

Chapter: Semiconducting Materials

Semiconductors are materials whose electrical conductivity lies between that of good conductors and insulators. Some examples of semiconductor include Antimony, arsenic, Boron, Carbon, Germanium, Silicon, etc.

A semiconductor acts as an insulator at 0 K, and conductivity increases as temperature increases.

Electrons and holes in Semiconductors:

The valence band is fully occupied and conduction band is empty in semiconductor at absolute zero temperature. When a valence electron gains energy greater than band gap (E_g), it crosses the band gap and enters to the conduction band. This electron is free to move throughout the solid and is called free electron. This free electron leaves behind a deficiency of electron in valence band, called hole. These electron - hole pairs are equally formed in an intrinsic semiconductor.

The electron – hole pair can be formed by various means like incident of photon, thermal energy etc. The sufficient thermal energy given to a semiconductor makes the covalent bond to overstretch. Due to which, the electron in the bond becomes free and is excited to conduction band.

Electrical Conduction in semiconductors:

When an external electric field is applied to a semiconductor, valence holes drift in the direction of the field and conduction electrons in opposite direction. The conventional current however, is in one direction.

Therefore, the total current density,

$$J = neV_{de} + peV_{dh}$$

Where, n = electron concentration in conduction band

p= hole concentration in valence band

V_{de} = drift velocity of electrons

V_{dh} = drift velocity of holes.

$$\text{Since, mobility, } \mu = \frac{V_d}{E} \Rightarrow V_d = \mu E$$

$$\therefore J = ne\mu_e E + pe\mu_h E$$

$$\text{or, } \sigma E = ne\mu_e E + pe\mu_h E \quad (\text{as } J = \sigma E, \sigma \text{ being conductivity})$$

$$\Rightarrow \sigma = n e \mu_e + p e \mu_h$$

Where, $\mu_e = \frac{e\tau_e}{m_e^*}$ is electron mobility

$\mu_h = \frac{e\tau_h}{m_h^*}$ is hole mobility

m_e^* & m_h^* are called effective mass of electron and hole

Electron concentration in intrinsic semiconductor:

Let, 'n' be the density of electrons in the conduction band, then in equilibrium condition,

$$n = \int_{E_c}^{\infty} Z(E) \cdot F(E) dE \dots \dots \dots \text{(i)}$$

Where, $Z(E) = \frac{4\pi}{h^3} (2m_e^*)^{\frac{3}{2}} (E - E_c)^{\frac{1}{2}}$ (ii) is the density of states in conduction band. E_c is the energy of bottom of conduction band.

The F-D distribution function is,

$$F(E) = \frac{1}{1 + e^{\frac{-(E-E_f)}{KT}}} \approx e^{\frac{-(E-E_f)}{KT}} \text{ for } E - E_f \gg KT$$

$$\therefore n = \frac{4\pi}{h^3} (2m_e^*)^{\frac{3}{2}} \int_{E_c}^{\infty} (E - E_c)^{\frac{1}{2}} \cdot e^{\frac{-(E-E_f)}{KT}} dE$$

$$\therefore n = \frac{4\pi}{h^3} (2m_e^*)^{\frac{3}{2}} \int_{E_c}^{\infty} (E - E_c)^{\frac{1}{2}} \cdot e^{\frac{E_f - E_c}{KT}} \cdot e^{\frac{-(E-E_c)}{KT}} dE$$

$$\therefore n = \frac{4\pi}{h^3} (2m_e^*)^{\frac{3}{2}} \cdot e^{\frac{E_f - E_c}{KT}} \int_{E_c}^{\infty} (E - E_c)^{\frac{1}{2}} \cdot e^{\frac{-(E-E_c)}{KT}} dE$$

$$\text{Let, } e^{\frac{(E-E_c)}{KT}} = x \Rightarrow E - E_c = KT \cdot x$$

$$\Rightarrow dE = KT dx$$

$$\text{When, } E \rightarrow E_c \Rightarrow x \rightarrow 0$$

$$\text{When, } E \rightarrow \infty \Rightarrow x \rightarrow \infty$$

$$\text{Also, } (E - E_c)^{\frac{1}{2}} = (KT)^{\frac{1}{2}} \cdot x^{\frac{1}{2}}$$

$$\text{Then, } n = \frac{4\pi}{h^3} (2m_e^*)^{\frac{3}{2}} \cdot e^{\frac{E_f - E_c}{KT}} \int_0^{\infty} (KT)^{\frac{1}{2}} \cdot (x)^{\frac{1}{2}} \cdot e^{-x} \cdot (KT) dx$$

$$n = \frac{4\pi}{h^3} (2m_e^* KT)^{\frac{3}{2}} \cdot e^{\frac{E_f - E_c}{KT}} \int_0^{\infty} (x)^{\frac{1}{2}} \cdot e^{-x} \cdot dx$$

Using integration table,

$$n = \frac{4\pi}{h^3} (2m_e^* KT)^{\frac{3}{2}} e^{\frac{E_f - E_c}{KT}} \frac{\sqrt{\pi}}{2}$$

$$n = 2 \left(\frac{2\pi m_e^* KT}{h^2} \right)^{\frac{3}{2}} e^{-\frac{(E_c - E_f)}{KT}}$$

$\Rightarrow n = N_c e^{-\frac{(E_c - E_f)}{KT}}$ (iii) \Rightarrow Required expression for e^- concentration.

Where, $N_c = 2 \left(\frac{2\pi m_e^* KT}{h^2} \right)^{\frac{3}{2}}$ is called effective density of state at the conduction band edge.

Hole concentration in intrinsic semiconductor:

As $F(E)$ gives the probability of occupation for an energy states. The probability that it may be unoccupied by an electron is $[1 - F(E)]$.

Therefore, the concentration of holes in valence band is,

$$p = \int_{-\infty}^{E_v} Z(E) [1 - F(E)] dE \dots \dots \dots \text{(i)}$$

Here, $1 - F(E) = 1 - \frac{1}{1 + e^{\frac{E - E_f}{KT}}} = \frac{e^{\frac{E - E_f}{KT}}}{1 + e^{\frac{E - E_f}{KT}}} \approx e^{\frac{E - E_f}{KT}}$ (As $E \ll E_f$. So the term $e^{\frac{E - E_f}{KT}}$ in the denominator can be neglected comparing to 1.)

$$\text{And, } Z(E) = \frac{4\pi}{h^3} (2m_h^*)^{\frac{3}{2}} (E_v - E)^{\frac{1}{2}}$$

where, m_h^* is the effective mass of a hole.

$$\text{So, } p = \frac{4\pi}{h^3} (2m_h^*)^{\frac{3}{2}} \int_{-\infty}^{E_v} (E_v - E)^{\frac{1}{2}} e^{\frac{E - E_f}{KT}} dE$$

$$p = \frac{4\pi}{h^3} (2m_h^*)^{\frac{3}{2}} \int_{-\infty}^{E_v} (E_v - E)^{\frac{1}{2}} e^{\frac{E_v - E_f}{KT}} e^{\frac{-(E_v - E)}{KT}} dE$$

$$\text{or, } p = \frac{4\pi}{h^3} (2m_h^*)^{\frac{3}{2}} e^{\frac{E_v - E_f}{KT}} \int_{-\infty}^{E_v} (E_v - E)^{\frac{1}{2}} e^{\frac{-(E_v - E)}{KT}} dE$$

$$\text{Let, } \frac{(E_v - E)}{KT} = x \Rightarrow E_v - E = xKT \Rightarrow dE = -KTdx$$

