

## Chapter 1 (linear Algebra)

### Assignment

1 Solve the linear equation by checking consistency.

$$x + 2y - z = 3$$

$$3x - y + 2z = 1$$

$$2x - 2y + 3z = 2$$

2 Check the consistency and solve by using Gauss elimination method

$$4y + 3z = -3$$

$$x + y - z = 9$$

$$-x + 2y - 3z = 20$$

3 Investigate for what values of p and q, the system of simultaneous equations

$$x + y + z = 6, x + 2y + 3z = 10, x + 2y + pz = q \text{ has}$$

(i) no solution (ii) a unique solution (iii) an infinite number of solutions

4 Define consistency of a system of equations. Check consistency of the equations:  $5x + 3y + 7z = 4$ ,  $3x + 26y + 2z = 9$ ,  $7x + 2y + 10z = 5$ . If it is consistent find its solution.

5 Define basis of a vector space over the field. Check the following vectors form a basis of  $\mathbb{R}^3$ .  $(1, 1, 1), (1, 3, 2), (-1, 0, 1)$ .

Define Eigen value and Eigen vectors of a square matrix A. Find Eigen value and

$$A = \begin{bmatrix} 6 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Eigen vector of  $\begin{bmatrix} 6 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$$= \lambda A \cdot$$

7 Find rank of the matrix.  
Find the rank of the matrix

0 1 3 1

8 Define eigen values and eigen vectors of the square matrix. Find eigen value

and vectors of matrix  $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 1 & 2 & 3 \end{bmatrix}$

3 1 1

Theorem and use it to find

9

the inverse of  $\begin{pmatrix} 1 & 3 & 7 & 4 & 2 & 3 & 1 & 2 & 1 \end{pmatrix}$

State Cayley Hamilton

10 Verify Cayley-Hamilton Theorem Where  $\begin{bmatrix} 3 & 1 \\ 4 \end{bmatrix}$

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$A = \begin{bmatrix} 5 & 0 & 2 & 6 & 0 & 0 \end{bmatrix}$

11 Find eigen value and eigen vector of  $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$

0 0 3

12 Short questions

1 2 3

+

a) Show that  $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{bmatrix} (6)$

$\begin{bmatrix} + & = & + \\ x & x & x \end{bmatrix}$

1 2 3 + x

the  
b) Find rank of  $A = \begin{bmatrix} 1 & 2 & 5 & 3 & 1 \\ 4 & 1 & 1 & 4 \end{bmatrix}$

c) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be a transformation which is defined by  $T(x, y) = (x + y, x - y)$ ,  
check the linearity of  $T$ .

d) Prove that:  $\begin{vmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 & 9 \end{vmatrix} = 0$

e) Show that the transformation  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  defined by  $T(x, y) = x + y$  is not  
linear.

f) Check the following transformation is linear or not  
 $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be defined by  $T(x, y) = (x + 3, y)$