# Expected Value

Tech Lead Data Science

Master en Data Science
2022-2023

Assembler
Institute of Technology

# Index

# Introduction to Expected Value

# Expected value

A computer is programmed to produce a sequence of integers, $X$, from 0 to 3 inclusive, with probabilities as shown below:


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$$\frac{frequency}{total\ frequency} = probability$$

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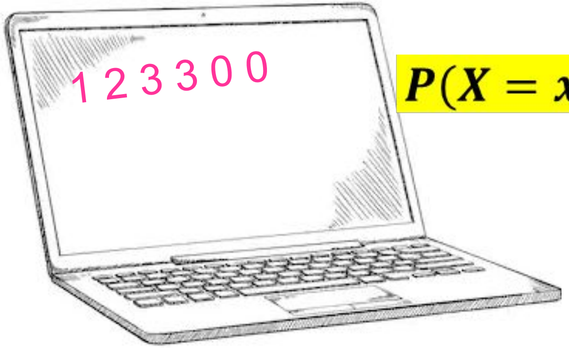


$$P(X = x_i) = p_i$$

| | $x_1$ | $x_2$ | $x_3$ | $x_4$ |
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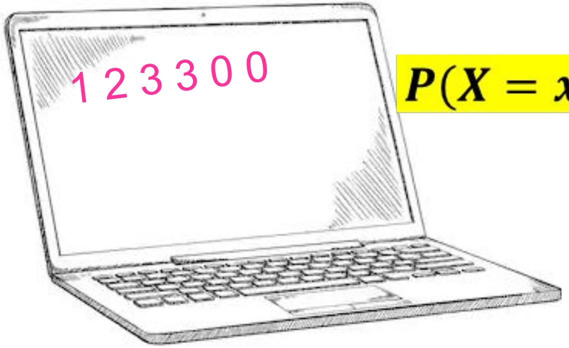


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$$\mu = E(X) = p_1 \times x_1 + p_2 \times x_2 + p_3 \times x_3 + p_4 \times x_4 =$$
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**Expectation or Expected value**

$$E(X) = \sum p_i x_i$$

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# Expected value

- In probability theory, **the expected value** is a generalization of the weighted average. Informally, the expected value is the arithmetic mean of a large number of independently selected outcomes of a random variable.
- The expected value of a random variable with a **finite number of outcomes** is a weighted average of all possible outcomes.

$$\mathrm{E}[X] = \sum_{i=1}^{\infty} x_i \, p_i \, ,$$

- In the case of a **continuum of possible outcomes**, the expectation is defined by integration. In the axiomatic foundation for probability provided by measure theory, the expectation is given by Lebesgue integration.

$$\mathrm{E}[X] = \int_{-\infty}^{\infty} x f(x) \, dx.$$

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# Properties

## PROPERTIES

- **Non-negativity:** If X ≥ 0 (a.s.), then E[ X ] ≥ 0 .
- **Linearity of expectation:**The expected value operator (or **expectation operator**) E [ · ] is linear in the sense that, for any random variables X and Y , and a constant a:
  - E [X + Y] = E [ X ] + E [ Y ]
  - E[ a X ] = a E [ X ]
- **Monotonicity:** If X ≤ Y, and both E[ X ] and E[ Y ] exist, then E[ X ] ≤ E[ Y ]
- **Constant:** If X is a random variable and P(X=c)=1, then E[X]=c