When, $E = -\infty, x = \infty$

When, $E = E_v, x = 0$

$$\text{Then, } p = \frac{4\pi}{h^3} (2m_h^*)^{\frac{3}{2}} e^{\frac{E_v - E_f}{KT}} \int_{-\infty}^0 (KT)^{\frac{1}{2}} x^{\frac{1}{2}} e^{-x} (-KT) dx$$

$$p = \frac{4\pi}{h^3} (2m_h^* KT)^{\frac{3}{2}} e^{\frac{E_v - E_f}{KT}} \int_0^\infty x^{\frac{1}{2}} e^{-x} dx$$

$$p = \frac{4\pi}{h^3} (2m_h^* KT)^{\frac{3}{2}} \cdot e^{\frac{E_v - E_f}{KT}} \cdot \frac{\sqrt{\pi}}{2} \quad (\text{as } \int_0^\infty x^{\frac{1}{2}} e^{-x} dx = \frac{\sqrt{\pi}}{2})$$

$$p = 2 \left(\frac{2\pi m_h^* KT}{h^2} \right)^{\frac{3}{2}} \cdot e^{\frac{-(E_f - E_v)}{KT}} \Leftrightarrow p = N_v \cdot e^{\frac{-(E_f - E_v)}{KT}} \Leftrightarrow \text{Required expression for hole concentration.}$$

Here, $N_v = 2 \left(\frac{2\pi m_h^* KT}{h^2} \right)^{\frac{3}{2}}$ is called Density of state at the upper boundary of the valence band.

Fermi level of intrinsic semiconductor:

$$\text{We have electron concentration, } n = N_c \cdot e^{-\frac{(E_c - E_f)}{KT}} \dots \text{(i)}$$

$$\text{Where, } N_c = 2 \left(\frac{m_e^* KT}{h^2} \right)^{\frac{3}{2}}$$

$$\text{And hole concentration, } p = N_v \cdot e^{-\frac{(E_f - E_v)}{KT}} \dots \text{(ii)}$$

$$\text{Where, } N_v = 2 \left(\frac{2\pi m_h^* KT}{h^2} \right)^{\frac{3}{2}}$$

Multiplying (i) and (ii), we get

$$n.p = N_v N_c \cdot e^{-\frac{(E_c - E_f)}{KT}} \cdot e^{-\frac{(E_f - E_v)}{KT}}$$

$$\text{or, } n.p = N_v N_c \cdot e^{-\frac{(E_c - E_V)}{KT}} \dots \text{(iii)} \Leftrightarrow \text{this equation is called mass action law.}$$

For intrinsic semiconductor, $n = p = n_i$, say

$$\text{Eq^n (iii) becomes, } n_i^2 = N_v N_c \cdot e^{-\frac{(E_c - E_V)}{KT}} \Leftrightarrow n_i = (N_v N_c)^{\frac{1}{2}} \cdot e^{-\frac{(E_c - E_V)}{2KT}} \dots \text{(iv)}$$

Now, $p = n_i$

$$N_v \cdot e^{-\frac{(E_f - E_v)}{KT}} = (N_v N_c)^{\frac{1}{2}} \cdot e^{-\frac{(E_c - E_V)}{2KT}}$$

$$e^{-\frac{(E_f - E_v)}{KT}} = \left(\frac{N_c}{N_v} \right)^{\frac{1}{2}} \cdot e^{-\frac{(E_c - E_V)}{2KT}}$$

Taking natural log on both sides,

$$\frac{-(E_f - E_v)}{KT} = \ln \left(\frac{N_c}{N_v} \right)^{\frac{1}{2}} - \frac{(E_c - E_V)}{2KT}$$

Using $N_c = 2\left(\frac{m_e^* KT}{h^2}\right)^{\frac{3}{2}}$ and $N_v = 2\left(\frac{2\pi m_h^* KT}{h^2}\right)^{\frac{3}{2}}$ we get

$$-(E_f - E_v) = \frac{3KT}{4} \cdot \ln\left(\frac{m_e^*}{m_h^*}\right) - \frac{(E_c - E_v)}{2}$$

For $m_e^* = m_h^*$ (eg. at 0 K), the first term in RHS is zero.

$$E_f - E_v = \frac{(E_c - E_v)}{2} \Leftrightarrow E_f = E_v + \frac{(E_c - E_v)}{2} = \frac{(E_c + E_v)}{2}$$

For intrinsic S.C. E_f is written as E_{fi}

Therefore, $E_{fi} = \frac{(E_c + E_v)}{2}$ i.e. the Fermi level lies between that of CB and VB.

Extrinsic Semiconductor:

When a pure semiconductor is doped with impurity atoms, the semiconductor so formed is called extrinsic semiconductor. Doping with impurity atoms increases the conductivity of the semiconductor.

Two types of extrinsic semiconductor are: n – type semiconductor and p – type semiconductor

- n – type semiconductor:** When a pentavalent impurity atom like As, P, Sb etc is added to a pure semiconductor, a n – type semiconductor is formed.

The impurity atom donates an excess electron from the covalent bond to the conduction band. So, it is called donor.

In n – type semiconductor, electrons are majority charge carriers and holes are minority charge carriers.

We need to show that the majority carrier concentration (electron concentration) is almost equal to the donor concentration.

so, let n and p be the concentration of electrons and holes. And N_d be the concentration of donors.

Then, $n = p + N_d$ (i)

But from mass action law, $n.p = n_i^2$. Here, n_i is the intrinsic carrier concentration .

$$\Leftrightarrow p = n_i^2/n$$

$$\text{Then eqn (i) becomes, } n = \frac{n_i^2}{n} + N_d$$

$$\begin{aligned} &\Leftrightarrow n^2 = n_i^2 + nN_d \\ &\Leftrightarrow n^2 - n \cdot N_d - n_i^2 = 0 \end{aligned}$$

($ax^2 + bx + c = 0$ form)

$$\text{i.e. } n = \frac{N_d \pm \sqrt{N_d^2 + 4n_i^2}}{2}$$

Since, $N_d^2 \gg 4n_i^2$. Also neglecting negative sign.

$$n \approx \frac{N_d}{2} + \frac{N_d}{2} = N_d$$

This shows that the electron concentration is almost equal to the donor concentration in n-type semiconductor.

2. P - type semiconductor:

when a pure semiconductor is doped with trivalent impurity atoms like Boron, Aluminum, Gallium, Indium etc, the extrinsic semiconductor so formed is called p - type semi conductor. The three valence electrons of these impurity atoms can form three complete covalent bonds with the pure semiconductor and there is one deficiency of electron to form fourth bond. That is, one vacancy of electron is left on fourth bond. This gives rise to a hole.

The impurity atom that contributes a hole to the semiconductor in valence band is called the acceptor .

We need to show that hole concentration in p - type semiconductor is almost equal to acceptor concentration.

For this, let n and p be the electron and hole concentration and N_a be the concentration of acceptor.

