Short circuit Admittance parameters (Y-parameters):-

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$
$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

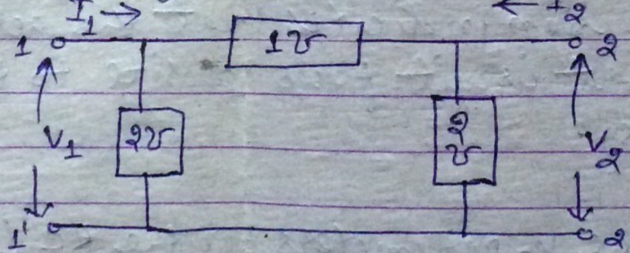
$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \qquad Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \qquad Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

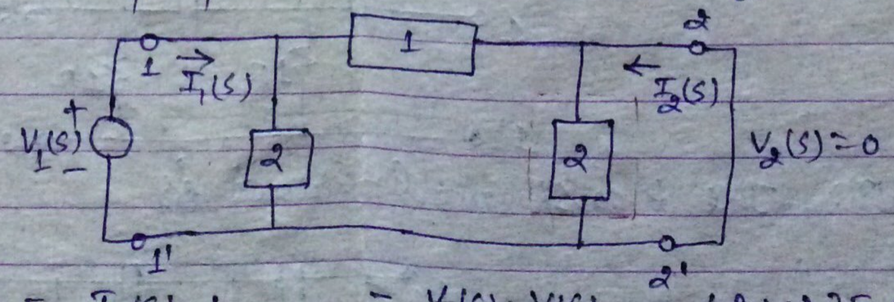
where, Y_{11} = input driving point admittance.
Y_{22} = Output driving point admittance.
Y_{12} = Reverse transfer admittance.
Y_{21} = Forward transfer admittance.

Examples:-

① Compute the short circuit admittance parameters for the resistive π-network given below:



Soln: With output port 2 short circuit i.e. $V_2(s) = 0$

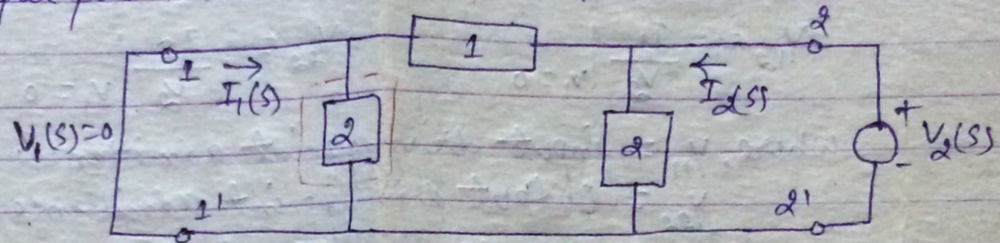


$$Y_{11} = \left.\frac{I_1(s)}{V_1(s)}\right|_{V_2(s)=0} = \frac{V_1(s)\cdot Y_e(s)}{V_1(s)} = (2+1)\,Ʊ = 3\,Ʊ$$

$$Y_{21} = \frac{I_2(s)}{V_1(s)}\Bigg|_{V_2(s)=0} = \frac{\left(-\frac{1}{2+1}\right) I_1(s)}{V_1(s)} = \frac{-1/3 \cdot 3V_1}{V_1} = -1\,℧$$

The -ve sign indicates that the actual current through the short circuit flows opposite to the reference direction of $I_2(s)$.

With input port 1 short circuit i.e $V_1(s)=0$

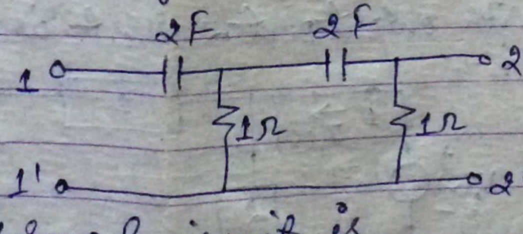


$$Y_{22} = \frac{I_2(s)}{V_2(s)}\Bigg|_{V_1(s)=0} = \frac{V_2(s)\cdot Y_{eq}(s)}{V_2(s)} = (2+1)\,℧ = 3\,℧$$

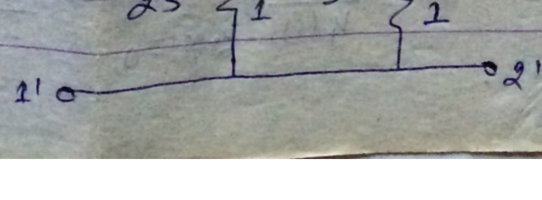
$$Y_{12} = \frac{I_1(s)}{V_2(s)}\Bigg|_{V_1(s)=0} = \frac{\left(-\frac{1}{1+2}\right) I_2(s)}{V_2(s)} = \frac{-1/3 \times 3V_2}{V_2} = -1\,℧$$

The -ve sign indicates that the actual current through the short circuit flows opposite to the reference direction of $I_1(s)$.

