EXAMPLE 12.38 *Find the open circuit impedance parameter of the circuit shown in Fig. E12.56. Also find the Y parameters.*

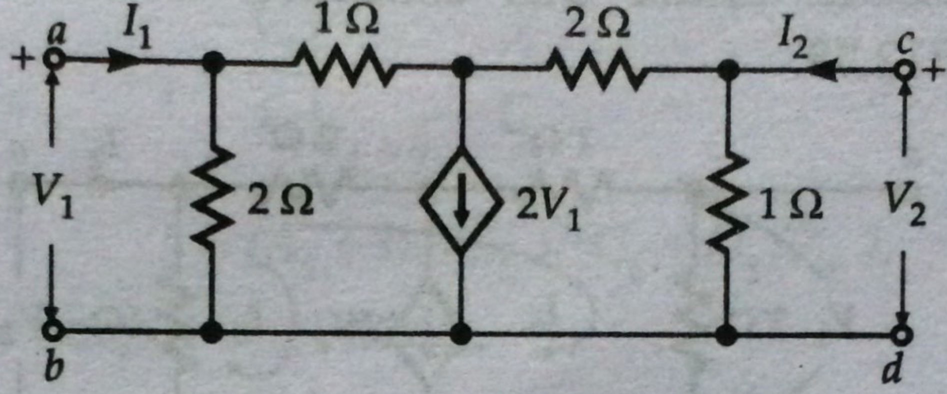


Fig. E12.56

SOLUTION. Assuming the output port (port *cd*) to the open-circuited, $I_2 = 0$. The circuit configuration alongwith loop currents has been shown in Fig. E12.57.

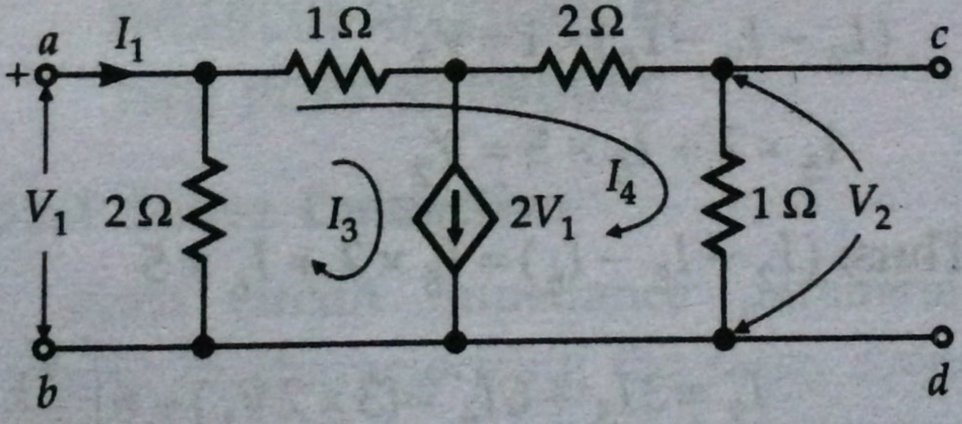


Fig. E12.57

Here,

$$I_4 \times 1 = V_2\ ;\ (I_1 - I_3 - I_4)2 = V_1\ ;$$

$$I_3 \times 1 + I_4 \times 4 = V_1.$$

Thus,

$$2I_1 - 2I_3 - 2I_4 = I_3 + 4I_4$$

or, $$I_1 = 1.5\,I_3 + 3\,I_4 = 1.5 \times 2V_1 + 3 \times V_2$$

$$= 3V_1 + 3\left(-\frac{1}{4}V_1\right) = \frac{9}{4}V_1.$$

$$\left[\because V_1 = 4I_4 + I_3 = 4V_2 + 2V_1 \text{ or } V_2 = -\frac{1}{4}V_1\right]$$

$$\therefore \quad \frac{V_1}{I_1} = Z_{11} = \frac{4}{9}\,\Omega.$$

Also, $$(2+1)\,I_4 + I_3 + V_2 = V_1$$

or $$3I_4 + I_3 + V_2 = V_1$$

or $$\left(\frac{V_2}{1}\right)3 + I_3 + V_2 = V_1 \qquad \left[\because I_4 = \frac{V_2}{1\Omega}\right]$$

or $$4V_2 + 2V_1 = V_1 \qquad [\because I_3 = 2\,V_1]$$

or $$4V_2 = -V_1 = -\frac{4}{9}I_1 \quad [\because 4I_1 = 9V_1]$$

or $$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = -\frac{1}{9}\,\Omega.$$

Let us now open the circuit of Fig. E12.58 and apply voltage at input port (port *a-b*). With $I_1 = 0$, the circuit under this condition is redrawn in Fig. E12.58.

