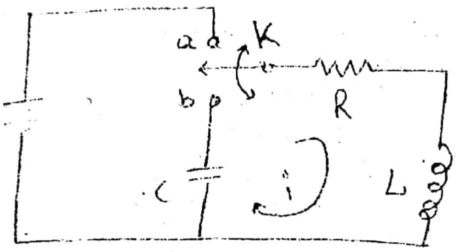
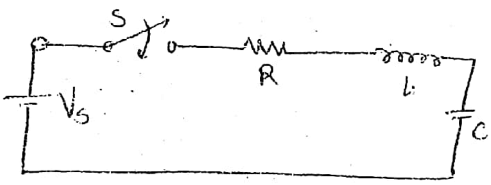
1. In the network of the figure, K is changed from position a to b at $t = 0$. Solve for i, $\frac{di}{dt}$ and $\frac{d^2i}{dt^2}$ at $t = 0^+$ if $R = 1000\Omega$, $L = 1H$, $C = 0.1\mu F$ and $V = 100V$.



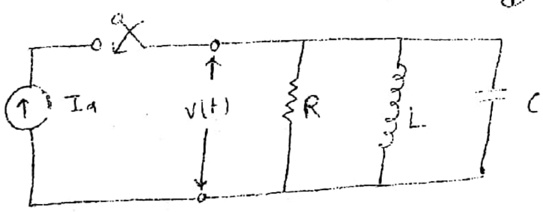
Ans:
$i(0^+) = 0.1 amp$
$\frac{di(0^+)}{dt} = -100 amp/sec$
$\frac{d^2i(0^+)}{dt^2} = -9 \times 10^5 A/sec^2$

2. Consider the RLC series circuit shown in figure below. $V_s = 2V$; $R = 6\Omega$, $L = 2H$, $C = 0.25F$. Determine $i(0^+)$; $\frac{di(0^+)}{dt}$, $\frac{d^2i(0^+)}{dt^2}$ ~~and i(t)~~.



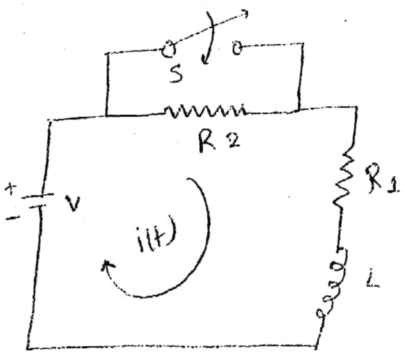
Ans:
$i(0^+) = 0 amp$
$\frac{di(0^+)}{dt} = 1 A/sec$
$\frac{d^2i(0^+)}{dt^2} = -3 A/s$

3. Consider the RLC parallel circuit shown in figure below. $I_s = 2A$, $R = \frac{1}{16}$, $L = \frac{1}{16}H$; $C = 4F$. Determine $v(0^+)$, $\frac{dv}{dt}(0^+)$, $\frac{d^2v(0^+)}{dt^2}$ ~~and v(t)~~.



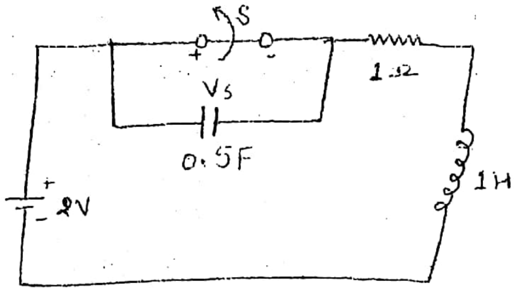
Ans:
$v(0^+) = 0$
$\frac{dv(0^+)}{dt} = \frac{1}{2} V/sec$
$\frac{d^2v(0^+)}{dt^2} = -2 V/sec$

4. In the network shown in figure below, switch S is closed at $t = 0$, a steady state current having previously been attained. Solve for the current as a function of time.



Ans:
$$i(t) = \frac{V}{R_1}\left[1 - \frac{R_2}{R_1 + R_2} e^{-\frac{R_1}{L}t}\right]$$

5. The circuit shown in figure is in the steady state with the switch closed. This switch is opened at $t = 0$. Determine voltage across the ... and $\frac{dV}{dt}$ at $t = 0^+$

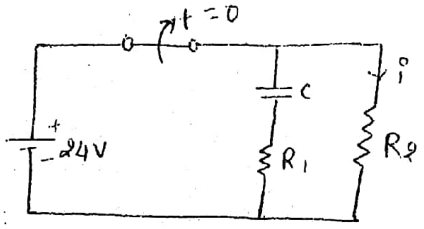


Ans:

$V_s(0^-) = 0$

$\frac{dV_s(0^+)}{dt} = 4 V/sec.$

6. The circuit was under steady state before the switch was opened. If $R_1 = 1\Omega$, $R_2 = 2\Omega$ and $C = 0.167F$, determine $V_c(0^-)$, $V_c(0^+)$ and $i(0^+)$.



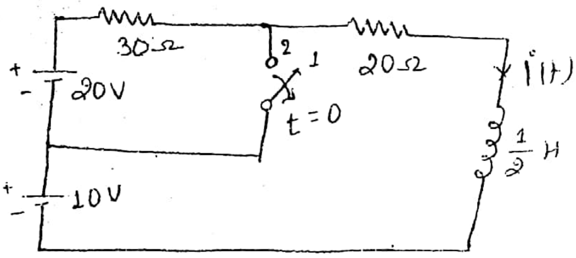
Ans:

$V_c(0^-) = 24V$

$V_c(0^+) = 24V$

$i(0^+) = 8A$

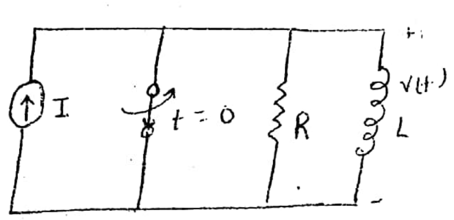
7. The circuit of given figure reaches a steady state in position 2 and at $t = 0$ the switch S is moved to position 1. Find $i(t)$.



Ans:

$i(t) = 0.6 - 0.1e^{-100t}$ A

8. The circuit shown in figure below has the switch S opened at $t = 0$. Solve for v, $\frac{dv}{dt}$ and $\frac{d^2v}{dt^2}$ at $t = 0^+$ if $I = 1$ amp, $R = 100\Omega$ and $L = 1H$. Also find the expression for $v(t)$.



Ans:

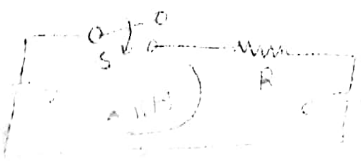
$v(0^+) = 100V$

$\frac{dv(0^+)}{dt} = -10^4 V/sec$

$\frac{d^2v(0^+)}{dt^2} = 10^6 V$

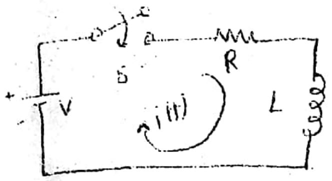
$v(t) = 100e^{-100t}$ V

9. In the circuit of the given figure, the switch S is closed at $t = 0$ with the capacitor uncharged. Find values for i, $\frac{di}{dt}$, $\frac{d^2i}{dt^2}$ at $t = 0^+$ for element values as follows: $V = 100V$, $R = 1000\Omega$ and $C = 1\mu F$



Ans:

Q10. In the given circuit S is closed at $t=0$ with zero current in the inductor. Find the values of i, $\frac{di}{dt}$ and $\frac{d^2i}{dt^2}$ at $t=0^+$, if $R=10\Omega$, $L=1H$ and $V=100V$.

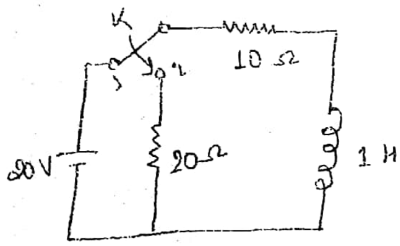


Ans:
$i(0^+) = 0$

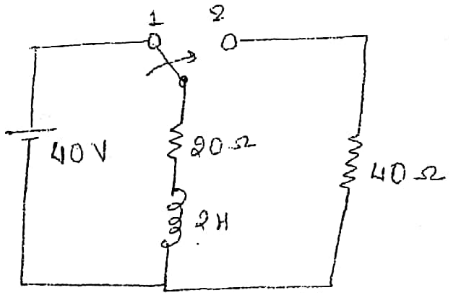
$\frac{di(0^+)}{dt} = 100$ A/sec.

$\frac{d^2i(0^+)}{dt^2} = -1000$ A/sec

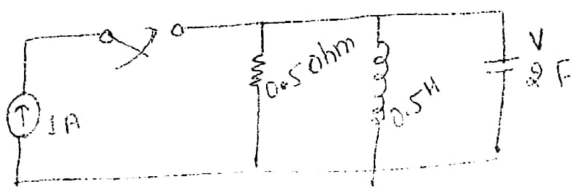
Q.11 In the given circuit switch K is changed from position 1 to 2 at $t=0$, steady state condition having reached before switching. Find, i, $\frac{di}{dt}$, $\frac{d^2i}{dt^2}$ at $t=0^+$. [2007 fall]



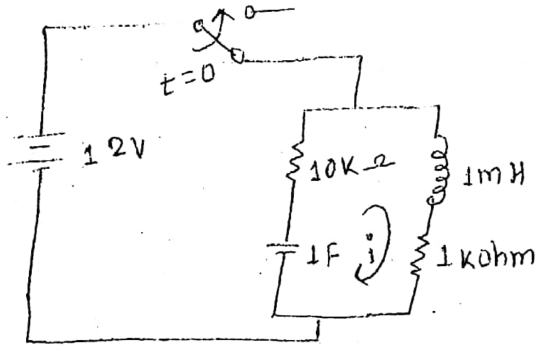
12. In the given circuit the switch is changed from position 1 to 2 at time $t=0$, steady state having reached previously. Find: $i(0^+)$, $\frac{di(0^+)}{dt}$, $\frac{d^2}{dt^2}i(0^+)$ [2008 Fall]



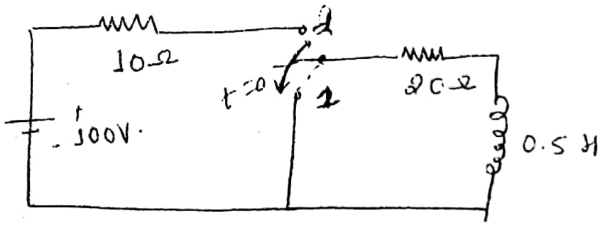
13 In the given RLC parallel circuit, determine $v(0^+)$, $\frac{dv}{dt}(0^+)$, $\frac{d^2}{dt^2}v(0^+)$. [2009 Fall]



Assume no initial energy is stored



3.15 Switch K is kept at position 2 and steady state condition is reached. At t = 0, the switch is moved to position 1. find, i, $\frac{di}{dt}$ and, $\frac{d^2i}{dt^2}$ at $t = 0$.



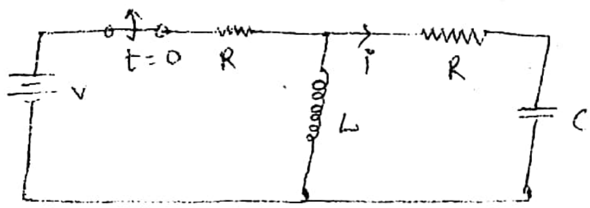
Chapter: 3

Assignment: 2

Q1 : A series circuit containing $R = 7\Omega$, $L = 1H$ and $C = 0.1F$ is connected to 20V dc supply at $t = 0$sec. Find the expression of voltage across capacitor at any time using classical method considering inductor has zero current and capacitor has zero voltage initially.

[2009 Fall 1.a]

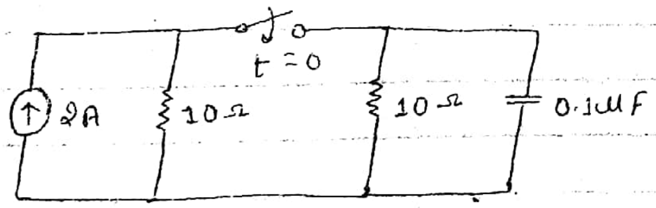
Q2 : In the given circuit, switch opens at $t = 0$.



solve $i(t)$ for $V = 12V$, $R = 4\Omega$, $C = 1/3 F$ and $L = 1H$.

[2009 spring 1.a]

Q.3 Find the total response $V_c(t)$ for the voltage across the capacitor in circuit given below using classical method. The switch S is closed at $t = 0$.

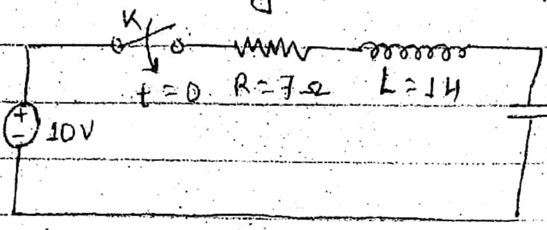


[2008 S 1.b]

Q.4 : A dc current of 5A is suddenly connected to a parallel RLC network at $t = 0$. Obtain the particular solution for voltage $v(t)$ across the circuit elements. Given that $R = \frac{1}{5}\Omega$, $L = 0.1H$ and $C = 4F$. Assume zero initial charge across capacitor and zero initial current through inductor. (2009 S 2.a)

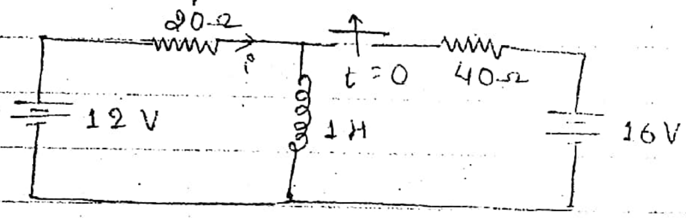
Q.5 An R-L circuit is connected to an ac voltage $e = 100\sin(500t + $ volts at $t = 0$. If $R = 5\Omega$ and $L = 0.01H$. Find the equation for the current using classical method. (2009 S 2 b)

Q.6 At $t=0$ switch K is closed. Find the value of current $t = 0.1$ sec. Using classical approach.



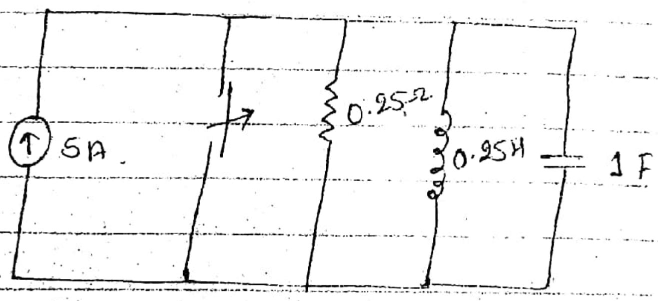
[2010 S 1b]

Q.7 Find $i(t)$ for $t>0$ in the circuit shown in figure below. Switch is opened at $t=0$.



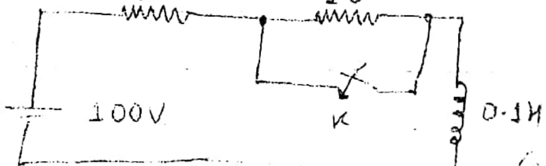
[2010 S 2b]

Q.8 For the network shown in figure below, determine the voltage response of the circuit when switch is opened at $t=0$.



[2011 F 1(a)]

Q.9 A dc voltage 100V is applied in the adjoining circuit and switch K is open. The switch K is closed at $t=0$. Find the complete expression for current.



(2012

Q.10 In the series RLC network, $V = 100V$, $L = 0.2H$, $R = 30\Omega$ and $C = 40\mu F$. Obtain the expression for $i(t)$ using classical approach. Assume there is no initial charge on the capacitor or current in the inductor.

[2014 F 1 9]

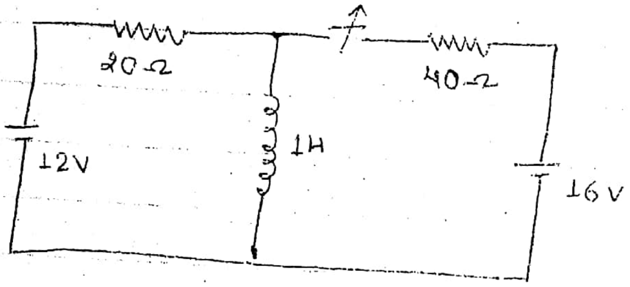
Everest Management
Sitapaila

CHAPTER : 4

Name :
Roll :
Subject :
Date :

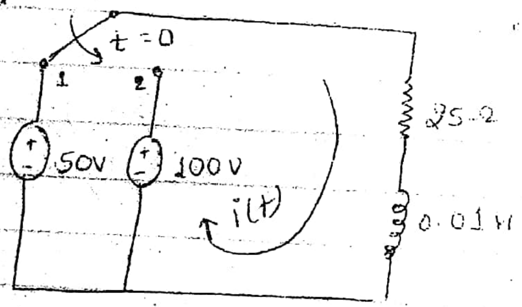
Assignment : 3

1. Find $i(t)$ for $t > 0$ in the circuit shown in figure below using Laplace transform. Switch is opened at $t = 0$.



(2014 Fall)

2. In the series R-L circuit shown below. Switch K is in position 1 for a long time and moved to position 2 at time $t = 0$. Find the resulting current $i(t)$ using Laplace transform method.



(2013 Fall)

3. What are the conditions necessary for the existence of Laplace Transform? Prove that the Laplace Transform of Delta function is unity. 2012 Fall

(2012 spring)

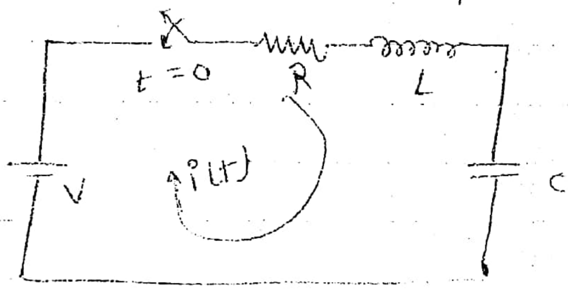
4. An exponential voltage $v(t) = 4e^{-2t}$ is suddenly at time $t = 0$ 2012 Spring

to a series RL circuit consisting of $R = 2\Omega$ and $L = 1H$ using Laplace transform method, find the particular solution for $i(t)$ through the circuit. Assume the current through the inductor is 3A before switching.

[2012 spring]

5. In the circuit below $V_0 = 10V$, $R = 5\Omega$, $L = 1H$ and $C = 0.25F$, the switch is closed at time $t = 0$, using Laplace technique to obtain the particular solution for $i(t)$ through the circuit. Assume that there is zero current through inductor and zero charge across capacitor before switching.

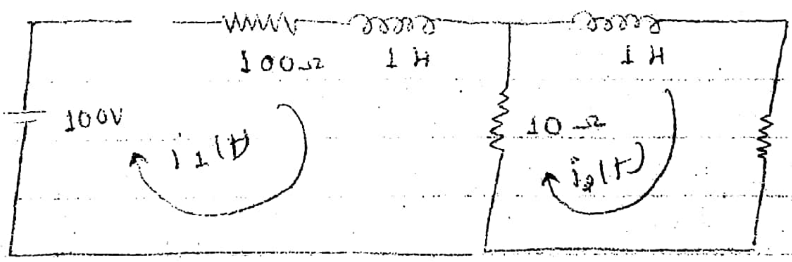
2012 fall



[2012 Fall]

6. Using Laplace transform in the given network, find $i_2(t)$. Assume zero current through inductor initially.

2014 spring



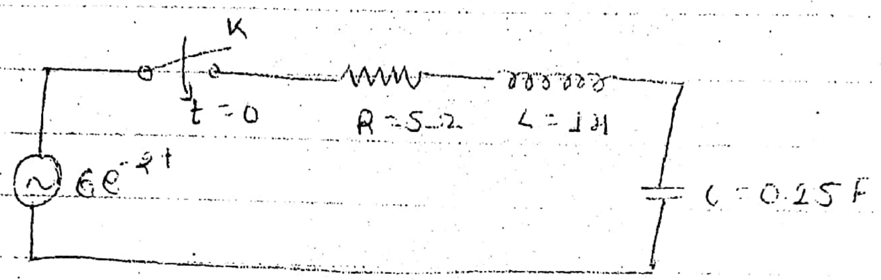
[2011 spring]

7. State initial and final value theorems for Laplace transform. Find $i(0)$ and steady state value $i(t)$ for the function.

$$I(S) = \frac{(S+1)}{S(S+2)}$$

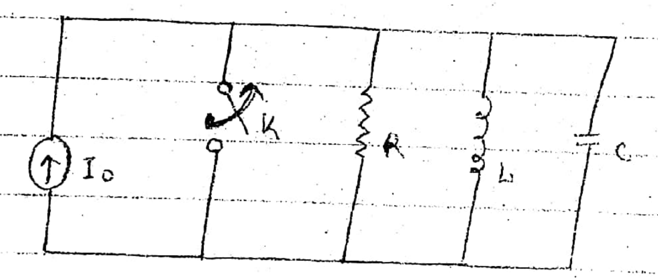
[2011 Spring]

8. Switch K is closed at $t = 0$. Find the time domain circuit current using Laplace transform for the following circuit.



