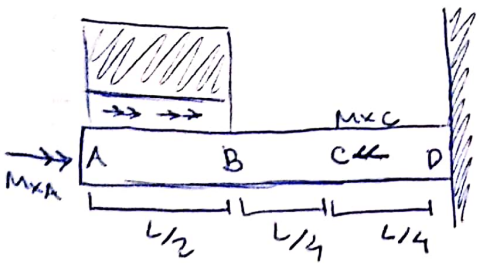
Lista de exercícios 6 - Letícia Lemos Diniz 201438

Legenda:

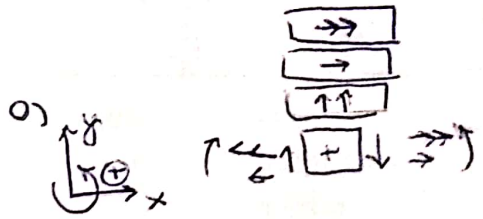
0) Sistemas de eixos e convenções
1) Equações diferenciais de equilíbrio
2) Equações de carregamento
3) Condições de contorno
4) Integração
5) Determinação das constantes
6) Equações finais
7) Análise numérica e diagramas

Parte 1: Problemas de Torção:



Dados:
$M_{xA} = 1500\,Nm$
$M_{xC} = 4500\,Nm$
$t_0 = 3000\,Nm/m$
$L = 2m$

a)



b) $\frac{dM_x(x)}{dx} = -t(x)$

2) $t(x) = +t_0\langle x-0\rangle^0 - t_0\langle x-L/2\rangle^0 - M_{xC}\langle x-3L/4\rangle^{-1}$

3) $M_x(x=0) = -M_{xA}$

4) $\frac{dM_x(x)}{dx} = -t_0\langle x-0\rangle^0 + t_0\langle x-L/2\rangle^0 + M_{xC}\langle x-3L/4\rangle^{-1}$

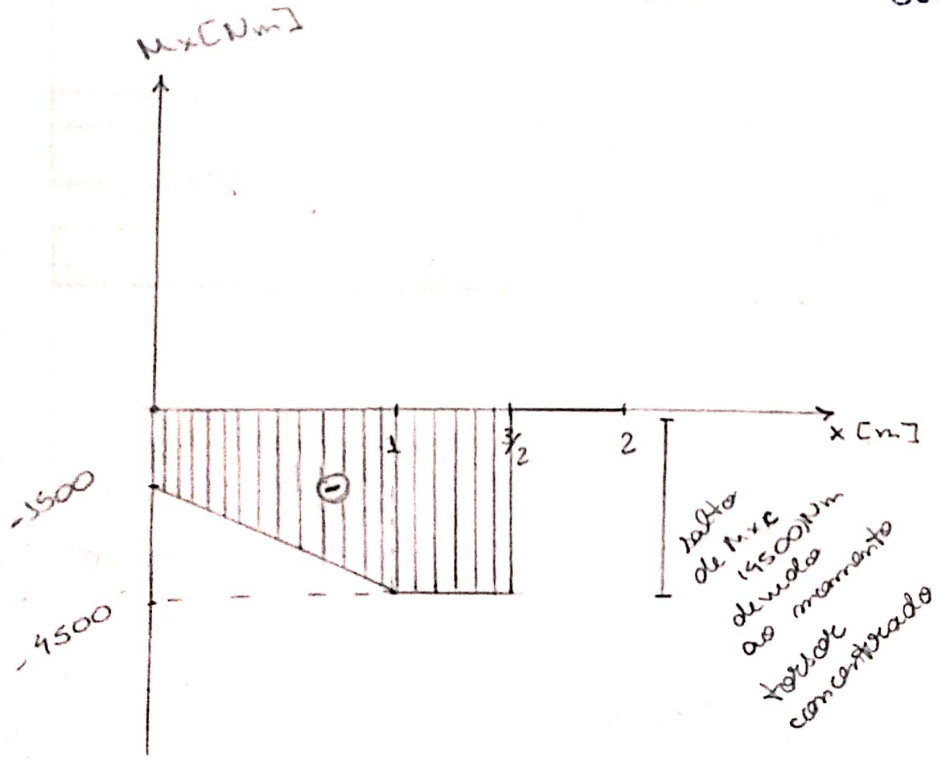
$M_x(x) = -t_0\langle x-0\rangle^1 + t_0\langle x-L/2\rangle^1 + M_{xC}\langle x-3L/4\rangle^0 + C_1$

5) $M_x(x=0) = -t_0\underbrace{\langle 0-0\rangle^1}_{=0} + t_0\underbrace{\langle 0-L/2\rangle^1}_{=0} + M_{xC}\underbrace{\langle 0-3L/4\rangle^0}_{=0} + C_1 = -M_{xA}$

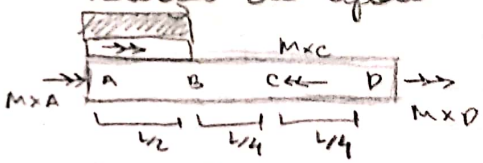
$M_x(x=0) = C_1 = -M_{xA} \Rightarrow \underline{C_1 = -M_{xA}}$

