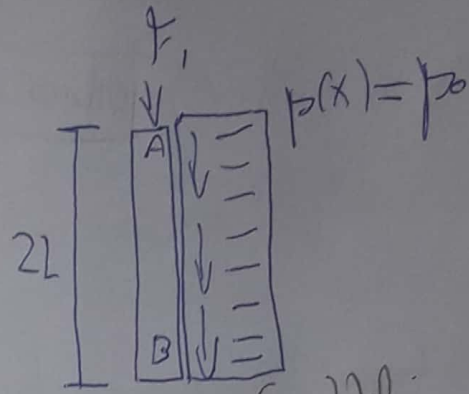


Ex - Equilif Equil - Axial - 01

Dados: $L = 2\text{ m}$
 $p_0 = 5000\text{ N/m}$
 $F_1 = 8000\text{ N}$



Eg. Dif.:

$$\frac{dN_x(x)}{dx} = -p(x)$$

$$p(x) = p_0$$

$$N_x(x=0) = -F_1$$

Eg. Dif.:

$$\frac{dM_x(x)}{dx} = -t(x)$$

$$t(x) = 0$$

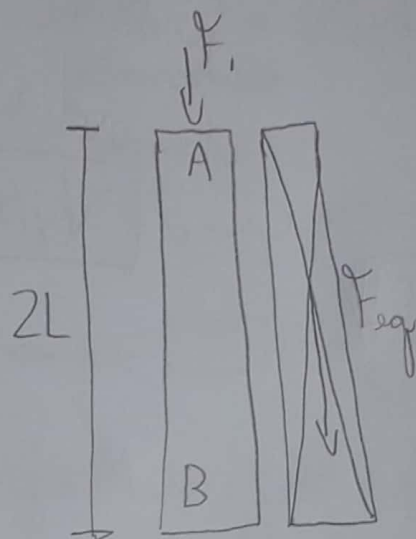
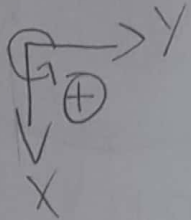
$$M_x(x=0) = 0$$

Eg. Dif.:

$$\frac{d^2 M_z(x)}{dx^2} = q(x)$$

$$q(x) = 0$$

$$\left. \frac{dM_z(x)}{dx} \right|_{x=0} = 0$$



$$N_x(x) = -\int p(x) dx + C_p = -\int_0^x p_0 dx + C_p$$

$$N_x(x) = -x \cdot p_0 + C_p \rightarrow N_x(x=0) = -F_1$$

$$N_x(x=0) = -8000\text{ N}$$

$$-F_1 = -0 \cdot p_0 + C_p \rightarrow C_p = -F_1$$

$$C_p = -8000\text{ N}$$

$$N_x(x=4) = -(4) \cdot 5000 - 8000$$

$$N_x(x=4) = -2800\text{ N}$$

$$N_x(x) = -x \cdot 5000 - 8000$$

$$M_x(x) = - \int t(x) \cdot dx + C_1 \rightarrow M_x(x) = - \int 0 \cdot dx + C_1$$

$$M_x(x) = C_1 \rightarrow M_x(0) = \boxed{0 \text{ N.m} = C_1} \quad \boxed{M_x(x) = 0 \text{ N}}$$

Eg. 2. Dif.: $\frac{dV_y(x)}{dx} = q(x) \rightarrow q(x) = 0 \rightarrow V_y(x) = 0 = \frac{dM_z(x)}{dx}$

$$V_y(x) = \int q(x) \cdot dx + C_1 \rightarrow V_y(x) = \int 0 \cdot dx + C_1 \rightarrow V_y(x) = C_1$$

$$V_y(x=0) = 0 \rightarrow V_y(x=0) = 0 = C_1 \rightarrow \boxed{V_y(x) = 0 \text{ N}}$$

$$\boxed{C_1 = 0 \text{ N}}$$

$$M_z(x) = \int \left[\int q(x) \cdot dx \right] dx + C_1 \cdot x + C_2 \rightarrow M_z(x) = C_2$$

