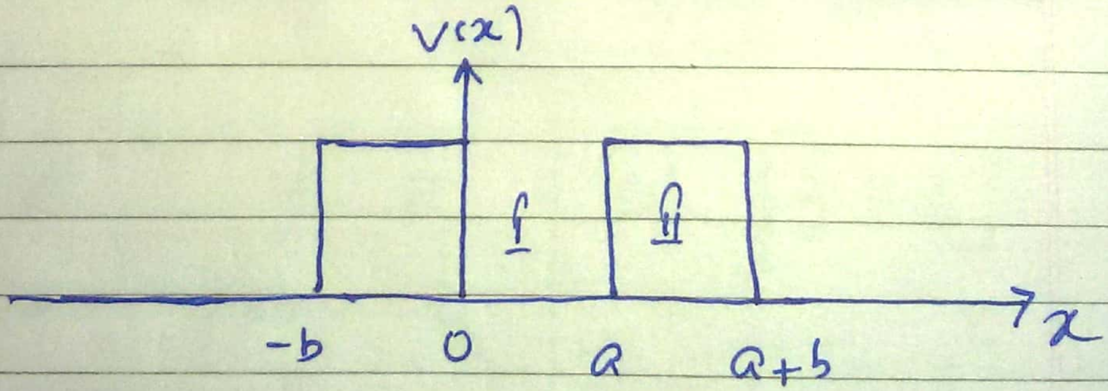
Kronig Penny Model

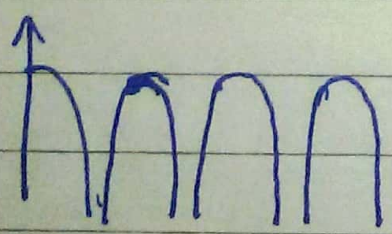
The free electron model implies that potential inside solid is uniform in metal.

It is the treatment of energy levels of an electron experiencing ~~to~~ one dimensional (1 D) periodic potential

Consider a periodic potential with period $L = a+b$ as shown in figure.

where a = width of well
b = " " barrier





The potential function for the particle can be expressed as

$V(x) = 0$ for region I

$V(x) = V$ for " II

For region I the Schrödinger wave eqn can be written as

$$\frac{d^2\psi_1}{dx^2} + \frac{2mE}{\hbar^2}\psi_1 = 0$$

$$\frac{d^2\psi_1}{dx^2} + \alpha^2\psi_1 = 0 \quad \text{——— (1)}$$

where $\alpha^2 = \frac{2mE}{\hbar^2}$

Since the potential is periodic, its solution can be written as

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$$\psi_1 = u_1 e^{ikx}$$

$$\frac{d\psi_1}{dx} = e^{ikx}\frac{du_1}{dx} + ik\, u_1 e^{ikx}$$

$$\frac{d^2\psi_2}{dx^2} = e^{ikx}\frac{d^2u_1}{dx^2} + ik\frac{du_1}{dx}e^{ikx} + ik\frac{du_1}{dx}e^{ikx} + (ik)^2 u_1 e^{ikx}$$

$$\frac{d^2\psi_1}{dx^2} = e^{ikx}\frac{d^2u_1}{dx^2} + 2ik\frac{du_1}{dx}e^{ikx} - k^2 u_1 e^{ikx}$$

$$\frac{d^2\psi_1}{dx^2} = e^{ikx}\left[\frac{d^2u_1}{dx^2} + 2ik\frac{du_1}{dx} - k^2 u_1\right]$$

Eqn ① becomes

$$e^{ikx}\left[\frac{d^2u_1}{dx^2} + 2ik\frac{du_1}{dx} - k^2u_1\right] + \alpha^2 u_1 e^{ikx} = 0$$

$$\frac{d^2u_1}{dx^2} + 2ik\frac{du_1}{dx} - k^2u_1 + \alpha^2 u_1 = 0 \quad \text{—— ②}$$

