1. Maximize $z = -x_1 + 3x_2$ subject to the constraints.

$x_1 + x_2 \le 6$, $-x_1 + x_2 \le 4$, $x_1, x_2 \ge 0$

→ soln,

let us introduce two slack variable s_1 and s_2. The normal form or standard form of given LPP is

$$z + x_1 - 3x_2 + 0\cdot s_1 + 0\cdot s_2 = 0$$

$$x_1 + x_2 + s_1 = 6$$

$$-x_1 + x_2 + s_2 = 4 \quad s_1, s_2 \ge 0$$

Initial feasible solution table (IBFS table)

z	x_1	x_2	s_1	s_2	b (R.H.S)	Minimum +ve ratio
1	1	-3	0	0	0	$0/-3 = 0$
0	1	1	1	0	6	$6/1 = 6$
0	-1	1	0	1	4	$4/1 = 4$ ↑ P.R
		↑ P.E				

In first row the greatest negative element is -3. So, x_2 is pivot column. Minimum +ve ratio is 4.

∴ R_2 is pivot row and 1 is pivot element.

Apply, $R_1 \to R_1 + 3R_3$ $\quad R_2 \to R_2 - R_3$

z	x_1	x_2	s_1	s_2	b (R.H.S)	Minimum +ve
1	-2	0	0	3	12	$12/-2 = -6$
0	2 ←	0	1	-1	2	$2/2 = 1$ ←
0	-1	1	0	1	4	$4/-1 = -4$

Apply $R_1 = R_1 + R_2$ $R_3 \to 2R_3 + R_2$

z	x_1	x_2	s_1	s_2	b	~~Min +ve ratio~~
1	0	0	1	2	14	
0	2	0	1	-1	2	
0	0	2	1	-1	10	

since there is no negative element in first row. so, it has optimal solution on equation

$z = 14$

$2x_1 = 2$ $2x_2 = 10$
$x_1 = 1$ $x_2 = 5$

Here,

Max value $(z_1) = 14$ at point $(1, 5)$

2.

soln,

Let us introduce two slack variable s_1 and s_2. The normal form or standard form of given LPP is

$$z - x_1 - 3x_2 + 0 \cdot s_1 + 0 \cdot s_2 = 0$$

$$x_1 + x_2 + s_1 = 7$$

$$x_1 + 2x_2 + s_2 = 10$$

Initial feasible solution table

Z	x_1	x_2	s_1	s_2	b	Min tve ratio
1	-1	-3	0	0	0	$0/-3 = 0$
0	1	1	1	0	7	$7/1 = 7$
0	1	2 ← P.E	0	1	10	$10/2 = 5$ ←

In first row the greatest negative element is -3 and x_2 is pivoit column. Min tve ratio is 5 and R_3 is pivoit row and 2 is pivoit element

Apply, $R_1 \to 2R_1 + 3R_3$

$R_2 \to 2R_2 - R_3$

z	x_1	x_2	s_1	s_2	b
2	1	0	0	3	30
0	1	0	2	-1	-4
0	1	2	0	1	10

since there is no negative element in first row - so it has optimal solution on equating

$2z = 30$

$\therefore z = 15$

$\therefore x_1 = 0$ $\qquad 2x_2 = 10$

$\therefore x_2 = 5$

max value $(z_1) = 15$ at $(0, 5)$

8.

soln) Let us introduce ~~two~~ 3 slack variable s_1, s_2 and s_3. The normal form or standard form of given LPP is

$$z - 5x_1 - 4x_2 + 0\cdot s_1 + 0\cdot s_2 + 0\cdot s_3 = 0$$
$$x_1 + x_2 + s_1 = 20$$
$$2x_1 + x_2 + s_2 = 35$$
$$-3x_1 + x_2 + s_3 = 12$$

Initial feasible solution table

z	x_1	x_2	s_1	s_2	s_3	b	Min tve ratio
1	-5	-4	0	0	0	0	0/-5 = 0
0	1	1	1	0	0	20	20/1 = 20
0	2 ← P.E	1	0	1	0	35	35/2 = 17.5 ←
0	-3	1	0	0	1	12	12/-3 = -4

