#### 香港考試及評核局 HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

#### 2022年香港中學文憑考試 HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2022

數學 延伸部分 單元二 (代數與微積分)
MATHEMATICS EXTENDED PART MODULE 2 (ALGEBRA AND CALCULUS)

#### 評卷參考 MARKING SCHEME

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Hong Kong Diploma of Secondary Education Examination Mathematics Extended Part Module 2 (Algebra and Calculus)

#### **General Marking Instructions**

- 1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct answer merits all the marks allocated to that part, unless a particular method has been specified in the question. Markers should be patient in marking alternative solutions not specified in the marking scheme.
- 2. In the marking scheme, marks are classified into the following three categories:

'M' marks awarded for correct methods being used; 'A' marks awarded for the accuracy of the answers;

Marks without 'M' or 'A' awarded for correctly completing a proof or arriving

at an answer given in a question.

In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, 'A' marks for the corresponding answers should NOT be awarded (unless otherwise specified).

- 3. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is still likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates' work. In general, marks for a certain step should be awarded if candidates' solution indicated that the relevant concept/technique had been used.
- 4. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.
- In the marking scheme, 'r.t.' stands for 'accepting answers which can be rounded off to' and 'f.t.' stands for 'follow through'. Steps which can be skipped are shaded whereas alternative answers are enclosed with rectangles. All fractional answers must be simplified.
- 6. Unless otherwise specified in the question, numerical answers not given in exact values should not be accepted.

Solution	Marks Remarks
1. $g(1+h)-g(1)$	
= 1 1	
$=\frac{1}{\sqrt{5(1+h)+4}}-\frac{1}{\sqrt{5(1)+4}}$	
$=\frac{3-\sqrt{5h+9}}{3\sqrt{5h+9}}$	
$=\frac{3^2-(5h+9)}{3\sqrt{5h+9}(3+\sqrt{5h+9})}$	
$=\frac{-5h}{3\sqrt{5h+9}(3+\sqrt{5h+9})}$	1
g'(1)	
$=\lim_{h\to 0}\frac{\mathrm{g}(1+h)-\mathrm{g}(1)}{h}$	1 <b>M</b>
$= \lim_{h \to 0} \frac{1}{h} \left( \frac{-5h}{3\sqrt{5h+9}(3+\sqrt{5h+9})} \right)$	
$= \lim_{h \to 0} \frac{1}{h} \left( \frac{3\sqrt{5h+9}(3+\sqrt{5h+9})}{3\sqrt{5h+9}(3+\sqrt{5h+9})} \right)$	
$= \lim_{h \to 0} \frac{-5}{3\sqrt{5h+9}(3+\sqrt{5h+9})}$	1M withhold 1M if this step is skippe
· · · · · · · · · · · · · · · · · · ·	Within the Title Stop is skipped
= \frac{-5}{54}	IA
J. <del></del>	(4)
tan A cot A	
(a) $\frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta}$	
$= \frac{\tan^2 \theta}{\tan \theta - 1} + \frac{1}{\tan \theta (1 - \tan \theta)}$	1M
	1141
$=\frac{\tan^3\theta-1}{\tan\theta(\tan\theta-1)}$	
$=\frac{(\tan\theta-1)(\tan^2\theta+\tan\theta+1)}{\tan\theta(\tan\theta-1)}$	1 <b>M</b>
$= \sec^2 \theta + \tan \theta$	
$={\tan \theta}$	
$=1+\frac{\sec^2\theta}{\tan\theta}$	
$\tan \theta$ $= 1 + \sec \theta \csc \theta$	
(b) $\frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 5$	
$1 + \sec\theta \csc\theta = 5$	
$\sin\theta\cos\theta = \frac{1}{4}$	
$\sin 2\theta = \frac{1}{2}$	1M
$2\theta = \frac{5\pi}{6}$	
$\theta = \frac{5\pi}{12}$	IA
12	(5)
	[3]
)22-DSE-MATH-EP(M2)-3	

Solution			Remarks
(a)	For $n = 1$ , L.H.S. = $(-1)(1^2) + (-1)^2(2^2) = 3$		
	R.H.S. = $(-1)(1) + (-1)(2) = 3$ R.H.S. = $(1)(2+1) = 3$		
	Therefore, the statement is true for $n=1$ .	1	
	Therefore, the statement is true for $n-1$ .	1	
	Assume that $\sum_{k=1}^{2m} (-1)^k k^2 = m(2m+1)$ , where m is a positive integer.	1M	
	$\sum_{k=1}^{2(m+1)} (-1)^k k^2$		
	$=\sum_{k=1}^{2m}(-1)^k k^2 - (2m+1)^2 + (2m+2)^2$		
	$= m(2m+1) - (2m+1)^2 + (2m+2)^2$ (by induction assumption)	1M	for using induction assumption
	$=2m^2+5m+3$		•
	=(m+1)(2m+3)		
	So, the statement is true for $n=m+1$ if it is true for $n=m$ . By mathematical induction, the statement is true for all positive integers $n$ .	I	
(b)	$\sum_{k=1}^{100} (-1)^k k^2$ $= \sum_{k=1}^{100} (-1)^k k^2 - \sum_{k=1}^{10} (-1)^k k^2$ $= 50(2(50) + 1) - 5(2(5) + 1)$ $= 4995$	1M 1M 1A (7)	
	-MATH-EP(M2)–4		

