5.7 r_e model

r_e model for CB, CE and CC configuration are shown in Figure 5.13, 5.14 and 5.15 respectively.

i) Common base

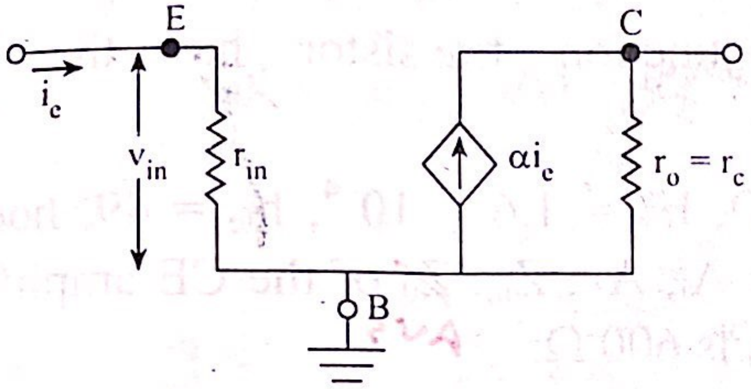


Fig. 5.13. r_e-model for CB configuration

$$\text{Here, } r_{in} = \frac{V_{in}}{i_e} \cong \frac{26\,mV}{I_E} = r_e.$$

ii) Common emitter

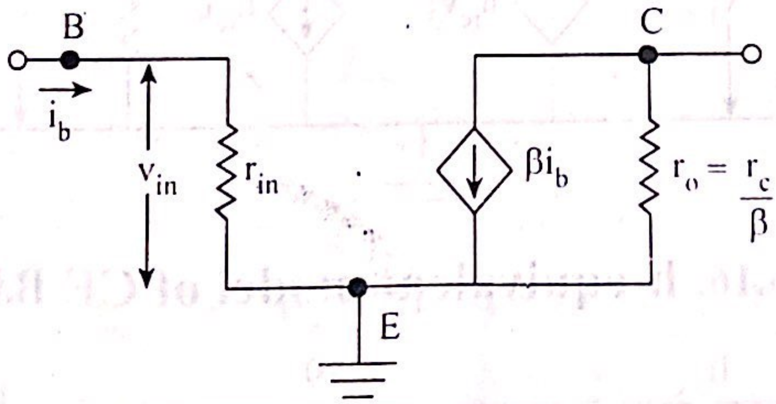


Fig. 5.14. r_e-model for CE configuration

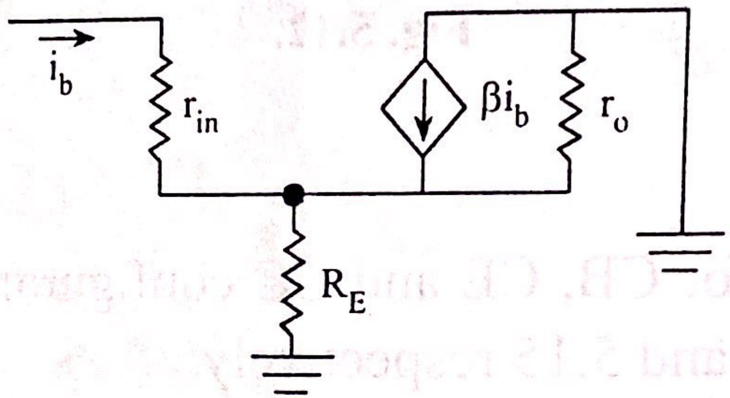
Here, $r_{in} = \frac{V_{in}}{i_b} = \frac{V_{in}}{\frac{i_e}{(\beta+1)}}$

or $r_{in} = (\beta + 1)\, r_e$

or $r_{in} \simeq \beta r_e.$

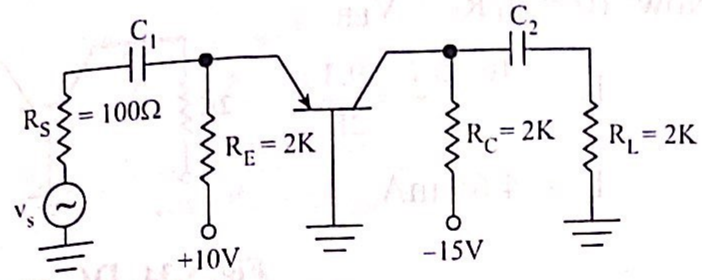
iii) Common collector

Here $r_{in} = \beta r_e$



Example 5.7. For the given common-base configuration transistor shown in the Figure, find r_e, r_{in}, $r_{instage}$, r_0, r_{0stage}, A_v, A_{vs}, A_I using r_e-model.

