. 12	MESTINICIED	Vide 1	P.2
	Solution	Marks	Remarks
•	(a) $(\sqrt{2(x+\Delta x)} - \sqrt{2x})(\sqrt{2(x+\Delta x)} + \sqrt{2x})$		
1.	(a) $ (\sqrt{2}(x + \Delta x) - \sqrt{2}x)(\sqrt{2}(x + \Delta x) + \sqrt{2}x) $ $ = 2(x + \Delta x) - 2x $		
	$= 2\Delta x$	1A	,
	(b) $\frac{d}{dx}\sqrt{2x} = \lim_{\Delta x \to 0} \frac{1}{\Delta x} (\sqrt{2(x + \Delta x)}) - \sqrt{2x})$	1A	•
	$\frac{dx}{dx}\sqrt{2x} - \Delta x \rightarrow 0 \overline{\Delta x} \left(\sqrt{2}\left(x + \Delta x\right) - \sqrt{2}x\right)$		
	$= \frac{1 \text{ im}}{\Delta x \to 0} \frac{1}{\Delta x} \left(\sqrt{2(x + \Delta x)} - \sqrt{2x} \right) \cdot \frac{\sqrt{2(x + \Delta x)}}{\sqrt{2(x + \Delta x)}}$	$)+\sqrt{2x}$	
	$\sqrt{2(x + \Delta x)}$	$) + \sqrt{2x}$	
	$= \frac{1 \text{ im}}{\Delta x \to 0} \frac{1}{\Delta x} \cdot \frac{2\Delta x}{\sqrt{2(x + \Delta x)} + \sqrt{2x}}$		•
			·
•	$= \frac{1 \text{ im}}{\Delta x \to 0} \frac{2}{\sqrt{2(x + \Delta x)} + \sqrt{2x}}$	1A .	
•	$\sqrt{2(x + \Delta x)} + \sqrt{2}x$		
	$= \frac{1}{\sqrt{2x}}$	1A	
<u></u>	√2 ×	_5	
2.	(a) $\frac{50}{4+3i} = \frac{50}{4+3i} (\frac{4-3i}{4-3i})$	1M	:
·	= 8 - 61	1A	
;			
	(b) $5z + 3\overline{z} = \frac{50}{4 + 3i}$		
	5(a + bi) + 3(a - bi) = 8 - 6i	1A	For $\overline{z} = a - bi$
	75a + 3a = 8		
	$\begin{cases} 5b - 3b = -6 \end{cases}$	1M	
	$\therefore a = 1, b = -3$	1A	
	z = 1 - 3i		•
3.	$\alpha + \beta = -p$, $\alpha\beta = q$	1A	
	$-q = (\alpha + 3) + (\beta + 3)$	1M	
•	$= (\alpha + \beta) + 6$		
	-q = -p + 6	1A	·
	$p = (\alpha + 3)(\beta + 3)$	1M	
	$= \alpha\beta + 3 (\alpha + \beta) + 9$		
	p = q - 3p + 9	1A	
	Solving the equations, $p = 1$, $q = -5$	1A .	
		_6	·
			I .

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/ 	Solution	Marks	Remarks
	Alternative solution	 	† 1
	α , β are the roots of $(x + 3)^2 + q(x + 3) + p = 0$	1M	
	$x^2 + (q+6)x + (p+3q+9) =$	0 1A	
	Comparing coefficient with $x^2 + px + q = 0$	1M	
	$\int p = q + 6$		
	$\begin{cases} q = p + 3q + 9 \end{cases}$	1A+1A	
	Solving the equations, $p = 1$, $q = -5$.	1A	
		 	
4.	$\sin\frac{2\pi}{3} + i\cos\frac{2\pi}{3}$	-	
	$=\frac{\sqrt{3}}{2}-\frac{1}{2}i$	1A	
	$=\cos\frac{\pi}{5}-i\sin\frac{\pi}{5}$	1A	(can be omitted)

	$=\cos(-\frac{\pi}{6})+i\sin(-\frac{\pi}{6})$	1A	or $\cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6}$ etc.
			Accept degree measures
	Alternative solution		<u> </u>
•	$\sin\frac{2\pi}{3} + i\cos\frac{2\pi}{3} = \cos(\frac{\pi}{2} - \frac{2\pi}{3}) + i\sin(\frac{\pi}{2} - \frac{2\pi}{3})$	1A	
	$= \cos(-\frac{\pi}{6}) + i\sin(-\frac{\pi}{6})$	2A	
*	$\sin \frac{2\pi}{3} + i\cos \frac{2\pi}{3} = \sin(\frac{\pi}{2} + \frac{\pi}{6}) + i\cos(\frac{\pi}{2} + \frac{\pi}{6})$	1A	
	$=\cos\frac{\pi}{6}-i\sin\frac{\pi}{6}$	1A	(can be omitted)
	$= \cos(-\frac{\pi}{6}) + i\sin(-\frac{\pi}{6})$	1 A	
<u> </u>	$(\sin\frac{2\pi}{3} + i\cos\frac{2\pi}{3})^{\frac{1}{3}} = [\cos(-\frac{\pi}{6}) + i\sin(-\frac{\pi}{6})]^{\frac{1}{3}}$		
	$=\cos\frac{2k\pi-\frac{\pi}{6}}{3}+i\sin\frac{2k\pi-\frac{\pi}{6}}{3},$	1M+1A	1M for De Moivre's Theorem
	where $k = -1, 0, 1,$	1A	1A if others correct or k = 0, 1, 2 or etc.
	27 2- 3		
	OR $(\sin\frac{2\pi}{3} + i\cos\frac{2\pi}{3})^{\frac{\pi}{3}} = \cos(-\frac{\pi}{18}) + i\sin(-\frac{\pi}{18}),$	1 <u>A</u>	
	$\cos\frac{11\pi}{18} + i\sin\frac{11\pi}{18},$	1A	
	$\cos(-\frac{13\pi}{18}) + i\sin(-\frac{13\pi}{18})$	1A	· .
