Lista de Exercícios 5 Letícia Levrin Diniz 201438

Legenda:

0) Sistemas de eixos e convenções

1) Equações diferenciais de equilíbrio

2) Equações de Carregamento

3) Condições de contorno e Restrições

4) Integração

5) Determinação das constantes e reações de apoio

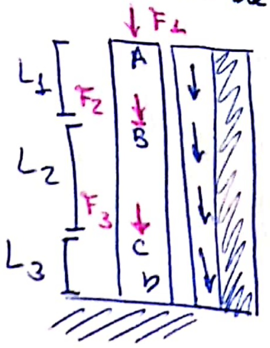
6) Equações finais

7) Análise e diagrama

Diniz 201450

Parte 01: Carregamento Axial

Ex - F1 - axial - 06:



Dados:

$po = 20kN/m$

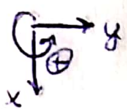
$L_1 = L_3 = 3m$

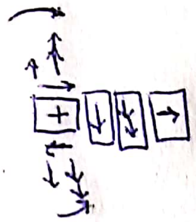
$L_2 = 6m$

$F_1 = 15kN$

$F_2 = 20kN$

$F_3 = 25kN$

0) 



1) $\dfrac{dN_x(x)}{dx} = -p(x)$

2) $p(x) = po\langle x-0\rangle^0 + F_2\langle x-L_1\rangle^{-1} + F_3\langle x-L_1-L_2\rangle^{-1}$

3) contorno:

$N_x(x=0) = -F_1$

restrições:

Não tem

4) $\dfrac{dN_x(x)}{dx} = -po\langle x-0\rangle^0 - F_2\langle x-L_1\rangle^{-1} - F_3\langle x-L_1-L_2\rangle^{-1}$

$\int \dfrac{dN_x(x)}{dx}dx = \int(-po\langle x-0\rangle^0)dx + \int(-F_2\langle x-L_1\rangle^{-1})dx + \int(-F_3\langle x-L_1-L_2\rangle^{-1})dx$

$N_x(x) = -po\langle x-0\rangle^1 - F_2\langle x-L_1\rangle^0 - F_3\langle x-L_1-L_2\rangle^0 + C_1$

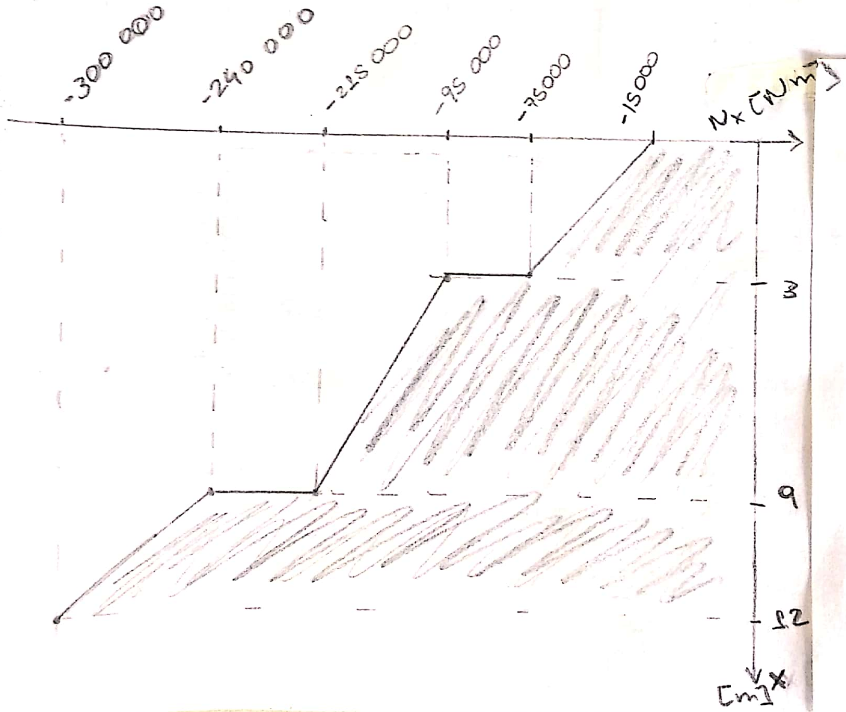
$N_x(x) = -pox - F_2\langle x-L_1\rangle^0 - F_3\langle x-L_1-L_2\rangle^0 + C_1$

