A4: Hodgkin Huxley Model

BM2102 - Modelling and Analysis of Physiological Systems
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1 Threshold

```
amp1 = 6;
width1 = 1;
hhmplot(0,50,0);
amp1 = 7;
hhmplot(0,50,1);
```

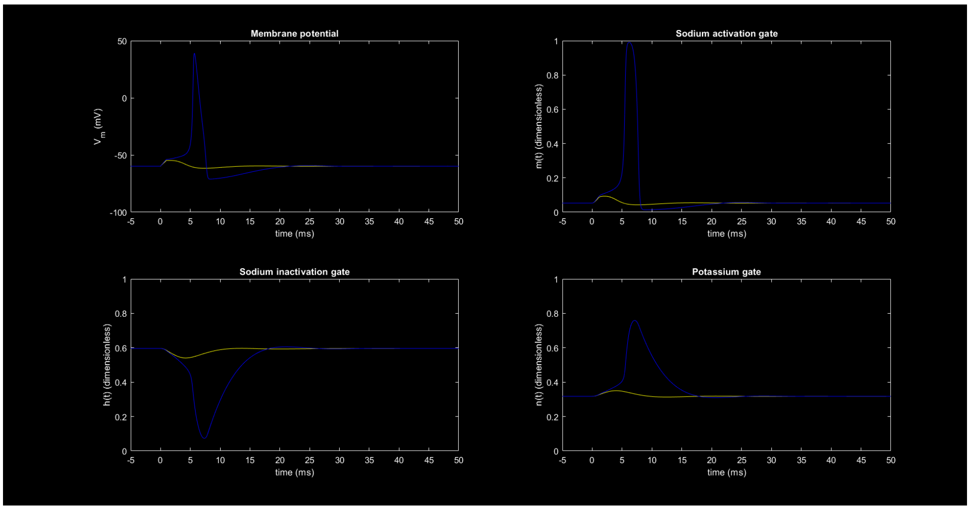

Figure 1: Membrane potential when the input stimulus intensities are $6\,\mu A\,cm^{-2}$ (yellow) and $7\,\mu A\,cm^{-2}$ (blue).

1.1 Question 1

```
amp1 = 6;
width1 = 1;

for i = 1:11
    hhmplot(0, 50, 1);
    amp1 = 6 + 0.1*i;

end
```

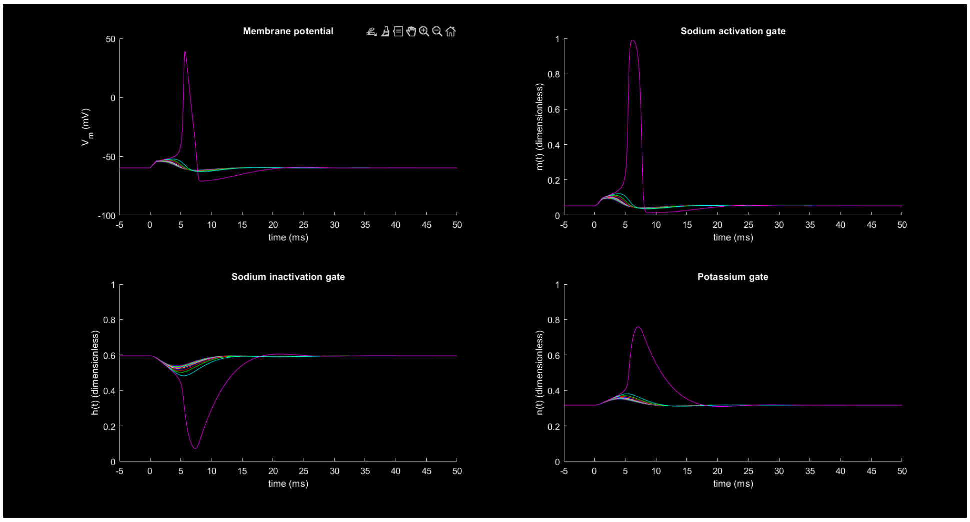

Figure 2: Action potential is triggered during the last iteration which corresponds to $6.9\,\mu\mathrm{A\,cm^{-2}}$

```
amp1 = 6.9;
width1 = 1;

for i = 1: 10
    hhmplot(0, 50, 1);
    amp1 = 6.9 + 0.01*i;
end
```

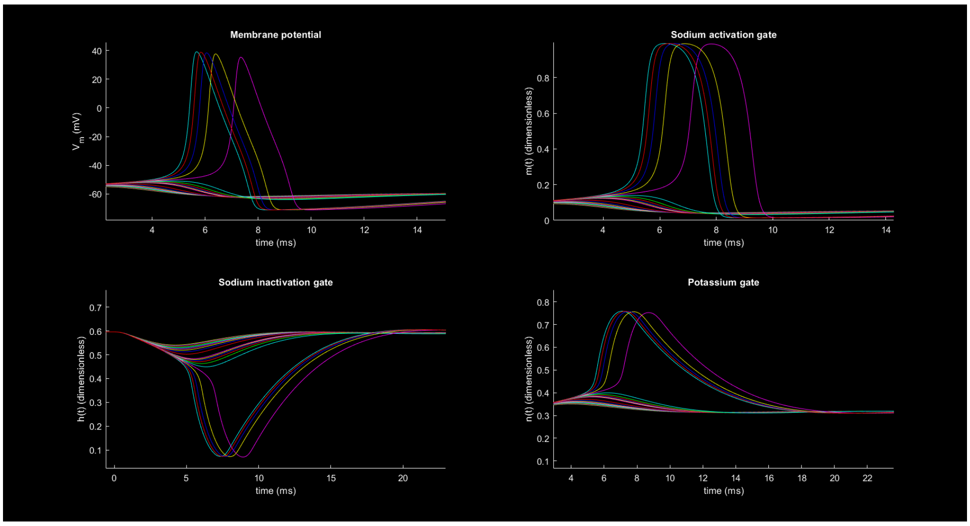

Figure 3: First Action potential is triggered during the 6th iteration(Cyan) $6.96\,\mu\mathrm{A\,cm^{-2}}$

- Using Figure 2 we can conclude that Action Potential lies between $6.9\,\mu\mathrm{A\,cm^{-2}}$ and $7\,\mu\mathrm{A\,cm^{-2}}$
- Using Figure 3 we can conclude that **threshold of trigger stimulus** is at $6.96\,\mu\mathrm{A\,cm^{-2}}$

1.2 Question 2

1.2.1 When stimulus is lower than the threshold

```
amp1 = 6.83;
width1 = 1;
[qna , qk , ql] = hhsplot(0, 50)

sum_Jk = qna + qk + ql
```

