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香港考試局 HONG KONG EXAMINATIONS AUTHORITY

一九八八年香港中學會考 HONG KONG CERTIFICATE OF EDUCATION EXAMINATION, 1988

> 附加數學 (卷一) Additional Mathematics (Paper I)

> > 評卷参考 Marking Scheme

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請在學校任教之閱卷員特別留意

本評卷參考並非標準答案,故極不宜 落於學生手中,以免引起誤會。

遇有學生求取此文件時, 閱卷員應嚴 予拒絕。閱卷員在任何情況下披露本 評卷參考內容, 均有違閱卷員守則及 「一九七七年香港考試局法例」。

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86 Add. Maths. 1	SOLUTIONS SOLUTIONS	MARKS	REMARKS
1. (a) $(\sqrt{x+1} + 1)$	$\frac{\Delta x - \sqrt{x+1}}{(\sqrt{x+1} + \Delta x + \sqrt{x+1})}$		
= (x + I + i	$\triangle x) - (x + 1)$		
= <u>\(\(\times \) \(\times \) \(\times \)</u>		1A	
(b) $\frac{\mathrm{d}y}{\mathrm{d}x} = \lim_{\Delta x \to c} \frac{\Delta y}{\Delta x}$	<u>x</u>	1A	-
= lim (-	$\frac{\sqrt{x+1+\Delta x}-\sqrt{x+1}}{\Delta x}$	lA	
= 1im 7	$\frac{1}{\kappa \div 1 \div \bot \kappa + \sqrt{\kappa \div 1}}$	1A	This can't be omitted.
$= \frac{1}{2\sqrt{x} + }$	1	1A 5	
2. Iferentiating	$y^2 = x^2y + 2$		
	$(2y)(y') = x^2y' + 2xy$	1M	
When $x = 1$,	$y^2 = y + 2$		
	y = 2 or -1	3	
At (1, 2),	$y^{\dagger} = \frac{4}{3} \dots$	1A	
	4x - 3y + 2 = 0	1A	
At (1, -1),	$y' = \frac{2}{3} \dots$	1 A	
	2x - 3y - 5 = 0	1A 5	
. (a) z, = 1 + 2i	, z ₂ = 1 + i, z ₃ = 3 + 2i	1A	
$\frac{z_1 z_2}{z_3} = \frac{-1 + z_2}{3 + z_3}$	$\frac{3i}{2i}$ or $\frac{7+4i}{13} \cdot (1+i)$	1 A	For $z_1 z_2$ or $\frac{z_1}{z_3}$
$= \frac{3}{13} +$	$\frac{11}{13}$ i	1.A	Accept 3 + 11i
(b) \(\(\text{LAOD} \) + \(\text{LBO} \)	OD - ZCOD		
= arg z ₁ + a	arg z ₂ - arg z ₃		
$= \arg \left(\frac{z_1 z_2}{z_3} \right)$	2	1 M	
$= \tan^{-1} \frac{11}{3}$			
$-$ can $\frac{3}{3}$		1	

