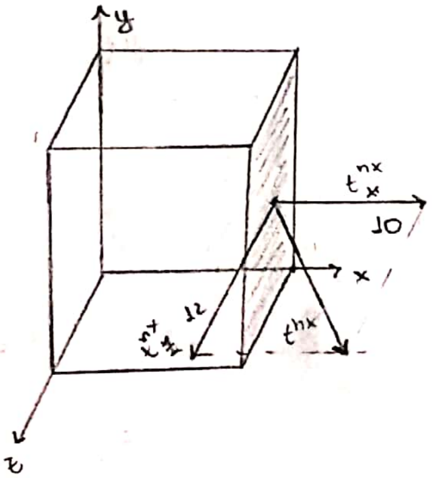
Lista de exercícios 7 - Letícia Levin Diniz 201938

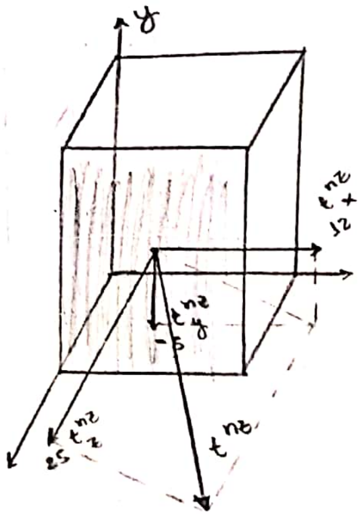
1) $[\sigma] = \begin{bmatrix} 10 & 0 & 12 \\ 0 & -15 & -5 \\ 12 & -5 & 25 \end{bmatrix} N/mm^2$

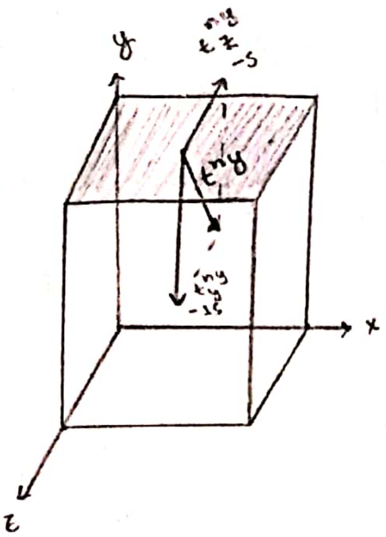
$$\{t^{nx}\} = \begin{Bmatrix} t_x^{nx} \\ t_x^{ny} \\ t_x^{nz} \end{Bmatrix} = \begin{bmatrix} 10 & 0 & 12 \\ 0 & -15 & -5 \\ 12 & -5 & 25 \end{bmatrix} \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 10 \\ 0 \\ 12 \end{Bmatrix} N/mm^2 \qquad |t^{nx}| = 15,620 \; N/mm^2$$

$$\{t^{ny}\} = \begin{Bmatrix} t_y^{nx} \\ t_y^{ny} \\ t_y^{nz} \end{Bmatrix} = \begin{bmatrix} 10 & 0 & 12 \\ 0 & -15 & -5 \\ 12 & -5 & 25 \end{bmatrix} \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ -15 \\ -5 \end{Bmatrix} N/mm^2 \qquad |t^{ny}| = 15,811 \; N/mm^2$$

$$\{t^{nz}\} = \begin{Bmatrix} t_z^{nx} \\ t_z^{ny} \\ t_z^{nz} \end{Bmatrix} = \begin{bmatrix} 10 & 0 & 12 \\ 0 & -15 & -5 \\ 12 & -5 & 25 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix} = \begin{Bmatrix} 12 \\ -5 \\ 25 \end{Bmatrix} N/mm^2 \qquad |t^{nz}| = 28,178 \; N/mm^2$$







2 - $[\sigma] = \begin{bmatrix} 10 & 0 & 12 \\ 0 & -15 & -5 \\ 12 & -5 & 25 \end{bmatrix} N/mm^2$ $n_1 = \begin{bmatrix} 0 \\ \sqrt{2}/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}$ $n_2 = \begin{bmatrix} 1/\sqrt{6} \\ 1/\sqrt{6} \\ -2/\sqrt{6} \end{bmatrix}$ $n_3 = \begin{bmatrix} -1/\sqrt{6} \\ -1/\sqrt{6} \\ 2/\sqrt{6} \end{bmatrix}$

