A5: Compartmental Modelling

BM2102 - Modelling and Analysis of Physiological Systems
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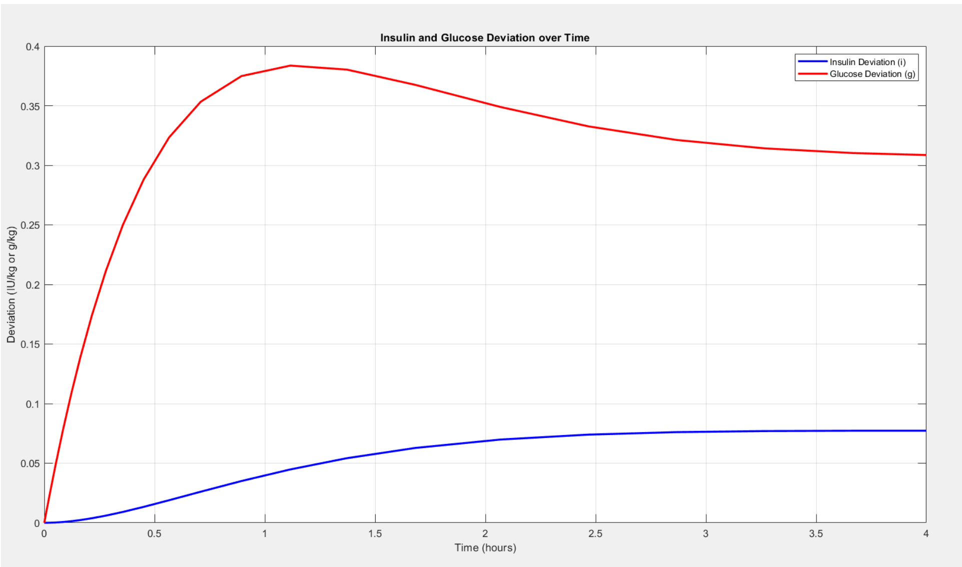
Contents

1 Part 1

1.1 Question 1

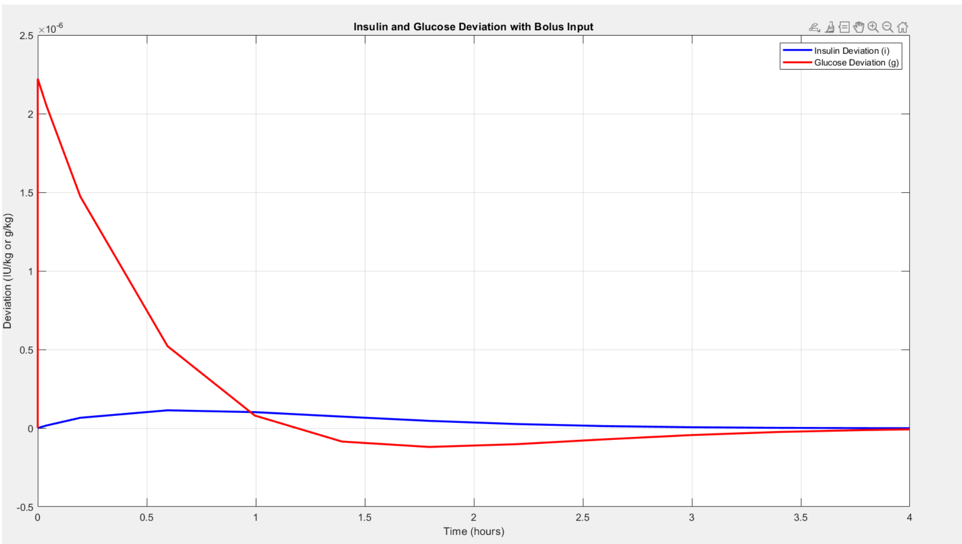
The code for the system as yp = ax + b format (for unit step input) can be found in the file glucose_insulin_step.m as a function. The following code will solve the system and plot the Glucose/Insulin levels against time.

```
% Simulation of glucose-insulin model with step input
tspan = [0, 4]; % Time span: 0 to 4 hours
y0 = [0; 0]; % Initial conditions: i(0) = 0, g(0) = 0

% Solve ODE
[t, y] = ode23(@glucose_insulin_step, tspan, y0);

% Plot results in a single graph
figure;
plot(t, y(:,1), 'b-', 'LineWidth', 2, 'DisplayName', 'Insulin
    Deviation (i)');
hold on;
plot(t, y(:,2), 'r-', 'LineWidth', 2, 'DisplayName', 'Glucose
    Deviation (g)');
hold off;
title('Insulin and Glucose Deviation over Time');
xlabel('Time (hours)');
ylabel('Deviation (IU/kg or g/kg)');
grid on;
legend('show');
```



The code for the system for a bolus input can be found in the file glucose_insulin_bolus.m as a function. The following code will solve the system and plot the Glucose/Insulin levels against time.

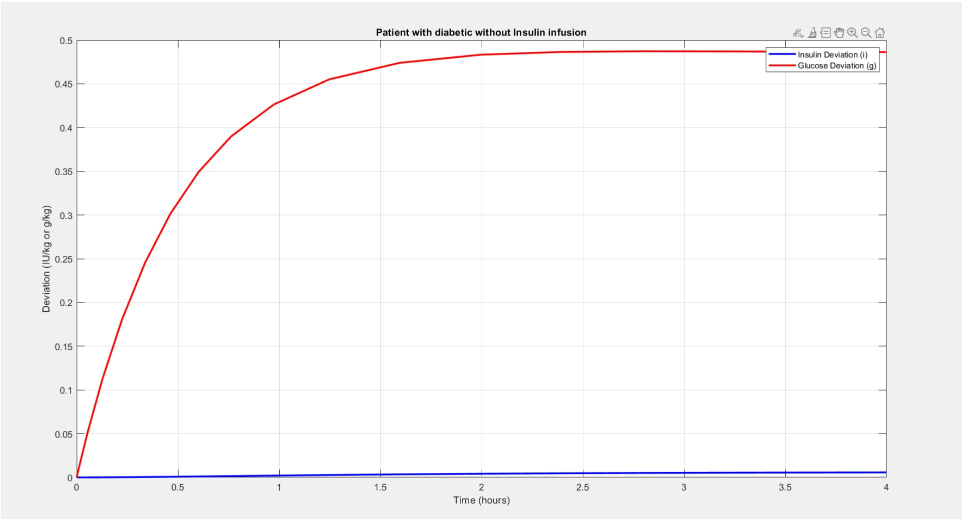
```
% Simulation of glucose-insulin model with bolus input
tspan = [0, 4]; % Time span: 0 to 4 hours
y0 = [0; 0]; % Initial conditions: i(0) = 0, g(0) = 0

% Solve ODE
[t, y] = ode23(@glucose_insulin_bolus, tspan, y0);

% Plot results in a single graph
figure;
plot(t, y(:,1), 'b-', 'LineWidth', 2, 'DisplayName', 'Insulin
    Deviation (i)');
hold on;
plot(t, y(:,2), 'r-', 'LineWidth', 2, 'DisplayName', 'Glucose
    Deviation (g)');
hold off;
title('Insulin and Glucose Deviation with Bolus Input');
xlabel('Time (hours)');
ylabel('Deviation (IU/kg or g/kg)');
grid on;
legend('show')
```



The code for the system with diabetic conditions (unit step input) can be found in the file diabetic_patient.m as a function. The following code will solve the system and plot the Glucose/Insulin levels against time.

```
% Simulation of glucose-insulin model with step input for
    diabetic patient
tspan = [0, 4]; % Time span: 0 to 4 hours
y0 = [0; 0]; % Initial conditions: i(0) = 0, g(0) = 0

% Solve ODE
[t, y] = ode23(@diabetic_patient, tspan, y0);

% Plot results in a single graph
```

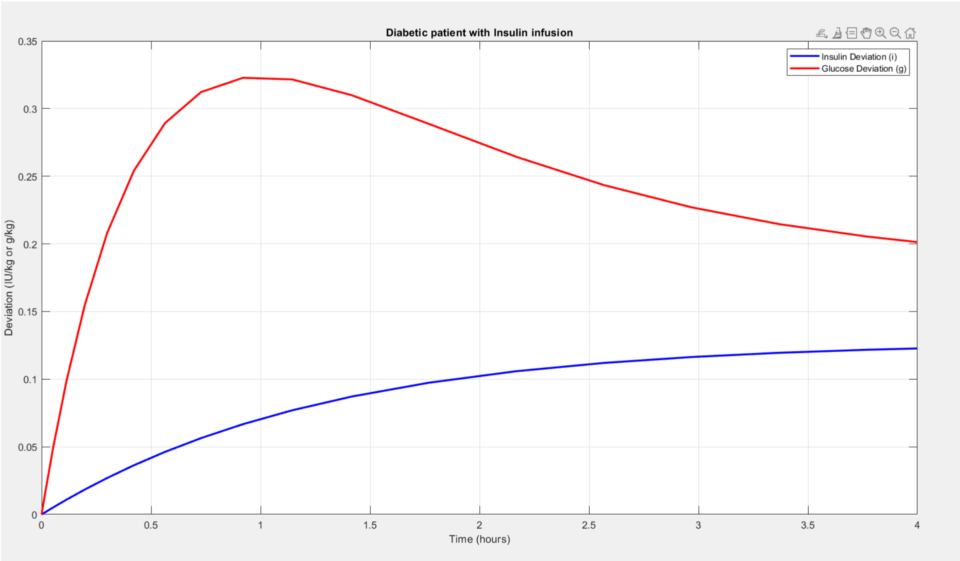
```
figure;
plot(t, y(:,1), 'b-', 'LineWidth', 2, 'DisplayName', 'Insulin Deviation (i)');
hold on;
plot(t, y(:,2), 'r-', 'LineWidth', 2, 'DisplayName', 'Glucose Deviation (g)');
hold off;
title('Insulin and Glucose Deviation over Time');
xlabel('Time (hours)');
ylabel('Deviation (IU/kg or g/kg)');
grid on;
legend('show');
```



The code for the system with diabetic conditions with insulin infusion (unit step input) can be found in the file diabetic_patient_insulin_infusion.m as a function. The following code will solve the system and plot the Glucose/Insulin levels against time.

```
% Simulation of glucose-insulin model with insulin infusion
tspan = [0, 4]; % Time span: 0 to 4 hours
y0 = [0; 0]; % Initial conditions: i(0) = 0, g(0) = 0

% Solve ODE
[t, y] = ode23(@diabetic_patient_insulin_infusion, tspan, y0);

% Plot results in a single graph
figure;
plot(t, y(:,1), 'b-', 'LineWidth', 2, 'DisplayName', 'Insulin Deviation (i)');
hold on;
plot(t, y(:,2), 'r-', 'LineWidth', 2, 'DisplayName', 'Glucose Deviation (g)');
hold off;
title('Diabetic patient with Insulin infusion');
xlabel('Time (hours)');
ylabel('Deviation (IU/kg or g/kg)');
```