88 Add. Maths. I RESIRICIED	八石	
SOLUTIONS	MARKS	REMARKS
4. (a) y' = cosx + 2sinx	1A	
y" = -sinx + 2cosx	1A	
(b) $y^{t} = 0$	1M	
$tanx = -\frac{1}{2}$		
x = 2.68 or 5.82	1A	Accept x = 153° or 333°
Testing for min.	1M	but pp-1.
x = 5.82 corr. to a min.		
Minimum value of $y = -2.24$	1A 6	
5. $D = (4m)^2 - 4(4m + 15)$	1A	
16m² - 16m - 60	1A	사다. 가요요 가요요
f(x) > 0 for all values of x		$f(x) = (x+2m)^2 + (15+4m-4m^2)$
' D < 0	1M	$ \begin{vmatrix} > 0 \\ (15 + 4m - 4m^2) > 0 \end{vmatrix} $
$16m^2 - 16m - 60 < 0$		
(2m + 3)(2m - 5) < 0	1A	
$-\frac{3}{2} < m < \frac{5}{2}$	1A 5	
6. $\underline{\mathbf{a}} \cdot \underline{\mathbf{c}} = 6 + 4\mathbf{k}$ $\underline{\mathbf{b}} \cdot \underline{\mathbf{c}} = 16 + 6\mathbf{k}$	1A 1A	If vector sign omitted, pp-
Let θ be the angle between \underline{a} and \underline{c} $\underline{a} \cdot \underline{c} = \underline{a} \underline{c} \cos \theta$ $= 5\sqrt{4 + k^2} \cos \theta$	lA	Alt. Solution (1):
$\underline{b} \cdot \underline{c} = 10 \sqrt{4 + k^2} \cos \theta \qquad \dots$	1A	OA: $4x - 3y = 0$ OB: $6x - 8y = 0$ 3x - 4y = 0
$\frac{6+4k}{5\sqrt{4+k^2}} = \frac{16+6k}{10\sqrt{4+k^2}}$	1M	$\frac{8 - 3k}{5} = \pm \frac{6 - 4k}{5}$ 1M+1
k = 2	1A 6	$k = \pm 2 \qquad 1$ rejecting $k = -2$, $k = 2 \qquad 1$
Alt. Solution (2): $\angle AOX = \tan^{-1} \frac{4}{3} = 53.13^{\circ}$) $\angle BOX = \tan^{-1} \frac{3}{4} = 36.87^{\circ}$)		
$\angle COX = \frac{\angle AOX + \angle BOX}{2}$ IM	,	
= 45° IA k = 2		
k = 2	1	

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er'	SOLUTIONS	MARKS	REMARKS
7. (i) $x \ge 3$, $\frac{x-2}{2}$	$\frac{3}{x} < 1$		
x -	3 < 2x		
	x > -3	1A	
Therefore,	x ≥ 3	1A	
(ii) $3 > x > 0$,	$\frac{3-x}{2x} < 1$		
	3 - x < 2x		
	x > 1	1A	
Therefore,	3 > x > 1	1A	
$(\underline{i}\underline{i}\underline{i})x < 0, \qquad \frac{3-}{2}$	$\frac{x}{x} < 1$		
3 -	x > 2x		
	1 > x	IA	3
Therefore,	x < 0	lA	
Combining the 3 ca	ses, x < 0 or x > 1	1 <u>A</u>	

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	SOLUTIONS	MARKS	REMARKS
3. (a)	Solving $y = \frac{x^2 + 4x - 2}{x^2 + 4}$ and $y = 1$,		
	$x^2 + 4x - 2 = x^2 + 4$		
	4x = 6		
	x = 1.5		
	P is the point (1.5, 1)	$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	
(b)	(i) put $y = 0$		
	$x^2 + 4x - 2 = 0$		
	$x = -2 \pm \sqrt{6} \text{ (or } -4.45, 0.45)$	1A+1A	
	put x = 0		
	y = -0.5	1A	
_	(ii) $y' = \frac{(x^2 + 4)(2x + 4) - (x^2 + 4x - 2)(2x)}{(x^2 + 4)^2}$	1M	
	$= \frac{-4x^2 + 12x + 16}{(x^2 + 4)^2}$	1A	,
	$= \frac{-4(x-4)(x+1)}{(x^2+4)^2}$		
	< 0	1M	Putting y' < 0
	x > 4 or $x < -1$	2A	
	(iii)y' = 0	1M	
	x = 4 or $x = -1$		
	(4, 1.5) is a maximum point.	1A	
	(-1, -1) is a minimum point.	1 <u>A</u>	
(-)		2	Shape
(c)	y		Shape
	2	IA	intercepts, end-points (3 out of 5)
	$(4,1.5)$ $(5,1\frac{14}{29})$	1A	max. & min. points
•	y = 1		
		-	
$(-5, \frac{3}{29})$	(-2+16,0)		
-5	0 / 5 x		
(-2-	(0,-05)		
`	-1		
	(-1, -1)		{
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88	Add. Maths. I	KES KI	CIED	リ古り又作 MARKS	REMARKS
s.	(d) (i) $y = 1 + \frac{4x}{x^2}$ If $x > 1$	$\frac{x-6}{5}$, $4x-6>0$)	1A	
	Therefore	$\frac{4x - 6}{x^2 + 4} > 0$ y > 1)	1	
	(ii) y 1 2:+			2	Curve cutting y = 1, dedu- l mark. Wrong position of startin point, deduct l mark.
	1 (5 ,	1 1/24)	y = 1		
•	9 . 5	25 50	75	x	·
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98 Add. Maths. I RESTRICTED 內部文件

