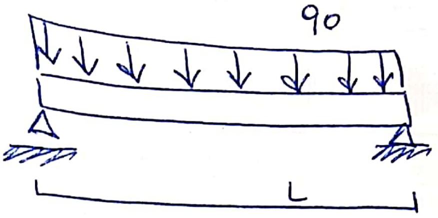
Letícia Levin Diniz 201438 Lista de exercícios 13

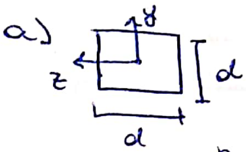
dim – viga – tensão – 01:



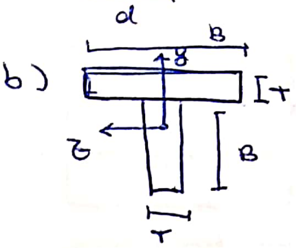
$q_0 = 3500 N/m$
$L = 3 m$
$\sigma_{xx max} = 15 N/mm^2$

1) Primeiramente, devemos conhecer as expressões de I_{zz}, W_z para cada figura.

a)



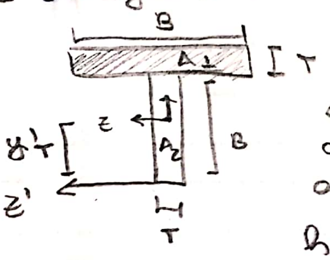
b)



$B = 5T$

figura a) $I_{zz} = \frac{d(d)^3}{12} = \frac{d^4}{12}$, $W_z = \frac{I_{zz}}{y_{max}} = \frac{\frac{d^4}{12}}{\frac{d}{2}} = \frac{d^3}{6}$

figura b) seja z':



$A_1 = BT = A_2$

alturas dos centroides de cada área parcial com relação a z':

$h_1' = B + \frac{T}{2}$ $h_2' = \frac{B}{2}$

Assim:

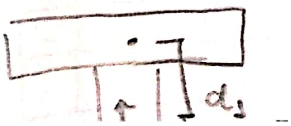
$$(A_1 + A_2) y_T' = A_1 h_1' + A_2 h_2'$$

$$2BT y_T' = BT\left(B + \frac{T}{2}\right) + BT \cdot \frac{B}{2}$$

$$2BT y_T' = \frac{3}{2} B^2 T + \frac{BT^2}{2}$$

$$y_T' = \frac{3}{4} B + \frac{T}{4}$$, seja $B = 5T$

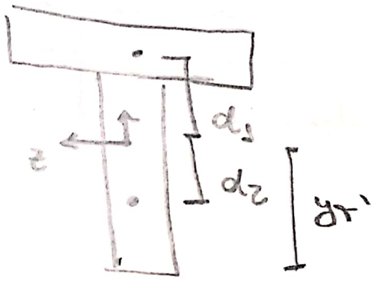
$$\underline{y_T' = 4T}$$



Com eixo ...

$y_T' = \frac{3}{4}B + \frac{T}{4}$, seja $B = 5T$

$y_T' = 4T$



Com isso, fazemos:

$d_1 = h_1' - y_T' = B + \frac{T}{2} - 4T = 5T + \frac{T}{2} - 4T = \frac{3T}{2}$

$d_2 = h_2' - y_T' = \frac{B}{2} - 4T = \frac{5T}{2} - 4T = -\frac{3T}{2}$

Sabendo que: $I_{zz_1} = \frac{BT^3}{12} = \frac{5T^4}{12}$ e $I_{zz_2} = \frac{TB^3}{12} = \frac{125T^4}{12}$

Concluímos:

Teorema da translação de eixos paralelos $\rightarrow$ $I_{zz} = I_{zz_1} + A_1 d_1^2 + I_{zz_2} + A_2 d_2^2$

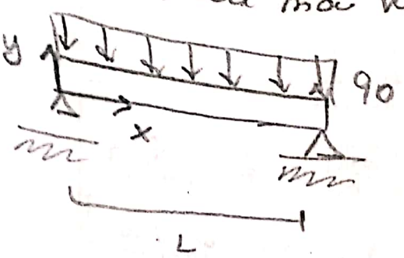
$I_{zz} = \frac{5T^4}{12} + 5T \cdot \left(\frac{3T}{2}\right)^2 + \frac{125T^4}{12} + 5T\left(-\frac{3T}{2}\right)^2$

$I_{zz} = \frac{5T^4}{12} + \frac{45T^4}{4} + \frac{125T^4}{12} + \frac{45T^4}{4} = \frac{400T^4}{12} = \frac{100T^4}{3}$