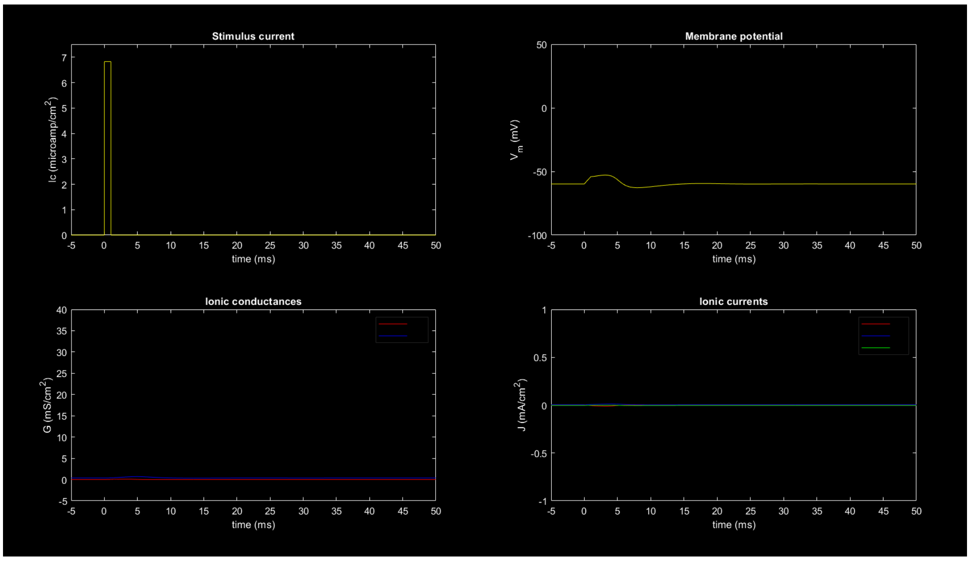

Figure 4: Input stimulus intensity is lesser than threshold

```
sum_Jk =

    6.8297
```

$$\int_{t_0}^{t_f} \sum_k J_k \, dt = \mathrm{Sum\_}Jk$$

The code calculates the above formula and as we can see there is no significant difference between Sum_Jk and stimulus amplitude(6.83).

1.2.2 When stimulus is higher than the threshold

```
amp1 = 7.05;
width1 = 1;
[qna , qk , ql] = hhsplot(0, 50)

sum_Jk = qna + qk + ql
```

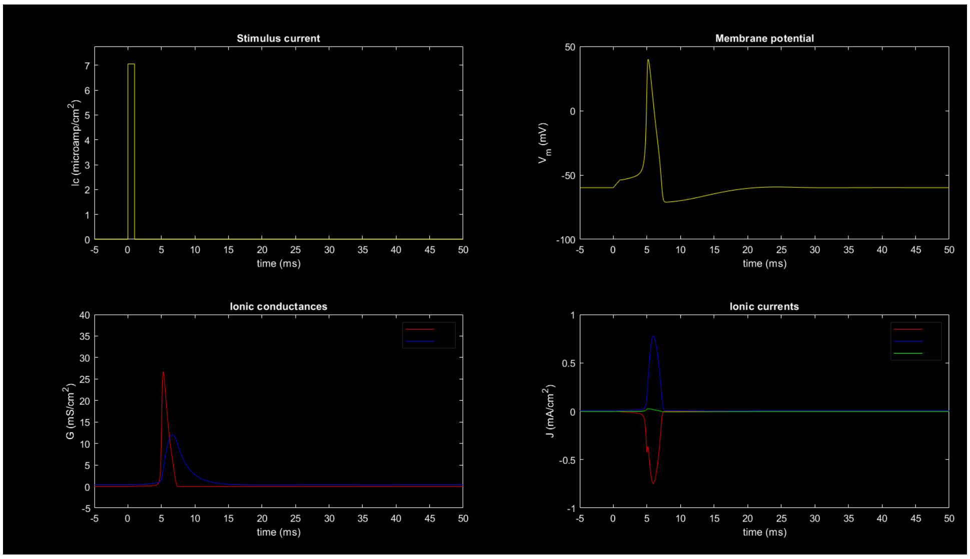

Figure 5: Input stimulus intensity is higher than threshold

```
sum_Jk =

    7.0501
```

$$\int_{t_0}^{t_f} \sum_k J_k \, dt = \text{Sum\_}Jk$$

The code calculates the above formula and as we can see there is no significant difference between Sum_Jk and stimulus amplitude(7.05).
So now we can confirm the relationship $\int_{t_0}^{tf} \sum_k J_k \, dt = \int_{t_0}^{tf} J_{ie} \, dt$.

2 Refractoriness

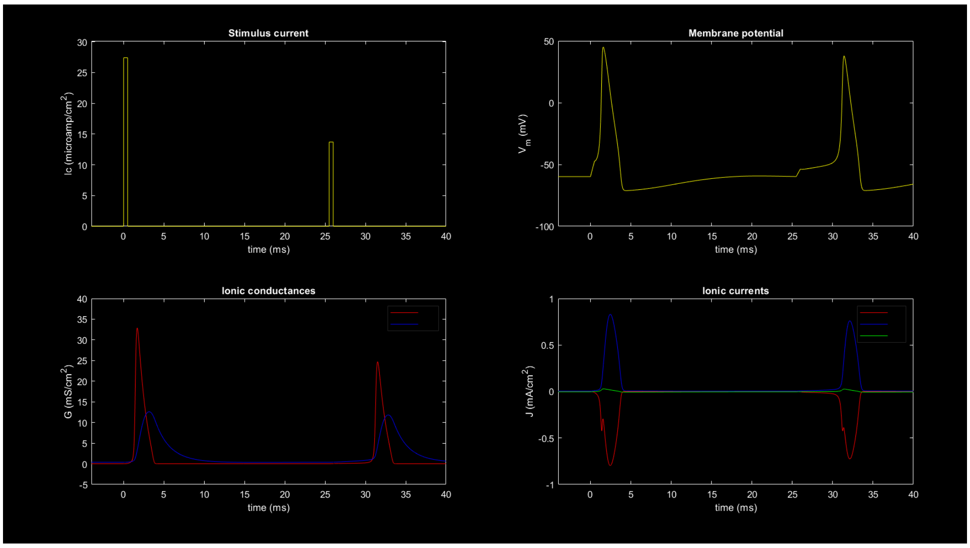

Figure 6: Activation after two pulses of different amplitudes

2.1 Question 3

2.1.1 delay = 20 ms

```
amp1 = 27.4;
width1 = 0.5;
delay2 = 20;
amp2 = 11;
width2 = 0.5;

for j =  1:7
    hhmplot(0, 30, 1);
    amp2 = amp2 + 0.1;
end
```