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	SOLUTIONS	MARKS	REMARKS
). (a)	$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$	1A	If vector sign ommitted, pp
	= -7 <u>i</u> - 4 <u>j</u>	1A	Alt. Solution:
	$\overrightarrow{AB} - \overrightarrow{BC} = -12\underline{i} + 6\underline{j}$		$\overrightarrow{AB} - \overrightarrow{BC}$
	$\overrightarrow{BC} = (-7\underline{i} - 4\underline{j}) - (-12\underline{i} + 6\underline{j}) \dots$	1M	$= (\overrightarrow{OB} - \overrightarrow{OA}) - (\overrightarrow{OC} - \overrightarrow{OB})$
	= 5 <u>i</u> - 10 <u>j</u>		= -12 <u>i</u> + 6 <u>j</u>
	$\overrightarrow{OC} - \overrightarrow{OB} = 5\underline{i} - 10\underline{j}$		$\overrightarrow{OC} = 2\overrightarrow{OB} - \overrightarrow{OA} - (-12\underline{i} + 6\underline{j})$
	$\overrightarrow{OC} = -\underline{i} - 12\underline{j}$	2 <u>A</u> 5	= - <u>i</u> - 12 <u>j</u>
(b)	$(i) \overrightarrow{AX} = k\overrightarrow{OX}$		
	$\overrightarrow{OX} - \overrightarrow{OA} = \overrightarrow{kOX}$		
	$(1 - k)\overrightarrow{OX} = \overrightarrow{OA}$	1A	
	$k \neq 1$, $\overrightarrow{OX} = \frac{1}{1-k} (\underline{i} + 2\underline{j})$	1A	Accept omitting $k \neq 1$.
	(ii) $\overrightarrow{BX} = \overrightarrow{OX} - \overrightarrow{OB}$		
	$= (\frac{1}{1-k} + 6) \underline{i} + (\frac{2}{1-k} + 2) \underline{j}$	1A	
	OX 1 BX		
	$\frac{1}{1-k} \left(\frac{1}{1-k} + 6 \right) + \frac{2}{1-k} \left(\frac{2}{1-k} + 2 \right) = 0$	IM	
	(7 - 6k) + 2(4 - 2k) = 0		
	$k = 1\frac{1}{2}$	2A	
	$\overrightarrow{OX} = -2\underline{i} - 4\underline{j}$	1A	Alt. Solution:
,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,	$\overrightarrow{AX} + \overrightarrow{BX} + \overrightarrow{CX}$		$\overrightarrow{AX} = -3\underline{i} - 6\underline{j})$
	$= (\overrightarrow{OX} - \overrightarrow{OA}) + (\overrightarrow{OX} - \overrightarrow{OB}) + (\overrightarrow{OX} - \overrightarrow{OC})$		$\overrightarrow{AX} = -3\underline{i} - 6\underline{j} \qquad)$ $\overrightarrow{BX} = 4\underline{i} - 2\underline{j} \qquad)$
	$= 3\overrightarrow{OX} - (\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC}) \qquad \dots$	2A	$\overrightarrow{CX} = -\underline{i} + 8\underline{j}$
	$= (-6\underline{i} - 12\underline{j}) - (-6\underline{i} - 12\underline{j})$		$\overrightarrow{AX} + \overrightarrow{BX} + \overrightarrow{CX} = \overrightarrow{0} \dots$
	= 0	1A	
	$\overrightarrow{AC} = -2\underline{i} - 14\underline{j}$, $\overrightarrow{AB} = -7\underline{i} - 4\underline{j}$		
		IM	Alt. Solution: M is the point $(-\frac{7}{2}, -7)$
	$\overrightarrow{AM} = \frac{1}{2} (\overrightarrow{AC} + \overrightarrow{AB})$ $= -\frac{9}{2} \mathbf{i} - 9 \mathbf{j}$		$\overrightarrow{AM} = -\frac{9}{2}\underline{i} - 9\underline{j} \qquad$
	$\frac{-}{2^{\perp}} = \frac{3}{2^{\perp}} = \frac{3}{2^{\perp}}$ $\overrightarrow{AX} = -3\underline{i} - 6\underline{j} \qquad \dots$	IA	Alt. Solution:
	$AX = -3\underline{1} - 6\underline{1}$ $= \frac{2}{3} \left(-\frac{9}{2}\underline{i} - 9\underline{j} \right)$	LA	$\frac{\text{Alc. Solderon.}}{\text{Slope of AX}} = \frac{-6}{-3} = 2$
	$= \frac{3}{3} \left(-\frac{1}{2} - 9 \right)$ $= \frac{2}{3} \overrightarrow{AM} \qquad \dots$	lA	Slope of AM = $\frac{-9}{-9}$ = 2
	j	1M	2
	Therefore, X lies on AM.	15	Slope of AX = slope of AM X lies on AM V

