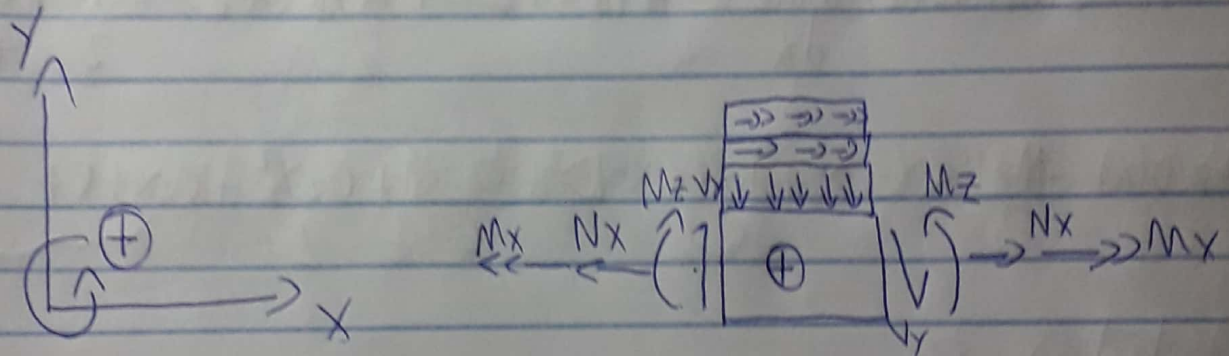
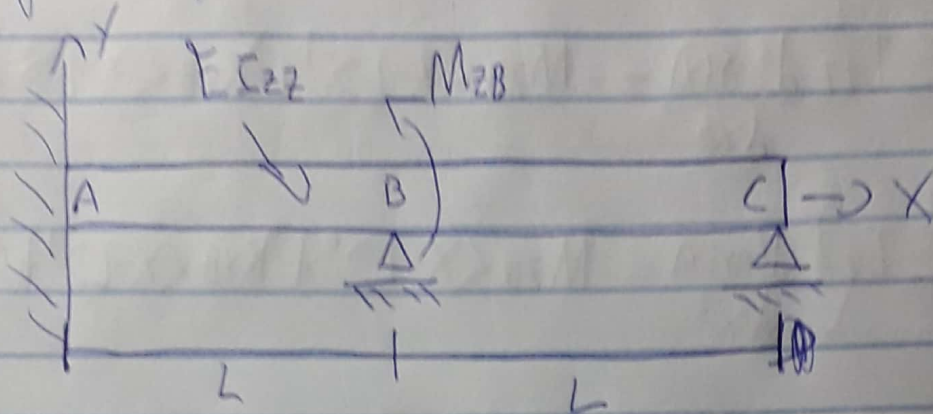


Lutu 15

Liga Rocio Olisi

e flex - viga - 29



Equação de Carregamento

$$EI_{zz} = \frac{d^4 v(x)}{dx^4} = q(x)$$

$$q(x) = EI_{zz} \frac{d^4 v(x)}{dx^4} = -M_{zB} \langle x-L \rangle^{-2} + R_{yB} \langle x-L \rangle^{-1}$$

Condições de contorno e restrições

Restrições: $v(x=0) = 0$

$$v(x=L) = 0 \quad \left| \quad \frac{dv(x=0)}{dx} = \theta_z(x=0) = 0$$

$$v(x=2L) = 0$$

$$M_z(x=2L) = 0$$

Integrando

$$EI_{zz} \frac{d^4 v(x)}{dx^4} = q(x) = -M_{zB} \langle x-L \rangle^{-2} + R_{yB} \langle x-L \rangle^{-1}$$

$$V_y(x) = EI_{zz} \frac{d^3 v(x)}{dx^3} = -M_{zB} \langle x-L \rangle^{-1} + R_{yB} \langle x-L \rangle^0 + C_1$$

$$M_z(x) = EI_{zz} \frac{d^2 v(x)}{dx^2} = -M_{zB} \langle x-L \rangle^0 + R_{yB} \langle x-L \rangle^1 + C_1 \cdot x + C_2$$

$$EI_{zz} \theta_z(x) = EI_{zz} \frac{dv(x)}{dx} = -M_{zB} \langle x-L \rangle^1 + \frac{R_{yB} \langle x-L \rangle^2}{2} + \frac{C_1 x^2}{2} + C_2 x + C_3$$

$$EI_{zz} v(x) = -\frac{M_{zB} \langle x-L \rangle^2}{2} + \frac{R_{yB} \langle x-L \rangle^3}{6} + \frac{C_1 x^3}{6} + \frac{C_2 x^2}{2} + C_3 x + C_4$$