	$(\text{or } \cos \frac{23\pi}{18} + i \sin \frac{23\pi}{18})$		
4		6	1

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	MESTRICIED	ヘハロに	P.4
,	Solution	Marks	Remarks
5.	$ -x^2+2x+3 \geq 5$		
	$-x^2 + 2x + 3 \ge 5$ or $= -x^2 + 2x + 3 \le -5$	2A	use 'and' or ' ,, and'
	$x^2 - 2x + 2 \le 0$ or $x^2 - 2x - 8 \ge 0$		(no mark)
	$(x-1)^2+1\leq 0$ or $(x+2)(x-4)\geq 0$	1A	For factorisation
	No solution or $x \ge 4$ or $x \le -2$	1A+1A	
	$\therefore x \le -2$ or $x \ge 4$	1A	cannot omit 'or'
	Alternative solution (1) $(-x^2 + 2x + 3)^2 \ge 5^2$	1A	
	$(-x^2 + 2x + 3 + 5)(-x^2 + 2x + 3 - 5) \ge 0$	1A	
	$(-x^2 + 2x + 8)(-x^2 + 2x - 2) \ge 0$		
•	$(x^2 - 2x + 2)(x^2 - 2x - 8) \ge 0$		
	$x^2 - 2x + 2 = (x - 1)^2 + 1 > 0$	1M	
	$(x^2-2x-8)\geq 0$	1A	
_	$(x-4)(x+2)\geq 0$	1A	
	$x \ge 4$ or $x \le -2$	1A	
			
	Alternative solution (2)		
	Consider the following cases:	IM	
	case 1 : x ≥ 3		
•	$(x+1)(x-3)\geq 5$	-~	
ř	$(x-4)(x+2)\geq 0$		
	$x \ge 4$ or $x \le -2$		
	since $x \ge 3$, $\therefore x \ge 4$	1A	
	case 2 : -1 < x < 3		
_	$-(x+1)(x-3)\geq 5$	-	
	$x^2-2x+2\leq 0$		
	$(x-1)^2+1\leq 0$		·
	no solution	1A	
	case 3 : $x \leq -1$.		
	$(x+1)(x-3)\geq 5$		
	$x \ge 4$ or $x \le -2$		
	since $x \le -1$, $\therefore x \le -2$	1A	
	Combining the 3 cases,		
	$x \le -2$ or $x \ge 4$	2A	
	i i		

Remarks
Remarks
Omit vector sign (pp-1)
Omit dot sign (pp-1)
1
†
For $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$
$\frac{d}{dx}(-2xy^2)$
1A for other terms
·.
or $y = -\frac{2}{11}x - \frac{7}{11}$
· ·

•	KESTKICTED	人にはいて	P.6
. sol	lution	Marks	Remarks
8. (a)	$\overrightarrow{OP} = \frac{\overrightarrow{a} + r\overrightarrow{b}}{1 + r}$	1A	Omit vector sign (pp-1)
	$\overrightarrow{OQ} = \frac{\overrightarrow{OP} + r\overrightarrow{OB}}{1 + r}$	1A	
·	$= \frac{\frac{1}{1+r}(\overrightarrow{a}+r\overrightarrow{b})+r\overrightarrow{b}}{1+r}$		
	$=\frac{\overrightarrow{d}+(r^2+2r)\overrightarrow{b}}{(1+r)^2}$	1A	
·	Alternative solutions for \overrightarrow{OO} $\overrightarrow{PO} = \frac{r}{r^2 + 2r + 1} \overrightarrow{AB}$	1A	
	$\overrightarrow{OQ} = \overrightarrow{OP} + \overrightarrow{PQ}$ $= \frac{\overrightarrow{a} + r\overrightarrow{b}}{1 + r} + \frac{r}{(1 + r)^2} (\overrightarrow{b} - \overrightarrow{a})$		
	$=\frac{\overrightarrow{a}+(r^2+2r)\overrightarrow{b}}{(1+r)^2}$	1A	
	$AQ : QB = (r^2 + 2r) : 1$	1A	
	$\overrightarrow{OQ} = \frac{\overrightarrow{d} + (r^2 + 2r)\overrightarrow{b}}{(1+r)^2}$	1A	
		_3	
(b)	$\overrightarrow{OT} = \frac{1}{1+r}\overrightarrow{b}$	1A -	
	$\overrightarrow{TQ} = \overrightarrow{OQ} - \overrightarrow{OT}$		
	$=\frac{\overrightarrow{a}+(r^2+2r)\overrightarrow{b}}{(1+r)^2}-\frac{1}{(1+r)}\overrightarrow{b}$	1M	
	$= \frac{\vec{d} + (r^2 + r - 1)\vec{b}}{(1 + r)^2}$	1	
(c)	Since $\overrightarrow{OA} \mid \overrightarrow{TQ}$,	3	
٠	$r^2+r-1=0$	2М	or $\frac{r^2+r-1}{(1+r)^2}=0$
	$r = \frac{-1 \pm \sqrt{5}}{2}$		
	Since $r > 0$, $\therefore r = \frac{-1 + \sqrt{5}}{2}$	1 <u>A</u>	

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Solution	Marks	Remarks
Alternative solution		
$AQ : QB = (r^2 + 2r) : 1$		
OT: TB = 1: r		
Since OA TB,		
$\frac{r^2+2r}{1}=\frac{1}{r}$	1M+1A	
$x^3 + 2x^2 - 1 = 0$		
$(r+1)(r^2+r-1)=0$		
$r = -1, \frac{-1 \pm \sqrt{5}}{2}$		•.
Since $r > 0$, $\therefore r = \frac{-1 + \sqrt{5}}{2}$	1A	
(d) (i) $\overrightarrow{a} \cdot \overrightarrow{a} = 4$	1A Omit	dot sign (pp-1)
		(PP 1)
$\vec{a} \cdot \vec{b} = 2(16)(\cos\frac{\pi}{3})$	1M	•
= 16	1A	
$(ii) \overrightarrow{OA} \cdot \overrightarrow{TQ} = 0$	1M	
٠ 		
$\overrightarrow{a} \cdot \left\{ \frac{\overrightarrow{a} + (r^2 + r - 1)\overrightarrow{b}}{(1 + r)^2} \right\} = 0$		
1 7 7		
$\frac{1}{(1+r)^2} [\vec{a} \cdot \vec{a} + (r^2 + r - 1) \vec{a} \cdot \vec{b}] = 0$		
$\frac{1}{(1+r)^2}[4+(r^2+r-1)16]=0$	1M	· · · · · · · · · · · · · · · · · · ·
· · · · · · · · · · · · · · · · · · ·		
$16r^2 + 16r - 12 = 0$	1A	
$r=\frac{1}{2}$ or $-\frac{3}{2}$ (rejected)		•
·		
$\therefore r = \frac{1}{2}$	1A	
	7	
		· ·

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Paper	1

	KESTRICTED M	加义	FF Paper P.8
Solu	tion	Marks	Remarks
9. (a)	ΔBCD ~ ΔBAE		\
	$\frac{BC}{BA} = \frac{BD}{BE}$	1M	
	$\frac{\sqrt{x^2+1}}{8} = \frac{x}{x+s} = \frac{1}{E} $	1 A	
	$s = \frac{8x}{\sqrt{1 + x^2}} - x$	1	
		_3	*
(b)	$\frac{ds}{dx} = \frac{8\sqrt{1 + x^2} - \frac{8x^2}{\sqrt{1 + x^2}}}{1 + x^2} - 1$	1M+1A	1M for quotient rule
	$=\frac{8}{(1+x^2)^{3/2}}-1$		
	$\frac{ds}{dx} = 0$	1M	
	$(1 + x^2)^{3/2} = 8$ $x = \pm \sqrt{3}$		
•	Since $x > 0$, $\therefore x = \sqrt{3}$ $\frac{d^2s}{dx^2} = \frac{-24x}{(1+x^2)^{5/2}}$	1A 1A	When $0 < x < \sqrt{3}$, $\frac{ds}{dx} > 0$
	At $s = \sqrt{3}$, $\frac{d^2s}{dx^2} \left(= -\frac{3\sqrt{3}}{4} \right) < 0$ s is a maximum	IM	When $\sqrt{3} < x \sqrt{3} \sqrt{7}$, $\frac{ds}{dx} < 0$ $\therefore s$ is a maximum M
- 1	$s_{\text{max}} = \frac{8\sqrt{3}}{\sqrt{1+3}} - \sqrt{3} = 3\sqrt{3}$	<u>1A</u>	Awarded if checking is omitted
(c)	(i) $P = Area \ of \ \Delta ABE - area \ of \ \Delta CBD$	7	
	$= \frac{1}{2} (s + x)(8) \sin \angle CBD - \frac{x}{2}$	1M	
	$= \frac{1}{2}(s + x)(8) \cdot \frac{1}{\sqrt{1 + x^2}} - \frac{x}{2}$	1A	
	$= \frac{1}{2} \left(\frac{8x}{\sqrt{1+x^2}} - x + x \right) \cdot \frac{8}{\sqrt{1+x^2}} - \frac{x}{2}$		
	$=\frac{32x}{1+x^2}-\frac{x}{2}$	1	

Solution		Marks	` ·	Remarks
			L .	