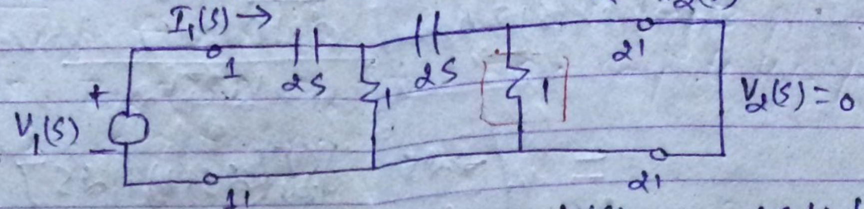
(ii) Find the y-parameters for the RC Ladder network.



Soln: The transformed circuit is



With port 2 short circuit i.e. $V_2(s) = 0$



$$Y_{11} = \left.\frac{I_1(s)}{V_1(s)}\right|_{V_2(s)=0} = \frac{V_1(s) \cdot Y_{eq}(s)}{V_1(s)} = 2s \,||\, (2s+1)$$

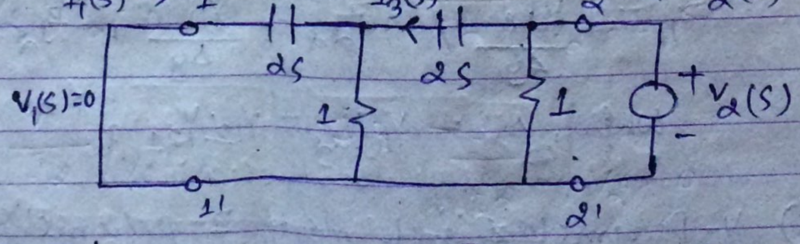
$$= \left(\frac{2s(2s+1)}{4s+1}\right)$$

$$Y_{21} = \left.\frac{I_2(s)}{V_1(s)}\right|_{V_2(s)=0} = \frac{-\left(\frac{2s}{1+2s}\right) I_1(s)}{V_1(s)}$$

$$= \frac{\left(-\frac{2s}{\cancel{2s+1}}\right) \times \frac{2s\cancel{(2s+1)}}{(4s+1)} \times \cancel{V_1}}{}$$

$$= \left(\frac{-4s^2}{4s+1}\right) \cancel{V_1}$$

With port 1 short circuit i.e. $V_1(s) = 0$



$$Y_{22} = \left.\frac{I_2(s)}{V_2(s)}\right|_{V_1(s)=0} = \frac{V_2(s) \cdot [(2s+1) \,||\, 2s] + 1}{V_2(s)}$$

$$= \left(\frac{2s(2s+1)}{4s+1} + 1\right) = \left(\frac{4s^2+6s+1}{4s+1}\right)$$

$$Y_{12} = \left.\frac{I_1(s)}{V_2(s)}\right|_{V_1(s)=0} = \frac{\left(\frac{-2s}{2s+1}\right) I_3(s)}{V_2(s)}$$

$$= \frac{\left(\frac{-2s}{2s+1}\right)\left\{\frac{[(2s+1)\,||\,2s]}{[[(2s+1)\,||\,2s]+1]}\right\} \times I_2(s)}{V_2(s)}$$

$$= \frac{\left(\frac{-2s}{2s+1}\right)\left[\frac{2s(2s+1)}{(4s+1)} \times \frac{(4s+1)}{(4s^2+6s+1)}\right]\left(\frac{4s^2+6s+1}{4s+1}\right)V_2}{V_2}$$

$$= \left(\frac{-4s^2}{4s+1}\right)$$

Expressions for Z-parameters in terms of Y-parameters and vice-versa:-

(i) Z-parameters in terms of Y-parameters:-

$$Z_{11} = \frac{Y_{22}}{\Delta y}, \quad Z_{22} = \frac{Y_{11}}{\Delta y}, \quad Z_{12} = -\frac{Y_{21}}{\Delta y} \text{ and } Z_{21} = \frac{-Y_{12}}{\Delta y}$$

Where, $\Delta y = Y_{11} Y_{22} - Y_{12} \cdot Y_{21}$

(ii) Y-parameters in terms of Z-parameters:-

$$Y_{11} = \frac{Z_{22}}{\Delta Z}, \quad Y_{22} = \frac{Z_{11}}{\Delta Z}, \quad Y_{12} = \frac{-Z_{21}}{\Delta Z} \text{ and } Y_{21} = \frac{-Z_{12}}{\Delta Z}$$

Where, $\Delta Z = Z_{11} \cdot Z_{22} - Z_{12} \cdot Z_{21}$

Examples:-

① A two-port network has the following Z-parameters: $Z_{11} = 10\Omega$, $Z_{22} = 12\Omega$, $Z_{12} = Z_{21} = 5\Omega$. Compute the Y-parameters for the same network.