[2011 Fall]

9. In the circuit shown below $C = 1F$, $R = 1/2\ \Omega$, $L = 1/2\ H$ and I_0. Switch K is opened at $t = 0$. Obtain particular solution for $v(t)$. Calculate the value of voltage V at $t = 0.1$ and 0.2 second. (use laplace transform)



[2008 Fall]

10. A sinusoidal voltage $25 \sin 10t$ is applied at time $t = 0$ to a series R-L circuit comprising $R = 5\ \Omega$ and $L = 1H$. By the method of Laplace transformation, find current $i(t)$. Assume zero current through inductor before application of voltage.

2013 Spring

[2007 Spring]

Chapter : 5

Assignment : 4

1. From pole zero plot, obtain the time response for $I(s) = \frac{4s}{(s+1)(s+3)}$ also determine the value of $i(t)$ for $t = 1$ sec.

[2014 Fall] [2010 spring]

2. Check the stability of the system whose characteristic equation is given by $s^5 + s^4 + 2s^3 + 2s^2 + 3s + 5 = 0$. Using Routh - Hurwitz criteria.

[2013 Fall]

3. Determine the range of 'K' for which the system is stable using Routh - Hurwitz criteria;
$g(s) = s^4 + s^3 + s^2 + s + K$.

[2012 spring]

4. Plot the poles and zeroes in s-plane and obtain T(t) for the transfer function of a network given by

$$T(s) = \frac{3s}{(s+2)(s^2+2s+2)}$$

[2012 Fall]

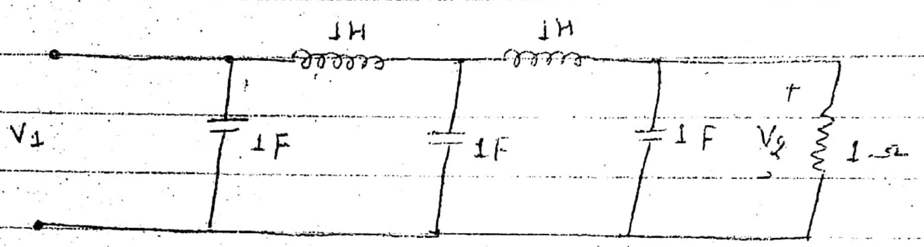
5. Check the stability ~~for~~ of the following system expressed in polynomial as, $g(s) = s^3 + 2s^2 + 2s + 40$ using Routh Hurwitz criteria.

[2012 Fall]

6. Show the pole-zero plot of the given network function $H(S) = \frac{10S}{(S+1)(S+3)}$, and obtain h(t) using pole-zero plot.

[2011 Spring]

7. Define transfer function. For the network shown below determine $\frac{V_2(S)}{V_1(S)}$

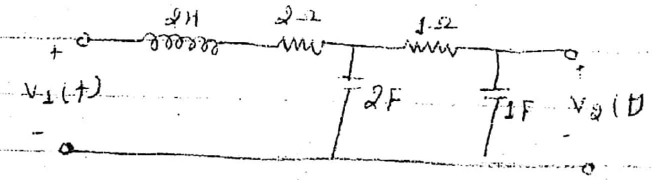


[2011 Fall]

8. The denominator polynomial of a network is given as $S^3 + 7S^2 + 10S + 10K = 0$. Determine the range of K for which the system is stable using Routh-Hurwitz criteria.

[2010 Fall]

9. Find $V_2(S)/V_1(S)$ for the following network.



[2009 Fall]

10. What does Routh-Hurwitz criteria state? Form Routh array for the following characteristic equation and state whether the system is stable or not.

$$G(S) = 5S^5 + 3S^4 + 2S^3 + 2S^2 + S + 1$$

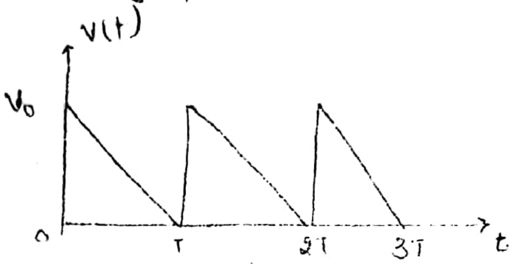
[2008 Fall]

Name :
Roll
Subject :

CHAPTER : 6

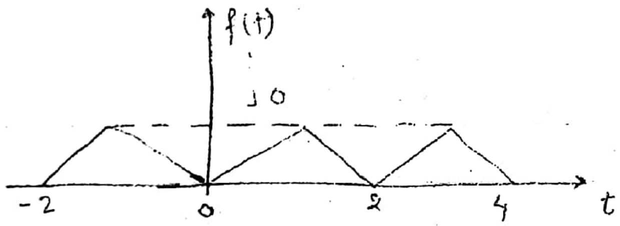
Assignment : 5

1. for the following periodic waveform, find the fourier series expansion.



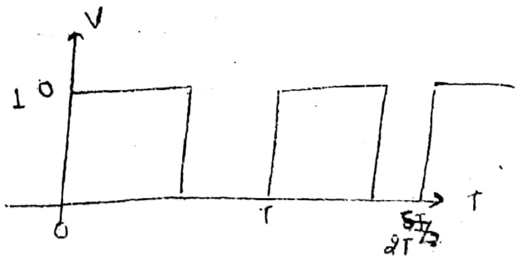
[2012 spring]
[2014 Fall]

2. Find the trigonometric fourier series for the waveform shown.



[2013 Fall]

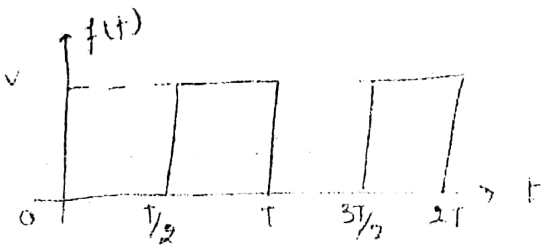
3. Determine the fourier series of the wave form given by



[2012 Fall]

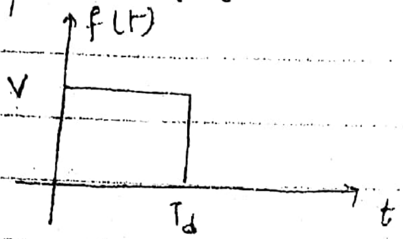
4. find the fourier series of the given waveform.

(i)



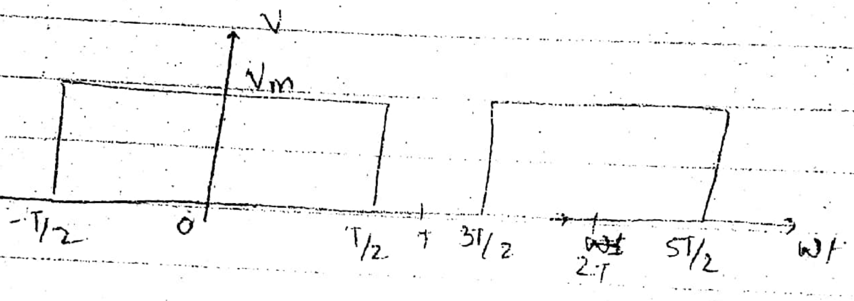
[2011 spring]

5. Determine the fourier transform of a pulse of duration T_d and amplitude V as shown in figure below:



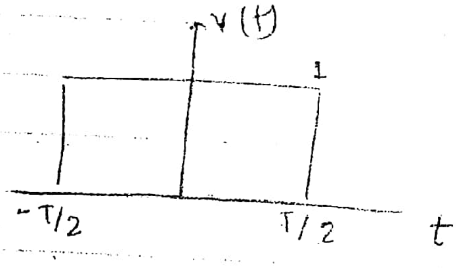
[2011 Fall]

6. Obtain the trigonometric form of the fourier series for following function.



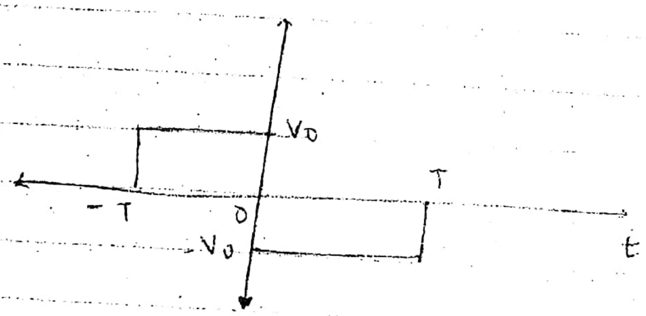
[2010 spring]

7. Obtain the fourier transform of the given waveform.



[2009 Fall]

8. Determine the fourier transform of the function shown below.

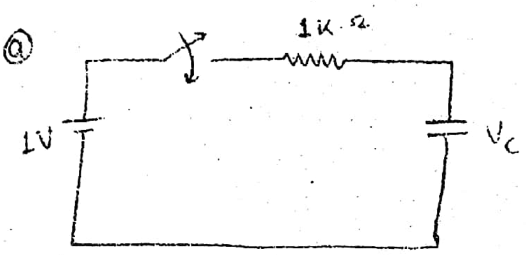


[2008 spring]

1/ Assignments:

Find, $V(0^-)$, $V(0^+)$, $V(\infty)$ and $i(0^-)$, $i(0^+)$ and $i(\infty)$

ⓐ



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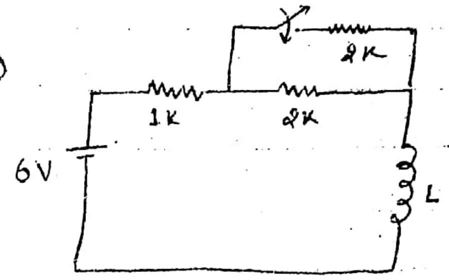
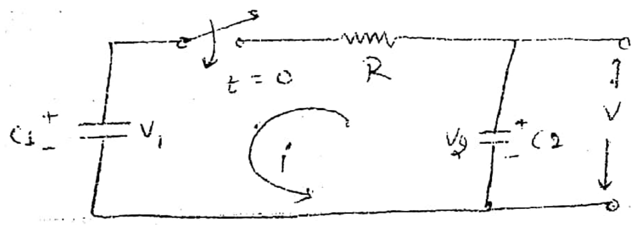


$V_c(0^-) = 0$	$i(0^-) = 0$	$V_L(0^-) = 0$	$i_L(0^-) = 2mA$
$V_c(0^+) = 0$	$i(0^+) = 1mA$	$V_L(0^+) = 2V$	$i_L(0^+) = 2mA$
$V_c(\infty) = 1V$	$i(\infty) = 0$	$V_L(\infty) = 0$	$i_L(\infty) = 3mA$

2) In the given circuit C_1 has initial voltage V_1 and C_2 has initial voltage V_2. At time $t=0$, switch K is closed. Find the expression for voltage 'v' across capacitor C_2 at $t>0$.



Soln: For $t>0$, apply KVL

$$V_{C_1} + V_R + V_{C_2} = 0$$

or, $\frac{1}{C_1}\int_{-\infty}^{t} i\,dt + iR + v = 0 \Rightarrow \frac{1}{C_1}\int_{-\infty}^{0} i\,dt + \frac{1}{C_1}\int_{0}^{t} i\,dt + iR + v = 0$

or, $-V_1 + \frac{1}{C_1}\int_0^t i\,dt + iR + v = 0$ .... ①

Since, $i = C_2\frac{dv}{dt}$ and when $t=0$, $v = V_2$ ...... ⑪

$t \to t$, $v = v$

from ① and ⑪

or, $-V_1 + \frac{C_2}{C_1}\int_{V_2}^{V} dV + RC_2\frac{dv}{dt} + V = 0$

or, $-V_1 + \frac{C_2}{C_1}[V - V_2] + RC_2\frac{dv}{dt} + V = 0$ ..... (iii)

or, $RC_2\frac{dv}{dt} + V + \frac{C_2}{C_1}V = V_1 + \frac{C_2}{C_1}V_2$

or, $RC_1C_2\frac{dv}{dt} + C_1V + C_2V = C_1V_1 + C_2V_2$

Since source is constant, force response is obtained by placing $p=0$ i.e. $\frac{d()}{dt} = 0$

or, $(C_1+C_2)V_f = C_1V_1 + C_2V_2$

$\therefore V_f = \frac{C_1V_1 + C_2V_2}{C_1+C_2}$ ..... (iv)

To find transient response, Corresponding homogeneous equation is,

$RC_2C_1\frac{dv}{dt} + (C_1+C_2)V = 0$

or, $\cancel{RC_1C_2P(V) + (C_1+C_2)V = 0}$

Let, $V = Ke^{st}$ be the form of solution

$RC_1C_2\frac{d}{dt}(Ke^{st}) + (C_1+C_2)Ke^{st} = 0$

or, $RC_1C_2\,sKe^{st} + (C_1+C_2)Ke^{st} = 0$

or, $Ke^{st}(RC_1C_2s + (C_1+C_2)) = 0$

since, $Ke^{st} \neq 0$

so, $RC_1C_2s + (C_1+C_2) = 0$

$$\therefore V_t(t) = K e^{-\left(\frac{C_1+C_2}{RC_1C_2}\right)t}$$

The complete voltage response is given by,

$$V_t = V_f(t) + V_t(t)$$

$$\text{i.e. } V_t = \frac{C_1V_1 + C_2V_2}{C_1+C_2} + K e^{-\left(\frac{C_1+C_2}{RC_1C_2}\right)t}$$

using initial condition.

for $v(0^+)$ at $t=0^-$ $V(0^-) = V_2$ so, $V(0^+) = V_2$

[$\because$ Since voltage across capacitor can not change instanteneously]

so,

$$V(0^+) = \frac{C_1V_1 + C_2V_2}{C_1+C_2} + K e^{-\left(\frac{C_1+C_2}{RC_1C_2}\right)(0^+)}$$

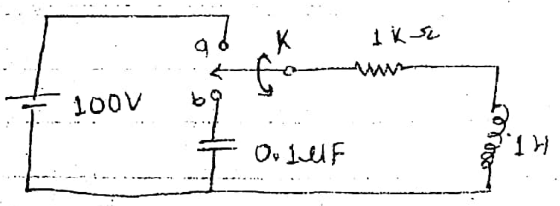
$$\text{or, } V_2 = \frac{C_1V_1 + C_2V_2}{(C_1+C_2)} + K$$

$$\text{or, } K = \frac{C_1V_2 + C_2V_2 - C_1V_1 - C_2V_2}{(C_1+C_2)} = \frac{C_1(V_2-V_1)}{C_1+C_2}$$

$\therefore$ Total solution is

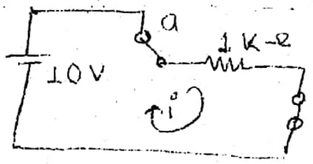
$$V(t) = \frac{C_1V_1 + C_2V_2}{C_1+C_2} + \frac{C_1(V_2-V_1)}{C_1+C_2} e^{-\left(\frac{C_1+C_2}{RC_1C_2}\right)t} \quad //$$

Q.1



At $t = 0^-$

Equivalent ckt



Initially switch is at position 'a'.

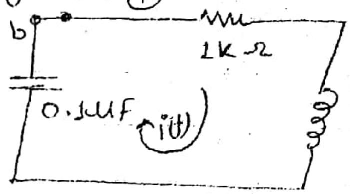
The steady state value of current,

$$i(0^-) = \frac{100}{1000} = 0.1\ A$$

$\therefore i(0^+) = i(0^-) = 0.1\ A$ [$\because$ Current through inductor can not change instantaneously]

For $t>0$, apply KVL

Equivalent circuit for $t>0$



$$1000\, i(t) + 1\frac{di(t)}{dt} + \frac{1}{0.1\times10^{-6}}\int_{-\infty}^{t} i(t)dt = 0 \quad \text{......} (1)$$

$$L \quad \frac{1}{C}\int i(t)dt = V_C$$

$$\text{or, } 1000\, i(t) + \frac{di(t)}{dt} + V_C = 0$$

At $t = 0^+$

$$1000\, i(0^+) + \frac{di(0^+)}{dt} + V_C(0^+) = 0$$

[$\because V_C(0^-) = 0 = V_C(0^+)$ Since voltage across capacitor can not change instantaneous

$$\frac{di(0^+)}{dt} = -100\ amp/sec.$$

Now, differentiating equation (1), we get

$$1000\frac{di(t)}{dt} + \frac{d^2i(t)}{dt^2} + \frac{1}{0.1\times10^{-6}}i(t) = 0$$

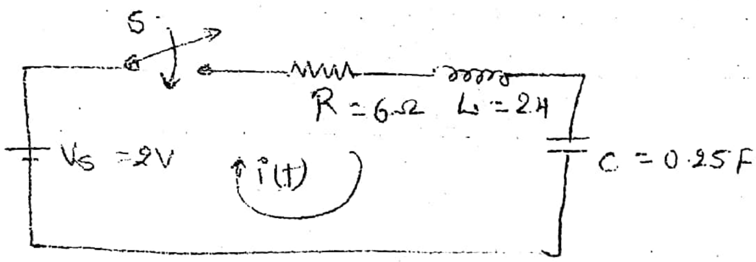
at $t = 0^+$

$$1000\frac{di(0^+)}{dt} + \frac{d^2i(0^+)}{dt^2} + \frac{i(0^+)}{0.1\times10^{-6}} = 0$$

$$\frac{di(0^+)}{dt} = -10^6 + 10^5 = -9\times10^5 \text{ Amp/sec}^2$$

Thus,

$i(0^+) = 0.1$ Amp, $\frac{di(0^+)}{dt} = -100$ amp/sec & $\frac{d^2i(0^+)}{dt^2} = -9\times10^5$ Amp/sec^2



At $t = 0^-$

$i(0^-) = 0$ and, $V_c(0^-) = 0$

so, $i(0^+) = i(0^-) = 0$ [∵ Current through inductor can not change instanteneously]

and, $V_c(0^+) = V_c(0^-) = 0$ [∵ Voltage across capacitor can not change instanteneously]

Now, Apply KVL for $t > 0$

$$2V = 6\,i(t) + 2\frac{di(t)}{dt} + \frac{1}{0.25}\int_0^t i(t)\,dt \quad \ldots \text{①}$$

At, $t = 0^+$

$$2 = 6\,i(0^+) + 2\frac{di(0^+)}{dt} + \frac{1}{0.25}\int_0^t i(0^+)\,dt \qquad \left[∵ V_c = \frac{1}{C}\int_0^t i\,dt\right]$$

$$2 = 6\times0 + 2\frac{di(0^+)}{dt} + \cancel{V_c(0^+)}^{\,0}$$

$$\frac{di(0^+)}{dt} = 1 \text{ amp/sec.}$$

Now, Differentiating equation ① w.r.to t,

at, $t = 0^+$

$$0 = 6\frac{di(0^+)}{dt} + 2\frac{d^2i(0^+)}{dt^2} + \frac{i(0^+)}{0.25}$$

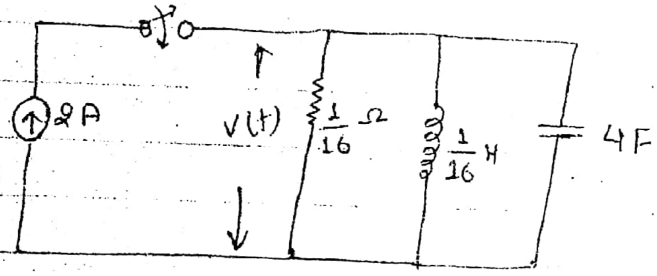
$$\Rightarrow 0 = 6 \times 1 + 2\frac{d^2i(0^+)}{dt^2}$$

or, $\frac{d^2i(0^+)}{dt^2} = \frac{-6}{2} = -3$ amp/sec^2

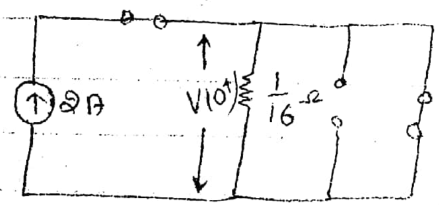
Thus,

$i(0^+) = 0$, $\frac{di(0^+)}{dt} = 1$ A/sec, $\frac{d^2i(0^+)}{dt^2} = -3$ A/sec^2