Then, $p = n + N_a \dots \text{(i)}$

Using mass action law, $n.p = n_i^2$. Here, n_i^2 is the intrinsic carrier concentration.

$$\therefore n = n_i^2/p$$

Then, equation (i) becomes, $p = \frac{n_i^2}{p} + N_a$

$$\begin{aligned} &\Rightarrow p^2 = n_i^2 + pN_a \\ &\Rightarrow p^2 - p.N_a - n_i^2 = 0 \end{aligned}$$

$(ax^2 + bx + c = 0 \text{ form})$

$$\text{i.e. } p = \frac{N_a \pm \sqrt{N_a^2 + 4n_i^2}}{2}$$

Since, $N_a^2 \gg 4n_i^2$. Also neglecting negative sign.

$$p \approx \frac{N_a}{2} + \frac{N_a}{2} = N_a$$

This shows that hole concentration is almost equal to the acceptor concentration in p - type semiconductor.

Compensation doping:

Doping of a semiconductor with both acceptors and donors to control its property is called compensation doping.

A p - type semiconductor formerly doped with acceptor can be converted to n - type semiconductor by again doping with donors until the donor concentration N_d exceeds acceptor concentration N_a . The holes due to acceptor doping will recombine with the electrons due to donor doping.

The new electron concentration, $n = N_d - N_a$

Using mass action law, $n.p = n_i^2$

$$\therefore p = n_i^2/n$$

Then, new hole concentration, $p = \frac{n_i^2}{N_d - N_a}$

In the similar manner, a n-type semiconductor formerly doped with donors can be converted to p-type semiconductor by again doping with acceptors until the acceptor concentration N_a exceeds donor concentration N_d . The electrons due to donor doping will recombine with the holes due to acceptor doping.

The new hole concentration, $p = N_a - N_p$

Using mass action law, $n.p = n_i^2$

Then, new electron concentration, $n = \frac{n_i^2}{N_a - N_d}$

Degenerate and non-degenerate semiconductors:

For an intrinsic semiconductor, the electron concentration,

$$n = N_c \cdot e^{-\frac{(E_C - E_f)}{KT}} \dots\dots\dots (i)$$

This equation is valid only when $n \ll N_c$.

Those semiconductors for which $n \ll N_c$ and $p \ll N_v$ are called non-degenerate semiconductors. They essentially follow all the normal properties of a semiconductor. They follow Maxwell-Boltzmann statistics rather than Fermi-Dirac Statistics, since electron (or hole) concentration is too less and Pauli exclusion principle is no more valid.

When a semiconductor is heavily doped with donors, then 'n' will be so large. In this case equation (i) is no more valid. In this case Pauli Exclusion principle is important. So, Fermi-Dirac statistics comes into play but not the Maxwell-Boltzmann statistics. Such type of semiconductor that has $n > N_c$ and $p > N_v$ are called degenerate semiconductors. The mass action law too, is no more valid and they behave like metal rather than semiconductor.

Degenerate semiconductors are used in laser diodes, Ohmic contacts in IC's and as metal gates in many microelectronics MOS devices.

Generation and Recombination:

Above absolute zero temperature, the thermal excitation of electrons from the VB to the CB continuously generates free electron-hole pairs.

When an electron in CB meets a hole in VB, it falls into it. This process is called recombination. The excess of energy E_g during recombination is released as a phonon or photon.

Two types of recombination are : direct recombination and indirect recombination.

i) Direct recombination:

It takes place in compound semiconductors like GaAs, in which the electron in CB directly meets a hole in VB and recombines with it, without any agent required. During this

recombination, the excess of energy between CB and VB is released as a photon of energy $E_g = E_c - E_v = hf$. That's why Gallium Arsanide(GaAs) is used as a light emitting diode (LED), utilizing this phenomenon.

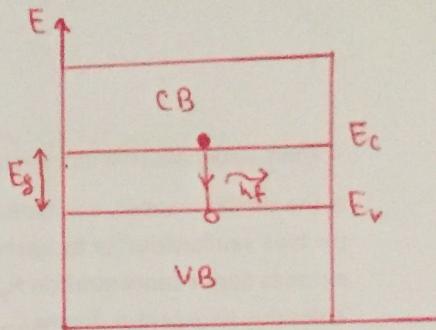


Fig. Direct recombination in GaAs

- ii) Indirect recombination: It takes place in elemental semiconductors like Si and Ge. The electron in CB recombines with a hole in VB through a recombination center. The recombination center may be an impurity atom or a crystal defect. Firstly, the electron finds the recombination center and locate there until it meets a hole from VB. The electron resting in recombination center when meets a hole from VB, recombination takes place, releasing a Phonon. A Phonon is a quantum of acoustic or vibration energy (sound) released during lattice vibration.

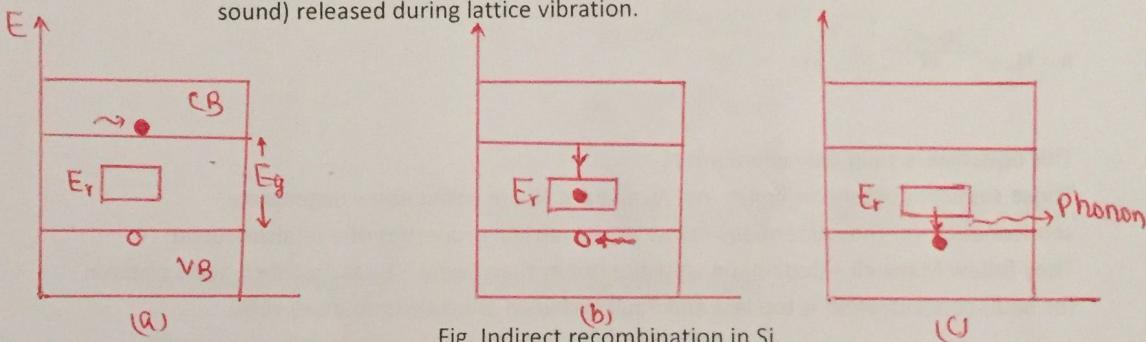


Fig. Indirect recombination in Si.

Photo-generation:

When a photon of energy greater than band gap is incident on a semiconductor, it is absorbed by the semiconductor. This phenomenon is called photo-generation. Due to photo-generation, electron in VB excites to CB absorbing the energy of the photon, creating an excess of electron-hole pairs.

We need to find the nature of rise and decay of excess electron or hole concentration when the light is illuminated and switched off respectively in photo-generation process.

So, we consider for a heavily doped n-type semiconductor. Let, n_0 and p_0 be the electron and hole concentration at thermal equilibrium condition. Due to photogeneration, let excess electron and hole concentration be Δn and Δp .

Then, total electron concentration, $n = n_0 + \Delta n$

and total hole concentration, $p = p_0 + \Delta p$

Since, there are equal no. of electron-hole pairs generated. $\therefore \Delta n = \Delta p$

So, we consider for the holes and the process holds good for electrons too.