Soln:

$$\Delta Z = Z_{11} \cdot Z_{22} - Z_{12} \cdot Z_{21} = Z_{11} \cdot Z_{22} - (Z_{12})^2 = 10 \times 12 - 5^2 = 95$$

Now,

$$Y_{11} = \frac{Z_{22}}{\Delta Z} = \frac{12}{95} = 0.1263 \, \mho$$

$$Y_{22} = \frac{Z_{11}}{\Delta Z} = \frac{10}{95} = 0.1053 \, \mho$$

$$Y_{12} = Y_{21} = -\frac{Z_{12}}{\Delta Z} = \frac{-5}{95} = -0.0526 \, \mho$$

Transmission Paremeters (A, B, C, D):-

$$V_1 = A \cdot V_2 - B \cdot I_2$$
$$I_1 = C \cdot V_2 - D \cdot I_2$$

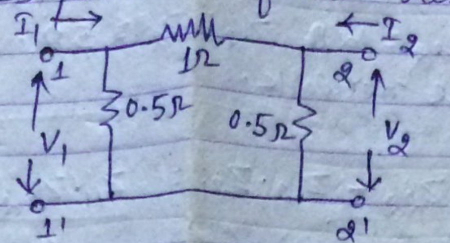
$$A = \left. \frac{V_1}{V_2} \right|_{I_2 = 0} \qquad C = \left. \frac{I_1}{V_2} \right|_{I_2 = 0}$$

$$B = -\left. \frac{V_1}{I_2} \right|_{V_2 = 0} \qquad D = \left. \frac{-I_1}{I_2} \right|_{V_2 = 0}$$

Here, A, B, C and D are the transmission parameters / ABCD parameters / chain parameters / general circuit parameters.

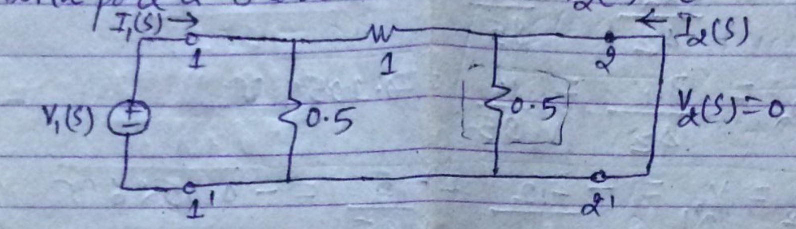
Examples:-

① Find out ABCD parameters for the resistive π- network:



Soln:

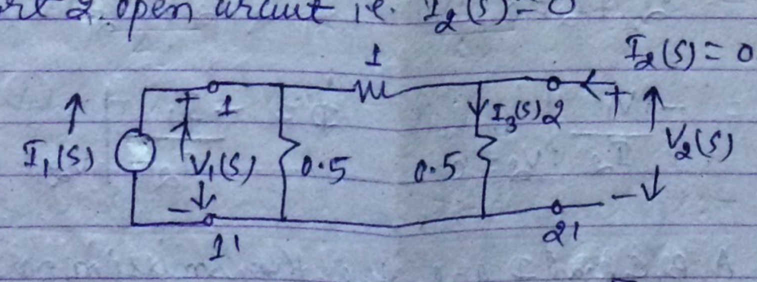
With port 2 short circuit i.e. $V_2(s) = 0$



$$B = -\left.\frac{V_1(s)}{I_2(s)}\right|_{V_2(s)=0} = -\frac{(1 \| 0.5)\, I_1(s)}{I_2(s)} = \frac{-\frac{0.5 \times I_1(s)}{1.5}}{-\frac{0.5}{1.5} \times I_1(s)} = 1\Omega$$

$$D = -\left.\frac{I_1(s)}{I_2(s)}\right|_{V_2(s)=0} = -\frac{I_1(s)}{-\frac{0.5}{1.5} \times I_1(s)} = 3$$