Given $\beta = 100$, $r_0 = 200\ k\Omega$.



Solution:

The ac equivalent circuit is shown in Figure 5.32. For this, all capacitors are shorted and all DC voltage sources are grounded.

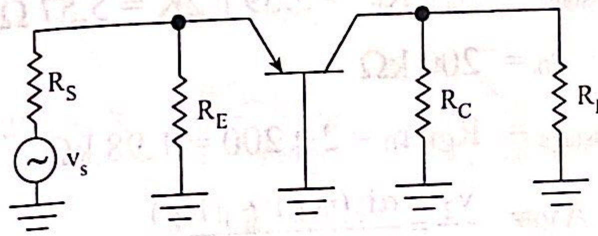


Fig. 5.32. The ac equivalent for the CB transistor

The transistor is replaced by its equivalent r_e-model in Figure 5.33.

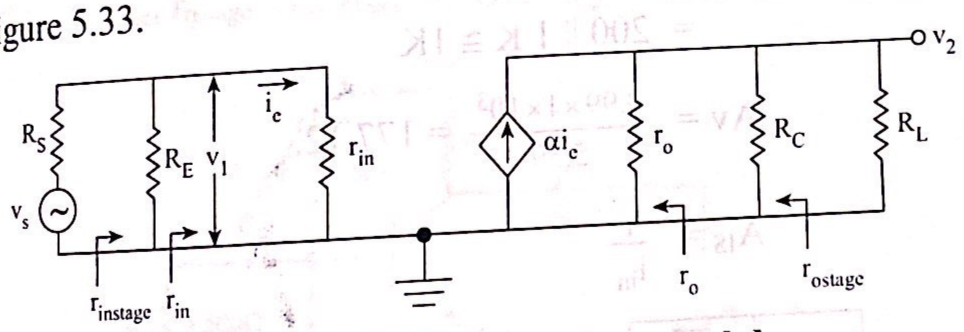


Fig. 5.33. The r_e-equivalent model

$$\text{Here, } r_{in} = \frac{V_1}{I_E} = \frac{26\text{mV}}{I_E}$$

Where I_E is DC emitter current. The DC equivalent circuit is shown in Figure 5.34.

Now, $10 = I_E R_E + V_{EB}$

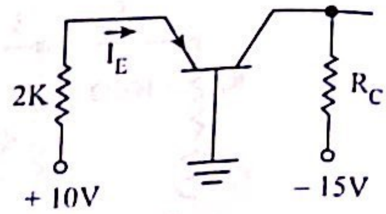


Fig. 5.34. **DC equivalent circuit**

$\therefore$ $$I_E = \frac{10-0.7}{2K} = \frac{9.3}{2K}$$

or $$I_E = 4.65 \text{ mA}$$

$\therefore$ $$r_{in} = \frac{26\text{mV}}{I_E} = \frac{26\times10^{-3}}{4.65\times10^{-3}}.$$

Thus, $r_e = r_{in} = 5.59\ \Omega.$

i) $$r_{instage} = r_{in} \,||\, R_E = 5.59 \,||\, 2K = 5.57\ \Omega.$$

ii) $$r_0 = 200\ \text{k}\Omega.$$

iii) $$r_{0stage} = R_C || r_0 = 2 \,||\, 200 = 1.98\ \text{k}\Omega.$$

iv) $$Av = \frac{v_2}{v_1} = \frac{\alpha i_e (r_0 \,||\, R_C \,||\, R_L)}{i_e r_e}$$