Apply, $R_1 \to 2R_1 + 5R_3$
$R_2 \to 2R_2 - R_3$
$R_4 \to 2R_4 + 3R_3$

z	x_1	x_2	s_1	s_2	s_3	b	Min tve ratio
2	0	-3	0	5	0	175	-58.33
0	0	1 P.E	2	-1	0	5	5 ←
0	2	1	0	1	0	35	35
0	0	5	0	3	2	106	21.2

x_2 is pivot column

R_2 is pivot row

1 is pivot element

Apply $R_1 \to R_1 + 3R_2$

$R_3 \to R_3 - R_2$

$R_4 \to R_4 - 5R_2$

z	x_1	x_2	s_1	s_2	s_3	b
2	0	0	6	2	0	190
0	0	1	2	-1	0	5
0	2	0	-2	2	0	30
0	0	0	-10	7	3	81

Since there is no negative element in first row. so, it has optimal solution on equation

$$2z = 190$$

$$\therefore z = 95$$

$2x_1 = 30$ $\therefore x_2 = 5$

$\therefore x_1 = 15$

$\therefore$ Max $z = 95$ at $(15, 5)$

4.

soln)

Let us introduce ~~two~~ 3 slack variable s_1, s_2 and s_3. The normal form or standard form of given LPP is

$$z - 6x_1 - 12x_2 + 0 \cdot s_1 + 0 \cdot s_2 + s_3 = 0$$

$$x_1 + s_1 = 4$$

$$x_2 + s_2 = 4$$

$$6x_1 + 12x_2 + s_3 = 72$$

Initial feasible solution table

z	x_1	x_2	s_1	s_2	s_3	b	min tue ratio
1	-6	-12	0	0	0	0	$-12/0 = 0$
0	1	0	1	0	0	4	$4/0 = 0$
0	0	1 ← P.E	0	1	0	4	$4/1 = 4$ ←
0	6	12	0	0	1	72	$72/12 = 6$

In first row the greatest negative element is -12 and minimum tue ratio is 4. R_3 is pivot row and 1 is pivot element

Apply → $R_1 \Rightarrow R_1 + 12R_3$
$R_4 \rightarrow R_4 - 12R_3$

z	x_1	x_2	s_1	s_2	s_3	b	Min two ratio
1	-6	0	0	12	0	48	$48/-6 = 8$
0	1 ←P.E	0	1	0	0	4	4 ←
0	0	1	0	1	0	4	0
0	6	0	0	12	1	24	4

x_1 = pivot column
R_2 = pivot row
1 = piot element

Apply, $R_1 \rightarrow R_1 + 6R_2$
$R_4 \rightarrow R_4 - 6R_2$

z	x_1	x_2	s_1	s_2	s_3	b
1	0	0	6	12	0	72
0	1	0	1	0	0	4
0	0	1	0	1	0	4
0	0	0	-6	12	1	0

since there is no negative element in first row. so, it has optimal solution on equatio

$\therefore z = 72$

$\therefore x_1 = 4$

$\therefore x_2 = 4$

7.

Soln)

Let us introduce two slack variable S_1 and S_2. The normal form or standard form of given LPP is

$$z - x_1 - x_2 - x_3 + 0 \cdot S_1 + 0 \cdot S_2 = 0$$

$$4x_1 + 5x_2 + x_3 + S_1 = 12$$

$$8x_1 + 5x_2 + 4x_3 + S_2 = 12$$

Initial feasible solution table

z	x_1	x_2	x_3	S_1	S_2	b	Min +ve ratio
1	-1	-1	-1	0	0	0	$0/-1 = 0$
0	4	5	1	1	0	12	$12/4 = 3$
0	8	5	4	0	1	12	$12/8 = 1.5$ ←

In first row the greatest element is -1. so x_1 is pivot column. Min +ve ratio is 1.5 and 8 is pivot element