With port 2 open circuit i.e. $I_2(s) = 0$



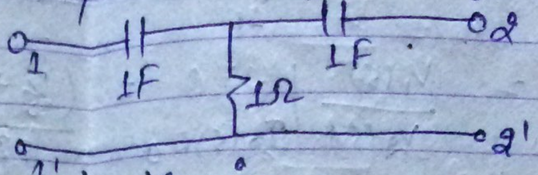
$$A = \left.\frac{V_1(s)}{V_2(s)}\right|_{I_2(s)=0} = \frac{[(1+0.5) \| 0.5]\, I_1(s)}{I_3(s) \times 0.5} = \frac{0.75/2\, I_1(s)}{\frac{0.5}{2} \times I_1(s) \times 0.5}$$

$$= 3$$

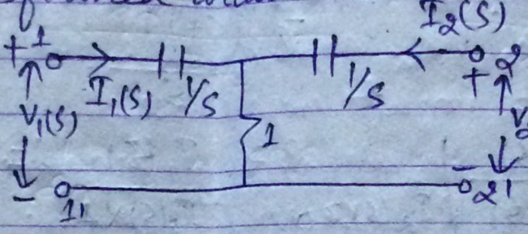
Bhattarai (4)

$$C = \left.\frac{I_1(s)}{V_2(s)}\right|_{I_2(s)=0} = \frac{I_1(s)}{I_2(s)\times 0.5} = \frac{I_1(s)}{\frac{0.5}{2}\times I_1(s)\times 0.5} = 8\,\mho$$

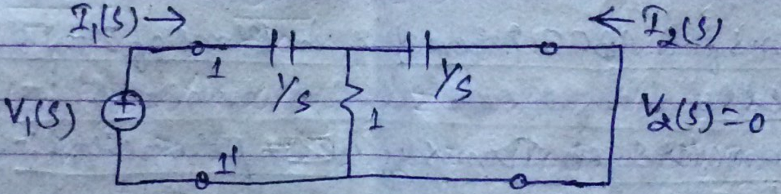
(2) Find the transmission parameters for the given RC network:



Soln: The transformed circuit is



With port 2 short circuit i.e. $V_2(s)=0$



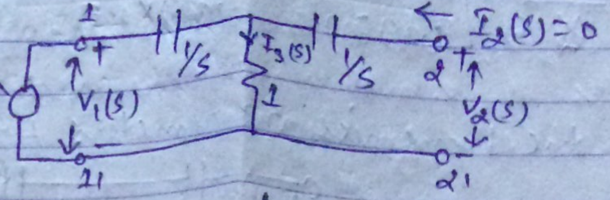
$$B = -\left.\frac{V_1(s)}{I_2(s)}\right|_{V_2(s)=0} = \frac{-\left[(1 \parallel 1/s) + 1/s\right] I_1(s)}{-\left(\frac{1}{1+1/s}\right) I_1(s)} = \frac{\frac{2s+1}{s(s+1)}}{\left(\frac{s}{s+1}\right)}$$

$$= \left(\frac{2s+1}{s^2}\right)$$

$$D = -\left.\frac{I_1(s)}{I_2(s)}\right|_{V_2(s)=0} = \frac{-I_1(s)}{-\left(\frac{1}{1+1/s}\right) I_1(s)} = \left(\frac{s+1}{s}\right)$$

Scanned with CamScanner

With port 2 open circuit i.e. $I_2(s) = 0$



$$A = \left.\frac{V_1(s)}{V_2(s)}\right|_{I_2(s)=0} = \frac{\left(\frac{1}{s}+1\right) I_1(s)}{I_3(s)\times 1} = \frac{\frac{(s+1)}{s}\, I_1(s)}{I_1(s)}$$

$$= \frac{(s+1)}{s}$$

$$C = \left.\frac{I_1(s)}{V_2(s)}\right|_{I_2(s)=0} = \frac{I_1(s)}{I_1(s)\times 1} = 1$$

Expressions for ABCD parameters in terms of Z-parameters and Y-parameters:-

(i) ABCD parameters in Terms of Z-parameters:-

$$A = \frac{Z_{11}}{Z_{21}}, \quad B = \frac{\Delta Z}{Z_{21}} = \frac{Z_{11}\cdot Z_{22} - Z_{21}\cdot Z_{12}}{Z_{21}}$$

$$C = \frac{1}{Z_{21}} \text{ and } D = \frac{Z_{22}}{Z_{21}}$$

(ii) ABCD parameters in terms of Y-parameters:-

$$A = -\frac{Y_{22}}{Y_{21}}, \quad B = -\frac{1}{Y_{21}}$$

$$C = -\frac{\Delta y}{Y_{21}} = -\left(\frac{Y_{11}\cdot Y_{22} - Y_{12}\cdot Y_{21}}{Y_{21}}\right) \text{ and } D = \frac{-Y_{11}}{Y_{21}}$$

Example:- The Z-parameters for a two port network are: $Z_{11} = 40\,\Omega$, $Z_{22} = 30\,\Omega$, $Z_{12} = Z_{21} = 20\,\Omega$. Compute the transmission parameters for the network. Hence, write the network equations using these two types of parameters.

