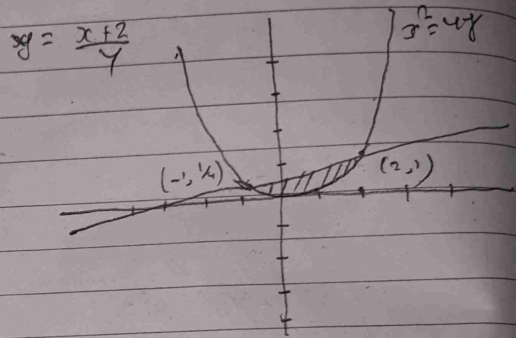
2012

The given curve is

$$x^2 = 4y \quad \text{--- (i)}$$

The given line is

$$x = 4y - 2 \quad \text{--- (ii)}$$



solving eqn (i) and (ii)

$$x^2 = x + 2$$

$$x^2 - x - 2 = 0$$

$$x^2 - 2x + x - 2 = 0$$

$$\text{or, } x(x-2) + 1(x-2) = 0$$

$$(x-2)(x+1) = 0$$

$$x = 2$$

$$x = -1$$

when $x = 2$, $y = 1$

when $x = -1$, $y = 1/4$

∴ point of intersection are $(2, 1)$ and $(-1, 1/4)$

Now

$$A = \int_{-1}^{2} y_1 - y_2 \, dx$$

$$= \int_{-1}^{2} \frac{x^2}{4} - \left(\frac{x+2}{4}\right) dx$$

$$= \frac{1}{4}\int_{-1}^{2} x^2 - x - 2 \, dx$$

$$= \frac{1}{4}\left[\left[\frac{x^3}{3}\right]_{-1}^{2} - \left[\frac{x^2}{2}\right]_{-1}^{2} - 2\left[x\right]_{-1}^{2}\right]$$

$$\frac{1}{4}\left[\frac{1}{3}(8+1) - \frac{1}{2}(4-1) - 2\times 3\right)$$

$$= \frac{1}{4}\left[3 - \frac{3}{2} - 6\right]$$

$$= \frac{1}{4}\left[\frac{6-3-12}{2}\right]$$

$$= \frac{1}{4} \times -9/2$$

$$= -9/4$$

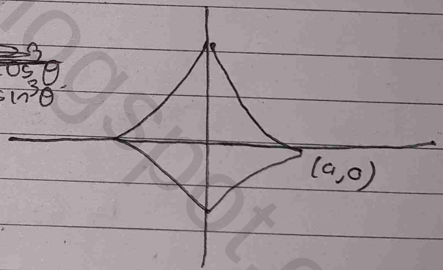
$$= 9/8 \text{ sq. unit.}$$

2012 (fall)

The given asteroid is

$$x^{2/3} + y^{2/3} = a^{2/3}$$

let $x = \cancel{a\sin^3\theta}$ $a\cos^3\theta$ and $y = \cancel{a\cos^3\theta}$ $a\sin^3\theta$
then.



$$V = \int_{\theta_1}^{\theta_2} \pi y^2 \frac{dx}{d\theta} \cdot d\theta$$

$$\cancel{2\pi\int_0^{\pi/2} a^6\cos^6\theta \cdot a\, 3\sin^2\theta \cdot \cos\theta}$$

$$= 2\pi\int_0^{\pi/2} a^2\sin^6\theta \cdot a \cdot 3(\cos^2\theta)(-\sin\theta)\, d\theta$$

$$= -2\pi\int_0^{\pi/2} 3a^3 \cdot \sin^7\theta \cdot \cos^2\theta\, d\theta$$

$$= 6\pi a^3 \int_0^{\pi/2} \sin^7\theta \cdot \cos^2\theta \, d\theta$$

$$= 6\pi a^3 \left[\frac{\Gamma\frac{7+1}{2}\,\Gamma\frac{2+1}{2}}{2\,\Gamma\frac{7+2+2}{2}} \right]_0^{\pi/2}$$