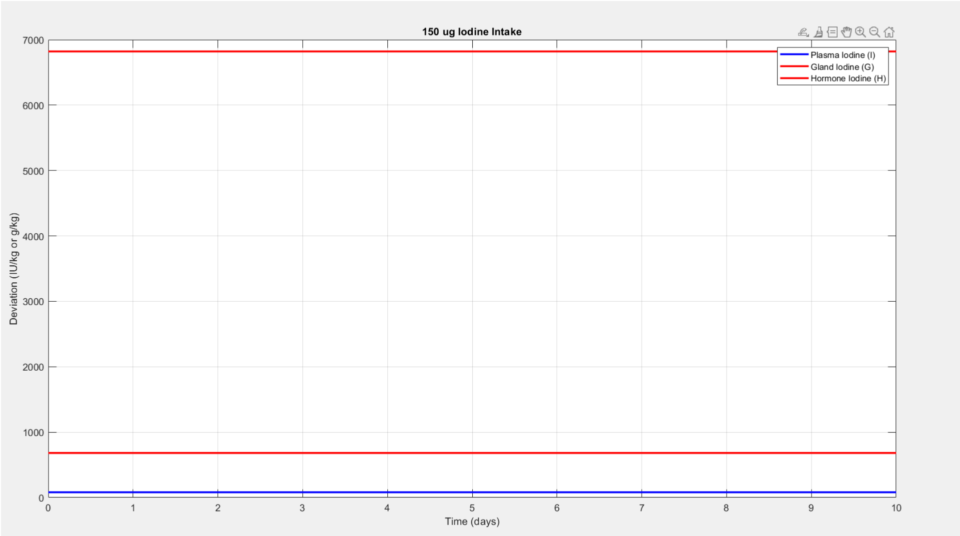
```
grid on;
legend('show');
```



1.2 Question 2

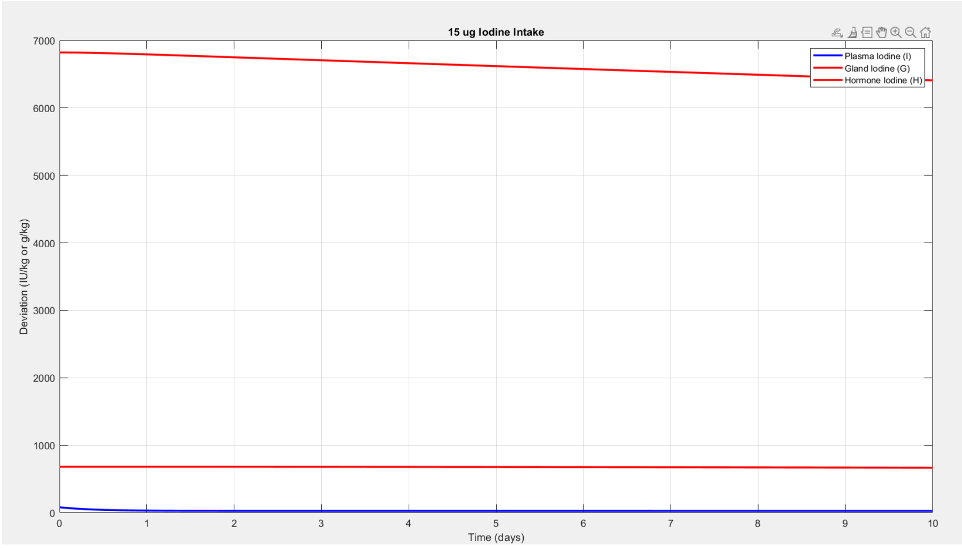
The code for the Rigg's iodine model with 150 μg iodine intake is included in riggs_iodine.m file. The following code simulates it for 10 days.

```
tspan = [0, 10]; % Time span: 0 to 10 days
y0 = [81.2, 6821, 682]'; % Initial conditions: i(0) = 81.2, g(0) = 6821, H(0) = 682

% Solve ODE
[t, y] = ode23(@riggs_iodine, tspan, y0);


% Plot results in a single graph
figure;
plot(t, y(:,1), 'b-', 'LineWidth', 2, 'DisplayName', 'Plasma Iodine (I)');
hold on;
plot(t, y(:,2), 'r-', 'LineWidth', 2, 'DisplayName', 'Gland Iodine (G)');
hold on;
plot(t, y(:, 3),  'r-', 'LineWidth', 2, 'DisplayName', 'Hormone Iodine (H)');
title('150 ug Iodine Intake');
xlabel('Time (days)');
ylabel('Deviation (IU/kg or g/kg)');
grid on;
legend('show');
```



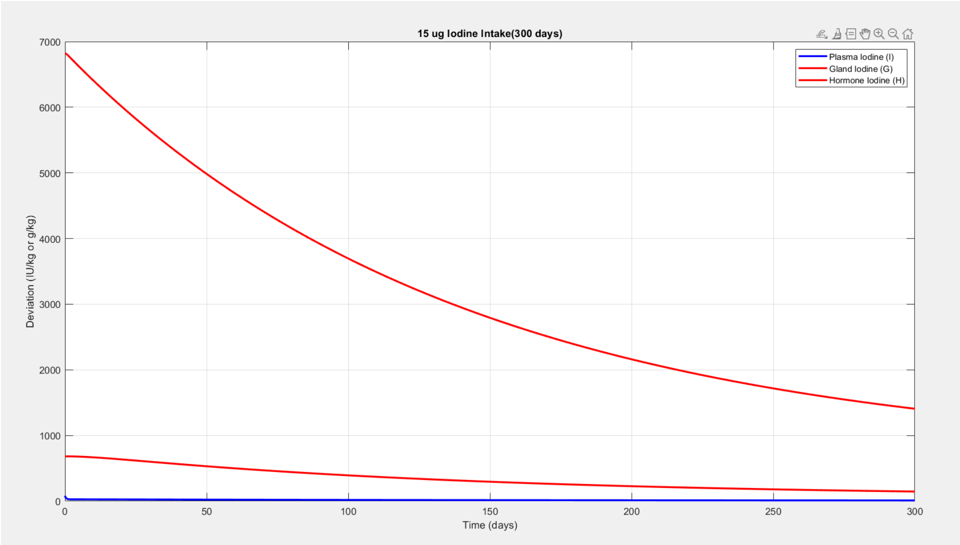
The code for the Rigg's iodine model with 15 μg iodine intake is included in riggs_iodine_15.m file. The following code simulates it for 10 days.

```
tspan = [0, 10]; % Time span: 0 to 10 days
y0 = [81.2, 6821, 682]'; % Initial conditions: i(0) = 81.2, g(0) = 6821, H(0) = 682

% Solve ODE
[t, y] = ode23(@riggs_iodine_15, tspan, y0);

% Plot results in a single graph
figure;
plot(t, y(:,1), 'b-', 'LineWidth', 2, 'DisplayName', 'Plasma Iodine (I)');
hold on;
plot(t, y(:,2), 'r-', 'LineWidth', 2, 'DisplayName', 'Gland Iodine (G)');
hold on;
plot(t, y(:, 3),  'r-', 'LineWidth', 2, 'DisplayName', 'Hormone Iodine (H)');
hold off;
title('15 ug Iodine Intake');
xlabel('Time (days)');
ylabel('Deviation (IU/kg or g/kg)');
grid on;
legend('show');
```



The code for the Rigg's iodine model with 15 μg iodine intake is included in riggs_iodine_15.m file. The following code simulates it for 300 days.

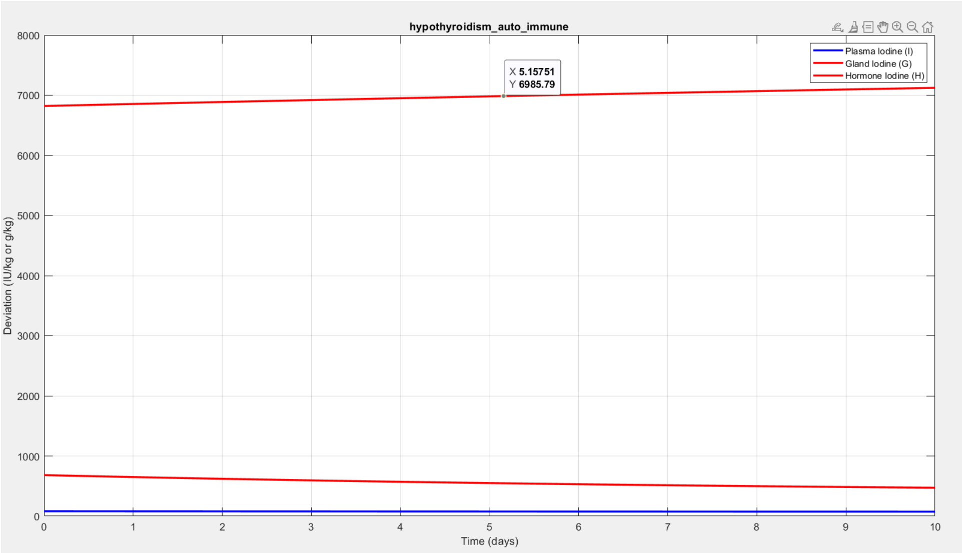
```
tspan = [0, 300]; % Time span: 0 to 10 days
y0 = [81.2, 6821, 682]'; % Initial conditions: i(0) = 81.2, g(0) = 6821, H(0) = 682

% Solve ODE
[t, y] = ode23(@riggs_iodine_15, tspan, y0);


% Plot results in a single graph
figure;
plot(t, y(:,1), 'b-', 'LineWidth', 2, 'DisplayName', 'Plasma Iodine (I)');
hold on;
plot(t, y(:,2), 'r-', 'LineWidth', 2, 'DisplayName', 'Gland Iodine (G)');
hold on;
plot(t, y(:, 3),  'r-', 'LineWidth', 2, 'DisplayName', 'Hormone Iodine (H)');
hold off;
title('15 ug Iodine Intake');
xlabel('Time (days)');
ylabel('Deviation (IU/kg or g/kg)');
grid on;
legend('show');
```



1.2.1 (a)

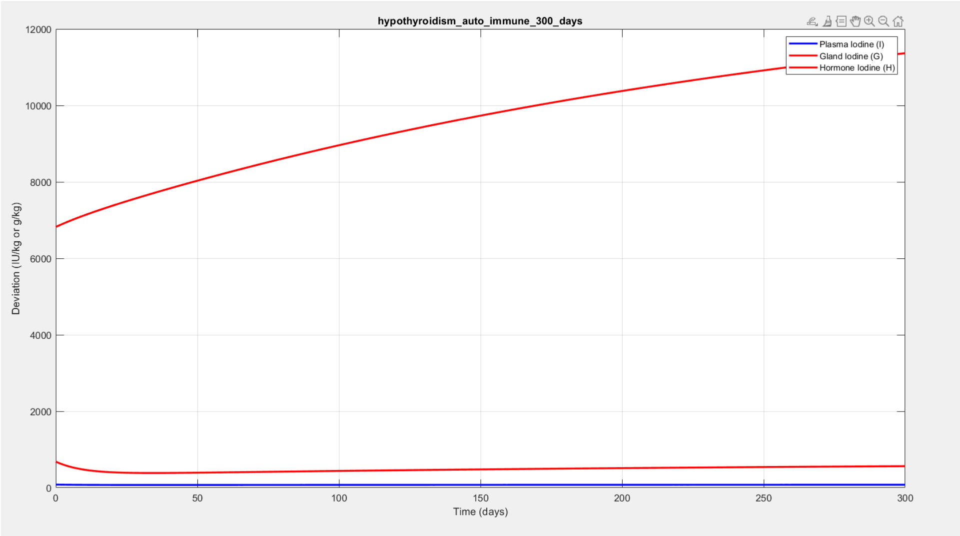
Hypothyroidism is the condition where there is a lack of thyroid hormornes compared to healthy level. Due to autoimmune disease, the cells producing thyroid hormornes die out. This results in a decline of thyroid hormorne production which can be reflected in the Riggs model by reducing k2 value to 0.005. This represents slowing the conversion of gland iodine to hormornal iodine. The code for the updated Riggs model is in hypothyroidism_auto_immune.m. The following code simulates it for 10 days.

```
tspan = [0, 10]; % Time span: 0 to 10 days
y0 = [81.2, 6821, 682]'; % Initial conditions: i(0) = 81.2, g(0) = 6821, H(0) = 682

% Solve ODE
[t, y] = ode23(@hypothyroidism_auto_immune, tspan, y0);


% Plot results in a single graph
figure;
plot(t, y(:,1), 'b-', 'LineWidth', 2, 'DisplayName', 'Plasma Iodine (I)');
hold on;
plot(t, y(:,2), 'r-', 'LineWidth', 2, 'DisplayName', 'Gland Iodine (G)');
hold on;
plot(t, y(:, 3),  'r-', 'LineWidth', 2, 'DisplayName', 'Hormone Iodine (H)');
title('hypothyroidism\_auto\_immune');
xlabel('Time (days)');
ylabel('Deviation (IU/kg or g/kg)');
grid on;
legend('show');
```



The following code simulates it for 300 days.