$$= \frac{\alpha r_L}{r_e}, \text{ where } r_L = r_0 \,||\, R_C \,||\, R_L$$

$$= 200 \,||\, 1\ K \cong 1K$$

$\therefore$ $$Av = \frac{0.99\times1\times10^3}{5.59} = 177.12.$$

v) $$A_{IS} = \frac{i_L}{i_{in}}$$

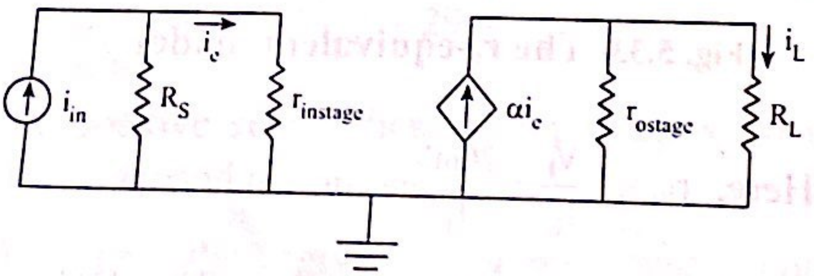


Fig. 5.35. The r_e-model for the CB configuration

From r_e-model shown in Figure 5.35, we can write

$$A_{IS} = \frac{i_L}{i_e} \times \frac{i_e}{i_{in}}$$

or

$$\text{Now, } i_L = \frac{\alpha i_e \times r_{ostage}}{(R_L + r_{ostage})}$$

$$\frac{i_L}{i_e} = \frac{\alpha r_{ostage}}{(R_L + r_{ostage})}$$

$$\text{Also, } i_e \cong \frac{i_{in} \times R_S}{(R_S + r_{instage})}$$

or

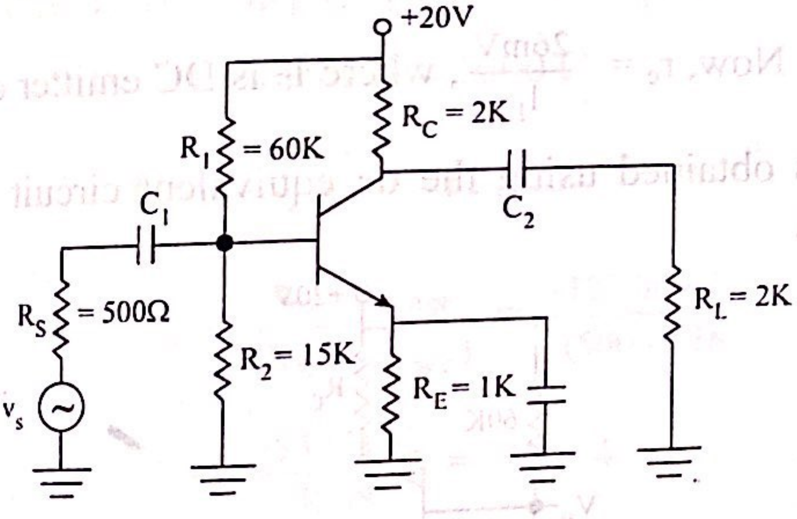
$$\frac{i_e}{i_{in}} = \frac{R_S}{(R_S + r_{instage})}$$

$$\text{Thus } A_{IS} \cong \frac{\alpha\, r_{ostage}}{(R_L + r_{ostage})} \times \frac{R_S}{(R_S + r_{instage})}$$

$$= \frac{1.98 \times 10^3}{2 \times 10^3 + 1.98 \times 10^3} \times \frac{100}{(100 + 5.57)} = 0.471.$$

vc (CE bypass)

Example 5.8. For the given voltage-divider (or self-bias) circuit, find r_{in}, $r_{instage}$, r_{ostage}, A_v, A_{vs}, A_I. Given, $\beta = 100$, $r_0 = 200\ k\Omega$.



Solution:

The ac equivalent circuit is shown in Figure 5.36. To draw the ac equivalent circuit, all capacitors are shorted and all DC voltage sources are grounded.

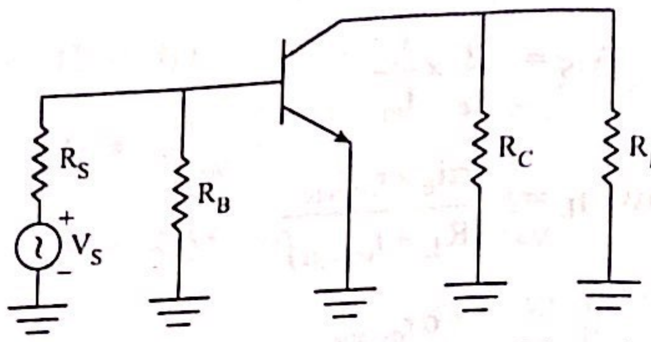


Fig. 5.36. The ac equivalent circuit

The re-model for the given voltage-divider circuit is shown in Figure 5.37.

