Chapter 5

# Small Signal Low Frequency Analysis Model of BJT

## Small Signal Parameters

# The small signal current gains of BJT are;

$$\beta = \frac{i_c}{i_b}\Big|_{V_{CE}=const} \qquad \alpha = \frac{i_c}{i_e}\Big|_{V_{CB}=const}$$

# Emitter resistance ($r_e$):
It is the resistance offered by the base-emitter junction to the small signal i/p signal.

$$r_e = \frac{i_c}{i_e}\Big|_{V_{CE}=const.} \qquad \alpha = \frac{i_c}{i_b}\Big|_{V_{CB}=const}$$

The small signal emitter-base resistance of BJT also called also called emitter resistance.

$$r_e = \frac{V_{BE}}{i_c}\Big|_{V_{CE}=const.}$$

Since BE junction of BJT resembles to a forward biased PN junction diode, we can calculate $r_e$ in the similar way of dynamic resistance ($r_D$) of PN junction that we calculated diode.

i.e. $r_D = \frac{\eta V_T}{I}$

Taking $\eta = 1$ & $V_T = 0.026$ at room tempr

$$r_D = \frac{0.026}{I}$$

$$\therefore r_e = \frac{0.026}{I_E}$$

# Small signal collector-base resistance of BJT, also called collector resistance ($r_c$) is given by,

$$r_c = \frac{V_{cb}}{i_c}\Big|_{I_E=const}$$

Since collector-base junction is reverse biased, the value of $r_c$ is very high (MΩ).

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# $r_e$-model of Common-Base Configuration (small signal low frequency Analysis of CB configuration)

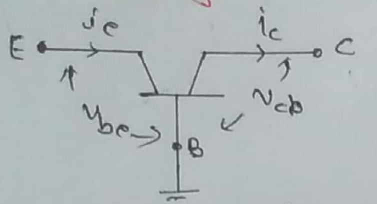



Fig: CB configuration

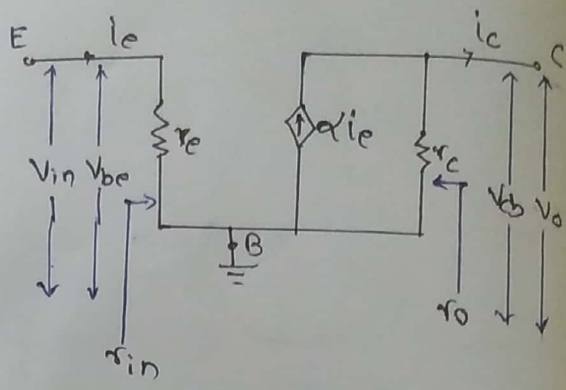



Fig: $r_e$-model of CB configuration.

Here, $V_{in} = V_{be}$

$V_o = V_{cb}$

$i_{in} = i_e$

$i_o = i_c$

i) I/P resistance ($r_{in}$)

$$r_{in} = \frac{V_{in}}{i_{in}} = \frac{V_{be}}{i_e} = r_e$$

ii) O/P resistance

$$r_o = \frac{V_o}{i_o} = \frac{V_{cb}}{i_c} = r_c$$

When biasing resistor & voltage are connected to CB configuration, its $r_e$-model is as given below,

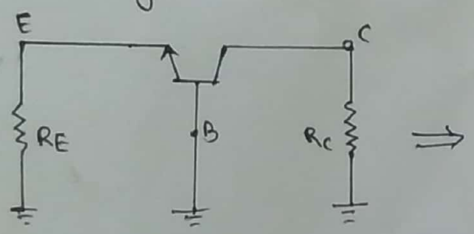



Fig: Biasing ckt

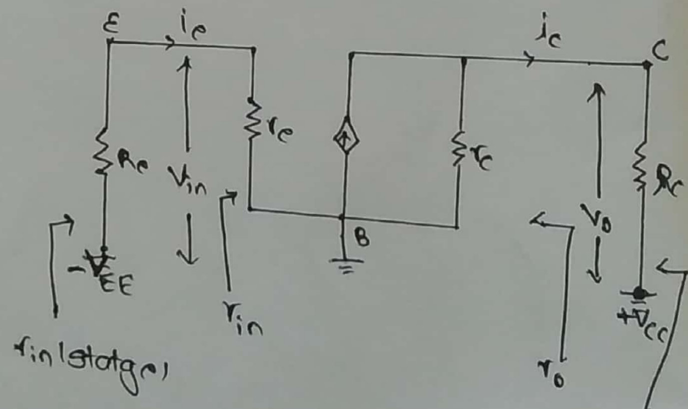



Fig: $r_e$-model of biasing ckt

iii) $r_{in}(\text{stage}) = R_E // r_e$

iv) $r_o(\text{stage}) = R_C // r_c \simeq R_C$ $(\because r_c >> R_C)$

v) Voltage Gain ($A_v$)

$$A_v = \frac{V_o}{V_{in}} = \frac{i_c R_C}{i_e r_e} = \frac{i_c}{i_e} \cdot \frac{R_C}{r_e} = \alpha \frac{R_C}{r_e}$$

Since $\alpha \simeq 1$

$$\therefore A_v = \frac{R_C}{r_e}$$

vi) Current Gain ($A_I$) $= \dfrac{i_o}{i_{in}} = \dfrac{i_c}{i_e} = \alpha \simeq 1$

When a transistor is driven by source voltage $V_s$ with resistance $r_s$ & load resistance $R_L$, then its $r_e$-model is,

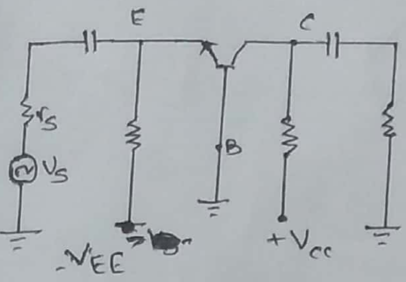



fig: CB amplifier ckt

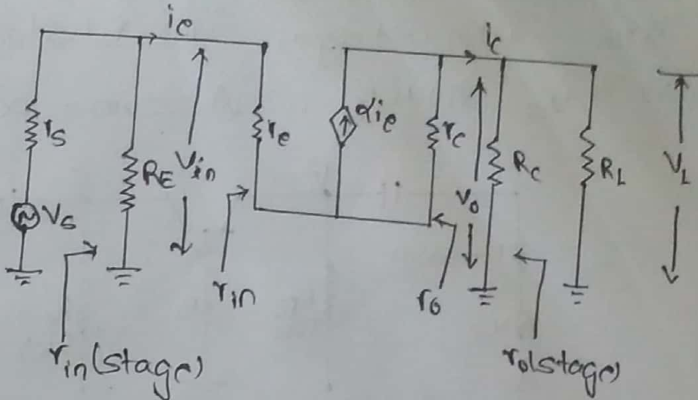
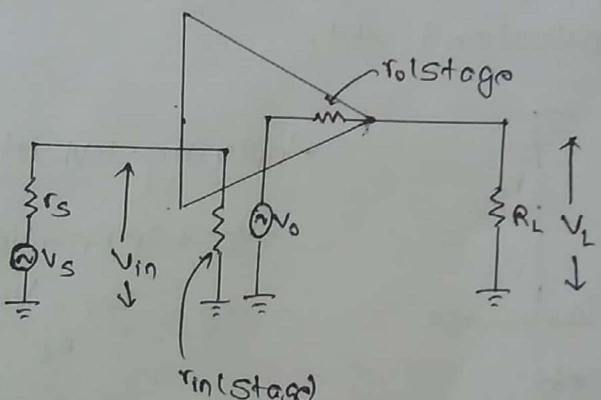



fig: $r_e$-model of CB amplier

The transistor amplifier can be represented by the transistor block as shown below,




Overall Gain (voltage) $(V_L/V_s)$

$$\frac{V_L}{V_s} = \frac{V_L}{V_o} \times \frac{V_o}{V_{in}} \times \frac{V_{in}}{V_s} = \frac{V_L}{V_o} \times A_V \times \frac{V_{in}}{V_s} = A_V \times \left(\frac{V_L}{V_o}\right) \times \left(\frac{V_{in}}{V_s}\right)$$

$$\therefore \frac{V_L}{V_s} = A_V \times \left(\frac{R_L}{r_{o(stage)} + R_L}\right) \times \left(\frac{r_{in(stage)}}{r_{in(stage)} + r_s}\right)$$