5) $N_x(x=0) = -po\cdot 0 - F_2\underbrace{\langle -L_1\rangle^0}_{0} - F_3\underbrace{\langle -L_1-L_2\rangle^0}_{0} + C_1 = -F_1$

$\therefore C_1 = -F_1$

6) $N_x(x) = -pox - F_2\langle x-L_1\rangle^0 - F_3\langle x-L_1-L_2\rangle^0 - F_1$

7) $N_x(x) = -20\cdot10^3x - 20\cdot10^3\langle x-3\rangle^0 - 25\cdot10^3\langle x-9\rangle^0 - 15\cdot10^3$ N



Reação em D:

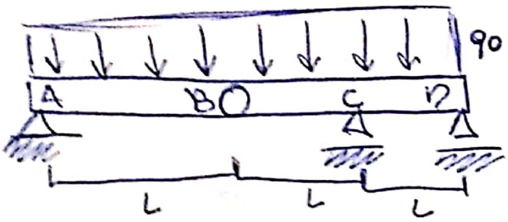
$R_D = N_x(x=12)$

$R_D = -300\ 000\ N$

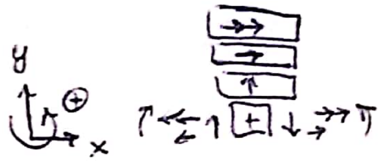
R_D ↳ sentido correto da reação ↑

Parte 02: Carregamento Transversal

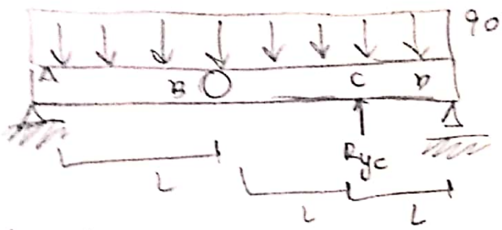
Ex - ekiga - 29:



Dados:
$q_0 = 10k\ N/m$
$L = 2m$

0) 

1) $\frac{d^2 M_z(x)}{dx^2} = q(x)$



2) $q(x) = -q_0 \langle x-0 \rangle^0 + R_{yc} \langle x - 2L \rangle^{-1}$

3) contorno:
$M_z(x=0) = 0$
$M_z(x=3L) = 0$

restrição:
$M_z(x=L) = 0$

4) $\frac{d^2 M_z(x)}{dx^2} = -q_0 \langle x-0 \rangle^0 + R_{yc} \langle x-2L \rangle^{-1}$

$\frac{dM_z(x)}{dx} = V_y(x) = \int (-q_0 \langle x-0 \rangle^0) dx + \int R_{yc} \langle x-2L \rangle^{-1} dx$

$\frac{dM_z(x)}{dx} = V_y(x) = -q_0 \langle x-0 \rangle^1 + R_{yc} \langle x-2L \rangle^0 + C_1 = -q_0 x + R_{yc} \langle x-2L \rangle^0 + C_1$

$M_z(x) = \int -q_0 x\, dx + \int R_{yc} \langle x-2L \rangle^0 dx + \int C_1 dx$

$M_z(x) = -q_0 \frac{x^2}{2} + R_{yc} \langle x-2L \rangle^1 + C_1 x + C_2$

5) $M_z(0) = -q_0 \cdot \frac{0}{2} + R_{yc} \underbrace{\langle -2L \rangle^1}_{0} + C_1 \cdot 0 + C_2 = 0 \Rightarrow \underline{C_2 = 0}$

$M_z(3L) = -q_0 \frac{9L^2}{2} + R_{yc} \underbrace{\langle L \rangle^1}_{=L} + C_1 3L = 0 \Rightarrow -q_0 L^2 \frac{9}{2} + R_{yc} L + 3L C_1 = 0$ (II)

