Hay Bridge :-

The Hay bridge is suited for the measurement of high-Q inductors especially for those inductors having a Q greater than 10.

The general equation for ac bridge balance is

$$Z_1 Z_x = Z_2 Z_3$$

where,

$$Z_1 = R_1 - \frac{j}{\omega C_1}; \quad Z_2 = R_2;$$

$$Z_3 = R_3; \quad Z_x = R_x + j\omega L_x$$

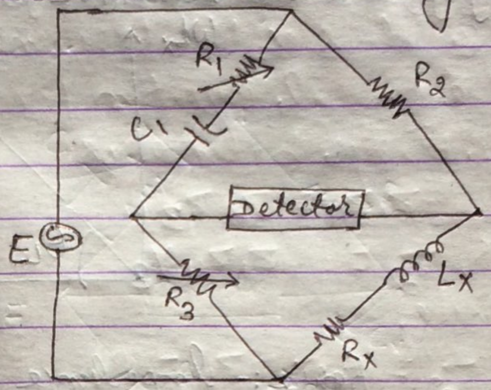

Fig. Hay Bridge for inductance measurements.

so, $\left(R_1 - \frac{j}{\omega C_1}\right)\left(R_x + j\omega L_x\right) = R_2 R_3$

or, $R_1 R_x + \frac{L_x}{C_1} - \frac{jR_x}{\omega C_1} + j\omega L_x R_1 = R_2 R_3$

Separating the real and imaginary terms,

$$R_1 R_x + \frac{L_x}{C_1} = R_2 R_3 \quad \text{—— (I)}$$

$$-\frac{R_x}{\omega C_1} + \omega L_x R_1 = 0$$

$$\text{or, } \frac{R_x}{\omega C_1} = \omega L_x R_1 \quad \text{—— (II)}$$

solving eqn (I) and (II), we get

$$R_x = \frac{\omega^2 C_1^2 R_1 R_2 R_3}{1 + \omega^2 C_1^2 R_1^2}$$

and,

$$L_x = \frac{R_2 R_3 C_1}{1 + \omega^2 C_1^2 R_1^2}$$

Also, the sum of the opposite sets of phase angles must be equal i.e. the inductive phase angle must be equal to the capacitive phase angle since the resistive angles are zero.

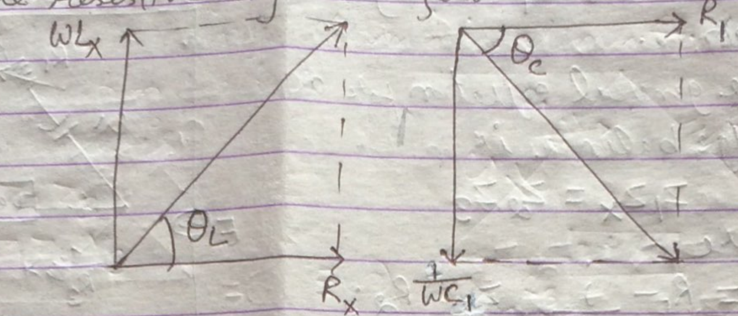

Figure :- Impedence triangles illustrate inductive and capacitive phase angles.

Now, the tangent of the inductive phase angle equals

$$\tan\theta_L = \frac{X_L}{R} = \frac{WL_X}{R_X} = Q = \text{Quality of coil.}$$

and that of capacitive is

$$\tan\theta_C = \frac{X_C}{R} = \frac{1}{WC_1R_1}$$

When two phase angles are equal, their tangents are also equal,

$$\tan\theta_L = \tan\theta_C$$

$$\text{or, } Q = \frac{1}{WC_1R_1}$$

$$\text{and hence, } L_X = \frac{R_2R_3C_1}{1+(1/Q)^2}$$

For a value of Q greater than ten, the term $(1/Q)^2$ can be neglected

$$\text{So, } L_X = R_2R_3C_1$$

For a value of Q smaller than ten, the term $(1/Q)^2$ becomes important and cannot be neglected. So, the Maxwell bridge is more suitable in this case.

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Capacitance Bridge:-

Schering Bridge:-

It is one of the most important ac bridges extensively used for the measurement of capacitors.