Apply, $R_1 = 8R_1 + R_3$

$R_2 \rightarrow 8R_2 - 4R_3$

z	x_1	x_2	x_3	S_1	S_2	b	Min +ve ratio
8	0	-3	-4	0	1	12	$12/-4 = -3$
0	0	20 ← P.E	-8	8	-4	48	2.4 ←
0	8	5	4	0	1	12	2.4

Apply, $R_1 = 20R_1 + 3R_2$
$R_3 \to 20R_3 - 5R_2$

Z	x_1	x_2	x_3	s_1	s_2	b	Min +ve ratio
160	0	0	-104	24	8	384	-3.69
0	0	20	-8	8	-4	48	-6
0	160	0	120	-40	40	0	0 ←

↑ P.E

In first ~~column,~~ row, the greatest negative element is -104. ∴ is pivot element and R_3 is pivot row.

Apply, $R_1 = 120R_1 + 104R_3$
$R_2 = 120R_2 + 8R_3$

Z	x_1	x_2	x_3	s_1	s_2	b
19200	16640	0	0	-1280	5120	46080
0	1280	2400	0	640	-160	5760
0	160	0	120	-40	40	0

Since there is no negative element in first row. so it has optimal solution on equating

$19200 z = 46080$

$\therefore z = 2.4$

$\therefore x_1 = 0$

$\therefore 2400 x_2 = 5760$

$\therefore x_2 = 2.4$

$\therefore x_3 = 0$

5.

soln,

Let us introduce 3 slack variables s_1, s_2 and s_3. such that

$$z - x_1 - x_2 + 0 \cdot s_1 + 0 \cdot s_2 + 0 \cdot s_3 = 0$$
$$2x_1 + 3x_2 + s_1 = 130$$
$$3x_1 + 8x_2 + s_2 = 300$$
$$4x_1 + 2x_2 + s_3 = 140$$

Initial feasible solution table

z	x_1	x_2	s_1	s_2	s_3	b	Min +ve ratio
1	-1	-1	0	0	0	0	0
0	2	3	1	0	0	130	65
0	3	8	0	1	0	300	100
0	4	2	0	0	1	140	35 ←

↑ P.E

In first row, the first negative element is -1 so pivot column is x_1 and pivot element is 4.

Apply
$$R_1 = 4R_1 + R_4$$
$$R_2 = 4R_2 - 2R_4$$
$$R_3 = 4R_3 - 3R_4$$

z	x_1	x_2	s_1	s_2	s_3	b	Min +ve ratio
4	0	-2	0	0	1	140	-70
0	0	8	4	0	-2	240	301 ←
0	0	26	0	4	-3	780	30
0	4	2	0	0	1	140	70

Apply, $R_1 \to 8R_1 + 2R_2$

$R_2 \to 8R_3 - 26R_2$

$R_4 \to 8R_4 - 2R_2$

z	x_1	x_2	s_1	s_2	s_3	b
32	0	0	8	0	4	~~1320~~ 1600
0	0	8	-4	0	-2	240
0	0	0	-140	32	28	0
4	32	0	-8	0	12	640

since there is no negative element in first row . so it has optimal solution on equati

$32z = 1600$

$\therefore z = 50$

$32x_1 = 640$

$\therefore x_1 = 20$

$8x_2 = 240$

$\therefore x_2 = 30$

6.

Soln)

Here,

Let us introduce slack variables x_3, x_4, x_5 such that

$$z - 90x_1 - 50x_2 + 0\cdot s_1 + 0\cdot s_2 + 0\cdot s_3 = 0$$

$$x_1 + 3x_2 + s_1 = 18$$

$$x_1 + x_2 + s_2 = 10$$

$$3x_1 + x_2 + s_3 = 24$$

Initial feasible solution is

z	x_1	x_2	s_1	s_2	s_3	b	Min tve value
1	-90	-50	0	0	0	0	0
0	1	3	1	0	0	18	18
0	1	1	0	0	0	10	10
0	3 ↑ P.E	1	0	0	1	24	8 ←

In first row, the first negative entries is -90. so pivot ~~equation~~ element is 3

