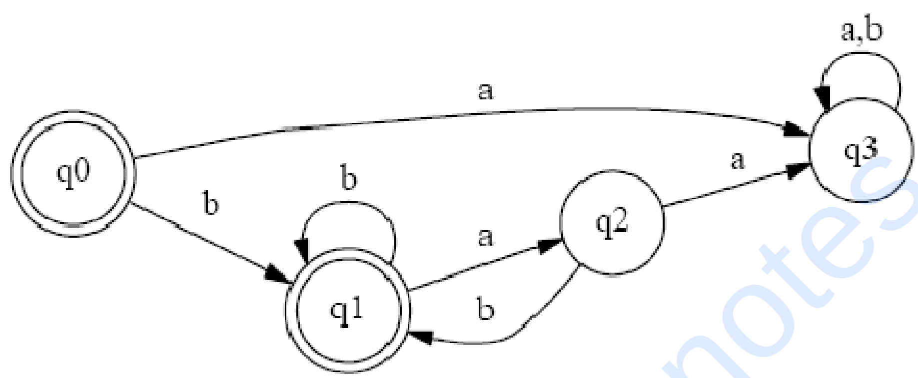
Theory of Computation Solutions
Tutorial No: 1

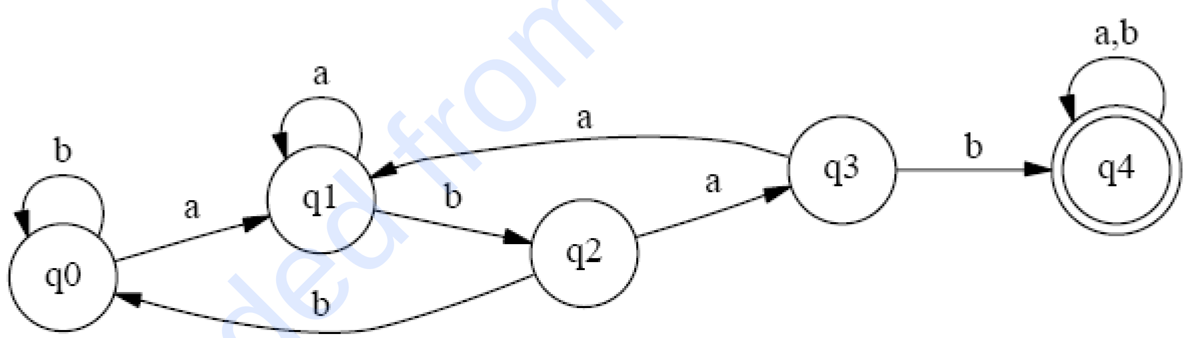
[A] Problems Related to DFA and NFA

- Construct deterministic finite automata accepting each of the following languages.

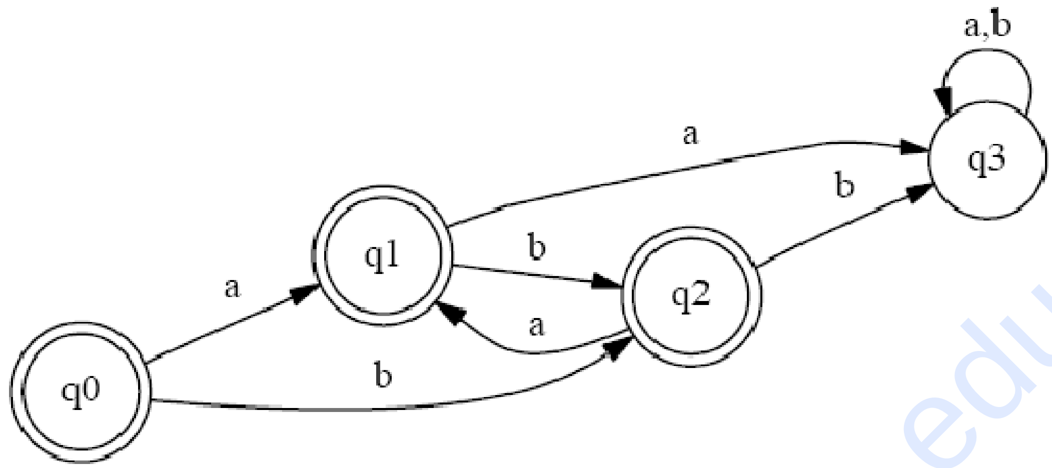
(a) {w ϵ {a, b} : each a in w is immediately preceded and immediately followed by a "*b*" }.



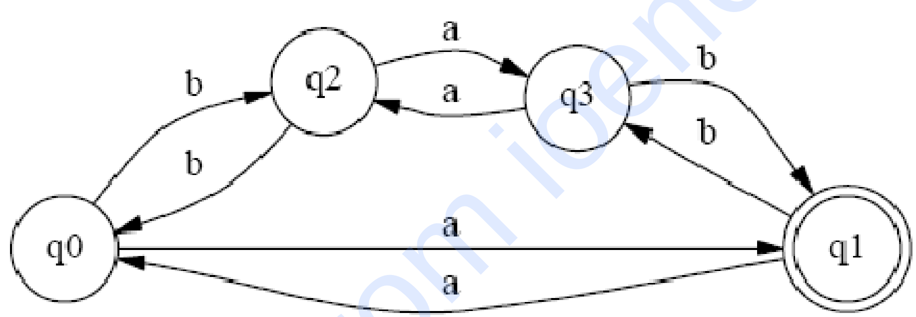
(b) {w ϵ {a, b} : w has abab as a substring}.



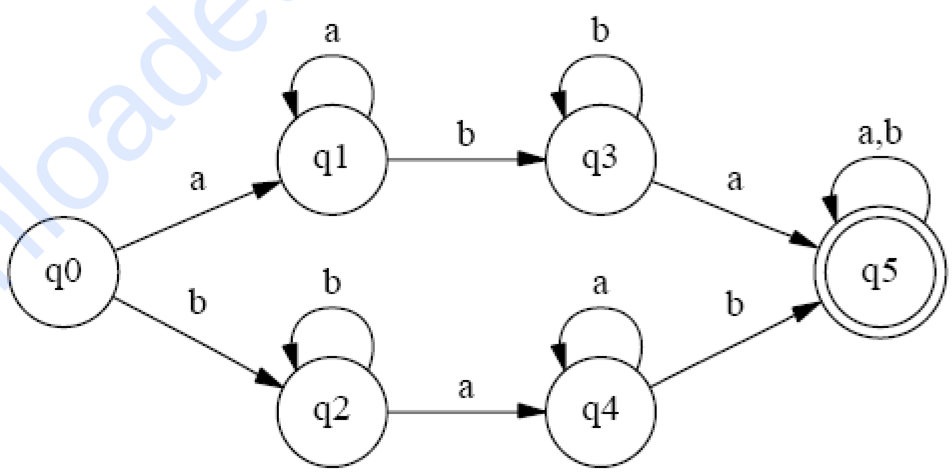
(c) {w ϵ {a, b} : w has neither aa nor bb as a substring}.