For region II

the Schrödinger wave eqn can be written as

$$\frac{d^2\psi_2}{dx^2} + \frac{2m(E-V)}{\hbar^2}\psi_2 = 0$$

$$\frac{d^2\psi_2}{d^2x^2} - \frac{2m(V-E)}{\hbar^2}\psi_2 = 0$$

$$\frac{d^2\psi_2}{dx^2} - \beta^2\psi_2 = 0 \quad \text{——} \ (3)$$

And its solution is

$$\psi_2 = U_2 e^{ikx}$$

proceeding as above

$$\frac{d^2U_2}{dx^2} + 2ik\frac{dU_2}{dx} - k^2U_2 - \beta^2U_2 = 0 \quad \text{——} \ (4)$$

The solution of (3) & (4)

$$U_1 = Ae^{i(\alpha-k)x} + Be^{-i(\alpha+k)x}$$

$$U_2 = Ce^{(\beta-ik)x} + De^{-(\beta+ik)x}$$

We have boundary conditions

$U_1(0) = U_2(0)$ | $U_1(a) = U_2(-b)$

$U_1'(0) = U_2'(0)$ | $U_1'(a) = U_2'(-b)$

$$A + B = C + D$$

$$Ae^{i(\alpha-k)a} + Be^{-i(\alpha+k)a} = Ce^{-(\beta-ik)b} + De^{(\beta+ik)b}$$

$$i(\alpha-k)A - i(\alpha+k)B = (\beta-ik)C - (\beta+ik)D$$

$$i(\alpha-k)Ae^{i(\alpha-k)a} - i(\alpha+k)Be^{-i(\alpha+k)a} = (\beta-ik)Ce^{-(\beta-ik)b} - (\beta+ik)De^{(\beta+ik)b}$$

To solve determinant form

$$0=\begin{vmatrix} 1 & 1 & 1 & 1 \\ e^{i(\alpha-k)a} & e^{-i(\alpha+k)a} & e^{-(\beta-ik)b} & e^{(\beta+ik)b} \\ i(\alpha-k) & -i(\alpha+k) & (\beta-ik) & -(\beta+ik) \\ i(\alpha-k)e^{i(\alpha-k)a} & -i(\alpha+k)e^{-i(\alpha+k)a} & (\beta-ik)e^{-(\beta-ik)b} & -(\beta+ik)e^{(\beta+ik)b} \end{vmatrix}$$

By expanding determinant, we get

$$\frac{\beta^2-\alpha^2}{2\alpha\beta}\operatorname{Sinh}\beta b \operatorname{Sin}\alpha a + \operatorname{Cosh}\beta b \operatorname{Cos}\alpha a = \operatorname{Cos}k(a+b)$$

when $V\to 0$ and $b\to 0$

but Vb will be finite

as $b\to 0$ $\operatorname{Sinh}\beta b \to \beta b$

and $\operatorname{Cosh}\beta b \to 1$

$$\frac{\beta^2-\alpha^2}{2\alpha\beta} \simeq \frac{\beta^2}{2\alpha\beta} = \frac{\beta}{2\alpha}$$