When the light is switched off, the holes recombine with electrons and it takes some time. If τ_h be the mean recombination time, then the rate of recombination of excess holes is $\Delta p / \tau_h$

If we consider the rate of photogeneration be G_{ph} then,

the rate of increase of excess hole concentration, $d\Delta p/dt$ is given by,

$$\frac{d(\Delta p)}{dt} = G_{ph} - \frac{\Delta p}{\tau_h} \dots\dots(i)$$

$$\text{or, } \frac{d(\Delta p)}{dt} = \frac{G_{ph}\tau_h - \Delta p}{\tau_h}$$

or, $\frac{d(\Delta p)}{G_{ph}\tau_h - \Delta p} = \frac{dt}{\tau_h} \Leftrightarrow \text{Integrating both sides we get,}$

$$\text{or, } \int \frac{-d(\Delta p)}{G_{ph}\tau_h - \Delta p} = \int \frac{-dt}{\tau_h} \Leftrightarrow \ln(G_{ph}\tau_h - \Delta p) = \frac{-t}{\tau_h} + C$$

when $t = 0$ then $\Delta p = 0 \Leftrightarrow C = \ln(G_{ph}\tau_h)$

This gives, $\ln(G_{ph}\tau_h - \Delta p) = \frac{-t}{\tau_h} + \ln(G_{ph}\tau_h)$

$$\text{or, } \ln(G_{ph}\tau_h - \Delta p) - \ln(G_{ph}\tau_h) = \frac{-t}{\tau_h}$$

$$\text{or, } \ln\left(\frac{G_{ph}\tau_h - \Delta p}{G_{ph}\tau_h}\right) = \frac{-t}{\tau_h}$$

$$\text{or, } \frac{G_{ph}\tau_h - \Delta p}{G_{ph}\tau_h} = e^{\frac{-t}{\tau_h}}$$

$$\text{or, } G_{ph}\tau_h - \Delta p = G_{ph}\tau_h \cdot e^{\frac{-t}{\tau_h}} \Leftrightarrow \Delta p = G_{ph}\tau_h \left[1 - e^{\frac{-t}{\tau_h}}\right] \dots\dots(ii)$$

Equation (ii) shows that the excess holes rise exponentially.

When $t \rightarrow \infty, \Delta p \rightarrow G_{ph}\tau_h$ (A steady state value)

For decay of excess holes: we switch off the light source, when the steady state value of excess holes is reached. Then, photo - generation is instantly stopped i.e. $G_{ph} = 0$

We see equation (i) and find it as, $\frac{d\Delta p}{dt} = -\frac{\Delta p}{\tau_h}$

or, $\frac{d\Delta p}{\Delta p} = -\frac{dt}{\tau_h} \Leftrightarrow \text{Integrating it, we get,}$

$$\int \frac{d\Delta p}{\Delta p} = -\frac{1}{\tau_h} \int dt \Leftrightarrow \ln(\Delta p) = -\frac{t}{\tau_h} + C$$

When $t \rightarrow 0, \Delta p \rightarrow G_{ph}\tau_h \Leftrightarrow C = \ln(G_{ph}\tau_h)$

$$\text{This gives, } \ln(\Delta p) = -\frac{t}{\tau_h} + \ln(G_{ph}\tau_h) \Leftrightarrow \Delta p = G_{ph}\tau_h \cdot e^{\frac{-t}{\tau_h}} \dots\dots(iii)$$

Equation (iii) shows that the decay is exponential too.

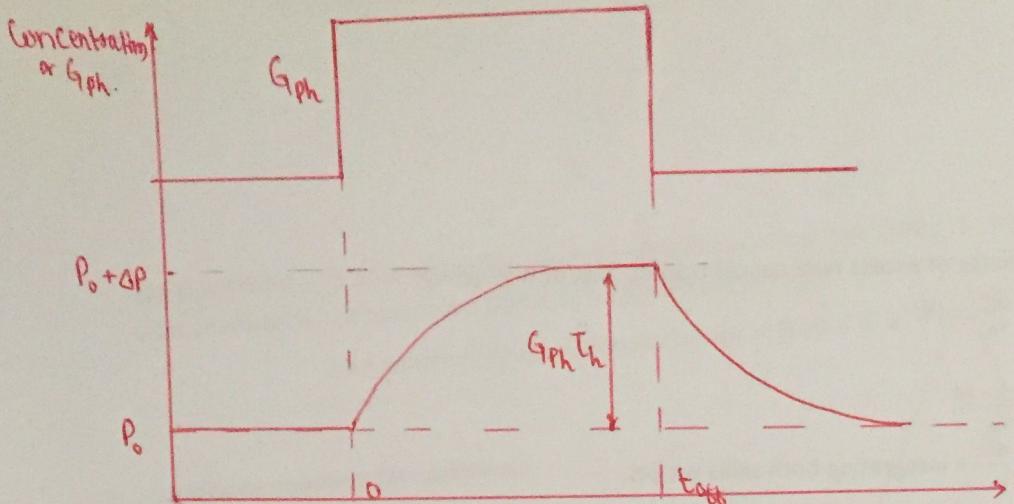


Fig. Rise and decay of excess holes due to photogeneration and light switch off condition

Continuity Equation for Carriers:

1. Continuity equation for holes:

Consider an elemental volume of a n-type semiconductor having cross-section A and length Δx . Due to recombination of holes into electrons, the current densities entering, $J(x)$ and leaving, $J(x+\Delta x)$ are different.

The rate of hole concentration that built up inside the volume $(\frac{\partial p}{\partial t})$ is equal to the rate of increase of hole concentration in volume $A\Delta x$ per unit time minus recombination rate.

$$\text{i.e. } \frac{\partial p}{\partial t} = \frac{1}{e} \left[\frac{J(x) - J(x+\Delta x)}{\Delta x} \right] - \frac{\Delta p}{\tau} \dots \text{(i)}$$

Using Taylor's series expansion

$$J(x + \Delta x) = J(x) + \frac{\partial J}{\partial x} \Delta x$$

$$\text{Also, } \Delta p = p - p_0 \Rightarrow \frac{\partial p}{\partial t} = \frac{\partial \Delta p}{\partial t}$$

$$\text{Therefore (i) becomes, } \frac{\partial(\Delta p)}{\partial t} = - \frac{1}{e} \frac{\partial J}{\partial x} - \frac{\Delta p}{\tau} \dots \text{(ii)}$$

Equation (ii) is called continuity equation for holes. The total current density,

$$J = J_{\text{drift}} + J_{\text{diffusion}} = pe\mu E - eD \frac{\partial p}{\partial x}$$

$$\text{Since drift is dominated by the diffusion, } J = -eD \frac{\partial p}{\partial x}$$

Now substituting J in (ii) we get

$$\frac{\partial(\Delta p)}{\partial t} = D \frac{\partial^2 p}{\partial x^2} - \frac{\Delta p}{\tau} \dots \text{(iii)} \Leftrightarrow \text{It is called Fick's 2nd law (or diffusion equation for holes).}$$

$$\text{Under steady state condition, } \frac{\partial(\Delta p)}{\partial t} = 0$$

$$D \frac{\partial^2 p}{\partial x^2} - \frac{\Delta p}{\tau} = 0$$

$$\Leftrightarrow D \frac{\partial^2 \Delta p}{\partial x^2} - \frac{\Delta p}{\tau} = 0$$

$$\Leftrightarrow \frac{\partial^2 \Delta p}{\partial x^2} - \frac{\Delta p}{D\tau} = 0$$

$$\frac{\partial^2 \Delta p}{\partial x^2} - \frac{\Delta p}{L^2} = 0 \dots \text{(iv)} \quad \text{Where, } L = \sqrt{D\tau} \text{ is called diffusion length for holes in n-type}$$

