area surrounding the instrument such as the effects of changes in temperature, humidity, barometric pressure or of magnetic or electrostatic fields.

Elimination:-

(i) By air conditioning, hermetically sealing certain components in the instrument, use of magnetic shields.

Example (1):-

A voltmeter, having a sensitivity of 1,000 Ω/V, reads 100 V on its 150 V scale when connected across an unknown resistor in series with a milliammeter. When the milli-ammeter reads 5 mA, calculate:

(a) the apparent resistance of the unknown resistor.

(b) the actual resistance of the unknown resistor.

(c) the error due to the loading effect of the voltmeter.

Soln:

(a) The total circuit Resistance equals

$$R_T = \frac{V_T}{I_T} = \frac{100V}{5mA} = 20k\Omega$$

Neglecting the resistance of the milli-ammeter, the value of the unknown resistor is $R_x = 20k\Omega$.

(b) The voltmeter resistance equals

$$R_V = 1000\,\Omega/V \times 150V = 150k\Omega$$

Since the voltmeter is in parallel with the unknown resistor, we have,

$$R_T = \frac{R_x R_V}{R_V + R_x} \quad \text{or,} \quad R_T R_V + R_T R_x = R_x R_V$$

or, unknown resistor value $(R_x) = \dfrac{R_T R_V}{R_V - R_T}$

$$= \frac{20 \times 150}{150 - 20} = \frac{300\cancel{0}}{13\cancel{0}} = 23.077\,k\Omega$$

(c) % error = $\frac{\text{actual} - \text{apparent value}}{\text{actual value}} \times 100\%$

$= \frac{23.077 - 20}{23.077} \times 100\% = 13.33\%$

Example (2):-

Repeat example (1) if the milliammeter reads 800 mA and the voltmeter reads 40V on its 150V scale.

Soln:

(a) $R_T = \frac{V_T}{I_T} = \frac{40V}{800 mA} = \frac{40V}{0.8 A} = 50 \Omega$

(b) $R_V = 1,000 V/\Omega \times 150 = 150 k\Omega$

$\therefore R_X = \frac{R_T R_V}{R_V - R_T} = \frac{50 \times 150 \times 10^3}{150 \times 10^3 - 50} = 50.017 \Omega$

(c) % error = $\frac{50.017 - 50}{50.017} \times 100\% = 0.034\%$

(c) Random Errors:-

These errors are due to unknown causes and occur even when all systematic errors have been accounted for. In well-designed experiments, few random errors usually occur but they become important in high accuracy work.

Elimination:-

(1) By increasing the number of readings and using statistical means to obtain the best approximation of the true value of the quantity under measurement.

Statistical Analysis:-

① Arithmetic mean :-

$$\bar{x} = \frac{x_1 + x_2 + x_3 + x_4 + \cdots + x_n}{n} = \frac{\Sigma x}{n}$$

Where, $\bar{x}$ = arithmetic mean

$x_1, x_2, \ldots, x_n$ = readings taken

n = number of readings

② Deviation from the Mean:-

$$d_1 = x_1 - \bar{x}$$
$$d_2 = x_2 - \bar{x}$$
$$\vdots$$
$$d_n = x_n - \bar{x}$$

Where, d_n is the deviations of the nth reading from the mean.

③ Average Deviation from the mean:-

Average deviation from the mean can be expressed as

$$D = \frac{|d_1| + |d_2| + |d_3| + \cdots + |d_n|}{n}$$

$$= \frac{\Sigma |d_i|}{n}$$

④ Standard Deviation :-

The standard deviation or the root-mean-square deviation of a finite number of data is given by

$$S.D. = S = \sqrt{\frac{d_1^2 + d_2^2 + d_3^2 + \cdots + d_n^2}{n-1}} = \sqrt{\frac{\Sigma d_t^2}{n-1}}$$

For infinite number of data,

$$S.D. = \sigma = \sqrt{\frac{\Sigma d_t^2}{n}}$$

Note:- When the number of observations is greater than 20, S.D. is denoted by symbol σ while if the no. of observations is less than 20, the symbol used is S.

