Heat and work.

A gas expands from an initial state where $P_1 = 550$ kpa and $V_1 = 0.1\,m^3$ to final state $P_2 = 1000$ kpa. The relationship between pressure and volume during the process is $PV^2 = C$. Determine the work done in kJ.

Solution:

$P_1 = 550$ kpa

$P_2 = 1000$ kpa

$V_1 = 0.1\,m^3$

$V_2 = ?$

Workdone $(W) = ?$

Given that,

$$PV^2 = C$$

$$P_1V_1^2 = P_2V_2^2$$

$$\frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^2$$

$$\frac{550}{1000} = \left(\frac{V_2}{0.1}\right)^2$$

$$V_2 = 0.074\ m^3$$

So,

$$\text{workdone} = \frac{1}{1-n}\left[P_2V_2 - P_1V_1\right]$$

$$= \frac{1}{1-2}\left[1000 \times 0.074 - 550 \times 0.1\right]$$

$$\text{workdone} = -19.0\ kJ$$

A mass of gas is compressed in a quasi-static process from 80 kpa, $0.1\,m^3$ to 0.4 MPa, $0.03\,m^3$. Assuming that the pressure and the volume are related by pv^n = constant. Find the workdone by the gas system.

sol^n,

=). Given,

$P_1 = 80$ kpa

$V_1 = 0.1\ m^3$

$P_2 = 0.4$ Mpa $= 0.4 \times 1000$ kpa

$V_2 = 0.03\ m^3$

workdone (w) = ?

Given, pv^n = constant

$$p_1 V_1^n = p_2 V_2^n$$

or $$\left(\frac{p_1}{p_2}\right) = \left(\frac{V_2}{V_1}\right)^n$$

Taking ln on both sides,

or $$\ln\left(\frac{p_1}{p_2}\right) = n \ln\left(\frac{V_2}{V_1}\right)$$

or $$\ln\left(\frac{80}{0.4 \times 1000}\right) = n \ln\left(\frac{0.03}{0.1}\right)$$

or $$-1.609 = n \cdot -1.203$$

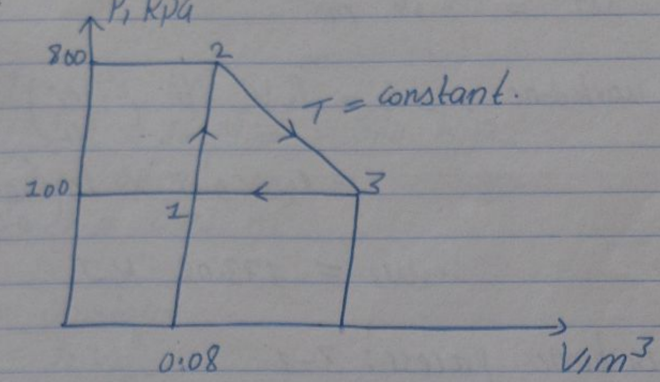
$\therefore$ $$n = 1.33$$

Now, work done by gas system $= \dfrac{P_2 V_2 - P_1 V_1}{1-n}$

$$= \frac{0.4 \times 1000 \times 0.03 - 80 \times 0.1}{1 - 1.33}$$

$\therefore$ $${}_1W_2 = -12.1212\ kJ$$

3. For the cycle shown in figure, determine work output and heat transfer.



Soln,

From figure, process is 1–2–3

where process 1-2 : Isochoric process.

process 2-3 : Isothermal process.

process 3-1 : Isobaric process

$P_2 = 800$ kPa

$P_1 = 100$ kPa $= P_3$

$V_1 = 0.08\ m^3 = V_2$.

a) Isochoric process (1-2):

$${}_1W_2 = \int p \cdot dv$$

$$= 0$$

Since, $V_1 = V_2$, change in volume is zero.

b. Isothermal process (2-3):

$$PV = C$$

$$P_2V_2 = P_3V_3$$

or $800 \times 0.08 = V_3 \times 100$

$\therefore V_3 = 0.64 \ m^3$

workdone, ${}_2W_3 = P_2 V_2 \ln\left(\frac{V_3}{V_2}\right)$

$= 800 \times 0.08 \times \ln\left(\frac{0.64}{0.08}\right)$

$\therefore {}_2W_3 = 133.08 \ kJ$

Isobaric process: 3-1

${}_3W_1 = + P_3 (V_1 - V_3)$.

$= 100 (0.08 - 0.64)$

$= -56 \ kJ$

So,

$\therefore$ net workdone $(W) = {}_1W_2 + {}_2W_3 + {}_3W_1$

$= -56 + 0 + 133.08 \ kJ$

$= 77.08 \ kJ$

Heat transfer = net workdone = 77.08 kJ.

According to 1st law of thermodynamics, for a cyclic process, net heat transfer is equal to net work done.

i.e, $\oint \delta Q = \oint \delta W$

4. A mass of gas is compressed in a quasi-static process from pressure of 72 kpa and volume of $0.1 m^3$ to pressure of 0.36 Mpa and volume of $0.03 \ m^3$. If the pressure and volume are related by PV^n = constant.

Find workdone by the gas system.

Soln,

Given,

=> Pressure (P_1) = 72 kpa

Final pressure (P_2) = 0.36 Mpa = 360 kpa.

Initial volume (V_1) = 0.1 m^3

Final volume (V_2) = 0.03 m^3

Given,

$$PV^n = \text{constant}$$

$$P_1V_1^n = P_2V_2^n$$

or $$\left(\frac{P_1}{P_2}\right) = \left(\frac{V_2}{V_1}\right)^n$$

or $$\ln\left(\frac{72}{360}\right) = n \ln\left(\frac{0.03}{0.1}\right)$$

or $$-1.61 = n.(-1.20)$$

$$\therefore n = 1.33$$

Now,

$$\text{workdone by system } (W) = \frac{P_2V_2 - P_1V_1}{1-n}$$

$$= \frac{360 \times 0.03 - 0.1 \times 72}{1-1.33}$$

$$= -10.90 \text{ kJ}$$

$\therefore$ Workdone by gas system is -10.90 kJ.

A gas expands from an initial state where $P_1 = 500$ kpa and $V_1 = 0.1 m^3$ to final state where $P_2 = 100$ kpa. The relationship between pressure and volume during the process is $PV = C$. Determine work in kJ.

Sol^n,

$\Rightarrow$ $P_1 = 500$ kpa

$V_1 = 0.1\ m^3$

$P_2 = 100$ kpa.

$V_2 = ?$

We know that,

$PV = C$

or $P_1V_1 = P_2V_2$

or $\frac{P_1}{P_2} = \frac{V_2}{V_1}$

or $\frac{500}{100} = \frac{V_2}{0.1}$

$\therefore V_2 = 0.5\ m^3$

Since this is isothermal process,

$\therefore$ work done $(W) = P_1V_1 \ln\left(\frac{P_1}{P_2}\right)$

$= 500 \times 0.1 \ln\left(\frac{500}{100}\right)$

$= 50 \times 1.609$

${}_1W_2 = 80.471$ kJ

6. A gas is compressed from $V_1 = 0.09\ m^3$, $P_1 = 1$ bar, to $V_2 = 0.03\ m^3$, $P_2 = 3$ bar. Pressure and volume are related linearly during the process for the gas. Find the workdone.

Sol^n,

$V_1 = 0.09\ m^3$

$P_1 = 1$ bar
$P_2 = 3$ bar
$V_2 = 0.03\ m^3$
Given, $PV = C$
So, this is isothermal process.
Then, workdone $= P_1V_1 \ln\left(\frac{P_1}{P_2}\right)$

$= 1 \times 0.09 \ln\left(\frac{1}{3}\right)$

$= -0.09 \times 1.09861$
$= -0.098875$ bar
$= -0.098875 \times 1 \times 10^5 J$
$= -9.887.510\ J$
$\therefore$ workdone $= -9.887$ kJ

7. 0.2 m^3 of an ideal gas at a pressure of 2 MPa and 600 k is expanded isothermally to 5 times the initial volume. It is then cooled to 300 k at constant volume and then compressed back polytropically to its initial state. Determine total work done during the cycle.

sol^n,
$\Rightarrow$ Initial condition, case ①:
$V_1 = 0.2\ m^3$
$P_1 = 2$ MPa $= 2 \times 1 \times 10^6$ pa
$T_1 = 600$ k

Final condition,
$T_2 = 600$ k

$V_2 = 5V_1 = 5 \times 0.2 = 1\ m^3$
$T_3 = 300 k$

workdone from 1 to 2 process,

$${}_1W_2 = P_1V_1 \ln\left(\frac{P_1}{P_2}\right)$$
$$= P_1V_1 \ln\left(\frac{V_2}{V_1}\right)$$
$$= 2\times10^6 \times 0.2 \times \ln\left(\frac{1}{0.2}\right)$$
$$\therefore {}_1W_2 = 643.78\ kJ$$

When gas is expanded isothermally 5 times the initial ~~value~~ volume, $V_2 = 1\ m^3$

case (ii):
It is cooled to 300k at constant volume, so this process is called isochoric process and workdone is 0.

$$\therefore {}_2W_3 = 0\ kJ$$

From case (i): isothermal process,
$$PV = C$$
or $$P_1V_1 = P_2V_2$$
or $$P_2 = 2\times10^6 \times \frac{0.2}{1}$$
$$\therefore P_2 = 0.4\times10^6\ pa$$

we have, $pV = nRT$

$V = \text{constant}$

$$\frac{P}{T} = C$$

Applying this on 2-3 process,

$$\frac{P_3}{T_3} = \frac{P_2}{T_2}$$

or $P_3 = 0.4 \times 10^6 \times \frac{300}{600}$

$\therefore P_3 = 0.2 \times 10^6$ pa

Applying $pV^n = C$ to polytropic process 3-1

$$P_3 V_3^n = P_1 V_1^n$$

or $\left(\frac{V_3}{V_1}\right)^n = \frac{P_1}{P_3}$

Taking log on both sides,

$$\ln\left(\frac{V_3}{V_1}\right)^n = \ln\left(\frac{P_1}{P_3}\right)$$

or $n \ln\left(\frac{V_3}{V_1}\right) = \ln\left(\frac{P_1}{P_3}\right)$

or $n \ln\left(\frac{1}{0.2}\right) = \ln\left(\frac{2 \times 10^6}{0.2 \times 10^6}\right)$

$\therefore n = 1.43$

So,

$${}_3W_1 = \frac{P_1V_1 - P_3V_3}{1-n}$$

$$= \frac{2\times10^6 \times 0.2 - 0.2\times1\times10^6}{1-1.431}$$

$\therefore\ {}_3W_1 = -464.04\ kJ$

Thus, net work done $= {}_1W_2 + {}_2W_3 + {}_3W_1$

$= 643.78 + 0 - 464.04$

$= 179.74\ kJ.$

8. A cylinder fitted with a piston contains propane gas at 100 kpa, 300k with a volume of $0.2\ m^3$. The gas is ~~always~~ slowly compressed according to the relation $pV^{1.4} = c$ to temperature of 340k.

a. What is final pressure?

b. How much work is done during the process?

Sol^n,

=> Given, Initial condition,

$P_1 = 100\ kpa$

$V_1 = 0.2\ m^3$

$T_1 = 300\ k.$

$T_2 = 340\ k$

$P_2 = ?$

Work done $(W) = ?$

We have,

$$pV^{1.4} = c$$

So, $P_1V_1^{1.4} = P_2V_2^{1.4}$

$$\left(\frac{P_1}{P_2}\right) = \left(\frac{V_2}{V_1}\right)^{1.4} \quad \text{---} \ (1)$$

or $\dfrac{P_2 V_2}{T_2} = \dfrac{P_1 V_1}{T_1}$

or $P_2 V_2 = \dfrac{P_1 V_1}{T_1} \times T_2$

or $P_2 V_2 = \dfrac{100 \times 10^3 \times 0.2}{300} \times 340$

$\therefore P_2 V_2 = 22666.7$ ——— (II)

From (I),

$\ln\left(\dfrac{P_1}{P_2}\right) = 1.1 \ln\left(\dfrac{V_2}{V_1}\right)$

or $P_1 V_1^{1.1} = P_2 V_2^{1.1}$

or $P_2 V_2^{1.1} = 100 \times 10^3 \times (0.2)^{1.1}$

$\therefore P_2 V_2^{1.1} = 17026.79$ ——— (III)

Dividing (III) by (II)

or $\dfrac{P_2 V_2^{1.1}}{P_2 V_2} = \dfrac{17026.79}{22666.67}$

or $V_2^{0.1} = \dfrac{17026.79}{22666.67}$

$\therefore V_2 = 0.0572\ m^3$

Replacing value of V_2 in eq^n (II),

$P_2 = \dfrac{22666.7}{0.0572}$

$\therefore P_2 = 396270.45$ pa

$$\text{work done } (W) = \frac{P_2V_2 - P_1V_1}{1-n}$$

$$= \frac{22666.67 - 100 \times 0.2 \times 10^3}{1 - 1.1}$$

$$\therefore \quad W = -26666.7 \text{ J}$$

9. A cylinder fitted with a movable piston contains 0.04 m^3 of air at 10 bar pressure and 400 K temperature. The air expands according to law $P = \frac{A}{V^2} - \frac{B}{V}$ to a final pressure of 1 bar and volume of 0.2 m^3. Determine work done.