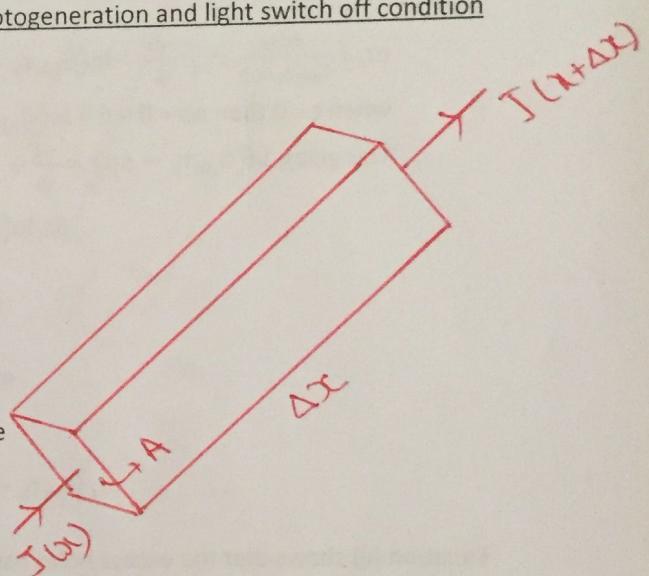


Fig.

semiconductor. It is defined as the length a hole diffuses in average before it recombines with an electron. L is also called minority carrier diffusion length.

The solution of equation (iv) is, $\Delta p = A e^{\frac{x}{L}} + B e^{-\frac{x}{L}}$

2. Continuity equation for electrons:

Consider an elemental volume of a p - type semiconductor having cross-section A and length Δx . Due to recombination of electrons into holes, the current densities entering, $J(x)$ and leaving, $J(x+\Delta x)$ are different.

The rate of electron concentration that built up inside the volume ($\frac{\partial n}{\partial t}$) is equal to the rate of increase of electron concentration in volume $A\Delta x$ per unit time minus recombination rate.

$$\text{i.e. } \frac{\partial n}{\partial t} = \frac{1}{-e} \left[\frac{J(x) - J(x+\Delta x)}{\Delta x} \right] - \frac{\Delta n}{\tau} \dots \text{(i)}$$

Using Taylor's series expansion

$$J(x + \Delta x) = J(x) + \frac{\partial J}{\partial x} \Delta x$$

$$\text{Also, } \Delta n = n - n_0 \Rightarrow \frac{\partial n}{\partial t} = \frac{\partial \Delta n}{\partial t}$$

$$\text{Therefore (i) becomes, } \frac{\partial(\Delta n)}{\partial t} = \frac{1}{e} \frac{\partial J}{\partial x} - \frac{\Delta n}{\tau} \dots \text{(ii)}$$

Equation (ii) is called continuity equation for holes. The total current density,

$$J = J_{\text{drift}} + J_{\text{diffusion}} = ne\mu E + eD \frac{\partial n}{\partial x}$$

$$\text{Since drift is dominated by the diffusion, } J = eD \frac{\partial n}{\partial x}$$

Now substituting J in (ii) we get

$$\frac{\partial(\Delta n)}{\partial t} = D \frac{\partial^2 n}{\partial x^2} - \frac{\Delta n}{\tau} \dots \text{(iii)} \Rightarrow \text{It is called Fick's 2nd law (or diffusion equation for electrons).}$$

Under steady state condition, $\frac{\partial(\Delta n)}{\partial t} = 0$

$$D \frac{\partial^2 n}{\partial x^2} - \frac{\Delta n}{\tau} = 0$$

$$\Rightarrow D \frac{\partial^2 \Delta n}{\partial x^2} - \frac{\Delta n}{\tau} = 0$$

$$\Rightarrow \frac{\partial^2 \Delta n}{\partial x^2} - \frac{\Delta n}{D\tau} = 0$$

$\frac{\partial^2 \Delta n}{\partial x^2} - \frac{\Delta n}{L^2} = 0 \dots \text{(iv)}$ Where, $L = \sqrt{D\tau}$ is called diffusion length for electrons in p - type semiconductor. It is defined as the length an electron diffuses in average before it recombines with a hole. L is also called minority carrier diffusion length.

The solution of equation (iv) is, $\Delta n = A e^{\frac{x}{L}} + B e^{-\frac{x}{L}}$

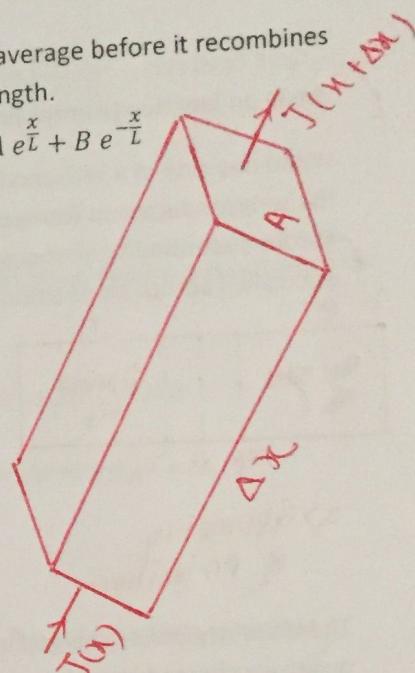


Fig.

Ideal pn junction (Abrupt pn junction):

When one side of a semiconductor is doped with donors and other side with acceptors, the semiconductor so formed is called pn junction semiconductor. There occurs an abrupt discontinuity of energy bands at the interface between the two regions so, it is also called abrupt pn junction.

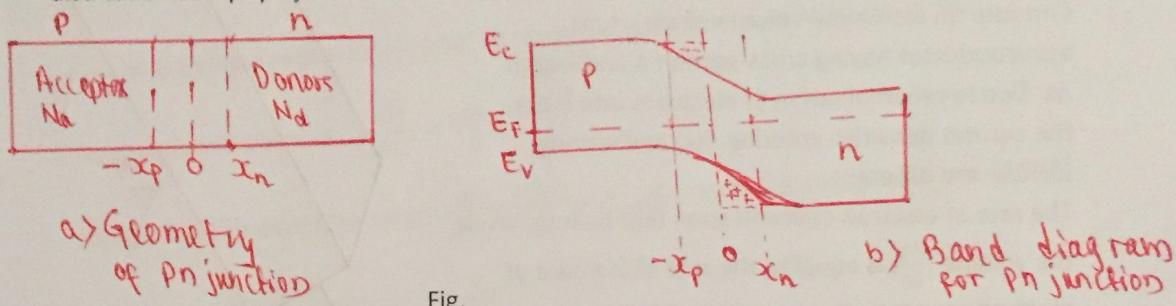


Fig.

There are negatively charged immobile acceptors in the p - side from $-x_p$ to 0 and positively charged immobile donors in n - side from 0 to x_n . So, an internal electric field and at the same time, a potential is built up there, called build in potential or barrier potential. This region is called transition or space charge region.