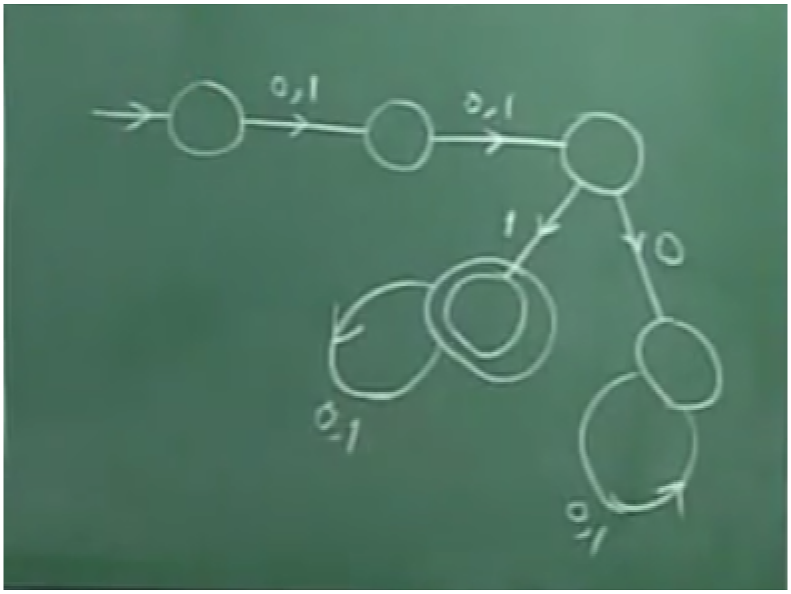
(c) {w ϵ {a, b} : w has an odd number of a's and an even number of b's}.



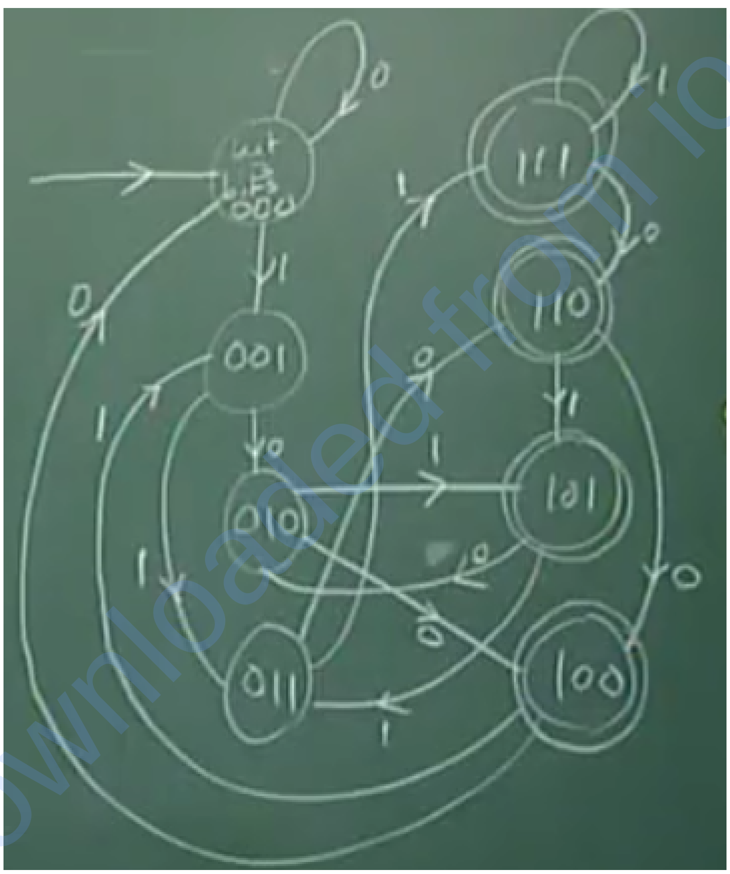
(d) {w ϵ {a, b} : w has both ab and ba as substrings}.



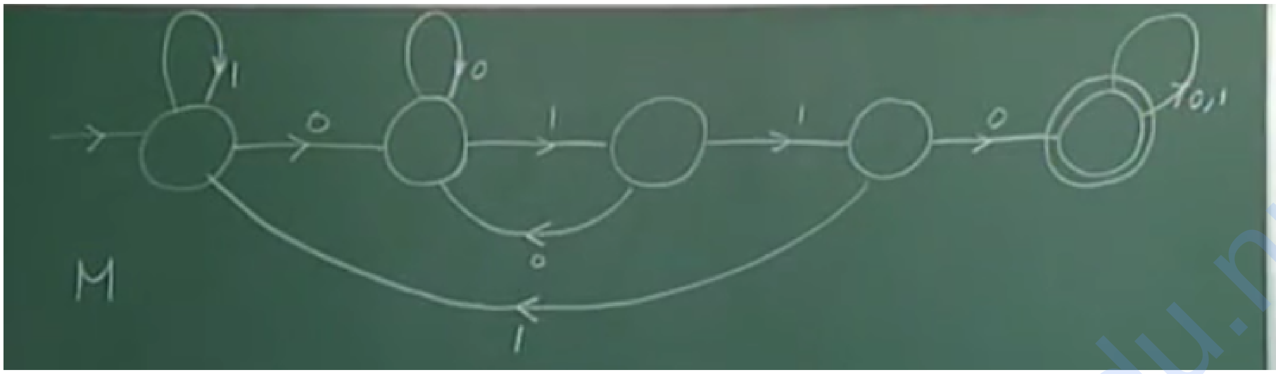
- Design a DFA that accepts the language L= { x ∈ {0,1}* : the third bit from the left is 1 in x}



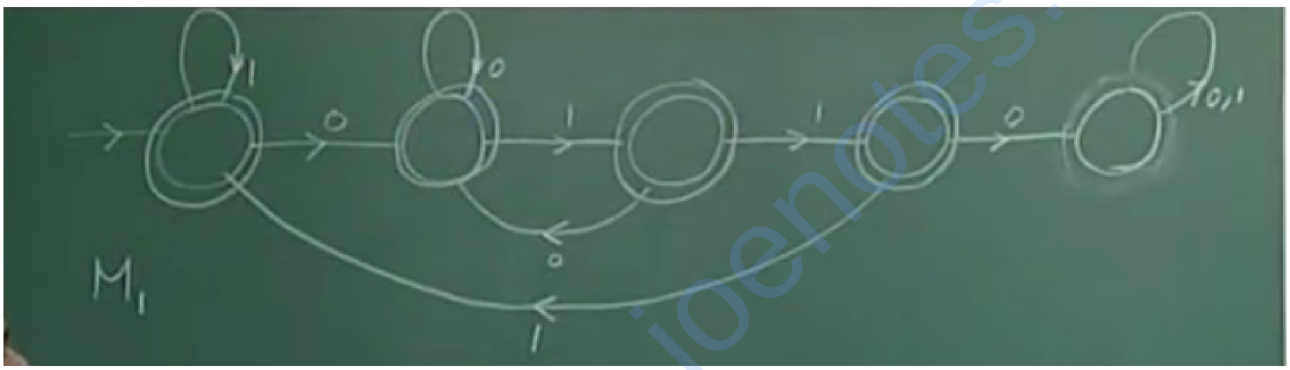
- Design a DFA that accepts the language L= { x ∈ {0,1}* : the third bit f x from its right end is 1 }