Q.3

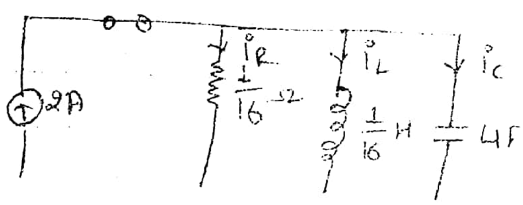


Here, at, $t = 0^-$ switch is opened. $i_R(0^-) = 0$, $i_L(0^-) = 0$, $i_C(0^-)$ =
at, $t = 0^+$, switch is closed and equivalent circuit is,



$v(0^+) = 0$ and, $i_L(0^+) = 0$ [∵ Current through inductor can not change instantaneously]

At $t > 0$, Equivalent circuit is



Now, using KCL

$$2 = i_R + i_L + i_C$$

$$\text{or, } 2 = \frac{V(t)}{\frac{1}{16}} + 4\frac{dV(t)}{dt} + i_L \quad \cdots \text{(1)}$$

at, $t = 0^+$

$$2 = 16\frac{V(0^+)}{\cancel{16}} + 4\frac{dV(0^+)}{dt} + i_L(0^+)$$

$$2 = 0 + 4\frac{dV(0^+)}{dt} + 0$$

$$\therefore \frac{dV(0^+)}{dt} = \frac{1}{2} \text{ V/sec.}$$

Now, Differentiating equation (1) w.r.to t.

$$0 = \frac{16}{\cancel{16}}\frac{dV(t)}{dt} + 4\frac{d^2V(t)}{dt^2} + \frac{di_L}{dt}$$

$$0 = \frac{16}{\cancel{16}}\frac{dV(t)}{dt} + 4\frac{d^2V(t)}{dt^2} + \frac{V_L}{L} \quad \left[\because i_L = \frac{1}{L}\int V_L\,dt\right]$$

at, $t = 0^+$

$$0 = \frac{16}{\cancel{16}}\frac{dV(0^+)}{dt} + 4\frac{d^2V(0^+)}{dt} + \frac{V_L(0^+)}{L}$$

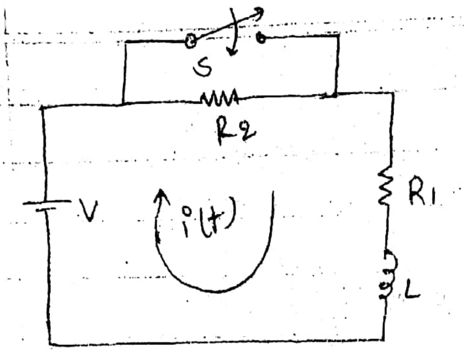
$$0 = \frac{16}{\cancel{16}} \times \frac{1}{2} + 4\frac{d^2V(0^+)}{dt} + 0 \quad [\because V_L(0^+) = V(0^+)]$$

$$\therefore \frac{d^2V(0^+)}{dt^2} = -2 \text{ V/sec}^2$$

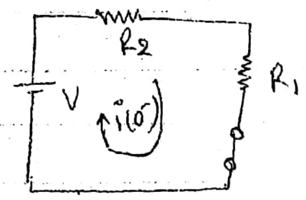
Thus,

$\therefore V(0^+) = 0$, $\frac{dV(0^+)}{dt} = \frac{1}{2}$ V/sec. and $\frac{d^2V(0^+)}{dt^2} = -2$ V/sec^2

Q.4



At, $t = 0^-$, steady state is reached. Equivalent circuit is,

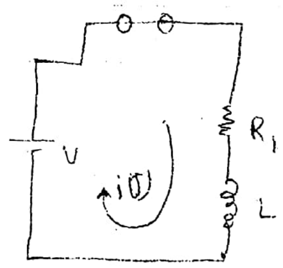


$$i(0^-) = \frac{V}{R_1 + R_2}$$

and,

$$i(0^+) = i(0^-) = \frac{V}{R_1+R_2}$$ [∵ current through inductor can not change instanteneously]

for $t > 0$, Equivalent circuit is,



$$V = R_1 i(t) + L\frac{di(t)}{dt} \quad \cdots (1)$$

In p-operator form

$$V = R_1 i(t) + LP i(t)$$

$$i(t) = \frac{V}{R_1 + LP} \quad \cdots (2)$$

Here, equation (1) is non-homogeneous equation. It contains two parts of solution.

$$i(t) = \frac{V}{R_1 + LP}$$

Here, source is constant so put $p=0$ for forced response

$$i_f(t) = \frac{V}{R_1}$$

② Transient solution

Homogeneous version of equation ① is,

$$R_1 i(t) + L\frac{di(t)}{dt} = 0$$

Characteristic equation is, $R_1 + LS = 0$

$$\Rightarrow S = -\frac{R_1}{L}$$

∴ Transient solution, $i_t(t) = ke^{st} = ke^{-\frac{R_1}{L}t}$

Total solution $i(t) = i_t(t) + i_f(t)$

$$= ke^{-\frac{R_1}{L}t} + \frac{V}{R_1}$$

Now, to find value of K, use initial conditions

$$i(0^+) = K + \frac{V}{R_1}$$

$$\frac{V}{R_1+R_2} - \frac{V}{R_1} = K \quad \Rightarrow K = \frac{VR_1 - VR_1 - VR_2}{R_1(R_1+R_2)} = -\frac{VR_2}{R_1(R_1+R_2)}$$

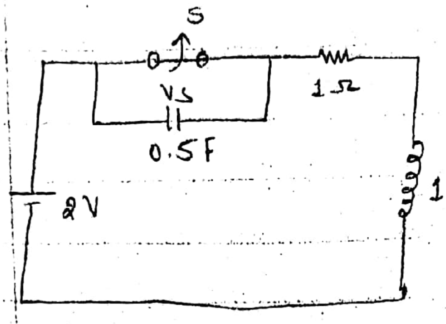
Now,

$$i(t) = \frac{V}{R_1} - \frac{V}{R_1}\left(\frac{R_2}{R_1+R_2}\right)e^{-\frac{R_1}{L}t}$$

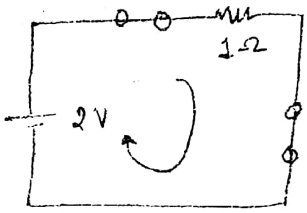
$$\boxed{i(t) = \frac{V}{R_1}\left[1 - \frac{R_2}{R_1+R_2}e^{-\frac{R_1}{L}t}\right]}$$

This is the required expression for i(t)

Q.5



At, $t = 0^-$, steady state is reached. Equivalent circuit is,



$i(0^-) = \frac{2}{1} = 2A$

$\therefore i(0^+) = i(0^-) = 2A$ [$\because$ Current through inductor can not change instanteneously]

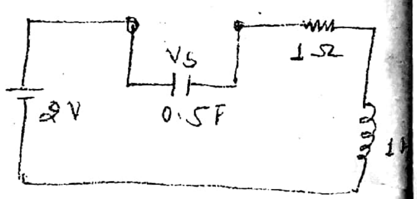
Circuit is in steady state with

When, switch is closed, capacitor is short circuited so,

$V_S(0^-) = 0$

$\therefore V_S(0^+) = V_S(0^-) = 0$ [$\because$ Voltage across capacitor can not change instanteneously]

Now, at $t > 0$, Equivalent circuit is.



$V_S(t) = \frac{1}{C} \int i(t)\, dt$

or, $i(t) = C \frac{dV_S(t)}{dt}$

$2 = \frac{1}{0.5} \int i(t)\,dt + i(t) + \frac{di(t)}{dt}$

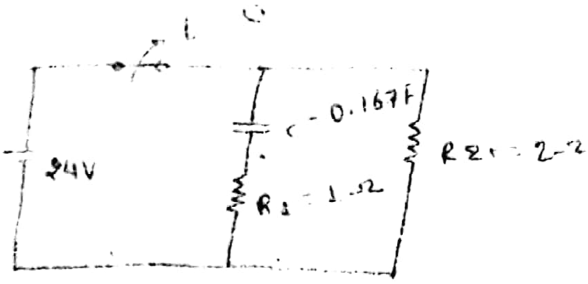
at, $t = 0^+$

$i(0^+) = 0.5 \frac{dV_S(0^+)}{dt}$

$2 = i(0^+) + \frac{di(0^+)}{dt}$

$\frac{2}{0.5} = \frac{dV_S(0^+)}{dt}$

$\therefore \frac{dV_S(0^+)}{dt} = 4$ V/sec.



Since At steady state capacitor behaves as an open circuit (i.e capacitor is fully charged)

so.

$$V_c(0^-) = 24V$$

$\therefore V_c(0^+) = V_c(0^-) = 24$ [$\because$ Voltage across capacitor can not change instantaneously]

At. $t > 0$, Apply KVL

$$V_C = V_{R_1} + V_{R_2}$$

$$V_c = i(t) R_1 + i(t) R_2$$

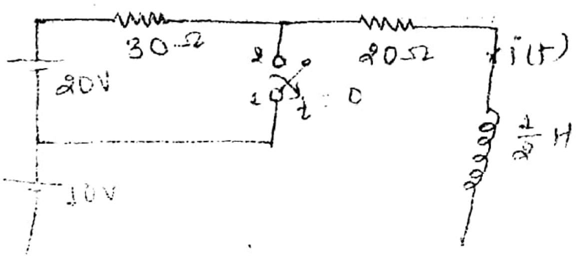
or, $i(t) = \dfrac{V_C}{R_1 + R_2}$

at, $t = 0^+$

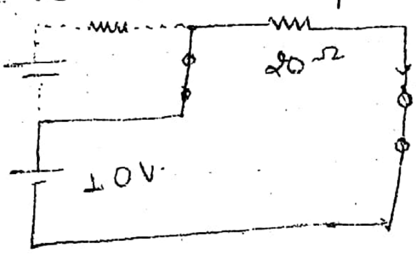
$$i(0^+) = \frac{V_c(0^+)}{1+2} = \frac{24}{3} = 8 \text{ amp.}$$

$\therefore V_c(0^-) = 24V$, $V_c(0^+) = 24V$ and $i(0^+) = 8$ Amp.

7.



At position 2, steady state is reached. Equivalent ckt is



$i(0^-) = \frac{10}{20} = 0.5$ amp.

$\therefore i(0^+) = i(0^-) = 0.5$ amp [current through inductor can not change instanteneously].

At $t>0$, apply KVL

$$30 = 50\,i(t) + \frac{1}{2}\frac{di(t)}{dt} \quad \cdots\cdots (1)$$

Equation (1) is non-homogeneous equation. It contains two parts of solution.

(1) Forced solution

Equation (1) in p-operator form

$$30 = 50\,i(t) + \frac{1}{2}p\,i(t)$$

$$\text{or, } i(t) = \frac{30}{50+\frac{1}{2}p}$$

Here, source is constant so put $p=0$ to find forced response,

$$i_f(t) = \frac{30}{50} = 0.6 \text{ amp.}$$

(2) Transient solution

Homogeneous version of equation (1) is,

$$\frac{1}{2}\frac{di(t)}{dt} + 50\,i(t) = 0$$

$\therefore$ Transient solution, $i_t(t) = Ke^{st} = Ke^{-100t}$

Total solution, $i(t) = i_f(t) + i_t(t)$
$= 0.6 + Ke^{-100t}$

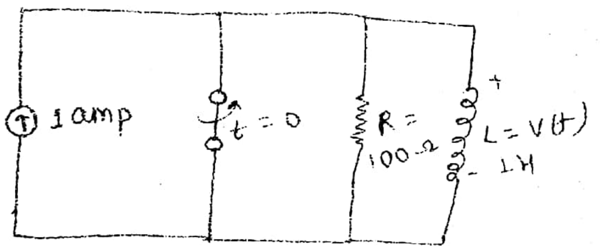
at, $t = 0^+$

$i(0^+) = 0.6 + K$

$0.5 = 0.6 + K$

$K = -0.1$

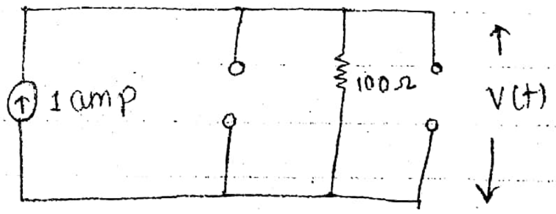
$\therefore$ Total solution, $\boxed{i(t) = 0.6 - 0.1e^{-100t}}$



At, $t = 0^+$,

$V(0^+) = IR = 100V.$

Equivalent circuit is,



At $t > 0$, Apply KCL

$1 = i_R + i_L$

or, $1 = \frac{V(t)}{R} + \frac{1}{L}\int V(t)\,dt$ ...... ①

Differentiating w.r.to t, we get

$0 = \frac{1}{}\frac{dv(t)}{} + v(t)$ ...... ②

at, $t = 0^+$

$$0 = \frac{1}{100}\frac{dv(0^+)}{dt} + v(0^+)$$

or, $$\frac{dv(0^+)}{dt} = -100 \times 100 = -10^4 \text{ V/sec.}$$

Differentiating eqⁿ ② w.r.to t, we get

$$0 = \frac{1}{100}\frac{d^2v(t)}{dt} + \frac{dv(t)}{dt}$$

at, $t = 0^+$

$$0 = \frac{1}{100}\frac{d^2v(0^+)}{dt} + \frac{dv(0^+)}{dt}$$

or, $$\frac{d^2v(0^+)}{dt} = -(-10^4) \times 100 = 10^6 \text{ V/sec}^2$$

Thus, $v(0^+) = 100\text{V}$, $\frac{dv(0^+)}{dt} = -10^4 \text{V/sec}$ and $\frac{d^2v(0^+)}{dt} = 10^6 \text{V/sec}^2$

Here, equation ② is homogeneous equation. So, its forced response will be zero. i.e. $V_f(t) = 0$

For transient solution :

Characteristic equation is,

$$\frac{1}{100}s + 1 = 0$$

$$s = -100$$

∴ Transient solution, $V_t(t) = ke^{st} = ke^{-100t}$

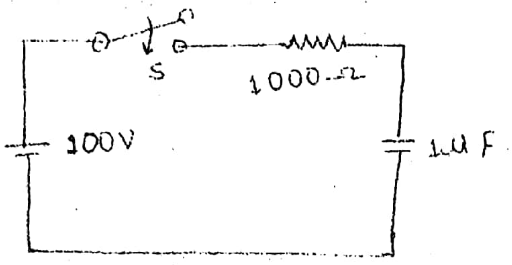
∴ Total solution, $V(t) = V_f(t) + V_t(t)$

$$= ke^{-100t}$$

To find value of k, use initial conditions,

at $t = 0^+$

... Total response $\boxed{V(t) = 100e^{-100t} \text{ Volt}}$



At, $t = 0^-$

$V_c(0^-) = 0$

$\therefore V_c(0^+) = V_c(0^-) = 0$ [$\because$ Voltage across capacitor can not change instanteneously].

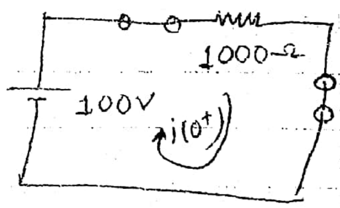
Apply KVL for $t>0$, we get

$$100V = 1000\, i(t) + \frac{1}{1\times10^{-6}}\int i(t)\, dt \quad \cdots ①$$

Differentiating w.r.to t, we get

$$0 = 1000\frac{di(t)}{dt} + \frac{i(t)}{10^{-6}} \quad \cdots ②$$

At, $t = 0^+$, Equivalent circuit is,



$$i(0^+) = \frac{100}{1000} = 0.1 \text{ Amp.}$$

Now, equation ② at, $t = 0^+$

$$0 = 1000\frac{di(0^+)}{dt} + i(0^+)\times10^6$$

or, $$\frac{di(0^+)}{dt} = -\frac{0.1\times10^6}{1000} = -100 \text{ amp/sec.}$$

Differentiating eqn. ② w.r.to t, we get

$$0 = 1000\frac{d^2 i(t)}{} + 10^6\frac{di(t)}{}$$

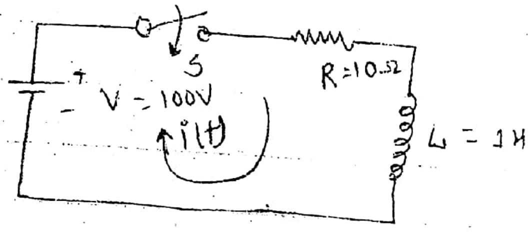
at, $t = 0^+$

$$0 = 1000 \frac{d^2 i(0^+)}{dt^2} + 10^6 \frac{d i(0^+)}{dt}$$

or, $\frac{d^2 i(0^+)}{dt^2} = \frac{-10^6 \times (-100)}{1000} = 10^5$ amp/sec^2

$\therefore i(0^+) = 0.1$ Amp, $\frac{di(0^+)}{dt} = -100$ Amp/sec and $\frac{d^2 i(0^+)}{dt^2} = 10^5$ amp/sec

Q.10



At, $t = 0^-$

$i(0^-) = 0$

$\therefore i(0^+) = i(0^-) = 0$ [$\because$ current through inductor can not change instanteneously]

Apply KVL for $t > 0$

$$V = R\, i(t) + L \frac{di(t)}{dt}$$

or, $100 = 10\, i(t) + 1 \frac{d}{dt} i(t)$ .... (1)

at, $t = 0^+$

$$100 = 10\, i(0^+) + \frac{di(0^+)}{dt}$$

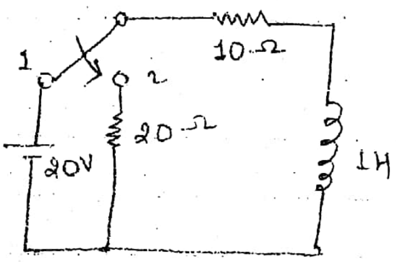
$\therefore \frac{di(0^+)}{dt} = 100$ Amp/sec.

Differentiating (1) w.r.to t

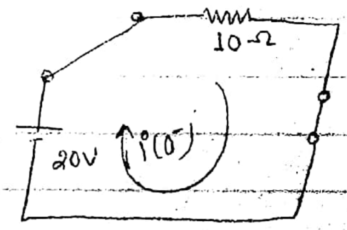
$$0 = 10 \frac{di(t)}{dt} + \frac{d^2 i(t)}{dt^2}$$

$\therefore$ $i(0^+) = 0$, $\frac{di(0^+)}{dt} = 100$ Amp/sec and $\frac{d^2 i(0^+)}{dt^2} = -1000$ Amp/sec^2

3.11.



At, $t = 0^-$, steady state is reached. Equivalent circuit is,



$i(0^-) = \frac{20V}{10\,\Omega} = 2A$

$\therefore$ $i(0^+) = i(0^-) = 2A$

($\because$ Current through inductor can not change instantaneously)

Now, Apply KVL at $t>0$, we get

$20\,i(t) + 10\,i(t) + 1\frac{di(t)}{dt} = 0$

or, $30\,i(t) + \frac{di(t)}{dt} = 0$ ...... ①

at, $t = 0^+$

$30\,i(0^+) + \frac{di(0^+)}{dt} = 0$

$\frac{di(0^+)}{dt} = -30 \times 2 = -60$ Amp/sec.