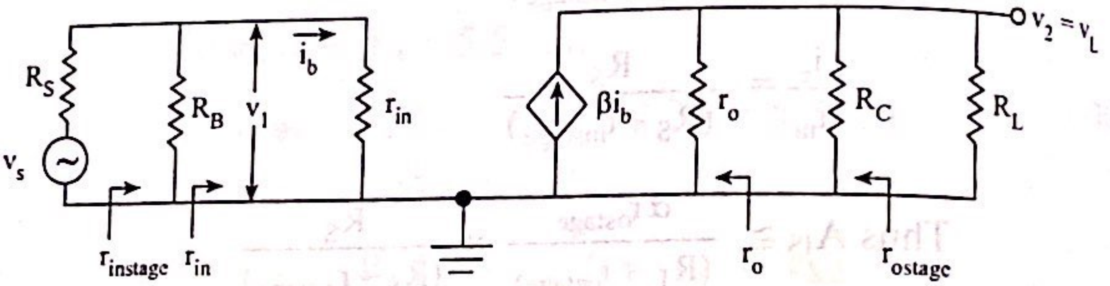


Fig. 5.37. The r_e-equivalent model for the CE transistor

Now,

a) $r_{in} = \dfrac{V_1}{I_b} = \dfrac{V_1}{\dfrac{I_E}{(\beta+1)}} = (\beta+1)\dfrac{V_1}{I_E} = \beta r_e$

Now, $r_e = \dfrac{26mV}{I_E}$, where I_E is DC emitter current.

I_E is obtained using the dc equivalent circuit shown in Figure 5.38.

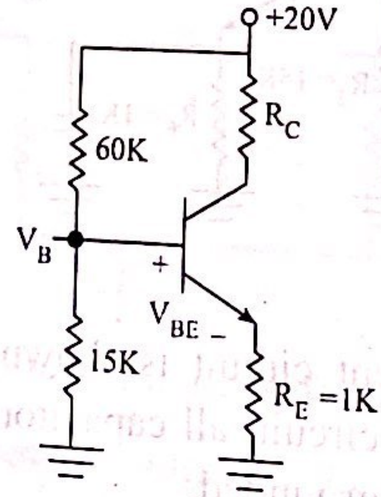


Fig. 5.38. The dc equivalent circuit

Here, $V_B = V_{BE} + I_E R_E$

But, $V_B = \dfrac{20\times15}{(60+15)} = \dfrac{20\times15}{75} = 4V.$

Then, $I_E = \dfrac{4-0.7}{1K} = 3.3$ mA.

Thus, $r_e = \dfrac{26}{3.3} = 7.87\ \Omega$

Hence, $r_{in} = \beta r_e = (7.87\ \Omega \times 100) = 787\ \Omega.$

And $R_B = R_1 \| R_2 = \dfrac{R_1 \times R_2}{R_1 + R_2}$

or $R_B = \dfrac{60\times15}{75} = 12$ kΩ.

b) $r_{instage} = R_B \| r_e = 12\times10^3 \| 787\ \Omega = 738\ \Omega.$

c) $r_{Ostage} = r_0 \| R_C \simeq R_C = 2$ kΩ.

d) $A_v = \dfrac{v_2}{v_1} = \dfrac{-\beta i_b (r_L)}{i_b \beta r_e}$

Where, $r_L = r_0 \| R_C \| R_L = 200K \| 2K \| 2K$

$= 0.99\ k\Omega \simeq 1000\ \Omega.$

$\therefore$ $Av \cong -\dfrac{1000}{7.87} = -127.06.$

e) $A_{vs} = \dfrac{A_v \times r_{instage}}{(R_S + r_{instage})} = \dfrac{-127.06\times738}{(500+738)}$