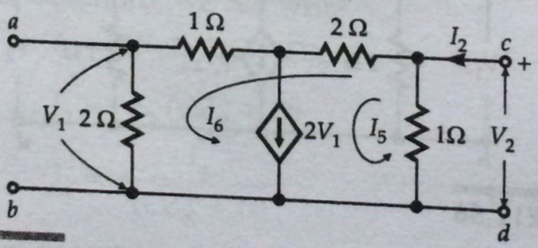


Fig. E12.58

Here, $I_5 = 2V_1; 2 \times I_6 = V_1$

Also utilising KVL,

$$(I_2 - I_5 - I_6) \times 1 = V_2$$

and $$I_5 \times 2 + I_6 \times 5 = V_2$$

Thus, $(I_2 - I_5 - I_6) = I_5 \times 2 + I_6 \times 5$

or $$I_2 = 3I_5 + 6I_6 = (3 \times 2V_1) + 6\left(\frac{V_1}{2}\right)$$

$$[\because I_5 = 2V_1 \text{ and } 2I_6 = V_1]$$

$$= 6V_1 + 3V_1 = 9V_1$$

$$\therefore \quad Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{1}{9}\text{ ohm}$$

Again, $5I_6 + 2I_5 = V_2$

or $$\frac{5}{2}V_1 + 4V_1 = V_2$$

$[\because I_5 = 2V_1 \text{ and } 2I_6 = V_1]$

or $$\frac{5}{2} \times \frac{I_2}{9} + 4\frac{I_2}{9} = V_2 \qquad \left[\because \frac{V_1}{I_2} = \frac{1}{9}\right]$$

or $$\frac{13}{18} I_2 = V_2$$

or $$\left.\frac{V_2}{I_2}\right|_{I_1=0} = Z_{22} = \frac{13}{18} \text{ ohm}$$

$\therefore$ Z parameters are given by,

$$Z = \begin{bmatrix} (4/9) & (1/9) \\ (-1/9) & (13/18) \end{bmatrix}$$

i.e., $$\text{Det}[Z] = \left(\frac{4}{9}\right)\left(\frac{13}{18}\right) + \left(\frac{1}{9}\right)\left(\frac{1}{9}\right) = \frac{1}{3}$$

Thus Y parameters are given as

$$Y_{11} = \frac{Z_{22}}{\text{Det}[Z]} = \frac{13/18}{1/3} = \frac{13}{6} \text{ mho.}$$

$$Y_{21} = -\frac{Z_{21}}{\text{Det}[Z]} = \frac{1/9}{1/3} = \frac{1}{3} \text{ mho}$$

$$Y_{12} = -\frac{Z_{12}}{\text{Det}[Z]} = -\frac{(1/9)}{1/3} = -\frac{1}{3} \text{ mho}$$

$$Y_{22} = \frac{Z_{11}}{\text{Det}[Z]} = \frac{4/9}{1/3} = \frac{4}{3} \text{ mho.}$$

EXAMPLE 12.39 *The Z-parameters of a circuit are given by*

EXAMPLE 12.61 *Obtain short circuit parameters of the network shown in Fig. E12.89.*

