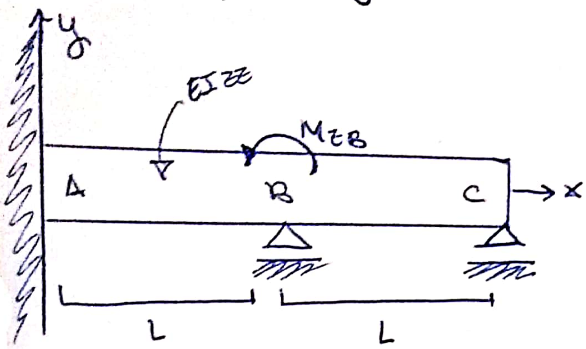
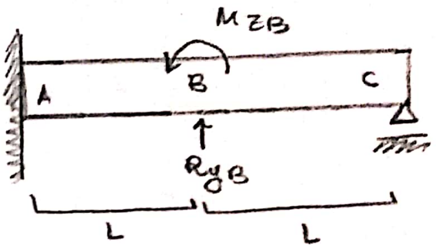
Lista de exercícios 15 Letícia Lertin Deniz 201438

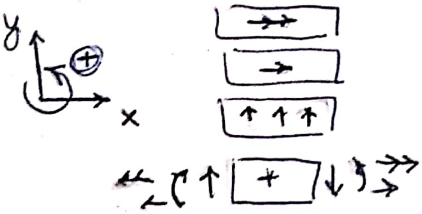
exercício eflex-viga-29:



a) montagem do problema:



0) Convenções:



1) Equação diferencial:

$$EI_{zz}\frac{d^4 v}{dx^4} = q(x)$$

2) Equação de carregamento:

$$q(x) = +R_{yB}\langle x-L\rangle^{-1} - M_{zB}\langle x-L\rangle^{-2}$$

3) Condições de contorno e restrição

$v(x=0) = 0$ $\quad v(x=2L) = 0$ $\quad v(x=L) = 0$

$\theta_z(x=0) = 0$ $\quad M_z(x=2L) = 0$ $\quad$ ↳ restrição do apoio deslizante

↳ contorno

b) reações de apoio:

$\theta_z(x=0)=0$ $\quad M_z(x=2L)=0$ $\quad$ apoio deslizante

→ contorno

b) reações de apoio:

Para obter as reações de apoio é necessário obter $V_y(x)$ e $M_z(x)$, assim:

4) Integração:

$$EI_{zz}\frac{d^4v}{dx^4} = R_{yB}\langle x-L\rangle^{-1} - M_{zB}\langle x-L\rangle^{-2}$$

$$EI_{zz}\frac{d^3v}{dx^3} = V_y(x) = R_{yB}\langle x-L\rangle^{0} - M_{zB}\langle x-L\rangle^{-1} + C_1 \quad (I)$$

$$EI_{zz}\frac{d^2v}{dx^2} = M_z(x) = R_{yB}\langle x-L\rangle^{1} - M_{zB}\langle x-L\rangle^{0} + C_1x + C_2 \quad (II)$$

$$EI_{zz}\frac{dv}{dx} = EI_{zz}\theta_z(x) = R_{yB}\frac{\langle x-L\rangle^2}{2} - M_{zB}\langle x-L\rangle^{1} + C_1\frac{x^2}{2} + C_2x + C_3 \quad (III)$$

$$EI_{zz}v = R_{yB}\frac{\langle x-L\rangle^3}{6} - M_{zB}\frac{\langle x-L\rangle^2}{2} + C_1\frac{x^3}{6} + C_2\frac{x^2}{2} + C_3x + C_4 \quad (IV)$$