$$\frac{\beta^2 b}{2\alpha} \sin \alpha a + \cos \alpha a = \cos ka$$

$$\frac{\beta^2 ba}{2\alpha a} \sin \alpha a + \cos \alpha a = \cos ka$$

Let $P = \frac{\beta^2 ab}{2}$ $\quad \beta^2 = \frac{2m(V-E)}{\hbar^2}$

$$P = \frac{2mV}{\hbar^2} \frac{ab}{2} = \frac{mV}{\hbar^2} ab$$

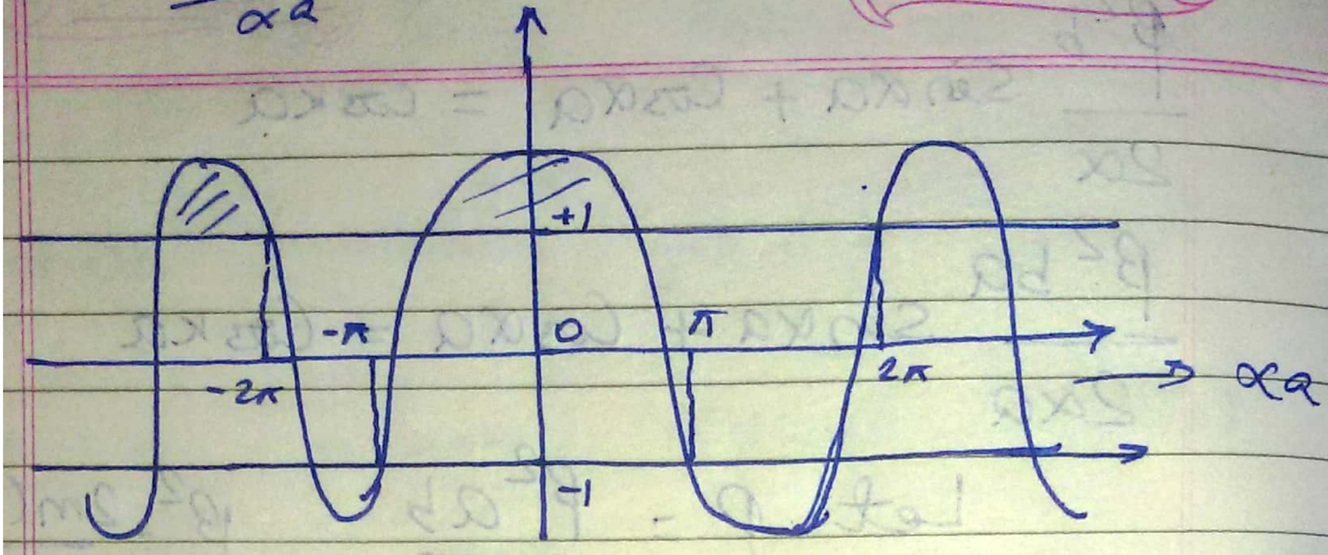
$$P = \frac{mV}{\hbar^2} ab$$

$$\frac{P \sin \alpha a}{\alpha a} + \cos \alpha a = \cos ka$$

It is tedious and complicated to solve

So, we plot the graph.