	Solution	Marks	Remarks
(a)	$\frac{\mathrm{d}y}{\mathrm{d}x}$		
	$dx = (7-4x)e^{-x} + (7x-2x^2)(e^{-x})(-1)$	13.4	
	$= (2x^2 - 11x + 7)e^{-x}$	1M 1A	
	-(2x-11x+7)e	IA	
	$d^2y$		
	$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$		
	$= (4x-11)e^{-x} + (2x^2-11x+7)(e^{-x})(-1)$		
	$= (-2x^2 + 15x - 18)e^{-x}$	ΙA	
	42,,		
(b)	$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = 0$		
	$(-2x^2 + 15x - 18)e^{-x} = 0$		
	$-2x^2 + 15x - 18 = 0$		
	$x = \frac{3}{2}$ or $x = 6$	1 <b>M</b>	
	2		
	$x = \left(-\infty, \frac{3}{2}\right) = \frac{3}{2} = \left(\frac{3}{2}, 6\right) = 6 = \left(6, \infty\right)$		
	THE RESIDENCE OF THE PROPERTY	1M	for testing
	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		
	Therefore, there are two points of inflexion of the graph		
	of $y = (7x - 2x^2)e^{-x}$ .		
	Thus, the claim is agreed.	1A (6)	f.t.

2022-DSE-MATH-EP(M2)-5

#### **CONFIDENTIAL (FOR MARKER'S USE ONLY)**

	Solution	Marks	Remarks
(a)	$(a+x)^n$		
	$= a^{n} + C_{1}^{n} a^{n-1} x + C_{2}^{n} a^{n-2} x^{2} + \dots + x^{n}$		
	$= a^{n} + a^{n-1}nx + \frac{a^{n-2}n(n-1)}{2}x^{2} + \dots + x^{n}$	1M	
	$\mu_2 = -10$		
	$\frac{a^{n-2}n(n-1)}{2} = -10$	IM	
	$a^{n-2}n(n-1) = -20$		
	Since $n$ is greater than $1$ , $a^{n-2}$ is a negative number. Thus, $a$ is a negative number and $n$ is an odd number.	I	
(b)	$(bx-1)^n$		
	$= (-1)^n + C_1^n (-1)^{n-1} (bx) + C_2^n (-1)^{n-2} (bx)^2 + \dots + (bx)^n$		
	$= (-1)^{n} + (-1)^{n-1}bnx + (-1)^{n-2}\frac{b^{2}n(n-1)}{2}x^{2} + \dots + b^{n}x^{n}$		
	$\lambda_0 = \mu_0$		
	$(-1)^n = a^n$		
	a = -1	IA	
	$\lambda_1 = 2\mu_1$		
	$(-1)^{n-1}bn = 2a^{n-1}n$		
	<i>b</i> = 2	IA	
	$a^{n-2}n(n-1) = -20$		
	$(-1)^{n-2}n(n-1) = -20$		
	n(n-1) = 20 (since <i>n</i> is an odd number)		
	$n^2 - n - 20 = 0$ (n-5)(n+4) = 0		
	n=5 or $n=-4$ (rejected)	1A	
		(6)	

2022-DSE-MATH-EP(M2)-6

Sc	olution	Marks	Remarks
(a) Note that $x^2 + 2x + 5 \equiv (x + 1)^2$	$(2)^2 + 2^2$ .		
Let $x+1=2\tan\theta$ .		1M	
So, we have $\frac{dx}{d\theta} = 2\sec^2\theta$ .			
$\int \frac{1}{x^2 + 2x + 5}  \mathrm{d}x$			
$= \int \frac{1}{4 \tan^2 \theta + 4} (2 \sec^2 \theta) d\theta$		IM	
$= \int \frac{1}{2}  \mathrm{d}  \theta$			
$=\frac{1}{2}\theta$ + constant			
$= \frac{1}{2} \tan^{-1} \left( \frac{x+1}{2} \right) + \text{constant}$		1	
(b) <i>y</i>			
$= \int \frac{2x+1}{x^2+2x+5}  \mathrm{d}x$			
$= \int \frac{2x+2}{x^2+2x+5}  \mathrm{d}x - \int \frac{1}{x^2+1}  \mathrm{d}x$			
$= \int \frac{1}{x^2 + 2x + 5}  \mathrm{d}(x^2 + 2x +$	•	IM	
$= \ln \left  x^2 + 2x + 5 \right  - \frac{1}{2} \tan^{-1} \left( \frac{x}{2} \right)^{-1}$	$\left(\frac{+1}{2}\right) + C$ , where C is a constant	1M	
When $x = -3$ , $y = \ln 2$ . $\ln 2 = \ln \left  (-3)^2 + 2(-3) + 5 \right  - \frac{1}{2}$	$\frac{1}{1+an^{-1}}\left(\frac{(-3)+1}{1+an^{-1}}\right) + C$		
	2 " ( 2 ) ( )		
$\ln 2 = \ln 8 - (\frac{1}{2})(\frac{-\pi}{4}) + C$		1M	
$C = -2\ln 2 - \frac{\pi}{8}$			
The equation of $G$ is $y = \ln x$	$\left x^2 + 2x + 5\right  - \frac{1}{2} \tan^{-1} \left(\frac{x+1}{2}\right) - 2 \ln 2$	$-\frac{\pi}{8}$ .	
When $x = -1$ ,			
$= \ln \left  (-1)^2 + 2(-1) + 5 \right  - \frac{1}{2} \tan \theta$	$1^{-1}\left(\frac{(-1)+1}{2}\right)-2\ln 2-\frac{\pi}{8}$	,	
$=\frac{-\pi}{8}$			
Thus, $G$ passes through the	point $\left(-1, \frac{-\pi}{8}\right)$ .	1A	f.t.
		[(7)	
22-DSE-MATH-EP(M2)-7			