soln,

=> $V_1 = 0.04 \text{ m}^3$

$P_1 = 10$ bar

$P_2 = 1$ bar

$V_2 = 0.2 \text{ m}^3$

$T_1 = 400$ K

By question,

$$P = \frac{A}{V^2} - \frac{B}{V}$$

When, $P_1 = 10$ bar and $V_1 = 0.04 \text{ m}^3$,

$$10 = \frac{A}{(0.04)^2} - \frac{B}{0.04}$$

$$10 = \frac{A - B(0.04)}{(0.04)^2}$$

$$A - 0.04B = 10\,(0.04)^2 \quad \text{——} \quad (1)$$

when $P_2 = 1$ bar and $V_2 = 0.2 \text{ m}^3$,

$$1 = \frac{A}{(0.2)^2} - \frac{B}{0.2}$$

or $$1 = \frac{A - 0.2B}{(0.2)^2}$$

or $$A - 0.2B = (0.2)^2 \quad \text{——} \quad (ii)$$

subtracting eqn (ii) from (i),

or $$A - (0.04)B - A + 0.2B = 10(0.04)^2 - (0.2)^2$$

or $$0.16B = -0.024$$

$$\therefore B = -0.15$$

Replacing value of B in eqn (ii),

$$A - (0.2)(-0.15) = (0.2)^2$$

$$\therefore A = 0.01$$

So, $$P = \frac{0.01}{V^2} - \left(-\frac{0.15}{V}\right) \text{ bar}$$

$$= \left[\frac{0.01}{V^2} + \frac{0.15}{V}\right] \times 10^5 \text{ pa}$$

Then,

$$_1W_2 = \int_1^2 P.dV$$

$$= \int_1^2 \left(\frac{0.01}{V^2} + \frac{0.15}{V}\right) \times 10^5 \, dV$$

$$= 10^5 \left[\int_1^2 \frac{0.01}{V^2} dV + \int_1^2 \frac{0.15}{V} dV\right.$$

$$= 10^5 \left[0.01 \int_1^2 \frac{1}{V^2} dV + 0.15 \int_1^2 \frac{dV}{V}\right]$$

$$= 10^5 \left[0.01 \left| \frac{-1}{V} \right|_{0.04}^{0.2} + 0.15 \left| \ln V \right|_{0.04}^{0.2} \right]$$

$$= 10^5 \left[0.01 \left(\frac{-1}{0.2} + \frac{1}{0.04} \right) + 0.15 \left(\ln 2 + \ln 0.04 \right) \right]$$

$$= 44141.57 \text{ J}$$

$$\therefore {}_1W_2 = 44.141 \text{ kJ}$$

11. A piston cylinder arrangement contains carbondioxide at 300 kpa, 100°C, with a volume of 0.2 m^3. weights are added to the piston such that the gas compresses according to the law $pV^{1.2} = C$. to a final temperature of 200°c. Determine work done.

Sol^n,

=> $P_1 = 300$ kpa

$T_1 = 100°C = 273 + 100$ K

$= 373$ K

$V_1 = 0.2 \, m^3$

By question,

$$pV^{1.2} = C$$

or $$P_1 V_1^{1.2} = P_2 V_2^{1.2}$$

$T_2 = 200°C = 200 + 273$ K

$= 473$ K.

workdone (w) = ?

We know,

$$P_1 V_1^{1.2} = P_2 V_2^{1.2}$$

$$300 \times (0.2)^{1.2} = P_2 V_2^{1.2}$$

or, $P_2V_2^{1.2} = 43.486$ —— (i)

Also, $\dfrac{P_1V_1}{T_1} = \dfrac{P_2V_2}{T_2}$

or, $\dfrac{300 \times 0.2}{373} = \dfrac{P_2V_2}{473}$

∴ $P_2V_2 = 76.085$ —— (ii)

Dividing (i) by (ii),

or $\dfrac{P_2V_2^{1.2}}{P_2V_2} = \dfrac{43.486}{76.085}$

or, $V_2^{0.2} = 0.571$

∴ $V_2 = 0.060 \ m^3$

Replacing value of V_2 in eqn (ii)

or, $P_2 \times 0.060 = 76.085$

∴ $P_2 = 1268.08$ kPa

workdone (w) $= \dfrac{P_2V_2 - P_1V_1}{1-n}$

$= \dfrac{1268.08 \times 0.060 - 300 \times 0.2}{1 - 1.2}$

∴ ${}_1W_2 = -80.4240$ kJ

12. 3 kg of air kept at an absolute pressure of 100 kpa and temperature of 300 k is compressed polytropically until the pressure and temperature becomes 1500 kpa and 500 k respectively. Evaluate the index of compression and work of compression.
$R = 287$ J/kgk.

soln,

=> Initial condition:

pressure (P_1) = 100 kpa = 100 X 1000 pa

volume (V_1) = ?

Temperature (T_1) = 300 k

weight of air = 3kg = m.

R = 287 J/kgk.

Final condition,

Temperature (T_2) = 500 k

pressure (P_2) = 1500 kpa
= 1500 X 1000 pa.

workdone (w) = ?

compression index (n) = ?

we know,

Equation of state for simple compressible substance is, $Pv = RT$

where, v is volume per unit mass.

so,

$$100000 \times \frac{V_1}{3} = 287 \times 300$$

$$V_1 = 2.583 \ m^3$$

We have, for polytropic process,

$PV^n = \text{constant}$

$P_1 V_1^n = P_2 V_2^n$

or $100000 \times V_1^n = 1500000 \times V_2^n$

or $\frac{100000}{1500000} = \left(\frac{V_2}{V_1}\right)^n$

$\therefore \left(\frac{V_2}{V_1}\right)^n = 0.066667$ —— (i)

Also,

$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$

or $\frac{100000 \times V_1}{300} = \frac{1500000 \times V_2}{500}$

$\therefore \frac{V_2}{V_1} = 0.11111$ —— (ii)

Replacing value of (ii) in (i),

or $(0.11111)^n = 0.066667$

Taking log,

$n \ln(0.11111) = \ln(0.066667)$

or, $n = \frac{-2.708}{-2.197}$

$\therefore n = 1.232$

Index of compression is 1.232

So, workdone $(w) = \dfrac{P_2V_2 - P_1V_1}{1-n}$

So,

$P_2V_2 = RT_2$

or $1500 \times 1000 \times \dfrac{V_2}{3} = 287 \times 500$

$\therefore \quad V_2 = 0.287 \ m^3$

Now,

$w = \dfrac{0.287 \times 1500000 - 2.583 \times 100000}{1-(1.23)}$

$= -742241.379 \ J$

$\therefore \quad W = -742.241 \ kJ$

13. A cylinder with a movable piston contains 9 m^3 of air at 1 bar and 400 k. The air expands according to the law $V = Ap^4 + Bp^3$ to a final pressure of 2 bar when the volume is 112 m^3. Calculate the workdone during the expansion.

Sol^n,

$\Rightarrow$. $V_1 = 9 \ m^3$

$p_1 = 1$ bar

$T_1 = 400$ k

By question, air expands according to the law,

$V = Ap^4 + Bp^3$.

$p_2 = 2$ bar

$V_2 = 112 \ m^3$

workdone (W_2) = ?

We know,

$$V = AP^4 + BP^3$$

when, $p_1 = 1$ bar, $V_1 = 9\ m^3$,

or $9 = A.1^4 + B.1^3$

$\therefore$ $9 = A + B$ ——— (1)

when, $P_2 = 2$ bar, $V_2 = 112\ m^3$,

$112 = A.2^4 + B.2^3$

or $112 = A.16 + 8B$

or $14 = 2A + B$

or $14 = 2A + (9 - A)$ [From eq^n (1)]

or $14 = 2A - A + 9$

or $14 - 9 = A$

$\therefore$ $A = 5$

$\therefore$ $B = 4$

Thus,

$$V = 5p^4 + 4p^3$$

Diff. w.r.to P,

$$\frac{dV}{dP} = 20p^3 + 12p^2$$

or $dV = (20p^3 + 12p^2)\,dp$.

Now,

workdone $= \int P.dV$.

or $W = \int_{P_1}^{P_2} P(20P^3 + 12P^2)\,dP$

$= \int_1^2 20P^4\,dP + \int_1^2 12P^3\,dP$

$= 20\left[\frac{P^5}{5}\right]_1^2 + 12\left[\frac{P^4}{4}\right]_1^2$

$= 4\left[2^5 - 1\right] + \frac{12}{4}\left[2^4 - 1\right]$

$= 124 + 45$

$\therefore W = 169 \times 10^5 J$

14. A nonflow reversible process occurs for which pressure and volume are correlated by expansion $P = \left(V^2 + \frac{6}{V}\right)$ where P is in bar and V is in m^3.
What amount of work will be done when volume changes from 2 to 4 m^3.

Soln,

$\Rightarrow$ workdone $(W) = ?$

$V_1 = 2\ m^3$

$V_2 = 4\ m^3$

By question,

$P = \left(V^2 + \frac{6}{V}\right)$

we have,

$W = \int P \cdot dV$

$= \int_{V_1}^{V_2} \left(V^2 + \frac{6}{V}\right) dV$

or $w = \int_2^4 v^2 \cdot dv + \int_2^4 \frac{6}{v}\, dv$

or $w = \left[\frac{v^3}{3}\right]_2^4 + 6\left[\ln v\right]_2^4$

or $w = \frac{1}{3}\left[4^3 - 2^3\right] + 6\left[\ln 4 - \ln 2\right]$

or $w = 18.666 + 4.158$

$\therefore$ $w = 22.82 \times 10^5$ J.

15. In a non flow process, a gas expands from volume 0.5 m^3 to a volume of 1 m^3 according to the law $P = 3v^2 + \frac{1}{v}$ where P is the pressure in bar and V is the volume in m^3. Determine pressure at the end of the expansion and workdone by the gas in the expansion process in KJ.

Soln,

$\Rightarrow$ $V_1 = 0.5\ m^3$

$V_2 = 1\ m^3$

By question, $P = 3v^2 + \frac{1}{v}$

workdone (w) $= \int P \cdot dv$

$= \int_{v_1}^{v_2} \left(3v^2 + \frac{1}{v}\right) dv$

$= \int_{0.5}^{1} 3v^2 dv + \int_{0.5}^{1} \frac{1}{v}\, dv$

$$= 3\left[\frac{V^3}{3}\right]_{0.5}^{1} + [\ln V]_{0.5}^{1}$$

$$= \frac{3}{3}\left[1 - 0.5^3\right] + (\ln 1 - \ln 0.5)$$

or $W = (0.875 + 0.6931) \times 10^5$ J

or $W = 1.56 \times 10^5$ J

$\therefore W = 156.814$ kJ

Pressure at the expansion $(P_2) = ?$

or $P_2 = 3V_2^2 + \frac{1}{V_2}$

$= 3 \times 1^2 + \frac{1}{1}$

$= 3 + 1$

$\therefore P_2 = 4$ bar

$\therefore P_2 = 4 \times 10^5$ pa.

16. Consider a two part process with an expansion from $0.1\ m^3$ to $0.2\ m^3$ at a constant pressure of 150 kpa followed by an expansion from $0.2\ m^3$ to $0.4\ m^3$ with a linearly rising pressure from 150 kpa ending at 300 kpa. Show the process in a P-V diagram & find boundary work.

Soln,

=> 1st process,

$V_1 = 0.1\ m^3$

$V_2 = 0.2\ m^3$

$P_1 = P_2 = 150$ kpa.

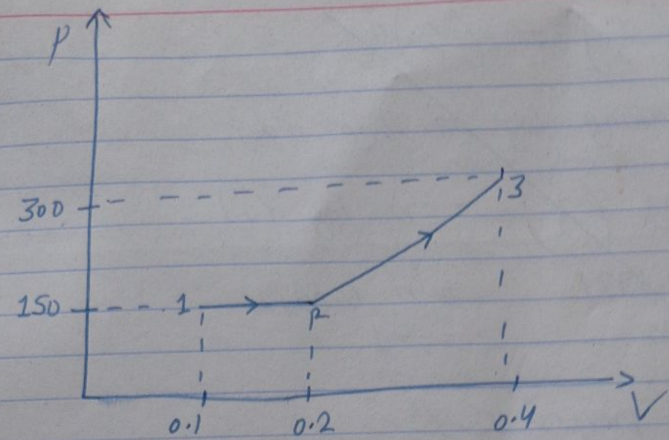


Fig: P–V diagram.

2nd process,

$V_3 = 0.2 = V_2$

$V_4 = 0.4\ m^3$

For process 1-2 :

pressure constant = Isobaric process.