	Alternative solution			1
•	Area of $\triangle ABE$: Area of $\triangle CBD$			
•	$= (s + x)^2 : x^2$	1M		
	$=\frac{64x^2}{1+x^2}:x^2$			
	$\frac{1}{1+x^2}$			
	Area of $\triangle ABE = \frac{32x}{1+x^2}$	1A		
•	Area of $\triangle CBD = \frac{x}{2}$			
	32 4			•
	$\therefore P = \frac{32x}{1+x^2} - \frac{x}{2}$	1		
	$P = \frac{1}{2} (1 + AE) s$	1M		
	$=\frac{1}{2}(1+\frac{s+x}{x})s$			
	$=\frac{1}{2}(1+\frac{1}{x})s$			
	$=\frac{1}{2}(1+\frac{8}{\sqrt{1+x^2}})(\frac{8x}{\sqrt{1+x^2}}-x)$	IA.	.	
	$\sqrt{1+x^2} \sqrt{1+x^2}$			
·				
	$= \frac{x}{2}(1 + \frac{8}{\sqrt{1 + x^2}})(\frac{8}{\sqrt{1 + x^2}} - 1)$			
				•
	$=\frac{32x}{1+x^2}-\frac{x}{2}$	1		
•	•		Ш	
-	$dn = 22/1 + v^2 = 22v/2v = 1$			
(ii)	$\frac{\mathrm{d}p}{\mathrm{d}x} = \frac{32(1+x^2)-32x(2x)}{(1+x^2)^2} - \frac{1}{2}$	1M	,	
	$32(1-x^2)$			
	$=\frac{32(1-x^2)}{(1+x^2)^2}-\frac{1}{2}$			
	From (b), s attains its maximum at $x = \sqrt{3}$		•	• **
	$a + \sqrt{3} dp = 9$	12		
	At $x = \sqrt{3}$, $\frac{\mathrm{d}p}{\mathrm{d}x} = -\frac{9}{2}$	1A		•
	Since $\frac{dp}{dx} \neq 0$ at $x = \sqrt{3}$, P does not attain			
÷	a maximum when s attains its maximum	1	,	
		6		
			I	

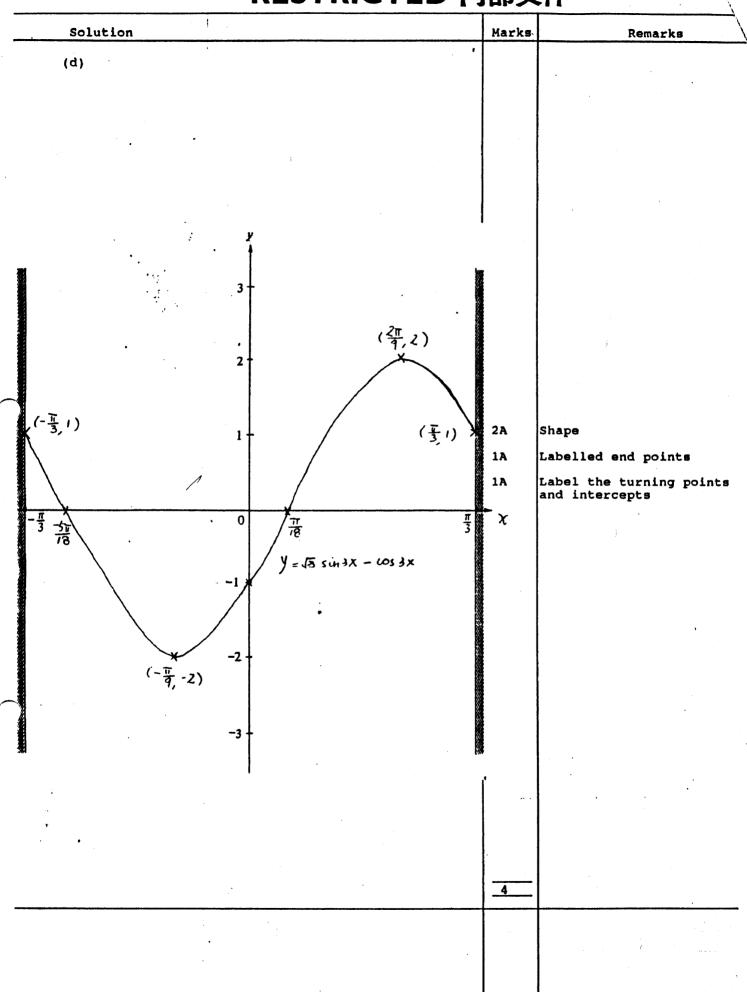
	Mai Mai Est	VADE	-11-
Sol	ution	Marks.	Remarks
10. (a)	Let α , β be the roots of the equation		
	$\frac{1}{k+1}[2x^2+(k+7)x+4]=0$		
	$\begin{cases} \alpha \cdot + \beta = -\frac{k+7}{2} \\ \alpha \beta = 2 \end{cases}$	1A	
	$PQ = \alpha - \beta = 1$	1M	
	$(\alpha - \beta)^2 = 1$	ŀ	
	$(\alpha + \beta)^2 - 4\alpha\beta = 1$	1M	
	$(-\frac{k+7}{2})^2 - 8 = 1$	1A	
	Alternative solution		
	$\frac{1}{k+1}[2x^2+(k+7)x+4]=0$		
_	$x = \frac{-(k+7) \pm \sqrt{(k+7)^2 - 32}}{4}$	1A	
	$PQ = \frac{-(k+7) + \sqrt{(k+7)^2 - 32}}{4} - \frac{-(k+7) - \sqrt{(k+7)^2 - 32}}{4}$	$\frac{7}{10^{2}-32}$	2М
	$1 = \frac{\sqrt{(k+7)^2 - 32}}{2}$	1A	
	$k^2 + 14k + 13 = 0$	1A	
	k = -1 or -13		(can be omitted)
•	$\therefore k \neq -1, \therefore k = -13 \qquad \bullet$	1A	
(b)	Discriminant = $\frac{(k+7)^2-32}{(k+1)^2} < 0$	1M+1A	Accept $\frac{(k+7)^2}{4} - 8 < 0$, $(k+7)^2 - 32 < 0$
•	$-7 - 4\sqrt{2} < k < -7 + 4\sqrt{2}$	2 <u>A</u>	$(k+7)^2-32<0$
•		4	
- 1			
		,	
		i	· ·

Solution (c) Put $k = 0$, C becomes $y = 2x^2 + 7x + 4 \dots (1)$ $k = 1$, C becomes $y = x^2 + 4x + 2 \dots (2)$ Solving (1) and (2), the points of intersection are $(-1, -1)$ and $(-2, -2)$ Put $x = -1$, $y = -1$ into C LHS = $y = -1$ RHS = $\frac{1}{k+1}[2 + (k+7)(-1) + 4] = -1 = LHS \lor k \not= -1$ Put $x = -2$, $y = -2$ into C LHS = $\frac{1}{k+1}[8 + (k+7)(-2) + 4] = -2 = LHS \lor k \not= -1$ C Always passes through $(-1, -1)$ and $(-2, -2)$ Alternative solution (c) $y = \frac{1}{k+1}[2x^2 + (k+7)x + 4]$ $(k+1)y = 2x^2 + (k+7)x + 4$ $(y - 2x^2 - 7x - 4) + k(y - x) = 0$ Calways passes through the intersection points of the 2 curves $y - 2x^2 - 7x - 4 = 0$ Solving, the 2 points are $(-1, -1)$ and $(-2, -2)$ Inhib (c) Let k_1 , k_2 be two distinct values of k $\begin{cases} (k_1 + 1)y = 2x^2 + (k_1 + 7)x + 4 & \dots (1) \\ (k_2 + 1)y = 2x^2 + (k_1 + 7)x + 4 & \dots (2) \\ (1) - (2) : (k_1 - k_2)y = (k_1 - k_2)x & \dots (2) \\ (1) - (2) : (k_1 - k_2)y = (k_1 - k_2)x & \dots (2) \\ (1) - (2) : (k_1 - k_2)y = (k_1 - k_2)x & \dots (3) \\ x = -1 & x - 2 \\ y + x = -1 & x - 2 \\ y$		WEST VICTED K	人后	·ff-	P.13