$$M_z(x=0) = 0 = C_2 \rightarrow \boxed{M_z(x) = 0 \text{ N}}$$

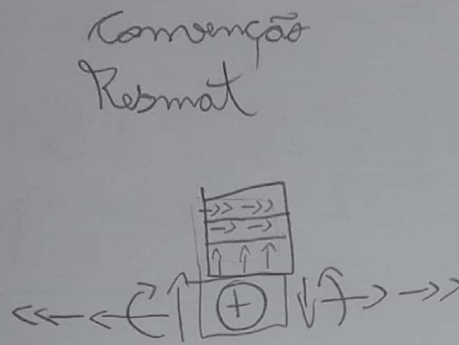
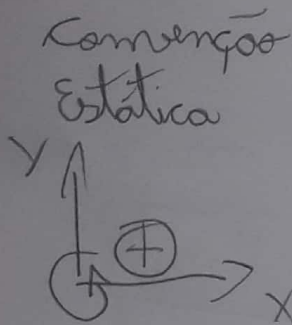
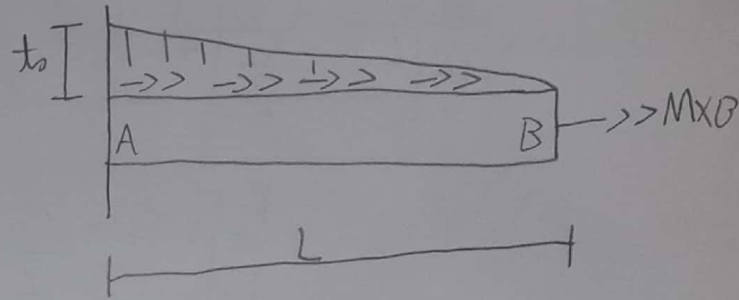
$$\boxed{C_2 = 0 \text{ N.m}}$$

Ex - Eq Dif Equil - Torção - 02

Dados: $M_{XB} = 1000 \text{ N.m}$

$t_0 = 500 \text{ N.m/m}$

$L = 2 \text{ m}$



$$\frac{d^2 M_z(x)}{dx^2} = q(x) \rightarrow q(x) = 0 \rightarrow \frac{dM_z(x)}{dx} \Big|_{x=0} = 0$$

$$\frac{dM_z(x)}{dx} = V_y(x) \rightarrow V_y(x=0) = 0$$

$$V_y = \int q(x) \cdot dx + C_1 = C_1 \rightarrow C_1 = V_y(x=0) = 0$$

$C_1 = 0 \text{ N}$ $V_y(x) = 0 \text{ N}$

$$M_z(x) = \int \left[\int q(x) \cdot dx \right] \cdot dx + C_1 \cdot x + C_2 \rightarrow M_z(x) = C_2$$

$$M_z(x=0) = 0 \rightarrow M_z(x=0) = C_2 = 0 \text{ N.m} \rightarrow M_z(x) = 0 \text{ N.m}$$

$$\frac{dM_x(x)}{dx} = -t(x) \rightarrow \frac{t_0 \cdot (L-x)}{L} = t(x) \Rightarrow t(x) = \frac{t_0 \cdot L - t_0 \cdot x}{L}$$

$$M_x(x=L) = M_{XB} \rightarrow M_x(x) = - \int t(x) \cdot dx + C_t$$

Eq. Dif

$$\frac{dN_x(x)}{dx} = -p(x)$$

$$p(x) = 0$$

$$N_x(x=0) = 0$$

$$N_x(x) = - \int p(x) \cdot dx + C_p$$

$$N_x(x) = +C_p$$

$$N_x(x=0) = C_p = 0 \text{ N}$$

$N_x(x) = 0 \text{ N}$

$$M_x(X) = - \int \left(\frac{t_0(L-X)}{L} \right) dX + Cx = - \frac{t_0}{L} \int (L-X) dX + Cx$$

$$M_x(X) = - \frac{t_0}{L} \left[L \cdot X - \frac{X^2}{2} \right] + Cx \rightarrow M_x(X=L) = M_{XB}$$

$$M_x(X=L) = M_{XB} = - \frac{t_0}{L} \left[L \cdot (L) - \frac{(L)^2}{2} \right] + Cx \rightarrow M_{XB} = - \frac{t_0 \cdot L^2}{L \cdot 2} + Cx$$

$$Cx = M_{XB} + \frac{t_0 \cdot L^2}{2} = 1000 + \frac{500(2)^2}{2} = 1500 \text{ N.m}$$

$$\boxed{Cx = 1500 \text{ N.m}}$$

$$M_x(X) = - \frac{t_0}{L} \left[L \cdot X - \frac{X^2}{2} \right] + Cx \rightarrow M_x(X=0) = Cx \rightarrow \boxed{M_x(X=0) = 1500 \text{ N.m}}$$

$$M_x(X=L) = - \frac{t_0}{L} \left[L \cdot X - \frac{X^2}{2} \right] + Cx = - \frac{t_0}{L} \left[L \cdot L - \frac{(L)^2}{2} \right] + Cx$$

$$M_x(X=L) = - \frac{t_0}{L} \left[\frac{L^2}{2} \right] + Cx = - 500 \left[\frac{2^2}{2} \right] + 2000 = 1000$$