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(5) Variance:-

The variance is square of standard Deviation i.e

$V = (S \cdot D)^2$

$= \sigma^2 = \frac{\sum d_i^2}{n}$ for no. of observations greater than 20

$= s^2 = \frac{\sum d_i^2}{n-1}$ for no. of observations less than 20.

Example:-

A circuit was tuned for resonance by eight different students and the values of resonant frequency in KHz were recorded as 532, 548, 543, 535, 546, 531, 543 and 536. Calculate (a) the arithmetic mean, (b) deviations from mean, (c) the average deviation, (d) the standard deviation and (e) variance.

Soln:

(a) The arithmetic mean of the readings is

$\bar{x} = \frac{\sum x}{n} = \frac{532+548+543+535+546+531+543+536}{8}$

$= 539.25$ KHz.

(b) The deviations from mean are

$d_1 = x_1 - \bar{x} = 532 - 539.25 = -7.25$ KHz

$d_2 = x_2 - \bar{x} = 548 - 539.25 = +8.75$ KHz

$d_3 = x_3 - \bar{x} = 543 - 539.25 = +3.75$ KHz

$d_4 = x_4 - \bar{x} = 535 - 539.25 = -4.25$ KHz

$d_5 = x_5 - \bar{x} = 546 - 539.25 = +6.75$ KHz

$d_6 = x_6 - \bar{x} = 531 - 539.25 = -8.25$ KHz

$d_7 = x_7 - \bar{x} = 543 - 539.25 = +3.75$ KHz

$d_8 = x_8 - \bar{x} = 536 - 539.25 = -3.25$ KHz

(c) The average deviation from the mean is

$$D = \frac{\Sigma |d|}{n} = \frac{7.25+8.75+3.75+4.25+6.75+8.25+3.75+3.25}{8}$$

$$= 5.75 \text{ KHz}.$$

(d) the standard deviation is

$$S.D. = S = \sqrt{\frac{\Sigma d_t^2}{n-1}}$$ since the no. of reading is 8 which is less than 20.

$$= \sqrt{\frac{(-7.25)^2+(8.75)^2+(3.75)^2+(-4.25)^2+(6.75)^2+(-8.25)^2+(3.75)^2+(-3.25)^2}{8-1}}$$

$$= 6.54 \text{ KHz}.$$

(e) variance $(v) = s^2 = 42.77 \ (\text{KHz})^2$.

Probability of Errors :-

(1) Normal Distribution of Errors:-

Table:-Tabulation of voltage Readings

Voltage Reading (Volts)	Number of Readings	
99.7	1	$P(99.7) = 1/50$
99.8	4	
99.9	12	
100.0	19	most probable value of true voltage.
100.1	10	
100.2	3	
100.3	1	
	50	

