ICPC Notebook - UNAL

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Índice

1. Miscellaneous

1.1. Miscellaneous

```
#define between(a, b, c) (a <= b && b <= c)
#define has_key(it, key) (it.find(key) != it.end())
```

```cpp
#define check_coord(x, y, n, m) (0 <=x && x < n && 0 <= y && y < m)

const int d4x[4] = {0, -1, 1, 0};
const int d4y[4] = {-1, 0, 0, 1};
const int d8x[8] = {-1, 0, -1, 1, -1, 1, 0, 1};
const int d8y[8] = {-1, -1, 0, -1, 1, 0, 1, 1};

#define endl '\n'
#define _ << ' ' <<
#define PB push_back
#define SZ(v) ((int) v.size())
#define trav(ref, ds) for(auto &ref: ds)
#define forn(i, b) for(int i = 0; i < int(b); ++i)
#define forr(i, b) for(int i = int(b)-1; i >= 0; i--)
#define rep(i, a, b) for(int i = int(a); i <= int(b); ++i)
#define rev(i, b, a) for(int i = int(b); i >= int(a); i--)

#define precise(n, k) fixed << setprecision(k) << n
// or at main()
cout << fixed << setprecision(9);

#define all(x) (x).begin(), (x).end()
#define rall(x) (x).rbegin(), (x).rend()
#define ms(arr, value) memset(arr, value, sizeof(arr))

template<typename T>
inline void unique(vector<T> &v) {
    sort(v.begin(), v.end());
    v.resize(distance(v.begin(), unique(v.begin(), v.end())));
}

#define infinity while(1)
#define unreachable assert(false && "Unreachable");

#pragma GCC optimize("O3,unroll-loops")
#pragma GCC target("avx2,bmi,bmi2,lzcnt,popcnt")

// THINGS TO KEEP IN MIND
//   * int overflow, time and memory limits
//   * Special case (n = 1?)
//   * Do something instead of nothing and stay organized
//   * Don't get stuck in one approach

// TIME AND MEMORY LIMITS
//   * 1 second is approximately 10^8 operations (c++)
//   * 10^6 Elements of 32 Bit (4 bytes) is equal to 4 MB
//   * 62x10^6 Elements of 32 Bit (4 bytes) is equal to 250 MB
//   * 10^6 Elements of 64 Bits (8 bytes) is equal to 8 MB
//   * 31x10^6 Elements of 64 Bit (8 bytes) is equal to 250 MB

ios::sync_with_stdio(0);
cin.tie(0);

// Lectura segun el tipo de dato (Se usan las mismas para imprimir):

scanf("%d", &value); //int
scanf("%ld", &value); //long y long int
scanf("%c", &value); //char
scanf("%f", &value); //float
scanf("%lf", &value); //double
scanf("%s", &value); //char*
scanf("%lld", &value); //long long int
scanf("%x", &value); //int hexadecimal
scanf("%o", &value); //int octal

// Impresion de punto flotante con d decimales, ejemplo 6 decimales:
printf("%.6lf", value);

// Genera un numero entero aleatorio en el rango [a, b]. Para ll usar
    "mt19937_64" y cambiar todo a ll.

mt19937 rng(chrono::steady_clock::now().time_since_epoch().count());
int rand(int a, int b) {
    return uniform_int_distribution<int>(a, b)(rng);
}

vector<string> split(string str, string separator) {
    vector<string> tokens;
    for ( auto tok = strtok(&str[0], separator.data());
            tok != NULL;
            tok = strtok(NULL, separator.data())) {
        tokens.push_back(tok);
    }
    return tokens;
}

// Custom hashing for secure unordered_map
struct custom_hash {
    size_t operator()(uint64_t x) const {
        static const uint64_t FIXED_RANDOM =
            chrono::steady_clock::now().time_since_epoch().count();
        x ^= FIXED_RANDOM;
        return x ^ (x >> 16);
    }
};

unordered_map<ll, int, custom_hash> safe_map;
gp_hash_table<ll, int, custom_hash> safe_hash_table;
safe_map.reserve(1024); // Power of 2
safe_map.max_load_factor(0.25);

// Python Read
from sys import stdin, stdout
list(map(func, stdin.readline().strip().split()))
```

1.2. Stress Testing Script

```
# A and B are executables you want to compare, gen takes int
# as command line arg. Usage: 'sh stress.sh'
for ((i = 1; ; ++i)); do # if they are same then will loop forever
    echo $i
    ./gen $i > int
    ./A < int > out1
    ./B < int > out2
    diff -w out1 out2 || break
    # diff -w <(./A < int) <(./B < int) || break
done
```

2. STD Library

2.1. Find Nearest Set

```
// Finds the element nearest to target
template<typename T>
T find_nearest(set<T> &st, T target) {
    assert(!st.empty());
    auto it = st.lower_bound(target);
    if (it == st.begin()) {
        return *it;
    } else if (it == st.end()) {
        it--; return *it;
    }
    T right = *it; it--;
    T left = *it;
    if (target-left < right-target)
        return left;
    // if they are the same distance, choose right
    // if you want to choose left change to <=
    return right;
}
```

2.2. Merge Vector

```
template<typename T> // To merge two vectors, the answer is an ordered
     vector
void merge_vector(vector<T> &big, vector<T> &small) {
    int n = (int) big.size();
    int m = (int) small.size();
    if(m > n) swap(small, big);
    if(!is_sorted(big.begin(), big.end()))
        sort(big.begin(), big.end());
    if(!is_sorted(small.begin(), small.end()))
        sort(small.begin(), small.end());
    vector<T> aux;
    merge(small.begin(), small.end(), big.begin(), big.end(),
         aux.begin());
    big = move(aux);
}
```

2.3. Shorter - Priority Queue

```
template<typename T, typename Sequence=vector<T>, typename
     Compare=less<T>>
using template_heap = priority_queue<T, Sequence, Compare>;

template<typename T>
using max_heap = template_heap<T>;

template<typename T>
using min_heap = template_heap<T, vector<T>, greater<T>>;
```

```cpp
#define pop_heap(heap) heap.top(); heap.pop();
```

2.4. Rope

```cpp
#include <ext/rope>
using namespace __gnu_cxx;
#define trav_rope(it, v) for(auto it=v.mutable_begin(); it!=
    v.mutable_end(); ++it)
#define all_rope(rp) (rp).mutable_begin(), (rp).mutable_end()
// trav_rope(it, v) cout << *it << " ";
// Use ‘crope‘ for strings
// push_back(T val):
//     This function is used to input a character at the end of the rope
//     Time Complexity: O(log2(n))
// pop_back():
//     this function is used to delete the last character from the rope
//     Time Complexity: O(log2(n))
// insert(int i, rope r): !!!!!!!!!!!!!!!WARING!!!!!!!!!!! Worst Case:
    O(N).
//     Inserts the contents of ‘r‘ before the i-th element.
//     Time Complexity: Best Case: O(log N) and Worst Case: O(N).
// erase(int i, int n):
//     Erases n elements, starting with the i-th element
//     Time Complexity: O(log2(n))
// substr(int i, int n):
//     Returns a new rope whose elements are the n elements starting at
    the position i-th
//     Time Complexity: O(log2(n))
// replace(int i, int n, rope r):
//     Replaces the n elements beginning with the i-th element with the
    elements in r
//     Time Complexity: O(log2(n))
// concatenate(+):
//     Concatenate two ropes using the +   symbol
//     Time Complexity: O(1)
```

2.5. Set Utilities

```cpp
template<typename T>
T get_min(set<T> &st) {
   assert(!st.empty());
   return *st.begin();
}
template<typename T>
T get_max(set<T> &st) {
   assert(!st.empty());
   return *st.rbegin();
}
template<typename T>
T erase_min(set<T> &st) {
   assert(!st.empty());
   T to_return = get_min(st);
   st.erase(st.begin());
   return to_return;
}
template<typename T>
T erase_max(set<T> &st) {
   assert(!st.empty());
   T to_return = get_max(st);
   st.erase(--st.end());
   return to_return;
}
#define merge_set(big, small) big.insert(small.begin(), small.end());
#define has_key(it, key) (it.find(key) != it.end())
```

2.6. To Reverse Utilities

```cpp
template<typename T>
class to_reverse {
private:
   T& iterable_;
public:
   explicit to_reverse(T& iterable) : iterable_{iterable} {}
   auto begin() const { return rbegin(iterable_); }
   auto end() const { return rend(iterable_); }
};
```

3. Data Structure

3.1. Disjoint Set Union

```cpp
struct DSU {
    vector<int> par, sizes;
    int size;
    DSU(int n) : par(n), sizes(n, 1) {
        size = n;
        iota(par.begin(), par.end(), 0);
    }
    // Busca el nodo representativo del conjunto de u
    int find(int u) {
        return par[u] == u ? u : (par[u] = find(par[u]));
    }
    // Une los conjuntos de u y v
    void unite(int u, int v) {
        u = find(u), v = find(v);
        if (u == v) return;
        if (sizes[u] > sizes[v]) swap(u,v);
        par[u] = v;
        sizes[v] += sizes[u];
        size--;
    }
    // Retorna la cantidad de elementos del conjunto de u
    int count(int u) { return sizes[find(u)]; }
};
```

3.2. Min - Max Queue

```cpp
// Permite hallar el elemento minimo para todos los subarreglos de un
    largo fijo en O(n). Para Max queue cambiar el > por <.
struct min_queue {
    deque<int> dq, mn;
    void push(int x) {
        dq.push_back(x);
        while (mn.size() && mn.back() > x) mn.pop_back();
        mn.push_back(x);
    }
    void pop() {
        if (dq.front() == mn.front()) mn.pop_front();
        dq.pop_front();
    }
    int min() { return mn.front(); }
};
```

3.3. Prefix Sum Immutable 2D

```cpp
template<typename T>
class PrefixSum2D {
public:
    int n, m;
    vector<vector<T>> dp;

    PrefixSum2D() : n(-1), m(-1) {}
    PrefixSum2D(vector<vector<T>>& grid) {
        n = (int) grid.size();
        assert(0 <= n);
        if(n == 0) { m = 0; return; }
        m = (int) grid[0].size();
        dp.resize(n+1, vector<T>(m+1, static_cast<T>(0)));

        for(int i = 1; i <= n; ++i)
            for(int j = 1; j <= m; ++j)
                dp[i][j] = dp[i][j-1] + grid[i-1][j-1];
        for(int j = 1; j <= m; ++j)
            for(int i = 1; i <= n; ++i)
                dp[i][j] += dp[i-1][j];
    }
    T query(int x1, int y1, int x2, int y2) {
        assert(0<=x1&&x1<n && 0<=y1&&y1<m);
        assert(0<=x2&&x2<n && 0<=y2&&y2<m);
        int SA = dp[x2+1][y2+1];
        int SB = dp[x1][y2+1];
        int SC = dp[x2+1][y1];
        int SD = dp[x1][y1];
        return SA-SB-SC+SD;
    }
};

// Prefix Sum Immutable 2D - Shorter code
const int N = 102;
const int M = 102;
const int inf = 1e9;

int n;
int a[N][M];
int sum[N][M];

int query(int x1, int z1, int x2, int z2){
    return sum[x2][z2] + sum[x1-1][z1-1] - sum[x1-1][z2] - sum[x2][z1-1];
}

// initialization / at main()

for(int i = 1; i <= n; ++i) {
    for(int j = 1; j <= m; ++j) {
        cin >> a[i-1][j-1]; // 0-Indexed
        sum[i][j] = sum[i-1][j] + sum[i][j-1] - sum[i-1][j-1] +
            a[i-1][j-1];
    }
}

// query(x1, z1, x2, z2)
```

3.4. Prefix Sum

```
template<typename T>
class PrefixSum {
public:
    int n;
    vector<T> dp;
    PrefixSum() : n(-1) {}
    PrefixSum(vector<T>& nums) {
        n = (int) nums.size();
        if(n == 0)
            return;
        dp.resize(n + 1);
        dp[0] = 0;
        for(int i = 1; i <= n; ++i)
            dp[i] = dp[i-1] + nums[i-1];
    }
    T query(int left, int right) {
        assert(0 <= left && left <= right && right <= n - 1);
        return dp[right+1] - dp[left];
    }
};
```

3.5. Segment Tree Lazy

```
using int64 = long long;
const int64 nil = 1e18; // for sum: 0, for min: 1e18, for max: -1e18
int64 op(int64 x, int64 y) { return min(x, y); }

struct segtree_lazy {
    segtree_lazy *left, *right;
    int l, r, m;
    int64 sum, lazy;

    segtree_lazy(int l, int r) : l(l), r(r), sum(nil), lazy(0) {
        if(l != r) {
            m = (l+r)/2;
            left = new segtree_lazy(l, m);
            right = new segtree_lazy(m+1, r);
        }
    }
    /// (l, l+1, l+2 .... r-1, r)
    /// x  x    x    x  x x x
    /// (cuantos tengo) * x
    /// r-l+1
    void propagate() {
        if(lazy != 0) {
            /// voy a actualizar el nodo
            sum += (r - l + 1) * lazy;
            if(l != r) {
                left->lazy += lazy;
                right->lazy += lazy;
            }
            /// voy a propagar a mis hijos
            lazy = 0;
        }
    }
    // void modify(int pos, int v) {
    //    if(l == r) {
    //        sum = v;
    //    } else {
    //        if(pos <= m) left->modify(pos, v);
    //        else right->modify(pos, v);
    //        sum = op(left->sum, right->sum);
    //    }
    // }
    void modify(int a, int b, int64 v) {
        propagate();
        if(a > r || b < l) return;
        if(a <= l && r <= b) {
            lazy = v; // lazy += v, for add
            propagate();
            return;
        }
        left->modify(a, b, v);
        right->modify(a, b, v);
        sum = op(left->sum, right->sum);
    }

    int64 query(int a, int b) {
        propagate();
        if(a > r || b < l) return nil;
        if(a <= l && r <= b) return sum;
        return op(left->query(a, b), right->query(a, b));
    }
};
```

