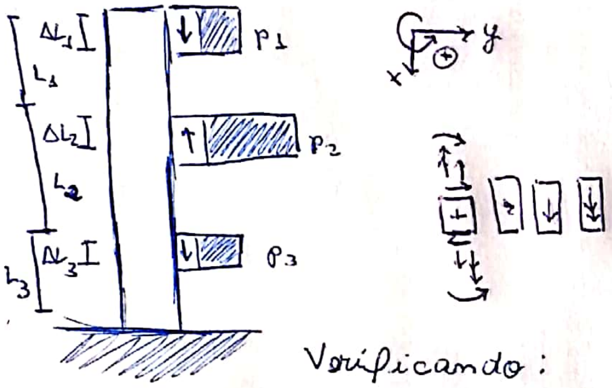
Lista de exercícios 4 — Letícia Lerrin Diniz 201438

Parte 01: Equações de Carregamento

Carregamento Axial:



A expressão que descreve esse carregamento é:

$$p(x) = +p_1\langle x-0\rangle^0 - p_1\langle x-\Delta L_1\rangle^0 - p_2\langle x-L_1\rangle^0 + p_2\langle x-(L_1+\Delta L_2)\rangle^0 + p_3\langle x-(L_1+L_2)\rangle^0 - p_3\langle x-(L_1+L_2+\Delta L_3)\rangle^0$$

Verificando:

a) $(0 < x < \Delta L_1)$: $p(x) = p_1\underbrace{\langle x-0\rangle^0}_{=(x-0)^0=1} - p_1\underbrace{\langle x-\Delta L_1\rangle^0}_{=0} - p_2\underbrace{\langle x-L_1\rangle^0}_{=0} + p_2\underbrace{\langle x-(L_1+\Delta L_2)\rangle^0}_{=0} + p_3\underbrace{\langle x-(L_1+L_2)\rangle^0}_{=0} - p_3\underbrace{\langle x-(L_1+L_2+\Delta L_3)\rangle^0}_{=0}$

$p(x) = p_1 \cdot 1 = p_1$

b) $(\Delta L_1 < x < L_1)$:

$p(x) = p_1\underbrace{\langle x-0\rangle^0}_{=1} - p_1\underbrace{\langle x-\Delta L_1\rangle^0}_{=1} - p_2\underbrace{\langle x-L_1\rangle^0}_{=0} + p_2\underbrace{\langle x-(L_1+\Delta L_2)\rangle^0}_{=0} + p_3\underbrace{\langle x-(L_1+L_2)\rangle^0}_{=0} - p_3\underbrace{\langle x-(L_1+L_2+\Delta L_3)\rangle^0}_{=0}$

$p(x) = p_1 1 - p_1 \cdot 1 = 0$

c) $(L_1 < x < L_1+\Delta L_2)$:

$p(x) = p_1\underbrace{\langle x-0\rangle^0}_{=1} - p_1\underbrace{\langle x-\Delta L_1\rangle^0}_{=1} - p_2\underbrace{\langle x-L_1\rangle^0}_{=1} + p_2\underbrace{\langle x-(L_1+\Delta L_2)\rangle^0}_{=0} + p_3\underbrace{\langle x-(L_1+L_2)\rangle^0}_{=0} - p_3\underbrace{\langle x-(L_1+L_2+\Delta L_3)\rangle^0}_{=0}$

$p(x) = p_1 1 - p_1 1 - p_2 1 = -p_2$

d) $(L_1+\Delta L_2 < x < L_1+L_2)$:

$p(x) = p_1\underbrace{\langle x-0\rangle^0}_{=1} - p_1\underbrace{\langle x-\Delta L_1\rangle^0}_{=1} - p_2\underbrace{\langle x-L_1\rangle^0}_{=1} + p_2\underbrace{\langle x-(L_1+\Delta L_2)\rangle^0}_{=1} + p_3\underbrace{\langle x-(L_1+L_2)\rangle^0}_{=0} - p_3\underbrace{\langle x-(L_1+L_2+\Delta L_3)\rangle^0}_{=0}$