So, workdone $(w) = P_1 (V_2 - V_1)$

$= 150 \times 1000 \times (0.2 - 0.1)$

$= 15000\ J$

$\therefore\ {}_1W_2 = 15\ kJ$

For process 2-3:
Temperature being constant: Isothermal process.

$$_2W_3 = P_2V_2 \ln\left(\frac{P_2}{P_3}\right)$$

$$= 150 \times 0.2 \ln\left(\frac{150}{300}\right)$$

$$_2W_3 = -20.794 \text{ kJ}$$

So, net work done $= {_1W_2} + {_2W_3}$

$$= -20.794 + 15 \text{ kJ}$$

$$\therefore {_1W_3} = -5.794 \text{ kJ}$$

17. A fluid at pressure 3 bar and with specific volume 0.18 m^3/kg contained in a cylinder behind a piston that expands reversibly to a pressure of 0.06 bar according to law: $p = \frac{c}{V^2}$, where c is constant. show expansion on P-V diagram and calculate net workdone.

soln,

=>. $P_1 = 3$ bar $= 3 \times 10^5$ pa.

specific volume $\left(\frac{V}{m}\right) = 0.18\ m^3/kg$.

$P_2 = 0.06$ bar
$= 0.06 \times 10^5$ pa.

By question,

$$p = \frac{c}{V^2}$$

or $c = PV^2$

i.e,

$$P_1 V_1^2 = P_2 V_2^2 = P_3 V_3^2 = C.$$

or $\left(\frac{P_1}{P_2}\right) = \left(\frac{V_2}{V_1}\right)^2$

or $\left(\frac{3 \times 10^5}{0.06 \times 10^5}\right) = \left(\frac{V_2}{V_1}\right)^2$

or $50 = \left(\frac{V_2}{V_1}\right)^2$

$\therefore \left(\frac{V_2}{V_1}\right) = 7.07\ m^3$

Since, process is reversible,

2. Heat Transfer

1. A brick of 450 mm thick is plastered with concrete 30mm thick. The thermal conductivity of the brick & the concrete are 0.7 w/mk and 0.92 w/mk respectively. If the temperature of the exposed brick face is 35°c and that of concrete is 10°c. determine heat loss per hour through the wall which is 5m long and 3.6 m high. Also, determine the interface temperature.

Sol^n,

⟹. For Brick,

thermal conductivity $(k_b) = 0.7$ w/mk

temperature $(T_b) = 35°C = 35 + 273\ k = 308\ k$

Heat loss per hour $(Q) = ?$

Interface temperature $= ?$

thickness of brick $(t_b) = 450$ mm

$= 0.45$ m

For concrete,

thickness of concrete $(t_c) = 30$ mm

$= 0.03$ m

temperature of concrete $(T_c) = 10°C = 283\ k$

thermal conductivity $(k_2) = 0.9$ w/mk.

We know that,

Heat conduction through composite plane slab is,

$$\therefore \quad \mathring{Q} = \frac{t_1 - t_2}{\frac{X_1}{k_1A_1} + \frac{X_2}{k_2A_2}}$$

or $Q = \dfrac{T_b - T_c}{\dfrac{t_b}{k_1A_1} - \dfrac{t_c}{k_2A_2}}$

$A_1 = 5 \times 3.6 = 18\ m^2 = A_2$.

So,

$$Q = \frac{35 - 10}{\dfrac{0.45}{18\times0.7} - \dfrac{0.03}{18\times0.9}}$$

$$= \frac{25}{0.0357 - 0.001851}$$

$\therefore\ Q = 738.592\ J/s$

Heat loss per hour $= 738.59 \times 60 \times 60\ J/hr$
$= 2658934.238\ J/hr$
$= 2658.934\ kJ/hr$.

2. An exterior wall of a house may be approximated by a 100 mm layer of common brick ($k = 0.7\ w/mk$) followed by a 40 mm layer of gypsum ~~salt~~ plaster ($k = 0.48\ w/mk$). What thickness of loosely packed rock wool insulation ($k = 0.065$) w/mk should be added to reduce the heat loss or gain through the wall by 80 % ?

Sol^n,

⇒ Considering no rock-wool insulation, we have,

$$Q = \frac{t_1 - t_2}{R_1 + R_2} \quad \text{———} \quad (1)$$

Considering rock-wool insulation,

$$\dot{Q}' = \frac{t_1 - t_2}{R_1 + R_2 + R_3} \quad \text{--- (ii)}$$

Dividing (i) by (ii),

or $$\frac{\dot{Q}}{\dot{Q}'} = \frac{R_1 + R_2 + R_3}{R_1 + R_2} \quad \text{--- (3)}$$

By question,

$$\dot{Q}' = \dot{Q} - 80\% \text{ of } \dot{Q}$$
$$= \dot{Q} - 0.8\,\dot{Q}$$
$$= 0.2\,\dot{Q}$$

$$\therefore \frac{\dot{Q}}{\dot{Q}'} = \frac{1}{0.2} = 5 \quad \text{--- (4)}$$

From (3) and (4),

$$\frac{R_1 + R_2 + R_3}{R_1 + R_2} = 5$$

or $$R_1 + R_2 + R_3 = 5R_1 + 5R_2$$

or $$R_3 = 4R_1 + 4R_2$$

or $$R_3 = 4(R_1 + R_2)$$

or $$\frac{x_3}{k_3 A} = 4\left(\frac{x_1}{k_1 A} + \frac{x_2}{k_2 A}\right)$$

or $$\frac{x_3}{k_3 A} = \frac{4}{A}\left(\frac{x_1}{k_1} + \frac{x_2}{k_2}\right)$$

or $$\frac{x_3}{k_3} = 4\left(\frac{x_1}{k_1} + \frac{x_2}{k_2}\right)$$

$$\therefore x_3 = 4\left(x_1 \cdot \frac{k_3}{k_1} + x_2 \frac{k_3}{k_2}\right)$$

$$= 4\left(100 \times \frac{0.065}{0.7} + 40 \times \frac{0.065}{0.48}\right)$$

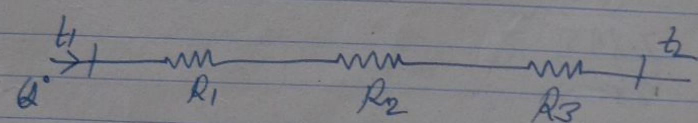
$$\therefore x_3 = 58.8 \text{ mm}$$

So, thickness of rock wool is 58.8 mm.

3. A steam pipe 18cm long inside diameter and 20 cm outside diameter is covered with two layers of insulation. The thickness of the first and second layers is 4cm and 8cm respectively and their corresponding thermal conductivities are 0.15 and 0.08 w/mk respectively. The conductivity of pipe material is 50 w/mk. The temperature of inner surface of the steam pipe is 350°C and that of the outer surface of insulation is 30°C. Find the quantity of heat lost per meter unit length of the pipe. Also, find out the percentage decrease in the heat loss with two layers of insulation compared to only first layer of insulation.

Sol^n,

=>



$$\text{diameter } (d_1) = 18 \text{ cm}$$

$$\text{radius} (r_1) = \frac{18}{2} = 9 \text{ cm}$$

$$r_2 = \frac{20}{2} \text{ cm} = 10 \text{ cm}$$

$$r_3 = r_2 + 4 = 10 + 4 = 14 \text{ cm}$$

$r_4 = r_3 + 8 = 22$ cm.
$k_1 = 50$ w/mk
$k_2 = 0.15$ w/mk
$k_3 = 0.08$ w/mk
$t_1 = 350°$ C
$t_2 = 30°$ C.

Considering both layers of insulation,
we have,

$$\dot{Q} = \frac{t_1 - t_2}{R_{th}}$$

Also, $R_{th} = R_1 + R_2 + R_3$

$$= \frac{\ln \frac{r_2}{r_1}}{2\pi k_1 l} + \frac{\ln \frac{r_3}{r_2}}{2\pi k_2 l} + \frac{\ln \frac{r_4}{r_3}}{2\pi k_3 l}$$

$$= \frac{1}{2\pi l}\left(\frac{\ln \frac{10}{9}}{50} + \frac{\ln \frac{14}{10}}{0.15} + \frac{\ln \frac{22}{14}}{0.08}\right)$$

$$= \frac{7.895}{2\pi l}$$

So,

$$\dot{Q} = \frac{350 - 30}{\frac{7.895}{2\pi l}}$$

or $$\dot{Q} = \frac{2\pi l \times 320}{7.895}$$

or $$\left(\frac{\dot{Q}}{l}\right) = \frac{2\pi \times 320}{7.895}$$

$\therefore \frac{\dot{Q}}{l} = 254.67$ W/m.

Considering single layer insulation only,

$$\dot{Q}'' = \frac{t_1 - t_2}{R_1 + R_2}$$

$$= \frac{350 - 30}{\frac{\ln(r_2/r_1)}{2\pi k_1 l} + \frac{\ln(r_3/r_2)}{2\pi k_2 l}}$$

$$\therefore \dot{Q}' = \frac{2\pi l \times 320}{\ln(10/9) + \ln(14/10)}$$

$$\therefore \left(\frac{\dot{Q}'}{l}\right) = \frac{2\pi \times 320}{\frac{\ln(10/9)}{50} + \frac{\ln(14/10)}{50}}$$

$$\therefore \frac{\dot{Q}'}{l} = 895.497 \text{ W/m}$$

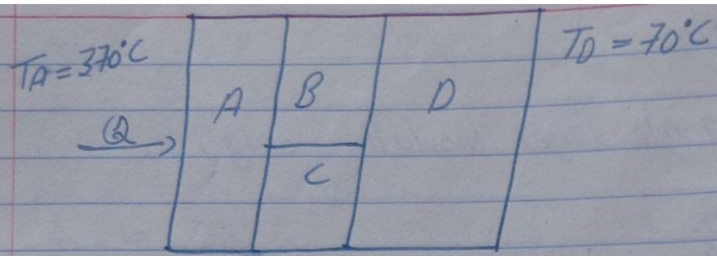
$$\%\text{ decrease in heat loss} = \frac{\Delta q}{q'} \times 100\,\%$$

$$= \frac{q' - q}{q'} \times 100\,\%$$

$$= \frac{895.497 - 254.67}{895.497} \times 100\,\%$$

$$= 71.56\,\%$$

4. Find heat transfer through composite wall shown in figure.



Given,

$k_A = 150\ w/m°C$ $\quad l_A = 2.5\ cm = 0.025\ m$

$k_B = 30\ w/m°C$ $\quad l_B = 7.5\ cm = 0.075\ m$

$k_C = 50\ w/m°C$ $\quad l_C = 7.5\ cm = 0.075\ m$

$k_D = 70\ w/m°C$ $\quad l_D = 5\ cm = 0.05\ m$

$A_A = A_D = 2A_B = 2A_C = 0.2\ m^2$

Heat transfer through composite wall is,

$$\dot{Q} = \frac{t_1 - t_2}{\frac{x_1}{k_1 A_A} + \frac{x_2}{k_2 A_B} + \frac{x_3}{k_3 A_C} + \frac{x_4}{k_4 A_D}}$$

$$= \frac{370 - 70}{\frac{0.025}{150 \times 0.2} + \frac{0.075}{30 \times 0.1} + \frac{0.075}{0.1 \times 50} + \frac{0.05}{70 \times 0.2}}$$

$$= \frac{300}{0.000833 + 0.025 + 0.015 + 0.003571}$$

or $\dot{Q} = \frac{300}{0.04441}$

$= 6756.09\ W$

$\therefore\ \dot{Q} = 6.756\ kW$

The equivalent thermal resistance for the parallel thermal resistance R_B & R_C is given by,

$$\frac{1}{R_{eq}} = \frac{1}{R_B} + \frac{1}{R_C}$$

or, $$\frac{1}{R_{eq}} = \frac{1}{\frac{x_2}{k_2 A_B}} + \frac{1}{\frac{x_3}{k_3 A_C}}$$

$$= \frac{1}{\frac{0.075}{30 \times 0.1}} + \frac{1}{\frac{0.075}{50 \times 0.1}}$$

or $$\frac{1}{R_{eq}} = 40 + 66.67$$

or $R_{eq} = 0.009375$

Now, total, thermal resistance is,

$$R_{total} = R_A + R_{eq} + R_D$$

$$= \frac{x_1}{k_1 A_A} + 0.009375 + \frac{x_4}{k_4 A_D}$$

$$= \frac{0.025}{150 \times 0.2} + 0.009375 + \frac{0.05}{70 \times 0.2}$$

$$= 0.000833 + 0.009375 + 0.003571$$

$$= 0.013779$$

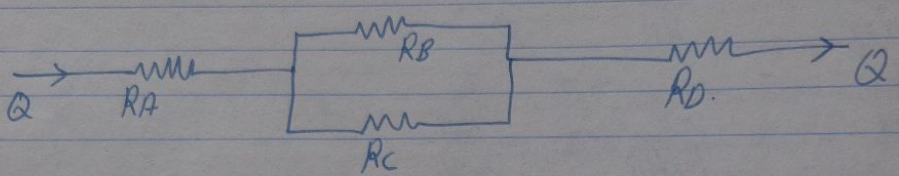
Hence,

$$\dot{Q} = \frac{t_1 - t_2}{R_{total}}$$

$$= \frac{370-70}{0.0137794}$$

$$= 21771.58 \text{ W}$$

$$\therefore Q = 21.771 \text{ kW}$$



5. A 150 mm steam pipe has inside diameter of 120mm and outside diameter of 160 mm. It is insulated at the outside with asbestos. The steam temperature is 150°C and air temperature is 20°C, (h = steam side = 100 W/m²K), (h air side = 30 W/m²K, k (asbestos) = 0.5 W/mK and k(steel) = 42 W/mK. How thick should the asbestos be provided in order to limit the heat loss to 2.1 kW/m².

