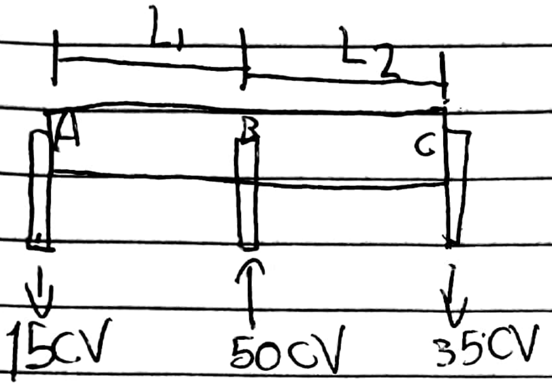
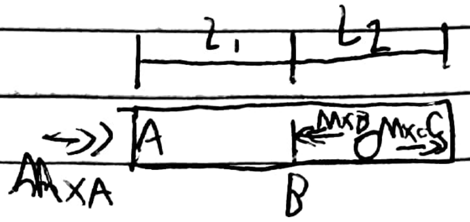
Lista 11

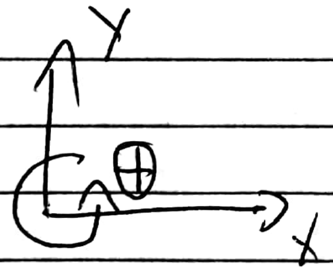


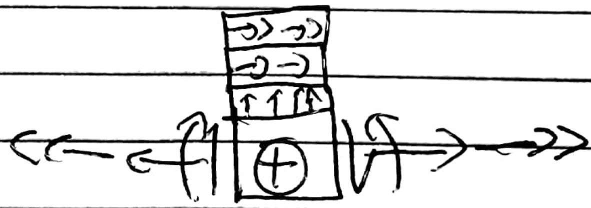


$L_1 = 600\ mm$ $L_2 = 800\ mm$ $G = 85\ GPa$

$N_m = 900\ RPM$ $\frac{d_I}{d_E} = 0{,}9 \rightarrow d_I = 0{,}9\, d_E$

$r_I = 0{,}9\, r_E$





1) $\tau_{máx} \leq 60\ N/mm^2$ 2) $\phi_{AC} \leq \frac{1}{500\pi}$ rad

3) Diagrama de momento torsor e $\phi(x)$

4) $\tau_{máx}$ e

$7023{,}5\, W\ [CV] = M_x\ [N.m] \cdot N_M\ [RPM]$

$$M_x\ (N.m) = \frac{7023{,}5\, W_{cv}\ (CV)}{N_M}$$

$W_A(CV) = +15\,CV \rightarrow M_{xA}(N.mm) = 117{,}1 \cdot 10^3\ N.mm$
$W_B(CV) = -50\,CV \rightarrow M_{xB}(N.mm) = -390{,}2 \cdot 10^3\ N.mm$
$W_C(CV) = +35\,CV \rightarrow M_{xC}(N.mm) = 273{,}1 \cdot 10^3\ N.mm$

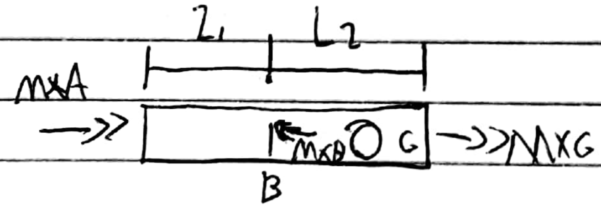
Para equilíbrio: $\sum M_x = 0 \rightarrow M_{xA} + M_{xB} + M_{xC} = 0$

$117{,}1 \cdot 10^3 - 390{,}2 \cdot 10^3 + 273{,}1 \cdot 10^3 = 0 \rightarrow$ Equilíbrio

Eq. Diferencial: $J_p \cdot G \dfrac{d^2\phi(x)}{dx^2} = \dfrac{dM_x(x)}{dx} = -t(x)$

$t(x) = -M_{xB} \langle x - L_1 \rangle^{-1}$. Velocidade angular constante

Condição de contorno:



$\phi(x=0) = 0$
$M(x = L_1 + L_2) = +M_{xC}$

$$\frac{dM_x(x)}{dx} = J_pG \frac{d^2\phi(x)}{dx^2} = -t(x) = +M_{xB} \langle x - L_1 \rangle^{-1}$$

$$J_pG \frac{d\phi(x)}{dx} = M_x(x) = M_{xB} \langle x - L_1 \rangle^0 + C_1$$

$$J_pG\, \phi(x) = M_{xB} \langle x - L_1 \rangle^1 + C_1 x + C_2$$

$$J_pG\,\phi(0) = 0 \rightarrow C_2 = 0 \rightarrow M_x(L_1 + L_2) = M_{xC}$$

$M_X(L_1+L_2) = M_{XB}\langle L_1+L_2-L_1\rangle^0 + C_1 = M_{XC}$

$C_1 = -M_{XB} + M_{XC}$

Momento Torsor: $M_X(X) = M_{XB}\langle X-L_1\rangle^0 - M_{XB} + M_{XC}$

$0 < X < L_1$: $M_X(X) = -M_{XB} + M_{XB}$

$L_1 < X < L_2 + L_1$: $M_X(X) = +M_{XB}\langle X-L_1\rangle^0 - M_{XB} + M_{XC}$

$$M_X(X) = +M_{XC}$$

Ângulo de torção relativo:

$J_pG\phi(X) = +M_{XB}\langle X-L_1\rangle^1 + (-M_{XB} + M_{XC})X$

Para $0 < X < L_1$; $J_pG\phi(X) = (-M_{XB} + M_{XC})X$

$$\phi(X) = \frac{(-M_{XB} + M_{XC})X}{J_pG}$$

Para $L_1 < X < L_2 + L_1$

~~$J_pG\phi(X) = (-M_{XB}L_1 + M_{XC})X$~~ ~~$\phi(X) = \frac{(-M_{XB}L_1 + M_{XC})X}{J_pG}$~~

$J_pG\phi(X) = -M_{XB}L_1 + M_{XC}X$

$$\phi = \frac{-M_{XB}L_1 + M_{XC}X}{J_pG}$$

Reação de Apoio: assumiu-se que não há rotação em $X=0$. Então, para calcular M_{XA}.

$\phi(0) = 0$

O valor de M_{xA} deve coincidir com o valor do momento associado da potência de 15CV e 900 rpm:

$M_x(X=0) = -M_{xB} + M_{xC} = -390,2 + 273,1$ [N.m]

$M_x(X=0) = -117,1$ N.m → Convenção de Resmat, sentido correto

$\tau_{máx} = 60/mm^2$; $G = 85$ GPa e $r_I = 0,9 r_E$

Para AB: $0 \leq X \leq L_1$ momento $= -117,1 \cdot 10^3$ N mm

Para BC: $0 < X \leq L_2 + L_1$ momento $= 273,1 \cdot 10^3$ N. mm

Momento máximo: $M_{x\,máx} = M_x(L_1 \leq X \leq L_1 + L_2) = 273,1 \cdot 10^3$ Nmm

$$\tau(r) = \frac{M_x(X)}{J_p(X)} r \rightarrow \cancel{J_p(X) = \frac{\pi}{2}(d_E^4 - d_i^4) \rightarrow d_i = 0,9 d_E}$$

$$J_p(X) = \frac{\pi}{2}(r_E^4 - r_I^4) = \frac{\pi}{2}(r_E^4 - (0,9 r_E)^4)$$

$$\cancel{J_p(X) = \frac{\pi}{32}}$$

$$J_p(X) = \frac{\pi}{2}(r_E^4 - 0,6561 r_E^4)$$

$$\tau_{máx} = \frac{M_{x\,máx}}{J_p(X)} \cdot r_E \rightarrow 60 = \frac{273,1 \cdot 10^3 \cdot r_E}{(\pi/2) \cdot 0,3439\, r_E^4}$$

$r_E = 20,35$ mm → $d_E = 2 r_E = 40,70$ mm

$r_I = 0,9 \cdot 20,35$ mm → $d_I = 2 r_I = 36,63$ mm

$r_I = 18,32$ mm

Também levando $\phi_{máx}$ nos cálculos: $\phi_{máx} = \frac{\pi}{500}$ rad

Calculando J_p → $J_p(X) = \frac{\pi}{2}((20,35)^4 - (18,32)^4) = 92449,3$ rad

$$\phi = \frac{-390,2 \cdot 10^3 \cdot 600 + 273,1 \cdot 10^3 \cdot 1400}{85000 \cdot 92449,3} = 0,01888 \text{ rad}$$