3.6. Segment Tree Standard

```
// Reference: descomUNAL's Notebook
using int64 = long long;
const int64 nil = 1e18; // for sum: 0, for min: 1e18, for max: -1e18
int64 op(int64 x, int64 y) { return min(x, y); }
struct segtree {
    segtree *left, *right;
    int l, r, m;
    int64 sum;
    segtree(int l, int r) : l(l), r(r), sum(nil) {
        if(l != r) {
            m = (l+r)/2;
            left = new segtree(l, m);
            right = new segtree(m+1, r);
        }
    }
    void modify(int pos, int64 v) {
        if(l == r) {
            sum = v;
        } else {
            if(pos <= m) left->modify(pos, v);
            else right->modify(pos, v);
            sum = op(left->sum, right->sum);
        }
    }
    int64 query(int a, int b) {
        if(a > r || b < l) return nil;
        if(a <= l && r <= b) return sum;
        return op(left->query(a, b), right->query(a, b));
    }
};
// Usage:
// segtree st(0, n);
// forn(i, n) {
//   cin >> val;
//   st.modify(i, val);
// }
```

3.7. Sparse Table

```
struct RMQ {
    vector<vector<int>> table;
    RMQ(vector<int> &v) : table(20, vector<int>(v.size())) {
        int n = v.size();
        for (int i = 0; i < n; i++)
            table[0][i] = v[i];
        for (int j = 1; (1<<j) <= n; j++)
            for (int i = 0; i + (1<<(j-1)) < n; i++)
                table[j][i] = min(table[j-1][i], table[j-1][i +
                    (1<<(j-1))]);
    }
    int query(int a, int b) {
        int j = 31 - __builtin_clz(b-a+1);
        return min(table[j][a], table[j][b-(1<<j)+1]);
    }
};
```

3.8. Tree Order Statistic

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>

using namespace std;
using namespace __gnu_pbds;

template <typename K, typename V, typename Comp = less<K>>
```

```cpp
using indexed_map = tree<K, V, Comp, rb_tree_tag,
    tree_order_statistics_node_update>;

template <typename K, typename Comp = less<K>>
using indexed_set = indexed_map<K, null_type, Comp>;
// Usage
//   auto it = any.find_by_order(idx); (0-indexed)
//   (*it).first, (*it).second
//   int index = any.order_of_key(key);
//   {1: 10, 2 :20, 5: 50}, order_of_key(3) -> return index 2
```

4. Graph

4.1. Articulation Points

```cpp
// Encontrar los nodos que al quitarlos, se deconecta el grafo

vector<vector<int>> adj;
vector<bool> visited;
vector<int> low;
// Order in which it was visited
vector<int> order;
vector<bool> points;
// Count the components
int counter = 0;
// Number of Vertex
int vertex;

void dfs(int node, int parent = -1) {
    visited[node] = true;
    low[node] = order[node] = ++counter;

    int children = 0;

    for(int &neighbour: adj[node]) {

        if(!visited[neighbour]) {
            children++;

            dfs(neighbour, node);

            low[node] = min(low[node], low[neighbour]);

            // Conditions #1
            if(parent != -1 && order[node] <= low[neighbour]) {
                points[node] = true;
            }
        } else {
            low[node] = min(low[node], order[neighbour]);
        }
    }

    // Conditions #2
    if(parent == -1 && children > 1) {
        points[node] = true;
    }
}

vector<int> build() {
    for(int node = 0; node < vertex; ++node)
        if(!visited[node]) dfs(node);

    vector<int> output;
    for(int node = 0; node < vertex; ++node)
        if(points[node]) output.push_back(node);
    return output;
}
```

4.2. Bellman Ford

```cpp
struct edge {
    int from, to;
    int64 cost;
};

int n, m;
const int N = 2505;
const int64 inf = 1e18;
vector<edge> edges;

vector<int64> bellman_ford(int u, bool &cycle) {
    vector<int64> dist(n, inf);
    dist[u] = 0LL;
    for(int i = 0; i < n + 1; ++i){
        for(const edge &e: edges) {
            if(dist[e.from] != inf && dist[e.from] + e.cost < dist[e.to]) {
                dist[e.to] = dist[e.from] + e.cost;
                if(i == n)
                    cycle = true; // There are negative edges
            }
        }
    }
    return dist;
    // Time Complexity: O(V*E), Space Complexity: O(V)
}

// cin >> l >> r >> cost, --l, --r;
// edges.push_back({l, r, cost});
// bool cycle = false;
// vector<int64> dist = bellman_ford(0, cycle);
```

4.3. BFS

```
// Busqueda en anchura sobre grafos. Recibe un nodo inicial u y visita
    todos los nodos alcanzables desde u.
// BFS tambien halla la distancia mas corta entre el nodo inicial u y los
    demas nodos si todas las aristas tienen peso 1.

const int mxN = 1e5+5; // Cantidad maxima de nodos
vector<int> adj[mxN]; // Lista de adyacencia
vector<int64> dist; // Almacena la distancia a cada nodo
int n, m; // Cantidad de nodos y aristas

void bfs(int u) {
    queue<int> Q;
    Q.push(u);
    dist[u] = 0;

    while (Q.size() > 0) {
        u = Q.front();
        Q.pop();
        for (auto &v : adj[u]) {
            if (dist[v] == -1) {
                dist[v] = dist[u] + 1;
                Q.push(v);
            }
        }
    }
}

void init() {
    dist.assign(n, -1);
    for (int i = 0; i <= n; i++) {
        adj[i].clear();
    }
}
```

4.4. Binary Lifting

```
const int mxN = 2e5 + 10;
const int LOG = 20;

vector<int> adj[mxN];
int up[mxN][LOG];
int tin[mxN];
int tout[mxN];
int depth[mxN];
int timer = 0;

void lifting(int node, int parent) {
    tin[node] = ++timer;
    up[node][0] = parent;
    for(int i = 1; i < LOG; ++i) {
        up[node][i] = up[ up[node][i-1] ][i-1];
        // up[node][i] = up[max(0, up[node][i-1])][i-1]; // to use the
            jump(node, k) function
    }
    for(auto &child: adj[node]) {
        if(child == parent) continue;
        depth[child] = depth[node] + 1;
        lifting(child, node);
    }
    tout[node] = ++timer;
}

bool is_ancestor(int left, int right) {
    return tin[left] <= tin[right] && tout[left] >= tout[right];
}

int lca(int left, int right) {
    if(is_ancestor(left, right)) {
        return left;
    } else if(is_ancestor(right, left)) {
        return right;
    }
    for(int i = LOG-1; i >= 0; i--) {
        if(!is_ancestor(up[left][i], right)) {
            left = up[left][i];
        }
    }
    return up[left][0];
}

// jump k levels up in the tree
int jump(int node, int k) {
    for(int i = 0; i < LOG; ++i) {
        if((k >> i) & 1 && node != -1) {
            node = up[node][i];
        }
    }
    return node;
}

// distance between 2 nodes -> O(lg(n))
// depth[left] + depth[right]  2*depth[ lca(left, right) ]

// lifting(0, -1); to use the jump(node, k) function
// lifting(0, 0); to use the lca(left, right) function
```

4.5. Bridges

```
// Encontrar las aristas que al quitarlas, el grafo queda desconectado

vector<vector<int>> adj;
vector<bool> visited;
vector<int> low;
// Order in which it was visited
vector<int> order;
// Answer:
vector<pair<int, int>> bridges;
// Number of Vertex
int vertex;
// Count the components
int cnt;

void dfs(int node, int parent = -1) {
    visited[node] = true;
    order[node] = low[node] = ++cnt;
    for (int neighbour: adj[node]) {
        if (!visited[neighbour]) {
            dfs(neighbour, node);
            low[node] = min(low[node], low[neighbour]);

            if (order[node] < low[neighbour]) {
                bridges.push_back({node, neighbour});
            }
        } else if (neighbour != parent) {
            low[node] = min(low[node], order[neighbour]);
        }
    }
}

vector<pair<int, int>> build() {
    cnt = 0;
    for (int node = 0; node < adj.size(); node++)
        if (!visited[node]) dfs(node);
    return bridges;
}
```

4.6. Dijkstra

```
// Dado un grafo con pesos no negativos halla la ruta de costo minimo
    entre un nodo inicial u y todos los demas nodos.

struct edge {
    int v; int64 cost;
    bool operator < (const edge &other) const {
        return other.cost < cost;
    }
};

const int64 inf = 1e18;
const int N = 1e5+5; // Cantidad maxima de nodos
vector<edge> adj[N]; // Lista de adyacencia
bool was[N];       // Marca los nodos ya visitados
int64 dist[N];     // Almacena la distancia a cada nodo
int pre[N];        // Almacena el nodo anterior para construir las rutas
```

```cpp
int n, m;          // Cantidad de nodos y aristas

void dijkstra(int u) {
    priority_queue<edge> Q;
    Q.push({u, 0});
    dist[u] = 0;

    while (!Q.empty()) {
        u = Q.top().v; Q.pop();
        if (!was[u]) {
            was[u] = true;
            for (auto &ed : adj[u]) {
                int v = ed.v;
                if (!was[v] && dist[v] > dist[u] + ed.cost) {
                    dist[v] = dist[u] + ed.cost;
                    pre[v] = u;
                    Q.push({v, dist[v]});
                }
            }
        }
    }
}

void init() {
    for (int i = 0; i <= n; i++) {
        adj[i].clear();
        was[i] = 0;
        dist[i] = inf;
    }
}
```

4.7. Floyd Warshall

```cpp
const int mxN = 500 + 10;
const int64 inf = 1e18;
int64 dp[mxN][mxN];

for(int i = 0; i < n; ++i)
    for(int j = 0; j < n; ++j)
        dp[i][j] = (i == j)? 0 : inf;

// Adding edges
// dp[from][to] = min(dp[from][to], cost);
// dp[to][from] = min(dp[to][from], cost);

for(int k = 0; k < n; ++k) {
    for(int i = 0; i < n; ++i) {
        for(int j = 0; j < n; ++j) {
            if(dp[i][k] < inf && dp[k][j] < inf) {
                dp[i][j] = min(dp[i][j], dp[i][k] + dp[k][j]);
            }
        }
    }
}
// answer: dp[from][to]
```

4.8. Merge Trick on Trees

```cpp
// Reference: https://usaco.guide/plat/merging?lang=cpp
const int mxN = 2e5 + 10;
vector<int> adj[mxN];
int colors[mxN];
set<int> cnt[mxN];
int answer[mxN];

void dfs(int node, int parent) { // O(n*lg^2(n))
    cnt[node].insert(colors[node]);

    for(auto &child: adj[node]) {
        if(child == parent) continue;

        dfs(child, node);

        // always make the child set the smallest
        if(cnt[child].size() > cnt[node].size())
            swap(cnt[child], cnt[node]); // O(1)

        // Merge
        for(auto &it: cnt[child]) {
            cnt[node].insert(it);
        }
        cnt[child].clear(); // if time is too high don't use, only use
            when giving MLE
    }
    answer[node] = (int) cnt[node].size();
}
```

```
// dfs(0, -1)
```

4.9. Kahn Algoritm

4.10. SCC - Kasaraju

```
vector<vector<int>> adj;
vector<vector<int>> radj;
vector<bool> visited;
stack<int> toposort;
vector<vector<int>> components; // Answer - SCC
int vertex; // Number of Vertex

// First
// Topological Sort
void toposort_dfs(int node) {
    visited[node] = true;
    for(int neighbour: adj[node]) {
        if(!visited[neighbour]) {
            toposort_dfs(neighbour);
        }
    }
    toposort.push(node);
}

// Second
// dfs Standard - Reverse Adj
void dfs(int node) {
    visited[node] = true;
    components.back().push_back(node);
    for(int neighbour: radj[node]) {
        if(!visited[neighbour]) {
            dfs(neighbour);
        }
    }
}

// Third
// Build Algorithm
vector<vector<int>> build() {
    // Topological Sort
    for(int node = 0; node < vertex; ++node)
        if(!visited[node]) toposort_dfs(node);

    // Reset - Visited
    fill(visited.begin(), visited.end(), false);

    // In the topological order run the reverse dfs
    while(!toposort.empty()) {
        int node = toposort.top();
        toposort.pop();
        if(!visited[node]) {
            components.push_back(vector<int>{});
            dfs(node);
        }
    }
    return components;
}
```

4.11. SCC - Tarjan

```
// Dado un grafo dirigido halla las componentes fuertemente conexas (SCC).

const int inf = 1e9;
const int MX = 1e5+5; // Cantidad maxima de nodos
vector<int> g[MX]; // Lista de adyacencia
stack<int> st;
int low[MX], pre[MX], cnt;
int comp[MX]; // Almacena la componente a la que pertenece cada nodo
int SCC; // Cantidad de componentes fuertemente conexas
int n, m; // Cantidad de nodos y aristas

void tarjan(int u) {
    low[u] = pre[u] = cnt++;
    st.push(u);
    for (auto &v : g[u]) {
        if (pre[v] == -1) tarjan(v);
        low[u] = min(low[u], low[v]);
    }
    if (low[u] == pre[u]) {
        while (true) {
            int v = st.top(); st.pop();
            low[v] = inf;
            comp[v] = SCC;
            if (u == v) break;
        }
        SCC++;
    }
}

void init() {
    cnt = SCC = 0;
    for (int i = 0; i <= n; i++) {
        g[i].clear();
        pre[i] = -1; // no visitado
    }
}
```

4.12. Topological Sort

```
class KahnTopoSort {
    vector<vector<int>> adj;
    vector<int> indegree;
    vector<int> toposort;
    int nodes;
    bool solved;
    bool isCyclic;
public:

    KahnTopoSort(int n) : nodes(n) {
        adj.resize(n);
        indegree.resize(n, 0);
        solved = false;
        isCyclic = false;
    }

    void addEdge(int from, int to) {
        adj[from].push_back(to);
        indegree[to]++;
        solved = false;
        isCyclic = false;
    }

    vector<int> sort() {
        if(solved) return toposort;
        toposort.clear();
        queue<int> Q;
        vector<int> in_degree(indegree.begin(), indegree.end());
        for(int i = 0; i < nodes; ++i) {
            if(in_degree[i] == 0) Q.push(i);
        }
        int count = 0;
        while(!Q.empty()) {
            int node = Q.front(); Q.pop();
            toposort.push_back(node);
            for(int neighbour: adj[node]) {
                in_degree[neighbour]--;
                if(in_degree[neighbour] == 0) {
                    Q.push(neighbour);
                }
            }
            count++;
        }
        solved = true;
        if(count != nodes) {
            // There exists a cycle in the graph
            isCyclic = true;
            return vector<int> {};
        }
        return toposort;
    }

    bool getIsCyclic() {
        sort();
        return isCyclic;
    }
};
```

4.13. Topological Sort - Dfs

```
vector<vector<int>> adj;
vector<bool> visited;
vector<bool> onstack;
vector<int> toposort;

// Implementation I
// Topological Sort - Detecting Cycles
void dfs(int node, bool &isCyclic) {
    if(isCyclic) return;
```

```cpp
    visited[node] = true;
    onstack[node] = true;
    for(int neighbour: adj[node]) {
        if (visited[neighbour] && onstack[neighbour]) {
            // There is a cycle
            isCyclic = true;
            return;
        }
        if(!visited[neighbour]) {
            dfs(neighbour, isCyclic);
        }
    }
    onstack[node] = false;
    toposort.push_back(node);
}
```

4.14. Tree Diameter

```cpp
// const int mxN = 1e5;
int dp[mxN];
int dfs(int node, int parent) {
    int mx = 0;
    int first = 0, second=0;
    for(int &child: adj[node]) {
        if(child == parent) continue;
        int factor = dfs(child, node) + 1;
        mx = max(mx, factor);
        if(factor >= first) {
            second = first;
            first = factor;
        } else if(factor >= second) {
            second = factor;
        }
    }
    dp[node] = first + second;
    return mx;
}
// n: number of nodes
// dfs(0, 0);
// int diameter = *max_element(dp, dp + n);
```

4.15. Tree Difference Array Technique on Trees

```cpp
int diff[mxN]; // Difference Array
int answer[mxN]; // Array after propagation of differences

void dfs(int node, int parent) {
    answer[node] = diff[node];
    for(int &child: adj[node]) {
        if(child == parent) continue;
        dfs(child, node);
        answer[node] += answer[child];
    }
}

// main()
for(int i = 0; i < m; ++i) {
    int l, r; cin >> l >> r, l--, r--;
    int anc = lca(l, r);
    diff[l]++;
    diff[r]++;
    diff[anc]--;
    if(anc != 0)
        diff[up[anc][0]]--;
}

dfs(0, -1);

for(int i = 0; i < n; ++i) {
    cout << answer[i] << " \n" [i == (n-1)];
}

//      [1] -1 -> parent of lca
//      / \
//    [2] [3] -1  -> lca
//        / \
//   +1 [4] [5] +1
```

4.16. Tree Euler Tour

```cpp
const int mxN = 2e5 + 10;
vector<int> adj[mxN];
int tour[2 * mxN]; // Euler Tour
int timer = 0;
```

```
void eulerTree(int node, int parent) {
    tour[timer++] = node;
    for(int &child: adj[node]) {
        if(child == parent) continue;
        eulerTree(child, node);
        tour[timer++] = node;
    }
}

//        1
//       / \
//      2   3
//         / \
//        4   5

// for(int i = 0; i < 2*n-1; ++i) cout << tour[i] + 1 << " \n" [i ==
     (2*n-2)];
// Euler Tour: 1 2 1 3 4 3 5 3 1
```

4.17. Tree Subtree Queries

```
const int mxN = 2e5 + 10;
vector<int> adj[mxN];
int n, q;

int ids[mxN]; // Node ID
// Subtree Range: [ids[node], tout[node]]
int tout[mxN]; // useful for calculating the query range
int sub[mxN]; // Subtree Size
int values[mxN]; // Node Values
int timer = 0;

void dfs(int node, int parent) {
    sub[node] = 1;
    ids[node] = timer++;
    for(int &child: adj[node]) {
        if(child == parent) continue;
        dfs(child, node);
        sub[node] += sub[child];
    }
    tout[node] = timer - 1;
}
// Start the query structure
//     forn(i, n) st.modify(ids[i], values[i]);
// Update a tree Node
//     st.modify(ids[idx], val);
// query on the subtree of a node, including the node
//     st.query(ids[node], tout[node])
// Run dfs
//     dfs(0, 0)
```

5. String

5.1. Hashing

```
// Convierte el string en un polinomio, en O(n), tal que podemos comparar
     substrings como valores numericos en O(1).
// Primero llamar calc_xpow() (una unica vez) con el largo maximo de los
     strings dados.
// Primes: 1000234999, 1000567999, 1000111997, 1000777121, 1001864327,
     1001265673
using ll = long long;
inline int add(int a, int b, const int &mod) { return a+b >= mod ?
     a+b-mod : a+b; }
inline int sub(int a, int b, const int &mod) { return a-b < 0 ? a-b+mod :
     a-b; }
inline int mul(int a, int b, const int &mod) { return 1LL*a*b % mod; }

const int X[] = {257, 359};
const int MOD[] = {(int)1e9+7, (int)1e9+9};
vector<int> xpow[2];

struct hashing {
    vector<int> h[2];

    hashing(string &s) {
        int n = s.size();
        for (int j = 0; j < 2; ++j) {
            h[j].resize(n+1);
            for (int i = 1; i <= n; ++i) {
                h[j][i] = add(mul(h[j][i-1], X[j], MOD[j]), s[i-1],
                     MOD[j]);
            }
        }
    }
    //Hash del substring en el rango [i, j)
    ll query(int l, int r) {
        int a = sub(h[0][r], mul(h[0][l], xpow[0][r-l], MOD[0]), MOD[0]);
        int b = sub(h[1][r], mul(h[1][l], xpow[1][r-l], MOD[1]), MOD[1]);
        return (ll(a)<<32) + b;
    }
};

void calc_xpow(int mxlen) {
    for (int j = 0; j < 2; ++j) {
        xpow[j].resize(mxlen+1, 1);
        for (int i = 1; i <= mxlen; ++i) {
            xpow[j][i] = mul(xpow[j][i-1], X[j], MOD[j]);
        }
    }
}

// Check palindrome: from - to
// auto hash1 = hash.query(from, to);
// auto hash2 = hash_reverse.query(n-to-1, n-from-1);
// hash1 == hash2
```

5.2. KMP Standard

```
// Use prefix_function

template <typename T>
vector<int> kmp(const T &text, const T &pattern) {
    int n = (int) text.size();
    int m = (int) pattern.size();
    vector<int> lcp = prefix_function(pattern);
    vector<int> occurrences;
    int matched = 0;
    for(int idx = 0; idx < n; ++idx){
        while(matched > 0 && text[idx] != pattern[matched])
            matched = lcp[matched-1];
        if(text[idx] == pattern[matched])
            matched++;
        if(matched == m) {
            occurrences.push_back(idx-matched+1);
            matched = lcp[matched-1];
        }
    }
    return occurrences;
}
//KMP - Knuth-Morris-Pratt algorithm
// Time Complexity: O(N), Space Complexity: O(N)
// N: Length of text
// Usage:
//   string txt = "ABABABAB";
//   string pat = "ABA";
//   vector<int> ans = search_pattern(txt, pat); {0, 2, 4}
```

5.3. Longest Common Prefix Array

```
// Longest Common Prefix Array

template <typename T>
vector<int> lcp_array(const vector<int>& sa, const T &S) {
    int N = int(S.size());
    vector<int> rank(N), lcp(N - 1);
    for (int i = 0; i < N; i++)
        rank[sa[i]] = i;

    int pre = 0;
    for (int i = 0; i < N; i++) {
        if (rank[i] < N - 1) {
            int j = sa[rank[i] + 1];
            while (max(i, j) + pre < int(S.size()) && S[i + pre] == S[j +
                pre]) ++pre;
            lcp[rank[i]] = pre;
            if (pre > 0)--pre;
        }
    }
    return lcp;
}
// La matriz de prefijos comunes más larga ( matriz LCP ) es una
    estructura de datos auxiliar
// de la matriz de sufijos . Almacena las longitudes de los prefijos
    comunes más largos (LCP)
// entre todos los pares de sufijos consecutivos en una matriz de sufijos
    ordenados
```

5.4. Minimum Expression

Dado un string s devuelve el indice donde comienza la rotación lexicograficamente menor de s.

```
// O(n)
int minimum_expression(string s) {
    s = s+s; // si no se concatena devuelve el indice del sufijo menor
    int len = s.size(), i = 0, j = 1, k = 0;
    while (i+k < len && j+k < len) {
        if (s[i+k] == s[j+k]) k++;
        else if (s[i+k] > s[j+k]) i = i+k+1, k = 0; // cambiar por < para
            maximum
        else j = j+k+1, k = 0;
        if (i == j) j++;
    }
    return min(i, j);
}
```

5.5. Manacher

```
template <typename T>
vector<int> manacher(const T &s) {
    int n = (int) s.size();
    if (n == 0)
        return vector<int>();
    vector<int> res(2 * n - 1, 0);
    int l = -1, r = -1;
    for (int z = 0; z < 2 * n - 1; z++) {
        int i = (z + 1) >> 1;
        int j = z >> 1;
        int p = (i >= r ? 0 : min(r - i, res[2 * (l + r) - z]));
        while (j + p + 1 < n && i - p - 1 >= 0) {
            if (!(s[j + p + 1] == s[i - p - 1])) break;
            p++;
        }
        if (j + p > r) {
            l = i - p;
            r = j + p;
        }
        res[z] = p;
    }
    // Time Complexity: O(N), Space Complexity: O(N)
    return res;
}
// res[2 * i] = odd radius in position i
// res[2 * i + 1] = even radius between positions i and i + 1
// s = "abaa" -> res = {0, 0, 1, 0, 0, 1, 0}
// in other words, for every z from 0 to 2 * n - 2:
// calculate i = (z + 1) >> 1 and j = z >> 1
// now there is a palindrome from i - res[z] to j + res[z]
// (watch out for i > j and res[z] = 0)

template <typename T>
vector<string> palindromes(const T &txt) {
    vector<int> res = manacher(txt);
    int n = (int) txt.size();
    vector<string> answer;
    for(int z = 0; z < 2*n-1; ++z) {
        int i = (z + 1) / 2;
        int j = z / 2;
        if (i > j && res[z] == 0)
            continue;
        int from = i - res[z];
        int to = j + res[z];
        string pal="";
        for(int i = from; i <= to; ++i)
            pal.push_back(txt[i]);
        answer.push_back(pal);
    }
    return answer;
}
```

5.6. Prefix Function

Te estan dando un string s de longitud n, la prefix function para este string esta definido como un array π de longitud n, donde $\pi[i]$ es la longitud del prefijo propio más largo de la subcadena $s[0..i]$ que también es un sufijo de esta subcadena. Un prefijo propio de una cadena es un prefijo que no es igual a la propia cadena. Por definición $\pi[0] = 0$

$$\pi[i] = max_{k=0...i} \quad k : s[0..k-1] = s[i-(k-1)..i]$$

Por Ejemplo la prefix function del string '*abcabcd*' is[0, 0, 0, 1, 2, 3, 0] y la prefix function del string '*aabaaab*' es [0, 1, 0, 1, 2, 2, 3]

```
template <typename T>
```

```
vector<int> prefix_function(const T &s) {
    int n = (int) s.size();
    vector<int> lps(n, 0);
    lps[0] = 0;
    int matched = 0;
    for(int pos = 1; pos < n; ++pos){
        while(matched > 0 && s[pos] != s[matched])
            matched = lps[matched-1];
        if(s[pos] == s[matched])
            matched++;
        lps[pos] = matched;
    }
    return lps;
}
// Longest prefix which is also suffix
// Time Complexity: O(N), Space Complexity: O(N)
// N: Length of pattern

// Naive Algorithm
vector<int> prefix_function(string s) {
    int n = (int)s.length();
    vector<int> pi(n);
    for (int i = 0; i < n; i++)
        for (int k = 0; k <= i; k++)
            if (s.substr(0, k) == s.substr(i-k+1, k))
                pi[i] = k;
    return pi;
}
```

5.7. Suffix Array

```
template <typename T>
vector<int> suffix_array(const T &S) {
    int N = int(S.size());
    vector<int> suffix(N), classes(N);
    for (int i = 0; i < N; i++) {
        suffix[i] = N - 1 - i;
        classes[i] = S[i];
    }
    stable_sort(suffix.begin(), suffix.end(), [&S](int i, int j) {return
        S[i] < S[j];});
    for (int len = 1; len < N; len *= 2) {
        vector<int> c(classes);
        for (int i = 0; i < N; i++) {
            bool same = i && suffix[i - 1] + len < N
                        && c[suffix[i]] == c[suffix[i - 1]]
                        && c[suffix[i] + len / 2] == c[suffix[i - 1] + len
                            / 2];
            classes[suffix[i]] = same ? classes[suffix[i - 1]] : i;
        }
        vector<int> cnt(N), s(suffix);
        for (int i = 0; i < N; i++){
            cnt[i] = i;
        }
        for (int i = 0; i < N; i++) {
            int s1 = s[i] - len;
            if (s1 >= 0) suffix[cnt[classes[s1]]++] = s1;
        }
    }
    return suffix;
}
/// Complexity: O(|N|*log(|N|))
// Usage:
//   Index:              012345
//   string some_string = "banana";
//   vector<int> suffix = suffix_array(some_string)