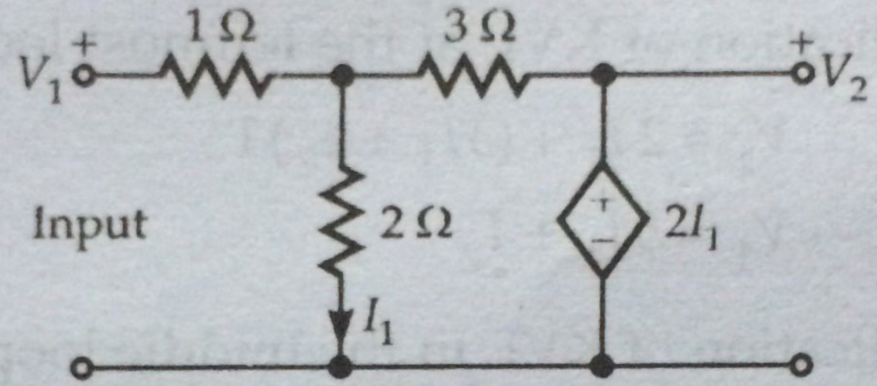


Fig. E12.89

SOLUTION. To obtain the short circuit parameters (*i.e.*, *Y*-parameters), short circuit is applied at the output port. (Fig. E12.90). Under this condition,

$$2I_1 = 0$$

i.e., $$I_1 = 0$$

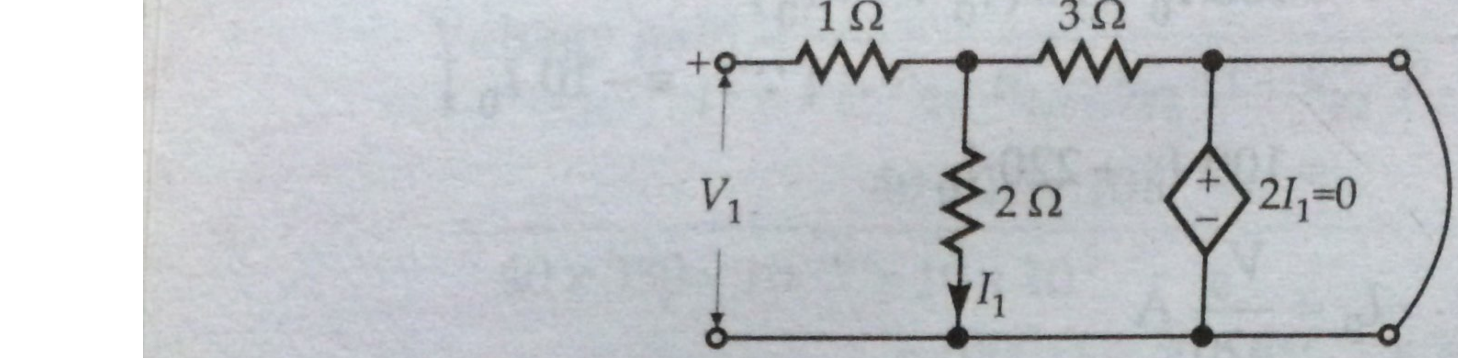


Fig. E12.90

However, I_1 should be a finite quantity as V_1, the voltage applied at the input port is a nonzero quantity. This apparent paradox indicates that the solution of the network though Y-parameter concept is not possible and *it can be concluded that the Y-parameters of the network do not exist.*

EXAMPLE 12.62 *Find Z-parameters of the network shown in Fig. E12.91.*

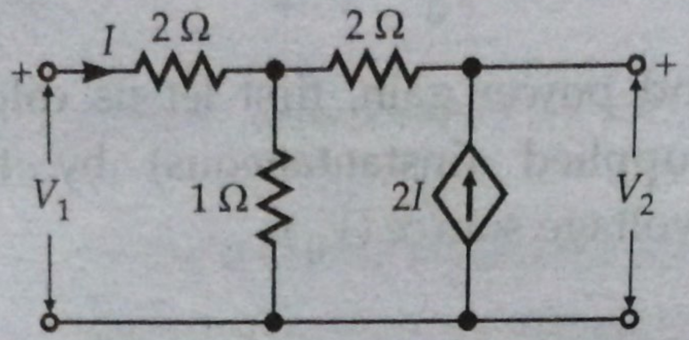


Fig. E12.91

SOLUTION. Let us redraw the given circuit with respective branch currents (Fig. E12.92).

