Lista de exercícios 9 - Letícia Lemos Diniz 201438

ex. 1: $\vec{u} = [(0{,}001x_1 + 0{,}009x_2 + 0{,}006x_3)\hat{a}_1 + (0{,}002x_1 + 0{,}007x_2 + 0{,}009x_3)\hat{a}_2$

$+ (0{,}001x_1 + 0{,}001x_2 - 0{,}008x_3)\hat{a}_3]$ (mm)

parte 1.

a) $[g_{ij}]$:

$$[g_{ij}] = \begin{bmatrix} \frac{\partial u_1}{\partial x_1} & \frac{\partial u_1}{\partial x_2} & \frac{\partial u_1}{\partial x_3} \\ \frac{\partial u_2}{\partial x_1} & \frac{\partial u_2}{\partial x_2} & \frac{\partial u_2}{\partial x_3} \\ \frac{\partial u_3}{\partial x_1} & \frac{\partial u_3}{\partial x_2} & \frac{\partial u_3}{\partial x_3} \end{bmatrix} = \begin{bmatrix} 0{,}001 & 0{,}009 & 0{,}006 \\ 0{,}002 & 0{,}007 & 0{,}009 \\ 0{,}001 & 0{,}001 & -0{,}008 \end{bmatrix}$$

b) $[\varepsilon_{ij}]$:

$$[\varepsilon_{ij}] = \begin{bmatrix} \frac{\partial u_1}{\partial x_1} & \left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right)\frac{1}{2} & \left(\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1}\right)\frac{1}{2} \\ \left(\frac{\partial u_2}{\partial x_1} + \frac{\partial u_1}{\partial x_2}\right)\frac{1}{2} & \frac{\partial u_2}{\partial x_2} & \left(\frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2}\right)\frac{1}{2} \\ \left(\frac{\partial u_3}{\partial x_1} + \frac{\partial u_1}{\partial x_3}\right)\frac{1}{2} & \left(\frac{\partial u_3}{\partial x_2} + \frac{\partial u_2}{\partial x_3}\right)\frac{1}{2} & \frac{\partial u_3}{\partial x_3} \end{bmatrix}$$

$$[\varepsilon_{ij}] = \begin{bmatrix} 0{,}001 & \dfrac{0{,}009+0{,}002}{2} & \dfrac{0{,}006+0{,}001}{2} \\ \dfrac{0{,}009+0{,}002}{2} & 0{,}007 & \dfrac{0{,}009+0{,}001}{2} \\ \dfrac{0{,}006+0{,}001}{2} & \dfrac{0{,}009+0{,}001}{2} & -0{,}008 \end{bmatrix} = \begin{bmatrix} 0{,}001 & 0{,}0055 & 0{,}0035 \\ 0{,}0055 & 0{,}007 & 0{,}005 \\ 0{,}0035 & 0{,}005 & -0{,}008 \end{bmatrix}$$

c) $[w_{ij}]$:

$$[w_{ij}] = \begin{bmatrix} 0 & \left(\dfrac{\partial u_1}{\partial x_2} - \dfrac{\partial u_2}{\partial x_1}\right)\dfrac{1}{2} & \left(\dfrac{\partial u_1}{\partial x_3} - \dfrac{\partial u_3}{\partial x_1}\right)\dfrac{1}{2} \\ \left(\dfrac{\partial u_2}{\partial x_1} - \dfrac{\partial u_1}{\partial x_2}\right)\dfrac{1}{2} & 0 & \left(\dfrac{\partial u_2}{\partial x_3} - \dfrac{\partial u_3}{\partial x_2}\right)\dfrac{1}{2} \\ \left(\dfrac{\partial u_3}{\partial x_1} - \dfrac{\partial u_1}{\partial x_3}\right)\dfrac{1}{2} & \left(\dfrac{\partial u_3}{\partial x_2} - \dfrac{\partial u_2}{\partial x_3}\right)\dfrac{1}{2} & 0 \end{bmatrix}$$

$$[\omega_{ij}] = \begin{bmatrix} 0 & \frac{0{,}009-0{,}002}{2} & \frac{0{,}006-0{,}001}{2} \\ \frac{0{,}002-0{,}009}{2} & 0 & \frac{0{,}009-0{,}001}{2} \\ \frac{0{,}001-0{,}006}{2} & \frac{0{,}001-0{,}009}{2} & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0{,}0035 & 0{,}0025 \\ -0{,}0035 & 0 & 0{,}004 \\ -0{,}0025 & -0{,}004 & 0 \end{bmatrix}$$