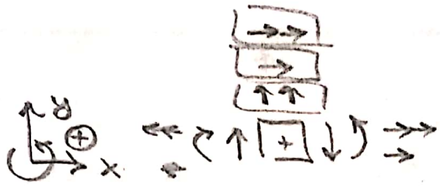
e $y_{max} = y_T' = 4T$

$W_z = \frac{I_{zz}}{y_{max}} = \frac{\frac{100T^4}{3}}{4T} = \frac{25T^3}{3}$

Em seguida, precisamos conhecer o momento fletor que atua na viga:



0) convenções:



1) eq. diferencial:

$$\frac{d^2M_z}{dx^2} = q(x)$$

2) eq. de carregamento: $q(x) = -q_0$

3) condições de contorno: $M_z(x=0) = 0$

$$M_z(x=L) = 0$$

4) Integração:

$$\frac{d^2M_z}{dx^2} = -q_0$$

$$\frac{dM_z}{dx} = -q_0x + C_1$$

$$M_z(x) = -\frac{q_0x^2}{2} + c_1x + C_2$$

5) Constantes:

$M_z(0) = C_2 = 0$ $\qquad$ $M_z(L) = -\frac{q_0L^2}{2} + c_1L = 0$

$$c_1 = \frac{q_0L}{2}$$

6) eq final:

$$M_z(x) = -\frac{q_0x^2}{2} + \frac{q_0L}{2}x$$

Sabe-se que: $\sigma_{xx}(y) = -\dfrac{M_z(x)\,y}{I_{zz}}$ e $W_z = \dfrac{I_{zz}}{y_{max}}$

$$\therefore\ \sigma_{xx\,max} = \frac{M_{z\,max}}{W_z}$$

Deseja-se que $\sigma_{xx\,max} = 15\ N/mm^2$

Além disso, sabe-se que o máximo de $M_z(x)$ ocorre em:

$$\frac{dM_z}{dx} = -q_0 x + q_0\frac{L}{2} = 0 \qquad \underline{M_{z\,max}} = M_z(L/2) = \underline{\underline{q_0\frac{L^2}{8}}}$$

$$x = \frac{L}{2}$$

Dessa forma, para a figura a):

seja $q_0 = 3{,}5\ N/mm$, $L = 3000\,mm$

$\sigma_{xx\,max} = 15\ N/mm^2$

$$\sigma_{xx\,max} = \frac{q_0\frac{L^2}{8}}{\frac{d^3}{6}} \Rightarrow d = \sqrt[3]{\frac{q_0 L^2\, 6}{8\,\sigma_{xx\,max}}} = \sqrt[3]{\frac{3{,}5\ N/mm\,(3000\,mm)^2\,6}{8\,(15\ N/mm^2)}}$$

temos: $d = 116{,}35\ mm$

E para a figura b):

$$\sigma_{xx\,max} = \frac{\frac{q_0 L^2}{8}}{\frac{25T^3}{3}} \Rightarrow T = \sqrt[3]{\frac{q_0 L^2 \cdot 3}{8\,\sigma_{xx\,max} \cdot 25}} = \sqrt[3]{\frac{(3{,}5\,N/mm)(3000\,mm)^2\,3}{8\,(15\,N/mm^2)\,25}}$$

$$T = 31{,}58\,mm$$

Digitalizado com CamScanner

2) Para a figura a):

$$I_{zz} = \frac{d^4}{12} \Bigg| = \frac{(116{,}35\,mm)^4}{12} = 15{,}27 \cdot 10^6\ mm^4$$

$d = 116{,}35\,mm$

$$\sigma_{xx}(x,y) = -\frac{M_z(x)\,y}{I_{zz}}$$

$$\sigma_{xx}(x,y) = -\frac{\left(-90\frac{x^2}{2} + 90\frac{L}{2}x\right)y}{15{,}27 \cdot 10^6}\ N/mm^2$$