Soln,

$\Rightarrow$ Radius of steam pipe $(r_1) = \frac{d}{2}$ inside

$$= \frac{120}{2} \text{ mm}$$

$$= 0.06 \text{ m}$$

Radius of outside pipe $(r_2) = \frac{160}{2}$ mm

$= 80\ mm$

$= 0.08\ m$

$k_A = 42\ w/m°C$

$k_B = 0.8\ w/m°C.$

Heat loss $= 2.1\ kw/m^2$

h (steam side) $= h_1 = 100\ w/m^2°C.$

h (air side) $= h_2 = 30\ w/m^2°C$

steam temperature $(t_1) = 150°C$

air temperature $(t_2) = 20°C.$

We know,

thickness of insulation (asbestos) is, $r_3 - r_2$.

$$\text{Area of heat transfer} = 2\pi r L = 2\pi\left(\frac{150}{2 \times 1000}\right) \times L = 0.94712\ L$$

$$\text{Now, Heat loss} = 2.1 \times \text{area kW} = 2.1 \times 0.4712\ L = 0.989\ L \times 10^3 \text{ watts}$$

Heat transfer rate,

$$Q = \frac{2\pi L\ (t_1 - t_2)}{\frac{1}{h_1 \cdot r_1} + \frac{\ln(r_2/r_1)}{k_A} + \frac{\ln(r_3/r_2)}{k_B} + \frac{1}{r_3 \cdot h_2}}$$

or $0.989L \times 10^3 = \dfrac{2\pi L\,(150-20)}{\left[\dfrac{1}{100\times 0.06} + \dfrac{\ln(0.08/0.06)}{42} + \dfrac{\ln(r_3/0.08)}{0.8} + \dfrac{1}{30\times r_3}\right]}$

or $\dfrac{\ln(r_3/0.08)}{0.8} + \dfrac{1}{30r_3} = \dfrac{816.81}{0.989\times 10^3} - (0.1666 + 0.00685)$

$= 0.6524$

or $1.25\ln(r_3/0.08) + \dfrac{1}{30r_3} = 0.6524$

or $1.25\ln(2.53\,r_3) + \dfrac{1}{30r_3} = 0.6524$

or $1.25\ln 2.53 + 1.25\ln r_3 + \dfrac{1}{30r_3} = 0.6524$

or $1.25\ln r_3 + \dfrac{1}{30r_3} = 0.6524 - 1.1602$

or $1.25\ln r_3 + \dfrac{1}{30r_3} = -0.5078$

$\therefore r_3 = 0.105$ m

$= 105$ mm

Thickness of insulation $= r_3 - r_2$

$= 105 - 80$

$= 25$ mm.

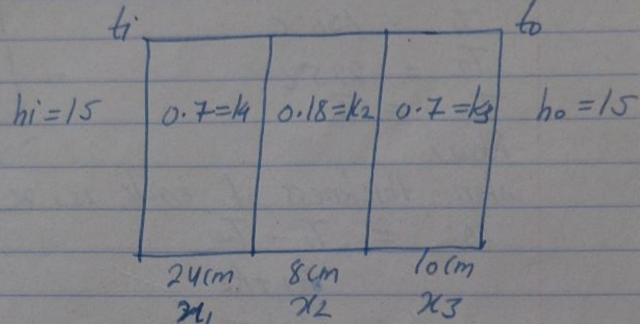
6. A house wall may be approximated as two 12 cm layers of fiber insulating board, a 8cm layer of loosely packed asbestos, and a 10 cm layer of common brick. Assuming convection heat transfer coeff of 15 w/mk on both sides of the wall, find overall heat transfer coeff for this arrangement.

Soln,

$K_{fib} = 0.7$ w/mk

$K_{asbestos} = 0.18$ w/mk

$K_{brick} = 0.7$ w/mk



$x_1 = 24$ cm $= 0.24$ m

$x_2 = 8$ cm $= 0.08$ m

$x_3 = 10$ cm $= 0.1$ m

$h_i = 15$ w/mk $= h_o$

$$\dot{Q} = \frac{t_i - t_o}{\frac{1}{h_iA} + \frac{x_1}{k_1A} + \frac{x_2}{k_2A} + \frac{x_3}{k_3A} + \frac{1}{h_oA}}$$

or $$\dot{Q} = \frac{(t_i - t_o)A}{\frac{1}{15} + \frac{0.24}{0.7} + \frac{0.08}{0.18} + \frac{0.1}{0.7} + \frac{1}{15}}$$

$\therefore \dot{Q} = 0.94\ (t_i - t_o)A$ —— (i)

We know,

$\dot{Q} = UA(t_i - t_f)$ —— (ii)

Equating (i) & (ii),

$0.94 = U$

$\therefore U = 0.94\ W/m^2K$.

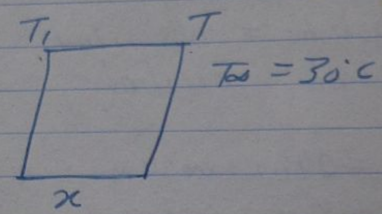
7. The temperature of inside and outside surfaces of brick work of a furnance have been noted to be 650° & 225°C. calculate percentage decrease in loss if thickness of brick by 100%. The ambident temperature i & assume thremal conductivity and convencient heat transfer coeff constant before & after increase in thickness?

Given,

$T_1 = 650°C$

$T_2 = 225°C$

$T_\infty = 30°$



Now,

when thickness of brick is, x,

$$\dot{Q} = \frac{T_1 - T_\infty}{R_1 + R_2}$$

$$= \frac{650 - 30}{\frac{x}{kA} + \frac{1}{hA}}$$

$$= \frac{620A}{\frac{x}{k} + \frac{1}{h}}$$

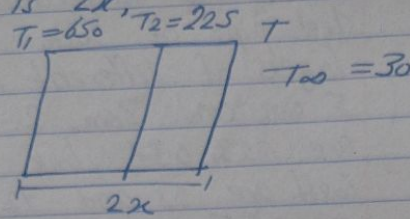
Also, $\dot{Q} = \frac{T_1 - T}{R_1} = \frac{T - T_\infty}{R_2}$

So, $\frac{(625-225)}{x/kA} = \frac{225-30}{1/hA}$

or $425 \times \frac{1}{hA} = 195 \times \frac{x}{kA}$

$\therefore \frac{1}{hA} = 0.459 \frac{x}{kA}$ —— (1)

Again, when thickness of block is $2x$,



$$\dot{Q}' = \frac{T_1 - T_\infty}{\frac{2x}{kA} + \frac{1}{hA}}$$

or $$\dot{Q}' = \frac{650 - 30}{\frac{2x}{kA} + \frac{1}{hA}}$$

or $$\dot{Q}' = \frac{620}{\frac{2x}{kA} + 0.459 \cdot \frac{x}{kA}}$$

$\therefore$ $$\dot{Q}' = \frac{620}{2.459 \cdot \frac{x}{kA}}$$

Then, $$\frac{\dot{Q}'}{\dot{Q}} = \frac{\frac{620}{2.459\, x/kA}}{425\, x/kA}$$

or $$\frac{\dot{Q}'}{\dot{Q}} = \frac{620}{2.459} \times \frac{1}{425}$$

$\therefore$ $$\frac{\dot{Q}'}{\dot{Q}} = 0.5932$$

Thus, % decrease on heat loss $(\% \Delta Q) = \left(1 - \frac{Q'}{Q}\right) \times 100\ \%$

$$= (1 - 0.5932) \times 100\ \%$$

$$= 40.68\ \%$$

8. Hot air at temperature of 60°C is flowing through a steel pipe of 10cm diameter. The pipe is covered with 2 layers of different insulating materials of thickness 5 cm and 3cm, & their corresponding thremal conductivities are 0.23, 0.37 w/mk. The inside & outside heat transfer coeff are 58 and 12 w/m²k. The atmosphere is at 25°C. Find rate of heat lost from a 50m long pipe. Neglect resistance of steel pipe.

Sol^n,

=) $r_1 = 10cm/2 = 5cm$

$r_2 = 5cm + 5cm = 10\ cm$

$r_3 = 10cm + 3cm = 13\ cm$

$k_1 = 0.23\ w/mk,$

$k_2 = 0.37\ w/mk$

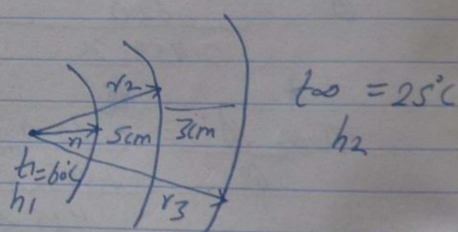
$h_1 = 58\ w/m^2k.$

$h_2 = 12\ w/m^2k$

$L = 50\ cm$

$t_i = 60^\circ\ C$

$t_\infty = 25^\circ C.$



$$\dot{Q} = \frac{t_i - t_\infty}{R_{th}}$$

$$= \frac{60-25}{R_{th}}$$

$$= \frac{35}{R_{th}}$$

But, $R_{th} = \dfrac{1}{h_1 A} + \dfrac{\ln(r_2/r_1)}{2\pi k_1 l} + \dfrac{\ln(r_3/r_2)}{2\pi k_2 l} + \dfrac{1}{h_2 A}$

$$= \frac{1}{h_1 2\pi r_1 l} + \frac{\ln(r_2/r_1)}{2\pi k_1 l} + \frac{\ln(r_3/r_2)}{2\pi k_2 l} + \frac{1}{h_2 2\pi r_3 l}$$

$$= \frac{1}{2\pi l}\left(\frac{1}{58 \times 0.05} + \frac{\ln(10/5)}{0.25} + \frac{\ln(13/10)}{0.37} + \frac{1}{12 \times 0.13}\right)$$

$$= \frac{1}{314}(0.3448 + 3.0136 + 0.709 + 0.64)$$

$$= \frac{4.708}{314}$$

$$= 0.0149$$

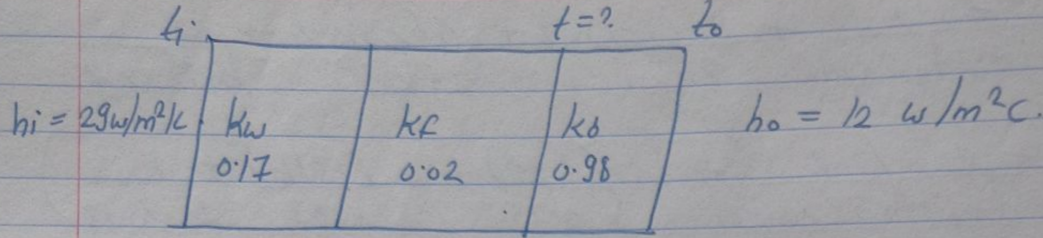
$$\therefore \dot{Q} = \frac{35}{0.0149}$$

$$= 2334.42 \text{ W}$$

$$= 2.334 \text{ Kw}.$$

9. A cold storage ~~man~~ room has walls made of 0.23m of brick on the outside 0.08 m of plastic foam & finally 1.5 cm of wood on the inside. The outside & inside air temperature are 20°C and −2°C respectively. If the inside and outside heat transfer coeff are respectively 29 w/m²k and 12 W/m²k & thermal conductivity of brick, foam and wood are 0.98, 0.02 & 0.17 w/mk resp. determine ① rate of heat removed be refrigeration if the total wall area is 90 m². the temperature of inside surface of the brick.

Soln,



Thickness of brick wall $(x_b) = 0{\cdot}23\,m$

" " plastic foam $(x_f) = 0{\cdot}08\,m$

" " wood $(x_w) = 0{\cdot}015\,m$.

$t_i = 2°C$

$t_o = 20°C$

$h_i = 29\ w/m^2k$

$h_o = 12\ w/m^2k$

$k_b = 0{\cdot}98\ w/mk$

$k_f = 0{\cdot}02\ w/mk$

$k_w = 0{\cdot}17\ w/mk$.

Area $(A) = 90\ m^2$

Now,

$$Q = \frac{t_o - t_i}{R_{th}} = \frac{20-(-2)}{R_{th}}.$$

$$R_{th} = \frac{1}{h_iA} + \frac{x_b}{k_bA} + \frac{x_f}{k_fA} + \frac{x_w}{k_wA} + \frac{1}{h_2A}.$$

$$= \frac{1}{A}\left(\frac{1}{29} + \frac{0{\cdot}23}{0{\cdot}98} + \frac{0{\cdot}08}{0{\cdot}02} + \frac{0{\cdot}015}{0{\cdot}17} + \frac{1}{12}\right)$$

$$= \frac{4{\cdot}40}{A}$$

$$= \frac{4{\cdot}40}{90}$$

$\therefore \dot{Q} = \dfrac{22}{4.89 \times 10^{-2}}$

$\therefore \dot{Q} = 449.36\ W$

Now

temperature at inside surface of brick wall = ?