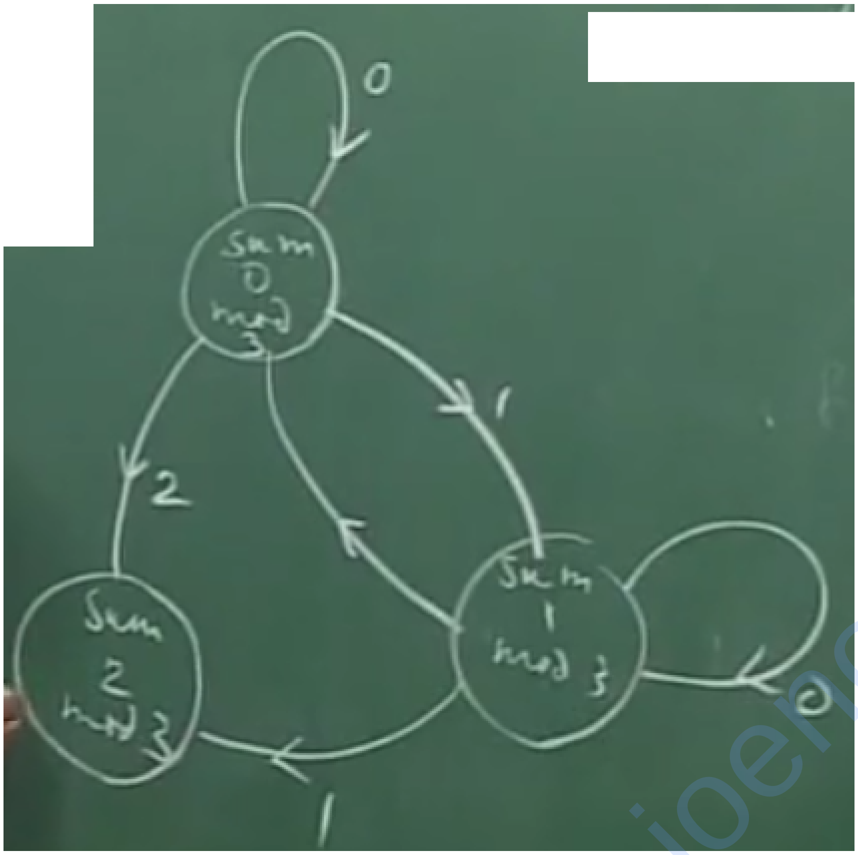
- Design a DFA that accepts the language L= { x ∈ {0,1}* : 0110 occurs as a substring in x}



- Design a DFA that accepts the language L= { x ∈ {0,1}* : 0110 does not occurs as a substring in x}

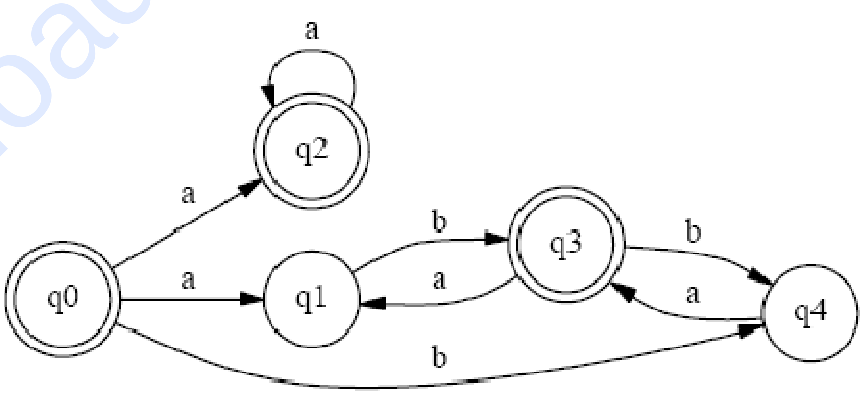


- Design a DFA that accepts the language L= { x ∈ {0,1,2} : the sum of digits in x is 2 mod 3 }

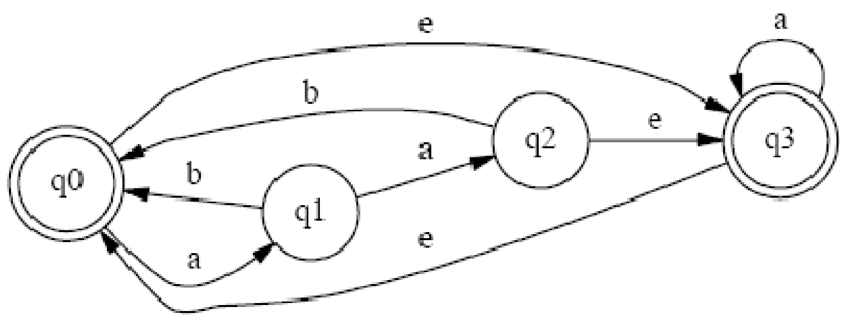


- Construct a DFA that accepts set of strings where the number of 0's in every string is multiple of three over alphabet {*0*, *1*}
- Construct a DFA that accepts set of strings where the number of 1's in every string is exactly 1 over alphabet {*0*, *1*}
- Design DFA for the language L= { $(01)^i 1^{2j}$: i ≥1, j ≥ 1};
- Draw state diagrams for nondeterministic finite automata that accept these languages. (You can use concept/techniques used in proving Closure properties of Regular Languages : Union, Concatenation, and Kleene Star)

