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1) $E = 80\ KN/mm^2$ $\nu = 0{,}25$ $u = \{3z \ \ 7y \ \ -2z\}^T$

$$\varepsilon = \frac{1}{2}\left(\frac{du_i}{dx_j} + \frac{du_j}{dx_i}\right) = \begin{bmatrix} 0 & 0 & 1{,}5 \\ 0 & 7 & 0 \\ 1{,}5 & 0 & -2 \end{bmatrix}$$

$u_1 = 3z$; $u_2 = 7y$; $u_3 = -2z$ e temos $x_1 = x$ $x_2 = y$ $x_3 = z$

$$G = \frac{E}{2(1+\nu)} = 32\ GPa \ ; \ C_{11} = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} = 96\ GPa$$

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{xz} \\ \sigma_{xy} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{12} & C_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{44} \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ 2\varepsilon_{yz} \\ 2\varepsilon_{xz} \\ 2\varepsilon_{xy} \end{bmatrix}$$

$$C_{12} = \frac{E\nu}{(1+\nu)(1-2\nu)} = 32\ GPa$$

$$C_{44} = \frac{1}{2}(C_{11} - C_{12}) = 32\ GPa$$

$$\sigma = \begin{bmatrix} 160 & 0 & 96 \\ 0 & 608 & 0 \\ 96 & 0 & 32 \end{bmatrix} 10^6\ Pa$$

$t = \sigma n = \{160 \ \ 0 \ \ 96\}^T\ 10^6\ Pa$

$t_n = \{t \cdot n\}[n] = \{160 \ \ 0 \ \ 0\}^T\ 10^6\ Pa$

$F_n = t_n . A = \{80 \ \ 0 \ \ 0\}^T\ 10^6\ N$

2) $E = 210\ GPa \quad G = 80\ GPa$

$$[\sigma] = \begin{bmatrix} 100 & 30 & -60 \\ 30 & -50 & 0 \\ -60 & 0 & -120 \end{bmatrix} MPa$$

$$E = 2G(1+\nu) \Rightarrow \nu = \frac{E}{2G} - 1 = \frac{210\,10^9}{160\,10^9} - 1 \Rightarrow \nu = 0,3125$$

$$\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{yz} \\ \varepsilon_{xz} \\ \varepsilon_{xy} \end{bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ -\nu & 1 & -\nu & 0 & 0 & 0 \\ -\nu & -\nu & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1+\nu & 0 & 0 \\ 0 & 0 & 0 & 0 & 1+\nu & 0 \\ 0 & 0 & 0 & 0 & 0 & 1+\nu \end{bmatrix} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{xz} \\ \sigma_{xy} \end{bmatrix}$$

$$\varepsilon_{xx} = \frac{1}{E}(\sigma_{xx} - \nu\sigma_{yy} - \nu\sigma_{zz}) \Rightarrow \varepsilon_{xx} = 7,29 \cdot 10^{-4}$$

$$\varepsilon_{yy} = \frac{1}{E}(-\nu\sigma_{xx} + \sigma_{yy} - \nu\sigma_{zz}) \Rightarrow \varepsilon_{yy} = -2,083\ 10^{-4}$$

$$\varepsilon_{zz} = \frac{1}{E}(-\nu\sigma_{xx} - \nu\sigma_{yy} + \sigma_{zz}) \Rightarrow \varepsilon_{zz} = -6,46 \cdot 10^{-4}$$

$$\varepsilon_{yz} = \frac{1+\nu}{E}\sigma_{yz} \Rightarrow \varepsilon_{yz} = 0 \; ; \; \varepsilon_{xz} = \frac{1+\nu}{E}\sigma_{xz} \Rightarrow \varepsilon_{xz} = -3,75 \cdot 10^{-4}$$

$$\varepsilon_{xy} = \frac{1+\nu}{E}\sigma_{xy} \Rightarrow \varepsilon_{xy} = 1,875\ 10^{-4}$$

$$\varepsilon = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix} = 10^{-4} \begin{bmatrix} 7,3 & 1,9 & -3,8 \\ 1,9 & -2,1 & 0 \\ -3,8 & 0 & -6,5 \end{bmatrix}$$

3) $E = 71 GPa$ $\nu = 0,27$ $\alpha = 22,5 \cdot 10^{-6}$ $L_x = 15 cm$
$L_y = 12 mm$

$$\varepsilon_{xx} = \frac{1}{E}\sigma_{xx} - \frac{\nu}{E}\sigma_{yy} - \frac{\nu}{E}\sigma_{zz} + \varepsilon_T$$

$\varepsilon_T = \alpha \Delta T$ $\sigma_{yy} = \sigma_{zz} = 0$ e $\varepsilon_{xx} = 0$

$$0 = \frac{1}{E}\sigma_{xx} - \frac{\nu}{E}0 - \frac{\nu}{E}.0 + \alpha\Delta T \Rightarrow \sigma_{xx} = -E\alpha\Delta T$$

$$\sigma_{xx} = -71 \cdot 10^{9} \cdot 22,5 \cdot 10^{-6} \cdot 50 = -79,875 MPa$$

$$F_x = \sigma_{xx} A_x = -79,875 \cdot 10^{6} \cdot 0,012 \cdot 0,012 = -11,5 \cdot 10^{3} N$$

$$\varepsilon_{yy} = -\frac{\nu}{E}\sigma_{xx} + \frac{1}{E}\sigma_{yy} - \frac{\nu}{E}\sigma_{zz} + \varepsilon_T = 1,43 \cdot 10^{-3}$$

$$\varepsilon_{zz} = -\frac{\nu}{E}\sigma_{xx} - \frac{\nu}{E}\sigma_{yy} + \frac{1}{E}\sigma_{zz} + \varepsilon_T = 1,43 \cdot 10^{-3}$$

$$\Delta L_y = \varepsilon_{yy} L_y = 1,43 \cdot 10^{-3}\ 0,012 = 1,72 \cdot 10^{-5} m$$

$$\Delta L_z = \varepsilon_{zz} L_z = 1,43 \cdot 10^{-3}\ 0,012 = 1,72 \cdot 10^{-5} m$$