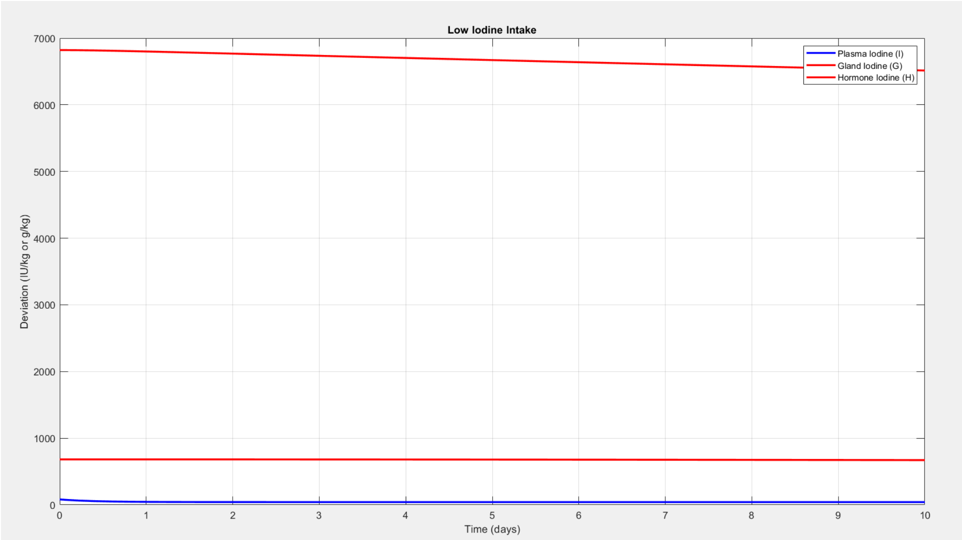
```
tspan = [0, 300]; % Time span: 0 to 300 days
y0 = [81.2, 6821, 682]'; % Initial conditions: i(0) = 81.2, g(0) = 6821, H(0) = 682

% Solve ODE
[t, y] = ode23(@hypothyroidism_auto_immune, tspan, y0);


% Plot results in a single graph
figure;
plot(t, y(:,1), 'b-', 'LineWidth', 2, 'DisplayName', 'Plasma Iodine (I)');
hold on;
plot(t, y(:,2), 'r-', 'LineWidth', 2, 'DisplayName', 'Gland Iodine (G)');
hold on;
plot(t, y(:, 3),  'r-', 'LineWidth', 2, 'DisplayName', 'Hormone Iodine (H)');
title('hypothyroidism\_auto\_immune\_300\_days');
xlabel('Time (days)');
ylabel('Deviation (IU/kg or g/kg)');
grid on;
legend('show');
```



1.2.2 (b)

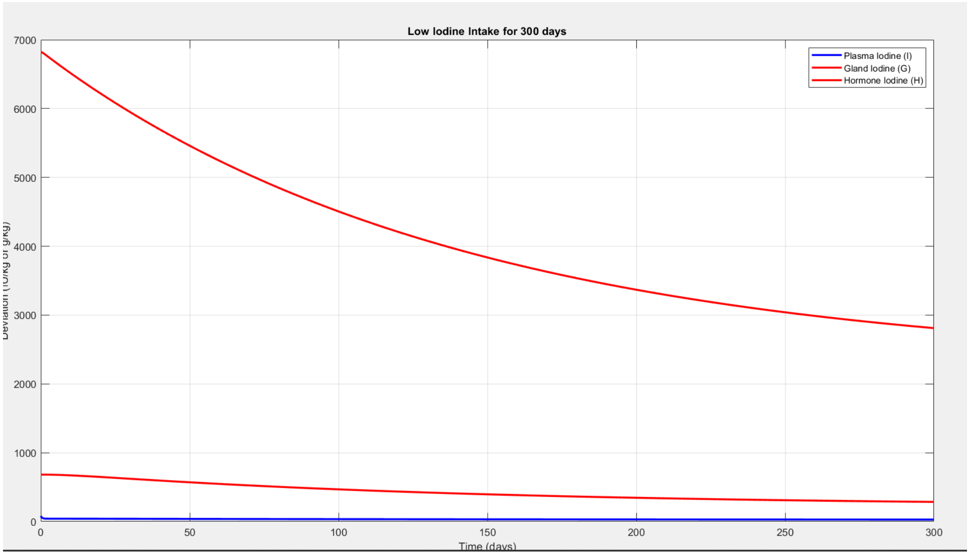
Hypothyroidism can also occur due to low iodine intake. This can be modelled using the Riggs model by loweirng the input B1(t) to 50. The code for the updated Riggs model is in riggs_low_iodine.m. The following code simulates it for 10 days.

```
tspan = [0, 10]; % Time span: 0 to 10 days
y0 = [81.2, 6821, 682]'; % Initial conditions: i(0) = 81.2, g(0) = 6821, H(0) = 682

% Solve ODE
[t, y] = ode23(@riggs_low_iodine, tspan, y0);


% Plot results in a single graph
figure;
plot(t, y(:,1), 'b-', 'LineWidth', 2, 'DisplayName', 'Plasma Iodine (I)');
hold on;
plot(t, y(:,2), 'r-', 'LineWidth', 2, 'DisplayName', 'Gland Iodine (G)');
hold on;
plot(t, y(:, 3),  'r-', 'LineWidth', 2, 'DisplayName', 'Hormone Iodine (H)');
title('Low Iodine Intake');
xlabel('Time (days)');
ylabel('Deviation (IU/kg or g/kg)');
grid on;
legend('show');
```



The following code simulates it for 300 days.

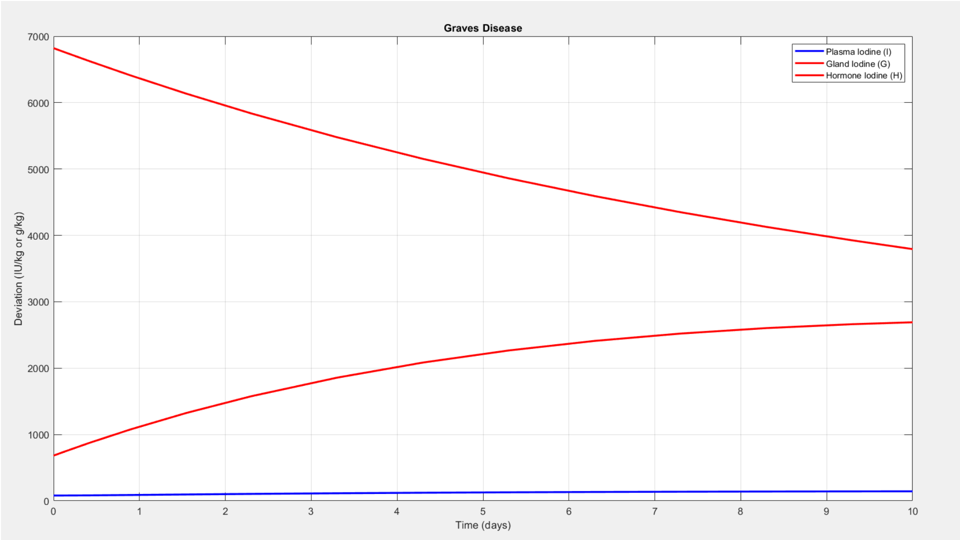
```
tspan = [0, 300]; % Time span: 0 to 300 days
y0 = [81.2, 6821, 682]'; % Initial conditions: i(0) = 81.2, g(0)
    = 6821, H(0) = 682

% Solve ODE
[t, y] = ode23(@riggs_low_iodine, tspan, y0);


% Plot results in a single graph
figure;
plot(t, y(:,1), 'b-', 'LineWidth', 2, 'DisplayName', 'Plasma
    Iodine (I)');
hold on;
plot(t, y(:,2), 'r-', 'LineWidth', 2, 'DisplayName', 'Gland
    Iodine (G)');
hold on;
plot(t, y(:, 3),  'r-', 'LineWidth', 2, 'DisplayName', 'Hormone
    Iodine (H)');
title('Low Iodine Intake for 300 days');
xlabel('Time (days)');
ylabel('Deviation (IU/kg or g/kg)');
grid on;
legend('show');
```



1.2.3 (c)

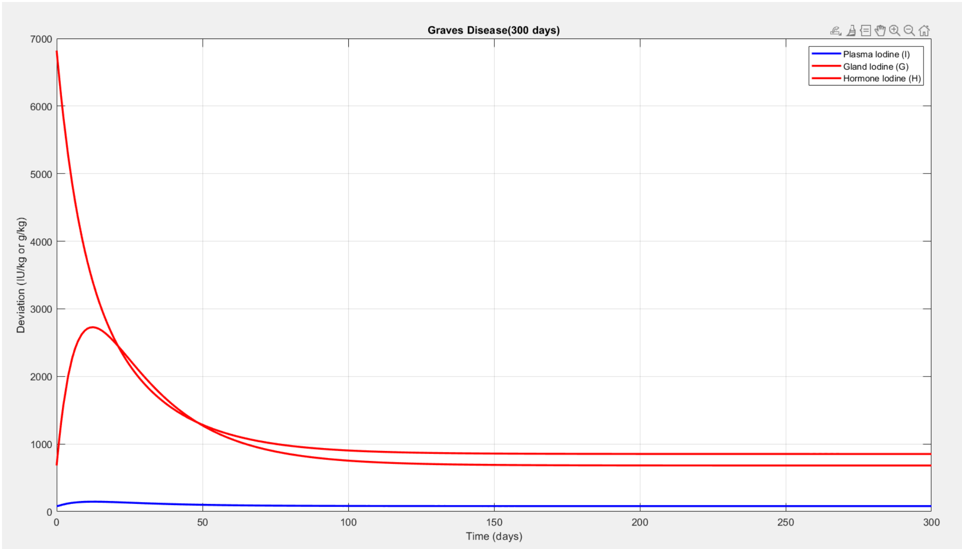
During Grave's disease, the thyroid glands produce too much Iodine. The Riggs model can be used to model this behaviour by increasing the k2 parameter upto 0.08. This represents an increased conversion of gland iodine to hormornal iodine. The code for the updated Riggs model is in hypothyroidism_graves.m . The following code simulates it for 10 days.

```
tspan = [0, 10]; % Time span: 0 to 10 days
y0 = [81.2, 6821, 682]'; % Initial conditions: i(0) = 81.2, g(0)
    = 6821, H(0) = 682

% Solve ODE
[t, y] = ode23(@hypothyroidism_graves, tspan, y0);


% Plot results in a single graph
figure;
plot(t, y(:,1), 'b-', 'LineWidth', 2, 'DisplayName', 'Plasma
    Iodine (I)');
hold on;
plot(t, y(:,2), 'r-', 'LineWidth', 2, 'DisplayName', 'Gland
    Iodine (G)');
hold on;
plot(t, y(:, 3),  'r-', 'LineWidth', 2, 'DisplayName', 'Hormone
    Iodine (H)');
title('Graves Disease');
xlabel('Time (days)');
ylabel('Deviation (IU/kg or g/kg)');
grid on;
legend('show');
```