	Solution	Marks	Remarks
(a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{x+2}$		
	The equation of L is $y = \frac{1}{h+2}(x-h) + \ln(h+2)$ .	1 M	
	A		
	$= \int_0^h \left( \frac{1}{h+2} (x-h) + \ln(h+2) - \ln(x+2) \right) dx$	1 <b>M</b>	
	$= \left[\frac{1}{h+2}\left(\frac{1}{2}x^2 - hx\right) + x\ln(h+2)\right]_0^h - \left[x\ln(x+2)\right]_0^h + \int_0^h \frac{x}{x+2} dx$		
	$= \frac{1}{h+2} \left( \frac{1}{2} h^2 - h^2 \right) + h \ln(h+2) - h \ln(h+2) + \int_0^h \left( 1 - \frac{2}{x+2} \right) dx$	1 <b>M</b>	
	$= \frac{-h^2}{2h+4} + \left[x-2\ln x+2 \right]_0^h$		
	$= \frac{h^2 + 4h}{2h + 4} - 2\ln(h + 2) + 2\ln 2$	1	
(b)	$h = 3^{-t} = e^{-t \ln 3}$ $\frac{dh}{dt} = (-\ln 3)e^{-t \ln 3} = (-\ln 3)3^{-t}$	1M	
	u,		
	$= \frac{\frac{\mathrm{d}A}{\mathrm{d}h}}{(2h+4)(2h+4) - (h^2+4h)(2)} - \frac{2}{h+2}$	1M	
	$=\frac{h^2}{2(h+2)^2}$		
	$\frac{\mathrm{d}A}{\mathrm{d}t}$		
	$= \frac{\mathrm{d}A}{\mathrm{d}h} \frac{\mathrm{d}h}{\mathrm{d}t}$		
	$= \frac{(3^{-t})^2}{2(3^{-t} + 2)^2} (-\ln 3)3^{-t}$	1M	
	$=\frac{-\ln 3}{2(3')(1+2(3'))^2}$		
	$\frac{\mathrm{d}A}{\mathrm{d}t}\Big _{t=1}$		
	$=\frac{-\ln 3}{2(3^1)(1+2(3^1))^2}$		
	$=\frac{-\ln 3}{294}$	1A	
	Thus, the rate of change of A is $\frac{-\ln 3}{294}$ when $t = 1$ .	(0)	
		(8)	
	-MATH-EP(M2)–8		

Solution	Marks	Remarks
(a) $\Delta$ $= \begin{vmatrix} a & 2 & -1 \\ -1 & a & 2 \\ 2 & -1 & a \end{vmatrix}$ $= a^3 + 6a + 7$	1 <b>M</b>	
$\Delta_{y}$ =\begin{array}{ccc} a & 4k & -1 \\ -1 & 4 & 2 \\ 2 & k^2 & a \end{array} = 4a^2 + 4ak - 2ak^2 + k^2 + 16k + 8  Note that $\Delta \neq 0$ .		
$= \frac{\Delta_y}{\Delta}$ $= \frac{4a^2 + 4ak - 2ak^2 + k^2 + 16k + 8}{a^3 + 6a + 7}$	1M+1A	lM for Cramer's rule
(b) As (E) has infinitely many solutions, $\Delta = 0$ $a^{3} + 6a + 7 = 0$ $(a+1)(a^{2} - a + 7) = 0$ $a = -1$	1M	
The augmented matrix of (E) becomes $ \begin{pmatrix} -1 & 2 & -1 &   & 4k \\ -1 & -1 & 2 &   & 4 \\ 2 & -1 & -1 &   & k^2 \end{pmatrix} \sim \begin{pmatrix} -1 & 2 & -1 &   & 4k \\ 0 & -3 & 3 &   & 4-4k \\ 0 & 0 & 0 &   & k^2+4k+4 \end{pmatrix} $	1M	
$k^{2} + 4k + 4 = 0$ $(k+2)^{2} = 0$ $k = -2$	1M	either one
Hence, the augmented matrix of (E) becomes $ \begin{pmatrix} -1 & 2 & -1 &   & -8 \\ 0 & -3 & 3 &   & 12 \\ 0 & 0 & 0 &   & 0 \end{pmatrix} \sim \begin{pmatrix} -1 & 2 & -1 &   & -8 \\ 0 & -1 & 1 &   & 4 \\ 0 & 0 & 0 &   & 0 \end{pmatrix} $		
Thus, the solution set of $(E)$ is $\{(t, t-4, t): t \in \mathbb{R} \}$ .	1A (7)	
22-DSE-MATH-EP(M2)–9		

	Solution	Marks	Remarks
(a)	The equation of the vertical asymptote is $x - 1 = 0$ .	IA	
	Note that $\frac{x^2 + 3x}{x - 1} = x + 4 + \frac{4}{x - 1}$ .	IM	
	Thus, the equation of the oblique asymptote is $y = x + 4$ .	1A (3)	
(b)	$f'(x) = 1 - \frac{4}{(x-1)^2}$	IM	
	f'(x) = 0		
	$1 - \frac{4}{(x-1)^2} = 0$		
	$x^2 - 2x - 3 = 0$		
	x = -1 or $x = 3$	1M	
	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	IM	
	t the state of the		
	Thus, the maximum point and minimum point of $H$ are $(-1, 1)$ and $(3, 9)$ respectively.	1A	
		(4)	
(c)			
	y = f(x) $y = x + 4$		
	y = f(x) $x = 1$	<b>&gt;</b> .x	
	.//	1M 1M 1A (3)	for shape for asymptotes for all correct