Overall current Gain $(i_L/i_s)$

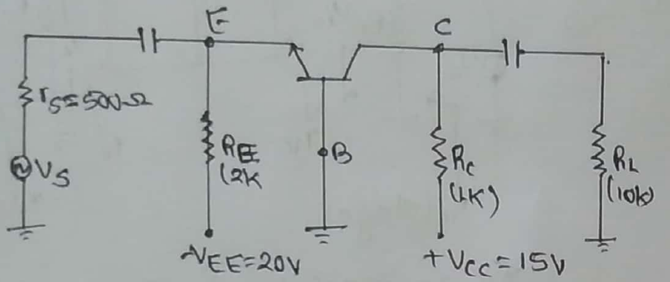
we have, $i_L = V_L/R_L$

$$\& \quad i_s = \frac{V_s}{r_s + r_{in(stage)}}$$

$$\therefore \frac{i_L}{i_s} = \frac{V_L/R_L}{\dfrac{V_s}{r_s + r_{in(stage)}}} = \frac{V_L}{V_s} \times \frac{r_s + r_{in(stage)}}{R_L}$$

$$\therefore \frac{i_L}{i_s} = \frac{V_L}{V_s}\left(\frac{r_s + r_{in(stage)}}{R_L}\right)$$

# For the transistor amplifier ckt shown below, draw $r_e$-model & find
i) $r_{in}$ ii) $r_{in}$ (stage). iii) $r_o$ (stage) iv) $A_V$
v) $V_L/V_s$ vi) $i_L/i_s$. Assume $\alpha = 1$.




Soln: We have, $r_e = \frac{0.026}{I_E}$

To find $I_E$, need to perform dc analysis of given ckt.
For this, draw dc equivalent ckt.

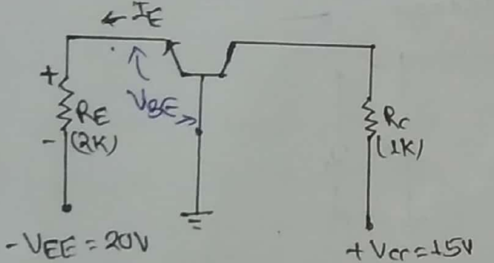



Fig: dc equt. ckt

Applying KVL at i/p loop,

$-V_{EE} + I_E R_E + V_{BE} = 0$

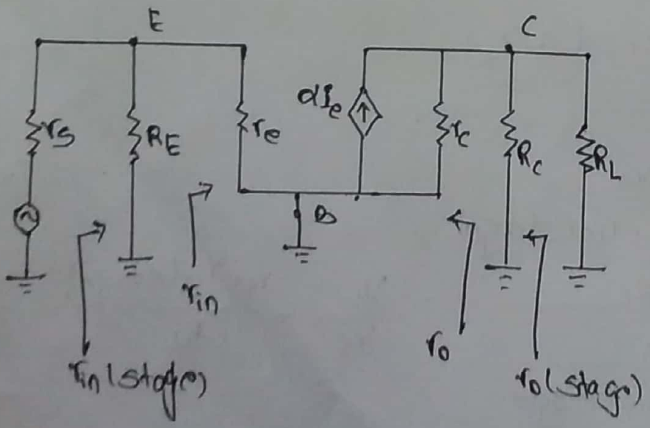
$\Rightarrow I_E = \frac{V_{EE} - V_{BE}}{R_E}$

or, $I_E = \frac{20 - 0.7}{2\times10^3}$

$\therefore I_E = 9.65 mA$

Thuse, $r_e = \frac{0.026}{9.65\times10^{-3}} = 2.69\Omega$

Now, $r_e$-model of given ckt is,




Here, i) $r_{in} = r_e$

$\therefore r_{in} = 2.69\Omega$

ii) $r_{in}(stage) = R_E // r_e = R_E // r_{in} = 2000 // 2.69$

$$= \frac{2000 \times 2.69}{2000 + 2.69} = 2.68\ \Omega$$

iii) $r_o(stage) = R_C // r_c \simeq R_C = 1K\Omega$

iv) $A_v = \alpha\left(\frac{R_C}{r_e}\right) = 1 \times \left(\frac{1000}{2.69}\right) = 371.74$

v) $$\frac{V_L}{V_S} = A_v\left(\frac{r_{in}(stage)}{r_s + r_{in}(stage)}\right)\left(\frac{R_L}{R_L + r_o(stage)}\right)$$

$$= 371.74 \times \left(\frac{2.68}{500 + 2.68}\right) \times \left(\frac{10}{10 + 1}\right)$$

$$= 1.798$$

vi) $$\frac{i_L}{i_S} = \frac{V_L}{V_S} \times \left(\frac{r_s + r_{in}(stage)}{R_L}\right)$$

$$= 1.798 \times \left(\frac{2.68 + 500}{10,000}\right)$$

$$= 0.09$$

## Small Signal Analysis of Common Emitter Configuration ($r_e$-model of CE configuration)

1) Emitter bypassed (with bypass capacitor)
2) Emitter unbypassed (without bypass capacitor)

### 1) Emitter Bypassed

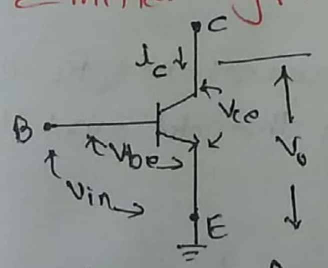



Fig: CE configuration

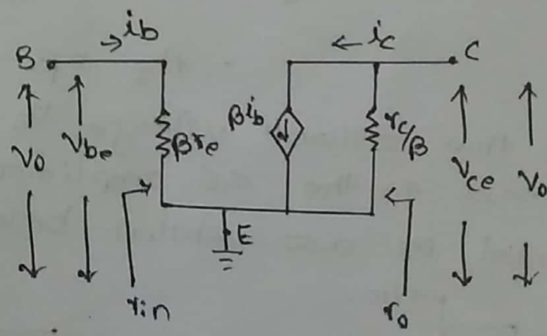



Fig: $r_e$-model of CE configuration

Here, $V_{in} = V_{be}$

$i_{in} = i_b$

$V_o = V_{ce}$

$i_o = -i_c$

Input resistance, $r_{in} = \frac{V_{in}}{i_{in}} = \frac{V_{be}}{i_b} = \frac{V_{be}}{i_e/\beta} = \beta \frac{V_{be}}{i_e} = \beta . r_e$

$\therefore r_{in} = \beta r_e$ $\quad (\because i_e \simeq i_c = \beta i_b \Rightarrow i_b = \frac{i_e}{\beta})$

Output resistance, $r_o = \frac{V_o}{i_o} = \frac{V_{ce}}{i_c} = \frac{V_{ce}}{\beta i_b}$

$$r_o = \frac{r_c}{\beta}$$

If the CE transistor is biased by dc voltage, then above ckt becomes,

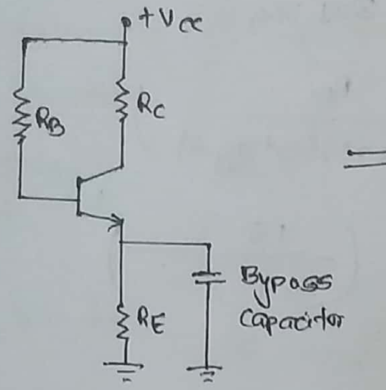



Fig: Emitter bypassed biasing ckt

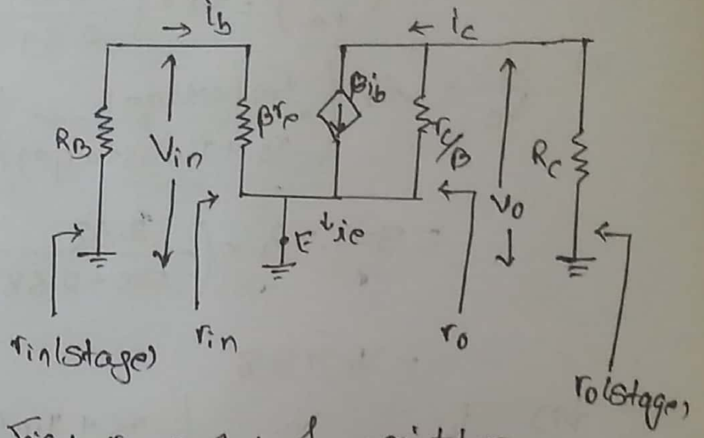



Fig: $r_e$-model of emitter bypassed CE configuration.

Here, $r_{in}(stage) = R_B // r_{in} = R_B // \beta r_e$

$r_o(stage) = R_c // r_c \approx R_c \quad (r_o >> R_c)$

Voltage gain $(A_v) = \frac{V_o}{V_{in}} = \frac{-i_c R_c}{i_b \beta r_e} = \frac{-\beta i_b R_c}{\beta r_e i_b} = \frac{-R_c}{r_e}$

$$\therefore A_v = -\frac{R_c}{r_e}$$

(Here -ve sign indicates i/p & o/p are out of phase)

Current gain $(A_I) = \frac{i_o}{i_{in}} = \frac{i_c}{i_b}$ (neglecting -ve sign i.e. $i_o = -i_c$)

$$\therefore A_I = \beta$$

When the source voltage $V_s$ & load resistance $R_L$ is connected to the CE amplifier ckt then the ckt and its $r_e$-model is as shown below.

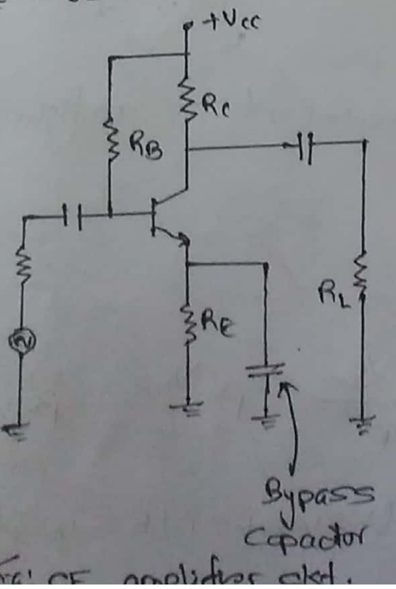



Fig: CE amplifier ckt.

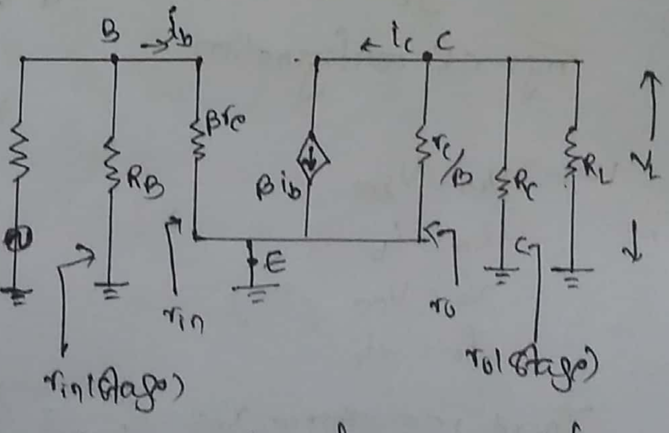
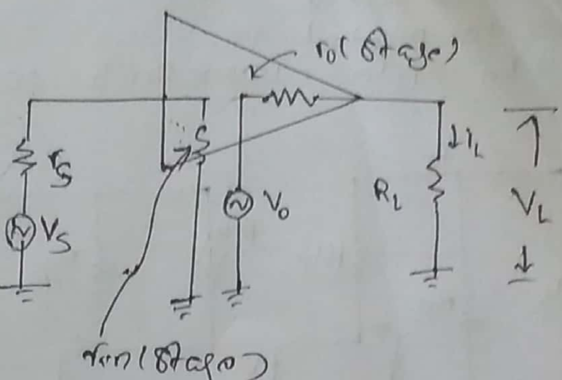



Fig: $r_e$-model of CE amplifier (Emitter bypassed)

Overall Voltage Gain ($V_L/V_s$)




Here,

$$\frac{V_L}{V_s} = \frac{V_L}{V_o} \times \frac{V_o}{V_{in}} \times \frac{V_{in}}{V_L}$$

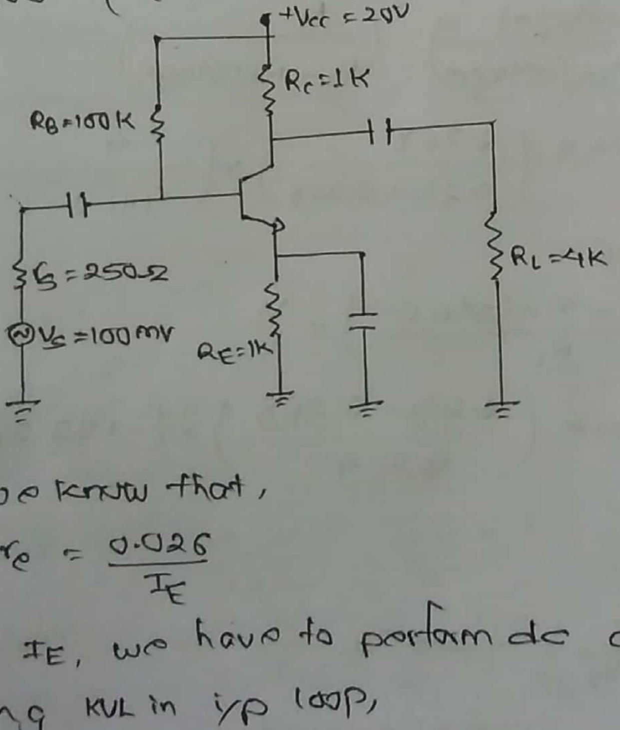
$$\therefore \frac{V_L}{V_s} = A_v\left(\frac{r_{in}(\text{Stage})}{r_s + r_{in}(\text{Stage})}\right) \times \left(\frac{R_L}{R_L + r_o(\text{Stage})}\right)$$

Overall current gain ($i_L/i_s$)

$$\frac{i_L}{i_s} ;\quad i_L = V_L/R_L \ \&\ i_s = \frac{V_s}{r_s + r_{in}(\text{Stage})} = \frac{V_L/R_L}{\left(\frac{V_s}{(r_s + r_{in}(\text{Stage}))}\right)}$$

$$\therefore \frac{i_L}{i_s} = \frac{V_L}{V_s}\left(\frac{r_s + r_{in}(\text{Stage})}{R_L}\right)$$

# For the transistor amplifier ckt shown below, find its $r_e$-model & $r_{in}$, $r_{in}$ (Stage), $A_v$, $V_L/V_s$ & Also find the value of $V_L$ if $V_s = 100$. (Take $\beta = 100$)




Soln: we know that,

$$r_e = \frac{0.026}{I_E}$$

To find $I_E$, we have to perform dc analysis of given ckt,

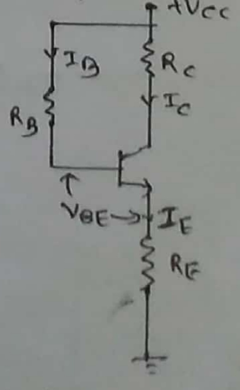



fig: dc equivalent ckt

Applying KVL in i/p loop,

$$V_{cc} = I_B R_B + V_{BE} + I_E R_E$$

$$= \frac{I_E}{\beta} R_B + V_{BE} + I_E R_E$$

$$\Rightarrow I_E = \frac{V_{cc} - V_{BE}}{R_B/\beta + R_E}$$

$$= \frac{20 - 0.7}{\frac{100}{100} + 1} = \frac{19.3}{2} = 9.65\,mA$$

Thus, $r_e = \dfrac{0.026}{9.65 \times 10^{-3}} = 2.69\,\Omega$

Now, its re-model is,

i) $r_{in} = \beta r_e = 100 \times 2.69 = 269\,\Omega = 0.269\,K\Omega$

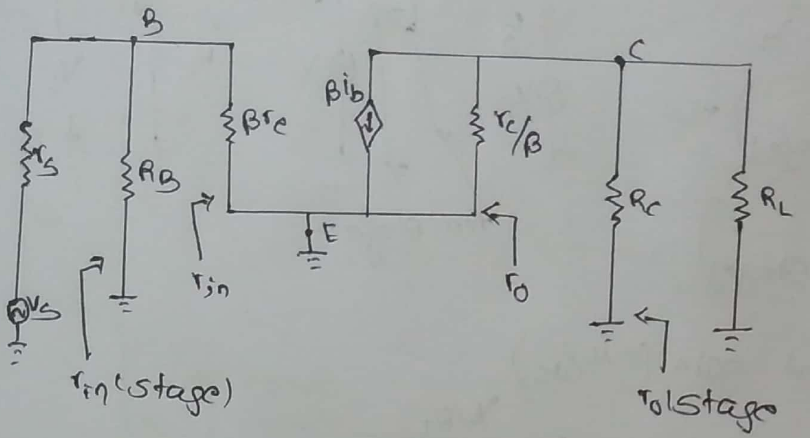



Fig: re-model of CE amplifier ckt

ii) $r_{in}(stage) = R_B // r_{in} = 100 // 0.269$
$= 0.268\,K\Omega$

iii) $r_o(stage) = R_C // r_c/\beta \simeq R_C = 1K$

iv) $A_v = -\dfrac{R_C}{r_e} = -\dfrac{1 \times 10^3}{2.69} = -371.74$

v) $\dfrac{V_L}{V_S} = A_v \left( \dfrac{r_{in}(stage)}{r_S + r_{in}(stage)} \right) \left( \dfrac{R_L}{R_L + R_o(stage)} \right)$

$= -371.74 \times \left( \dfrac{0.268}{0.25 + 0.268} \right) \times \left( \dfrac{4}{4+1} \right)$

$= -153.26$

vi) $\dfrac{i_L}{i_S} = \cancel{A_v} \left( \dfrac{r_S + r_{in}(stage)}{R_L} \right) \times \dfrac{V_L}{V_S}$

$= \cancel{-371.74 \times} \left( \dfrac{0.25 + 0.268}{\cancel{0.2}\ 4} \right) \times (-153.26)$

$= -19.84$

vii) $V_S = 100\,mV$

we have, $\dfrac{V_L}{V_S} = -153.26$

$\Rightarrow V_L = -153.26 \times 100$

$= -15.326\,V$ //

## 2) Emitter Unbypassed

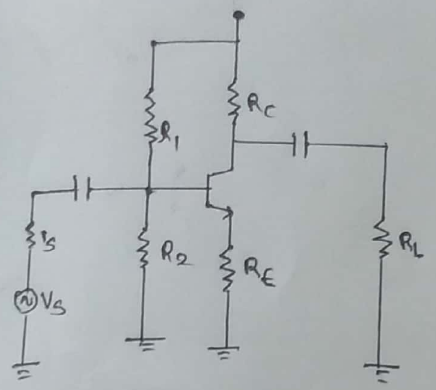



Fig: Emitter unbypassed CE configuration

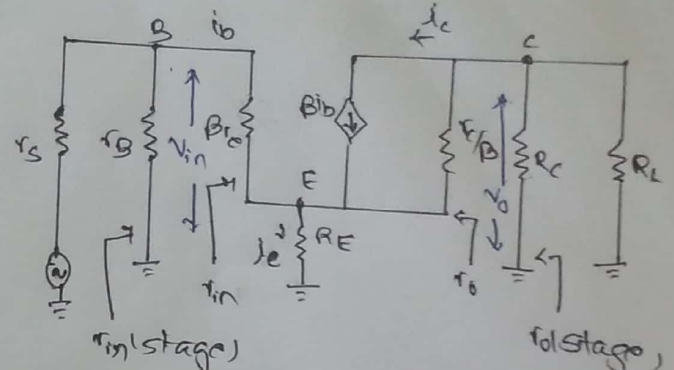



Fig: re-model of emitter unbypassed CE configuration.

Here, Input voltage $(V_{in}) = i_b \cdot \beta r_e + i_e R_E$

$$= \beta i_b r_e + \beta i_b R_E$$

$$= \beta i_b (r_e + R_E)$$

Output voltage $(V_o) = -i_c R_c$

i) Input resistance $(r_{in}) = \dfrac{V_{in}}{i_{in}} = \dfrac{\beta i_b (r_e + R_E)}{i_b} = \beta(r_e + R_E)$

ii) Output resistance $(r_o) = r_c/\beta$

iii) $r_o(\text{stage}) = R_c \parallel r_o = R_c \parallel r_c/\beta \simeq R_c$

iv) $r_{in}(\text{stage}) = R_B \parallel r_{in} = R_B \parallel \beta(r_e + R_E)$

v) Voltage gain, $A_V = \dfrac{V_o}{V_{in}} = \dfrac{-i_c R_c}{\beta i_b (r_e + R_E)} = -\dfrac{\beta i_b R_c}{\beta i_b (r_e + R_E)}$

$$= \frac{-R_c}{r_e + R_E}$$

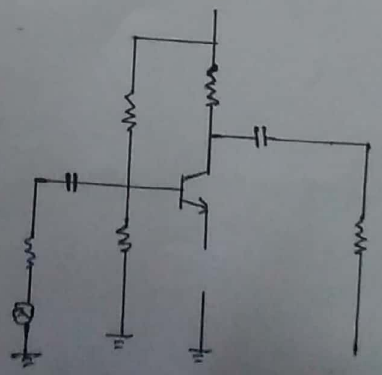
vi) Current gain, $A_I = \dfrac{i_o}{i_{in}} = \dfrac{i_c}{i_b}$ $\quad$ ($\because i_o = -i_c$, & neglect negative sign)

$$= \beta$$

vii) Overall voltage gain, $\dfrac{V_L}{V_S} = A_V \left( \dfrac{r_{in}(\text{stage})}{r_s + r_{in}(\text{stage})} \right) * \left( \dfrac{R_L}{r_o(\text{stage}) + R_L} \right)$

viii) Overall current gain, $\dfrac{i_L}{i_s} = \dfrac{V_L}{V_S} \times \left( \dfrac{r_s + r_{in}(\text{stage})}{R_L} \right)$

# For the ckt diagram shown, find $r_{in}$, $r_{in}$ (stage), $r_o$(stage) And $\frac{V_L}{V_S}$.
(Take $\beta = 100$)



i) Input resistance $(r_{in}) = \frac{V_{in}}{i_{in}}$

where, $V_{in} = I_b \cdot \beta r_e + i_e \cdot R_E = i_b \beta r_e + \beta i_b R_E$

$= \beta i_b (r_e + R_E)$

& $i_{in} = i_b$

$\therefore r_{in} = \frac{\beta i_b (r_e + R_E)}{i_b} = \beta (r_e + R_E)$

ii) $r_{in}(stage) = R_B // r_{in} = R_B // \beta (r_e + R_E)$

iii) $r_o = r_c/\beta$

iv) $r_o(stage) = R_c // r_o = R_c // r_c/\beta \simeq R_c \quad (R_c << r_c/\beta)$

v) voltage gain $(A_v) = \frac{V_o}{V_{in}} = \frac{-i_c R_c}{\beta i_b (r_e + R_E)} = \frac{-\beta i_b R_c}{\beta i_b (r_e + R_E)}$

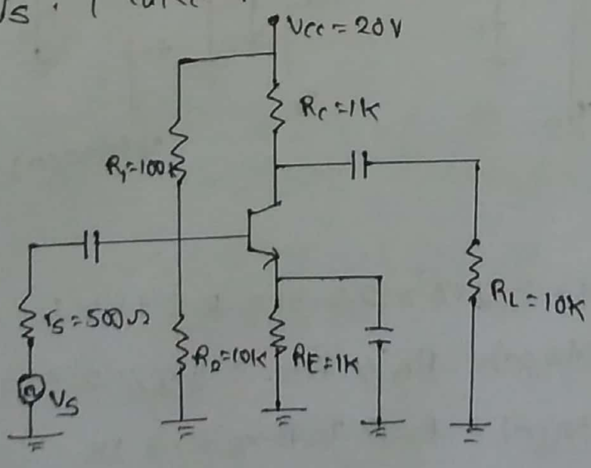
$\therefore A_v = \frac{-R_c}{r_e + R_E}$

vi) Current gain $(A_I) = \frac{i_o}{i_{in}} = \frac{i_c}{i_b} = \beta$

vii) overall voltage gain $\left(\frac{V_L}{V_S}\right) = A_v \left(\frac{r_{in}(stage)}{r_s + r_{in}(stage)}\right)\left(\frac{R_L}{R_L + r_o(stage)}\right)$

viii) Overall current gain, $\frac{I_L}{i_s} = \frac{V_L}{V_S}\left(\frac{r_s + r_{in}(stage)}{R_L}\right)$