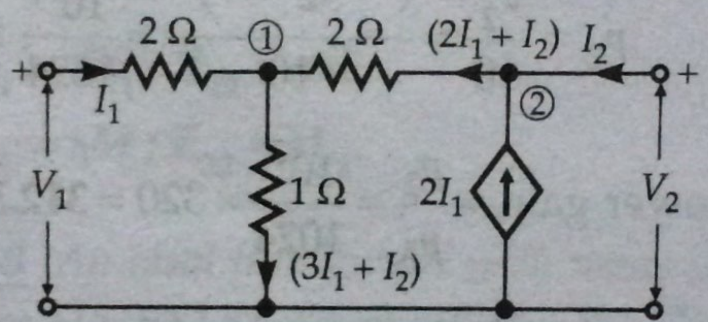


Fig. E12.92

Application of KVL at the leftmost loop yields

$$V_1 = 2I_1 + (3I_1 + I_2)1$$

or $$V_1 = 5I_1 + I_2 \qquad ...(A)$$

Application of KVL in the middle loop [*i.e.*, the loop involving the nodes (1) and (2)] gives

$$+2(2I_1 + I_2) + (3I_1 + I_2)1 - V_2 = 0$$

or $$V_2 = +7I_1 + 3I_2 \qquad ...(B)$$

From equations (*A*) and (*B*),

$$\mathbf{Z_{11} = 5\,\Omega,\ Z_{12} = 1\,\Omega,\ Z_{21} = 7\,\Omega,\ Z_{22} = 3\,\Omega.}$$

EXAMPLE 12.62 *Find Z-parameters of the network shown in Fig. E12.91.*

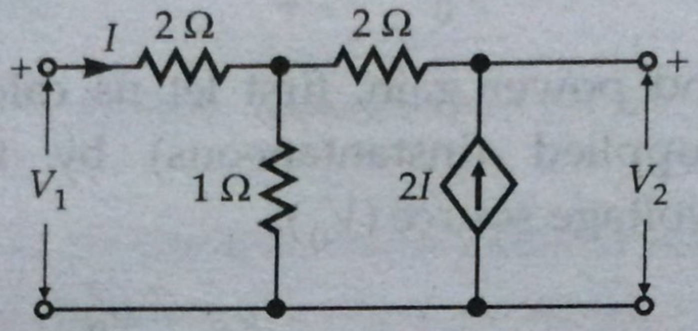


Fig. E12.91

SOLUTION. Let us redraw the given circuit with respective branch currents (Fig. E12.92).

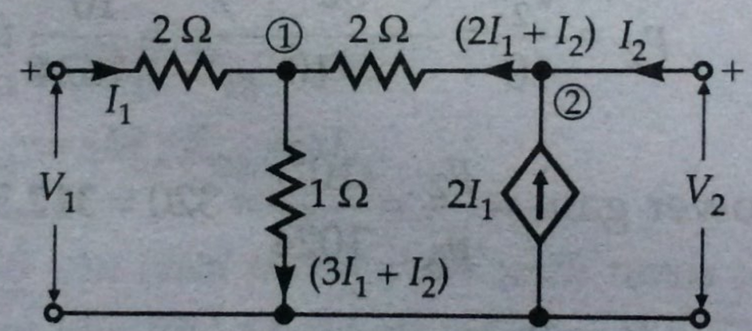


Fig. E12.92

Application of KVL at the leftmost loop yields

$$V_1 = 2I_1 + (3I_1 + I_2)1$$

or $$V_1 = 5I_1 + I_2 \qquad ...(A)$$

Application of KVL in the middle loop [*i.e.*, the loop involving the nodes (1) and (2)] gives

$$+2(2I_1 + I_2) + (3I_1 + I_2)1 - V_2 = 0$$

or $$V_2 = +7I_1 + 3I_2 \qquad ...(B)$$

From equations (*A*) and (*B*),

$$\mathbf{Z_{11} = 5\,\Omega,\ Z_{12} = 1\,\Omega,\ Z_{21} = 7\,\Omega,\ Z_{22} = 3\,\Omega.}$$