$$\frac{P\sin\alpha a}{\alpha a} + \cos\alpha a$$



$\cos ka = \pm 1 = \cos n\pi$

$ka = n\pi$

$\Rightarrow k = \frac{n\pi}{a}$

$n = 1, 2, 3, \dots$

$\alpha^2 = \frac{2mE}{\hbar^2}$

$E = \frac{\hbar^2 \alpha^2}{2m}$

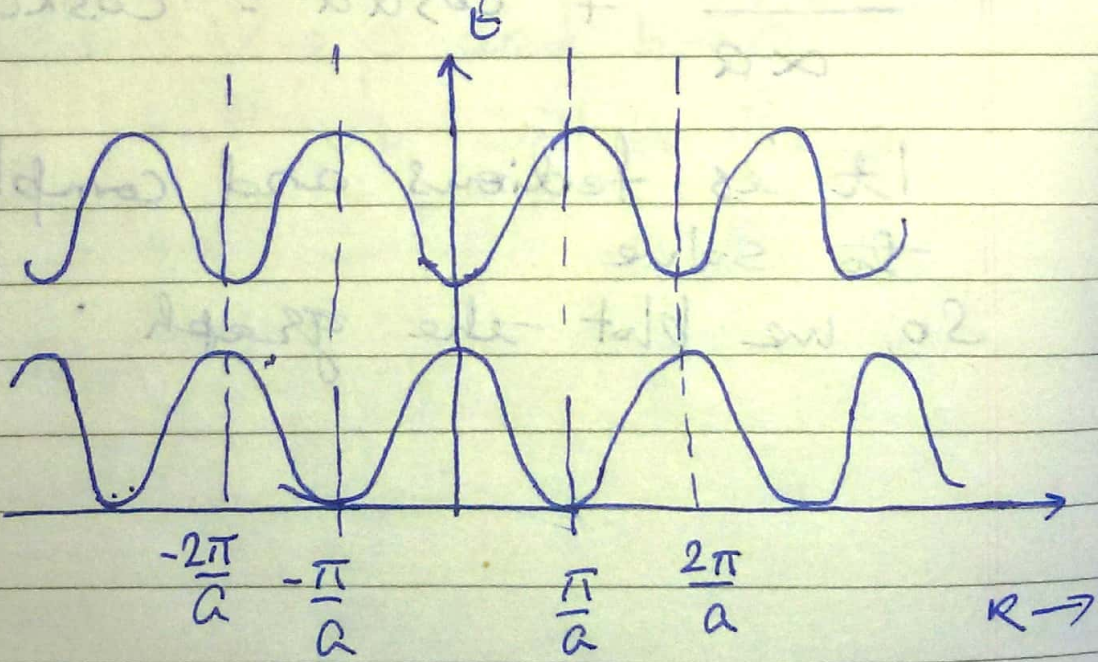


Fig: E-K Curve for 1D lattice structure from K-P model

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Case I

As $P \to \infty$, the band reduces to a single energy level i.e. discrete energy spectrum for isolated atoms which is the actual case so, P is proportional to the potential barrier of the given system.

$$\sin \alpha a = 0$$

$$\alpha a = n\pi$$

$$\alpha^2 a^2 = n^2 \pi^2$$

$$\frac{2mE}{\hbar^2} a^2 = n^2 \pi^2$$

$$\boxed{E = \frac{n^2 \pi^2 \hbar^2}{2ma^2}}$$

which is exactly same as the equation of an electron in an infinite potential well of width a.

Case II

when $P \to 0$ $\cos \alpha a = \cos ka$

$$\alpha a = ka$$

$$\alpha^2 = k^2$$

$$\frac{2mE}{\hbar^2} = k^2$$

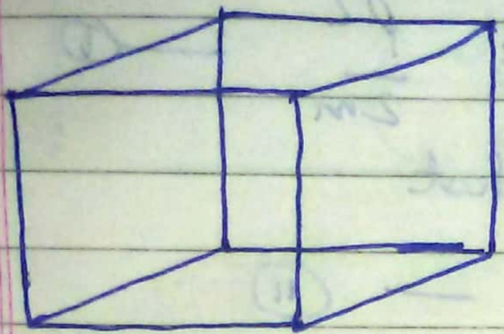
$$\boxed{E = \frac{\hbar^2 k^2}{2m}}$$

Eqn is exactly same as that of eqn for a free electron.

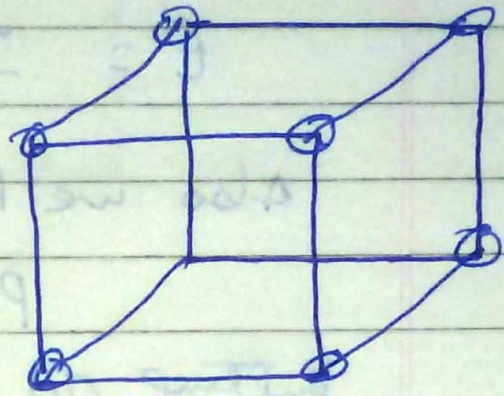
So, the variation in the value of P from zero to infinity accounts for the whole range of conditions in the crystalline solid.

$$m^* = \frac{\hbar^2}{d^2E/dk^2}$$

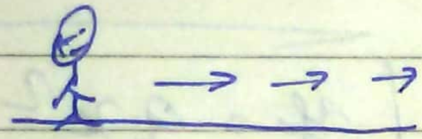
Effective Mass

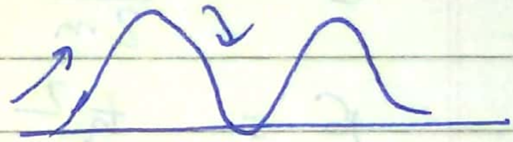


free space



crystal lattice





Effective mass is that mass which a particle (electron) seems to have when it is present in a crystal of periodic potential.

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$$\boxed{m^* = \frac{\hbar^2}{\left(\frac{d^2E}{dk^2}\right)}}$$

$$K.E = \frac{1}{2} m v^2$$

$$E = \frac{m^2 v^2}{2m} = \frac{p^2}{2m} \quad \text{— (i)}$$

Also we know that

$$p = \hbar k \quad \text{— (ii)}$$

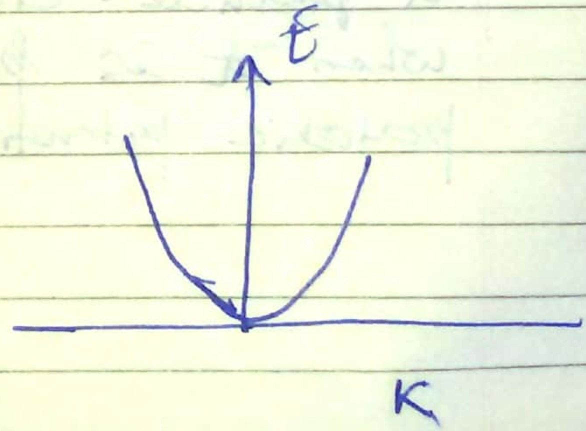
putting (ii) in (i) we get

$$E = \frac{\hbar^2 k^2}{2m}$$

$$E = \frac{\hbar^2}{2m} k^2 \qquad \boxed{y = a x^2}$$

parabolic eq^n

Diff w.r.to k



$$\frac{dE}{dk} = \left(\frac{\hbar^2}{2m}\right) 2k$$

$$\frac{dE}{dk} = \frac{\hbar^2}{m} k$$

$$\frac{d^2E}{dk^2} = \frac{\hbar^2}{m}$$

$\therefore$ for crystal $m \rightarrow m^*$

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$$\boxed{m^* = \frac{\hbar^2}{\left(\frac{d^2E}{dk^2}\right)}}$$

$$E = \frac{\hbar^2 k^2}{2m}$$

$$\frac{dE}{dk} = \frac{\hbar^2 k}{m}$$

$$\frac{dE}{dk} = \frac{\hbar(\hbar k)}{m}$$

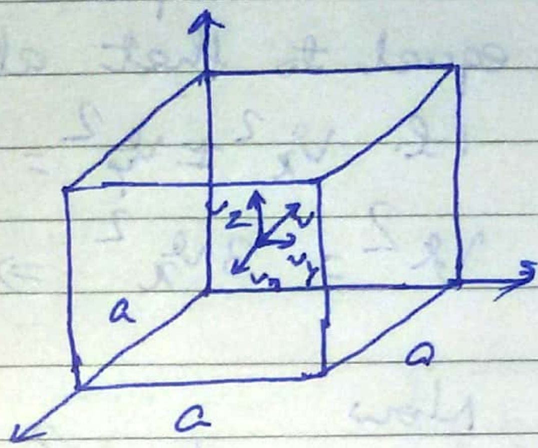
$$\frac{dE}{dk} = \frac{\hbar \cancel{m} v}{\cancel{m}}$$

$$\boxed{\text{velocity } v = \frac{1}{\hbar}\frac{dE}{dk}}$$

The velocity of particle in the crystal.

Thermal Velocity of Electron at Equilibrium

Consider an electron 'e' of mass 'm' is free to move inside a box of length 'a'.