Now, Differentiating equation ① w.r.to t, we get

$$30\frac{di(t)}{dt} + \frac{d^2 i(t)}{dt^2} = 0$$

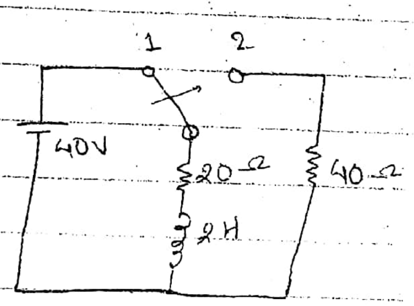
at, $t = 0^+$

$$30\frac{di(0^+)}{dt} + \frac{d^2 i(0^+)}{dt^2} = 0$$

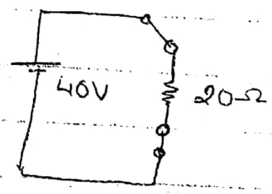
$$\frac{d^2 i(0^+)}{dt^2} = -30 \times -60 = 1800 \text{ Amp/sec}^2$$

$\therefore i(0^+) = 2$ Amp, $\frac{di(0^+)}{dt} = -60$ Amp/sec and $\frac{d^2 i(0^+)}{dt^2} = 1800$ Amp/sec^2

Q.12



At, $t = 0^-$ steady state is reached. Equivalent circuit is,



$$i(0^-) = \frac{40V}{20\Omega} = 2 \text{ Amp.}$$

$\therefore i(0^+) = i(0^-) = 2$ Amp

[$\because$ Current through inductor can not change instantaneously]

Now, apply KVL at $t > 0$, we get

$$20i(t) + 40i(t) + 2\frac{di(t)}{dt} = 0$$

$$60\,i(t) + 2\frac{di(t)}{dt} = 0 \quad \ldots (1)$$

at, $t = 0^+$

$$60\,i(0^+) + 2\frac{di(0^+)}{dt} = 0$$

$$2\frac{di(0^+)}{dt} = -60 \times 2 = -120$$

$$\therefore \frac{di(0^+)}{dt} = -60 \text{ Amp/sec.}$$

Now, Differentiating equation (1) w.r.to t,

$$60\frac{di(t)}{dt} + 2\frac{d^2 i(t)}{dt^2} = 0$$

at, $t = 0^+$

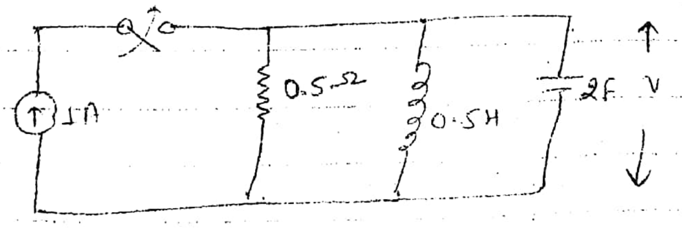
$$60\frac{di(0^+)}{dt} + 2\frac{d^2 i(0^+)}{dt^2} = 0$$

$$2\frac{d^2 i(0^+)}{dt^2} = -60 \times (-60) = 3600$$

$$\therefore \frac{d^2 i(0^+)}{dt^2} = 1800 \text{ Amp/sec}^2$$

$\therefore$ $i(0^+) = 2$ Amp, $\frac{di(0^+)}{dt} = -60$ Amp/sec and $\frac{d^2 i(0^+)}{dt^2} = 1800$ amp/sec

Q.13

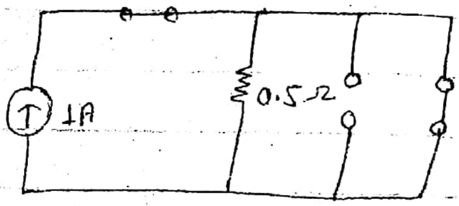


Here, at, $t = 0^-$ switch is opened so, $i_R(0^-) = 0$, $i_L(0^-) = 0$ and $i_C(0^-) = 0$

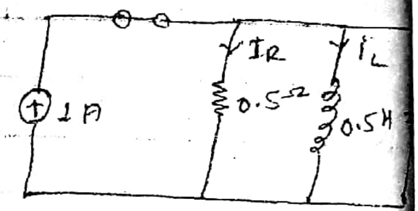
$\therefore i_L(0^+) = i_L(0^-) = 0$ [$\because$ Current through inductor can not change instanteneously.]

at, $t = 0^+$, Equivalent circuit

$V(0^+) = 0.$



Now, at $t > 0$, Equivalent circuit



Apply KCL, we have

$$1 = i_R + i_L + i_C$$

or, $1 = \frac{V(t)}{R} + i_L + \cancel{\frac{1}{L}} C\frac{dV(t)}{dt}$ .... ①

or, $1 = \frac{V(t)}{R} + \frac{1}{L}\int V(t)\,dt + C\frac{dV(t)}{dt}$ ... ②

Now,

at, $t = 0^+$ Equation ① becomes

$$1 = \frac{V(0^+)}{0.5} + i_L(0^+) + 2\frac{dV(0^+)}{dt}$$

$$1 = 0 + 0 + 2\frac{dV(0^+)}{dt}$$

$\therefore \frac{dV(0^+)}{dt} = \frac{1}{2}$ ~~Amp~~ V/sec.

Now, Differentiating equation ② w.r.to t we get

$$\therefore 0 = \frac{1}{R}\frac{dv(t)}{dt} + \frac{v(t)}{L} + C\frac{d^2v(t)}{dt}$$

at, $t = 0^+$

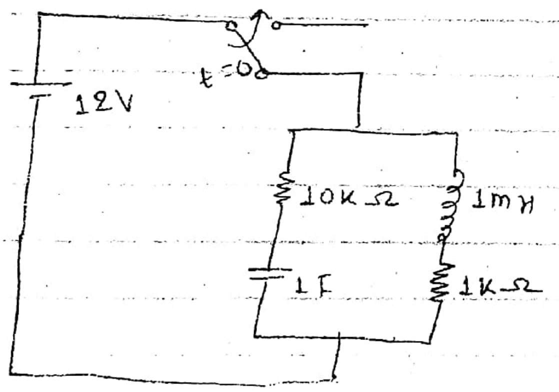
$$0 = \frac{1}{0.5}\frac{dv(0^+)}{dt} + \frac{v(0^+)}{0.5} + 2\frac{d^2v(0^+)}{dt^2}$$

or, $0 = 2\times\frac{1}{2} + 0 + 2\frac{d^2v(0^+)}{dt^2}$

or, $\frac{d^2v(0^+)}{dt^2} = -\frac{1}{2}$ Volt/sec^2

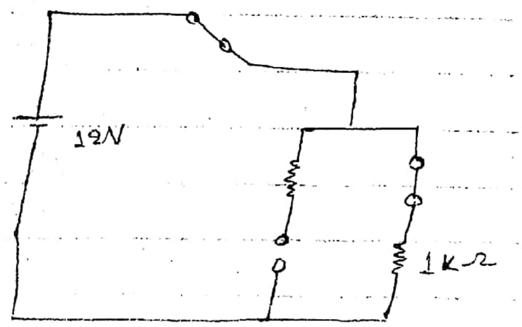
$\therefore v(0^+) = 0$, $\frac{dv(0^+)}{dt} = \frac{1}{2}$ V/sec and, $\frac{d^2v(0^+)}{dt^2} = -\frac{1}{2}$ V/sec^2

.14



At $t = 0^-$, steady state is reached. Equivalent circuit is

$i(0^-) = \frac{12V}{1K\Omega} = 0.012$ Amp
$= 0.012$ Amp



$i(0^+) = i(0^-) = 0.012$ Amp

[$\because$ Current through inductor can not change instantaneously.]

$V_c(0^-) = 12V. = V_c(0^+)$ ($\because$ Voltage across capacitor can not change instantaneou

Apply KVL for $t > 0$, we get

$$10K\, i(t) + 1m \frac{di(t)}{dt} + 1K\, i(t) + \frac{1}{1}\int i(t)\,dt = 0$$

$$11{,}000\, i(t) + 10^{-3}\frac{di(t)}{dt} + \int i(t)\,dt = 0 \quad \cdots\cdots (1)$$

$$11{,}000\, i(t) + 10^{-3}\frac{di(t)}{dt} + V_c = 0 \quad \cdots\cdots (2)$$

at, $t = 0^+$

$$11{,}000\, i(0^+) + 10^{-3}\frac{di(0^+)}{dt} + V_c(0^+) = 0$$

$$11{,}000 \times 0.012 + 10^{-3}\frac{di(0^+)}{dt} + 12 = 0$$

$$11 \times 12 + 10^{-3}\frac{di(0^+)}{dt} = -12$$

$$\frac{di(0^+)}{dt} = \frac{-12 - 132}{10^{-3}} = -\cancel{144}\,120 \times 10^{3} \text{ amp/sec.}$$

Differentiating eq^n (1) w.r.to t

$$11{,}000\frac{di(t)}{dt} + 10^{-3}\frac{d^2 i(t)}{dt} + i(t) = 0$$

at, $t = 0^+$

$$11{,}000\frac{di(0^+)}{dt} + 10^{-3}\frac{d^2 i(0^+)}{dt^2} + i(0^+) = 0$$

$$\Rightarrow 11{,}000 \times (-\cancel{144}\,120 \times 10^{3}) + 10^{-3}\frac{d^2 i(0^+)}{dt^2} + 0.012 = 0$$

$$\frac{d^2 i(0^+)}{dt^2} = \frac{\cancel{1584} \overset{132}{} \times 10^{\cancel{4}7} + 0.012}{10^{-3}}$$

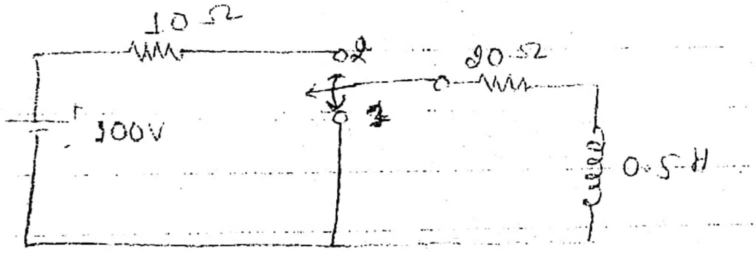
$$= \cancel{1584 \times 10^9} \text{ amp/sec}^2$$

$$= 132 \times 10^{10}$$

$i(0^+) = 0.012$ amp , $\frac{di(0^+)}{dt} = -120 \times 10^3$ amp/sec,

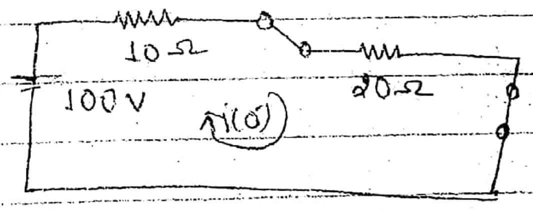
and, $\frac{d^2 i(0^+)}{dt} = \cancel{1584 \times 10^9}$ amp/sec^2
132×10^{10}

Q.15



at, $t = 0^-$, Steady state is reached at position (2)
equivalent circuit is,

$\therefore i(0^-) = \frac{100V}{10+20} = \frac{10}{3}$ amp.



$\therefore i(0^+) = i(0^-) = \frac{10}{3}$ Amp [∵ Current through inductor can not change instantaneously.]

Apply KVL at $t > 0$.

$$20\, i(t) + 0.5 \frac{di(t)}{dt} = 0 \quad \cdots (1)$$

at, $t = 0^+$

$$20\, i(0^+) + 0.5 \frac{di(0^+)}{dt} = 0$$

$$20 \times \frac{10}{3} + 0.5 \frac{di(0^+)}{dt} = 0$$

$$\frac{di(0^+)}{dt} = \frac{-200}{3 \times 0.5} = -\frac{400}{3} \text{ amp/sec.}$$

Differentiating eq^n ① w.r.t t, we get

$$20 \frac{di(t)}{dt} + 0.5 \frac{d^2 i(t)}{dt^2} = 0$$

at, $t = 0^+$

$$20 \frac{di(0^+)}{dt} + 0.5 \frac{d^2}{dt^2}(0^+) = 0$$

$$0.5 \frac{d^2 i(0^+)}{dt^2} = -20 \times -\frac{400}{3} = \frac{800}{3}$$

$$\therefore \frac{d^2 i(0^+)}{dt^2} = \frac{1600}{3} \text{ amp/sec}^2$$

$\therefore i(0^+) = \frac{10}{3}$ amp, $\frac{di(0^+)}{dt} = -\frac{400}{3}$ amp/sec.

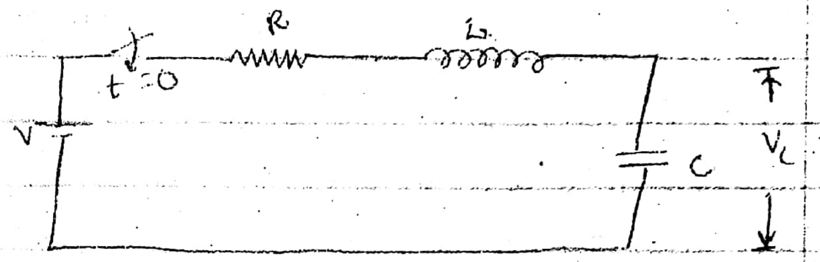
and, $\frac{d^2 i(0^+)}{dt^2} = \frac{1600}{3}$ amp/sec^2.

//.

Prepared by: Praladh Chapagai

Solution (Assignment 2) Ch.3

Q.1 Here, $R = 7\Omega$, $L = 1H$ and $C = 0.1F$
$V = 20V$



Initially, $i_L(0^-) = 0 = i_L(0^+)$ [∵ Current through inductor can not change instanteneously.]

$V_c(0^-) = 0 = V_c(0^+)$ [∵ Voltage across capacitor can not change instanteneously].

Now, at $t>0$, Using KVL

$$V = Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt \quad \text{①}$$

$$20 = 7i + \frac{di}{dt} + 10\int_{-\infty}^{t} i\,dt \quad \text{①}$$

$$20 = 7i + \frac{di}{dt} + \cancel{10\int_{-\infty}^{0} i\,dt} + 10\int_{0}^{t} i\,dt \quad V_c(0^+) = 0$$

$$20 = 7i + \frac{di}{dt} + 10\int_0^t i\,d(t)$$

Differentiating with respect to t

$$0 = 7\frac{di}{dt} + \frac{d^2i}{dt^2} + 10i$$

$$\frac{d^2i}{dt^2} + 7\frac{di}{dt} + 10i = 0 \quad \text{②}$$

Equation ② is homogeneous second order differential

2.

equation. So, forced response becomes zero.

Thus, $i(t) = i_f(t) + i_t(t)$

$= i_t(t)$

Now, for transient response $i_t(t)$:

characteristic equation is, $s^2 + 7s + 10 = 0$

$$s_1, s_2 = \frac{-7 \pm \sqrt{49-40}}{2}$$

$$= \frac{-7 \pm 3}{2} = -2, -5$$

Thus, $i(t) = k_1 e^{s_1 t} + k_2 e^{s_2 t}$

$= k_1 e^{-2t} + k_2 e^{-5t}$ - - - - (3)

Now, using initial condition at $t = 0^+$

$i(0^+) = k_1 e^0 + k_2 e^0 \Rightarrow 0 = k_1 + k_2$

$\therefore k_1 = -k_2$

Now, Equation (1) at $t = 0^+$

$$20 = 7i(0^+) + \frac{di(0^+)}{dt} + v_c(0^+)$$

$$\frac{di(0^+)}{dt} = 20$$

Differentiating eq^n. (3) we get

$$\frac{di(t)}{dt} = -2k_1 e^{-2t} - 5k_2 e^{-5t}$$

at $t = 0^+$

$di(0^+) = -2k_1 e^0 - 5k_2 e^0$

$20 = -2k_1 - 5k_2$

or, $20 = -2k_1 + 5k_1$

$\therefore k_1 = \frac{20}{3}$ and, $k_2 = -\frac{20}{3}$

Thus, $i(t) = \frac{20}{3}e^{-2t} - \frac{20}{3}e^{-5t}$

$$\therefore V_c(t) = \frac{1}{C}\int_{-\infty}^{t} i(t)\,dt$$

$$= \frac{1}{C}\int_{-\infty}^{0} i(t)\,dt + \frac{1}{C}\int_{0}^{t} i(t)\,dt$$

$$= \frac{1}{C}\int_{0}^{t}\left[\frac{20}{3}e^{-2t} - \frac{20}{3}e^{-5t}\right]$$

$$= 10\left\{\frac{20}{3}\left[\frac{e^{-2t}}{-2}\right]_0^t - \frac{20}{3}\left[\frac{e^{-5t}}{-5}\right]_0^t\right\}$$

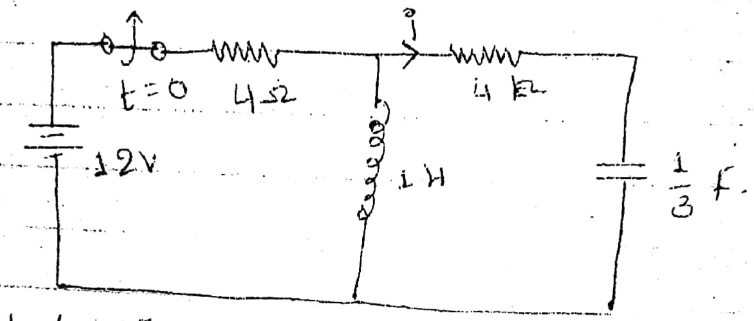
$$= \frac{200}{3}\left\{\frac{e^{-2t}-1}{-2}\right\} - \frac{200}{3}\left\{\frac{e^{-5t}-1}{-5}\right\}$$

$$= -\frac{200}{3}e^{-2t} + \frac{100}{3} + \frac{40}{3}e^{-5t} - \frac{40}{3}$$

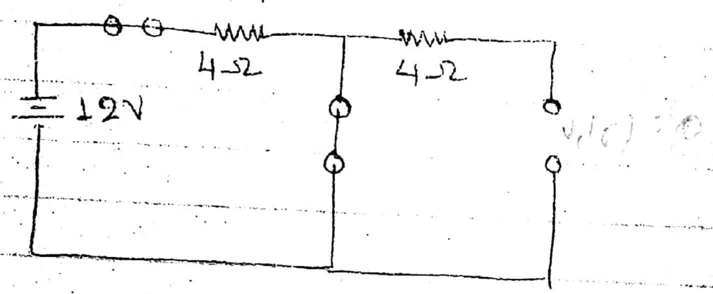
$$= -\frac{100}{3}e^{-2t} + \frac{40}{3}e^{-5t} + 20$$

$$\therefore V_c(t) = -\frac{100}{3}e^{-2t} + \frac{40}{3}e^{-5t} + 20$$

Q.2 Give circuit is,



At, $t = 0^-$ Equivalent circuit is,

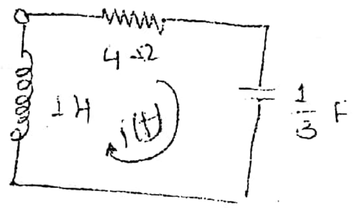


$$i(0^-) = \frac{12V}{4\Omega} = 3A$$

$\therefore i(0^+) = i(0^-) = 3A$ ($\because$ Current through inductor can no[t] instanteneously.

$V_c(0^-) = 0 = V_c(0^+)$ ($\because$ Voltage across capacitor ca[n't] change instanteneously.)

At $t > 0$, Equivalent circuit is,



Using KVL,

or, $4i(t) + \frac{di(t)}{dt} + 3V_c(t) = 0$ ...... ①

or, $4i(t) + \frac{di(t)}{dt} + 3\int_0^t i(t)dt = 0$ ...... ②

Differentiating w.r.to t,

$4\frac{di(t)}{dt} + \frac{d^2i(t)}{dt^2} + 3i(t) = 0$ ...... ③

Characteristic equation is,

$s^2 + 4s + 3 = 0$

$s_1, s_2 = \frac{-4 \pm \sqrt{16-12}}{2} = \frac{-4 \pm 2}{2} = -3, -1$

Equation ③ is homogeneous equation so, it has forced response $i_f(t) = 0$.

$\therefore i(t) = i_t(t) + \cancel{i_f(t)}^{0}$

$= i_t(t)$

$\therefore$ Transient response, $i_t(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t}$

$i_t(t) = K_1 e^{-3t} + K_2 e^{-t}$

$\therefore i(t) = K_1 e^{-3t} + K_2 e^{-t}$ ...... ④

at, $t = 0^+$

$i(0^+) = K_1 e^0 + K_2 e^0$

$\Rightarrow 3A = K_1 + K_2$

$\Rightarrow K_1 = \cancel{K_2}\ 3 - K_2$ ...... ⑤

Now, differentiating at $t = 0^+$ eq^n ① becomes

$4i(0^+) + \frac{di(0^+)}{dt} + 3\cancel{V_c(0^+)}^{0} = 0$

$\frac{di(0^+)}{dt} = -4 \times 3 = -12$ amp/sec

③

6.

Differentiating eqn (4)

$$\frac{di(t)}{dt} = -3K_1 e^{-3t} - K_2 e^{-t}$$

at, $t = 0^+$

$$\frac{di(0^+)}{dt} = -3K_1 e^{0} - K_2 e^{0}$$

$$-12 = -3K_1 - K_2$$

or, $-12 = -3(3-K_2) - K_2$

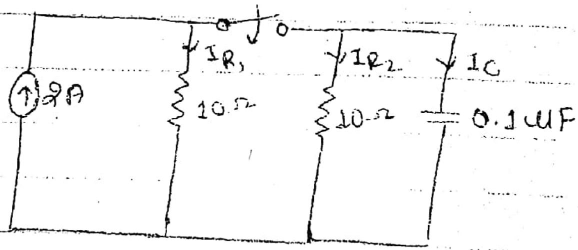
or, $-12 = -9 + 3K_2 - K_2$

$\therefore K_2 = -\frac{3}{2}$ and, $K_1 = 3 + \frac{3}{2} = \frac{9}{2}$

Thus,

$$i(t) = \cancel{-\frac{3}{2}e^{-3t}} + \cancel{\frac{9}{2}e^{-t}} \quad \frac{9}{2}e^{-3t} - \frac{3}{2}e^{-t}$$

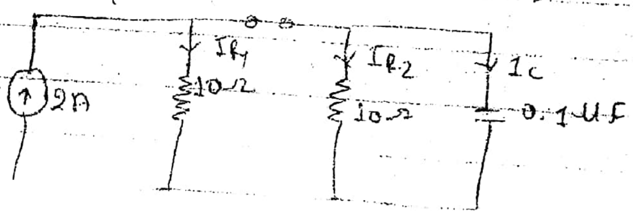
Q.3 Here,



From figure,

$v_c(0^-) = 0 = v_c(0^+)$ [$\because$ voltage across Capacitor can not change instanteneously]

at, $t > 0$, Equivalent circuit is,



Using KCL we have,

$$2A = I_{R1} + I_{R2} + I_{C1}$$

$$2 = \frac{V}{10} + \frac{V}{10} + C\frac{dv}{dt}$$

$$2 = \frac{V}{5} + 0.1\frac{dv}{dt} \quad \ldots \text{(1)}$$

Equation (1) is non-homogeneous equation. It has two parts of solution.

(1) Forced solution

from (1), in p operator form

$$2 = \frac{V}{5} + 0.1PV$$

$$V(t) = \frac{2}{\left(\frac{1}{5} + 0.1p\right)}$$

Here, source is constant so put $p = 0$ to find $V_f(t)$

$$V_f(t) = \frac{2}{1/5} = 10\,V.$$

(2) Transient solution : Homogeneous version of (1) is, $0.1\frac{dv}{dt} + \frac{V}{5} =$

from (1), characteristic equation is,

$$0.1S + \frac{1}{5} = 0$$

$$S = -\frac{1}{0.5} = -2$$

∴ Transient solution, $V_1(t) = K_1 e^{-2t}$

8

$\therefore$ Complete response, $v(t) = v_f(t) + v_r(t)$

$$= 10 + K_1 e^{-2t}$$

at, $t = 0^+$

$$v(0^+) = 10 + K_1 e^{-2t}$$

$$0 = 10 + K_1 e^{-2 \cdot 0}$$

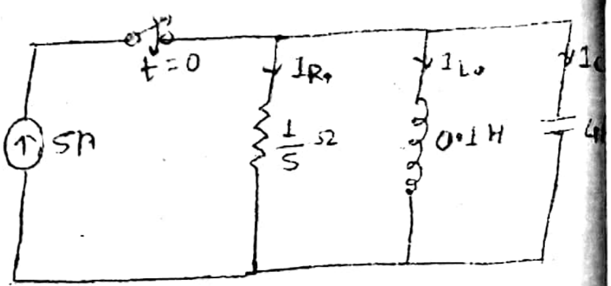
$$K_1 = -10$$

Thus, $V(t) = 10 - 10 e^{-2t}$

$$= 10(1 - e^{-2t})$$

Q.4 Here,

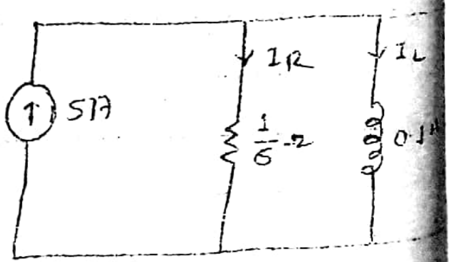
$I = 5A$, $R = \frac{1}{5}\,\Omega$, $L = 0.1H$, $C = 4F$.



At, $t = 0^-$, $V_c(0^-) = 0$

$V_c(0^+) = 0 = V_c(0^-)$ ($\because$ Voltage across capacitor can not change instanteneously.)

For $t > 0$, equivalent circuit is,



Using KCL,

$5A = I_R + I_L + I_C$

Differentiating w.r. to t

$$0 = 5\frac{dv(t)}{dt} + 10\,v(t) + 4\frac{d^2v(t)}{dt} \quad \cdots ②$$

Equation ② is homogeneous equation. It has forced response zero. So,

$$v(t) = \cancel{v_f(t)}^{0} + v_t(t)$$
$$= v_t(t)$$

For transient response,
Characteristic equation is,

$$4s^2 + 5s + 10 = 0$$

$$s_1, s_2 = \frac{-5 \pm \sqrt{25 - 160}}{8} = \frac{-5 \pm \sqrt{-135}}{8}$$

$$s_1 = \frac{-5 + j\sqrt{135}}{8}$$

$$s_2 = \frac{-5 - j\sqrt{135}}{8}$$

Transient response, $v_t(t) = K_1 e^{\left(\frac{-5+j\sqrt{135}}{8}\right)t} + K_2 e^{\left(\frac{-5-j\sqrt{135}}{8}\right)t} \quad \cdots ③$

$v(t) = v_t(t)$

Now, at $t = 0^+$

$$v(0^+) = K_1 e^0 + K_2 e^0$$
$$0 = K_1 + K_2$$
$$\therefore K_1 = -K_2$$

From eq^n ① at $t = 0^+$

$$5 = \frac{\cancel{v(0^+)}^{0}}{R} + \cancel{i_L(0^+)}^{0} + c\frac{dv(0^+)}{dt} \Rightarrow \frac{dv(0^+)}{dt} = \frac{5}{4} \quad ④$$

10

Now, differentiating eqⁿ 3

$$\frac{dv(t)}{dt} = \left(\frac{-5+j\sqrt{135}}{8}\right) k_1 e^{\left(\frac{-5+j\sqrt{135}}{8}\right)t} + \left(\frac{-5-j\sqrt{135}}{8}\right) k_2 e^{\left(\frac{-5-j\sqrt{135}}{8}\right)t}$$

at, $t = 0^+$

$$\frac{dv(0^+)}{dt} = \left(\frac{-5+j\sqrt{135}}{8}\right) k_1 + \left(\frac{-5-j\sqrt{135}}{8}\right) k_2$$

$$\frac{5}{4} = \cancel{-\frac{5}{8}k_1} + \frac{j\sqrt{135}}{8} k_1 + \cancel{\frac{5}{8}k_1} + \frac{j\sqrt{135}}{8} k_2$$

$$\frac{5}{\cancel{4}} = \frac{j\sqrt{135}}{\cancel{4}} k_2$$

$$k_2 = \frac{5}{j\sqrt{135}} = -\frac{j5}{\sqrt{135}} = -k_1$$

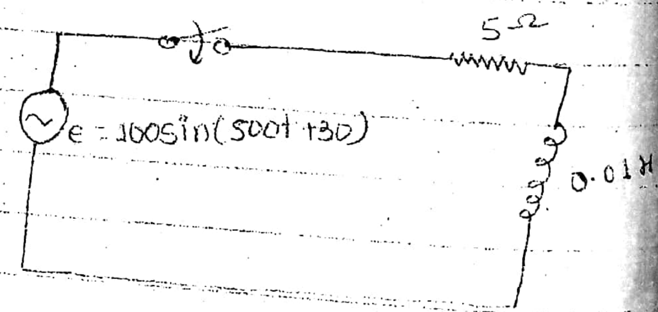
$$\therefore V(t) = \frac{5j}{\sqrt{135}} e^{\left(\frac{-5+j\sqrt{135}}{8}\right)t} - \frac{5j}{\sqrt{135}} e^{\left(\frac{-5-j\sqrt{135}}{8}\right)t}$$

Q.5. Here,

$e = 100\sin(500t + 30)$

$R = 5\Omega$

$L = 0.01H$



At, $t = 0^-$

$i(0^-) = 0$

$\therefore i_L(0^+) = i(0^-) = 0$ ($\because$ Current through inductor can not change instantaneously)

Now, at t>0, apply KVL

$$e = iR + L\frac{di}{dt}$$

$$100\sin(500t+30) = 5\,i(t) + 0.01\frac{di(t)}{dt} \quad \cdots\cdots (1)$$

Equation (1) is non-homogeneous equation so it has two parts of solution.

(1) Forced solution

from eqn. (1) in p-operator form

$$5i(t) + 0.01\,p\,i(t) = 100\sin(500t+30)$$

$$i(t)\,[5+0.01P] = 100\sin(500t+30)$$

$$i(t) = \frac{\mathrm{Im}\left[100e^{j(500t+30)}\right]}{5+0.01P}$$

Here, source is exponential so replace p by s and s by j500

$$i_f(t) = \frac{\mathrm{Im}\left[100e^{j(500t+30)}\right]}{5+j5}$$

$$= \mathrm{Im}\left[\frac{100\,e^{j(500t+30)}}{7.07\angle 45^\circ}\right]$$

$$= \mathrm{Im}\left[14.14\,e^{j(500t-15)}\right]$$

$$= 14.14\sin(500t-15)$$

(2) Transient response

Homogeneous version of equation ① is,

$$0.01\frac{di(t)}{dt} + 5i(t) = 0$$

Characteristic equation is, $0.01S + 5 = 0$

$S = -500$

∴ transient solution, $i_t(t) = ke^{S_1 t}$

$= Ke^{-500t}$

∴ Total solution, $i(t) = i_f(t) + i_t(t)$

$= 14.14\sin(500t - 15) + Ke^{-500t}$

at, $t = 0^+$

$i(0^+) = 14.14\sin(0 - 15) + Ke^0$

$0 = -3.7 + K$

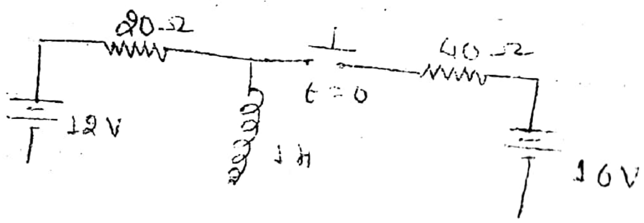
$K = 3.7$

Thus, $i(t) = 14.14\sin(500t - 15^\circ) + 3.7e^{-500t}$

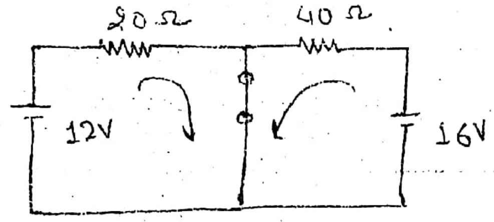
Q.6. Same as Q.1

Hints: Find, i(t) only. and i (0.1) by putting t = 0.1

Q.7 Here,



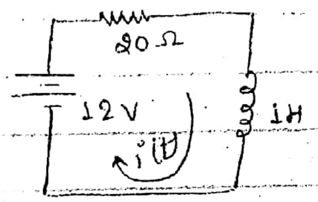
At, $t = 0^-$



$$i(0^-) = \frac{12}{20} + \frac{16}{40} = \frac{40}{40} = 1\ A.$$

So, $i(0^+) = i(0^-) = 1$ Amp [∵ Current through inductor cannot change instanteneously.]

At, $t>0$, Equivalent circuit



$$12 = 20\,i(t) + \frac{di(t)}{dt} \quad \ldots (1)$$

Equation (1) is non-homogeneous equation. It has two parts of solution.

(1) Forced solution

Equation (1) in p operator form

$$12 = 20\,i(t) + p\,i(t)$$

$$i(t) = \frac{12}{20+p}$$

Here, source is constant. So, put $p=0$ for forced response.

$$i_f(t) = \frac{12}{20} = \frac{3}{5}\ \text{Amp.}$$

(2) Transient solution

Homogeneous version of equation (1) is,

$$\frac{di(t)}{dt} + 20\,i(t) = 0$$

Characteristic equation is,

$$s + 20 = 0$$

$$s = -20$$

$\therefore$ transient solution, $i_t(t) = Ke^{st}$

$$= Ke^{-20t}$$

Thus, total solution,

$$i(t) = i_f(t) + i_t(t)$$

$$= \frac{3}{5} + Ke^{-20t}$$

at, $t = 0^+$

$$i(0^+) = i_f(t) + i_t(t)$$

$$1 = \frac{3}{5} + Ke^0$$

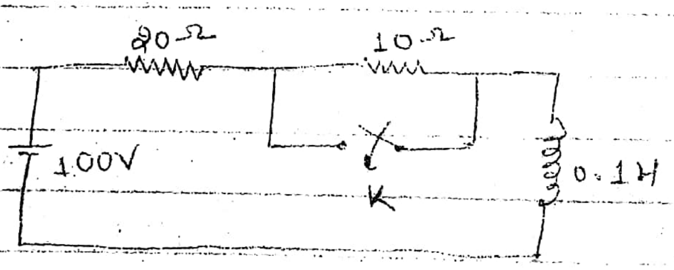
$$K = 1 - \frac{3}{5} = \frac{2}{5}$$

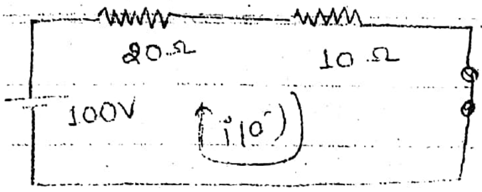
$$\boxed{\therefore\ i(t) = \frac{3}{5} + \frac{2}{5}e^{-20t}}$$

Q.8

Same as Q.4.

Q.9 Here,

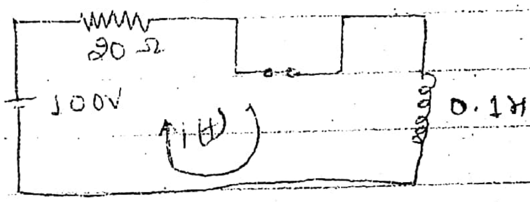




$i(0^-) = \frac{100}{30} = \frac{10}{3}$ Amp.

$i(0^+) = i(0^-) = \frac{10}{3}$ ($\because$ Current through inductor can not change instanteneously.)

At $t > 0$, Equivalent circuit



$20\,i(t) + 0.1\frac{di(t)}{dt} = 100$ ... ①

equation ① is non-homogeneous equation. It has two parts of solution.

① Forced solution

from eqⁿ ①, in p-operator form

$20\,i(t) + 0.1\,p\,i(t) = 100$

$i(t) = \frac{100}{20 + 0.1p}$ Here source is constant, so put $p = 0$ for forced response.

$i_f(t) = \frac{100}{20} = 5$ Amp

② Transient solution

Homogeneous version of eqⁿ ① is,

$20\,i(t) + 0.1\frac{di(t)}{dt} = 0$

characteristic equation is,

$$20 + 0.1s = 0$$

$$s = -200$$

$\therefore$ transient response $i_t(t) = Ke^{st}$

$$= Ke^{-200t}$$

$\therefore$ Total response $i(t) = i_f(t) + i_t(t)$

$$= 5 + Ke^{-200t}$$

at, $t = 0^+$

$$i(0^+) = 5 + Ke^0$$

$$\frac{10}{3} = 5 + K \Rightarrow K = \frac{10}{3} - 5 = -\frac{5}{3}$$

$$\therefore i(t) = 5 - \frac{5}{3} e^{-200t}$$

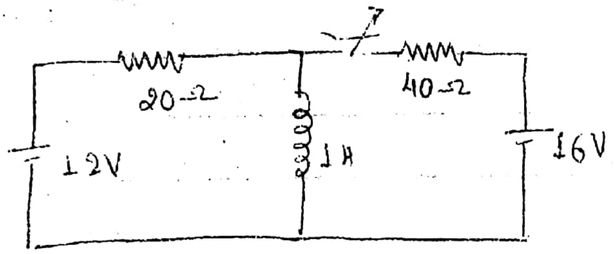
Q.10 Same as Q.6

only values are different.

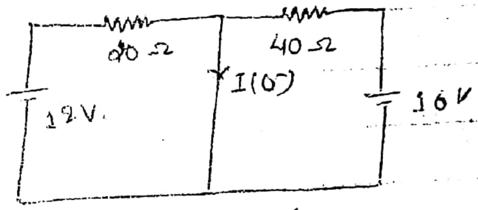
do yourself ☺ !!

Ch-4
Assignment 3: Solution

Q. 1



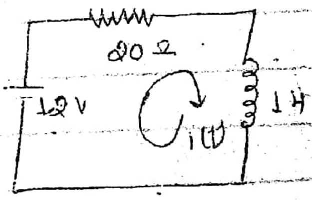
At, $t = 0^-$, Equivalent circuit is



$i(0^-) = \frac{12}{20} + \frac{16}{40} = 1$ Amp.

$i(0^+) = i(0^-) = 1$ ($\because$ Current through inductor can not change instanteneously)

Now, Equivalent circuit at $t > 0$



$$20\,i(t) + \frac{di(t)}{dt} = 12$$

Taking Laplace transform on both side we get

$$20I(S) + SI(S) - i(0^+) = \frac{12}{S}$$

$$I(S)(S+20) = \frac{12}{S} + 1 = \frac{(12+S)}{S}$$

$$I(S) = \frac{12+S}{S(S+20)}$$

Using partial fraction expansion,

$$I(S) = \frac{12+S}{S(S+20)} = \frac{A}{S} + \frac{B}{S+20}$$

18

$$A = \frac{12+s}{s(s+20)} \times s \Big|_{s=0} = \frac{12}{20} = \frac{3}{5}$$

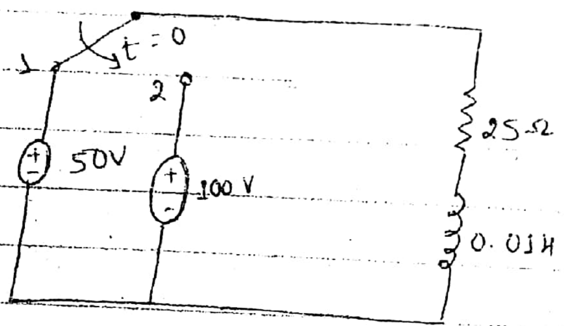
$$B = \frac{12+s}{s(s+20)} \times (s+20) \Big|_{s=-20} = \frac{12-20}{-20} = \frac{-8}{-20} = \frac{2}{5}$$

$$\therefore I(s) = \frac{3}{5s} + \frac{2}{5(s+20)}$$

Taking inverse laplace transform, we have

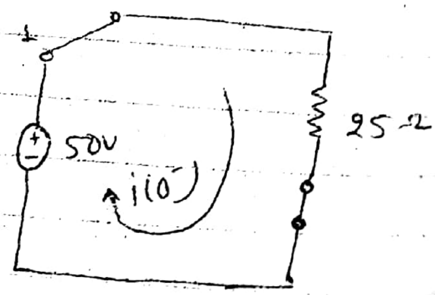
$$i(t) = \frac{3}{5} + \frac{2}{5} e^{-20t}$$

Q.2.



At $t = 0^-$

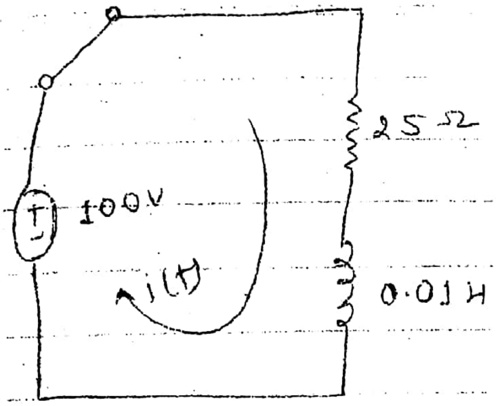
Equivalent circuit is,



$$i(0^-) = \frac{50}{25} = 2 \text{ Amp.}$$

NOW,

$i(0^+) = i(0^-) = 2$ Amp. [$\because$ Current through inductor can not change instanteneously.]



$$25\,i(t) + 0.01\frac{di(t)}{dt} = 100$$

Taking Laplace transform on both side, we get

$$25\,I(S) + 0.01\,S\,I(S) - 0.01\,i(0) = \frac{100}{S}$$

or, $$I(S)\,[0.01S + 25] = \frac{100}{S} + 0.02$$

or, $$I(S) = \frac{0.02S + 100}{S(0.01S + 25)}$$

or, $$I(S) = \frac{0.02\,[S + 5000]}{0.01\,S\,[S + 2500]} = \frac{2\,(S + 5000)}{S\,(S + 2500)}$$

Using partial fraction expansion

$$I(S) = \frac{2\,(S + 5000)}{S(S + 2500)}$$

$$\Rightarrow 2\,\frac{S + 5000}{S(S + 2500)} = \frac{A}{S} + \frac{B}{S + 2500}$$

$$A = 2\frac{(S + 5000)}{S(S + 2500)} \times S\Big|_{S=0} = 4$$

$\cancel{B} =$

$$B = \frac{2(s+5000)}{s(s+2500)} \times (s+2500) \Bigg|_{s=-2500} = \frac{2(-2500+5000)}{-2500}$$

$$= -2$$

$$\therefore I(s) = \frac{4}{s} - \frac{2}{s+2500}$$

Taking Inverse laplace transform we get

$$i(t) = 4 - 2e^{-2500t}$$

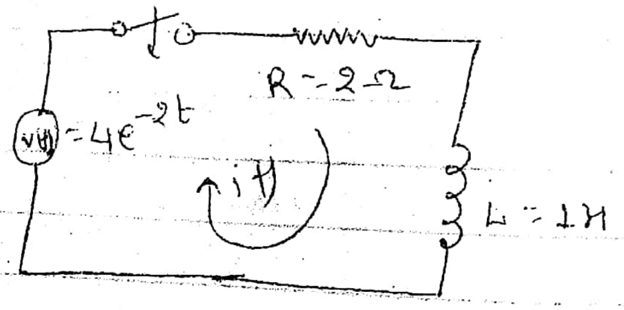
Q.4 Here,

$V(t) = 4e^{-2t}$

$R = 2\Omega$

$L = 1H$

$i_L(0^-) = 3A$



$\therefore i(0^+) = i(0^-) = 3A$ [$\because$ Current through inductor can not change instanteneously.]

Now, at $t>0$, apply KVL

$$2i(t) + \frac{di(t)}{dt} = 4e^{-2t}$$

Using Laplace transform on both side, we get

21)

~~$s[I(s)]$~~

$$I(s)[s+2] - 3 = \frac{4}{s+2}$$

$$I(s) = \frac{4+3s+6}{(s+2)^2}$$

$$I(s) = \frac{10+3s}{(s+2)^2}$$

Using partial fraction expansion:

$$I(s) = \frac{10+3s}{(s+2)^2} = \frac{A}{s+2} + \frac{B}{(s+2)^2}$$

$$\Rightarrow \frac{10+3s}{(s+2)^2} = \frac{A(s+2)+B}{(s+2)^2}$$

$$\Rightarrow 10+3s = As + (2A+B)$$

Comparing coefficient,

$A = 3$

$B = 10 - 2 \times 3 = 4$

Thus,

$$I(s) = \frac{3}{s+2} + \frac{4}{(s+2)^2}$$

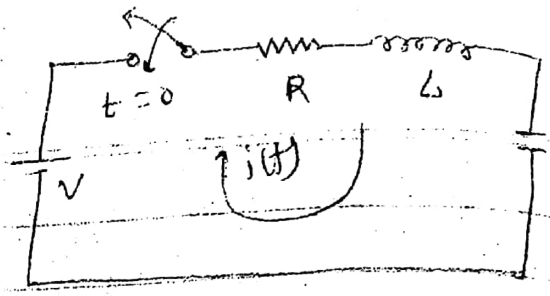
Taking inverse Laplace transform

$$i(t) = 3e^{-2t} + 4te^{-2t}$$

25. Here,

$V_0 = 10V$, $R = 5\Omega$, $L = 1H$ and $C = 0.25F$

22



$i(0^-) = 0$ and $V_c(0^-) = 0$

$\therefore i(0^+) = i(0^-) = 0$ ($\because$ current through inductor can not ch instanteneously)

$\therefore V_c(0^+) = V_c(0^-) = 0$ ($\because$ Voltage across capacitor can not cha instanteneously)

Now, applying KVL at $t > 0$

$$Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int_{-\infty}^{t} i(t)dt = V$$

or, $$Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int_{-\infty}^{0} i(t)dt + \frac{1}{C}\int_{0}^{t} i(t)dt = V$$

or, $$5i(t) + \frac{di(t)}{dt} + \cancel{V_c(0^+)}^{0} + 4\int_{0}^{t} i(t)dt = 10 \quad \ldots \ldots (1)$$

Differentiating w.r.to t we get

$$5\frac{di(t)}{dt} + \frac{d^2i(t)}{dt^2} + 4i(t) = 0 \quad \ldots \ldots (2)$$

from (1)

$$5i(t) + \frac{di(t)}{dt} + 4V_c(t) = 10$$

at $t = 0^+$

$$5i(0^+) + \frac{di(0^+)}{dt} + 4V_c(0^+) = 10$$

$$\frac{di(0^+)}{dt} = 10 \text{ amp/sec.}$$

Now, Taking Laplace transform of equation ②, we get

$$5s\,I(s) - i(0) + s^2 I(s) - s\,i(0) - i'(0) + 4\,I(s) = 0$$

or, $I(s)\left[5s + s^2 + 4\right] - 0 - s \cdot 0 - 10 = 0$

or, $I(s)\left[s^2 + 5s + 4\right] = 10$

or, $I(s) = \dfrac{10}{s^2+5s+4} = \dfrac{10}{(s+4)(s+1)}$

Now, using partial fraction expansion,

$$I(s) = \frac{10}{s^2+5s+4} = \frac{10}{(s+4)(s+1)} = \frac{A}{s+4} + \frac{B}{s+1}$$

Now,

$$A = \left.\frac{10}{(s+4)(s+1)} \times (s+4)\right|_{s=-4} = -\frac{10}{3}$$

$$B = \left.\frac{10}{(s+4)(s+1)} \times (s+1)\right|_{s=-1} = \frac{10}{3}$$

Thus, $I(s) = \dfrac{-10}{3(s+4)} + \dfrac{10}{3(s+1)}$

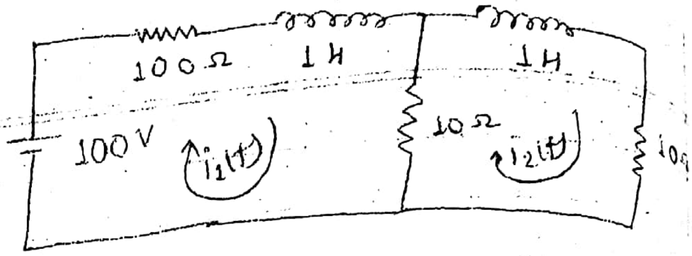
Taking inverse Laplace transform, we get

$$i(t) = -\frac{10}{3} e^{-4t} + \frac{10}{3} e^{-t}$$

④

27

Q.6 Here



$i_L(0^-) = 0$

$\therefore I_L(0^+) = i_L(0^-) = 0$ ($\because$ Current through inductor cannot change instanteneously.)

Applying KVL, at loop 1

$$100\, i_1(t) + 1\frac{di_1(t)}{dt} + 10(i_1(t) - i_2(t)) = 100$$

Taking Laplace transform

$$100 I_1(S) + S I_1(S) - \cancel{I_L(0)}^{0} + 10 I_1(S) - 10 I_2(S) = \frac{100}{S}$$

or, $I_1(S)[100 + S + 10] I_1(S) - 10 I_2(S) = \frac{100}{S}$

or, $(110 + S) I_1(S) - 10 I_2(S) = \frac{100}{S}$ ...... (1)

Applying KVL, at loop 2

$$10 i_2(t) + 1\frac{di_2(t)}{dt} + 10(i_2(t) - i_1(t)) = 0$$

Taking Laplace transform,

$$10 I_2(S) + S I_2(S) - I_L(0) + 10 I_2(S) - 10 I_1(S) = 0$$

or, $I_2(S)[10 + S + 10] = 10 I_1(S)$

from eq^n ① and ②

$$\frac{(110+s)(20+s)}{10} I_2(s) - 10\, I_2(s) = \frac{100}{s}$$

or, $I_2(s)\left[(110+s)(20+s) - 100\right] = \frac{1000}{s}$

or, $I_2(s)\left[2200 + 130s + s^2 - 100\right] = \frac{1000}{s}$

or, $I_2(s)\left[s^2 + 130s + 2100\right] = \frac{1000}{s}$

or, $I_2(s) = \frac{1000}{s(s+18.9)(s+111.1)}$

Now, taking Partial fraction expansion,

$$I_2(s) = \frac{1000}{s(s+18.9)(s+111.1)} = \frac{A}{s} + \frac{B}{s+18.9} + \frac{C}{s+111.1}$$

$$A = \left.\frac{1000}{s(s+18.9)(s+111.1)}\right|_{s=0} = \frac{1000}{18.9\times 111.1}$$

$$= 0.48$$

$$B = \left.\frac{1000}{s(s+18.9)(s+111.1)} \times (s+18.9)\right|_{s=-18.9} = \frac{1000}{-18.9\times(-18.9+111.1)}$$

$$= -0.57$$

$$C = \left.\frac{1000}{s(s+18.9)(s+111.1)} \times (s+111.1)\right|_{s=-111.1} = \frac{1000}{-111.1\,(-111.1+18.9)}$$

$$= 0.097$$

Thus, $I(S) = \frac{0.48}{S} - \frac{0.57}{S+18.9} + \frac{0.097}{S+111.1}$

Taking Inverse Laplace transform

$i(t) = 0.48 - 0.57e^{-18.9t} + 0.097e^{-111.1t}$

Q.7 Initial value theorem

If $f(t)$ and its first derivative $f'(t)$ [illegible] Laplace transformable then,

$$f(0) = \lim_{t \to 0} f(t) = \lim_{S \to \infty} SF(S)$$

Final value theorem

If $f(t)$ and its first derivative $f'(t)$ [illegible] Laplace transformable then,

$$f(\infty) = \lim_{t \to \infty} f(t) = \lim_{S \to 0} SF(S)$$

Numerical part

Given,

$$I(S) = \frac{(S+1)}{S(S+2)}$$

For $i(0)$

use Initial value theorem

$$= \lim_{s \to \infty} \frac{s(1 + 1/s)}{s(1 + 2/s)}$$

$$= \lim_{s \to \infty} \frac{1 + 1/s}{1 + 2/s}$$

$$= \frac{1}{1} = 1$$

for steady state value of $i(t)$ i.e. $i(\infty)$

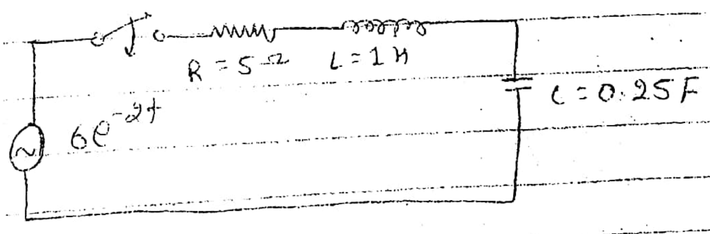
use final value theorem,

$$i(\infty) = \lim_{s \to 0} sI(s) = \lim_{s \to 0} \frac{\cancel{s}(s+1)}{\cancel{s}(s+2)}$$

$$= \frac{1}{2}$$

$\therefore$ $i(0) = 1$ amp and $i(\infty) = \frac{1}{2}$ amp

Q.8 Given, circuit



Here, $i(0^-) = 0$

$V_c(0^-) = 0$

so, $i(0^+) = i(0^-) = 0$ [Current through inductor can not change instanteneously.]

$V_c(0^+) = V_c(0^-) = 0$ [Voltage across capacitor can not c instanteneously]

Now, using KVL at $t>0$, we get

$$5i(t) + \frac{di(t)}{dt} + 4\int_{-\infty}^{t} i(t)\,dt = 6e^{-2t}$$

or, $$5i(t) + \frac{di(t)}{dt} + 4\int_{-\infty}^{0} i(t)\,dt + 4\int_{0}^{t} i(t)\,dt = 6e^{-2t}$$

or, $$5i(t) + \frac{di(t)}{dt} + \cancel{4V_c(0^+)}^{0} + 4\int_{0}^{t} i(t)\,dt = 6e^{-2t} \quad (1)$$

at, $t = 0^+$

$$\cancel{5i(0^+)}^{0} + \frac{di(0^+)}{dt} + \cancel{4V_c(0^+)}^{0} = 6e^{-0}$$

$$\frac{di(0^+)}{dt} = 6$$

Now, Differentiating eq^n (1) w.r.to t, we get

$$5\frac{di(t)}{dt} + \frac{d^2 i(t)}{dt^2} + 4i(t) = -12e^{-2t}$$

Now, Taking Laplace transform we get

$$5SI(S) - i(0) + S^2 I(S) - Si(0) - i'(0) + 4I(S) = \frac{-12}{(S+2)}$$

or, $$I(S)\left[5S + S^2 + 4\right] - 0 - 0 - 6 = \frac{-12}{(S+2)}$$

$$I(s)\left[(s+4)(s+1)\right] = \frac{6s}{(s+2)}$$

$$I(s) = \frac{6s}{(s+1)(s+2)(s+4)}$$

Now, using partial fraction expansion, we have

$$I(s) = \frac{6s}{(s+1)(s+2)(s+4)} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+4}$$

$$A = \frac{6s}{(s+1)(s+2)(s+4)} \times (s+1)\Big|_{s=-1} = \frac{-6}{3} = -2$$

$$B = \frac{6s}{(s+1)(s+2)(s+4)} \times (s+2)\Big|_{s=-2} = \frac{-12}{-1\times 2} = 6$$

$$C = \frac{6s}{(s+1)(s+2)(s+4)} \times (s+4)\Big|_{s=-4} = \frac{-24}{-3\times -2} = -4$$

Thus,

$$I(s) = -\frac{2}{s+1} + \frac{6}{s+2} - \frac{4}{s+4}$$

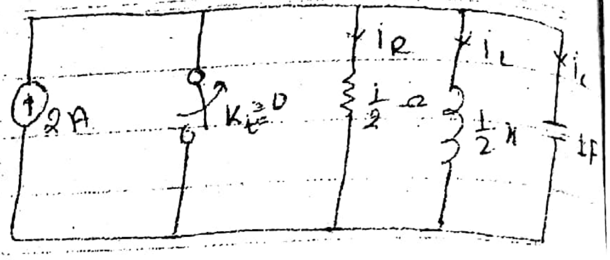
Now, taking inverse Laplace transform we get

$$i(t) = -2e^{-t} + 6e^{-2t} - 4e^{-4t}$$

Q.9 Here,

$C = 1F$, $R = \frac{1}{2}\,\Omega$, $L = \frac{1}{2}\,H$ and $I_0 = 2A$

3o



Here,

$i_R(0^-) = i_L(0^-) \pm i_c(0^-) = 0$

$V_c(0^-) = 0$

$\therefore V_c(0^+) = V_c(0^-) = 0$ ($\because$ Voltage across capacitor can not change instanteneously)

Now, use KCL at $t>0$

$$2A = i_R + i_L + i_c$$

$$\text{or, } 2 = \frac{V}{R} + \frac{1}{L}\int v\,dt + C\frac{dv}{dt}$$

$$\text{or, } 2 = 2V(t) + 2\int_{-\infty}^{t} v(t)dt + \frac{dv(t)}{dt}$$

$$\text{or, } 2 = 2V(t) + 2\int_{-\infty}^{0} v(t)dt + 2\int_{0}^{t} v(t)dt + \frac{dv(t)}{dt}$$

$$\text{or, } 2V(t) + i_L(0^+) + 2\int_{0}^{t} v(t)dt + \frac{dv(t)}{dt} = 2$$

Since, $i_L(0^+) = i_L(0^-) = 0$ ($\because$ current through inductor can not change instanteneously)

so,

$$2V(t) + 2\int_{0}^{t} v(t)\,dt + \frac{dv(t)}{dt} = 2 \qquad (1)$$

at, $t = 0^+$

$$2\cancel{V(0^+)}^{0} + \cancel{2 i_L(0^+)}^{0} + \frac{dv(0^+)}{dt} = 2$$

$$\frac{dv(0^+)}{dt} = 2$$

Differentiating equation ① we get

$$2\frac{dv(t)}{dt} + 2v(t) + \frac{d^2v(t)}{dt} = 0$$

Taking Laplace transform, we get

$$2sV(s) - 2V(0) + 2V(s) + s^2V(s) - sV(0) - V'(0) = 0$$

$$V(s)[2s+2+s^2] - 0 - 0 - 2 = 0$$

$$V(s)[s^2+2s+2] = 2$$

$$V(s) = \frac{2}{s^2+2s+2}$$

$$V(s) = \frac{2}{(s+1)^2+(1)^2}$$

$$V(s) = 2\frac{1}{(s+1)^2+1^2}$$

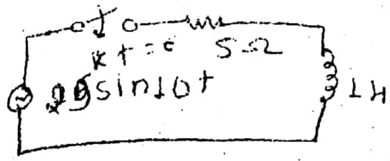
Now, taking inverse Laplace transform, we get

$$v(t) = 2e^{-t}\sin t$$

Here, $V = 25\sin 10t$, $R = 5\Omega$ and $L = 1H$.

$i(0^-) = 0$

$i(0^+) = i(0^-) = 0$ [current through inductor can not change instanteneously]



Apply KVL at $t > 0$

$$5i(t) + \frac{di(t)}{dt} = 25\sin 10t$$

32 Using Laplace transform we get

$$5I(s) + sI(s) - i(0) = 25\,\frac{10}{s^2+100}$$

$$I(s)[s+5] = \frac{250}{(s^2+100)}$$

$$I(s) = \frac{250}{(s+5)(s^2+100)}$$

Using partial fraction expansion

$$I(s) = \frac{250}{(s+5)(s^2+100)} = \frac{A}{s+5} + \frac{Bs+C}{s^2+100}$$

or, $$\frac{250}{(s+5)(s^2+100)} = \frac{As^2+100A+Bs^2+Cs+5Bs+5C}{(s+5)(s^2+100)}$$

or, $$250 = (A+B)^2 + (C+5B)s + (5C+100A)$$

Comparing coefficient.

