r s$	ing	41	`	•			
			$\tan \theta \frac{d\mathbf{r}}{d\theta} + \mathbf{r}$. 12				
$\sum_{i=1}^{n} f_i$	•		$\frac{dr}{d\theta} - r \tan \theta$	••••••	••••••	• • • • • •	•••••		1	•
- }	tan y	tan(g) – 9)	· · · · · · · · · · · · · · · · · · ·					I	1 10
;	,	tan9	$\frac{dr}{d\theta} + r = -\tan\theta$							
		49 _	r cana							
		$\frac{\mathbf{t}}{\mathbf{d}}$	$an\theta \frac{dr}{d\theta} + r$ $\frac{r}{\theta} - r \tan\theta$	=0	1.					
			. •							•
		$\frac{r(1+\frac{dr}{dr})}{\frac{dr}{dr}}$	+ tan ² 0)			•				
•			+ tan-e)							
	•	$-\frac{r}{\left(\frac{dr}{d\theta}\right)}$.								
		1 44)		••••••••	••••••••••	•••••	•••••		1	
			•			_			- 1	
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G- 342	・・・・・・・・・・・・・・・・・・・・・・・・・・・・・・・・・・・・・		P:10		Later National Parking National Parkin	
	SOLUTIONS	MAR	KS REMARKS			ب
5. 2 (b)	(i) Substituting $r = 2$ in C_1 , $2 = 2(1 - \cos\theta)$			- 1	SULUTIONS REMA	R.
	cos9 = 0	1		٠ پي	25. (b) (ii) $-4\sqrt{2}\int_{0}^{2}\sqrt{1-\cos\theta} \ d\theta$	-
A	$\theta = \frac{\tau}{2} \text{ or } \frac{3\tau}{2}$	1			$= 8 \int_{0}^{\pi} \sin \frac{\theta}{2} d\theta$	
	the points of intersection are $(2, \frac{\pi}{2})$ and $(2, \frac{3\pi}{2})$.			· *	$= 16 \left\{-\cos\frac{\theta}{2}\right\}_0^{\frac{\pi}{2}}$:
	At $\theta = \frac{\pi}{2}$, the tangent to C_2 is parallel to the x-axis			10 A	$=8(2-\sqrt{2})$ (# 4.69)	
	For C_1 , $\frac{dr}{d\theta} = 2 \text{ sin}\theta$				10	
1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1	- 2		1			
! 		}				
Γ.	$\tan \gamma = \frac{r}{\frac{dr}{dr}} = \frac{2}{2} = 1$			•	Alternatively	
	The angle between C_1 and C_2 is $=\frac{\pi}{2} - \tan^{-1} 1$				(a) From the diagram	
	$\frac{1}{2}$ and $\frac{1}{2}$ is $\frac{1}{2}$ - can 1				PR ÷ Δr	
	By cympany the same and same a				QR ÷ r 49	
	By symmetry, the angle between C_1 and C_2 at $\theta = \frac{3\pi}{2}$ is also $\frac{\pi}{2}$				$\tan \theta = \frac{\Gamma \Delta \theta}{\Delta \Gamma}$	
·	4	1			$\therefore \tan \psi = \lim_{\Delta \theta \to 0} r \frac{\Delta \theta}{\Delta r} \qquad 0$	
]	C1				1	
	C.				$\frac{r}{dr}$	
					(b) (ii) $ds^2 = (rd\theta)^2 + (dr)^2$	
l					Length of C_1 inside $C_2 = \int ds$	
,		1+1		-	$= \int \int r^2 + \left(\frac{dr}{d\theta}\right)^2 d\theta$	
•	For any point (r, 6) of C ₁ lying inside C ₂			F	, - etc	
·	r = 2(1 - cos9) , r < 2				et _c	
-	∴ 2(1 - cosθ) < 2					
1	0 < cosθ					
	$0 < \theta < \frac{\pi}{2} \text{or} \frac{3\pi}{2} < \theta < 2\pi$					
	Length of C ₁ inside C ₂	1	or for lim: of integral belo	F		
_	$= \int_0^{\frac{\pi}{2}} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2} d\theta + \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2} d\theta$		-			
	$\frac{10 \int \frac{dQ}{dQ} + \frac{dQ}{dQ} = \frac{dQ}{dQ} = \frac{10}{2} \left(\frac{dQ}{dQ} \right)^2 + \left(\frac{dQ}{dQ} \right)^2 dQ}{10 \left(\frac{dQ}{dQ} \right)^2 + \left(\frac{dQ}{dQ} \right)^2 dQ}$	i				
	$= 2 \int_{0}^{\frac{\pi}{2}} \int \left(\frac{dx}{d\theta}\right)^{2} + \left(\frac{dy}{d\theta}\right)^{2} d\theta \text{by symmetry}$,			
	$= 2 \int_{0}^{\frac{\pi}{2}} \int (\cos^{\theta} \frac{dr}{d\theta} - r \sin \theta)^{2} + (\sin \theta \frac{dr}{d\theta} + r \cos \theta)^{2} d\theta$		en e			
	$=2\int_{0}^{t}\sqrt{\left(\frac{dr}{d\theta}\right)^{2}+r^{2}}d\theta$	1-				
	= $2\int_{-\infty}^{\infty} \int 4 \sin^2\theta + 4(\Gamma - \cos\theta)^2 d\theta$			H		