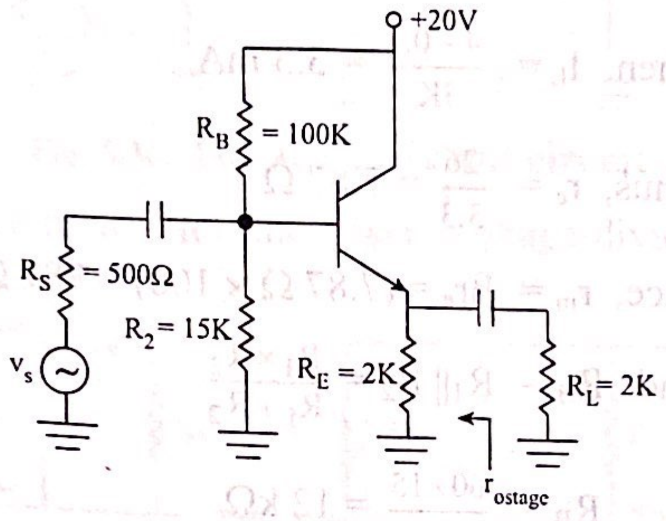
$= \dfrac{-93773.82}{1238} = -75.74.$

Here, the negative sign indicates that there is a 180° phase shift between the input and output wave form.

f) $A_I \cong \beta = 100.$

re (CC) or emitter follower

Example 5.9. For the given common-collector transistor amplifier shown in the figure, find A_v, A_I, r_{in}, $r_{instage}$, r_{Ostage}. Given, $r_e = 20\ \Omega$, $r_0 = 200\ k\Omega$, $\beta = 100$.



Solution:

The equivalent r_e-model is shown in Figure 5.39.

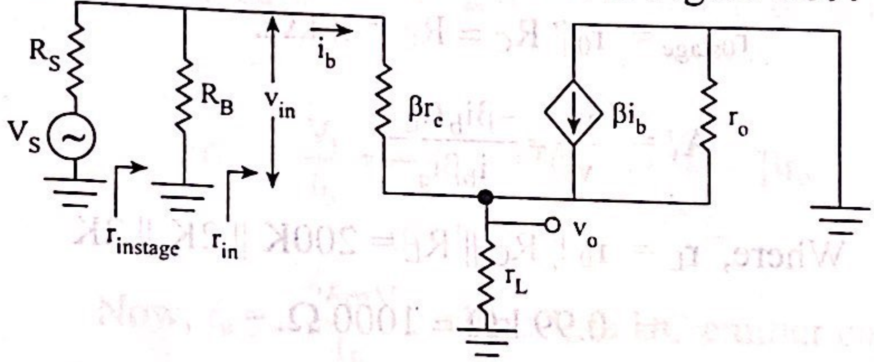


Fig. 5.39. The r_e-equivalent model for the emitter follower

i)
$$r_{in} = \frac{V_{in}}{i_b} = \frac{i_b \beta r_e + (i_b + \beta i_b) r_L}{i_b}$$

$$= \beta r_e + (\beta + 1)\, r_L.$$

Where, $r_L = R_E \parallel R_L = 2K \parallel 2K = 1\ k\Omega$.

or
$$r_{in} \simeq \beta(r_e + r_L) = 100\,(20 + 1000) = 102\ k\Omega.$$

Thus, the input resistance of the CC transistor is very high.

ii)
$$r_{instage} = R_B \parallel r_{in}$$

$$\text{Where } R_B = 100\text{ K} \parallel 15 = 13.04\text{ k}\Omega$$

$$r_{instage} = 13.04 \parallel 102 = 11.56\text{ k}\Omega.$$

$\therefore$

iii) $$A_v = \frac{V_0}{V_{in}} = \frac{(i_b + \beta i_b)r_L}{i_b \beta r_e + (\beta+1)i_b r_L}$$

$$\cong \frac{(\beta+1)r_L}{\beta r_e + (\beta+1)r_L}$$

$$= \frac{101 \times 1 \times 10^3}{100 \times 20 + 101 \times 1 \times 10^3}$$

$$= 0.98 \simeq 1.$$

Since the voltage gain is approximately equal to unity and with a positive sign, the common-collector transistor is also called the emitter follower.

