1. Maximize $z=-x_1+3x_2$ subject to the constraints.

$x_1+x_2\le 6,\quad -x_1+x_2\le 4,\quad x_1,x_2\ge 0$

2. Maximize $z=x_1+3x_2$ subject to the constraints

$x_1+x_2\le 7, x_1+2x_2\le 10, x_1\ge 0, x_2\ge 0.$

3. Maximize $z=5x_1+4x_2$ subject to the constraints

$x_1+x_2\le 20, 2x_1+x_2\le 35, -3x_1+x_2\le 12,\ x_1,x_2\ge 0.$

4. Maximize $z=6x_1+12x_2$ subject to the constraints

$0\le x_1\le 4, 0\le x_2\le 4, 6x_1+12x_2\le 72.$

5. Maximize the daily output in producing x_1 glass plates by a process P_1 and x_2 glass plates by a process P_2 subject to the constraints

$2x_1+3x_2\le 130, 3x_1+8x_2\le 300, 4x_1+2x_2\le 140$

Hints: The daily output is given by $z=x_1+x_2$.

6. Maximize the daily profit in producing x_1 metal frames F_1 (profit \$90 per frame) and x_2 frames F_2 (profit \$50 per frame) subject to the constraints

$$x_1 + 3x_2 \le 18 \qquad \text{(material)}$$
$$x_1 + x_2 \le 10 \qquad \text{(machine hours)}$$
$$3x_1 + x_2 \le 24 \qquad \text{(labour)}$$

Hints: The daily profit is $z = 90x_1 + 50x_2$.

7. Maximize the total output $z = x_1 + x_2 + x_3$ subject to the constraints
   $4x_1 + 5x_2 + 8x_3 \le 12, 8x_1 + 5x_2 + 4x_3 \le 12.$

8. Maximize $z = 5x_1 + 6x_2 + x_3$ subject to the constraints
   $9x_1 + 3x_2 - 2x_3 \le 5, 4x_1 + 2x_2 - x_3 \le 2, x_1 - 4x_2 + x_3 \le 3,\ x_1, x_2, x_3 \ge 0.$

9. Maximize $z = x_1 + 3x_2$ subject to the constraints
   $x_1 - 3x_2 \ge -6, x_1 + x_2 \le 6, x_1 \ge 0, x_2 \ge 0.$

10. Maximize $z = x_1 + 2x_2$ subject to the constraints
    $x_1 + x_2 \le 9, x_1 - x_2 \ge 1, x_1 \ge 0, x_2 \ge 0.$

11. Maximize $z = 2x_1 + x_2$ subject to the constraints
    $x_1 + x_2 \le 12, x_1 + 2x_2 \le 20, -x_1 + x_2 \ge 2, x_1 \ge 0, x_2 \ge 0.$

12. Maximize $z = 2x_1 + x_2$ subject to the constraints
    $x_1 + x_2 \le 6, -x_1 + x_2 \ge 4, x_1 \ge 0, x_2 \ge 0.$

13. Minimize $z = 3x_1 + 6x_2$ subject to the constraints
    $-x_1 + x_2 \ge 6, x_1 + x_2 \ge 10, x_1 \ge 0, x_2 \ge 0.$

14. Minimize $z = 5x_1 - 20x_2$ subject to the constraints
    $-2x_1 + 10x_2 \le 5, 2x_1 + 5x_2 \le 10.$

15. Find the duals of the following linear programming problems
    (a) Maximize $z = 2x_1 + 3x_2$ subject to
    $x_1 + x_2 \le 6, -x_1 + x_2 \le 4,\ x_1, x_2 \ge 0.$
    (b) Maximize $z = x_1 - x_2 + 4x_3$ subject to
    $x_1 + x_2 + x_3 \le 9, x_1 - 2x_2 + x_3 \ge 6, x_1, x_2, x_3 \ge 0.$
    (c) Maximize $z = 3x_1 + 8x_2$ subject to
    $x_1 + 2x_2 \le 8,\ x_1 + 6x_2 \le 12,\ x_1, x_2 \ge 0.$

(d) Minimize $z = 8x_1 + 9x_2$ subject to

$x_1 + x_2 \geq 5, 3x_1 + x_2 \geq 21, x_1 \geq 0, x_2 \geq 0.$

(e) Minimize $z = 4x_1 + 4x_2 + 6x_3$ subject to

$x_1 - x_2 - x_3 \leq 3, x_1 - x_2 + x_3 \geq 3$

16. Solve the following by using Dual Simplex Method

(a) Minimize $z = 4x_1 + 7x_2$ subject to

$x_1 + x_2 \geq 5, 3x_1 + x_2 \geq 21, x_1 \geq 0, x_2 \geq 0.$

(b) Minimize $z = 8x_1 + 12x_2$ subject to

$2x_1 + 2x_2 \geq 1, x_1 + 3x_2 \geq 2, \ x_1, x_2 \geq 0$

(c) Minimize $z = x_1 + 8x_2 + 5x_3$ subject to

$x_1 + x_2 + x_3 \geq 8, -x_1 + 2x_2 + x_3 \geq 2, x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$

(d) Maximize $z = 4x_1 - x_2 - x_3$ subject to

$3x_1 + x_2 - x_3 \leq 4, x_1 + x_2 + x_3 \leq 2, x_1 \geq 0, x_3 \geq 0, x_2 \geq 0.$

Answers

1. $z = 14, x_1 = 1, x_2 = 5$
2. $z = 15, x_1 = 0, x_2 = 5$
3. $z = 95, x_1 = 15, x_2 = 5.$
4. $z = 72, x_1 = 4, x_2 = 4$
5. $z = 50, x_1 = 20, x_2 = 30$
6. $z = 780, x_1 = 7, x_2 = 3$
7. $z = 2.4, x_1 = 0, x_2 = 2.4, x_3 = 0.$
8. No optimum solution (unbounded)
9. $z = 12, x_1 = 3, x_2 = 3$
10. $z = 13, x_1 = 5, x_2 = 4$
11. $z = 17, x_1 = 5, x_2 = 7$
12. $z = 7, x_1 = 1, x_2 = 5$
13. $z = 54, x_1 = 2, x_2 = 8$
14. $z = -10, x_1 = 0, x_2 = 0.5$

15. (a) Minimize $w = 6y_1 + 4y_2$ subject to

$y_1 - y_2 \geq 2, y_1 + y_2 \geq 3, y_1, y_2 \geq 0.$

(b) Minimize $w = 9y_1 + 6y_2$ subject to

$y_1 + y_2 \geq 1, y_1 - 2y_2 \geq -1, y_1 + y_2 \geq 4, y_1, y_2 \geq 0.$

(c) Minimize $w = 8y_1 + 12y_2$ subject to

$y_1 + y_2 \geq 3, 2y_1 + 6y_2 \geq 8, y_1, y_2 \geq 0.$

(d) Maximize $w = 5y_1 + 21y_2$ subject to

$y_1 + 3y_2 \leq 8, y_1 + y_2 \leq 9, y_1, y_2 \geq 0.$

(e) Maximize $w = -3y_1 + 3y_2$ subject to

$-y_1 + y_2 \leq 4, y_1 - y_2 \leq 4, y_1 + y_2 \leq 6, y_1, y_2 \geq 0.$

16. (a) Min. value of C=28 at $x_1 = 7, x_2 = 0$.

(b) Min. value of z=8 at $x_1 = 0, x_2 = \frac{2}{3}$.

(c) z=28 when $x_1 = 3, x_2 = 0, x_3 = 5$.

(d) Max. value of z=5.5 at $x_1 = 1.5, x_2 = 0, x_3 = 0.5$.

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