Apply: $R_1 \to R_1 + 30R_4$ $R_2 \to 3R_2 - R_4$ $R_3 : 3R_3 - R_4$

z	x_1	x_2	s_1	s_2	s_3	b	Min tve value
1	0	-20	0	0	30	720	-36
0	0	8	3	0	-1	30	3.75
0	0	2 ← P.E	0	3	-1	6	3 ←
0	3	1	0	0	1	24	24

Apply, $R_1 \to R_1 + 10R_3$
$R_2 \to R_2 - 4R_3$
$R_4 \to 2R_4 - R_3$

Z	x_1	x_2	S_1	S_2	S_3	b
1	0	0	0	30	20	780
0	0	0	3	-12	3	6
0	0	2	0	3	-1	6
0	6	0	0	-3	3	42

Since there is no negative element in 1st row so it has optimal solution on equating.

$\therefore z = 780$

$\therefore 6x_1 = 42$
$x_1 = 7$

$\therefore 2x_2 = 6$
$\therefore x_2 = 3$

8.

Soln,

Let us introduce slack variables s_1, s_2 and s_3.

such that

$$z - 5x_1 - 6x_2 - x_3 + 0 \cdot s_1 + 0 \cdot s_2 + 0 \cdot s_3 = 0$$

$$9x_1 + 3x_2 - 2x_3 + s_1 = 5$$

$$4x_1 + 2x_2 - x_3 + s_2 = 2$$

$$x_1 - 4x_2 + x_3 + s_3 = 3$$

Initial feasible solation is

z	x_1	x_2	x_3	s_1	s_2	s_3	b	Min tve
1	-5	-6	-1	0	0	0	0	0
0	9	3	-2	1	0	0	5	1.67
0	4	2 ←P.E	-1	0	1	0	2	1 ←
0	1	-4	1	0	0	1	3	-0.75

Apply $R_1 \to 2R_1 + 6R_3$ $R_4 \to 2R_4 + 4R_3$

$R_2 \to 2R_2 - 3R_3$

z	x_1	x_2	x_3	s_1	s_2	s_3	b	min tve ratio
2	14	0	-8	0	6	0	12	-1.5
0	6	0	-1	2	-3	0	4	-4
0	4	2	-1	0	1	0	2	-2
0	18	0	-2	0	2	4	16	-8

Hence, all minimum ratio are negative so, it has a unbounded solution.

9.

soln,

Let us introduce slack variable s_1 and s_2 such that

$$z - x_1 - 3x_2 + 0 \cdot s_1 + 0 \cdot s_2 = 0$$

$$x_1 - 3x_2 + s_1 = -6$$

$$x_1 + x_2 + s_2 = 6$$

Initial feasible solution is

z	x_1	x_2	s_1	s_2	b	Min +ve ratio
1	-1	-3	0	0	0	0
0	1	-3 ←P.E	1	0	-6	2 ←
0	1	1	0	1	6	6

Apply, $R_1 \to R_1 - R_2$ $R_3 \to 3R_3 + R_2$

z	x_1	x_2	s_1	s_2	b	Min +ve ratio
1	-2	0	-1	0	6	-3
0	1	-3	1	0	-6	-6
0	4 ↑ P.E	0	1	3	12	3 ←

Apply, $R_1 \to 4R_1 + 2R_3$ $R_2 \to 4R_2 - R_3$

z	x_1	x_2	s_1	s_2	b
4	0	0	-2	6	48
0	0	-12	3	-3	-36
0	4	0	1	3	12

In row first, there is no negative element. so, it has optimal solution on equating.

$\therefore 4z = 48$

$\therefore z = 12$

$\therefore 4x_1 = 12$ $\qquad$ $\therefore -12x_2 = -36$

$x_1 = 3$ $\qquad$ $\therefore x_2 = 3$

$\therefore$ Max $z = 12$ at $(3,3)$

13. Soln: Let us introduce two slack variable s_1 and s_2.

$$z - 3x_1 - 6x_2 + 0 \cdot s_1 + 0 \cdot s_2 = 0$$
$$-x_1 + x_2 + s_1 = 6$$
$$x + x_2 + s_2 = 10$$

Initial feasible solution is

z	x_1	x_2	s_1	s_2	b	Min tue ratio
1	-3	-6	0	0	0	0
0	-1	1 ← P.E	1	0	6	6 ←
0	1	1	0	1	10	10

In row first, the maxm ~~negative~~ negative element is -6. The min tue ratio is 6 and pivot element is 1.