//   suffix{5, 3, 1, 0, 4, 2}
//   5:a, 3:ana, 1:anana, 0:banana, 4:na, 2:nana
```

5.8. Trie Automaton

```
const int ALPHA = 26; // alphabet letter number
const char L = 'a'; // first letter of the alphabet

struct TrieNode {
    int next[ALPHA];
    bool end : 1;

    TrieNode() {
        fill(next, next + ALPHA, 0);
        end = false;
    }
    int& operator[](int idx) {
        return next[idx];
    }
```

```
};

class Trie {
public:

    int nodes;
    vector<TrieNode> trie;

    Trie() : nodes(0) {
        trie.emplace_back();
    }

    void insert(const string &word) {
        int root = 0;
        for(const char &ch :word) {
            int c = ch - L;
            if(!trie[root][c]) {
                trie.emplace_back();
                trie[root][c] = ++nodes;
            }
            root = trie[root][c];
        }
        trie[root].end = true;
    }

    bool search(const string &word) {
        int root = 0;
        for(const char &ch :word) {
            int c = ch - L;
            if(!trie[root][c])
                return false;
            root = trie[root][c];
        }
        return trie[root].end;
    }

    bool startsWith(const string &prefix) {
        int root = 0;
        for(const char &ch : prefix) {
            int c = ch - L;
            if(!trie[root][c])
                return false;
            root = trie[root][c];
        }
        return true;
```

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A	C	B	A	C	D	A	C	B	A	C	B	A	C	D	A
–	0	0	2	0	0	5	0	0	7	0	0	2	0	0	1

```
    }
};
```

5.9. Z Algorithm

El Z-Array z de un string s de longitud n continene para cada $k = 0,\ 1,\ ,2,\ \ldots,\ n-1$ la longitud del mas largo substring de s que inicia en la posición k y es un prefijo de s.

Por lo tanto, $z[k] = p$ nos dice que $s\,[0..p-1]$ es igual a $s\,[k..k+p-1]$

Por Ejemplo el Z-Array de $ACBACDACBACBACDA$ es el siguiente:

Es este caso, para el ejemplo, $z[6] = 5$, porque el substring $ACBAC$ de longitud 5 es un prefijo de s, pero para el substring $ACBACB$ de longitud 6 no es un prefijo de s.

```
// z_array=length of the longest substring starting from s[i] which is
    also a prefix of s
vector<int> z_algorithm(const string &s) {
    int n = (int) s.size();
    vector<int> z_array(n);
    int left=0, right=0;
    z_array[0] = 0;
    for(int idx = 1; idx < n; ++idx) {
        z_array[idx] = max(0, min(z_array[idx-left], right-idx+1));
        while (idx+z_array[idx] < n && s[z_array[idx]] ==
            s[idx+z_array[idx]]) {
            left = idx;
            right = idx + z_array[idx];
            z_array[idx]++;
        }
    }
    return z_array;
}
```

5.10. Aho Corasick

```
// El trie (o prefix tree) guarda un diccionario de strings como un arbol
    enraizado.
// Aho corasick permite encontrar las ocurrencias de todos los strings
    del trie en un string s.

const int alpha = 26; // cantidad de letras del lenguaje
const char L = 'a'; // primera letra del lenguaje

struct node {
    int next[alpha], end;
    int link, exit, cnt;
    int& operator[](int i) { return next[i]; }
};

vector<node> trie = {node()};

void add_str(string &s, int id = 1) {
    int u = 0;
    for (auto ch : s) {
        int c = ch-L;
        if (!trie[u][c]) {
            trie[u][c] = trie.size();
            trie.push_back(node());
        }
        u = trie[u][c];
    }
    trie[u].end = id; //con id > 0
    trie[u].cnt++;
}

// aho corasick
void build_ac() {
    queue<int> q; q.push(0);
    while (q.size()) {
        int u = q.front(); q.pop();
        for (int c = 0; c < alpha; ++c) {
            int v = trie[u][c];
            if (!v) trie[u][c] = trie[trie[u].link][c];
            else q.push(v);
            if (!u || !v) continue;
            trie[v].link = trie[trie[u].link][c];
                       trie[v].exit = trie[trie[v].link].end ?
                           trie[v].link : trie[trie[v].link].exit;
                trie[v].cnt += trie[trie[v].link].cnt;
            }
        }
    }

vector<int> cnt; //cantidad de ocurrencias en s para cada patron

void run_ac(string &s) {
    int u = 0, sz = s.size();
    for (int i = 0; i < sz; ++i) {
        int c = s[i]-L;
        while (u && !trie[u][c]) u = trie[u].link;
        u = trie[u][c];
        int x = u;
        while (x) {
            int id = trie[x].end;
            if (id) cnt[id-1]++;
            x = trie[x].exit;
        }
    }
}
```

6. Math

6.1. Diophantine

```
// Use extgcd
template<typename T>
bool diophantine(T a, T b, T c, T & x, T & y, T & g) {
    if (a == 0 && b == 0) {
        if (c == 0) {
            x = y = g = 0;
            return true;
        }
        return false;
    }
    auto [g1, x1, y1] = extgcd(a, b);
    if (c % g1 != 0)
        return false;
    g = g1;
    x = x1 * (c / g);
    y = y1 * (c / g);
    return true;
}
// Usage
// int x, y, g;
// bool can = diophantine(a, b, c, x, y, g);

// a*x + b*y = c -> If and only if gcd(a, b) is a divisor of c
```

6.2. Divisors

```
template<typename T>
vector<T> divisors(T number) {
    vector<T> ans;
    for (T i = 1; i*i <= number; ++i) {
        if (number % i == 0) {
            if (number/i == i) {
                // if i*i == number
                ans.push_back(i);
            } else {
                // x=i, y=number/i, if x*y==number
                ans.push_back(i);
                ans.push_back(number/i);
            }
        }
    }
    return ans;
}
```

6.3. Ext GCD

```
template<typename T>
tuple<T, T, T> extgcd(T a, T b) {
    if (a == 0)
        return {b, 0, 1};
    T p = b / a;
    auto [g, y, x] = extgcd(b - p * a, a);
    x -= p * y;
    return {g, x, y};
}
// Usage:
//  auto [g, x, y] = extgcd(a, b);

// a*x    1 (mod m) -> If and only if gcd(a, m) == 1
// a*x + m*y = 1

// auto [g, x, y] = extgcd(a, m);

// a*x + b*y = gcd(a, b)
```

6.4. GCD

```
template<class T>
T gcd(T a, T b) {
    return (b == 0)?a:gcd(b, a % b);
}
```

6.5. LCM

```
template<class T>
T lcm(T a, T b) {
    return (a*b)/gcd<T>(a, b);
}
```

6.6. Matrix

```
// Estructura para realizar operaciones de multiplicacion y
    exponenciacion modular sobre matrices.

const int mod = 1e9+7;

struct matrix {
    vector<vector<int>> v;
    int n, m;

    matrix(int n, int m, bool o = false) : n(n), m(m), v(n,
        vector<int>(m)) {
        if (o) while (n--) v[n][n] = 1;
    }

    matrix operator * (const matrix &o) {
        matrix ans(n, o.m);
        for (int i = 0; i < n; i++)
            for (int k = 0; k < m; k++) if (v[i][k])
                for (int j = 0; j < o.m; j++)
                    ans[i][j] = (1LL * v[i][k] * o.v[k][j] + ans[i][j]) %
                        mod;
        return ans;
    }

    vector<int>& operator[] (int i) { return v[i]; }
};

matrix pow(matrix b, ll e) {
    matrix ans(b.n, b.m, true);
    while (e) {
        if (e&1) ans = ans*b;
        b = b*b;
        e /= 2;
    }
    return ans;
}
```

6.7. Lineal Recurrences

```
// Calcula el n-esimo termino de una recurrencia lineal (que depende de
    los k terminos anteriores).
// * Llamar init(k) en el main una unica vez si no es necesario
    inicializar las matrices multiples veces.
// Este ejemplo calcula el fibonacci de n como la suma de los k terminos
    anteriores de la secuencia (En la secuencia comun k es 2).
// Agregar Matrix Multiplication con un construcctor vacio.

matrix F, T;

void init(int k) {
    F = {k, 1}; // primeros k terminos
    F[k-1][0] = 1;
    T = {k, k}; // fila k-1 = coeficientes: [c_k, c_k-1, ..., c_1]
    for (int i = 0; i < k-1; i++) T[i][i+1] = 1;
    for (int i = 0; i < k; i++) T[k-1][i] = 1;
}

/// O(k^3 log(n))
int fib(ll n, int k = 2) {
    init(k);
    matrix ans = pow(T, n+k-1) * F;
    return ans[0][0];
}
```

6.8. Phi Euler

```
template<typename T>
T phi_euler(T number) {
    T result = number;
    for(T i = static_cast<T>(2); i*i <= number; ++i) {
        if(number % i != 0)
            continue;
        while(number % i == 0) {
            number /= i;
        }
        result -= result / i;
    }
    if(number > 1)
        result -= result / number;
    return result;
}
```

6.9. Primality Test

```
template<typename T>
bool is_prime(T number) {
    if(number <= 1)
        return false;
    else if(number <= 3)
        return true;
    if(number%2==0 || number%3==0)
        return false;
    for(T i = 5; i*i <= number; i += 6) {
        if(number%i==0 || number%(i+2)==0)
            return false;
    }
    return true;
    // Time Complexity: O(sqrt(N)), Space Complexity: O(1)
}
```

6.10. Primality Test Miller Rabin

```
// Reference: notebook_descomUNAL
ll mul (ll a, ll b, ll mod) {
    ll ret = 0;
    for(a %= mod, b %= mod; b != 0;
      b >>= 1, a <<= 1, a = a >= mod ? a - mod : a) {
        if (b & 1) {
            ret += a;
            if (ret >= mod) ret -= mod;
        }
    }
    return ret;
}
ll fpow (ll a, ll b, ll mod) {
    ll ans = 1;
    for (; b; b >>= 1, a = mul(a, a, mod))
        if (b & 1)
            ans = mul(ans, a, mod);
    return ans;
}
bool witness (ll a, ll s, ll d, ll n) {
    ll x = fpow(a, d, n);
    if (x == 1 || x == n - 1) return false;
    for (int i = 0; i < s - 1; i++) {
        x = mul(x, x, n);
        if (x == 1) return true;
        if (x == n - 1) return false;
    }
    return true;
}
ll test[] = {2, 3, 5, 7, 11, 13, 17, 19, 23, 0};
bool is_prime (ll n) {
    if (n < 2) return false;
    if (n == 2) return true;
    if (n % 2 == 0) return false;
    ll d = n - 1, s = 0;
    while (d % 2 == 0) ++s, d /= 2;
    for (int i = 0; test[i] && test[i] < n; ++i)
        if (witness(test[i], s, d, n))
            return false;
    return true;
}
```

6.11. Prime Factos

```
template<class T>
map<T, int> prime_factors(T number) {
    map<T, int> factors;
    while (number % 2 == 0) {
        factors[2]++;
        number = number / 2;
    }
    for (T i = 3; i*i <= number; i += 2) {
        while (number % i == 0) {
            factors[i]++;
            number = number / i;
        }
    }
    if (number > 2)
        factors[number]++;
    return factors;
}
//  for n=100, { 2: 2, 5: 2}
//  2*2*5*5 = 2^2 * 5^2 = 100
```

6.12. Sieve

```
using int64 = long long;

const int mxN = 1e6;
bool marked[mxN+1];
vector<int> primes;
/// O(mxN log(log(mxN)))
void sieve() {
    marked[0] = marked[1] = true;
    for (int i = 2; i <= mxN; i++) {
        if (marked[i]) continue;
        primes.push_back(i);
        for (int64 j = 1LL * i*i; j <= mxN; j += i)
            marked[j] = true;
    }
}
```

6.13. Math Utils

```
#define PI  3.1415926535897932384626433832795028841L // acos(-1);
#define E   2.71828182845904523536028747135266249 7L
#define eps 1e-9

template<typename T>
int cmp(const T &a, const T &b) {
    return ( (a + eps < b)? -1 :( (b + eps < a )? 1 : 0) );
}

template<typename T>
T ceiling_division(T numerator, T denominator) {
    assert(denominator != static_cast<T>(0));
    return (numerator+denominator-1)/denominator;
}

// How much does it need to add to n so that it is divisible by k
template<typename T>
T distance_divisible(T n, T k) {
    assert(0 < k); if(n < k) return k - n % k;
    return n % k;
}
```

7. Dynamic Programming

7.1. Diameter dp on tree

```
const int mxN = 2e5 + 10;
vector<int> adj[mxN];
int n;

int dist[mxN];
int dp[mxN];

int dfs(int node, int parent) {
    dist[node] = 0;
    int mx_dist = 0;
    int first = -1, second = -1;
    for(auto &child: adj[node]) {
        if(child == parent)
            continue;
            mx_dist = max(mx_dist, dfs(child, node) + 1);
            if(dist[child] >= first) {
                if(first != -1) second = first;
                first = dist[child];
            } else if(dist[child] >= second) {
                second = dist[child];
            }
        }
        dist[node] = mx_dist;
        dp[node] = first + second + 2;
        return mx_dist;
}
// undigraph
// dfs(0, -1);
// int diameter = *max_element(dp, dp + n);
```

7.2. DP on Directed Acyclic Graph