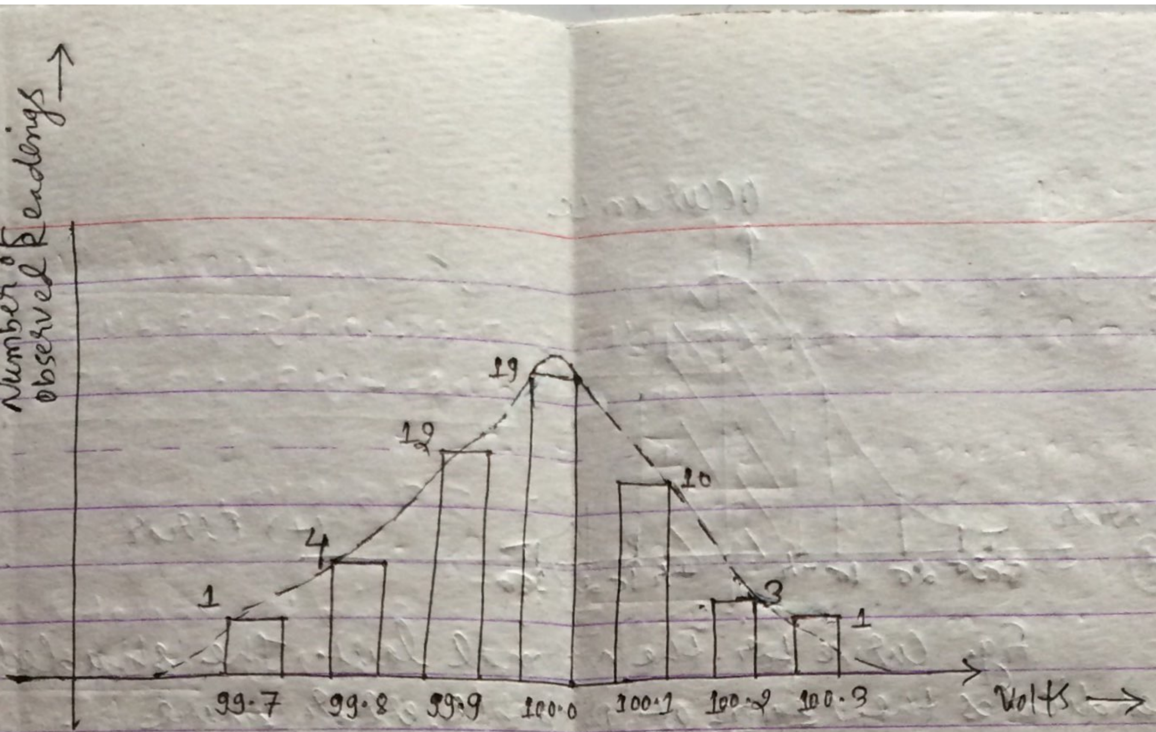

Fig. Histogram showing the frequency of occurance of the 50 voltage readings. The dashed curve represents the limiting case of the histogram when a large no. of reading as small increments are taken.

The Gaussian or Normal law of errors forms the basis of the analytical study of random effects. The following qualitative statements are based on the normal law:

ⓐ All observations include small disturbing effects, called random errors.

ⓑ Random errors can be positive or negative.

ⓒ There is an equal probability of positive and negative random errors.

The possibilities as to form of the error distribution curve can be stated as:

ⓐ Small errors are more probable than large errors.

ⓑ Large errors are very improbable.

ⓒ There is an equal probability of plus and minus errors so that the probability of a given error will be symmetrical about the zero value.

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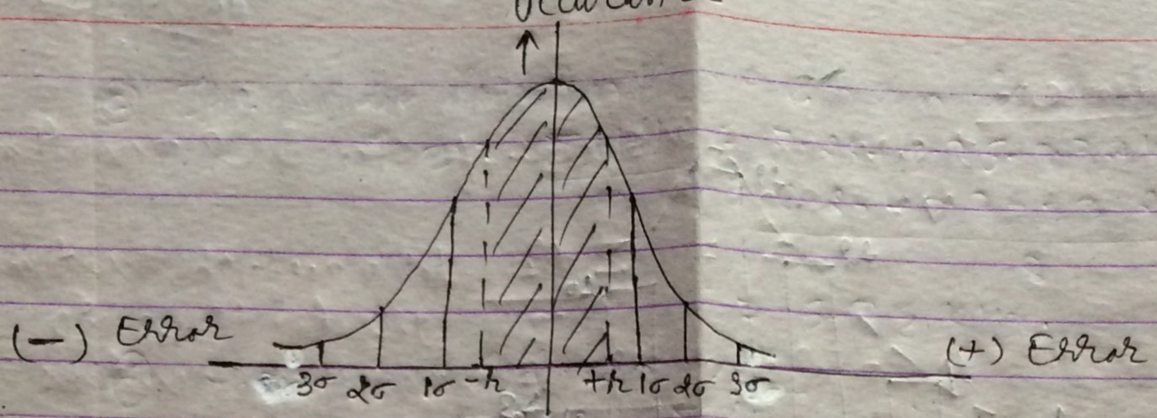

Fig. Curve for the Normal law. The shaded portion indicates the region of probable error where
$r = \pm 0.6745\sigma$.

