Question 2

From Bolie's model

$$\frac{dG}{dt} = k_5 + A(t) - k_4 G - k_6 I + k_{10} G_n(t)$$

$$\frac{dI}{dt} = k_2 + k_3 G + B(t) - k_1 I$$

$$\frac{dG_n}{dt} = k_8 + C(t) + k_9 G - k_7 G_n$$

$G \rightarrow$ glucose

$G_n \rightarrow$ Glucogon

$I \rightarrow$ Insulin

At equillibrium

$$\frac{dG}{dt} = \frac{dI}{dt} = \frac{dG_n}{dt} = 0$$

$$\frac{dG}{dt} = 0 \Rightarrow \quad k_5 = k_4 G_0 + k_6 I_0 - k_{10} G_{n0}$$

$$\frac{dI}{dt} = 0 \Rightarrow \quad k_2 = k_1 I_0 - k_3 G_0$$

$$\frac{dG_n}{dt} = 0 \Rightarrow \quad k_8 = k_7 G_{n0} - k_9 G_0$$

$A(t) = a.u(t)$, $B(t) = 0$, $C(t) = 0$;

$$\frac{dg}{dt} = -k_4 g - k_6 i - k_{10} g_n + a.u(t) \qquad [g = G - G_0]$$

$$\frac{di}{dt} = k_3 g - k_1 i \qquad [i = I - I_0]$$

$$\frac{dg_n}{dt} = k_9 g - k_7 i \qquad [g_n = G_n - G_{n0}]$$

$$\begin{pmatrix} \frac{dg}{dt} \\ \frac{di}{dt} \\ \frac{dg_n}{dt} \end{pmatrix} = \begin{pmatrix} -k_4 & -k_6 & k_{10} \\ k_3 & -k_1 & 0 \\ k_9 & 0 & -k_8 \end{pmatrix} \begin{pmatrix} g \\ i \\ g_n \end{pmatrix} + \begin{pmatrix} a \cdot u(t) \\ 0 \\ 0 \end{pmatrix}$$