Lista de exercícios 8 - Letícia Lervin Dимек   201438

Exercício e - deformação 10:

$\vec{u} = (0,001x_1 + 0,009x_2 + 0,006x_3)\hat{a_1} + (0,002x_1 + 0,007x_2 + 0,009x_3)\hat{a_2}$
$+ (0,001x_1 + 0,001x_2 - 0,008x_3)\hat{a_3}$

a) $[g_{ij}]$:

$$[G] = \begin{bmatrix} \dfrac{\partial u_1}{\partial x_1} & \dfrac{\partial u_1}{\partial x_2} & \dfrac{\partial u_1}{\partial x_3} \\[2mm] \dfrac{\partial u_2}{\partial x_1} & \dfrac{\partial u_2}{\partial x_2} & \dfrac{\partial u_2}{\partial x_3} \\[2mm] \dfrac{\partial u_3}{\partial x_1} & \dfrac{\partial u_3}{\partial x_2} & \dfrac{\partial u_3}{\partial x_3} \end{bmatrix} = \begin{bmatrix} 0,001 & 0,009 & 0,006 \\[2mm] 0,002 & 0,007 & 0,009 \\[2mm] 0,001 & 0,001 & -0,008 \end{bmatrix}$$

b) $[\varepsilon_{ij}]$:

$$[\varepsilon] = \begin{bmatrix} \dfrac{\partial u_1}{\partial x_1} & \dfrac{1}{2}\left(\dfrac{\partial u_1}{\partial x_2} + \dfrac{\partial u_2}{\partial x_1}\right) & \dfrac{1}{2}\left(\dfrac{\partial u_1}{\partial x_3} + \dfrac{\partial u_3}{\partial x_1}\right) \\[3mm] \dfrac{1}{2}\left(\dfrac{\partial u_2}{\partial x_1} + \dfrac{\partial u_1}{\partial x_2}\right) & \dfrac{\partial u_2}{\partial x_2} & \dfrac{1}{2}\left(\dfrac{\partial u_2}{\partial x_3} + \dfrac{\partial u_3}{\partial x_2}\right) \\[3mm] \dfrac{1}{2}\left(\dfrac{\partial u_3}{\partial x_1} + \dfrac{\partial u_1}{\partial x_3}\right) & \dfrac{1}{2}\left(\dfrac{\partial u_3}{\partial x_2} + \dfrac{\partial u_2}{\partial x_3}\right) & \dfrac{\partial u_3}{\partial x_3} \end{bmatrix}$$

$$[\varepsilon] = \begin{bmatrix} 0,001 & \dfrac{0,009 + 0,002}{2} & \dfrac{0,006 + 0,001}{2} \\[3mm] \dfrac{0,009 + 0,002}{2} & 0,007 & \dfrac{0,009 + 0,001}{2} \\[3mm] \dfrac{0,006 + 0,001}{2} & \dfrac{0,009 + 0,001}{2} & -0,008 \end{bmatrix} = \begin{bmatrix} 0,001 & 0,0055 & 0,0035 \\[3mm] 0,0055 & 0,007 & 0,005 \\[3mm] 0,0035 & 0,005 & -0,008 \end{bmatrix}$$

e) $[\omega_{ij}]$:

$$[\omega] = \begin{bmatrix} 0 & \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} - \frac{\partial u_2}{\partial x_1}\right) & \frac{1}{2}\left(\frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1}\right) \\ \frac{1}{2}\left(\frac{\partial u_2}{\partial x_1} - \frac{\partial u_1}{\partial x_2}\right) & 0 & \frac{1}{2}\left(\frac{\partial u_2}{\partial x_3} - \frac{\partial u_3}{\partial x_2}\right) \\ \frac{1}{2}\left(\frac{\partial u_3}{\partial x_1} - \frac{\partial u_1}{\partial x_3}\right) & \frac{1}{2}\left(\frac{\partial u_3}{\partial x_2} - \frac{\partial u_2}{\partial x_3}\right) & 0 \end{bmatrix} = \begin{bmatrix} 0 & \frac{0,009 - 0,002}{2} & \frac{0,006 - 0,001}{2} \\ \frac{0,002 - 0,009}{2} & 0 & \frac{0,009 - 0,001}{2} \\ \frac{0,001 - 0,006}{2} & \frac{0,001 - 0,009}{2} & 0 \end{bmatrix}$$

$$[\omega] = \begin{bmatrix} 0 & 0,0035 & 0,0025 \\ -0,0035 & 0 & 0,004 \\ -0,0025 & -0,004 & 0 \end{bmatrix}$$

a) Ω_i:

$$\frac{1}{2}\vec{\nabla}\times\vec{u} = \frac{1}{2}\left(\frac{\partial u_3}{\partial x_2} - \frac{\partial u_2}{\partial x_3}\right)\hat{a_1} + \frac{1}{2}\left(\frac{\partial u_1}{\partial x_3} - \frac{\partial u_2}{\partial x_1}\right)\hat{a_2} + \frac{1}{2}\left(\frac{\partial u_2}{\partial x_1} - \frac{\partial u_1}{\partial x_2}\right)\hat{a_3}$$

$$\frac{1}{2}\vec{\nabla}\times\vec{u} = \frac{1}{2}(0,001 - 0,009)\hat{a_1} + \frac{1}{2}(0,006 - 0,001)\hat{a_2} + \frac{1}{2}(0,002 - 0,009)\hat{a_3}$$
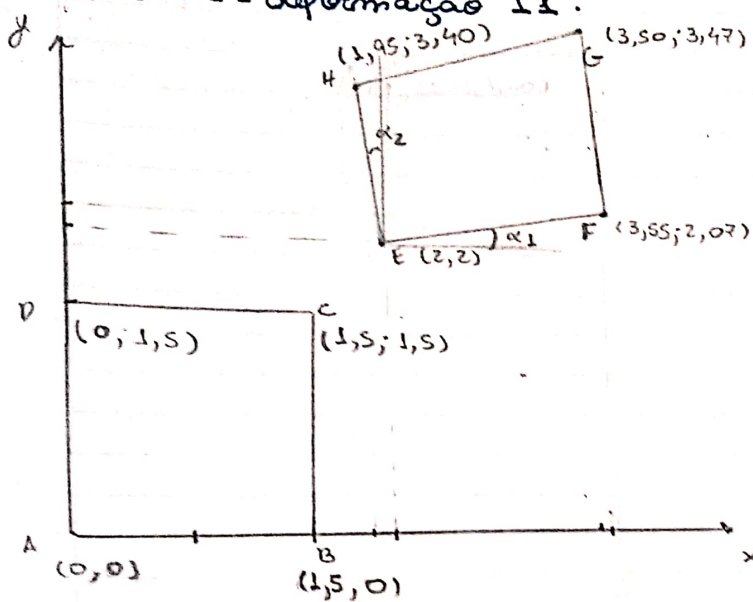
$$\vec{\Omega} = \frac{1}{2}\vec{\nabla}\times\vec{u} = -0,004\,\hat{a_1} + 0,0025\,\hat{a_2} - 0,0035\,\hat{a_3}$$

$$\Omega_{x_1} = -0,004 \qquad \Omega_{x_2} = 0,0025 \qquad \Omega_{x_3} = -0,0035$$

e) Δ:

$$\Delta = \varepsilon_{x_1 x_1} + \varepsilon_{x_2 x_2} + \varepsilon_{x_3 x_3} = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3} = 0,001 + 0,007 - 0,008 = 0$$

Exercício e - deformação 11:

1) deslocamentos:

$$u_A = x_E - x_A = 2 - 0 = 2 \qquad u_B = x_F - x_B = 3,55 - 1,5 = 2,05$$

$$v_A = y_E - y_A = 2 - 0 = 2 \qquad v_B = y_F - y_B = 2,07 - 0 = 2,07$$

$$u_C = x_G - x_C = 3,50 - 1,5 = 2 \qquad u_D = x_H - x_D = 1,95 - 0 = 1,95$$

$$v_C = y_G - y_C = 3,47 - 1,5 = 1,97 \qquad v_D = y_H - y_D = 3,40 - 1,5 = 1,9$$

2) tensor de deformações em torno de A:

$L_{AB} = x_B - x_A = 1,5$

$L_{EF} = x_F - x_E = 1,55$

$L_{AD} = y_D - y_A = 1,5$

$L_{EH} = y_H - y_E = 1,4$

$\alpha_1 = \dfrac{y_F - y_E}{L_{AB}} = 0,04\overline{6}$

$\alpha_2 = \dfrac{x_H - x_E}{L_{AD}} = -0,0\overline{3}$

$\varepsilon_{xxA} = \dfrac{L_{EF} - L_{AB}}{L_{AB}} = 0,0\overline{3}$

$\varepsilon_{yyA} = \dfrac{L_{EH} - L_{AD}}{L_{AD}} = -0,0\overline{6}$

$\varepsilon_{xyA} = \varepsilon_{yxA} = \dfrac{\alpha_1 + \alpha_2}{2}$

$\varepsilon_{xyA} = \varepsilon_{yxA} = 0,00667$

$$[\varepsilon] = \begin{bmatrix} 0,0334 & 0,00667 & 0 \\ 0,00667 & -0,0667 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$