$M_z(L) = -q_0 \frac{L^2}{2} + R_{yc} \underbrace{\langle -L \rangle^1}_{0} + C_1 L = 0 \Rightarrow -q_0 \frac{L^2}{2} + C_1 L = 0$ (I)

$\underline{C_1 = q_0 \frac{L}{2}}$

Substituindo I em II:

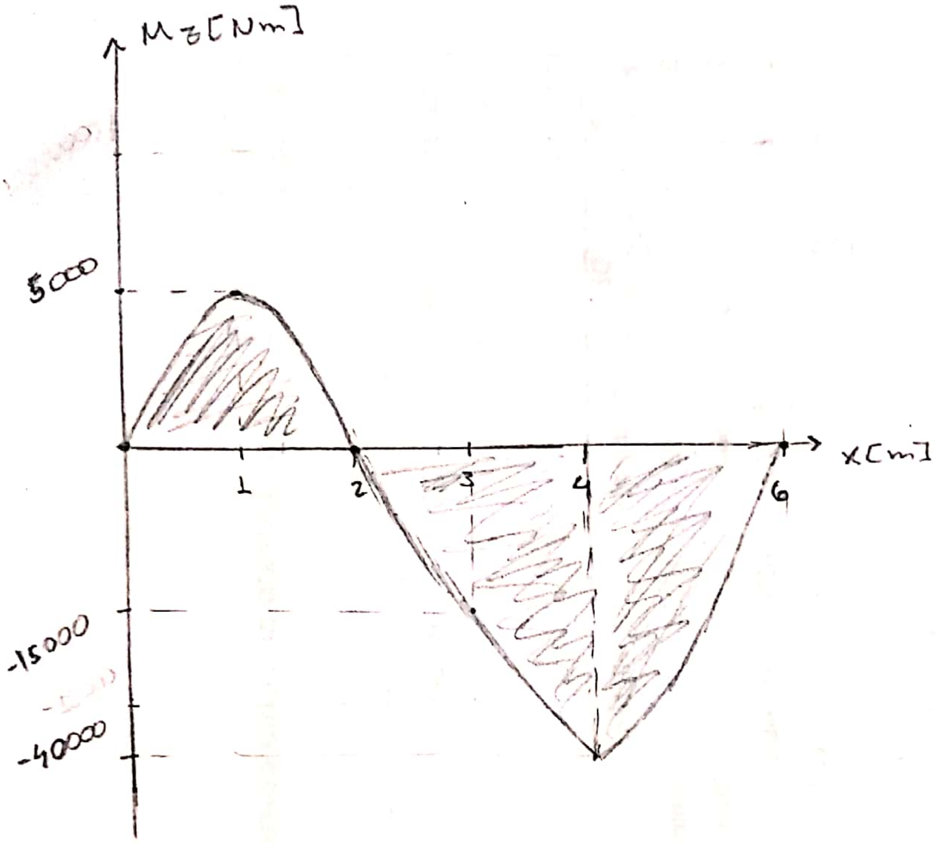
$-9 q_0 \frac{L^2}{2} + R_{yc} L + \frac{3L^2 q_0}{2} = 0 \Rightarrow \underline{R_{yc} = 3L q_0}$

6) $M_z(x) = -q_0 \frac{x^2}{2} + 3L q_0 \langle x - 2L \rangle^1 + q_0 \frac{L}{2} x$

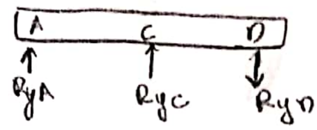
$V_y(x) = -q_0 x + 3L q_0 \langle x - 2L \rangle^0 + q_0 \frac{L}{2}$

7) $M_z(x) = -5000 x^2 + 60000 \langle x - 4 \rangle^1 + 10000 x$

$V_y(x) = -10000 x + 60000 \langle x - 4 \rangle^0 + 10000$



Reações de apoio:



carga interna em x = 0:

$V_y(0) = 10000\,N = R_{yA}$

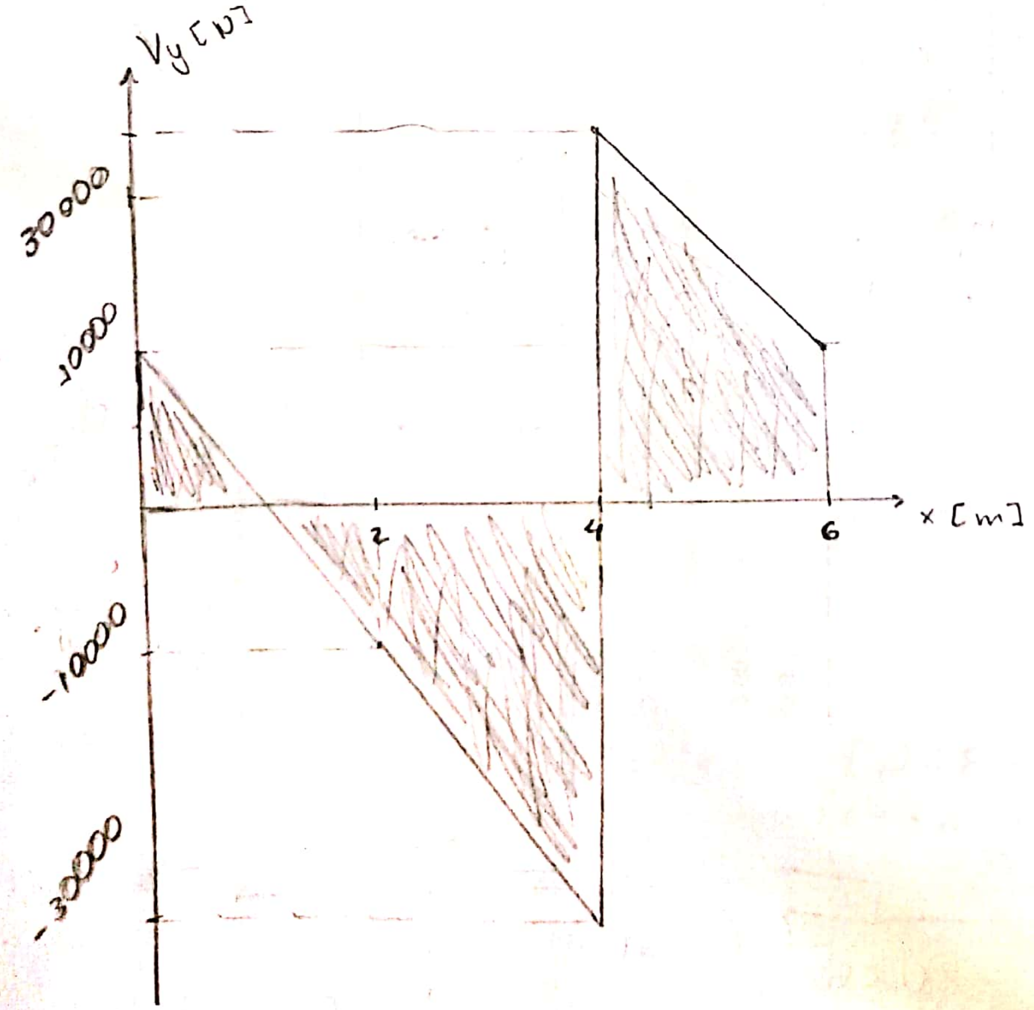
R_{yC} já foi determinada:

$R_{yC} = 3Lq_0 = 60000\,N$

carga interna em x = 6:

$V_y(6) = 10000\,N = R_{yD}$

↳ todas as reações foram desenhadas no sentido correto

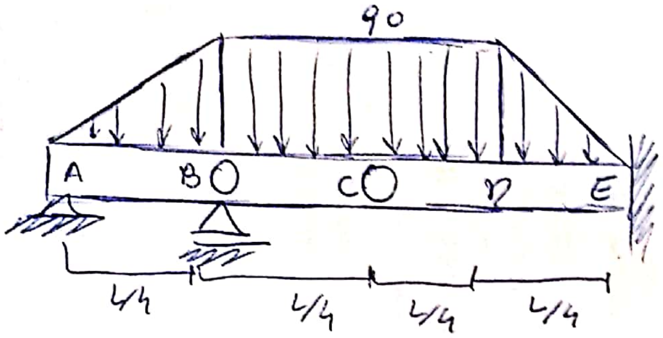


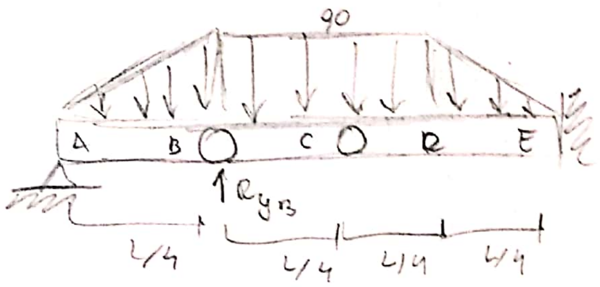
Ex_viga_29:

Dados:

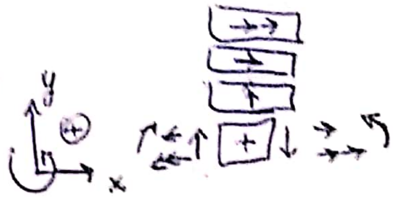
$L = 6\ m$

$q_0 = 15000\ N/m$





0)



1) $$\frac{d^2 M_z(x)}{dx^2} = q(x)$$

2) $$q(x) = -\frac{q_0 4}{L}\langle x-0\rangle^1 + \frac{4q_0}{L}\langle x - L/4\rangle^1 + \frac{q_0 4}{L}\langle x - 3L/4\rangle^1 + R_{yB}\langle x - L/4\rangle^{-1}$$

3) contorno: $M_z(x=0) = 0$

restrição: $M_z(x = L/4) = 0$

$M_z(x = L/2) = 0$

4) $$\frac{d^2 M_z(x)}{dx^2} = -\frac{q_0 4}{L}\langle x-0\rangle^1 + \frac{4q_0}{L}\langle x - L/4\rangle^1 + \frac{q_0 4}{L}\langle x - 3L/4\rangle^1 + R_{yB}\langle x - L/4\rangle^{-1}$$

$$\frac{dM_z(x)}{dx} = -\frac{2q_0x^2}{L} + \frac{2q_0}{L}\langle x - L/4\rangle^2 + \frac{q_0 2}{L}\langle x - 3L/4\rangle^2 + R_{yB}\langle x - L/4\rangle^0 + C_1 = V_y(x)$$

$$M_z(x) = -\frac{2q_0}{3L}x^3 + \frac{2q_0}{3L}\langle x - L/4\rangle^3 + \frac{q_0 2}{3L}\langle x - 3L/4\rangle^3 + R_{yB}\langle x - L/4\rangle^1 + C_1 x + C_2$$

5) $M_z(0) = C_2 = 0$

$$M_z(L/4) = -\frac{q_0L^2}{96} + C_1 L/4 = 0 \Rightarrow C_1 = \frac{q_0 L}{24} \quad \text{(I)}$$

$$M_z(L/2) = -\frac{q_0L^2}{12} + \frac{q_0L^2}{96} + R_{yB}\frac{L}{4} + C_1\frac{L}{2} = 0 \quad \text{(II)}$$