Soln:

$$A = \frac{Z_{11}}{Z_{21}} = \frac{40}{20} = 2$$

$$B = \frac{\Delta Z}{Z_{21}} = \frac{Z_{11}Z_{22} - Z_{21}Z_{12}}{Z_{21}} = \frac{40\times30 - 20^2}{20} = \frac{800}{20} = 40\,\Omega$$

$$C = \frac{1}{Z_{21}} = \frac{1}{20} = 0.05\,℧$$

$$D = \frac{Z_{22}}{Z_{21}} = \frac{30}{20} = 1.5$$

Network equations using Z-parameters are:

$$V_1 = Z_{11}I_1 + Z_{12}I_2 = 40I_1 + 20I_2$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2 = 20I_1 + 30I_2$$

Similarly, network equations using ABCD parameters are:

$$V_1 = AV_2 - BI_2 = 2V_2 - 40I_2$$
$$I_1 = CV_2 - DI_2 = 0.05V_2 - 1.5I_2$$

Note:-

(i) Z-parameters in terms of ABCD parameters:-

$$Z_{11} = A/C,\ Z_{12} = \frac{AD-BC}{C},\ Z_{21} = 1/C \text{ and } Z_{22} = D/C.$$

(ii) Y-parameters in terms of ABCD parameters:-

$$Y_{11} = \frac{D}{B},\ Y_{12} = -\left(\frac{AD-BC}{B}\right),\ Y_{21} = -1/B \text{ and } Y_{22} = A/B.$$

Hybrid parameters (h-parameters):-

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}, \quad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} \text{ and } h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

Here, h_{11}, h_{21}, h_{12} and h_{22} are the h-parameters.

h_{11} = input impedance.

h_{21} = Forward current gain.

h_{12} = Reverse voltage gain.

h_{22} = output admittance.

Inverse Hybrid parameters (g-parameters):-

$$I_1 = g_{11} \cdot V_1 + g_{12} I_2$$
$$V_2 = g_{21} \cdot V_1 + g_{22} I_2$$

$g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0}$ = open circuit input admittance.

$g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0}$ = open circuit forward voltage ratio/gain.

$g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0}$ = Short circuit reverse current gain.

$g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0}$ = Short circuit output impedance.

ROLL No. [redacted] NAME

Bhattarai (5)

SUBJECT

Note: STREAM

(i) h-parameters in terms of Z-parameters:-

$$h_{11} = \frac{Z_{11}Z_{22} - Z_{21}Z_{12}}{Z_{22}}, \quad h_{12} = \frac{Z_{12}}{Z_{22}}$$

$$h_{21} = -\frac{Z_{21}}{Z_{22}} \text{ and } h_{22} = \frac{1}{Z_{22}}$$

(ii) h-parameters in terms of Y-parameters:-

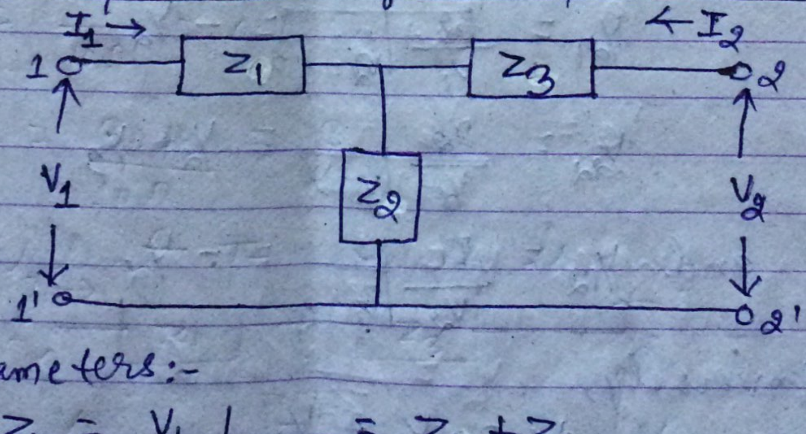
$$h_{11} = \frac{1}{Y_{11}}, \quad h_{12} = -\frac{Y_{12}}{Y_{11}}, \quad h_{21} = \frac{Y_{21}}{Y_{11}} \text{ and}$$

$$h_{22} = \frac{Y_{11}Y_{22} - Y_{12}Y_{21}}{Y_{11}}$$

(iii) h-parameters in terms of ABCD parameters:-

$$h_{11} = B/D, \quad h_{12} = \frac{AD - BC}{D}, \quad h_{21} = -1/D \text{ and } h_{22} = C/D$$

T-section Representation of a Two-port Network:-



Z-parameters:-

$$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0} = Z_1 + Z_2$$

$$Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0} = Z_2$$

$$Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0} = Z_2 + Z_3$$

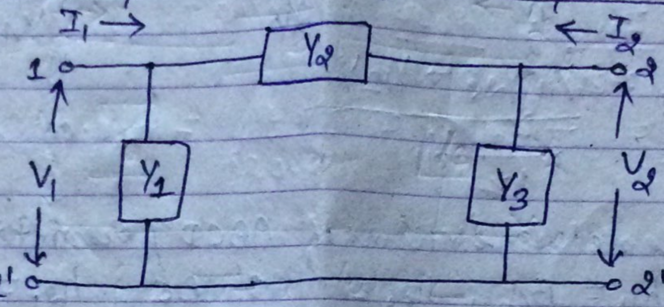
Scanned with CamScanner

and $Z_{12} = \left.\dfrac{V_1}{I_2}\right|_{I_1=0} = Z_2$

Conversely,

$$Z_2 = Z_{12} = Z_{21}$$
$$Z_1 = Z_{11} - Z_{12} \text{ and}$$
$$Z_3 = Z_{22} - Z_{12}$$

π- section representation of a Two-port network:-



Y-parameters:-

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = Y_1 + Y_2$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = -Y_2$$

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = Y_2 + Y_3$$

and $Y_{12} = \left.\dfrac{I_1}{V_2}\right|_{V_1=0} = -Y_2$

Conversely,

$$Y_1 = Y_{11} + Y_{21}$$
$$Y_2 = -Y_{21} \text{ and}$$

$$Y_3 = Y_{22} + Y_{21}$$

Example:-

The Z-parameters of a two-port network are: $Z_{11} = 20\,\Omega$, $Z_{22} = 30\,\Omega$, $Z_{12} = Z_{21} = 10\,\Omega$. Find the Y and ABCD parameters of the network. Also, find its equivalent T-network.

Soln:

Y-parameters:-

$$Y_{11} = \frac{Z_{22}}{\Delta Z} = \frac{Z_{22}}{Z_{11}\cdot Z_{22} - Z_{12}Z_{21}} = \frac{30\,\Omega}{(20\times 30 - 10^2)\,\Omega^2} = \frac{30\,\Omega}{500\,\Omega^2} = 0.06\,\mho$$

$$Y_{22} = \frac{Z_{11}}{\Delta Z} = \frac{20\,\Omega}{500\,\Omega^2} = 0.04\,\mho$$

$$Y_{21} = Y_{12} = \frac{-Z_{21}}{\Delta Z} = \frac{-10\,\Omega}{500\,\Omega^2} = -0.02\,\mho$$

ABCD parameters:-

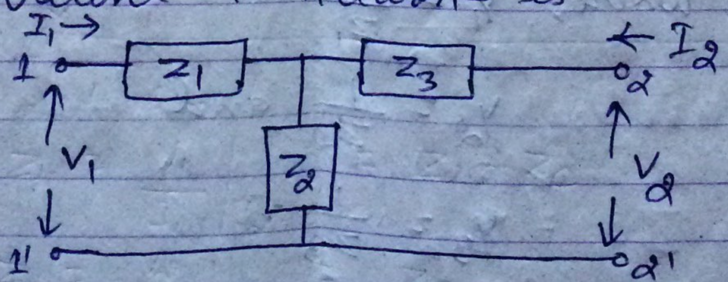
$$A = \frac{Z_{11}}{Z_{21}} = \frac{20\,\Omega}{10\,\Omega} = 2$$