d) Ω_i:

$$\Omega_i = \frac{1}{2}\vec{\nabla}\times\vec{u} = \frac{1}{2}\begin{bmatrix} \hat{a}_1 & \hat{a}_2 & \hat{a}_3 \\ \frac{\partial}{\partial x_1} & \frac{\partial}{\partial x_2} & \frac{\partial}{\partial x_3} \\ u_1 & u_2 & u_3 \end{bmatrix}$$

$$\Omega_i = \left[\left(\frac{\partial u_3}{\partial x_2} - \frac{\partial u_2}{\partial x_3}\right)\frac{1}{2}\hat{a}_1 + \left(\frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1}\right)\frac{1}{2}\hat{a}_2 + \left(\frac{\partial u_2}{\partial x_1} - \frac{\partial u_1}{\partial x_2}\right)\frac{1}{2}\hat{a}_3\right]$$

$$\Omega_i = \left[(0{,}001-0{,}009)\tfrac{1}{2}\hat{a}_1 + (0{,}006-0{,}001)\tfrac{1}{2}\hat{a}_2 + (0{,}002-0{,}009)\tfrac{1}{2}\hat{a}_3\right]$$

$$\Omega_i = [-0{,}004\hat{a}_1 + 0{,}0025\hat{a}_2 - 0{,}0035\hat{a}_3]$$

e) Δ:

$$\Delta = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3} = 0{,}001 + 0{,}007 - 0{,}008 = 0$$

parte 2: $E = 210k\ N/mm^2$ $G = 80k\ N/mm^2$

f) $[\sigma_{ij}]$:

$$E = 2G(1+\nu) \rightarrow \nu = \frac{E}{2G} - 1 = 0{,}3125$$

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{xz} \\ \sigma_{yz} \end{Bmatrix} = \left(\frac{E}{1+\nu}\right) \begin{bmatrix} 1+\frac{\nu}{1-2\nu} & \frac{\nu}{1-2\nu} & \frac{\nu}{1-2\nu} & 0 & 0 & 0 \\ \frac{\nu}{1-2\nu} & 1+\frac{\nu}{1-2\nu} & \frac{\nu}{1-2\nu} & 0 & 0 & 0 \\ \frac{\nu}{1-2\nu} & \frac{\nu}{1-2\nu} & 1+\frac{\nu}{1-2\nu} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{xy} \\ \varepsilon_{xz} \\ \varepsilon_{yz} \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{xz} \\ \sigma_{yz} \end{Bmatrix} = 10^3 \begin{bmatrix} 293,\bar{3} & 133,\bar{3} & 133,\bar{3} & 0 & 0 & 0 \\ 133,\bar{3} & 293,\bar{3} & 133,\bar{3} & 0 & 0 & 0 \\ 133,\bar{3} & 133,\bar{3} & 293,\bar{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 160 & 0 & 0 \\ 0 & 0 & 0 & 0 & 160 & 0 \\ 0 & 0 & 0 & 0 & 0 & 160 \end{bmatrix} \begin{Bmatrix} 0{,}001 \\ 0{,}007 \\ -0{,}008 \\ 0{,}0055 \\ 0{,}0035 \\ 0{,}005 \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{xz} \\ \sigma_{yz} \end{Bmatrix} = \begin{Bmatrix} 160 \\ 1120 \\ -1280 \\ 880 \\ 560 \\ 800 \end{Bmatrix} \qquad [\sigma] = \begin{bmatrix} 160 & 880 & 560 \\ 880 & 1120 & 800 \\ 560 & 800 & -1280 \end{bmatrix} N/mm^2$$

g) $t_i^n \quad , \{n\} = 1/\sqrt{3} \begin{Bmatrix} -1 \\ 1 \\ -1 \end{Bmatrix}$

$$\{t\} = \begin{Bmatrix} 160 & 880 & 560 \\ 880 & 1120 & 800 \\ 560 & 800 & -1280 \end{Bmatrix} 1/\sqrt{3} \begin{Bmatrix} -1 \\ 1 \\ -1 \end{Bmatrix} = \begin{Bmatrix} 160/\sqrt{3} \\ -560/\sqrt{3} \\ 1520/\sqrt{3} \end{Bmatrix} = \begin{Bmatrix} 92,376 \\ -323,316 \\ 877,572 \end{Bmatrix} N/mm^2$$