$\dot{Q} = \dfrac{t - t_o}{\dfrac{x_b}{k_b A} + \dfrac{1}{hA}}$

or $449.36 = \dfrac{t - 20}{\left(\dfrac{0.23}{0.98} + \dfrac{1}{12}\right)\dfrac{1}{90}}$

or $1.588 = t - 20$

$\therefore t = 21.58\ ^\circ C.$

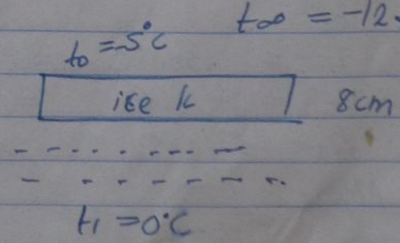
10\. A lake surface is covered by a 8cm thick layer of ice ($k = 2.23$ w/mk) when ambient air temperature is $-12°C$. A thermocouple embedded on upper surface of ice layer indicates temperature of $-5°C$. Assuming steady state conditions in ice & no liquid subcooling at bottom of ice layer, find heat transfer coefficient at upper surface. Find the heat lost per unit area.

Soln,

thickness of ice layer

$(x_i) = 8cm = 0.08\ m$

$t_\infty = -12.5°C.$



$k_i = 2.23\ w/mk$

$t_\infty = -12.5°C$

$t_o = -5°C.$

We know, For steady process,

$\dot{Q}_{conv} = \dot{Q}_{cond}$

or $hA(t_0 - t_\infty) = \frac{kA}{x}(t_i - t_0)$

or $h(7.5^\circ) = \frac{2.23}{0.08}(5)$

∴ $h = 18.58\ w/m^2k$

Again,

$\dot{Q} = (t_i - t_\infty)\frac{kA}{x}$

or $\frac{\dot{Q}}{A} = \frac{5 \times 2.23}{0.08}$

∴ $\dot{Q} = 139.375\ w/m^2$.

12. A surface having an area of $1.5m^2$ & maintained at 30°C exchanges heat by radiation with another surface at 40°C. The value of factor due to geometric location & emissivity is 0.52. determine,

a) heat lost by radiation.

b) value of thermal resistance.

Sol^n,

$T_1 = 300^\circ C = 573\ k$

$T_2 = 40^\circ C = 313\ k$

$f_{12} = 0.52$

Now,

$$Q_{12} = \sigma f_{12} (T_1^4 - T_2^4)$$
$$= 5.67 \times 10^{-8} (0.52 \times 1.5 \times ((573)^4 - (313)^4$$
$$= 4343.0819 \text{ W}$$

$R_{th} = ?$

$$Q = \frac{T_1 - T_2}{R_{th}}$$

$$R_{th} = \frac{260}{4343.0819}$$

$\therefore$ $R_{th} = 0.05986$ k/w.

c. $h_{equ} = ?$

$$Q = (T_1 - T_0) h A$$

or $$h = \frac{4343.0819}{260 \times 1.5}$$

$\therefore$ $h = 11.136$ w/m^{2}k.

13. A cylindrical rod, 1.5m long 2cm in diameter is heated electrically & positioned in a vaccum furnance which has interior walls at 800k temperature. A controlled amount of current passed through rod & its surface is maintained at 1000k. calculate power supplied to the heating rod if its surface has emissivity of 0.9.

Soln,

$T_1 = 800$k

length (l) = 1.5 m

diameter (d) = 2cm

r = 1cm = 0.01 m

length (l) = 1.5 m

T_2 = 1000 k.

Now,

$Q_{rad} = \sigma \varepsilon A (T_2^4 - T_1^4)$

$= 5.67 \times 10^{-8} \times 0.9 \times 2\pi \times 0.01 \times 1.5 (1000^4 - 800^4)$

$\therefore \dot{Q} = 2838.068$ watt

Pure substance:

1. A two phase liquid vapour mixture of H_2O is at 30 bar. If on heating at fixed volume, critical point is attained. Determine quality at initial state.

Sol^n,

=> At initial state,

$P_1 = 30$ bar

At final state, critical point is obtained from steam table,

Sp volume at final state $(v_2) = 0.00311\ m^3/kg$.

Then,

$v_1 = v_2$ (since process is constant volume process)

$\therefore v_1 = v_2 = 0.00311\ m^3/kg. = v$

Now,

From steam table at 3000 kpa,

$v_f = 0.001217\ m^3/kg$

$v_{fg} = 0.06544\ m^3/kg$

So,

$$v = v_f + x \cdot v_{fg}$$

$$\text{or}\quad x = \frac{v - v_f}{v_{fg}}$$

$$= \frac{0.00311 - 0.01217}{0.06544}$$

$$\therefore x = 0.0289.$$

2. A vessel having a volume of $0.4\ m^3$ contains 2 kg of liquid water and water vapour state mixture in equilibrium at a pressure of 0.1 Mpa. Calculate

a) volume and mass of liquid.

b) " and mass of vapor.

Sol^n,

=). mass of the mixture (m) = 2 kg
pressure (P) = 0.6 Mpa = 600 kpa.
Volume of mixture (v) = 0.4 m^3
we know that,
sp. volume of mixture = $V_s = \frac{V}{m}$

$= \frac{0.4}{2}$

$V_s = 0.2\ m^3/kg$

At 600 kpa,
from steam table,
$V_f = 0.001101$
$V_{fg} = 0.3145$
$V_g = 0.31560$
Since, the mixture contains water and steam at the equilibrium temperature, we have,

$V = V_f + X \cdot V_{fg}$

or $0.2 = 0.001101 + x \cdot 0.3145$

or $x = \frac{0.198899}{0.3145}$

$\therefore$ $x = 0.6324$

Also,

$X = \frac{m_g}{m}$

or $m_g = x \times m$

$= 0.6324 \times 2$

$\therefore \; m_g = 1.264 \text{ kg}$

and,

$m_f = m - m_g$

$= 2 - 1.264$

$= 0.735 \text{ kg}$

Where,

m_f = mass of liquid

m_g = mass of vapour.

x = dryness factor.

Specific volume, $v_s = \frac{V}{m}$ $\quad \therefore V = v m$

Then,

Volume of liquid, $V_g = v \cdot m_g$

$= 0.31560 \times 1.264$

$= 0.398 \text{ m}^3.$

Volume of liquid $V_f = v \, m_f$

$= 0.001101 \times 0.735$

$= 0.000809 \text{ m}^3$

3. A vessel contains 1 kg of steam which contains 1/3 liquid and 2/3 vapour by volume. The temerature of the steam is 151.86°C. Find the quality, sp. volume & sp. enthalpy of the mixture.

Sol^n,

$\Rightarrow$ let, V be the total volume of the mixture.

Then, Volume of liquid $(V_f) = \frac{V}{3}$

volume of vapour $(V_g) = \frac{2V}{3}$

we have,

total mass of mixture $(m) = m_f + m_g$

$$\text{or} \quad \frac{V}{v} = \frac{V_f}{v_f} + \frac{V_g}{v_g}$$

$$\text{or,} \quad \frac{V}{v} = \frac{V/3}{v_f} + \frac{2V/3}{v_g}$$

$$\text{or,} \quad \frac{V}{v} = \frac{V}{3}\left[\frac{1}{v_f} + \frac{2}{v_g}\right]$$

$$\text{or,} \quad 1 = \frac{v}{3}\left[\frac{1}{v_f} + \frac{2}{v_g}\right]$$

$$\therefore v = \frac{3}{\left[\frac{1}{v_f} + \frac{2}{v_g}\right]} \quad \text{—— (1)}$$

At 151.86° C,

From steam table,

$v_f = 0.00109 \ m^3/kg$

$v_{fg} = 0.3738 \ m^3/kg$

$v_g = 0.3749 \ m^3/kg.$

So, (1) becomes,

$$v = \frac{3}{\frac{1}{0.001090} + \frac{1}{0.3749}}$$

$$v = 3.26 \times 10^{-3} \ m/kg$$

Then,

$$v = v_f + v_{fg} \times x$$

or $$x = \frac{v - v_f}{v_{fg}}$$

$$= \frac{3.26 \times 10^{-3} - 0.00109}{0.3738}$$

$\therefore$ $x = 0.00579$

Again,

From steam table,

$h_f = 640.38$ kJ/kg

$h_{fg} = 2108.2$ kJ/kg

$h_g = 2748.6$ kJ/kg

Then,

$$h = h_f + x \cdot h_{fg}$$

$$= 640.38 + 0.00579 \times 2108.2$$

$\therefore$ $h = 652.60$ kJ/kg.

4. A vessel having a volume of $0.6 m^3$ contains 3 kg of liquid water and water vapour mixture in equilibrium at a pressure of 0.5 MPa. Calculate,

a) volume and mass of liquid.

b) volume and mass of steam.

Sol^n,

$\Rightarrow$ Volume of mixture $(v) = 0.6 m^3$

mass $(m) = 3$ kg

Equilibrium pressure $(P) = 0.5$ MPa

$P = 500$ kpa.

Then,

specific volume of mixture, $V_s = \frac{V}{m}$

$= \frac{0.6}{3}$

$= 0.2\ m^3/kg$

From steam table,

$V_f = 0.001093\ m^3/kg$

$V_g = 0.3749\ m^3/kg$

$V_{fg} = 0.3738\ m^3/kg$.

Then,

$V_s = V_f + x \cdot V_{fg}$.

or $0.2 = 0.001093 + x \times 0.3738$

$\therefore x = 0.5321$

Also,

$x = \frac{m_g}{m}$

or $x \cdot m = m_g$

or $m_g = 3 \times 0.532$

$\therefore m_g = 1.596$ kg

Also, $m = m_f + m_g$

$m_f = 3 - 1.596$

$\therefore m_f = 1.403$ kg.

But $V = \frac{v}{m}$

or $v = Vm$

Then,

$v_f = V_f \times m_f$

$= 0.001093 \times 1.403$

$= 0.00153 \ m^3$

$V_g \times m_g = v_g$

or $v_g = 0.3749 \times 1.596$

$= 0.598 \ m^3$

6. A closed system containing dry saturated steam undergoes expansion according to the law $pv^n = C$ from an initial pressure of 10 bar to a final pressure of 2 bar. If the steam is finally wet with dryness fraction of 0.85. If the steam is finally wet with dryness fraction evaluate index of expansion and work done.

Sol^n,

=> At initial state,

$p_1 = 10$ bar

$= 1000$ kpa.

Final state,

$p_2 = 2$ bar

$= 200$ kpa.

$x = 0.85$

At 200 kpa,

$V_f = 0.001060 \ m^3/kg$

$V_g = 0.8859 \ m^3/kg$

$V_{fg} = 0.8848 \ m^3/kg$

Now,

$$V = V_f + x \cdot V_{fg}$$
$$= 0.001060 + 0.85 \times 0.8848$$
$$= 0.753 \ m^3/kg$$

$V_2 = V = 0.753 \ m^3/kg$

At 1000 kpa,

$V_f = 0.001127 \ m^3/kg$

$V_g = 0.1944 \ m^3/kg$

Here, system contains dry saturated steam initially,

so, $V_1 = V_g = 0.1944 \ m^3/kg$.

Since the process is polytropic,

$$p_1 V_1^n = p_2 V_2^n$$

$$\ln\left(\frac{p_1}{p_2}\right) = n \ln\left(\frac{V_2}{V_1}\right).$$

$$\ln\left(\frac{1000}{200}\right) = n \ln\left(\frac{0.753}{0.1944}\right)$$

$$1.60 = n \times 1.35$$

$$n = 1.19$$

Hence,

$$W = \frac{P_2V_2 - P_1V_1}{1-n}$$

$$= \frac{200 \times 1000 \times 0.753 - 1000 \times 1000 \times 0.1944}{1 - 1.19}$$

$$\therefore W = 230.52 \ kJ/kg$$

7. A vessel contains wet steam at a pressure of 2 bar 12 % of whose mass is in liquid. Find,

a. temperature
b. enthalpy
c. dryness fraction.
d. specific enthalpy.
e. specific internal energy.

Soln,

=> $P = 2$ bar $= 200$ kpa

% of mass of liquid = 12 %

Then,

% of vapour in the wet steam = 100-12 %
= 88 %

Then,

$$x = \frac{m_g}{m}$$

$$= \frac{88\% \text{ of } m}{m}$$

$x = 88/100 \frac{X m}{m}$

$\therefore x = 0.88$

At 200 kpa,

$V_f = 0.001060 \ m^3/kg$
$V_{fg} = 0.8848 \ m^3/kg$
$V_g = 0.8859 \ m^3/kg$

Then, $V = V_f + x \cdot V_{fg}$

$= 0.001060 + 0.88 \times 0.8848$

$= 0.7796 \ m^3/kg$

From steam table,

$t = 120.24°C$

$h_f = 504.80 \ kJ/kg$
$h_g = 2706.5 \ kJ/kg$

$h_{fg} = 2201.7 \ kJ/kg$

Then,

$h = h_f + x \cdot h_{fg}$

$= 504.80 + 0.88 \times 2201.7$

$= 2442.29 \ kJ/kg.$

$U_f = 504.59$ kJ/kg
$U_{fg} = 2024.8$ kJ/kg
$U_g = 2529.4$ kJ/kg.