We need to find the built in potential and width of transition region. For this, we use the condition that the drift electron current is balanced by the electron diffusion current, at equilibrium condition. So, we can write

current density, $J = J_{\text{drift}} + J_{\text{diffusion}} = 0$

$$\text{i.e. } ne\mu E + eD \frac{dn}{dx} = 0$$

$$\Rightarrow -E = \frac{D}{n\mu} \cdot \frac{dn}{dx} \Rightarrow \frac{dV}{dx} = \frac{D}{n\mu} \cdot \frac{dn}{dx} \quad (\text{as } E = -\frac{dV}{dx})$$

$$\text{Using Einstein's relation, } \frac{D}{\mu} = \frac{KT}{e}$$

$$\frac{dV}{dx} = \frac{KT}{ne} \cdot \frac{dn}{dx} \Rightarrow dV = \frac{KT}{e} \cdot \frac{dn}{n}$$

$$\text{Integrating from } x = -x_p \text{ to } x = x_n \text{ we get, } \int_{-x_p}^{x_n} dV = \frac{KT}{e} \int_{-x_p}^{x_n} \frac{dn}{n}$$

$$\Rightarrow V(x)|_{-x_p}^{x_n} = [\ln n]|_{-x_p}^{x_n} \Rightarrow V(x_n) - V(-x_p) = \frac{KT}{e} \cdot [\ln(x_n) - \ln(-x_p)]$$

$$\Rightarrow V_0 = \frac{KT}{e} \cdot [\ln n(x_n) - \ln n(-x_p)] \dots \text{(i) Where, } V_0 = V(x_n) - V(-x_p) \text{ is built in potential.}$$

Let, all the impurities are ionized, so hole concentration in p - region, $p(-x_p) = N_a$

And the electron concentration in n - region, $n(x_n) = N_d$

$$\text{Using, } p(-x_p) \cdot n(-x_p) = n_i^2 \Rightarrow n(-x_p) = \frac{n_i^2}{p(-x_p)} = \frac{n_i^2}{N_a}$$

$$\text{Equation (i) becomes, } V_0 = \frac{KT}{e} \cdot \left[\ln N_d - \ln \frac{n_i^2}{N_a} \right] \Rightarrow V_0 = \frac{KT}{e} \cdot \ln \left[\frac{N_a N_d}{n_i^2} \right] \dots (\text{ii}) \text{ Req'd Exp^n}$$

Width of the Transition region:

We need to show that width of the transition region depends upon doping concentration on either side of the junction.

For this, we use Poisson's equation, $\frac{dE}{dx} = \frac{\rho}{\epsilon} \Rightarrow -\frac{d^2V}{dx^2} = \frac{\rho}{\epsilon} \dots (\text{i})$ (as $E = -dv/dt$)

For the transition region in p - side (i.e. from $-x_p$ to 0) charge density $\rho = -e N_a$

So equation (i) becomes, $-\frac{d^2V}{dx^2} = \frac{-e N_a}{\epsilon}$

Integrating this equation we get, $\frac{dV}{dx} = \frac{e N_a x}{\epsilon} + A \dots (\text{ii})$ Here, A is integration constant

At $x = -x_p$ we have $\frac{dV}{dx} = 0 \Rightarrow A = -\frac{e N_a x_p}{\epsilon}$

Equation (ii) becomes $\frac{dV}{dx} = \frac{e N_a x}{\epsilon} - \frac{e N_a x_p}{\epsilon}$ Again integrating it, we get

$$V(x) = \frac{e N_a x^2}{2\epsilon} - \frac{e N_a x_p x}{\epsilon} + B$$

$$\text{Using } x = -x_p \text{ we get } V(-x_p) = \frac{e N_a x_p^2}{2\epsilon} - \frac{e N_a x_p^2}{\epsilon} + B \Rightarrow V(-x_p) = -\frac{e N_a x_p^2}{2\epsilon} + B \dots (\text{iii})$$

Similarly, for the transition region in n - side (i.e. from 0 to x_n) charge density $\rho = e N_d$

So equation (i) becomes, $\frac{d^2V}{dx^2} = -\frac{e N_d}{\epsilon}$

Integrating this equation we get, $\frac{dV}{dx} = -\frac{e N_d x}{\epsilon} + C \dots (\text{iv})$ Here, C is integration constant

At $x = x_n$ we have $\frac{dV}{dx} = 0 \Rightarrow C = \frac{e N_d x_n}{\epsilon}$

Equation (iv) becomes $\frac{dV}{dx} = -\frac{e N_d x}{\epsilon} + \frac{e N_d x_n}{\epsilon}$ Again integrating it we get

$$V(x) = -\frac{e N_d x^2}{2\epsilon} + \frac{e N_d x_n x}{\epsilon} + D$$

$$\text{Using } x = x_n \text{ we get } V(x_n) = -\frac{e N_d x_n^2}{2\epsilon} + \frac{e N_d x_n^2}{\epsilon} + D \Rightarrow V(x_n) = \frac{e N_d x_n^2}{2\epsilon} + D \dots (\text{v})$$

Subtracting (iii) from (v) and using B = D as V is continuous.

$$\text{We get, built in potential, } V_0 = V(x_n) - V(-x_p) = \frac{eN_d x_n^2}{2\epsilon} + \frac{eN_a x_p^2}{2\epsilon} \dots (\text{vi})$$

We get, built in potential, $V_0 = V(x_n) - V(-x_p) = \frac{eN_d x_n^2}{2\epsilon} + \frac{eN_a x_p^2}{2\epsilon} \dots (\text{vi})$

For overall charge neutrality, the total charge per unit area on the left hand side must be equal to that on right hand side. i.e. $N_a x_p = N_d x_n \Rightarrow x_p = \frac{N_d}{N_a} x_n \dots (\text{a})$ and $x_n = \frac{N_a}{N_d} x_p \dots (\text{b})$

$$\text{Using (a) in (vi) we get, } V_0 = \frac{eN_d x_n^2}{2\epsilon} + \frac{eN_a \left[\frac{N_d}{N_a} x_n \right]^2}{2\epsilon} = \frac{eN_d}{2\epsilon} \left[1 + \frac{N_d}{N_a} \right] x_n^2$$

$$\Rightarrow x_n = \left[\frac{2V_0 \epsilon}{eN_d \left(1 + \frac{N_d}{N_a} \right)} \right]^{\frac{1}{2}}$$

$$\text{Similarly using (b) in (vi) and solving, we get, } x_p = \left[\frac{2V_0 \epsilon}{eN_a \left(1 + \frac{N_a}{N_d} \right)} \right]^{\frac{1}{2}}$$

Therefore, total width of transition region,

$$W = x_n - (-x_p) = x_n + x_p = \left[\frac{2V_0 \epsilon}{eN_d \left(1 + \frac{N_d}{N_a} \right)} \right]^{\frac{1}{2}} + \left[\frac{2V_0 \epsilon}{eN_a \left(1 + \frac{N_a}{N_d} \right)} \right]^{\frac{1}{2}}$$

$$\text{or, } W = \left[\frac{2V_0 \epsilon}{e} \right]^{\frac{1}{2}} \left[\frac{1}{\left\{ N_a \left(1 + \frac{N_a}{N_d} \right) \right\}^{\frac{1}{2}}} + \frac{1}{\left\{ N_d \left(1 + \frac{N_d}{N_a} \right) \right\}^{\frac{1}{2}}} \right] = \left[\frac{2V_0 \epsilon}{e} \right]^{\frac{1}{2}} \left[\frac{N_d^{\frac{1}{2}}}{N_a^{\frac{1}{2}} (N_d + N_a)^{\frac{1}{2}}} + \frac{N_a^{\frac{1}{2}}}{N_d^{\frac{1}{2}} (N_a + N_d)^{\frac{1}{2}}} \right]$$

$$\text{or, } W = \left[\frac{2V_0 \epsilon}{e} \right]^{\frac{1}{2}} \left[\frac{(N_d + N_a)}{N_a^{\frac{1}{2}} N_d^{\frac{1}{2}} (N_d + N_a)^{\frac{1}{2}}} \right] = \left[\frac{2V_0 \epsilon}{e} \right]^{\frac{1}{2}} \left[\frac{(N_d + N_a)^{\frac{1}{2}}}{N_a^{\frac{1}{2}} N_d^{\frac{1}{2}}} \right]$$

Therefore, $W = \left[\frac{2V_0 \epsilon}{e} \left(\frac{1}{N_d} + \frac{1}{N_a} \right) \right]^{\frac{1}{2}} \Rightarrow$ Required expression for the width of transition region. It shows that width decreases as the impurity concentrations increase.