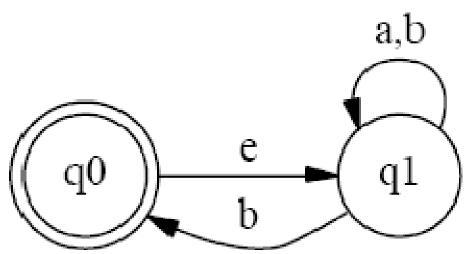
  (a) *(ab)* (ba)** U *aa**



  (b) *((ab* U *aab)*a*)**

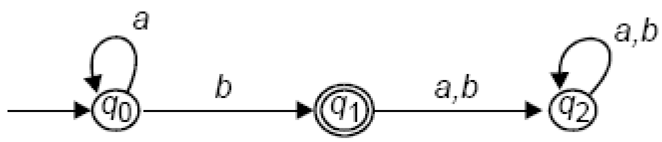


(c) *((a*b*a*)*b)**

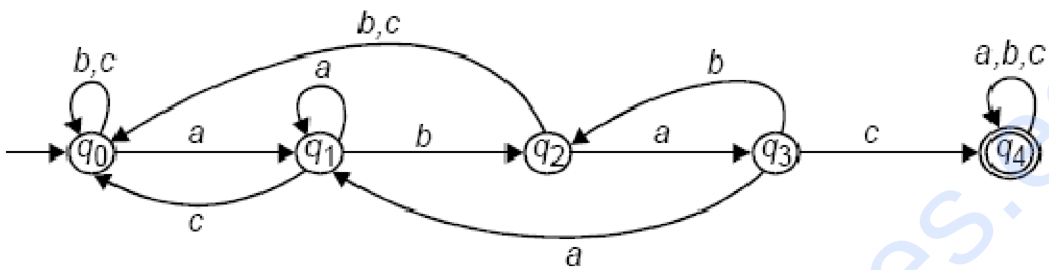


(d) *(baUb)*U(bbUa)**

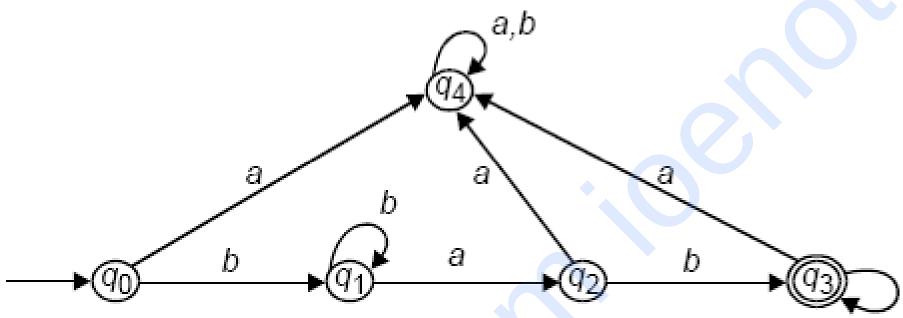
- Using the construction in the proofs of *Theorem: Regular languages are closed under Union, Concatenation, and Kleene Star , etc.*) construct finite automata accepting these languages.
  (a) a*(abUbaUe)b*
  (b) ((aU b)*(e U c)*)*
  (c) ((ab)* U (bc)*)ab
- Construct a simple nondeterministic finite automaton to accept the language (ab U aba) * a. Then apply to it the construction of Keene star of the proof of Theorem *Theorem: Regular languages are closed under Union, Concatenation, and Kleene Star , etc.*) to obtain a nondeterministic finite automaton accepting
  ((ab U aba)*a)*.
- Design a NFA that accepts the language L= { x ∈ {0,1}* : x has a substring either 010 or 11}
- Design a DFA that accepts the language L= { x ∈ {0,1}* : the fourth bit of x from its right end is 1 }
- Construct a NFA that accepts the language L={ $0^i1^j2^K$: i,j,k >=0} and the convert it into DFA
- Design a DFA that accepts the language L = {w | w contains at least one 1 and a even number of 0s follow the last 1}
- Design a DFA, the language recognized by the Automaton being $L = \{a^n b : n \geq 0\}$



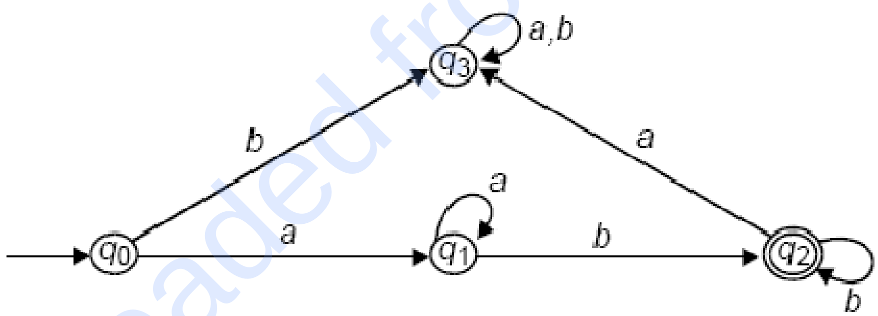
- Obtain the DFA that accepts/recognizes the language $L(M) = \{w \mid w = \{a, b, c\}^*$ and w contains the pat tern $abac\}$



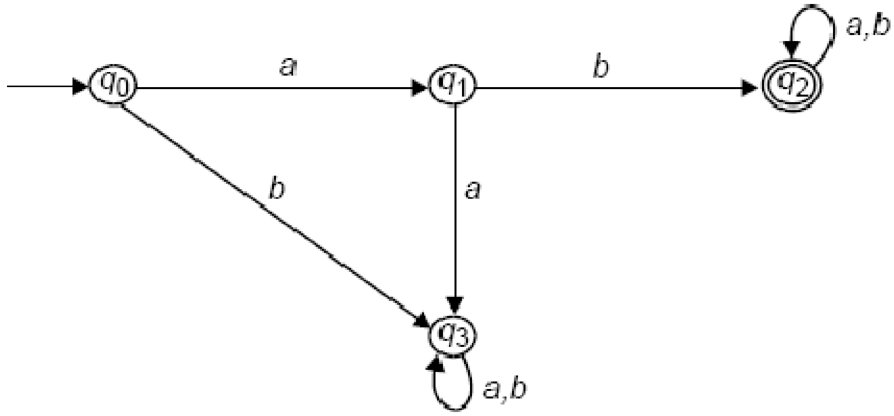
- Given $\sum= \{a, b\}$, construct a DFA that shall recognize the language $L = \{b^m ab^n : m, n \geq 0\}$.



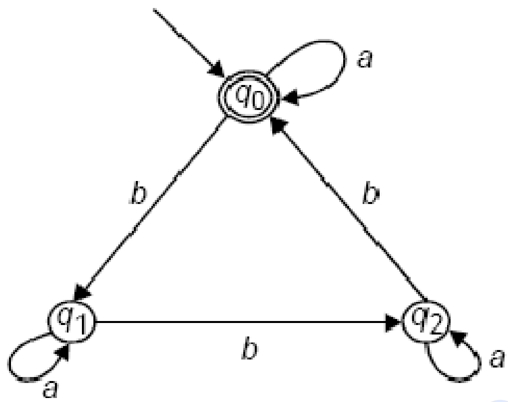
- Given $\sum=\{a, b\}$, construct a DFA which recognize the language $L = \{a^m b^n : m, n \geq 0\}$.



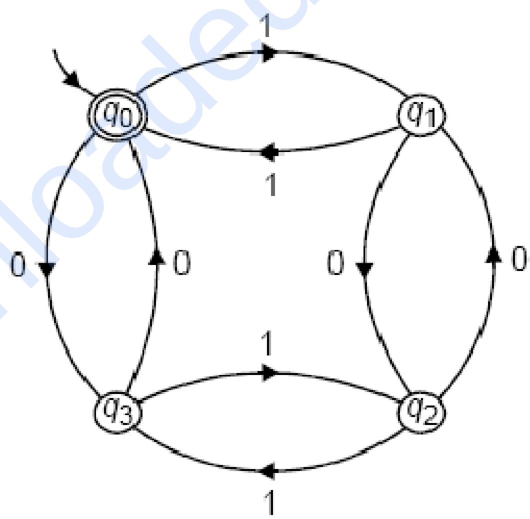
- Construct a DFA which recognizes the set of all strings on alphabet Σ= {a, b} starting with the prefix 'ab'.



- Determine the DFA that will accept those words from alphabets Σ= {*a*, *b*} where the number of *b*'s is divisible by three. Sketch the state table diagram of the finite Automaton *M* also.



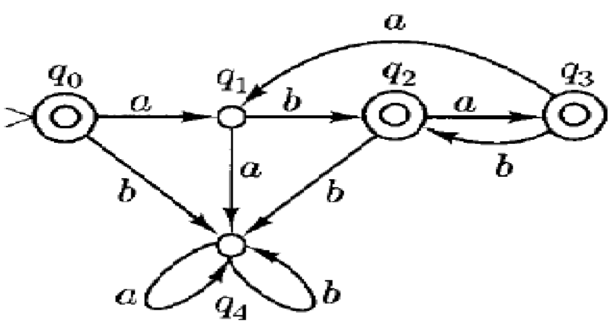
- Construct a finite automaton accepts all strings over {0, 1}
  a. having odd number of 0's
  b. having even number of 0's and even number of 1's.



