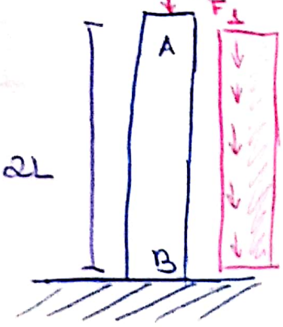
Lista de Exercícios – 1

Letícia Levin Diniz
201438

Ex_ReApoio_Axial_01:

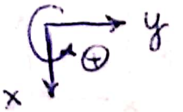


$p(x) = p_0$

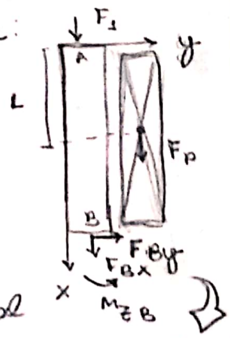
$L = 2\,m$

$p_0 = 5000\,N/m$

$F_1 = 8000\,N$



DCL:



reações do apoio

seja: $F_p = p(x)2L$

$F_p = p_0 2L$

↳ força pontual equivalente ao carregamento distribuído

Para o equilíbrio: (no plano)

$\vec{F} = 0$, $\vec{M} = 0$

$\{F_x, F_y\}^T = 0$ $\quad \Sigma M_z = 0$

$\Sigma F_x = 0$

$F_1 + F_{Bx} + F_p = 0$

$F_{Bx} = -F_1 - F_p = -F_1 - p_0 2L$

$\Sigma F_y = 0$

$F_{By} = 0$

$\Sigma M_z(A) = 0$

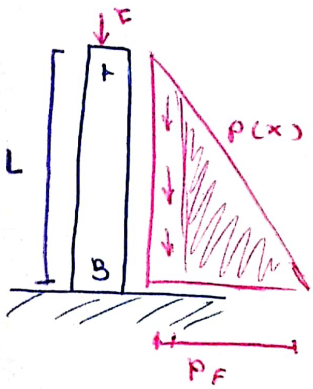
$\cancel{F_{By}}(2L) + M_{zB} = 0$

$M_{zB} = 0$

$\therefore F_{Bx} = -F_1 - p_0 2L$ $\quad F_{By} = 0$ $\quad M_{zB} = 0$

$F_{Bx} = -28000\,N$

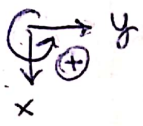
Ex – Reação de Apoio – Axial – 02:



$L = 3\,m$

$p_0 = 1000\,N/m$

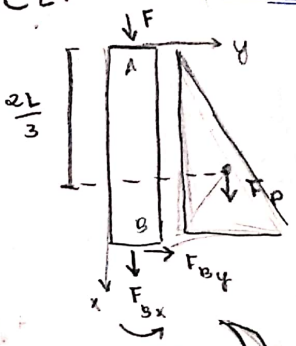
$F = 2000\,N$



Equilíbrio no plano:

$\Sigma F_x = 0 \quad \Sigma F_y = 0 \quad \Sigma M_z = 0$

DCL:



seja: $F_p = \frac{p_F \cdot L}{2}$

↳ força pontual equivalente ao carregamento distribuído

reações do apoio

$\Sigma F_x = 0$

$F + F_p + F_{Bx} = 0$

$F_{Bx} = -F - F_p = -F - \frac{p_F L}{2}$

$\Sigma F_y = 0$

$F_{By} = 0$

$\Sigma M_z(A) = 0$

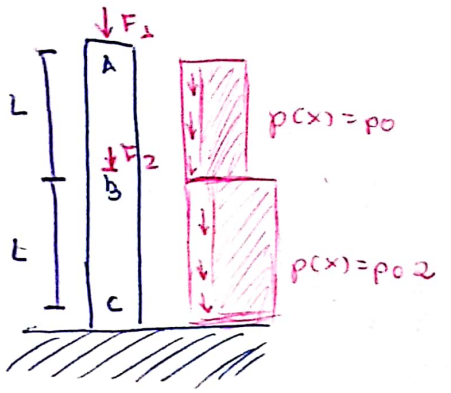
$M_{zB} + F_{By} L = 0$

$M_{zB} = 0$

$\therefore \quad F_{Bx} = -F - \frac{p_F L}{2} \qquad F_{By} = 0 \qquad M_{zB} = 0$

