SS3843 Final Project – Second Component

Due: Monday, December 13th, 2021 at 8:30am

Upload a PDF file to to Gradescope. Any questions and/or question parts that are incorrectly uploaded to Gradescope will not be graded and will receive a grade of 0. Make sure to show all your code and the output from your code. Marks may be lost for not showing sufficient work.

Note: This component is to be completed individually and independently. It is worth 10% out of the total 25%.

1) Chelsea loves sour cream! Sour cream is made in the same way as yogurt except that cream is substituted for milk. What makes sour cream or yogurt is the addition of the lactobacilli bacterium to the cream or milk. This is done by adding an existing yogurt culture to the cream or milk. The number of bacteria in the final sour cream is measured by the acidity of the resulting sour cream. Chelsea conducts an experiment for making sour cream (described below) in a 5 × 5 Latin square.

   Five containers of cream were placed in a yogurt maker (essentially a cooker) containing five recessed holes and a yogurt culture was added to each container. The process was repeated five times (i.e., five batches were produced). The initial yogurt culture used as the starter was one of five brands of commercial yogurt (labelled C, H, M, N, and Y). The data and the layout of the Latin square are as follows.

Batch	**Position**				
	1	**2**	**3**	**4**	**5**
1	M: 8.04	H: 6.61	N: 11.99	Y: 7.78	C: 8.40
2	N: 9.58	Y: 6.58	C: 6.66	M: 5.34	H: 7.92
3	Y: 7.98	M: 7.98	H: 8.98	C: 7.94	N: 11.32
4	H: 9.74	C: 9.46	M: 9.14	N: 12.00	Y: 9.32
5	C: 9.66	N: 11.28	Y: 8.04	H: 8.12	M: 6.72

   a. **DO NOT USE R:** Complete the analysis of variance table by hand. Test for differences between the batches, the positions, and the yogurt cultures.

   b. Construct a Q-Q plot to examine the assumption of normality. Interpret the plot.

2) To determine the effect of cool-zone oxygen level and preheat oxygen level on the coefficient of variation in resistance on a 20-k Ω resistor, five cool-zone oxygen levels and two preheat oxygen levels were used. Three observations were taken per experiment. The results follow:

		Cool-Zone Oxygen Level (in ppm)				
		5	**10**	**15**	**20**	**25**
Preheat Oxygen (in ppm)	**5**	6.45	2.51	4.71	11.47	10.69
		2.71	4.42	4.91	9.31	9.25
		3.08	4.20	8.19	12.04	10.00
	25	2.86	5.43	5.35	8.92	13.13
		5.51	8.37	4.20	7.57	12.01
		5.66	3.54	6.49	7.14	11.71

a. What type of experimental design is this?

b. What type of model should be used (e.g., fixed effects model, random effects model, mixed effects model)?

c. Write out the model with all constraints and distributional assumptions. Give the specific limits on all the indices. Describe all terms used in the model.

d. **DO NOT USE R:** Complete the analysis of variance table by hand and test for differences between preheat levels, the cool-zone levels and their interaction.

e. Construct a Q-Q plot to examine the assumption of normality. Interpret the plot.

f. Make a plot of cell averages (using ggplot in R) to determine if there is an interaction between cool-zone oxygen and preheat oxygen. Interpret the plot.

g. Assume there is no interaction. Complete the analysis of variance table and perform all necessary tests. Does this change your results from part (d)?

h. Is it possible to use a model that only contains the overall mean and interaction term? Why or why not?

3) Boeing 737 airplanes are often used for commercial flights. The airplane has two main landing gear and a single nose gear. Each gear is made up of a two-wheels, making up 6 wheels in total. The integrity of these wheels is very important since any serious problems with them could lead to catastrophic issues. A test is run at the parts producer (Collins Aerospace) located in Oakville, Ontario to determine the effect of four factors on the wheels.