```
// Problemas clasicos con DAG
const int INF = 1e9;
const int MAX = 1000;
int init, fin;
int dp[MAX];
vector<int> g[MAX]; // USADO PARA ARISTAS NO PONDERADAS
vector<pair<int, int>> gw[MAX]; // PARA ARISTAS PONDERADAS First: Nodo
    vecino. Second = Peso de la arista
// Funcion para calcular el numero de formas de ir del nodo u al nodo end
// LLamar para nodo inicial (init)
int ways(int u){
    if(u == fin) return 1;
    int &ans = dp[u];
    if(ans != -1) return ans;
    ans = 0;
    for(auto v: g[u]){
        ans += ways(v);
    }
    return ans;
}
// MINIMO CAMINO DESDE U HASTA END. LLAMAR PARA INIT
int min_way(int u){
    if(u == fin) return 0;
    int &ans = dp[u];
    if(ans != -1) return ans;
    ans = INF;
    for(auto v: gw[u]){
        ans = min(ans, min_way(v.first) + v.second);
    }
    return ans;
}
```

7.3. Edit Distance

```
int edit_dist(string &s1, string &s2, int m, int n) {
    // If first string is empty, the only option is to
    // insert all characters of second string into first
    if (m == 0) return n;

    // If second string is empty, the only option is to
    // remove all characters of first string
    if (n == 0)
        return m;

    // If last characters of two strings are same, nothing
    // much to do. Ignore last characters and get count for
    // remaining strings.
    if (s1[m - 1] == s2[n - 1])
        return edit_dist(s1, s2, m - 1, n - 1);

    // If last characters are not same, consider all three
    // operations on last character of first string,
    // recursively compute minimum cost for all three
    // operations and take minimum of three values.
    return 1 + min({
        edit_dist(s1, s2, m, n - 1), // Insert
        edit_dist(s1, s2, m - 1, n), // Remove
        edit_dist(s1, s2, m - 1, n - 1) // Replace
    });
}
```

7.4. Elevator Rides

There are n people who want to get to the top of a building which has only one elevator. You know the weight of each person and the maximum allowed weight in the elevator. **What is the minimum number of elevator rides?**

```
Input:
n x
w1 w2 ... wn
-----------
Input:
4 10
4 8 6 1
Output:
2
```

```
int n; // the number of people
ll x; // maximum allowed weight in the elevator
vector<ll> W; // w1, w2, ..., wn: the weight of each person.

struct Answer {
    int rides; // is the minimum number of rides for a subset mask
    ll lastW; // is the minimum weight of the last ride

    bool operator<(const Answer &other) const {
        if(rides == other.rides) return lastW < other.lastW;
        return rides < other.rides;
    }

    bool operator>(const Answer &other) const {
        if(rides == other.rides) return lastW > other.lastW;
        return rides > other.rides;
    }
};

// at main()
vector<Answer> dp((1 << n) + 2);
dp[0] = {1, 0}; // We set the value for an empty group as follows:
for(int mask = 1; mask < (1 << n); ++mask) {
    dp[mask] = {inf, inf};
    for(int p = 0; p < n; ++p) {
        if(mask & (1 << p)) {
            Answer option = dp[mask ^ (1 << p)];
            if(option.lastW + W[p] <= x) {
                // add p to an existing ride
                option.lastW += W[p];
            } else {
                // reserve a new ride for p
                option.rides++;
                option.lastW = W[p];
            }
            dp[mask] = min(dp[mask], option);
        }
    }
}

cout << dp[(1 << n) - 1].rides << endl;
```

7.5. Snapsack

```
vector<vector<int64>> dp;

int64 knapsack(vector<int64> &val, vector<int64> &wt, int item, int
     capacity) {
    // Casos base
    if(item <= 0 || capacity <= 0) return 0;

    if(dp[item][capacity] != -1) return dp[item][capacity];

    int itemCurr = item - 1;
    // Maximos items acumulado
    int64 lastMax = knapsack(val, wt, item-1, capacity);
    int64 currMax = 0;

    if(wt[itemCurr] <= capacity) {
        // Valor del item actual + el mejor item que cabe en la mochila
        currMax = val[itemCurr] + knapsack(val, wt, item - 1,
             capacity-wt[itemCurr]);
    }

    dp[item][capacity] = max(lastMax, currMax);
    return dp[item][capacity];
}
// vector<int> val{10, 40, 30, 50};
// vector<int> wt{5, 4, 6, 3};
// int n = val.size();
// int w = 10;
// knapsack(val, wt, n, w)
```

7.6. Longest Common Subsecuence

```
// Longest Common Subsecuence
int lcs(string X, string Y, int m, int n) {
    if (m == 0 || n == 0) {
        return 0;
    }
    if (X[m - 1] == Y[n - 1]) {
        return 1 + lcs(X, Y, m - 1, n - 1);
    }
    return max(lcs(X, Y, m, n - 1), lcs(X, Y, m - 1, n));
}
```

7.7. Longest Increasing Subsecuence - DP

```
int lis(int arr[], int i, int n, int prev) {
    // Base case: nothing is remaining
    if (i == n) {
        return 0;
    }
    int excl = lis(arr, i + 1, n, prev);
    int incl = 0;
    if (arr[i] > prev) {
        incl = 1 + lis(arr, i + 1, n, arr[i]);
    }
    return max(incl, excl);
}
```

7.8. Longest Increasing Subsecuence - Optimization

```
// Longest Increasing Subsequence O(n*lg(n))
template <typename T>
int lis(const vector<T> &a) {
    vector<T> u;
    for (const T &x : a) {
        auto it = lower_bound(u.begin(), u.end(), x);
        if (it == u.end()) {
            u.push_back(x);
        } else {
            *it = x;
        }
    }
```

```cpp
    return (int) u.size();
}
// LIS O(nlog(n)) Para longest non-decreasing cambiar lower_bound por
     upper_bound
int lis(){
    LIS.clear();
    for(int i = 0; i < N; i++){
        auto id = lower_bound(LIS.begin(), LIS.end(), A[i]);
        if(id == LIS.end()){
            LIS.push_back(A[i]);
            dp[i] = LIS.size();
        }
        else{
            int idx = id - LIS.begin();
            LIS[idx] = A[i];
            dp[i] = idx + 1;
        }
    }
    return LIS.size();
}
// METODO PARA RECONSTRUIR LIS. Para non-decreasing cambiar < por <=
stack<int> rb;
void build(){
    int k = LIS.size();
    int cur = oo;
    for(int i = N - 1; i >= 0, k; i--){
        if(A[i] < cur && k == dp[i]){
            cur = A[i];
            rb.push(A[i]);
            k--;
        }
    }
}
```

8. Search

8.1. Binary Search - I

```cpp
int n = oo;
int low = 0, high = n, mid;
while (high - low > 1) {
    mid = low + (high - low) / 2;
    if(!ok(mid)) {
        low = mid;
    } else {
        high = mid;
    }
}
// low or high
```

8.2. Binary Search - II

```cpp
int n = oo;
int index = -1;
for(int jump = n+1; jump >= 1; jump /= 2) {
    while(jump+index<n && !ok(jump+index)) {
        index += jump;
    }
}
// index + 1
```

8.3. Binary Search on Real Values - I

```cpp
double eps = 1e-9;
double n = inf;
double low = 0.0, high = n, mid;
while ((high - low) > eps) {
    mid = double(high + low) / 2.0;
    if(!ok(mid)) {
        low = mid;
    } else {
        high = mid;
    }
}
// low or high
```

8.4. Binary Search on Real Values - II

```cpp
double n = inf;
double low = 0.0, high = n, mid;
int iter = 0;
```

```cpp
while(iter < 300) {
    mid = double(high + low) / 2.0;
    if(!ok(mid)) {
        low = mid;
    } else {
        high = mid;
    }
    iter++;
}
// low or high
```

8.5. Ternary Search - I

```cpp
double ternary_search(const function<double(double)> &func, double low,
     double high) {
    int it = 0;
    while (it < 100) { // with 50 iterations it has precision for 1e-9
        double diff = (high - low) / 3.0;
        double mid1 = low + diff;
        double mid2 = high - diff;

        double f1 = func(mid1);
        double f2 = func(mid2);

        if (f1 > f2) // change to < to find the maximum
            low = mid1;
        else
            high = mid2;
        it++;
    }
    return func(low);
}
// Usage:
// double ans = ternary_search(funct1, low, high);
```

8.6. Ternary Search - II

```cpp
// This version is slower than the iterations version.
double ternary_search(const function<double(double)> &func, double low,
     double high) {
    double eps = 1e-9;
    while (high - low > eps) {
        double diff = (high - low) / 3.0;
        double mid1 = low + diff;
        double mid2 = high - diff;

        double f1 = func(mid1);
        double f2 = func(mid2);

        if (f1 > f2) // change to < to find the maximum
            low = mid1;
        else
            high = mid2;
    }
    return func(low);
}
// Usage:
// double ans = ternary_search(funct1, low, high);
```

8.7. Merge Sort

```cpp
void merge(vector<int> &v, int left, int mid, int right) {
    vector<int> ordered(right-left+1);
    int i = left, j = mid + 1, idx = 0;
    while(i <= mid || j <= right) {
        if(i <= mid && j <= right) {
            if(v[i] < v[j]) {
                ordered[idx++] = v[i++];
            } else if(v[i] > v[j]) {
                ordered[idx++] = v[j++];
            } else {
                ordered[idx++] = v[i++];
                ordered[idx++] = v[j++];
            }
        } else if(i <= mid) {
            ordered[idx++] = v[i++];
        } else if(j <= right) {
            ordered[idx++] = v[j++];
        }
    }
    for(idx=0, i = left; i <= right; i++)
        v[i] = ordered[idx++];
}
void merge_sort(vector<int> &v, int left, int right) {
    if(left == right) {
        return;
    } else if(left < right) {
        int mid = (left+right)/2;
        merge_sort(v, left, mid);
        merge_sort(v, mid+1, right);
        merge(v, left, mid, right);
    }
}
void merge_sort(vector<int> &v) {
    merge_sort(v, 0, (int) v.size() - 1);
}
// Usage:
// Vector<int> A { ... };
// merge_sort(A);
```

9. Techniques

9.1. Divide and Conquer

```
void divide(int left, int right) {
    if(left == right) {
        return;
    } else if(left < right) {
        int mid = (left + right) / 2;
        divide(left, mid);
        divide(mid+1, right);
    }
}
```

9.2. Mo's Algorithm

```
// Complexity: O(|N+Q|*sqrt(|N|)*|add+del|)

struct Query {
    int left, right, index;
    Query (int l, int r, int idx)
        : left(l), right(r), index(idx) {}
};
int S; // S = sqrt(n);

bool cmp (const Query &a, const Query &b) {
    if (a.left/S != b.left/S)
        return a.left/S < b.left/S;
    return a.right > b.right;
}

// global functions
void add(int idx) {

}
void del(int idx) {

}
auto get_answer() {

}

// at main()
vector<Query> Q;
Q.reserve(q+1);
int from, to;
for(int i = 0; i < q; ++i){
    cin >> from >> to; // don't forget (from--, to--) if it's 1-indexed
    Q.push_back(Query(from, to, i));
}

S = sqrt(n); // n = size of array
sort(Q.begin(), Q.end(), cmp);

vector<int> ans(q);
int left = 0, right = -1;

for (int i = 0; i < (int) Q.size(); ++i) {
    while (right < Q[i].right)
        add(++right);
    while (left > Q[i].left)
        add(--left);
    while (right > Q[i].right)
        del(right--);
    while (left < Q[i].left)
        del(left++);

    ans[Q[i].index] = get_answer();
}
```

9.3. Sliding Windows

```
// sequence: [a1, a2, a3, a4, a5, a6, a7, ..., an]
//               |<- sliding window ->|
//               v                  v
//            [start]-->          [end]-->

// int n = (int) any.size();
// int start=0, end=0;
// map<int, int> counter;
// int ans = 0;
// while(end < n) {
//    counter[any[end]]++;
//    while(condition(start, end) && start <= end) {
//        counter[any[start]]--;
//        process_logic1(start,end);
//        start++;
//    }
//    process_logic2(start,end);
//    ans = max(ans, end - start + 1);
//    end++;
// }
// print(ans);
```

9.4. Sweep Line

```
struct Event {
    int time, delta, idx;
    bool operator<(const Event &other) const { return time < other.time; }
};

// Usage:
// vector<Event> events;
// events.reserve(2*n);
// int from, to;
// for(int i = 0; i < n; ++i) {
//    read from and to values
//    events.push_back(Event{from, 1, i});
//    events.push_back(Event{to, -1, i});
// }
// sort(events.begin(), events.end());
// for(const auto &event: events) {
//    process_logic(event.delta); for example
//    total += event.delta;
//    best = max(best, total);
// }
```

9.5. Two Pointer Left Right Boundary

```
// sequence: [a1, a2, a3, a4, ..., an]
//    [left] ->->          <-<-<- [right]

// int left=0, right=n-1;
// while(left < right) {
//    if(left_condition(left)) {
//        left++;
//    }
//    if(right_condition(right)) {
//        right--;
//    }
//    process_logic(left, right);
// }
```

9.6. Two Pointer1 Pointer2

```
// seq1: [a1, a2, a3, ..., an]
// [p1] ->->->->->

// seq2: [b1, b2, b3, ..., bn]
// [p2] ->->

// int n = (int) seq1.size();
// int m = (int) seq2.size();
// int p1=0, p2=0; // or seq1[0], seq2[0]
// while(p1 < n && p2 < m) {
//    if(p1_condition(p1)) {
//        p1++;
//    }
//    if(p2_condition(p2)) {
//        p2++;
//    }
//    process_logic(p1, p2);
// }
```

9.7. Two Pointers Old And New State

```
// sequence:           [ a1,  a2,  a3, ...]
//                       |     |    |
//                       v     v    v
// new state:    [new0, new1, new2, new3, ...]
//                  |     |     |
//                  v     v     v
// new state: [old0, old1, old2, old3, ...]

// new state:     [old0, old1, old2, old3, ...]
//                  |     |     |
//                  v     v     v
// new state: [new0, new1, new2, new3, ...]