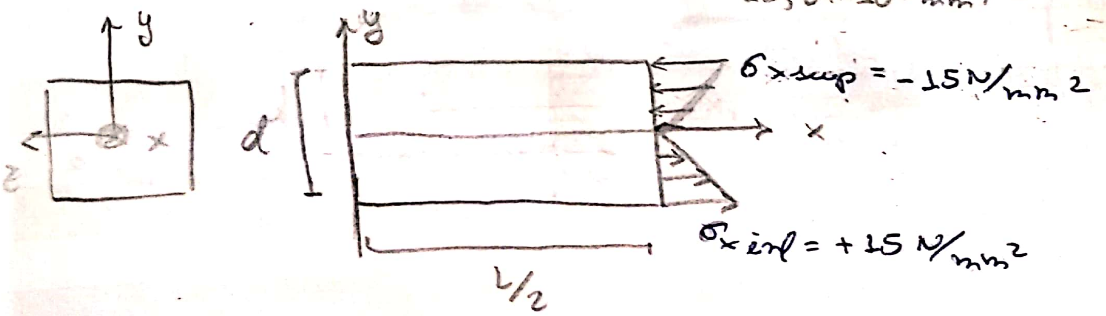
para um x fixo; seja $x = \frac{L}{2}$

$$\sigma_{xx}(y) = -\frac{M_z(L/2)\,y}{I_{zz}}$$

$$M_z(L/2) = 90\frac{L^2}{8} = \frac{(3{,}5\ N/mm)(3000\,mm)^2}{8}$$

$$M_z(L/2) = 3{,}9375 \cdot 10^6\ Nmm$$

$$\sigma_{xx} = -\frac{3{,}9375 \cdot 10^6\ Nmm}{15{,}27 \cdot 10^6\ mm^4}\,y = -0{,}2578\,y\ N/mm^2$$



Para a figura b):

$$I_{zz} = \left.\frac{100T^4}{3}\right|_{T=31,58\,mm} = 33,153 \cdot 10^6\ mm^4$$

$$\sigma_{xx}(y) = -\frac{\left(-\frac{90x^2}{2} + \frac{90Lx}{2}\right)}{33,153 \cdot 10^6}\,y\ N/mm^2$$

Para um x fixo: seja $x = L/2$

$$\sigma_{xx}(y) = -\frac{M_z(L/2)\,y}{I_{zz}}$$

$$\sigma_{xx}(y) = -\frac{3,9375 \cdot 10^6\ Nmm}{33,153 \cdot 10^6\ mm^4}\,y$$

$$\sigma_{xx}(y) = -0,1187y\ N/mm^2$$

Para $y_{superior} = 6T - y_T' = 6T - 4T = 2T$

$$\sigma_{xx_{sup}} = -0{,}1187 \cdot 2 \cdot 31{,}58\,mm$$

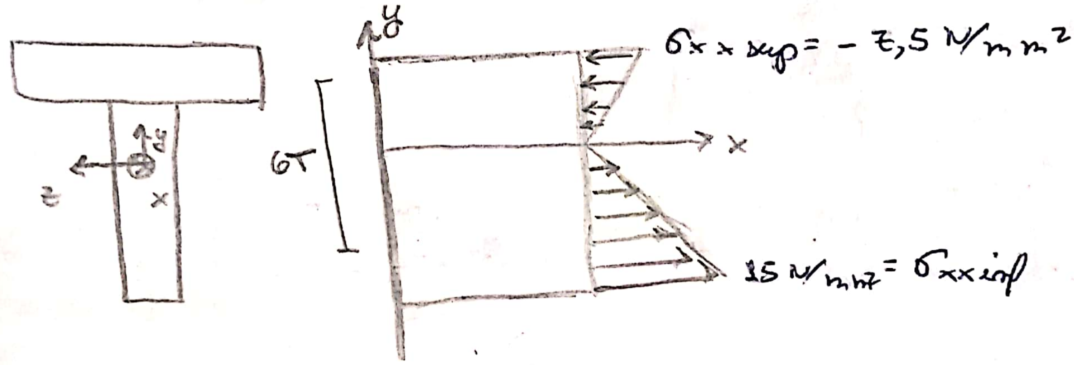
$$\sigma_{xx_{sup}} = -7{,}5\ N/mm^2$$

Para $y_{inferior} = -y_T' = -4T$

$$\sigma_{xx_{inf}} = -0{,}1187(-4 \cdot 31{,}58\,mm)$$

$$\sigma_{xx_{inf}} = 15\ N/mm^2$$

Assim:



3) Para a figura a):