The following code simulates it for 300 days.

```
tspan = [0, 300]; % Time span: 0 to 300 days
y0 = [81.2, 6821, 682]'; % Initial conditions: i(0) = 81.2, g(0)
    = 6821, H(0) = 682

% Solve ODE
[t, y] = ode23(@hypothyroidism_graves, tspan, y0);


% Plot results in a single graph
figure;
plot(t, y(:,1), 'b-', 'LineWidth', 2, 'DisplayName', 'Plasma
    Iodine (I)');
hold on;
plot(t, y(:,2), 'r-', 'LineWidth', 2, 'DisplayName', 'Gland
    Iodine (G)');
hold on;
plot(t, y(:, 3),  'r-', 'LineWidth', 2, 'DisplayName', 'Hormone
    Iodine (H)');
title('Graves Disease(300 days)');
xlabel('Time (days)');
ylabel('Deviation (IU/kg or g/kg)');
grid on;
legend('show');
```



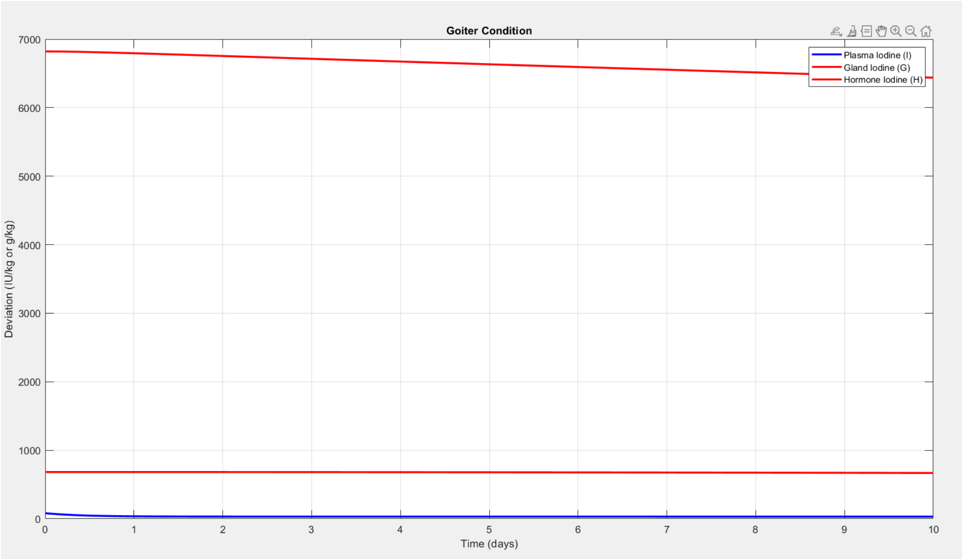
1.2.4 (d)

Causes of Goiter

Goiter is mainly caused by a lack of iodine in the diet. Due to low inodine intake the thyroid gland enlarges trying to absorb more iodine. In the Riggs model, the input B1(t) represents Iodine intake. By lowering the value of B1(t) from 150 to 25 we can mimic the condition for Goiter.

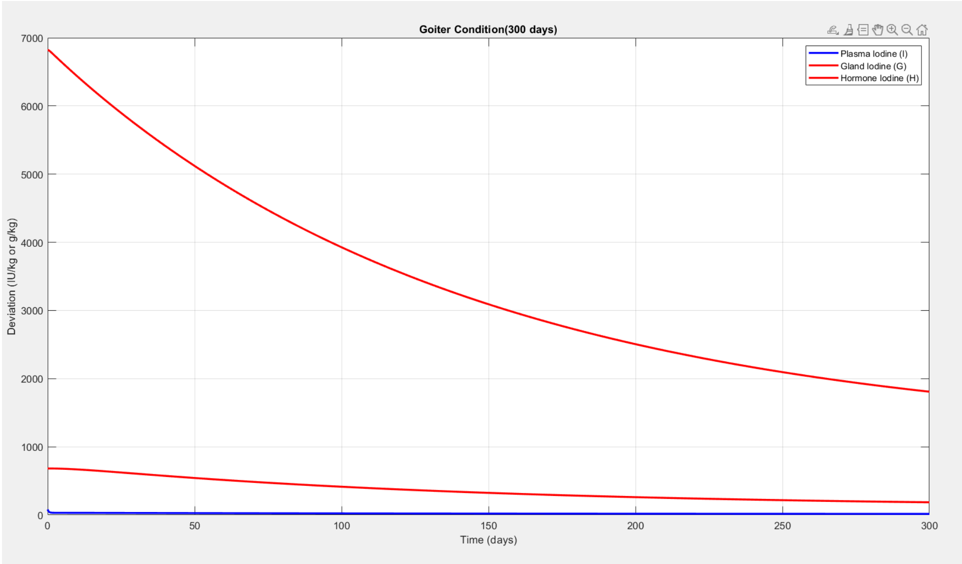
The code for the updated Riggs model is in riggs_goiter.m. The following code simulates it for 10 days.

```
tspan = [0, 10]; % Time span: 0 to 10 days
y0 = [81.2, 6821, 682]'; % Initial conditions: i(0) = 81.2, g(0) = 6821, H(0) = 682

% Solve ODE
[t, y] = ode23(@riggs_goiter, tspan, y0);


% Plot results in a single graph
figure;
plot(t, y(:,1), 'b-', 'LineWidth', 2, 'DisplayName', 'Plasma Iodine (I)');
hold on;
plot(t, y(:,2), 'r-', 'LineWidth', 2, 'DisplayName', 'Gland Iodine (G)');
hold on;
plot(t, y(:, 3),  'r-', 'LineWidth', 2, 'DisplayName', 'Hormone Iodine (H)');
title('Goiter Condition');
xlabel('Time (days)');
ylabel('Deviation (IU/kg or g/kg)');
grid on;
legend('show');
```



The following code simulates it for 300 days.

```
tspan = [0, 300]; % Time span: 0 to 300 days
y0 = [81.2, 6821, 682]'; % Initial conditions: i(0) = 81.2, g(0) = 6821, H(0) = 682

% Solve ODE
[t, y] = ode23(@riggs_goiter, tspan, y0);


% Plot results in a single graph
figure;
plot(t, y(:,1), 'b-', 'LineWidth', 2, 'DisplayName', 'Plasma Iodine (I)');
hold on;
plot(t, y(:,2), 'r-', 'LineWidth', 2, 'DisplayName', 'Gland Iodine (G)');
hold on;
plot(t, y(:, 3),  'r-', 'LineWidth', 2, 'DisplayName', 'Hormone Iodine (H)');
title('Goiter Condition(300 days)');
xlabel('Time (days)');
ylabel('Deviation (IU/kg or g/kg)');
grid on;
legend('show');
```

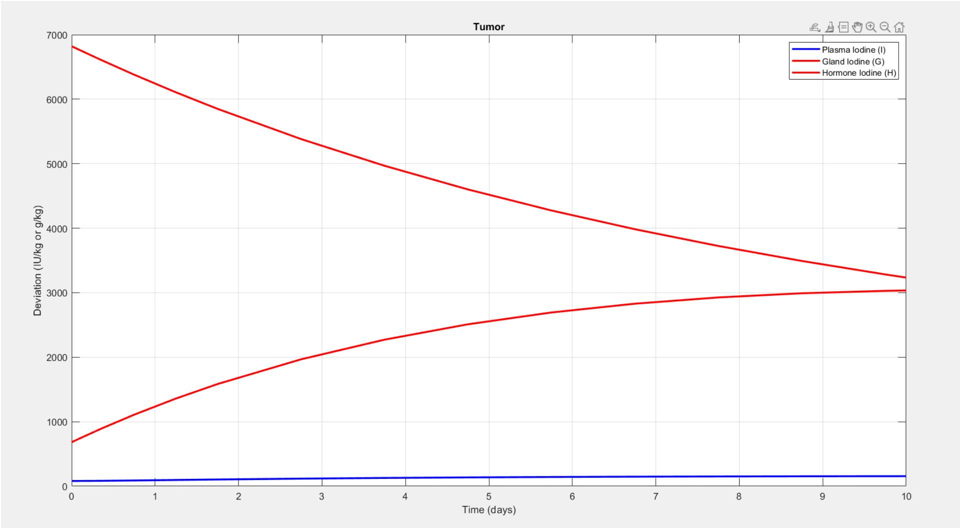


Causes of Tumor

Tumors grown in the Thyroid gland (thryoid nodules) cause excess production of thyroid hormornes. This means the conversion of glan iodine to hormornal iodine is higher, signified by increasing the k2 parameter in the Riggs model. Updating the k2 value from 0.01 to 0.1 will mimic this condition.

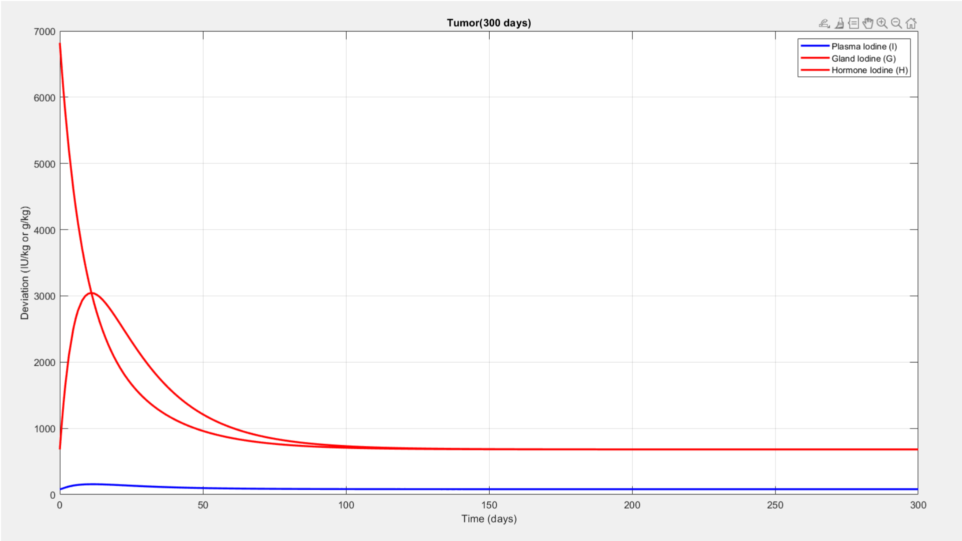
The code for the updated Riggs model is in riggs_tumor.m. The following code simulates it for 10 days.

```
tspan = [0, 10]; % Time span: 0 to 10 days
y0 = [81.2, 6821, 682]'; % Initial conditions: i(0) = 81.2, g(0)
    = 6821, H(0) = 682

% Solve ODE
[t, y] = ode23(@riggs_tumor, tspan, y0);


% Plot results in a single graph
figure;
plot(t, y(:,1), 'b-', 'LineWidth', 2, 'DisplayName', 'Plasma
    Iodine (I)');
hold on;
plot(t, y(:,2), 'r-', 'LineWidth', 2, 'DisplayName', 'Gland
    Iodine (G)');
hold on;
plot(t, y(:, 3),  'r-', 'LineWidth', 2, 'DisplayName', 'Hormone
    Iodine (H)');
title('Tumor');
xlabel('Time (days)');
ylabel('Deviation (IU/kg or g/kg)');
grid on;
legend('show');
```



The following code simulates it for 300 days.