// int last = default_val1;
// int now = default_val2;
// for(int i = 0; i < n; ++i){
//    last = now;
//    now = process_logic(element, old)
// }
```

9.8. Two Pointers Slow Fast

```
// sequence: [a1, a2, a3, ..., an]
// slow runner: [slow] ->->
// fast runner: [fast] ->->->->

// int slow = 0;
// for(int fast = 0; fast < n; ++fast){
//    if(slow_condition(slow)) {
//        slow = slow.next;
//        slow += 1;
//    }
//    process_logic(slow, fast);
// }
```

10. Combinatorics

10.1. All Combinations Backtracking

```
vector<vector<int>> answer;
vector<int> combination;
void combinations_backtraking(const int &n, const int &k, int idx) {
    if(idx == k) {
        answer.push_back(combination);
        return;
    }
    int start = (combination.size()==0)?1:combination.back()+1;
    for(int i = start; i <= n; ++i) {
        combination.push_back(i);
        combinations_backtraking(n, k, idx+1);
        combination.pop_back();
    }
}
```

10.2. Binomial Coefficient

Calcula el coeficiente binomial nCr, entendido como el numero de subconjuntos de r elementos escogidos de un conjunto con n elementos.

```
// O(min(r, n-r))
int64 nCr(int64 n, int64 r) {
    if (r < 0 || n < r) return 0;
    r = min(r, n-r);
    int64 ans = 1;
    for (int i = 1; i <= r; i++) {
        ans = ans * (n-i+1) / i;
    }
    return ans;
}
```

10.3. Kth Permutation

```
vector<int> kth_permutation(vector<int> perm, int k) {
    int64_t factorial = 1LL;
    int n = (int) perm.size();
    for(int64_t num = 2; num < n; ++num)
```

```
        factorial *= num; // (n-1)!
    k--; // k-th to 0-indexed
    vector<int> answer; answer.reserve(n);
    while(true) {
        answer.push_back(perm[k / factorial]);
        perm.erase(perm.begin()+(k/factorial));
        if((int) perm.size() == 0)
            break;
        k %= factorial;
        factorial /= (int) perm.size();
    }
    return answer;
}

vector<int> kth_permutation(int n, int k, int start=0) {
    vector<int> perm(n);
    iota(perm.begin(), perm.end(), start);
    return kth_permutation(perm, k);
}

string kth_perm_string(int n, int k) {
    assert(1 <= n && n <= 26);
    vector<int> perm = kth_permutation(n, k);
    string alpha = "";
    for(char i='a'; i <= ('a'+n); ++i)
        alpha.push_back(i);
    string answer="";
    for(int &idx: perm)
        answer.push_back(alpha[idx]);
    return answer;
}
```

10.4. Next Combination

this works for 1 ¡= k ¡= n ¡= 20 approximately Complexity: worst case $O(2^n)$ approximately

```
bool next_combination(vector<int> &comb, int n) {
    int k = (int) comb.size();
    for (int i = k - 1; i >= 0; i--) {
        if (comb[i] <= n - k + i) {
            ++comb[i];
            while (++i < k) {
                comb[i] = comb[i - 1] + 1;
            }
            return true;
        }
    }
    return false;
}

void all_combinations(int n, int k) {
    vector<int> comb(k);
    iota(comb.begin(), comb.end(), 1);
    do {
        for (const int &v : comb) {
            cout << v << " ";
        }
        cout << endl;
    } while (next_combination(comb, n));
}
```

11. Numerics

11.1. Fastpow

```
template<typename T, typename U>
T fastpow(T a, U b) {
    assert(0 <= b);
    T ans = static_cast<T>(1);
    while (b > 0) {
        if (b & 1) ans = ans*a;
        a *= a;
        b >>= 1;
    }
    return ans;
}
```

11.2. Numeric Mod

```
const int MOD = int(1e9+7);

template<typename T>
T sub(T a, T b) {
```

```cpp
    return (1LL*(a-b)%MOD + MOD) % MOD;
}
template<typename T>
T add(T a, T b) {
    return (1LL*(a%MOD) + 1LL*(b%MOD)) % MOD;
}
template<typename T>
T mul(T a, T b) {
    return (1LL*(a%MOD) * (b%MOD)) % MOD;
}
template<typename T, typename U>
T fastpow(T a, U b) {
    assert(0 <= b);
    T answer = static_cast<T>(1);
    while (b > 0) {
        if (b & 1) {
            answer = mul(answer, a);
        }
        a = mul(a, a);
        b >>= 1;
    }
    return answer;
}
template<typename T>
T inverse(T a) {
    a %= MOD;
    if (a < 0) a += MOD;
    T b = MOD, u = 0, v = 1;
    while (a) {
        T t = b / a;
        b -= t * a; swap(a, b);
        u -= t * v; swap(u, v);
    }
    assert(b == 1);
    if (u < 0) u += MOD;
    return u;
}
template<typename T>
T division(T a, T b) {
    return mul(a, inverse(b));
}
```

12. Bit Mask

12.1. Tricks

```cpp
int zeros_left(int num) {return (num==0)?32:__builtin_clz(num);}
int zeros_right(int num) {return (num==0)?0:__builtin_ctz(num);}
int count_ones(int num) {return __builtin_popcount(num);}
int parity(int num) {return __builtin_parity(num);}
int LSB(int num) {return __builtin_ffs(num);} // Least Significant Bit [0
    if num == 0]

int64_t zeros_left(int64_t num) {return
    (num==0LL)?64LL:__builtin_clzll(num);}
int64_t zeros_right(int64_t num) {return
    (num==0LL)?0LL:__builtin_ctzll(num);}
int64_t count_ones(int64_t num) {return __builtin_popcountll(num);}
int64_t parity(int64_t num) {return __builtin_parityll(num);}
int64_t LSB(int64_t num) {return __builtin_ffsll(num);} // Least
    Significant Bit [0 if number == 0]

template<typename T>
int hamming(const T &lhs, const T &rhs) {
   if(is_same<T, int64_t>::value) return __builtin_popcountll(lhs ^ rhs);
   return __builtin_popcount(lhs ^ rhs);
}

// 1LL for 64-bits

// x & 1          : Check if x is odd
// x & (1 << i)   : Check if the i-th bit is HIGH
// x = x | (1<<i) : Set HIGH i-th bit
// x = x & ~(1<<i) : Set LOW i-th bit
// x = x ^ (1<<i) : Flip i-th bit
// x = ~x         : Flip all the bits
// x & -x         : returns the number of the first HIGH bit from right
    to left (power of 2, not the index)
// log2(x & -x)   : Return position of first bit HIGH from right to left
    (0-index [..., 3, 2, 1, 0])
// ~x & (x+1)     : Returns the number of the first LOW bit from right to
    left (power of 2, not the index)
// log2(~x & (x+1)) : Returns position of the first LOW bit from right to
    left (0-index [..., 3, 2, 1, 0])
// x = x | (x+1)  : Set HIGH of first bit from right to left
// x = x & (x-1)  : Set LOW of first bit from right to left
// x = x & ~y     : Set LOW in x the HIGH bits in y

// Iterates over the indices of the high bits in a mask
/// O(#bits_encendidos)
// for (int x = mask; x; x &= x-1) {
//    int i = __builtin_ctz(x);
// }

// Iterate all the submasks of a mask. (Iterate all submasks of all masks
    is O(3^n)).
/// O(2^(#bits_encendidos))
// for (int sub = mask; sub; sub = (sub-1)&mask) {
// }
```

13. Geometry

13.1. Geometry Template

```
const lf eps = 1e-9;
typedef double T;
struct pt {
  T x, y;
  pt operator + (pt p) { return {x+p.x, y+p.y}; }
  pt operator - (pt p) { return {x-p.x, y-p.y}; }
  pt operator * (pt p) { return {x*p.x-y*p.y, x*p.y+y*p.x}; }
  pt operator * (T d) { return {x*d, y*d}; }
  pt operator / (lf d) { return {x/d, y/d}; } /// only for floating point
  bool operator == (pt b) { return x == b.x && y == b.y; }
  bool operator != (pt b) { return !(*this == b); }
  bool operator < (const pt &o) const { return y < o.y || (y == o.y && x
      < o.x); }
  bool operator > (const pt &o) const { return y > o.y || (y == o.y && x
      > o.x); }
};
int cmp (lf a, lf b) { return (a + eps < b ? -1 :(b + eps < a ? 1 : 0)); }
/** Already in complex **/
T norm(pt a) { return a.x*a.x + a.y*a.y; }
lf abs(pt a) { return sqrt(norm(a)); }
lf arg(pt a) { return atan2(a.y, a.x); }
ostream& operator << (ostream& os, pt &p) {
  return os << "("<< p.x << "," << p.y << ")";
}
/***/
istream &operator >> (istream &in, pt &p) {
    T x, y; in >> x >> y;
    p = {x, y};
    return in;
}
T dot(pt a, pt b) { return a.x*b.x + a.y*b.y; }
T cross(pt a, pt b) { return a.x*b.y - a.y*b.x; }
T orient(pt a, pt b, pt c) { return cross(b-a,c-a); }
//pt rot(pt p, lf a) { return {p.x*cos(a) - p.y*sin(a), p.x*sin(a) +
    p.y*cos(a)}; }
//pt rot(pt p, double a) { return p * polar(1.0, a); } /// for complex
//pt rotate_to_b(pt a, pt b, lf ang) { return rot(a-b, ang)+b; }
pt rot90ccw(pt p) { return {-p.y, p.x}; }
pt rot90cw(pt p) { return {p.y, -p.x}; }
pt translate(pt p, pt v) { return p+v; }
pt scale(pt p, double f, pt c) { return c + (p-c)*f; }
bool are_perp(pt v, pt w) { return dot(v,w) == 0; }
int sign(T x) { return (T(0) < x) - (x < T(0)); }
pt unit(pt a) { return a/abs(a); }

bool in_angle(pt a, pt b, pt c, pt x) {
  assert(orient(a,b,c) != 0);
  if (orient(a,b,c) < 0) swap(b,c);
  return orient(a,b,x) >= 0 && orient(a,c,x) <= 0;
}

//lf angle(pt a, pt b) { return acos(max(-1.0, min(1.0,
    dot(a,b)/abs(a)/abs(b)))); }
//lf angle(pt a, pt b) { return atan2(cross(a, b), dot(a, b)); }
/// returns vector to transform points
pt get_linear_transformation(pt p, pt q, pt r, pt fp, pt fq) {
  pt pq = q-p, num{cross(pq, fq-fp), dot(pq, fq-fp)};
  return fp + pt{cross(r-p, num), dot(r-p, num)} / norm(pq);
}

bool half(pt p) { /// true if is in (0, 180]
  assert(p.x != 0 || p.y != 0); /// the argument of (0,0) is undefined
  return p.y > 0 || (p.y == 0 && p.x < 0);
}
bool half_from(pt p, pt v = {1, 0}) {
  return cross(v,p) < 0 || (cross(v,p) == 0 && dot(v,p) < 0);
}
bool polar_cmp(const pt &a, const pt &b) {
  return make_tuple(half(a), 0) < make_tuple(half(b), cross(a,b));
```

```
}

struct line {
  pt v; T c;
  line(pt v, T c) : v(v), c(c) {}
  line(T a, T b, T c) : v({b,-a}), c(c) {}
  line(pt p, pt q) : v(q-p), c(cross(v,p)) {}
  T side(pt p) { return cross(v,p)-c; }
  lf dist(pt p) { return abs(side(p)) / abs(v); }
  lf sq_dist(pt p) { return side(p)*side(p) / (lf)norm(v); }
  line perp_through(pt p) { return {p, p + rot90ccw(v)}; }
  bool cmp_proj(pt p, pt q) { return dot(v,p) < dot(v,q); }
  line translate(pt t) { return {v, c + cross(v,t)}; }
  line shift_left(double d) { return {v, c + d*abs(v)}; }
  pt proj(pt p) { return p - rot90ccw(v)*side(p)/norm(v); }
  pt refl(pt p) { return p - rot90ccw(v)*2*side(p)/norm(v); }
};

bool inter_ll(line l1, line l2, pt &out) {
  T d = cross(l1.v, l2.v);
  if (d == 0) return false;
  out = (l2.v*l1.c - l1.v*l2.c) / d;
  return true;
}
line bisector(line l1, line l2, bool interior) {
  assert(cross(l1.v, l2.v) != 0); /// l1 and l2 cannot be parallel!
  lf sign = interior ? 1 : -1;
  return {l2.v/abs(l2.v) + l1.v/abs(l1.v) * sign,
          l2.c/abs(l2.v) + l1.c/abs(l1.v) * sign};
}

bool in_disk(pt a, pt b, pt p) {
  return dot(a-p, b-p) <= 0;
}
bool on_segment(pt a, pt b, pt p) {
  return orient(a,b,p) == 0 && in_disk(a,b,p);
}
bool proper_inter(pt a, pt b, pt c, pt d, pt &out) {
  T oa = orient(c,d,a),
  ob = orient(c,d,b),
  oc = orient(a,b,c),
  od = orient(a,b,d);
  /// Proper intersection exists iff opposite signs
  if (oa*ob < 0 && oc*od < 0) {
    out = (a*ob - b*oa) / (ob-oa);
    return true;
  }
  return false;
}
set<pt> inter_ss(pt a, pt b, pt c, pt d) {
  pt out;
  if (proper_inter(a,b,c,d,out)) return {out};
  set<pt> s;
  if (on_segment(c,d,a)) s.insert(a);
  if (on_segment(c,d,b)) s.insert(b);
  if (on_segment(a,b,c)) s.insert(c);
  if (on_segment(a,b,d)) s.insert(d);
  return s;
}
lf pt_to_seg(pt a, pt b, pt p) {
  if(a != b) {
    line l(a,b);
    if (l.cmp_proj(a,p) && l.cmp_proj(p,b)) /// if closest to projection
      return l.dist(p); /// output distance to line
  }
  return min(abs(p-a), abs(p-b)); /// otherwise distance to A or B
}
lf seg_to_seg(pt a, pt b, pt c, pt d) {
  pt dummy;
  if (proper_inter(a,b,c,d,dummy)) return 0;
  return min({pt_to_seg(a,b,c), pt_to_seg(a,b,d),
              pt_to_seg(c,d,a), pt_to_seg(c,d,b)});
}

enum {IN, OUT, ON};
struct polygon {
  vector<pt> p;
  polygon(int n) : p(n) {}
  int top = -1, bottom = -1;
  void delete_repetead() {
    vector<pt> aux;
    sort(p.begin(), p.end());
    for(pt &i : p)
      if(aux.empty() || aux.back() != i)
        aux.push_back(i);
    p.swap(aux);
  }
  bool is_convex() {
    bool pos = 0, neg = 0;
    for (int i = 0, n = p.size(); i < n; i++) {
```