Tensão da superfície 1:

$$\{t_1\} = \begin{bmatrix} 10 & 0 & 12 \\ 0 & -15 & -5 \\ 12 & -5 & 25 \end{bmatrix} \begin{Bmatrix} 0 \\ \sqrt{2}/\sqrt{3} \\ 1/\sqrt{3} \end{Bmatrix} = \begin{Bmatrix} 12/\sqrt{3} \\ \frac{-15\sqrt{2}-5}{\sqrt{3}} \\ \frac{-5\sqrt{2}+25}{\sqrt{3}} \end{Bmatrix} = \begin{Bmatrix} 6,928 \\ -15,134 \\ 10,351 \end{Bmatrix} N/mm^2$$

A componente normal da tensão na superfície 1 é:

$$\{t_{n_1}\} = (\{t_1\} \cdot \{n_1\})\{n_1\} = \left(\frac{12}{\sqrt{3}} \cdot 0 + \frac{\sqrt{2}}{\sqrt{3}}\left(\frac{-15\sqrt{2}-5}{\sqrt{3}}\right) + \frac{1}{\sqrt{3}}\left(\frac{-5\sqrt{2}+25}{\sqrt{3}}\right)\right) \begin{Bmatrix} 0 \\ \sqrt{2}/\sqrt{3} \\ 1/\sqrt{3} \end{Bmatrix}$$

$$\{t_{n_1}\} = \left(\frac{-5-10\sqrt{2}}{3}\right) \begin{Bmatrix} 0 \\ \sqrt{2}/\sqrt{3} \\ 1/\sqrt{3} \end{Bmatrix} = \begin{Bmatrix} 0 \\ \frac{-5\sqrt{2}-20}{3\sqrt{3}} \\ \frac{-5-10\sqrt{2}}{3\sqrt{3}} \end{Bmatrix} = \begin{Bmatrix} 0 \\ -5,209 \\ -3,684 \end{Bmatrix} N/mm^2$$

componente tangencial

$$\{t_{t_1}\} = \{t_1\} - \{t_{n_1}\} = \begin{Bmatrix} 12/\sqrt{3} \\ \frac{-15\sqrt{2}-5}{\sqrt{3}} \\ \frac{-5\sqrt{2}+25}{\sqrt{3}} \end{Bmatrix} - \begin{Bmatrix} 0 \\ \frac{-5\sqrt{2}-20}{3\sqrt{3}} \\ \frac{-5-10\sqrt{2}}{3\sqrt{3}} \end{Bmatrix} = \begin{Bmatrix} 12/\sqrt{3} \\ \frac{-40\sqrt{2}+5}{3\sqrt{3}} \\ \frac{-5\sqrt{2}+80}{3\sqrt{3}} \end{Bmatrix} = \begin{Bmatrix} 6,928 \\ -9,924 \\ 14,035 \end{Bmatrix} N/mm^2$$

$|t_1| = 19,60\ N/mm^2$ $|t_{n_1}| = 6,380\ N/mm^2$ $|t_{t_1}| = 18,532\ N/mm^2$

Tensão da superfície 2:

$$\{t_2\} = \begin{bmatrix} 10 & 0 & 12 \\ 0 & -15 & -5 \\ 12 & -5 & 25 \end{bmatrix} \begin{Bmatrix} 1/\sqrt{6} \\ 1/\sqrt{6} \\ -2/\sqrt{6} \end{Bmatrix} = \begin{Bmatrix} -14/\sqrt{6} \\ -5/\sqrt{6} \\ -43/\sqrt{6} \end{Bmatrix} = \begin{Bmatrix} -5,715 \\ -2,041 \\ -17,555 \end{Bmatrix} \ N/mm^2$$

$$\{t_{n2}\} = (\{t_2\} \cdot \{n_2\})\{n_2\} = \left(-\frac{14}{\sqrt{6}} \cdot \frac{1}{\sqrt{6}} - \frac{5}{\sqrt{6}} \cdot \frac{1}{\sqrt{6}} - \frac{43}{\sqrt{6}}\left(-\frac{2}{\sqrt{6}}\right)\right) \begin{Bmatrix} 1/\sqrt{6} \\ 1/\sqrt{6} \\ -2/\sqrt{6} \end{Bmatrix}$$