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	Solution	Marks	Remarks
	$f(x) = 10$ $\frac{x^2 + 3x}{x - 1} = 10$ $x^2 - 7x + 10 = 0$ $x = 2 \text{ or } x = 5$ The required volume $= \pi \int_{2}^{5} (f(x) - 10)^2 dx$ $= \pi \int_{2}^{5} \left(x + 4 + \frac{4}{x - 1} - 10\right)^2 dx$	IM+1A	
	$= \pi \int_{2}^{5} \left(x - 6 + \frac{4}{x - 1}\right)^{2} dx$ $= \pi \int_{2}^{5} \left((x - 6)^{2} + \frac{8(x - 6)}{x - 1} + \frac{16}{(x - 1)^{2}}\right) dx$ $= \pi \int_{2}^{5} \left((x - 6)^{2} + 8 - \frac{40}{x - 1} + \frac{16}{(x - 1)^{2}}\right) dx$		
	$= \pi \left[ \frac{(x-6)^3}{3} + 8x - 40 \ln x-1  - \frac{16}{x-1} \right]_2^5$ $= \pi (57 - 80 \ln 2)$	1A (3)	
<b>2</b> "DSF	MATH-EP(M2)–11		

	Solution	Marks	Remarks
0. (a)	$\int g(x) dx$		
	$= \frac{\sin 2x \cos^2 x}{2} - \int \frac{\sin 2x}{2} (2\cos x)(-\sin x) dx$	lM	
	$= \frac{\sin 2x \cos^2 x}{2} + \frac{1}{2} \int \sin^2 2x  dx$	1	
	2 2 J		
	$\frac{\mathrm{d}}{\mathrm{d}x} \left( \frac{\sin 2x \cos^2 x}{2} \right)$		
	$= \frac{1}{2}(\cos 2x)(2)(\cos^2 x) + \frac{1}{2}(\sin 2x)(2\cos x)(-\sin x)$		
	$=\cos^2 x \cos 2x - \frac{1}{2}\sin^2 2x$		
	$\therefore \cos^2 x \cos 2x = \frac{d}{dx} \left( \frac{\sin 2x \cos^2 x}{2} \right) + \frac{1}{2} \sin^2 2x$	1M	
	$\int \cos^2 x \cos 2x  dx = \int \left( \frac{d}{dx} \left( \frac{\sin 2x \cos^2 x}{2} \right) + \frac{1}{2} \sin^2 2x \right) dx$		
	$\int g(x) dx = \frac{\sin 2x \cos^2 x}{2} + \frac{1}{2} \int \sin^2 2x dx$	1	
		(2)	
(b)	$\int_0^{\pi} g(x) dx$		
	$= \left[ \frac{\sin 2x \cos^2 x}{2} \right]_0^{\pi} + \frac{1}{2} \int_0^{\pi} \sin^2 2x  dx \qquad \text{(by (a))}$	1 <b>M</b>	IM for using (a)
	$= \left[ \frac{\sin 2x \cos^2 x}{2} \right]_0^{\pi} + \frac{1}{2} \int_0^{\pi} \frac{1 - \cos 4x}{2} dx$		
	0		
	$=0+\frac{1}{4}\left[x-\frac{\sin 4x}{4}\right]_0^{\pi}$		
	$=\frac{\pi}{4}$	1A (2)	
		!	
)22-DSE	E-MATH-EP(M2)-12	l	

Solution	Marks	Remarks
(c) Let $u = \pi - x$ .	1M	
$\int_0^{\pi} x g(x) dx$		
$= \int_{\pi}^{0} -(\pi - u)\cos^{2}(\pi - u)\cos 2(\pi - u) du$	MI	
$= \int_0^{\pi} (\pi - u) \cos^2 u \cos 2u  \mathrm{d}u$		
$= \int_0^{\pi} (\pi - x) \cos^2 x \cos 2x  \mathrm{d}x$		
$= \pi \int_0^{\pi} g(x) dx - \int_0^{\pi} x g(x) dx$		
$\int_0^\pi x g(x)  \mathrm{d}x$		
$=\frac{\pi}{2}\int_0^{\pi} g(x) dx$	1 <b>M</b>	
$=\frac{\pi}{2}\left(\frac{\pi}{4}\right)$		
$=\frac{\pi^2}{8}$		
= 8	1 A	

	Solution	Marks	Remarks
(d) $(-x)g(-x)$ $= (-x)\cos^2(-x)\cos 2x$ $= -x\cos^2 x \cos 2x$ $= -xg(x)$ $\therefore xg(x) \text{ is an odd fu}$		1M	
So, we have $\int_{-\pi}^{\pi} xg(x)$	dx = 0 .		
Let $y = x - \pi$ . $\int_{\pi}^{2\pi} x g(x) dx$		1M	
$= \int_0^{\pi} (\pi + y) \cos^2(\pi + y)$ $= \int_0^{\pi} (\pi + y) \cos^2 y \cos^2 y$			
$= \int_0^{\pi} (\pi + x) \cos^2 x \cos^2 $	s2xdx		
$= \pi \int_0^{\pi} g(x) dx + \int_0^{\pi} x g(x) dx + \int_$	( by (b) and (c) )	IM	
$\int_{-\pi}^{2\pi} x g(x) dx$ $= \int_{-\pi}^{\pi} x g(x) dx + \int_{\pi}^{2\pi} x g(x) dx$	g(x) dx		
$=0+\frac{3\pi^2}{8}$ $=\frac{3\pi^2}{8}$		1A	
		(4)	
DSE-MATH-EP(M2)–14			