5) Constantes:

$$v(x=0) = 0 = \frac{R_{yB}}{6}\underbrace{\langle 0-L\rangle^3}_{=0} - M_{zB}\underbrace{\frac{\langle 0-L\rangle^2}{2}}_{=0} + C_4 \Rightarrow \therefore C_4 = 0$$

$$\theta_z(x=0) = 0 = R_{yB}\underbrace{\frac{\langle 0-L\rangle^2}{2}}_{=0} - M_{zB}\underbrace{\langle 0-L\rangle^1}_{=0} + C_3 \Rightarrow \therefore C_3 = 0$$

$$v(x=2L)=0=R_{yB}\frac{\langle 2L-L\rangle^3}{6}-M_{zB}\frac{\langle 2L-L\rangle^2}{2}+C_1\frac{(2L)^3}{6}+C_2\frac{(2L)^2}{2}$$

$$0=\frac{R_{yB}L^3}{6}-\frac{M_{zB}L^2}{2}+C_1\frac{4}{3}L^3+C_2 2L^2$$

$$\frac{M_{zB}L^2}{2}=\frac{R_{yB}L^3}{6}+C_1\frac{4}{3}L^3+C_2 2L^2 \quad (V)$$

$$M_z(x=2L)=0=R_{yB}\langle 2L-L\rangle^1-M_{zB}\langle 2L-L\rangle^0+C_1(2L)+C_2$$

$$0=R_{yB}L-M_{zB}+2LC_1+C_2$$

$$M_{zB}=R_{yB}L+C_1 2L+C_2 \quad (VI)$$

$$v(x=L)=0=R_{yB}\frac{\langle L-L\rangle^3}{6}-M_{zB}\frac{\langle L-L\rangle^2}{2}+C_1\frac{(L)^3}{6}+C_2\frac{(L)^2}{2}$$

$$0=C_1\frac{L^3}{6}+C_2\frac{L^2}{2} \quad (VII)$$

Resolvendo o sistema com as equações V, VI e VII:

$$-\frac{C_1 L^3}{6} = C_2 \frac{L^2}{2}$$

$$-\frac{C_1 L}{3} = C_2$$

substituindo em V e VI:

$$M_{zB}\frac{L^2}{2} = R_{yB}\frac{L^3}{6} + \frac{2}{3}L^3 C_1 \quad (VIII)$$

$$M_{zB} = R_{yB} L + \frac{5L}{3} C_1 \quad (IX)$$

multiplicando VIII por 6, IX por L^2 e subtraindo IX de VIII:

$$3M_{zB} L^2 = R_{yB} L^3 + 4L^3 C_1$$

$$-\left(L^2 M_{zB} = R_{yB} L^3 + \frac{5L^3}{3} C_1\right)$$

$$2L^2 M_{zB} = 0 + \frac{7}{3} L^3 C_1$$

$$\therefore C_1 = \frac{2L^2 M_{zB}}{\frac{7L^3}{3}} = \frac{6 M_{zB}}{7L}$$

substituindo em IX e em VII:

$$R_{yB} = \frac{1}{L}\left(M_{zB} - \frac{5L}{3} \cdot \frac{6 M_{zB}}{7L}\right)$$

$$\therefore R_{yB} = \frac{-3 M_{zB}}{7L}$$

e $$C_2 = -\frac{L}{3}\left(\frac{6 M_{zB}}{7L}\right)$$

$$\therefore C_2 = -\frac{2}{7} M_{zB}$$

Assim, temos:

$$V_y(x) = -\frac{3M_{zB}}{7L}\langle x-L\rangle^0 - M_{zB}\langle x-L\rangle^{-1} + \frac{6M_{zB}}{7L} \qquad (X)$$

$$M_z(x) = -\frac{3M_{zB}}{7L}\langle x-L\rangle^1 - M_{zB}\langle x-L\rangle^0 + \frac{6M_{zB}}{7L}x - \frac{2M_{zB}}{7} \qquad (XI)$$

$$EI_{zz}\,\theta_z(x) = -\frac{3M_{zB}\langle x-L\rangle^2}{14L} - M_{zB}\langle x-L\rangle^1 + \frac{6M_{zB}}{14L}x^2 - \frac{2M_{zB}}{7}x \qquad (XII)$$

$$EI_{zz}\,v(x) = -\frac{M_{zB}\langle x-L\rangle^3}{14L} - \frac{M_{zB}}{2}\langle x-L\rangle^2 + \frac{M_{zB}x^3}{7L} - \frac{M_{zB}}{7}x^2 \qquad (XIII)$$