No nosso problema $\rightarrow$ $\phi_{máx} \leq \frac{\pi}{500}$ rad; $\phi_{máx} \leq 6{,}3 \cdot 10^{-3}$ rad

J_p para $\pi/500 = \phi$

$$\phi_{máx} = \frac{(-390{,}2 \cdot 10^3 \cdot 600 + 273{,}1 \cdot 10^3 \cdot 1400)}{J_p \cdot 85000} = \frac{\pi}{500}$$

$J_p = 277{,}53 \cdot 10^3$ rad

$$J_p = \frac{\pi}{2}\left(r_E^4 - (0{,}9 r_E)^4\right) \rightarrow \frac{J_p \cdot 2}{\pi} = r_E^4 - 0{,}6561 r_E^4$$

$$r_E^4 = \frac{J_p 2}{\pi(1 - 0{,}6561)} \Rightarrow r_E = 26{,}83 \text{ mm}$$

$d_E = 53{,}66$ mm

$\tau(r_E) < \tau_{máx}$? $\quad \tau(r = r_E) = \frac{M_{x\,máx}}{J_p(x)} \cdot r_E$

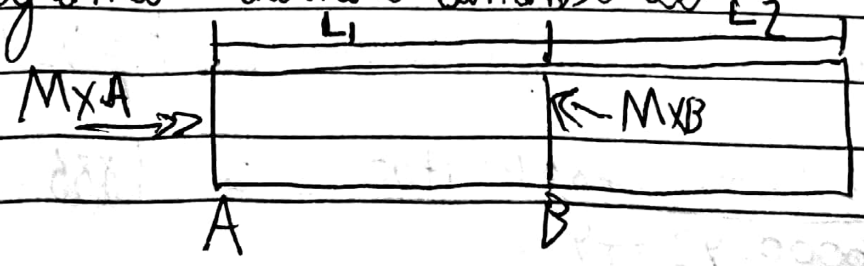
$$\tau(r_E) = \frac{273{,}1 \cdot 10^3 \cdot 26{,}83}{277{,}53 \cdot 10^3} = 26{,}40 \text{ N/mm}^2$$

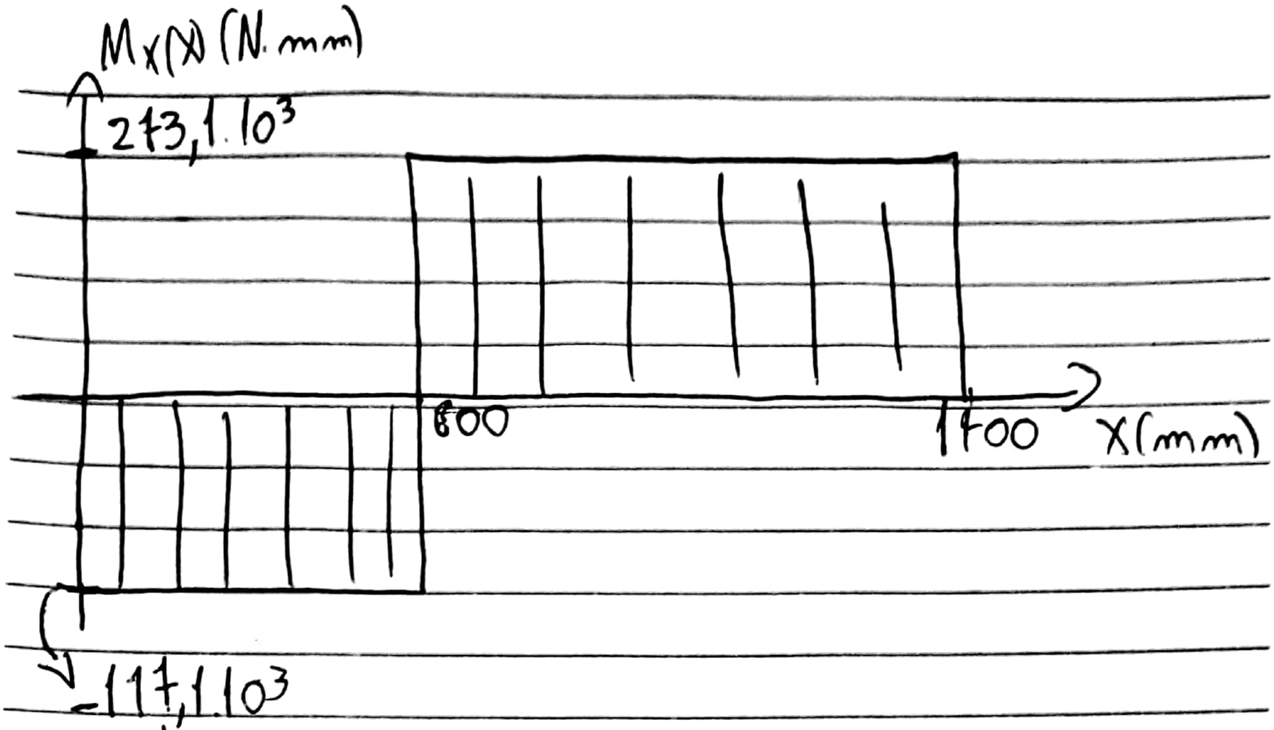
$\tau(r_E) < \tau_{máx}$: pode-se adotar esse valor.