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	Solution	Marks	Remarks
I. (a) (i)	$(I - A)(I + A + A^{2} + \dots + A^{n})$ $= I + A + A^{2} + \dots + A^{n} - (A + A^{2} + A^{3} + \dots + A^{n+1})$ $= I - A^{n+1}$	1A	
(ii)	(1) $I - A$ $= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$ $= \begin{pmatrix} 1 - \cos \theta & \sin \theta \\ -\sin \theta & 1 - \cos \theta \end{pmatrix}$		
	$ I - A $ $= (1 - \cos \theta)^2 + \sin^2 \theta$ $= 2(1 - \cos \theta)$	IM	
	$(I - A)^{-1}$ $= \frac{1}{2(1 - \cos \theta)} \begin{pmatrix} 1 - \cos \theta & -\sin \theta \\ \sin \theta & 1 - \cos \theta \end{pmatrix}$	1M	
	$= \frac{1}{2} \begin{pmatrix} 1 & \frac{-\sin\theta}{1-\cos\theta} \\ \frac{\sin\theta}{1-\cos\theta} & 1 \end{pmatrix}$ $= \frac{1}{2} \begin{pmatrix} 1 & \frac{-2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}} \\ \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}} & 1 \end{pmatrix}$ $= \frac{1}{2\sin\frac{\theta}{2}} \begin{pmatrix} \sin\frac{\theta}{2} & -\cos\frac{\theta}{2} \\ \cos\frac{\theta}{2} & \sin\frac{\theta}{2} \end{pmatrix}$	1	
	2 ( 2 2 )		

# CONFIDENTIAL (FOR MARKER'S USE ONLY)

Solution	Marks	Remarks
Solution	IVIGINS	Nomans
(2) $I + A + A^2 + \dots + A^n$ = $(I - A)^{-1} (I - A^{n+1})$	1M	
$= \frac{1}{2\sin\frac{\theta}{2}} \begin{pmatrix} \sin\frac{\theta}{2} & -\cos\frac{\theta}{2} \\ \cos\frac{\theta}{2} & \sin\frac{\theta}{2} \end{pmatrix} \begin{pmatrix} 1 - \cos(n+1)\theta & \sin(n+1)\theta \\ -\sin(n+1)\theta & 1 - \cos(n+1)\theta \end{pmatrix}$		
$= \frac{1}{2\sin\frac{\theta}{2}} \begin{pmatrix} \sin\frac{\theta}{2} & -\cos\frac{\theta}{2} \\ \cos\frac{\theta}{2} & \sin\frac{\theta}{2} \end{pmatrix} \begin{pmatrix} 2\sin\frac{(n+1)\theta}{2} & \cos\frac{(n+1)\theta}{2} \\ -\cos\frac{(n+1)\theta}{2} & \sin\frac{\theta}{2} \end{pmatrix}$	$\frac{(n+1)\theta}{2}$ $\frac{(n+1)\theta}{2}$ $1M$	
$= \frac{\sin\frac{(n+1)\theta}{2}}{\sin\frac{\theta}{2}} \left( \frac{\sin\frac{\theta}{2}\sin\frac{(n+1)\theta}{2} + \cos\frac{\theta}{2}\cos\frac{(n+1)\theta}{2}}{\cos\frac{\theta}{2}\sin\frac{(n+1)\theta}{2} - \sin\frac{\theta}{2}\cos\frac{(n+1)\theta}{2}} - \sin\frac{\theta}{2}\cos\frac{(n+1)\theta}{2} - \cos\frac{\theta}{2}\cos\frac{(n+1)\theta}{2} - \cos\frac{\theta}{2}\cos\frac$	$s\frac{(n+1)\theta}{2} - c$ $s\frac{(n+1)\theta}{2} + c$	$\cos\frac{\theta}{2}\sin\frac{(n+1)\theta}{2}$ $\sin\frac{\theta}{2}\sin\frac{(n+1)\theta}{2}$
$= \frac{\sin\frac{(n+1)\theta}{2}}{\sin\frac{\theta}{2}} \begin{pmatrix} \cos\frac{n\theta}{2} & -\sin\frac{n\theta}{2} \\ \sin\frac{n\theta}{2} & \cos\frac{n\theta}{2} \end{pmatrix}$	1	
	(7)	
(b) (i) $I + A + A^2 + \dots + A^n$	of Advention and	
$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} + \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}^2 + \cdots$		
$+ \left( \frac{\cos \theta - \sin \theta}{\sin \theta - \cos \theta} \right)^n$		
$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} + \begin{pmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} + \cdots$		
$+\begin{pmatrix} \cos n\theta & -\sin n\theta \\ \sin n\theta & \cos n\theta \end{pmatrix}$		
$= \begin{pmatrix} 1 + \cos \theta + \cos 2\theta + \dots + \cos n\theta & -\sin \theta - \sin 2\theta - \dots - \sin n\theta \\ \sin \theta + \sin 2\theta + \dots + \sin n\theta & 1 + \cos \theta + \cos 2\theta + \dots + \cos n\theta \end{pmatrix}$	1M	
$\therefore 1 + \cos \theta + \cos 2\theta + \dots + \cos n\theta = \frac{\sin \frac{(n+1)\theta}{2} \cos \frac{n\theta}{2}}{\sin \frac{\theta}{2}}$	1M	
Putting $\theta = \frac{5\pi}{18}$ and $n = 90$ ,		
$\cos \frac{5\pi}{18} + \cos \frac{5\pi}{9} + \cos \frac{5\pi}{6} + \dots + \cos 25\pi$		
• • • • • • • • • • • • • • • • • • • •		
$= \frac{\sin\frac{445\pi}{36}\cos\frac{25\pi}{2}}{\sin\frac{5\pi}{36}} - 1$		
=-1	1A	