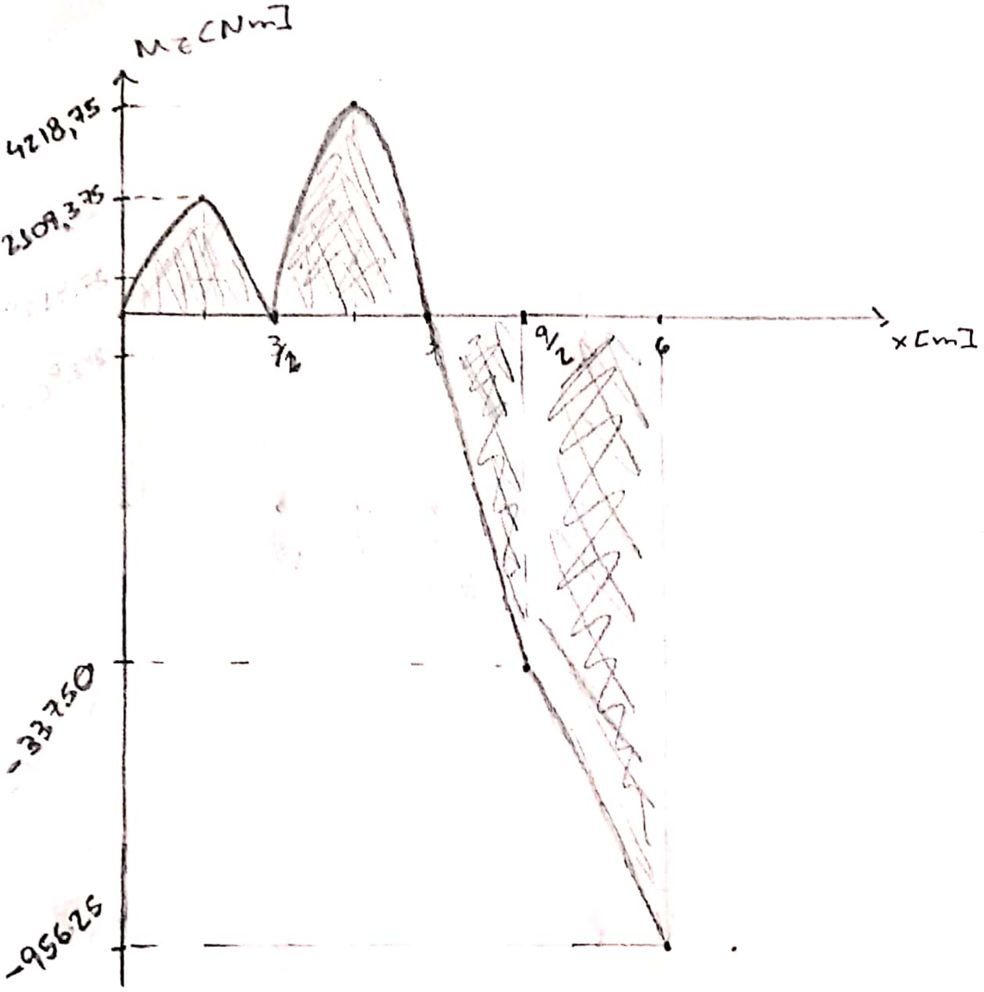
Substituindo I em II: $R_{yB} = \dfrac{5q_0L}{24}$