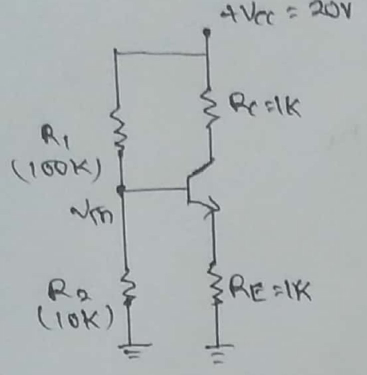
# For the ckt diagram given find $r_{in}$, $r_{in}(stage)$, $r_o(stage)$ $A_v$ & $V_L/V_S$. (Take $\beta = 100$)



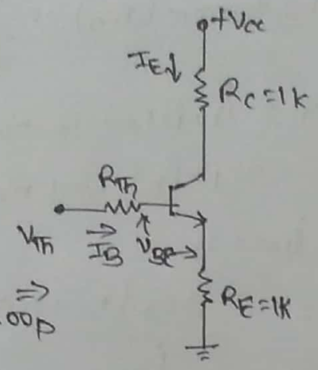

We know that,

$r_e = \frac{0.026}{I_E}$

To find $I_E$, we need to perform dc analysis of given ckt.




Thevenins equt ckt $\longrightarrow$




Here, $R_{Th} = R_1 // R_2 = 100 // 10 = 9.8 K\Omega$

$$V_{Th} = \frac{R_2}{R_1 + R_2} \times V_{cc} = \frac{10}{10+100} \times 20 = 1.81\,V$$

Applying KVL at i/p loop,

$$V_{Th} = I_B R_{Th} + V_{BE} + I_E R_E$$

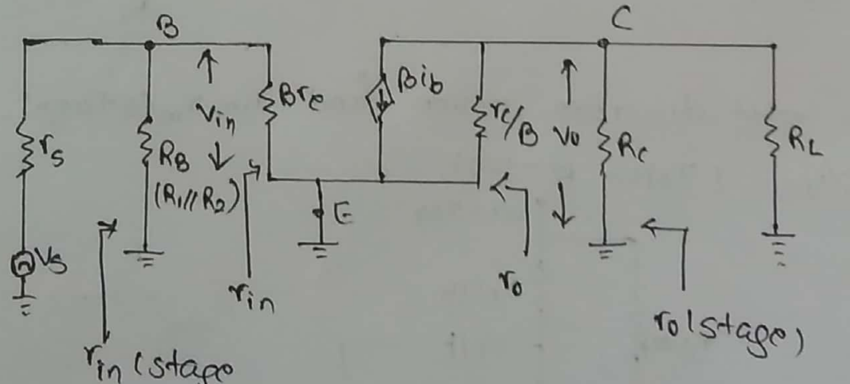
$$= \frac{I_E}{\beta} R_{Th} + V_{BE} + I_E R_E$$

$$\Rightarrow I_E = \frac{V_{Th} - V_{BE}}{R_{Th}/\beta + R_E} = \frac{1.81 - 0.7}{\frac{9.8}{100} + 1} = 1.01\,mA$$

Thus,

$$r_e = \frac{0.026}{1.01 \times 10^{-3}} = 25.71\,\Omega$$

Now, the re-model of given ckt is,




Here, i) $r_{in} = \beta r_e = 100 \times 25.71 = 2.571\,K\Omega$

ii) $r_{in}(stage) = R_B // \beta r_e = 9.8 // 2.571 = 2.03K$

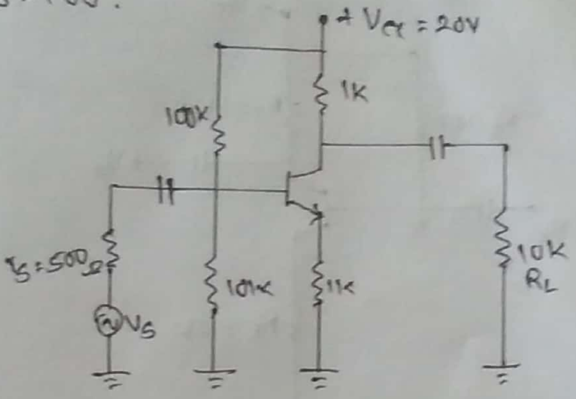
iii) $r_o(stage) = R_C // r_c/\beta \simeq R_C = 1K$

iv) $A_V = -\frac{R_C}{r_e} = \frac{-1000}{25.71} = -38.8$

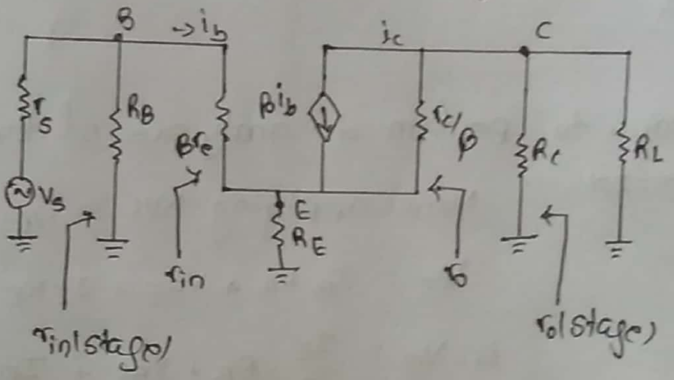
v) $$\frac{V_L}{V_S} = A_V \left(\frac{r_{in}(stage)}{r_s + r_{in}(stage)}\right)\left(\frac{R_L}{R_L + r_o(stage)}\right)$$

$$= -38.8 \times \left(\frac{2.03}{0.5 + 2.03}\right) \times \left(\frac{10}{10+1}\right) = -28.38$$

# Draw $r_e$-model & find $r_{in}$, $r_{in}(stage)$, $r_o(stage)$, $A_v$, $V_L/V_s$ & $i_L/i_s$. Take $\beta = 100$.




Soln: Here, $r_e = \dfrac{0.026}{I_E} = \dfrac{0.026}{1.01\times10^{-3}} = 25.71\,\Omega$




i) $r_{in} = \beta(r_e + R_E)$

$= 100(25.71 + 1000) = 102.57\,K\Omega$

ii) $r_{in}(stage) = R_B \parallel r_{in} = (R_1 \parallel R_2) \parallel r_{in} = 9.8K \parallel 102.57$

$= 8.94$

iii) $r_o(stage) = R_c \parallel r_c/\beta \simeq R_c = 1K$

iv) $A_v = \dfrac{-R_c}{r_e + R_E} = \dfrac{1000}{25.71+1000} \approx -0.97$

v) $\dfrac{V_L}{V_s} = A_v\left(\dfrac{r_{in}(stage)}{r_s + r_{in}(stage)}\right) \times \left(\dfrac{R_L}{r_o(stage) + R_L}\right)$

$= -0.97 \times \left(\dfrac{8.94}{0.5+8.94}\right) \times \left(\dfrac{10}{10+1}\right)$

$= -0.83$.

vi) $\dfrac{i_L}{i_s} = \dfrac{V_L}{V_s}\left(\dfrac{r_s + r_{in}(stage)}{R_L}\right)$

$= -0.83\left(\dfrac{0.5+8.94}{10}\right) = -0.78$

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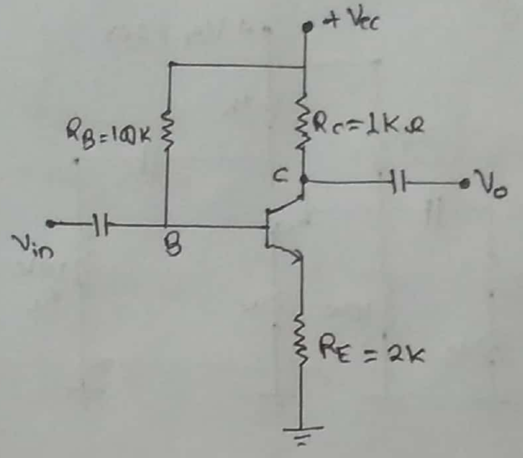
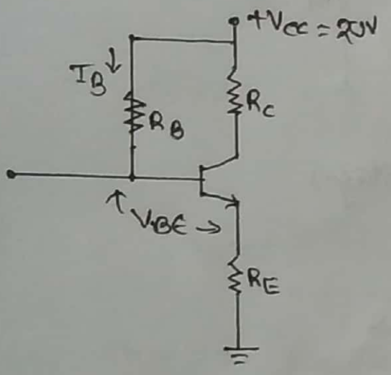
# Draw $r_e$-model & find $r_{in}$, $r_{in}(stage)$, $r_o(stage)$, $A_v$. Take $\beta = 100$.




Soln: We know that,

$$r_e = \frac{0.026}{I_E}$$

To find $I_E$, we have to perform dc analysis of the ckt.




Now, applying KVL in i/p loop

$$V_{cc} = I_B R_B + V_{BE} + I_E R_E$$

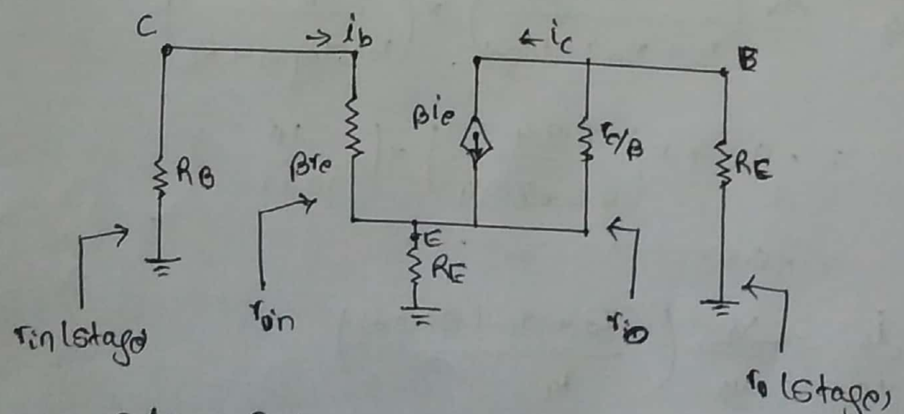
$$\text{or, } V_{cc} = \frac{I_E}{\beta} R_B + V_{BE} + I_E R_E$$

$$\text{or, } I_E = \frac{V_{cc} - V_{BE}}{R_B/\beta + R_E}$$

$$\text{or, } I_E = \frac{20 - 0.7}{100/100 + 2}$$

$$= 6.43 mA$$

$$\therefore r_e = \frac{0.026}{6.43 \times 10^{-3}} = 4.04 \Omega$$




i) $r_{in} = \beta(r_e + R_E)$

$= 100 \times (4.04 + 2000)$

$= 200.404 k\Omega$

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ii) $r_{in}(\text{stage}) = R_B // r_{in} = 100 // 200.404$

$= 66.71\,K\Omega$

iii) $r_o(\text{stage}) \simeq R_C // r_c/\beta = R_C = 1K\Omega$

iv) $A_V = -\dfrac{R_C}{r_e - R_E} = -\dfrac{1000}{4.04 + 2000} = -0.4989$

## $r_e$ - model of Common Collector Configuration

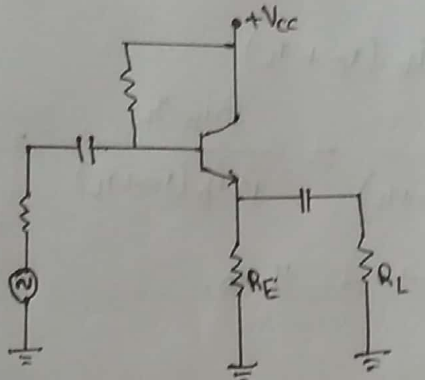
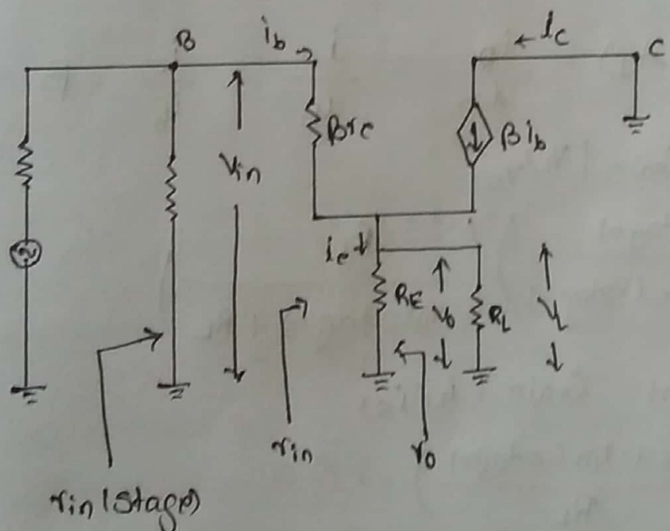



Fig: Common Collector Configuration




* If o/p is taken from RE side if no resistor in Vcc side, then common collector configuration

Fig: $r_e$ - model

i) Input resistance ($r_{in}$)

$r_{in} = \dfrac{V_{in}}{I_{in}}$

$V_{in} = i_b \cdot \beta r_e + i_e (R_E // R_L)$ $\quad (\because r_L = R_E // R_L)$

$= i_b \beta r_e + i_e \cdot r_L$

$= i_b \beta r_e + \beta i_b r_L$

$V_{in} = \beta i_b (r_e + r_L)$

$I_{in} = i_b$

Thus, $r_{in} = \dfrac{\beta i_b (r_e + r_L)}{i_b} = \beta (r_e + r_L)$

ii) $r_{in}(stage) = R_B // r_{in} = R_B // \beta(r_e + r_L)$

iii) $r_o(stage) = R_E // \left( \frac{r_s // R_B + \beta r_e}{\beta} \right)$

iv) Voltage Gain $(A_V) = \frac{V_o}{V_{in}} = \cancel{\frac{i_e (R_E // R_L)}{i_e r_L}} = \cancel{\frac{\beta i_b r_L}{\beta i}}$

$$v_o = i_e (R_E // R_L)$$
$$= i_e . r_L$$
$$\& \; V_{in} = \beta i_b (r_e + r_L)$$

$$A_V = \frac{i_e . r_L}{\beta i_b (r_e + r_L)} = \frac{\beta i_b . r_L}{\beta i_b (r_e + r_L)} = \frac{r_L}{r_e + r_L}$$

If $R_L$ is not connected then,

$$r_L = R_E$$

$$\therefore A_V = \frac{R_E}{r_e + R_E}$$

v) Current Gain $(A_I) = \frac{i_o}{i_{in}} = \frac{i_e}{i_b} = \frac{i_c}{i_b} = \beta$

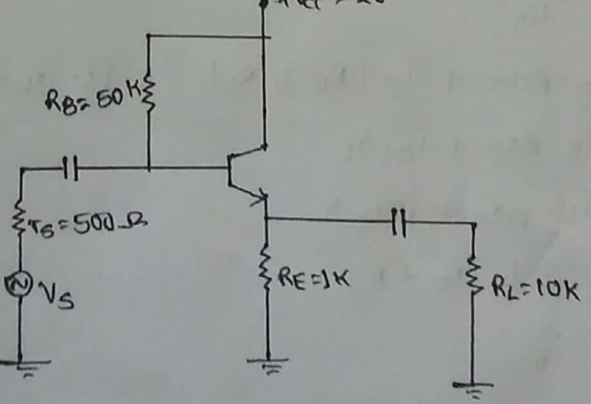
vi) Overall Voltage Gain $(V_L/V_S)$

$$\frac{V_L}{V_S} = A_V \left( \frac{r_{in}(stage)}{r_s + r_{in}(stage)} \right) \left( \frac{R_L}{r_o(stage) + R_L} \right)$$

vii) Overall Current Gain $(i_L / i_s)$

$$\frac{i_L}{i_s} = \frac{V_L}{V_S} \left( \frac{r_s + r_{in}(stage)}{R_L} \right)$$