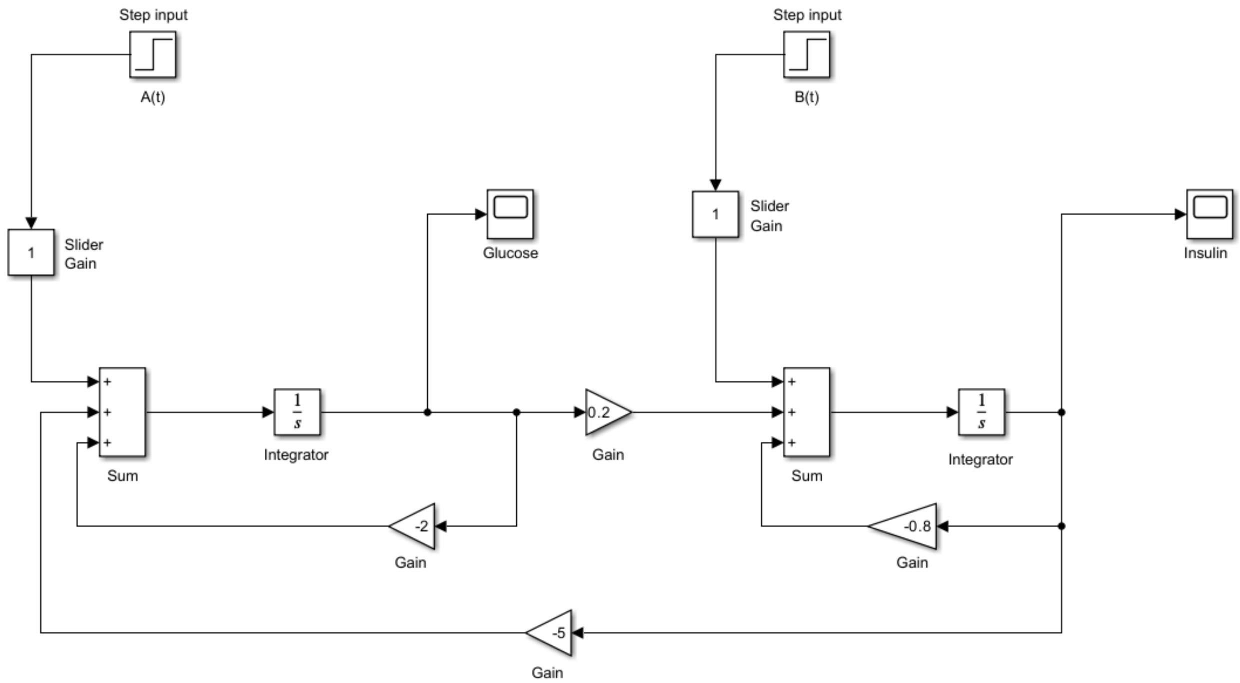
```

tspan = [0, 300]; % Time span: 0 to 300 days
y0 = [81.2, 6821, 682]'; % Initial conditions: i(0) = 81.2, g(0)
    = 6821, H(0) = 682

% Solve ODE
[t, y] = ode23(@riggs_tumor, tspan, y0);


% Plot results in a single graph
figure;
plot(t, y(:,1), 'b-', 'LineWidth', 2, 'DisplayName', 'Plasma
    Iodine (I)');
hold on;
plot(t, y(:,2), 'r-', 'LineWidth', 2, 'DisplayName', 'Gland
    Iodine (G)');
hold on;
plot(t, y(:, 3),  'r-', 'LineWidth', 2, 'DisplayName', 'Hormone
    Iodine (H)');
title('Tumor(300 days)');
xlabel('Time (days)');
ylabel('Deviation (IU/kg or g/kg)');
grid on;
legend('show');
```



2 Part 2

2.1 Question 1

2.1.1 Model 1 - Same system as in part 1



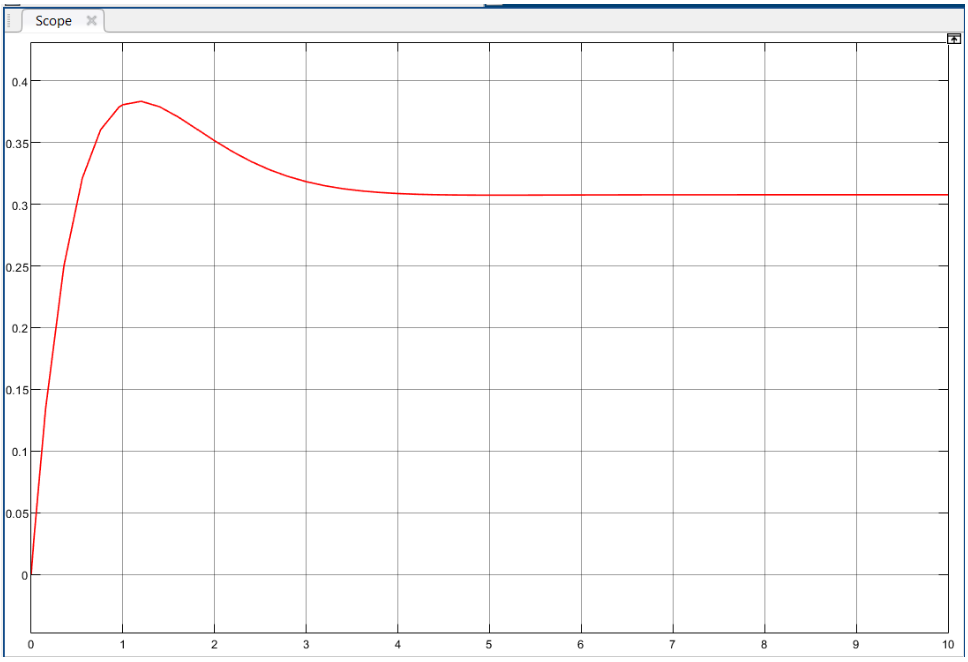


Figure 1: Glucose level

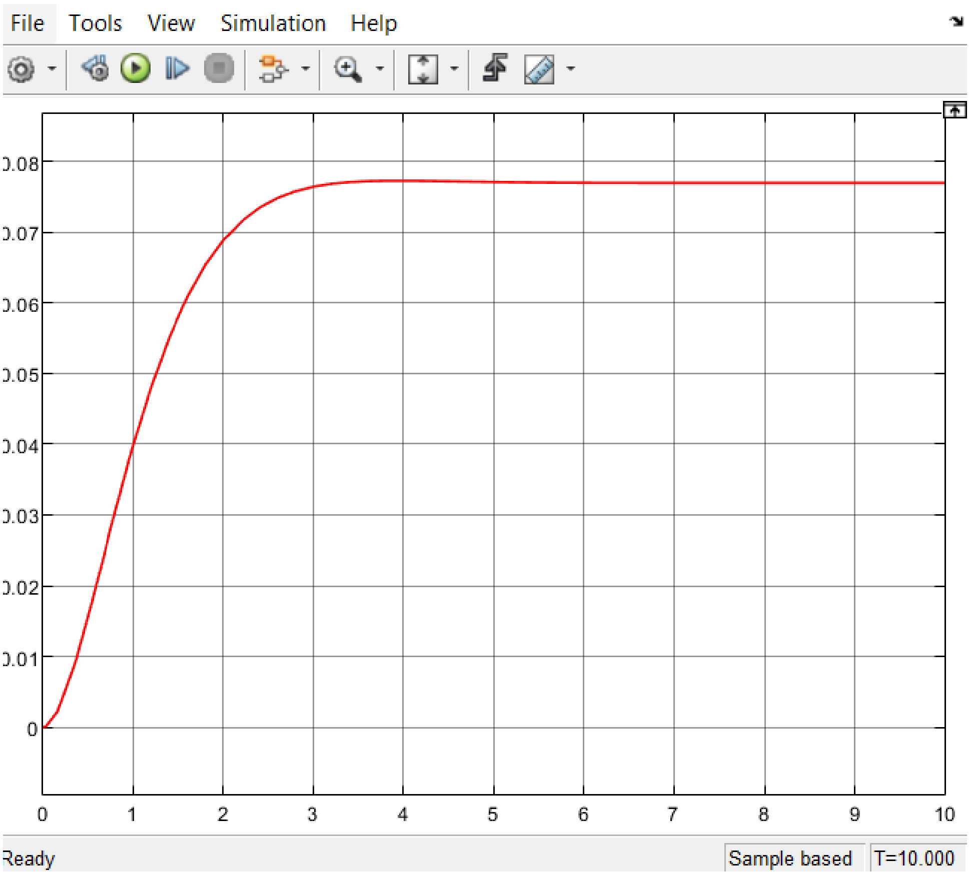
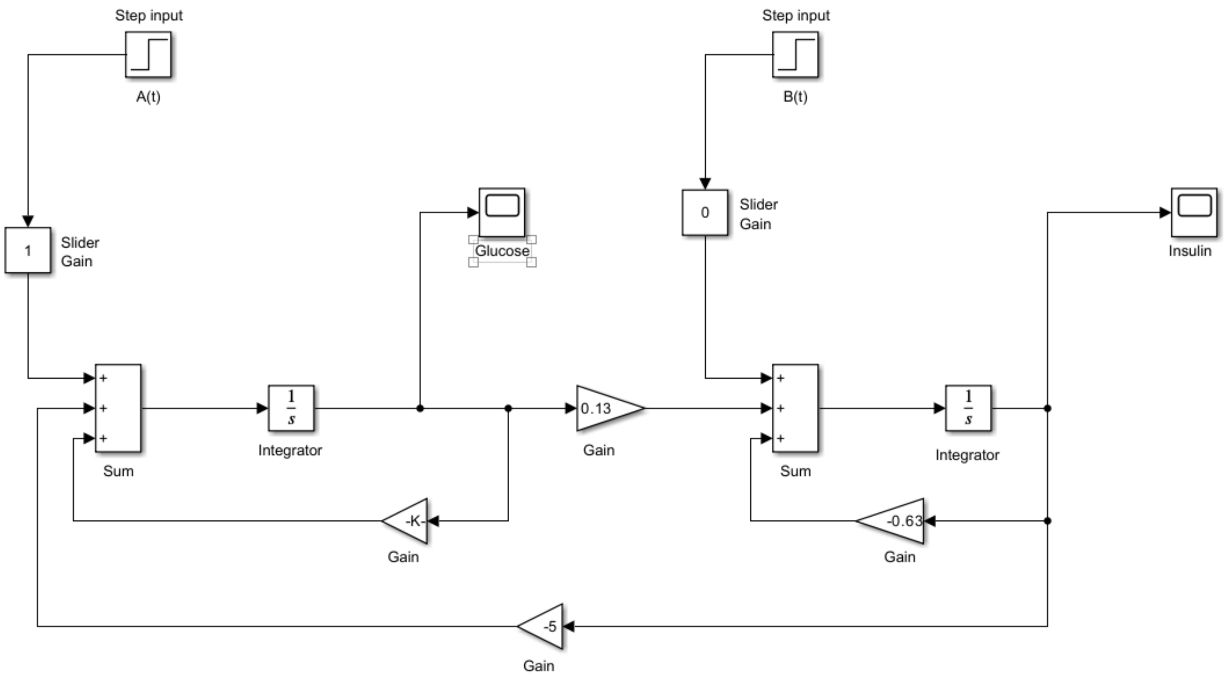