6) $M_x(x) = -t_0\langle x-0\rangle^1 + t_0\langle x-L/2\rangle^1 + M_{xC}\langle x-3L/4\rangle^0 - M_{xA}$

7) $M_x(x) = -3000\langle x-0\rangle^1 + 3000\langle x-1\rangle^1 + 4500\langle x-\tfrac{3}{2}\rangle^0 - 1500 \quad Nm$



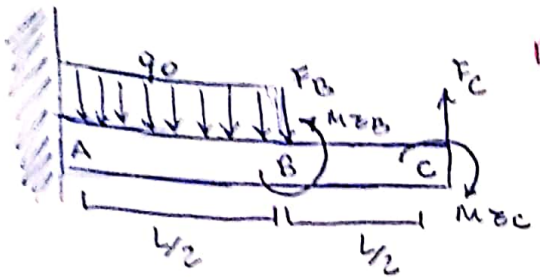
Reações de apoio:



$M_x(x=2) = 0 = M_{xD}$

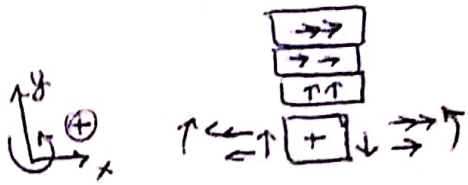
↳ reação em D é nula

Exercício 2: Problema de Flexão:



Dados:
$L = 2m$
$F_B = F_C = 1000N$
$M_{zB} = M_{zC} = 500Nm$
$q_0 = 800\ N/m$

a)



1) $\frac{d^2 M_z(x)}{dx^2} = q(x)$

2) $q(x) = -q_0\langle x-0\rangle^0 + q_0\langle x-L/2\rangle^0 - F_B\langle x-L/2\rangle^{-1} - M_{zB}\langle x-L/2\rangle^{-2}$

3) $M_z(x=L) = -M_{zC}$
$V_y(x=L) = \frac{dM_z}{dx}(x=L) = -F_C$

4) $\frac{d^2 M_z(x)}{dx^2} = -q_0\langle x-0\rangle^0 + q_0\langle x-L/2\rangle^0 - F_B\langle x-L/2\rangle^{-1} - M_{zB}\langle x-L/2\rangle^{-2}$

$\frac{dM_z(x)}{dx} = V_y(x) = -q_0\langle x-0\rangle^1 + q_0\langle x-L/2\rangle^1 - F_B\langle x-L/2\rangle^0 - M_{zB}\langle x-L/2\rangle^{-1} + C_1$

$M_z(x) = -\frac{q_0}{2}\langle x-0\rangle^2 + \frac{q_0}{2}\langle x-L/2\rangle^2 - F_B\langle x-L/2\rangle^1 - M_{zB}\langle x-L/2\rangle^0 + C_1 x + C_2$

5) $V_y(x=L) = -q_0\langle L-0\rangle^1 + q_0\langle L-L/2\rangle^1 - F_B\langle L-L/2\rangle^0 - M_{zB}\langle L-L/2\rangle^{-1} + C_1 = -F_C$

$V_y(x=L) = -q_0L + q_0L - q_0L/2 - F_B + C_1 = -F_C$

$\therefore C_1 = -F_C + F_B + q_0L/2$

podemos desprezar o impulso, porque não sabemos como o momento fletor foi criado

$M_z(x=L) = -\frac{q_0}{2}\langle L-0\rangle^2 + \frac{q_0}{2}\langle L-L/2\rangle^2 - F_B\langle L-L/2\rangle^1 - M_{zB}\langle L-L/2\rangle^0 + C_1 L + C_2 = -M_{zC}$

$M_z(x=L) = -\frac{q_0L^2}{2} + \frac{q_0L^2}{8} - F_B\frac{L}{2} - M_{zB} + (-F_C + F_B + q_0L/2)L + C_2 = -M_{zC}$

$\therefore C_2 = -M_{zC} + M_{zB} + F_CL - \frac{F_BL}{2} - \frac{q_0L^2}{8}$

6) $V_y(x) = -q_0\langle x-0\rangle^1 + q_0\langle x-L/2\rangle^1 - F_B\langle x-L/2\rangle^0 - M_{zB}\langle x-L/2\rangle^{-1} - F_C + F_B + q_0L/2$ N

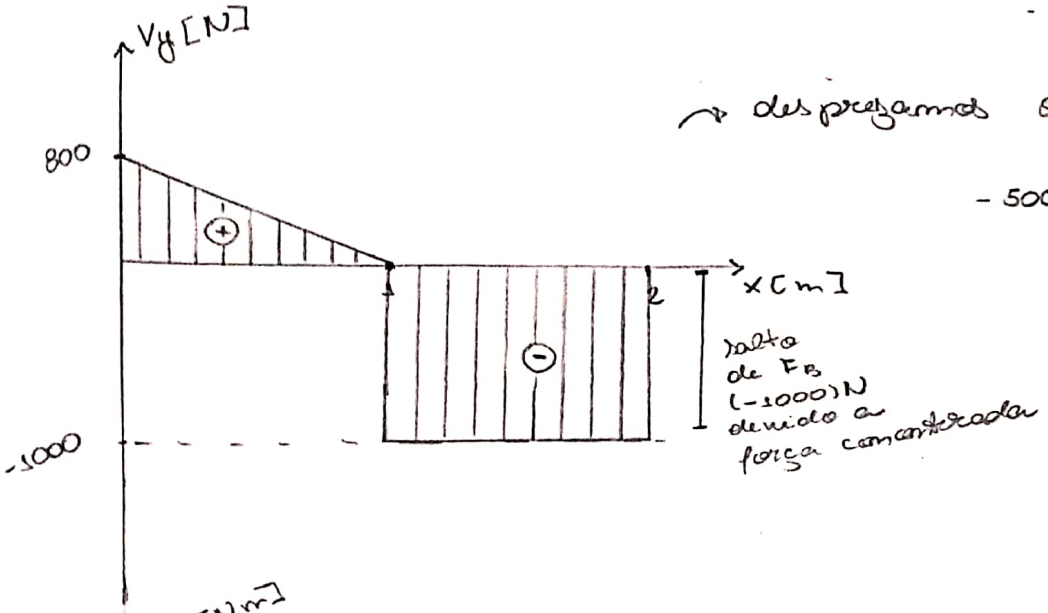
(o último termo é desprezível)

$M_z(x) = -\frac{q_0}{2}\langle x-0\rangle^2 + \frac{q_0}{2}\langle x-L/2\rangle^2 - F_B\langle x-L/2\rangle^1 - M_{zB}\langle x-L/2\rangle^0 + (-F_C + F_B + q_0L/2)x$
$-M_{zC} + M_{zB} + F_CL - \frac{F_BL}{2} - \frac{q_0L^2}{8}$ Nm

7) $V_y(x) = -800\langle x-0\rangle^1 + 800\langle x-1\rangle^1 - 1000\langle x-1\rangle^0 - 500\langle x-1\rangle^{-1} + 800$ N

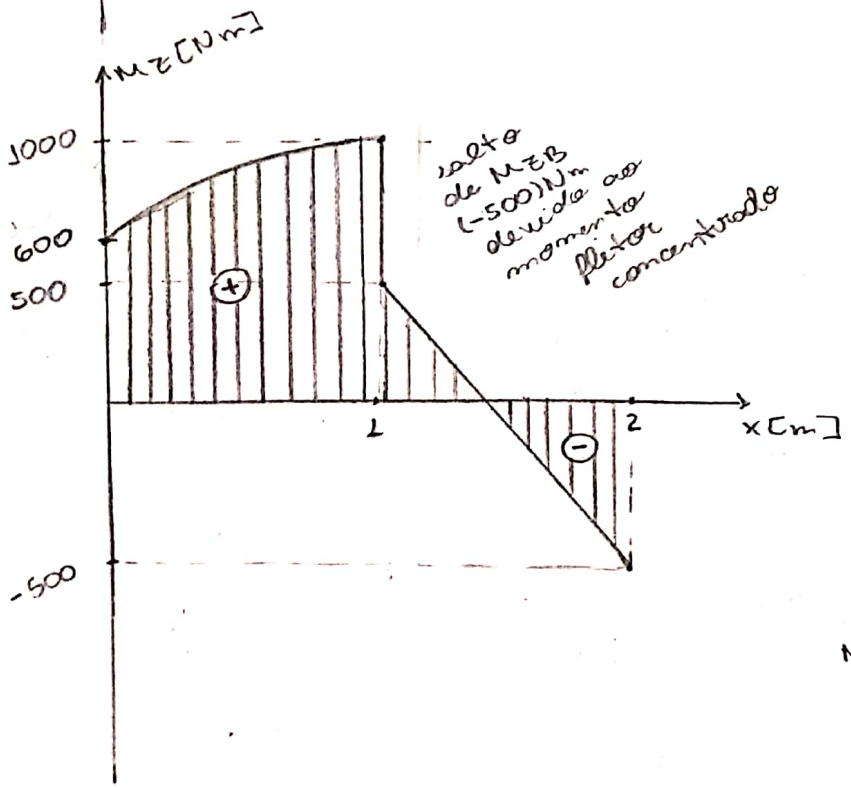
(o último termo é desprezível)

$M_z(x) = -400\langle x-0\rangle^2 + 400\langle x-1\rangle^2 - 1000\langle x-1\rangle^1 - 500\langle x-1\rangle^0 + 800x + 600$ Nm

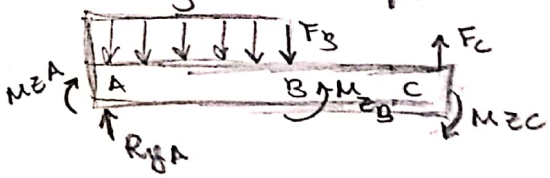


↳ desprezamos o impulso/delta de Dirac

$-500\langle x-1\rangle^{-1}$



Reações de apoio:



$R_{yA} = V_y(x=0) = +800\ N$

$M_{zA} = M_z(x=0) = +600\ Nm$