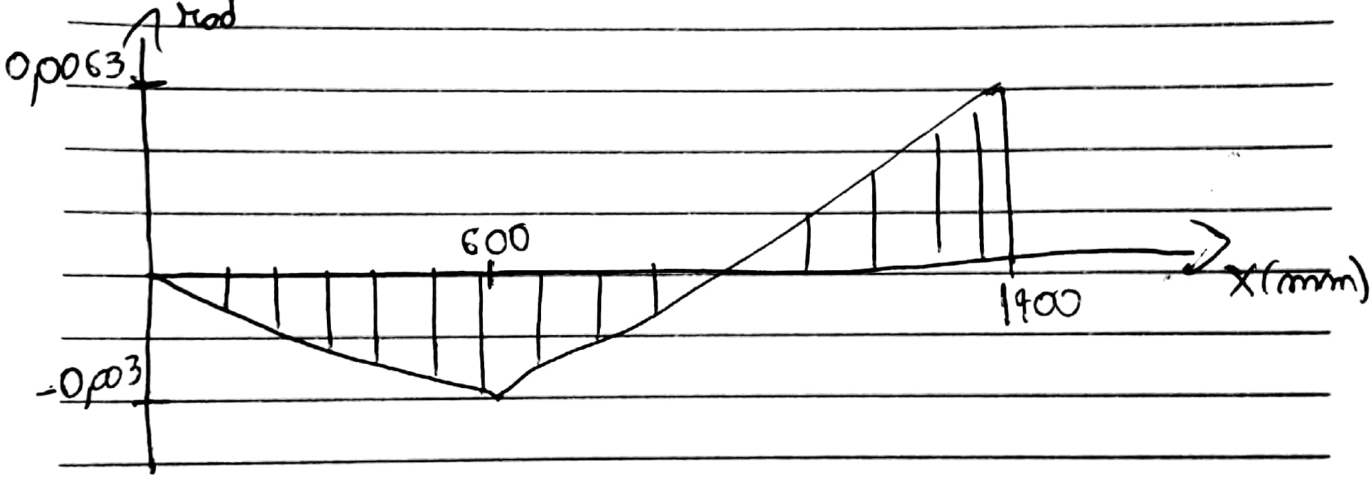
$d_E = 53{,}66$ mm $\rightarrow$ $d_I = 0{,}9\, d_E = 48{,}30$ mm

$r_I = 0{,}9\, r_E = 0{,}9 \cdot 26{,}83 = 24{,}15$ mm

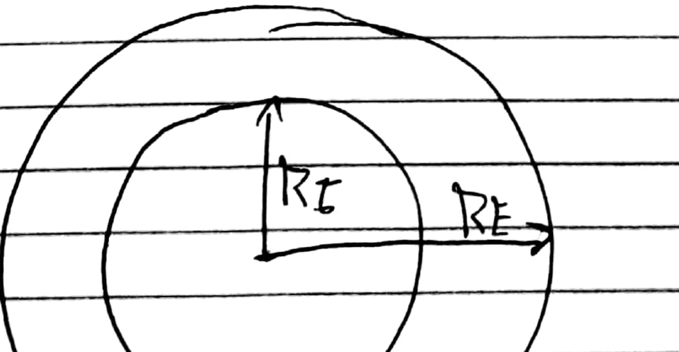
Diagrama diametro dimensionado







$\tau_{máx}$



$\tau(R_E) = \tau_{máx} = 26,40\ N/mm^2$