$$= 6\pi a^3 \left[\frac{\Gamma 4 \; \Gamma\frac{3}{2}}{2\,\Gamma\frac{11}{2}} \right]$$

$$= 6\pi a^3 \left[\frac{3\cdot 2\cdot 1 \times \frac{1}{2}\Gamma\frac{1}{2}}{2 \times \frac{9}{2} \times \frac{7}{2} \times \frac{5}{2} \times \frac{3}{2} \times \frac{1}{2}\Gamma\frac{1}{2}} \right]$$

$$= \frac{\cancel{6}^{2}\pi a^3 \times \cancel{3} \times 16}{3\cancel{8} \times 7 \times 5 \times \cancel{3}}$$

$$= \frac{32}{105}\pi a^3 \text{ cubic unit}$$

2013 (Fall)

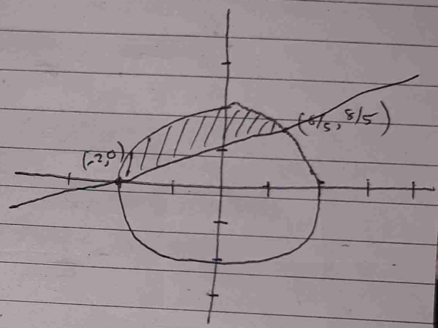
4.a) The given circle is

$$x^2+y^2=4 \quad \text{— (i)}$$

The given line is

$$x-2y=-2 \quad \text{— (ii)}$$

$$y=\frac{x+2}{2}$$



$$\therefore \quad x^2+\left(\frac{x+2}{2}\right)^2=4$$

$$\text{or,} \quad x^2+\frac{x^2+4x+4}{4}=4$$

$$\text{or,} \quad 4x^2+x^2+4x+4=16$$

$$\text{or,} \quad 5x^2+4x-12=0$$

$$\text{or,} \quad 5x^2+10x-6x-12=0$$

$$\text{or,} \quad 5x(x+2)-6(x+2)=0$$

$$(x+2)(5x-6)$$

$$\therefore \quad x=-2 \text{ and } x=6/5$$

when $x=-2$, $y=0$

when $x=6/5$, $y=8/5$

point of intersection are $(-2,0)$ and $(6/5, 8/5)$

Now

Area of the region is

$$A=\int_{-2}^{6/5}(y_1-y_2)\,dx$$

$$=\int_{-2}^{6/5}\left(\sqrt{4-x^2}-\frac{x+2}{2}\right)dx$$

$$= \frac{48}{50} + \cancel{73.72} + 3.14 + \frac{16}{25} - \frac{16}{5}$$

$$0.96 \cancel{+ 73.72 + 3.14} + 0.64 - 3.2$$

$$: \left[\frac{x\sqrt{4-x^2}}{2} + \frac{4}{2}\sin^{-1}\frac{x}{2} - \frac{1}{2}\left(\frac{x^2}{2} + 2x\right) \right]_{-2}^{6/5}$$

$$: \left\{ \frac{6}{10}\sqrt{4 - \frac{36}{25}} + 2\sin^{-1}\left(\frac{6}{10}\right) - \frac{1}{2}\left(\frac{36}{50} + \frac{12}{5}\right) \right\}$$

$$- \left\{ 0 + 2\sin^{-1}(-1) - \frac{1}{2}\left(2 - 4\right) \right\}$$

$$= 0.96 + 2\sin^{-1}(0.86) - 3.12) - \left(2 \times \frac{-\pi}{2} + 1\right)$$

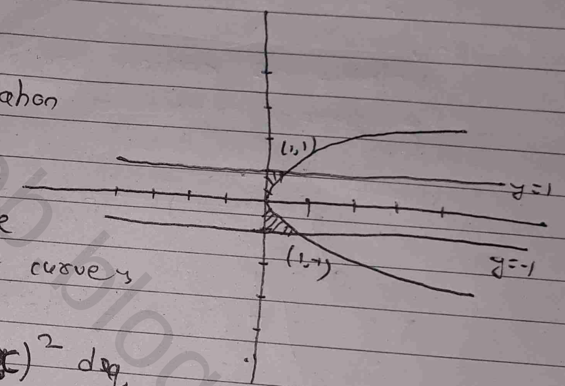
$0.96 + 2 \times \sin^{-1}(0.6) - 3.12 + \pi - 1$

$= 73.72$ sq. unit

2013 (fall)

4.a) Or

Here the curve $x = y^2$ is bounded by $x=0, y=-1$ and $y=1$



The limit of integration is $y=0$ to $y=1$

Now, volume of the region bounded by curves

$$V = 2\int_0^1 \pi (x_2 - x)^2 - (x_1 - x)^2 \, dy$$

$$= 2\pi \int_0^1 (y^2 - 0)^2 - (0-0)^2 \, dy$$

$$= 2\pi \int_0^1 y^4 \, dy$$

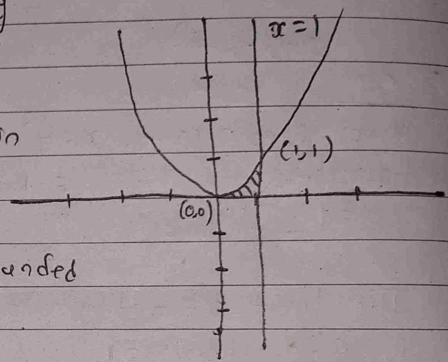
$$= 2\pi \left[\frac{y^5}{5} \right]_0^1$$

$$= \frac{2\pi}{5} \text{ cubic unit}$$

2013 (spring)

3.b)

Here the curve is ~~bounded~~ $y=x^2$ is bounded by x-axis (i.e. $y=0$) and $x=1$



The limit of integration is $y=0$ to $y=1$

Now

volume of region bounded by given curve is

$$\cancel{V = \int_0^1 \pi (y_2-y)^2 - (y_1-y)^2 \, d}$$

$$V = \int_0^1 \pi (x_2-x)^2 - (x_1-x)^2 \, dy$$

$$= \pi \int_0^1 (y^{1/2} - (-1))^2 - (1-(-1))^2 \, dy$$

$$= \pi \int_0^1 (y^{1/2}+1)^2 - 4 \, dy$$

$$= \pi \int_0^1 y + 2y^{1/2} + 1 - 4 \, dy$$

$$= \pi \int_0^1 y + 2y^{1/2} - 3 \, dy$$

$$= \pi \left[\cancel{1} \frac{y^2}{2} + 2 \cdot \frac{y^{3/2}}{3/2} - 3y \right]_0^1$$

$$= \pi \left[\left\{ \frac{1}{2} + \frac{4 \cdot 1}{3} - 3 \cdot 1 \right\} - \left\{ 0 + 0 - 0 \right\} \right]$$

$$= \pi \left[\frac{1}{2} + \frac{4}{3} - 3 \right]$$

$$= \pi \left[\frac{3 + 8 - 18}{6} \right]$$

$$= \pi \left[\frac{-7}{6} \right]$$

$= \cancel{\frac{7\pi}{6} \text{ cubic}}$ unit.

$$= \frac{7\pi}{6} \text{ cubic unit.}$$

2014 (fall)

4.a) The given curve is

$$x^2 = 4y \quad \text{— (i)}$$

The given line is

$$y = |x| = \begin{cases} x & x > 0 \\ -x & x < 0 \\ 0 & x = 0 \end{cases} \quad \text{— (ii)}$$

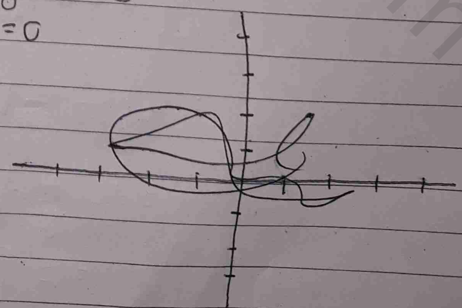
from (i) and (ii)

$\therefore x^2 = 4x$

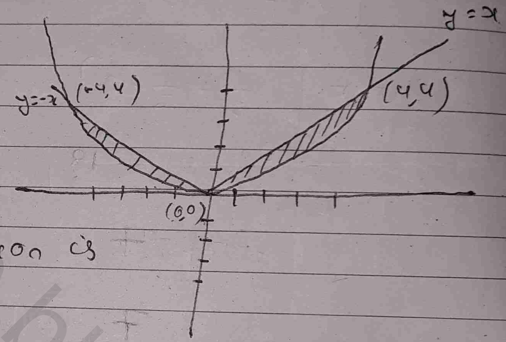
$x^2 - 4x = 0$

$x(x-4) = 0$

$\therefore x = 0, \; x = 4$



when $x=0$, $y=0$
when $x=4$, $y=4$
point of intersection are $(0,0)$ and $(4,4)$



Area of the region is

$$A = 2\int_0^4 (y_1 - y_2)\,dx$$

$$= 2\int_0^4 \frac{x^2}{4} - |x|$$

$$= 2\int_0^4 \frac{x^2}{4} - x$$

$$= 2\left[\frac{1}{4}\left[\frac{x^3}{3}\right]_0^4 - \left[\frac{x^2}{2}\right]_0^4\right]$$

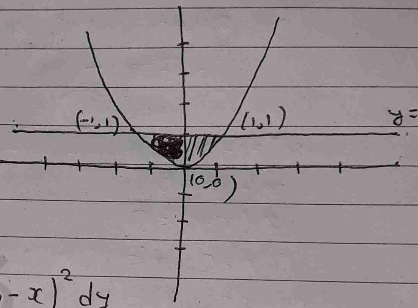
$$= 2\left[\frac{1}{4}\times\frac{1}{3}\left[64-0\right] - \frac{1}{2}\left[16-0\right]\right]$$

$$= 2\left[\frac{1}{\cancel{12}\,3}\times\cancel{64}^{16} - \frac{1}{2}\times 16\right]$$

$$= 2\left[\frac{32-48}{6}\right] = 2\left[-\frac{\cancel{16}^{8}}{\cancel{6}_{3}}\right] = \frac{16}{3} \text{ sq. unit}$$

'2014 (fall) OR

The curve $y=x^2$ is bounded by ~~line~~ y-axis (i.e $x=0$) and $y=1$



The limit of integration is $y=0$ to $y=1$

Now,

The volume of region bounded by curve is

$$V = \int_0^1 \pi (x_2 - x_0)^2 - (x_1 - x)^2 \, dy$$

$$= \pi \int_0^1 (\sqrt{y} - 3/2)^2 - (0 - 3/2)^2 \, dy$$

$$= \pi \int_0^1 \left(y - 2\cdot\sqrt{y}\cdot\frac{3}{2} + \frac{9}{4} - \frac{9}{4}\right) dy$$

$$= \pi \int_0^1 y - 3y^{1/2} \, dy$$

$$= \pi \left[\frac{y^2}{2} - \frac{3 \cdot y^{3/2}}{3/2}\right]_0^1 = 2\pi \left[\frac{y^2}{2} - 2y^{3/2}\right]_0^1$$

$$= \pi \left[\frac{1}{2} - 2 - (0 - 0)\right]$$

$$= \pi \left[\frac{1}{2} - 2\right] = \pi \left[\frac{-3}{2}\right] = \frac{3\pi}{2} \text{ cubic unit}$$

2014 (spring)

4.a) The given circle is

$$x^2+y^2=1 \quad \text{--- (1)}$$

The given parabola is

$$y^2=1-x \quad \text{--- (ii)}$$

from (1) and (ii)

$x^2+1-x=1$

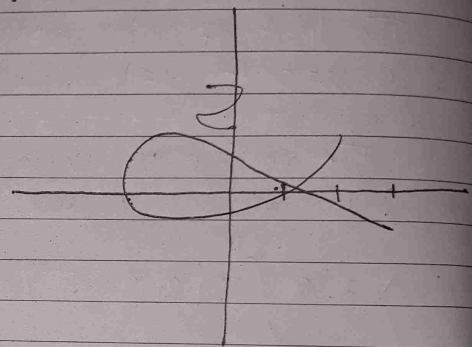
$x^2-x=0$

$x(x-1)=0$

$\therefore x=0,\ x=1$

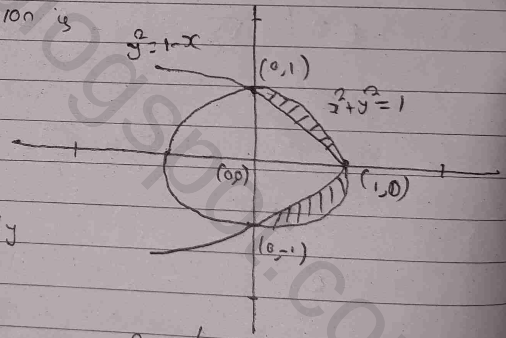
when $x=0$, $y=1$

when $x=1$, $y=0$



Now,

Area of shaded region is



$$A = 2\int_0^1 (x_1-x_2)\,dy$$

$$=2\int_0^1 \left(\sqrt{1-y^2} - (1-y^2)\right) dy$$

$$=2\left[\frac{y\sqrt{1-y^2}}{2} + \frac{1}{2}\sin^{-1}\frac{y}{1} - y + \frac{y^3}{3}\right]_0^1$$

$$=2\left[0 + \frac{1}{2}\sin^{-1}(1) - 1 + \frac{1}{3} - (0+0-0+0)\right]$$

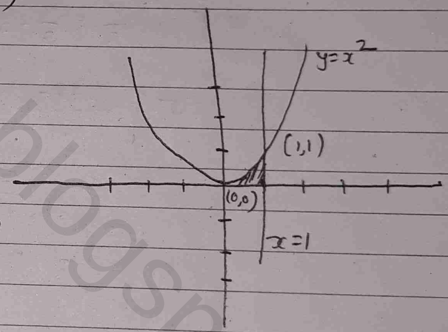
$$=2\left[\frac{1}{2}\times\frac{\pi}{2} - 1 + \frac{1}{3}\right]$$

$$= 2\left[\frac{\pi}{4} + \frac{2}{3}\right]$$

$$A = \frac{\pi}{2} + \frac{4}{3} \text{ sq. unit.}$$

2014 (spring) OR

Here, the curve $y = x^2$ is bounded by x-axis (i.e $y=0$) and $x = 1$



The limit of integration is $y=0$ to $y=1$

Now,

volume of the ~~given~~ region bounded by curve is

$$V = \int_0^1 \pi\, (x_2 - x)^2 - (x_1 - x)^2\, dy$$

$$= \pi \int_0^1 \left[\left(\sqrt{y} - (-1)\right)^2 - \left(1 - (-1)\right)^2\right] dy$$

$$= \pi \int_0^1 \left[\left(\sqrt{y} + 1\right)^2 - 2^2\right] dy$$

$$= \pi \int_0^1 \left(y + 2\cdot\sqrt{y}\cdot 1 + 1 - 4\right) dy$$

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Page

$$= \pi \int_0^1 (y + 2y^{1/2} - 3)\, dy$$

$$= \pi \left[\frac{y^2}{2} + \frac{2 \cdot y^{3/2}}{3/2} - 3y \right]_0^1$$

$$= \pi \left[\frac{1}{2} + \frac{4}{3} - 3 - (0 + 0 - 0) \right]$$

$$= \pi \left[\frac{1}{2} + \frac{4}{3} - 3 \right]$$

$$= \pi \left[\frac{3+8-18}{6} \right]$$

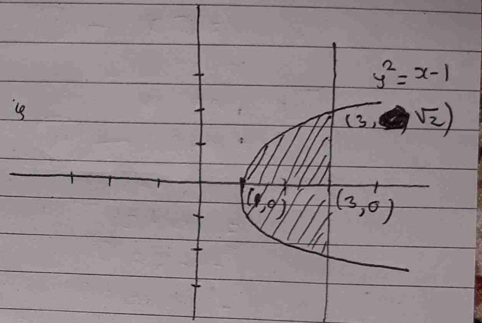
$$= \pi \left[\frac{-7}{6} \right]$$

$= 7\pi/6$ cubic unit.

$x = y^2+1$
$y^2 = 2$
$y = \sqrt{2}$

2015 (fall)

4.a) Here, the ~~curve~~ parabola $x = y^2+1$ is bounded by line $x = 3$



The limit of integration is $y = 0$ to $y = \sqrt{2}$

Now,
volume of the region bounded by curve is

$$V = 2\int_0^{\sqrt{2}} \pi (x_2 - x)^2 - (x_1 - x)^2 \, dy$$

$$= 2\pi \int_0^{\sqrt{2}} (y^2+1-3)^2 - (3-3)^2 \, dy$$

$$= 2\pi \int_0^{\sqrt{2}} (y^2-2)^2 \, dy$$

$$= 2\pi \int_0^{\sqrt{2}} (y^4 - 4y^2 + 4) \, dy$$

$$= 2\pi \left[\frac{y^5}{5} - 4 \cdot \frac{y^3}{3} + 4y \right]_0^{\sqrt{2}}$$

$$= 2\pi \left[\left(\frac{(\sqrt{2})^5}{5} - \frac{4(\sqrt{2})^3}{3} + 4\sqrt{2} \right) - 0 \right]$$

$$= 2\pi \left[\frac{4\sqrt{2}}{5} - \frac{4 \times 2\sqrt{2}}{3} + 4\sqrt{2} \right]$$

$$= 2\pi \left[\frac{12\sqrt{2} - 40\sqrt{2} + 60\sqrt{2}}{15} \right]$$

$$= 2\pi \left[\frac{32\sqrt{2}}{15} \right]$$

$$= \frac{64\sqrt{2}}{15} \pi \text{ cubic units}$$

2015 (spring)

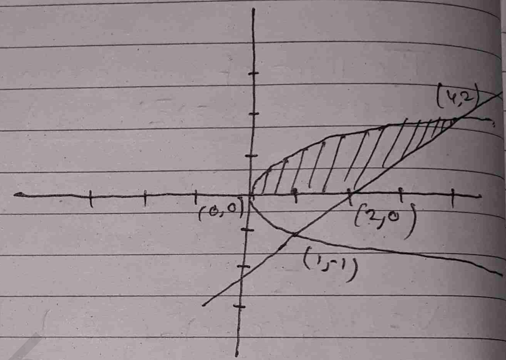
4.b) The given curve is

$y = \sqrt{x}$

or, $y^2 = x$ — (i)

The line is

$y = x - 2$ — (ii)



from (i) and (ii)