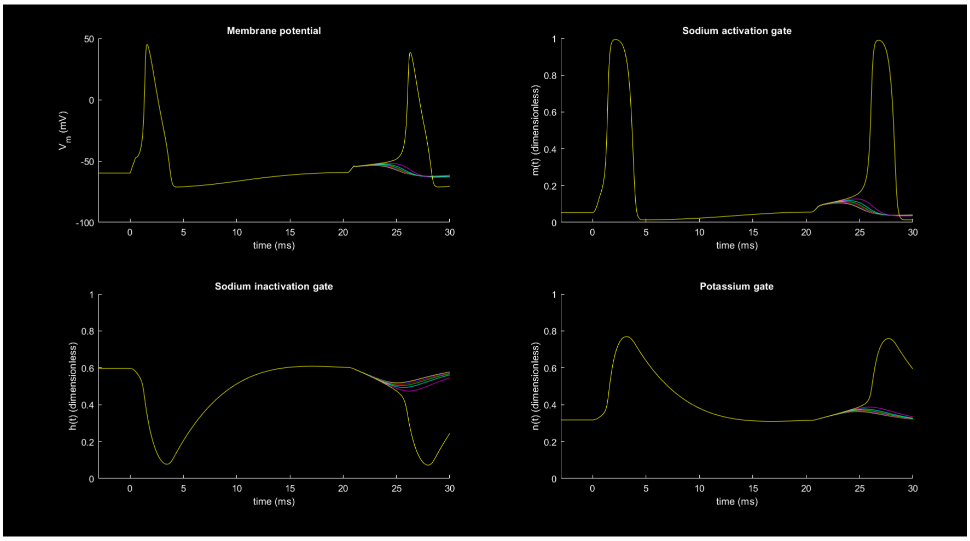

Figure 7: Second pulse with a delay of 20 ms

Action potential is triggered at $I_{2\text{th}} = 11.6\,\mu\text{A}\,\text{cm}^{-2}$. We can conclude that the threshold for the second pulse is this value.

2.1.2 delay = 18 ms

```
amp1 = 27.4;
width1 = 0.5;
delay2 = 18;
amp2 = 11;
width2 = 0.5;

for j =  1:4
    hhmplot(0, 30, 1);
    amp2 = amp2 + 0.1;
end
```

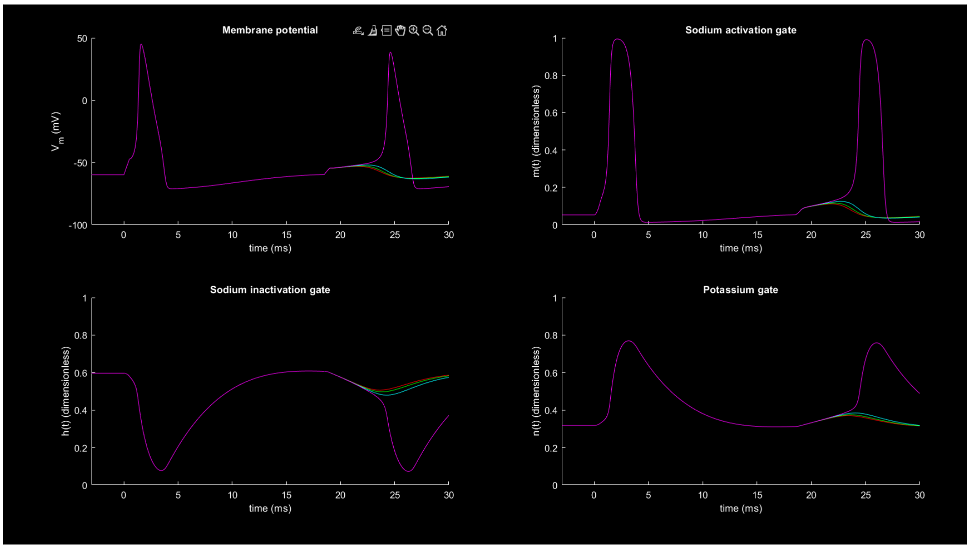

Figure 8: Second pulse with a delay of 18 ms

Action potential is triggered at $I_{2\text{th}} = 11.3\,\mu\text{A}\,\text{cm}^{-2}$. We can conclude that the threshold for the second pulse is this value.

2.1.3 delay = 16 ms

```
amp1 = 27.4;
width1 = 0.5;
delay2 = 16;
amp2 = 12;
width2 = 0.5;

for j =  1:8
    hhmplot(0, 30, 1);
    amp2 = amp2 + 0.1;
end
```

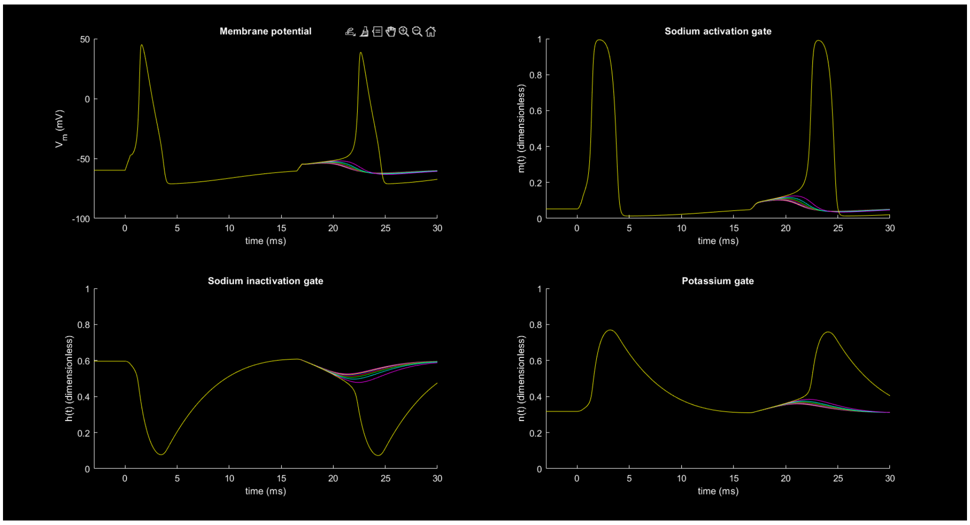

Figure 9: Second pulse with a delay of 16 ms

Action potential is triggered at $I_{2\text{th}} = 12.7\,\mu\text{A}\,\text{cm}^{-2}$. We can conclude that the threshold for the second pulse is this value.

2.1.4 delay = 14 ms

```
amp1 = 27.4;
width1 = 0.5;
delay2 = 14;
amp2 = 16;
width2 = 0.5;

for j =  1:10
    hhmplot(0, 30, 1);
    amp2 = amp2 + 0.1;
end
```

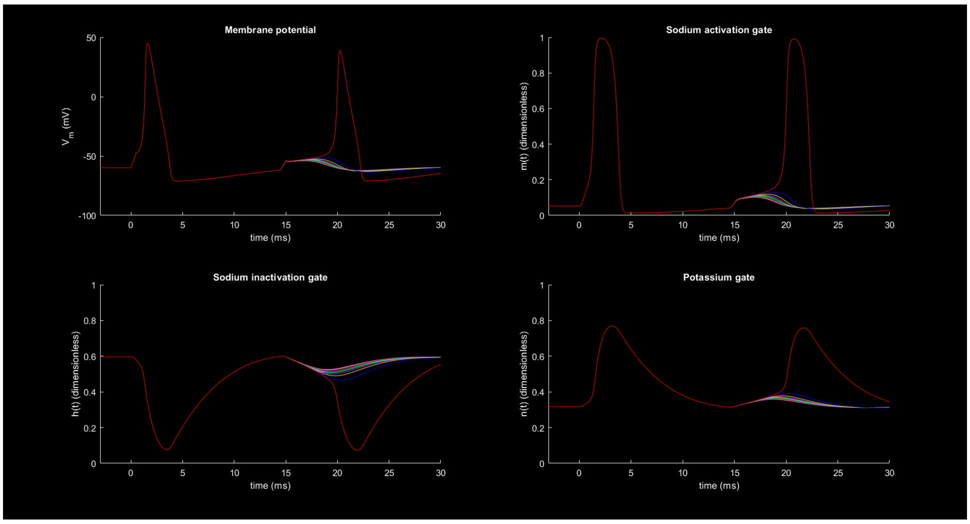

Figure 10: Second pulse with a delay of 14 ms

Action potential is triggered at $I_{2\text{th}} = 16.9\,\mu\text{A}\,\text{cm}^{-2}$. We can conclude that the threshold for the second pulse is this value.