$$B = \frac{\Delta Z}{Z_{21}} = \frac{500\,\Omega^2}{10\,\Omega} = 50\,\Omega$$

$$C = \frac{1}{Z_{21}} = \frac{1}{10\,\Omega} = 0.1\,\mho$$

$$D = \frac{Z_{22}}{Z_{21}} = \frac{30\,\Omega}{10\,\Omega} = 3$$

The equivalent T-network is



where, $Z_1 = Z_{11} - Z_{12} = 20 - 10 = 10\,\Omega$

$Z_2 = Z_{12} = Z_{21} = 10\,\Omega$ and

$Z_3 = Z_{22} - Z_{12} = 30 - 10 = 20\,\Omega$

T to π and π to T Transformation:-

(i) T to π Transformation:-

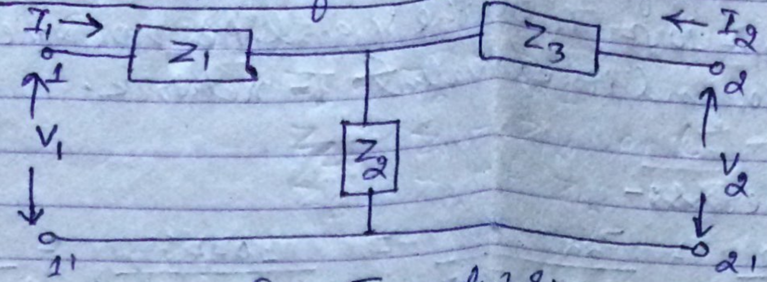


Fig. T- network

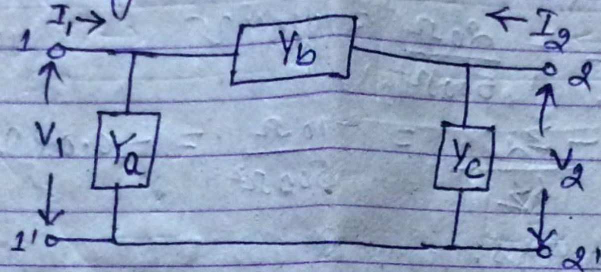


Fig. π- network

$$Y_a = \frac{Y_1 Y_2}{Y_1 + Y_2 + Y_3}$$

$$Y_b = \frac{Y_1 Y_3}{Y_1 + Y_2 + Y_3}$$

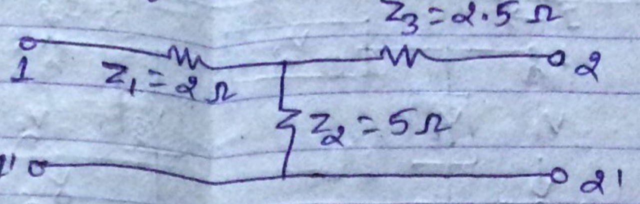
$$Y_c = \frac{Y_3 Y_2}{Y_1 + Y_2 + Y_3}$$

where, $Y_1 = \frac{1}{Z_1}$

$Y_2 = \frac{1}{Z_2}$ and

$Y_3 = \frac{1}{Z_3}$.

Example:- Find the equivalent π- network for the given T- network.



Soln:

Let the equivalent π- network has Y_b as the series admittence and Y_a and Y_c as the shunt/parallel admittances of ports 1 and 2 respectively i.e.

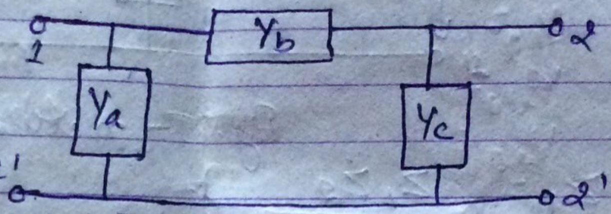


Fig. π- network.

Then, $Y_1 = \frac{1}{Z_1} = \frac{1}{2} = 0.5\,S$

$Y_2 = \frac{1}{Z_2} = \frac{1}{5} = 0.2\,S$

$Y_3 = \frac{1}{Z_3} = \frac{1}{2.5} = 0.4\,S$

Now, $Y_a = \frac{Y_1 Y_2}{Y_1 + Y_2 + Y_3} = \frac{0.5 \times 0.2}{0.5 + 0.2 + 0.4} = \frac{0.1}{1.1} = \frac{1}{11}\,S$

$$Y_b = \frac{Y_1 Y_3}{Y_1 + Y_2 + Y_3} = \frac{0.5 \times 0.4}{1.1} = \frac{0.2}{1.1} = \frac{2}{11}\,S$$

$$Y_c = \frac{Y_3 Y_2}{Y_1 + Y_2 + Y_3} = \frac{0.4 \times 0.2}{1.1} = \frac{0.8}{1.1} = \frac{8}{11}\,S$$