The four factors are temperature (A), rubber content (B), titanium alloy content (C), and assembly method (D). Two replicates of a 2^4 design are run and the integrity of the wheel (scale from 0 – 2.5) subjected to a standard test is measured. The data are shown in the table below:

A	*B*	*C*	*D*	Treatment Combination	Replicate	
					I	II
–	–	–	–	(1)	1.71	2.01
+	–	–	–	*a*	1.42	1.58
–	+	–	–	*b*	1.35	1.63
+	+	–	–	*ab*	1.67	1.65
–	–	+	–	*c*	1.23	1.48
+	–	+	–	*ac*	1.25	1.36
–	+	+	–	*bc*	1.46	1.52
+	+	+	–	*abc*	1.29	1.37
–	–	–	+	*d*	2.04	2.29
+	–	–	+	*ad*	1.86	1.95
–	+	–	+	*bd*	1.79	2.05
+	+	–	+	*abd*	1.42	1.69
–	–	+	+	*cd*	1.81	2.02
+	–	+	+	*acd*	1.34	1.39
–	+	+	+	*bcd*	1.46	1.63
+	+	+	+	*abcd*	0.85	1.00

a. Estimate the factor effects for *D*, *BC*, and *ABCD*.

b. Conduct an analysis for *D*, *BC*, and *ABCD*. You are given that SSE = 0.29405.

4) Five dentists were chosen at random to take part in an experiment to examine the hardness of gold fillings made by the dentists. Two types of gold alloys were used (called 97-1-1-1 and AuCa). The fillings were each sintered at three temperatures using one of three methods of condensation. A single run of the experiment consisted of a single dentist preparing a filling using one of the gold alloys sintered at one of the temperatures, using one of the methods of condensation to fill a 2 × 2 mm hole in an ivory block. The diamond pyramid hardness was measured on each of the fillings produced. The data are as follows:

Dentist	**Method**	**Alloy 97-1-1-1**			**Alloy AuCa**		
		1500E F	**1600E F**	**1700E F**	**1500E F**	**1600E F**	**1700E F**
1	**1**	813	792	792	907	792	835
	2	782	698	665	1115	835	870
	3	752	620	835	847	560	585
2	**1**	715	803	813	858	907	882
	2	772	782	743	933	792	824
	3	835	715	673	698	734	681
3	**1**	743	627	752	858	762	724
	2	813	743	613	824	847	782
	3	743	681	743	715	824	681
4	**1**	792	743	762	894	792	649
	2	690	882	772	813	870	858
	3	493	707	289	715	813	312
5	**1**	707	698	715	772	1048	870
	2	803	665	752	824	933	835
	3	421	483	405	536	405	312

a. Write out the <u>complete</u> model with all constraints and distributional assumptions. Give the specific limits on all the indices. Describe all terms used in the model.

b. State the degrees of freedom for the following: dentist, method, alloy, temperature, all interactions, total and error.

5) A company manufactures a spectrophotometer for use in medical laboratories. A new model is being tested prior to its release for sale. It is necessary to determine whether the new design determined spectral properties over the required range of serum glucose standards. To test the machine, a factorial design was used with "concentrations" of glucose and "days" as factors. The serum samples were enhanced with three different levels of glucose to cover the range of glucose concentrations the machine should be able to analyze. The experiment was run on three days and all three concentrations were analyzed on each day. On each day of the experiment, two runs (or replications) of the experiment were done. The day are as follows:

Concentration	**Day 1**		**Day 2**		**Day 3**	
	Run 1	**Run 2**	**Run 3**	**Run 4**	**Run 5**	**Run 6**
1	41.2	41.2	39.8	41.5	41.9	45.5
	42.6	41.4	40.3	43.0	42.7	44.7
2	135.7	143.0	132.4	134.4	137.4	141.1
	136.8	143.3	130.3	130.0	135.2	139.1
3	163.2	181.4	173.6	174.9	166.6	175.0
	163.3	180.3	173.9	175.6	165.5	172.0

The initial expression for the model for this experiment is

$$y_{ijkl} = \mu + \alpha_i + \beta_j + \gamma_{k(j)} + (\alpha\beta)_{ij} + (\alpha\gamma)_{ik(j)} + \varepsilon_{(ijk)l}$$

where α_i is a fixed effect corresponding to concentrations, β_j is a random effect corresponding to days, $\gamma_{k(j)}$ is a random effect corresponding to runs which are nested within days and $\varepsilon_{(ijk)l}$ is the random error.

a. Using the statistical software R, fit the above model and provide the summary output.

b. Perform all valid tests.