EXAMPLE 12.66 *Find Z parameters of the following circuit (Fig. E12.95).*

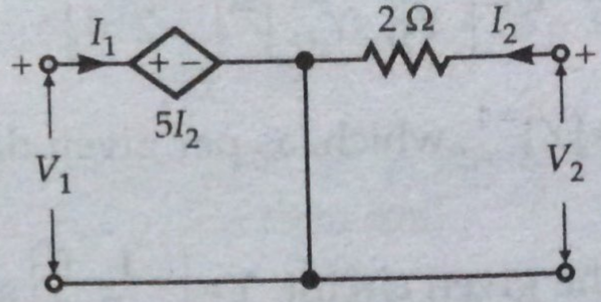


Fig. E12.95

SOLUTION. Applying KVL in the left handed loop,

$$V_1 = 0.I_1 + 5I_2 \quad \text{...(1)}$$

Applying KVL in the right handed loop,

$$V_2 = 2I_2 + 0.(I_1 + I_2)$$
$$= 0.I_1 + 2I_2 \quad \text{...(2)}$$

From (1) and (2), we get

$$Z_{11} = 0\,\Omega,\ Z_{12} = 5\,\Omega$$
$$Z_{21} = 0,\ Z_{22} = 2\,\Omega$$

It may be noted that for this circuit,

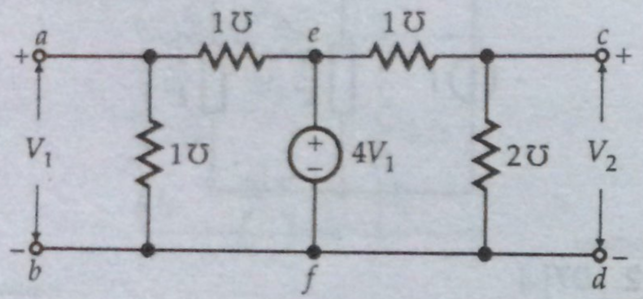
$$Z = \begin{bmatrix} 0 & 5 \\ 0 & 2 \end{bmatrix} \text{ and } Y = [Z]^{-1}$$

However, it is evident that with the given values of Z, Y $(=[Z]^{-1})$ does not exist.

∴ For the given circuit, Y parameter does not exist. Also, it can be observed that in this circuit, V_1 and V_2 can not be chosen independently justifying the absence of Y parameter.

CHAPTER 12

2. Find Y parameters of the network shown in Fig. P12.2.



g. P12.2

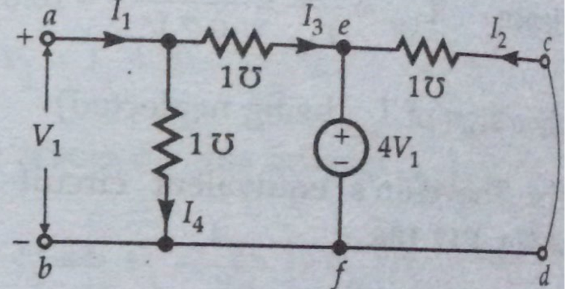


Fig. P12.3

[**Hint.** In the first step *c-d* is short-circuited (Refer to Fig. P12.3)

$$4V_1 = -(I_2)/1\,\text{mho} \quad \text{or, } I_2 = -4\,V_1$$

$$V_1 = 4\,V_1 + \frac{I_3}{1\Omega} \quad \text{or, } I_3 = -3\,V_1$$

$$I_1 = I_3 + I_4 = -3\,V_1 + V_1/1 = -2\,V_1$$

$\therefore \quad Y_{11} = -2$ mho

$Y_{21} = -4$ mho

Next, short terminal a-b (refer to Fig. P12.4)

Note that $4\,V_1 = 0$ as $V_1 = 0$ due to s.c. at terminal (a-b)

$$V_2 = I_2\,(2 + 1) = 3\,I_2 \qquad \therefore\ Y_{22} = 3 \text{ mho}$$

also $\quad V_2 = -\,I_1 \times 1 \qquad \therefore\ Y_{12} = -1$ mho]