Then,

$$U = U_f + x\,U_{fg}$$

$$= 504.59 + 0.88 \times 2024.8$$

$$\therefore U = 2286.414 \text{ kJ/kg}$$

8. Steam of 10 bar pressure and 0.9 dryness fraction is cooled at constant volume at 160°C. What will be its final condition.

Sol^n,

=) Initial condition,

$P_1 = 10$ bar
$= 1000$ kPa.
$x = 0.9$

Final condition,
$t_2 = 160°C$
process is constant volume process.

At 1000 kPa,
From steam table,
$V_f = 0.001127$ m^3/kg
$V_g = 0.1944$ m^3/kg
$V_{fg} = 0.1933$ m^3/kg

So, $v = v_f + x \cdot v_{fg}$
$= 0.001127 + 0.9 \times 0.1933$
$= 0.175097 \ m^3/kg$.

$v = v_1 = v_2 = 0.175097 \ m^3/kg$.

At 160°C,
From steam table,
$v_f = 0.001102 \ m^3/kg$

$v_g = 0.3071 \ m^3/kg$

$v_{fg} = 0.3060 \ m^3/kg$.

Then,
$v = v_f + v_{fg} \times x$
or $0.175 = 0.001102 + x \cdot 0.3060$
$\therefore x = 0.57$
Also,
$v_f < v < v_g$, so,

$\therefore x = 0.57$ dryness.

9. Steam at 3 bar and 0.9 dry expands in a cylinder till the volume is four times that at the commencement. The law of expansion is $pv^{1.1} = C$. Determine the quality of steam after expansion.

Sol^n,

Initial condition,

$p_1 = 3$ bar $= 3 \times 10^5$ pa

$x_1 = 0.9$

At 300 kpa,

From steam table,

$V_f = 0.001073 \; m^3/kg$

$V_{fg} = 0.6048 \; m^3/kg$

Then,

$V_1 = V_f + x \cdot V_{fg}$

$= 0.001070 + (0.9 \times 0.6048)$

$= 0.5453 \; m^3/kg$

Final condition,

$V_2 = 4V_1$

process follows expansion law,

$pV^{1.1} = C$

or $p_1 V_1^{1.1} = p_2 V_2^{1.1}$

or $3 \times 10^5 \times \left(\frac{V_1}{V_2}\right)^{1.1} = P_2$

or $P_2 = 3 \times 10^5 \times \left(\frac{V_1}{4V_1}\right)^{1.1}$

$\therefore$ $P_2 = 65.29$ kpa.

$\therefore V_2 = 4V_1$

$= 4 \times 0.5453$

$= 2.1812 \ m^3/kg.$

At 65.29 kpa, properties of ~~syster~~ steam can not be directly obtained from steam table, so we have to determine properties by interpating,

Pressure (kpa)	Vf	Vfg (m^3/kg)
60	0.001033	2.7314
65.29	?	?
70	0.001036	2.3644

For Vf, we get,

$$V_f - 0.001033 = \frac{0.001036 - 0.001033}{70-60}(65-60)$$

$\therefore V_f = 1.03 \times 10^{-3} \ m^3/kg.$

For Vfg,

$$V_{fg} - 2.731 = \frac{2.3644 - 2.7314}{70-60}(65.29-60)$$

or $V_{fg} = 2.537 \ m^3/kg.$

At 65.29 kpa,

$\therefore V_2 = V_f + x \cdot V_{fg}$

$\therefore x = 0.859.$

1st law of thermodynamics:

1. Following table gives data in kJ, for a system undergoing a thermodynamic cycle consisting of four process in series. For cycle, kinetic and potential energy effects can be neglected. Determine,

   a. missing table entries each in kJ.

   b. whether cycle is a power cycle or a refrigeration cycle?

Process	ΔU	Q	W
1-2	600		-600
2-3			-1300
3-4	-700	0	
4-1		500	700

Sol^n,

$\Rightarrow$ For process 1-2, $Q = \Delta U + W$

$$Q_1 = 600 - 600$$

$$Q = 0.$$

process, 3-4, $Q = \Delta U + W$

$$0 = -700 + W$$

$$\therefore W = 700 \text{ kJ}$$

cyclic process, $\oint \Delta U = 0$

process 4-1, $Q = \Delta U + W$

$$500 = \Delta U + 700$$

$$\therefore W = -200 \text{ kJ}$$

$\oint \Delta U = 0$

i.e, $600 + x - 700 - 200 = 0$

$x = 300$

process 2-3, $Q = 300 - 1300$

$\therefore Q = -1000$

Total work output, $= -600 - 1300 + 700 + 700$

$= -500$ kJ.

total workdone output is -ve, so given cycle is refrigeration cycle.

2. Steam enters a horizontally aligned well insulated nozzle of inlet area $0.15\ m^2$ at the velocity of 65 m/s. At the inlet, specific enthalpy of steam is 3000 kJ/kg, specific volume $0.187\ m^3/kg$ and outlet specific enthalpy is 2762 kJ/kg. specific volume $0.498\ m^3/kg$. Determine,

i) velocity of steam at the outlet.

ii) mass flow rate of steam.

iii) Exit area of nozzle.

Sol^n,

At inlet,

Area $(A_1) = 0.15\ m^2$

Velocity $(v_1) = 65$ m/s.

Specific enthalpy $(h_1) = 3000$ kJ/kg

$= 3000000$ J/kg

Specific volume $(v_1) = 0.187\ m^3/kg$.

At outlet,
specific enthalpy $(h_2) = 2762$ kJ/kg
$= 2762000$ J/kg

specific volume $(v_2) = 0.498\ m^3/kg$.

Applying STFEE to inlet and outlet of nozzle, we get,

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$

or $$3000000 + \frac{65^2}{2} = 2762 \times 10^3 + \frac{V_2^2}{2}$$

or $$V_2^2 = 480225$$

$\therefore$ $$V_2 = 692.98\ m/s.$$

ii) mass flow rate of steam $(\dot{m}) = \dfrac{A_1 V_1}{v_1}$

$$= \frac{0.15 \times 65}{0.187}$$

$$= 52.13\ kg/s.$$

iii) For mass balance, $\dfrac{A_1 V_1}{v_1} = \dfrac{A_2 V_2}{v_2}$

or $$\frac{0.15 \times 65}{0.187} = A_2 \times \frac{692.98}{0.498}$$

or $A_2 = \frac{4.8556}{12958}$

$\therefore$ $A = 0.0375 \ m^2$

3. Nitrogen is heated while it flows steadily through a constant area tube. At the inlet nitrogen is 200 kpa, 100°C and has velocity of 175 m/s. It is heated to 300°C. At the outlet pressure is 115 kpa. At the inlet, $u_1 = 227$ kJ/kg and $h_1 = 387$ kJ/kg at the exit $u_2 = 428$ kJ/kg and $h_2 = 598$ kJ/kg. Find heat transfer per kg of nitrogen.

Soln,

$\Rightarrow$ At inlet,

$p_1 = 200$ kpa

$t_1 = 100°c$

$v_1 = 175$ m/s

$u_1 = 227$ kJ/kg

$h_1 = 387$ kJ/kg

At outlet,

$p_2 = 115$ kpa

$u_2 = 428$ kJ/kg

$h_2 = 598$ kJ/kg

$t_2 = 300°c$

Applying SFEE to inlet and outlet,

$$\dot{m}\left(h_1 + \frac{V_1^2}{2} + gZ_1\right) + \dot{Q} = \dot{m}\left(h_2 + \frac{V_2^2}{2} + gZ_2\right) + \dot{W}_x$$

or $h_1 + \frac{V_1^2}{2} + gZ_1 + q = h_2 + \frac{V_2^2}{2} + gZ_2 + w_x$

put, $W_x = 0$ & $Z_1 = Z_2$

∴ $h_1 + \frac{V_1^2}{2} + q = h_2 + \frac{V_2^2}{2}$ —— (I)

mass flow rate is same,

$\dot{m}_1 = \dot{m}_2$

or $\frac{A_1 V_1}{v_1} = \frac{A_2 V_2}{v_2}$ (∵ uniform area tube)

or $\frac{V_1}{v_1} = \frac{V_2}{v_2}$ —— (II)

At outlet,

$h_2 = u_2 + p_2 v_2$

or $598 = 428 + 115 \times v_2$

∴ $v_2 = 1.47\ m^3/kg$.

At inlet,

$h_1 = u_1 + p_1 v_1$

$387 = 227 + 200 \times v_1$

∴ $v_1 = 0.8\ m^3/kg$

Then,

$$\frac{v_1}{v_2} = \frac{V_1}{V_2}$$

or $$\frac{175}{v_2} = \frac{0.8}{1.47}$$

∴ $v_2 = 321.56$ m/s

In eqn (1)

$$h_1 + \frac{v_1^2}{2} + q = h_2 + \frac{v_2^2}{2}$$

or $$387\times10^3 + \frac{175^2}{2} + q = 598\times10^3 + \frac{(322.56)^2}{2}$$

or $402312.5 + q = 649700.42$

∴ $q = 247.38$ kJ/kg.

4. A turbine operating under steady flow conditions receives 5000 kg of steam per hour. The steam enters the turbine at a velocity of 3000 m/min at an elevation of 5m and specific enthalpy of 278 kJ/kg. It leaves the turbine at a velocity of 6000 m/min, at an elevation of 1 m and specific enthalpy of 2259 kJ/kg. Heat losses from the turbine to the surrounding amounts to 16736 kJ/kg. Determine power output of the turbine.

Soln,

At inlet,

m° = 5000 kg per hour

$= \frac{5000}{60 \times 60}$ kg per sec

$= 1.38$ kg/sec

At outlet,

$V = 6000$ m/min

$= 100$ kg/sec.

$Z_1 = 7m$

$\dot{m} = 1.38$ kg/s

$h_2 = 2259$ kJ/kg

$= 2259 \times 10^3$ J/kg.

For inlet, $Z_1 = 5m$

$h_1 = 278J$ kJ/kg

$= 278 \times 10^3$ J/kg

$\dot{Q} = -16736$ kJ/kg.

$= -16736 \times 10^3$ J/kg.

Applying SFEE to the turbine, we get,

or, $\dot{m}\left(h_1 + \frac{V_1^2}{2} + gZ_1\right) + \dot{Q} = \dot{m}\left(h_2 + \frac{V_2^2}{2} + gZ_2\right) + \dot{w}_x$

or, $h_1 + \frac{V_1^2}{2} + gZ_1 + \dot{Q} = h_2 + \frac{V_2^2}{2} + gZ_2 + w_x$

or, $\left(278 \times 10^3 + \frac{50^2}{2} + 10 \times 5\right) + (-16736 \times 10^3) = \left(2259 \times 10^3 + \frac{100^2}{2} +\right.$

$+10\times1) + \dot{W}x$

or, $385434 + (-16736\times10^3) = 3124333.8 + \dot{W}x$

or $368698 = 3124333.8 + \dot{W}x$

$\therefore$ $\dot{W}x = -19.47$ mw.

$\therefore$ Power developed by turbine $= -19.47$ MW

6\. 1 kg of air can be expanded between two states as it does 20 kJ of work and receives 16 kJ of heat. A 2nd kind of expansion can be found between the same initial and final states which require a heat input of only 9 kJ. What is change of internal energy in the first expansion and what is the work done by the air in the second expansion?

soln,

=> For 1st expansion,

$_1W_2 = 20$ kJ

$\delta Q = 16$ kJ.

For 2nd expansion,

$\delta Q = 9$ kJ

Then, from 1st law of thermodynamic system,

$\delta Q = U_2 - U_1 + {}_1W_2$

on 1st expansion,

$16 = U_2 - U_1 + 20$

$\therefore U_2 - U_1 = -4 \text{ kJ}$

For 2nd expansion, $9 = \Delta U + {}_1W_2$

$9 = -4 + {}_1W_2$

$\therefore {}_1W_2 = 13 \text{ kJ}$

7. Steam enters a steam turbine at a pressure of 1 Mpa, a temperature of 300°C and a velocity of 50 m/s. The steam leaves the turbine at a pressure of 150 kpa and velocity of 200 m/s. Determine work done per kg of steam flowing through the turbine, assuming the process to be reversible and adiabatic.

Sol^n,

=) At inlet,

$p_1 = 1 \text{ Mpa}$

$= 1 \times 10^6 \text{ pa}$

$t_1 = 300°C = 573 \text{ K}$

$V_1 = 50 \text{ m/s}$.

At outlet,

$P_2 = 150 \text{ kpa} = 150 \times 10^3 \text{ pa}$

$V_2 = 200 \text{ m/s}$

Then,

$w_x = ?$

Now,

From SFEE based on unit mass, we have,

$$h_1 + \frac{V_1^2}{2} + gZ_1 + q = h_2 + \frac{V_2^2}{2} + gZ_2 + w$$

or, Assuming no change in P.E and process to be adiabatic.
i.e, $q = 0$, then, above eqn becomes,

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} + w \quad \text{— (1)}$$

To find w, we have to find values at inlet and outlet
i.e, h_1 and h_2.