SOUTHINGS SOUTHINGS SOUTHINGS SOUTHINGS RECORD 15 (cb) (11) $-4\sqrt{2} \int_{0}^{1} \frac{1}{1 - \cos \theta} d\theta$ $-8 \int_{0}^{1} \sin \frac{\theta}{2} d\theta$ $-16 (-\cos \frac{\theta}{2}) \frac{1}{4}$ $-8(2-\sqrt{2})$ ($+4.69$) Alternatively (a) From the diagram PR $\div \Delta \theta$ $\tan \theta \div \frac{\tau \Delta \theta}{\Delta r}$ $\cot \eta + 1 \sin r \frac{\Delta \theta}{\Delta r}$ 1 (b) (11) $ds^{2} - (rd\theta)^{2} + (dr)^{2}$ Length of C_{1} incide $C_{2} - \int ds$ $-\sqrt{r^{2}} + (\frac{dr}{d\theta})^{2} d\theta$ 1 $-\frac{\pi c}{dt}$ etc.	
(b) (ii) $= 4\sqrt{2} \int_{0}^{2} \sqrt{1 - \cos \theta} \ d\theta$ $= 8 \int_{0}^{2} \sin \frac{\theta}{2} \ d\theta$ $= 16 \left[-\cos \frac{\theta}{2} \right]_{0}^{\frac{1}{2}}$ $= 8(2 - \sqrt{2}) \ (4 + 4.69)$ $= 8 \int_{0}^{2} \sin \frac{\theta}{2} \ d\theta$	SOLUTIONS 97 MARKS - DOL
$= 8 \int_{0}^{\pi} \sin \frac{\theta}{2} d\theta$ $= 16 \left[-\cos \frac{\theta}{2}\right]_{0}^{\frac{\pi}{4}}$ $= 8(2 - \sqrt{2}) (\neq 4.69)$ $= 8(2 - \sqrt{2}) (\neq 4.69)$ $= 10$ Alternatively $PR \div \Delta r$ $QR \div r \Delta \theta$ $tan \theta \div \frac{r \Delta \theta}{\Delta r}$ $tan \psi = \lim_{\Delta \theta \to 0} r \frac{\Delta \theta}{\Delta r}$ $(b) (11) ds^{2} = (rd\theta)^{2} + (dr)^{2}$ $Length of C_{1} inside C_{2} = \int ds$ $= \int r^{2} + (\frac{dr}{d\theta})^{2} d\theta$ $= 1$	25. (b) (11) = $4\sqrt{2}$ $\sqrt{1 - \cos\theta}$ d0
$= 16 \left[-\cos\frac{\theta}{2}\right]_{0}^{\frac{\pi}{4}}$ $= 8(2 - \sqrt{2}) (4 \cdot 4.69)$ $\frac{1}{10}$ Alternatively $(a) \text{From the diagram}$ $PR \stackrel{?}{+} \Delta r$ $QR \stackrel{?}{+} r \Delta \theta$ $\tan \theta \stackrel{?}{+} \frac{r \Delta \theta}{\Delta r}$ $\tan \psi = \lim_{\Delta \theta \neq 0} r \frac{\Delta \theta}{\Delta r}$ $(b) (11) ds^{2} = (rd\theta)^{2} + (dr)^{2}$ $\text{Length of } C_{1} \text{inside } C_{2} = \int_{0}^{\pi} ds$ $= \int_{0}^{\pi} \int_{0}^{\pi^{2}} + (\frac{dr}{d\theta})^{2} d\theta$ $= \text{etc}$	= 8 (f stn = 40
$= 8(2 - \sqrt{2}) (4 \text{ 4.69})$ $\frac{1}{10}$ Alternatively $(a) \text{From the diagram}$ $PR \div \Delta r$ $QR \div r \Delta \theta$ $\tan \theta \div \frac{r \Delta \theta}{\Delta r}$ $\tan \psi = \lim_{n \to \infty} r \frac{\Delta \theta}{\Delta r}$ $\frac{r}{d\theta}$ $(b) (11) ds^2 = (rd\theta)^2 + (dr)^2$ $Length of C_1 inside C_2 = \int ds$ $= \int r^2 + (\frac{dr}{d\theta})^2 d\theta$ $= \text{etc}$	
Alternatively (a) From the diagram $PR \div \Delta r$ $QR \div r \Delta \theta$ $tan $	
Alternatively (a) From the diagram $PR \div \Delta \tau$ $QR \div r \Delta \theta$ $tan \theta \div \frac{r \Delta \theta}{\Delta r}$ $tan \psi = \lim_{\Delta \theta \to 0} r \frac{\Delta \theta}{\Delta r}$ $\frac{1}{1}$ $-\frac{r}{dr}$ $(b) (ii) ds^2 = (rd\theta)^2 + (dr)^2$ $Length of C1 inside C2 = \int ds = \int r^2 + (\frac{dr}{d\theta})^2 d\theta = etc$	