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52 } ,	SOLUTIONS	MARKS	REMARKS
10.(a)	(i) f(x) = 0		Alt. Solution:
	$x^2 + 2x - 1 = 0 \qquad \dots$	IM	$x^2 + 2x - 1 = 0$
	$x = -1 \pm \sqrt{2}$ (Accept -2.41 or 0.41)	1A	$PQ = x_1 - x_2 $
	$PQ = 2\sqrt{2} \text{ (Accept } \sqrt{8} \text{ or } 2.83)$	1A	$= \sqrt{(x_1 - x_2)^2}$
	g(x) = 0		$= \sqrt{(x_1 + x_2)^2 - 4x_1 x_2}$
	$-x^2 + 2kx - k^2 + 6 = 0$		$= \sqrt{(-2)^2 - 4(-1)}$
	$x^2 - 2kx + k^2 - 6 = 0$		$=2\sqrt{2}$
	$x = k \pm \sqrt{k^2 - (k^2 - 6)}$		g(x) = 0
	$= k \pm \sqrt{6} \qquad \dots$	1A	$RS = \sqrt{(2k)^2 - 4(k^2 - 6)}$
_	RS = $2\sqrt{6}$ (Accept $\sqrt{24}$ or 4.90)	1A	= 2√6
	(ii) x-coordinate of the mid-point of RS		
ž	$= \frac{(k + \sqrt{6}) + (k - \sqrt{6})}{2}$	1M	This can be omitted.
	= k	1A	
	x-coordinate of the mid-point of $PQ = -1$	1.A	This can be omitted.
	k = -1	1A 9	
(ð)	$x^2 + 2x - 1 = -x^2 + 2kx - k^2 + 6$	1	
	$2x^2 + (2 - 2k)x + (k^2 - 7) = 0$		
	D = 0		
_	$(2 - 2k)^2 - 4(2)(k^2 - 7) = 0$	1M	
	$k^2 + 2k - 15 = 0$		
	k = 3 or -5	2A	
	For $k = 3$, $2x^2 - 4x + 2 = 0$		
	$x^2 - 2x + 1 = 0$		
	x = 1 y = 2)	1A	
	The point is $(1, 2)$.		
	For $k = -5$, $2x^2 + 12x + 18 = 0$		
	$x^2 + 6x + 9 = 0$		
	x = -3)	1A	
	The point is $(-3, 2)$.	6	