$$t^n = (t \cdot n)\, n$$

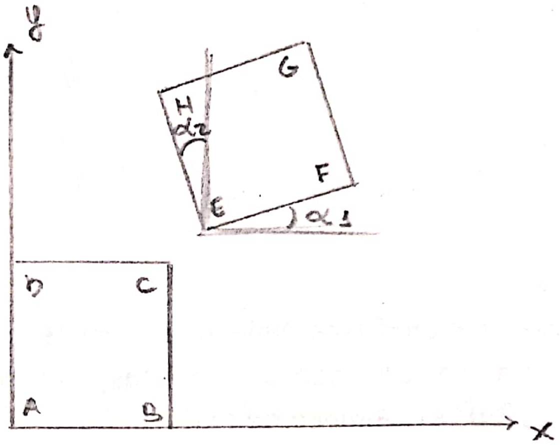
$$t \cdot n = \frac{160}{\sqrt{3}} \cdot \left(-\frac{1}{\sqrt{3}}\right) - \frac{560}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}} + \frac{1520}{\sqrt{3}}\left(-\frac{1}{\sqrt{3}}\right) = -\frac{2240}{3}$$

$$\{t^n\} = -\frac{2240}{3} \cdot \frac{1}{\sqrt{3}} \begin{Bmatrix} -1 \\ 1 \\ -1 \end{Bmatrix} = \begin{Bmatrix} 431{,}088 \\ -431{,}088 \\ 431{,}088 \end{Bmatrix} N/mm^2$$

$$\{t^t\} = \{t\} - \{t^n\} = \begin{Bmatrix} 160/\sqrt{3} - \frac{2240}{3\sqrt{3}} \\ -560/\sqrt{3} - \left(-\frac{2240}{3\sqrt{3}}\right) \\ 1520/\sqrt{3} - \frac{2240}{3\sqrt{3}} \end{Bmatrix} = \begin{Bmatrix} -338{,}712 \\ 107{,}772 \\ 446{,}484 \end{Bmatrix} N/mm^2$$

ex2.

A (0;0)
B (1,5 ; 0)
C (1,5 ; 1,5)
D (0; 1,5)
E (2 ; 2)
F (3,55 ; 2,07)
G (3,50 ; 3,47)
H (1,95 ; 3,40)



parte 1:

a) deslocamentos:

$$\left.\begin{aligned} u_A &= x_E - x_A = 2 - 0 = 2 \\ v_A &= y_E - y_A = 2 - 0 = 2 \end{aligned}\right\}$$ deslocamento do ponto A

$$\left.\begin{aligned} u_B &= x_F - x_B = 3,55 - 1,5 = 2,05 \\ v_B &= y_F - y_B = 2,07 - 0 = 2,07 \end{aligned}\right\}$$ deslocamento do ponto B

$$\left.\begin{aligned} u_C &= x_G - x_C = 3,50 - 1,5 = 2 \\ v_C &= y_G - y_C = 3,47 - 1,5 = 1,97 \end{aligned}\right\}$$ deslocamento do ponto C

$$\left.\begin{aligned} u_D &= x_H - x_D = 1,95 - 0 = 1,95 \\ v_D &= y_H - y_D = 3,40 - 1,5 = 1,9 \end{aligned}\right\}$$ deslocamento do ponto D

b) $[\varepsilon_{ij}]$ em torno de A :

$$L_{AB} = x_B - x_A = 1{,}5 - 0 = 1{,}5$$

$$L_{EF} = x_F - x_E = 3{,}55 - 2 = 1{,}55$$

$$\varepsilon_{xx} = \frac{L_{EF} - L_{AB}}{L_{AB}} = \frac{1{,}55 - 1{,}5}{1{,}5} = 0{,}03\overline{3}$$

$$L_{AD} = y_D - y_A = 1{,}5 - 0 = 1{,}5$$

$$L_{EH} = y_H - y_E = 3{,}40 - 2 = 1{,}40$$

$$\varepsilon_{yy} = \frac{L_{EH} - L_{AD}}{L_{AD}} = \frac{1{,}4 - 1{,}5}{1{,}5} = -0{,}06\overline{6}$$

$$tg\,\alpha_1 \approx \alpha_1 = \frac{y_F - y_E}{x_B - x_A} = \frac{2{,}07 - 2}{1{,}5 - 0} = 0{,}04\overline{6}$$

$$tg\,\alpha_2 \approx \alpha_2 = \frac{x_H - x_E}{y_D - y_A} = \frac{1{,}95 - 2}{1{,}5 - 0} = -0{,}03\overline{3}$$

