

Ex. Método-Corial-01

Dados: $L = 2\text{ m}$

$p_0 = 5000\text{ N/m}$

$F_1 = 8000\text{ N}$

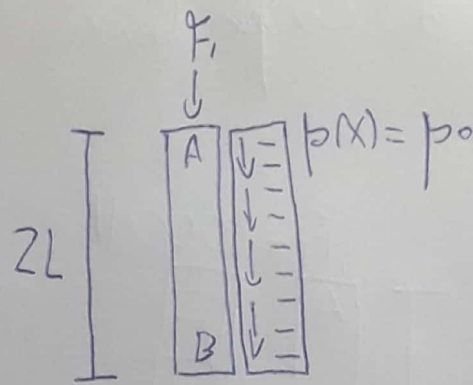
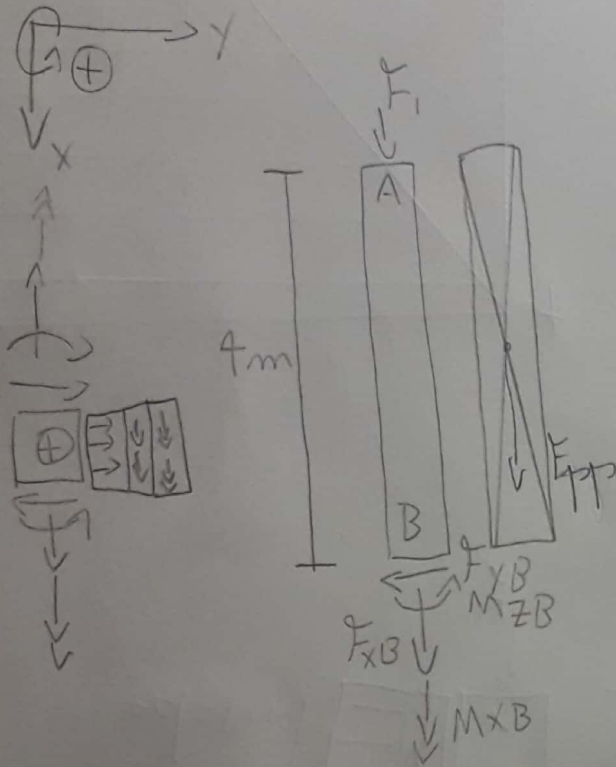


Diagrama de Corpo Livre



$$F_{pp} = p_0 \cdot X$$

Número de Seções: 1

$$0 < X < 2L$$

Seção AB: $(0 < X < 2L)$

$$\sum M_z = 0 \rightarrow M_z(x) = 0\text{ N}\cdot\text{m}$$

$$\sum M_x = 0 \rightarrow M_x(x) = 0\text{ N}\cdot\text{m}$$

$$\sum F_y = 0 \rightarrow V_y(x) = 0\text{ N}$$

$$\sum F_x = 0 \rightarrow F_1 + N_x + F_{pp} = 0$$

$$F_1 + N_x + X \cdot p_0 = 0 \rightarrow N_x = -F_1 - X \cdot p_0$$

$$N_x = -8000 - X \cdot 5000$$

$$N_x(0) = -8000\text{ N}$$

$$N_x(4) = -28000\text{ N}$$

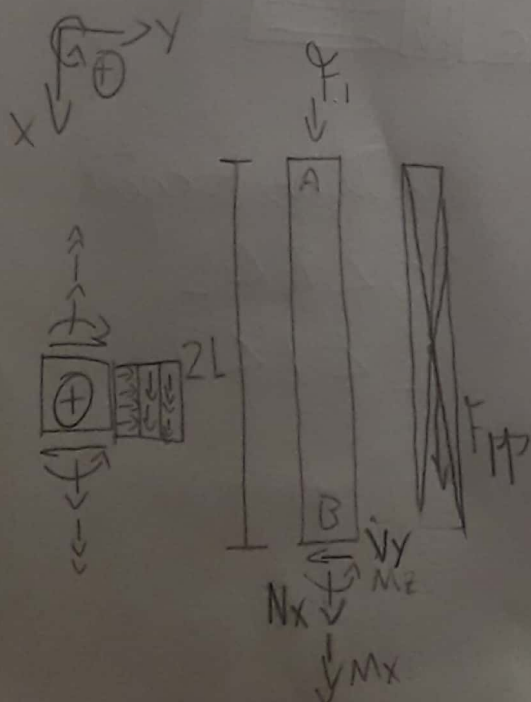
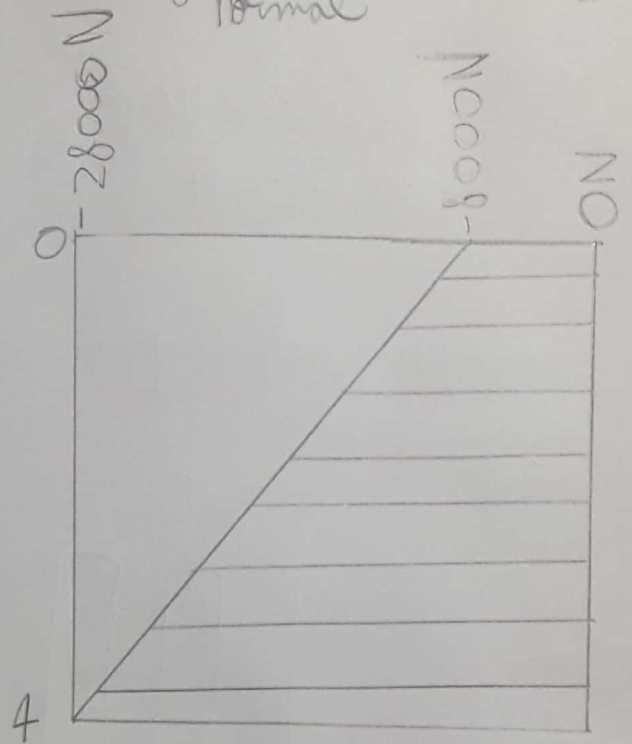
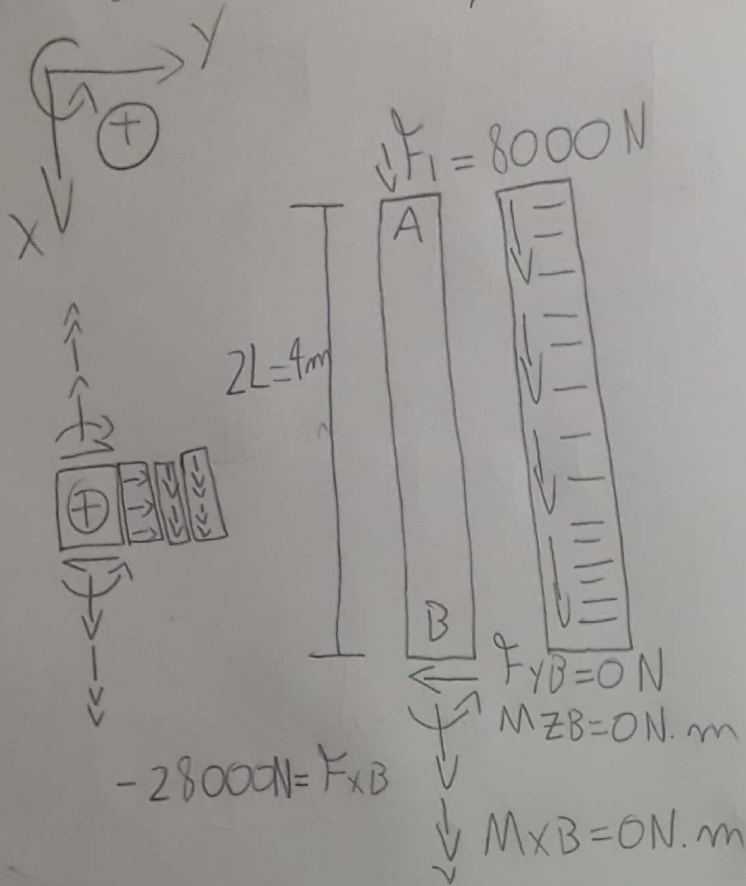