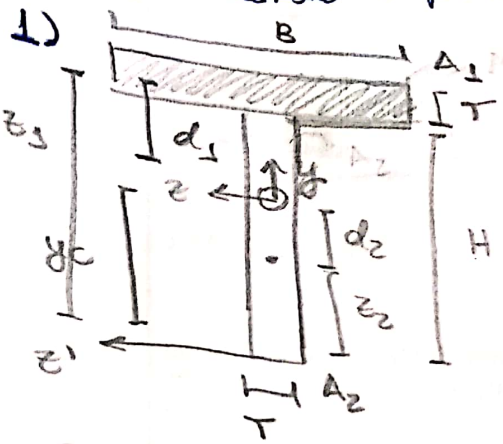
$$A_T = d^2 = (116{,}35\,mm)^2 = 13{,}537 \cdot 10^3\ mm^2$$

Para a figura b):

$$A_T' = (BT)2 = 2 \cdot 5T \cdot T = 10(31{,}58\,mm)^2 = 9{,}973 \cdot 10^3\ mm^2$$

Ex. Viga tensão e flexão 03. – 2° exercício:

1)



$B = 9T$ e $H = 11T$

Sejam:

$A_1 = BT = 9T^2$

$A_2 = HT = 11T^2$

Além disso:

$z_2 = H/2 = 11T/2$ $\quad z_1 = H + T/2 = 11T + T/2$

$z_1 = 23T/2$

Assim, sabe-se que:

$y_c(A_1 + A_2) = A_1 z_1 + z_2 A_2$

$y_c \, 20T^2 = 9T^2 \cdot \frac{23T}{2} + 11T^2 \cdot \frac{11T}{2}$

$y_c = \frac{328T^3}{40T^2} = \frac{82}{10}T = \frac{41}{5}T$

Assim, temos:

$d_1 = z_1 - y_c = \frac{23T}{2} - \frac{41T}{5} = \frac{33}{10}T$

$d_2 = z_2 - y_c = \frac{11T}{2} - \frac{41T}{5} = -\frac{27}{10}T$

Sabe-se que:

$I_{zz_1} = \frac{BT^3}{12} = \frac{9T^4}{12} = \frac{3T^4}{4}$ $\quad I_{zz_2} = \frac{TH^3}{12} = \frac{1331T^4}{12}$

Assim:

Assim:

$$I_{zz} = I_{zz_1} + A_1 d_1^2 + I_{zz_2} + A_2 d_2^2$$ ← pelo Teorema de Translação de eixos paralelos

$$I_{zz} = \frac{3T^4}{4} + 9T^2\left(\frac{33T}{10}\right)^2 + \frac{1331T^4}{12} + 11T^2\left(-\frac{27}{10}T\right)^2$$

$$I_{zz} = \frac{3T^4}{4} + \frac{9801T^4}{100} + \frac{1331T^4}{12} + \frac{8019T^4}{100} = \frac{34784T^4}{120} = \frac{4348T^4}{15}$$

Além disso,

$$y_{max} = y_c = \frac{41T}{5}$$

$$W_z = \frac{I_{zz}}{y_{max}} = \frac{\frac{4348T^4}{15}}{\frac{41T}{5}} = \frac{4348T^3}{123}$$

2) $T = 15mm$

$M_z = 1{,}5 \cdot 10^6 \, Nmm$

$\sigma_{xx}(y) = -\frac{M_z}{I_{zz}} y$

Como M_z e I_{zz} são constantes:

$|\sigma_{xx\,mín}| \rightarrow y_{min} = 0$

$|\sigma_{xx}|_{mín} = 0$

e $|\sigma_{xx\,máx}| \rightarrow |y_{max}| = \frac{41T}{5}$

$|\sigma_{xx\,max}| = \left| \frac{-M_z}{I_{zz}} y_{max} \right|$

seja $T = 15 \cdot 10^{-3} \, m$:

$I_{zz} = \frac{4348}{15} T^4 = \frac{4348 (15mm)^4}{15} = 14{,}674 \cdot 10^6 \, mm^4$

$y_{max} = -\frac{41T}{5} = -\frac{41(15mm)}{5} = -123 \, mm$

Assim:

$|\sigma_{xx\,max}| = \frac{(-1{,}5 \cdot 10^6 Nmm)(-123mm)}{(14{,}674 \cdot 10^6 mm^4)} = 12{,}573 \, N/mm^2$

Além disso, na extremidade superior:

$y_{superior} = T + H - y_c = 12T - \frac{41T}{5} = 19T/5$

temos:

$$\sigma_{xx\,superior} = \frac{+M_z}{I_{zz}} \cdot \frac{19T}{5} = -\frac{(1,5 \cdot 10^6 N\,mm)\, 19 \cdot (15\,mm)}{5 \cdot 14,674 \cdot 10^6 mm^4} = -5,826\ N/mm^2$$

sinal indica o sentido

Assim: