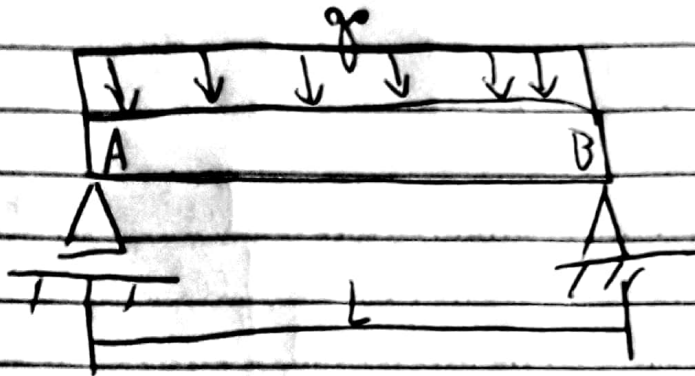


Eng. R. Oliva

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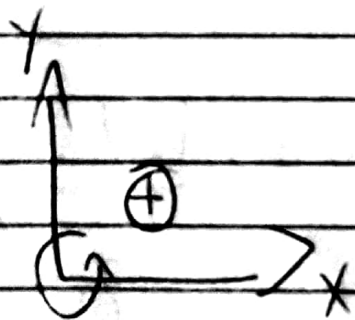
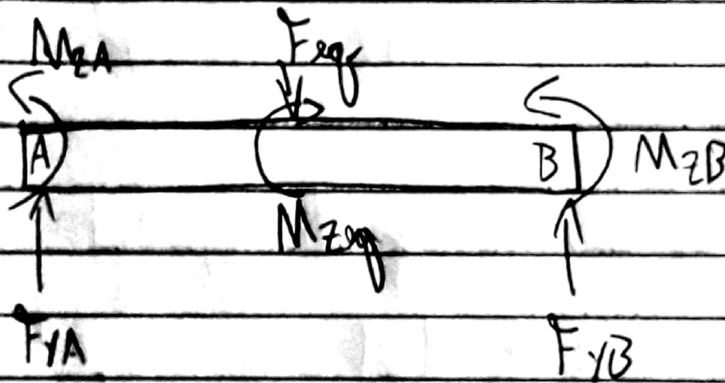
# Exercício de vigas - torção - 01



$$L = 3$$

$$q_0 = 3500 \text{ N/m}$$

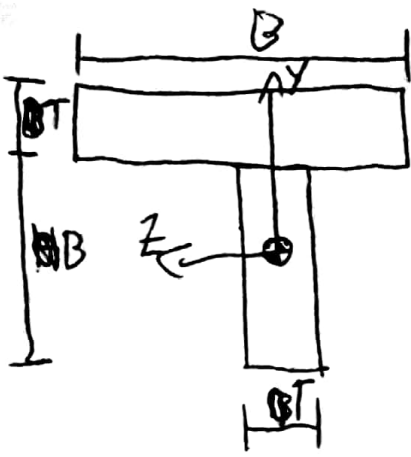
$$\sigma_{xx \text{ max}} = 15 \text{ N/mm}^2$$



$$\sum M_Z = 0 \rightarrow M_{ZA} - M_{Zeq} = 0 \rightarrow F_{YA}L - q_0L \frac{L}{2} = 0$$

$$F_{YA} \cdot L = q_0 \frac{L^2}{2} \rightarrow F_{YA} = \frac{q_0 L}{2} = \frac{3500 \cdot 3}{2} =$$

$$F_{YA} = 5250 \text{ N}$$



$$J_2 = B + \frac{I}{2}$$

$$J_1 = \frac{B}{2}$$

$$B = 5T$$

$$J_1 = \frac{5T}{2}$$

$$J_2 = 5T + \frac{T}{2}$$

$$J_2 = 10T + \frac{T}{2}$$

$$J_2 = \frac{11T}{2}$$

$$A_{p1} = A_{p2} = B \cdot T = 5T \cdot T = 5T^2$$

$$A = A_{p2} + A_{p1} = 10T^2$$

$$W_z = \frac{I_{zz}}{y_{max}}$$

$$W_z = \frac{M_{z_{max}}}{\sigma_{x_{max}}}$$

$$\bar{Y} \cdot A = A_{p1} \cdot \bar{d}_1 + A_{p2} \cdot \bar{d}_2 \rightarrow \bar{Y} \cdot 10T^2 = 5T^2 \cdot \frac{5T}{2} + 5T^2 \cdot \frac{11T}{2}$$