$A+B=0 \Rightarrow A=-B$

$C+5B=0 \Rightarrow C=-5B$

$5C+100A=250$

or, $-25B-100B=250$

$\Rightarrow B=-2$

$\Rightarrow A=2$

$\Rightarrow C=10$

$$\therefore I(s) = \frac{2}{s+5} + \frac{2s-10}{s^2+100}$$

$$= \frac{2}{s+5} - 2\frac{s}{s^2+10^2} + \frac{10}{s^2+10^2}$$

Taking inverse Laplace transform we get

$$i(t) = 2e^{-5t} - 2\cos 10t + \sin 10t$$

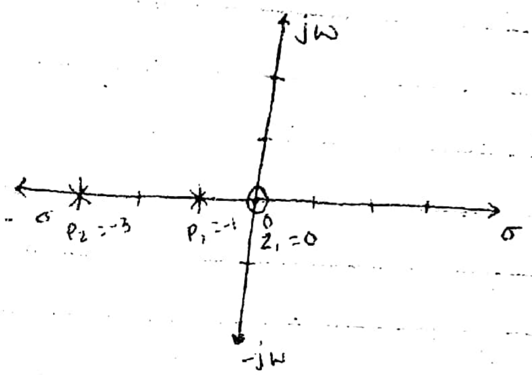
Ch 5

Assignment 4 : Solution

Q1 Given, $I(s) = \frac{4s}{(s+1)(s+3)}$

It has zeros at $s = 0$

and poles at $s = -1, -3$



Now,

$$I(s) = \frac{K_1}{(s+1)} + \frac{K_2}{(s+3)}$$

For pole at $s = -1$

$$K_1 = \frac{K(P_1 - Z_1)}{(P_1 - P_2)} = \frac{4(-1-0)}{(-1+3)} = \frac{-4}{2} = -2$$

For pole at $s = -3$

$$K_2 = \frac{K(P_2 - Z_1)}{(P_2 - P_1)} = \frac{4(-3-0)}{(-3+1)} = \frac{-12}{2} = 6$$

$$\therefore I(s) = \frac{-2}{s+1} + \frac{6}{s+3}$$

Taking inverse Laplace transform

$$i(t) = -2e^{-t} + 6e^{-3t}$$

$\therefore i(1) = -2e^{-1} + 6e^{-3} = -0.44$ $\quad \therefore |i(1)| = 0.44$ sec.

Q2 Given, characteristic equation:

$$s^5 + s^4 + 2s^3 + 2s^2 + 3s + 5 = 0$$

33

34.

R-H array

s^5	1	2	3
s^4	1	2	5
s^3	$b_1 = 0$	$b_2 = -2$	0
s^2	can not determine		
s^1			
s^0			

$$b_1 = -\frac{1}{1}\begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} = 0$$

$$b_2 = -\frac{1}{1}\begin{vmatrix} 1 & 3 \\ 1 & 5 \end{vmatrix} = -2$$

Here, First column of third row has value zero. So, othe values below third row can not be determined. In such case we replace 0 by small valued number near to zero, say ϵ

Then stability depends of ϵ

Now, R-H array becomes

s^5	1	2	3
s^4	1	2	5
s^3	ϵ	-2	0
s^2	$c_1 = 2+\frac{2}{\epsilon}$	5	
s^1	$d_1 = -\left(\frac{5\epsilon^2+4\epsilon+4}{2\epsilon+2}\right)$	0	
s^0	5		

$$c_1 = -\frac{1}{\epsilon}\begin{vmatrix} 1 & 2 \\ \epsilon & -2 \end{vmatrix} = -\frac{1}{\epsilon}\left|-2-2\epsilon\right|$$

$$= \frac{2\epsilon+2}{\epsilon} = 2+\frac{2}{\epsilon}$$

$$d_1 = -\frac{\epsilon}{2\epsilon+2}\begin{vmatrix} \epsilon & -2 \\ 2+\frac{2}{\epsilon} & 5 \end{vmatrix}$$

$$= -\frac{\epsilon}{2\epsilon+2}\left[5\epsilon+4+\frac{4}{\epsilon}\right]$$

$$= -\frac{\epsilon}{2\epsilon+2}\left[\frac{5\epsilon^2+4\epsilon+4}{\epsilon}\right]$$

$$= -\left(\frac{5\epsilon^2+4\epsilon+4}{2\epsilon+2}\right)$$

For -ve value of ϵ, 1^{st} column of 3^{rd} row has -ve value so sign changes occus and system is unstable.

For +ve value of ϵ, 1^{st}

Hence above system is unstable.

Q3. Here,

$Q(s)$ $s^4+s^3+s^2+s+k$

R-H array

s^4	1	1	K
s^3	1	1	0
s^2	$b_1 = 0$	K	
s^1	can not determine		
s^0			

$b_1 = -\frac{1}{1}\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0$

Here first column of third row has value 0. so, fourth and fifth row can not be determined. so, we replace 0 by small number near to zero. say ϵ

Now, R-H array becomes

s^4	1	1	K
s^3	1	1	0
s^2	ϵ	K	
s^1	$c_1 = 1 - \frac{K}{\epsilon}$	0	
s^0	K		

$c_1 = -\frac{1}{\epsilon}\begin{vmatrix} 1 & 1 \\ \epsilon & K \end{vmatrix}$

$= -\frac{1}{\epsilon}(K-\epsilon) = 1 - \frac{K}{\epsilon}$

For stability, first column has same sign i.e. it has no sign changes. ~~(a) if ϵ is positive~~

so, $1 - \frac{K}{\epsilon} > 0 \Rightarrow \epsilon - K > 0 \Rightarrow \epsilon > K$

and $K > 0$ ~~(b) If ϵ is -ve~~

so, $\epsilon > K > 0$ $\quad 1 + \frac{K}{\epsilon} > 0$ ~~$\Rightarrow -\epsilon < K$~~

Here, ϵ should be positive value. so, ~~$-\epsilon < K > 0$~~

(2)

Q.4 Given,

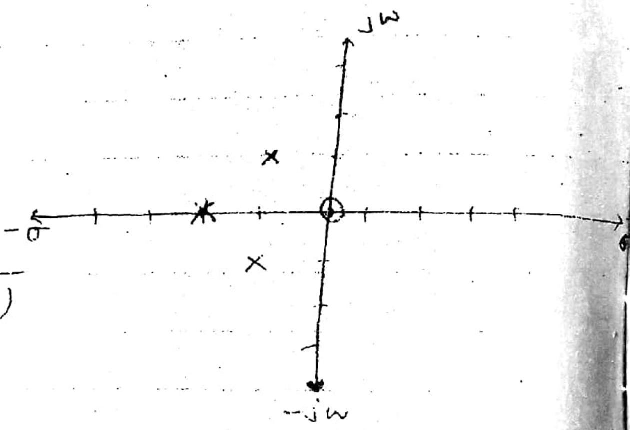
$$T(s) = \frac{3s}{(s+2)(s^2+2s+2)}$$

$$= \frac{3s}{(s+2)\{(s+1)^2+1\}}$$

$$= \frac{3s}{(s+2)\{(s+1)^2-j^2\}} = \frac{3s}{(s+2)(s+1\pm j)}$$

Poles are at, $s = -2,\ -1 \pm j$

Zero at, $s = 0$



Now,

$$T(s) = \frac{K_1}{s+2} + \frac{K_2}{(s+1+j)} + \frac{K_3}{(s+1-j)}$$

For pole at $P_1 = -2$

$$K_1 = \frac{K(P_1 - Z_1)}{(P_1 - P_2)(P_1 - P_3)} = \frac{3(-2-0)}{(-2+1+j)(-2+1-j)}$$

$$= \frac{-6}{\cancel{(-4+j)(-3+j)}(-1+j)(-1-j)} = \frac{-6}{\cancel{9+1}\ 1+1} = \cancel{-0.6}\ -3$$

For pole at $P_2 = -1-j$

$$K_2 = \frac{K(P_2 - Z_1)}{(P_2 - P_1)(P_2 - P_3)} = \frac{3(-1-j-0)}{(-1-j+2)(-1-j+1-j)}$$

$$= \frac{-3-3j}{(1-j)(-2j)} = \frac{3+3j}{2j+2}$$

$$= \frac{3+3j}{2+2j} \times \frac{2-2j}{2-2j} = \frac{6-6j+6j+6}{\ldots} = \frac{12}{8} = 1.5$$

For pole at $P_3 = -1+j$

$$K_3 = \frac{K(P_3 - Z_1)}{(P_3 - P_1)(P_3 - P_2)} = \frac{3(-1+j-0)}{(-1+j+2)(-1+j+1+j)}$$

$$= \frac{-3+3j}{(1+j)2j} = \frac{-3+3j}{-2+2j}$$

$$= \frac{-3+3j}{-2+2j} \times \frac{-2-2j}{-2-2j} = \frac{6+6j-6j+6}{4+4} = \frac{12}{8} = 1.5$$

$$\therefore T(s) = -\frac{3}{s+2} + \frac{1.5}{(s+1+j)} + \frac{1.5}{(s+1-j)}$$

Taking inverse Laplace transform

$$T(t) = -3e^{-2t} + 1.5e^{-(1+j)t} + 1.5e^{-(1-j)t}$$

[NOTE: K_3 can be calculated directly as,

$K_3 = K_2^* = 1.5$

(K_2 and K_3 are complex conjugate]

Q.5 Given,

$$Q(s) = s^3 + 2s^2 + 2s + 40$$

R-H array

s^3	1	2
s^2	2	40
s^1	$b_1 = -18$	0
s^0	40	

$$b_1 = -\frac{1}{2}\begin{vmatrix} 1 & 2 \\ 2 & 40 \end{vmatrix}$$

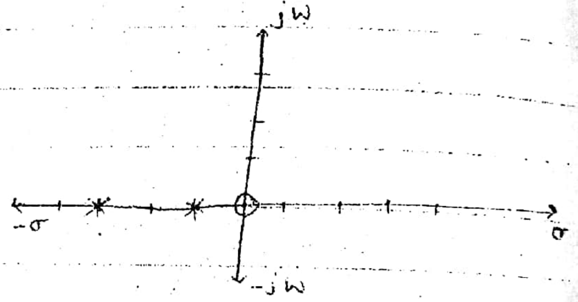
$$= -\frac{1}{2}(40-4) = -18$$

Here, 1^{st} column has two sign changes. Therefore two poles are lies on Right half of S plane so, the System is unstable.

Q.6. Here, $H(S) = \frac{10S}{(S+1)(S+3)}$

Poles are at, $S = -1, -3$

zero at, $S = 0$



Now,

$$H(S) = \frac{K_1}{S+1} + \frac{K_2}{S+3}$$

For pole at, $S = -1$

$$K_1 = \frac{K(P_1 - Z_1)}{(P_1 - P_2)} = \frac{10(-1-0)}{(-1+3)} = -5$$

For pole at, $S = -3$

$$K_2 = \frac{K(P_2 - Z_1)}{(P_2 - P_1)} = \frac{10(-3-0)}{(-3+1)} = 15$$

$$\therefore H(S) = \frac{-5}{S+1} + \frac{15}{S+3}$$

Taking inverse Laplace transform

$$h(t) = -5e^{-t} + 15e^{-3t}$$

~~Q.7 Refer note~~

~~Same as Q.4 of chapter 5.~~

Q.8 Given,

$$s^3 + 7s^2 + 10s + 10K = 0$$

R-H array

s^3	1	10
s^2	7	10K
s^1	$b_1 = \frac{70 - 10K}{7}$	0
s^0	$c_1 = 10K$	

$$b_1 = -\frac{1}{7}\begin{vmatrix} 1 & 10 \\ 7 & 10K \end{vmatrix} = \frac{1}{7}(70 - 10K) = 10 - \frac{10K}{7}$$

The system to be stable, first column should have same sign. for this, $10K > 0$ i.e. $K > 0$

and, $10 - \frac{10K}{7} > 0$ i.e. $70 > 10K$

$\therefore 7 > K$

$\therefore$ Required Range of K is

$0 < K < 7$

Now, check for $K = 7$

R-H array

s^3	1	10
s^2	7	70
s^1	0	
s^0	can not determine	

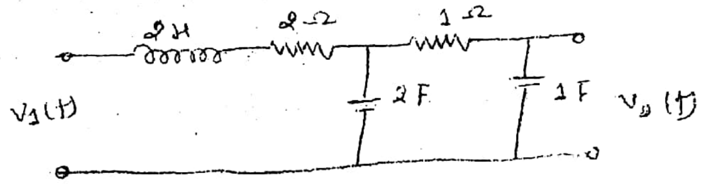
Replace 0 by small number ϵ then,

R-H array

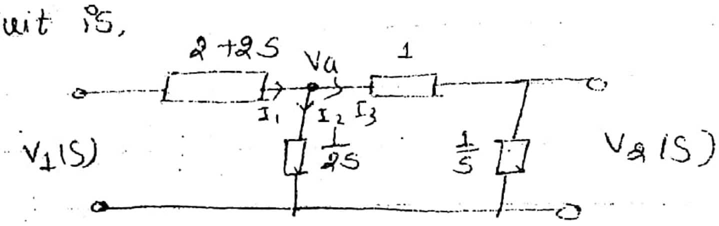
s^3	1	10
s^2	7	70
s^1	ϵ	0
s^0	70	

stability depends on G. If $G > 0$, system is stable for A = . if $G < 0$, system is unstable for $A = 7$.

Q.9 Given circuit,



Transformed circuit is,



Applying KCL

$$I_1 = I_2 + I_3$$

$$\frac{V_1(S) - V_a}{2+2S} = V_a \times 2S + V_a - V_2(S)$$

$$\frac{V_1(S)}{(2+2S)} - \frac{V_a}{2+2S} = V_a \times 2S + V_a - V_2(S)$$

$$V_a\left[\frac{1}{2+2S} + 2S + 1\right] = \frac{V_1(S)}{2+2S} + V_2(S)$$

$$V_a\left[\frac{1 + 4S + 4S^2 + 2 + 2S}{2+2S}\right] = \frac{V_1(S) + (2+2S)V_2(S)}{(2+2S)}$$

$$V_a = \frac{V_1(S) + (2+2S)V_2(S)}{(4S^2 + 6S + 3)}$$

$$\therefore V_2(S) = \frac{\frac{1}{S} \times V_a(S)}{1 + \frac{1}{S}} \quad \therefore \frac{V_a(S)}{(S+1)}$$

$$V_a(S) = V_1(S) + (2+2S)V_2(S)$$

$$V_2(S)\left[1 - \frac{2+2S}{(S+1)(4S^2+6S+3)}\right] = \frac{V_1(S)}{(S+1)(4S^2+6S+3)}$$

$$V_2(S)\left[\frac{4S^3+6S^2+3S+4S^2+6S+3-2-2S}{(S+1)(4S^2+6S+3)}\right] = \frac{V_1(S)}{(S+1)(4S^2+6S+3)}$$

$$\frac{V_2(S)}{V_1(S)} = \frac{1}{4S^3+10S^2+7S+1}$$

Q10. R-H criteria states " The network or system described by a transfer function for which P(S) is the denominator polynomial, is stable if there are no changes of sign in first column of the array."

Given,

$$Q(S) = 5S^5+3S^4+2S^3+2S^2+S+1$$

R-H array

S^5	5	2	1
S^4	3	2	1
S^3	$b_1 = -\frac{4}{3}$	$b_2 = -\frac{2}{3}$	0
S^2	$c_1 = \frac{1}{2}$	1	
S'	$d_1 = 2$	0	
S^0	1		

$$b_1 = -\frac{1}{3}\begin{vmatrix} 5 & 2 \\ 3 & 2 \end{vmatrix} = -\frac{1}{3}(10-6) = -\frac{4}{3}$$

$$b_2 = -\frac{1}{3}\begin{vmatrix} 5 & 1 \\ 3 & 1 \end{vmatrix} = -\frac{2}{3}$$

$$c_1 = \frac{3}{4}\begin{vmatrix} 3 & 2 \\ -\frac{4}{3} & -\frac{2}{3} \end{vmatrix} = \frac{3}{4}\left(-2+\frac{8}{3}\right) = \frac{3}{4}\times\frac{2}{3} = \frac{1}{2}$$

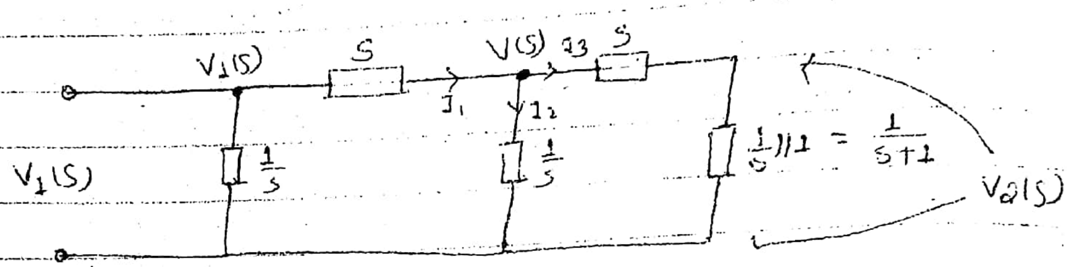
$$d_1 = -2\begin{vmatrix} -4/3 & -2/3 \\ 1/2 & 1 \end{vmatrix} = -2\left(-\frac{4}{3}+\frac{2}{6}\right) = -2\times\frac{-6}{6} = 2$$

First column of R-H array has two sign changes so, two poles are lies on right half of S-plane so the system is unstable.

(5)

42

Q 7. Transformed circuit is,



Apply KCL,

$I_1 = I_2 + I_3$

$$\frac{V_1(S) - V(S)}{S} = \frac{V(S)}{1/S} + \frac{V(S) - V_2(S)}{S}$$

$$\frac{V(S)}{(S)} + S\,V(S) + \frac{V(S)}{S} = \frac{V_1(S) + V_2(S)}{S}$$

$$V(S)\left[\frac{1}{S} + S + \frac{1}{S}\right] = \frac{V_1(S) + V_2(S)}{S}$$

$$V(S) = \frac{V_1(S) + V_2(S)}{(S^2 + 2)}$$

Now,

$$V_2(S) = \frac{\left(\frac{1}{S+1}\right) V(S)}{S + \frac{1}{(S+1)}} = \frac{1}{(S^2 + S + 1)} V(S)$$

$$V_2(S) = \frac{1}{(S^2 + S + 1)} \frac{V_1(S) + V_2(S)}{(S^2 + 2)}$$

$$V_2(S) = \frac{1}{S^4 + 2S^2 + S^3 + 2S + S^2 + 2}\left[V_1(S) + V_2(S)\right]$$

$$V_2(S) = \frac{1}{S^4 + S^3 + 3S^2 + 2S + 2}\left[V_1(S) + V_2(S)\right]$$

$$V_2(S)\left[1-\frac{1}{S^4+S^3+3S^2+2S+2}\right]=\frac{V_1(S)}{(S^4+S^3+3S^2+2S+2)}$$

$$V_2(S)\left[\frac{S^4+S^3+3S^2+2S+2-1}{S^4+S^3+3S^2+2S+2}\right]=\frac{V_1(S)}{(S^4+S^3+3S^2+2S+2)}$$

$$\frac{V_2(S)}{V_1(S)}=\frac{1}{S^4+S^3+3S^2+2S+1}$$

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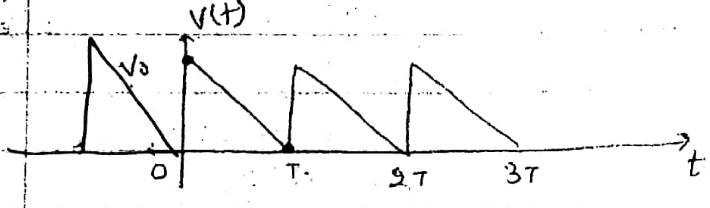
Assignment 5 : Solution

Q.1.



$(0, V_0)$

$(T, 0)$

$y - V_0 = \frac{0 - V_0}{T - 0}(x - 0)$

$y = -\frac{V_0}{T} t + V_0$

correct as

Here Period, $T = T$

$\omega_0 = \frac{2\pi}{T}$

and, $v(t) = \begin{cases} -\frac{V_0}{T} t + V_0 & \text{for } 0 \text{ to } T \end{cases}$

Now, Trigonometric fourier series expansion is given as,

$$v(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos n\omega_0 t + b_n \sin n\omega_0 t]$$

where $a_0 = \frac{1}{T}\int_0^T v(t)\,dt = \frac{1}{T}\int_0^T -\frac{V_0}{T} t\,dt$

$$= -\frac{V_0}{T^2}\left[\frac{t^2}{2}\right]_0^T = -\frac{V_0}{T^2}\left[\frac{T^2}{2} - 0\right]$$

$$= -\frac{V_0}{2}$$

$a_n = \frac{2}{T}\int_0^T v(t)\cos n\omega_0 t\,dt = \frac{2}{T}\int_0^T -\frac{V_0}{T} t \cos n\omega_0 t\,dt$

$$= -\frac{2V_0}{T^2}\left[\frac{t\,\sin n\omega_0 t}{n\omega_0} + \frac{\cos n\omega_0 t}{n^2\omega_0^2}\right]_0^T$$

$$= -\frac{2V_0}{T^2}\left[\frac{T \sin n\frac{2\pi}{T}\times T}{n\frac{2\pi}{T}} + \frac{\cos n\frac{2\pi}{T}\times T}{n^2\omega_0^2} - 0 - \frac{\cos 0}{n^2\omega_0^2}\right]$$

$$= -\frac{2V_0}{T^2}\left[\frac{\overset{0}{T\sin 2n\pi}}{\frac{2n\pi}{T}} + \frac{\cos 2n\pi}{n^2\omega_0^2} - \frac{1}{n^2\omega_0^2}\right]$$

45

$$= -\frac{2V_0}{T}\left[\frac{1}{n^2\omega_0^2} \quad \frac{1}{n^2\omega_0^2}\right]$$

$$= 0$$

$$b_n = \frac{2}{T}\int_0^T v(t)\sin n\omega_0 t\, dt$$

$$= \frac{2}{T}\int_0^T -\frac{V_0}{T}\, t\, \sin n\omega_0 t\, dt$$

$$= -\frac{2V_0}{T^2}\int_0^T t\sin n\omega_0 t\, dt$$

$$= -\frac{2V_0}{T^2}\left[-\frac{t\cos n\omega_0 t}{n\omega_0} + \frac{\sin n\omega_0 t}{n^2\omega_0^2}\right]_0^T$$

$$= -\frac{2V_0}{T^2}\left[-\frac{T\cos 2n\pi}{n\omega_0} + \frac{\sin 2n\pi}{n^2\omega_0^2} + 0 - \frac{\sin 0}{n^2\omega_0^2}\right]$$

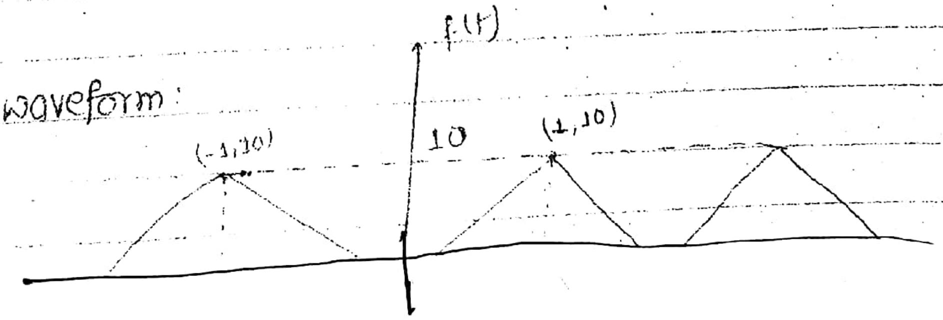
$$= -\frac{2V_0}{T^2}\left[-\frac{T}{n\omega_0} + 0\right]$$

$$= \frac{2V_0}{Tn \times \frac{2\pi}{T}}$$

$$= \frac{V_0}{n\pi}$$

$$\therefore V(t) = -\frac{V_0}{2}\sum_{n=1}^{\infty}\left[\frac{V_0}{n\pi}\sin\frac{2\pi n t}{T}\right]$$

Q.2 Given waveform:



Time period, $T = 2$

$$\omega_0 = \frac{2\pi}{T} = \pi$$

$$f(t) = \begin{cases} -10t & \text{for } -1 \text{ to } 0 \\ 10t & \text{for } 0 \text{ to } 1 \end{cases}$$

Here function $f(t)$ is even function so, it has $b_n = 0$.

Now, trigonometric fourier series becomes,

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos \omega_0 n t$$

where, $a_0 = \frac{2}{T}\int_0^{T/2} f(t)\,dt$

$$= \frac{2}{2}\int_0^1 10t\,dt = 10\left[\frac{t^2}{2}\right]_0^1$$

$$= 5$$

Now, $a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos \omega_0 n t\,dt$

$$= \frac{4}{2}\int_0^1 10t \cos n\pi t\,dt$$

$$= 20\int_0^1 t\cos n\pi t\,dt$$

$$= 20\left[\frac{t\sin n\pi t}{n\pi} + \frac{\cos n\pi t}{n^2\pi^2}\right]_0^1$$

$$= 20\left[\frac{\sin n\pi}{n\pi}^{\to 0} + \frac{\cos n\pi}{n^2\pi^2} - 0 - \frac{\cos 0}{n^2\pi^2}\right]$$

$$= \frac{20}{n^2\pi^2}\left[(-1)^n - 1\right]$$

47 When n is odd.

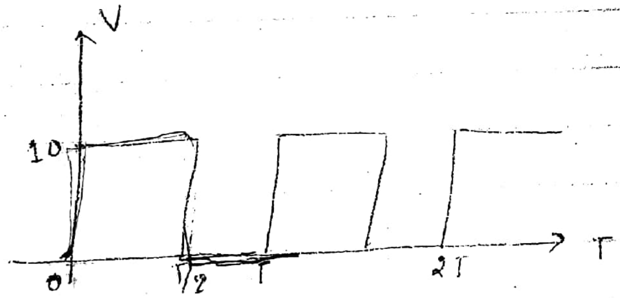
$a_n = -\frac{40}{n^2\pi^2}$

When n is even, $a_n = 0$

$\therefore f(t) = 5 + \sum_{n=1}^{\infty}\left(-\frac{40}{n^2\pi^2}\right)\cos n\pi t$ for n is odd

$= 5$ for n is even

Q.



Sol^n: Here, Time period, T = T

$w_0 = \frac{2\pi}{T}$ and, $v(t) = \begin{cases} 10 & \text{for } 0 \text{ to } T/2 \\ 0 & \text{for } T/2 \text{ to } T \end{cases}$

Trigonometric fourier series is given by

$v(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos w_0 n t + b_n \sin w_0 n t$

Where, $a_0 = \frac{1}{T}\int_0^T v(t)\,dt$

$= \frac{1}{T}\int_0^{T/2} 10\,dt = \frac{10}{T}\left[t\right]_0^{T/2} = \frac{10}{T}\times\frac{T}{2}$

$= 5$

Name :
Roll :

$$a_n = \frac{2}{T}\int_0^T v(t)\cos w_0 nt\, dt$$

$$= \frac{2}{T}\int_0^{T/2} 10\cos w_0 nt\, dt$$

$$= \frac{20}{T}\left[\frac{\sin w_0 nt}{w_0 n}\right]_0^{T/2}$$

$$= \frac{20}{T\times\frac{2\pi}{T}\times n}\left[\sin\frac{2n\pi}{T}\times\frac{T}{2} - \sin 0\right]$$

$$= \frac{10}{n\pi}\left[\sin n\pi - 0\right] = 0$$

$$b_n = \frac{2}{T}\int_0^T v(t)\sin w_0 nt\, dt$$

$$= \frac{2}{T}\int_0^{T/2} 10\sin w_0 nt\, dt$$

$$= \frac{20}{T}\left[-\frac{\cos w_0 nt}{w_0 n}\right]_0^{T/2}$$

$$= \frac{20}{T\times\frac{2\pi}{T}\times n}\left[-\cos\frac{2n\pi}{T}\times\frac{T}{2} + \cos 0\right]$$

$$= \frac{10}{n\pi}\left[-\cos n\pi + 1\right]$$

$$= \frac{10}{n\pi}\left[1-(-1)^n\right]$$

When n is odd, $b_n = \frac{20}{n\pi}$

When n is even, $b_n = 0$

$$= \frac{2}{T} \cdot V \int_{T/2}^{T} \cos \omega_0 n t \, dt$$

$$= \frac{2V}{T} \left[\frac{\sin \omega_0 n t}{\omega_0 n} \right]_{T/2}^{T}$$

$$= \frac{2V}{T \times \frac{2\pi}{T} \times n} \left[\sin \frac{2n\pi}{T} \times T - \sin \frac{2n\pi}{T} \times \frac{T}{2} \right]$$

$$= \frac{V}{n\pi} \left[\sin 2n\pi - \sin n\pi \right] = 0$$

$$b_n = \frac{2}{T} \int_0^T f(t) \sin \omega_0 n t \, dt$$

$$= \frac{2V}{T} \int_{T/2}^{T} \sin \omega_0 n t \, dt$$

$$= \frac{2V}{T} \left[- \frac{\cos \omega_0 n t}{\omega_0 n} \right]_{T/2}^{T}$$

$$= \frac{2V}{T} \left[\frac{- \cos \frac{2n\pi}{T} \times T + \cos \frac{2n\pi}{T} \times \frac{T}{2}}{\frac{2\pi}{T} \times n} \right]$$

$$= \frac{V}{n\pi} \left[- \cos 2n\pi + \cos n\pi \right]$$

$$= \frac{V}{n\pi} \left[-1 + (-1)^n \right]$$

$$= \frac{V}{n\pi} \left[(-1)^n - 1 \right]$$

When n is even, $b_n = 0$

When n is odd, $b_n = -\frac{2V}{n\pi}$ (4)

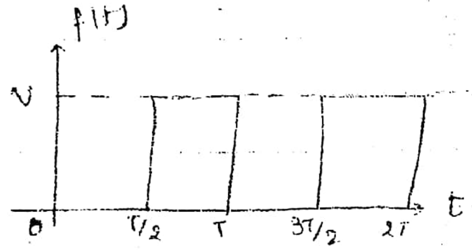
19

Thus

$$v(t) = 5 + \sum_{n=1}^{\infty} \left(\frac{20}{n\pi}\right) \sin n\omega_0 t \qquad \text{when } n \text{ is odd}$$

$$= 5 \qquad \text{when } n \text{ is even}$$

Q. 4



Sol^n: Time period, $T = T$

$$\omega_0 = \frac{2\pi}{T} \text{ and, } f(t) = \begin{cases} 0 & \text{for } 0 \text{ to } T/2 \\ V & \text{for } T/2 \text{ to } T \end{cases}$$

We have,

Trigonometric fourier series expansion,

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos \omega_0 n t + b_n \sin \omega_0 n t$$

where, $a_0 = \frac{1}{T}\int_0^T f(t)\, dt$

$$= \cancel{\frac{1}{T}\int_0^{T/2} 0\, dt} + \frac{1}{T}\int_{T/2}^{T} V\, dt$$

$$= \frac{V}{T}\left[t\right]_{T/2}^{T} = \frac{V}{T}\left[T - \frac{T}{2}\right] = \frac{V}{T} \times \frac{T}{2}$$

$$= \frac{V}{2}$$

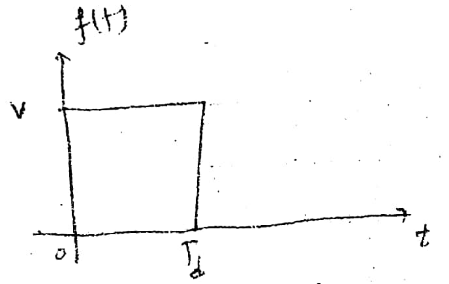
$$a_n = \frac{2}{T}\int^{T} f(t) \cos \omega_0 n t\, dt$$

5)

$$f(t) = 5 + \sum_{n=1}^{\infty} \left(-\frac{2V}{n\pi}\right) \sin w_0 n t \quad \text{when } n \text{ is odd}$$

$$= 5 \quad \text{when } n \text{ is even.}$$

Q 5



sol^n: Here,

$$f(t) = \begin{cases} V & \text{for } 0 \text{ to } T_d \\ 0 & \text{otherwise} \end{cases}$$

We have

$$F(j\omega) = F[f(t)] = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

$$= \int_0^{T_d} V e^{-j\omega t} dt$$

$$= V \left[\frac{e^{-j\omega t}}{-j\omega}\right]_0^{T_d} = -\frac{V}{j\omega}\left[e^{-j\omega T_d} - 1\right]$$

$$= V\left(\frac{1 - e^{-j\omega T_d}}{j\omega}\right) = V\left[\frac{e^{j\omega \frac{T_d}{2}} \cdot e^{-j\omega \frac{T_d}{2}} - e^{-j\omega \frac{T_d}{2}} \cdot e^{-j\omega \frac{T_d}{2}}}{j\omega}\right]$$

$$= \frac{2V \cdot e^{-j\omega T_d/2}}{\omega}\left[\frac{e^{j\omega T_d/2} - e^{-j\omega T_d/2}}{2j}\right]$$

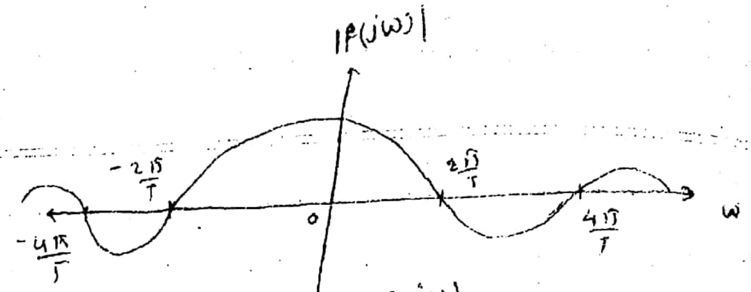
$$= \frac{2V}{\omega} e^{-j\omega T_d/2} \sin(\omega T_d/2)$$

Everest Engi...

additional | not required in this ques...

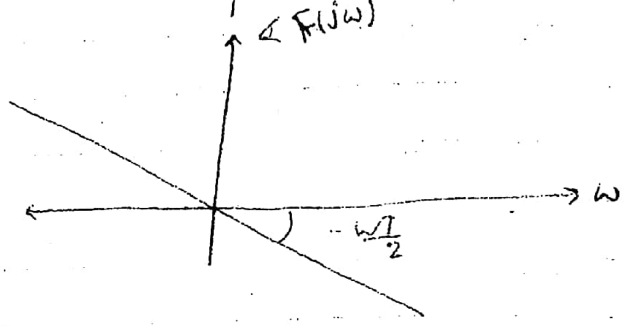
S2

Amplitude response:

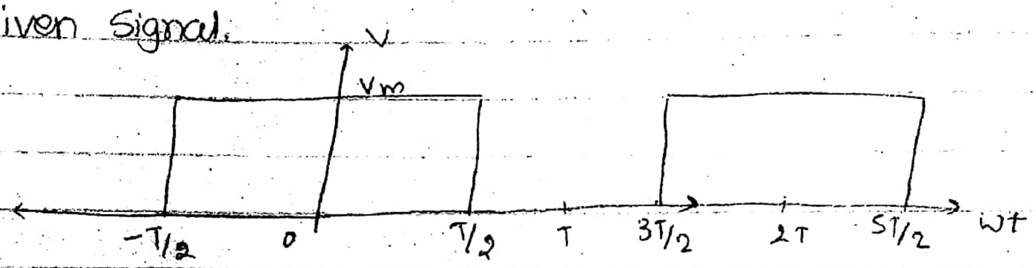


phase response

$$\angle F(j\omega) = -\left(\frac{\omega T}{2}\right)$$



6. Given Signal.



Here, Time period, $T = 2T$

$$\omega = \frac{2\pi}{T} = \frac{\pi}{T}$$

$$V(t) = \begin{cases} V_m & \text{for } -T/2 \text{ to } T/2 \\ 0 & \text{~~otherwise~~ } T/2 \text{ to } 3T/2 \end{cases}$$

Trigonometric fourier series expansion,

$$V(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos \omega_0 nt + b_n \sin \omega_0 nt$$

Where, $a_0 = \frac{1}{T} \int_0^T V(t) dt$

$$= \frac{1}{2T} \int_{-T/2}^{T/2} V_m \, dt$$

$$= \frac{V_m}{2T} \left[t \right]_{-T/2}^{T/2} = \frac{V_m}{2T} \left[\frac{T}{2} + \frac{T}{2} \right] = \frac{V_m}{2}$$

$\frac{T}{2}$)

$$a_n = \frac{2}{T}\int_0^T f(t)\cos w_0 nt\,dt$$

$$= \frac{2}{2T}\int_{-T/2}^{T/2} V_m \cos w_0 nt\,dt$$

$$= \frac{V_m}{T}\left[\frac{\sin w_0 nt}{w_0 n}\right]_{-T/2}^{T/2}$$

$$= \frac{V_m}{T \times \frac{\pi}{T} \times n}\left[\sin \frac{\pi}{T} n \frac{T}{2} - \sin \frac{\pi}{T} * n * \left(\frac{-T}{2}\right)\right]$$

$$= \frac{V_m}{n\pi}\left[\sin \frac{n\pi}{2} + \sin \frac{n\pi}{2}\right]$$

$$= \frac{V_m}{n\pi}\left[2 \sin \frac{n\pi}{2}\right]$$

when n is odd, $a_n = \frac{2V_m}{n\pi}\left[\sin \frac{n\pi}{2}\right]$

when n is even, $a_n = 0$

$$b_n = \frac{2}{T}\int_0^T v(t)\sin w_0 nt\,dt$$

$$= \frac{2}{2T}\int_{-T/2}^{T/2} V_m \sin w_0 nt\,dt$$

$$= \frac{V_m}{T}\left[-\frac{\cos w_0 nt}{w_0 n}\right]_{-T/2}^{T/2}$$

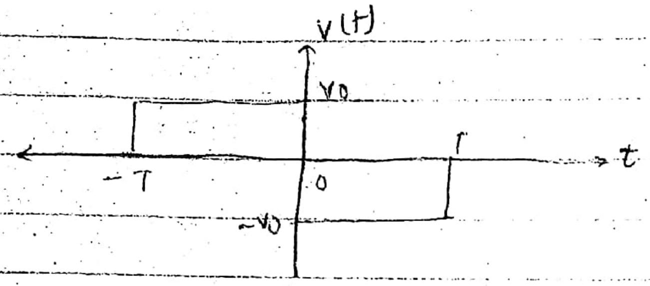
$$= \frac{V_m}{\quad}\left[-\cos \frac{\pi}{2} \times n \times \frac{T}{2} + \cos \frac{\pi}{2} \times n \times -\frac{T}{2}\right] \Rightarrow \frac{V_m}{n\pi}\left[-\cos \frac{n\pi}{2} + \cos\right.$$

$$\therefore V(t) = \frac{V_m}{2} + \sum_{n=1}^{\infty} \left(\frac{2V_m}{n\pi} \sin \frac{n\pi}{2} \right) \cos \omega_0 n t \quad \text{for } n \text{ is odd}$$

$$= \frac{V_m}{2} \quad \text{for } n \text{ is even}$$

Q.7 Refer note [same question as qus. no. 2

Q.8



Here, $v(t) = \begin{cases} V_0 & \text{for } -T \text{ to } 0 \\ -V_0 & \text{for } 0 \text{ to } T \end{cases}$

We have fourier transform of $v(t)$

$$V(j\omega) = F(v(t)) = \int_{-\infty}^{\infty} v(t) e^{-j\omega t} dt$$

$$= \int_{-T}^{0} V_0 e^{-j\omega t} dt + \int_{0}^{T} -V_0 e^{j\omega t} dt$$

$$= V_0 \left[\frac{e^{-j\omega t}}{-j\omega} \right]_{-T}^{0} - V_0 \left[\frac{e^{-j\omega t}}{-j\omega} \right]_{0}^{T}$$

$$= V_0 \left[\frac{1 - e^{j\omega T}}{-j\omega} \right] - V_0 \left[\frac{e^{-j\omega T} - 1}{-j\omega} \right]$$

$$= \frac{V_0}{-j\omega} \left[1 - e^{j\omega T} - e^{-j\omega T} + 1 \right]$$

S.S.

$$= \frac{V_0}{-j\omega}\left[2 - \left(e^{j\omega T} + e^{-j\omega T}\right)\right]$$

$$= \frac{2V_0}{-j\omega} + \frac{2V_0}{j\omega}\left[\frac{e^{j\omega T} + e^{-j\omega T}}{2}\right]$$

$$= \frac{2V_0}{j\omega}\left[-1 + \cos\omega T\right]$$

$$= \frac{2V_0}{j\omega}\left[\cos\omega T - 1\right].$$