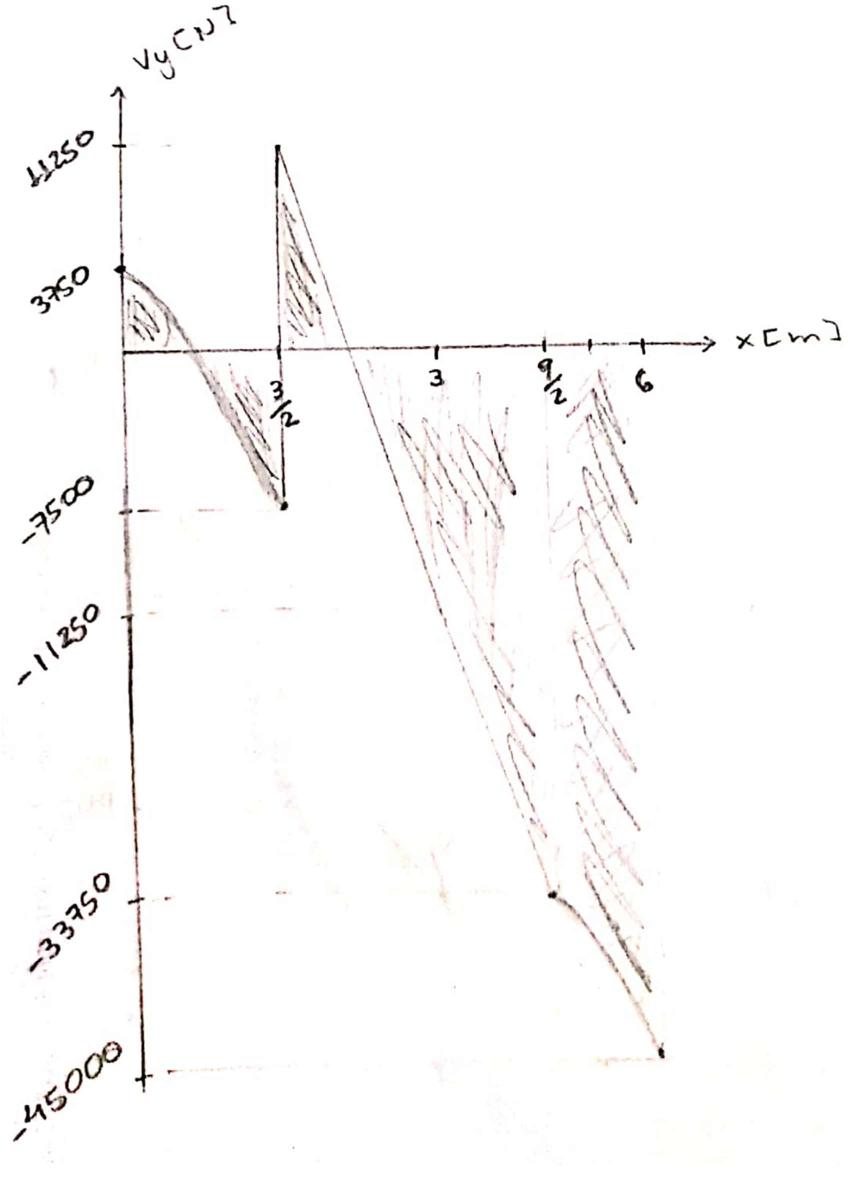
6) $$M_z(x) = -\frac{2q_0}{3L}x^3 + \frac{2q_0}{3L}\langle x - L/4\rangle^3 + \frac{2q_0}{3L}\langle x - 3L/4\rangle^3 + \frac{5q_0L}{24}\langle x - L/4\rangle^1 + \frac{q_0L}{24}x$$

$$V_y(x) = -\frac{2q_0x^2}{L} + \frac{2q_0}{L}\langle x - L/4\rangle^2 + \frac{2q_0}{L}\langle x - 3L/4\rangle^2 + \frac{5q_0L}{24}\langle x - L/4\rangle^0 + \frac{q_0L}{24}$$

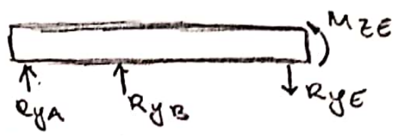
7) $$M_z(x) = -\frac{10000}{6}x^3 + \frac{10000}{6}\langle x - 3/2\rangle^3 + \frac{10000}{6}\langle x - 9/2\rangle^3 + 18750\langle x - 3/2\rangle^1 + 3750x$$

$$V_y(x) = -5000x^2 + 5000\langle x - 3/2\rangle^2 + 5000\langle x - 9/2\rangle^2 + 18750\langle x - 3/2\rangle^0 + 3750$$





Reações de apoio:



$R_{yA} = V_y(0) = 3750N$

R_{yB} já foi determinada:

$R_{yB} = \frac{5 q_0 L}{24} = 18750N$

$R_{yE} = V_y(6) = -45000N$

$M_{zE} = M_z(6) = -95625Nm$

R_{yA}, R_{yB} estão desenhadas no sentido correto, mas a reação em E está no sentido errado, a força que a produz tem sentido ↑ e seu momento