Forward Biased pn junction:

If the positive terminal of a battery is connected to p - side and negative terminal to n - side, the pn junction is said to be forward biased.

Applying potential V reduces the built in potential V_0 to $V_0 - V$. Since, built in potential is reduced, diffusion of electrons towards p - side and holes towards n - side takes place.

Let, $P_n(0)$ be the hole concentration just outside the transition region in n - side (at $x = x_n$) due to diffusion from p - side. Then, $P_n(0) = p_p \cdot e^{\frac{-e(V_0-V)}{KT}}$ (i) Where, p_p is the total hole concentration in P - side.

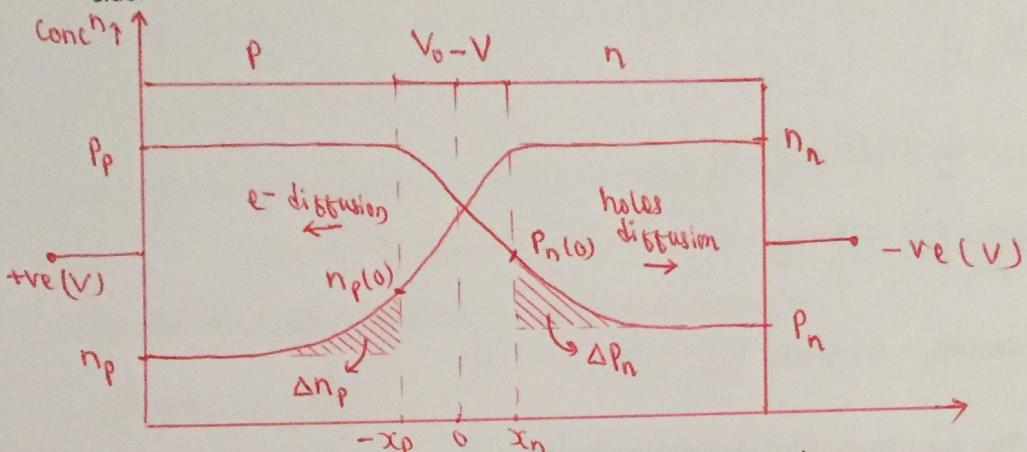


Fig. Carrier concentration in pn junction when forward biased.

We have for ideal case, $\frac{p_p}{p_n} = e^{\frac{eV_0}{KT}} \Rightarrow p_p = p_n \cdot e^{\frac{eV_0}{KT}}$... (ii) Here, p_n is the concentration of holes in n - region.

Using (ii) in (i), we get $P_n(0) = p_n \cdot e^{\frac{eV_0}{KT}} \cdot e^{\frac{-e(V_0-V)}{KT}} = p_n \cdot e^{\frac{eV}{KT}}$... (iii) This eqⁿ is called law of junction.

The excess holes in p - side, $\Delta p_n = P_n(0) - P_n$

Using Eqⁿ (iii) we get $\Delta p_n = p_n \cdot e^{\frac{eV}{KT}} - p_n = p_n \left[e^{\frac{eV}{KT}} - 1 \right]$... (iv)

The current density for holes is given by,

$$J = p_e \mu E - eD \frac{dp}{dx}$$

Since, the diffusion dominates drift, the first term can be neglected.

$$\text{i.e. } J = -eD \frac{dp}{dx} = -eD \frac{d\Delta p}{dx}$$

If the length of p - side and n - side are large enough, then the excess hole concentration $\Delta p_n(x)$ in n - region falls exponentially from equilibrium value $\Delta p(0)$ given by,

$$\Delta p_n = \Delta p(0) e^{-x/L} \Leftrightarrow \frac{d(\Delta p_n)}{dx} = \frac{-1}{L} \Delta p(0) e^{-x/L} \Rightarrow \frac{d(\Delta p_n)}{dx} = -\frac{1}{L} \Delta p_n$$

Therefore, current density, $J = \frac{eD}{L} \Delta p_n$

Substituting Δp_n from (iv) we get, , $J = \frac{eD}{L} \cdot p_n \left[e^{\frac{eV}{KT}} - 1 \right]$

But we have, $p_n \cdot n_n = n_i^2 \Rightarrow p_n = n_i^2 / n_n = n_i^2 / N_d$

$$\text{Therefore, } J = \frac{eD}{L} \frac{n_i^2}{N_d} \left[e^{\frac{eV}{KT}} - 1 \right]$$

$$\text{For holes we can write, } J_h = \frac{eD_h}{L_h} \frac{n_i^2}{N_d} \left[e^{\frac{eV}{KT}} - 1 \right] \dots(v)$$

$$\text{And for electrons, } J_e = \frac{eD_e}{L_e} \frac{n_i^2}{N_a} \left[e^{\frac{eV}{KT}} - 1 \right] \dots(vi)$$

$$\text{Total diffusion current density, } J = J_e + J_h = e n_i^2 \left(\frac{D_e}{L_e N_a} + \frac{D_h}{L_h N_d} \right) \left[e^{\frac{eV}{KT}} - 1 \right]$$

$$\Rightarrow J = J_s \left[e^{\frac{eV}{KT}} - 1 \right] \dots(vii) \text{ This equation is called ideal diode equation.}$$

Where, $J_s = e n_i^2 \left(\frac{D_e}{L_e N_a} + \frac{D_h}{L_h N_d} \right)$ is called reverse saturation current density.

Reverse Biased pn junction:

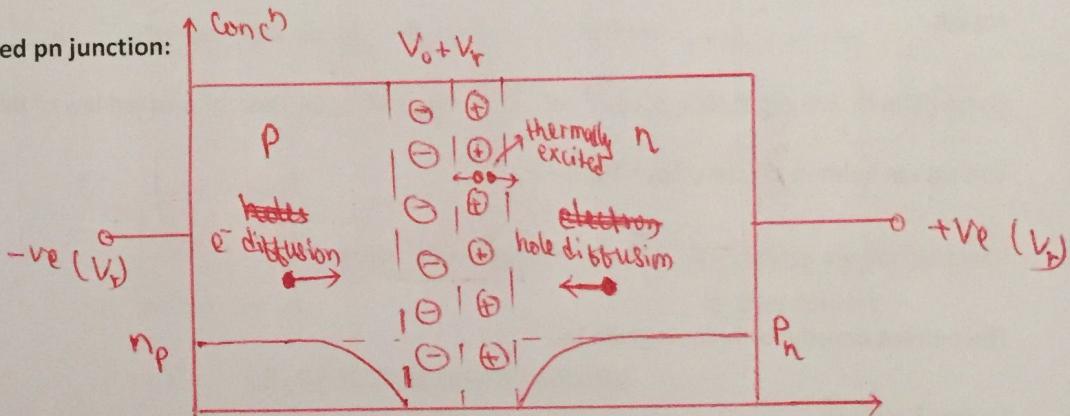


Fig. Reversed biased pn junction: minority carrier profile.