2.1.5 delay = 12 ms

```
amp1 = 27.4;
width1 = 0.5;
delay2 = 12;
amp2 = 25;
width2 = 0.5;

for j =  1:4
    hhmplot(0, 30, 1);
    amp2 = amp2 + 0.1;
end
```

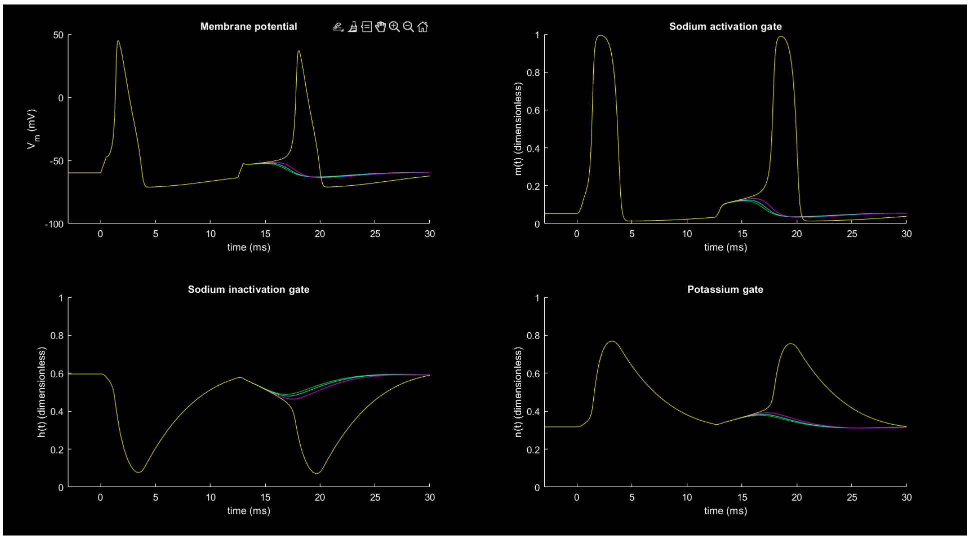

Figure 11: Second pulse with a delay of 12 ms

Action potential is triggered at $I_{2\text{th}} = 25.3\,\mu\text{A}\,\text{cm}^{-2}$. We can conclude that the threshold for the second pulse is this value.

2.1.6 delay = 10 ms

```
amp1 = 27.4;
width1 = 0.5;
delay2 = 10;
amp2 = 40;
width2 = 0.5;

for j =  1:6
    hhmplot(0, 30, 1);
    amp2 = amp2 + 0.1;
end
```

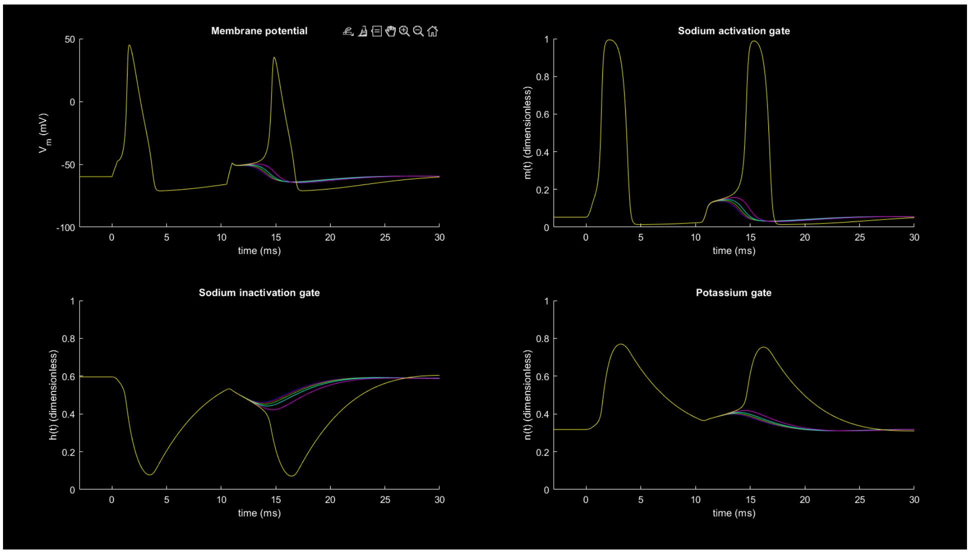

Figure 12: Second pulse with a delay of 10 ms

Action potential is triggered at $I_{2\text{th}} = 40.5\,\mu\text{A}\,\text{cm}^{-2}$. We can conclude that the threshold for the second pulse is this value.

2.1.7 delay = 8 ms

```
amp1 = 27.4;
width1 = 0.5;
delay2 = 8;
amp2 = 69;
width2 = 0.5;

for j =  1:7
    hhmplot(0, 30, 1);
    amp2 = amp2 + 0.1;
end
```

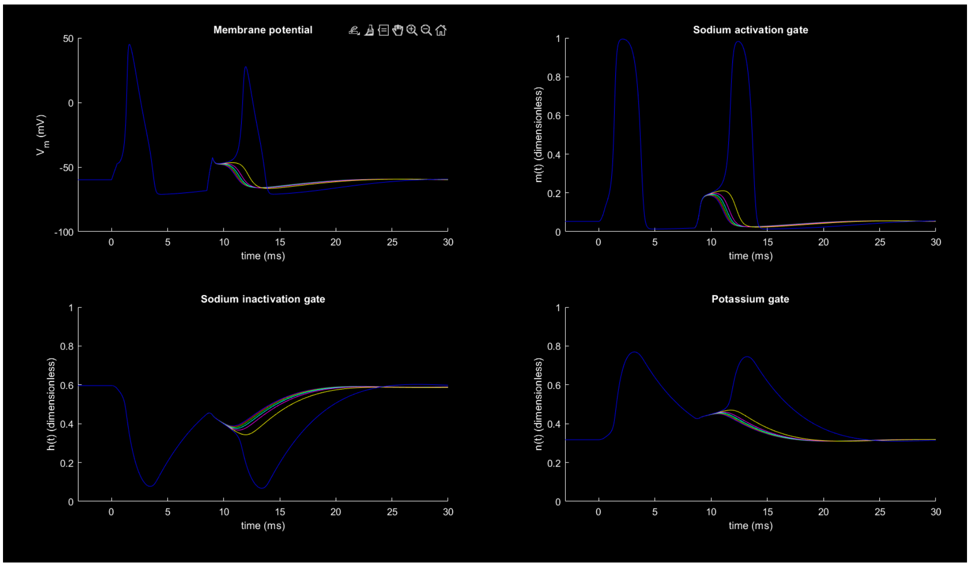

Figure 13: Second pulse with a delay of 8 ms

Action potential is triggered at $I_{2\text{th}} = 69.6\,\mu\text{A}\,\text{cm}^{-2}$. We can conclude that the threshold for the second pulse is this value.