At inlet, At 1000 kpa,
From steam table,
$t_s = 179.92°C$
$t_1 = 300°C$.
Since, $t_1 > t_s$, steam table at inlet must be superheated.
Now, from steam table, we have,
$h_1 = 3050.6$ KJ/kg.

$S_1 = 7.1219$ KJ/kgk.
Since, process is adiabatic and reversible,
it must be isentropic.
$S_1 = S_2 = 7.1219$ KJ/kg.
So,
at outlet, $P_2 = 150$ kpa
$S_2 = 7.1219$ kJ/kgk.

But at P_2, 150 kpa.
From steam table,

$S_f = 1.4338$ kJ/kgK
$S_{fg} = 5.7894$ kJ/kgK
$S_g = 7.2232$ kJ/kgK.

$h_f = 467.18$ kJ/kg
$h_{fg} = 2226.2$ kJ/kg
$h_g = 2693.4$ kJ/kg.

Since, $S_f < S_2 < S_g$, the steam at the outlet must be wet one.
let, x_2 be it its dryness fraction.

$$S_2 = S_f + X_2 S_{fg}.$$

or $7.1219 = 1.4338 + x_2 \times 5.7894$
$\therefore x_2 = 0.9825$
Similarly,

$$h_2 = h_f + x_2 h_{fg}$$
or $h_2 = 467.18 + 0.9825 \times 2226.2$

$\therefore h_2 = 2654.92$ kJ/kg.
So from (i),

$$3050.6 \times 10^3 + \frac{50^2}{2} = 2654.42 \times 10^3 + \frac{200^2}{2} + W$$

or $W = 377430$ kW

$= 163.35 + (0.8 \times 2408.3)$

$= 2089.99$ kJ/kg.

Then,

(1) becomes,

$5[2801700 + 2812.5 + 49] - 5000 = 5[2089990 + 12800] + W_x$

or $14022807.5 - 5000 = 10513999 + W_x$

$\therefore W_x = 3.5$ MW

10. 0.3 kg of nitrogen gas at 100 kpa and 40°C is contained in a cylinder. The piston is moved compressing nitrogen until the pressure becomes 2 mpa and temperature becomes 160°C. The work done during the process is 60 kJ. calculate heat transferred from the nitrogen to the surroundings. (Cv for nitrogen = 0.75 kJ/kg.

Sol^n,

=) Cv for Nitrogen = 0.75 kJ/kg.

mass of nitrogen (m) = 0.3 kg.

pressure (P_1) = 100 kpa

= 100000 pa

pressure (P_2) = 2 Mpa

= 2000000 pa.

t_1 = 40°C + 273 = 313 K.

temperature (t_2) = 160°C = 433 K.

workdone (w) = −60 kJ

Heat transferred from nitrogen to surrounding (Q) = ?

We have,

From 1st law,

$$\delta Q = du + \delta W$$

Integrating,

$$Q = U_2 - U_1 + {}_1W_2$$

or $Q = -60 + m\,C_v(dt)$

or $Q = -60 + 0.3 \times 0.75 \times (160^\circ - 40^\circ)$

or $Q = -33$ kJ

$-$ve sign indicates that heat is being transferred from ~~surrounding~~ nitrogen to surrounding.

11. The properties of a system during a reversible constant pressure non-flow process at $P = 1.6$ bar changes from $V_1 = 0.3\ m^3/kg$, $T_1 = 20^\circ C$ to $V_2 = 0.55\ m^3/kg$, $T_1 = 260^\circ C$. The specific heat of the fluid is given by

$$C_p = 1.5 + \frac{75}{(T+45))}\ kJ/kg^\circ C.$$

Determine:

a. Heat added / kg.

b. Change in internal energy

Change in enthalpy / kg.

Sol^n,

=> $P = 1.6 \times 10^5$ pa

$V_1 = 0.3\ m^3/kg$

$V_2 = 0.55\ m^3/kg.$

$T_1 = 20°C = 293\ K$

$T_2 = 260°C = 533\ K.$

Then,

(i) Heat added per kg,

We have, total heat added = total change in enthalpy.

Also,

$$Cp = \frac{dh}{dt}$$

$\therefore$ $dH = Cp\,dt$

${}_1Q_2 = Cp\,dt$ [for constant pressure process]

Integrating both sides wrto T,

$${}_1Q_2 = \int_{T_1}^{T_2} Cp \cdot dt$$

$$= \int_{T_1}^{T_2} 1.5 + \frac{75}{(T+45)}\ dt$$

$$= 1.5\,[t]_{T_1}^{T_2} + 75\,[\ln(T+45)]_{T_1}^{T_2}$$

$$= 1.5\,[T]_{20}^{260} + 75\,[\ln(T+45)]_{20}^{260}$$

$= 1.5(260-20) + 75(\ln(260+45) - \ln(20+45))]$

$q = 475.04$ kJ/kg.

(ii) work done during constant pressure non-flow process is $W_{1-2} = p(V_2 - V_1)$

$= 1.6 \times 10^5 \times (0.55 - 0.3)$

$\therefore {}_1W_2 = 40000$ J.

(iii) Change in internal energy,
From 1st law of thermodynamic,

${}_1Q_2 = U_2 - U_1 + {}_1W_2$

or $U_2 - U_1 = {}_1Q_2 - {}_1W_2$

$= 475.04 \times 10^3 - 40000$

$\therefore U_2 - U_1 = 4359.40$ J

iv Change in Enthalpy:

$H_2 - H_1\, \cancel{U_2 - U_1} = {}_1Q_2$

$= 475.9.40$ J

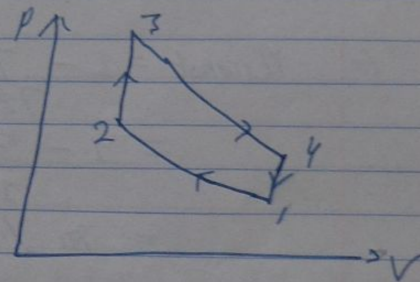
since, change in enthalpy in a constant pressure process is equal to change in ~~enthalpy~~. internal energy.

Cycle.

1. $\gamma = 1.4$
$R = 287\ J/kg$.

$C_v = 718\ J/kgK$
$C_p = 1005\ J/kgK$.

1. An ideal otto cycle has a compression ratio of 8. The minimum and maximum temperature during the cycle are 300 K and 1500 K respectively. Find,
a) Heat added per kg of air.
b) thermal efficiency.
c) efficiency of a carnot cycle.



soln,
→ compression ratio(γ) = 8
$T_1 = 300\ K$
$T_2 = 1500\ K$.

For reversible and adiabatic compression, 1-2, we have,

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}.$$

or, $T_1\, r^{\gamma-1} = \frac{T_2}{T_1}$

or $\frac{T_2}{} = 300 \times 8^{1.4-1}$

$\therefore\ T_2 = 689.219\ K.$

Now,

Heat added per kg of air, q_1

a. $q_1 = C_v (T_3 - T_2)$
$= 718 (1500 - 689.219)$
$= 582.14$ kJ/kg.

b. $\eta = 1 - \frac{1}{r^{\gamma - 1}}$
$= 1 - \frac{1}{8^{1.4-1}}$
$= 56.47\ \%$

c. $\eta_{carnot} = 1 - \frac{T_4}{T_3}$
$= 1 - \frac{300}{1500}$
$= 80\ \%$

2. The compression ratio of an ideal otto cycle is 8.5. At the beginning of the compression stroke, air is at 100 kpa and 27°C. The pressure is doubled during the constant volume heat addition process.

Find:

a). Heat added per kg of air.
b. net work output per kg of air.
c. thermal efficiency.
d. mean effective pressure.

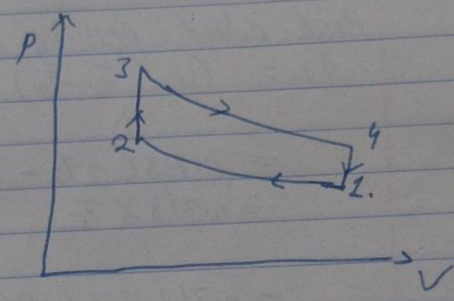
Soln,

Compression ratio $(r) = 8.5$

$P_1 = 100$ kpa

$T_1 = 273 + 27 = 300k$

$P_3 = 2P_2$



For reversible & adiabatic compression, 1-2.

$P_2 V_2^{\gamma} = P_1 V_1^{\gamma}$

or $P_2 = P_1 \left(\frac{V_1}{V_2}\right)^{\gamma}$

$= 100 \times r^{\gamma}$

$= 100 \times 8.5^{1.4}$

$\therefore P_2 = 2000.72$ kpa.

$T_2 V_2^{\gamma-1} = T_1 V_1^{\gamma-1}$

or $T_2 = T_1 r^{\gamma-1}$

$= 300 \times 8.5^{1.4-1}$

$= 706.136$ k

For isothoric process, 2-3,

$\frac{P_3}{T_3} = \frac{P_2}{T_2}$

or $T_3 = \frac{P_3}{P_2} \times T_2$

$= \frac{2P_2}{P_2} \times T_2$

$= 2T_2$

$= 2 \times 706.136$

$= 1412.272\ K.$

Heat added per kg of air is,

$q_1 = C_v (T_3 - T_2)$

$= 0.718(2T_2 - T_2)$

$= 0.718 \times T_2$

$= 0.718 \times 706.136$

$= 507.005\ kJ/kg.$

$$\eta = 1 - \frac{1}{r^{\gamma-1}}$$

$$= 1 - \frac{1}{(8.5)^{1.4-1}}$$

$= 0.5752$

$= 57.52\ \%$

But,

$$\eta = \frac{W_{net}}{q_1}$$

or $W_{net} = \eta \times q_1$

$= 0.5752 \times 507.005$

$= 291.629\ kJ/kg.$

Now,

$p_1 V_1 = RT_1$

$$V_1 = \frac{RT_1}{p_1}$$

$$= \frac{287 \times 300}{100 \times 10^3}$$

$$= 0.861$$

So,

mean effective pressure, $P_m = \frac{W_{net}}{V_1 - V_2}$

$$= \frac{W_{net}}{V_1(1 - V_2/V_1)}$$

$$= \frac{W_{net}}{V_1(1 - \frac{1}{r})}$$

$$= \frac{291.829}{0.861(1 - 1/8.5)}$$

$\therefore$ P_m. = 383.87 kpa.

3. The properties of air at the beginning of compression stoke in an air standard diesel cycle are 100 kpa and 300 k. The air at the beginning of the expansion stoke is 6500 kpa and 2000 k. Find.

a. compression ratio.
b. thermal efficiency
c. mean effective pressure.

soln,

Given,

$P_1 = 100$ kpa

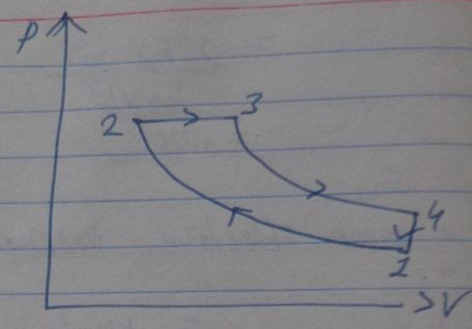
$P_3 = 6500$ kpa.

$T_1 = 300\ K$

$T_3 = 2000\ K.$

Since, pressure remains constant during heating,

$P_2 = P_3 = 6500\ kPa$



For reversible adiabatic compression process 1-2,

$$P_2 V_2^{\gamma} = P_1 V_1^{\gamma}$$

or $$\left(\frac{V_1}{V_2}\right)^{\gamma} = \frac{P_2}{P_1}$$

or $$r^{\gamma} = \frac{P_2}{P_1}$$

or, $$r = \left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}}$$

$$= \left(\frac{6500}{100}\right)^{1/1.4}$$

$$\therefore r = 19.7214$$

Also,

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\therefore T_2 = T_1 \times \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$= 300 \times \left(\frac{6500}{100}\right)^{\frac{1.4-1}{1.4}}$$

$$\therefore T_2 = 988.77\,K.$$

For constant pressure process 2-3,

$$\frac{V_3}{T_3} = \frac{V_2}{T_2}$$

$$\text{or, } \frac{V_3}{V_2} = \frac{T_3}{T_2}$$

$$\text{or } \alpha = \frac{T_3}{T_2}$$

$$= \frac{2000}{988.77}$$

$$\therefore \alpha = 2.0227$$

$$\therefore \eta = 1 - \frac{1}{\gamma} \cdot \frac{1}{r^{\gamma-1}} \cdot \frac{\alpha^{\gamma}-1}{\alpha-1}$$

$$= 1 - \frac{1}{1.4} \times \frac{1}{(19.7214)^{1.4-1}} \times \frac{(2.0227)^{1.4}-1}{2.0227-1}$$