When a pn junction is connected to a voltage source with p – region connected to negative terminal of the battery and n – region to positive terminal, the pn junction is said to be reverse biased.

The built in potential widens from V_0 to $V_0 + V_r$, where V_r is the applied voltage.

There will be a small diffusion current through the junction due to holes diffusing from depletion layer to n – region, and electrons to p – region.

The reverse current is given by

$$\Rightarrow J = - J_s \left[e^{\frac{eV}{KT}} - 1 \right] \dots(i)$$

Where, $J_s = en_i^2 \left(\frac{D_e}{L_e N_a} + \frac{D_h}{L_h N_d} \right)$... (ii) is called reverse saturation current density.

Here, eqⁿ (ii) is called Shockley equation.

Considering the effect of thermal generation eqⁿ (ii) is modified to,

$$J_{s,rev} = en_i^2 \left(\frac{D_e}{L_e N_a} + \frac{D_h}{L_h N_d} \right) + \frac{en_i W}{\tau_g}$$

Where, the 2nd term is the current density due to thermal generation.

τ_g is the mean thermal generation time to generate electron hole pairs.

Width of transition region for reverse biased condition modifies to,

$$W = \left[\frac{2(V_0 + V)}{e} \left(\frac{1}{N_d} + \frac{1}{N_a} \right) \right]^{\frac{1}{2}}$$

Metal - semiconductor junction:

When a metal and a semiconductor are brought into contact, we can observe two situations: the work function of metal (Φ_m) is greater than that of semiconductor (Φ_s) or vice versa.

i) Consider the case $\Phi_m > \Phi_s$ (Schottky Junction):

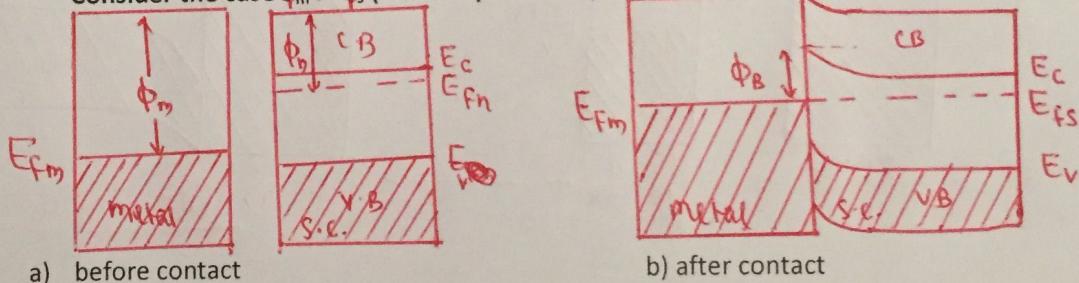


Fig. metal semiconductor junction

Here we consider a n - type semiconductor. Obviously, $E_{F_s} > E_{F_m}$.

So, electrons are transferred from semiconductor to metal. So, it behaves as a rectifier that allows flow of electrons in one direction.

At equilibrium, the Fermi levels of both materials line up. That is, $E_{F_m} = E_{F_s}$

There are two current components due to the electrons flowing through the junction.

1. The current due to electrons being thermally emitted from metal to conduction band of the semiconductor which faces Schottky barrier $\Phi_B = \Phi_m - \Phi_s$.
i.e. $J_1 = C_1 e^{\frac{-\Phi_B}{kT}}$... (i) Where C_1 is a constant
2. The current due to electrons being thermally emitted from the conduction band of the semiconductor to the metal at cost of built in potential V_0 .

$$\text{i.e. } J_2 = C_2 e^{\frac{-eV_0}{KT}} \dots (\text{ii})$$

At equilibrium, $J_1 = J_2$

- a) Forward biasing: It supports the flow of electrons from semiconductor to metal. The built in potential reduces to $V_0 - V$.

Therefore, the forward current density,

$$J_2^{\text{For.}} = C_2 e^{\frac{-e(V_0-V)}{KT}}$$

The net current density, $J = J_2^{\text{For.}} - J_2$ (as $J_1 = J_2$)

$$\text{Therefore, } J = C_2 e^{\frac{-e(V_0-V)}{KT}} - C_2 e^{\frac{-eV_0}{KT}} = C_2 e^{\frac{-eV_0}{KT}} (e^{\frac{eV}{KT}} - 1)$$

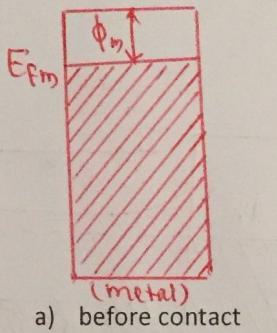
$J = J_0 \left(e^{\frac{eV}{KT}} - 1 \right) \dots (\text{iii})$ Where $J_0 = C_2 e^{\frac{-eV_0}{KT}}$ is reverse saturation current density.

- b) When reversed biased, it resists the flow of electron from semiconductor to metal. The built in potential becomes $V_0 + V_r$. The corresponding current density is,

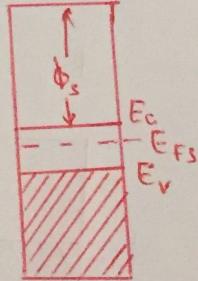
$$J_2^{\text{rev.}} = C_2 e^{\frac{-e(V_0+V_r)}{KT}}$$

$$\phi_s > \phi_m$$

- b) Consider the case $\phi_s > \phi_m$ (Ohmic Junction):



a) before contact



b) After contact

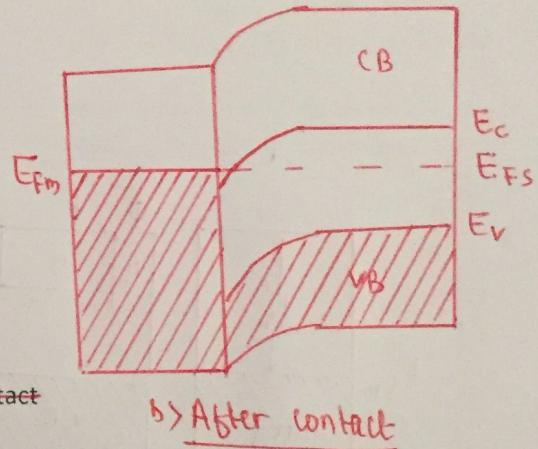


Fig. Ohmic contact

An ohmic contact allows the flow of current in both forward and reverse direction. It is formed when a metal having smaller work function is brought in contact with a semiconductor having higher Fermi level. Larger work function.

Obviously, E_{Fm} is greater than E_{Fs} . So, electrons move from metal side to semiconductor side. These electrons will accumulate in the region of semiconductor near to contact.

The current is determined by the resistance of the bulk region. The current density is simply $J = \sigma E$

Where, σ is conductivity of semiconductor

E = applied field.
