Lista de Exercícios 2 Letícia Lenin Diniz 201438

e.

Legenda:

1) Número de seções

2) Convenções

3) Reações de apoios

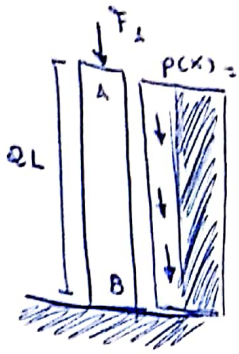
4) Método das seções

5) Diagramas

Lista de Exercícios 2

Letícia Levin Diniz 201438

Ex - Met Sec - Axial - 01:



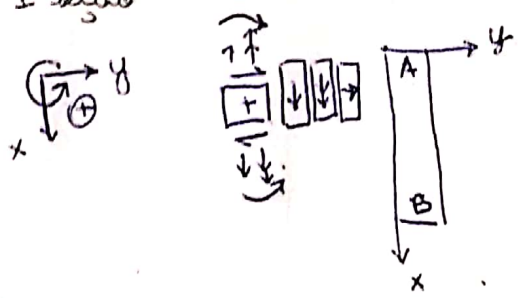
$L = 2m$

$p_0 = 5000 N/m$

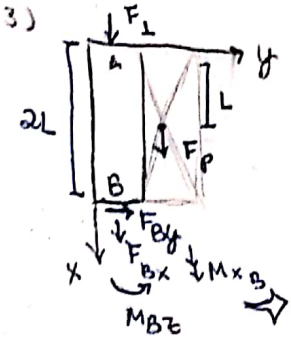
$F_1 = 8000 N$

1) 1 seção → AB: $0 < x < 2L$

2)



3)



seja $F_p = p_0(2L)$ a força pontual equivalente ao carregamento distribuído

$\Sigma F_x = 0$

$F_1 + F_{Bx} + F_p = 0$

$F_{Bx} = -F_1 - F_p$

$F_{Bx} = -F_1 - p_0 2L = -28000 N$

$\Sigma F_y = 0$

$F_{By} = 0$

$\Sigma M_z(B) = 0$

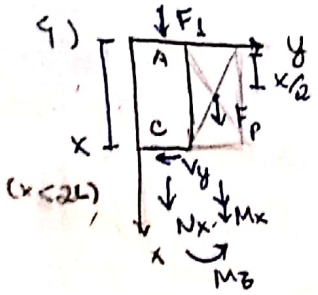
$M_{Bz} = 0$

$\Sigma M_x(B) = 0$

$M_{xB} = 0$

⇨ reações do vínculo B

4)



seja $F_p = p_0 x$

$\Sigma F_y = 0$

$V_y = 0$

$\Sigma M_z(C) = 0$

$M_z = 0$

$\Sigma M_x(C) = 0$

$M_x = 0$

$\Sigma F_x = 0$

$F_1 + F_p + N_x = 0$

$N_x(x) = -F_1 - F_p = -F_1 - p_0 x$

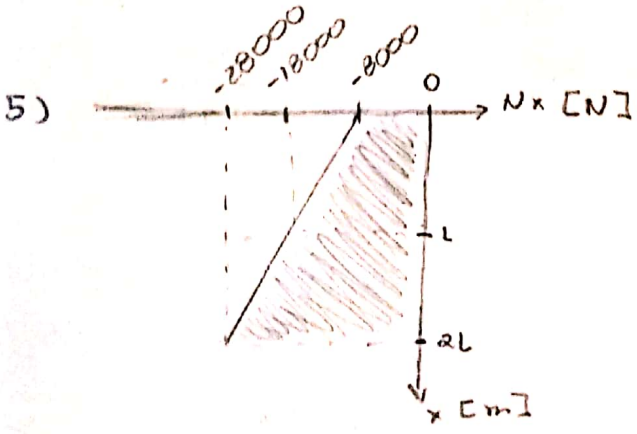
∴ Resultados: $V_y = 0$ $M_z = 0$ $M_x = 0$ $N_x(x) = -F_1 - p_0 x$

$N_x(x) = -8000 - 5000x$

Verificando:

$N_x(2L) = -28000 N = F_{Bx}$

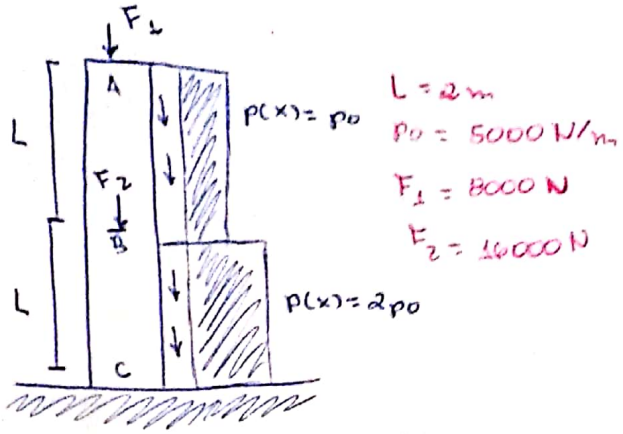
5)



Legenda:

1) Número de seções
2) Convenções
3) Reações de apoio
4) Método das seções
5) Diagramas

Ex - Mat Sec - Axial - 04:



$L = 2\,m$

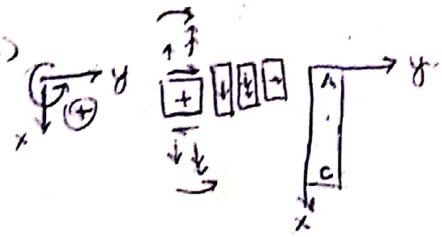
$p_0 = 5000\,N/m$

$F_1 = 8000\,N$

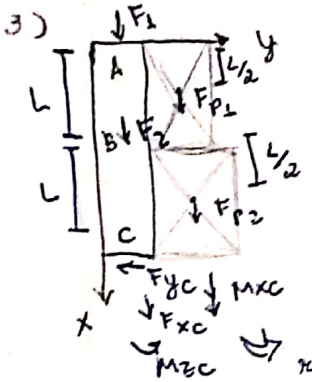
$F_2 = 16000\,N$

1) 2 seções: AB: $0 < x < L$; BC: $L < x < 2L$

2)



3)



sejam F_{P1} e F_{P2} as forças equivalentes de $p(x) = p_0$ e $p(x) = 2p_0$, respectivamente

$F_{P1} = p_0 L$

$F_{P2} = p_0 L 2$

reações do vínculo C

$\Sigma F_x = 0$

$F_1 + F_2 + F_{xc} + F_{P1} + F_{P2} = 0$

$F_{xc} = -F_1 - F_2 - F_{P1} - F_{P2}$

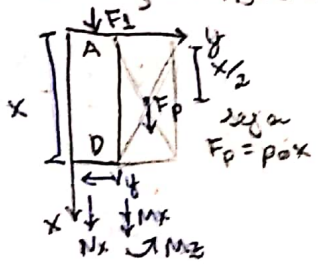
$F_{xc} = -F_1 - F_2 - 3p_0 L = -54000\,N$

$\Sigma F_y = 0 \quad \Sigma M_z(C) = 0 \quad \Sigma M_x(C) = 0$

$F_{yc} = 0 \quad M_{zc} = 0 \quad M_{xc} = 0$

4)

1ª seção: AB: $0 < x < L$



$\Sigma F_y = 0 \quad \Sigma M_x(D) = 0 \quad \Sigma M_z(D) = 0$

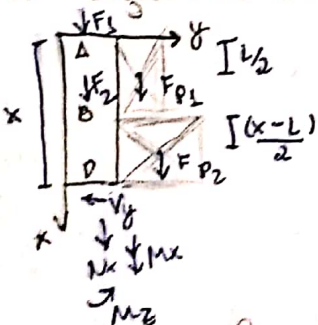
$V_y = 0 \quad M_x = 0 \quad M_z = 0$

sejam $F_P = p_0 x$

$\Sigma F_x = 0$

$F_1 + F_P + N_x = 0 \Rightarrow N_x(x) = -F_1 - F_P = -F_1 - p_0 x$

2ª seção: BC: $L < x < 2L$



sejam:

$F_{P1} = p_0 L$

$F_{P2} = (2p_0)(x - L)$

$\Sigma F_y = 0 \quad \Sigma M_x(D) = 0 \quad \Sigma M_z(D) = 0$

$V_y = 0 \quad M_x = 0 \quad M_z = 0$

$\Sigma F_x = 0$

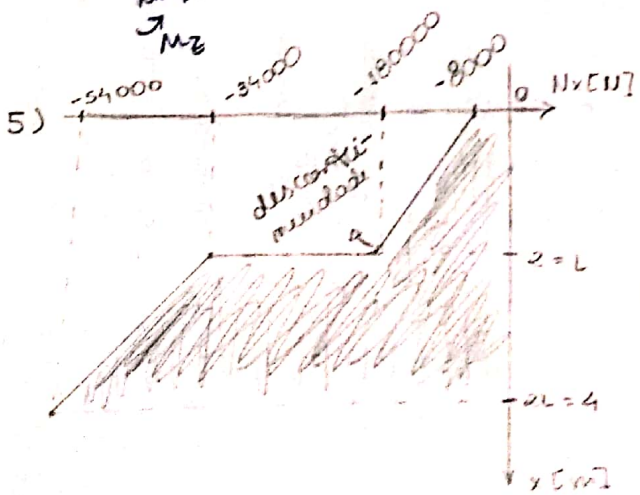
$F_1 + F_{P1} + F_{P2} + F_2 + N_x = 0$

$N_x = -F_1 - F_2 - F_{P1} - F_{P2}$

$N_x(x) = -F_1 - F_2 - p_0 L - 2p_0(x - L)$

$N_x(x) = -F_1 - F_2 + p_0 L - 2p_0 x$

5)



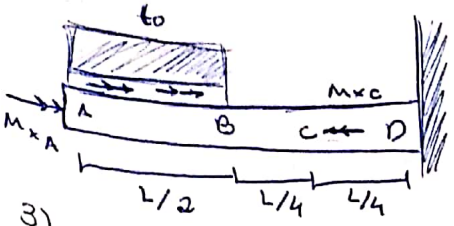
∴ Resultados:

$V_y = 0 \quad M_x = 0 \quad M_z = 0$

$N_x(x) = \begin{cases} -F_1 - p_0 x = -8000 - 5000x, & p/\ x < L = 2m \\ -F_1 - F_2 + p_0 L - 2p_0 x = -14000 - 10000x, & p/\ L < x < 2L \\ & 2 < x < 4\ m \end{cases}$

verificando: $N_x(2L) = -54000\,N = F_{xc}$

Ex - MatSec - Torção - 03:

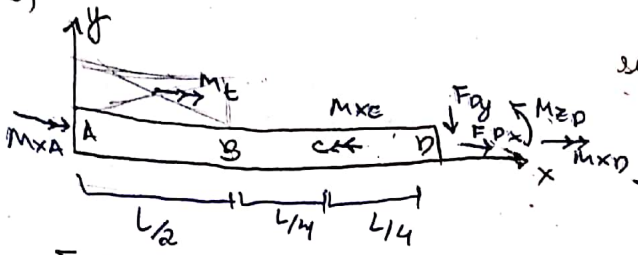


$L = 4\,m$
$to = 3000\ Nm/m$
$M_{xA} = 3000\ Nm$
$M_{xC} = 4500\ Nm$

1) 3 seções { AB: $0 < x < L/2$; BC: $L/2 < x < 3L/4$; CD: $3L/4 < x < L$

2)

3)



