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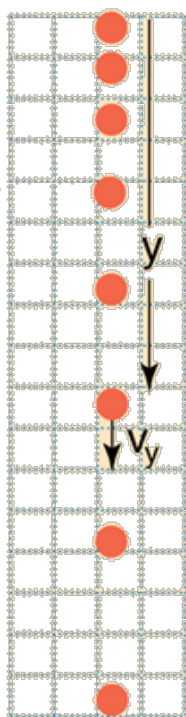
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Freefall

Images of an object in freefall at constant time intervals. Note that the distance traveled in each successive interval is larger.

$g = 9.8 \text{ m/s}^2$
so that the velocity increases 9.8 m/s each second.



In the absence of frictional drag, an object near the surface of the earth will fall with the constant acceleration of gravity g . Position and speed at any time can be calculated from the [motion equations](#).

Illustrated here is the situation where an object is released from rest. It's position and speed can be predicted for any time after that. Since all the quantities are directed downward, that direction is chosen as the positive direction in this case.

$$v_y = gt \quad \text{Taking } g = 9.8 \text{ m/s}^2 = 32.15 \text{ ft/s}^2$$

$$y = \frac{1}{2} g t^2$$

At time $t =$ s after being

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dropped,

the speed is $v_y =$ m/s =

ft/s ,

The distance from the starting point will be

$y =$ m =

ft

Enter data in any box and click outside the box.

Note that you can enter a distance (height) and click outside the box to calculate the freefall time and impact velocity in the absence of air friction. But the calculation assumes that the gravity acceleration is the surface value $g = 9.8 \text{ m/s}^2$, so the height is great enough for gravity to have changed significantly the results will be incorrect.

[Free fall with air friction](#)

[Free fall from great height](#)

[Free fall experiment with spark timer](#)

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Peak at

m at

t=

s

Vertical Trajectory

Vertical motion under the influence of gravity can be described by the basic motion equations. Given the constant acceleration of gravity g , the position and speed at any time can be calculated from the motion equations:

$$v_y = v_{0y} - gt$$

$$y = v_{0y} t - \frac{1}{2} g t^2$$

$$\text{Taking } g = 9.8 \text{ m/s}^2 \\ = 32.15 \text{ ft/s}^2$$

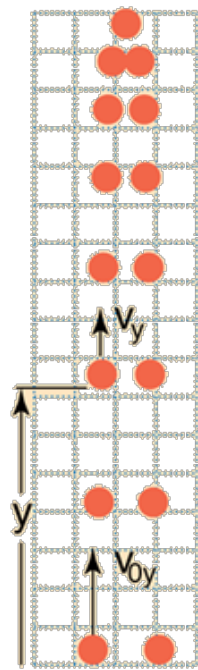
You may enter values for launch velocity and time in the boxes below and click outside the box to perform the calculation.

For launch speed $v_{0y} =$ m/s =

ft/s

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and time $t =$ s ,

The values below are output values; those boxes will not accept input for calculation. The velocity will be

$v_y =$ m/s = ft/s

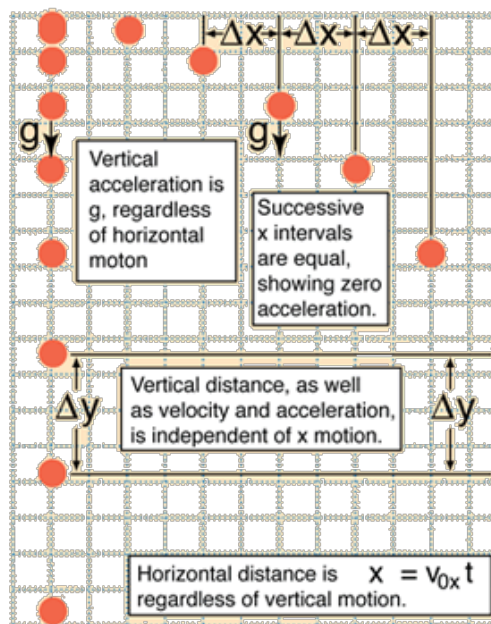
and the height will be $y =$ m =

ft

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Trajectories can be described by the general **motion equations** for constant acceleration. The key idea is that the horizontal and vertical motions can be separated. The motion equations obtained constitute a complete description of the motion, given the initial conditions.