(a) From the diagram $PR \stackrel{?}{=} \Delta r$ $QR \stackrel{?}{=} r \Delta \theta$ $tan \theta \stackrel{?}{=} \frac{r}{\Delta r}$ $tan \psi = \lim_{\Delta \theta \neq 0} r \frac{\Delta \theta}{\Delta r}$ $= \frac{r}{\frac{dr}{d\theta}}$ (b) (ii) $ds^2 = (rd\theta)^2 + (dr)^2$ $Length of C1 inside C2 = \int_{\Gamma} ds = \int_{\Gamma} r^2 + (\frac{dr}{d\theta})^2 d\theta = etc$	10
(a) From the diagram $PR \stackrel{?}{=} \Delta r$ $QR \stackrel{?}{=} r \Delta \theta$ $tan \theta \stackrel{?}{=} \frac{r}{\Delta r}$ $tan \psi = \lim_{\Delta \theta \neq 0} r \frac{\Delta \theta}{\Delta r}$ $= \frac{r}{\frac{dr}{d\theta}}$ (b) (ii) $ds^2 = (rd\theta)^2 + (dr)^2$ $Length of C1 inside C2 = \int_{\alpha \theta} ds = \int_{\alpha \theta} r^2 + (\frac{dr}{d\theta})^2 d\theta = etc$	
(a) From the diagram $PR \stackrel{?}{=} \Delta r$ $QR \stackrel{?}{=} r \Delta \theta$ $tan \theta \stackrel{?}{=} \frac{r}{\Delta r}$ $tan \psi = \lim_{\Delta \theta \neq 0} r \frac{\Delta \theta}{\Delta r}$ $= \frac{r}{\frac{dr}{d\theta}}$ (b) (ii) $ds^2 = (rd\theta)^2 + (dr)^2$ $Length of C1 inside C2 = \int_{\alpha \theta} ds = \int_{\alpha \theta} r^2 + (\frac{dr}{d\theta})^2 d\theta = etc$	Alternatively
PR = Δr QR = $r \Delta \theta$ $tan \theta = \frac{r \Delta \theta}{\Delta r}$ $tan \psi = \lim_{\Delta \theta \to 0} r \frac{\Delta \theta}{\Delta r}$ $\frac{r}{dr}$ (b) (ii) $ds^2 = (rd\theta)^2 + (dr)^2$ Length of C_1 inside $C_2 = \int ds$ $= \int r^2 + (\frac{dr}{d\theta})^2 d\theta$ 1 - etc	
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	
tan $\emptyset = \frac{r \Delta \theta}{\Delta r}$ $tan \psi = \lim_{\Delta \theta \to 0} r \frac{\Delta \theta}{\Delta r}$ $= \frac{r}{\frac{dr}{d\theta}}$ (b) (ii) $ds^2 = (rd\theta)^2 + (dr)^2$ Length of C_1 inside $C_2 = \int ds$ $= \int r^2 + (\frac{dr}{d\theta})^2 d\theta$ $= etc$	
$\tan \psi = \lim_{\Delta \theta \to 0} r \frac{\Delta \theta}{\Delta r}$ $= \frac{r}{\frac{dr}{d\theta}}$ (b) (ii) $ds^2 = (rd\theta)^2 + (dr)^2$ Length of C_1 inside $C_2 = \int ds$ $= \int \int r^2 + (\frac{dr}{d\theta})^2 d\theta$ $= etc$	QR ÷ r 49
$\tan \psi = \lim_{\Delta \theta \to 0} r \frac{\Delta \theta}{\Delta r}$ $= \frac{r}{\frac{dr}{d\theta}}$ (b) (ii) $ds^2 = (rd\theta)^2 + (dr)^2$ Length of C_1 inside $C_2 = \int ds$ $= \int \int r^2 + (\frac{dr}{d\theta})^2 d\theta$ $= etc$	$\tan \theta \div \frac{\tau \Delta \theta}{\Delta \tau}$
(b) (ii) $ds^2 = (rd\theta)^2 + (dr)^2$ Length of C_1 inside $C_2 = \int ds$ $= \int \int r^2 + (\frac{dr}{d\theta})^2 d\theta$ $= etc$	
(b) (ii) $ds^2 = (rd\theta)^2 + (dr)^2$ Length of C_1 inside $C_2 = \int ds$ $= \int \int r^2 + (\frac{dr}{d\theta})^2 d\theta$ 1 - etc	Δ5-70 ΔΤ
(b) (ii) $ds^2 = (rd\theta)^2 + (dr)^2$ Length of C_1 inside $C_2 = \int ds$ $= \int \int r^2 + (\frac{dr}{d\theta})^2 d\theta$ 1 - etc	- r dr
Length of C_1 inside $C_2 = \int ds$ $= \int \int r^2 + (\frac{dr}{d\theta})^2 d\theta$ $= etc$	d0
$= \int \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta$ $= e^{tc}$	
$= \int \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta$ $= e^{tc}$	Length of C_1 inside $C_2 = \int ds$
- etc	
etc	1 db , db , db
etc	
	etc