$p(x) = p_1 - p_1 - p_2 + p_2 = 0$

e) $(L_1+L_2 < x < L_1+L_2+\Delta L_3)$:

$p(x) = p_1\underbrace{\langle x-0\rangle^0}_{=1} - p_1\underbrace{\langle x-\Delta L_1\rangle^0}_{=1} - p_2\underbrace{\langle x-L_1\rangle^0}_{=1} + p_2\underbrace{\langle x-(L_1+\Delta L_2)\rangle^0}_{=1} + p_3\underbrace{\langle x-(L_1+L_2)\rangle^0}_{=1} - p_3\underbrace{\langle x-(L_1+L_2+\Delta L_3)\rangle^0}_{=0}$

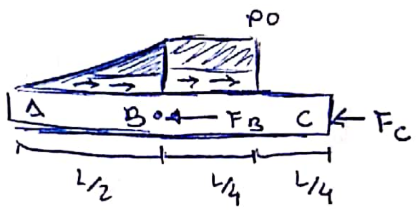
$p(x) = p_1 - p_1 - p_2 + p_2 + p_3 = p_3$

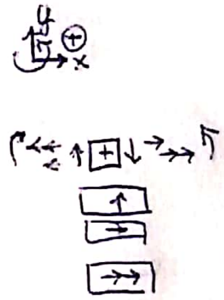
f) $(L_1 + L_2 + \Delta L_3 < x < L_1 + L_2 + L_3)$:

$$p(x) = p_1 \underbrace{\langle x - 0 \rangle^0}_{=1} - p_1 \underbrace{\langle x - \Delta L_1 \rangle^0}_{=1} - p_2 \underbrace{\langle x - L_1 \rangle^0}_{=1} + p_2 \underbrace{\langle x - (L_1 + \Delta L_2) \rangle^0}_{=1}$$

$$+ p_3 \underbrace{\langle x - (L_1 + L_2) \rangle^0}_{=1} - p_3 \underbrace{\langle x - (L_1 + L_2 + \Delta L_3) \rangle^0}_{=1}$$

$$p(x) = p_1 - p_1 - p_2 + p_2 + p_3 - p_3 = 0$$





A expressão que descreve esse carregamento é:

$$p(x) = +\frac{p_0 2}{L}\langle x-0\rangle^1 - \frac{p_0 2}{L}\langle x-L/2\rangle^1 - p_0\langle x-3L/4\rangle^0 - F_B\langle x-L/2\rangle^{-1}$$

Verificando:

a) $(0 < x < L/2)$:

$$p(x) = \frac{p_0 2}{L}\underbrace{\langle x-0\rangle^1}_{=x} - \frac{p_0 2}{L}\underbrace{\langle x-L/2\rangle^1}_{=0} - p_0\underbrace{\langle x-3L/4\rangle^0}_{=0} - F_B\underbrace{\langle x-L/2\rangle^{-1}}_{=0} = \frac{p_0 2x}{L}$$

$$\begin{cases} p(0)=0 \\ p(L/2)=p_0 \end{cases}$$

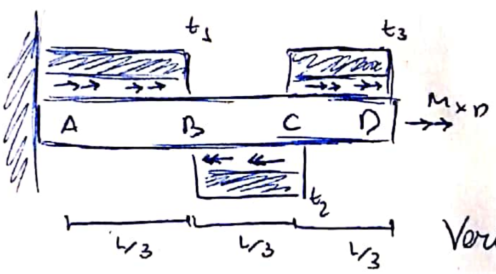
b) $(L/2 < x < 3L/4)$:

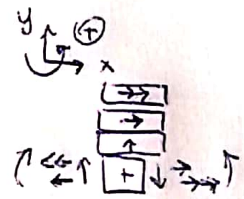
$$p(x) = \frac{p_0 2}{L}\underbrace{\langle x-0\rangle^1}_{=x} - \frac{p_0 2}{L}\underbrace{\langle x-L/2\rangle^1}_{=x-L/2} - p_0\underbrace{\langle x-3L/4\rangle^0}_{=0} - F_B\underbrace{\langle x-L/2\rangle^{-1}}_{=\delta(x-L/2)} = p_0 - F_B\,\delta(x-L/2)$$

c) $(3L/4 < x < L)$:

$$p(x) = \frac{2p_0}{L}\underbrace{\langle x-0\rangle^1}_{=x} - \frac{p_0 2}{L}\underbrace{\langle x-L/2\rangle^1}_{=x-L/2} - p_0\underbrace{\langle x-3L/4\rangle^0}_{=1} - F_B\underbrace{\langle x-L/2\rangle^{-1}}_{=\delta(x-L/2)} = -F_B\,\delta(x-L/2)$$

Carregamento Torcional:





A expressão que descreve esse carregamento é:

$$t(x) = t_1\langle x-0\rangle^0 - t_1\langle x-L/3\rangle^0 - t_2\langle x-L/3\rangle^0 + t_2\langle x-\tfrac{2L}{3}\rangle^0 + t_3\langle x-2L/3\rangle^0$$

Verificação:

a) $(0 < x < L/3)$:

$$t(x) = t_1\underbrace{\langle x-0\rangle^0}_{=1} - t_1\underbrace{\langle x-L/3\rangle^0}_{=0} - t_2\underbrace{\langle x-L/3\rangle^0}_{=0} + t_2\underbrace{\langle x-2L/3\rangle^0}_{=0} + t_3\underbrace{\langle x-2L/3\rangle^0}_{=0} = t_1$$

b) $(L/3 < x < 2L/3)$:

$$t(x) = t_1\underbrace{\langle x-0\rangle^0}_{=1} - t_1\underbrace{\langle x-L/3\rangle^0}_{=1} - t_2\underbrace{\langle x-L/3\rangle^0}_{=1} + t_2\underbrace{\langle x-2L/3\rangle^0}_{=0} + t_3\underbrace{\langle x-2L/3\rangle^0}_{=0}$$

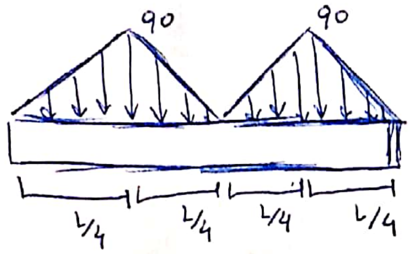
$$t(x) = t_1 - t_1 - t_2 = -t_2$$

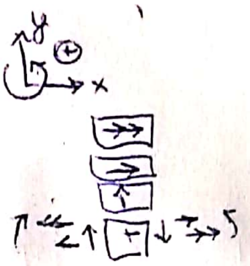
c) $(2L/3 < x < L)$:

$$t(x) = t_1\underbrace{\langle x-0\rangle^0}_{=1} - t_1\underbrace{\langle x-L/3\rangle^0}_{=1} - t_2\underbrace{\langle x-L/3\rangle^0}_{=1} + t_2\underbrace{\langle x-2L/3\rangle}_{=1} + t_3\underbrace{\langle x-2L/3\rangle^0}_{=1}$$

$$t(x) = t_1 - t_1 - t_2 + t_2 + t_3 = t_3$$

Carregamento transversal:





A expressão do carregamento é:

$$q(x) = -\frac{90\,4}{L}\langle x-0\rangle^1 + \frac{90\,8}{L}\langle x-L/4\rangle^1 - \frac{90\,8}{L}\langle x-L/2\rangle^1 + \frac{90\,8}{L}\langle x-3L/4\rangle^1$$

