

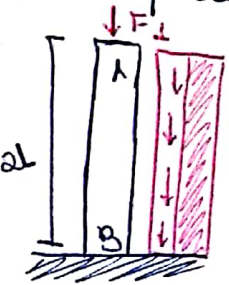
Lista de exercícios 3 - Letícia Arven Dines  
201438

## \* Legenda:

- 0) Convenções
- 1) Equação diferencial
- 2) Equação de carregamento
- 3) Condições de contorno
- 4) Integração
- 5) Conhecimento das constantes
- 6) Equação final
- 7) Verificação e Análise (7b) Reações de Apoio)
- 8) Diagrama e Valores Numéricos

# Lista de exercícios 3 - Letícia Arvin Dimes 201438

Ex - Eq Dif Equil - Axial - Ol:

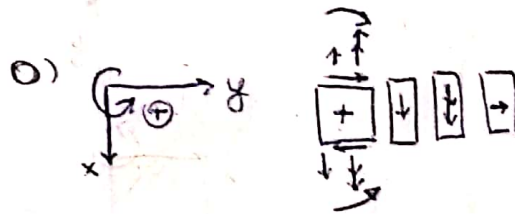


Dados:

$$L = 2\text{ m}$$

$$p_0 = 5000\text{ N/m}$$

$$F_1 = 8000\text{ N}$$



$$1) \frac{dN_x(x)}{dx} = -p(x)$$

$$2) p(x) = +p_0$$

$$3) N_x(x=0) = -F_1$$

$$4) \frac{dN_x(x)}{dx} = -p_0$$

$$\int dN_x(x) = \int -p_0 dx$$

$$N_x(x) = -p_0 x + C_1$$

$$5) N_x(x=0) = -p_0 \cdot 0 + C_1 = -F_1$$

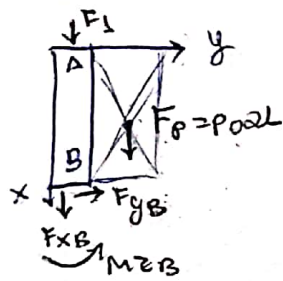
$$\rightarrow C_1 = -F_1$$

$$6) N_x(x) = -p_0 x - F_1$$

$$7) N_x(x) \rightarrow \text{comportamento linear; } \begin{cases} N_x(0) = -F_1 \\ N_x(2L) = -p_0 2L - F_1 \end{cases}$$

$$7b) \text{ reação em B: } N_x(x=2L) = -p_0 2L - F_1 = F_{xB}$$

↳ pode-se conferir:



$$\Sigma F_y = 0$$

$$F_{yB} = 0$$

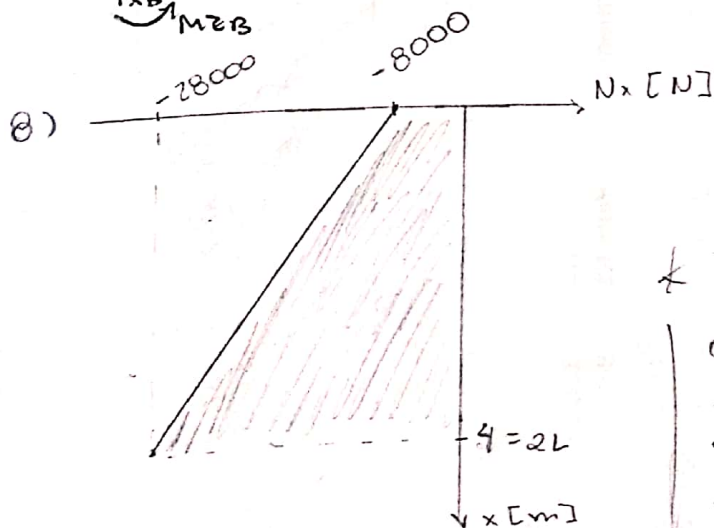
$$\Sigma M_z(B) = 0$$

$$M_{zB} = 0$$

$$\Sigma F_x = 0$$

$$F_1 + F_p + F_{xB} = 0$$

$$F_{xB} = -F_1 - F_p = -F_1 - p_0 2L$$



∴ Resultados:

$$N_x(x) = -p_0 x - F_1 = -5000x - 8000$$

$$F_{xB} = -F_1 - p_0 2L = -28000\text{ N}$$

Legenda:

0) Convenções

1) Equação diferencial

2) Equação de carregamento

3) Condições de contorno

4) Integração

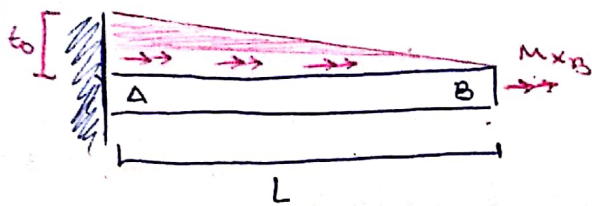
5) Conhecimento das constantes

6) Equação final

7) Verificação e Análise (7b) Reações de Apoio

8) Diagrama e Valores Numéricos

# Ex - Eq Dif Equil - Torção - 02:

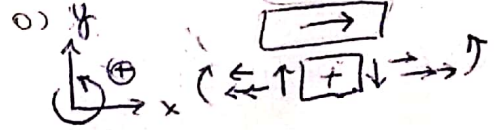


Dados:

$$L = 2\text{m}$$

$$t_0 = 500\text{Nm/m}$$

$$M_{xB} = 1000\text{Nm}$$



$$1) \frac{dM_x(x)}{dx} = -t(x) \quad 2) t(x) = -\frac{t_0}{L}x + t_0 \quad 3) M_x(x=L) = +M_{xB}$$

$$4) \frac{dM_x(x)}{dx} = +\frac{t_0 x}{L} - t_0 \Rightarrow \int dM_x(x) = \int (\frac{t_0 x}{L} - t_0) dx$$

$$M_x(x) = \frac{t_0 x^2}{2L} - t_0 x + C_1$$

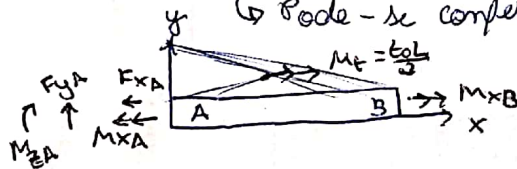
$$5) M_x(x=L) = \frac{t_0 L^2}{2L} - t_0 L + C_1 = M_{xB} \rightarrow C_1 = M_{xB} + \frac{t_0 L}{2}$$

$$6) M_x(x) = \frac{t_0 x^2}{2L} - t_0 x + (M_{xB} + \frac{t_0 L}{2})$$

7)  $M_x$  tem comportamento parabólico

$$7b) M_x(x=0) = M_{xB} + \frac{t_0 L}{2} = M_{xA} \rightarrow \text{reação de apoio em A}$$

$M_x(x=L) = M_{xB}$  (Pode-se conferir:)



$$\sum F_x = 0$$

$$-F_{xA} = 0$$

$$\sum F_y = 0$$

$$F_{yA} = 0$$

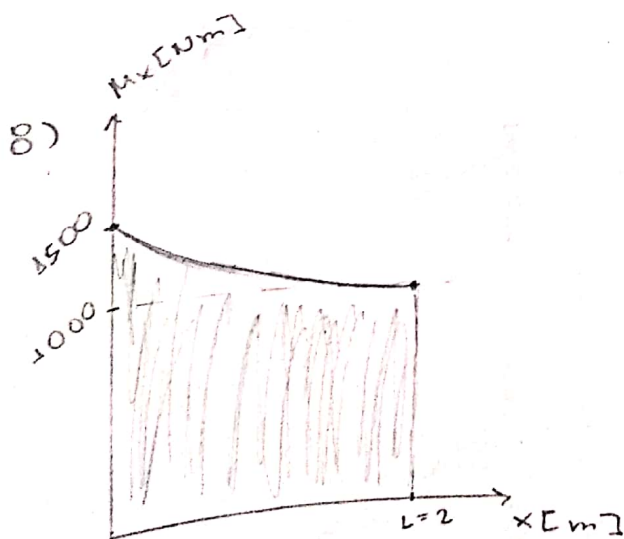
$$\sum M_x(A) = 0$$

$$-M_{xA} + \frac{t_0 L}{2} + M_{xB} = 0$$

$$M_{xA} = \frac{t_0 L}{2} + M_{xB}$$

$$\sum M_z(A) = 0$$

$$-M_{zA} = 0$$



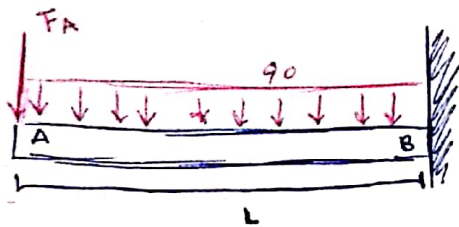
∴ Resultado:

$$M_x(x) = \frac{t_0 x^2}{2L} - t_0 x + M_{xB} + \frac{t_0 L}{2} = 125x^2 - 500x + 1500$$

$$M_{xA} = \frac{t_0 L}{2} + M_{xB} = 1500\text{Nm}$$



# Ex - Eq Dif Equil - Flexão - Ol:

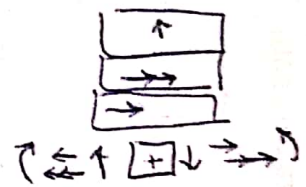
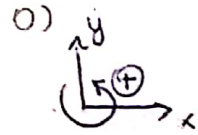


Dados:

$$L = 3 \text{ m}$$

$$F_A = 15000 \text{ N}$$

$$q_0 = 8000 \text{ N/m}$$



$$1) \frac{d^2 M_z(x)}{dx^2} = q(x)$$

$$2) q(x) = -q_0 \quad 3) M_z(x=0) = 0$$

$$\frac{dM_z(x)}{dx} = V_y(x)$$

$$\frac{dM_z(x=0)}{dx} = V_y(x=0) = -F_A$$

$$4) \frac{d^2 M_z(x)}{dx^2} = -q_0 \Rightarrow \frac{dM_z(x)}{dx} = \int -q_0 dx = -q_0 x + C_1$$

$$M_z(x) = \int (-q_0 x + C_1) dx = -\frac{q_0 x^2}{2} + C_1 x + C_2$$

$$5) M_z(x=0) = C_2 = 0$$

$$\frac{dM_z}{dx} = -q_0 x + C_1 = V_y(x) \Rightarrow V_y(x=0) = C_1 = -F_A$$

$$6) M_z(x) = -\frac{q_0 x^2}{2} - F_A x$$

$$\frac{dM_z}{dx} = V_y(x) = -q_0 x - F_A$$

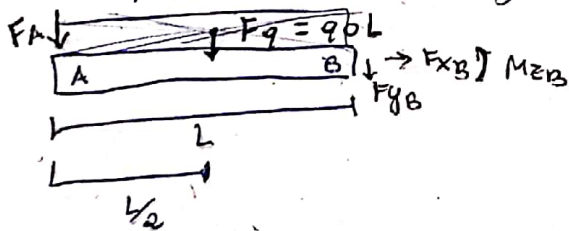
$$7) M_z(0) = 0$$

$$M_z(L) = -\frac{q_0 L^2}{2} - F_A L = M_{zB}$$

$$V_y(0) = -F_A$$

$$V_y(L) = -q_0 L - F_A = F_{yB}$$

Conferindo as reações em B:



$$\sum F_x = 0$$

$$F_{xB} = 0$$

$$\sum F_y = 0$$

$$-F_A - F_q - F_{yB} = 0$$

$$F_{yB} = -F_A - q_0 L$$

$$\sum M_z(B) = 0$$

$$F_A L + F_q \frac{L}{2} + M_{zB} = 0$$

$$M_{zB} = -F_A L - F_q \frac{L}{2}$$

$$M_{zB} = -F_A L - \frac{q_0 L^2}{2}$$

8)  $M_z [Nm]$

$V_y [N]$

$L = 3$

$L = 3$

$x [m]$

$x [m]$

Resultados:

$$M_z(x) = -\frac{q_0 x^2}{2} - F_A x$$

$$M_z(x) = -4000x^2 - 15000x$$

$$M_{zB} = -F_A L - \frac{q_0 L^2}{2}$$

$$M_{zB} = -81000 \text{ Nm}$$

$$V_y(x) = -q_0 x - F_A$$

$$V_y(x) = -8000x - 15000$$

$$F_{yB} = -F_A - q_0 L$$

$$F_{yB} = -39000 \text{ N}$$