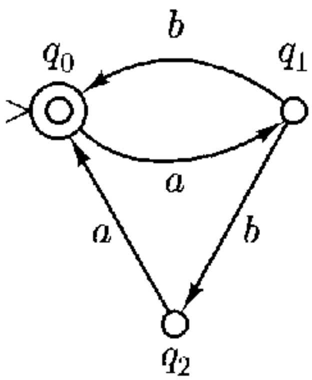
- Design a DFA for the language L=(ab∪aba)* and Convert it into NFA without e-transitions and with e-transitions

Solutions

To see that a nondeterministic finite automaton can be a much more convenient device to design than a deterministic finite automaton, consider the language L = (ab U aba)*, which is accepted by the deterministic finite automaton illustrated in below



L is accepted by the simple nondeterministic device shown in fig below



Fog: NFA for L = (ab U aba)*

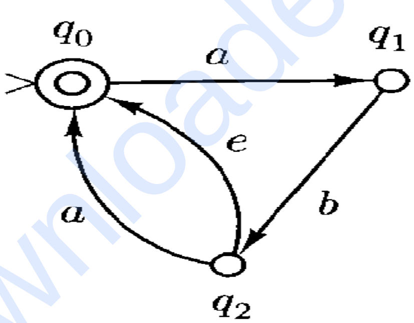
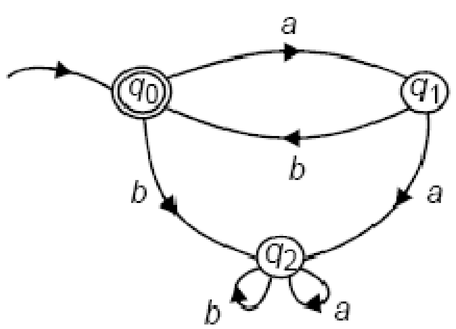


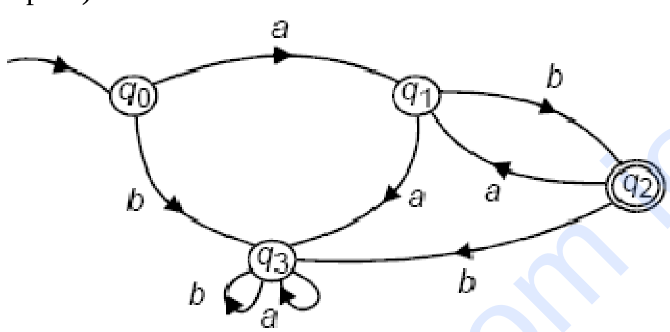
Fig: NFA for L = (ab U aba)* with e-transition

- Construct a DFA that accepts set of strings either starts with 01 or end with 01 over alphabet Σ= {*0*, *1*}

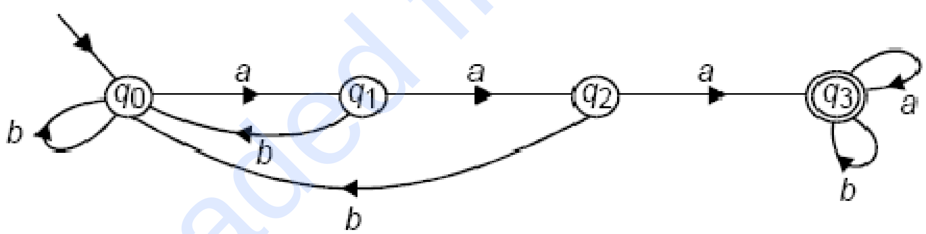
- Construct a NFA accepting language $L = (ab)^*(ba)^* \cup aa^*$, and Convert the designed NFA into DFA.
- Determine the DFA if $\Sigma = \{a, b\}$ for Language generated $L_A = (ab)^* = (ab)^{n\,:}$, $n \geq 0$ (e-not accepted)



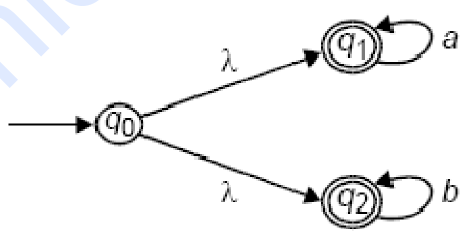
- Determine the DFA if $\Sigma = \{a, b\}$ for Language generated $L_B = (ab)^* = (ab)^{n\,:}$, $n \geq 1$ (e-not accepted)



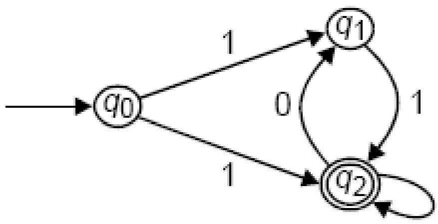
- Determine the DFA with the set of strings having ‘aaa’ as a subword.



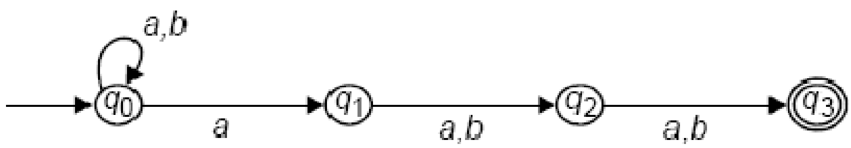
- Determine an NFA accepting the language L= $\{a^* \cup b^*\}$

(b)


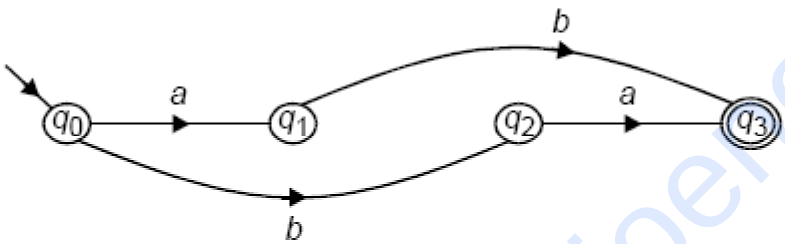
- Determine an NFA accepting all strings over {0,1} which end in 1 but does not contain the substring 00.



- Obtain an NFA which should accept a language L_A, given by $L_A = \{ x \in \{a, b\}^* : |x| \geq 3$ and third symbol of x from the right is { 'a'}.

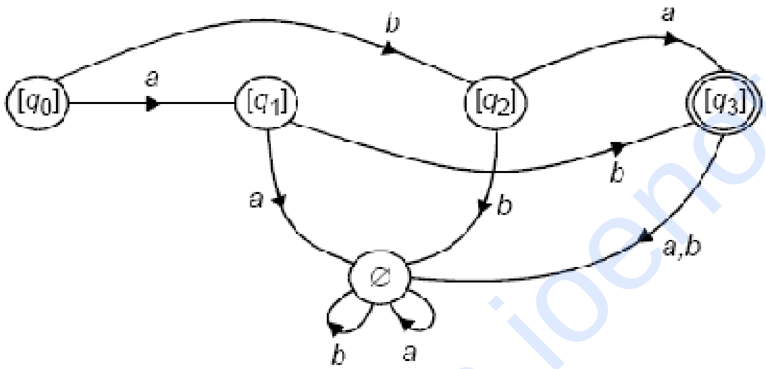


- Determine a NFA accepting {ab, ba} and use it to find a DFA accepting it.