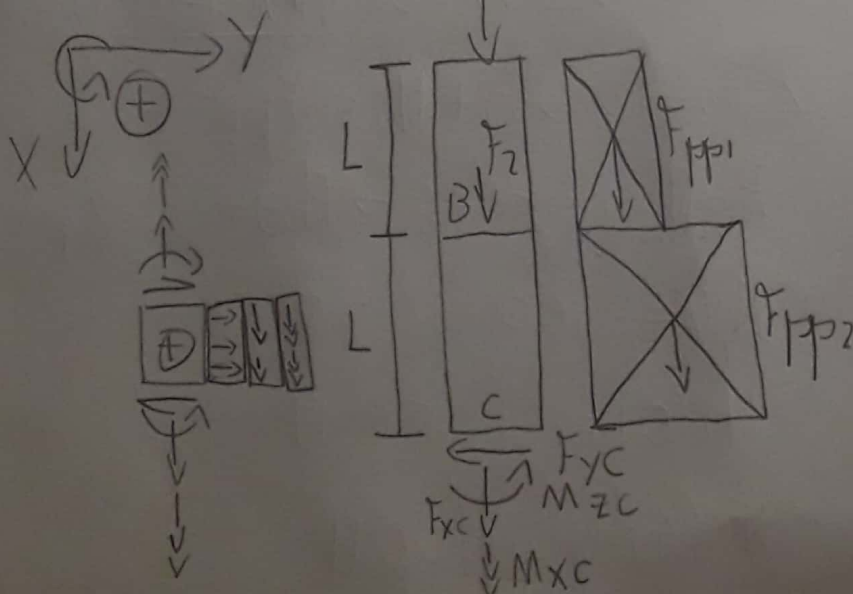
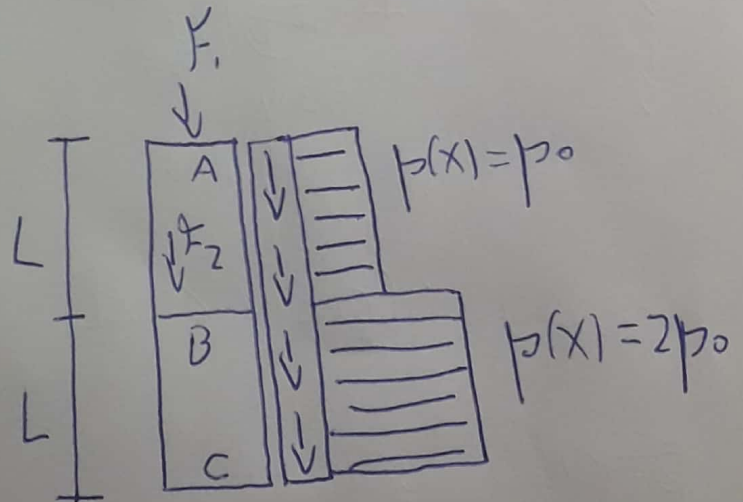
Diagrama de Corpo Livre

Diagrama de Esforço Normal



Ex. Método Axial - 04

Dados: $L = 2m$
 $p_0 = 5000 N/m$
 $F_1 = 8000 N$
 $F_2 = 16000 N$



Quas seções

$$0 < x < L$$

$$L < x < 2L$$

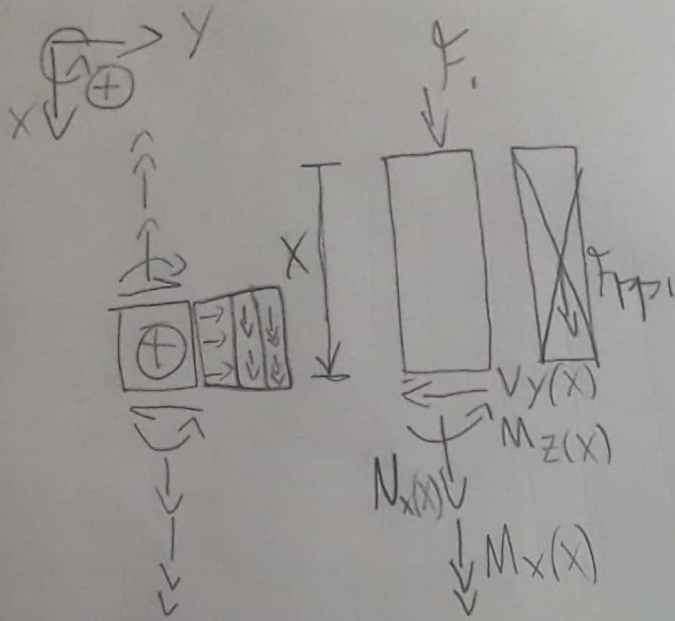
$$F_{pp1} = p_0 \cdot x$$

$$F_{pp2} = 2p_0 \cdot x$$

Seção AB: ($0 < x < L$) $\Sigma F_y = 0 \rightarrow \boxed{V_y(x) = 0 \text{ N}}$ ③

$$\Sigma M_z = 0 \rightarrow \boxed{M_z(x) = 0 \text{ N.m}}$$

$$\Sigma M_x = 0 \rightarrow \boxed{M_x(x) = 0 \text{ N.m}}$$



$$\Sigma F_x = 0 \rightarrow F_1 + F_{pp1} + N_x = 0$$

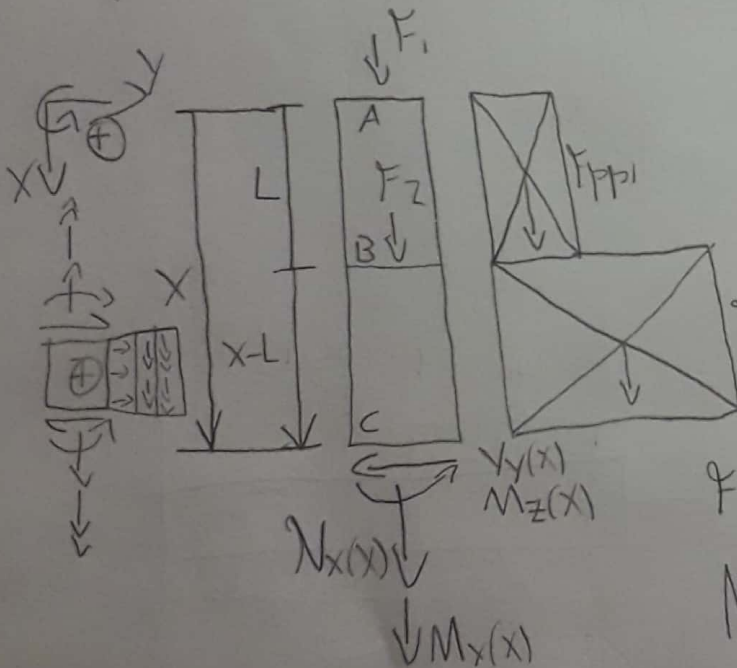
$$N_x = -F_1 - F_{pp1} = -8000 - p_0 \cdot x$$

$$\boxed{N_x(x) = -8000 - 5000x}$$

$$\boxed{N_x(0) = -8000 \text{ N}}$$

$$\boxed{N_x(L) = -18000 \text{ N}}$$

Seção BC: ($L < x < 2L$)



$$\Sigma F_y = 0 \rightarrow \boxed{V_y = 0 \text{ N}}$$

$$\Sigma M_z = 0 \rightarrow \boxed{M_z(x) = 0 \text{ N.m}}$$

$$\Sigma M_x = 0 \rightarrow \boxed{M_x(x) = 0 \text{ N.m}}$$

$$\Sigma F_x = 0$$

$$F_{pp2} + F_1 + F_2 + F_{pp1} + N_x = 0$$

$$N_x = -F_1 - F_2 - F_{pp1} - F_{pp2}$$

$$N_x = -8000 - (6000 \cdot p_0 \cdot L - 2p_0(x-L))$$

$$N_x(x) = -24000 - p_0 \cdot L - 2p_0(x-L)$$

$$N_x(x=L) = -34000 - 2p_0(L-L)$$

$$\boxed{N_x(x=L) = -34000 \text{ N}}$$

$$N_x(x=2L) = -34000 - 2p_0(2L-L)$$

$$N_x(x) = -24000 - 5000 \cdot 2 - 2p_0(x-L)$$

$$\boxed{N_x(x) = -34000 - 2p_0(x-L)}$$

$$N_x(x=2L) = -34000 - 2 \cdot 5000 \cdot 2 = \boxed{-54000 \text{ N}}$$

Diagrama de Corpo Livre

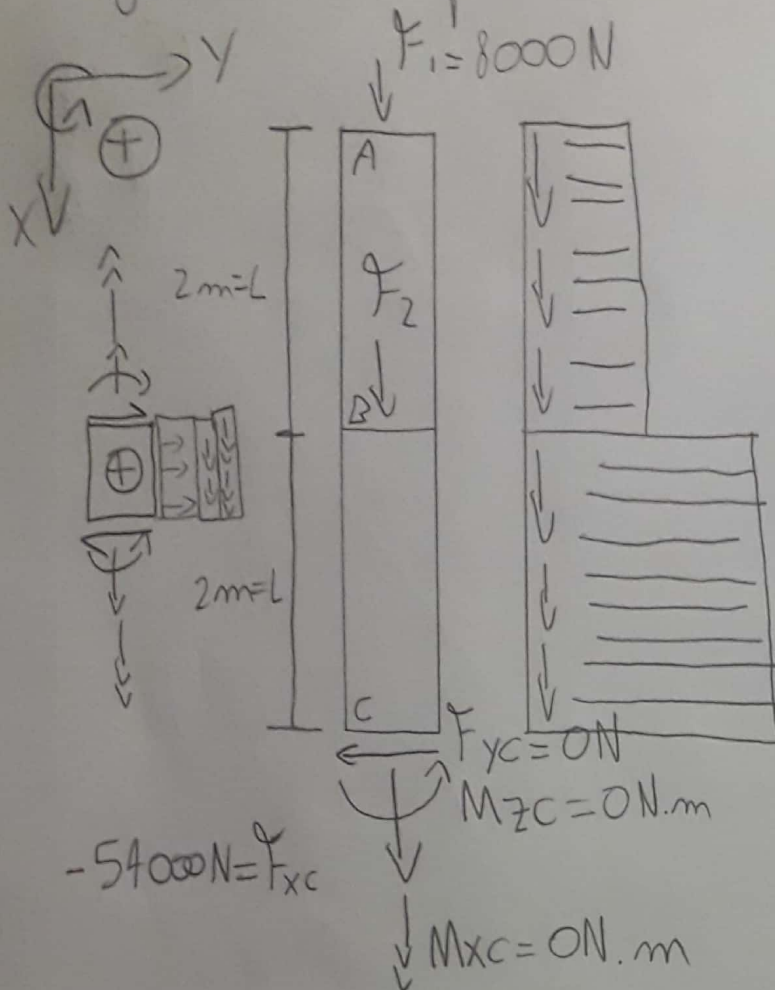
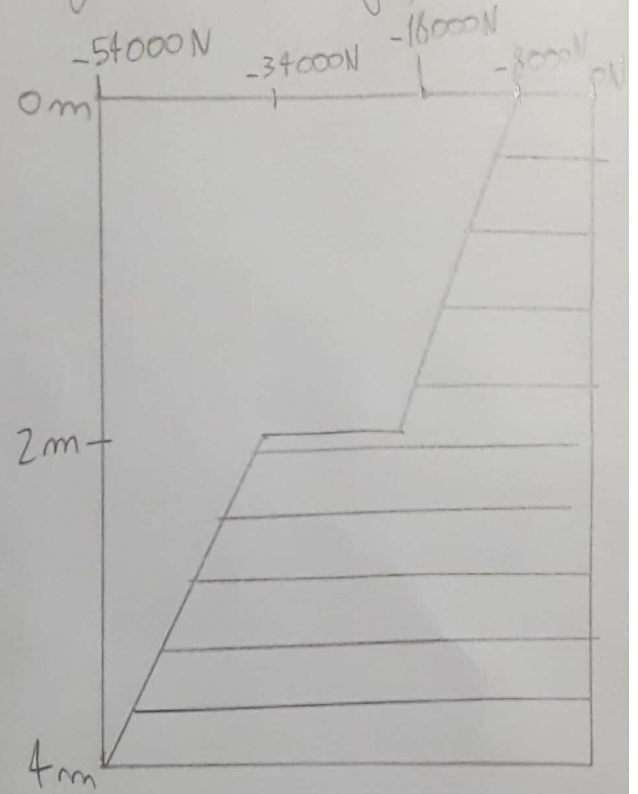


Diagrama de Esforço Normal



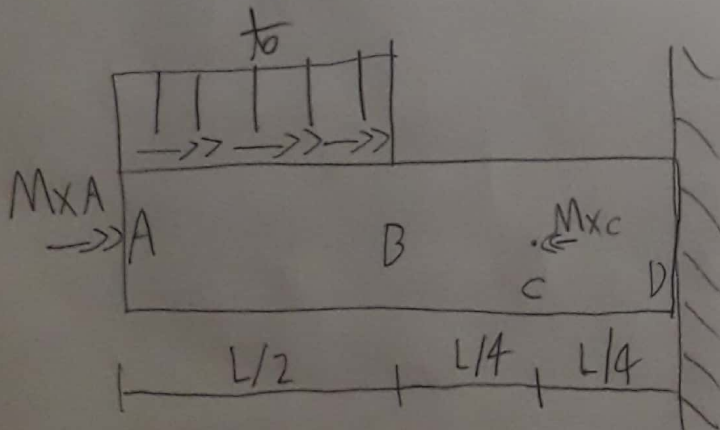
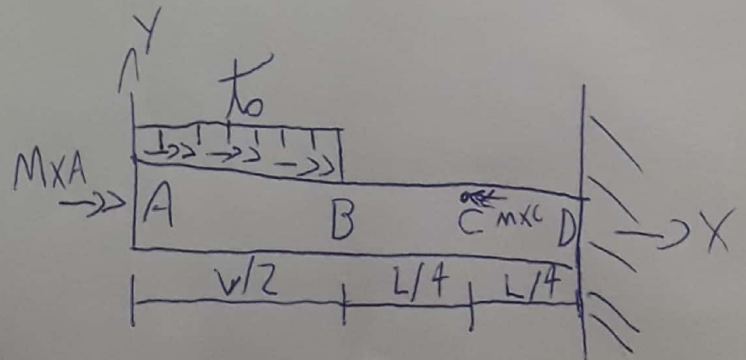
Ex. Métodos - Torção - 03

Dados: $L = 4\text{ m}$

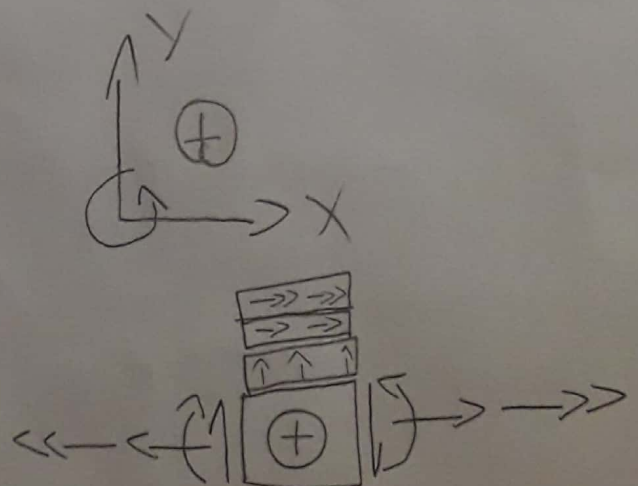
$$t_0 = 3000\text{ N.m/m}$$

$$M_{xA} = 3000\text{ N.m}$$

$$M_{xC} = 4500\text{ N.m}$$



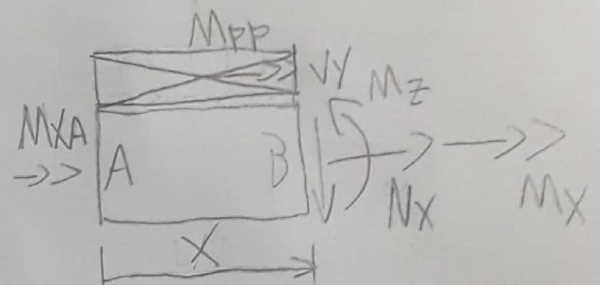
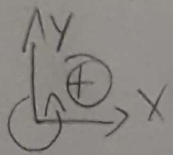
$$M_{pp} = t_0 \cdot X$$



Número de Seções: 3 \rightarrow $0 < x < L/2$;
 $L/2 < x < 3L/4$;
 $3L/4 < x < L$

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Seção AB: ($0 < x < L/2$)



$$\sum F_x = 0 \rightarrow N_x = 0 \text{ N}$$

$$\sum F_y = 0 \rightarrow V_y = 0 \text{ N}$$

$$\sum M_z = 0 \rightarrow M_z = 0 \text{ N.m}$$

$$\sum M_x = 0 \rightarrow +M_{xA} + M_{pp} + M_x = 0$$

$$M_x = -M_{xA} - M_{pp}$$

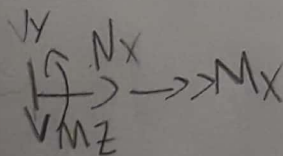
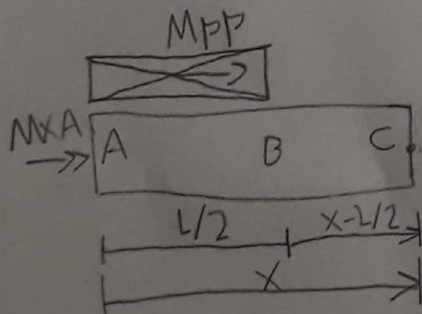
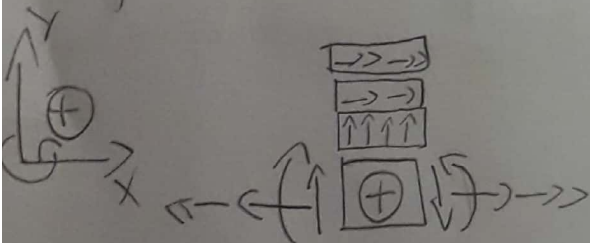
$$M_x(x) = -3000 - 3000 \cdot x$$

$$M_x(x=0) = -3000 \text{ N.m}$$

$$M_x(x=L/2) = -3000 - 3000(4/2)$$

$$M_x(x=L/2) = -9000 \text{ N.m}$$

Seção BC: ($L/2 < x < 3L/4$)



$$\sum F_x = 0 \rightarrow N_x(x) = 0 \text{ N}$$

$$\sum F_y = 0 \rightarrow V_y(x) = 0 \text{ N}$$

$$\sum M_z = 0 \rightarrow M_z(x) = 0 \text{ N.m}$$

$$\sum M_x = 0$$

$$+M_{xA} + M_{pp} + M_x = 0$$

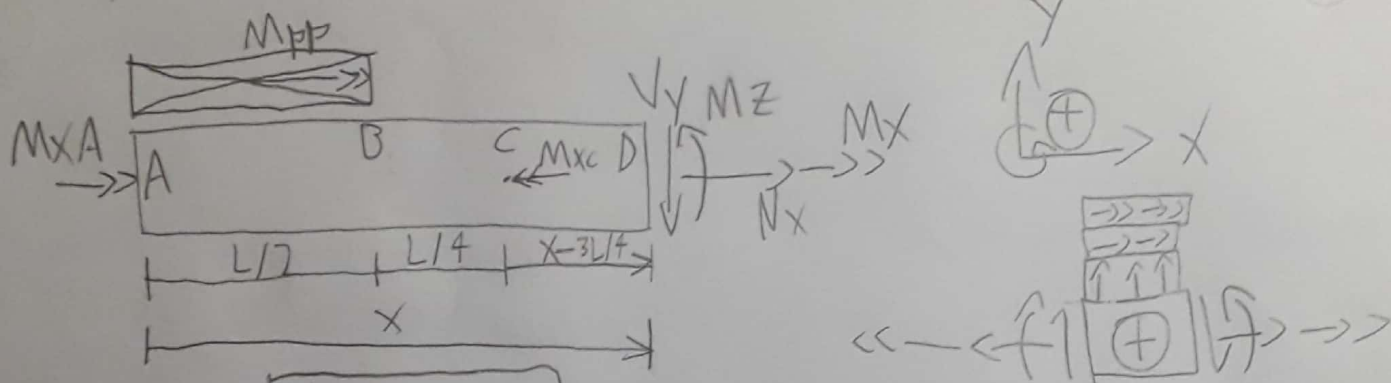
$$M_x = +M_{xA} - M_{pp}$$

$$M_x = -3000 + 3000 \cdot L/2$$

$$M_x(x) = -9000 \text{ N.m}$$

Seção CD. ($3L/4 < x < L$)

