Exercício deformações 10

$u_1 = 0{,}001 X_1 + 0{,}009 X_2 + 0{,}006 X_3$

$u_2 = 0{,}002 X_1 + 0{,}007 X_2 + 0{,}009 X_3$

$u_3 = 0{,}001 X_1 + 0{,}001 X_2 - 0{,}008 X_3$

ⓐ Matriz gradiente

$$[g_{ij}] = [u_{i,j}] = \left[\frac{\partial u_i}{\partial x_j}\right] = \begin{bmatrix} 0{,}001 & 0{,}009 & 0{,}006 \\ 0{,}002 & 0{,}007 & 0{,}009 \\ 0{,}001 & 0{,}001 & -0{,}008 \end{bmatrix}$$

ⓑ
$$[\varepsilon_{ij}] = \left[\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right] = \frac{1}{2}\begin{bmatrix} \frac{\partial u_1}{\partial x_1} + \frac{\partial u_1}{\partial x_1} & \frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1} & \frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1} \\ \frac{\partial u_2}{\partial x_1} + \frac{\partial u_1}{\partial x_2} & \frac{\partial u_2}{\partial x_2} + \frac{\partial u_2}{\partial x_2} & \frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2} \\ \frac{\partial u_3}{\partial x_1} + \frac{\partial u_1}{\partial x_3} & \frac{\partial u_3}{\partial x_2} + \frac{\partial u_2}{\partial x_3} & \frac{\partial u_3}{\partial x_3} + \frac{\partial u_3}{\partial x_3} \end{bmatrix}$$

Tensor de deformações infinitesimais de Cauchy

$$[\varepsilon_{ij}] = \frac{1}{2}\begin{bmatrix} 0{,}001 + 0{,}001 & 0{,}009 + 0{,}002 & 0{,}006 + 0{,}001 \\ 0{,}002 + 0{,}009 & 0{,}007 + 0{,}007 & 0{,}009 + 0{,}001 \\ 0{,}001 + 0{,}006 & 0{,}001 + 0{,}009 & -0{,}008 - 0{,}008 \end{bmatrix}$$

$$[\varepsilon_{ij}] = \begin{bmatrix} 0{,}001 & 0{,}0055 & 0{,}0035 \\ 0{,}0055 & 0{,}007 & 0{,}005 \\ 0{,}0035 & 0{,}005 & -0{,}008 \end{bmatrix}$$

(c) Tensor de rotações infinitesimais

$$[w_{ij}] = \left[\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right)\right] = \frac{1}{2}\begin{bmatrix} 0{,}001-0{,}001 & 0{,}009-0{,}002 & 0{,}006-0{,}001 \\ 0{,}002-0{,}009 & 0{,}007-0{,}007 & 0{,}009-0{,}00 \\ 0{,}001-0{,}006 & 0{,}001-0{,}009 & -0{,}008+0{,}008 \end{bmatrix}$$

$$[w_{ij}] = \begin{bmatrix} 0 & 0{,}0035 & 0{,}0025 \\ -0{,}0035 & 0 & 0{,}004 \\ -0{,}0025 & -0{,}004 & 0 \end{bmatrix}$$

(d) Vetor de rotações infinitesimais

$$\Omega_i = \frac{1}{2}\nabla \times \vec{u} = \frac{1}{2}\begin{bmatrix} \hat{e}_1 & \hat{e}_2 & \hat{e}_3 \\ \partial/\partial x_1 & \partial/\partial x_2 & \partial/\partial x_3 \\ u_1 & u_2 & u_3 \end{bmatrix}$$

$$\Omega_i = \frac{1}{2}\left[\left(\frac{\partial u_3}{\partial x_2} - \frac{\partial u_2}{\partial x_3}\right)\hat{e}_1 + \left(\frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1}\right)\hat{e}_2 + \left(\frac{\partial u_2}{\partial x_1} - \frac{\partial u_1}{\partial x_2}\right)\hat{e}_3\right]$$