2.1.8 delay = 6 ms

```
amp1 = 27.4;
width1 = 0.5;
delay2 = 6;
amp2 = 143;
width2 = 0.5;

for j =  1:6
    hhmplot(0, 30, 1);
    amp2 = amp2 + 0.1;
end
```

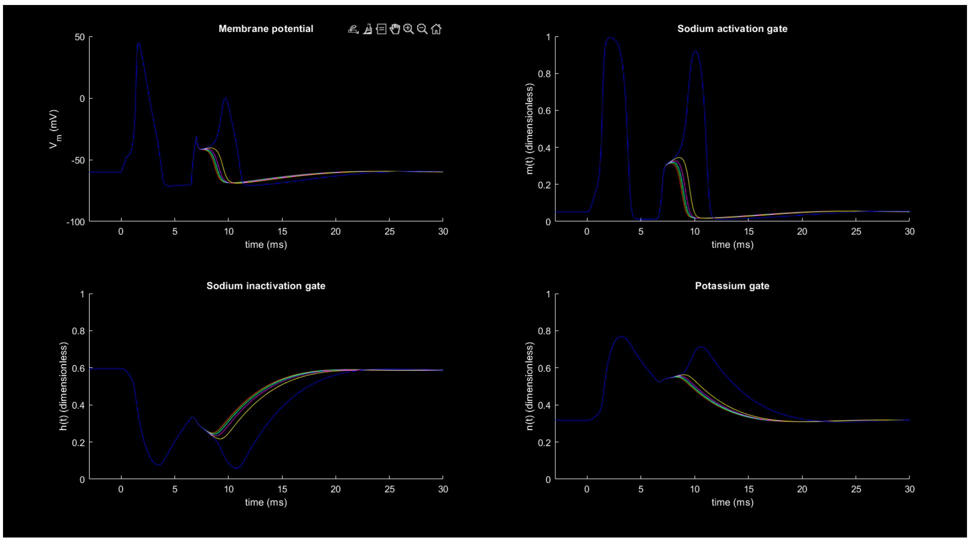

Figure 14: Second pulse with a delay of 6 ms

Action potential is triggered at $I_{2\text{th}} = 143.5\,\mu\text{A}\,\text{cm}^{-2}$. We can conclude that the threshold for the second pulse is this value.

2.2 Question 4

```
delays = [6, 8, 10, 12, 14, 16, 18, 20, 25];
I2ths = [143.6, 69.6, 40.5, 25.3, 16.9, 12.7, 11.3, 11.6, 13.7];
ratios = I2ths / 27.4;

t = linspace(4, 25, 1000);
f = spline(delays, ratios, t);

plot(t, f, 'LineWidth', 2);
yline(1, 'r--', 'LineWidth', 1);
grid on;
xlabel('Delay (ms)');
ylabel('I_2/I_1 Ratio');
title('I_2/I_1 Ratio vs Delay');
```

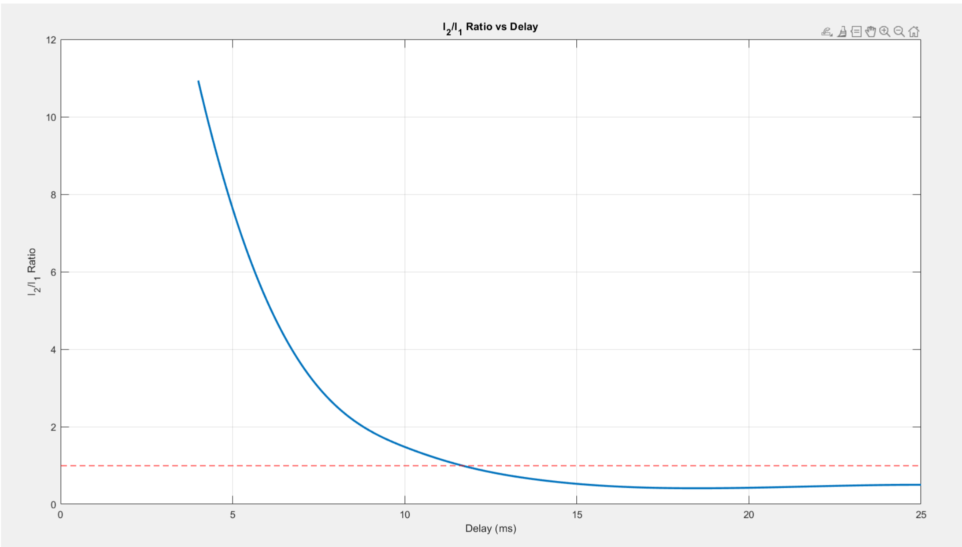

Figure 15: I2th/I1th over delay

- For delays below 4 ms, the secondary impulse must be at least 10x greater than the first in order to trigger an action potential. Therefore, the interval from 0 ms to 4 ms can be identified as the **absolute refractory period**.
- For delays greater than 12 ms, an action potential is triggered even when the secondary impulse is a fraction of the first. The red dashed line indicates the point at which $I_{2\text{th}} < I_{1\text{th}}$. We can thus conclude that the **relative refractory period** lies approximately between 12 ms and 17 ms.

3 Repetitive activity

3.1 Question 5

3.1.1 Intensity $5\,\mu\mathrm{A\,cm^{-2}}$

```
amp1 = 5;
width1 = 80;
delay2 = 0;
amp2 = 0;
width2 = 0;

hhmplot(0, 100, 0);
```