4) $L_x = 500\,mm$ $L_y = 80\,mm$ $L_z = 100\,mm$ $F_x = 224\,kN$
$F_y = 22004\,kN$, $E = 71\,GPa$ $G = 26\,GPa$

a) $\sigma_{xx} = \frac{F_x}{A_{yz}} = \frac{224\,10^3}{80.100.10^{-6}} = 28\,MPa$ $\sigma_{yy} = \frac{F_y}{A_{xz}} = 44\,MPa$

$$[\sigma] = \begin{bmatrix} 28 & 0 & 0 \\ 0 & 44 & 0 \\ 0 & 0 & 0 \end{bmatrix} MPa$$

b) $E = 2G(1+\nu) \Rightarrow \frac{E}{2G} = 1+\nu \Rightarrow \nu = \frac{E}{2G} - 1 = \frac{71}{2.26} - 1 = \boxed{0{,}37}$

c) $\varepsilon_{xx} = (\sigma_{xx} - \nu\sigma_{yy} - \nu\sigma_{zz})/E \Rightarrow \varepsilon_{xx} = 1{,}65.10^{-4}$
$\varepsilon_{yy} = (-\nu\sigma_{xx} + \sigma_{yy} - \nu\sigma_{zz})/E \Rightarrow \varepsilon_{yy} = 4{,}74.10^{-4}$
$\varepsilon_{zz} = (-\nu\sigma_{xx} - \nu\sigma_{yy} - \sigma_{zz})/E \Rightarrow \varepsilon_{zz} = -3{,}75\,10^{-4}$
$\varepsilon_{yz} = (1+\nu)\sigma_{yz}/E = 0$
$\varepsilon_{xy} = \varepsilon_{xz} = 0$

$$\varepsilon = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix} = 10^{-4} \begin{bmatrix} 1{,}65 & 0 & 0 \\ 0 & 4{,}74 & 0 \\ 0 & 0 & -3{,}75 \end{bmatrix}$$

d) $\Delta L_x = \varepsilon_{xx} L_x = 1{,}65.10^{-4}.\,500.10^{-3} = 8{,}25.10^{-5}\,m$

$\Delta L_y = \varepsilon_{yy} L_y = 4{,}74\,10^{-4}.\,80\,10^{-3} = 3{,}79\,10^{-5}\,m$

$\Delta L_z = \varepsilon_{zz} L_z = -3{,}75\,10^{-4}.\,100.10^{-3} = -3{,}75\,10^{-5}\,m$

5) $\nu = 0{,}3 \quad E = 21 \cdot 10^3 \, N/mm^2$

a) $E(2,1)$; $F(3{,}08,\ 1{,}07)$; $G(3{,}17,\ 2{,}19)$; $H(2{,}09,\ 2{,}12)$

b) $\varepsilon_{xx} = \dfrac{\Delta u}{\Delta x}$; $\Delta u = EF_x - AB_x = (3{,}08-2)-1 = 0{,}08$

$\varepsilon_{xx} = \dfrac{0{,}08}{1} = 0{,}08$

$\varepsilon_{yy} = \dfrac{\Delta v}{\Delta y}$; $\Delta v = EH_y - AD_y = (2{,}12-1)-1 = 0{,}12$

$\tan \alpha_1 \doteq \alpha_1 = \dfrac{EF_y}{EF_x} = \dfrac{0{,}07}{1{,}08} = 0{,}065^\circ$

$\tan \alpha_2 \doteq \alpha_2 = \dfrac{EH_x}{EH_y} = \dfrac{0{,}09}{1{,}12} = 0{,}08$ $\quad [\varepsilon] = \begin{bmatrix} 0{,}08 & 0{,}073 \\ 0{,}073 & 0{,}12 \end{bmatrix}$

$\varepsilon_{xy} = \varepsilon_{yx} = \dfrac{1}{2}(\alpha_1 + \alpha_2) = 0{,}073$

c) $\sigma_{xx} = \dfrac{1+\nu}{1-2\nu}\varepsilon_{xx} + \dfrac{\nu}{1-2\nu}\varepsilon_{yy} + \dfrac{\nu}{1-\nu}\cancel{\varepsilon_{zz}}^{\,0} = 0{,}14 + 0{,}09 = 0{,}023$

$\sigma_{yy} = \dfrac{\nu}{1-2\nu}\varepsilon_{xx} + \dfrac{1+\nu}{1-2\nu}\varepsilon_{yy} = 0{,}06 + 0{,}21 = 0{,}27$

$\sigma_{xy} = \sigma_{yx} = \varepsilon_{xy} = \varepsilon_{yx} = 0{,}073 \quad [\sigma] = \begin{bmatrix} 0{,}23 & 0{,}073 \\ 0{,}073 & 0{,}27 \end{bmatrix} N/mm^2$

d) $[\sigma]\{n\} = \{t\}$

$$\begin{bmatrix} 0{,}23 & 0 & 0 \\ 0{,}073 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{Bmatrix} = \begin{Bmatrix} t_x \\ t_y \\ t_z \end{Bmatrix} \rightarrow \begin{Bmatrix} t_x \\ t_y \\ t_z \end{Bmatrix} = \begin{Bmatrix} 0{,}175 \\ 0{,}202 \\ 0 \end{Bmatrix} N/mm^2$$