$$\Omega_i = \frac{1}{2}\left[(0{,}001-0{,}009)\hat{e}_1 + (0{,}006-0{,}001)e_2 + (0{,}002-0{,}009)\hat{e}_3\right]$$

$$\Omega_i = -0{,}004\hat{e}_1 + 0{,}0025\hat{e}_2 - 0{,}0035\hat{e}_3$$

Parte 2

Módulo de elasticidade longitudinal (Young) $E = 210 KN/mm^2$

Módulo de cisalhamento $G = 80 KN/mm^2$

f) O tensor de tensões $[\sigma_{ij}]$

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} = \sigma_{yx} \\ \sigma_{xz} = \sigma_{zx} \\ \sigma_{yz} = \sigma_{zy} \end{Bmatrix} = \begin{bmatrix} 2G+\lambda & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & 2G+\lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & 2G+\lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & 2G & 0 & 0 \\ 0 & 0 & 0 & 0 & 2G & 0 \\ 0 & 0 & 0 & 0 & 0 & 2G \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{xy} = \varepsilon_{yx} \\ \varepsilon_{xz} = \varepsilon_{zx} \\ \varepsilon_{yz} = \varepsilon_{zy} \end{Bmatrix}$$

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)} \qquad \nu = \frac{E}{2G} - 1 \qquad \nu = \frac{210^3}{2.80.10^3} - 1 = 0,3125$$

$$\lambda = \frac{210.10^3 . 0,3125}{(1+0,3125)(1-2.0,3125)} = 133,\bar{3}.10^3$$

$$\sigma_{xx} = (2G+\lambda)\varepsilon_{xx} + \lambda.\varepsilon_{yy} + \lambda.\varepsilon_{zz} = 1,6.10^2 \quad N/mm^2$$

$$\sigma_{yy} = \lambda.\varepsilon_{xx} + (2G+\lambda)\varepsilon_{yy} + \lambda.\varepsilon_{zz} = 1,12.10^3 \quad N/mm^2$$

$$\sigma_{zz} = \lambda.\varepsilon_{xx} + \lambda.\varepsilon_{yy} + \varepsilon_{zz}(2G+\lambda) = -1,28.10^3 \quad N/mm^2$$

$$\sigma_{xy} = \sigma_{yx} = 2G\varepsilon_{xy} = 2G\varepsilon_{yx} = 8,8.10^2 \quad N/mm^2$$

$$\sigma_{zx} = \sigma_{xz} = 2G\varepsilon_{xz} = 2G\varepsilon_{zx} = 5,6.10^2 \quad N/mm^2$$

$$\sigma_{yz} = \sigma_{zy} = 2G\varepsilon_{zy} = 2G\varepsilon_{yz} = 8.10^2 \quad N/mm^2$$

② Dilatação Cúbica $\rightarrow \Delta = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}$

$$\Delta = (0{,}001) + (0{,}007) + (-0{,}008) = 0 \rightarrow \boxed{\Delta = 0}$$

Exercícios deformações 11

① (u_a, v_a) (u_b, v_b) (u_c, v_c) (u_d, v_d)

$u_a = 2 - 0 = 2$

$v_a = 2 - 0 = 2$

$\boxed{(u_a, v_a) = (2, 2)}$

$u_b = 3{,}55 - 1{,}5 = 2{,}05$

$v_b = 2{,}07 - 0 = 2{,}07$

$\boxed{(u_b, v_b) = (2{,}05;\ 2{,}07)}$

$u_c = 3{,}5 - 1{,}5 = 2$

$v_c = 3{,}47 - 1{,}5 = 1{,}97$

$\boxed{(u_c, v_c) = (2;\ 1{,}97)}$

$u_d = 1{,}95 - 0 = 1{,}95$ $\quad v_d = 3{,}4 - 1{,}5 = 1{,}9$

$\boxed{(u_d, v_d) = (1{,}95;\ +1{,}9)}$

② Componentes de deformação em torno do ponto A

Deformação e_{xxa}

$L_{ab} = X_b - X_a = 1{,}5 - 0 = 1{,}5$

$L_{fe} = X_f - X_e = 3{,}55 - 2 = 1{,}55$

$\Delta_{ab} = L_{fe} - L_{ab} = 1{,}55 - 1{,}5$

$\Delta_{ab} = 0{,}05$

$$e_{xxa} = \frac{\Delta_{ab}}{L_{ab}} = \frac{0{,}05}{1{,}5} \cong \boxed{0{,}03}$$

Deslocamento angular θV_{xa}

$\Delta V_{ab} = Y_f - Y_e = 2{,}07 - 2 = 0{,}07$

$$\theta V_{xa} = \frac{\Delta V_{ab}}{L_{ab}} = \frac{0{,}07}{1{,}5} \cong \boxed{0{,}05}$$

Deformação e_{yya}

$L_{ad} = 1{,}5 - 0 = 1{,}5$

$L_{eh} = 3{,}4 - 2 = 1{,}4$

$\Delta_{ad} = 1{,}4 - 1{,}5 = -0{,}1$

$$e_{yya} = \frac{\Delta_{ad}}{L_{ad}} = \frac{-0{,}1}{1{,}5} \cong \boxed{-0{,}07}$$

Deslocamento angular $\theta u y a$

$\Delta u a d = X h - X e = 1,95 - 2$

$\Delta u a d = -0,05$

$\theta u y a = \frac{\Delta u a d}{L a b} = \frac{-0,05}{1,5}$

$\boxed{\theta u y a \cong -0,03}$

Componente de deformação angular $e_{yx} = e_{xy}$

$e_{xya} = \frac{(\theta v x a + \theta u y a)}{2}$

$e_{xya} = \frac{(-0,03 + 0,05)}{2} = \boxed{0,01}$

Rotação de corpo rígido do ponto A em torno do eixo Z, Ωz

$\Omega z = \frac{(\theta v x a - \theta u y a)}{2} = \frac{0,05 - (-0,03)}{2} = \boxed{0,04}$

Determinação do tensor de tensão e_{ij}

$$e_{ij} = \begin{Bmatrix} 0,03 & 0,01 & 0 \\ 0,01 & -0,07 & 0 \\ 0 & 0 & 0 \end{Bmatrix}$$

$$[\sigma_{ij}] = \begin{bmatrix} 0{,}16 & 0{,}88 & 0{,}56 \\ 0{,}88 & +1{,}12 & 0{,}8 \\ 0{,}56 & 0{,}8 & -1{,}28 \end{bmatrix} \ KN/mm^2$$

9) $$\vec{t}_i = \sigma_{ij}\,\vec{n}_j = \begin{bmatrix} 0{,}16 & 0{,}88 & 0{,}56 \\ 0{,}88 & 1{,}12 & 0{,}8 \\ 0{,}56 & 0{,}8 & -1{,}28 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix} \frac{1}{\sqrt{3}}$$

$$\vec{t}_i = \begin{bmatrix} 92{,}37 \\ -323{,}3 \\ 877{,}6 \end{bmatrix} N/mm^2 = \boxed{\begin{bmatrix} 0{,}09237 \\ -0{,}3233 \\ 0{,}8776 \end{bmatrix} KN/mm^2}$$

$$\vec{t}^n = (\vec{t}_i \cdot \vec{n})\,\vec{n}$$

$$\vec{t}^n = \left(\begin{bmatrix} 92{,}37 \\ -323{,}3 \\ 877{,}6 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix} \frac{1}{\sqrt{3}} \right) \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix} \frac{1}{\sqrt{3}} = \begin{bmatrix} 431{,}1 \\ -431{,}1 \\ 431{,}1 \end{bmatrix}$$

$$\boxed{\vec{t}^n = \begin{bmatrix} 0{,}4311 \\ -0{,}4311 \\ 0{,}4311 \end{bmatrix} KN/mm^2}$$

$$\vec{t}_i^n = \vec{t}_i - \vec{t}^n$$

$$\vec{t}_i^n = \begin{bmatrix} 92{,}37 \\ -323{,}3 \\ 877{,}6 \end{bmatrix} - \begin{bmatrix} 431{,}1 \\ -431{,}1 \\ 431{,}1 \end{bmatrix} \rightarrow \boxed{\vec{t}_i^n = \begin{bmatrix} -0{,}3387 \\ 0{,}1078 \\ 0{,}4465 \end{bmatrix} KN/mm^2}$$