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···ESTRICTED 內部文件

	SOLUTIONS 86	MARKS	REMAI
6. (a)	$f(x) = x^3 - 3x^2 + 4$		
]	$f'(x) = 3x^2 - 6x = 3x(x - 2)$		
Same and the contraction	f''(x) = 6x - 6 = 6(x - 1)	1 - 1	-2
•	the stationary points are at $x = 0$ and $x = 2$.	1.	

3

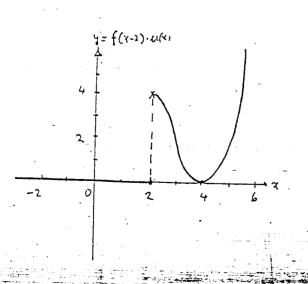
×	-1	0	1	2	1
f(x)	0	4	2	0	1 4
f'(x)	+	0	_	0	+
f''(x)			0 :		•

- : (0, 4) is a maximum point,
 - (2, 0) is a minimum point,

and (1, 2) is the only point of inflexion.

(b)
$$h(x) = f(x-2) \cdot u(x) = \begin{cases} 0 & \text{when } x < 2 \\ f(x-2) & \text{when } x \ge 2. \end{cases}$$

Translating the graph of f(x) ($x \ge 0$) horizontally to the right by 2 units, one obtains the graph of h(x) for $x \ge 2$.