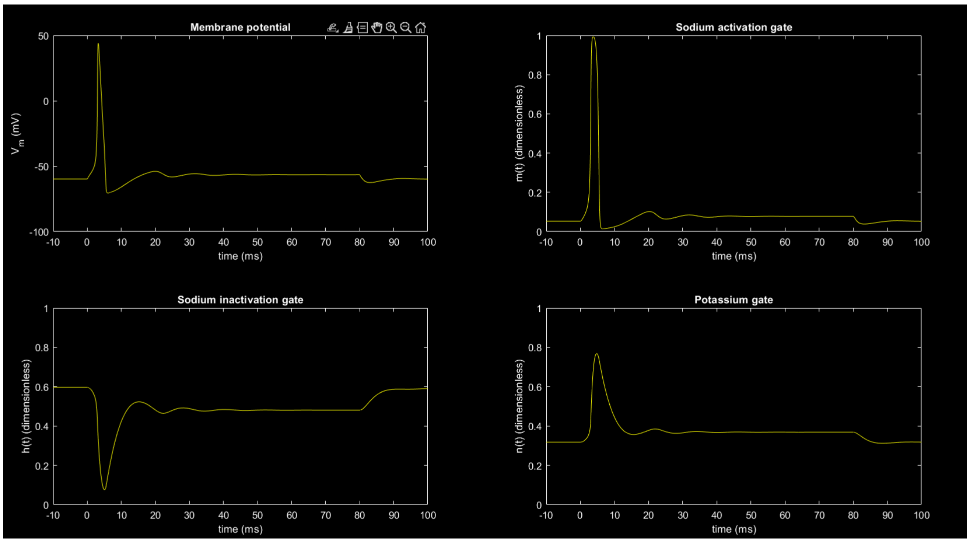

Figure 16: Intensity $5\,\mu\mathrm{A\,cm^{-2}}$

3.1.2 Intensity $10\,\mu\mathrm{A\,cm^{-2}}$

```
amp1 = 10;
width1 = 80;
delay2 = 0;
amp2 = 0;
width2 = 0;

hhmplot(0, 100, 0);
```

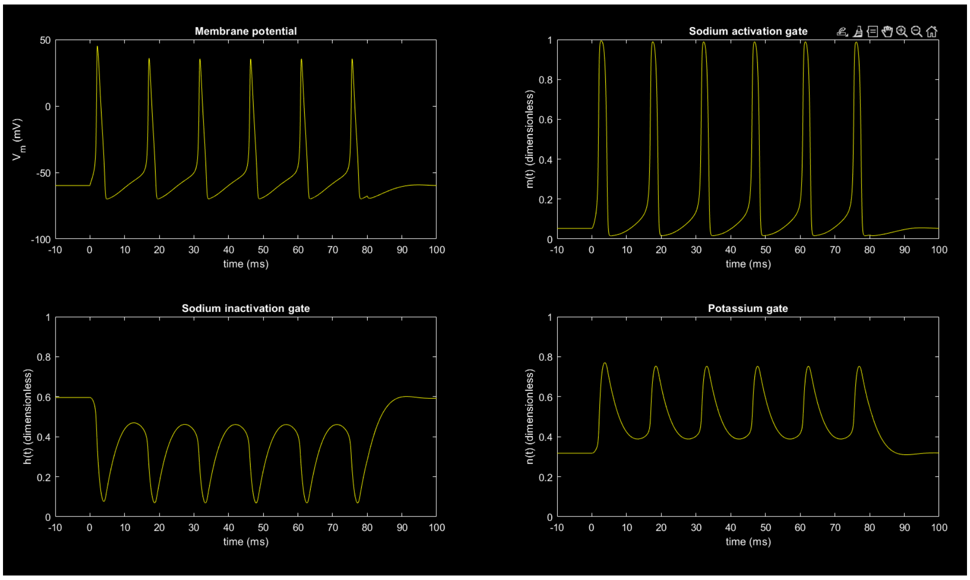

Figure 17

3.1.3 Intensity $20\,\mu\mathrm{A\,cm^{-2}}$

```
amp1 = 20;
width1 = 80;
delay2 = 0;
amp2 = 0;
width2 = 0;

hhmplot(0, 100, 0);
```

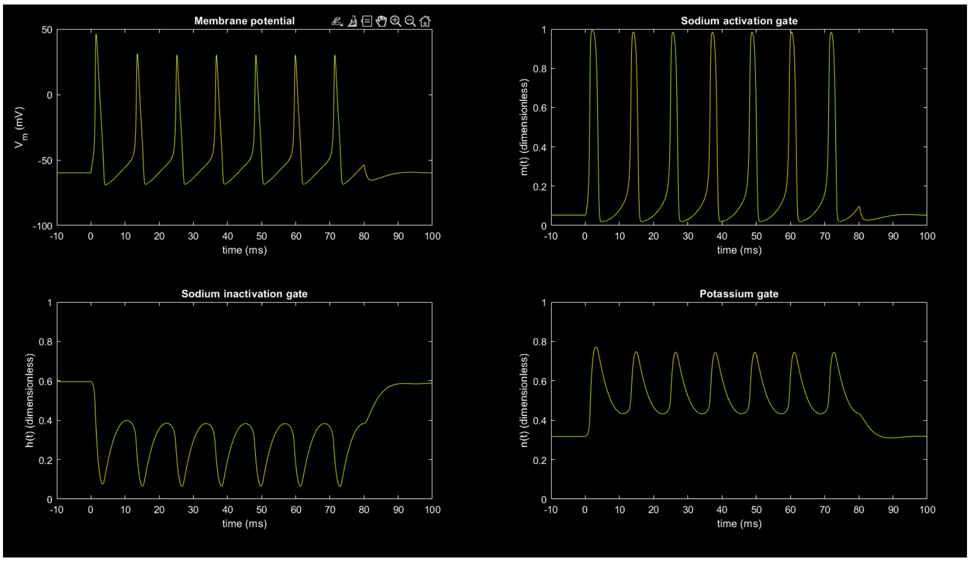

Figure 18

3.1.4 Intensity $30\,\mu\mathrm{A\,cm^{-2}}$

```
amp1 = 30;
```

```
width1 = 80;
delay2 = 0;
amp2 = 0;
width2 = 0;

hhmplot(0, 100, 0);
```

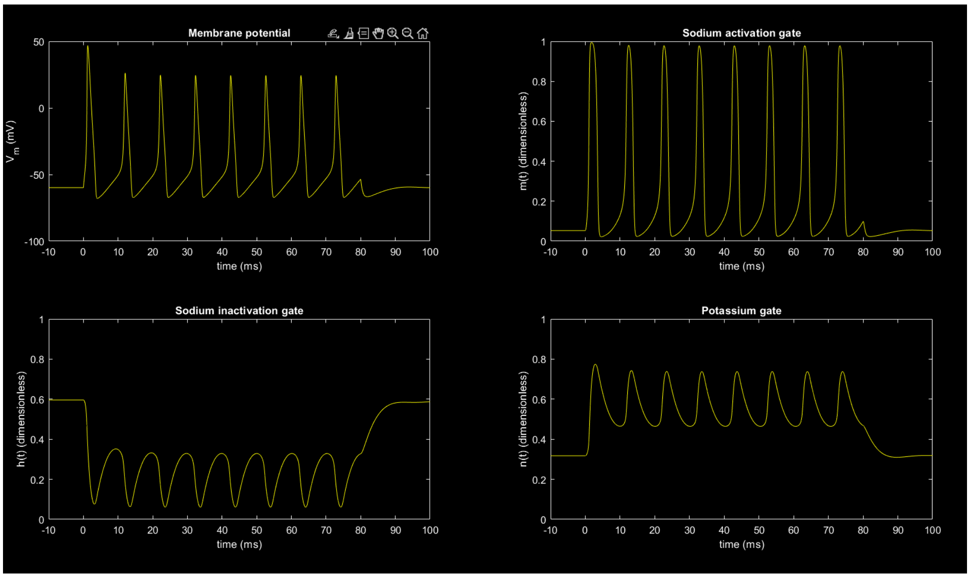

Figure 19

3.1.5 Intensity $50\,\mu\mathrm{A\,cm^{-2}}$

```
amp1 = 50;
width1 = 80;
delay2 = 0;
amp2 = 0;
width2 = 0;

hhmplot(0, 100, 0);
```