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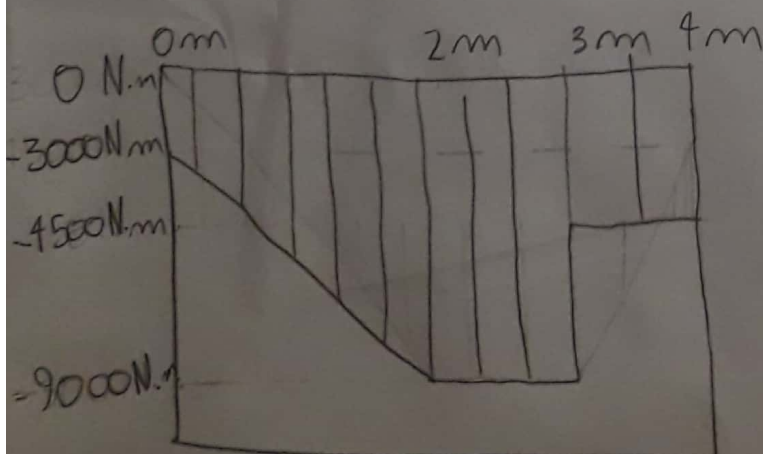
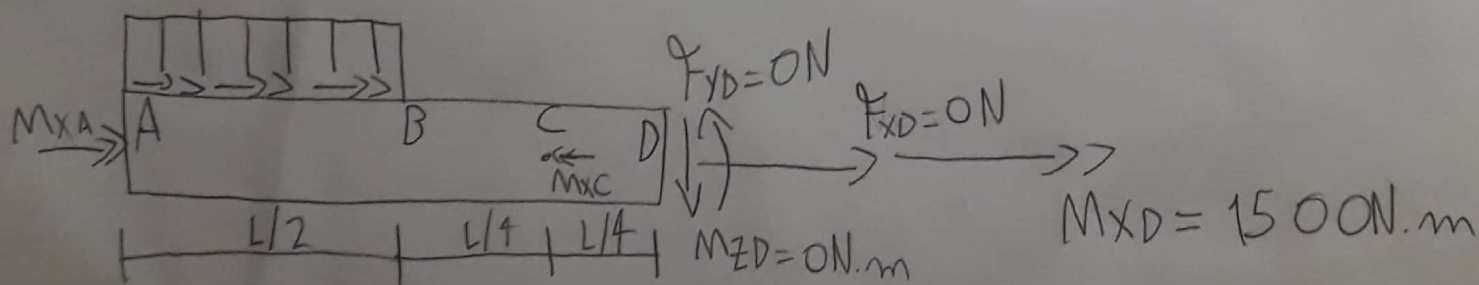
$$\sum F_x = 0 \rightarrow N_x(x) = 0 \text{ N} \quad \sum F_y = 0 \rightarrow V_y(x) = 0 \text{ N}$$

$$\sum M_z = 0 \rightarrow M_z(x) = 0 \text{ N.m}$$

$$\sum M_x = 0 \rightarrow +M_{xA} - M_{xC} + M_x + M_{pp} = 0$$

$$M_x = -M_{xA} + M_{xC} - M_{pp} = -3000 + 4500 - 3000 \cdot (L/2)$$

$$M_x(x) = 1500 - 3000 \cdot (L/2) \rightarrow M_x(x) = -4500 \text{ N.m}$$

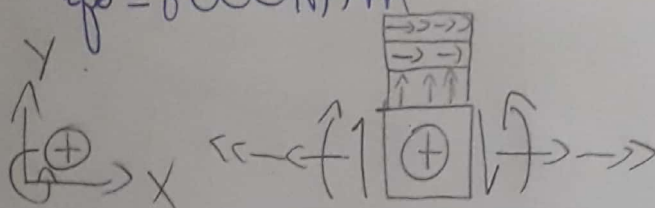


Ex. método dos cortes - 01

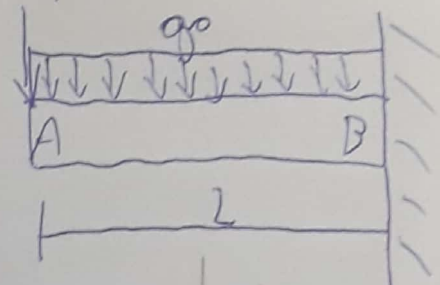
Dados: $L = 3\text{ m}$

$$F_A = 15000\text{ N}$$

$$q_0 = 8000\text{ N/m}$$



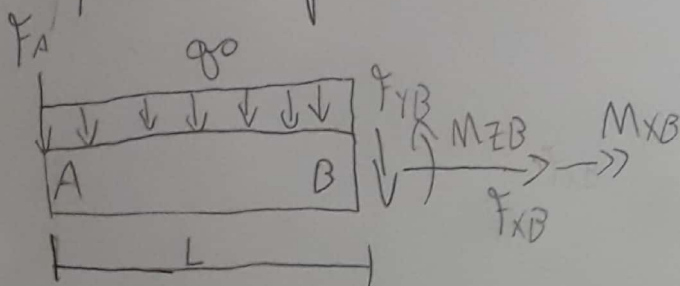
F_A



Número de seções: 1

$$0 < x < L$$

Reações de apoio



$$\sum M_x = 0 \rightarrow M_{xB} = 0\text{ N.m}$$

$$\sum F_y = 0$$

$$-F_A - F_{eq} - F_{yB} = 0$$

$$F_{yB} = -F_A - F_{eq}$$

$$F_{yB} = -15000 - 8000 \cdot 3$$

$$F_{yB} = -39000\text{ N}$$

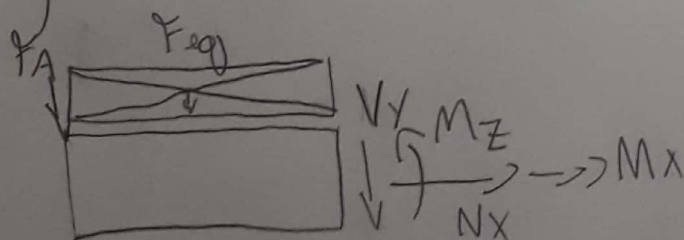
$$\sum M_z = 0$$

$$M_{zA} + M_z + M_{zB} = 0$$

$$M_{zB} = -M_{zA} - M_z$$

$$M_{zB} = -15000 \cdot L + 24000 \cdot \frac{L}{2} \Rightarrow L = 3 \Rightarrow M_{zB} = 81000\text{ N.m}$$