(ii) π to T Transformation:-

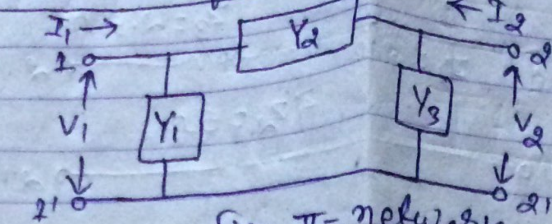


Fig. π- network

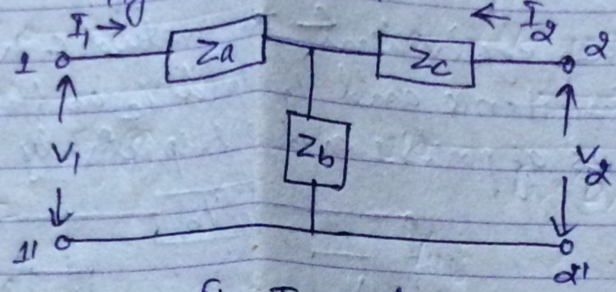


Fig. T- network

$$Z_a = \frac{Z_1 Z_2}{Z_1 + Z_2 + Z_3}$$

$$Z_b = \frac{Z_1 Z_3}{Z_1 + Z_2 + Z_3}$$

$$Z_c = \frac{Z_2 Z_3}{Z_1 + Z_2 + Z_3}$$

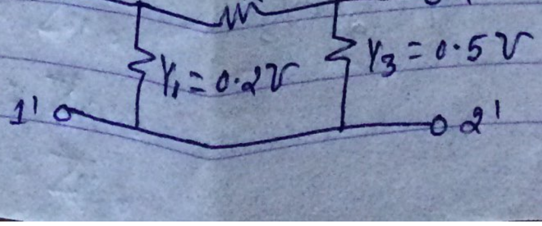
Where, $Z_1 = 1/Y_1$

$Z_2 = \frac{1}{Y_2}$ and

$Z_3 = \frac{1}{Y_3}$.

Example:-

① Find the equivalent T- network for the given π- network.



Soln:

Let the equivalent T-network has impedance Z_b in the parallel / shunt arm and impedances Z_a and Z_c in the series arm of ports 1 and 2 respectively i.e.

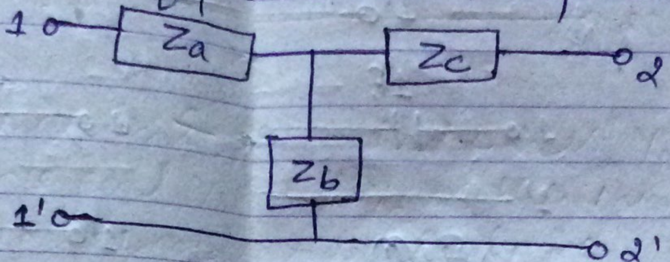


fig. T- network

Then,

$$Z_1 = \frac{1}{Y_1} = \frac{1}{0.25} = 5\,\Omega$$

$$Z_2 = \frac{1}{Y_2} = \frac{1}{1S} = 1\,\Omega$$

$$Z_3 = \frac{1}{Z_3} = \frac{1}{0.5S} = 2\,\Omega$$

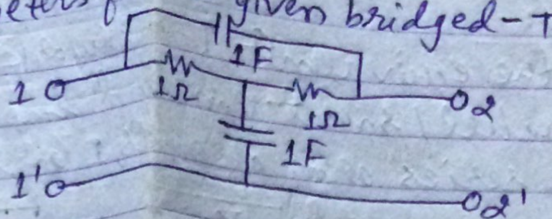
Now,
$$Z_a = \frac{Z_1 Z_2}{Z_1 + Z_2 + Z_3} = \frac{5 \times 1}{5+1+2} = \frac{5}{8} = 0.625\,\Omega$$

$$Z_b = \frac{Z_1 Z_3}{Z_1 + Z_2 + Z_3} = \frac{5 \times 2}{8} = 10/8 = 1.25\,\Omega$$

$$Z_c = \frac{Z_2 Z_3}{Z_1 + Z_2 + Z_3} = \frac{1 \times 2}{8} = 2/8 = 0.25\,\Omega$$

Scanned with CamScanner

(11) Find the y-parameters for the given bridged-T RC network.



Soln:

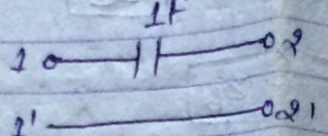


Fig (a)

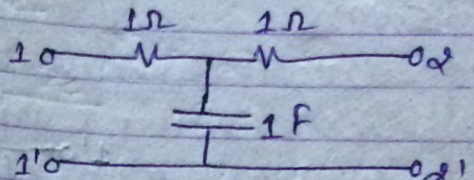


Fig (b)

$Y_{11a} =$
$Y_{21a} =$
$Y_{22a} =$
$Y_{12a} =$

$Y_{11b} =$
$Y_{21b} =$
$Y_{22b} =$
$Y_{12b} =$

$$\therefore \; Y_{11eq} = Y_{11a} + Y_{11b}$$
$$Y_{21eq} = Y_{21a} + Y_{21b}$$
$$Y_{22eq} = Y_{22a} + Y_{22b}$$
$$Y_{12eq} = Y_{12a} + Y_{12b}$$

Note:-

In Fig (a) since I_1 and I_2 are not independent i.e as soon as V_1 will be applied, I_1 will flow causing I_2 to flow as $(-I_1)$, Z-parameters cannot be determined.

Hence, $Z_{11eq} = Z_{11b}$
$Z_{21eq} = Z_{21b}$
$Z_{22eq} = Z_{22b}$
$Z_{12eq} = Z_{12b}$