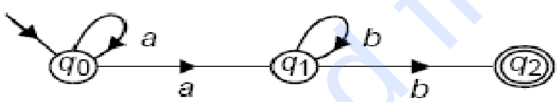
The state table corresponding to the DFA is derived by using subset construction. State table for DFA is as shown below.

	a	b
$[q_0]$	$[q_1]$	$[q_2]$
$[q_1]$	$\varnothing$	$[q_3]$
$[q_2]$	$[q_3]$	$\varnothing$
$[q_3]$	$\varnothing$	$\varnothing$
$\varnothing$	$\varnothing$	$\varnothing$



The DFA is as shown above.

- Given the NDFA as shown in Fig. , Determine the equivalent DFA for the above given NDA.

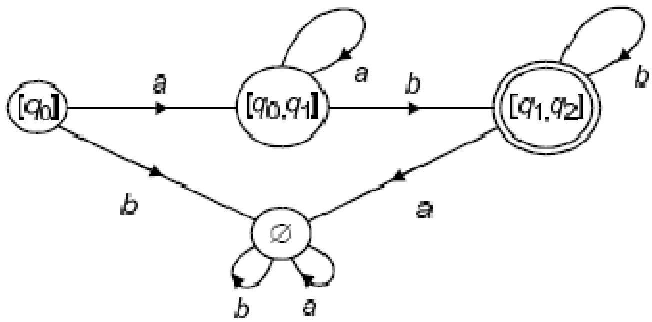


Solutions

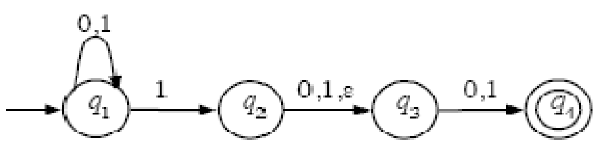
Conversion of NDA to DFA is done through subset construction as shown in the State table diagram below.

	a	b
$[q_0]$	$[q_0, q_1]$	$\varnothing$
$[q_0, q_1]$	$[q_0, q_1]$	$[q_1, q_2]$
$[q_1, q_2]$	$\varnothing$	$[q_1, q_2]$
$\varnothing$	$\varnothing$	$\varnothing$

The corresponding DFA is shown below. Please note that here any subset containing q_2 is the final state.



- An NFA that accepts all strings over {0, 1} that contain a 1 either at the third position from the end or at the second position from the end is given below, Determine the equivalent DFA .



Solutions

Conversion to DFA

The state set consists of: $\emptyset$, $\{q_1\}$, $\{q_2\}$, $\{q_3\}$, $\{q_4\}$, $\{q_1, q_2\}$, $\{q_1, q_3\}$, $\{q_1, q_4\}$, $\{q_2, q_3\}$, $\{q_2, q_4\}$, $\{q_3, q_4\}$, $\{q_1, q_2, q_3\}$, $\{q_1, q_2, q_4\}$, $\{q_1, q_3, q_4\}$, $\{q_2, q_3, q_4\}$, $\{q_1, q_2, q_3, q_4\}$.

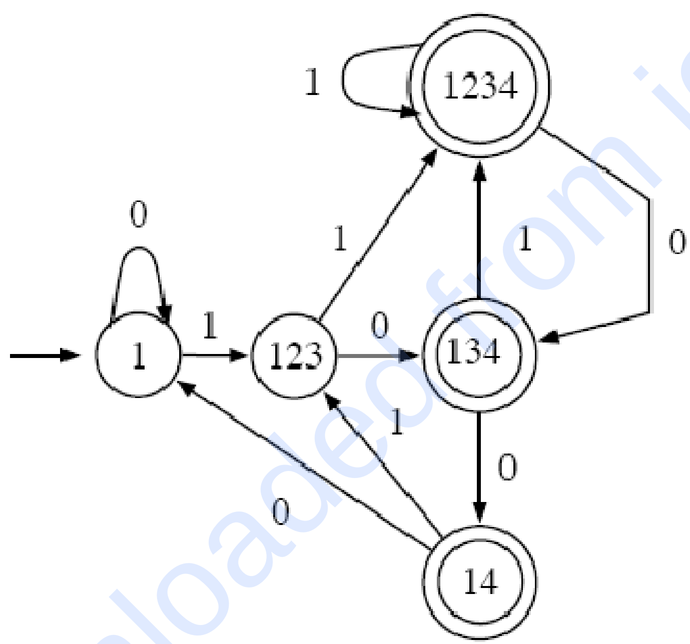
F consists of: $\{q_4\}, \{q_1, q_4\}, \{q_2, q_4\}, \{q_3, q_4\}, \{q_1, q_2, q_4\}, \{q_1, q_3, q_4\}, \{q_2, q_3, q_4\}, \{q_1, q_2, q_3, q_4\}$.

The initial state is $\{q_1\}$.

Transition

State	0	1
$\{q_1\}$	$\{q_1\}$	$\{q_1, q_2, q_3\}$
$\{q_1, q_2, q_3\}$	$\{q_1, q_3, q_4\}$	$\{q_1, q_2, q_3, q_4\}$
$\{q_1, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_1, q_2, q_3, q_4\}$
$\{q_1, q_4\}$	$\{q_1\}$	$\{q_1, q_2, q_3\}$
$\{q_1, q_2, q_3, q_4\}$	$\{q_1, q_3, q_4\}$	$\{q_1, q_2, q_3, q_4\}$

The other states are unreachable from the initial state.



[B] Problems Related To State Minimization

1. Minimise the following DFA represnted as transition table,

Current State	Input symbol	
	0	1
$\longrightarrow q_0$	q_1	q_2
q_1	q_2	q_3
q_2	q_2	q_4
$*q_3$	q_3	q_3
$*q_4$	q_4	q_4
q_5	q_5	q_4

2. Minimise the following DFA

Current state	input symbol	
	a	b
→q0	q5	q1
q1	q2	q6
*q2	q2	q0
q4	q5	q7
q5	q6	q2
q6	q4	q6
q7	q2	q6
q3	q6	q2

Step 1: Eliminate any state that can't be reached from the start state

In above, the state q3 can't be reached. So remove the corresponding to q3 from the transition table. Now the new transition table is

Current state	input symbol	
	a	b
→q0	q5	q1
q1	q2	q6
*q2	q2	q0
q4	q5	q7
q5	q6	q2
q6	q4	q6
q7	q2	q6

Step 2: Divided the rows of the table into 2 sets as

1. one set containing only rows which starts from non final states

Set 1

q0	q5	q1
q1	q2	q6
q4	q5	q7
q5	q6	q2
q6	q4	q6
q7	q2	q6

2. another set containing those rows which start from final states

* q2	q2	q0

Step 3a: Consider the set 1

q0	q5	q1	Row1
q1	q2	q6	Row2
q4	q5	q7	Row3
q5	q6	q2	Row4
q6	q4	q6	Row5
q7	q2	q6	Row6