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	SOLUTIONS
10.(c)	$f(x) \ge g(x)$
	$x^2 + 2x - 1 > -x^2 + 2kx - k^2 + 6$
	$2x^2 + (-2k + 2)x + k^2 - 7 > 0$
	This is true for any real value of x,
	$(2-2k)^2-4(2)(k^2-7)<0$
	$k^2 + 2k - 15 > 0$
	k >3 or k < -5
	Alt. Solution:
	f(x) > g(x)
	$x^2 + 2x - 1 > -x^2 + 2kx - k^2 + 6$
	$2x^2 + (-2k + 2)x + k^2 - 7 > 0$ 1A
	$x^2 + (1 - k)x + \frac{1}{2}(k^2 - 7) > 0$
	$(x + \frac{1-k}{2})^2 + \frac{1}{2}(k^2 - 7) - (\frac{1-k^2}{2})^2 > 0$ IM
	$\frac{1}{2}(k^2 - 7) - (\frac{1-k}{2})^2 > 0$
	$k^2 + 2k - 15 > 0$
	k > 3 or $k < -5$

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11.(a) (i) $z = \cos\theta + i\sin\theta$ $z^{n} = \cos\theta + i\sin\theta$ $\frac{1}{z^{n}} = \cos\theta - i\sin\theta$ $z^{n} + \frac{1}{z^{n}} = 2\cos\theta$ $z^{n} + \frac{1}{z^{n}} = 2\cos\theta$ $z^{n} - \frac{1}{z^{n}} = 2i\sin\theta$ 1A $\frac{1}{z^{n}} = \cos(-n\theta) + i\sin(-n\theta)$ 1b $\frac{1}{z^{n}} = \frac{1}{z^{n}} = 2i\sin\theta$ 1c $\frac{1}{z^{n}} = \frac{1}{z^{n}} = 2i\sin\theta$ 1d $\frac{1}{z^{n}} = \frac{1}{z^{n}} = 2i\sin\theta$ 1d $\frac{1}{z^{n}} = \frac{1}{z^{n}} = $	9	110.011	SOLUTIONS		MARKS	REMARKS
$z^{n} = \cos n\theta + i \sin n\theta$ $\frac{1}{z^{n}} = \cos n\theta - i \sin n\theta$ $z^{n} + \frac{1}{z^{n}} = 2 \cos n\theta$ $z^{n} + \frac{1}{z^{n}} = 2 \sin n\theta$ $z^{n} - \frac{1}{z^{n}} = 2 \sin n\theta$ $z^{n} $	11 . 2)	(i)				
$\frac{1}{2^{n}} = \cos \theta - i \sin \theta$ $z^{n} + \frac{1}{z^{n}} = 2 \cos \theta$ $z^{n} - \frac{1}{z^{n}} = 2 i \sin \theta$ $\frac{1}{z^{n}} = = 2 i \sin \theta$	11. (4)	(1)			1A	
$ z^{n} + \frac{1}{z^{n}} = 2 \cos \theta $ $ z^{n} - \frac{1}{z^{n}} = 2 \sin \theta $ $ z^{n} - \frac{1}{z^{n}} = 2 \sin \theta $ $ (iii) \frac{(z^{2} - \frac{1}{z^{2}})i}{z^{2} + \frac{1}{z^{2}}} = \frac{24^{2} \sin 2\theta}{2 \cos 2\theta} $ $ = - \tan 2\theta $ $ = - \tan 2\theta $ $ z = \cos \left(\frac{n\pi}{2} + \frac{\pi}{6} \right) $ $ z = \cos \left(\frac{n\pi}{2} + \frac{\pi}{6} \right) + i \sin \left(\frac{n\pi}{2} + \frac{\pi}{6} \right) $ $ z = \cos \left(\frac{n\pi}{2} + \frac{\pi}{6} \right) + i \sin \left(\frac{n\pi}{2} + \frac{\pi}{6} \right) $ $ z = \cos \left(\frac{n\pi}{2} + \frac{\pi}{6} \right) + i \sin \left(\frac{n\pi}{2} + \frac{\pi}{6} \right) $ $ z = \cos \left(\frac{n\pi}{2} + \frac{\pi}{6} \right) + i \sin \left(\frac{n\pi}{2} + \frac{\pi}{6} \right) $ $ z = \frac{1}{2} - \frac{3}{2} + \frac{1}{2} - \frac{1}{2} + \frac{\sqrt{3}}{2} + \frac{1}{2} + $			•		IA	$Accept \frac{1}{z^n} = z^{-n}$
$\tan 2\theta = \sqrt{3}$ $\tan 2\theta = \pi \pi + \frac{\pi}{3}$ $\theta = \frac{n\pi}{2} + \frac{\pi}{6}$ $z = \cos(\frac{n\pi}{2} + \frac{\pi}{5}) + i\sin(\frac{n\pi}{2} + \frac{\pi}{6})$ $\sinh 2\theta = 0, 1, 2, 3$ $\sinh 2\theta = \frac{\sqrt{3}}{2} + \frac{1}{2}, -\frac{1}{2} + \frac{\sqrt{3}}{2}i, -\frac{\sqrt{3}}{2} - \frac{1}{2},$ $\ln \frac{1}{2} + \frac{1}{2} +$,		1	$= \cos(-n\theta) + i\sin(-n\theta)$