The electron has momentum → $+mv_x$ while moving along +ve x-dir^n

& $-mv_x$ — - - - -ve - - after its reflection from wall of the box

∴ Change in momentum $\Delta P = 2mv_x$

time interval between collision $\Delta t = \frac{2a}{v_z}$

∴ Force on the wall $F = \frac{dP}{dt} = \frac{mv_x^2}{a}$

Pressure on the wall $= \frac{\text{Force}}{\text{Area}} = \frac{mu_n^2}{a \cdot a^2}$

$$= \frac{mv_x^2}{a^3}$$

Total pressure due to N electrons

$$P = \frac{Nmv_x^3}{a^3}$$

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for acceleration

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{1}{\hbar}\frac{dE}{dk}\right)$$

$$a = \frac{1}{\hbar}\frac{d}{dt}\left(\frac{dE}{dk}\right)$$

$$\frac{d}{dt} = \frac{dk}{dt}\frac{d}{dk}\cancel{\left(dE\right)}$$

$$a = \frac{1}{\hbar}\frac{dk}{dt}\frac{d}{dk}\left(\frac{dE}{dk}\right)$$

$$\boxed{a = \frac{1}{\hbar}\frac{dk}{dt}\frac{d^2E}{dk^2}}$$

The mean square velocity

$$v^2 = v_x^2 + v_y^2 + v_z^2$$

Since mean square velocity along x-dir^n is equal to that along y & z direction.

i.e. $v_x^2 \pm v_y^2 = v_z^2$

$$v^2 = 3v_x^2 \Rightarrow v_x^2 = \frac{v^2}{3}$$

Now

$$P = \frac{Nmv^2}{3a^3}$$

$$P = \frac{Nmv^2}{3V} \quad \left[\because V = a^3\right]$$

$$PV = \frac{\cancel{3}\, Nmv^2}{3} \quad \text{——— (1)}$$

we have, ideal gas eq^n $PV = nRT$ —(

$$\frac{Nmv^2}{3} = nRT$$

$$v^2 = \frac{3nRT}{Nm}$$

$$N = n N_A$$

Total no. of electron = no. of moles x no. of electron in 1 mole.

$$u^2 = \frac{3\cancel{n}RT}{\cancel{n}N_A m}$$

$$u^2 = \frac{3RT}{N_A m}$$

$$u = \sqrt{\frac{3RT}{N_A m}} = \sqrt{\frac{3kT}{m}}$$

$$K = \frac{R}{N_A} = 1.38 \times 10^{-23} \, J \, K^{-1}$$

Electron Mobility, Conductivity and Resistivity

When a conductor doesnot have applied electric field across it, its conduction electron move randomly. Since there is no net flow of charge so there no current. When a potential difference is applied across it, the free electron tend to move in the direction opposite to that of applied electric field with a speed called drift speed V_d.

Let an electron of mass 'm' and charge 'e' is placed in an electric field E. The electric force experienced by it is

$$F = eE$$

Also from Newton's law

$$F = ma$$

Hence $ma = eE$

$$\boxed{a = \frac{eE}{m}}$$

The average ~~speed~~ drift speed of electron is $V_d = a\tau$ where $\tau \rightarrow$ average time between collision called mean free time or relaxation time

$$\therefore V_d = \frac{eE\tau}{m} \quad \text{——— (1)}$$

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Resistivity $\rho = \frac{1}{\sigma}$

$$\boxed{\rho = \frac{m}{ne^2\tau}}$$

Here

$$\sigma = \frac{J}{E}$$

$$\sigma = \frac{\frac{ne^2\tau \cancel{E}}{m}}{\cancel{E}}$$

$$\sigma = \frac{e\tau}{m} en$$

$$\boxed{\sigma = \mu en}$$

The electron mobility is defined as the drift speed per unit applied electric field.

Electron mobility $\mu = \frac{V_d}{E} = \frac{eE\tau}{Em}$

$$\boxed{\mu = \frac{e\tau}{m}} \quad \text{—— (2)}$$

Current density $J = \frac{I}{A} = \frac{neVA}{A}$

$$J = neV_d$$

$$\boxed{J = \frac{ne^2\tau}{m}E} \quad \left[V_d = \frac{e\tau}{m}E \right.$$

—— (3)

Also $J = \sigma E$ —— (4)

Comparing (3) & (4)