Seção AB ($0 < x < L$)



$$\sum F_x = 0 \rightarrow N_x(x) = 0\text{ N}$$

$$\sum M_x = 0 \rightarrow M_x(x) = 0\text{ N.m}$$

$$\sum F_y = 0 \rightarrow -F_A - V_y - F_{eq} = 0 \rightarrow V_y = -F_A - F_{eq} = -8000 \cdot x - 15000$$

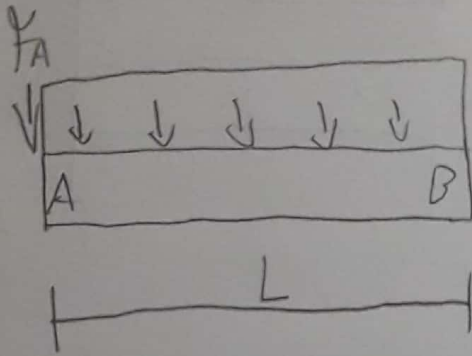
$$V_y(x=0) = -15000\text{ N}$$

$$V_y(x=L) = -39000\text{ N}$$

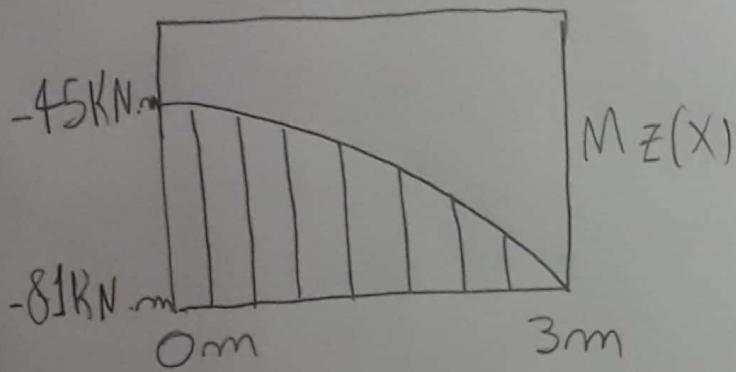
$$\sum M_z = 0 \rightarrow M_{zA} + M_{zeg} + M_z = 0 \rightarrow M_z = -M_{zA} - M_{zeg}$$

$$M_z = -F_A \cdot L - q_0 \cdot \frac{x^2}{2} \Rightarrow M_z(x) = -4500 - 4000x^2$$

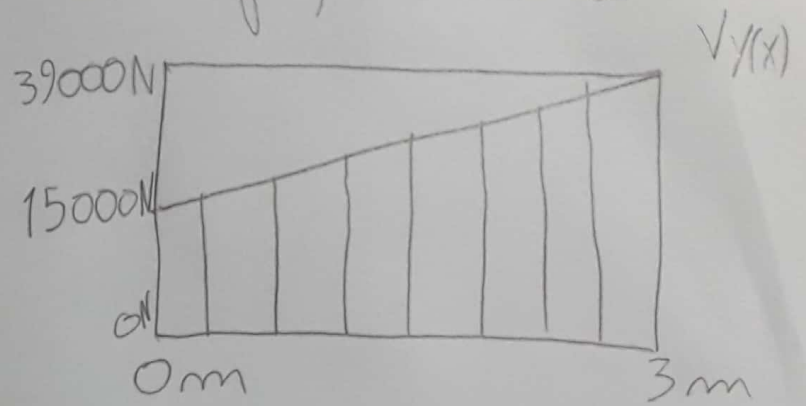
$$M_z(x=0) = -45000 \text{ N} \cdot \text{m} \quad M_z(x=3) = -81000 \text{ N} \cdot \text{m}$$



Momento fletor

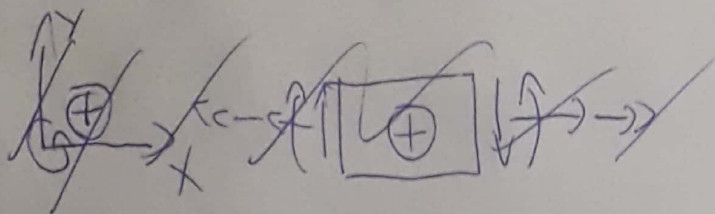


Esforço Cortante

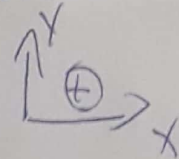


Ex - Método Flexão - 02

9



Sistemas de coordenadas da estática



Convenções de Resmat

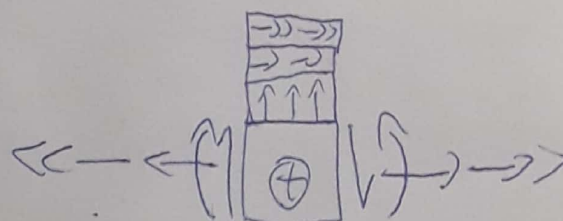
Dados: $L = 2\text{ m}$

$M_B = 5000\text{ N.m}$

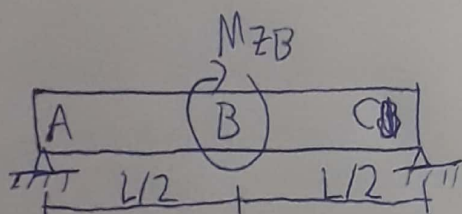
Número de trechos: 2

AB ($0 < x < L/2$)

BC ($L/2 < x < L$)