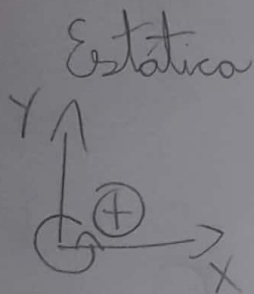
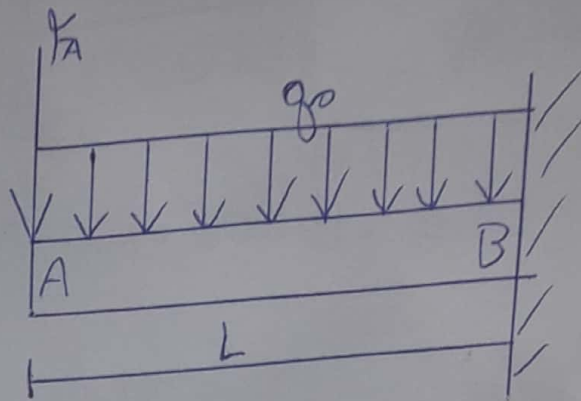
$$\boxed{M_x(X=L) = 1000 \text{ N.m}}$$

Ex - E) 2º Equil - Flexão - 01

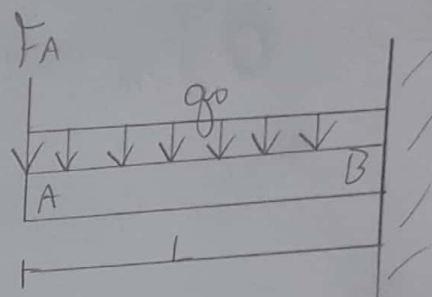
Dados: $L = 3\text{ m}$

$$F_A = 15000\text{ N}$$

$$q_0 = 8000\text{ N/m}$$



Resistência
dos Materiais



$$\frac{dN_x(x)}{dx} = -p(x) \rightarrow p(x) = 0 \rightarrow N_x(x=0) = 0\text{ N}$$

$$N_x(x) = -\int p(x) \cdot dx + C_p = C_p \rightarrow N_x(x) = C_p \rightarrow N_x(x=0) = 0 = C_p$$

$$C_p = 0\text{ N} \rightarrow \boxed{N_x(x) = 0\text{ N}}$$

$$\frac{dM_x}{dx} = -t(x) \rightarrow t(x) = 0 \rightarrow M_x(x=0) = 0\text{ N.m}$$

$$M_x(x) = -\int t(x) \cdot dx + C_t = C_t \rightarrow M_x(x) = C_t \rightarrow M_x(x=0) = 0 = C_t$$

$$C_t = 0\text{ N.m} \rightarrow \boxed{M_x(x) = 0\text{ N.m}}$$

$$\frac{dV_y(x)}{dx} = q(x) \rightarrow q(x) = q_0 \rightarrow V_y(x=0) = -F_A$$

$$V_y(x) = \int q(x) \cdot dx + C_1 = q_0 \cdot x + C_1 \rightarrow V_y(x=0) = q_0 \cdot (0) + C_1$$

$$C_1 = -F_A \rightarrow \boxed{C_1 = -15000\text{ N}} \quad \boxed{V_y(x) = -8000 \cdot x - 15000\text{ N}}$$

$$V_y(X=0) = -15000 \text{ N}$$

$$V_y(X=L) = -15000 - 8000 \cdot 3 = -39000 \text{ N} = V_y(X=L)$$

$$\frac{dM_z(X)}{dx} = V_y(X) \rightarrow V_y(X) = -15000 - 8000X = -F_A - q_0 \cdot X$$

$$M_z(X=0) = 0 \text{ N.m} \rightarrow M_z(X) = \int V_y(X) dX + C_2$$

$$M_z(X) = -F_A \cdot X - q_0 \cdot \frac{X^2}{2} + C_2 \Rightarrow M_z(X=0) = C_2 = 0$$

$$C_2 = 0 \rightarrow C_2 = 0 \rightarrow C_2 = 0 \text{ N.m}$$

$$M_z(X) = -F_A \cdot X - q_0 \cdot \frac{X^2}{2} + C_2 = -15000 \cdot X - 4000X^2 - 0$$

$$M_z(X=0) = 0 \text{ N.m}$$

$$M_z(X=L) = -15000 \cdot (3) - \frac{8000 \cdot 3^2}{2}$$

$$M_z(X=L) = -81000 \text{ N.m}$$