Horizontal Motion →

$$\begin{aligned} a_x &= 0 \\ v_x &= v_{0x} \\ x &= v_{0x} t \end{aligned}$$

Vertical Motion ↓

$$\begin{aligned} a_y &= -g \\ v_y &= v_{0y} - gt \\ y &= v_{0y} t - \frac{1}{2}gt^2 \end{aligned}$$

↑ Upward chosen as positive direction, so the y values will be negative.

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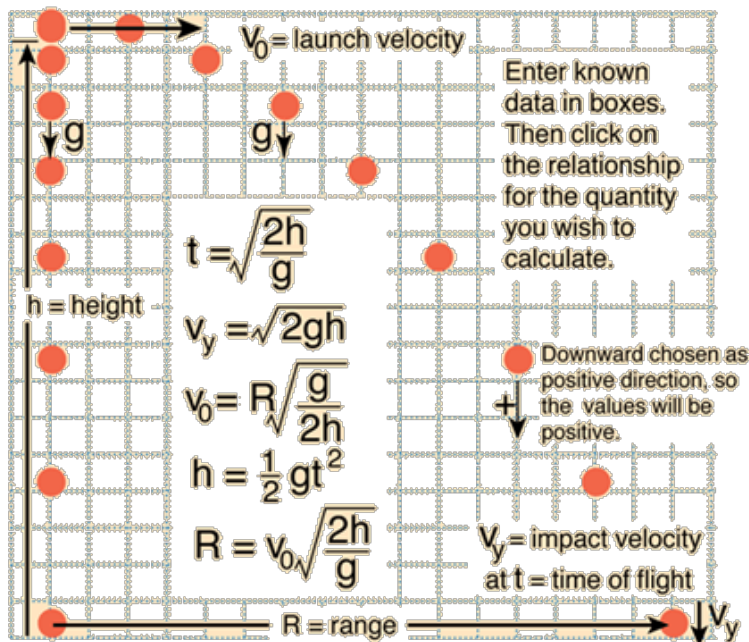
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Horizontal Launch

All the parameters of a horizontal launch can be calculated with the [motion equations](#), assuming a downward acceleration of gravity of 9.8 m/s^2 .



Time of flight
 $t =$

s

Vertical impact
velocity

$v_y =$

m/s

Launch velocity

$v_0 =$

m/s

Height of launch

$h =$

m

Horizontal range

$R =$

m

Calculation is initiated by clicking on the formula in the illustration for the quantity you wish to calculate.

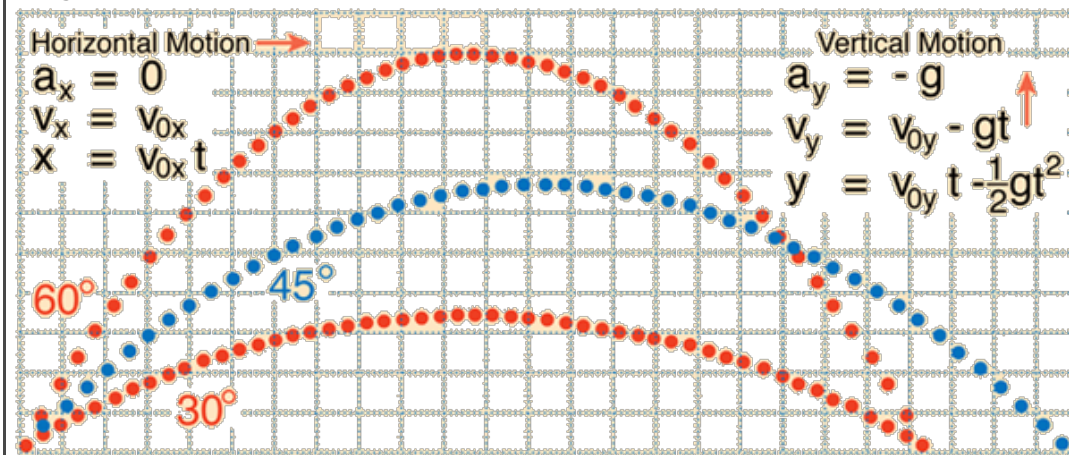
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General Ballistic Trajectory

The motion of an object under the influence of gravity is determined completely by the acceleration of gravity, its launch speed, and launch angle provided air friction is negligible. The horizontal and vertical motions may be separated and described by the general [motion equations](#) for constant acceleration. The initial vector components of the velocity are used in the equations. The diagram shows trajectories with the same launch speed but different launch angles. Note that the 60 and 30 degree trajectories have the same range, as do any pair of launches at complementary angles. The launch at 45 degrees gives the maximum range.