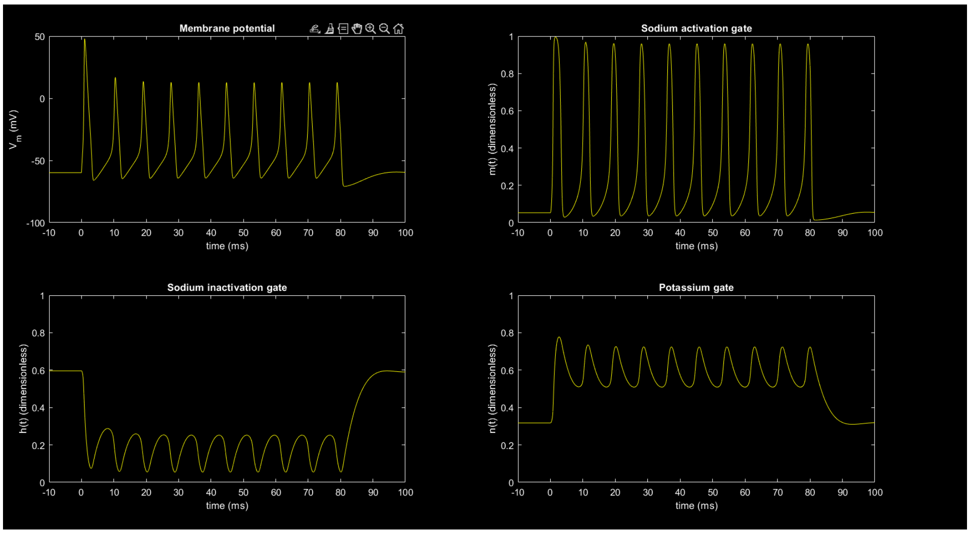

Figure 20

3.1.6 Intensity $70\,\mu\mathrm{A\,cm^{-2}}$

```
amp1 = 70;
width1 = 80;
delay2 = 0;
amp2 = 0;
width2 = 0;

hhmplot(0, 100, 0);
```

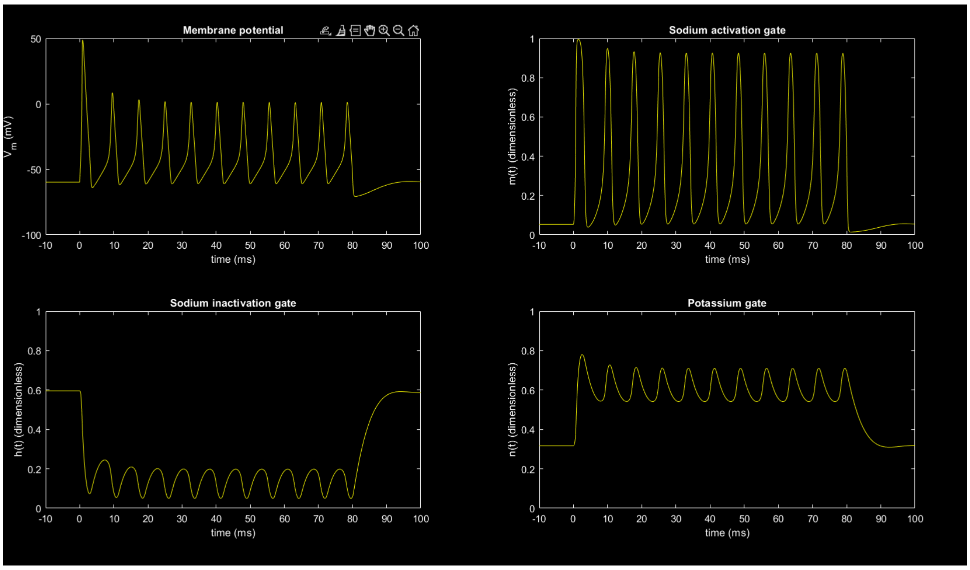

Figure 21

3.1.7 Intensity $100\,\mu\mathrm{A\,cm^{-2}}$

```
amp1 = 100;
width1 = 80;
```

```
delay2 = 0;
amp2 = 0;
width2 = 0;

hhmplot(0, 100, 0);
```

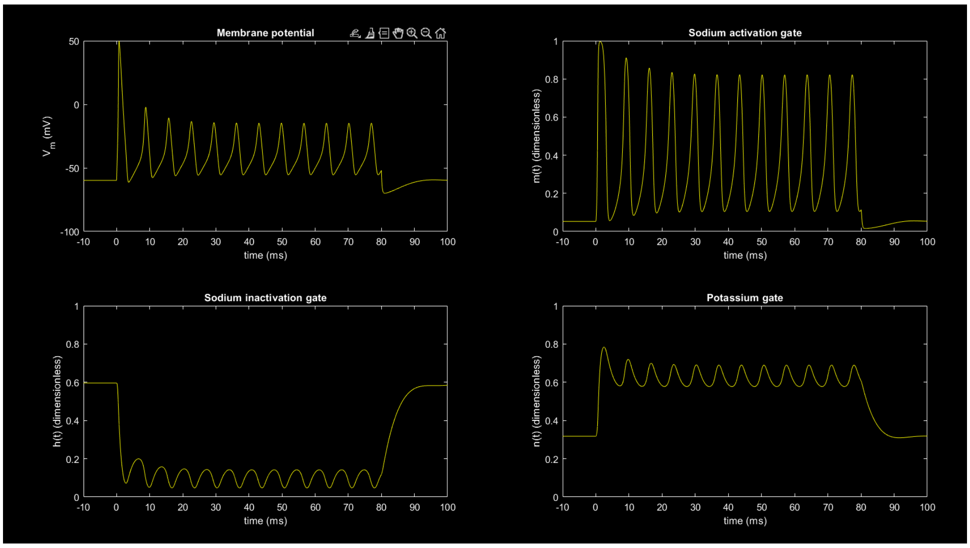

Figure 22

Action Potential ($\mu A/cm^2$)	Number of Triggered Action Potentials
5	1
10	6
20	7
30	8
50	10
70	11
100	12

Table 1: Action potential strength vs. number of triggered action potentials

The following code was used to plot the values of frequency of APs triggered against stimulus amplitude.

```
amplitudes = [5 10 20 30 50 70 100];
frequencies = [1 6 7 8 10 11 12];

x = linspace(min(amplitudes), max(amplitudes), 1000);
f = spline(amplitudes, frequencies, x);

figure;
plot(x, f, 'b-', 'LineWidth', 2);
hold on;
plot(amplitudes, frequencies, 'ro', 'MarkerSize', 8,
    'LineWidth', 2);
xlabel('Amplitude (\muA/cm^2)');
```

```
ylabel('Frequency (Number of Action Potentials Triggered)');
title('Amplitude vs Frequency of Triggered Action Potentials');
ylim([0 15]);
grid on;
```