Parte 2

3)

$$[\sigma_{ij}] \rightarrow \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} = \sigma_{yx} \\ \sigma_{xz} = \sigma_{zx} \\ \sigma_{yz} = \sigma_{zy} \end{Bmatrix} = \begin{bmatrix} 2G+\lambda & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & 2G+\lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & 2G+\lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & 2G & 0 & 0 \\ 0 & 0 & 0 & 0 & 2G & 0 \\ 0 & 0 & 0 & 0 & 0 & 2G \end{bmatrix} \begin{Bmatrix} e_{x} \\ e_{yy} \\ e_{zz} \\ e_{xy} = e_{yx} \\ e_{xz} = e_{zx} \\ e_{yz} = e_{zy} \end{Bmatrix}$$

$$\nu = \frac{E}{2G} - 1 = \frac{50.10^3}{2.85.10^3} - 1 = -0,15$$

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)} = -1,15.10^4$$

$$\sigma_{xx} = (2G+\lambda)e_{xx} + \lambda e_{yy} + \lambda e_{zz} = 3,46 \quad KN/mm^2$$

$$\sigma_{yy} = \lambda . e_{xx} + e_{yy}(2G+\lambda) + e_{zz}(\lambda) = -6,54 \quad KN/mm^2$$

$$\sigma_{zz} = \lambda . e_{xx} + e_{xy} . \lambda + (2G+\lambda) e_{zz} = 0,462 \quad KN/mm^2$$

$$\sigma_{xy} = \sigma_{yx} = 2.G.e_{xy} = 2G e_{yx} = 1 \quad KN/mm^2$$

$$\sigma_{xz} = \sigma_{zx} = 2G.e_{zx} = 2G e_{xz} = 0 \quad KN/mm^2$$

$$\sigma_{zy} = \sigma_{yz} = 2G e_{zy} = 2G e_{yz} = 0 \quad KN/mm^2$$

$$\boxed{[\sigma_{ij}] = \begin{bmatrix} 3,46 & 1 & 0 \\ 1 & -6,54 & 0 \\ 0 & 0 & 0,462 \end{bmatrix} KN/mm^2}$$

④ $\vec{t_i} = \sigma_{ij} \vec{n_j}$

$$\vec{t_i} = \begin{bmatrix} 3{,}46 & 1 & 0 \\ 1 & -6{,}54 & 0 \\ 0 & 0 & 0{,}462 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \frac{1}{\sqrt{2}} \Rightarrow \boxed{\vec{t_i} = \begin{bmatrix} -1{,}741 \\ -5{,}330 \\ 0 \end{bmatrix} KN/mm^2}$$

$$\vec{t_n^n} = (\vec{t_i} \cdot \vec{n})\vec{n}$$

$$\vec{t_n^n} = \left(\begin{bmatrix} -1{,}741 \\ -5{,}330 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \frac{1}{\sqrt{2}} \right) \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \frac{1}{\sqrt{2}} = \boxed{\begin{bmatrix} 0{,}5280 \\ -0{,}5280 \\ 0 \end{bmatrix} KN/mm^2}$$

$$\vec{t_i^m} = \vec{t_i} - \vec{t_n^n} = \begin{bmatrix} -1{,}741 \\ -5{,}330 \\ 0 \end{bmatrix} - \begin{bmatrix} 0{,}5280 \\ -0{,}5280 \\ 0 \end{bmatrix}$$

$$\boxed{\vec{t_i^m} = \begin{bmatrix} -2{,}269 \\ -4{,}803 \\ 0 \end{bmatrix} KN/mm^2}$$