$F_{Bx} = -3500\,N$

Ex_ReApoio_Axial_04:



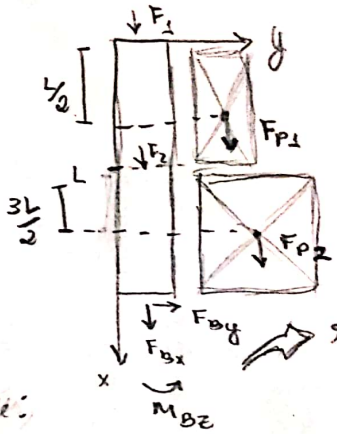
$L = 2\,m$

$p_0 = 5000\,N/m$

$F_1 = 8000\,N$

$F_2 = 16000\,N$

DCL:



reações de apoio

Equilíbrio no plano:

$\Sigma F_x = 0$

$\Sigma F_y = 0$

$\Sigma M_z = 0$

Sejam F_{P_1} e F_{P_2} forças resultantes tais que:

- $F_{P_1} = p_0 L$ → equivalente ao carregamento $p(x) = p_0$
- $F_{P_2} = (p_0 2)L$ → equivalente ao carregamento $p(x) = 2p_0$

$\Sigma F_x = 0$

$F_1 + F_{P_1} + F_{P_2} + F_2 + F_{Bx} = 0$

$F_{Bx} = -F_1 - F_2 - F_{P_1} - F_{P_2}$

$F_{Bx} = -F_1 - F_2 - p_0 L - 2p_0 L$

$F_{Bx} = -F_1 - F_2 - 3p_0 L$

$\Sigma F_y = 0$

$F_{By} = 0$

$\Sigma M_z(A) = 0$

$M_{Bz} + F_{By}(2L) = 0$ (A: 0)

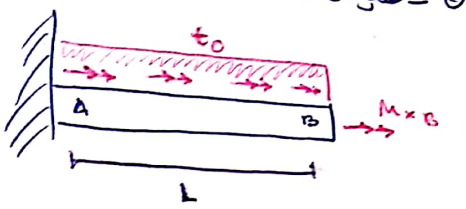
$M_{Bz} = 0$

$\therefore F_{Bx} = -F_1 - F_2 - 3p_0 L \qquad F_{By} = 0 \qquad M_{Bz} = 0$

$F_{Bx} = -54000\,N$

DCL:

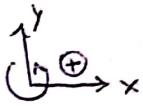
Ex – Re Apoio – Torção – 01:



$L = 1\,m$

$t_0 = 1500\ N\,m/m$

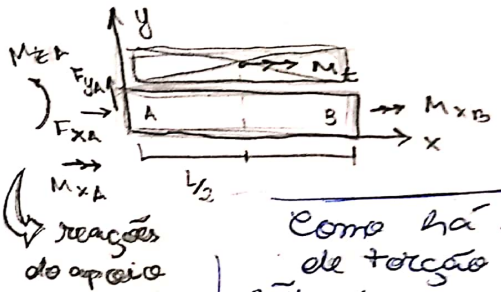
$M_{xB} = 2000\ N\,m$



seja: $M_t = t_0 L$

↳ momento torsional equivalente ao esforço distribuído $t(x) = t_0$

DCL:



Como há esforços de torção, as equações de equilíbrio se tornam:

$\Sigma F_x = 0 \quad \Sigma F_y = 0$

$\Sigma M_x = 0 \quad \Sigma M_z = 0$

$\sum F_x = 0$

$F_{xA} = 0$

$\sum F_y = 0$

$F_{yA} = 0$

$\Sigma M_z(B) = 0$

$M_{zA} - F_{yA} L = 0$

$M_{zA} = 0$

$\Sigma M_x(B) = 0$

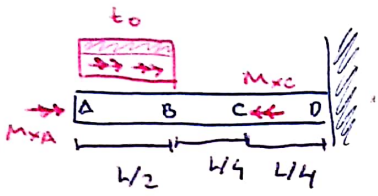
$M_{xA} + M_{xB} + M_t = 0$

$M_{xA} = -M_{xB} - M_t$

$M_{xA} = -M_{xB} - t_0 L$

$\therefore F_{xA} = 0 \quad F_{yA} = 0 \quad M_{zA} = 0 \quad M_{xA} = -M_{xB} - t_0 L = -3500\ N\,m$

Ex - Reapoio - Torção - 03:



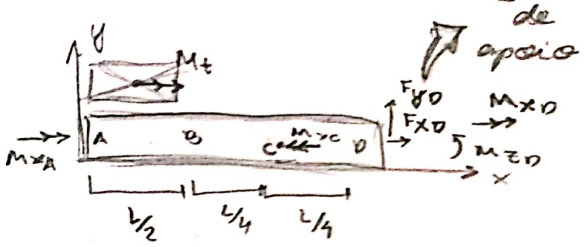
$L = 4\,m$

$t_0 = 3000\,N\,m/m$

$M_{xA} = 3000\,N\,m$

$M_{xc} = 4500\,N\,m$

DCL:



Seja: $M_t = t_0 \frac{L}{2}$

↳ momento torcional equivalente ao esforço distribuído

Como há esforços de torção, as equações de equilíbrio são:

$\Sigma F_x = 0$ $\quad \Sigma M_z = 0$

$\Sigma F_y = 0$ $\quad \Sigma M_x = 0$

$\Sigma F_x = 0$

$F_{xD} = 0$

$\Sigma F_y = 0$

$F_{yD} = 0$

$\Sigma M_z (A) = 0$

$M_{zD} + F_{yD} L = 0$

$M_{zD} = 0$

$\Sigma M_x (A) = 0$

$M_{xA} + M_t - M_{xc} + M_{xD} = 0$

$M_{xD} = -M_{xA} + M_{xc} - M_t$

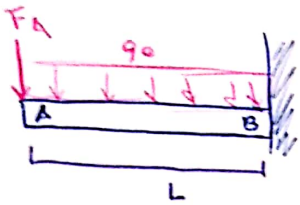
$M_{xD} = -M_{xA} + M_{xc} - t_0 L/2$

$\therefore F_{xD} = 0 \quad F_{yD} = 0 \quad M_{zD} = 0$

$M_{xD} = -M_{xA} + M_{xc} - t_0 L/2$

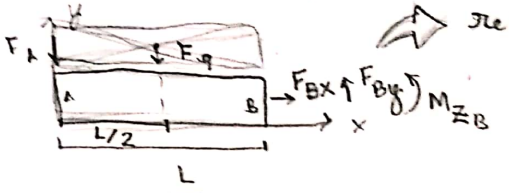
$M_{xD} = -4500\,Nm$

Ex - Reapoio - Flexão - 01:



L = 3 m
FA = 15000 N
q0 = 8000 N/m

DCL:



reações de apoio

$\Sigma F_x = 0 \quad \Sigma F_y = 0 \quad \Sigma M_z = 0$

Equações de equilíbrio (no plano)

seja: $F_q = q_0 L$ → força pontual equivalente ao carregamento distribuído

$\Sigma F_x = 0$

$F_{Bx} = 0$

$\Sigma F_y = 0$

$-F_A + F_{By} - F_q = 0$

$F_{By} = F_q + F_A$

$F_{By} = F_A + q_0 L$

$\Sigma M_z (B) = 0$

$F_A L + F_q \frac{L}{2} + M_{zB} = 0$

$M_{zB} = -F_A L - (q_0 L)\frac{L}{2}$

$M_{zB} = -F_A L - q_0 \frac{L^2}{2}$

$\therefore F_{Bx} = 0$

$F_{By} = F_A + q_0 L$

$F_{By} = 39000 N$

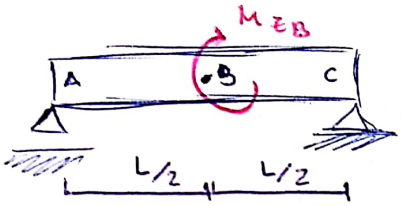
$M_{zB} = -F_A L - q_0 \frac{L^2}{2}$

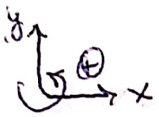
$M_{zB} = -81000 Nm$

Ex_ Re Apoio – Flexão – 02 :

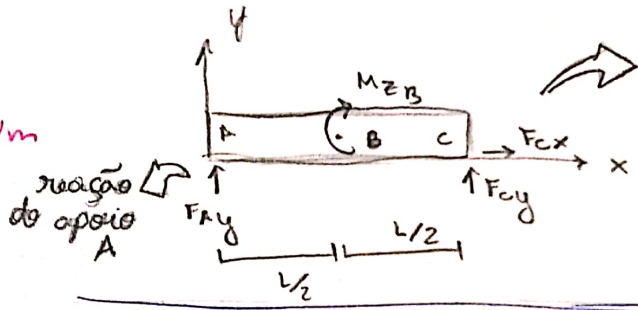
$L = 2\,m$

$MB_A = 5000\,Nm$





DCL:



Equilíbrio no plano:

$\Sigma F_x = 0 \quad \Sigma F_y = 0 \quad \Sigma M_z = 0$

$\Sigma F_x = 0$

$F_{cx} = 0$

$\Sigma F_y = 0$

$F_{Ay} + F_{cy} = 0$ (I)

$\Sigma M_z (A) = 0$

$-M_{zB} + F_{cy} L = 0$

$F_{cy} = \frac{M_{zB}}{L}$ (II)

Aplicando II em I:

$F_{Ay} + \frac{M_{zB}}{L} = 0 \Rightarrow F_{Ay} = -\frac{M_{zB}}{L}$

$\therefore F_{cx} = 0$

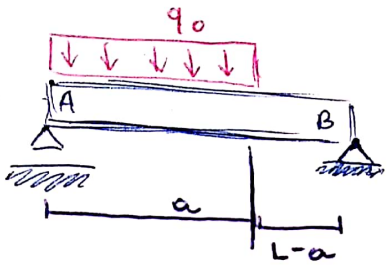
$F_{cy} = \frac{M_{zB}}{L}$

$F_{cy} = 2500\,N$

$F_{Ay} = -\frac{M_{zB}}{L}$

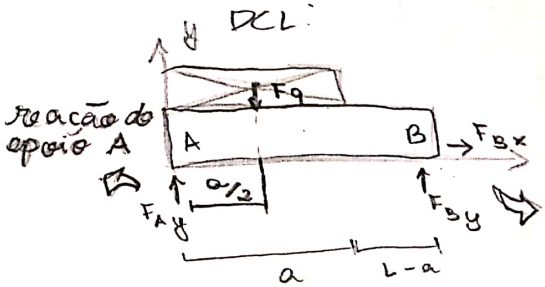
$F_{Ay} = -2500\,N$

Ex - Reapoio - Flexão - 03:



L = 3 m
a = 2,2 m
q0 = 8000 N/m

DCL:



Equilíbrio no plano:
$\Sigma F_x = 0 \quad \Sigma F_y = 0 \quad \Sigma M_z = 0$

seja: $F_q = q_0 a$
↳ força pontual equivalente ao carregamento distribuído

$\Sigma F_x = 0$
$F_{Bx} = 0$

$\Sigma F_y = 0$
$F_{Ay} - F_q + F_{By} = 0$
$F_{Ay} + F_{By} = F_q$
$F_{Ay} + F_{By} = q_0 a$ (I)