seja M_t o momento torçor equivalente ao esforço distribuído: $M_t = to\,L/2$

⇒ reações do vínculo D

$\Sigma F_x = 0 \quad F_{dx} = 0$

$\Sigma F_y = 0 \quad F_{dy} = 0$

$\Sigma M_z(D) = 0 \quad M_{zd} = 0$

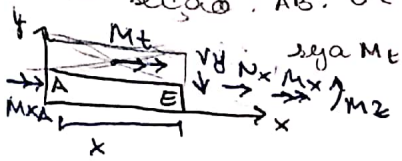
$\Sigma M_x(D) = 0$

$M_{xA} + M_t - M_{xC} + M_{xd} = 0$

$M_{xd} = M_{xC} - M_{xA} - M_t$

$M_{xD} = M_{xC} - M_{xA} - to\,L/2 = -4500\ Nm$

4) 1ª seção: AB: $0 < x < L/2$



seja $M_t = to\,x$

$\Sigma F_x = 0 \quad N_x = 0$

$\Sigma F_y = 0 \quad V_y = 0$

$\Sigma M_z(E) = 0 \quad M_z = 0$

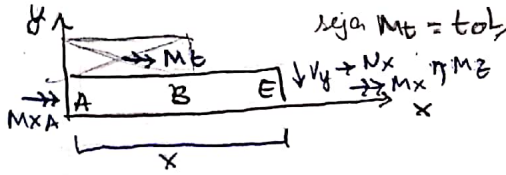
$\Sigma M_x(E) = 0$

$M_{xA} + M_t + M_x = 0$

$M_x(x) = -M_{xA} - M_t$

$M_x(x) = -M_{xA} - to\,x$

2° seção: BC: $L/2 < x < 3L/4$



seja $M_t = to\,L/2$

$\Sigma F_x = 0 \quad N_x = 0$

$\Sigma F_y = 0 \quad V_y = 0$

$\Sigma M_z(E) = 0 \quad M_z = 0$

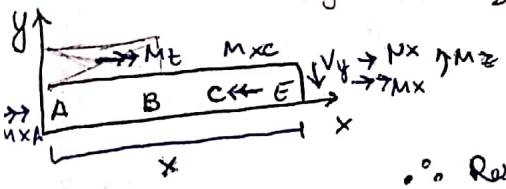
$\Sigma M_x(E) = 0$

$M_{xA} + M_t + M_x = 0$

$M_x = -M_{xA} - M_t$

$M_x(x) = -M_{xA} - to\,L/2$

3ª seção: CD: $3L/4 < x < L$



seja $M_t = to\,L/2$

$\Sigma F_x = 0 \quad N_x = 0$

$\Sigma F_y = 0 \quad V_y = 0$

$\Sigma M_z(E) = 0 \quad M_z = 0$

$\Sigma M_x(E) = 0$

$M_{xA} + M_t + M_x - M_{xC} = 0$

$M_x = M_{xC} - M_{xA} - M_t$

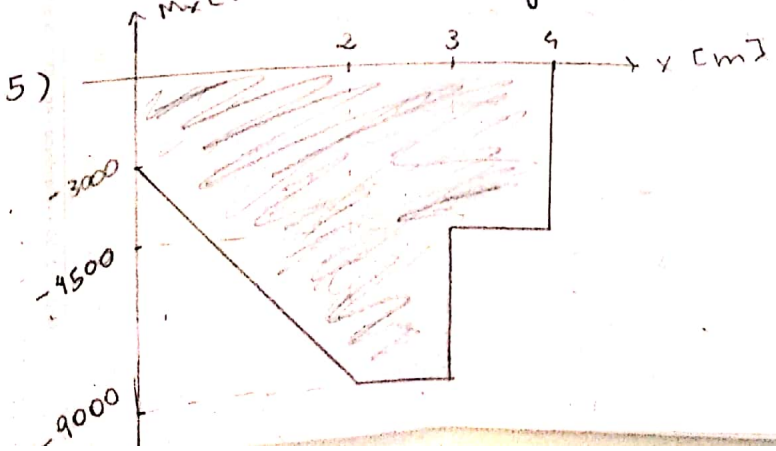
$M_x(x) = M_{xC} - M_{xA} - to\,L/2$

∴ Resultados:

$N_x = 0 \quad V_y = 0 \quad M_z = 0$

$M_x(x) = \begin{cases} -M_{xA} - to\,x = -3000 - 3000\,x \\ \quad \text{p/ } x < L/2 = 2 \\ -M_{xA} - to\,L/2 = -9000\ Nm \\ \quad \text{p/ } L/2 < x < 3L/4 \\ \quad 2 < x < 3 \\ M_{xC} - M_{xA} - to\,L/2 = -4500\ Nm \\ \quad \text{p/ } 3L/4 < x < L \\ \quad 3 < x < 4 \end{cases}$

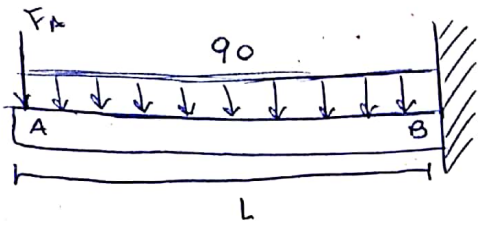
5)



↳ verificando:

$M_x(L) = -4500\ Nm = M_{xD}$

Ex - MetSec - Flexão - 01:



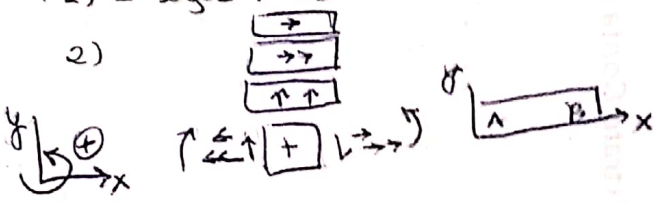
$L = 3\,m$

$F_A = 15000\,N$

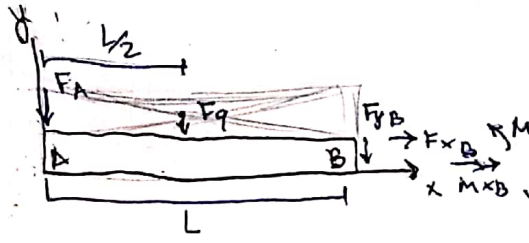
$q_0 = 8000\,N/m$

1) 1 região: AB: $0 < x < L$

2)