```
    int o = orient(p[i], p[(i+1)%n], p[(i+2)%n]);
    if (o > 0) pos = 1;
    if (o < 0) neg = 1;
  }
  return !(pos && neg);
}
lf area(bool s = false) {
  lf ans = 0;
  for (int i = 0, n = p.size(); i < n; i++)
    ans += cross(p[i], p[(i+1)%n]);
  ans /= 2;
  return s ? ans : abs(ans);
}
lf perimeter() {
  lf per = 0;
  for(int i = 0, n = p.size(); i < n; i++)
    per += abs(p[i] - p[(i+1)%n]);
  return per;
}
bool above(pt a, pt p) { return p.y >= a.y; }
bool crosses_ray(pt a, pt p, pt q) {
  return (above(a,q)-above(a,p))*orient(a,p,q) > 0;
}
int in_polygon(pt a) {
  int crosses = 0;
  for(int i = 0, n = p.size(); i < n; i++) {
    if(on_segment(p[i], p[(i+1)%n], a)) return ON;
    crosses += crosses_ray(a, p[i], p[(i+1)%n]);
  }
  return (crosses&1 ? IN : OUT);
}
void normalize() { /// polygon is CCW
  bottom = min_element(p.begin(), p.end()) - p.begin();
  vector<pt> tmp(p.begin()+bottom, p.end());
  tmp.insert(tmp.end(), p.begin(), p.begin()+bottom);
  p.swap(tmp);
  bottom = 0;
  top = max_element(p.begin(), p.end()) - p.begin();
}
int in_convex(pt a) {
  assert(bottom == 0 && top != -1);
  if(a < p[0] || a > p[top]) return OUT;
  T orientation = orient(p[0], p[top], a);
  if(orientation == 0) {
    if(a == p[0] || a == p[top]) return ON;
```

```
    return top == 1 || top + 1 == p.size() ? ON : IN;
  } else if (orientation < 0) {
    auto it = lower_bound(p.begin()+1, p.begin()+top, a);
    T d = orient(*prev(it), a, *it);
    return d < 0 ? IN : (d > 0 ? OUT: ON);
  }
  else {
    auto it = upper_bound(p.rbegin(), p.rend()-top-1, a);
    T d = orient(*it, a, it == p.rbegin() ? p[0] : *prev(it));
    return d < 0 ? IN : (d > 0 ? OUT: ON);
  }
}
polygon cut(pt a, pt b) {
  line l(a, b);
  polygon new_polygon(0);
  for(int i = 0, n = p.size(); i < n; ++i) {
    pt c = p[i], d = p[(i+1)%n];
    lf abc = cross(b-a, c-a), abd = cross(b-a, d-a);
    if(abc >= 0) new_polygon.p.push_back(c);
    if(abc*abd < 0) {
      pt out; inter_ll(l, line(c, d), out);
      new_polygon.p.push_back(out);
    }
  }
  return new_polygon;
}
void convex_hull() {
  sort(p.begin(), p.end());
  vector<pt> ch;
  ch.reserve(p.size()+1);
  for(int it = 0; it < 2; it++) {
    int start = ch.size();
    for(auto &a : p) {
      /// if colineal are needed, use < and remove repeated points
      while(ch.size() >= start+2 && orient(ch[ch.size()-2], ch.back(),
          a) <= 0)
        ch.pop_back();
      ch.push_back(a);
    }
    ch.pop_back();
    reverse(p.begin(), p.end());
  }
  if(ch.size() == 2 && ch[0] == ch[1]) ch.pop_back();
  /// be careful with CH of size < 3
  p.swap(ch);
```

```
}
vector<pii> antipodal() {
  vector<pii> ans;
  int n = p.size();
  if(n == 2) ans.push_back({0, 1});
  if(n < 3) return ans;
  auto nxt = [&](int x) { return (x+1 == n ? 0 : x+1); };
  auto area2 = [&](pt a, pt b, pt c) { return cross(b-a, c-a); };
  int b0 = 0;
  while(abs(area2(p[n - 1], p[0], p[nxt(b0)])) >
      abs(area2(p[n - 1], p[0], p[b0])))
    ++b0;
  for(int b = b0, a = 0; b != 0 && a <= b0; ++a) {
    ans.push_back({a, b});
    while (abs(area2(p[a], p[nxt(a)], p[nxt(b)])) >
        abs(area2(p[a], p[nxt(a)], p[b]))) {
      b = nxt(b);
      if(a != b0 || b != 0) ans.push_back({ a, b });
      else return ans;
    }
    if(abs(area2(p[a], p[nxt(a)], p[nxt(b)])) ==
     abs(area2(p[a], p[nxt(a)], p[b]))) {
      if(a != b0 || b != n-1) ans.push_back({ a, nxt(b) });
      else ans.push_back({ nxt(a), b });
    }
  }
  return ans;
}
pt centroid() {
  pt c{0, 0};
  lf scale = 6. * area(true);
  for(int i = 0, n = p.size(); i < n; ++i) {
    int j = (i+1 == n ? 0 : i+1);
    c = c + (p[i] + p[j]) * cross(p[i], p[j]);
  }
  return c / scale;
}
ll pick() {
  ll boundary = 0;
  for(int i = 0, n = p.size(); i < n; i++) {
    int j = (i+1 == n ? 0 : i+1);
    boundary += __gcd((ll)abs(p[i].x - p[j].x), (ll)abs(p[i].y -
        p[j].y));
  }
  return area() + 1 - boundary/2;
```

```
  }
  pt& operator[] (int i){ return p[i]; }
};

struct circle {
  pt c; T r;
};

circle center(pt a, pt b, pt c) {
  b = b-a, c = c-a;
  assert(cross(b,c) != 0); /// no circumcircle if A,B,C aligned
  pt cen = a + rot90ccw(b*norm(c) - c*norm(b))/cross(b,c)/2;
  return {cen, abs(a-cen)};
}
int inter_cl(circle c, line l, pair<pt, pt> &out) {
  lf h2 = c.r*c.r - l.sq_dist(c.c);
  if(h2 >= 0) {
    pt p = l.proj(c.c);
    pt h = l.v*sqrt(h2)/abs(l.v);
    out = {p-h, p+h};
  }
  return 1 + sign(h2);
}
int inter_cc(circle c1, circle c2, pair<pt, pt> &out) {
  pt d=c2.c-c1.c; double d2=norm(d);
  if(d2 == 0) { assert(c1.r != c2.r); return 0; } // concentric circles
  double pd = (d2 + c1.r*c1.r - c2.r*c2.r)/2; // = |O_1P| * d
  double h2 = c1.r*c1.r - pd*pd/d2; // = h2
  if(h2 >= 0) {
    pt p = c1.c + d*pd/d2, h = rot90ccw(d)*sqrt(h2/d2);
    out = {p-h, p+h};
  }
  return 1 + sign(h2);
}

int tangents(circle c1, circle c2, bool inner, vector<pair<pt,pt>> &out) {
  if(inner) c2.r = -c2.r;
  pt d = c2.c-c1.c;
  double dr = c1.r-c2.r, d2 = norm(d), h2 = d2-dr*dr;
  if(d2 == 0 || h2 < 0) { assert(h2 != 0); return 0; }
  for(double s : {-1,1}) {
    pt v = (d*dr + rot90ccw(d)*sqrt(h2)*s)/d2;
    out.push_back({c1.c + v*c1.r, c2.c + v*c2.r});
  }
  return 1 + (h2 > 0);
```

