Victor Paganotto RA: 206586 EM423 B 2 sem 2020 Lista 10

2) a)

$$G J_p \frac{d^2\phi(x)}{dx^2} = -t(x) \quad ; \quad t(x) = M_{xB} \langle x - L/2 \rangle^{-1}$$

$$\phi(0) = 0 \quad \phi(L) = 0$$

$$G J_p \frac{d^2\phi(x)}{dx^2} = -M_{xB} \langle x - \tfrac{L}{2} \rangle^{-1}$$

$$G J_p \frac{d\phi(x)}{dx} = -M_{xB} \langle x - L/2 \rangle^{0} + C_1$$

$$G J_p \, \phi(x) = -M_{xB} \langle x - L/2 \rangle^{1} + C_1 x + C_2$$

$$\phi(0) = 0 \Rightarrow C_2 = 0$$

$$\phi(L) = 0 \Rightarrow C_1 = M_{xB}/2$$

$$K_B = \frac{M_x(L/2)}{\phi(L/2)} = \frac{M_{xB}/2}{M_{xB}\, L/4 G J_p} = \frac{2 G J_p}{L} \Rightarrow K_B = 3{,}56 \cdot 10^{2}\ Nm$$

$$J_p = \left(\frac{\pi}{2}\right)\left(\frac{d}{2}\right)^4 = 1{,}27 \cdot 10^{-6}\ m^4$$

$$M_{B2} = 2 M_B = 7{,}12 \cdot 10^{2}\ Nm \quad ; \quad A_1 = \frac{A_1}{2} = 1{,}414 \cdot 10^{-3}\ m^2$$

$$A_1 = \pi \cdot \left(\frac{d_1}{2}\right)^2 \Rightarrow d_1 = 42{,}43\ mm$$

$$J_{p2} = \frac{\pi}{2}\left[\left(\frac{d_2}{2}\right)^4 - \left(\frac{d_1}{2}\right)^4\right] = \frac{M_{B2} L}{2G} = 7{,}54 \cdot 10^{-6}$$

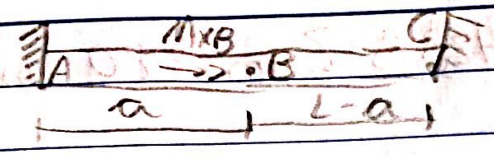
$$\left(\frac{d_2}{2}\right)^4 - \left(\frac{d_1}{2}\right)^4 = 4{,}619 \cdot 10^{-6} \rightarrow d_2 = 84{,}13\ mm$$

b) $\tau_{MAX} = \frac{Mx_{MAX}}{J_{pmin}} \cdot \frac{d}{2}$

$\tau_{MAX} = \frac{5 \cdot 10^3}{1,27 \cdot 10^{-6}} \cdot 30 \cdot 10^{-3} = 118,11 MPa$

Para o caso vazado:

$\tau_{MAX} = \frac{Mx_{MAX}}{J_{pmin}} \cdot \frac{de}{2} = \frac{5 \cdot 10^3}{2,54 \cdot 10^{-6}} \cdot 84,85 \cdot 10^{-3} = 167,03 MPa$

3) 

$GJ_p \frac{d^2\phi(x)}{dx^2} = -t(x)$ (1) ; $t(x) = M_{xB}\langle x-a\rangle^{-1}$ (2)

Substituindo (2) em (1) e integrando uma vez:

$GJ_p \frac{d\phi(x)}{dx} = -M_{xB}\langle x-a\rangle^0 + C_1 = M_x(x)$ $\Rightarrow$ Integrando novamente

(3)

$GJ_p\phi(x) = -M_{xB}\langle x-a\rangle^1 + C_1x + C_2$; $\phi(x=0)=0$ (4)
$\phi(x=L)=0$

Aplicando (4) em (3):

$GJ_p\phi(x=0) = -M_{xB}\langle \cancel{x-a}\rangle^1 + C_1 0 + C_2 = 0 \Rightarrow C_2 = 0$

$GJ_p\phi(x=L) = -M_{xB}\langle L-a\rangle^1 + C_1L \Rightarrow C_1 = \frac{M_{xB}(L-a)}{L}$

$M_x(x) = -M_{xB}\langle x-a\rangle^0 + \frac{M_{xB}(L-a)}{L}$

$GJ_p\phi(x) = -M_{xB}\langle x-a\rangle^1 + \frac{M_{xB}}{L}(L-a)x$

$K_{\phi a} = \frac{M_{xB}}{\phi(x=a)}$, em que $\phi(x=a) = \frac{1}{GJ_p}\left[\frac{M_{xB}(L-a)a}{L}\right]$

$K_{\phi a} = GJ_pL/(La-a^2)$, para $a = L/2 \Rightarrow K_\phi = 4GJ_p/L$,

$K_{\phi a} = \frac{GJ_pL}{La-a^2}$; $3K_\phi = 12\frac{GJ_p}{L} \Rightarrow \frac{GJ_pL}{La-a^2} = 12\frac{GJ_p}{L} \Rightarrow \frac{12a^2}{L} - 12a + L = 0$

$\Rightarrow \boxed{a = \frac{L}{2} \pm \frac{\sqrt{6}L}{6}}$