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$$\begin{aligned} 1) a) \quad u &= 0,001x + 0,009y + 0,006z \\ v &= 0,002x + 0,007y + 0,001z \\ w &= 0,001x + 0,001y - 0,008z \end{aligned}$$

Os estados de deformação podem ser calculados a partir de:

$$\epsilon_{xx} = \frac{du}{dx} = 0,001 \quad \epsilon_{yy} = \frac{dv}{dy} = 0,007 \quad \epsilon_{zz} = \frac{dw}{dz} = -0,008$$

Em deformação volumétrica é:

$$K = \epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz} = 0,001 + 0,007 - 0,008 = 0$$

$$\begin{aligned} b) \quad u &= 0,001x + 0,002y + 0,006z \\ v &= 0,002x + 0,003y + 0,004z \\ w &= 0,006x + 0,004y - 0,004z \end{aligned}$$

$$\epsilon_{xx} = \frac{du}{dx} = 0,001 \quad \epsilon_{yy} = \frac{dv}{dy} = 0,003 \quad \epsilon_{zz} = \frac{dw}{dz} = -0,004$$

$$K = \epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz} = 0,001 + 0,003 - 0,004 = 0$$



$$2) \vec{u}(\vec{x}) = x\hat{i} + y\hat{j} + z\hat{k}$$

$$[\varepsilon] = \begin{bmatrix} \frac{du}{dx} & \frac{1}{2} \left( \frac{du}{dy} + \frac{dv}{dx} \right) & \frac{1}{2} \left( \frac{du}{dz} + \frac{dw}{dx} \right) \\ \frac{1}{2} \left( \frac{du}{dy} + \frac{dv}{dx} \right) & \frac{dv}{dy} & \frac{1}{2} \left( \frac{dv}{dz} + \frac{dw}{dy} \right) \\ \frac{1}{2} \left( \frac{du}{dz} + \frac{dw}{dx} \right) & \frac{1}{2} \left( \frac{dv}{dz} + \frac{dw}{dy} \right) & \frac{dw}{dz} \end{bmatrix}$$

Temos que  $\frac{du}{dx} = \frac{dv}{dy} = \frac{dw}{dz} = 1$  o resto é nulo

$$[\varepsilon] = \begin{bmatrix} 1 & 1/2 & 1/2 \\ 1/2 & 0 & 0 \\ 1/2 & 0 & 0 \end{bmatrix}$$

$$\vec{u}(\vec{x}) = (3x^2y + 6) \cdot 10^{-11} \hat{i} + (y^2 + 6xz) \cdot 10^{-11} \hat{j} + (6z^2 + 2xy + 10) \cdot 10^{-11} \hat{k}$$

$$\varepsilon_{ij} = \frac{1}{2} \left( \frac{du_i}{dx_j} + \frac{du_j}{dx_i} \right) \Rightarrow \begin{aligned} \varepsilon_{11} &= (3x^2y + 6) \cdot 10^{-11} \\ \varepsilon_{12} &= (y^2 + 6xz) \cdot 10^{-11} \\ \varepsilon_{13} &= (6z^2 + 2xy + 10) \cdot 10^{-11} \end{aligned}$$

$$\varepsilon = \varepsilon_{ij} = \begin{bmatrix} 6xy - \frac{3}{2}x^2 + 3z & 0 & 0 \\ \frac{3}{2}x^2 + 3z & 2y & 3x + z \\ 0 & 3x + z & 12z + 2y \end{bmatrix} \cdot 10^{-11}$$



$$3) L_1 = L_2 = 1 \text{ m} \quad \Delta T_1 = \Delta T_2 = 50^\circ \text{C}$$

$$\alpha_1 = 11 \cdot 10^{-6} \text{ } ^\circ\text{C}^{-1} \quad \alpha_2 = 23 \cdot 10^{-6} \text{ } ^\circ\text{C}^{-1}$$

$$E\bar{\epsilon} = \alpha \Delta T \quad \Delta u = \frac{l_p - L}{L} \quad l_p = L(1 + \alpha \Delta T) \quad \begin{matrix} \text{Ferro} \\ \text{Curdido} \end{matrix}$$

$$L_{p1} = L_1(1 + \alpha_1 \Delta T_1) = 1(1 + 11 \cdot 10^{-6} \cdot 50^\circ) = \boxed{5,5 \cdot 10^{-4} \text{ m}}$$

$$L_{p2} = L_2(1 + \alpha_2 \Delta T_2) = 1(1 + 23 \cdot 10^{-6} \cdot 50^\circ) = \boxed{1,15 \cdot 10^{-3} \text{ m}}$$

Alumínio

$$4) A(0;0) \longrightarrow E(1,5;0,5)$$

$$B(1;0) \longrightarrow F(2,55;0,56)$$

$$C(1;1) \longrightarrow G(2,58;1,59)$$

$$D(0;1) \longrightarrow H(1,54;1,53)$$

Deformação pequena e somente no plano xy

$$a) \Delta u = EF_x - AB_x = (2,55 - 1,5) - (1 - 0) = 0,05$$

$$\Delta x = AB_x = 1$$

$$\Delta v = EH_y - AD_y = (1,53 - 0,5) - (1 - 0) = 0,03$$

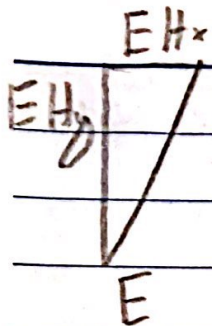
$$\Delta y = AD_y = 1$$

$$\tan \alpha_1 = \alpha_1 = \frac{EF_y}{EF_x} = \frac{0,06}{1,05} = 0,057$$

$$\alpha_1 = 0,057$$







$$\tan \alpha_2 = \alpha_2 = \frac{EH_x}{EH_y} = \frac{0,04}{1,03} = 0,039$$

$$\alpha_2 = 0,039$$

$$b) [\epsilon] = \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} \\ \epsilon_{yx} & \epsilon_{yy} \end{bmatrix}$$

$$\epsilon_{xx} = \frac{\Delta u}{\Delta x} = \frac{0,05}{1} = 0,05$$

$$\epsilon_{yy} = \frac{\Delta v}{\Delta y} = \frac{0,03}{1} = 0,03$$

$$\epsilon_{yx} = \epsilon_{xy} = \frac{1}{2} \gamma_{xy} = \frac{1}{2} (\alpha_1 + \alpha_2) = \frac{1}{2} (0,057 + 0,039)$$

$$\epsilon_{yx} = \epsilon_{xy} = 0,048$$

$$[\epsilon] = \begin{bmatrix} 0,05 & 0,048 \\ 0,048 & 0,03 \end{bmatrix}$$