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Solution  Marks  Remarks  (ii) $\cos^2 \frac{\pi}{7} + \cos^2 \frac{2\pi}{7} + \cos^2 \frac{7\pi}{7} + \cdots + \cos^2 7\pi$ $= \frac{1}{2} \left( 1 + \cos \frac{2\pi}{7} \right) + \frac{1}{2} \left( 1 + \cos \frac{4\pi}{7} \right) + \frac{1}{2} \left( 1 + \cos \frac{5\pi}{7} \right) + \cdots$ $+ \frac{1}{2} \left( 1 + \cos \frac{2\pi}{7} + \cos \frac{4\pi}{7} + \cos \frac{6\pi}{7} + \cdots + \cos 14\pi \right)$ IM $\cos^2 \frac{\pi}{7} + \cos^2 \frac{4\pi}{7} + \cos^2 \frac{6\pi}{7} + \cdots + \cos 14\pi$ $= \frac{\sin \frac{5\pi}{7}}{\sin \frac{\pi}{7}} - 1  (\text{putting } \theta = \frac{2\pi}{7} \text{ and } n = 49 \text{ in (b)(i)})$ $= 0$ $\therefore \cos^2 \frac{\pi}{7} + \cos^2 \frac{2\pi}{7} + \cos^2 \frac{2\pi}{7} + \cdots + \cos^2 7\pi = \frac{49}{2}$ 1A  1A  2022-DSE-MATH-EP(M2)-17	CONFIDENTIAL (FOR MARKER'S U	SE ONL	<b>Y</b> )
$= \frac{1}{2} \left( 1 + \cos \frac{2\pi}{7} \right) + \frac{1}{2} \left( 1 + \cos \frac{4\pi}{7} \right) + \frac{1}{2} \left( 1 + \cos \frac{6\pi}{7} \right) + \cdots$ $+ \frac{1}{2} \left( 1 + \cos \frac{4\pi}{7} + \cos \frac{4\pi}{7} + \cos \frac{6\pi}{7} + \cdots + \cos \frac{14\pi}{7} \right)$ $= \frac{1}{2} \left( 49 + \cos \frac{2\pi}{7} + \cos \frac{6\pi}{7} + \cdots + \cos \frac{14\pi}{7} \right)$ $= \cos \frac{2\pi}{7} + \cos \frac{7\pi}{7} + \cos \frac{6\pi}{7} + \cdots + \cos \frac{14\pi}{7}$ $= \frac{\sin \frac{50\pi}{7} \cos 7\pi}{\sin \frac{\pi}{7}} - 1 \qquad (\text{putting } \theta = \frac{2\pi}{7} \text{ and } n = 49 \text{ in (b)(i)})$ $= 0$ $\therefore \cos^2 \frac{\pi}{7} + \cos^2 \frac{2\pi}{7} + \cos^2 \frac{3\pi}{7} + \cdots + \cos^2 7\pi = \frac{49}{2}$ $1A$ (6)	Solution	Marks	Remarks
$= \frac{1}{2} \left( 1 + \cos \frac{2\pi}{7} \right) + \frac{1}{2} \left( 1 + \cos \frac{4\pi}{7} \right) + \frac{1}{2} \left( 1 + \cos \frac{6\pi}{7} \right) + \cdots$ $+ \frac{1}{2} \left( 1 + \cos \frac{4\pi}{7} + \cos \frac{4\pi}{7} + \cos \frac{6\pi}{7} + \cdots + \cos \frac{14\pi}{7} \right)$ $= \frac{1}{2} \left( 49 + \cos \frac{2\pi}{7} + \cos \frac{6\pi}{7} + \cdots + \cos \frac{14\pi}{7} \right)$ $= \cos \frac{2\pi}{7} + \cos \frac{7\pi}{7} + \cos \frac{6\pi}{7} + \cdots + \cos \frac{14\pi}{7}$ $= \frac{\sin \frac{50\pi}{7} \cos 7\pi}{\sin \frac{\pi}{7}} - 1 \qquad (\text{putting } \theta = \frac{2\pi}{7} \text{ and } n = 49 \text{ in (b)(i)})$ $= 0$ $\therefore \cos^2 \frac{\pi}{7} + \cos^2 \frac{2\pi}{7} + \cos^2 \frac{3\pi}{7} + \cdots + \cos^2 7\pi = \frac{49}{2}$ $1A$ (6)	(ii) $\cos^2 \frac{\pi}{-} + \cos^2 \frac{2\pi}{-} + \cos^2 \frac{3\pi}{-} + \dots + \cos^2 7\pi$		
$ \frac{1}{2}(1 + \cos 14\pi) $ $ = \frac{1}{2}(49 + \cos \frac{2\pi}{7} + \cos \frac{4\pi}{7} + \cos \frac{6\pi}{7} + \dots + \cos 14\pi) $ $ \cos \frac{2\pi}{7} + \cos \frac{4\pi}{7} + \cos \frac{6\pi}{7} + \dots + \cos 14\pi $ $ = \frac{\sin \frac{50\pi}{7} \cos 7\pi}{\sin \frac{\pi}{7}} - 1  (\text{putting } \theta = \frac{2\pi}{7} \text{ and } n = 49 \text{ in (b)(i)}) $ $ = 0 $ $ \therefore \cos^2 \frac{\pi}{7} + \cos^2 \frac{2\pi}{7} + \cos^2 \frac{3\pi}{7} + \dots + \cos^2 7\pi = \frac{49}{2} $ $ 1A $ (6)	, , , , , , , , , , , , , , , , , , , ,		
$= \frac{1}{2} \left( 49 + \cos \frac{2\pi}{7} + \cos \frac{4\pi}{7} + \cos \frac{6\pi}{7} + \dots + \cos 14\pi \right)$ $\cos \frac{2\pi}{7} + \cos \frac{4\pi}{7} + \cos \frac{6\pi}{7} + \dots + \cos 14\pi$ $= \frac{\sin \frac{50\pi}{7} - \cos 7\pi}{\sin \frac{\pi}{7}} - 1 \qquad \text{( putting } \theta = \frac{2\pi}{7} \text{ and } n = 49 \text{ in (b)(i))}$ $= 0$ $\therefore \cos^2 \frac{\pi}{7} + \cos^2 \frac{2\pi}{7} + \cos^2 \frac{3\pi}{7} + \dots + \cos^2 7\pi = \frac{49}{2}$ $1A$ (6)			
$\cos \frac{2\pi}{7} + \cos \frac{4\pi}{7} + \cos \frac{6\pi}{7} + \dots + \cos 14\pi$ $= \frac{\sin \frac{50\pi}{7} \cos 7\pi}{\sin \frac{\pi}{7}} - 1 \qquad (\text{ putting } \theta = \frac{2\pi}{7} \text{ and } n = 49 \text{ in (b)(i)})$ $= 0$ $\therefore \cos^2 \frac{\pi}{7} + \cos^2 \frac{2\pi}{7} + \cos^2 \frac{3\pi}{7} + \dots + \cos^2 7\pi = \frac{49}{2}$ $1A$ $(6)$	Z	:	
$= \frac{\sin \frac{50\pi}{7} \cos 7\pi}{\sin \frac{\pi}{7}} - 1 \qquad (putting \ \theta = \frac{2\pi}{7} \text{ and } n = 49 \text{ in (b)(i)})$ $= 0$ $\therefore \cos^2 \frac{\pi}{7} + \cos^2 \frac{2\pi}{7} + \cos^2 \frac{3\pi}{7} + \dots + \cos^2 7\pi = \frac{49}{2}$ $1A$ $(6)$	$= \frac{1}{2} \left( 49 + \cos \frac{2\pi}{7} + \cos \frac{4\pi}{7} + \cos \frac{6\pi}{7} + \dots + \cos 14\pi \right)$	Mi	
$\therefore \cos^{2} \frac{\pi}{7} + \cos^{2} \frac{2\pi}{7} + \cos^{2} \frac{3\pi}{7} + \dots + \cos^{2} 7\pi = \frac{49}{2}$ $1A \qquad (6)$	, , , , , , , , , , , , , , , , , , , ,		
$\therefore \cos^{2} \frac{\pi}{7} + \cos^{2} \frac{2\pi}{7} + \cos^{2} \frac{3\pi}{7} + \dots + \cos^{2} 7\pi = \frac{49}{2}$ $1A \qquad (6)$	$= \frac{\sin \frac{50\pi}{7} \cos 7\pi}{\sin \frac{\pi}{7}} - 1 \qquad \text{(putting } \theta = \frac{2\pi}{7} \text{ and } n = 49 \text{ in (b)(i))}$	IМ	
	,		
	$\therefore \cos^2 \frac{\pi}{7} + \cos^2 \frac{2\pi}{7} + \cos^2 \frac{3\pi}{7} + \dots + \cos^2 7\pi = \frac{49}{2}$		
2022-DSE-MATH-EP(M2)–17		(6)	
2022-DSE-MATH-EP(M2)–17			
2022-DSE-MATH-EP(M2)—17			
2022-DSE-MATH-EP(M2)–17			
2022-DSE-MATH-EP(M2)–17			
2022-DSE-MATH-EP(M2)—17			
2022-DSE-MATH-EP(M2)-17			
2022-DSE-MATH-EP(M2)—17			
2022-DSE-MATH-EP(M2)—17			
2022-DSE-MATH-EP(M2)–17			
2022-DSE-MATH-EP(M2)-17			
2022-DSE-MATH-EP(M2)–17			
	2022-DSE-MATH-EP(M2)–17		