$y^2 = y + 2$

or, $y^2 - y - 2 = 0$

or, $y^2 - 2y + y - 2 = 0$

or, $y(y-2) + 1(y-2) = 0$

$(y-2)(y+1) = 0$

$\therefore y = 2,\ y = -1$

when $y = 2,\ x = 4$

when $y = -1,\ x = 1$

Points of intersection are $(1,-1)$ and $(4,2)$

now,

Area of shaded region is

$A = \int_0^2 (x_1 - x_2)\, dy$

$= \int_0^2 [y^2 - (y+2)]\, dy$

$= \int_0^2 y^2 - y - 2\, dy$

$= \left[\frac{y^3}{3} - \frac{y^2}{2} - 2y\right]_0^2$

$= \left[\left(\frac{8}{3} - \frac{4}{2} - 4\right) - 0\right]$

$= \left[\frac{8}{3} - 2 - 4\right]$

$= \frac{8}{3} - 6$

$= \frac{8-18}{3}$

$= -10/3$

$= 10/3$ sq. unit

2016 (fall)

4.a) The given circle is
$x^2+y^2=4$ — (i)
The given line is
$x-2y=-2$ — (ii)

from (i) and (ii)
$(2y-2)^2+y^2=4$
$4y^2-8y+4$

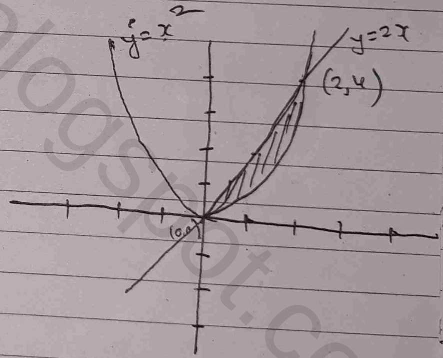
same as 2013 (fall)

2016 (fall) OR

Here, the region is bounded by parabola $y=x^2$ (i) and line $y=2x$ — (ii)

For (ii)

x	0	1	2
y	0	2	4



Now,
Area of shaded region is

$$A=\pi\int_0^4 (x_1-x_2)\,dy$$

$$=\pi\int_0^4 (\sqrt{y})^2-\left(\frac{y}{2}\right)^2\,dy$$

$$=\pi\int_0^4\left(y-\frac{y^2}{4}\right)dy$$

$$2\underline{\left|\,12, 2\right.}$$
$$6, 1$$

$$= \pi\left[\frac{y^2}{2} - \frac{1}{4}\frac{y^3}{3}\right]_0^4$$

$$= \pi\left[\frac{y^2}{2} - \frac{y^3}{12}\right]_0^4$$

$$= \pi\left[\frac{16}{2} - \frac{64}{12} - 0\right]$$

$$= \pi\left[\frac{96-64}{12}\right]$$

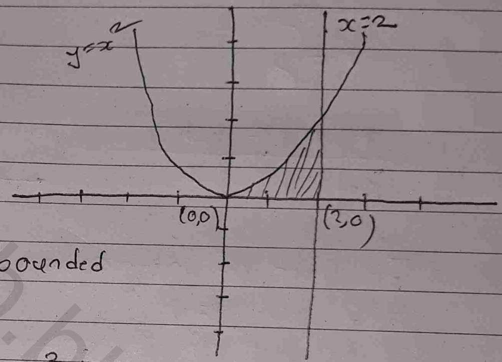
$$= \pi\left[\frac{\cancel{32}\,8}{\cancel{12}\,3}\right]$$

$$= \frac{8\pi}{3} \text{ sq. unit.}$$

2017 (fall)

4.a) Here, the region is bounded by parabola $y = x^2$, x-axis (i.e $y=0$) and $x = 2$