tan20 = $\sqrt{3}$ $2\theta = n\pi + \frac{\pi}{3}$ $\theta = \frac{n\pi}{2} + \frac{\pi}{6}$ $z = \cos(\frac{n\pi}{2} + \frac{\pi}{5}) + i\sin(\frac{n\pi}{2} + \frac{\pi}{6})$) where $n = 0, 1, 2, 3$ [Accept $n = 0, 1, 2, 3,/n$ is an integer.] $0R = z = \frac{\sqrt{3}}{2} + \frac{1}{2}, -\frac{1}{2} + \frac{\sqrt{3}}{2}i, -\frac{\sqrt{3}}{2} - \frac{1}{2}, \frac{1}{2}$ $\frac{1}{2} - \frac{\sqrt{3}}{2}i$ Positions should be either specified by coordinates or unit modulus and angles. Positions should be either specified by coordinates or unit modulus and angles. NOTE: If one or more roots missing, award i main strength in the points. NOTE: If positions not specified, deduct i mark.		(ii)	$\frac{(z^2 - \frac{1}{z^2})i}{z^2 + \frac{1}{z^2}} = \frac{2i^2 \sin 2\theta}{2\cos 2\theta}$		1A	
$2\theta = n\pi + \frac{\pi}{3}$ $\theta = \frac{n\pi}{2} + \frac{\pi}{6}$ $z = \cos\left(\frac{n\pi}{2} + \frac{\pi}{6}\right) + i\sin\left(\frac{n\pi}{2} + \frac{\pi}{6}\right)$ where $n = 0, 1, 2, 3$ $0R z = \frac{\sqrt{3}}{2} + \frac{i}{2}, -\frac{1}{2} + \frac{\sqrt{3}}{2}i, -\frac{\sqrt{3}}{2} - \frac{i}{2},$ $\frac{1}{2} - \frac{\sqrt{3}}{2}i.$ Positions should be either specified by coordinates or unit modulus and angles. Positions and angles. Positions should be either specified by coordinates or unit modulus and angles. NOTE: If one or more root: nissing, award 1 max. 1A+1A 1A For two points. 2A For four points. NOTE: If positions not specified, deduct 1 mark.	_		= -tan29	• • • • • • • • • • • • • • • •	1A	
$\theta = \frac{n\pi}{2} + \frac{\pi}{6}$ $z = \cos\left(\frac{n\pi}{2} + \frac{\pi}{6}\right) + i\sin\left(\frac{n\pi}{2} + \frac{\pi}{6}\right)$ where $n = 0, 1, 2, 3$ $[Accept n = 0, 1, 2, 3, \dots/n \text{ is an integer.}] OR z = \frac{\sqrt{3}}{2} + \frac{i}{2}, -\frac{1}{2} + \frac{\sqrt{3}}{2}i, -\frac{\sqrt{3}}{2} - \frac{i}{2}, \frac{1}{2} \frac{1}{2} - \frac{\sqrt{3}}{2}i. Positions should be either specified by coordinates or unit modulus and angles. Positions and angles. Positions should be either specified by coordinates or unit modulus and angles. NOTE: If positions not specified, deduct 1 mark.$			$\tan 2\theta = \sqrt{3}$		1M	
$\frac{1}{2} - \frac{\sqrt{3}}{2} i$ $\frac{1}{2} - \frac{\sqrt{3}}{2} i$ $\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}$			$\theta = \frac{n\pi}{2} + \frac{\pi}{6}$ $z = \cos(\frac{n\pi}{2} + \frac{\pi}{6}) + isi$ where n = 0, 1, 2, 3 [Accept n = 0, 1, 2, 3,	/n is an integer.]	2A	
(b), (i) $x = \frac{1 \pm \sqrt{1 - 4}}{2}$ $= \frac{1 \pm \sqrt{3}i}{2}$ $= \cos \frac{\pi}{3} + i \sin \frac{\pi}{3}$ 1A	-		$\frac{1}{2} - \frac{\sqrt{3}}{2} i$ Imaginary $\frac{3}{30}$ $\frac{3}{30}$ Real	Positions should be either specified by coordinates or unit	1A+1A	2A For four points. NOTE: If positions not specified, deduct 1
3 3	(ъ) _.		$x = \frac{1 \pm \sqrt{1 - 4}}{2}$		10	
or $\cos(-\frac{\pi}{3}) + i\sin(-\frac{\pi}{3})$ [Accept $\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}$] 1A Accept $\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}$			$= \cos \frac{\pi}{3} + i \sin \frac{\pi}{3}$		1A	
			or $\cos(-\frac{\pi}{3}) + i\sin(-\frac{\pi}{3})$	[Accept $\cos \frac{\pi}{3} - i \sin \frac{\pi}{3}$]	1A	Accept $\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3}$