$$\bar{Y} \cdot 10T^2 = \frac{25T^3}{2} + \frac{55T^3}{2} = \bar{Y} = 4T \rightarrow y_{max}$$

$$d_1 = \left(5T + \frac{T}{2}\right) - \bar{Y} = \frac{11T}{2} - 4T = \frac{3T}{2} \quad ; \quad d_2 = \frac{5T}{2} - 4T = -\frac{3T}{2}$$

$$I_{zz} = \frac{B \cdot (T)^3}{12} + (5T)(T) \cdot d_1^2 + \frac{T \cdot (5T)^3}{12} + (5T)(T) \cdot d_2^2$$

$$I_{zz} = \frac{5T \cdot T^3}{12} + 5T^2 \cdot \left(\frac{3T}{2}\right)^2 + \frac{125T^3 \cdot T}{12} + 5T^2 \cdot \left(-\frac{3T}{2}\right)^2$$

$$I_{zz} = \frac{5 \cdot T^4}{12} + \frac{45 \cdot T^4}{4} + \frac{125 \cdot T^4}{12} + \frac{45 \cdot T^4}{4}$$

$$I_{zz} = \frac{T^4}{12} (5 + 135 + 125 + 135) = \frac{T^4}{12} \cdot 400 = \frac{T^4 \cdot 200}{6}$$

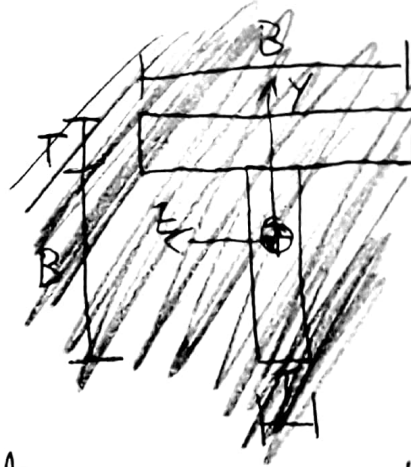
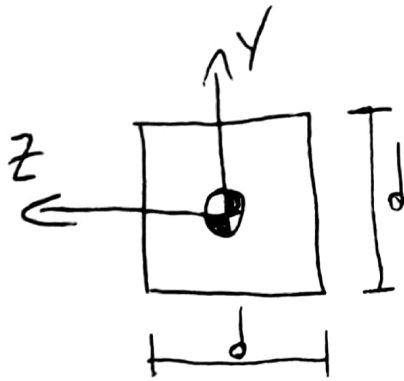
$$I_{zz} = \frac{T^4 \cdot 100}{3} \rightarrow W_z = \frac{I_{zz}}{y_{max}} = \left( \frac{100 \cdot T^4}{3} \right) \left( \frac{1}{4T} \right)$$

$$W_z = \frac{25T^3}{3} \Rightarrow \frac{M_{zmax}}{\sigma_{xxmax}} = W_z \rightarrow \frac{25T^3}{3} = \frac{M_{zmax}}{\sigma_{xxmax}}$$

$$T = \sqrt[3]{\frac{3 \cdot M_{zmax}}{\sigma_{xxmax} \cdot 25}} = \sqrt[3]{\frac{3 \cdot 3937500}{25 \cdot 15}}$$

$$\cancel{T = 14,659}$$

$$T = 31,582 \text{ mm}$$



$$I_{zz} = \frac{B \cdot (H^3)}{12} = \frac{d \cdot d^3}{12} = \frac{d^4}{12} \rightarrow y_{max} = \frac{d}{2}$$

$$W_z = \frac{I_{zz}}{y_{max}} = \left( \frac{d^4}{12} \right) \left( \frac{2}{d} \right) = \frac{d^3}{6}$$

$$\sigma_{xx_{max}} = \frac{M_{z_{max}}}{W_z} \rightarrow \text{circled } W_z =$$

