Problem 3

a.

This is the probability that at most one parent have contracted influenza. So the probability either the mother alone, the father alone, or neither parent has contracted the flu.

b.

Let F be the event that the father contracted cryptosporidiosis and M be the event that the mother contracted cryptosporidiosis. Using this notation, we are given P$(F \cup M) = 0.21$, P$(F \cap M) = 0.05$, and P(F) = 0.09. Note that the event in which either the mother or the father has contracted cryptosporidiosis is denoted by $F \cup M$, or equivalently, the event in which least one parent contracted the disease. We are given P$(F \cup M)$ is 0.21.

c.

Using the same notation as in part b,

$M \cap F^c = \{$mother contracted cryptosporidiosis, but father has not$\}$

We know that

P$(M \cap F^c) = P(M) - P(M \cap F) = 0.17 - 0.05 = 0.12$

d.

$P(F \cup M) = P(F) + P(M) - P(F \cap M)$.
Thus, solving for P(M), we get

$P(M) = P(F \cap M) + P(F \cap M) - P(F) = 0.21 + 0.05 - 0.09 = 0.17$

e.

$P(F^c \cap M^c) = 1 - P(F \cup M) = 1 - 0.21 = 0.79$

f.

$P(F \cap M^c) = P(F) - P(M \cap F) = 0.09 - 0.05 = 0.04$

Problem 7

(1) Since h(x)>0 for all x,$\sum h(i) > 0$ and thus p(x)>0.

(2)

$$\begin{aligned}\sum_{x=1}^{I} p(x) &= \sum_{x=1}^{I} h(x) / \sum_{i=1}^{I} h(i) \\ &= \frac{\sum h(x)}{\sum h(x)} \\ &= 1\end{aligned}$$

So p(x) is a valid pmf.

Problem 8

1. For all x, h(x) >0 and c>0, thus f(x)>0

2.

$$\begin{aligned}\int_{-\infty}^{\infty} f(x) &= \int_{-\infty}^{\infty} h(x)/c \\ &= \frac{\int_{-\infty}^{\infty} h(x)}{\int_{-\infty}^{\infty} h(x)} \\ &= 1\end{aligned}$$