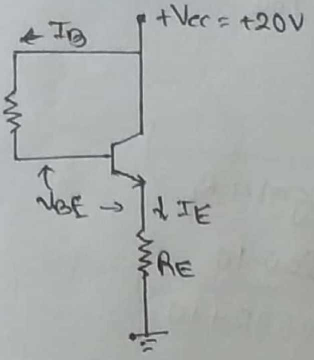
# For the transistor amplifier ckt shown below, draw its $r_e$-model & find $r_{in}$, $r_{in}(stage)$, $r_o(stage)$, $A_V$ & $V_L/V_S$. $(\beta = 100)$




Soln: We know that

$$r_e = \frac{0.026}{I_E}$$

To find $I_E$, we need to perform dc analysis of given ckt,




Applying KVL in i/p loop,

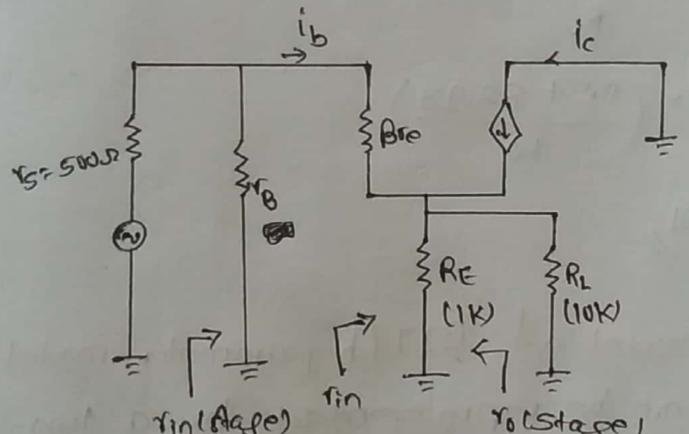
$$V_{cc} = I_B R_B + V_{BE} + I_E R_E$$

$$= \frac{I_E}{\beta} R_B + V_{BE} + I_E R_E$$

$$\Rightarrow I_E = \frac{V_{cc} - V_{BE}}{R_B/\beta + R_E} = \frac{20-0.7}{50/100 + 1}$$

$$= \frac{19.3}{1.5} = 12.86mA$$

Thus, $r_e = \frac{0.026}{12.86\times10^{-3}} = 2.02\Omega$




$r_e$ - model

Now,

i) $r_{in} = \beta(r_e + r_L)$

where, $r_L = R_E // R_L = 1//10 = 0.98K\Omega$

$r_{in} = 100\ (2.02 + 980) = 98.202K\Omega$

ii) $r_{in}(stage) = R_B // r_{in}$

$= 50 // 98.202 = 82.93K$

iii) $r_o\ (stage) = R_E // \left(\frac{r_s // R_B + \beta r_e}{\beta}\right)$

$= 1 // \left(\frac{0.5//50 + 100 \times 2.02\times10^{-3}}{100}\right)$

$= 1 // \left(\frac{0.495 + 0.202}{100}\right)$

$= 1 // 0.00697$

$= 0.00692\ K\Omega$

v) Voltage gain ($A_v$) $= \dfrac{r_L}{r_e + r_L} = \dfrac{980}{2.02 + 980} = 0.9979$

vi) Overall Voltage Gain ($V_L/V_S$)

$$\frac{V_L}{V_S} = A_v \left( \frac{r_{in}(\text{stage})}{r_s + r_{in}(\text{stage})} \right) \left( \frac{R_L}{r_o(\text{stage}) + R_L} \right)$$

$$= 0.99 \left( \frac{82.93}{500\Omega + 82.93} \right) \left( \frac{\cancel{980}\ 10}{0.00692 + 10} \right)$$

$$= 0.99 \left( \frac{82.93}{0.5 + 82.93} \right) \times \left( \frac{10}{10 + 0.00692} \right)$$

$$= 0.983$$

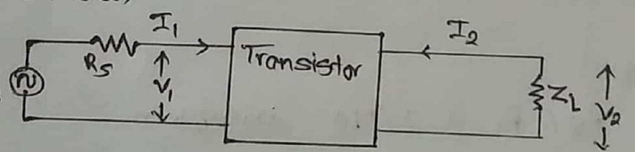
vii) $\dfrac{i_L}{i_S} = \dfrac{V_L}{V_S} \left( \dfrac{r_s + r_{in}(\text{stage})}{R_L} \right)$

$$= 0.983 \times \left( \frac{0.5 + 82.93}{10} \right)$$

$$= 8.201$$

## Hybrid Parameter model of BJT (h-parameter model of BJT)

A BJT amplifier ckt can be represented by a two-port n/w as shown below,




Here, $V_1$ = I/p voltage

$I_1$ = I/p current

$V_2$ = O/p voltage

$I_2$ = O/p current

$Z_L$ = load impedance

$R_S$ = Source resistance

$V_S$ = source voltage

The h-parameter eqn for above 2-port n/w is given by,

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

where, $h_{11} = h_i \rightarrow$ i/p imedance (resistance)

$h_{12} = h_r \rightarrow$ reverse voltage gain

$h_{21} = h_f \rightarrow$ forward current gain

$h_{22} = h_o \rightarrow$ o/p admittance

Thus, $\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_i & h_r \\ h_f & h_o \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$

i.e, $V_1 = h_i I_1 + h_r V_2$ ——— (i)

& $I_2 = h_f I_1 + h_o V_2$ ——— (ii)

From above two-port n/w,

$V_2 = -I_2 Z_L$ ——— (iii)

From above eqns we can draw h-parameter mode of transistor as below:

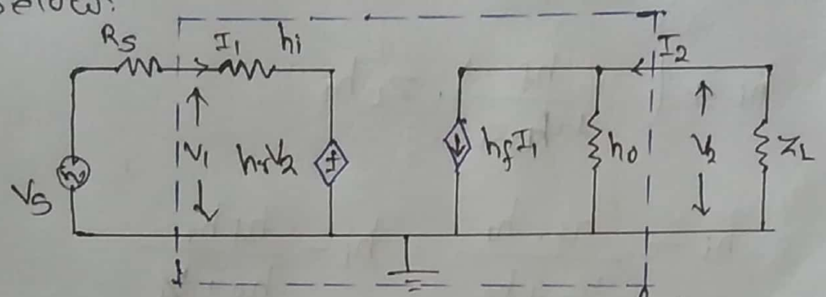



Fig: h-parameter model of transistor

Here, i) Current gain $(A_I) = -\frac{I_2}{I_1}$ ——— (iv)

From eqn (2), $I_2 = h_f I_1 + h_o V_2$

or, $I_2 = h_f I_1 + h_o(-I_2 Z_L)$

or, $I_2 = h_f I_1 - h_o I_2 Z_L$

or, $I_2(1 + h_o Z_L) = h_f I_1$

or, $\frac{I_1}{I_2} = \frac{1 + h_o Z_L}{h_f}$ or, $\frac{I_2}{I_1} = \frac{h_f}{1 + h_o Z_L}$

$\therefore A_I = -\left(\frac{h_f}{1 + h_o Z_L}\right)$

2) Voltage Gain $(A_V) = \left(\frac{V_o}{V_{in}}\right) = \frac{V_2}{V_1} = \frac{-I_2 Z_L}{V_1}$

$= -\frac{(-A_I I_1) Z_L}{V_1}$ ($\because$ From eqn (iv) $A_I = -\frac{I_2}{I_1}$)

$= \frac{A_I . Z_L}{V_1 / I_1}$

$\therefore A_V = \frac{A_I Z_L}{Z_i}$

where, $Z_i = \frac{V_1}{I_1} \Rightarrow$ i/p impedance.