$$\varepsilon_{xy} = \varepsilon_{yx} = \frac{\alpha_1 + \alpha_2}{2}$$

$$\varepsilon_{xy} = \varepsilon_{yx} = \frac{0{,}04\overline{6} - 0{,}03\overline{3}}{2}$$

$$\varepsilon_{xy} = \varepsilon_{yx} = 0{,}00\overline{6}$$

$$[\varepsilon] = \begin{bmatrix} 0{,}0334 & 0{,}0066 & 0 \\ 0{,}0066 & -0{,}0667 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

parede 2: $E = 85 kN/mm^2$ $G = 50 k N/mm^2$

c) $[\sigma]$:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{xz} \\ \sigma_{yz} \end{Bmatrix} = \left(\frac{E}{1+\nu}\right) \begin{bmatrix} 1+\frac{\nu}{1-2\nu} & \frac{\nu}{1-2\nu} & \frac{\nu}{1-2\nu} & 0 & 0 & 0 \\ \frac{\nu}{1-2\nu} & 1+\frac{\nu}{1-2\nu} & \frac{\nu}{1-2\nu} & 0 & 0 & 0 \\ \frac{\nu}{1-2\nu} & \frac{\nu}{1-2\nu} & 1+\frac{\nu}{1-2\nu} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{xy} \\ \varepsilon_{xz} \\ \varepsilon_{yz} \end{Bmatrix}$$

$E = 2G(1+\nu)$

$\hookrightarrow \nu = \frac{E}{2G} - 1 = -0{,}15$

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{xz} \\ \sigma_{yz} \end{Bmatrix} = \begin{bmatrix} 88461{,}53 & -11538{,}46 & -11538{,}46 & 0 & 0 & 0 \\ -11538{,}46 & 88461{,}53 & -11538{,}46 & 0 & 0 & 0 \\ -11538{,}46 & -11538{,}46 & 88461{,}53 & 0 & 0 & 0 \\ 0 & 0 & 0 & 100000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 100000 & 0 \\ 0 & 0 & 0 & 0 & 0 & 100000 \end{bmatrix} \begin{Bmatrix} 0{,}0334 \\ -0{,}0667 \\ 0 \\ 0{,}0066 \\ 0 \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{xz} \\ \sigma_{yz} \end{Bmatrix} = \begin{Bmatrix} 3724{,}230 \\ -6285{,}768 \\ 384{,}230 \\ 660 \\ 0 \\ 0 \end{Bmatrix}$$

$$[\sigma] = \begin{bmatrix} 3724{,}230 & 660 & 0 \\ 660 & -6285{,}768 & 0 \\ 0 & 0 & 384{,}230 \end{bmatrix} N/mm^2$$

d) t_i^n; $\{n\} = \frac{1}{\sqrt{2}} \begin{Bmatrix} -1 \\ 1 \\ 0 \end{Bmatrix}$

$$\{t\} = \begin{Bmatrix} 3724{,}230 & 660 & 0 \\ 660 & -6285{,}768 & 0 \\ 0 & 0 & 384{,}230 \end{Bmatrix} \frac{1}{\sqrt{2}} \begin{Bmatrix} -1 \\ 1 \\ 0 \end{Bmatrix} = \begin{Bmatrix} -3064{,}23/\sqrt{2} \\ -6945{,}768/\sqrt{2} \\ 0 \end{Bmatrix}$$

$$\{t\} = \begin{Bmatrix} -2166{,}737 \\ -4911{,}399 \\ 0 \end{Bmatrix} N/mm^2$$

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$$\{t^n\} = (t \cdot n)\{n\}$$

$$(t \cdot n) = -\frac{1}{\sqrt{2}}\left(\frac{-3064{,}23}{\sqrt{2}}\right) + \frac{1}{\sqrt{2}}\left(\frac{-6945{,}768}{\sqrt{2}}\right) = -1940{,}769$$

$$\{t^n\} = \frac{-1940{,}769}{\sqrt{2}}\begin{Bmatrix} -1 \\ 1 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 1372{,}331 \\ -1372{,}331 \\ 0 \end{Bmatrix}$$

$$\{t^t\} = \{t\} - \{t^n\}$$

$$\{t^t\} = \begin{Bmatrix} \frac{-3064{,}23}{\sqrt{2}} - \frac{1940{,}769}{\sqrt{2}} \\ \frac{-6945{,}768}{\sqrt{2}} - \left(-\frac{1940{,}769}{\sqrt{2}}\right) \\ 0 - 0 \end{Bmatrix} = \begin{Bmatrix} -3539{,}068 \\ -3539{,}068 \\ 0 \end{Bmatrix}$$