Row 2 and Row 6 are similar since q1 and q7 transit to same states on inputs a and b

so remove one of them (for instance q7) and replace q7 with q1 in rest we get

Set 1

q0	q5	q1	Row1
q1	q2	q6	Row2
q4	q5	q1	Row3
q5	q6	q2	Row4
q6	q4	q6	Row5

Now Row 1 and Row 3 are similar. So remove one of them (for instance q4) and

replace q4 with q0 in the rest we get

Set 1

q0	q5	q1	Row1
q1	q2	q6	Row2
q5	q6	q2	Row3
q6	q0	q6	Row4

Now there are no more similar rows

3b. Consider the set 2

Set 2

*q2	q2	qq0

Do the same process for set 2

But it contains only one row .It is already minimized

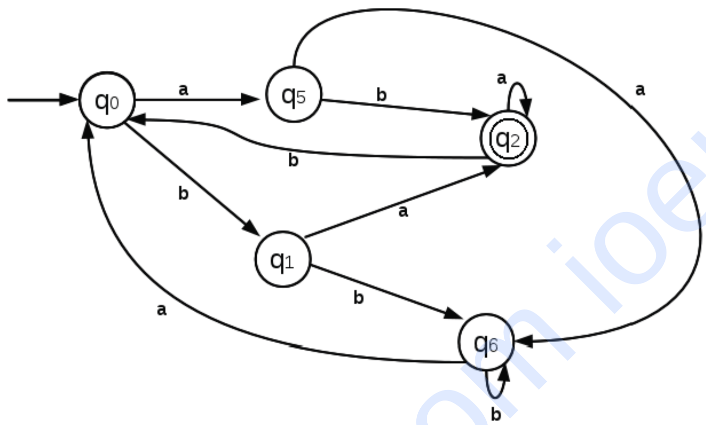
Step 4

Combine set 1 and set 2 we get

Current state	input symbol	
	a	b
→q0	q5	q1
q1	q2	q6
q5	q6	q2
q6	q0	q6
*q2	q2	q0

Now this is minimized DFA

The transition diagram is



[C] Problem Related to Pumping Lemma:

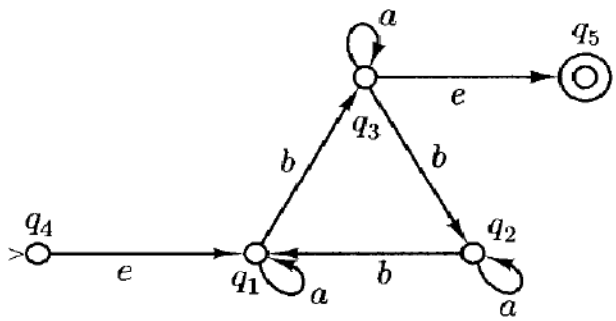
Use Pumping Lemma and Prove that:

- $L = \{w \mid w \in \{0, 1\}^*$ and has an equal number of 0s and 1s $\}$ is not regular.
- The language $L = \{vv \mid v \in \{0, 1\}^*\}$ is not regular (F is the language of all even length strings over {0, 1} whose first half is identical to the second half).
- $L = \{0^i 1^j : i > j\}$ is not regular
- $L = \{1^{n2:}\ n \geq 0\}$ is not regular.
- $L = \{1^n : n$ is a prime number$\}$ is not regular.
- $L = \{a^n\ b\ a^n$ for n=0,1,2……. $\}$ in not regular
- $L = \{0^n 1^{2n} : n \geq 0\}$ is not regular.
- $L = \{a^n b^n : n \geq 0\}$ is not regular.
- $L = \{a^n b2^n : n \geq 0\}$ is not regular.

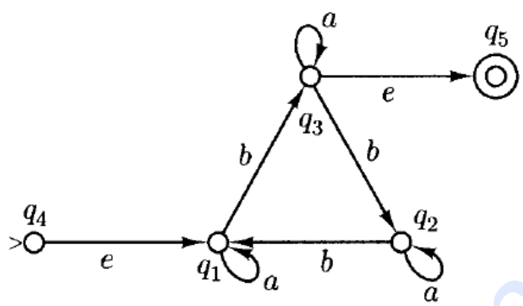
- $L = \{ a^{n!} : n \geq 1 \}$ is not regular

[D] Problems related to Regular Expressions

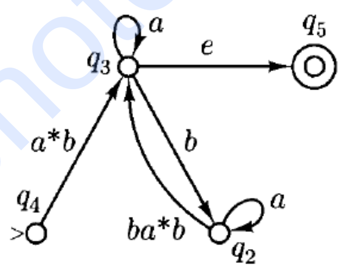
1. Find the regular expression from the NFA given below



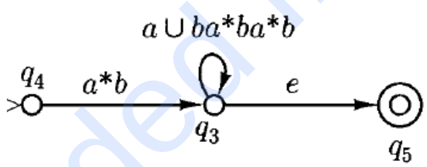
Solutions



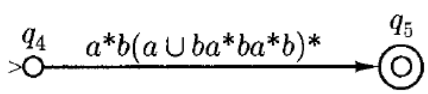
(a)



(b)



(c)



(d)

2. Obtain the regular expressions for the following sets:
   a. The set of all strings over {a, b} beginning and ending with ‘a’.
   b. $\{b^2, b^5, b^8, \ldots..\}$
   c. $\{a^{2n+1} \mid n > 0\}$

Solution

(a) The regular expression for 'the set of all strings over $\{a, b\}$ beginning and ending with 'a' is given by:

$$a\,(a+b)^*a$$

(b) The regular expression for $\{b^2, b^5, b^8, \ldots\ldots\}$ is given by:

$$bb\,(bbb)^*$$

(c) The regular expression for $\{a^{2n+1} \mid n > 0\}$ is given by:

$$a\,(aa)^*$$

3. Obtain the regular expressions for the languages given by:

(a) $L_1 = \{a^{2n}b^{2m+1} \mid n \geq 0, m \geq 0\}$

(b) $L_2 = \{a, bb, aa, abb, ba, bbb, \ldots\ldots\}$

(c) $L_3 = \{w \in \{0,1\}^* \mid w$ has no pair of consecutive zeros$\}$

(d) $L_4 = \{$strings of 0's and 1's ending in 00$\}$

Solutions

(a) $L_1 = \{a^{2n}b^{2m+1} \mid n \geq 0, m \geq 0\}$ denotes the regular expression

$$(aa)^*(bb)^*b$$

(b) The regular expression for the language $L_2 = \{a, bb, aa, abb, ba, bbb, \ldots\ldots\}$

$$(a+b)^*\,(a+bb)$$

(c) The regular expression for the language $L_3 = \{w \in \{0,1\}^* \mid w$ has no pair of consecutive zeros$\}$ is given by

$$(1^*011^*)^*(0+\lambda)+1^*(0+\lambda)$$

(d) The regular expression for the language $L_4 = \{$strings of 0's and 1's beginning with 0 and ending with 1$\}$ is given by

$$0\,(0+1)^*1$$

4. Find regular expressions over S = {a, b} for the language defined as follows:

(a) $L_1 = \{a^m b^m : m > 0\}$

(b) $L_2 = \{b^m a b^n : m > 0, n > 0\}$

(c) $L_3 = \{a^m b^m, m > 0, n > 0\}$

Solutions

(a) Given $L_1 = \{a^m b^m : m > 0\}$,

L_1 has those words beginning with one or more a's followed by one or more b's.