3)



seja F_q a força resultante equivalente ao carregamento distribuído

⇒ reação do vínculo B

$\Sigma F_x = 0 \quad \Sigma M_x(B) = 0$

$F_{xB} = 0 \quad M_{xB} = 0$

$\Sigma F_y = 0$

$-F_A - F_q - F_{yB} = 0$

$F_{yB} = -F_A - F_q$

$F_{yB} = -F_A - q_0 L = -39000\,N$

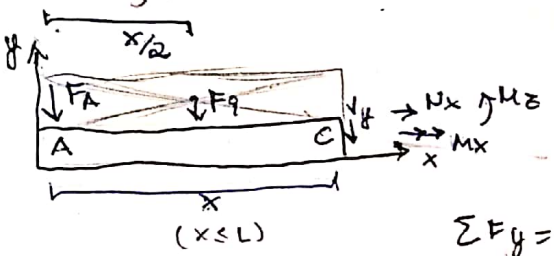
$\Sigma M_z(B) = 0$

$F_A L + F_q \frac{L}{2} + M_{zB} = 0$

$M_{zB} = -F_A L - q_0 \frac{L^2}{2}$

$M_{zB} = -81000\,Nm$

4) 1ª região: AB: $0 < x < L$



seja $F_q = q_0 x$

$\Sigma F_x = 0 \quad \Sigma M_x(C) = 0$

$N_x = 0 \quad M_x = 0$

$\Sigma F_y = 0$

$-F_A - F_q - V_y = 0$

$V_y = -F_A - F_q$

$V_y(x) = -F_A - q_0 x$

$\Sigma M_z(C) = 0$

$F_A x + F_q \frac{x}{2} + M_z = 0$

$M_z = -F_A x - F_q \frac{x}{2}$

$M_z(x) = -F_A x - q_0 \frac{x^2}{2}$

∴ Resultados:

$N_x = 0 \quad M_x = 0$

$V_y(x) = -F_A - q_0 x$

$V_y(x) = -15000 - 8000x$

▷ verificando:

$V_y(L) = -39000\,N = F_{yB}$

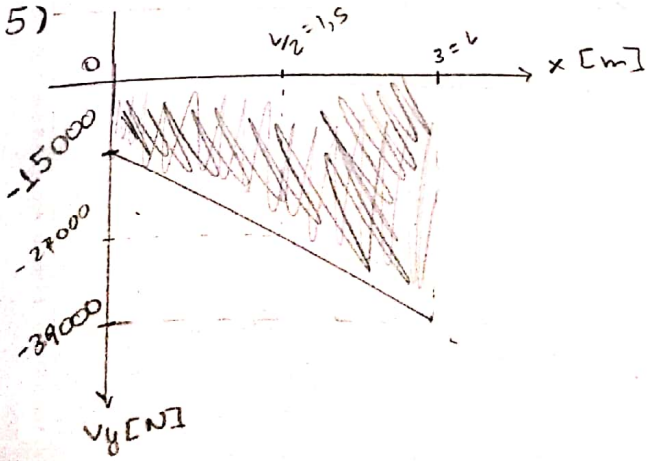
$M_z(x) = -F_A x - q_0 \frac{x^2}{2}$

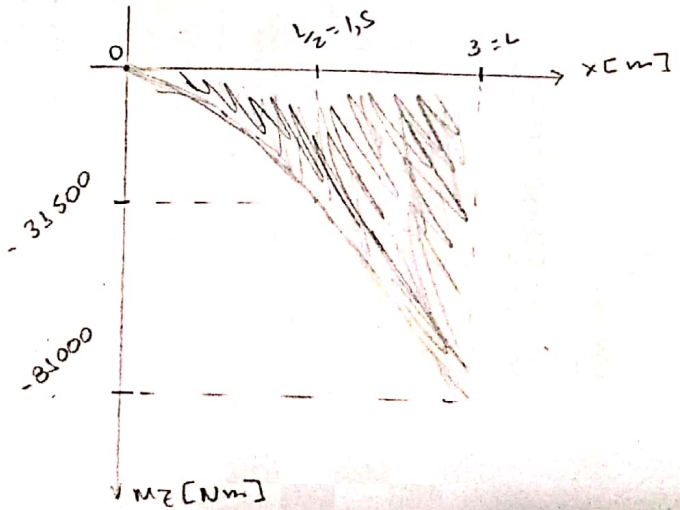
$M_z(x) = -15000x - 8000\frac{x^2}{2}$

▷ verificando:

$M_z(L) = -81000\,Nm = M_{zB}$

5)