RESTRICTED 內部文件

ACC INCIDENTIAL TO	ESTRICTED 內部	文件		P.1
	SOLUTIONS		MARKS	PEVAN
6. (c) $I_n = \int_0^n e^{-x} h(x)$	dx			REMARKS
$\int_{2}^{n} e^{-x} f(x-x)$	2) de			
$-\int_{0}^{2} e^{-(t+2)}$	(1) 2)	•••••	.1	
「	t(t) dt t=x-v	••• • • • • • • • • • • • • • • • • • •		
, e	t (t ³ - 3t ² + 4) dt	• • • • • • • • •	1	
Now $\int te^{-t}dt = -t$	•		1	
-(ı	t + I)e ^{-t} + c			
t ² e ^{-t} dt = -t	$t^2e^{-t} + 2\int_{te^{-t}dt} e^{-t}$			
($(t^2 + 2t + 2)e^{-t} + c$	1	- 1	
$\int t^3 e^{-t} dt = -t$	$3e^{-t} + 3\int t^2 e^{-t} dt$			
	$t^3 + 3t^2 + 6t + 6)e^{-t} + c$		•	
	- JC + 00 + b)e + c	_ 1	1	
n e - e	$(t^3 + 3t^2 + 6t + 6) - 3(t^2 + 2t)$	$+ 2) + 4] \begin{vmatrix} n-2 \\ 0 \end{vmatrix}$	1	W 11 .
$= -e^{-2}e^{-t}$	c ³ + 4) 0 0			
	$((n-2)^3 + 4) - 4]$		1	
By L'Hospital's rul				
$\lim_{t\to\infty}\frac{t^3}{e^t}=\lim_{t\to\infty}\frac{3t^2}{e^t}=$	$\lim_{t \to \infty} \frac{6t}{e^t} = \lim_{t \to \infty} \frac{6}{e^t} = 0$	たきょん ↓		•
1im I = -e ⁻² ($\lim_{n \to \infty} e^{-(n-2)}(n-2)^3 + \lim_{n \to \infty} e^{-(n-1)}$	2),	1	
47∞ - 4e ⁻²	17 co 17 co	7700		177.
		·····	1	
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LESIKICIED 内部文作	TO RE MATHS II
SOLUTIONS 86 MARKS REMARKS	
7. (a) (1) T: F. 1 - P Position vector 1 T	7. (b) π:x+y+
1: F = a + t b	(x, y,
Putting (2) in (1) $(\vec{a} + t \vec{b}) \cdot \vec{n} = P$	By (a)(11),
And the second s	vector m
$c = \frac{\rho - \vec{a} \cdot \vec{n}}{\vec{b} \cdot \vec{n}} (\vec{b} \cdot \vec{n} \neq 0)$	Since M is
intersects T at the point with position vect	P' is given b
$\frac{1}{a} + \left(\frac{p - a \cdot \hat{\eta}}{b \cdot \hat{\eta}}\right) \hat{b}$	$\overrightarrow{p'} = 2\overrightarrow{m} - \overrightarrow{p} .$
(11) Let the position vector of R_0 be $\vec{r}_0 = x_0 \vec{l} + y_0 \vec{j} + z_0 \vec{k}$.	$= \vec{p} + \frac{2(1-\vec{p})}{\vec{p}}$
The equation of the idea of t	- (α , β , δ)
The equation of the line through R_0 and perpendicular to \vec{r} is $\vec{r} = \vec{r_0} + t \cdot \vec{n}$, $t \in \mathbb{R}$	i.e., the coor
By (1), the nosteton many continues of the state of the s	
By (i), the position vector of the foot of the Candidate:	$y = \beta + \frac{2}{3}$
perpendicular from R_0 to T is $r_0 + \frac{\rho - r_0 \cdot \vec{n}}{\vec{n} \cdot \vec{n}} = \frac{1}{5}$	$z = y + \frac{2}{z}$
2: = a+tb	$\frac{3}{\text{Now } \lambda : x}$
* R. (xo, yo, zo)	, _ _
	If P(α, ρ, 1)
T: F. \(\frac{1}{\pi} = \rho\)	Substituting in
	$x = (1 + \epsilon)$
	y = (2 + 2e)
	z = (3 + 3t)
·	the locus o
	$[or \overrightarrow{r} = (-\frac{7}{3})]$
	$\begin{bmatrix} 0 & r & -(-\frac{3}{3}), \\ 1 & 1 \end{bmatrix}$
	17 23 4 4 1 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2