Figure 2: Insulin level

2.1.2 Model 2 - Model corresponding to different procedure in original paper



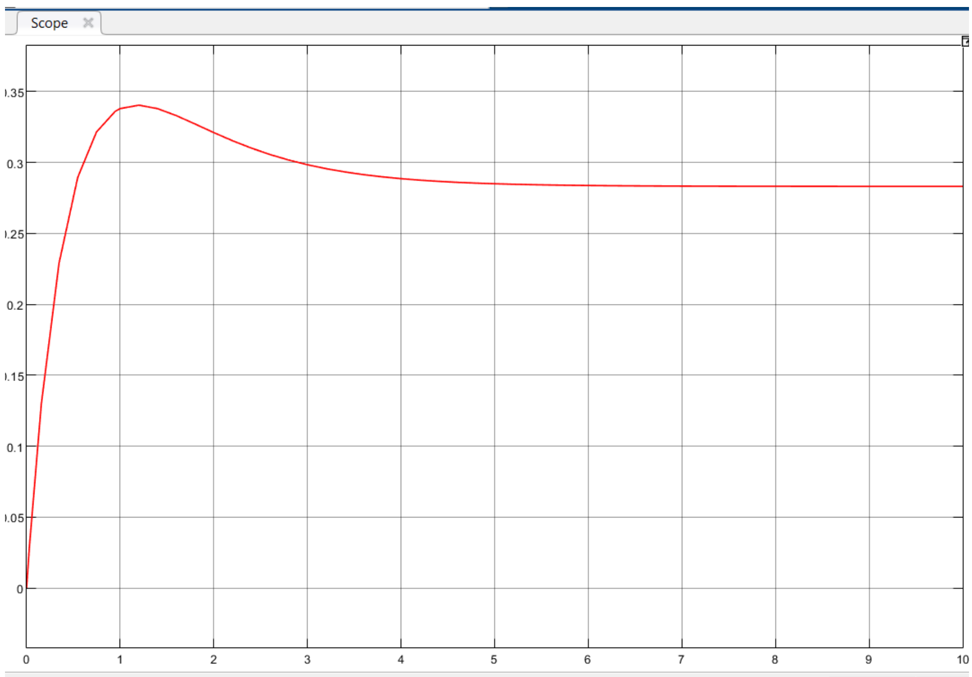


Figure 3: Glucose level

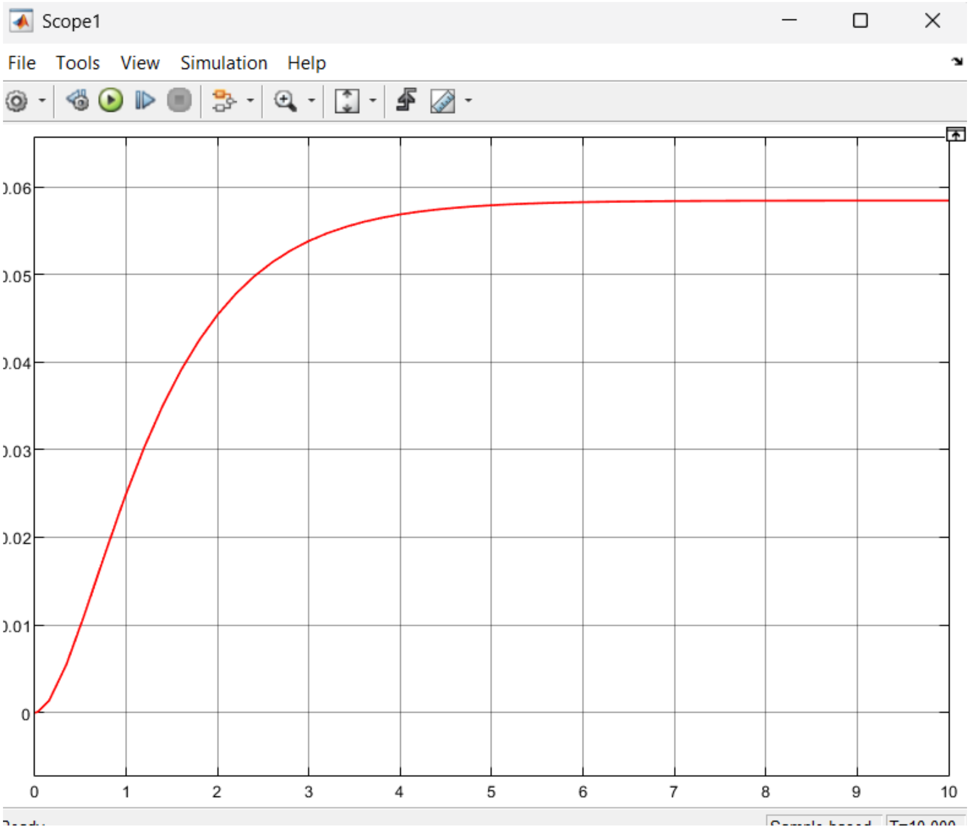
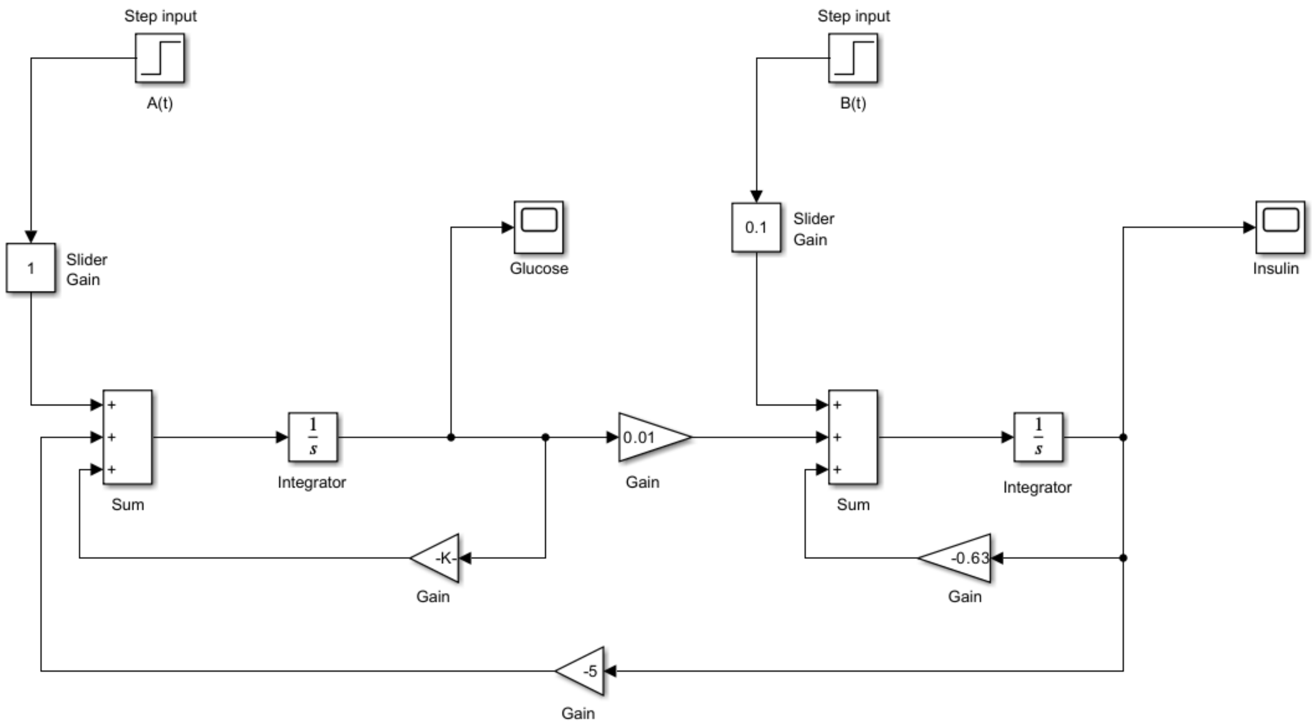