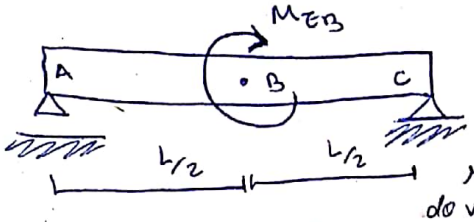
Ex_MatSec_Flexão_02:

$L = 2\,m$

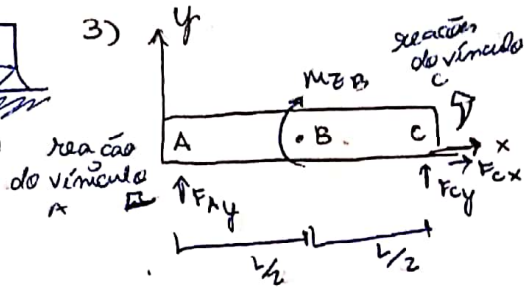
$M_{z_B} = 5000\,N\,m$

1) 2 seções $\begin{cases} AB: 0 < x < L/2 \\ BC: L/2 < x < L \end{cases}$

2)



3)



$\Sigma F_x = 0 \quad F_{cx} = 0$

$\Sigma F_y = 0 \quad F_{Ay} + F_{cy} = 0 \quad (I)$

$F_{cy} = -F_{Ay}$

$\Sigma M_z(B) = 0$

$-F_{Ay}\frac{L}{2} + F_{cy}\frac{L}{2} - M_{z_B} = 0 \quad (II)$

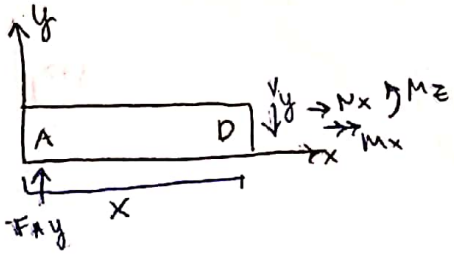
aplicando I em II:

$F_{cy}\frac{L}{2} + F_{cy}\frac{L}{2} = M_{z_B} \Rightarrow F_{cy} L = M_{z_B}$

$F_{cy} = \frac{M_{z_B}}{L} = 2500\,N$

$F_{Ay} = -2500\,N$

4) 1ª seção: $AB: 0 < x < L/2$



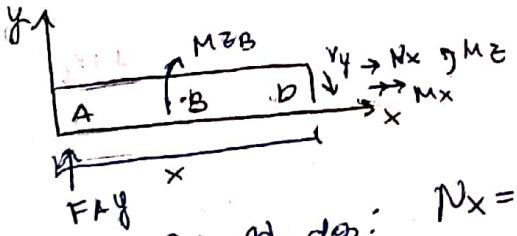
$\Sigma F_x = 0 \quad N_x = 0$

$\Sigma M_x(D) = 0 \quad M_x = 0$

$\Sigma F_y = 0 \quad +F_{Ay} - V_y = 0 \quad V_y(x) = F_{Ay}$

$\Sigma M_z(D) = 0 \quad -F_{Ay}x + M_z = 0 \quad M_z(x) = +F_{Ay}\cdot x$

2ª seção: $BC: L/2 < x < L$



$\Sigma F_x = 0 \quad N_x = 0$

$\Sigma M_x(D) = 0 \quad M_x = 0$

$\Sigma F_y = 0 \quad +F_{Ay} - V_y = 0 \quad V_y(x) = +F_{Ay}$

$\Sigma M_z(D) = 0 \quad -F_{Ay}x - M_{z_B} + M_z = 0 \quad M_z(x) = +F_{Ay}\cdot x + M_{z_B}$

∴ Resultados: $N_x = 0 \quad M_x = 0 \quad V_y(x) = F_{Ay}$

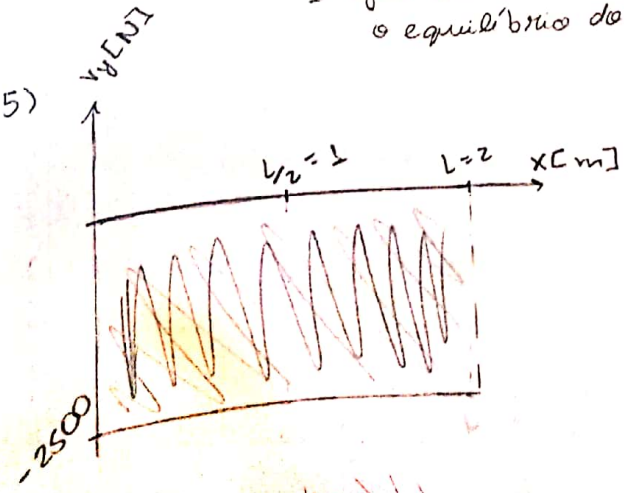
$M_z(x) = \begin{cases} F_{Ay}x = -2500\,x, & p/\ x < \frac{L}{2} = 1 \\ +F_{Ay}x + M_{z_B} = -2500\,x + 5000, & p/\ \frac{L}{2} < x < L \\ & 1 < x < 2 \end{cases}$

