

1.

Transient Response

Steady State

1. It is the response of a system to a change from an equilibrium

It is an equilibrium condition of a circuit or network that occurs as the effects of transients are no longer important

2. System is unstable during transient response.

During steady state a system is in relative stability.

3. It provides information about

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i) Initialization (when system responded to input)

i) Steady State error
ii) Nature of error (constant or varying w.r.t. time)

ii) Rate of rise of output w.r.t. time

iii) Accuracy of system

iii) Nature of response
: exponential or oscillatory

Q. Steps to find out Transient Response Solution

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The source is made to zero then the differential equation converts to homogeneous version.

Generalized procedure for finding transient solution

General form of diff. eq. is given by

$$D(P). Y(t) = N(P). f(t) \quad \text{--- (1)}$$

eq. (1) represents the non-homogeneous diff. eq. in operator form.

homogeneous version of diff. eq. is obtained by setting $f(t) = 0$ i.e.

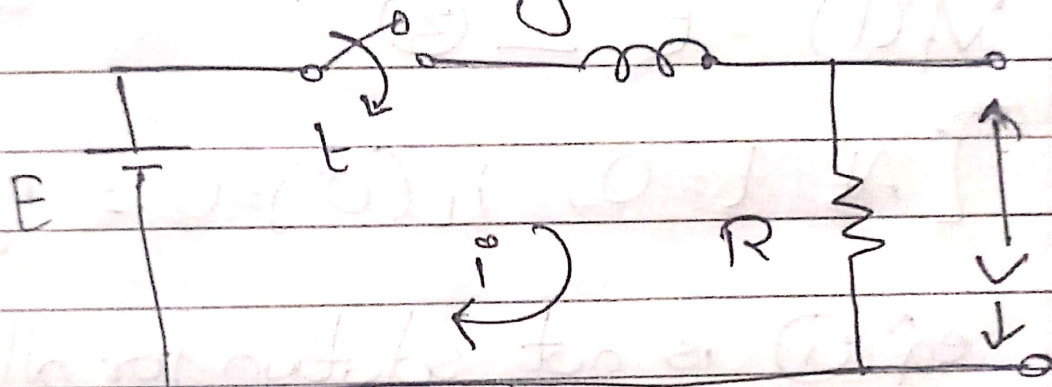
$$D(P). Y(t) = 0 \quad \text{--- (2)}$$

Since, only exponential function qualifies as a solution to the diff. eq., the operator can be replaced by the algebraic multiplier 's'. So,

$$D(s)Y(t) = 0 \quad \text{--- (3)}$$

$D(s)$ is the denominator polynomial of the network function $G(s)$ and $D(s) = 0$ represents the characteristics eqⁿ.

Consider following circuit:



Applying KVL when switch is closed
i.e. at $t = 0^+$

$$E = L \frac{di}{dt} + Ri \quad \text{--- (1)}$$

Now,

$$V = \frac{E * R}{L \frac{di}{dt} + R} \Rightarrow VLP + VR = ER$$

$$\Rightarrow \frac{L}{R} \frac{dV}{dt} + V = E$$

$$\Rightarrow \frac{L}{R} \frac{dV}{dt} + V = E \quad \text{--- (2) which is}$$

non-homogeneous eqⁿ

At steady state (i.e. $t \rightarrow \infty$). for constant source put $p=0$

i.e. $\frac{dV}{dt} = 0$

$$V_f(t) = E \quad \text{--- (3)}$$

$$[\text{At } t=0, i_L(0^-)=0 = i_L(0^+)]$$

Thus, eqⁿ (1) is not solution for all time

Now, for transient response, homogeneous version of diff. eqⁿ

$$\frac{1}{R} \frac{dV}{dt} + V = 0 \quad \text{--- (4)}$$

Since, only the exponential form satisfies the above condition So,

Let, $V_t = K e^{st}$ be the transient solution

From eqn (4)

$$\frac{1}{R} \frac{d}{dt} (ke^{st}) + ke^{st} = 0$$

$$\Rightarrow \frac{L}{R} k s e^{st} + k e^{st} = 0$$

$$\Rightarrow k e^{st} \left(\frac{L}{R} s + 1 \right) = 0$$

$$k e^{st} \neq 0$$

$$\text{So, } \frac{L}{R} s + 1 = 0$$

$$\text{or, } s = -\frac{R}{L}$$

$$\therefore V_t = k e^{-\frac{R}{L} t}$$

Then total solution ~~will~~ would be

$$V = V_f + V_t$$

$$= E + k e^{-\frac{R}{L} t} \quad (5)$$

Here k is unknown and can be found

by using initial conditions.

Initial condition:

At $(t = 0^-)$

$$i(0^-) = 0, \quad V(0^-) = 0 \quad [\text{switch open}]$$

At $(t = 0^+)$,

$i(0^+) = i(0^-) = 0$ (current through inductor cannot change instantaneously)

$$\therefore V(0^+) = 0$$

Now, for $t = 0^+$, eqⁿ (5) becomes

$$V(0^+) = E + Ke^{-\frac{R}{L}t}(0^+)$$

$$\Rightarrow 0 = E + K$$

$$\therefore K = -E$$

Hence, eqⁿ (5) becomes,

$$V = E - Ee^{-\frac{R}{L}t} = E(1 - e^{-\frac{R}{L}t})$$

which is the required solution.