Verificação:

a) $(0 < x < L/4)$:

$$q(x) = -\frac{90\,4}{L}\underbrace{\langle x-0\rangle^1}_{=x} + \frac{90\,8}{L}\underbrace{\langle x-L/4\rangle^1}_{=0} - \frac{90\,8}{L}\underbrace{\langle x-L/2\rangle^1}_{=0} + \frac{90\,8}{L}\underbrace{\langle x-3L/4\rangle^1}_{=0}$$

$$q(x) = -\frac{90\,4x}{L} \quad \begin{cases} q(0) = 0 \\ q(L/4) = -90 \end{cases}$$

b) $(L/4 < x < L/2)$:

$$q(x) = -\frac{90\,4}{L}\underbrace{\langle x-0\rangle^1}_{=x} + \frac{90\,8}{L}\underbrace{\langle x-L/4\rangle^1}_{=x-L/4} - \frac{90\,8}{L}\underbrace{\langle x-L/2\rangle^1}_{=0} + \frac{90\,8}{L}\underbrace{\langle x-3L/4\rangle^1}_{=0}$$

$$q(x) = -\frac{4\,90x}{L} + \frac{8\,90x}{L} - 2\,90 = \frac{4\,90x}{L} - 2\,90 \quad \begin{cases} 90(L/4) = -90 \\ 90(L/2) = 0 \end{cases}$$

c) $(L/2 < x < 3L/4)$:

$$q(x) = -\frac{90\,4}{L}\underbrace{\langle x-0\rangle^1}_{=x} + \frac{90\,8}{L}\underbrace{\langle x-L/4\rangle^1}_{=x-L/4} - \frac{90\,8}{L}\underbrace{\langle x-L/2\rangle^1}_{=x-L/2} + \frac{90\,8}{L}\underbrace{\langle x-3L/4\rangle^1}_{=0}$$

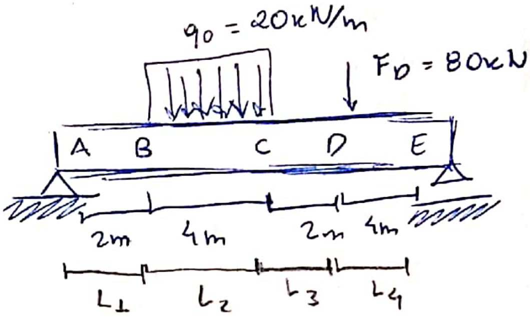
$$q(x) = -\frac{90\,4x}{L} + \frac{8\,90x}{L} - 2\,90 - \frac{8\,90x}{L} + 4\,90 = -\frac{4\,90x}{L} + 2\,90 \quad \begin{cases} 90(L/2) = 0 \\ 90(3L/4) = -90 \end{cases}$$

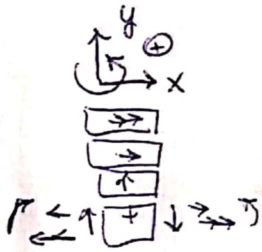
d) $(3L/4 < x < L)$:

$$q(x) = -\frac{90\,4}{L}\underbrace{\langle x-0\rangle^1}_{=x} + \frac{90\,8}{L}\underbrace{\langle x-L/4\rangle^1}_{=x-L/4} - \frac{90\,8}{L}\underbrace{\langle x-L/2\rangle^1}_{=x-L/2} + \frac{90\,8}{L}\underbrace{\langle x-3L/4\rangle^1}_{=x-3L/4}$$

$$q(x) = -\frac{90\,4x}{L} + \frac{8\,90x}{L} - 2\,90 - \frac{8\,90x}{L} + 4\,90 + \frac{8\,90x}{L} - 6\,90 \quad \begin{cases} 90(3L/4) = -90 \\ 90(L) = 0 \end{cases}$$

$$q(x) = \frac{4x\,90}{L} - 4\,90$$





A equação para esse carregamento é:

$$q(x) = -q_0\langle x - L_1\rangle^0 + q_0\langle x - (L_1+L_2)\rangle^0 - F_D\langle x - (L_1+L_2+L_3)\rangle^{-1}.$$