$\Sigma M_z(A) = 0$
$-F_q \frac{a}{2} + F_{By} L = 0$
$F_{By} = \frac{F_q a}{2L} = \frac{q_0 a^2}{2L}$ (II)

Aplicando II em I:
$F_{Ay} = q_0 a - F_{By} = q_0 a - \frac{q_0 a^2}{2L}$

$\therefore F_{Bx} = 0$

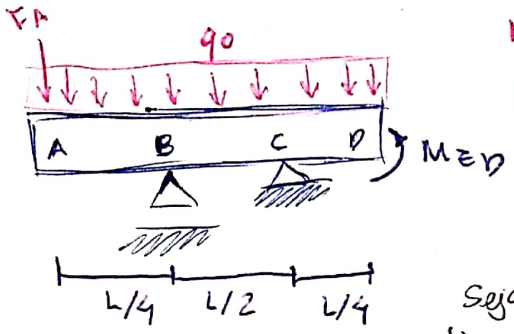
$F_{By} = \frac{q_0 a^2}{2L}$

$F_{By} = 6453,34\ N$

$F_{Cy} = q_0 a - \frac{q_0 a^2}{2L}$

$F_{Cy} = 11146,67\ N$

Ex - Reapoio - Flexão - 05:

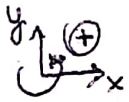


$L = 3m$

$q_0 = 8000\ N/m$

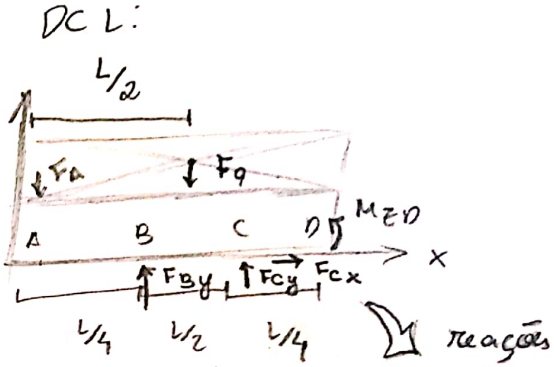
$F_A = 12000\ N$

$M_{zD} = 15000\ Nm$



Seja: $F_q = q_0 L$
↳ a força pontual equivalente ao carregamento distribuído

DCL:



reações dos apoios B e C

Equilíbrio no plano:

$\Sigma F_x = 0 \quad \Sigma F_y = 0 \quad \Sigma M_z = 0$

$\Sigma F_x = 0$

$F_{cx} = 0$

$\Sigma F_y = 0$

$F_{By} + F_{cy} - F_q - F_A = 0$

$F_{By} + F_{cy} = F_q + F_A = q_0 L + F_A$ (I)

$\Sigma M_z(C) = 0$

$-F_{By}\frac{L}{2} + F_q\frac{L}{4} + F_A\frac{3L}{4} + M_{zD} = 0$

$F_{By} = \frac{2}{L}\left(F_q\frac{L}{4} + F_A\frac{3L}{4} + M_{zD}\right)$

$F_{By} = \frac{q_0 L}{2} + M_{zD}\frac{2}{L} + \frac{F_A 3}{2}$ (II)

Aplicando II em I:

$\frac{q_0 L}{2} + M_{zD}\frac{2}{L} + \frac{F_A 3}{2} + F_{cy} = q_0 L + F_A$

$F_{cy} = \frac{q_0 L}{2} - M_{zD}\frac{2}{L} - \frac{F_A}{2}$

$\therefore F_{cx} = 0$

$F_{cy} = \frac{q_0 L}{2} - M_{zD}\frac{2}{L} - \frac{F_A}{2}$

$F_{cy} = -4000N$

$F_{By} = \frac{q_0 L}{2} + M_{zD}\frac{2}{L} + \frac{F_A 3}{2}$

$F_{By} = 40000\ N$