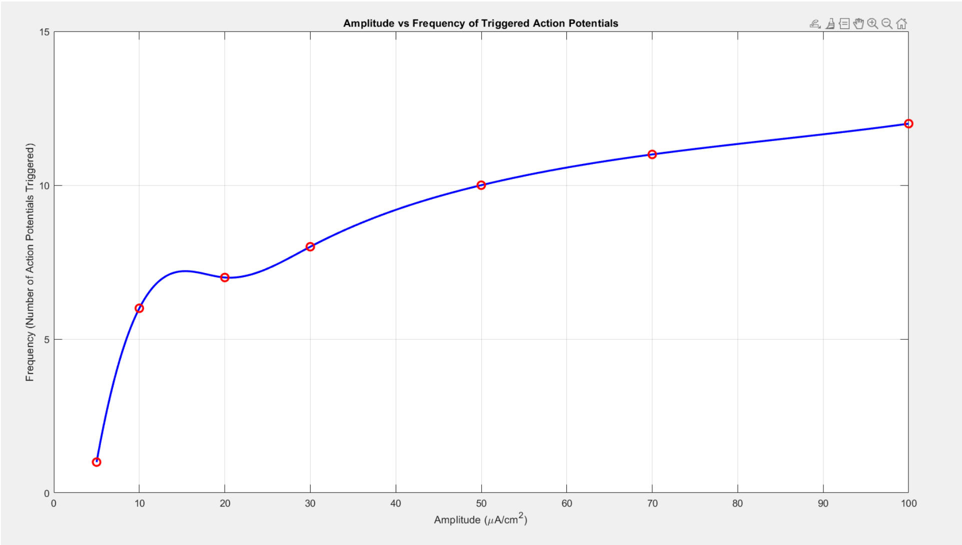

Figure 23: AP frequency over stimulus amplitude

The frequency of APs triggered rises with increasing stimulus amplitude, showing a steep increase at lower amplitudes, then gradually slowing down.

3.2 Question 6

```
amp1 = 200;
width1 = 80;
delay2 = 0;
amp2 = 0;
width2 = 0;
hhmplot(0, 100, 0);
```

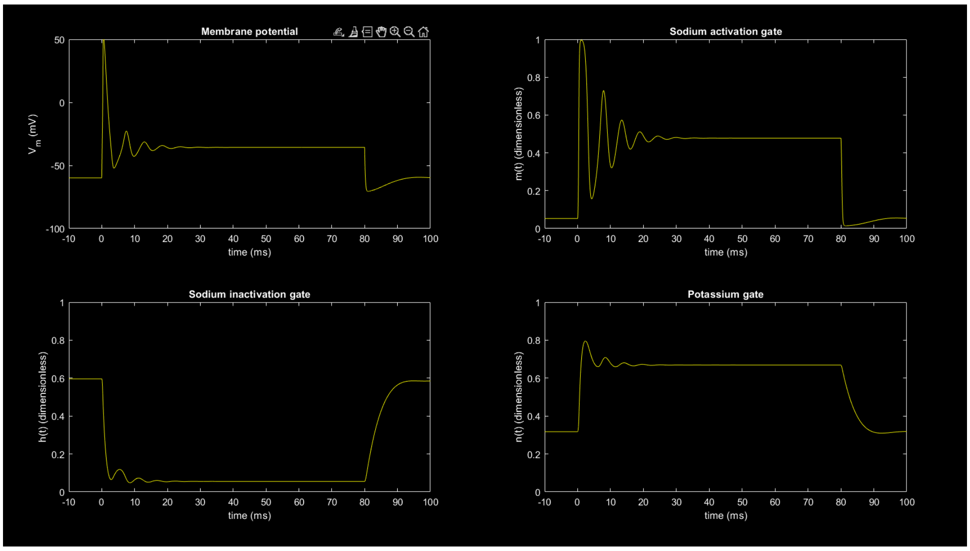

Figure 24: stimulus amplitude at 200 (µA/cm^2)

From the results above, the triggered action potentials exhibit a decaying sinusoidal pattern. A high-frequency oscillatory behavior is observed. Specifically:

- The Na^+ channels, represented by variables m and h, open more as depolarization increases, resulting in a larger inward Na^+ current during the upstroke of the action potential.
- However, as the stimulus amplitude increases, the amplitude of the action potential tends to decrease.
- Meanwhile, the K^+ channels, represented by n, become more active with membrane depolarization.
- If the K^+ conductance does not sufficiently offset the increased Na^+ influx, it can negatively impact the action potential amplitude.

4 Temperature dependence

4.1 Question 7

```
vclamp = 0;
amp1 = 20;
width1 = 0.5;

temp = [0, 5, 10, 15, 20, 24, 25, 26, 30];

for i = 1:length(temp)
    tempc = temp(i);
    hhmplot(0, 30, 1);
    legend('show');
end
```

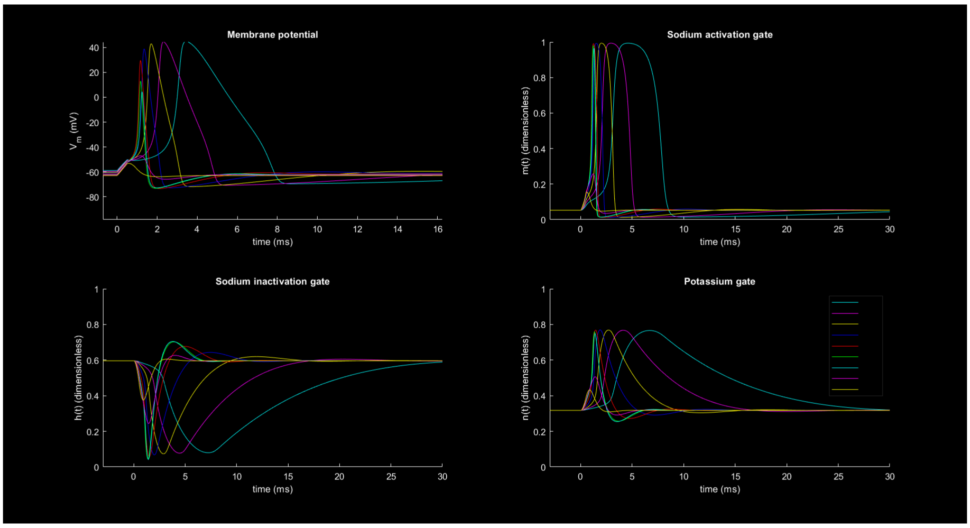

Figure 25: AP change with temperature

- As temperature increases, the duration of the action potential decreases, indicating higher conduction velocities.
- The peak of the membrane potential reduces with rising temperature.
- Both the absolute refractory period and relative refractory period shorten due to the faster completion of the action potential.