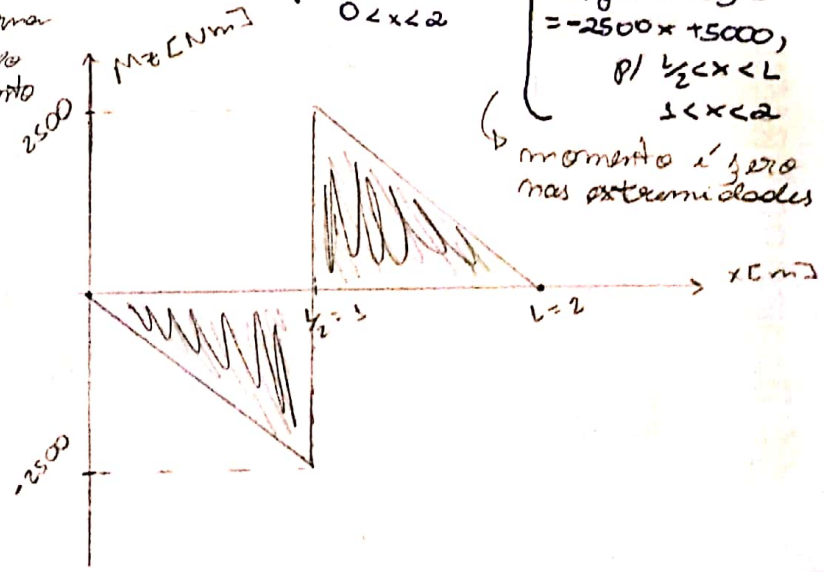
momento é zero nas extremidades

verificando: $V_y(x) = -2500\,N$ p/ $0 < x < L$, $0 < x < 2$

quando em C aplica-se $F_{cy} = 2500$, a reação interna é $V_y = -2500$, mantendo o equilíbrio do ponto

5)

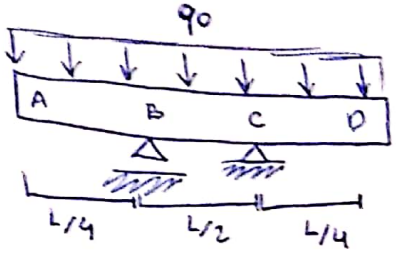




Ex_MetSec_Flexão_05:

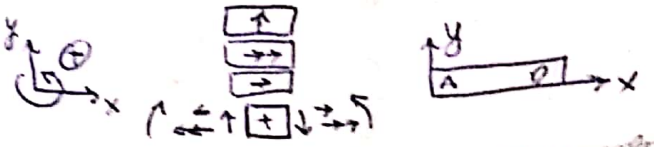
$L = 4\,m$

$q_0 = 8000\,N/m$

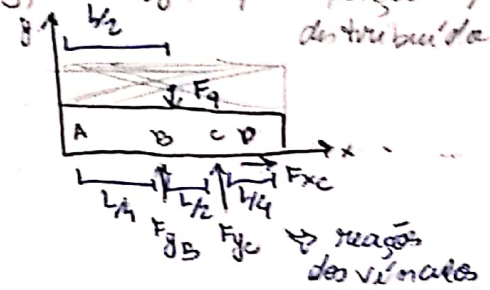


1) 3 seções { AB: $0 < x < L/4$; BC: $L/4 < x < 3L/4$; CD: $3L/4 < x < L$

2)



3) seja F_q a força equivalente ao carregamento distribuído ($F_q = q_0 L$)



$\Sigma F_x = 0$

$F_{xc} = 0$

$\Sigma F_y = 0$

$F_{yB} + F_{yc} - F_q = 0$

$F_{yB} + F_{yc} = q_0 L$ (I)

$\Sigma M_z(A) = 0$

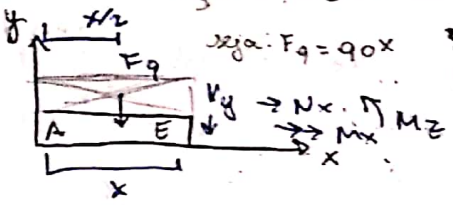
$F_{yB}\frac{L}{4} + F_{yc}\frac{3L}{4} - F_q\frac{L}{2} = 0 \Rightarrow F_{yB}\frac{L}{4} + F_{yc}\frac{3L}{4} = q_0\frac{L^2}{2}$ (II)