Apply: $R_1 \rightarrow R_1 + 6R_2$ $\quad R_3 \rightarrow R_3 - R_2$

z	x_1	x_2	s_1	s_2	b	Min tuo ratio
1	-9	0	6	0	36	-4
0	-1	1	1	0	6	-6
0	2 ↑ P.E	0	-1	1	4	2 ←

Apply, $R_1 \to 2R_1 + 9R_3$
$R_2 \to 2R_2 + R_3$

Z	x_1	x_2	s_1	s_2	b
2	0	0	3	9	108
0	0	2	1	1	16
0	2	0	-1	1	4

Since, there is no negative element in row, so it has optimal solution on equating

$2z = 108$
$\therefore z = 54$

$2x_1 = 4$ $\qquad$ $2x_2 = 16$
$x_1 = 2$ $\qquad$ $\therefore x_2 = 8$

14. Soln, Let us introduce two slack variable s_1 and s_2

$$z - 5x_1 + 20x_2 + 0 \cdot s_1 + 0 \cdot s_2 = 0$$
$$-2x_1 + 10x_2 + s_1 = 5$$
$$2x_1 + 5x_2 + s_2 = 10$$

Initial feasible solution is

z	x_1	x_2 ↓	s_1	s_2	b	Min the ratio
				0	0	0
1	-5	20	0			
0	-2	10	1	0	5	0.5 ←
0	2	5	0	1	10	2

In first row, the first positive entry is 20. So, column of x_2 is pivot colum and R_2 is pivot row. 10 is pivot element

Apply, $R_1 \to R_1 - 2R_2$
$R_3 \to 2R_3 - R_2$

z	x_1	x_2	s_1	s_2	b
1	-1	0	-2	0	-10
0	-2	10	1	0	5
0	6	0	-1	2	15

since, there is no negative element in first row. so, it has a optimal solution by equating

$\therefore z = -10$

$\therefore x_1 = 0$

$\therefore 10x_2 = 5$

$\therefore x_2 = 0.5$

$\therefore \max(z) = -10$ at $(x_1, x_2) = (0, 0.5)$

a) Maximize $z = 2x_1 + 3x_2$ subject to

$x_1 + x_2 \le 6$, $-x_1 + x_2 \le 4$ $x_1, x_2 \ge 0$

→ soln,

we have to maximize

$$Z = 2x_1 + 3x_2$$

s.t

$$x_1 + x_2 \le 6$$
$$-x + x_2 \le 4$$
$$x_1, x_2 \ge 0$$

x_1	x_2	constant	
1	1	6	(y_1)
-1	1	4	(y_2)

Let y_1 and y_2 be the dual variable then

$$\text{Min } z' = 6y_1 + 4y_2$$

y_1	y_2	c
1	-1	2
1	1	3

s.t

$$y_1 - y_2 \ge 2$$
$$y + y_2 \ge 3$$
$$y_1 y_2 \ge 0$$

b.

Soln,

$$\text{Maximize } (Z) = x_1 - x_2 + 4x_3$$

s.t

$$x_1 + x_2 + x_3 \le 9$$

$$x_1 - 2x_2 + x_3 \ge 6 , \quad x_1, x_2, x_3 \ge 0$$

showing constraints in table

x_1	x_2	x_3	b	
1	1	1	9	(x_1)
1	-2	1	6	(x_2)

Dual : Let y_1 and y_2 be dual

y_1	y_2	y_3	c
1	1	1	9
1	-2	1	6
1	-1	4	

Minimize ~~Maximize~~ $(Z') = 9y_1 + 6y_2$

s.t

$$y_1 + y_2 \ge 1$$

$$y_1 - 2y_2 \ge -1$$

$$y_1 + y_2 \ge 4$$

c.