$$\{t_{n2}\} = \frac{67}{6} \begin{Bmatrix} 1/\sqrt{6} \\ 1/\sqrt{6} \\ -2/\sqrt{6} \end{Bmatrix} = \begin{Bmatrix} 67/6\sqrt{6} \\ 67/6\sqrt{6} \\ -67/3\sqrt{6} \end{Bmatrix} = \begin{Bmatrix} 4,558 \\ 4,558 \\ -9,117 \end{Bmatrix} \ N/mm^2$$

$$\{t_{t2}\} = \{t_2\} - \{t_{n2}\} = \begin{Bmatrix} -14/\sqrt{6} \\ -5/\sqrt{6} \\ -43/\sqrt{6} \end{Bmatrix} - \begin{Bmatrix} 67/6\sqrt{6} \\ 67/6\sqrt{6} \\ -67/3\sqrt{6} \end{Bmatrix} = \begin{Bmatrix} -151/6\sqrt{6} \\ -97/6\sqrt{6} \\ -62/3\sqrt{6} \end{Bmatrix} = \begin{Bmatrix} -10,274 \\ -6,600 \\ -8,437 \end{Bmatrix} \ N/mm^2$$

$|t_2| = 18,574 \ N/mm^2$ $\quad |t_{n2}| = 11,165 \ N/mm^2$ $\quad |t_{t2}| = 14,842 \ N/mm^2$

Tensão da superfície 3:

$$\{t_3\} = \begin{bmatrix} 10 & 0 & 12 \\ 0 & -15 & -5 \\ 12 & -5 & 25 \end{bmatrix} \begin{Bmatrix} -1/\sqrt{6} \\ -1/\sqrt{6} \\ 2/\sqrt{6} \end{Bmatrix} = \begin{Bmatrix} 14/\sqrt{6} \\ 5/\sqrt{6} \\ 43/\sqrt{6} \end{Bmatrix} = \begin{Bmatrix} 5,715 \\ 2,041 \\ 17,555 \end{Bmatrix} N/mm^2$$

$$\{t_{n_3}\} = (\{t_3\}\cdot\{n_3\})\{n_3\} = \left(\tfrac{14}{\sqrt{6}}\left(-\tfrac{1}{\sqrt{6}}\right) + \tfrac{5}{\sqrt{6}}\left(-\tfrac{1}{\sqrt{6}}\right) + \tfrac{43}{\sqrt{6}}\left(\tfrac{2}{\sqrt{6}}\right)\right) \begin{Bmatrix} -1/\sqrt{6} \\ -1/\sqrt{6} \\ 2/\sqrt{6} \end{Bmatrix}$$

$$\{t_{n_3}\} = \frac{67}{6} \begin{Bmatrix} -1/\sqrt{6} \\ -1/\sqrt{6} \\ 2/\sqrt{6} \end{Bmatrix} = \begin{Bmatrix} -67/6\sqrt{6} \\ -67/6\sqrt{6} \\ 67/3\sqrt{6} \end{Bmatrix} = \begin{Bmatrix} -4,558 \\ -4,558 \\ 9,117 \end{Bmatrix} N/mm^2$$

$$\{t_{t_3}\} = \{t_3\} - \{t_{n_3}\} = \begin{Bmatrix} 14/\sqrt{6} \\ 5/\sqrt{6} \\ 43/\sqrt{6} \end{Bmatrix} - \begin{Bmatrix} -67/6\sqrt{6} \\ -67/6\sqrt{6} \\ 67/3\sqrt{6} \end{Bmatrix} = \begin{Bmatrix} 151/6\sqrt{6} \\ 97/6\sqrt{6} \\ 62/3\sqrt{6} \end{Bmatrix} = \begin{Bmatrix} 10,274 \\ 6,600 \\ 8,437 \end{Bmatrix} N/mm^2$$

$|t_3| = 18,574\ N/mm^2$ $\quad |t_{n_3}| = 11,165\ N/mm^2$ $\quad |t_{t_3}| = 14,842\ N/mm^2$

Na superfície 1 atua a maior força de cisalhamento, pois $|t_{t_1}|$ é o maior entre os módulos das componentes tangenciais.