CONFIDENTIAL (FOR MARKE)		
Solution	Marks	Remarks
12. (a) (i) $\overrightarrow{OD}$ $= \frac{b}{b+c} \overrightarrow{OB} + \frac{c}{b+c} \overrightarrow{OC}$	1M	
$\overrightarrow{AD}$ $= \overrightarrow{OD} - \overrightarrow{OA}$ $= \frac{b}{b+c} \overrightarrow{OB} + \frac{c}{b+c} \overrightarrow{OC} - \overrightarrow{OA}$ $= -\overrightarrow{OA} + \frac{b}{b+c} \overrightarrow{OB} + \frac{c}{b+c} \overrightarrow{OC}$	1	
(ii) $\overrightarrow{AJ}$ $= \overrightarrow{OJ} - \overrightarrow{OA}$ $= \frac{a}{a+b+c} \overrightarrow{OA} + \frac{b}{a+b+c} \overrightarrow{OB} + \frac{c}{a+b+c} \overrightarrow{OC} - \overrightarrow{OA}$ $= \frac{-(b+c)}{a+b+c} \overrightarrow{OA} + \frac{b}{a+b+c} \overrightarrow{OB} + \frac{c}{a+b+c} \overrightarrow{OC}$		
$= \left(\frac{b+c}{a+b+c}\right)\left(-\overrightarrow{OA} + \frac{b}{b+c}\overrightarrow{OB} + \frac{c}{b+c}\overrightarrow{OC}\right)$ $= \frac{b+c}{a+b+c}\overrightarrow{AD}$	1M	
Since $0 < \frac{b+c}{a+b+c} < 1$ , J lies on AD.	1 <b>M</b>	withhold 1M if this step is skippe
$\overrightarrow{OE}$ $= \frac{a}{a+c} \overrightarrow{OA} + \frac{c}{a+c} \overrightarrow{OC}$		
$ \overrightarrow{BE} $ $ = \overrightarrow{OE} - \overrightarrow{OB} $ $ = \frac{a}{a+c} \overrightarrow{OA} - \overrightarrow{OB} + \frac{c}{a+c} \overrightarrow{OC} $		
$= \overrightarrow{OJ} - \overrightarrow{OB}$ $= \frac{a}{a+b+c} \overrightarrow{OA} + \frac{b}{a+b+c} \overrightarrow{OB} + \frac{c}{a+b+c} \overrightarrow{OC} - \overrightarrow{OB}$		
$= \frac{a}{a+b+c} \overrightarrow{OA} - \frac{a+c}{a+b+c} \overrightarrow{OB} + \frac{c}{a+b+c} \overrightarrow{OC}$ $= \left(\frac{a+c}{a+b+c}\right) \left(\frac{a}{a+c} \overrightarrow{OA} - \overrightarrow{OB} + \frac{c}{a+c} \overrightarrow{OC}\right)$ $= \frac{a+c}{a+c} \overrightarrow{OA} + \overrightarrow{OB} + \frac{c}{a+c} \overrightarrow{OC}$		
$= \frac{a+c}{a+b+c} \overrightarrow{BE}$	1M	
Since $0 < \frac{a+c}{a+b+c} < 1$ , $J$ lies on $BE$ . Thus, $AD$ and $BE$ intersect at $J$ .	1(7)	
022-DSE-MATH-EP(M2)18	-	

Solution CONFIDENTIAL (FOR WARNER'S U	Marks	Remarks
(b) (i) Define $BC = a$ , $AC = b$ and $AB = c$ . $\begin{vmatrix} a \\ =  \overrightarrow{OC} - \overrightarrow{OB}  \\ =  (-3\mathbf{j} + \mathbf{k}) - (40\mathbf{i} - 3\mathbf{j} + \mathbf{k})  \\ =  -40\mathbf{i}  \\ = 40 \end{vmatrix}$ $= \begin{vmatrix} b \\ =  \overrightarrow{OC} - \overrightarrow{OA}  \\ =  (-3\mathbf{j} + \mathbf{k}) - (35\mathbf{i} + 9\mathbf{j} + \mathbf{k})  \\ =  -35\mathbf{i} - 12\mathbf{j}  \end{vmatrix}$	1M	
$=  \overrightarrow{OB} - \overrightarrow{OA} $ $=  (40\mathbf{i} - 3\mathbf{j} + \mathbf{k}) - (35\mathbf{i} + 9\mathbf{j} + \mathbf{k}) $ $=  5\mathbf{i} - 12\mathbf{j} $ $= 13$ $\overrightarrow{OI}$ $= \frac{40}{40 + 37 + 13} \overrightarrow{OA} + \frac{37}{40 + 37 + 13} \overrightarrow{OB} + \frac{13}{40 + 37 + 13} \overrightarrow{OC}$ $= 32\mathbf{i} + \frac{7}{3}\mathbf{j} + \mathbf{k}$	1A	by putting $J = I$ in (a)(ii)
2022-DSE-MATH-EP(M2)19		

CONFIDENTIAL (FOR WARKER'S USE ONLY)			
Solution	Marks	Remarks	
(ii) $\overrightarrow{AI}$ $= \overrightarrow{OI} - \overrightarrow{OA}$			
$= \left(32\mathbf{i} + \frac{7}{3}\mathbf{j} + \mathbf{k}\right) - \left(35\mathbf{i} + 9\mathbf{j} + \mathbf{k}\right)$			
$=-3\mathbf{i}-\frac{20}{3}\mathbf{j}$			
$\overrightarrow{AI} \times \overrightarrow{AB}$ $= \left(-3\mathbf{i} - \frac{20}{3}\mathbf{j}\right) \times (5\mathbf{i} - 12\mathbf{j})$			
$ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & \frac{-20}{3} & 0 \\ 5 & -12 & 0 \end{vmatrix} $			
$=\frac{208}{3}\mathbf{k}$	1M		
The radius of the inscribed circle $= AI \sin \angle BAI$			
$= \frac{(AI)(AB)\sin \angle BAI}{AB}$			
$=\frac{\left \overrightarrow{AI}\times\overrightarrow{AB}\right }{c}$	1 M		
$=\frac{16}{3}$	1A (5)		
2022-DSE-MATH-EP(M2)–20			