iv) $$A_{vs} = A_v \times \frac{r_{instage}}{(R_S + r_{instage})} = \frac{1 \times 11.56}{(0.5 + 11.56)} = 0.95.$$

v) $$r_{ostage} = R_E \parallel \left(\frac{\beta r_e + R_S \parallel R_B}{\beta}\right)$$

or $$r_{ostage} = R_E \parallel \left(r_e + \frac{R_S \parallel R_B}{\beta}\right)$$

$$\text{Now, } R_S \parallel R_B = 500 \parallel 13.04 \times 10^3$$

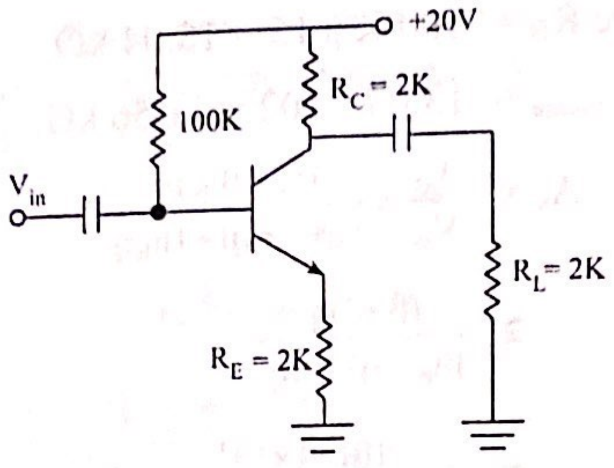
$$= 481\ \Omega$$

$\therefore$ $$r_{ostage} = 2 \times 10^3 \parallel \left(20 + \frac{481}{100}\right)$$

$$= 24.80\ \Omega \simeq 25\ \Omega.$$

The output impedance of CC configuration is low.

Example 5.10. Find A_I, A_V, $r_{instage}$ and r_{ostage} for the given the CE transistor amplifier shown in the Figure. Assume $r_e = 20\ \Omega$ and $\beta = 100$.



Solution:

The equivalent r_e-model is shown in Figure 5.40.

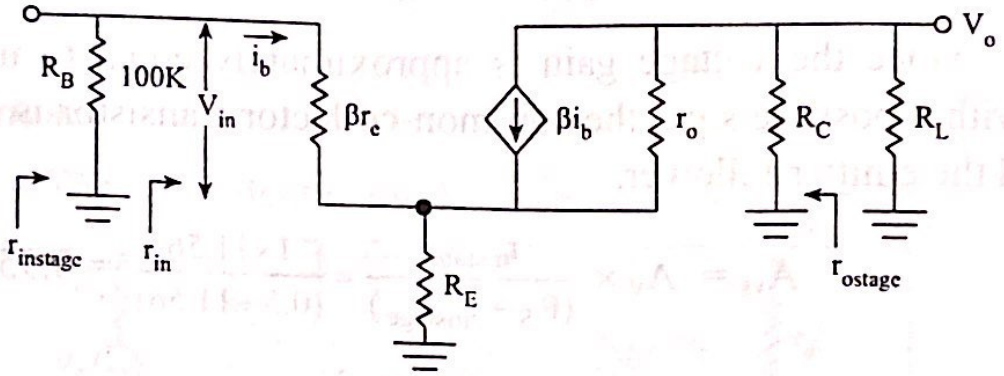


Fig. 5.40. The r_e equivalent model for the CE transistor

$$r_{in} = \frac{V_{in}}{i_b} = \frac{i_b\beta r_e + (\beta+1)i_b R_E}{i_b} \simeq \beta(r_e + R_E)$$

$$= 100(20 + 2000) = 202 \text{ k}\Omega.$$

$$r_{instage} = r_{in} \,||\, R_B = 202 \text{ K} \,||\, 100 \text{ K} = 66.88 \text{ K}.$$

$$r_{ostage} \cong r_O \,||\, R_C = R_C = 2\text{K}.$$

$$A_v = \frac{V_0}{V_{in}} = \frac{-\beta i_b(r_L)}{i_b\beta r_e + (\beta+1)i_b R_E}$$

$$\therefore \quad A_v = \frac{-r_L}{(r_e + R_E)}$$

where, $r_L = R_C \,||\, R_L = 2\text{K} \,||\, 2\text{ K} = 1\text{ K}.$

$$\therefore \quad Av = \frac{-1\times 10^3}{20 + 2\times 10^3} = -\,0.495.$$

When the emitter resistance R_E is unbypassed, then its input resistance increases and voltage gain reduces. The low voltage gain is due to the signal drop across the R_E.