GOVERNMENT COLLEGE OF ENGINEERING,KALAHANDI

DEPARTMENT OF ELECTRICAL ENGINEERING

Lecture notes on Network Theory

Submitted By

Soudamini Behera

BEES2211 **Network Theory**

MODULE- I

1. NETWORK TOPOLOGY: Graph of a network, Concept of tree, Incidence matrix, Tie-set matrix, Cut-set matrix, Formulation and solution of network equilibrium equations on loop and node basis.
2. NETWORK THEOREMS & COUPLED CIRCUITS: Substitution theorem, Reciprocity theorem, Maximum power transfer theorem, Tellegen's theorem, Millman's theorem, Compensation theorem, Coupled Circuits, Dot Convention for representing coupled circuits, Coefficient of coupling, Band Width and Q-factor for series and parallel resonant circuits.

MODULE- II

3. LAPLACE TRANSFORM & ITS APPLICATION: Introduction to Laplace Transform, Laplace transform of some basic functions, Laplace transform of periodic functions, Inverse Laplace transform, Application of Laplace transform: Circuit Analysis (Steady State and Transient).
4. TWO PORT NETWORK FUNCTIONS & RESPONSES
: z, y, ABCD and **h**-parameters, Reciprocity and Symmetry, Interrelation of two-port parameters, Interconnection of two-port networks, Network Functions, Significance of Poles and Zeros, Restriction on location of Poles and Zeros, Time domain behaviour from PoleZero plots.

MODULE- III

5. FOURIER SERIES & ITS APPLICATION: Fourier series, Fourier analysis and evaluation of coefficients, Steady state response of network to periodic signals, Fourier transform and convergence, Fourier transform of some functions, Brief idea about network filters (Low pass, High pass, Band pass and Band elimination) and their frequency response.
6. NETWORK SYNTHESIS
: Hurwitz polynomial, Properties of Hurwitz polynomial, Positive real functions and their properties, Concepts of network synthesis, Realization of simple R-L, R-C and L-C functions in Cauer-I,Cauer-II, Foster-I and Foster-II forms.

-: NETWORK THEORY:-

Any branch in a network may be substituted by a different branch without disturbing the voltage and current in the entire network provided the new branch has same set of terminal voltage and current as the original branch.

This theorem states that components can be interchanged as long as the terminal voltage and current are maintained.

NOTE.

This theorem is a general theorem and is applicable for any network. the modified network must be a unique solution. This theorem is very important in circuit analysis of network having non-linear elements.

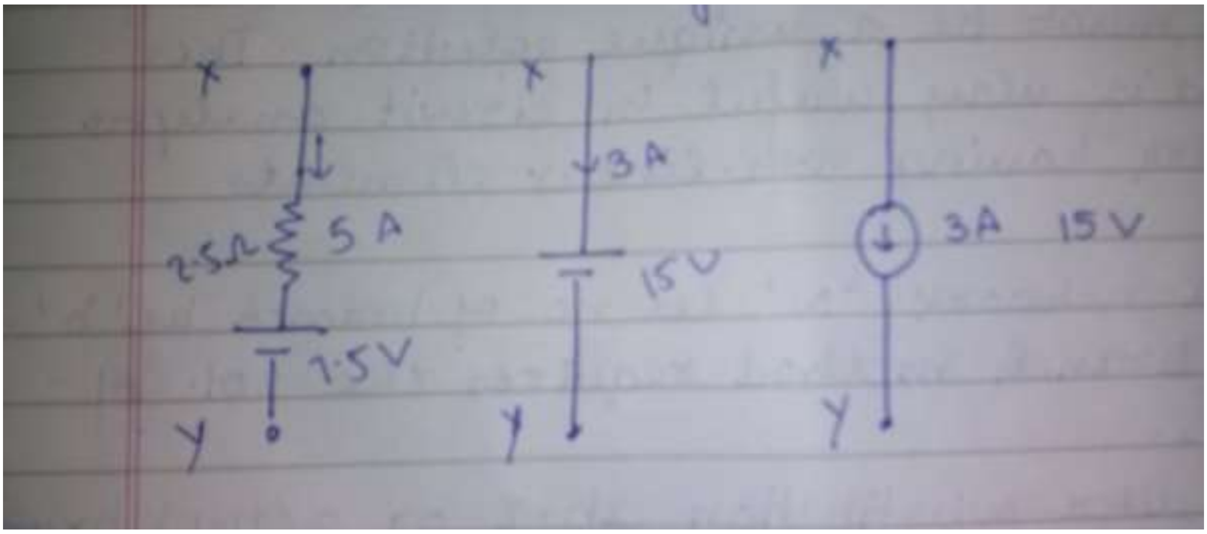
PROOF.

In a network 'N' let no of branches is 'b'. The branchy method requires the solution of '2b' equations. Now after substitution 2b-2 or 2 (b-1) branch equations remain unaltered. However as a branch voltage and current remain same in it. This means that the set of 2b equation will be satisfied with the same current and voltage as before.

EXPLANATION

Let's take an example of a simple networkwhere we see the branch equivalent of the load resistance.

It is observed that a known potential difference and current in a branch can be replaced by an ideal current and voltage source respectively.



LIMITATION

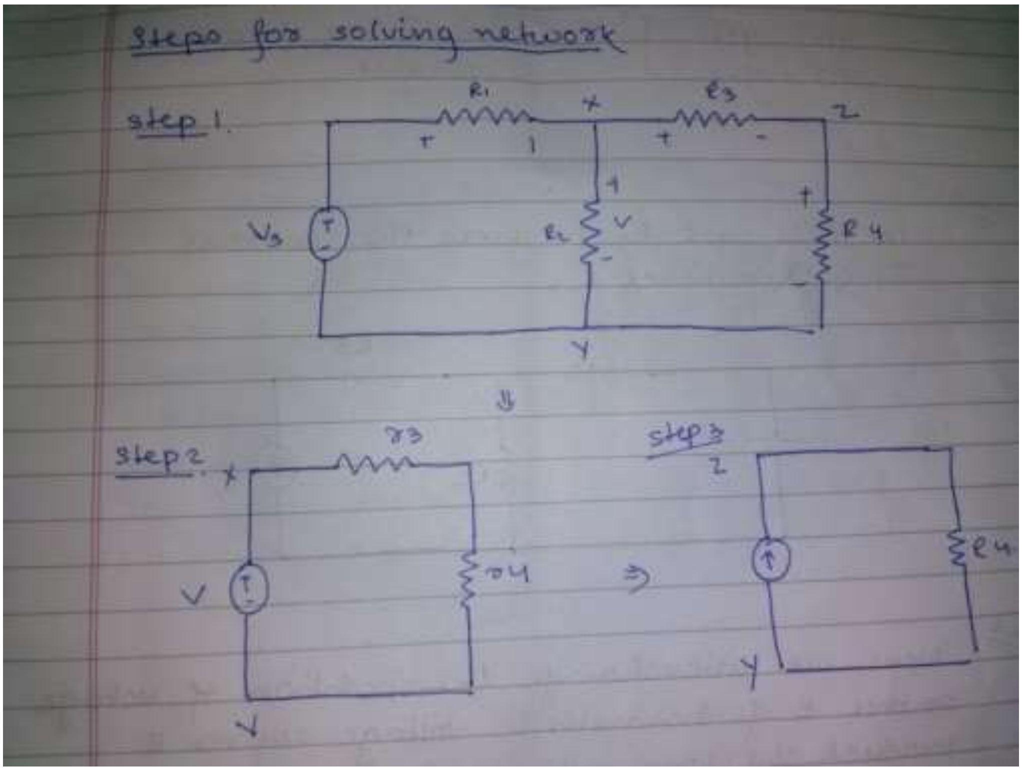
The theorem can't be used to solve the network containing two or more sources that are not in series or parallel.

- First obtain the concerned branch voltage and through current given by V_{xy} I_{xy}
- The branch may be substituted by independent voltage source or current source shown in fig.respectively.

RECIPROCTY THEORUM

In a linear bilateral network if current flowing through any branch is I due to voltage source E, then the same current will flow when the position of voltage and ammeter are interchanged.in other case E and I are mutually transferable.

Now transfer resistance =E/I



$$[Y] = \begin{pmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{pmatrix}$$

$$Y_{12} = Y_{21}$$

$$Y_{13} = Y_{31}$$

$$Y_{23} = Y_{32}$$

For reciprocity network reciprocity theorem is applicable

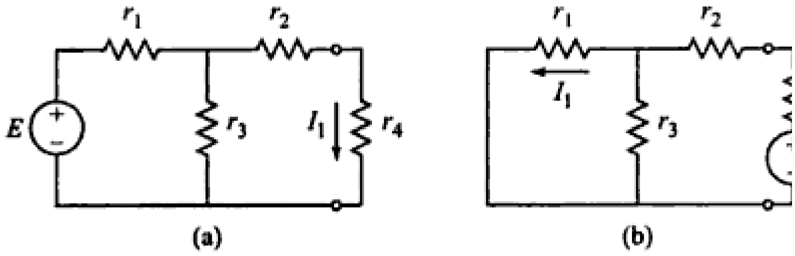


Fig. (a) In this case E produces the current I_1 in resistance R_2.

Fig. (b) Now we interchange the position of voltage source E and ammeter that means voltage source E produce the same current (I) in resistance R_1.

Linear circuit – it is a one in which parameters remain constant i.e. they do not change w.r.t. current or voltage

Non-linear circuit – its properties change w.r.t. to current and voltage.

Bilateral circuit – it is one whose properties or characteristics are same in either direction.

Unilateral circuit – it is the circuit whose properties change w.r.t. direction of operation.

Passive network – it is a network which contains no source of emf or voltage source.

Active network – it is a network which contains one or more source of emf.

Dependent or Controlled source – in these type of sources voltage or current source is not fixed but is dependent on a voltage or current fixed at some other part of the circuit.

Ideal voltage source – it is a circuit element where the voltage across it is independent of current. In analysis a voltage source supplies a constant AC or DC voltage b/w its terminal where any current flow through it.

An ideal voltage source has internal resistance 0.

It is able to supply or absorb any amount of current.

Voltage drop in the source is 0.

Does not consume any power.

Eg. DC source, ideal battery, generator etc.

Ideal current source - – it is a circuit element where the current through it is independent of voltage across it. An independent current source with 0 current is identical to an ideal open circuit, for this reason the internal resistance of an ideal current source is infinite.

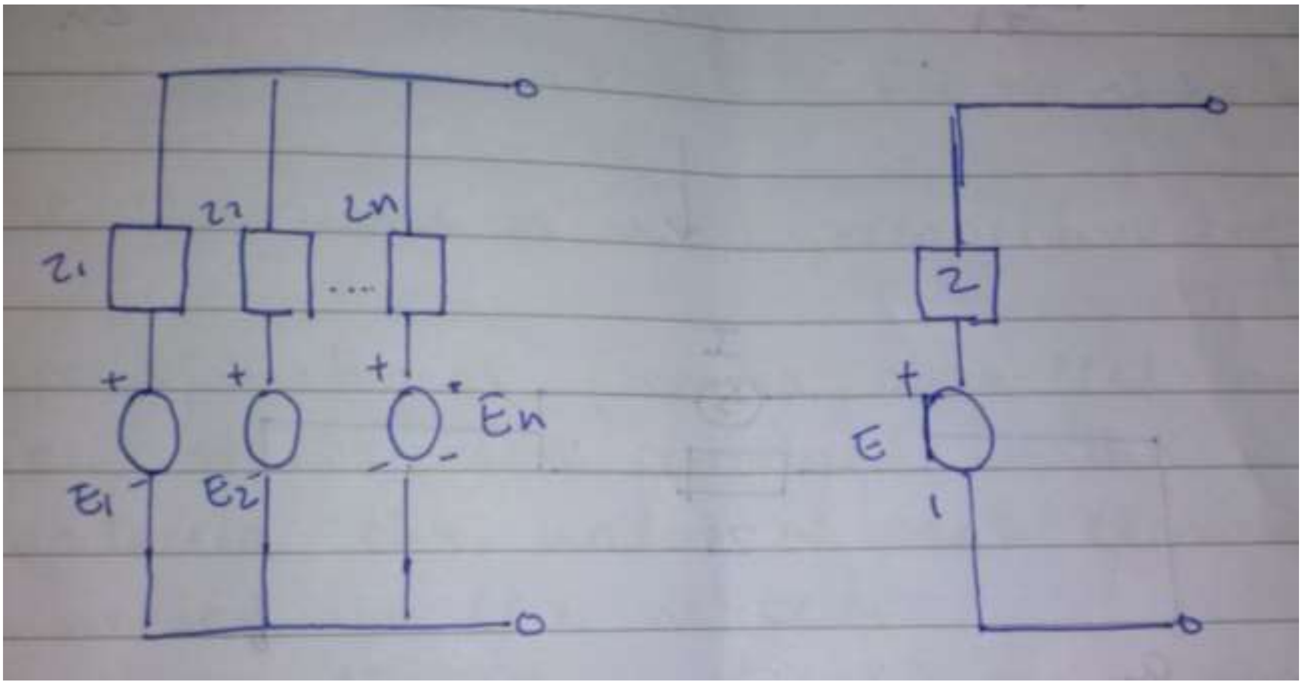
It is capable of supplying infinite power.

E.g.Battery

Independent ideal voltage source – an ideal independent voltage source is a two terminal circuit element that maintains a constant terminal voltage whatever be the value of current flowing through it.

MILLMAN'S THEORUM

This theorem states that if several ideal voltage sources (E_1 E_2 E_3…….)in series with impedances (z_1,z_2,z_3….)are connected in parallel then the circuit can be replaced by a single ideal voltage source E in series with an impedances Z such that

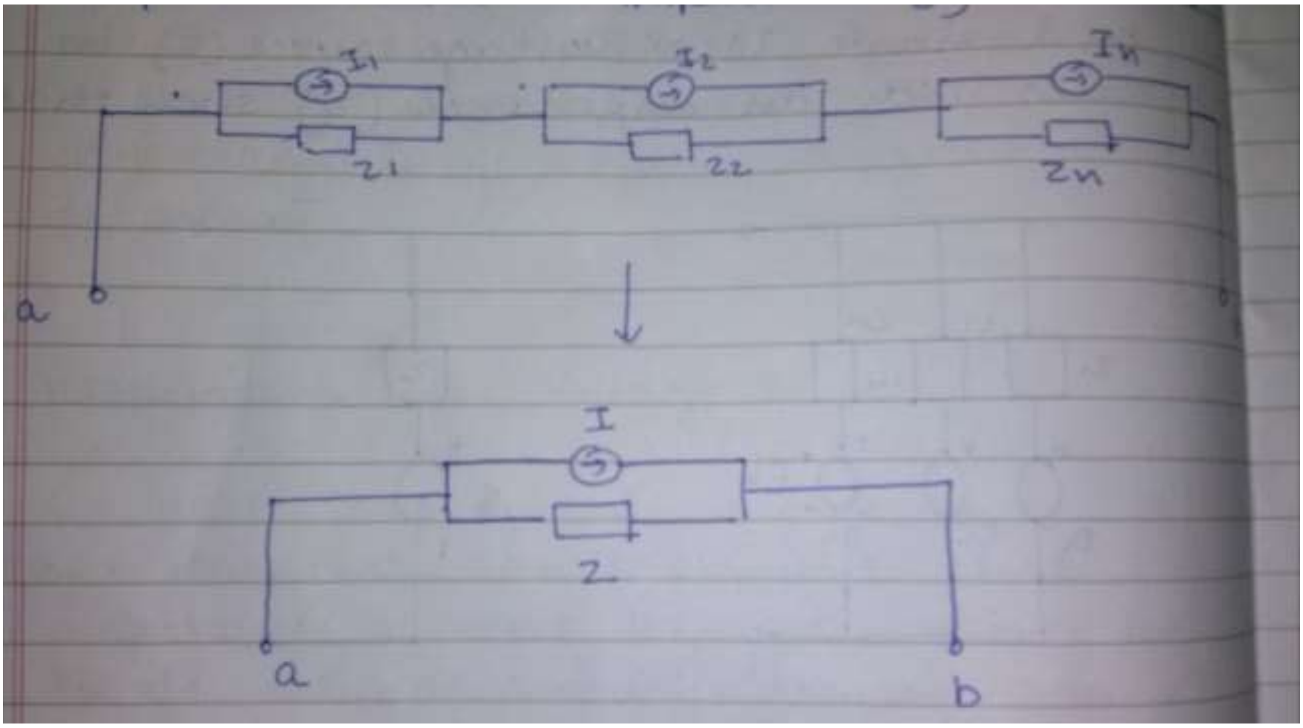


$$E = \frac{\Sigma IiZi}{\Sigma Zi}$$

$$Z = \frac{1}{\Sigma Yi}$$

<u>DUAL MILL MANS THEORUM</u>

If several ideal current sources (I1 I_2 ….), in parallel with several impedance $z_1 z_2$ z_3Connected in series then the circuit can be replaced by a single ideal current IIn parallel with an impedance (z) such that



$$I = \frac{\Sigma IiZi}{\Sigma Zi}$$

Z=ΣZi

TELLEGEN'S THEORUM

In any network the sum of instantaneous power absorbed by various elements is always equal to 0. Therefore the total power delivered by various active sources is equal to total power absorbed by various passive element by branches of the network.

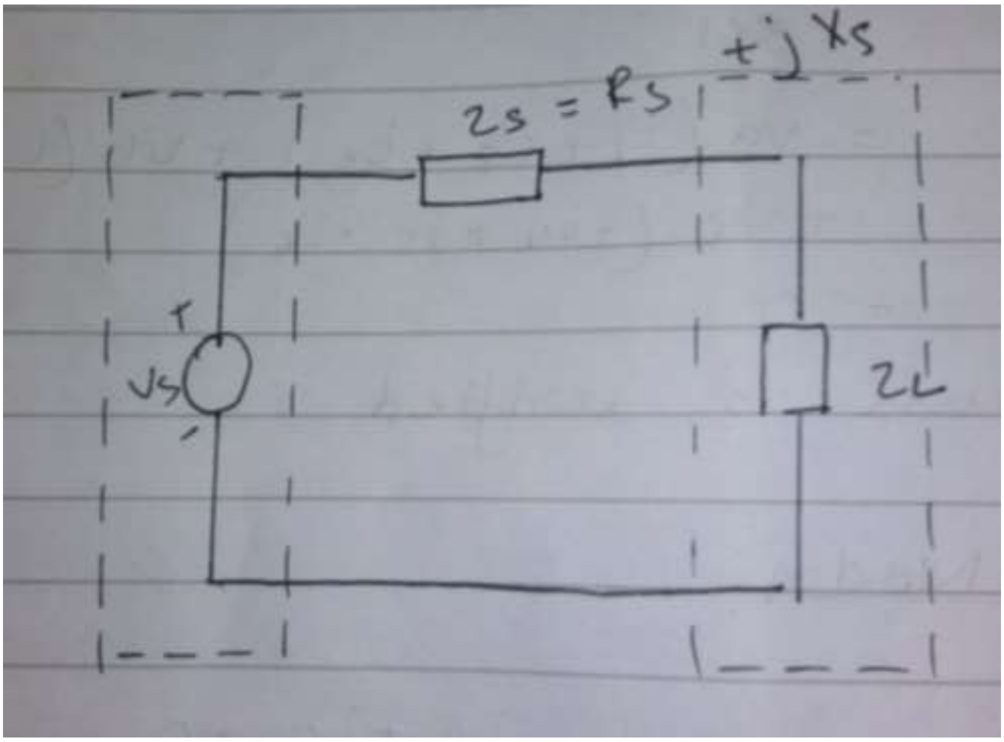
$$\sum_{k=1}^{b} \mathrm{V}i\, ik \;= 0$$

The theorem is valid irrespective of

- Type of network. (Series, parallel, any).
- Type of element contained in the network(R, L, C) .
- Whatever the value of each element in the network.
- Whatever the type of voltage or current sources present in the network.
- So long as the KVL and KCL equation are applicable to the network.
- Generally applicable for linear bilateral network.

MAXIMUM POWER TRANSFER THEORY

According to this theory, maximum power can be transferred from the source to node where the node impedance is equal to the complex conjugate of internal impedance of the circuit.



.Proof:-

Let V_s is the supply voltage

Z_s=Internal impedance/source impedance of the circuit

$=R_s+jX_s$

Z_l=Load impedance of the circuit

$=R_L+jX_L$

P_L+ Power consumed by load

In the above circuit

$I = \frac{Vs}{Zs+Zl}$

$= \frac{Vs}{(Rs+jXs)+(Rl+jXl)}$

$= \frac{Vs}{(Rs+Rl)+j(Xs+Xl)}$

$P_L = I^2 Z_L$

$= \frac{Vs^2}{\{(Rs+Rl)+j(Xs=Xl)\}^2}$

<u>According to AC Analysis:-</u>

For maximum power transformation X_L and X_S are two variables which vary with frequencies where R_L and R_s remain constant.

Now $P_L=P_{Lmax}$ when denominator is minimum.

For denominator to be minimum

$X_s+X_L=0$

$=>X_L=-X_S$

<u>According to DC analysis:-</u>

If complex part is reduced to 0

$P_L = \frac{Vs^2}{(Rs+Rl)^2} * R_L$

In this case load resistance R_L is variable in nature so that $P_{L=}P_{LMax}$

$\frac{\partial Pl}{\partial Rl}=0$

$=> \frac{\partial}{\partial Rl}\left[\frac{Vs^2Rl}{(Rs+Rl)^2}\right] = 0$

$=> \frac{(Rs+Rl)^2 * V^2 - V^2Rl(2Rs+2Rl)}{(Rs+Rl)^4}$

$=> R_s=R_L$

$Z_L=R_L+jX_L$

$Z_L=R_L-jX_S$

But, $Z_S= R_S+jX_S$

$Z_S^* =R_S-jX_S$

Hence $Z_L = Z_S^*$

So theorem is proved.

$P_{LMax}=\frac{Vs^2Rl}{(Rs+Rl)^2} = \frac{Vs^2Rl}{4(Rl)^2}$

$P_{LMax}= \frac{Vs^2}{4(Rl)^2/S}$

Efficiency (ɳ) $=\frac{outpur\ power}{Input\ power}$

$= \frac{Pl}{Ps+Pl^1}$

$=\frac{I^2Rl}{I^2Rs+I^2Rl}$

$=\frac{Rl}{Rs+Rl}$

In case of maximum power transfer

$R_S = R_L$

$\eta = \frac{Rl}{2Rl} = \frac{1}{2} = 50\%$

Hence maximum 50% of power can be transferred from source to load.

Case-1:-

$Z_S = R_S + jX_S$

$Z_L = R_L + jX_L$

Condition for maximum transfer is:-

$R_S = R_L$

$X_L = X_S$

$Z_L = Z_S^*$

Case 2:-

Condition for maximum power transfer

$R_L = R_S$

THEVENIN'S THEORUM:-

It states that in linear active two terminalnetwork containing impedances (for ac circuits) or resistance (for dc circuit) and voltage source can be replaced by a single voltage (Vth) in series with impedance (Zth).

Vth = thevenin's voltage.

Zth = thevenin's equivalent impedance.

PROCEDURE OF SOLVING PROBLEM

Steps 1- remove the resistance RL across which current is to be found.

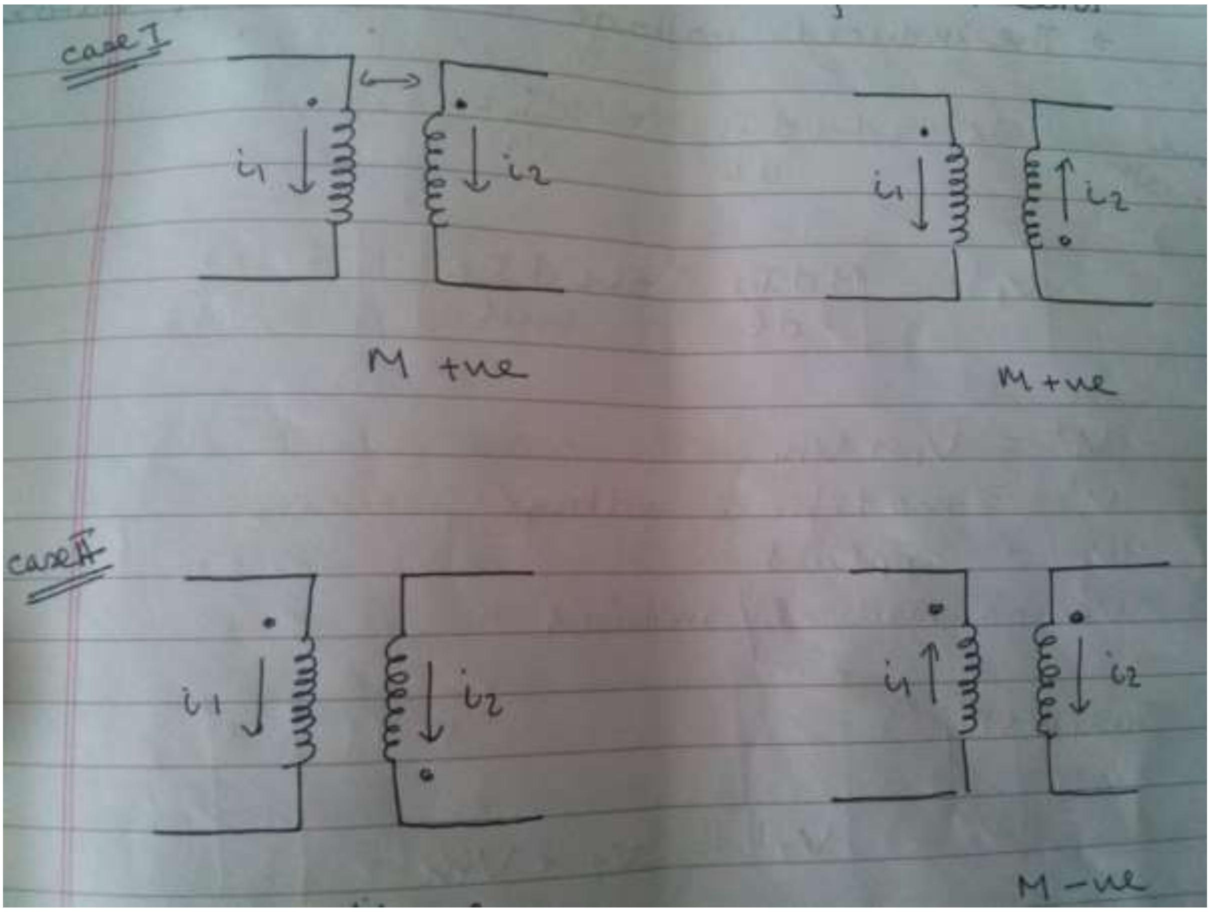
Step2 – calculate the open circuit voltage across the open terminal voltage.

Step3- calculate the Rth across the open circuit terminal. Ifthere is a source then replace by its internal resistance.

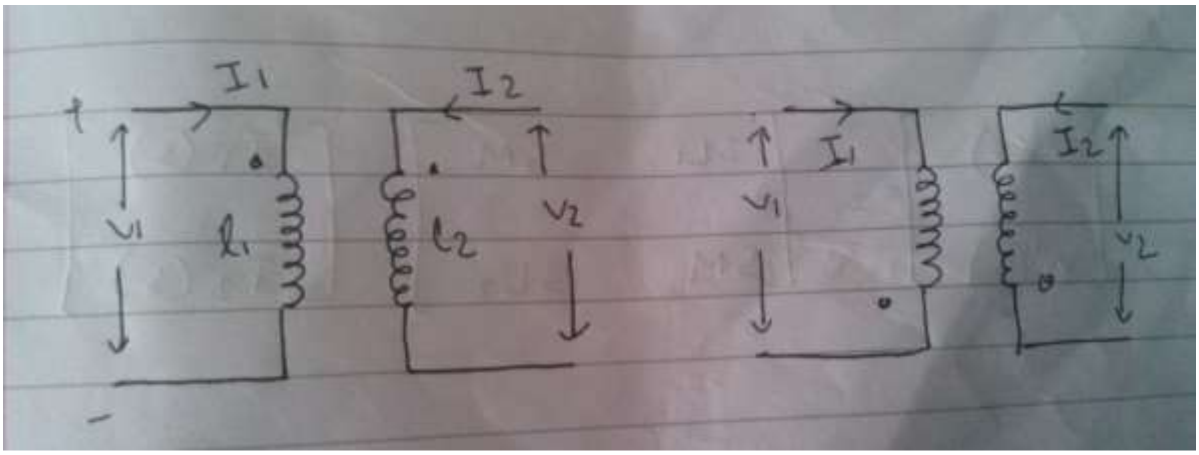
Step4- if there is a current source it can be replaced by an open circuit.

Step5 – connect Rth in series withVth that indicates equivalent circuit

DOT CONVENTION IN TRANSFORMER COILS



Electrical equivalence for magnetic coupled circuit



For same polarity -> effect of mutual inductance

➔ The induced voltage is additive in nature

$$V_1' = L1\frac{dI1}{dt} + L2\frac{dI2}{dt}$$

$V_1' = V_1 + V_m$

V_1' =modified voltage

V_1 =applied voltage

V_m = mutual/induced voltage

Similarly,

$V_2' = V_2 + V_m$

$V_1(S) = SL_1.I_1(S) + SM.I_2(S)$

$V_1(S) = SM.I_1(S) + SL_2.I_2(S)$

$$\begin{pmatrix} V_1'(S) \\ V_2'(S) \end{pmatrix} = \begin{pmatrix} SL_1 & SM \\ SM & SL_2 \end{pmatrix} \begin{pmatrix} I_1(S) \\ I_2(S) \end{pmatrix}$$

In case of sinusoidal circuit excitation when(s=jw)

$$\begin{pmatrix} V_1' \\ V_2' \end{pmatrix} = \begin{pmatrix} jWL_1 & jWM \\ jWM & jWL_2 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$

Case-2:-

When M is –ve

The induced voltage is subtractive in nature.

$$V_1' = L\frac{dI1}{dt} - M\frac{dI2}{dt}$$

$$V_2' = -M\frac{dI1}{dt} + L2\frac{dI2}{dt}$$

$V_1^l = V_1 - V_m$

V_1^l =modified voltage

V_1 =applied voltage

V_m = mutual/induced voltage

Similarly,

$$V_2^l = V_2 - V_m$$

$V_1(S)=SL_1.I_1(S)-SM.I_2(S)$

$V_1(S)=SM.I_1(S)+SL_2.I_2(S)$

$$\begin{pmatrix} V_1^l(S) \\ V_2^l(S) \end{pmatrix} = \begin{pmatrix} SL_1 & -SM \\ -SM & SL_2 \end{pmatrix} \begin{pmatrix} I_1(S) \\ I_2(S) \end{pmatrix}$$

In case of sinusoidal circuit excitation when(s=jw)

$$\begin{pmatrix} V_1^l \\ V_2^l \end{pmatrix} = \begin{pmatrix} jWL & -jWM \\ -jW & jWL_2 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$

CO-EFFICIENT OF COUPLING

Consider two coils having self-inductance L_1 and L_2 placed very close to each other. Let the number of turns of the two coils be N_1 and N_2 respectively. Let coil 1 carries current i_1 and coil 2 carries current i_2.

Due to current i_1, the flux produced is Φ_1 which links with both the coils. Then from the previous knowledge mutual inductance between two coils can be written as

$$M = N_1 \Phi_{21}/i_1 \qquad \text{............ (1)}$$

where Φ_{21} is the part of the flux Φ_1 linking with coil 2. Hence we can write, $\Phi_{21} = k_1 \Phi_1$.

$$\therefore \quad M = N_1 (k1\ \Phi_1)/i_1 \qquad \text{.......... (2)}$$

Similarly due to current i_2, the flux produced is Φ_2 which links with both the coils. Then the mutual inductance between two coils can be written as

$$M = N_2 \Phi_{21}/i_2 \qquad \text{.......... (3)}$$

Where Φ_{21} is the part of the flux Φ_2 linking with coil 1. Hence we can write $\Phi_{21} = k_2 \Phi_2$.

$$\therefore \quad M = N_2 (k_2 \Phi_2)/i_2 \qquad \text{........... (4)}$$

Multiplying equations (2) and (4),

$$M^2 = \frac{N_1(k_1\phi_1)}{i_1} \cdot \frac{N_2(k_2\phi_2)}{i_2}$$

$$\therefore \quad M^2 = k_1 k_2 \left[\frac{N_1\phi_1}{i_1}\right]\left[\frac{N_2\phi_2}{i_2}\right]$$

But $N_1\Phi_1/i_1$ = Self-induced of coil 1 = L_1

$N_2\Phi_2/i_2$ = Self-induced of coil 2 = L_2

$\therefore \quad M^2 = k_1k_2L_1L_2$

$\therefore \quad M = \sqrt{(k_1k_2)}\ \sqrt{(L_1L_2)}$

Let $\quad k = \sqrt{(k_1k_2)}$

$$\therefore \quad M = k \sqrt{(L_1L_2)} \qquad \text{......... (5)}$$

Where 'k' is called coefficient of coupling.

$\therefore \quad k = M/ (\sqrt{(L_1L_2)})$

a. When k = 1, it is called critical coupling and the network is called coupled network.
b. When k > 1, it is called tight coupling.
c. When k < 1 ,it is called looser coupling,

RESONANCE IN A COUPLED CIRCUIT

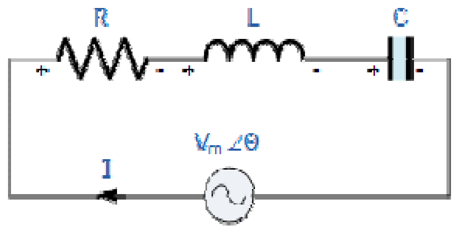
Resonance is typical state or condition of a system during which the frequency of oscillation produced by an external forcing function matches with the natural frequency of the system by causing a response of maximum amplitude.

Resonance doesn't take place on the steady state condition and for its occurrence in a system there must be some disturbances from outside the system that introduces oscillation in the system.

As the system continues with these oscillation interchange of energy takes place b/w two independent energy storing component present within the system (components like inductor and capacitor)

In some cases resonance occurs when the inductive reactance and capacitive reactance of the circuit are equal in magnitude then resonance occurs.

SERIES RESONANCE



Electrical resonance occurs in an AC circuit when the two reactance which are opposite and equal cancel each other out as XL = XC and the point on the graph at which this happens is where the two reactance curves cross each other. In a series resonant circuit, the resonant frequency, ƒr point can be

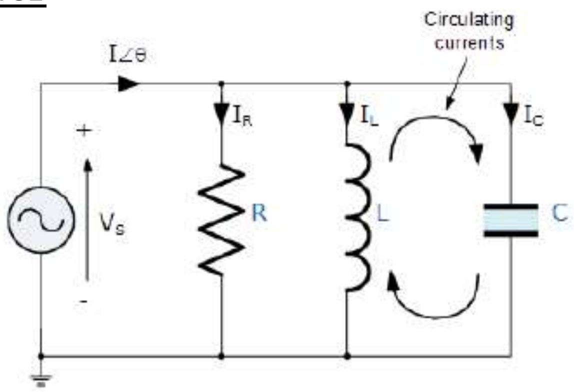
$$X_L = X_C \quad \Rightarrow \quad 2\pi f L = \frac{1}{2\pi f C}$$

$$f^2 = \frac{1}{2\pi L \times 2\pi C} = \frac{1}{4\pi^2 LC} = \sqrt{\frac{1}{4\pi^2 LC}}$$

$$\therefore\ f_r = \frac{1}{2\pi\sqrt{LC}}\ (\text{Hz}) \quad \text{or} \quad \omega_r = \frac{1}{\sqrt{LC}}\ (\text{rads})$$

calculated as follows

PARALLEL RESONANCE



$$Y = \frac{1}{R} + \frac{1}{j\omega L} + j\omega C$$

or

$$Y = \frac{1}{R} + \frac{1}{2\pi f L} + 2\pi f C$$

Resonance occurs when XL = XC and the imaginary parts of Y become zero. Then:

$$X_L = X_C \quad \Rightarrow \quad 2\pi f L = \frac{1}{2\pi f C}$$

$$f^2 = \frac{1}{2\pi L \times 2\pi C} = \frac{1}{4\pi^2 LC} = \sqrt{\frac{1}{4\pi^2 LC}}$$

$$\therefore \; f_r = \frac{1}{2\pi\sqrt{LC}} \, (\mathrm{Hz}) \quad \text{or} \quad \omega_r = \frac{1}{\sqrt{LC}} \, (\mathrm{rads})$$

NETWORK TOPOLOGY

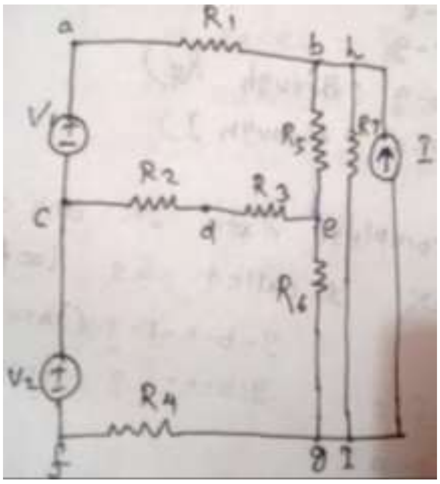
Defination: - Topology refers to the science of place. In mathematics, topology is a branch of geometry in which figure are considered perfectly elastic.

Network topology refers to the properties that relates to the geometry of the network, which remains unaffected, when the graph is twisted or folded; provided that no parts are cut & no new connections are made.

Why to study network topology?

Ans: to suppress the nature of the circuit elements that make up the network

Terminology:-



Node: - it is an equipotential point at which two or more circuit elements are joined.

From the above figure nodes are:- a,b,c,d,e,f,g,h

Junction: - It is that point of a network where three or more circuit elements are joined.

From the above figure junctions are:-c, e, b, g

Essential node:- A node that joins three or more circuit elements.

From the above figure junctions are:-c, e, b, g

Branch:- It is a path of a network that connects two nodes.

From the above figure branches are:- v1, v2, R1, R2, R3, R4, R5, R6, R7.

Essential branch:- It is a path of a network that lies between two junction points.

From the above figure Essential branches are:- c-a-b, c-d-e, c-f-g, b-e, e-g, b-g(through R7), b-g(through I)

Loop:- A loop is a complete path or any closed path in a network is called as loop.

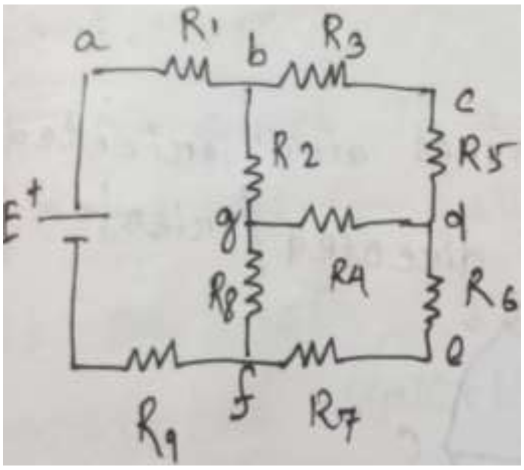
From the above figure Essential branches are:- a-b-e-c-a, c-d-e-g-f -c, a-b-e-g-f-c-a, c-d-e-g-f-c, g-b-h-i-g(through R7), g-b-h-i-g(through I).

Mesh:- An independent loop is a mesh. **OR** It is a special type of loop which does not contain any other loops within it.

From the above figure mesh are:- a-b-e-c-a, c-d-e-g-f –c, g-b-h-i-g, i-h-i.

ASSIGNMENT:

Find the nodes, Junction, essential nodes, circuit elements, branches, essential branches of the following network?



Graph of Network:-

A linear graph is defined as a collections of points called nodes, and line segment called branches, the nodes being joined together by the branches.

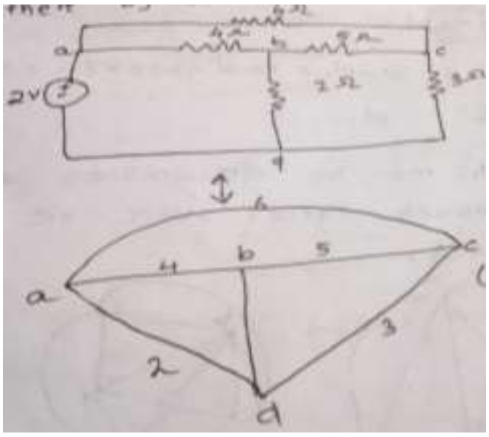
While drawing the graph of a given network, the following rules are to be noted.

- All the passive elements of network i.e. R, L, C are represented by straight lines.
- An ideal voltage source & by short circuit & ideal current sources is represented by their internal resistance i.e. (Voltage source by short circuit and current sources by open circuit) if they are accompanied by passive element i.e. a shunt admittance in a current source and a series impedance in voltage source.
- If the sources are not accompanied by passive elements an arbitrary impedance is assume to accompany the sources and finally we find the result by letting the impedance R->0 or R->+∞.

Let us consider a graph shown in this figure.

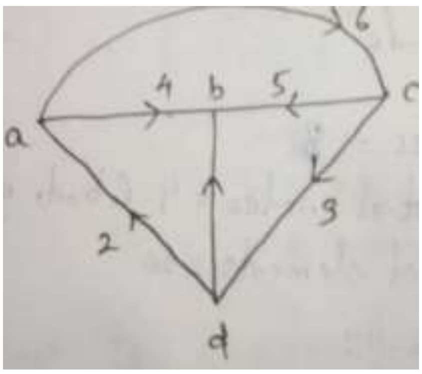
Nodes= a, b, c, d

Branches= 1, 2, 3,4,5,6



ORIENTED GRAPH:-

A graph whose is branches are oriented or directed is called a directed/oriented graph.



FULLY CONNECTED GRAPH:-

It is a graph where each node is connected to all other nodes.

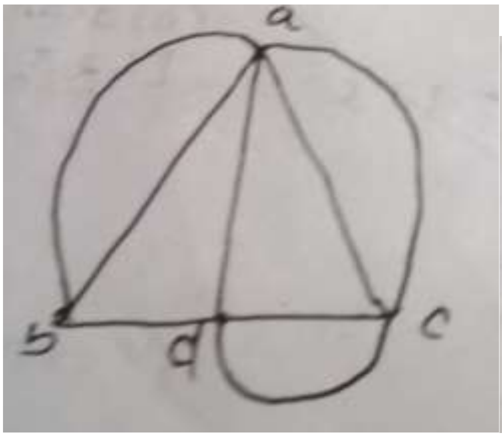
->Direction of current in any branch can be shown below.

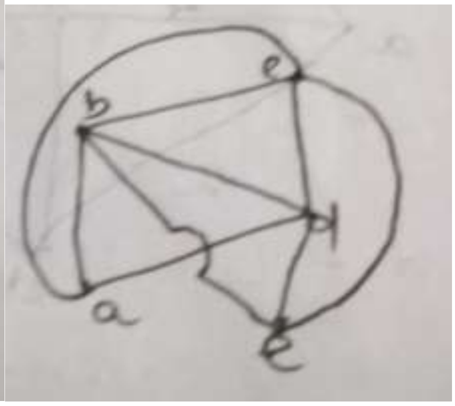
RANK OF A GRAPH:-

The rank of a graph =no. of nodes-1=n-1

PLANAR GRAPH:-

A graph which may be drawn on a plane surface such that no branch passes over or under any other branch.



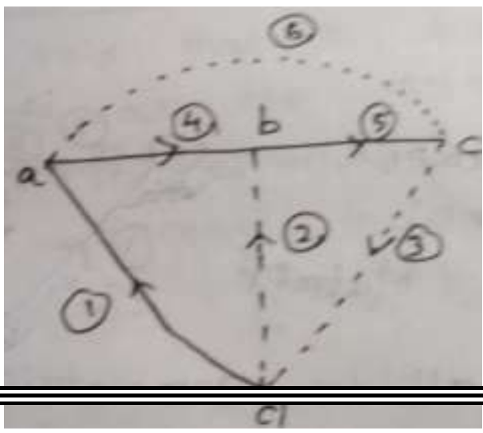


SUB GRAPH:-

A sub graph is a subset of the branches and nodes of a fully connected graph. The sub graph is said to be proper if it consists of strictly less than fully connected graph.

PATH:- A path is a particular sub graph where only two branches are incident at every node except the internal node that is starting and finishing node. At internal nodes, only one branch is incident.

CONCEPT OF TREE:-



Solid line= tree

Dotted line= co-tree

Tree=[1,4,5]

Co-tree= [2,3,6]

Tree- A Tree is a set of branches having no closed loops, but connect all the elements in the network. A tree should contain all the nodes. A graph have more than one tree.

Twigs-The branches of tree is called twigs. In the above figure twigs are 1,4,5.

No of twigs=n-1,where n=no. of nodes

Links-if a graph for a network is known then the remaining branches are referred as link.

No of links=total no of branches – total no twigs=b-n+1

Co- tree- The collection of links is called co- tree.

- No of possible tree of a graph= det[A.A']

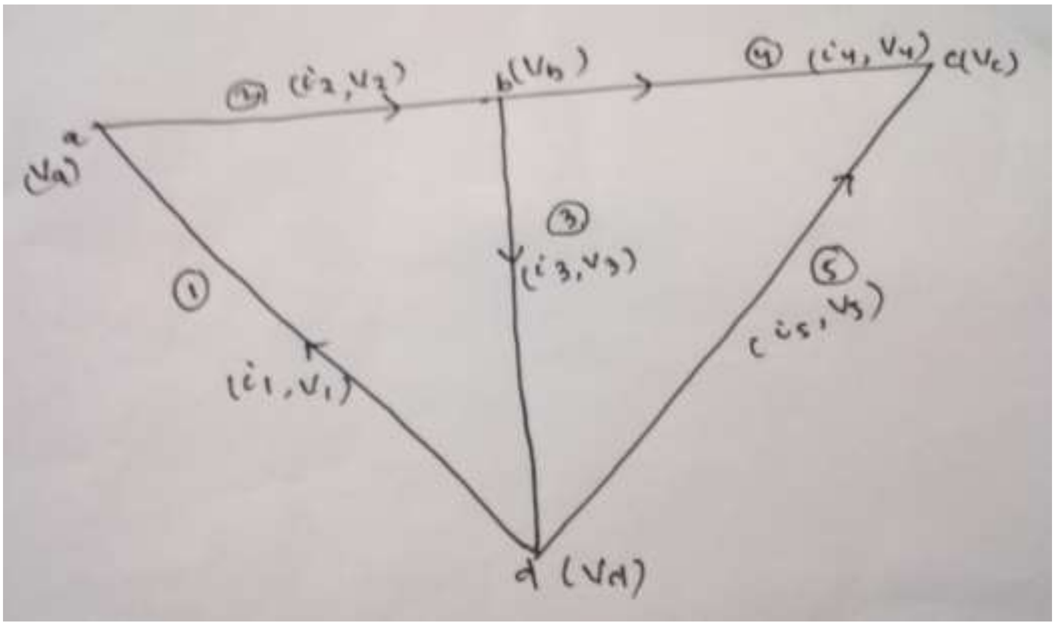
Where A = reduced incidence matrix and A'=transpose of A

- For a fully connected graph no. of trees= n^(n-2)
- No. of total branch of a fully connected graph =nC2

Properties of tree:-

- In a tree, there exists one and only one path between any pairs of nodes.
- Every connected graph has at least one tree.
- A tree contains all the nodes of the graph.
- There is no closed loop in a tree and hence tree is circuit less.

INCIDENCE MATRIX (Aa):-



- It symbolically describes a network.
- The matrix which translates all the geometrical features of the graph into an algebraic expression.
- Every graph has incidence matrix & vice-versa.
- It facilitates the testing & identification of the independent variable. It is a matrix which represent the graph uniquely.
- In incidence matrix the nodes are represented in rows & branches are represented in columns.
- Each column of the matrix has one entry of +1 & another entry of -1. So, the each column of matrix is 0.
- The convention of current taken arbitrarily.
  Let's us take the current coming towards the node is –ve & current going out from the node is +ve. So, the incidence matrix of the above figure is :

$$
\begin{array}{c}
a \\ b \\ c \\ d
\end{array}
\begin{bmatrix}
-1 & 1 & 0 & 0 & 0 \\
0 & -1 & 1 & 1 & 0 \\
0 & 0 & 0 & -1 & -1 \\
1 & 0 & -1 & 0 & 1
\end{bmatrix}
\quad 4*5
$$

Properties of Incidence matrix:-

- The determinant of the incident matrix of closed loop is zero.
- Order of incidence matrix is [n*b]
- Rank of incidence matrix is (n-1).

REDUCED INCIDENCE MATRIX:-

For this matrix one node of the graph is taken as reference node. Delete the row corresponding to that node to obtain the reduced incidence matrix.

Order of reduced incidence matrix is = [(n-1)*b]

This matrix is used to find total no of trees for fully connected graph.

Let b is reference node. So, reduced incidence matrix is

$$\begin{array}{c} a \\ c \\ d \end{array}\begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 \\ 1 & 0 & -1 & 0 & 1 \end{bmatrix}_{3*5}$$

Equilibrium equation from incidence matrix :-

Incidence matrix & KCL:-

- Row wise the sum of all branch currents is equal to zero corresponding to each node.
- Total no of Equilibrium equation from incidence matrix or KCL equation = no. of nodes.

At node a; $-i1+i2=0$

At node b; $-i2+i3+i4=0$

At node c; $-i4-i5=0$

At node d; $i1-i3+i5=0$

In matrix form

$$\begin{array}{c} a \\ b \\ c \\ d \end{array}\begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & -1 & -1 \\ 1 & 0 & -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} i1 \\ i2 \\ i3 \\ i4 \end{bmatrix} = 0$$

OR

$$A_a . I_b = 0$$

Incidence matrix & KVL:-

- Column wise the sum of all branch voltages are corresponding to nodal voltage.
- Total no of Equilibrium equation from incidence matrix or KVL equation = no. of branches.

At node 1; $V_1 = -V_a + V_d$

At node 2; $V_2 = V_a - V_b$

At node 3; $V_3 = V_b - V_c$

At node 4; $V_4 = V_b - V_c$

At node 5; $V_5 = -V_c + V_d$

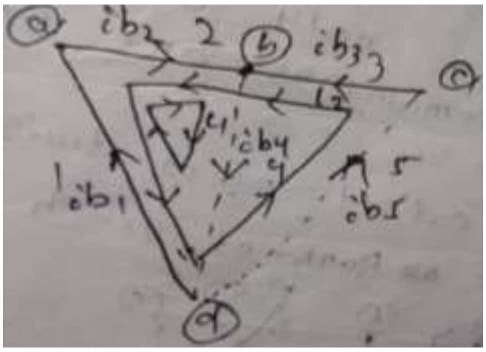
In matrix form

$$\begin{array}{c} a \\ b \\ c \\ d \\ e \end{array}\begin{bmatrix} -1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} * \begin{bmatrix} va \\ vb \\ vc \\ vd \end{bmatrix} = \begin{bmatrix} v1 \\ v2 \\ v3 \\ v4 \\ v5 \end{bmatrix}$$

OR

$$(A_a)'. V_n = V_b$$

F-tie set :-



- F-tie set matrix represents a group of branches in a closed loop. Each tie set contains only one link or cut , remaining are tree branches.
- The direction of each F-tie set matrix is same as that of link.
- No of fundamental F-tie set = no of links = no. of loops = b-n+1 = no. of KVL equation

F-tie set 1=[1,2,4]; 4 is link

F-tie set 2=[1,2,3,5]; 5 is link

F-tie set matrix:-

$$\begin{array}{c} L1 \\ L2 \end{array} \begin{bmatrix} +1 & +1 & 0 & +1 & 0 \\ -1 & -1 & +1 & 0 & +1 \end{bmatrix}$$

F-tie set matrix & KCL:-

From the above graph there is 2 loops i.e. L1,L2.

Branch currents are ib1, ib2, ib3, ib4, ib5.

Loop currents are iL1, iL2.

Then , the branch current are represents in terms of loop currents as follows:

ib1= iL1-iL2

ib2=iL1-iL2

ib3= iL2

ib4= iL1

ib5= iL2

F-tie set matrix & KVL :-

From the graph branch voltages are Vb1, Vb2, Vb3, Vb4, Vb5.

For loop 1 ; VL1= Vb1+Vb2+Vb4=0

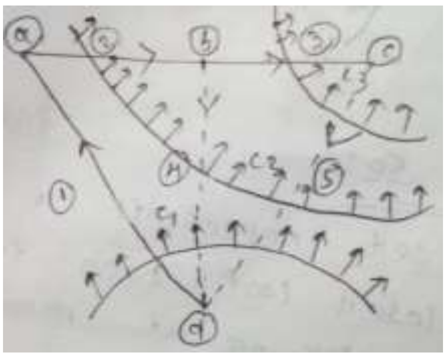
For loop 2 ; VL2= -Vb1-Vb2+Vb3+Vb5=0

In matrix form

$$\begin{array}{c} L1 \\ L2 \end{array}\begin{bmatrix} +1 & +1 & 0 & +1 & 0 \\ -1 & -1 & +1 & 0 & +1 \end{bmatrix} * \begin{bmatrix} Vb1 \\ Vb2 \\ Vb3 \\ Vb4 \\ Vb5 \end{bmatrix} = 0$$

OR $B_a . V_b = 0.$

F-cut set:-



- It is a group of branch which must be called so that the entire group is divided into two parts.
- Each F-cut set contain only one tree branches or twigs , remaining are links or chords.

- No of fundamental F-cut set = no of twigs = n-1 = no. of KCL equation= Rank of the graph

C_1=[1,4,5] ; 1 is Twig

C_2=[2,4,5] ; 2 is twig

C_3=[3,5] ; 3 is twig

F-cut set Matrix (Qc):-

$$\begin{array}{c} C_1 \\ C_2 \\ C_3 \end{array} \begin{bmatrix} 1 & 0 & 0 & -1 & -1 \\ 0 & 1 & 0 & -1 & -1 \\ 0 & 0 & 1 & 0 & -1 \end{bmatrix}$$

F-cut set Matrix & KCL:-

From the above graph, there are 3 cut-sets i.e. C_1,C_2,C_3 ; 5 branch currents i.e Ib_1,Ib_2,Ib_3,Ib_4,Ib_5

The branch current can be represented as

$Ic_1= Ib_1-Ib_4-Ib_5=0$

$Ic_2= Ib_2-Ib_4-Ib_5 =0$

$Ic_3= Ib_3-Ib_5 =0$

In matrix form

$$\begin{bmatrix} 1 & 0 & 0 & -1 & -1 \\ 0 & 1 & 0 & -1 & -1 \\ 0 & 0 & 1 & 0 & -1 \end{bmatrix} * \begin{bmatrix} Ib1 \\ Ib2 \\ Ib3 \\ Ib4 \\ Ib5 \end{bmatrix} = 0$$

OR Qc. Ib =0.

F-cut set Matrix & KVL:-

From the graph tree branch voltages are Vt1,Vt2,Vt3.

Branch voltages are Vb1,Vb2,Vb3,Vb4,Vb5.

The relationship between twig voltages and branch voltages are as follows:-

$Vb_1=Vt_1$

$Vb_2=Vt_2$

$Vb_3=Vt_3$

$Vb_1=-Vt_1-Vt_2$

$Vb_5=-Vt_1-Vt_2-Vt_3$

In matrix form:-

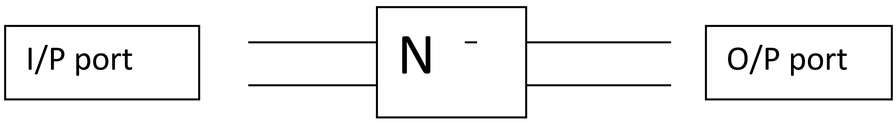
$$\begin{bmatrix} Vb_1 \\ Vb_2 \\ Vb_3 \\ Vb_4 \\ Vb_5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -1 & 0 \\ -1 & -1 & -1 \end{bmatrix} \times \begin{bmatrix} Vt1 \\ Vt2 \\ Vt3 \end{bmatrix}$$

$Vb = (Qc)'.Vt$ Or $Vb = (Aa)'.Vn$ Vn=Nodal voltage matrix

TWO PORT NETWORK

TWO PORT NETWORK FUNCTION AND RESOURCES

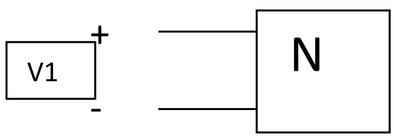
1. The general nature of a network can be represented by rectangular box.
2. A port is pair of nodes across which a device can be connected. These pairs are entry (or exist) point of the network.



3. The voltage is measured across the pair of nodes.
4. The current going into one node is same as the current coming out from the other node in the port.
5. In any electrical system there are two type of network:
    - Single port network
    - Two port network

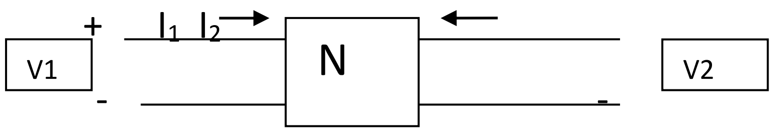
Single Port Network:

It is consistof two terminals. One terminal is positive with respect to other terminal.



Two Port Network:

This type of network has four terminals out of these two terminals are positive and other two ate negative.



In above diagram current I1 entering to the network but I2 flowing away from the network but we are considering that all currents entering to the junction so that I1 current is exporting in nature and I2 current importing in nature. That means exporting and importing takes place in a single phase is called **Port.**

Two Port network Parameters:

1. Open circuit parameter (z-parameter)
2. Short circuit parameter (y-parameter)
3. Transmission parameter (ABCD-parameter/T-parameter)
4. Inverse transmission parameter(T ’-parameter)
5. Hybrid parameter (H-parameter)
6. Inverse hybrid parameter (H ’-parameter)

Open circuit parameter (z-parameter):

In a z-parameter analysis the input voltage ‘V1’ and output voltage ‘V2’ can be expressed as:

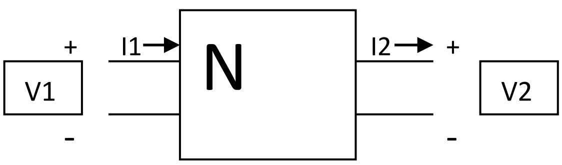
$$V1=Z_{11}I_1 + Z_{12}I_2 \longrightarrow \quad eq^n \ (1)$$

$$V2=Z_{21}I_1 + Z_{22}I_2 \longrightarrow \quad eq^n \ (2)$$

The impedence parameter matrix may be written as:

$$\begin{bmatrix} V1 \\ V2 \end{bmatrix} = \begin{bmatrix} Z11 & Z12 \\ Z21 & Z22 \end{bmatrix} \begin{bmatrix} I1 \\ I2 \end{bmatrix}$$

dependent variable   z-parameter matrix   independent variable



Step 1: Calculation of Z11 and Z21 from eq^n (1)

Put I2=0 (secondary side is open circuited)

$$Z11 = \frac{V1}{I2} \qquad Z21 = \frac{V2}{I2}$$

Driving point impedence   Transfer impedence

Step 2: Calculation for Z12 and Z22 from eq^n (2)

$$\boxed{Z12 = \frac{V1}{I2}} \qquad \boxed{Z22 = \frac{V2}{I2}}$$

Transfer impedence   Driving point impedence

Driving point function: (Z11, Z22, Y11, Y22)

If a function relates the transform of a quantity at one-port to the transform of another quantity at same port, it may be regarded as Driving point function.

Example: 1. Driving point input immitance function:

$$Z11(s) = \frac{V1(s)}{I1(s)} \qquad Y11(s) = \frac{I1(s)}{V1(s)}$$

2. Driving point output immitance:

$$Z22 = \frac{V2(s)}{I2(s)} \qquad Y22(s) = \frac{I2(s)}{V2(s)}$$

Transfer function:(Z12, Z21, Y12, Y21)

If a function relates transform of a quantity at one port to the transform of another quantity at other port, it may be regarded as Transfer function.

Example:

1. Transfer impedence function:

$$Z12(s) = \frac{V1(s)}{I2(s)} \qquad Z21(s) = \frac{V2(s)}{I1(s)}$$

2. Transfer admittance function:

$$Y12(s) = \frac{I1(s)}{V2(s)} \qquad Y21(s) = \frac{I2(s)}{V1(s)}$$

3. Transfer voltage ratio:

$$G12(s) = \frac{V1(s)}{V2(s)} \qquad G21(s) = \frac{V2(s)}{V1(s)}$$

Network function: (N(s))

A network function exhibits the relationship between input (excitation) and output (response) for electrical network.

Network function

- Driving point immitance
  - Driving point impedence — Ratio at same port
  - Driving point admittance — Ratio at different port
- Transfer function
  - Z12(s), Z21(s)
  - Y12(s), Y21(s)
  - G12(s), G21(s)

Short circuit parameter: (Y-parameter/Admittance parameter)

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \qquad \text{eq}^n \ (1) \ \text{KCL equation}$$

$$I_2 = Y_{21} V1 + Y_{22} V_2 \qquad \text{eq}^n \ (2) \ \text{KCL equation}$$

In matrix form:

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \underbrace{\begin{bmatrix} Y_{11} & Y_{22} \\ Y_{21} & Y_{22} \end{bmatrix}}_{\text{Admittance parameter matrix}} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Step 1: calculation for Y_{11} and Y_{21}

Put $V_2 = 0$ (secondary side is short circuited)

$$Y_{11} = \frac{I_1}{V_1} \quad \text{(Driving point admittance)}$$

$$Y_{21} = \frac{I_2}{V_1} \quad \text{(Transfer admittance)}$$

Step 2: calculation for Y_{12} and Y_{22}

Put $V_1 = 0$ (primary side is short circuited)

$$Y_{12} = \frac{I_1}{V_2} \quad \text{(Transfer admittance)}$$

$$Y_{22} = \frac{I_2}{V_2} \quad \text{(Driving point admittance)}$$

<u>Transmission parameter: (ABC-parameter/T-parameter)</u>

$$V= = A V_2 – B I_2 \longrightarrow (1)$$

$$I_1 = C V_2 - D I_2 \longrightarrow (2)$$

In matrix form:

$$\begin{pmatrix} V_{1} \\ I_1 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} V_2 \\ -I_2 \end{pmatrix}$$

V_1 = Sending end voltage

I_1 = Sending end current

V_2 = Receiving end voltage

I_2 = Receiving end current

Step 1: set I_2 = 0 (i.e. secondary side is open circuited)

$$A = \left.\frac{V1}{V2}\right|_{I2=0}$$ (reverse voltage ratio(unit less))

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$$ (transfer admittance (unit is mho))

Step 2: Put V2=0 (i.e. secondary side us short circuited)

$$B = \frac{-V1}{I2}$$ (Transfer impedance (ohm))

$$D = \frac{-I1}{I2}$$ (reverse current ratio (unit less))

These are known as transmission parameter as in transmission line. The current enter in one end and link at other end and we need to know a relation between the sending end quantities and receiving end quantities.

Hybrid parameters: (h-parameter)

General equations are:

$$V_1 = h_{11}\, I_1 + h_{12}\, V_2 \longrightarrow \quad (1)$$

$$I_2 = h_{21}\, I_1 + h_{22}\, V_2 \longrightarrow \quad (2)$$

In matrix form:

$$\begin{pmatrix} V_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} \begin{pmatrix} I_1 \\ V_2 \end{pmatrix}$$

Step 1: Set V2=0 (i.e. secondary is short circuited)

$$h11 = \frac{V1}{I1} \quad \text{(i/p impedance (ohm))}$$

$$h_{21} = \frac{I_2}{I_1} \quad \text{(forward current gain(unit less))}$$

Step 2: $I_1=0$ (i.e. primary side is open circuited)

$$h_{12} = \frac{V_1}{V_2} \quad \text{(reverse voltage gain (unit less))}$$

$$h_{22} = \frac{I_2}{V_2} \quad \text{(o/p admittance (mho))}$$

In above steps, parameter calculation, we conform that both short circuit and open circuit occurred in same network. So it is called Hybrid parameter.

Condition for a network to be Symmetrical or Reciprocal:

Reciprocal

- $Z_{12} = Z_{21}$
- $Y_{12} = Y_{21}$
- $AD - BC = 1$
- $h_{12} = -h_{21}$

Symmetrical

- $Z_{11} = Z_{22}$
- $Y_{11} = Y_{22}$
- $A = D$
- $h_{11} h_{22} - h_{12} h_{21} = 0$

Inter-Relationship between 2-port parameter

	[Z]	[Y]	[ABC]	[h]
[Z]	$\begin{matrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{matrix}$	$\begin{matrix} \frac{Y_{22}}{\Delta Y} & \frac{-Y_{12}}{\Delta Y} \\ \frac{-Y_{21}}{\Delta Y} & \frac{Y_{11}}{\Delta Y} \end{matrix}$	$\begin{matrix} \frac{A}{C} & \frac{\Delta T}{C} \\ \frac{1}{C} & \frac{D}{C} \end{matrix}$	$\begin{matrix} \frac{\Delta h}{h_{22}} & \frac{h_{12}}{h_{22}} \\ \frac{-h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{matrix}$
[Y]	$\begin{matrix} \frac{Z_{22}}{\Delta Z} & \frac{-Z_{12}}{\Delta Z} \\ \frac{-Z_{21}}{\Delta Z} & \frac{Z_{11}}{\Delta Z} \end{matrix}$	$\begin{matrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{matrix}$	$\begin{matrix} \frac{D}{B} & \frac{-\Delta T}{B} \\ \frac{-1}{B} & \frac{A}{B} \end{matrix}$	$\begin{matrix} \frac{1}{h_{11}} & \frac{-h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta h}{h_{11}} \end{matrix}$
[ABC]	$\begin{matrix} \frac{Z_{11}}{Z_{21}} & \frac{\Delta Z}{Z_{21}} \\ \frac{1}{Z_{21}} & \frac{Z_{22}}{Z_{21}} \end{matrix}$	$\begin{matrix} \frac{-Y_{22}}{Y_{21}} & \frac{-1}{Y_{21}} \\ \frac{-\Delta Y}{Y_{21}} & \frac{-Y_{11}}{Y_{21}} \end{matrix}$	$\begin{matrix} A & B \\ C & D \end{matrix}$	$\begin{matrix} \frac{-\Delta h}{h_{21}} & \frac{-h_{11}}{h_{21}} \\ \frac{-h_{22}}{h_{21}} & \frac{-1}{h_{21}} \end{matrix}$
[h]	$\begin{matrix} \frac{\Delta Z}{Z_{22}} & \frac{Z_{12}}{Z_{22}} \\ \frac{-Z_{21}}{Z_{22}} & \frac{1}{Z_{22}} \end{matrix}$	$\begin{matrix} \frac{C_1}{Y_{11}} & \frac{-Y_{12}}{Y_{11}} \\ \frac{Y_{21}}{Y_{11}} & \frac{\Delta Y}{Y_{11}} \end{matrix}$	$\begin{matrix} \frac{B}{D} & \frac{\Delta T}{D} \\ \frac{-1}{D} & \frac{C}{D} \end{matrix}$	$\begin{matrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{matrix}$

Where $\Delta Y = Y_{11} Y_{22} - Y_{12} Y_{21}$

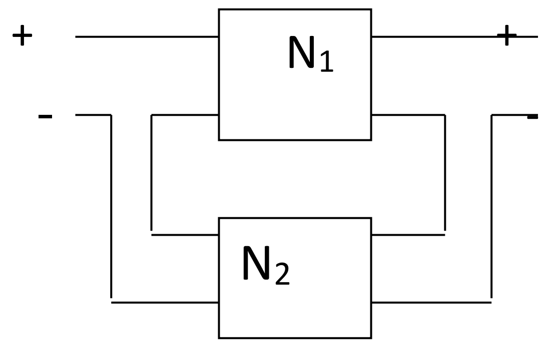
$\Delta Z = Z_{11} Z_{22} - Z_{12} Z_{21}$

$\Delta T = AD - BC$

$\Delta h = h_{11} h_{22} - h_{12} h_{21}$

Inter connection between various 2-port network:

Case 1: Series network



O/P voltage α O/P current

Feedback output voltage and input voltage area in series voltage.

Equivalent impedance:

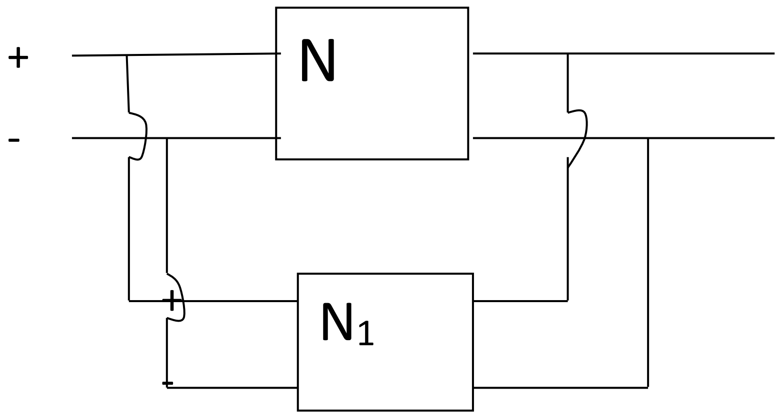
$$[Z] = \begin{bmatrix} Z'11 & Z'12 \\ Z'21 & Z'22 \end{bmatrix} + \begin{bmatrix} Z''11 & Z''12 \\ Z''21 & Z''22 \end{bmatrix}$$

$$[Z] = \begin{bmatrix} Z'11 + Z''11 & Z'12 + Z''12 \\ Z'21 + Z''21 & Z'22 + Z''22 \end{bmatrix}$$

Conclusion:

When two networks connected in series at i/p and o/p also, then the equivalent [Z]- parameter, then z-parameter of respective network are added.

Case 2: Parallel network



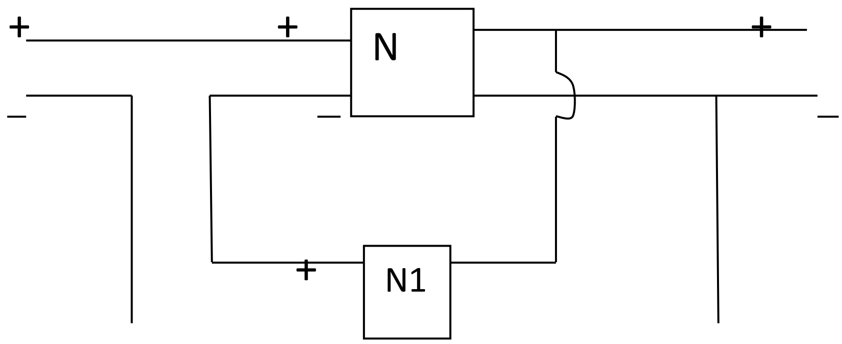
$$[Y]=\begin{bmatrix} Y11 & Y12 \\ Y21 & Y22 \end{bmatrix} + \begin{bmatrix} Y'11 & Y'12 \\ Y'21 & Y'22 \end{bmatrix}$$

$$[Y] = \begin{bmatrix} Y11 + Y'11 & Y12 + Y'12 \\ Y21 + Y'21 & Y22 + Y'22 \end{bmatrix}$$

Comment:

When two network are connected in parallel at i/p as well as at o/p then the Y-parameters of respective network added.

Case 3: Series-Parallel network



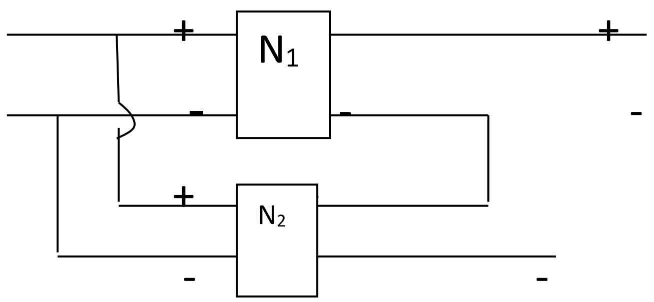
$$[h] = \begin{bmatrix} h'11 & h'12 \\ h'21 & h'22 \end{bmatrix} + \begin{bmatrix} h"11 & h"12 \\ h"21 & h"22 \end{bmatrix}$$

$$[h] = \begin{bmatrix} h'11 + h"11 & h'12 + h"12 \\ h'21 + h"21 & h'22 + h"22 \end{bmatrix}$$

Comment:

When two networks are connected in series at i/p and in parallel at o/p, then the [h] parameter of respective networks are added.

Case 4: Parallel-Series network



$$[G] = \begin{bmatrix} G'11 + G"11 & G'12 + G"12 \\ G'21 + G"21 & G'22 + G"22 \end{bmatrix}$$

NOTES:

$I_1 = g_{11} V_1 + g_{12} I_2$ ⟶ eq^n (1) (KCL equation)

$V_2 = g_{21} V_1 + g_{22} I_2$ ⟶ eq^n (2) (KCL equation)

Step 1: Put I_2=0, (secondary side is short circuited)

$g_{11} = \dfrac{I1}{V1}$

$g_{21} = \dfrac{V2}{V1}$

Step 2: Put V1=0 (primary side is open circuited)

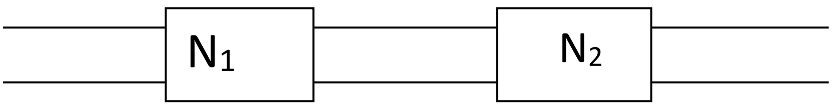
$g_{12} = \dfrac{I1}{I2}$

$g_{22} = \dfrac{V2}{I2}$

Comment:

When two networks are connected parallel at i/p and series at o/p, then the g-parameter of respective networks are added.

<u>Case-5</u>(Cascaded Network)



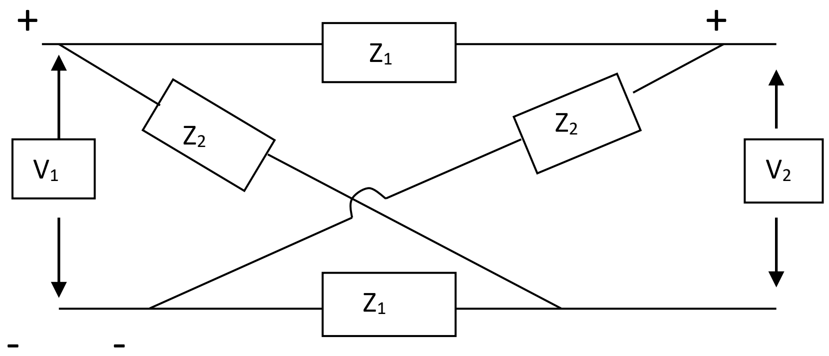
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \times \begin{bmatrix} A'' & B'' \\ C'' & D'' \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A'A'' + B'C'' & A'B'' + B'D'' \\ C'A'' + D'C'' & C'B'' + D'D'' \end{bmatrix}$$

Comments

When 2 networks are connected in cascaded, then ABCD parameters of the respective networks are multiplied.

Case-6(Lattice Network)



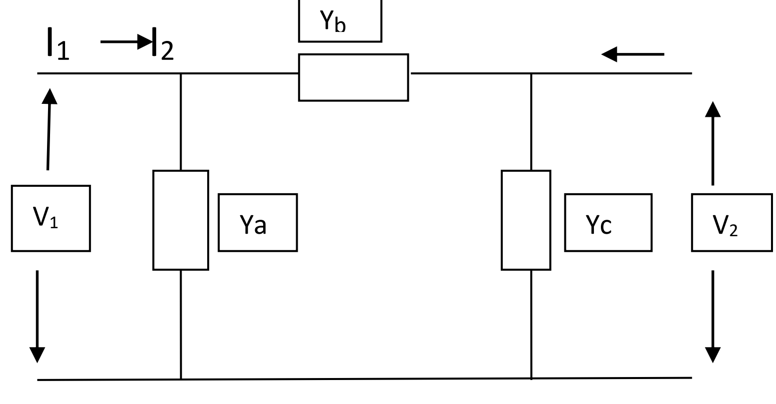
Z_1= straight arm impedance

Z_2= cross arm impedance

Comments

1. A lattice network is said to be symmetrical if the two straight arm impedances are equal.
2. A lattice network is said to be reciprocal if the two cross arm impedances are equal.

Π or Δ Network:



Q. To find the y-parameter

Ans: Put $V_2=0$

$$Y_{11}=\frac{I1}{V1}$$

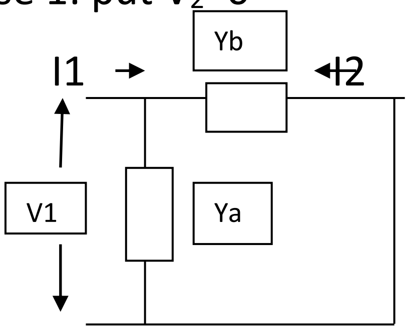
$$Y_{21}=\frac{I2}{V1}$$

Put $V_1=0$,

$$Y_{12}=\frac{I1}{V2}$$

$$Y_{22}=\frac{I2}{V2}$$

Case 1: put $V_2=0$



$I_{ya}= I_1 + I_2$

$\Rightarrow\ I_1+I_2=V1Y_a(1) \longrightarrow$

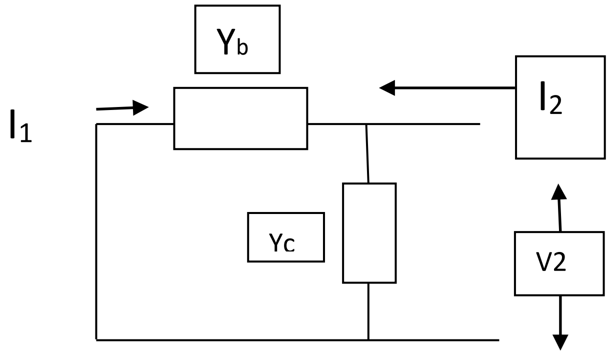
$-I_2= V_1 Y_b(2) \longrightarrow$

$$\Rightarrow -Y_b=\frac{I_2}{V_1}= Y_{21} \longrightarrow (3)$$

$$I_1= V_1(Y_a+Y_b)$$

$$\Rightarrow Y_a+Y_b=\frac{I_1}{V_1}= Y_{11} \longrightarrow (4)$$

Case 2 (Put $V_1=0$)



$I_1+I_2=V_2Y_c$ (5)

$-I_1=V_2Y_b$ (6)

$\Rightarrow -Y_b=\frac{I_1}{V_2}=Y_{12}$ (7)

Equation(1)+Equation(2) is

$\Rightarrow I2=V2(Y_b+Y_c)$

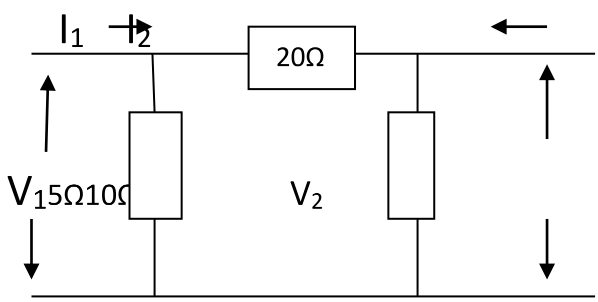
$\Rightarrow Y_a+Y_b=\frac{I_2}{V_2}=Y_{22}$(8)

$$[Y]=\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{21} \end{bmatrix}+\begin{bmatrix} Ya+Yb & -Yb \\ -Yb & Yb+Yc \end{bmatrix}$$

Comments:

- Hence the network is reciprocal (as$Y_{12}=Y_{21}$), but not symmetrical (as$Y_{22}\neq Y_{11}$).
- Any general π- network is always reciprocal network but may not be symmetrical.
- The general π- network can be symmetrical network when two shunt-arm admittances are equal. i.e. Ya=Yc

Q1. Find out the Y-parameter.



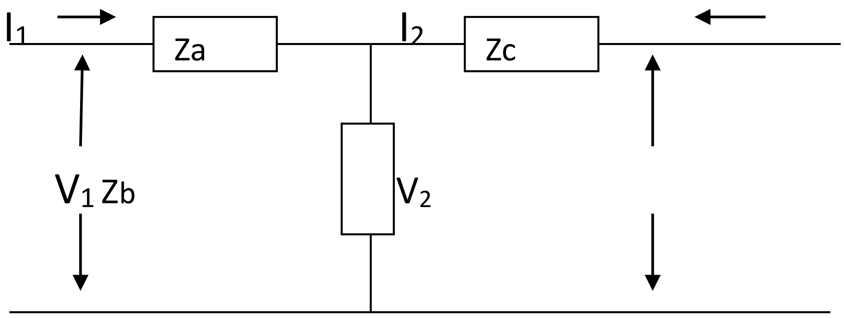
Ans: [Y]=$\begin{bmatrix} Ya+Yb & -Yb \\ -Yb & Yb+Yc \end{bmatrix}$

$$=\begin{bmatrix} \frac{1}{20}+\frac{1}{5} & \frac{-1}{20} \\ \frac{-1}{20} & \frac{1}{20}+\frac{1}{10} \end{bmatrix}$$

$$=\begin{bmatrix} \frac{1}{4} & \frac{-1}{20} \\ \frac{-1}{20} & \frac{3}{20} \end{bmatrix}$$

$$[Y] =\begin{bmatrix} 0.25 & -0.05 \\ -0.05 & 0.15 \end{bmatrix} \text{ (Ans)}$$

T-Network or * network:



Find out the Z-parameter:

Case 1: Put V_2=0

$V_1=I_1 Z_a+(I_1+I_2)Z_b$

$\Rightarrow V_1=(Z_a+Z_b)I1 + I_2 Z_b$ (1) $\longrightarrow$

Case 2: Put $V_1=0$

$$V_2=I_2 Z_c + (I_1+I_2)Z_b$$

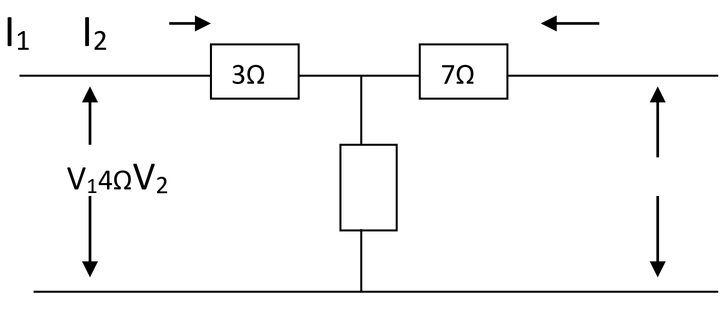
$$\Rightarrow V_2=I_1 Z_b + (Z_b + Z_c)I_2$$

$$[Z]=\begin{bmatrix} Za+Zb & Zb \\ Zb & Zb+Zc \end{bmatrix}$$

Comments:

- It is the reciprocal network (as $Z_{12}=Z_{21}$), but not symmetrical network (as$Z_{11}\neq Z_{22}$).
- Any general network is always reciprocal T-network but may not be a symmetrical network.
- The general T-network can be symmetrical if the two series arm impedances are equal (Za=Zb).

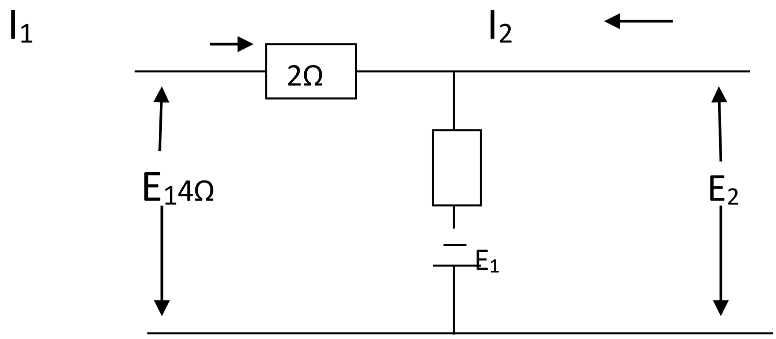
Q. Find out Z-parameter.



Ans: $[Z] = \begin{bmatrix} Za+Zb & Zb \\ Zb & Zb+Zc \end{bmatrix}$

$$[Z] = \begin{bmatrix} 9 & 6 \\ 6 & 13 \end{bmatrix} (ans)$$

Q. Find out the Z-parameter.



Ans: Applying KVL

$E_1 - 2I_1 - 4I_1 - 4I_2 + 10E_1 = 0$

$\Rightarrow 2I_1 + 4(I_1+I_2) = 11E_1$

$\Rightarrow 6I_1 + 4I_2 = 11E_1$ (1)

Similarly

$E_2 - 4(I_1+I_2) + 10E_1 = 0$

$\Rightarrow 4(I_1 + I_2) = E_2 + 10E_1$ (2)

When I2=0,

6I1 = 11E1

$$\boxed{\Rightarrow \frac{E_1}{I_1} = Z_{11} = \frac{6}{11}}$$

When I1=0,

4I2 = 11E1

$$\boxed{\Rightarrow \frac{E_1}{I_2} = \frac{4}{11} = Z_{12}}$$

Put the value of E_1 from equation (1) into the equation 2

$\Rightarrow 4\,(I_1+I_2) = E_2 + 10(\frac{6}{11}I_1 + \frac{4}{11}I_2)$

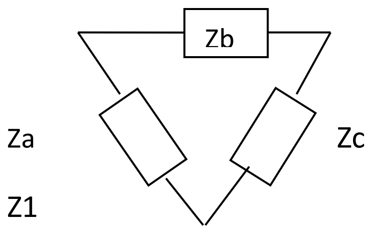
$\Rightarrow 4\,(I_1+I_2) = E_2 + \frac{60}{11}I_1 + \frac{40}{11}I_2$

$\Rightarrow (4-\frac{60}{11})I_1 + (4-\frac{40}{11})I_2 = E_2$

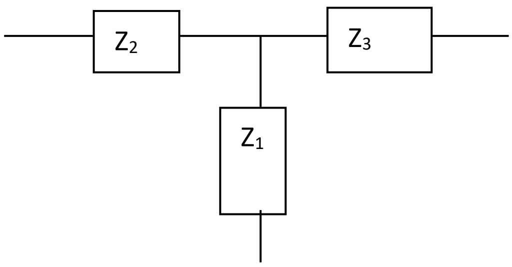
$\Rightarrow E_2 = \underbrace{\frac{-16}{11}}_{\boxed{Z_{21}}} I_1 + \underbrace{\frac{4}{11}}_{\boxed{Z_{22}}} I_2$

$$[z] = \begin{bmatrix} \frac{6}{11} & \frac{4}{11} \\ \frac{-16}{11} & \frac{4}{11} \end{bmatrix} \text{(Ans)}$$

Delta (Δ) to Star (*) conversion:



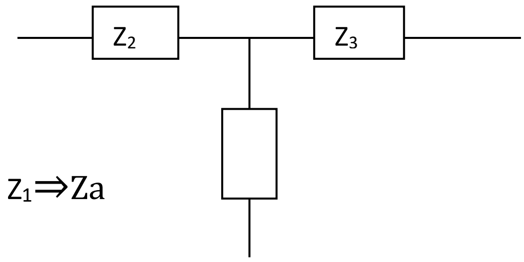
$\Rightarrow$

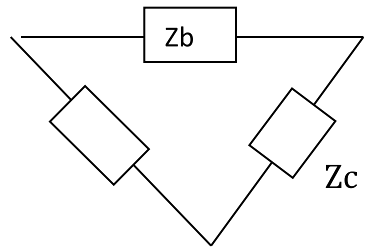


$\boxed{Z_1 = \frac{Za\,Zc}{\sum Z}}$ $\boxed{Z_2 = \frac{Za\,Zb}{\sum Z}}$ $\boxed{Z_3 = \frac{Zb\,Zc}{\sum Z}}$

Where, $\sum Z = Za+Zb+Zc$

Star (*) to Delta (Δ) conversion:





$$Za=\frac{\sum Z_1 Z_2}{Z_3}$$

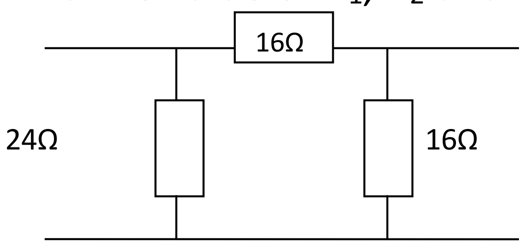
$$Zb=\frac{\sum Z_2 Z_3}{Z_1}$$

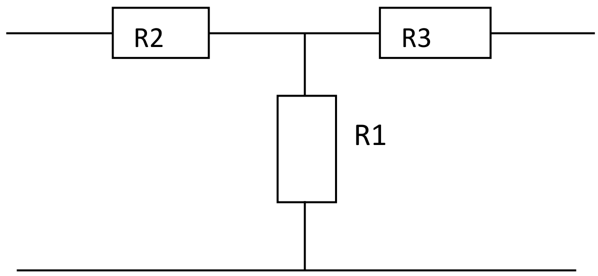
$$Zc=\frac{\sum Z_3 Z_1}{Z_2}$$

Where, $\sum Z_1 Z_2=\sum Z_2 Z_3=\sum Z_3 Z_1 = Z_1 Z_2+Z_2 Z_3+Z_3 Z_1$

Example:

Find the value of R_1, R_2 and R_3



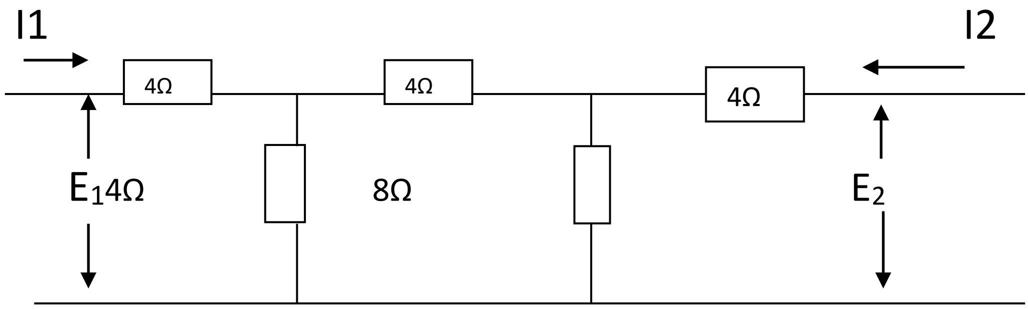


$$R_1=\frac{24 \times 24}{64}= 9\Omega$$

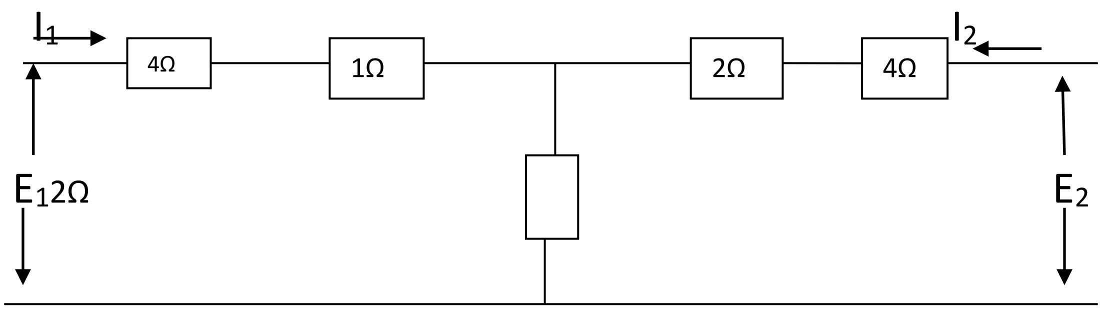
$$R_2=\frac{16\times 24}{64}= 6\Omega$$

$$R_3=\frac{16\times 24}{64}= 6\Omega$$

Q. Using Delta to Star conversion find out the h-parameter



Ans:



Applying KVL

$E_1=5I_1 + 2I_1 + 2I_2$

$E_1 = 7I_1 +2I_2 \quad (1)$

Similarly

$E_2= 2I_1 + 8I_2 \quad (2)$

From these equations, we find that

$Z_{11}=7$, $Z_{12}=2$, $Z_{21}=2$, $Z_{22}=8$

$\Delta Z=\begin{vmatrix} 7 & 2 \\ 2 & 8 \end{vmatrix} = 56 - 4=52$

So, $h_{11}=\frac{\Delta Z}{Z_{22}}= \frac{52}{8}= 6.5$ $\qquad$ $h_{12}=\frac{Z_{12}}{Z_{22}}=\frac{2}{8}= 0.25$

$h_{21}=\frac{-Z_{21}}{Z_{22}}=\frac{-2}{8}=-0.25$ $h_{22}=\frac{1}{Z_{22}}=\frac{1}{8}=0.125$

Physical significance of Poles and Zeroes:

For,

$$N(s)=\frac{2(s+1)(s+3)}{s(s+2)(s+4)} \quad (1)$$

2= Scale factor

Zeroes: numerator polynomial will be zero.

i.e. (s+1) (s+3) = 0

⇒s=-1, s=-3

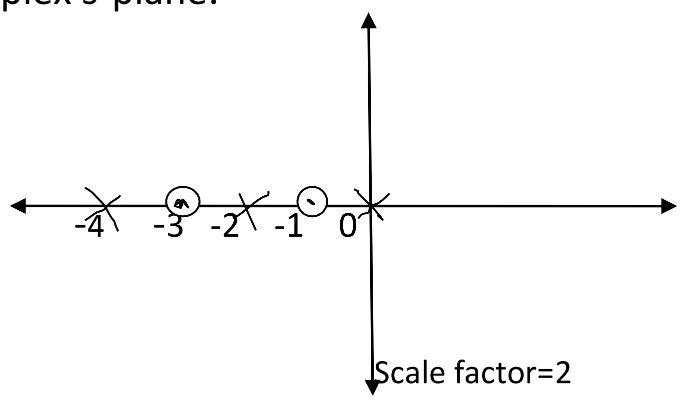
Poles: denominator polynomial equal to zero

⇒s(s+2) (s+4) = 0

⇒s=0, s=-2, s=-4

- Zeroes and Poles are also called as singularities and also called complex frequencies.

Complex s-plane:



Pole-zero for N(s)

Equatin (1) can be written as :

$$N(s)=\frac{A}{s}+\frac{B}{s}+\frac{C}{s}$$

A = Residue of pole at s=0

B = residue of pole at s=-2

C= Residue of pole at s=-4

Applying Laplace Transformation,

$$\boxed{L[N(s)] = A + Be^{-2t} + Ce^{-4t}}$$

Notes:

1. The zeroes represent the complex frequencies for which the numerator polynomial of a given function becomes zero.
2. Zero controls the magnitude of response of given network corresponding to particular input.
3. The poles are complex frequencies at which the denominator polynomial becomes zero or the network function itself becomes infinity.
4. The poles of network function control the type or shape of the response of a give network.
5. For any network function all the poles lie on the left half of the s-plane for its stabling.
6. No multiple poles can exist along jω-axis.
7. No poles can exist the right of the s-plane.
8. For driving point immittance function all the zeroes lie on the left half of the s-plane, whereas for transfer function some of the zeroes may also lie on the right half of the s-plane.
9. In any network, no. of zeroes and no. of poles are always equal.

10. If any network has either only zeroes or pole, the network behaves as a dead network. Since it response always remains constant and won't respond to any external excitation.