soln,

~~Minim~~ Maximize $(z) = 3x_1 + 8x_2$

s.t

$x_1 + 2x_2 \le 8$

$x_1 + 6x_2 \le 12$

$x_1, x_2 \ge 0$

showing constraints in table

x_1	x_2	b	
1	2	8	(y_1)
1	6	12	(y_2)
3	8		

Dual = Let y_1 and y_2 and dual

y_1 —— y_2

Minimize $(z') = 8y_1 + 12y_2$

s.t

$y_1 + y_2 \ge 3$

$2y_1 + 6y_2 \ge 8$

$y_1, y_2 \ge 0$

d.

Soln,

Minimize $(z) = 8x_1 + 9x_2$

s.t: $x_1 + x_2 \geq 5$

$3x_1 + x_2 \geq 21$

$x_1 \geq 0, x_2 \geq 0$

showing constraints in table

x_1	x_2	b	
1	1	5	(y_1)
3	1	21	(y_2)

Let y_1 and y_2 be dual

x_1	x_2	b
1	3	8
1	1	9

Max $(z') = 5y_1 + 21x_2$

s.t

$y_1 + 3y_2 \leq 8$

$y_1 + y_2 \leq 9$

$y_1, y_2 \geq 0$

16.

a.

soln)

Minimize $(z) = 4x_1 + 7x_2$

s.t

$x_1 + x_2 \geq 5$

$3x_1 + x_2 \geq 21$

$x_1 \geq 0,\ x_2 \geq 0$

Showing constraints in table

x_1	x_2	b	
1	1	5	(y_1)
3	1	21	(y_2)

Dual: Let y_1 and y_2 are dual variable

y_1	y_2	c
1	3	4
1	1	7

Max $(z') = 5y_1 + 21y_2$

s.t

$y_1 + 3y_2 \leq 4$ $\quad y_1, y_2 \geq 0$

$y_1 + y_2 \leq 7$

Let us introduce two slack variable s_1 and s_2

$$z' - 5y_1 - 21y_2 + 0 \cdot s_1 + 0 \cdot s_2 = 0$$
$$y_1 + 3y_2 + s_1 = 4$$
$$y_1 + y_2 + s_2 = 7$$

z'	y_1	y_2	s_1	s_2	b	min tve ratio
1	-5	-21	0	0	0	0
0	1	3	1	0	4	4/3
0	1	1	0	1	7	7

In first row, the negative element is -21
y_2 is pivot column and 3 is pivot elements

Apply $R_1 \to R_1 + 7R_2$ $R_3 \to 3R_3 - R_2$

z'	y_1	y_2	s_1	s_2	b
1	2	0	7	0	28/3
0	1	3	1	0	4/3
0	1	0	-1	3	19.337

Since there is no negative element in first row. so it has a optimal solution by equating.

$\therefore$ Max $(z) = 28/3$

$\therefore y_1 = 0$

$\therefore 3y_2 = 4/3$

$\therefore y_2 = 4$

Again,

$\therefore x_1 = 7$

$\therefore x_2 = 0$

$\therefore \text{Min}(z) = 4x_1 + 7x_2$
$= 4 \times 7 + 7 \times 0$
$= 28$

$\therefore$ Min value $= 28$ at $(7, 0)$

b. soln,

$$\text{Min } (z) = 8x_1 + 12x_2$$

s.t

$$2x_1 + 2x_2 \geq 1$$

$$x_1 + 3x_2 \geq 2$$

$$x_1, x_2 \geq 0$$

showing constraints in table

x_1	x_2	b	
			(y_0)
2	2	1	
			(y_1)
1	3	2	

Let y_1 and y_2 are dual variabe

y_1	y_2	c
2	1	8
2	3	12

$$\therefore \text{Max } (z') = y_1 + 2y_2$$

s.t

$$2y_1 + y_2 \leq 8$$

$$2y_1 + 3y_2 \leq 12$$

$$y_1, y_2 \geq 0$$

Let us introduce two slack variable S_1 and S_2.