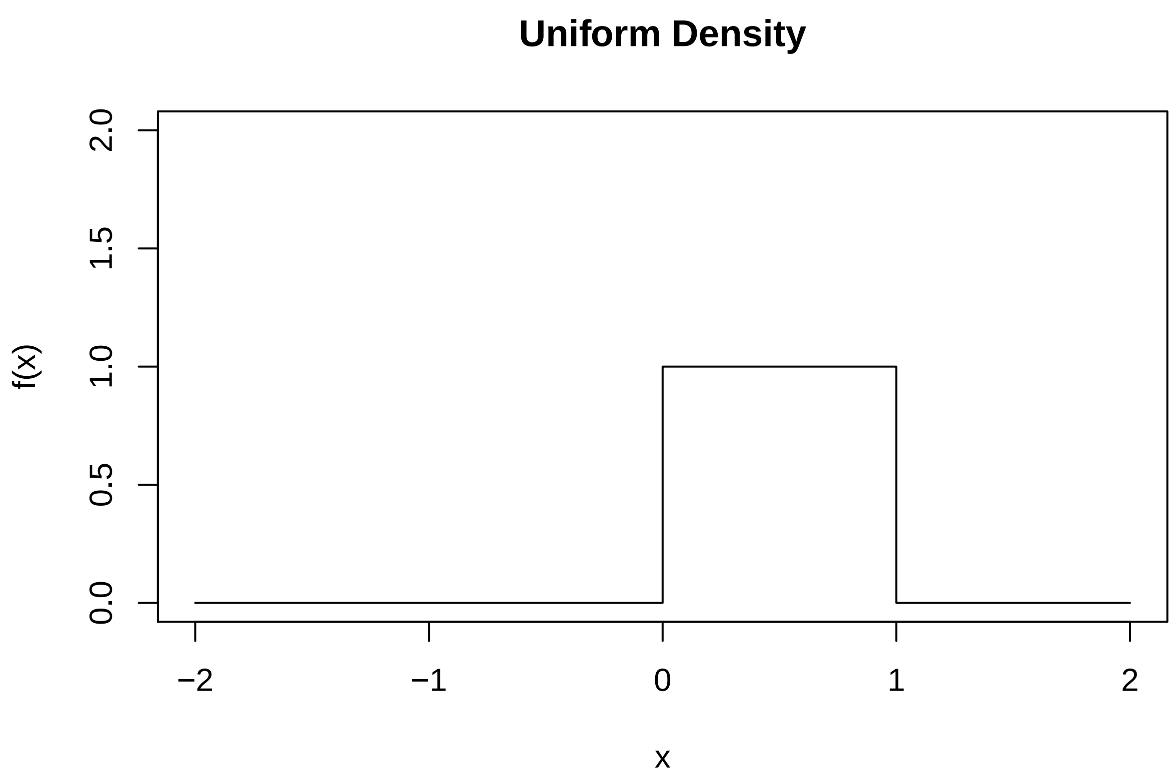
Problem 9

a.

$$\begin{aligned}\int_0^1 f(x)dx &= kx|_0^1 \\ &= k = 1\end{aligned}$$

So k = 1.

```r
x <- c(-2, 0, 0, 1, 1, 2)
y <- c(0, 0, 1, 1, 0, 0)
plot(x,y, "l", xlim = c(-2,2), ylim = c(0,2),
main = "Uniform Density", xlab = "x", ylab = "f(x)")
```



b.

$p = \int_{0.1}^{0.7} f(x)\,dx = 0.7 - 0.1 = 0.6$

c.

```r
lb <- 0.1 # Lowerbound
ub <- 0.7 # Upperbound
punif(ub) - punif(lb)
```

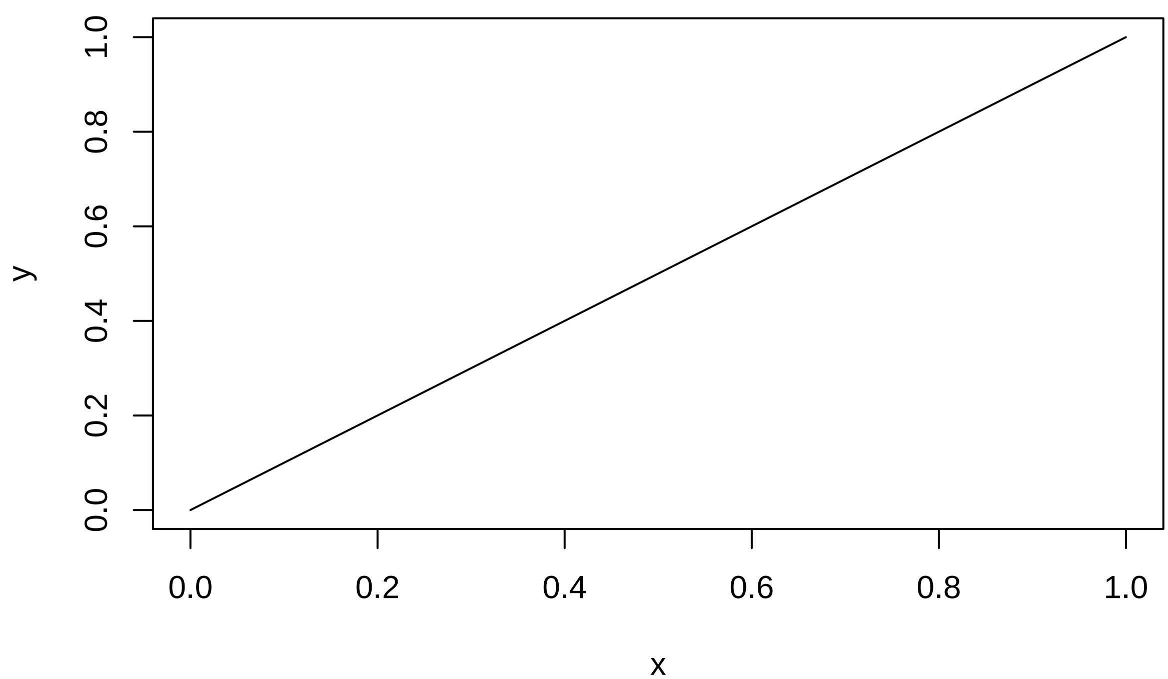
```
## [1] 0.6
```

For general a and b, we have P = b - a.