Electron Conductivity

$$\boxed{\sigma = \frac{ne^2\tau}{m}} \quad \text{—— (5)}$$

Einstein relation between Mobility and diffusion Coefficient

$$\frac{D}{\mu} = V_T$$

V_T = Thermal voltage

$$V_T = \frac{KT}{q}$$

$K = 1.38 \times 10^{-23}$ J/K (Boltzman constant)

$q = 1.6 \times 10^{-19}$ C

T = Kelvin $= 273.15$ K

$$\boxed{V_T = 23.5 \text{ mVolt}}$$

At room tempr

$V_T = 25$ mV

$$\boxed{\frac{D}{\mu} = \frac{KT}{q}}$$ Einstein relation

$$\boxed{\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = \frac{KT}{q}}$$

Here, -ve sign indicates that diffusion occurs towards decreasing concentration gradient.

Drift Current

$$J \propto E$$

$$J = \sigma E$$

$\boxed{\sigma = ne\mu}$

Extrinsic

$J = nq\mu_n E$ $\quad$ $J = pq\mu_p E$

$I = (nq\mu_n E)A$ $\quad$ $I = (pq\mu_p E)A$

Intrinsic

$$J = \sigma_i E$$

$$J = n_i q(\mu_n + \mu_p) E$$

$\boxed{\frac{I}{A} = \frac{q}{t \cdot A} = \phi}$

using ...
6. To determine the ...
7. To determine the low resis...
foster bridge
8. To plot a graph current and frequency in an L...
ii) the quality factor

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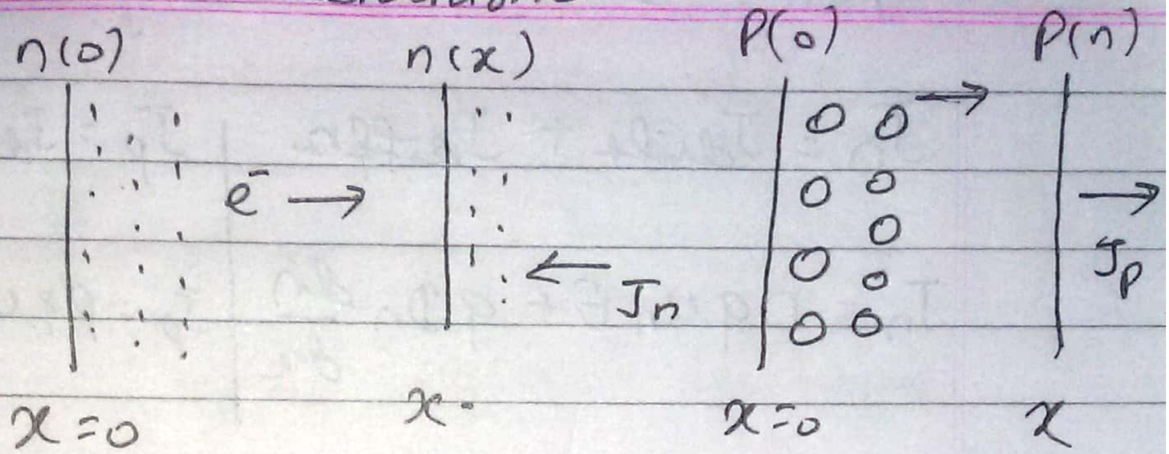
Diffusion mechanism



$J_n \propto \frac{dn}{dx}$

$J_n = q D_n \frac{dn}{dx}$ —①

Current is flowing in -ve x-dir^n.

$D_n \rightarrow$ diffusion coefficient of free electron

$J_p \propto \frac{dp}{dx}$

$J_p = -q D_p \left(\frac{dp}{dx}\right)$ —②

$D_p \rightarrow$ diffusion coefficient of holes.