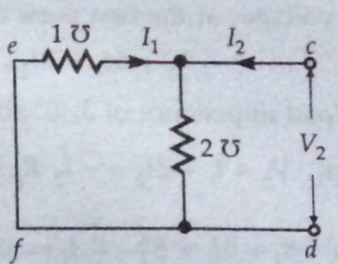


Fig. P12.4

3. Find Z-parameters for the attenuator network shown in Fig. P12.5.

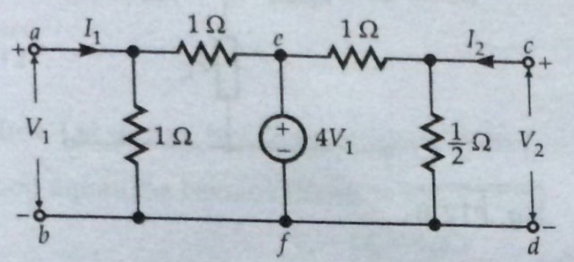


Fig. P12.5

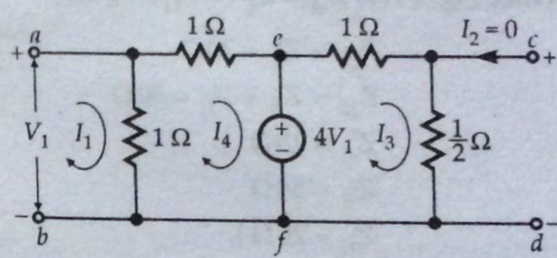


Fig. P12.6

[**Hint.** Open circuit output port (Fig. P12.6)

$$V_2 = \frac{1}{2} \times I_3 \quad \text{or} \quad V_2 = \frac{I_3}{2} - 4\,V_1 + I_3\left(1 + \frac{1}{2}\right) = 0$$

or
$$I_3 = \frac{4}{1.5}\,V_1$$

$$4V_1 + I_4\,(1 + 1) - I_1 \times 1 = 0 \qquad \text{or} \qquad 4V_1 + 2I_4 - I_1 = 0$$

$$V_1 = I_1 \times 1 - I_4 \times 1 \ \text{ or } \quad V_1 = I_1 - I_4$$

From the above four equations find Z_{11} and Z_{21}.

Next make terminal a-b open (Fig. P12.7),

$$V_1 = I_5 \times 1$$

$$2\,I_5 = 4\,V_1$$

$$\frac{1}{2}(I_6 - I_2) + I_6 \times 1 + 4\,V_1 = 0$$

$$\frac{1}{2}(I_2 - I_6) = V_2$$

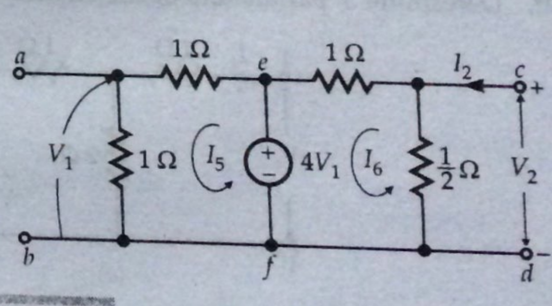


Fig. P12.7

From these equations, Z_{22} and Z_{12} can be found out.]

$_{22} + Y_{12}$; $Y_3 = -Y_{12}$].

12. Determine Y parameters of the network shown in Fig. P12.12.

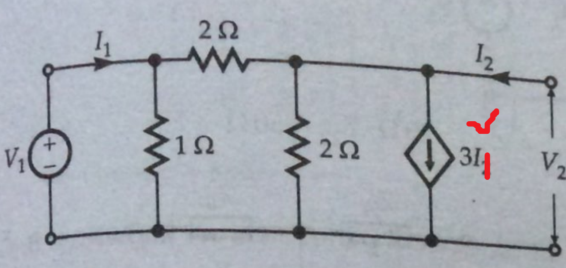


Fig. P12.12

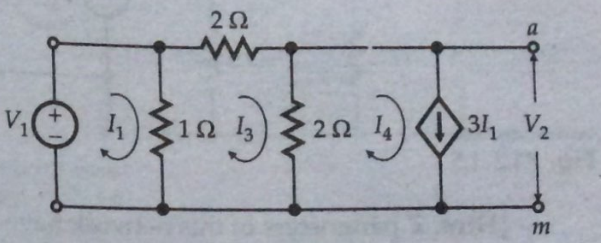


Fig. P12.13

[**Hint.** Let us first find Z parameters, With $I_2 = 0$,

Loop equations become (Refer to Fig. P12.13)

$$V_1 = (I_1 - I_3)\,1 \qquad \text{...(1)}$$

$$0 = (I_3 - I_1)\,1 + 2\,I_3 + (I_3 - I_4)\,2 \qquad \text{...(2)}$$

$$V_2 = 2(I_3 - I_4) \qquad \text{...(3)}$$

$$I_4 = 3\,I_1 \qquad \text{...(4)}$$

From (2), utilising (4),

$$0 = I_3 - I_1 + 2\,I_3 + 2\,I_3 - 6\,I_1 \quad \text{or} \quad 5\,I_3 = 7\,I_1$$

$$\therefore \quad I_3 = \frac{7}{5}\,I_1$$

$\therefore$ From (1) $\quad V_1 = I_1 - \frac{7}{5}\,I_1 = -\frac{2}{5}\,I_1$

$$\therefore \quad Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2 = 0} = -0.4\,\Omega$$