Figure 4: Insulin level

2.1.3 Model 3 - diabetic patient with B(t) = 0.1



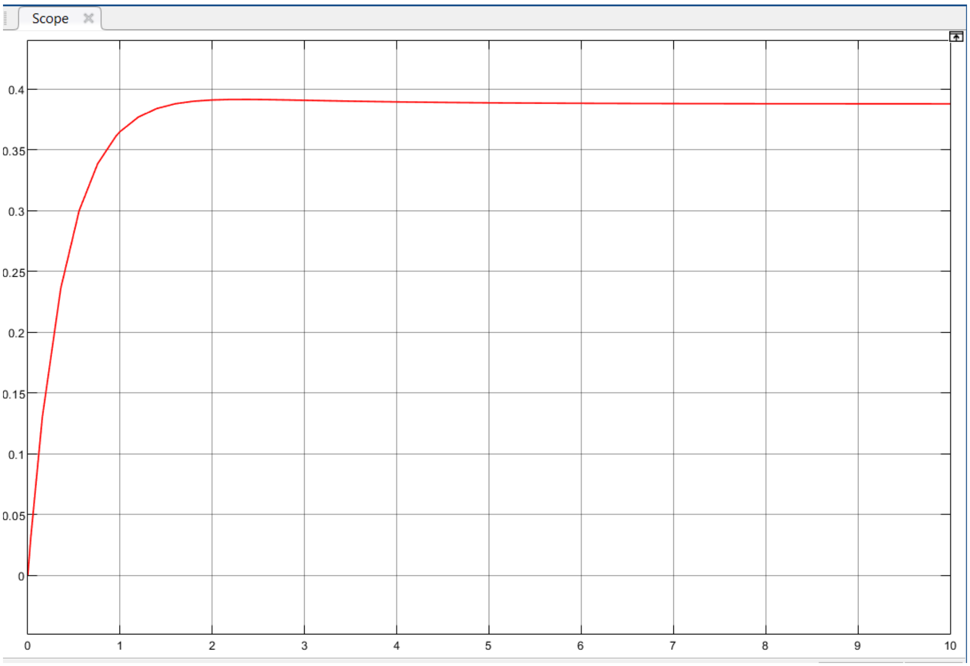


Figure 5: Glucose level

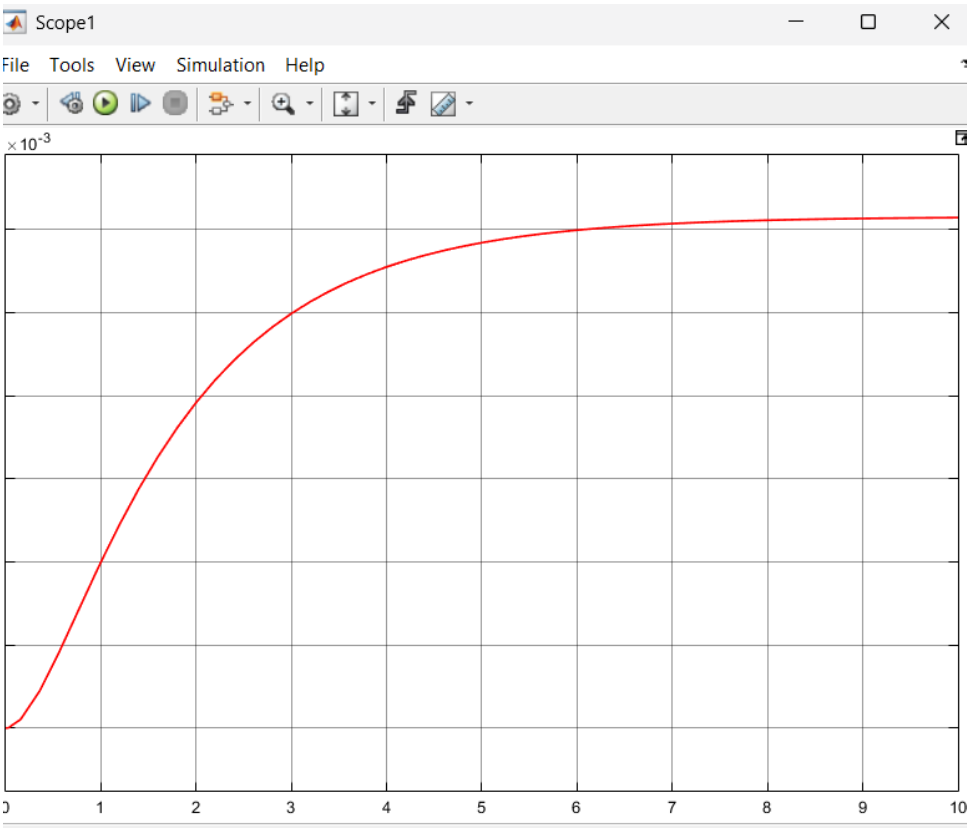
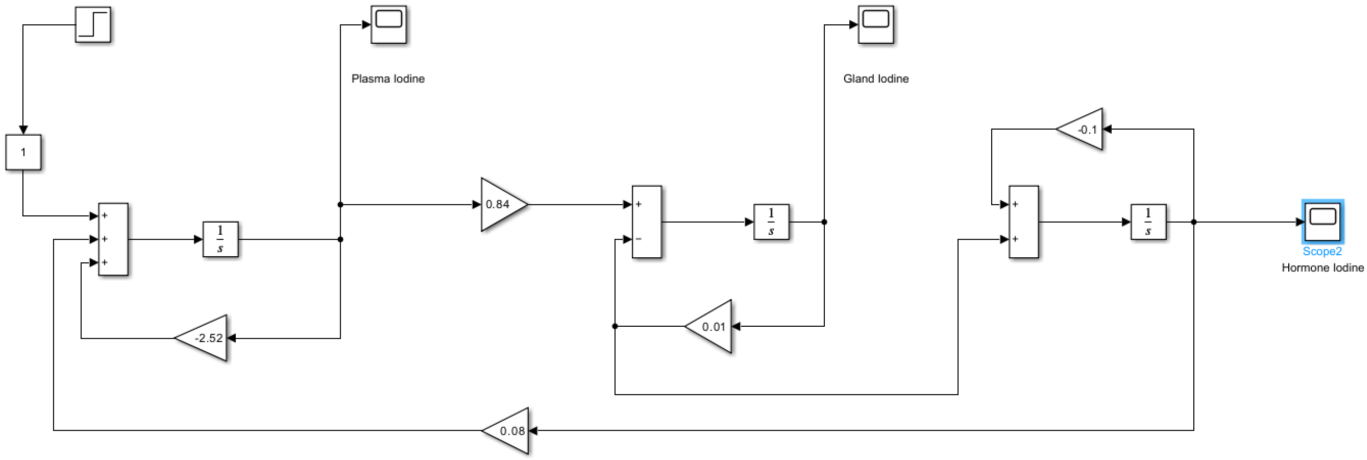


Figure 6: Insulin level

2.2 Question 2

Riggs Iodine Model



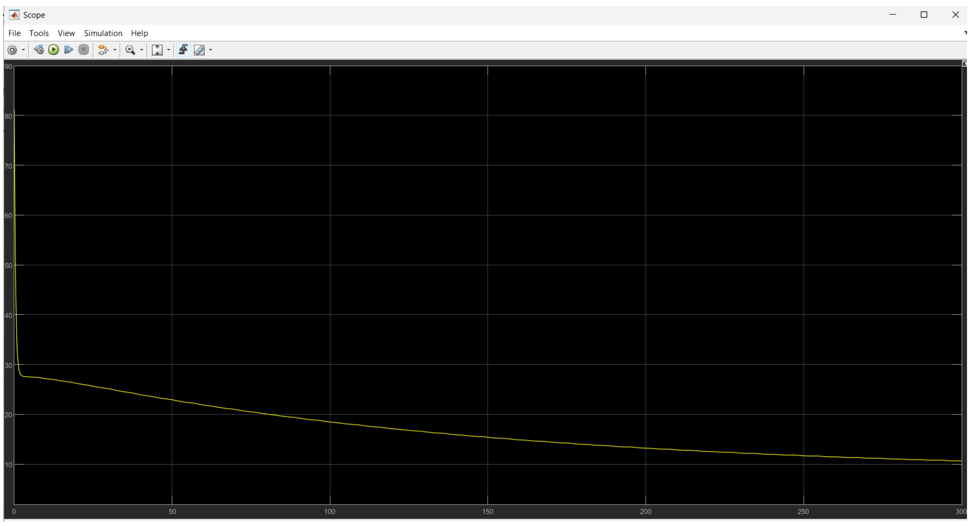


Figure 7: Plasma Iodine

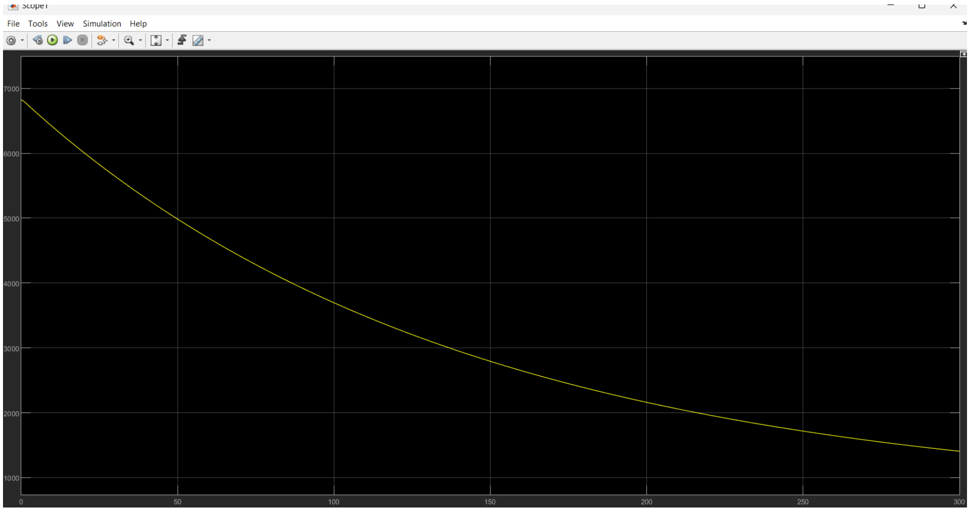


Figure 8: Gland Iodine

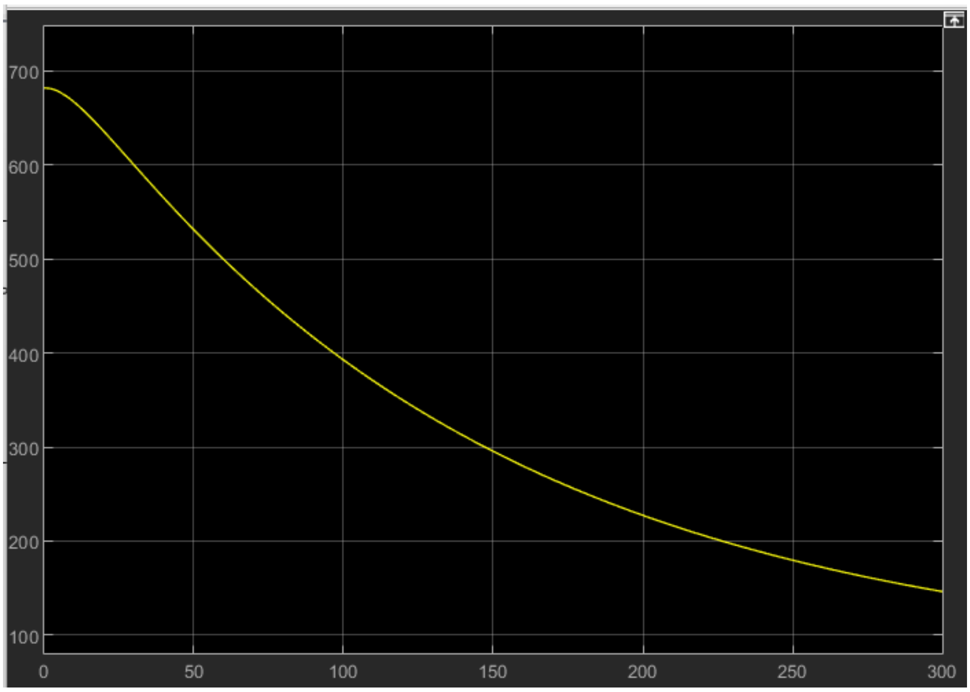


Figure 9: Hormone Iodine

3 Part 3

3.1 Question 1

Part 3 - Question 1

$$\frac{dg}{dt} = -k_4 g - k_6 i + A(t) \quad - (i)$$

$$\frac{di}{dt} = k_3 g - k_1 i + B(t) \quad - (ii)$$

$$\frac{d^2g}{dt^2} = -k_4 \frac{dg}{dt} - k_6 \frac{di}{dt} + \frac{d}{dt} A(t)$$

Let $A(t) = a.u(t)$ then from (i)

$$\frac{d^2g}{dt^2} = -k_4 \frac{dg}{dt} - k_6 \frac{di}{dt} + a.\frac{du(t)}{dt}$$