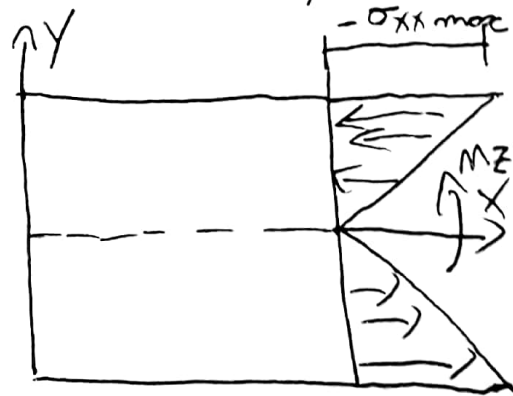
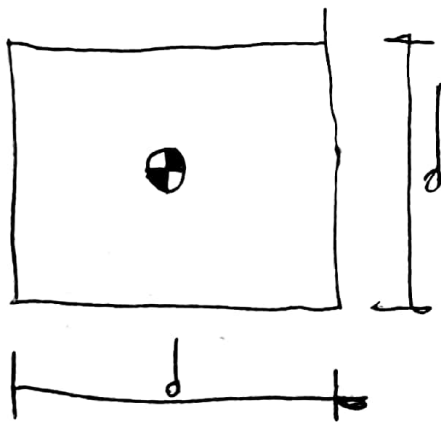
$$\hookrightarrow W_z = \frac{M_{z_{max}}}{\sigma_{xx_{max}}}$$

$$W_z = \frac{d^3}{6} \rightarrow \frac{d^3}{6} = \frac{M_{z_{max}}}{\sigma_{xx_{max}}}$$

$$d = \sqrt[3]{\frac{6 \cdot M_{z_{max}}}{\sigma_{xx_{max}}}} = \sqrt[3]{\frac{6 \cdot 3937500}{15}} \approx 116,348$$

$$\boxed{d \approx 116,348 \text{ mm}}$$

Esboço das tensões normais devido aos efeitos transversais



$$-\sigma_{xx \max} = -15 \text{ N/mm}^2$$

$$\sigma_{xx \max} = 15 \text{ N/mm}^2$$

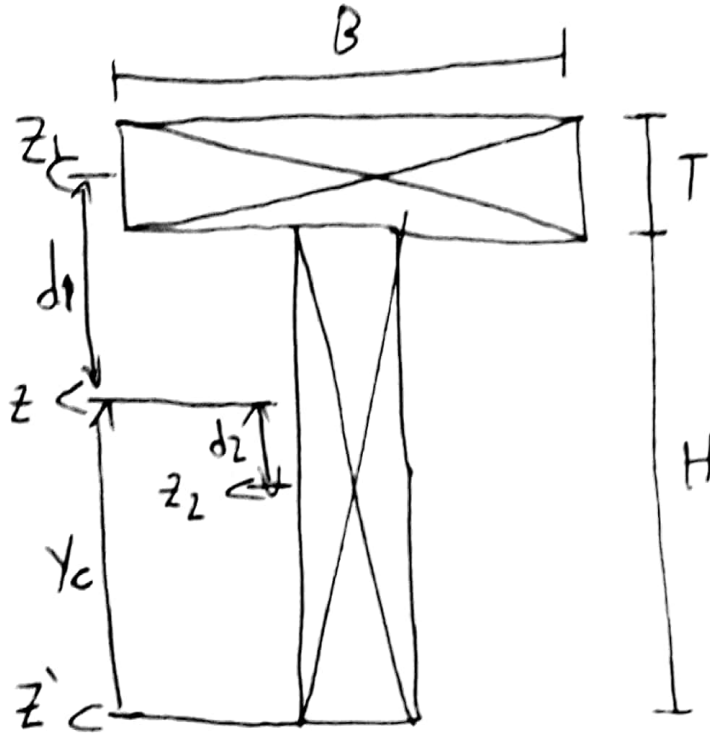
$$\sigma_{xx}(Y) = -\frac{M_{z \max}}{I_{zz}} Y = -\frac{M_{z \max} \cdot 3 \cdot Y}{100 \cdot T^4} = \frac{-3934500 \cdot Y \cdot 3}{100 (3182)^3}$$

$$\sigma_{xx}(Y) = -0,1187 Y \left[ \text{N/mm}^2 \right]$$

$$\textcircled{a} A = d^2 = 116,348^2 \Rightarrow A = 13536,936 \text{ mm}^2$$

$$\textcircled{b} A = 10T^2 = 10 \cdot (31,562)^2 \Rightarrow A = 9974,100 \text{ mm}^2$$