Also, from (3) and (4),

$$V_2 = 2(I_3 - 3\,I_1) = 2\left(\frac{7}{5}\,I_1 - 3\,I_1\right) = -\frac{16}{5}\,I_1 \quad \therefore \; Z_{21} = -3.2\,\Omega$$

Next input port is opened (Fig. P12.14), $I_1 = 0$,

$$V_2 = (I_2 - I_5)\,2 \qquad \text{...(5)}$$

and

$$0 = 2(I_5 - I_2) + 3\,I_5 \qquad \text{...(6)}$$

or

$$I_5 = \frac{2}{5}\,I_2$$

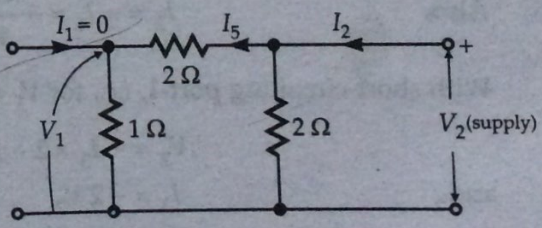


Fig. P12.14

Thus from (5), $V_2 = \left(I_2 - \frac{2}{5}\,I_2\right)2 = 2\,I_2 - \frac{4}{5}\,I_2 = \frac{6}{5}\,I_2$

$$\therefore \quad Z_{22} = \frac{6}{5} = 1.2\,\Omega$$

Also, $\quad V_1 = I_5 \times 1 = \frac{2}{5}\,I_2 \qquad \therefore \quad \frac{V_1}{I_2} = Z_{12} = \frac{2}{5} = 0.4\,\Omega$

Thus $\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} -0.4 & 0.4 \\ -3.2 & 1.2 \end{bmatrix}$

From these values of Z, referring to Art 12.11, the value of Y parameters can be determined].

EXAMPLE 12.54 *Find Y-parameters of the network shown in Fig. E12.80.*

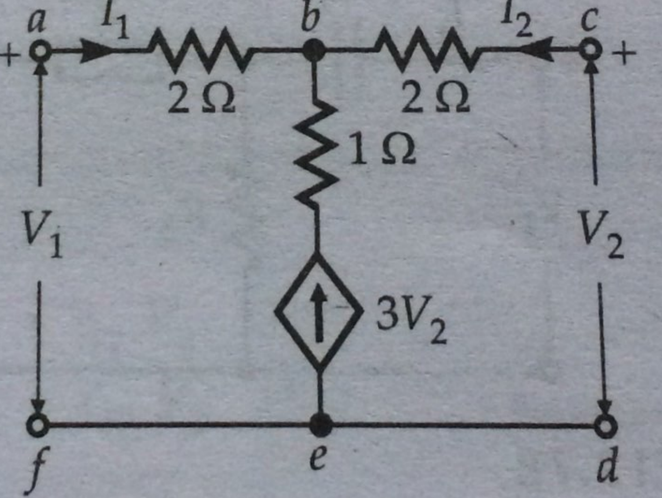


Fig. E12.80

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SOLUTION. Let us first apply KVL at the outer loop of the given circuit (*abcdef*), assuming the currents I_1 and I_2 entering the circuit from the input and output sides with application of voltages V_1 and V_2 at the input and output respectively.