② Probable error:-

Probable error, $r = \pm 0.6745\sigma$

Example:- ①

The following 10 observations were recorded when measuring a voltage: 41.7, 42.0, 41.8, 42.0, 42.1, 41.9, 42.0, 41.9, 42.5 and 41.8 volt. Find (i) the mean (ii) the standard deviation, (iii) the probable error of one reading, (iv) the probable error of mean and (v) the range.

Soln:

Measured voltage (x)	Deviation (d)	d^2
41.7	−0.27	0.0729
42.0	+0.03	0.0009
41.8	−0.17	0.0289
42.0	+0.03	0.0009
42.1	+0.13	0.0169
41.9	−0.07	0.0049
42.0	+0.03	0.0009
41.9	−0.07	0.0049
42.5	+0.53	0.2809
41.8	−0.17	0.0289
$\Sigma x = 419.7$		$\Sigma d^2 = 0.441$

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(i) mean $(\bar{x}) = \frac{\Sigma x}{n} = \frac{419.7}{10} = 41.97$ volt.

(ii) standard Deviation is

$$S.D. = S = \sqrt{\frac{\Sigma d_t^2}{n-1}}$$ since the no. of observation is less than 20

$$= \sqrt{\frac{0.441}{10-1}} = 0.22 \text{ Volt.}$$

(iii) Probable error of one reading is

$$r_1 = 0.6745 S$$
$$= 0.6745 \times 0.22 = 0.15 \text{ Volt.}$$

(iv) Probable error of mean is

$$r_m = \frac{r_1}{\sqrt{n-1}} = \frac{0.15}{\sqrt{9}} = 0.05 \text{ Volt.}$$

(v) Range $= 42.5 - 41.7 = 0.8$ Volt.

Note:-

(i) if $n < 20$,

$$r_m = \frac{r_1}{\sqrt{n-1}}$$ and

if $n > 20$, $r_m = \frac{r_1}{\sqrt{n}}$

(ii) standard deviation of standard deviation is

$$\sigma_\sigma = \frac{\sigma}{\sqrt{2n}} = \frac{\sigma_m}{\sqrt{2}}$$

(iii) standard deviation of mean is

$$\sigma_m = \frac{\sigma}{\sqrt{n}}$$

Where, n is the number of observations.

Limiting Errors:-

In most indicating instruments, the accuracy is guaranteed to a certain percentage of full-scale reading. Circuit components such as capacitors, resistors etc. are guranteed within a certain percentage of their rated value. The limits of these deviations from the specified value are known as limiting errors or guarantee errors. For example, if the resistance of a resistor is given as 500Ω ± 10%, the manufacture guarantees that the resistance falls between the limits 450Ω and 550Ω. The maker is not specifying a standard deviations or a probable error but promises that the error is no greater than the limits set.

Example 1:-

A 0–150V voltmeter has a guaranteed accuracy of 1% full scale reading. The voltage measured by this instrument is 83V. Calculate the limiting error in percentage.

Soln:

The magnitude of the limiting error is

$$1\% \text{ of } 150\ V = \frac{150}{100} = 1.5\ V$$

∴ % error at a meter indication of 83V is

$$\frac{1.5}{83} \times 100\% = 1.81\%$$

Example 2:-

The voltage generated by a circuit is equally dependent on the value of three resistors and is given by the following eqn:

$$V_{out} = \frac{R_1 R_2}{R_3}$$

If the tolerance of each resistor is 0.1%, what is the maximum error of the generated voltage?

Soln:

The highest resulting voltage occurs when R_1 and R_2 are at the maximum value allowed by the tolerance while R_3 is at the lowest value allowed by the tolerance. For a variation of 0.1% the highest value of resistor is 1.001 times the normal value while the lowest value is 0.999 times the normal value.

