BST 140.652 Midterm Exam

Notes:

- You may use your one 8.5 by 11 formula sheet.
- Please use only the basic mathematical functions on your calculator.
- Show your work on all questions. Simple "yes" or "no" answers will be graded as if blank.
- Please be neat and write legibly. Use the back of the pages if necessary.
- Good luck!

____

signature

____

printed name

1. A clinical trial is conducted where a antacid treatment is given to 100 patients with heartburn while a placebo is given to another 100 with heartburn. Of the treated, 73 reported an improvement in symptoms. Of the controls 63 reported an improvement. Perform a hypothesis test that the treatment is effective. State your hypotheses defining any notation that you use and report a P-value. Interpret your results.

null. $u_1 - u_2 = 0$

$$\frac{(0.73 - 0.63)}{S_p\left(\frac{1}{n_1} + \frac{1}{n_2}\right)^{\frac{1}{2}}}$$

$\begin{cases} 1 & 0.73 \\ 0 & \end{cases}$

$$S_p = \frac{(n_1 - 1) S_1^2 + (n_2 - 1) S_2^2}{n_1 + n_2 - 2}$$

$\begin{cases} 1 & 0.63 \\ 0 & \end{cases}$

$= 0.2151$

CI : $0.1 \pm 0.2151 \left(\frac{2}{100}\right)^{\frac{1}{2}}$ $\quad (-0.07, \underset{0.1}{\wedge} 0.13)$

fail to reject null : $u_1 = u_2$

2. Refer to the previous problem. Give and interpret confidence intervals for the relative risk and odds ratio.

A 73 27 100

B 63 37 100

$\left(\frac{0.73}{0.63}\right)$

RR: $\log\left(\frac{p_1}{p_2}\right) \pm z_{1-\alpha/2}\left(\frac{1-p_1}{p_1 n_1} + \frac{1-p_2}{p_2 n_2}\right)^{\frac{1}{2}}$

$\Downarrow$ exponentiate

CI ( )

OR: $\frac{p_1/(1-p_1)}{p_2/(1-p_2)} = \frac{73 \times 37}{63 \times 27}$

$\downarrow \pm z_{1-\alpha/2}\sqrt{\frac{1}{73} + \frac{1}{37} + \frac{1}{63} + \frac{1}{27}}$

$\Downarrow e^{(\ )}$

CI ( )

one-tail:
pbinom (3, 5, 0.6, lower.tail)

3. A friend gives you a coin that is slightly bent and claims that the probability of a head is $.6$. You would like to test this hypothesis. You flip it 5 times and obtain 4 heads. Perform the relevant hypothesis test, stating your hypotheses (defining any notation that you use), report a P-value and interpret your results.

H 0.6 p

$\hat{H}$: 0.8

T 0.4

$$p(x \geq 4) = \binom{5}{4} p^4 (1-p)^1 + \binom{5}{5} p^5 (1-p)^0 = 0.33696 > 0.05$$

$$0.6 \pm qt(0.975, 4)(0.6 \times 0.4)/\sqrt{5}$$

$$= (0.3, 0.89)$$

0.8 ✓

4. Refer to the previous problem. Suppose that in 200 flips of the coin there are 123 heads. Test the hypothesis using this new sample and report and interpret a P-value.

pbinom(122, 200, 0.6, lower.tail = FALSE)

$= 0.36 > 0.05$

5. Let $X_1, \ldots, X_{N_1}$ be iid random variables from a population with mean μ_1. Let $\bar{X}$ and S_x be the sample mean and standard deviation, respectively. Use the delta method to obtain a confidence interval for $\log(\mu_1^2)$.

$$\log(u_1^2) = 2\log u_1$$

$$\frac{\hat{\theta} - \theta}{S\hat{E}\hat{\theta}} \to N(0,1)$$

$$\frac{\bar{x} - u_1}{S_x} \to N(0,1)$$

$$\frac{f(\hat{\theta}) - f(\theta)}{f'(\hat{\theta})\, S\hat{E}\,\hat{\theta}} \to N(0,1)$$

$$\frac{2\log(\bar{x}) - 2\log(u_1)}{\frac{2}{\bar{x}}\, S_x} \to N(0,1) \quad (z_{0.05}, z_{0.975})$$

$$2\log(\bar{x}) - 2\log u_1 \in \frac{2}{\bar{x}} S_x (z_{0.05}, z_{0.975})$$

$$\log(u_1^2) = 2\log u_1 \in \left(2\log\bar{x} - \frac{2}{\bar{x}} S_x \cdot z_{0.975},\ 2\log\bar{x} - \frac{2}{\bar{x}} S_x \cdot z_{0.05}\right)$$

6. Refer to the previous problem. Let $Y_1, \ldots, Y_{N_2}$ be iid random variables froma population with mean μ_2. Let $\bar{Y}$ and S_y be the associated sample mean and standard deviations. Using your answer from the previous question, obtain a confidence interval for $\log(\mu_2^2/\mu_1^2)$. (If you do not have an answer for the previous problem define notation for the parts that you are missing.)

$2\log\left(\frac{u_2}{u_1}\right)$

$D(x) = E(x^2) - E^2(x)$

$S = D\left(\frac{Y}{X}\right) = E\left(\left(\frac{Y}{X}\right)^2\right) - E^2\left(\frac{Y}{X}\right)$

$= \frac{E(Y^2)}{E(X^2)} - E^2\left(\frac{Y}{X}\right)$

$= \frac{S_Y + E^2Y}{S_X + E^2X} - \left(\frac{EY}{EX}\right)^2$

$\frac{\left(\frac{\bar{Y}}{\bar{X}}\right) - \left(\frac{u_2}{u_1}\right)}{S} \to N(0,1)$

$\frac{2\log\left(\frac{\bar{Y}}{\bar{X}}\right) - 2\log\left(\frac{u_2}{u_1}\right)}{\frac{2}{\left(\frac{\bar{Y}}{\bar{X}}\right)} \cdot S} \to N(0,1)$

7. Researchers would like to test whether or not the mean systolic blood pressure in a particular obese population is greater than 135 mmHg. She conjectures that it may be as high as 138 mmHg. This population is known to have a standard deviation of SBP of 5 mmHg. She intends to take a sample of 100 subjects. What is the probability that she rejects her hypothesis if her conjecture is correct and she uses a .05 type I error rate?

$$1-\beta = P\left(\frac{\bar{X} - 135}{5/\sqrt{100}} > Z_{0.975} \,\middle|\, u = 138 \right)$$

$$+ P\left(\frac{\bar{X} - 135}{5/\sqrt{100}} < Z_{0.025} \,\middle|\, u = 138 \right)$$

$$= P\left(\frac{\bar{X} - 138 + 138 - 135}{5/\sqrt{100}} > Z_{0.975} \,\middle|\, u = 138 \right) + (\ldots)$$

8. Refer to the previous problem. After collecting data (100 subjects), the mean was $137mmHg$ with a standard deviation of $7mmHg$. Perform and interpret the relevant test. State your hypotheses defining any notation that you use. Report a P-value.

$P(X > 137)$?

$\bar{X} = 137$

$S = 7$

$$\frac{137 - 135}{7/\sqrt{100}}$$