Exercício Viga Tensão e flexão 03



$$A = A_1 + A_2 = HT + BT$$

$$A = 11T^2 + 9T^2 = 20T^2$$

$$\bar{Y}_c A = A_1 \cdot (H/2) + A_2 \left( H + \frac{T}{2} \right)$$

$$Y_c \cdot 20T^2 = 11T^2 \cdot \frac{11T}{2} + 9T^2 \left( 11T + \frac{T}{2} \right)$$

$$Y_c \cdot 20T^2 = \frac{121T^3}{2} + \frac{20T \cdot T^3}{2}$$

$$Y_c = \frac{164T^3}{20T^2} = \frac{82}{10} T = \frac{41}{5} T \rightarrow Y_c = \frac{41 \cdot T}{5} = 8,2T$$

$$d_2 = \frac{H}{2} - Y_c = \frac{11T}{2} - \frac{41T}{5} = \frac{55T - 82T}{10} = \frac{-27T}{10} = -2,7T$$

$$d_2 = \frac{11 \cdot 5T - 41 \cdot 2T}{10} = \frac{(55 - 82)T}{10} = \frac{-27}{10} T = -2,7T$$

$$d_1 = \left( H + \frac{T}{2} \right) - Y_c = \frac{23T}{2} - \frac{41T}{5} = \frac{(23 \cdot 5 - 41 \cdot 2)T}{10} = \frac{33T}{10} = 3,3T$$



$$I_{zz} = \frac{BT^3}{12} + B.T.(d_1)^2 + \frac{TH^3}{12} + H.T.(d_2)^2$$

$$I_{zz} = \frac{9.T^4}{12} + 9.T^2\left(\frac{33T}{10}\right)^2 + \frac{T.(11T)^3}{12} + (11.T^2)(-27T/10)^2$$

$$I_{zz} = \frac{9.T^4}{1200} + \frac{9801.T^4}{100} + \frac{1331T^4}{12} + \frac{8019.T^4}{100}$$

$$I_{zz} = \frac{T^4}{1200} (900 + 9801.12 + 1331.100 + 8019.12)$$

$$I_{zz} = \frac{341840}{1200} T^4 = 289,87 T^4$$

$$W_z = \frac{I_{zz}}{Y_c} = \frac{289,87 T^4}{84T/5} = \frac{289,87.5.T^3}{41} = 35,35 T^3$$

$$W_z = 35,35.T^3$$

$$I_{zz} = 289,87.T^4$$

$$Y_{max} = -Y_c = -\frac{41T}{5} = -8,2T \quad Y_{sup} = H + T - Y_c = +3,8T$$

~~$$I_{zz} = 289,87.(15^4) = 1189593,15 \text{ mm}^4$$~~

$$W_z = 35,35.(15^3) = 119306,25 \text{ mm}^3$$

~~$$\sigma_{zz}(Y) = \frac{-M_z}{I_{zz}} Y = \frac{-1,5.10^6}{1189593,15}$$~~

$$I_{zz} = 289,87.(15^4) = 14674668,15 \text{ mm}^4$$

$$\sigma_{zz}(Y) = -\frac{Mz}{I_{zz}} \quad Y = \frac{-1,5 \cdot 10^6}{1467468,75} \quad Y = -0,1022 \text{ m}$$

$$Y_{\max} = -8,215 = -123 \text{ mm}$$

$$\sigma_{zz}(Y_{\max}) = -0,1022 \cdot (-123) = 12,573 \text{ N/mm}^2$$

$$Y_{\text{sup}} = 3,815 = 57 \text{ mm}$$

$$\sigma_{zz}(Y_{\text{sup}}) = -0,1022 \cdot 57 = -5,8254 \text{ N/mm}^2$$