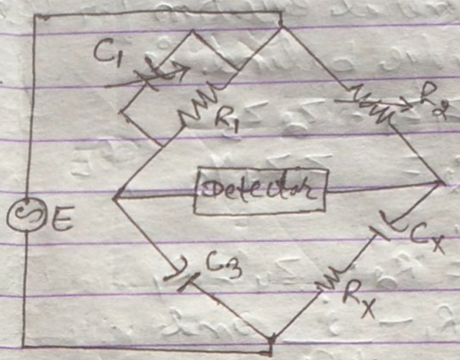

Fig. Schering Bridge for the measurement of capacitance.

The general equation for ac bridge balance is

$$Z_1 Z_x = Z_2 Z_3$$

$$\Rightarrow Z_x = Z_2 Z_3 Y_1$$

where,

$Z_2 = R_2$; $Z_3 = -j/wC_3$

$Y_1 = 1/R_1 + jwC_1$ and

$$Z_x = R_x - \frac{j}{wC_x}$$

or, $$Z_x = R_x - \frac{j}{wC_x} = R_2\left(\frac{-j}{wC_3}\right)\left(1/R_1 + jwC_1\right)$$

$$= \frac{R_2 C_1}{C_3} - \frac{jR_2}{wC_3R_1}$$

Equating the real and imaginary terms,

$$R_x = \frac{R_2C_1}{C_3} = R_2\frac{C_1}{C_3} \text{ and}$$

$$C_x = C_3\frac{R_1}{R_2}$$

Wien Bridge:-

It is an ac bridge used to measure the frequency. A wien Bridge is used as a notch filter in the harmonic distortion analyzer. The wien bridge also finds application in audio and HF oscillators as the frequency determining element.

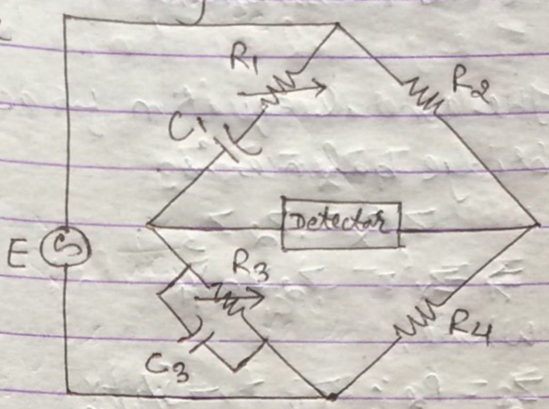

fig. Frequency measurement with the wien Bridge.

The general equation for the ac bridge balance is

$$Z_1 Z_4 = Z_2 Z_3$$

or, $Z_2 = Z_1 Z_4 Y_3$

Where,

$Z_2 = R_2$; $Z_4 = R_4$

$Z_1 = R_1 - \frac{j}{wC_1}$ and

$Y_3 = 1/R_3 + jwC_3$

Now, $Z_2 = R_2 = \left(R_1 - \frac{j}{wC_1}\right)(R_4)\left(\frac{1}{R_3} + jwC_3\right)$

$$= \frac{R_1 R_4}{R_3} + \frac{R_4 C_3}{C_1} + jwC_3 R_1 R_4 - \frac{jR_4}{wC_1 R_3}$$

Equating Real and Imaginary terms,

$$R_2 = \frac{R_1 R_4}{R_3} + \frac{R_4 C_3}{C_1}$$

Which reduces to

$$\frac{R_2}{R_4} = \frac{R_1}{R_3} + \frac{C_3}{C_1} \quad \text{—— (1)}$$

and $wC_3 R_1 R_4 - \frac{R_4}{wC_1 R_3} = 0$

or, $wC_3 R_1 R_4 = \frac{R_4}{wC_1 R_3}$

or, $w^2 = \frac{1}{C_1 C_3 R_1 R_3}$ where, $w = 2\pi f$ and solving

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for f, we get,

$$\omega = \frac{1}{\sqrt{C_1C_3R_1R_3}}$$

$$or,\ f = \frac{1}{2\pi\sqrt{C_1C_3R_1R_3}} \quad \text{--- ②}$$

In most wien Bridge circuits, the components are chosen such that $R_1 = R_3$ and $C_1 = C_3$. so, eqn ① and ② becomes

$$\frac{R_2}{R_4} = 2 \quad \text{and}$$

$$f = \frac{1}{2\pi RC}$$ is the general expression for the frequency of the wien bridge.