```
}

int tangent_through_pt(pt p, circle c, pair<pt, pt> &out) {
  double d = abs(p - c.c);
        if(d < c.r) return 0;
  pt base = c.c-p;
  double w = sqrt(norm(base) - c.r*c.r);
  pt a = {w, c.r}, b = {w, -c.r};
  pt s = p + base*a/norm(base)*w;
  pt t = p + base*b/norm(base)*w;
  out = {s, t};
  return 1 + (abs(c.c-p) == c.r);
}
```

14. Formulas

14.1. ASCII Table

Caracteres ASCII con sus respectivos valores numéricos.

No.	ASCII	No.	ASCII
0	NUL	16	DLE
1	SOH	17	DC1
2	STX	18	DC2
3	ETX	19	DC3
4	EOT	20	DC4
5	ENQ	21	NAK
6	ACK	22	SYN
7	BEL	23	ETB
8	BS	24	CAN
9	TAB	25	EM
10	LF	26	SUB
11	VT	27	ESC
12	FF	28	FS
13	CR	29	GS
14	SO	30	RS
15	SI	31	US

No.	ASCII	No.	ASCII
32	(space)	48	0
33	!	49	1
34	”	50	2
35	#	51	3
36	$	52	4
37	%	53	5
38	&	54	6
39	’	55	7
40	(	56	8
41	)	57	9
42	*	58	:
43	+	59	;
44	,	60	¡
45	-	61	=
46	.	62	¿
47	/	63	?

No.	ASCII	No.	ASCII
64	@	80	P
65	A	81	Q
66	B	82	R
67	C	83	S
68	D	84	T
69	E	85	U
70	F	86	V
71	G	87	W
72	H	88	X
73	I	89	Y
74	J	90	Z
75	K	91	[
76	L	92	\
77	M	93	]
78	N	94	ˆ
79	O	95	_

No.	ASCII	No.	ASCII
96	‘	112	p
97	a	113	q
98	b	114	r
99	c	115	s
100	d	116	t
101	e	117	u

102	f	118	v
103	g	119	w
104	h	120	x
105	i	121	y
106	j	122	z
107	k	123	{
108	l	124	|
109	m	125	}
110	n	126	~
111	o	127	

14.2. Summations

- $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$
- $\sum_{i=1}^{n} i^3 = (\frac{n(n+1)}{2})^2$
- $\sum_{i=1}^{n} i^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$
- $\sum_{i=1}^{n} i^5 = \frac{(n(n+1))^2(2n^2+2n-1)}{12}$
- $\sum_{i=0}^{n} x^i = \frac{x^{n+1}-1}{x-1}$ para $x \neq 1$

14.3. Misellanious Formulas

PERMUTACIÓN Y COMBINACIÓN	
Combinación (Coeficiente Binomial)	Número de subconjuntos de k elementos escogidos de un conjunto con n elementos. $\binom{n}{k} = \binom{n}{n-k} = \dfrac{n!}{k!(n-k)!}$
Combinación con repetición	Número de grupos formados por n elementos, partiendo de m tipos de elementos. $CR_m^n = \binom{m+n-1}{n} = \dfrac{(m+n-1)!}{n!(m-1)!}$
Permutación	Número de formas de agrupar n elementos, donde importa el orden y sin repetir elementos $P_n = n!$
Permutación múltiple	Elegir r elementos de n posibles con repetición n^r
Permutación con repetición	Se tienen n elementos donde el primer elemento se repite a veces , el segundo b veces , el tercero c veces, ... $PR_n^{a,b,c...} = \dfrac{P_n}{a!b!c!...}$
Permutaciones sin repetición	Núumero de formas de agrupar r elementos de n disponibles, sin repetir elementos $\dfrac{n!}{(n-r)!}$
DISTANCIAS	
Distancia Euclideana	$d_E(P_1,P_2) = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
Distancia Manhattan	$d_M(P_1,P_2) = \|x_2-x_1\| + \|y_2-y_1\|$
CIRCUNFERENCIA Y CÍRCULO	

Continúa en la siguiente columna

Continúa en la siguiente columna

Considerando r como el radio, α como el ángulo del arco o sector, y (R, r) como radio mayor y menor respectivamente.	
Área	$A = \pi * r^2$
Longitud	$L = 2 * \pi * r$
Longitud de un arco	$L = \frac{2 * \pi * r * \alpha}{360}$
Área sector circular	$A = \frac{\pi * r^2 * \alpha}{360}$
Área corona circular	$A = \pi(R^2 - r^2)$

TRIÁNGULO

Considerando b como la longitud de la base, h como la altura, letras minúsculas como la longitud de los lados, letras mayúsculas como los ángulos, y r como el radio de círcunferencias asociadas.	
Área conociendo base y altura	$A = \frac{1}{2}b * h$
Área conociendo 2 lados y el ángulo que forman	$A = \frac{1}{2}b * a * sin(C)$
Área conociendo los 3 lados	$A = \sqrt{p(p-a)(p-b)(p-c)}$ con $p = \frac{a+b+c}{2}$

Continúa en la siguiente columna

Área de un triángulo circunscrito a una circunferencia	$A = \frac{abc}{4r}$
Área de un triángulo inscrito a una circunferencia	$A = r(\frac{a+b+c}{2})$
Área de un triangulo equilátero	$A = \frac{\sqrt{3}}{4}a^2$

RAZONES TRIGONOMÉTRICAS

Considerando un triangulo rectángulo de lados a, b y c, con vértices A, B y C (cada vértice opuesto al lado cuya letra minuscula coincide con el) y un ángulo α con centro en el vertice A. a y b son catetos, c es la hipotenusa:
$sin(\alpha) = \frac{cateto\ opuesto}{hipotenusa} = \frac{a}{c}$
$cos(\alpha) = \frac{cateto\ adyacente}{hipotenusa} = \frac{b}{c}$
$tan(\alpha) = \frac{cateto\ opuesto}{cateto\ adyacente} = \frac{a}{b}$
$sec(\alpha) = \frac{1}{cos(\alpha)} = \frac{c}{b}$

Continúa en la siguiente columna

$csc(\alpha) = \frac{1}{sin(\alpha)} = \frac{c}{a}$	
$cot(\alpha) = \frac{1}{tan(\alpha)} = \frac{b}{a}$	
PROPIEDADES DEL MÓDULO (RESIDUO)	
Propiedad neutro	(a % b) % b = a % b
Propiedad asociativa en multiplicación	(ab) % c = ((a % c)(b % c)) % c
Propiedad asociativa en suma	(a + b) % c = ((a % c) + (b % c)) % c
CONSTANTES	
Pi	$\pi = acos(-1) \approx 3{,}14159$
e	$e \approx 2{,}71828$
Número áureo	$\phi = \frac{1+\sqrt{5}}{2} \approx 1{,}61803$

14.4. Time Complexity

Aproximación del mayor número n de datos que pueden procesarse para cada una de las complejidades algoritmicas. Tomar esta tabla solo como referencia.

Complexity	**n**
$O(n!)$	11
$O(n^5)$	50
$O(2^n * n^2)$	18
$O(2^n * n)$	22
$O(n^4)$	100
$O(n^3)$	500
$O(n^2 \log_2 n)$	1.000
$O(n^2)$	10.000
$O(n \log_2 n)$	10^6
$O(n)$	10^8
$O(\sqrt{n})$	10^{16}
$O(\log_2 n)$	-
$O(1)$	-

14.5. Theorems

- There is always a prime between numbers n^2 and $(n+1)^2$, where n is any positive integer
- There is an infinite number of pairs of the from $\{p, p+2\}$ where both p and $p+2$ are primes.
- Every even integer greater than 2 can be expressed as the sum of two primes.
- Every integer greater than 2 can be written as the sum of three primes.

14.6. Numbers of Divisors

- $\tau(n) = \prod_{i=1}^{k}(\alpha_i + 1)$

14.7. Euler Totient Properties

- $\phi(p) = p - 1$
- $\phi(p^e) = p^e(1 - \frac{1}{p})$
- $\phi(n * m) = \phi(n) * \phi(m)$ si $gcd(n, m) = 1$
- $\phi(n) = n(1 - \frac{1}{p_1})(1 - \frac{1}{p_2})...(1 - \frac{1}{p_k})$ donde p_i es primo y divide a n

14.8. Fermat Theorem

Let m be a prime and x and m coprimes, then:

- $x^{m-1} \mod m = 1$
- $x^k \mod m = x^{k \mod (m-1)} \mod m$
- $x^{\phi(m)} \mod m = 1$

14.9. Product of Divisors of a Number

$\mu(n) = n^{\frac{\tau(n)}{2}}$

- if p is a prime, then: $\mu(p^k) = p^{\frac{k(k+2)}{2}}$
- if a and b are coprimes, then: $\mu(ab) = \mu(a)^{\tau(b)}\mu(b)^{\tau(a)}$

14.10. Sum of Divisors of a Number

- $\sigma(n) = \prod_{i=1}^{k}(1 + p_i + ... + p_i^{\alpha_i}) = \prod_{i=1}^{k} \frac{p_i^{\alpha_i+1}-1}{p_i-1}$

14.11. Catalan Numbers

- $C_n = \frac{1}{n+1}\binom{2n}{n} = \frac{(2n)!}{(n+1)!n!}$ con $n \geq 0$, $C_0 = 1$ y $C_{n+1} = \frac{2(2n+1)}{n+2}C_n$
- 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, 208012, 742900, 2674440, 9694845, 35357670

14.12. Combinatorics

- Distribute N objects among K people
  $\left(\binom{n}{k}\right) = \binom{n+k-1}{k} = \frac{(n+k-1)!}{k!(n-1)!}$
- Hockey-stick identity
  $\sum_{i=r}^{n}\binom{i}{r} = \binom{n+1}{r+1}$

14.13. Burnside's Lema

$\#orbitas = \frac{1}{|G|}\sum_{g\in G}|fix(g)|$

1. **G**: Las acciones que se pueden aplicar sobre un elemento, incluyendo la identidad, eg. Shift 0 veces, Shift 1 veces...
2. **Fix(g)**: Es el número de elementos que al aplicar g vuelven a ser ser ellos mismos
3. **Órbita**: El conjunto de elementos que pueden ser iguales entre si al aplicar alguna de las acciones de G

14.14. DP Optimizations Theorems

Name	Original Recurrence	Sufficient Condition		
CH 1	$dp[i] = min_{j<i}\{dp[j] + b[j] * a[i]\}$	$b[j] \geq b[j+1]$Optionally $a[i] \leq a[i+1]$	$O(n^2)$	$O(n)$
CH 2	$dp[i][j] = min_{k<j}\{dp[i-1][k] + b[k] * a[j]\}$	$b[k] \geq b[k+1]$ Optionally $a[j] \leq a[j+1]$	$O(kn^2)$	$O(kn)$
D&Q	$dp[i][j] = min_{k<j}\{dp[i-1][k] + C[k][j]\}$	$A[i][j] \leq A[i][j+1]$	$O(kn^2)$	$O(kn\log n$
Knuth	$dp[i][j] = min_{i<k<j}\{dp[i][k] + dp[k][j]\} + C[i][j]$	$A[i,j-1] \leq A[i,j] \leq A[i+1,j]$	$O(n^3)$	$O(n^2)$

Notes:

- $A[i][j]$ - the smallest k that gives the optimal answer, for example in $dp[i][j] = dp[i-1][k] + C[k][j]$
- $C[i][j]$ - some given cost function
- We can generalize a bit in the following way $dp[i] = \min_{j<i}\{F[j] + b[j] * a[i]\}$, where $F[j]$ is computed from $dp[j]$ in constant time

14.15. 2-SAT Rules

- $p \rightarrow q \equiv \neg p \vee q$
- $p \rightarrow q \equiv \neg q \rightarrow \neg p$
- $p \vee q \equiv \neg p \rightarrow q$
- $p \wedge q \equiv \neg(p \rightarrow \neg q)$
- $\neg(p \rightarrow q) \equiv p \wedge \neg q$
- $(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$
- $(p \rightarrow q) \vee (p \rightarrow r) \equiv p \rightarrow (q \vee r)$
- $(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$
- $(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \vee q) \rightarrow r$
- $(p \wedge q) \vee (r \wedge s) \equiv (p \vee r) \wedge (p \vee s) \wedge (q \vee r) \wedge (q \vee s)$

14.16. Great circle distance or geographical distance

Great circle distance or geographical distance

- d = great distance, ϕ = latitude, λ = longitude, Δ = difference (all the values in radians)
- σ = central angle, angle form for the two vector
- $d = r * \sigma$, $\sigma = 2 * \arcsin(\sqrt{\sin^2(\frac{\Delta\phi}{2}) + \cos(\phi_1)\cos(\phi_2)\sin^2(\frac{\Delta\lambda}{2})})$

14.17. Heron's Formula

- $s = \frac{a+b+c}{2}$
- $Area = \sqrt{s(s-a)(s-b)(s-c)}$
- a, b, c there are the lenghts of the sides

14.18. Interesting theorems

- $a^d \equiv a^{d \mod \phi(n)} \mod n$
  if $a \in Z^{n_*}$ or $a \notin Z^{n_*}$ and $d \mod \phi(n) \neq 0$
- $a^d \equiv a^{\phi(n)} \mod n$
  if $a \notin Z^{n_*}$ and $d \mod \phi(n) = 0$
- thus, for all a, n and d (with $d \geq \log_2(n)$)
  $a^d \equiv a^{\phi(n)+d \mod \phi(n)} \mod n$

14.19. Law of sines and cosines

- a, b, c: lenghts, A, B, C: opposite angles, d: circumcircle
- $\frac{a}{sin(A)} = \frac{b}{sin(B)} = \frac{c}{sin(C)} = d$
- $c^2 = a^2 + b^2 - 2ab\cos(C)$

14.20. Pythagorean triples ($a^2 + b^2 = c^2$)

- Given an arbitrary pair of integers m and n with $m > n > 0$:
  $a = m^2 - n^2$, $b = 2mn$, $c = m^2 + n^2$
- The triple generated by Euclid's formula is primitive if and only if m and n are coprime and not both odd.
- To generate all Pythagorean triples uniquely:
  $a = k(m^2 - n^2)$, $b = k(2mn)$, $c = k(m^2 + n^2)$
- If m and n are two odd integer such that $m > n$, then:
  $a = mn$, $b = \frac{m^2-n^2}{2}$, $c = \frac{m^2+n^2}{2}$
- If $n = 1$ or 2 there are no solutions. Otherwise
  n is even: $((\frac{n^2}{4} - 1)^2 + n^2 = (\frac{n^2}{4} + 1)^2)$
  n is odd: $((\frac{n^2-1}{2})^2 + n^2 = (\frac{n^2+1}{2})^2)$

14.21. Sequences

Listado de secuencias mas comunes y como hallarlas.

Estrellas octangulares	0, 1, 14, 51, 124, 245, 426, 679, 1016, 1449, 1990, 2651, ...
	$f(n) = n * (2 * n^2 - 1)$.
Euler totient	1, 1, 2, 2, 4, 2, 6, 4, 6, 4, 10, 4, 12, 6,...
	$f(n)$ = Cantidad de números naturales $\leq n$ coprimos con n.
Números de Bell	1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, 115975, ...
	Se inicia una matriz triangular con f[0][0] = f[1][0] = 1. La suma de estos dos se guarda en f[1][1] y se traslada a f[2][0]. Ahora se suman f[1][0] con f[2][0] y se guarda en f[2][1]. Luego se suman f[1][1] con f[2][1] y se guarda en f[2][2] trasladandose a f[3][0] y así sucesivamente. Los valores de la primera columna contienen la respuesta.
Números de Catalán	1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, ...
	$f(n) = \frac{(2n)!}{(n+1)!n!}$
Números de Fermat	3, 5, 17, 257, 65537, 4294967297, 18446744073709551617, ...

Continúa en la siguiente columna

	$f(n) = 2^{(2^n)} + 1$
Números de Fibonacci	0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, ...
	$f(0) = 0$; $f(1) = 1$; $f(n) = f(n-1) + f(n-2)$ para $n > 1$
Números de Lucas	2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, ...
	$f(0) = 2$; $f(1) = 1$; $f(n) = f(n-1) + f(n-2)$ para $n > 1$
Números de Pell	0, 1, 2, 5, 12, 29, 70, 169, 408, 985, 2378, 5741, 13860, ...
	$f(0) = 0; f(1) = 1; f(n) = 2f(n-1) + f(n-2)$ para $n > 1$
Números de Tribonacci	0, 0, 1, 1, 2, 4, 7, 13, 24, 44, 81, 149, 274, 504, ...
	$f(0) = f(1) = 0; f(2) = 1; f(n) = f(n-1) + f(n-2) + f(n-3)$ para $n > 2$
Números factoriales	1, 1, 2, 6, 24, 120, 720, 5040, 40320, 362880, ...
	$f(0) = 1; f(n) = \prod_{k=1}^{n} k$ para $n > 0$.
Números piramidales cuadrados	0, 1, 5, 14, 30, 55, 91, 140, 204, 285, 385, 506, 650, ...
	$f(n) = \frac{n * (n+1) * (2 * n + 1)}{6}$
Números primos de Mersenne	3, 7, 31, 127, 8191, 131071, 524287, 2147483647, ...
	$f(n) = 2^{p(n)} - 1$ donde p representa valores primos iniciando en $p(0) = 2$.
Números tetraedrales	1, 4, 10, 20, 35, 56, 84, 120, 165, 220, 286, 364, 455, ...
	$f(n) = \frac{n * (n+1) * (n+2)}{6}$

Continúa en la siguiente columna

Números triangulares	0, 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, 105, ...
	$f(n) = \frac{n(n+1)}{2}$
OEIS A000127	1, 2, 4, 8, 16, 31, 57, 99, 163, 256, 386, 562, ...
	$f(n) = \frac{(n^4 - 6n^3 + 23n^2 - 18n + 24)}{24}$.
Secuencia de Narayana	1, 1, 1, 2, 3, 4, 6, 9, 13, 19, 28, 41, 60, 88, 129, ...
	$f(0) = f(1) = f(2) = 1; f(n) = f(n-1) + f(n-3)$ para todo $n > 2$.
Secuencia de Silvestre	2, 3, 7, 43, 1807, 3263443, 10650056950807, ...
	$f(0) = 2; f(n+1) = f(n)^2 - f(n) + 1$
Secuencia de vendedor perezoso	1, 2, 4, 7, 11, 16, 22, 29, 37, 46, 56, 67, 79, 92, 106, ...
	Equivale al triangular(n) + 1. Máxima número de piezas que se pueden formar al hacer n cortes a un disco. $f(n) = \frac{n(n+1)}{2} + 1$
Suma de los divisores de un número	1, 3, 4, 7, 6, 12, 8, 15, 13, 18, 12, 28, 14, 24, ...
	Para todo $n > 1$ cuya descomposición en factores primos es $n = p_1^{a_1} p_2^{a_2} ... p_k^{a_k}$ se tiene que: $f(n) = \frac{p_1^{a_1+1} - 1}{p_1 - 1} * \frac{p_2^{a_2+1} - 1}{p_2 - 1} * ... * \frac{p_k^{a_k+1} - 1}{p_k - 1}$

14.22. Simplex Rules

The simplex algorithm operated on linear programs in standard form:
Maximixe : $c^T \cdot x$
Subject to : $Ax \leq b, x_i \geq 0$

- $x = (x_1, ..., x_n)$ the variables of the problem
- $c = (c_1, ..., c_n)$ are the coefficients of the objective function
- A is a $p \times n$ matrix and $b = (b_1, ..., b_p)$ constants with $b_j \geq 0$