Reações de apoio:

$$V_y(x=0) = \frac{6M_{zB}}{7L} = F_{yA}$$

$$R_{yB} = -\frac{3M_{zB}}{7L}$$

$$M_z(x=0) = -\frac{2M_{zB}}{7} = M_{zA}$$

$$F_{yC} = V_y(x=2L) = -\frac{3M_{zB}}{7L}\cdot 1 - \cancel{M_{zB}\langle L\rangle^{-1}} + \frac{6M_{zB}}{7L}$$

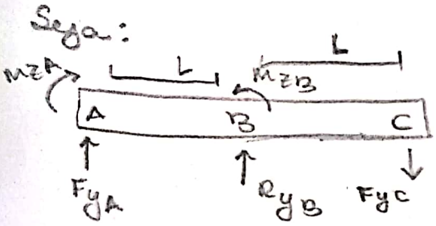
desconsidera-se, pois não se conhece a origem do momento

$$F_{yC} = V_y(x=2L) = +\frac{3M_{zB}}{7L}$$

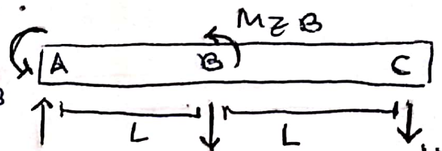
Seja:

$$F_{yC} = V_y(x=2L) = +\frac{3M_{zB}}{7L}$$

Seja:



, temos os sentidos corretos dados por:



(figura A)

$|M_{zA}| = \frac{2M_{zB}}{7}$

$|F_{yA}| = \frac{6M_{zB}}{7L}$ $\quad |R_{yB}| = \frac{3M_{zB}}{7L}$ $\quad |F_{yC}| = \frac{3M_{zB}}{7L}$

Pode-se conferir o equilíbrio estático pelos vetores da figura A:

$\sum F_y = 0$ (convenção: y, x, $\oplus$)

$$+\frac{6M_{zB}}{7L} - \frac{3M_{zB}}{7L} - \frac{3M_{zB}}{7L} = 0$$

$$0 = 0 \quad \text{(ok!)}$$

$\sum M_{z\,em\,A} = 0$

$$+\frac{2M_{zB}}{7} + M_{zB} - \frac{3M_{zB}}{7L}\cdot L - \frac{3M_{zB}}{7L}(2L) = 0$$

$$\frac{9M_{zB}}{7} - \frac{9M_{zB}}{7} = 0$$

$$0 = 0 \quad \text{(ok!)}$$

c) rigidez rotacional no ponto B, $K_{\varphi B} = K_{\varphi}(x=L) = \frac{M_{zB}}{\theta_{zB}} = \frac{M_{zB}}{\theta_z(x=L)}$:

$$K_{\varphi B} = \frac{M_{zB}}{\theta_z(x=L)}$$, utilizando XII:

$$EI_{zz}\,\theta_z(x=L) = -\frac{3M_{zB}}{14L}\langle L-L\rangle^2 - M_{zB}\langle L-L\rangle^1 + \frac{6M_{zB}}{14L}(L)^2 - \frac{2M_{zB}}{7}(L)$$

$$\therefore EI_{zz}\,\theta_z(x=L) = \frac{6M_{zB}L}{14} - \frac{2M_{zB}L}{7}$$

$$\theta_z(x=L) = \frac{M_{zB}L}{7EI_{zz}}$$

Assim:

$$K_{\varphi B} = \frac{M_{zB}}{\frac{M_{zB}L}{7EI_{zz}}} = \frac{7EI_{zz}}{L}$$