$$z' - y_1 - 2y_2 + 0 \cdot s_1 + 0 \cdot s_2 = 0$$
$$2y_1 + y_2 + s_1 = 8$$
$$2y_1 + 3y_2 + s_2 = 12$$

Table:

z'	y_1	y_2	s_1	s_2	b	Min tve ratio
1	-1	-2	0	0	0	0
0	2	1	1	0	8	8
0	2	3	0	1	12	4

In first row, the negative element is -2. y_2 is the pivot colum and 3 is pivot element

Apply $R_1 \to 3R_1 + 2R_3$
$R_2 \to 3R_2 - R_3$

z'	y_1	y_2	s_1	s_2	b
3	1	0	0	2	8
0	4	0	3	-1	20
0	0	3	0	1	4

Apply $R_1 \to R_1/3$

z'	y_1	y_2	s_1	s_2	b
1	1/3	0	0	2/3	8/3
0	4	0	3	-1	20
0	0	3	0	1	4

Since, there is no negative element in first row. So, it has a optimal solution is given

$$Max(z') = 8/3$$

$\therefore y_2 = 4/3$ $\therefore y_1 = 0$

Again,

$x_1 = 0$

$x_2 = 2/3$

$$Min(z) = 8x_1 + 12x_2$$

$$= 8 \times 0 + \cancel{12}^{4} \times 2/\cancel{3}$$

$$= 8$$

soln

$$\text{Min } (z) = x_1 + 8x_2 + 5x_3$$

s.t

$$x_1 + x_2 + x_3 \geq 8$$

$$-x_1 + 2x_2 + x_3 \geq 2$$

$$x_1, x_2, x_3 \geq 0$$

Showing constraints in table

x_1	x_2	x_3	b	
1	1	1	8	(y_1)
-1	2	1	2	(y_2)

Let y_1 and y_2 are dual variable

$$\text{Max } (z') = 8y_1 + 2y_2$$

s.t

$$y_1 - y_2 \leq 1$$

$$y_1 + 2y_2 \leq 8$$

$$y_1 + y_2 \leq 5$$

Let us introduce two slack variable s_1, ~~and~~ s_2 and s_3

$z' - 8y_1 - 2y_2 + 0 \cdot s_1 + 0 \cdot s_2 + 0 \cdot s_3 = 0$

$y_1 - y_2 + s_1 = 1$

$y_1 + 2y_2 + s_2 = 8$

$y + y_2 + s_3 = 5$

$s_1, s_2, s_3 \geq 0$

Table:

z'	y_1	y_2	s_1	s_2	s_3	b	Min +ve
1	-8	-2	0	0	0	0	0
0	1 ← P.E	-1	1	0	0	1	1 ←
0	1	2	0	1	0	8	8
0	1	1	0	0	1	5	5

In first row, the negative element is -8. y_1 is pivot column and 1 is the pivot element

Apply : $R_1 \to R_1 + 8R_2$

$R_3 \to R_3 - R_2$

$R_4 \to R_4 - R_2$

z'	y_1	y_2	s_1	s_2	s_3	b	Min +ve
1	0	-10	8	0	0	8	-0.8
0	1	-1	1	0	0	1	-1
0	0	3	-1	1	0	7	2.3
0	0	2 (P.E)	-1	0	1	2	2 ←

Apply $R_1 \rightarrow R_1 + 5R_4$, $R_2 \rightarrow 2R_2 + R_4$ $R_3 \rightarrow 2R_3 - 3R_4$

z'	y_1	y_2	s_1	s_2	s_3	c
1	0	0	3	0	5	28
0	2	0	1	0	1	6
0	0	0	1	2	-3	2
0	0	2	-1	0	1	4

$Max(z') = 28$

$2y_1 = 6$ ∴ $2y_2 = 4$

∴ $y_1 = 3$ ∴ $y_2 = 2$

∴ $x_1 = 3$ $x_2 = 0$ $x_3 = 5$

∴ $Min(z) = 3 + 0 + 5 \times 5$

$= 28$