$\boxed{\text{Unit } m^2/sec \text{ or } cm^2/sec}$

N-type

$J_{drift} = \sigma_n E$

$J_{drift} = nq\mu_n E$

$J_{diffusion} = q D_n \frac{dn}{dx}$

P-type

$J_{drift} = \sigma_p E$

$= pq\mu_p E$

$J_{diffusion} = -q D_p \frac{dp}{dx}$

Total Current density

$J_n = J_{drift} + J_{diffn}$ | $J_p = J_{drift} + J_{diffn}$

$J_n = nq\mu_n E + qD_n \frac{dn}{dx}$ | $J_p = pq\mu_p E - qD_p \frac{dp}{dx}$

For most materials the diffusion coefficient is found to vary with tempr as

$$D = D_0 e^{-\frac{Q}{KT}}$$

$$D_0 = \frac{1}{2} a^2 f$$

$a \rightarrow$ lattice constant

$f \rightarrow$ vibrational freq of lattice

$Q \rightarrow$ activation energy

6. To determine ...
7. To determine the low ... foster bridge
8. To plot a graph current and frequency in an ... ii) the quality factor

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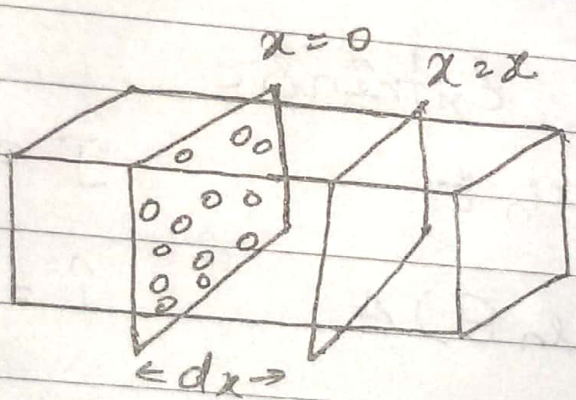
Diffusion of Electrons

If the number of either electron or holes is greater in one area compare to another area then carriers tend to move from region of higher concentration to the region of lower concentration even in the absence of applied voltage. This process is called diffusion. The electric current produced due to this process is known as diffusion current.



Natural process

Concentration gradient $= \frac{dn}{dx}$

Fick's Law (Adolf Fick)

The net rate of particles moving through an area (carrier flux) is directly proportional to concentration gradient.

$$\phi \propto \frac{dn}{dx} \Rightarrow \phi = -D \frac{dn}{dx}$$

$D \rightarrow$ diffusion coefficient

Alternative method

$$J = J_{drift} + J_{diffusion}$$

$$= \sigma E + eD\frac{dn}{dx}$$

$$J = neuE + eD\frac{dn}{dx}$$

Under equilibrium condition, total current zero.

$$neuE + eD\frac{dn}{dx} = 0$$

$$\Rightarrow \cancel{e}D\frac{dn}{dx} = -n\cancel{e}\mu E$$

$$\frac{dn}{n} = -\frac{\mu E}{D}dx$$

integrating

$$\ln(n) = -\frac{\mu E x}{D} + \ln(A)$$

$\ln(A) \to$ const

$$\ln\left(\frac{n}{A}\right) = -\frac{\mu E x}{D}$$

$$n = Ae^{-\frac{\mu E x}{D}} \quad \text{—— (1)}$$

from Boltzman statistics, ~~for an~~ for an electric field

$$n = Ae^{-\frac{eEx}{kT}} \quad \text{—— (2)}$$

$$\frac{\mu E x}{D} = \frac{eEx}{kT} \Rightarrow \boxed{\frac{D}{\mu} = \frac{kT}{e}}$$

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$$\mu = \frac{u_d}{E} = \frac{m/s}{volt/m} \qquad \Big| \; E = -\frac{dV}{dx}$$

$$\mu = \frac{m^2}{volt \cdot s}$$

$$\frac{D}{\mu} = V_T$$

$$D = \mu \cdot V_T$$

$$= \frac{m^2}{\cancel{volt} \cdot sec} \cdot \cancel{volt}$$

$$\boxed{D = m^2/sec}$$

6. To de...
7. To determine ...
foster bridge
8. To plot a graph current and frequ...
ii) the quality factor

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