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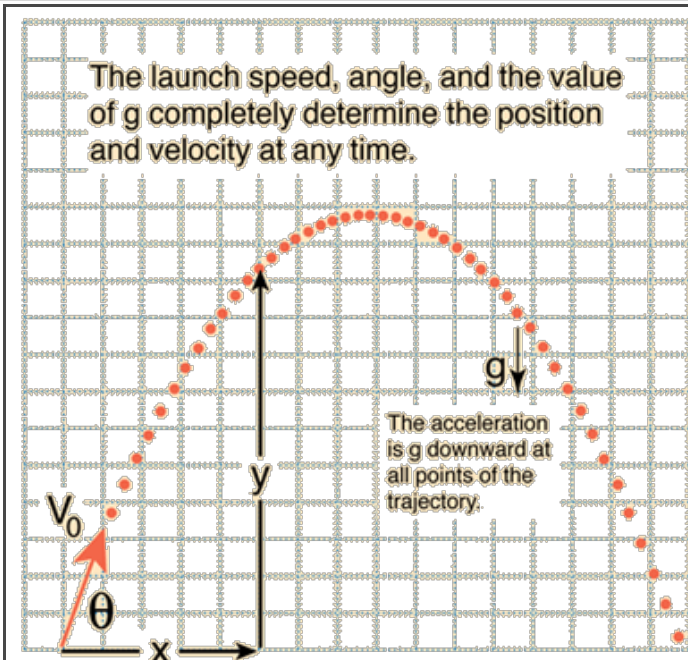
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At time $t =$

sec:

Horizontal Motion →

$$a_x = 0$$

$$v_x = v_{0x}$$

Horizontal velocity

$v_x =$ m/s.

$$x = v_{0x} t$$

Horizontal distance

$x =$ m.

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For launch velocity $v_0 =$ m/s,
 launch angle $\theta =$ degrees:

Vertical Motion $\uparrow +$

Upward chosen as positive
 direction for y motion.

$$a_y = -g = -9.8 \text{ m/s}^2$$

$$v_y = v_{0y} - gt$$

Vertical velocity

$$v_y =$$
 m/s.

$$y = v_{0y} t - \frac{1}{2} gt^2$$

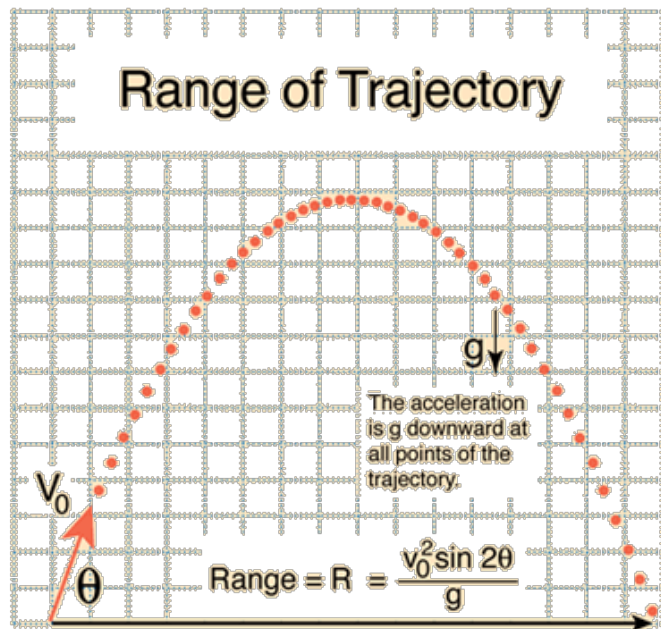
Vertical position

$$y =$$
 m.

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The basic **motion equation**

$$x = v_{0x} t$$

can be used to find the range.
 By symmetry, the total **time of flight** is equal to twice the time at the peak:

$$t_{\text{range}} = 2t_{\text{peak}} = \frac{2v_{0y}}{g}$$

This gives:

$$R = \frac{2v_{0x} v_{0y}}{g}$$

$$R = \frac{2v_0^2 \sin \theta \cos \theta}{g}$$

$$R = \frac{v_0^2 \sin 2\theta}{g}$$

using the **trig identity**:
 $\sin 2\theta = 2 \sin \theta \cos \theta$.

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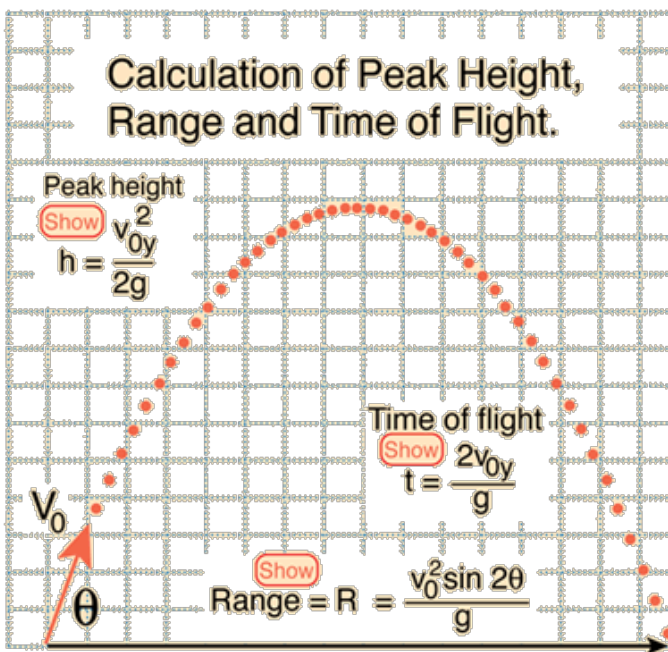
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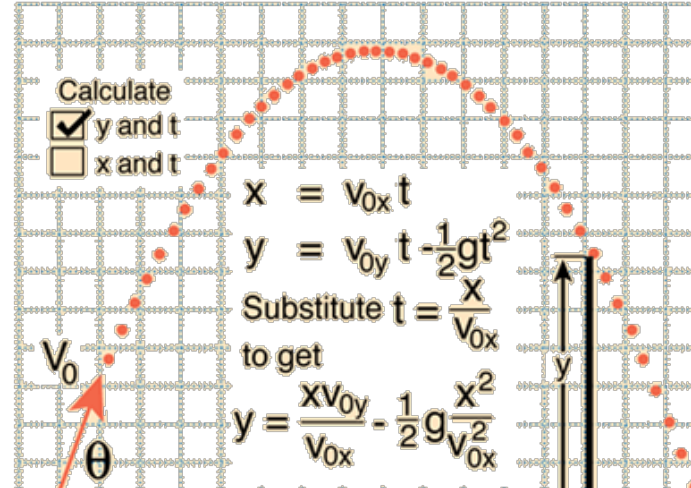
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<h3>Height of Trajectory</h3> <p>Vertical component of velocity is zero at the peak.</p> <p>$v_y = 0$</p> <p>$v_{0x} = v_0 \cos \theta$</p> <p>$v_{0y} = v_0 \sin \theta$</p> <p>$h = \frac{v_{0y}^2}{2g} = \frac{v_0^2 \sin^2 \theta}{2g}$</p> <p>Average vertical velocity is half the vertical component of the launch velocity.</p>	<p>The basic motion equation</p> $y = \bar{v}_y t$ <p>can be used to find the height. The average vertical speed is:</p> $\bar{v}_y = \frac{v_{0y} + 0}{2} = \frac{v_{0y}}{2}$ <p>The time at the peak is obtained by solving for the time at zero vertical speed:</p> $0 = v_{0y} - gt_{\text{peak}}$ <p>This gives:</p> $t_{\text{peak}} = \frac{v_{0y}}{g}$ <p>and substituting:</p> $h = y_{\text{peak}} = \frac{v_{0y}^2}{2g}$ <p>Calculation</p>	<p>Index</p> <p>Trajectory concepts</p>
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<h3>Time of Flight</h3> <p>The equation for t has two solutions since there are two times when the projectile passes through height h.</p>	<p>The basic motion equation</p> $h = v_{0y} t - \frac{1}{2} g t^2$ <p>can be used to find the time of flight at height h, giving:*</p> $t = \frac{v_{0y}}{g} \pm \sqrt{\frac{v_{0y}^2}{g^2} - \frac{2h}{g}}$ <p>Note that there is no real solution if</p> $\frac{2h}{g} > \frac{v_{0y}^2}{g^2} \text{ or } h > \frac{v_{0y}^2}{2g}$ <p>since such values of h are above the peak of the trajectory. For the value $h=0$:</p> $t = 0 \text{ and } t = \frac{2v_{0y}}{g}$ <p>Calculation</p> <p>*quadratic formula</p>	<p>Index</p> <p>Trajectory concepts</p>
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<p>Calculation of Peak Height, Range and Time of Flight.</p>  <p>Peak height <input type="button" value="Show"/> $h = \frac{v_{0y}^2}{2g}$</p> <p>Time of flight <input type="button" value="Show"/> $t = \frac{2v_{0y}}{g}$</p> <p>Range = <input type="button" value="Show"/> $R = \frac{v_0^2 \sin 2\theta}{g}$</p>	<p>For launch velocity $v_0 =$ <input type="text"/> m/s, launch angle $\theta =$ <input type="text"/> degrees, The horizontal range is $R =$ <input type="text"/> m. The total time of flight is $t =$ <input type="text"/> s. The peak height is $h =$ <input type="text"/> m.</p>	<p>Index</p> <p>Trajectory concepts</p>
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<h2>Will it clear the fence?</h2>		
<p>The basic motion equations can be solved simultaneously to express y in terms of x.</p>		
 <p>Calculate <input checked="" type="checkbox"/> y and t <input type="checkbox"/> x and t</p> <p>$x = v_{0x} t$ $y = v_{0y} t - \frac{1}{2} g t^2$ Substitute $t = \frac{x}{v_{0x}}$ to get $y = \frac{x v_{0y}}{v_{0x}} - \frac{1}{2} g \frac{x^2}{v_{0x}^2}$</p>	<p>For launch velocity $v_0 =$ <input type="text"/> m/s $=$ <input type="text"/> ft/s, launch angle $\theta =$ <input type="text"/> degrees, and horizontal range $x =$ <input type="text"/></p>	<p>Index</p> <p>Trajectory concepts</p>