$k = 1, \ C \text{ becomes } y = x^2 + 4x + 2 \dots (2)$ Solving (1) and (2), the points of intersection $\{-1, -1\}$ and $\{-2, -2\}$ Put $x = -1$, $y = -1$ into C LHS = $y = -1$ $RHS = \frac{1}{k+1}[2 + (k+7)(-1) + 4] = -1 = LHS \lor k \not= -1$ Put $x = -2$, $y = -2$ into C LHS = -2 $RHS = \frac{1}{k+1}[8 + (k+7)(-2) + 4] = -2 = LHS \lor k \not= -1$ $\therefore C \text{ always passes through } (-1, -1) \text{ and } (-2, -2)$ Alternative solution (c) $y = \frac{1}{k+1}[2x^2 + (k+7)x + 4]$ $(k+1)y = 2x^2 + (k+7)x + 4$ $(y - 2x^2 - 7x - 4) + k(y - x) = 0$ C always passes through the intersection points of the 2 curves $y - 2x^2 - 7x - 4 = 0$ and $y - x = 0$ Solving, the 2 points are $(-1, -1)$ and $(-2, -2)$ (c) Let k_1, k_2 be two distinct values of k $\begin{cases} (k_1 + 1)y = 2x^2 + (k_1 + 7)x + 4 \dots (1) \\ (k_2 + 1)y = 2x^2 + (k_2 + 7)x + 4 \dots (2) \\ (1) - (2) : (k_1 - k_2)y = (k_1 - k_2)x \\ y = x \text{ (since } k_1 \not= k_2) \end{cases}$ Subs. into (1): $(k_1 + 1)x = 2x^2 + (k_1 + 7)x + 4 \dots (2)$ when $x = -1$, $y = -2$ when $x = -1$, $y = -2$ $\therefore C$ always passes through 2 fixed points		Solution	Marks	Remarks	
Solving (1) and (2), the points of intersection are $(-1, -1)$ and $(-2, -2)$ Put $x = -1$, $y = -1$ into C LHS = $y = -1$ RHS = $\frac{1}{k+1}I^2 + (k+7)(-1) + 4\} = -1 = LHS \lor k \neq -1$ Put $x = -2$, $y = -2$ into C LHS = -2 RHS = $\frac{1}{k+1}[8 + (k+7)(-2) + 4] = -2 = LHS \lor k \neq -1$ $\therefore C$ always passes through $(-1, -1)$ and $(-2, -2)$ Alternative solution (c) $y = \frac{1}{k+1}[2x^2 + (k+7)x + 4]$ $(k+1)y = 2x^2 + (k+7)x + 4$ $(y - 2x^2 - 7x - 4) + k(y - x) = 0$ C always passes through the intersection points of the 2 curves $y - 2x^2 - 7x - 4 = 0$ and $y - x = 0$ $\begin{cases} y - 2x^2 - 7x - 4 = 0 \\ y - x = 0 \end{cases}$ Solving, the 2 points are $(-1, -1)$ and $(-2, -2)$ (c) Let k_1, k_2 be two distinct values of k $\begin{cases} (k_1 + 1)y = 2x^2 + (k_1 + 7)x + 4 \dots (1) \\ (k_2 + 1)y = 2x^2 + (k_1 + 7)x + 4 \dots (2) \end{cases}$ (d) Let k_1, k_2 be two distinct values of k $\begin{cases} (k_1 + 1)x = 2x^2 + (k_1 + 7)x + 4 \dots (2) \\ (1) - (2) : (k_1 - k_2)y = (k_1 - k_2)x \\ y = x \end{cases}$ Subs. into (1): $(k_1 + 1)x = 2x^2 + (k_1 + 7)x + 4 $ $2x^2 + 6x + 4 = 0$ $x = -1 \text{ or } -2$ when $x = -1$, $y = -1$ $y = -2$, $y = -2$ $\therefore C$ always passes through 2 fixed points		·	2 M	or any other values	
Put $x = -1$, $y = -1$ into C LHS = $y = -1$ RHS = $\frac{1}{k+1}[2 + (k+7)(-1) + 4] = -1 = LHS \lor k \neq -1$ Put $x = -2$, $y = -2$ into C LHS = $\frac{1}{k+1}[8 + (k+7)(-2) + 4] = -2 = LHS \lor k \neq -1$ Put $x = -2$, $y = -2$ into C LHS = $\frac{1}{k+1}[8 + (k+7)(-2) + 4] = -2 = LHS \lor k \neq -1$ $\therefore C$ always passes through $(-1, -1)$ and $(-2, -2)$ Alternative solution (c) $y = \frac{1}{k+1}[2x^2 + (k+7)x + 4]$ $(k+1)y = 2x^2 + (k+7)x + 4$ $(y-2x^2-7x-4) + k(y-x) = 0$ C always passes through the intersection points of the 2 curves $y-2x^2-7x-4=0$ and $y-x=0$ (y - 2x^2 - 7x - 4 = 0 y - x = 0 (y - 2x^2 - 7x - 4 = 0 y - x = 0 (c) Let k_1 , k_2 be two distinct values of k $\begin{cases} (k_1 + 1)y = 2x^2 + (k_1 + 7)x + 4 & \dots & (1) \\ (k_2 + 1)y = 2x^2 + (k_1 + 7)x + 4 & \dots & (2) \\ (1) - (2) : (k_1 - k_2)y = (k_1 - k_2)x & \dots & y = x \\ (since k_1 \neq k_2) Subs. into (1): (k_1 + 1)x = 2x^2 + (k_1 + 7)x + 4 2x^2 + 6x + 4 = 0 x = -1 or -2 when x = -1, y = -1 y = -2, y = -2 \therefore C always passes through 2 fixed points$					
Put $x = -1$, $y = -1$ into C LHS = $y = -1$ RHS = $\frac{1}{k+1}[2 + (k+7)(-1) + 4] = -1 = LHS \lor k \neq -1$ Put $x = -2$, $y = -2$ into C LHS = $\frac{1}{k+1}[8 + (k+7)(-2) + 4] = -2 = LHS \lor k \neq -1$ $\therefore C$ always passes through $(-1, -1)$ and $(-2, -2)$ Alternative solution (c) $y = \frac{1}{k+1}[2x^2 + (k+7)x + 4]$ $(k+1)y = 2x^2 + (k+7)x + 4$ $(y - 2x^2 - 7x - 4) + k(y - x) = 0$ C always passes through the intersection points of the 2 curves $y - 2x^2 - 7x - 4 = 0$ And $y - x = 0$ Solving, the 2 points are $(-1, -1)$ and $(-2, -2)$			ł		