The resulting voltage is

$$V_{out} = \frac{(1.001 R_1)(1.001 R_2)}{0.999 R_3} = 1.003 \frac{R_1 R_2}{R_3}$$

The lowest resulting voltage occurs when the value of R_3 is highest, and R_1 and R_2 are the lowest value.

$$V_{out} = \frac{(0.999 R_1)(0.999 R_2)}{1.001 R_3} = 0.997 \frac{R_1 R_2}{R_3}$$

Hence the total variation of the resulting voltage is ± 0.3%. which is the algebric sum of the individual tolerances.

$R = 100 \pm 0.2\Omega$

$= 100 \pm 0.2\%$

$I = 2 \pm 0.01 A$

$= 2 \pm 0.5\%$

$P = I^2 R$

Example 3:-

The current passing through a resistor of $100 \pm 0.2 \Omega$ is 2.00 ± 0.01 A. Using the relationship $P = I^2 R$, calculate the limiting error in the computed value of power dissipation.

Soln:

Expressing the guaranteed limits of both current and resistance in percentages instead of units, we obtain

$I = 2.00 \pm 0.01 A = 2.00 \pm 0.5\%$

$R = 100 \pm 0.2 \Omega = 100 \pm 0.2\%$

$P = I^2 R$

For highest power dissipation, the highest value of resistance and current is used. i.e.

$$P = (2.01)^2 \times 100.2 = 404.82 W$$

since the power dissipation without tolerance is $P = I^2 R = 2^2 \times 100 = 400 W$, % limiting error of power dissipation is

$$\frac{400 - 404.82}{400} \times 100\% = -\frac{4.82}{400} \times 100\% = -1.205\% \simeq -1.2\%$$

For the lowest power dissipation, lowest value of resistance and current is used i.e $P = (1.99)^2 \times 99.8$

$= 395.22 \, W$

Here, % limiting error of power dissipation is

$$\frac{400 - 395.22}{400} \times 100\%$$

$$= \frac{4.78}{400} \times 100\% = 1.195 \simeq 1.2\%$$

Hence, the error is 1.2% which is two times the 0.5% error of the current plus 0.2% error of the resistor.

Example 4:-

The resistance of an unknown resistor is determined by the Wheatstone bridge method. The solution for the unknown resistance is stated as $R_x = R_1 R_2 / R_3$, where

$R_1 = 500\Omega \pm 1\%$

$R_2 = 615\Omega \pm 1\%$

$R_3 = 100\Omega \pm 0.5\%$

Calculate (a) the nominal value of the unknown resistor

(b) the limiting error in Ω of the unknown resistor and

(c) the limiting error in percentage of the unknown resistor.

Soln:

(a) The nominal value of the unknown resistor is

$$R_x = \frac{R_1 R_2}{R_3} = \frac{500 \times 615}{100} = 3075\,\Omega$$

(b) The limiting error in ohms of the unknown resistor is for highest value of unknown resistor R_x, the value of R_1 and R_2 must be high and the value of R_3 should be low i.e.

$$R_x = \frac{(500+5)(615+6.15)}{(100-0.5)} = \frac{(505)(621.15)}{99.5} = 3152.57\,\Omega$$

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Now, the limiting error is Nominal value minus highest value of unknown resistor i.e.

$$\text{limiting error} = (3075 - 3152.57)\,\Omega$$
$$= -77.57\,\Omega$$

Hence, the limiting error of unknown resistor is $\pm 77.57\,\Omega$

(c) % limiting error of unknown resistor is

$$\frac{\text{limiting error}}{\text{Nominal value}} \times 100\,\%$$
$$= \pm \frac{77.57}{3075} \times 100\,\%$$
$$= \pm 2.52\,\%$$

Performance Parameters:-

It is divided into two categories:

1. Static characteristics
2. Dynamic characteristics

Static characterestics:-

The static characteristics of a measurement system are those that must be considered when the system or instrument is used to measure a condition not varying with time. The following are the static characteristics:

(a) Accuracy:-

It is the closeness with which an instrument reading approaches the true value of the variable being measured.

(b) Sensitivity:-

It is the ratio of output signal or response of the instrument to a change of input or measured variable.