We thus obtain,

$$-V_1 + 2I_1 - 2I_2 + V_2 = 0 \quad \text{...}(a)$$

Applying KCL at node (*b*),

$$I_1 + I_2 + 3V_2 = 0$$

or $$I_1 = -I_2 - 3V_2 \quad \text{...}(b)$$

i.e., $$I_2 = -I_1 - 3V_2 \quad \text{...}(c)$$

Substituting the value of I_2 from (*c*) in (*a*),

$$-V_1 + 2I_1 - 2(-I_1 - 3V_2) + V_2 = 0$$

or $$-V_1 + 4I_1 + 7V_2 = 0$$

or $$I_1 = -\frac{7}{4}V_2 + \frac{1}{4}V_1 \quad \text{...}(d)$$

Again substituting the value of I_1 from (*b*) in (*a*),

$$-V_1 + 2(-I_2 - 3V_2) - 2I_2 + V_2 = 0$$

or $$-V_1 - 2I_2 - 6V_2 - 2I_2 + V_2 = 0$$

or $$-V_1 - 4I_2 - 5V_2 = 0$$

$\therefore$ $$I_2 = -\frac{1}{4}V_1 - \frac{5}{4}V_2 \quad \text{...}(e)$$

Rearranging equations (*d*) and (*e*),

$$I_1 = \frac{1}{4}V_1 - \frac{7}{4}V_2$$

$$I_2 = -\frac{1}{4}V_1 - \frac{5}{4}V_2$$

Comparing these equations with the standard form of the *Y*-parameter equation,

$$Y_{11} = \frac{1}{4} \text{ mho}$$

$$Y_{12} = -\frac{7}{4} \text{ mho}$$

$$Y_{21} = -\frac{1}{4} \text{ mho}$$

$$Y_{22} = -\frac{5}{4} \text{ mho}.$$

EXAMPLE 12.55 Find *Z*-parameters of the circuit shown

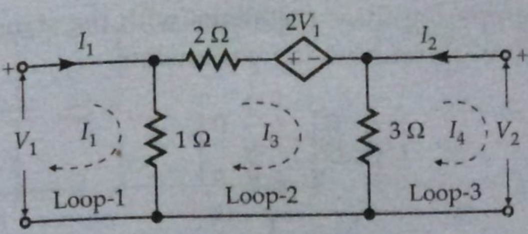

Fig. E12.81

For *loop-1*

$$I_1 - I_3 = V_1 \qquad \text{...}(a)$$

For *loop-2*

$$2I_3 + 2V_1 + 3(I_3 - I_4) + 1(I_3 - I_1) = 0$$

or $$-I_1 + 2V_1 + 6I_3 - 3I_4 = 0$$

or $$-I_1 + 3I_2 + 6I_3 = -2V_1, \qquad \text{...}(b)$$

$$[\because I_4 \equiv -I_2]$$

For *loop-3*

$$3(I_4 - I_3) + V_2 = 0$$

or $$V_2 = 3(I_3 - I_4)$$

or $$V_2 = 3I_3 + 3I_2 \qquad \text{...}(c)$$

Thus the equations are rearranged as

$$I_1 + 0.\mathrm{I}_2 - I_3 = V_1 \qquad \text{...}(a)$$

$$-I_1 + 3I_2 + 6I_3 = -2V_1 \qquad \text{...}(b)$$

$$0.I_1 + 3I_2 + 3I_3 = V_2 \qquad \text{...}(c)$$

From (*a*), $I_3 = I_1 - V_1$

Then from (*b*),

$$-I_1 + 3I_2 + 6(I_1 - V_1) = -2V_1$$

or $$5I_1 + 3I_2 = 4V_1$$

or $$V_1 = \frac{5}{4}I_1 + \frac{3}{4}I_2 \qquad \text{...}(d)$$

Again, using (*a*) in (*c*),

$$+3I_2 + 3(I_1 - V_1) = V_2$$

or $$+3I_2 + 3I_1 - 3V_1 = V_2$$

or $$V_2 = 3I_2 + 3I_1 - 3\left[\frac{5}{4}I_1 + \frac{3}{4}I_2\right]$$

$\therefore$ $$V_2 = 3I_1 - \frac{15}{4}I_1 + 3I_2 - \frac{9}{4}I_2$$

or $$V_2 = -\frac{3}{4}I_1 + \frac{3}{4}I_2 \qquad \text{...}(e)$$

Comparing (*d*) and (*e*) with the standard equations of Z-parameter,

$$Z_{11} = \frac{5}{4}\,\Omega; \quad Z_{12} = \frac{3}{4}; \quad Z_{21} = -\frac{3}{4}; \quad Z_{22} = \frac{3}{4}$$