RHS = $\frac{1}{k+1}[2 + (k+7)(-1) + 4] = -1 = LHS \lor k \neq -1$ Put x = -2 , y = -2 into C LHS = -2 RHS = $\frac{1}{k+1}[8 + (k+7)(-2) + 4] = -2 = LHS \lor k \neq -1$ $\therefore C$ always passes through $(-1, -1)$ and $(-2, -2)$ Alternative solution (c) $y = \frac{1}{k+1}[2x^2 + (k+7)x + 4]$ $(k+1)y = 2x^2 + (k+7)x + 4$ $(y - 2x^2 - 7x - 4) + k(y - x) = 0$ C always passes through the intersection points of the 2 curves $y - 2x^2 - 7x - 4 = 0$ and $y - x = 0$ $\begin{cases} y - 2x^2 - 7x - 4 = 0 \\ y - x = 0 \end{cases}$ Solving, the 2 points are $(-1, -1)$ and $(-2, -2)$ IA+IA (c) Let k_1, k_2 be two distinct values of k $\begin{cases} (k_1 + 1)y = 2x^2 + (k_1 + 7)x + 4 & \dots & (1) \\ (k_2 + 1)y = 2x^2 + (k_1 + 7)x + 4 & \dots & (2) \\ (1) - (2) : (k_1 - k_2)y = (k_1 - k_2)x \\ y = x & \text{(since } k_1 \neq k_2) \end{cases}$ Subs. into (1): $(k_1 + 1)x = 2x^2 + (k_1 + 7)x + 4$ $2x^2 + 6x + 4 = 0$ $x = -1, y = -1$ when $x = -1, y = -1$ $y = -2, y = -2$ $\therefore C$ always passes through 2 fixed points					
Put $x = -2$, $y = -2$ into C LHS = -2 RHS = $\frac{1}{k+1}\{8 + (k+7)(-2) + 4\} = -2 = \text{LHS } \forall k \neq -1$ $\therefore C$ always passes through $(-1, -1)$ and $(-2, -2)$ Alternative solution (c) $y = \frac{1}{k+1}\{2x^2 + (k+7)x + 4\}$ $(k+1)y = 2x^2 + (k+7)x + 4$ $(y - 2x^2 - 7x - 4) + k(y - x) = 0$ C always passes through the intersection points of the 2 curves $y - 2x^2 - 7x - 4 = 0$ and $y - x = 0$ $\begin{cases} y - 2x^2 - 7x - 4 = 0 \\ y - x = 0 \end{cases}$ Solving, the 2 points are $(-1, -1)$ and $(-2, -2)$ latha (c) Let k_1, k_2 be two distinct values of k $\begin{cases} (k_1 + 1)y = 2x^2 + (k_1 + 7)x + 4 \dots (1) \\ (k_2 + 1)y = 2x^2 + (k_2 + 7)x + 4 \dots (2) \end{cases}$ (1) $-(2)$: $(k_1 - k_2)y = (k_1 - k_2)x$ $y = x$ (since $k_1 \neq k_2$) Subs. into (1): $(k_1 + 1)x = 2x^2 + (k_1 + 7)x + 4$ $2x^2 + 6x + 4 = 0$ $x = -1$ or -2 when $x = -1$, $y = -1$ $y = -2$, $y = -2$ \therefore C always passes through 2 fixed points		LHS = y = -1			
LHS = $\frac{1}{k+1}$ [8 + (k + 7)(-2) + 4] = -2 = LHS \(\nabla k \neq -1 \) 1 ∴ C always passes through (-1, -1) and (-2, -2) $\frac{1}{6}$ Alternative solution (c) $y = \frac{1}{k+1} (2x^2 + (k+7)x + 4)$ $(k+1)y = 2x^2 + (k+7)x + 4$ $(y - 2x^2 - 7x - 4) + k(y - x) = 0$ C always passes through the intersection points of the 2 curves $y - 2x^2 - 7x - 4 = 0$ and $y - x = 0$ $\begin{cases} y - 2x^2 - 7x - 4 = 0 \\ y - x = 0 \end{cases}$ Solving, the 2 points are (-1, -1) and (-2, -2) IA+IA (c) Let k_1, k_2 be two distinct values of k $\begin{cases} (k_1 + 1)y = 2x^2 + (k_1 + 7)x + 4 & \dots & (1) \\ (k_2 + 1)y = 2x^2 + (k_2 + 7)x + 4 & \dots & (2) \\ (1) - (2) : (k_1 - k_2)y = (k_1 - k_2)x & \text{IM} \\ y = x & (\text{since } k_1 \neq k_2) \end{cases}$ Subs. into (1): $(k_1 + 1)x = 2x^2 + (k_1 + 7)x + 4$ $2x^2 + 6x + 4 = 0$ $x = -1 \text{ or } -2$ when $x = -1, y = -1$ $y = -2, y = -2$ ∴ C always passes through 2 fixed points	,	RHS = $\frac{1}{k+1}[2 + (k+7)(-1) + 4] = -1 = LHS \ \forall k \neq -1$	1		
RHS = $\frac{1}{k+1} \{8 + (k+7)(-2) + 4\} = -2 = LHS \ \forall k \neq -1$ $\therefore C \text{ always passes through } (-1, -1) \text{ and } (-2, -2)$ Alternative solution (c) $y = \frac{1}{k+1} \{2x^2 + (k+7)x + 4\}$ $(k+1)y = 2x^2 + (k+7)x + 4$ $(y - 2x^2 - 7x - 4) + k(y - x) = 0$ $C \text{ always passes through the intersection points of the 2 curves } y - 2x^2 - 7x - 4 = 0$ $and y - x = 0$ $\begin{cases} y - 2x^2 - 7x - 4 = 0 \\ y - x = 0 \end{cases}$ Solving, the 2 points are $(-1, -1)$ and $(-2, -2)$ lahla (c) Let k_1, k_2 be two distinct values of k $\begin{cases} (k_1 + 1)y = 2x^2 + (k_1 + 7)x + 4 \dots (1) \\ (k_2 + 1)y = 2x^2 + (k_2 + 7)x + 4 \dots (2) \end{cases}$ $(1) - (2) : (k_1 - k_2)y = (k_1 - k_2)x $ $y = x \text{(since } k_1 \neq k_2 \text{)}$ Subs. into (1): $(k_1 + 1)x = 2x^2 + (k_1 + 7)x + 4 $ $2x^2 + 6x + 4 = 0$ $x = -1 \text{or } -2$ when $x = -1, y = -1$ $y = -2, y = -2$ $\therefore C \text{ always passes through 2 fixed points}$	•	Put $x = -2$, $y = -2$ into C		• •	
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Alternative solution (c) $y = \frac{1}{k+1} [2x^2 + (k+7)x + 4]$ $(k+1) y = 2x^2 + (k+7)x + 4$ $(y-2x^2-7x-4) + k(y-x) = 0$ C always passes through the intersection points of the 2 curves $y-2x^2-7x-4=0$ and $y-x=0$ Solving, the 2 points are $(-1, -1)$ and $(-2, -2)$ $\begin{cases} y-2x^2-7x-4=0 \\ y-x=0 \end{cases}$ Solving, the 2 points are $(-1, -1)$ and $(-2, -2)$ $\begin{cases} (k_1+1)y=2x^2+(k_1+7)x+4 \dots (1) \\ (k_2+1)y=2x^2+(k_2+7)x+4 \dots (2) \end{cases}$ (1) $-(2): (k_1-k_2)y=(k_1-k_2)x$ $y=x$ (since $k_1\neq k_2$) Subs. into (1): $(k_1+1)x=2x^2+(k_1+7)x+4$ $2x^2+6x+4=0$ $x=-1$ or -2 when $x=-1, y=-1$ $y=-2, y=-2$ $\therefore C$ always passes through 2 fixed points		RHS = $\frac{1}{k+1}$ [8 + (k + 7)(-2) + 4] = -2 = LHS $\forall k \neq -1$	1		
(c) $y = \frac{1}{k+1} \{2x^2 + (k+7)x + 4\}$ $(k+1)y = 2x^2 + (k+7)x + 4$ $(y-2x^2-7x-4) + k(y-x) = 0$ IM+1A C always passes through the intersection points of the 2 curves $y-2x^2-7x-4=0$ and $y-x=0$ $\begin{cases} y-2x^2-7x-4=0\\ y-x=0 \end{cases}$ Solving, the 2 points are $(-1,-1)$ and $(-2,-2)$ IA+1A (c) Let k_1 , k_2 be two distinct values of k $\begin{cases} (k_1+1)y=2x^2+(k_1+7)x+4&\dots(1)\\ (k_2+1)y=2x^2+(k_2+7)x+4&\dots(2)\\ (1)-(2):(k_1-k_2)y=(k_1-k_2)x\\ y=x&(\text{since }k_1\neq k_2) \end{cases}$ IM Subs. into (1): $(k_1+1)x=2x^2+(k_1+7)x+4$ IM $2x^2+6x+4=0$ $x=-1 \text{ or } -2$ when $x=-1$, $y=-1$ $y=-2$, $y=-2$ $\therefore C \text{ always passes through 2 fixed points}$		∴ C always passes through (-1, -1) and (-2, -2)	_6		
$(k+1) y = 2x^{2} + (k+7)x + 4$ $(y-2x^{2}-7x-4) + k(y-x) = 0$ C always passes through the intersection points of the 2 curves $y-2x^{2}-7x-4=0$ $\begin{cases} y-2x^{2}-7x-4=0 \\ y-x=0 \end{cases}$ Solving, the 2 points are $(-1, -1)$ and $(-2, -2)$ 1A+1A (c) Let k_{1} , k_{2} be two distinct values of k $\begin{cases} (k_{1}+1)y=2x^{2}+(k_{1}+7)x+4(1) \\ (k_{2}+1)y=2x^{2}+(k_{2}+7)x+4(2) \end{cases}$ $(1)-(2): (k_{1}-k_{2})y=(k_{1}-k_{2})x $ $y=x $		Alternative solution	,		
$ (y-2x^2-7x-4)+k(y-x)=0 $ C always passes through the intersection points of the 2 curves $y-2x^2-7x-4=0$ $ \begin{cases} y-2x^2-7x-4=0 \\ y-x=0 \end{cases} $ Solving, the 2 points are $(-1,-1)$ and $(-2,-2)$ $ \begin{cases} (k_1+1)y=2x^2+(k_1+7)x+4&\dots \\ (k_2+1)y=2x^2+(k_2+7)x+4&\dots \\ (k_2+1)y=2x^2+(k_1+7)x+4&\dots \\ (k_1+1)x=2x^2+(k_1+7)x+4&\dots \\ (k_1+1)x=2x^2+(k_1+7)x+4&\dots$		(c) $y = \frac{1}{k+1} [2x^2 + (k+7)x + 4]$			
C always passes through the intersection points of the 2 curves $y-2x^2-7x-4=0$ and $y-x=0$ $\begin{cases} y-2x^2-7x-4=0 \\ y-x=0 \end{cases}$ Solving, the 2 points are $(-1,-1)$ and $(-2,-2)$ [C) Let k_1 , k_2 be two distinct values of k $\begin{cases} (k_1+1)y=2x^2+(k_1+7)x+4\dots(1) \\ (k_2+1)y=2x^2+(k_2+7)x+4\dots(2) \end{cases}$ IM $y=x (\text{since } k_1\neq k_2)$ Subs. into (1): $(k_1+1)x=2x^2+(k_1+7)x+4$ $2x^2+6x+4=0$ $x=-1 \text{or } -2$ when $x=-1$, $y=-1$ $y=-2$, $y=-2$ $\therefore C \text{ always passes through 2 fixed points}$		$(k+1)y = 2x^2 + (k+7)x + 4$	·		
points of the 2 curves $y - 2x^2 - 7x - 4 = 0$ and $y - x = 0$ $\begin{cases} y - 2x^2 - 7x - 4 = 0 \\ y - x = 0 \end{cases}$ Solving, the 2 points are $(-1, -1)$ and $(-2, -2)$ $\begin{cases} (c) \text{ Let } k_1, k_2 \text{ be two distinct values of } k \end{cases}$ $\begin{cases} (k_1 + 1)y = 2x^2 + (k_1 + 7)x + 4 \dots (1) \\ (k_2 + 1)y = 2x^2 + (k_2 + 7)x + 4 \dots (2) \end{cases}$ $(1) - (2) : (k_1 - k_2)y = (k_1 - k_2)x $ $y = x \text{(since } k_1 \neq k_2 \text{)}$ Subs. into (1): $(k_1 + 1)x = 2x^2 + (k_1 + 7)x + 4 $ $2x^2 + 6x + 4 = 0$ $x = -1 \text{or } -2$ when $x = -1, y = -1$ $y = -2, y = -2$ $\therefore C \text{ always passes through 2 fixed points}$		$(y-2x^2-7x-4)+k(y-x)=0$	1M+1A		
Solving, the 2 points are $(-1, -1)$ and $(-2, -2)$ 1A+1A (c) Let k_1 , k_2 be two distinct values of k $ \begin{cases} (k_1 + 1)y = 2x^2 + (k_1 + 7)x + 4 \dots (1) \\ (k_2 + 1)y = 2x^2 + (k_2 + 7)x + 4 \dots (2) \end{cases} $ (1) - (2): $(k_1 - k_2)y = (k_1 - k_2)x$ $y = x (since k_1 \neq k_2) Subs. into (1): (k_1 + 1)x = 2x^2 + (k_1 + 7)x + 4 2x^2 + 6x + 4 = 0 x = -1 \text{or } -2 when x = -1, y = -1 y = -2, y = -2 \therefore C \text{ always passes through 2 fixed points}$		points of the 2 curves $y - 2x^2 - 7x - 4 = 0$	2M		•
Solving, the 2 points are $(-1, -1)$ and $(-2, -2)$ 1A+1A (c) Let k_1 , k_2 be two distinct values of k $ \begin{cases} (k_1 + 1)y = 2x^2 + (k_1 + 7)x + 4 \dots (1) \\ (k_2 + 1)y = 2x^2 + (k_2 + 7)x + 4 \dots (2) \end{cases} $ (1) - (2): $(k_1 - k_2)y = (k_1 - k_2)x$ $y = x (since k_1 \neq k_2) Subs. into (1): (k_1 + 1)x = 2x^2 + (k_1 + 7)x + 4 2x^2 + 6x + 4 = 0 x = -1 \text{or } -2 when x = -1, y = -1 y = -2, y = -2 \therefore C \text{ always passes through 2 fixed points}$		$\begin{cases} y - 2x^2 - 7x - 4 = 0 \\ y - x = 0 \end{cases}$	· .		
$\begin{cases} (k_1 + 1)y = 2x^2 + (k_1 + 7)x + 4 & \dots & (1) \\ (k_2 + 1)y = 2x^2 + (k_2 + 7)x + 4 & \dots & (2) \end{cases}$ $(1) - (2) : (k_1 - k_2)y = (k_1 - k_2)x$ $y = x \qquad (since k_1 \neq k_2)$ Subs. into (1): $(k_1 + 1)x = 2x^2 + (k_1 + 7)x + 4$ $2x^2 + 6x + 4 = 0$ $x = -1 \text{or } -2$ when $x = -1$, $y = -1$ $y = -2$, $y = -2$ $\therefore C \text{ always passes through 2 fixed points}$			1A+1A		
$\begin{cases} (k_2 + 1)y = 2x^2 + (k_2 + 7)x + 4 & \dots & (2) \\ (1) - (2) : (k_1 - k_2)y = (k_1 - k_2)x & & \text{IM} \\ y = x & (\text{since } k_1 \neq k_2) & & \text{IA} \\ \end{cases}$ Subs. into (1): $(k_1 + 1)x = 2x^2 + (k_1 + 7)x + 4$ $2x^2 + 6x + 4 = 0$ $x = -1 \text{ or } -2$ when $x = -1$, $y = -1$ $y = -2$, $y = -2$ $\therefore C \text{ always passes through 2 fixed points}$		(c) Let k_1 , k_2 be two distinct values of k		†	
(1) - (2) : $(k_1 - k_2)y = (k_1 - k_2)x$ $y = x$ (since $k_1 \neq k_2$) Subs. into (1) : $(k_1 + 1)x = 2x^2 + (k_1 + 7)x + 4$ $2x^2 + 6x + 4 = 0$ $x = -1$ or -2 when $x = -1$, $y = -1$ $y = -2$, $y = -2$ $\therefore C$ always passes through 2 fixed points		1 1	1M		
$y = x \qquad (since \ k_1 \neq k_2)$ Subs. into (1): $(k_1 + 1)x = 2x^2 + (k_1 + 7)x + 4$ $2x^2 + 6x + 4 = 0$ $x = -1 \text{or } -2$ when $x = -1$, $y = -1$ $y = -2$, $y = -2$ $\therefore C \text{ always passes through 2 fixed points}$	•				
Subs. into (1): $(k_1 + 1)x = 2x^2 + (k_1 + 7)x + 4$ $2x^2 + 6x + 4 = 0$ $x = -1 \text{ or } -2$ when $x = -1$, $y = -1$ $y = -2$, $y = -2$ $\therefore C \text{ always passes through 2 fixed points}$	*				
$(k_1 + 1)x = 2x^2 + (k_1 + 7)x + 4$ $2x^2 + 6x + 4 = 0$ $x = -1 \text{ or } -2$ when $x = -1$, $y = -1$ $y = -2$, $y = -2$ $\therefore C \text{ always passes through 2 fixed points}$			IA		
$2x^{2} + 6x + 4 = 0$ $x = -1 \text{ or } -2$ $\text{when } x = -1, y = -1$ $y = -2, y = -2$ $\therefore C \text{ always passes through 2 fixed points}$			114		
x = -1 or $-2when x = -1, y = -1y = -2$, $y = -2\therefore C always passes through 2 fixed points$					
when $x = -1$, $y = -1$ $y = -2$, $y = -2$ $\therefore C \text{ always passes through 2 fixed points}$ 1A		1	·		
y = -2, $y = -2$ C always passes through 2 fixed points			1A		
C always passes through 2 fixed points					

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	Sol	ution	Marks	Remarks
11.	(a)	Put $x = 0$, $f(x) = -1$	1A	
		∴ The y-intercept is -1.		
		Put $f(x) = 0$, $\sqrt{3} \sin 3x - \cos 3x = 0$		
		$\tan 3x = \frac{1}{\sqrt{3}}$	1A	
		$x = \frac{\pi}{18} \text{ or } \frac{-5\pi}{18}$	1A+1A N	o mark for degrees
		\therefore The x-intercepts are $\frac{\pi}{18}$ and $\frac{-5\pi}{18}$		
		(1) 1 (1) 1	4	
	(b)	$f'(x) = 3\sqrt{3}\cos 3x + 3\sin 3x$	1A	
		$f''(x) = -9\sqrt{3}\sin 3x + 9\cos 3x$	1 <u>A</u> 2	
	(c)	$f'(x) = 3\sqrt{3}\cos 3x + 3\sin 3x = 0$	1м	e •
	•	$tan3x = -\sqrt{3}$		
		$x = \frac{2\pi}{9} \text{or} -\frac{\pi}{9}$	1A+1A	
		$f''(-\frac{\pi}{9})$ (= 18) > 0 \therefore it is a minimum	1M	
		The minimum point is $(-\frac{\pi}{9}, -2)$	1A	
		$f''(\frac{2\pi}{9})$ (= -18) < 0it is a maximum		
		The maximum point is $(\frac{2\pi}{9}, 2)$	1A	
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Solution	Marks	Remarks
Alternative solution		
$(a) f(x) = 2\sin(3x - \frac{\pi}{6})$		or $f(x) = -2\cos(3x + \frac{\pi}{3})$
f(0) = -1	1A	
∴ The y-intercept is -1		
Put $f(x) = 0$, $\sin(3x - \frac{\pi}{6}) = 0$	1A c	$\operatorname{pr} \cos(3x + \frac{\pi}{3}) = 0$
$x = \frac{\pi}{18} \text{ or } \frac{-5\pi}{18}$		
The x-intercepts are $\frac{\pi}{18}$ or $\frac{-5\pi}{18}$	1A+1A	
	4	
(b) $f'(x) = 6\cos(3x - \frac{\pi}{6})$	1A	
6		
$f''(x) = -18\sin(3x - \frac{\pi}{6})$	1A	
	2	
c) $f'(x) = 6\cos(3x - \frac{\pi}{6}) = 0$	1M	
$x = \frac{2\pi}{9} \text{ or } -\frac{\pi}{9}$	1A+1A	
		<i>(</i>
$f''(-\frac{\pi}{9})$ (= 18) > 0 it is a minimum	1M	
The minimum point is $(-\frac{\pi}{9}, -2)$	1A	
$f''(\frac{2\pi}{9})$ (= -18) < 0 : it is a maximum		
The maximum point is $(\frac{2\pi}{9}, 2)$	1A	
OR		
$f(x) = 2\sin(3x - \frac{\pi}{6})$		
$f(x)$ is maximum when $\sin(3x - \frac{\pi}{6}) = 1$	1M	
$x = \frac{2\pi}{8}$	1A	
The maximum point is $(\frac{2\pi}{9},2)$	1A	•
$f(x)$ is minimum when $\sin(3x - \frac{\pi}{6}) = -1$	1M	
$x = -\frac{\pi}{9}$	1A	
The minimum maint is (# 2)		
\therefore The minimum point is $(-\frac{\pi}{9}, -2)$	<u>1A</u>	• • • • • • • • • • • • • • • • • • • •
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<u> </u>	KESTRICTED P	人们	·TT
Solu	ition	Marks	. Remarks
12.(a)	$z + 1 = \cos\theta + i\sin\theta$	1A	
	$ z+1 (= \sqrt{\cos^2\theta + \sin^2\theta}) = 1$	1	·
	1 Imaginary		For airela
	c	j	For circle
	$\langle 1/ \rangle$	ł	For centre at $z = -1$
	-i O Real	1A	For radius = 1
		5	
(b)	$\tan 2\theta_1 = \frac{\sin \theta_1}{\cos \theta_1 - 1}$	1A	
	$\frac{2\sin\theta_1\cos\theta_1}{2\cos^2\theta-1} = \frac{\sin\theta_1}{\cos\theta_1-1}$	1M	
	$\sin\theta_1(2\cos\theta_1 - 1) = 0$		
	$\cos\theta_1 = \frac{1}{2}$ or $\sin\theta = 0$ (rejected $0 < \theta < \frac{\pi}{2}$)	1A+1A	
	$\theta_1 = \frac{\pi}{3}$	1A	
	Alternative solution		
	Let G be the centre of C and H be a point on the positive real axis		
	$\angle OGP_1 = \theta_1 \qquad \qquad \frac{\langle \theta_1 \rangle \langle \theta_2 \rangle}{\langle G_1 \rangle \langle G_2 \rangle}$	1A	
	$\angle HOP_1 = 2\theta_1$	1A	
	Since $GP_1 = GO$, ΔGOP_1 is isosceles.		·
	$\angle GOP_1 = \frac{\pi - \theta_1}{2}$	1A	$\angle OP_{1}G = \pi - 2\theta_{1}$
	$\frac{\pi-\theta_1}{2}+2\theta_1=\pi$	1 A	$(\pi - 2\theta_1) \times 2 + \theta_1 = \pi$
	$\theta_1 = \frac{\pi}{3}$	1A	
	$\tau_{\rm c} = \cos^{\pi} \tau_{\rm c} + 1 + i\sin^{\pi} \tau_{\rm c}$	11/	
	$z_1 = \cos\frac{\pi}{3} - 1 + i\sin\frac{\pi}{3}$	1M	٠.
i	$=-\frac{1}{2}+\frac{\sqrt{3}}{2}i$	1A	
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Solution		Marks	Remarks
(c)	$z_2 = \cos(\frac{\pi}{3} + \pi) - 1 + i\sin(\frac{\pi}{3} + \pi)$	1M+1M-	1A 1M for using ${\cal C}$
	$=-\frac{3}{2}-\frac{\sqrt{3}}{2}i$	1A	
	Alternative solutions P_1P_2 is a diameter of the circle C . Let P_2 represent the complex no. $x + yi$	1A	
	$\frac{x-\frac{1}{2}}{2}=-1 \ , \ \frac{y+\frac{\sqrt{3}}{2}}{2}=0$	2M	
	$x = -\frac{3}{2} \qquad y = -\frac{\sqrt{3}}{2}$		
	$\therefore P_2$ represents $-\frac{3}{2} - \frac{\sqrt{3}}{2}i$	1A	
	$\begin{vmatrix} z_2 \end{vmatrix} = \sqrt{3}$ $\angle GOP_2 = \frac{\pi}{6}$	1A	
	$\angle GOP_2 = \frac{\pi}{6}$	1M	
	$\operatorname{Arg} z_2 = \frac{\pi}{6} - \pi$	1M	Accept $\pi + \frac{\pi}{6}$
	$=\frac{-5\pi}{6}$		7π 6
	$\therefore z_2 = \sqrt{3} \left(\cos \frac{-5\pi}{6} + i \sin \frac{-5\pi}{6} \right)$		
	$=-\frac{3}{2}-\frac{\sqrt{3}}{2}i$	1A	
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