					P.
S	OLUTIONS	The second secon	97.	MARKS	
7. (b) $T: x + y + z - 1 =$	O can be written		- OC	3	REMAR
	1) - 1 777				
By (a) (11), pp' inter-	sects Th at the nes		-		
vector $\overrightarrow{m} = \overrightarrow{p} + \frac{1}{\overrightarrow{n}}$	<u>p</u> - <u>n</u> <u>p</u> who = =	vith posi	tion		
Since M is the mid-p	oint of pp!	(1, 1, 1)	•••••	1	
P' is given by	产拉一			-	
$\overrightarrow{p'} = 2\overrightarrow{m} - \overrightarrow{p}$	2	The second second			
$= \vec{p} + \frac{2(1-\vec{p}\cdot\vec{n})}{\vec{p}}\vec{n}$	*************	••••••	••••	1	
$ (\alpha, \beta, \delta) + \left(\frac{2(1-1)^{-1}}{(1, 1)^{-1}}\right) $	$(\frac{(+\beta+3)}{(1,1)}$, 1, 1)		. ,	
the coordinates o	f P' are given b	у		1	
$\int x = x + \frac{2}{3} (1 - x -$		•	4	.	
$\int y = \beta + \frac{2}{3} (1 - \alpha - \alpha)$	B- Y)				
$z = 3 + \frac{2}{3}(1 - \alpha -$	β- ¥)	(3)	/		
Now $\lambda : \frac{x-1}{1} = \frac{y-1}{2}$	$\frac{2-z-3}{3}$ can be	written as	!	.	
$\bullet \overline{r} = (1, 2, 3)$	1 + +(1 2 2)			,	•
If P(a, p, 1) lies on	1 , ~ ~ = 1 + c	er e		•	•
	/> = 2 + 2				
9	Y = 3 + 3t				
Substituting in (3)					
$x = (1 + t) + \frac{2}{3}(1 - t)$	1 - t - 2 - 2t - 3	$-3t) = -\frac{7}{3}$	3 t		
$y = (2 + 2t) + \frac{2}{3}(-5 - 1)$	6t) = $-\frac{4}{3}$ - 2t				
$z = (3 + 3t) + \frac{2}{3}(-5 - t)$	6t) = $-\frac{1}{3}$ - t		- :		
the locus of P' is	$\ell': \frac{x+\frac{7}{3}}{2} = \frac{y}{2}$	$+\frac{4}{3}$ $z + \frac{1}{3}$			
:	-	- 1			
[or $\vec{r} = (-\frac{7}{3}, -\frac{4}{3}, -\frac{1}{3}]$) + t(3, 2, 1)	ter].	<u> </u>	_	
			- 9	- .	

$= SOLUTIONS = -86$ $= 8 (a) \cdot E'(x) = \lambda g'(\lambda x + (1 - \lambda)a) - \lambda g'(x)$	MARKS	KEM
$\lambda x + (1 - \lambda)a = x$		100
$F'(a) = 0$ $\sum_{\lambda \in \{i-\lambda\} a, \lambda} \frac{1}{\lambda x + (i-\lambda) x} \cdot \lambda$ $\forall x < a, \lambda x + (1-\lambda)a > x $	1	•
\Rightarrow F'(x) ≥ 0 as g' is increasing	1	
F(x) attains its greatest value at x = a.		
(b) (1) By (a), $F(x) \leq F(a)$	1 4	
For $m = 2$, let $\lambda = \lambda_1^{1/2} \cdot (1 - \lambda_1) \cdot (1 - \lambda_2) \cdot (1 - \lambda_3) \cdot (1 - \lambda_$,2	
$\frac{g(\lambda_1 \chi_1 + \lambda_2 \chi_2) - [\lambda_1 g(\chi_1) + \lambda_2 g(\chi_2)] \leq 0}{g(\lambda_1 \chi_1 + \lambda_2 \chi_2)} \leq 0$	ton	
1.e. $g(\lambda x_1 + \lambda_2 x_2) \le \lambda_1 g(x_1) + \lambda_2 g(x_2)$ Suppose the given statement is true for $2 \le n < k$.	1	•
Let $\lambda_1 + \lambda_2 + \dots + \lambda_{k-1} = \lambda$, $1 - \lambda = \lambda_k$		
$x = \frac{1}{\lambda} \left(\lambda_1 x_1 + \lambda_2 x_2 + \dots + \lambda_{k-1} x_{k-1} \right)$ Then $g(\lambda_1 x_1 + \lambda_2 x_2 + \dots + \lambda_k x_k)$	2	
$= g(\lambda x + (1 - \lambda)x_k)$ $\sqrt{\leq \lambda g(x) + (1 - \lambda)g(x_k)}$		
$= \lambda g(\frac{\lambda_1}{\lambda} x_1 + \frac{\lambda_2}{\lambda} x_2 + \dots + \frac{\lambda_{t-1}}{\lambda} x_{k-1}) + (1 - \lambda)g(x_k)$		
$\{\lambda_1 g(x_1) + \lambda_2 g(x_2) + \dots + \lambda_{k-1} g(x_{k-1}) + \lambda_k g(x_k) \}$ the statement is true for m=k and hence $\forall n \neq 2 \dots$	2	
(ii) Let $g(x) = e^{x}$, which is differentiable and		
g'(x) = e ^x is increasing.		
For any positive numbers a1, a2,, am,		
let a ₁ = e ^x [(1 < 1 < m)	. .	
Then by (1), $ \begin{array}{ccccccccccccccccccccccccccccccccccc$		