$$\therefore \eta = 0.64526 \times 100$$

$$= 64.526\,\%$$

Now,

Heat supplied per kg of air is given by,

$$q_1 = C_p\,(T_3 - T_2)$$

$$= 1.005 \times (2000 - 988.77)$$

$= 1016.286 \text{ KJ/kg}$

$\therefore \eta = \dfrac{W_{net}}{q_1}$

or $W_{net} = \eta \times q_1$

$= 0.64526 \times 1016.286$

$= 655.7687 \text{ kJ/kg}.$

Also,

$V_1 = \dfrac{RT_1}{P_1}$

$= \dfrac{287 \times 300}{100 \times 10^3}$

$= 0.861 \text{ m}^3/\text{kg}.$

mean effective pressure, $P_m = \dfrac{W_{net}}{V_1 - V_2}$

$= \dfrac{W_{net}}{V_1\left(1 - \dfrac{V_2}{V_1}\right)}$

$= \dfrac{W_{net}}{V_1\left(1 - \dfrac{1}{r}\right)}$

$= \dfrac{655.7687}{0.861\left(1 - \dfrac{1}{19.7214}\right)}$

$\therefore P_m = 802.318 \text{ kpa}$

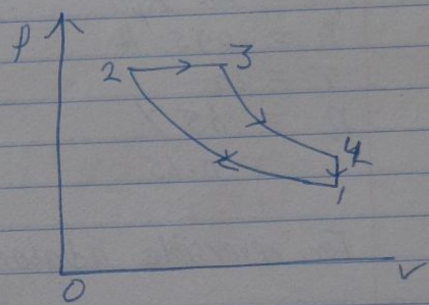
4. An engine working on a diesel cycle has a compression ratio of 16 and cut off takes place at 8 % of the stoke. Determine its air standard efficiency

soln,

=). Compression ratio, $\gamma = 16$

$$\frac{V_1}{V_2} = 16$$

$$\therefore V_1 = 16 V_2 \quad \text{—(i)}$$



By question,

$$V_3 - V_2 = 8\% \text{ of } (V_1 - V_2)$$

$$\text{or } V_3 - V_2 = 0.08 \times (V_1 - V_2) \quad \text{—(ii)}$$

$$\text{or } V_3 = V_2 + 0.08 V_2 \times 15$$

$$\text{or } V_3 = 2.2 V_2$$

$$\therefore \frac{V_3}{V_2} = 2.2$$

Cut-off ratio, $\alpha = \frac{V_3}{V_2} = 2.2$

Also,

$$\eta = 1 - \frac{1}{\gamma} \cdot \frac{1}{\gamma^{\gamma - 1}} \cdot \frac{\alpha^{\gamma} - 1}{\alpha - 1}$$

$$= 1 - \frac{1}{1.4} \cdot \frac{1}{16^{1.4-1}} \times \frac{2.2^{1.4} - 1}{2.2 - 1}$$

$$= 0.6042$$

$$\eta = 60.42\ \%$$

5. Air at beginning of compression stroke in an diesel cycle is at 100 kpa and 295 K. and the compression ratio is 20. Determine maximum temperature during cycle to have an efficiency of 65%.

Sol^n,

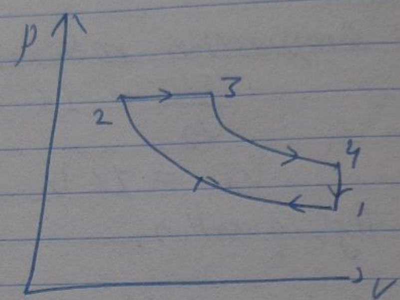
$p_1 = 100$ kpa

$T_1 = 295$ K

$r = 20$

$\eta = 65\%$

$= 0.65$



For reversible adiabatic compression 1-2,

$$T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$= T_1 \, r^{\gamma-1}$$

$$= 295 \times (20)^{1.4-1}$$

$$= 977.76 \text{ K.}$$

For an ideal Diesel cycle,

$$\eta = 1 - \frac{1}{\gamma} \cdot \frac{1}{r^{\gamma-1}} \cdot \frac{\alpha^{\gamma}-1}{\alpha-1}$$

$$0.65 = 1 - \frac{1}{1.4} \cdot \frac{1}{20^{1.4-1}} \cdot \frac{\alpha^{1.4}-1}{\alpha-1}$$

or, $\frac{1}{1.4 \times 20^{1.4-1}} \cdot \frac{\alpha^{1.4}-1}{\alpha-1} = 0.35$

or $\alpha^{1.4} - 1.6214\,\alpha - 1 + 1.6241 = 0$

or $\alpha^{1.4} - 1.6241\,\alpha + 0.6241 = 0$

Solving,

$\therefore \alpha = 1.928$

For process, 2-3,

$$\frac{P_3V_3}{T_3} = \frac{P_2V_2}{T_2}$$

or $$\frac{V_3}{T_3} = \frac{V_2}{T_2}$$

or $T_3 = T_2 \times \alpha$

$= 977.76 \times 1.92897$

$\therefore T_3 = 1886.06$ K

6. The compression ratio of air standard otto cycle is 8. At the beginning of the compression stroke, pressure is 0.1 Mpa. and temperature is 15°C. The heat transferred to the air per cycle is 1800kJ/kg. Determine:

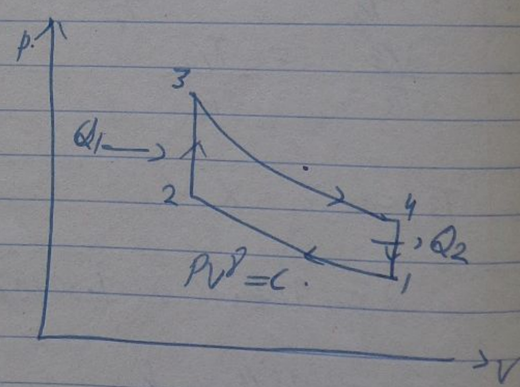
a. Pressure and temperature at end of each process
b. thermal effiency.
c. mean effective pressure.
Soln,

⟹ Compression ratio $(r) = 8$

$$\frac{V_1}{V_2} = 8$$

$P_1 = 0.1 \times 10^6$ pa
$t_1 = 15°C$
$= 288$ k.
$Q_1 = 1800 \times 10^3$ kJ/kg.

Thermal efficiency,



$$\eta = \frac{W_{net}}{Q_1}$$

$$= \frac{Q_1 - Q_2}{Q_1}$$

$$= 1 - \frac{Q_2}{Q_1}$$

Also,

$$\eta = 1 - \frac{1}{r^{\gamma-1}}$$

$$= 1 - \frac{1}{8^{1.4-1}}$$

$$\therefore \eta = 56.25 \%$$

Also,

$$PV = mRT$$

$$\frac{PV}{m} = RT$$

$$P_1 V_1 = RT_1$$

$$V_1 = \frac{RT_1}{P_1}$$

$$= \frac{287 \times 288}{0.1 \times 10^6}$$

$$\therefore V_1 = V_4 = 0.82656 \ m^3/kg.$$

$$V_2 = \frac{V_1}{8}$$

$$= \frac{0.82656}{8}$$

$$= 0.10332 \ m^3/kg = V_3$$

For adiabatic process 1-2,

$$PV^\gamma = C$$

or $P_1 V_1^\gamma = P_2 V_2^\gamma$

or $P_1 \left(\frac{V_1}{V_2}\right)^\gamma = P_2$

or $P_2 = P_1 r^\gamma$

or $P_2 = 0.1 \times 10^6 \times (8)^{1.4}$

$\therefore P_2 = 1837917.368$ pa

Also $P_2 V_2 = R T_2$

or $1837917.36 \times 0.10332 = 287 \times T_2$

$\therefore$ $T_2 = 661.65$ K.

Now,

i) $Q_1 = m C_v (T_3 - T_2)$

or, $1800 \times 10^3 = 718 (T_3 - T_2)$

or $1800 \times 10^3 = 718 (T_3 - 661.65)$

or, $T_3 = 3168.61$ K.

Again, $\dfrac{P_2 V_2}{T_2} = \dfrac{P_3 V_3}{T_3}$

or $P_3 = \dfrac{P_2 \cancel{V} T_3}{T_2}$

$= \dfrac{3168.61 \times 1837917.36}{661.65}$

or $P_3 = 8801708.288$ pa

$\therefore$ $P_3 = 8.80$ Mpa.

For adiabatic process, 3-4

ii) $PV^{\gamma} = C$

$P_3 V_3^{\gamma} = P_4 V_4^{\gamma}$

or $P_3 = P_4 \left(\dfrac{V_4}{V_3}\right)^{\gamma}$

$$= P_4 \left(\frac{V_1}{V_2}\right)^{\gamma}$$

or $P_3 = P_4 \, r^{\gamma}$

or, $880/708.288 = P_4 \times (8)^{1.4}$

∴ $P_4 = 478895.75$ pa

$= 0.4789$ Mpa.

Since,

$P_4 V_4 = R T_4$

or $478895.75 \times 0.82656 = 287 \times T_4$

∴ $T_4 = 1379.22$ K.

(iii) mean effective pressure:

$$mep = \frac{W_{net}}{stoke} = \frac{W_{net}}{V_2 - V_1}$$

$Q_2 = m \, C_v \, (T_4 - T_1)$

Also, $\eta = \frac{W_{net}}{Q_1}$

or, $W_{net} = \eta \times Q_1$

$= 0.565 \times 1800 \times 10^3$

$= 1017000$ KJ

$$mep = \frac{1017000}{0.10332 - 0.82658}$$

$$= 1406172.225 \ pa$$

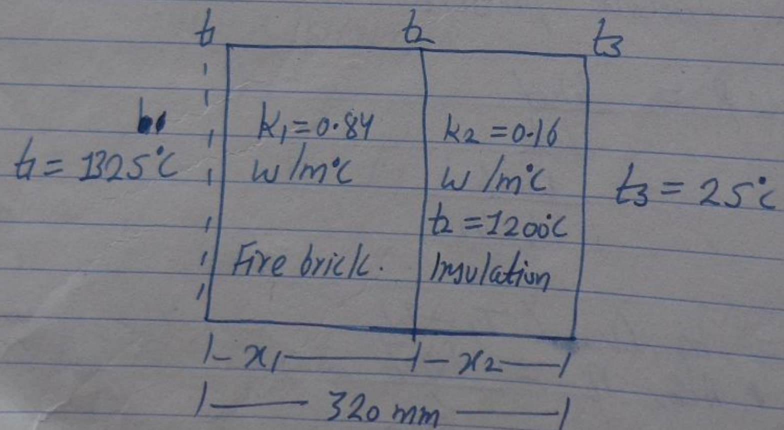
$$\therefore mep = 1406.17 \ kpa$$

8. A reactor's wall 320 mm thick, is made up of an inner layer of fire brick ($k = 0.84 \ w/m°c$) covered with a layer of insulation ($k = 0.16 \ w/m°c$). The reactor operates at a temperature of 1325°c at the atmospheric ~~pressure~~ temperature of 25°c.

(i) calculate thickness of brick and insulation.
(ii) Heat loss per unit area presuming that the insulating materials has a maximum temperature of 1200°c

soln,

=>



Given,

$t_1 = 1325°c$

$t_2 = 1200°c$

$t_3 = 25°C$

$x_A + x_B = x_1 + x_2 = L = 320\ mm = 0.32\ m$

$\therefore x_2 = (0.32 - x_1)$ — ①

$k_1 = 0.84\ W/m°C$

$k_2 = 0.16\ W/m°C.$

under steady state condition,

$$\dot{Q} = \frac{t_1 - t_3}{\frac{x_1}{k_1} + \frac{x_2}{k_2}} \quad \text{— (II)}$$

$$\Rightarrow \frac{t_1 - t_2}{\frac{x_1}{k_1}} = \frac{t_2 - t_3}{\frac{x_2}{k_2}}$$

From (II) and $\frac{t_1 - t_2}{x/k_1}$

$$\text{or,}\quad \frac{(1325 - 25)}{\frac{x_1}{0.84} + \frac{x_2}{0.16}} = \frac{(1325 - 1200)}{\frac{x_1}{0.84}}$$

$$\text{or,}\quad \frac{1300}{1.190\,x_1 + 6.25(0.32 - x_1)} = \frac{105}{x_1}$$

$$\text{or,}\quad \frac{1300}{1.190\,x_1 + 2 - 6.25x_1} = \frac{105}{x_1}$$

or $\dfrac{1300}{2-5.06\,x_1} = \dfrac{105}{x_1}$

or, $1300\,x_1 = 105(2-5.06\,x_1)$

or, $1300\,x_1 = 210 - 531.3\,x_1$

or $x_1 = \dfrac{210}{(1300+531.3)}$

$\therefore x_1 = 0.1146$ m

$\therefore x_2 = 320 - 114.6$
$= 205.4$ mm

Thickness of insulation is 205.4 mm.

ii) Heat loss per unit area, q ;

or $\dfrac{Q}{A} = \dfrac{t_1 - t_2}{x_1/k_1}$

$= \dfrac{1325-1200}{\dfrac{0.1146}{0.84}}$

$\therefore q = 916.23\ \text{W/m}^2$

6th-Feb-2016

We Also Have :

1) *Electronic Device Chapter wise Note for 3rd Semester*

2)*LOGIC CIRCUIT Board Exam solutions 3rd semester*

3)Network Theory Board Exam + probable Important Questions solution
4)Electrical Engineering Material
5) Engineering Chemistry
6)OOPS Board Exam Solutions+ probable important Questions solutions
7)C programming BOOK
8)C Programing Programs 70 Examples

And Many More for HSEB , SLC and Bachelor Students