Aplicando I em II:

$q_0\frac{L^2}{4} - F_{yc}\frac{L}{4} + F_{yc}\frac{3L}{4} = q_0\frac{L^2}{2} \Rightarrow F_{yc} = \frac{q_0 L}{2} = 16000\,N$

$F_{yB} = q_0 L - \frac{q_0 L}{2} = \frac{q_0 L}{2} = 16000\,N$

4) 1ª seção: AB: $0 < x < L/4$



seja: $F_q = q_0 x$

$\Sigma F_x = 0$ $\quad N_x = 0$

$\Sigma M_x(E) = 0$ $\quad M_x = 0$

$\Sigma F_y = 0$ $\quad -V_y - F_q = 0$

$V_y(x) = -F_q = -q_0 x$

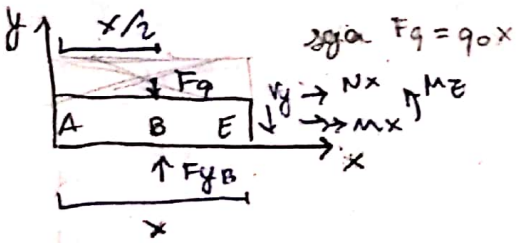
$\Sigma M_z(E) = 0$

$F_q\frac{x}{2} + M_z = 0$

$M_z = -F_q\frac{x}{2}$

$M_z(x) = -q_0\frac{x^2}{2}$

2ª seção: BC: $L/4 < x < 3L/4$



seja $F_q = q_0 x$

$\Sigma F_x = 0$ $\quad N_x = 0$

$\Sigma M_x(E) = 0$ $\quad M_x = 0$

$\Sigma F_y = 0$ $\quad -F_q + F_{yB} - V_y = 0$

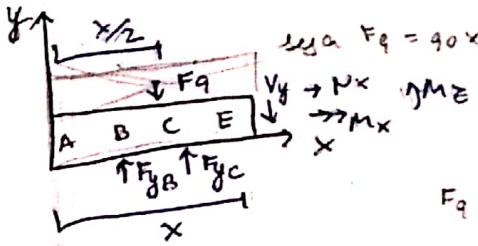
$V_y(x) = -q_0 x + F_{yB}$

$\Sigma M_z(E) = 0$

$F_q\frac{x}{2} - F_{yB}(x - L/4) + M_z = 0$

$M_z(x) = -\frac{q_0 x^2}{2} + F_{yB}(x - L/4)$

3ª seção: CD: $3L/4 < x < L$



seja $F_q = q_0 x$

$\Sigma F_x = 0$ $\quad N_x = 0$

$\Sigma M_x(E) = 0$ $\quad M_x = 0$

$\Sigma F_y = 0$

$F_{yB} + F_{yc} - F_q - V_y = 0$

$V_y(x) = F_{yB} + F_{yc} - q_0 x$

$\Sigma M_z(E) = 0$

$F_q\frac{x}{2} - F_{yB}(x - L/4) - F_{yc}(x - 3L/4) + M_z = 0 \Rightarrow M_z(x) = -\frac{q_0 x^2}{2} + F_{yB}(x - L/4) + F_{yc}(x - 3L/4)$

$\therefore N_x = 0 \qquad M_x = 0$

$$V_y(x) = \begin{cases} -q_0 x = -8000x, & x < L/4 = 1 \\ +F_{yB} - q_0 x = 16000 - 8000x, & p/\ \frac{L}{4} < x < \frac{3L}{4} \\ & 1 < x < 3 \\ +F_{yB} + F_{yC} - q_0 x = 32000 - 8000x, & p/\ \frac{3L}{4} < x < L \\ & 3 < x < 4 \end{cases}$$

$$M_z(x) = \begin{cases} -\frac{q_0 x^2}{2} = -4000x^2, & p/\ x < \frac{L}{4} = L \\ -\frac{q_0 x^2}{2} + F_{yB}(x - L/4) = -4000x^2 + 16000x - 16000, & p/\ \frac{L}{4} < x < \frac{3L}{4} \\ & 1 < x < 3 \\ -\frac{q_0 x^2}{2} + F_{yB}(x - L/4) + F_{yC}(x - 3L/4) = -4000x^2 + 32000x - 64000, & p/\ \frac{3L}{4} < x < L \\ & 3 < x < 4 \end{cases}$$

5)