Verificação:

a) $(0 < x < L_1)$:

$$q(x) = -q_0\underbrace{\langle x - L_1\rangle^0}_{=0} + q_0\underbrace{\langle x - (L_1+L_2)\rangle^0}_{=0} - F_D\underbrace{\langle x - (L_1+L_2+L_3)\rangle^{-1}}_{=0}$$

$$q(x) = 0$$

b) $(L_1 < x < L_1+L_2)$:

$$q(x) = -q_0\underbrace{\langle x - L_1\rangle^0}_{=1} + q_0\underbrace{\langle x - (L_1+L_2)\rangle^0}_{=0} - F_D\underbrace{\langle x - (L_1+L_2+L_3)\rangle^{-1}}_{=0}$$

$$q(x) = -q_0$$

c) $(L_1+L_2 < x < L_1+L_2+L_3)$:

$$q(x) = -q_0\underbrace{\langle x - L_1\rangle^0}_{=1} + q_0\underbrace{\langle x - (L_1+L_2)\rangle^0}_{=1} - F_D\underbrace{\langle x - (L_1+L_2+L_3)\rangle^{-1}}_{=0}$$

$$q(x) = -q_0 + q_0 = 0$$

d) $(L_1+L_2+L_3 < x < L_1+L_2+L_3+L_4)$:

$$q(x) = -q_0\underbrace{\langle x - L_1\rangle^0}_{=1} + q_0\underbrace{\langle x - (L_1+L_2)\rangle^0}_{=1} - F_D\underbrace{\langle x - (L_1+L_2+L_3)\rangle^{-1}}_{=\delta(x-(L_1+L_2+L_3))}$$

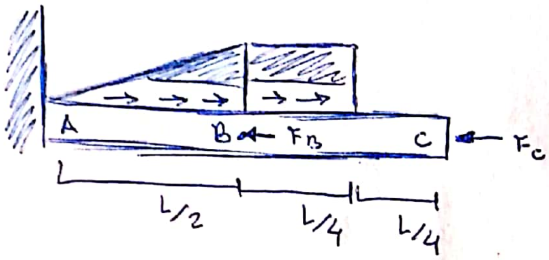
$$q(x) = -q_0 + q_0 - F_D\delta(x-(L_1+L_2+L_3)) = -F_D\,\delta(x-(L_1+L_2+L_3))$$

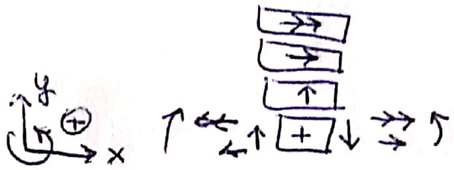
$$\therefore\ q(x) = \begin{cases} 0, & p/\ 0 < x < 2 \\ -20k\ N/m, & p/\ 2 < x < 6 \\ 0, & p/\ 6 < x < 8 \\ -80k\,\delta(x-8)\ N/m, & p/\ 8 < x < 12 \end{cases}$$

Digitalizado com CamScanner

Parte 02: Modelagem

Carregamento Axial:



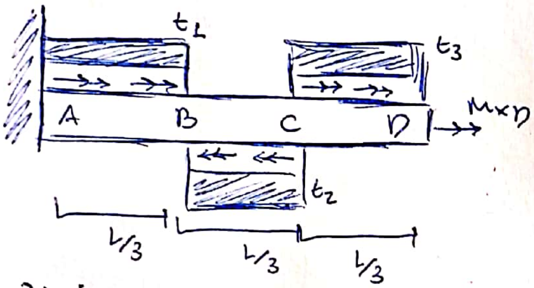
0) 

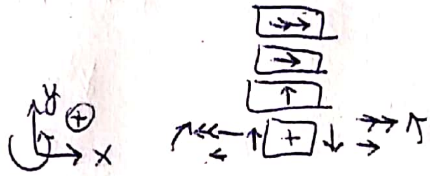
1) $\frac{dN_x(x)}{dx} = -p(x)$

2) $p(x) = +\frac{p_0 2}{L}\langle x-0\rangle^1 - \frac{p_0 2}{L}\langle x-L/2\rangle^1 - p_0\langle x-3L/4\rangle^0 - F_B\langle x-L/2\rangle^{-1}$

3) $N_x(x=L) = -F_c$

Carregamento torcional:

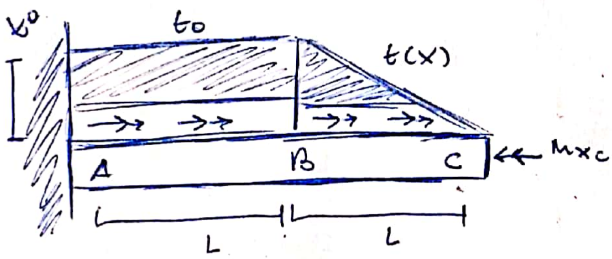


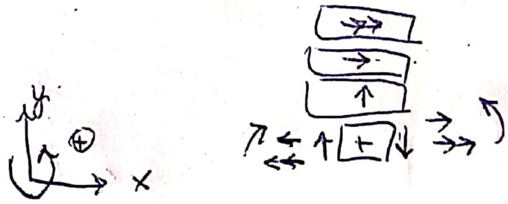
0) 

1) $\frac{dM_x(x)}{dx} = -t(x)$

2) $t(x) = t_1\langle x-0\rangle^0 - t_1\langle x-L/3\rangle^0 - t_2\langle x-L/3\rangle^0 + t_2\langle x-2L/3\rangle^0 + t_3\langle x-2L/3\rangle^0$

3) $M_x(x=L) = +M_{xD}$



0) 

1) $\frac{dM_x(x)}{dx} = -t(x)$

2) $t(x) = t_0\langle x-0\rangle^0 - \frac{t_0}{L}\langle x-L\rangle^1$

↳ verificação:

a) $(0 < x < L)$:

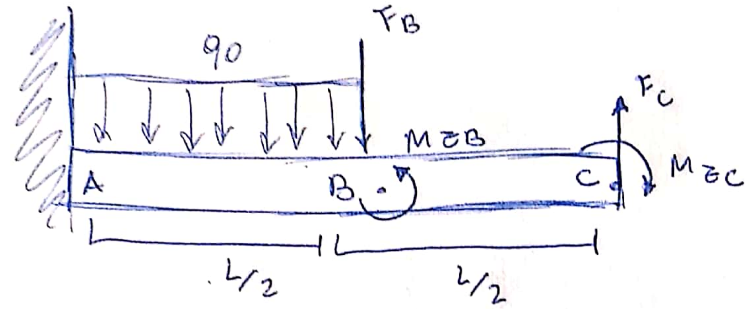
$t(x) = t_0\underbrace{\langle x-0\rangle^0}_{=1} - \frac{t_0}{L}\underbrace{\langle x-L\rangle^1}_{=0} = t_0$

b) $(L < x < 2L)$:

$t(x) = t_0\underbrace{\langle x-0\rangle^0}_{=1} - \frac{t_0}{L}\underbrace{\langle x-L\rangle^1}_{=x-L} = t_0 - \frac{t_0 x}{L} + t_0$

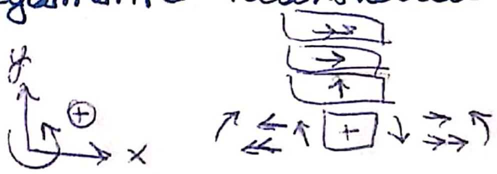
$t(x) = -\frac{t_0 x}{L} + 2t_0 \quad \begin{cases} t(L) = t_0 \\ t(2L) = 0 \end{cases}$

3) $M_x(x=2L) = -M_{xC}$



Carregamento transversal:

0)



1) $$\frac{d^2 M_z(x)}{dx^2} = q(x)$$

$$\frac{dV_y}{dx} = q(x)$$

2) $$q(x) = -q_0\langle x - 0\rangle^0 + q_0\langle x - L/2\rangle^0 - F_B\langle x - L/2\rangle^{-1} - M_{zB}\langle x - L/2\rangle^{-2}$$

3) $$M_z(x = L) = -M_{zC}$$

$$\left.\frac{dM_z}{dx}\right|_{x=L} = V_y(x = L) = -F_C$$