d.

$$
\begin{aligned}
F(x) &= p(X < x) \\
&= \int_0^x f(y)\,dy \\
&= y|_0^x = x
\end{aligned}
$$

```r
x = c(0,1)
y = c(0,1)
plot(x,y,'l')
```



e.

Let m denote the median. By definition of median, P(X < m) = F(m) = 0.5. From part (d) above, we know that F(x) = x, implying that m = 0.5. In the context of our problem, the median of 0.5 means that the probability of individuals having less than 50% of their skin covered in freckles is 0.5.

f.

The 95th percentile is the value x such that F(x) = 0.95. From part (d), we know that F(x) = x, implying that the 95th percentile is given by x = 0.95. In the context of this problem, the 95th percentile means that the probability of a randomly drawn subject having less that 95% of their skin covered in freckles is 0.95.

g.

The uniform distribution is probably not a reasonable distribution for the proportion of freckles on subjects in a given population as it should not be equally likely to have over 50% of your skin covered in freckles as it is to have less than 50% of your skin covered in freckles.

Problem 12

a.

1. $e^x > 0 for all x, and 1 + e^x > 0, thus pdf > 0.$
2.

$$\begin{aligned}\int_{-\infty}^{\infty} \frac{e^{-x}}{(1+e^{-x})^2} dx &= \int_{\infty}^{1} -\frac{1}{u^2} du \\ &= \frac{1}{u}|_{\infty}^{1} \\ &= 1 - 0 = 1\end{aligned}$$

b.

Same as a:

$$F(x) = \frac{1}{u}|_{\infty}^{1+e^{-x}} = \frac{1}{1+e^{-x}}$$

c.

$$F(0) = \frac{1}{e^{-0}} = 1/2$$

So 0 is the median of this distribution.In the context of this problem, a median of 0 means that the median cancer risk is 0.

d.

To find to value x such that F(x) = p:

$$\begin{aligned}\frac{1}{1+e^{-x}} &= p \\ e^{-x} &= 1/p - 1 \\ x &= log(\frac{p}{1-p})\end{aligned}$$

Problem 14

a.

$$\begin{aligned}\int_0^1 cx^k dx &= \frac{c}{k+1} x^{k+1}|_0^1 \\ &= \frac{c}{k+1} = 1 \\ c &= k+1\end{aligned}$$

b.

$$\begin{aligned} F(x) &= \int_0^x (k+1)y^k dy \\ &= x^{k+1}|_0^x \\ &= x^{k+1} \end{aligned}$$

c.

$p = x^{k+1} \to x = p^{1/(k+1)}$

d.

$$P(a < x < b) = \int_a^b (k+1)x^k dx = x^{k+1}|_a^b = b^{k+1} - a^{k+1}$$

Problem 15

a.

$1 = \int_0^\infty cexp(\frac{-x}{2.5})dx = -2.5cexp(\frac{-x}{2.5})|_0^\infty = 2.5c$
So we know c = 2/5.

b.

$F(x) = \int_0^x \frac{1}{2.5}exp(\frac{-y}{2.5})dy = -exp(\frac{-y}{2.5})|_0^x = 1 - exp(-x/2.5)$

c.

$S(x) = 1 - F(x) = exp(-x/2.5)$

d.

$S(11) = exp(\frac{-11}{2.5}) = 0.012$

e.

$F(x) = 0.5 \to 0.5 = exp(\frac{-x}{2.5}) \to x = -2.5log0.5$