Therefore the regular expression is

$$aa^+bb^+ \text{ (or) } a^*ab^*b$$

(b) Given $L_2 = \{b^m ab^n : m > 0, n > 0\}$. This language has those words w whose letters are all b except for one 'a' that is not the first or last letter of w.

Therefore the regular expression is

$$bb^*abb^*$$

(c) Given $L_3 = \{a^m b^m, m > 0\}$.

There is no regular expression for this beginning as L_3 is not regular.

5. Determine all strings in L((a + b)* b(a + ab)*) of length less than four.

Solutions

b, ab, bb, ba, aab, abb, bab, bbb, baa, bba, aba

6. Find the regular expressions for the languages defined by

(i) $L_1 = \{a^n b^m : n \geq 1, m \geq 1, nm \geq 3\}$

(ii) $L_2 = \{ab^n w : n \geq 3, w \in \{a, b\}^+\}$

(iii) $L_3 = \{vwv : v, w \in \{a, b\}^*, |v| = 2\}$

(iv) $L_4 = \{w : |w| \bmod 3 = 0\}$

Solutions

(i) Regular Expression for $L_1 = \{a^n b^m : n \geq 1, m \geq 1, nm \geq 3\}$ is given by

$$aa(a^*)b(b^*) + a(a^*)bb(b^*)$$

(ii) Regular Expression for $L_2 = \{ab^n w : n \geq 3, w \in \{a, b\}^+\}$ is given by

$$abbb(b^*)(a+b)(a+b)^*$$

(iii) Regular Expression for $L_3 = \{vwv : v, w \in \{a, b\}^*, |v| = 2\}$ is given by

$$(a+b)(a+b)(a+b)^*(a+b)(a+b)$$

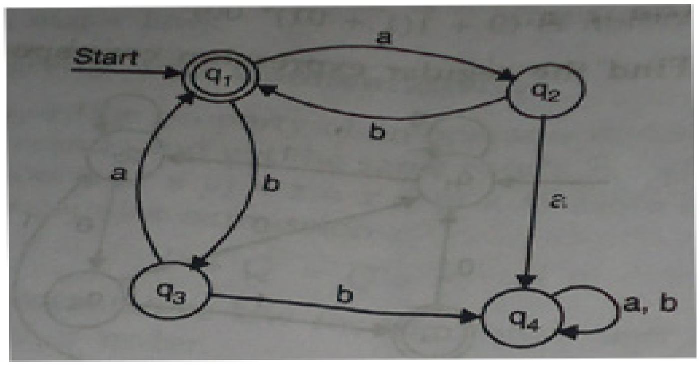
(iv) The regular expression for $L_4 = \{w : |w| \bmod 3 = 0\}$ is given by

$$(aaa + bbb + ccc + aab + aba + abb + bab + bba + cab + cba + cbb + caa)^*$$

7. Determine the NFA for regular expression a. $(a+b)^*$.b. b - **See on AK Panday Book**

8. Construct the e-NFA for the regular expression $(0+1)^*$ (0+1) AK Panday Book

9. Find the regular expression for the DFAs given below (Use Arden's Theorem)- AK Panday Book



Solution

Here we write equations for every state.

We write,

$q_1 = q_2b + q_3a + \varepsilon$

The term q_2b because there is an arrow from q_2 to $q_1$on input symbol b.

The term q_3a because there is an arrow from q_3 to $q_1$on input symbol a.

The term ε because $q_1$is the start state.

$q_2 = q_1a$

The term q_1a because there is an arrow from q_1 to $q_2$on input symbol a.

$q_3 = q_1b$

The term q_1b because there is an arrow from q_1 to $q_3$on input symbol b.

$q_4 = q_2a + q_3b + q_4a + q_4b$

The term q_2a because there is an arrow from q_2 to $q_4$on input symbol a.

The term $q_3 b$ because there is an arrow from q_3 to $q_4$on input symbol b.

The term $q_4 b$ because there is an arrow from q_4 to $q_4$on input symbol b.

The final state is q_1.

Putting $q_2$and q_3 in the first equation (corresponding to the final state), we get,

$q_1 = q_1 ab + q_1 ba + \varepsilon$

$q_1 = q_1(ab + ba) + \varepsilon$

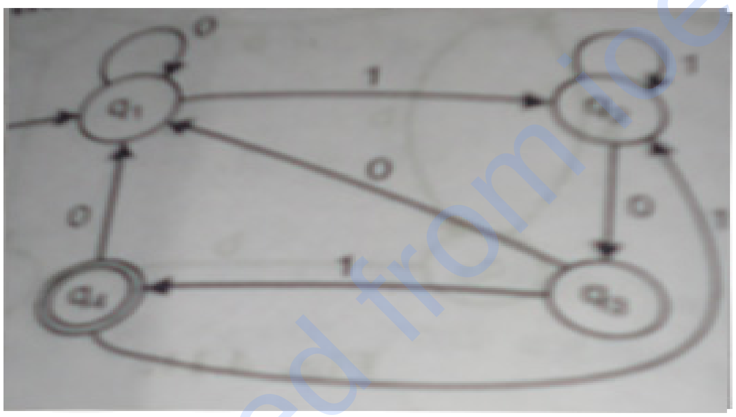
$q_1 = \varepsilon + q_1(ab + ba)$

From Arden's theorem,

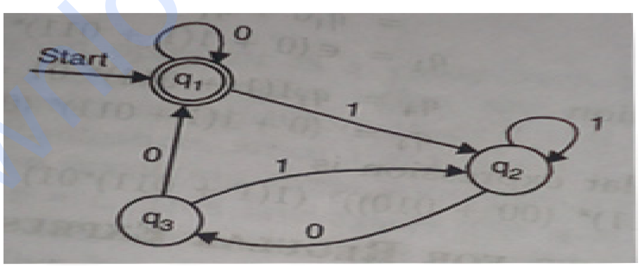
$q_1 = \varepsilon(ab + ba)^*$

$q_1 = (ab + ba)^*$

So the regular expression is, $((ab)/(ba))^*$



Solutions : Do yourself



Solutions

Let us write the equations

$$q_1 = q_1 0 + q_3 0 + \varepsilon$$

$$q_2 = q_1 1 + q_2 1 + q_3 1$$

$$q_3 = q_2 0$$

$$q_2 = q_1 1 + q_2 1 + q_3 1$$

$$q_2 = q_1 1 + q_2 1 + (q_2 0)1$$

$$q_2 = q_1 1 + q_2 (1 + 01)$$

$q_2 = q_1 1(1 + 01)^*$ (From Arden's theorem)

Consider the equation corresponding to final state,

$$q_1 = q_1 0 + q_3 0 + \varepsilon$$

$$q_1 = q_1 0 + (q_2 0)0 + \varepsilon$$

$$q_1 = q_1 0 + (q_1 1(1 + 01)^*)0)0 + \varepsilon$$

$$q_1 = q_1 (0 + 1(1 + 01)^*)00) + \varepsilon$$

$$q_1 = \varepsilon + q_1 (0 + 1(1 + 01)^*)00)$$

$$q_1 = \varepsilon (0 + 1(1 + 01)^*)00)^*$$

$$q_1 = (0 + 1(1 + 01)^*)00)^*$$

Since q_1 is a final state, the regular expression is,

$$(0/(1(1 + 01)^*)00))^*$$