Example:- ① The impedances of the basic ac bridge are given as follows:

$Z_1 = 100\Omega \angle 80^\circ$ (inductive impedance)

$Z_2 = 250\Omega$ (pure resistance)

$Z_3 = 400\Omega \angle 30^\circ$ (inductive impedance)

Z_4 = unknown.

Determine the constents of unknown arm.

Soln:

The general equation for ac bridge balance is

$$Z_1 Z_4 = Z_2 Z_3$$

$$or,\ |Z_1|\, Z_4 \angle \theta_1 = |Z_2||Z_3| \angle \theta_2 + \theta_3$$

$$\therefore Z_4 = \frac{|Z_2||Z_3|}{|Z_1|} \frac{\angle \theta_2 + \theta_3}{\angle \theta_1} = \frac{|Z_2||Z_3|}{|Z_1|} \angle \theta_2 + \theta_3 - \theta_1$$

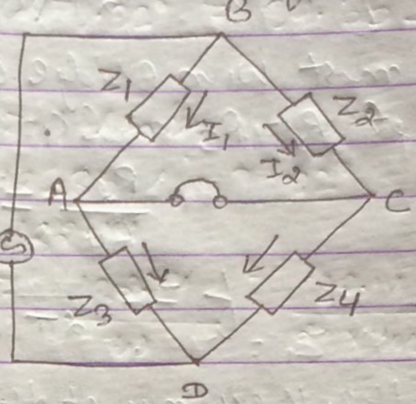
$$= \frac{250 \times 400}{100} \angle (0^\circ + 30^\circ - 80^\circ)$$

$= 1{,}000\Omega \angle -50^\circ$ indicates that the element is capacitor.

Example:-

② The ac bridge is in balance with the following constants: arm AB, $R = 450\,\Omega$; arm BC, $R = 300\,\Omega$ in series with $C = 0.265\,\mu F$; arm CD, unknown; arm DA, $R = 200\,\Omega$ in series with $L = 15.9\,mH$. The oscillator frequency is 1 KHz. Find the constants of arm CD.

Soln:



The general equation for ac bridge balance is

$$Z_1 Z_4 = Z_2 Z_3$$

Where,

$Z_1 = R = 450\,\Omega$

$Z_2 = R - j/\omega C = (300 - j600)\,\Omega$

$Z_3 = R + j\omega L = (200 + j100)\,\Omega$

Z_4 = unknown

Now, $$Z_4 = \frac{Z_2 Z_3}{Z_1} = \frac{(300 - j600)(200 + j100)}{450}$$

$$= \frac{60000 + j30000 - j120000 + 60000}{450}$$

$$= \frac{120000 - j90000}{450} = (266.67 - j200)\,\Omega$$
$$= (R_4 - j/\omega C_4)\,\Omega$$

This result indicates that Z_4 has a resistor $266.67\,\Omega$ in series with capacitor at a frequency of 1 KHz. Since capacitive reactance $X_{C_4} = \frac{1}{2\pi f C_4}$

or, $\frac{1}{2\pi f C_4} = 200$ or, $C_4 = \frac{1}{2\pi \times 1000 \times 200}$

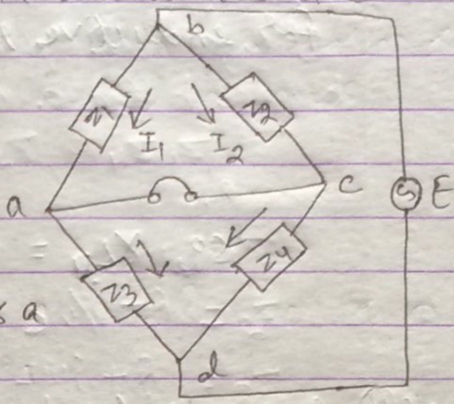
$$= \frac{1}{1256637.06}$$

$$= 0.7958\,\mu F.$$

and $R_4 = 266.67\,\Omega$.

Example ③ An a.c. bridge circuit working at 1000 Hz is shown. Arm ab is a 0.2 μF pure capacitor; arm bc is a 500 Ω pure resistance; arm cd contains an unknown impedance and arm da has a 300 Ω resistance in parallel with a 0.1 μF capacitor. Find the R and C or L contains of arm cd considering it as a series ckt.