x	1	2
y	1	4



limit of integration is $y=0$ to $y=4$

Now,
volume of region bounded by curve is

$$V = \int_0^4 \pi (x_2 - x)^2 - (x_1 - x)^2 \, dy$$

$$= \pi \int_0^4 (\sqrt{y} - 0)^2 - (2-0)^2 \, dy$$

$$= \pi \int_0^4 y - 4 \, dy$$

$$= \pi \left[\frac{y^2}{2} - 4y \right]_0^4$$

$$= \pi \left[\frac{1}{2}(16) - 16 - 0 \right]$$

$$= \pi [8 - 16]$$

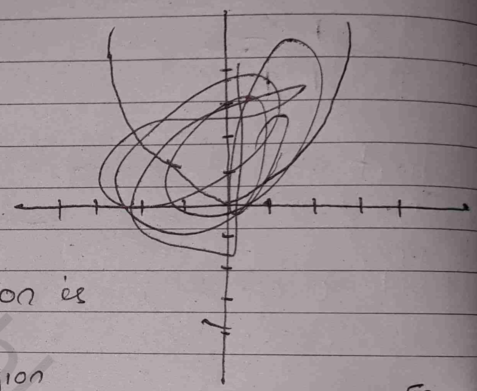
$$= -8\pi = 8\pi \text{ cubic units}$$

4.a) Here, the region is bounded by parabola $x = \sqrt{y}$ and line $y = x$

$x = \sqrt{y}$

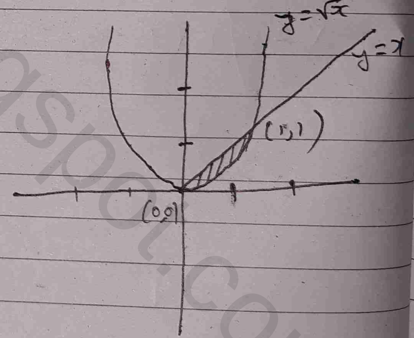
$x^2 = y$

x	1	2	3
y	1	4	9



The limit of integration is $y = 0$ to $y = 1$

now, volume of the region bounded by curve is



$$V = \int_0^1 \pi (x_2 - x)^2 - (x_1 - x)^2 \, dy$$

$$= \pi \int_0^1 (\sqrt{y} - 0)^2 - (y - 0)^2 \, dy$$

$$= \pi \int_0^1 y - y^2 \, dy$$

$$= \pi \left[\frac{y^2}{2} - \frac{y^3}{3} \right]_0^1$$

$$= \pi \left[\frac{1}{2} - \frac{1}{3} - (0) \right]$$

$$= \pi \left[\frac{3-2}{6} \right] = \pi/6 \text{ cubic units}$$

2018 (spring)

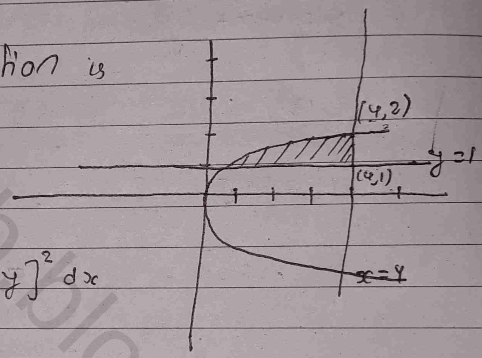
sol^n

Here,

given curve is $y = \sqrt{x}$ and bounded by $y = 1$ and $x = 4$

The limit of integration is

$x = 1$ to $x = 4$



Now

$$V = \int_1^4 \pi [y_2 - y_0]^2 - [y_1 - y]^2 \, dx$$

$$= \pi \int_1^4 [(\sqrt{x} - 1)^2 - (1 - 1)] \, dx$$

$$= \pi \int_1^4 (\sqrt{x} - 1)^2 \, dx$$

$$= \pi \int_1^4 (x - 2\sqrt{x} + 1) \, dx$$

$$= \pi \left[\frac{x^2}{2} - \frac{4}{3} x^{3/2} + x \right]_1^4$$

$$= \pi \left[8 - \frac{32}{3} + 4 - \frac{1}{2} + \frac{4}{3} - 1 \right]$$

$$= \pi \left[11 - \frac{32}{3} - \frac{1}{2} + \frac{4}{3} \right]$$

$$= \pi \left[\frac{66 - 64 - 3 + 8}{6} \right] = \frac{7\pi}{6} \text{ cubic unit.}$$