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iii) I/p impedance ($Z_i$)

$$Z_i = \frac{\text{i/p voltage}}{\text{i/p current}} = \frac{V_1}{I_1}$$

From eqn (a),

$$V_1 = I_1 h_i + h_r V_2 = I_1 h_i + h_r(-I_2 . Z_L)$$

$$= I_1 h_i - h_r I_2 Z_L = I_1 h_i - h_r(-A_I I_1) Z_L$$

$$= I_1 h_i + A_I I_1 Z_L h_r$$

or, $V_1 = I_1 (h_i + A_I Z_L h_r)$

or, $$\frac{V_1}{I_1} = h_i + \left( \frac{-h_f}{1+h_o Z_L} \right) . Z_L h_r$$

$$= h_i - \frac{h_f . h_r}{1/Z_L + h_o} = h_i - \frac{h_f h_r}{h_o + Y_L}$$

where, $Y_L = 1/Z_L$

$$\therefore Z_i = \frac{V_1}{I_1} = h_i - \frac{h_f h_r}{h_o + Y_L}$$

iv) O/p admittance ($Y_o$) & O/p impedance ($Z_o$)

$$Y_o = \frac{\text{o/p current}}{\text{o/p voltage}} \text{ with } V_s = 0$$

$$= \frac{I_2}{V_2} \text{ with } V_s = 0 \text{ (neglecting -ve sign)}$$

We have, $I_2 = h_f I_1 + h_o V_2$

So, $$Y_o = \frac{h_f I_1 + h_o V_2}{V_2} = h_f \frac{I_1}{V_2} + h_o = h_o + h_f \frac{I_1}{V_2}$$

To find $I_1/V_2$, apply KVL in i/p side of the h-model.

$$V_s = I_1 R_s + I_1 h_i + h_r V_2$$

or, $0 = I_1 (R_s + h_i) + h_r V_2$

or, $I_1 (R_s + h_i) = -h_r V_2$

or, $$\frac{I_1}{V_2} = -\frac{h_r}{R_s + h_i}$$

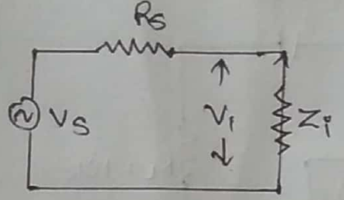
$$\therefore Y_o = h_o + h_f \left( -\frac{h_r}{R_s + h_i} \right)$$

& $Z_0 = 1/Y_0$

v) Overall Voltage Gain, $(A_{VS}) = \frac{V_2}{V_S} = \frac{V_2}{V_1} \times \frac{V_1}{V_S}$

$$= A_V \left( \frac{V_1}{V_S} \right)$$

— To find $V_1/V_S$, let's draw Thevenins eqvt ckt of i/p side.




$$\therefore V_1 = \frac{Z_i}{Z_i + R_S} \times V_S$$

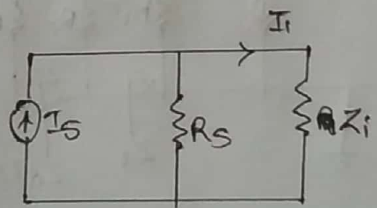
$$\text{or, } \frac{V_1}{V_S} = \frac{Z_i}{Z_i + R_S}$$

— Thus, $A_{VS} = A_V \times \left( \frac{Z_i}{Z_i + R_S} \right)$

vi) Overall current gain $(A_{IS}) = \frac{-I_2}{I_S} = -\frac{I_2}{I_1} \times \frac{I_1}{I_S}$

$$= A_I \times \frac{I_1}{I_S}$$

— To find $I_1/I_S$, draw Norton's-eqvt ckt




Here, $I_1 = \frac{R_S}{R_S + Z_i} \times I_S$

$$\text{or, } \frac{I_1}{I_S} = \frac{R_S}{R_S + Z_i}$$

$$\therefore A_{IS} = A_I \times \left( \frac{R_S}{R_S + Z_i} \right)$$

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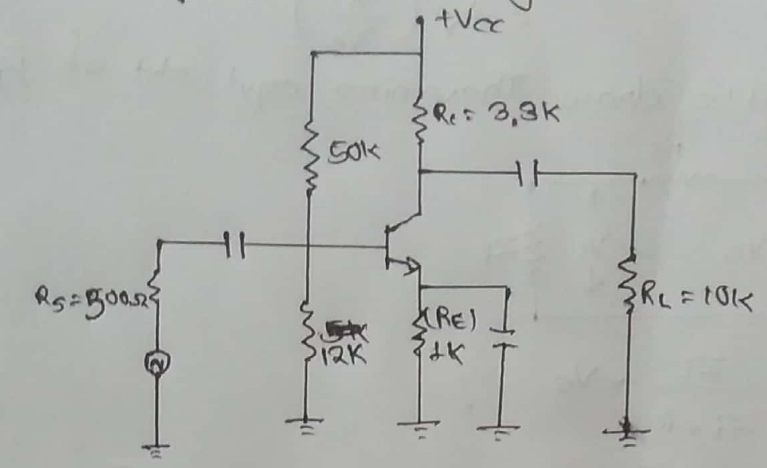
# For the given transistor amplifier ckt, the h-parameters are, $h_{ie} = 1600\,\Omega$, $h_{fe} = 80$, $h_{re} = 20\times10^{-4}$ & $h_{oe} = 20\mu\mho$. Find $Z_i$, $Z_i$ (stage), $Z_o$, $Z_o$ (stage), $A_I$, $A_V$, $A_{VS}$ & $A_{IS}$.




$h_{re}, h_{fe}, h_{oe}$
↳ common emitter

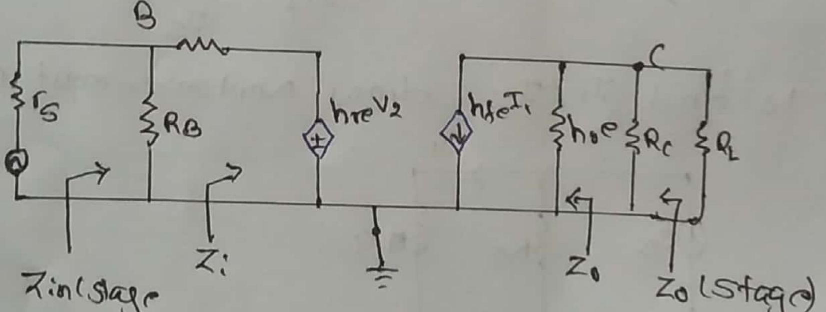
Soln: Given, $h_{ie} = 1600\,\Omega$

$h_{fe} = 80$

$h_{re} = 20\times10^{-4}$

$h_{oe} = 20\mu S$

The h-parameter model of given ckt is,




Here,

$Z_L = R_C \,//\, R_L = 3.3 \,//\, 10 = 2.48\,K\Omega$

$R_B = R_1 \,//\, R_2 = 50 \,//\, 12 = 9.8\,K\Omega$

Now, i) $Z_i = h_{ie} - \dfrac{h_{fe} * h_{re}}{h_{oe} + Y_L}$

where, $Y_L = 1/Z_L = \dfrac{1}{2.48\times10^{3}} = 4.03\times10^{-4}$ Ser

$\therefore Z_i = 1600 - \dfrac{80\times20\times10^{-4}}{20\times10^{-6} + 4.03\times10^{-4}} = 1221.95\,\Omega$

ii) $Z_i(\text{stage}) = R_B \,//\, Z_i$

$= 9800 \,//\, 1221.95$

$= 1086.47\,\Omega$

iii) $Z_o = 1/Y_o$

where, $Y_o = h_{oe} - \dfrac{h_{fe} \cdot h_{re}}{h_{ie} + R_S // R_B}$

$$= 20 \times 10^{-6} - \left( \frac{80 \times 20 \times 10^{-4}}{1600 + (500 // 9800)} \right)$$

$$= -5.7 \times 10^{-5} \, S$$

$$= 5.7 \times 10^{-5} \text{ (neglecting -ve sign)}$$

$$\therefore Z_o = \frac{1}{Y_o} = \frac{1}{5.7 \times 10^{-5}} = 17.518 \, K\Omega$$

iv) $Z_o(\text{stage}) = R_C // Z_o$

$$= 3.3 // 17.518$$

$$= 2.77 \, K\Omega$$

v) $A_I = \dfrac{-h_{fe}}{1 + h_{oe} Z_L} = \dfrac{-80}{1 + 20 \times 10^{-6} \times 2.48 \times 10^{3}} = -76.22$

vi) $A_V = \dfrac{A_I \cdot Z_L}{Z_i} = \dfrac{-76.22 \times 2480}{1221.95} = 154.67$

vii) $A_{VS} = -A_V \left( \dfrac{Z_i(\text{stage})}{Z_i(\text{stage}) + R_S} \right)$

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