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	9. A) For any $x \in I$ $f(x) = f(x) - f(0)$ $= \begin{cases} x & \text{f'}(t) & \text{dt} \end{cases}$		REMARKS
P	$\leq \int_0^\infty f'(x) dt \text{as } f' \text{ is increasing } f(x) > f'(t) \forall t$	[e[0,1)	
	$= x E'(x) \int f(x) dx$ $\frac{\partial f(x) + \partial f(x)}{\partial f(x)} dx = 2E(x) \int (1 + E'(x))^2 dx$	$\frac{1}{3}$	_
P	$\begin{vmatrix} 2f(x) & f'(x) \end{vmatrix}$		
F		1	
P	$F'(x) = f'(x)\sqrt{x^2 + f(x)^2} + \frac{f(x)}{2} \frac{(2x + 2f(x)f'(x))}{\sqrt{x^2 + f(x)^2}}$ $= \frac{xf(x) + x^2f'(x) + 2f(x)^2f'(x)}{\sqrt{x^2 + f(x)^2}}$		
F	$[F'(x)]^{2} - [G(x)]^{2}$ $= \frac{x^{2}f(x)^{2} + x^{4}f'(x)^{2} + 4f(x)^{4}f'(x)^{2} + 2x^{3}f(x)f'(x) + 4xf(x)^{3}f'(x) + 4x^{2}}{x^{2} + f(x)^{2}}$	56.) 751.6	
F	$-4t^{-}(x)(1+t^{-}(x)^{2})$		<u>) </u>
F	$\frac{1}{x^{2}+f(x)^{2}}\left[-\frac{3x^{2}f(x)}{2}+x^{4}f(x)^{2}+2\mathcal{G}^{2}f(x)f(x)+4\mathcal{G}^{2}f(x)^{3}f(x)-4f(x)\right]}{\left(2x^{2}+f(x)^{2}\right)^{2}+x^{2}f(x)^{2}+2x^{2}f(x)^{2}+4f(x)^{3}f(x)-4f(x)\right)^{2}}$		•
•	$f(x) \leq \chi f'(x)$		37.7
	From (a) $f'(x) \ge 0$, $F'(x) \ge 0$ as all quantities involved are non-negative.	1	
	F'(x) $\geq G(x)$,	
「 •	(c) $S = 2\pi \int_0^a f(x) \sqrt{1 + (f'(x))^2} dx$	2	•. *
r • :	$= \pi \int_0^a \frac{G(x)}{G(x)} dx \left(G(x) \ge 2f(x) f(x) \right) \left(F(x) \ge G(x) \right)$ $\therefore 2\pi \int_0^a f(x) f'(x) dx \le S \le \pi \int_0^a f'(x) dx$		•
	$\pi^{f}(x)^{2}_{0}^{a} \leq s \leq \pi^{f}(x)_{0}^{a}$	1	
	$\pi [f(a)]^2 \le S \le \pi f(a) \sqrt{a^2 + [f(a)]^2}$	1 + 1	,
	Provid	ea by	<u>ase.111e</u>