m =

 ft,

the calculated
height is

y =

m =

 ft.

The time of flight is

t =

s.

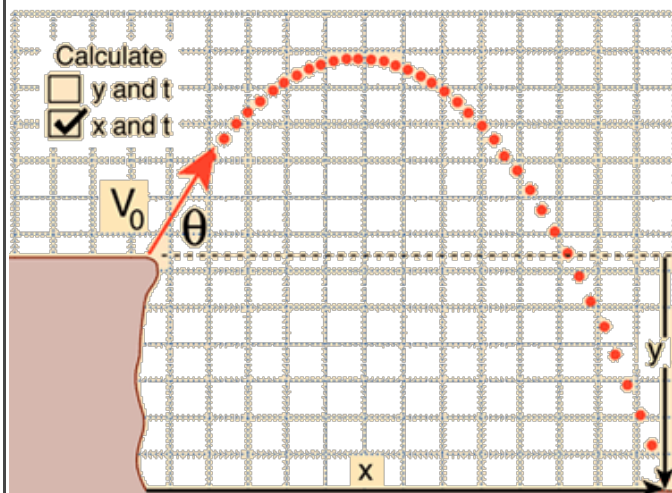
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Where will it land?

The basic motion equations give the position components x and y in terms of the time. Solving for the horizontal distance in terms of the height y is useful for calculating ranges in situations where the launch point is not at the same level as the landing point.



$$x = v_{0x} t$$

$$y = v_{0y} t - \frac{1}{2} g t^2$$

Using the **quadratic formula** to solve for t gives two values of time for a given value of y:

$$t = \frac{v_{0y}}{g} \pm \sqrt{\frac{v_{0y}^2}{g^2} - \frac{2y}{g}}$$

Substitution of the two time values gives the two values of x corresponding to a given height y.