Let $B(t) = 0$ and substitute $\frac{di}{dt}$ from (ii)

$$\frac{d^2g}{dt^2} = -k_4 \frac{dg}{dt} - k_6 \left[k_3 g - k_1 i + 0\right] + a.\frac{du(t)}{dt}$$

$$= -k_4 \frac{dg}{dt} - k_3 k_6 g + k_1 k_6 i + a\frac{du(t)}{dt}$$

substitute for $k_6 i$ from (i)

$$\frac{d^2g}{dt^2} = -k_4 \frac{dg}{dt} - k_3 k_6 g + k_1 \left[-k_4 g + a.u(t) - \frac{dg}{dt}\right] + a.\frac{du(t)}{dt}$$

$$= -(k_4 + k_1)\frac{dg}{dt} - (k_3 k_6 + k_1 k_4) g - k_1 a u(t) + a\frac{du(t)}{dt}$$

$$\frac{d^2g}{dt^2} + (k_1 + k_4)\frac{dg}{dt} + (k_1 k_4 + k_3 k_6) g = k_1 a + a\frac{du(t)}{dt}$$

Subtitute typical values

$k_1 = 0.8\ h^{-1}$ $\quad k_3 = 0.2$ lu/h/g $\quad K_4 = 2\ h^{-1}$ $\quad K_6 = 5$ g/h/lu

$a = 1$ g/l/h $\quad \frac{du(t)}{dt} = 0$ for $t > 0$

$$\frac{d^2g}{dt^2} + (0.8+2)\frac{dg}{dt} + (0.8\times 2 + 0.2\times 5)g = 0.8\times 1 + 1\times 0$$

$$\frac{d^2g}{dt^2} + 2.8\frac{dg}{dt} + 2.6g = 0.8$$

$$g(t) = g_c(t) + g_p(t)$$

complementary soln:

$$m^2 + 2.8m + 2.6 = 0 \Rightarrow m = -1.4 \pm 0.8i$$

$$g_c(t) = c_1 e^{(-1.4+0.8i)t} + c_2 e^{(-1.4-0.8i)t}$$

$$= e^{-1.4t}\left[A\cos(0.8t) + B\sin(0.8t)\right]$$

Particular soln:

$$g_p(t) = k$$

$$0 + 0 + 2.6k = 0.8 \Rightarrow k = 4/13$$

$$g(t) = e^{-1.4t}\left[A\cos(0.8t) + B\sin(0.8t)\right] + 4/13$$

using initial conditions

$$g(0) = 0 \Rightarrow A + 4/13 = 0 \Rightarrow A = -4/13$$

$$g'(0) = 1 \Rightarrow 0.8M - 1.4N = 1 \Rightarrow N = \frac{37}{52}$$

$$g(t) = e^{-1.4t}\left[-\tfrac{4}{13}\cos(0.8t) + \tfrac{37}{52}\sin(0.8t)\right] + \tfrac{4}{13}u(t)$$

Substituting g in (i)

$$\frac{dg}{dt} = -2g - 5i + 1$$

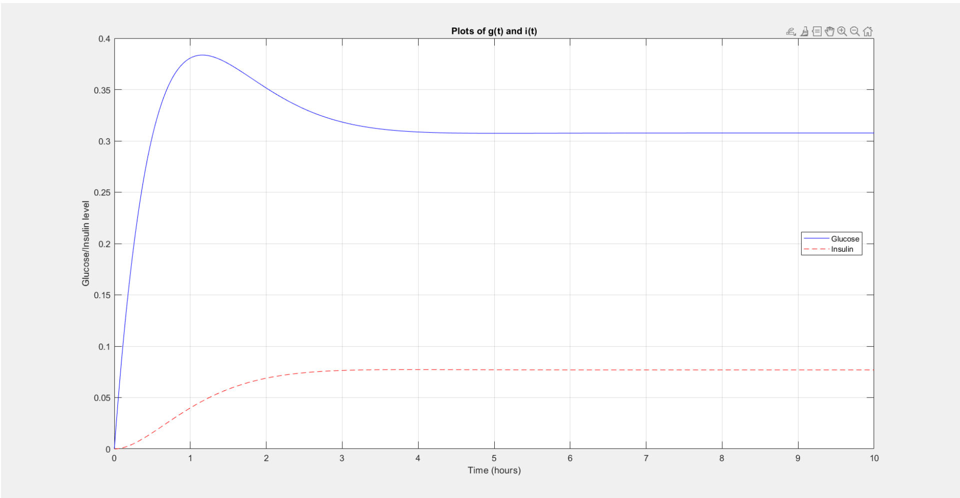
$$5i = -2g - \frac{dg}{dt} + 1$$

$$5i = -2 \cdot e^{-1.4t}\left[-\frac{4}{13}\cos(0.8t) + \frac{37}{52}\sin(0.8t) + \frac{4}{13}\right] - \frac{dg}{dt} + 1$$

$$\frac{dg}{dt} = e^{-1.4t}\left[\frac{4}{13}\times\frac{8}{10}\sin(0.8t) + \frac{37}{52}\times\frac{8}{10}\cos(0.8t)\right] - 1.4e^{-1.4t}\left[-\frac{4}{13}\cos(0.8t) + \frac{37}{52}\sin(0.8t) + \frac{4}{13}\right]$$

$$i(t) = e^{-1.4t}\left[-\frac{1}{13}\cos(0.8t) - \frac{7}{52}\sin(0.8t)\right] + \frac{1}{13}u(t)$$

```
t = 0:0.01:10;

% Analytical solutions
g_t = exp(-1.4.*t).*((-4/13)*cos(0.8.*t) + (37/52)*sin(0.8.*t)) + 4/13;
i_t = exp(-1.4.*t).*(-(1/13)*cos(0.8.*t) - (7/52)*sin(0.8.*t)) + 1/13;

% Plot solutions
figure;
plot(t, g_t, 'b-', t, i_t, 'r--');  % Optional: add line styles for clarity
hold on;
grid on;
legend('Glucose', 'Insulin', 'Location', 'best');
xlabel('Time (hours)');
ylabel('Glucose/Insulin level');
title('Plots of g(t) and i(t)');
```



Insulin, secreted by pancreatic beta cells, converts glucose into glycogen, lowering blood glucose levels. A slight delay occurs between the peak of glucose and the rise in insulin, as beta cells require time to sense elevated glucose levels and respond by releasing insulin. At steady state, both glucose and insulin levels stabilize.

3.2 Question 2

Question 2

From Bolie's model

$$\frac{dG}{dt} = k_5 + A(t) - k_4 G - k_6 I + k_{10} G_n(t)$$

$$\frac{dI}{dt} = k_2 + k_3 G + B(t) - k_1 I$$

$$\frac{dG_n}{dt} = k_5 + C(t) + k_9 G - k_7 G_n$$

$G \rightarrow$ glucose

$G_n \rightarrow$ Glucogon

$I \rightarrow$ Insulin

At equillibrium

$$\frac{dG}{dt} = \frac{dI}{dt} = \frac{dG_n}{dt} = 0$$

$$\frac{dG}{dt} = 0 \Rightarrow \quad k_3 = k_4 G_0 + k_6 I_0 - k_{10} G_{n_0}$$

$$\frac{dI}{dt} = 0 \Rightarrow \quad k_2 = k_1 I_0 - k_3 G_0$$

$$\frac{dG_n}{dt} = 0 \Rightarrow \quad k_8 = k_7 G_{n_0} - k_4 G_0$$

$A(t) = a.u(t)$, $B(t) = 0$, $C(t) = 0$;

$$\frac{dg}{dt} = -k_4 g - k_6 i - k_{10} g_n + a.u(t) \qquad [g = G - G_0]$$

$$\frac{di}{dt} = k_3 g - k_1 i \qquad [i = I - I_0]$$

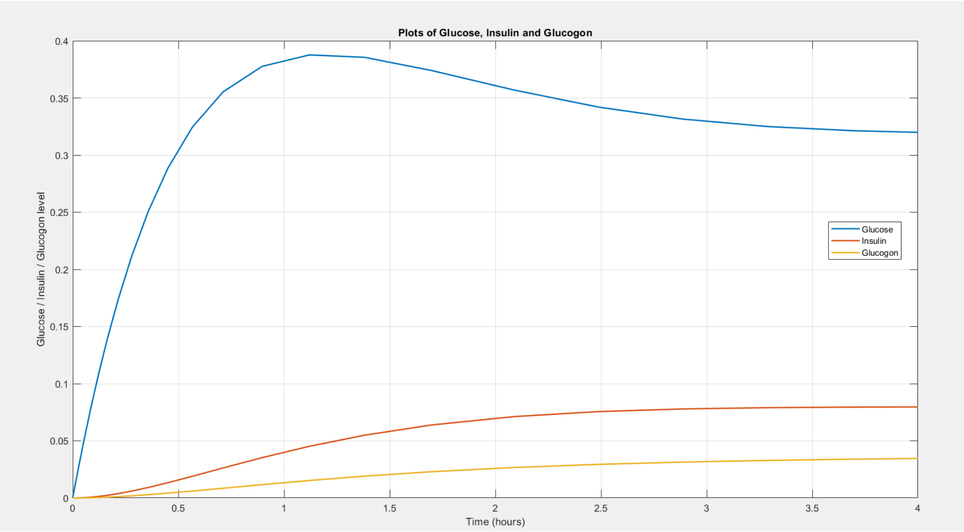
$$\frac{dg_n}{dt} = k_9 g - k_7 i \qquad [g_n = G_n - G_{n_0}]$$

$$\begin{pmatrix} \frac{dg}{dt} \\ \frac{di}{dt} \\ \frac{dg_n}{dt} \end{pmatrix} = \begin{pmatrix} -k_4 & -k_6 & k_{10} \\ k_3 & -k_1 & 0 \\ k_9 & 0 & -k_8 \end{pmatrix} \begin{pmatrix} g \\ i \\ g_n \end{pmatrix} + \begin{pmatrix} a \cdot u(t) \\ 0 \\ 0 \end{pmatrix}$$

Bolies Model

```
function yp = bolies_model(t, y)
    % Typical values for transfer rates
    k1 = 0.8; k3 = 0.2; k4 = 2; k6 = 5; a = 1;

    % Parameters for Glycogen
    k7 = 0.5; k9 = 0.06; k10 = 1;

    yp = [-k4 -k6 k10;
           k3 -k1   0;
           k9   0 -k7] * y + [a 0 0]';
end
```

```
% Solve the system
[t, y] = ode23('bolies_model', [0, 4], [0, 0, 0]);

% Plot the results
figure;
plot(t, y, 'LineWidth', 1.5);
grid on;

% Add legend and labels
legend('Glucose', 'Insulin', 'Glucogon', 'Location', 'best');
xlabel('Time (hours)');
ylabel('Glucose / Insulin / Glucogon level');

% Add title
title('Plots of Glucose, Insulin and Glucogon');
```



- The graph illustrates the dynamic relationship between **insulin** and **glucagon** in regulating blood glucose levels.
- **After a significant glucose intake**, insulin levels rise sharply, peaking shortly after glucose levels. This occurs as the pancreas releases insulin to manage the glucose surge.

- The slight **delay in insulin peak** is due to the time required for pancreatic cells to detect elevated glucose levels and respond accordingly.
- When **glucagon is present**, the glucose level at the endpoint remains higher compared to the scenario without glucagon.
- This is because glucagon counteracts insulin by signaling the liver to release stored glucose, thereby **maintaining elevated blood sugar levels**.
- Although insulin secretion increases in the presence of glucagon, the **hormonal balance** results in a higher final glucose concentration.
- This behavior **validates Bolie's model**, demonstrating the interplay between insulin and glucagon in glucose homeostasis.