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11.(b) (ii) Product of roots =
$$\left(\frac{\alpha}{\beta}\right)^k \left(\frac{\beta}{\alpha}\right)^k$$

= 1

1A

$$\frac{2}{\sqrt{2}} = \frac{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}{\cos(\frac{-\pi}{3}) + i\sin(\frac{-\pi}{3})} \text{ or } \frac{\cos(\frac{-\pi}{3}) + i\sin(\frac{-\pi}{3})}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}$$

$$= \cos\frac{2\tau}{3} + i\sin\frac{2\pi}{3} \text{ or } \cos(\frac{-2\pi}{3}) + i\sin(\frac{-2\pi}{3}) \text{ 1A}$$

Sum of roots =
$$\left(\frac{\alpha}{\beta}\right)^k + \left(\frac{\beta}{\alpha}\right)^k$$

= $z^k + \left(\frac{1}{z}\right)^k$
= $2\cos\frac{2k\pi}{3}$...

2A

Required equation is:

$$x^2 - 2\cos\frac{2k\pi}{3} \cdot x + 1 = 0$$

1A

1A

This can be omitted.

(1) When k = 3n,

$$\frac{2k\, T\!\!\!\!/}{3}$$
 is a multiple of $2\, T\!\!\!\!/$,

equation becomes $x^2 - 2x + 1 = 0$.

(2) When k = 3n + 1,

$$\cos \frac{2k\pi}{3} = \cos(2n\pi + \frac{2\pi}{3}) = -\frac{1}{2}$$

equation becomes $x^2 + x + 1 = 0$

1A

(3) When
$$k = 3n + 2$$
,

$$\cos \frac{2k\pi}{3} = \cos(2n\pi + \frac{4\pi}{3}) = -\frac{1}{2}$$

equation becomes $x^2 + x + I = 0$

1A 10

Alt. Solution:

(b) (ii) (1) k = 3n,

sum of roots =
$$\left(\frac{3}{3}\right)^{3n} + \left(\frac{3}{3}\right)^{3n}$$

= $2\cos\frac{2(3n\pi)}{3}$ IA

= 2 1A

Equation:
$$x^2 - 2x + 1' = 0$$
 1A

(2) k = 3n + 1,

Equation: $x^2 + x + 1 = 0$ 1A

(3)
$$k = 3n + 2$$
,

sum of roots = -1 IA

Equation: $x^2 + x + 1 = 0$

1A

88 Add. Maths. I RESTRICTED 內部文件

· j	SOLUTIONS	MARKS	REMARKS
12.(a)	(i) $2\pi r = 2\emptyset$		
	$r = \frac{\cancel{10}}{\cancel{2\pi}} \dots$	1A	
	(ii) Let h be the height of the cone.		
	$h^2 = \chi^2 - r^2$		
	$= \chi^2 - \frac{\chi^2 0^2}{4 \pi^2}$	1M	For Pythagoras' Theorem.
	Volume of the cone = $\frac{1}{3} \pi r^2 h$		
	$= \frac{1}{3}\pi \cdot \frac{1^2 0^2}{4 \pi^2} \cdot \sqrt{1^2 - \frac{1^2 0^2}{4 \pi^2}}$	1M	For substitution.
	$v^2 = \frac{\sqrt{6}}{576 \pi^4} [4\pi^2 0^4 - 0^6]$		
	$= k(4\pi^2\emptyset^4 - \emptyset^6) \dots$	1	
	$(iii)\frac{d(V^2)}{d\emptyset} = k(16\pi^2 0^3 - 60^5)$	1A	
- .	= 0	IM	
	$\emptyset \neq 0, \qquad \emptyset^2 = \frac{8\pi^2}{3}$		25
	$\emptyset > 0, \qquad \emptyset = \frac{2\sqrt{6}}{3} \pi \text{(or 5.13) (or 1.63$$\pi$)}$	2A	Accept $\emptyset = 0$ or $\pm \frac{2\sqrt{6}}{3}\pi$
	$\frac{d^2(V^2)}{d\theta^2} = k(48 T^2 \theta^2 - 30 \theta^4)$		
	$= 6k\emptyset^2(3\pi^2 - 5\emptyset^2)$		
	$\frac{d^{2}(\nabla^{2})}{d\theta^{2}}\Big _{\dot{\phi}^{2} = \frac{3\pi^{2}}{3}} = 6k \cdot \frac{8\pi^{2}}{3} (8\pi^{2} - 5 \cdot \frac{8\pi^{2}}{3})$		
	< 0	1M	
	V^2 is a maximum when $\emptyset = \frac{2\sqrt{6}}{3}\pi$)	1A	
•	$\forall \text{ is a maximum when } \emptyset = \frac{2\sqrt{6}}{3}\pi \qquad)$	10	
(1)	(i) $\lambda = r - r\cos\theta$	1A	
	(ii) $A = \frac{1}{2}(r^2)(2\theta) - \frac{1}{2}r^2 \sin 2\theta$		
	$= r^2\theta - \frac{1}{2} r^2 \sin 2\theta \dots$	2A	or $r^2\theta - r^2\sin\theta\cos\theta$
	$(iii)\frac{dA}{d\theta} = r^2 - r^2\cos 2\theta \qquad \dots$	1 A	or $2r^2\sin^2\theta$
	$\frac{d\theta}{d\ell} = \frac{1}{r\sin\theta}$	1A	
	$\frac{dA}{dt} = \frac{dA}{d\theta} - \frac{d\theta}{d\lambda} \cdot \frac{d\lambda}{dt}$	1M	•
	$= (r^2 - r^2 \cos 2\theta) \cdot \frac{1}{r \sin \theta} \cdot u \dots$	1M	
	$=\frac{ru(1-cos2\theta)}{sin \theta}$	1A	Accept $\frac{u(r - r\cos 2\theta)}{\sin \theta}$
	= $2 \operatorname{ru} \sin \theta$	1 A	
	When $\theta = \frac{\pi}{6}$, $\frac{dA}{dt} = 2ru \sin \frac{\pi}{6}$		
	= ru	1 10	