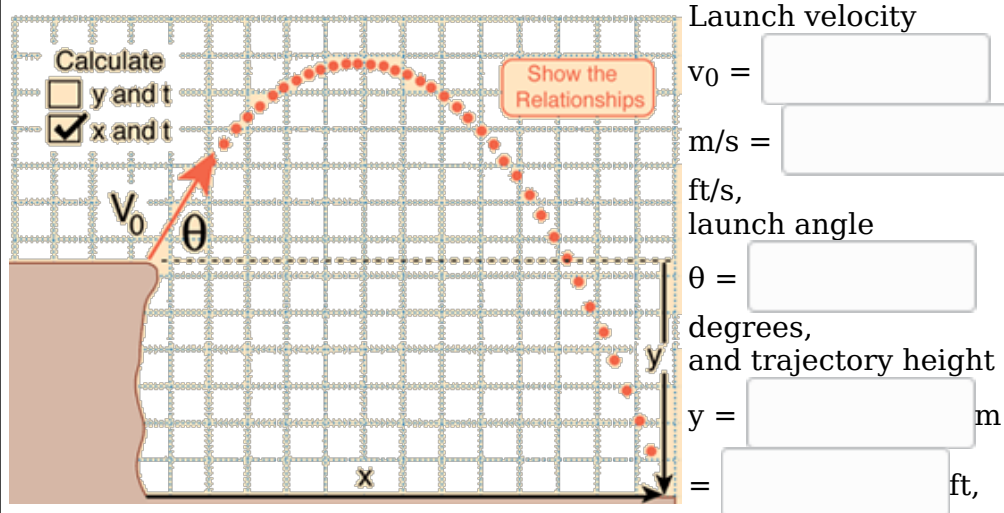
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Where will it land?

The basic motion equations give the position components x and y in terms of the time. Solving for the horizontal distance in terms of the height y is useful for calculating ranges in situations where the launch point is not at the same level as the landing point.



The two calculated times are

$t_1 =$ s and

$t_2 =$ s.

The corresponding ranges are

$x_1 =$ m =

ft

and

$x_2 =$ m =

ft.

Note that the value y in the illustration is downward and it is presumed that upward is positive. To reproduce the scenario in the diagram, the input value of y should be negative.

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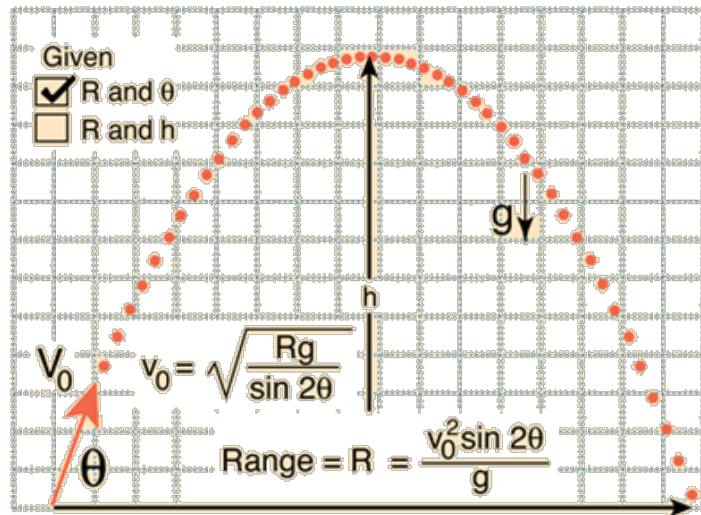
Launch Velocity

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The launch velocity of a projectile can be calculated from the range if

the angle of launch is known. It can also be calculated if the maximum height and range are known, because the angle can be determined.



From the [range relationship](#), the launch velocity can be calculated. For range

R = m =

ft,

and launch angle

$\theta =$ degrees,

the launch velocity is

$v_0 =$ m/s

= ft/s.

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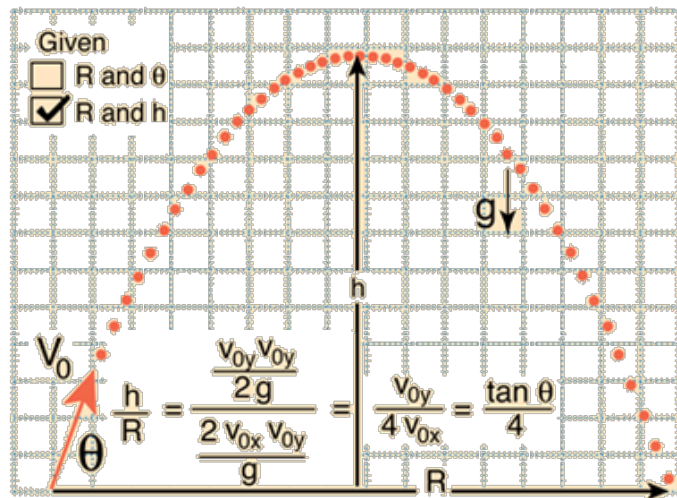
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Launch Velocity

The launch velocity of a projectile can be calculated from the range if the angle of launch is known. It can also be calculated if the maximum height and range are known, because the angle can be determined.