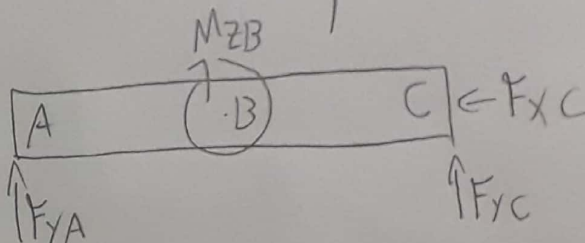
Reações de apoio



$$\sum F_x = 0 \rightarrow F_{xC} = 0\text{ N}$$

$$\sum F_y = 0 \rightarrow F_{yA} = -F_{yC}$$

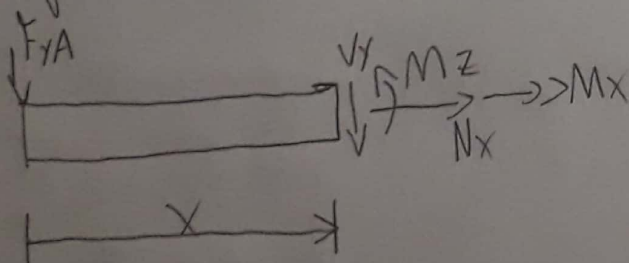
$$F_{yA} = -2500\text{ N}$$



$$\sum M_z = 0 \rightarrow M_{zB} + F_{yC} \cdot L = 0$$

$$F_{yC} = \frac{M_{zB}}{L} = \frac{5000}{2} = 2500\text{ N}$$

Esforços internos: Trecho AB: ($0 < x < L/2$)



$$\sum F_x = 0 \rightarrow N_x(x) = 0\text{ N}$$

$$\sum M_x = 0 \rightarrow M_x(x) = 0\text{ N.m}$$

$$\sum F_y = 0 \rightarrow -F_{yA} - V_y(x) = 0 \rightarrow V_y = -2500\text{ N}$$

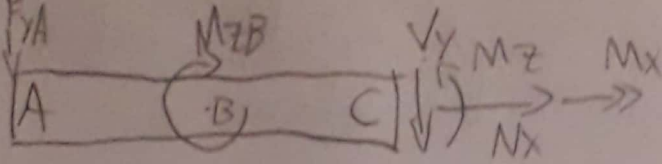
$$\sum M_z(x) = 0 \rightarrow M_z + F_{yA} \cdot x = 0 \rightarrow -F_A \cdot x = M_z(x) = -2500x\text{ N.m}$$

$$M_z(0) = 0\text{ N.m}$$

$$M_z(1) = -2500\text{ N.m}$$

Seção BC: ($L/2 < X < L$)

$$\sum F_x = 0 \rightarrow N_x(x) = 0 \text{ N}$$



$$\sum M_x = 0 \rightarrow M_x(x) = 0 \text{ N.m}$$

$$\sum F_y = 0 \rightarrow -V_y - F_{YA} = 0 \rightarrow V_y(x) = -2500 \text{ N}$$

$$\sum M_z = 0 \rightarrow M_z - M_{ZB} + F_{YA} \cdot X = 0 \rightarrow M_z = -2500X + 5000$$

$$M_z(1) = 2500 \text{ N.m}$$

$$M_z(2) = 0 \text{ N.m}$$

Diagrama de esforços

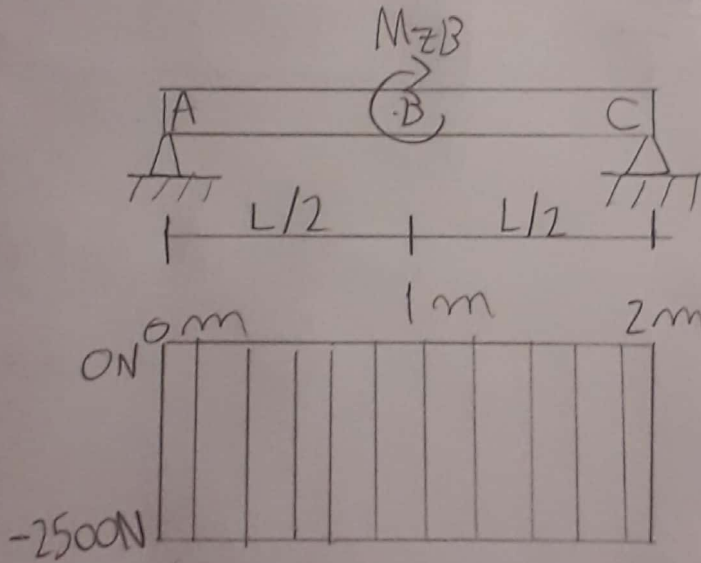


Diagrama da
Força Cortante
na direção Y

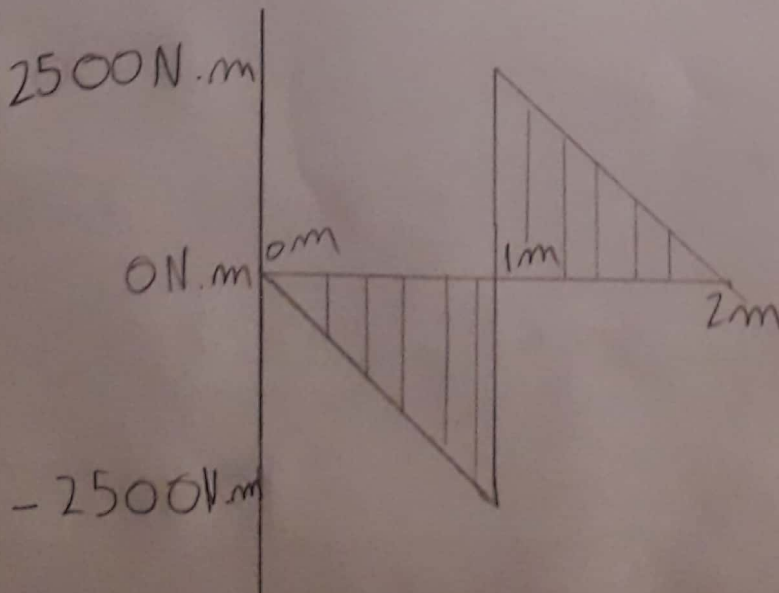


Diagrama da
Momento em torno
do eixo Z