Soln:



Impedance of arm ab is

$$Z_1 = \frac{1}{2\pi f C_1} = \frac{1}{2\pi \times 1000 \times 0.2 \times 10^{-6}}$$

$$= 795.77\,\Omega$$

$\therefore Z_1 = 795.77\,\Omega \angle -90^\circ$ since it is a pure capacitance.

Impedance of arm bc is

$$Z_2 = 500\,\Omega$$

$\therefore Z_2 = 500\,\Omega \angle 0^\circ$ since it is a pure resistance.

Impedance of arm cd is

$$Z_4 = \text{unknown}$$

and impedance of arm da is

$$Z_3 = R_3 \,||\, X_{C_3}$$

or, $Y_3 = 1/R_3 + j\omega C_3 = \frac{1}{300} + j2\pi \times 1000 \times 0.1 \times 10^{-6}$

$$= 3.33 \times 10^{-3} + j\,6.28 \times 10^{-4}$$

$$= 3.39 \times 10^{-3} \angle 10.68^\circ$$

$\therefore Z_3 = 1/Y_3 = 294.99 \angle -10.68^\circ$

Now, for ac bridge balance,

$$Z_1 Z_4 = Z_2 Z_3 \Rightarrow Z_4 = \frac{Z_2 Z_3}{Z_1} = \frac{500\angle 0^\circ \times 294.99 \angle -10.68^\circ}{795.77 \angle -90^\circ}$$

$$= 185.35 \angle 0^\circ - 10.68^\circ + 90^\circ$$

$$\therefore Z_4 = 185.35 \angle 79.32^\circ$$

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The positive angle for impedence indicates that the branch consists of a series R-L circuit.

For resistance,

$$R_4 = |Z_4| \cos\theta_4$$
$$= 185.35 \cos 79.32^\circ = 34.35\,\Omega$$

For inductive reactance,

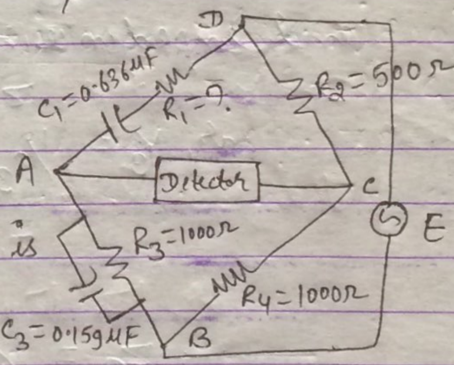
$$X_{L_4} = |Z_4| \sin\theta_4$$
$$= 185.35 \sin 79.32^\circ$$
$$= 182.14$$

So, $X_{L_4} = \omega L_4 \Rightarrow L_4 = \frac{182.14}{\omega} = \frac{182.14}{2\pi \times 1000} = 28.99\,mH \simeq 29\,mH$

<u>Example ④</u> An a.c. bridge has the following constants: arm AB, $R = 1000\,\Omega$ in parallel with $C = 0.159\,\mu F$; BC, $R = 1,000\,\Omega$; CD, $R = 500\,\Omega$; DA, $C = 0.636\,\mu F$ in series with an unknown resistance. Find the frequency for which this bridge is in balance and determine the value of the resistance in arm DA to produce this balance.

Soln:

If we draw the sketch of the given a.c. bridge, we find it is Wien's Bridge.



So, the frequency for Wien Bridge is

$$f = \frac{1}{2\pi\sqrt{C_1 C_3 R_1 R_3}} \quad \text{—— ①}$$

and

$$\frac{R_2}{R_4} = \frac{R_1}{R_3} + \frac{C_3}{C_1} \Rightarrow \frac{500}{1000} = \frac{R_1}{1000} + \frac{0.159 \times 10^{-6}}{0.636 \times 10^{-6}}$$

$$\text{or, } 0.5 = \frac{R_1}{1000} + 0.25 \Rightarrow R_1 = 0.25 \times 1000 = 250\,\Omega$$

and $$f = \frac{1}{2\pi\sqrt{0.636 \times 10^{-6} \times 0.159 \times 10^{-6} \times 250 \times 1000}} = 1000.97\,Hz.$$