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From the range and peak relationships:

$$R = \frac{v_0^2 \sin 2\theta}{g} \text{ and } h = \frac{v_{0y}^2}{2g}$$

the angle of launch can be determined, leading to:

$$v_0 = \sqrt{\frac{Rg}{\sin 2\theta}} \quad \text{Show}$$

For range

R = m = ft,

and peak height

h = m = ft,

the launch velocity is

v_0 m/s = ft/s.

The required launch angle is

θ = degrees.

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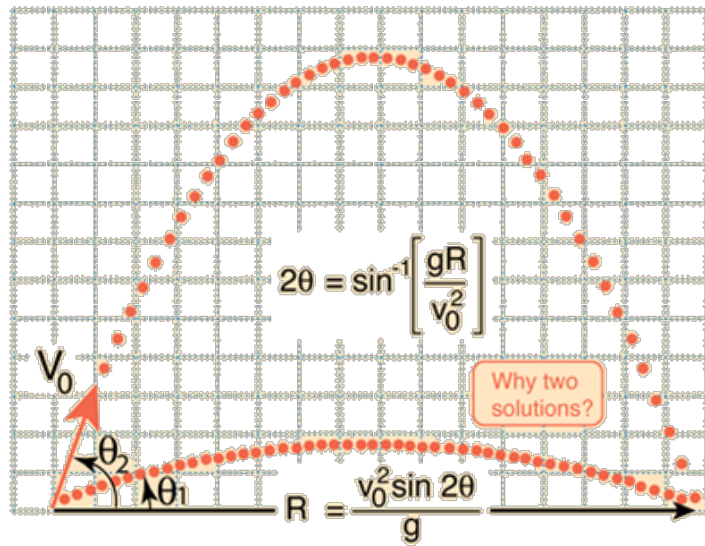
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Angle of Launch

Variation of the launch angle of a projectile will change the range. If the launch velocity is known, the required angle of launch for a desired range can be calculated from the [motion equations](#).

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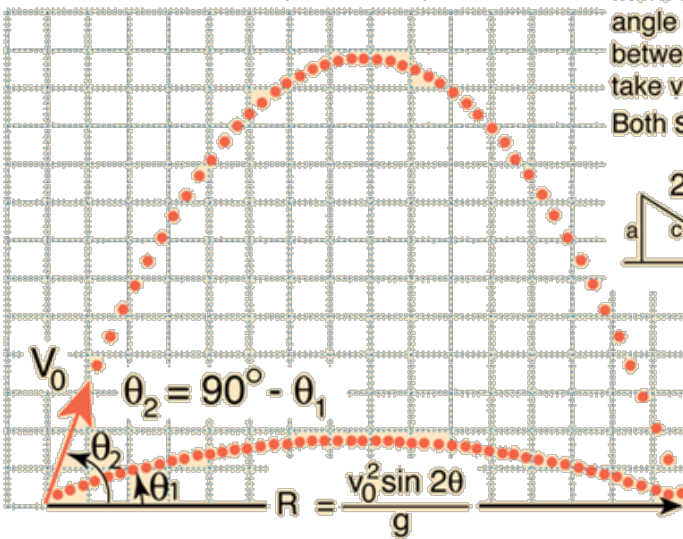

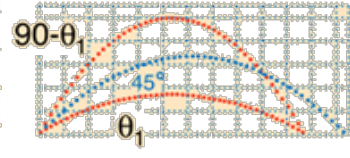
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From the [range relationship](#), the angle of launch can be determined. For range

$R =$

$m =$

<p>When the launch angle is calculated from the relationship $2\theta = \sin^{-1} \left[\frac{gR}{v_0^2} \right]$</p> <p style="text-align: center;">Calculation</p>  <p>there are two solutions. If the launch angle is envisioned as an angle between 0° and 90°, then 2θ can take values between 0° and 180°. Both $\sin 2\theta_1$ and $\sin 2\theta_2$ are equal to a/c, so they are both valid.</p>  <p>The two complementary launch angles approach each other at 45°</p> 	<p>Index</p> <p>Trajectory concepts</p>
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