Constantes

$$EI_{zz} v(x=0) = -\frac{M_{zB} \langle 0-L \rangle^2}{2} + \frac{R_{yB} \langle 0-L \rangle^3}{6} + \frac{C_1 \cdot 0}{6} + \frac{C_2 \cdot 0}{2} + C_3 \cdot 0 + C_4$$

$$EI_{zz} \theta_z(0) = -M_{zB} \langle 0-L \rangle^1 + \frac{R_{yB} \langle 0-L \rangle^2}{2} + \frac{C_1 \cdot 0}{2} + C_2 \cdot 0 + C_3 = 0$$

$$C_3 = 0$$

$$EI_{zz} v(x=L) = -\frac{M_{zB} \langle L-L \rangle^2}{2} + \frac{R_{yB} \langle L-L \rangle^3}{6} + \frac{C_1 L^3}{6} + \frac{C_2 L^2}{2} = 0$$

$$\frac{C_1 \cdot L^3}{6} + \frac{C_2 \cdot L^2}{2} = 0 \rightarrow \frac{C_1 \cdot L}{3} + C_2 = 0$$

$$EI_{ZZ} v''(x=2L) = \frac{-M_{ZB} \langle 2L-L \rangle^2}{2} + \frac{R_{YB} \langle 2L-L \rangle^3}{6} + \frac{C_1 (2L)^3}{6} + \frac{C_2 (2L)^2}{2} = 0$$

$$\frac{-M_{ZB} L^2}{2} + \frac{R_{YB} L^3}{6} + \frac{4C_1 L^3}{3} + 2C_2 L^2 = 0$$

$$\frac{R_{YB} L}{6} - \frac{M_{ZB}}{2} + \frac{4C_1 L}{3} + 2C_2 = 0$$

$$M_Z(x=2L) = 0$$

$$M_Z(x=2L) = -M_{ZB} \langle 2L-L \rangle^0 + R_{YB} \langle 2L-L \rangle^1 + C_1 \cdot 2L + C_2$$

$$R_{YB} L + 2C_1 L + C_2 = M_{ZB}$$

$$\left\{ \begin{array}{l} C_1 L + C_2 = 0 \rightarrow C_2 = -\frac{C_1 L}{3} \end{array} \right.$$

$$R_{YB} L + 2C_1 L + C_2 = M_{ZB}$$

$$\left\{ \begin{array}{l} \frac{-M_{ZB}}{2} + \frac{R_{YB} L}{6} + \frac{4C_1 L}{3} + 2C_2 = 0 \end{array} \right.$$

$$R_{YB} L + 2C_1 L + \left(-\frac{C_1 L}{3} \right) = M_{ZB}$$

$$R_{YB} L + \frac{5C_1 L}{3} = M_{ZB}$$

$$\frac{-M_{ZB}}{2} + \frac{R_{YB} L}{6} + \frac{4C_1 L}{3} + 2\left(-\frac{C_1 L}{3} \right) = 0$$

$$\frac{-M_{ZB}}{2} + \frac{R_{YB} L}{6} = 0 \rightarrow \frac{R_{YB} L}{3} = M_{ZB} \rightarrow R_{YB} = \frac{M_{ZB}}{3L}$$

$$\frac{C_1 L}{3} \cdot 5 = M_{ZB} - R_{YB} L = M_{ZB} - 3 M_{ZB} = -2 M_{ZB}$$

$$C_1 = \frac{-6 M_{ZB}}{5L}$$

$$C_2 = \frac{-C_1 L}{3} = \frac{2 M_{ZB}}{5}$$

Reação de Apoio

Carregamento

$$V_Y(X) = -M_{ZB} \langle X-L \rangle^1 + \frac{M_{ZB}}{3L} \langle X-L \rangle^0 - \frac{6 M_{ZB}}{5L}$$

$$M_Z(X) = -M_{ZB} \langle X-L \rangle^0 + \frac{M_{ZB}}{3L} \langle X-L \rangle^1 - \frac{3 M_{ZB} X}{5L} + \frac{2 M_{ZB}}{5}$$

Reação $X=0$

$$R_{YA} = \frac{-6 M_{ZB}}{5L}$$

$$M_{ZA} = \frac{2 M_{ZB}}{5}$$

$X=2$

$$R_{YC} = \frac{-3 M_{ZB}}{5L}$$

$$M_Z(X) = -M_{ZB} + \frac{M_{ZB}}{5L} \cdot L - \frac{3 M_{ZB} \cdot 2L}{5L} + \frac{2 M_{ZB}}{5}$$

$$M_Z(X) = -M_{ZB} + \frac{M_{ZB}}{5} - \frac{6 M_{ZB}}{5} + \frac{2 M_{ZB}}{5} = \frac{2 M_{ZB}}{5} - 2 M_{ZB}$$

$$M_Z(X) = \frac{-8 M_{ZB}}{5}$$

Rigidez $\theta(X=L)$

$$EI_{zz}\theta(X) = -M_{zB} \langle X-L \rangle' + \frac{R_{yB}}{2} \langle X-L \rangle^2 + \frac{C_1 X^2}{2} + C_2 X$$

$$EI_{zz}\theta(X=L) = \frac{C_1 L^2}{2} + C_2 L$$

$$EI_{zz}\theta(X=L) = \frac{-3M_{zB}L}{5} + \frac{2M_{zB}L}{5} = \frac{-M_{zB}L}{5}$$

$$\theta(X=L) = \frac{-M_{zB}L}{5EI_{zz}}$$

$$R_{\theta B} = \frac{5EI_{zz}}{L}$$