Mechanical Engineering

PRECISION MECHANISMS

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Requirements

- The mechanism must have 2 degrees of freedom (DOF)
- The accuracy is 10 micrometers over an area of 25 x 25 mm (on x y plane)
- The system should not exceed dimensions: 100 x 75 x 120 mm (L x W x H)
- The mechanism needs to move a cylinder 18 mm in diameter and 30 mm in height with a mass of 45 g

Degrees of freedom

The design of the mechanism that performs the smallest movements must be performed. The chosen design should have two DOF's and consists of wire flexures and rigid bodies connecting them. As can be seen in Fig 3, our design consists of exactly two rigid bodies. The first has a single DOF in x and is fixed to the mechanism of large stroke movement with five wire flexures that would become the fixed world. Two wire flexures are connected at the back of the rigid body in the y-direction and the other three are connected at the top in the z-direction. Through these connections it is possible to restrict five movements of the six possible, leaving only the translation in the x-direction. In order to get the translation in y, it was necessary to adapt a second body connected in such a way that it restricts all movements except the translation in y. Thus in combination, we would obtain the expected result. The second body is connected to the first by two wire flexures in the x-direction and to the fixed world with 3 wire flexures at the top in the z-direction.

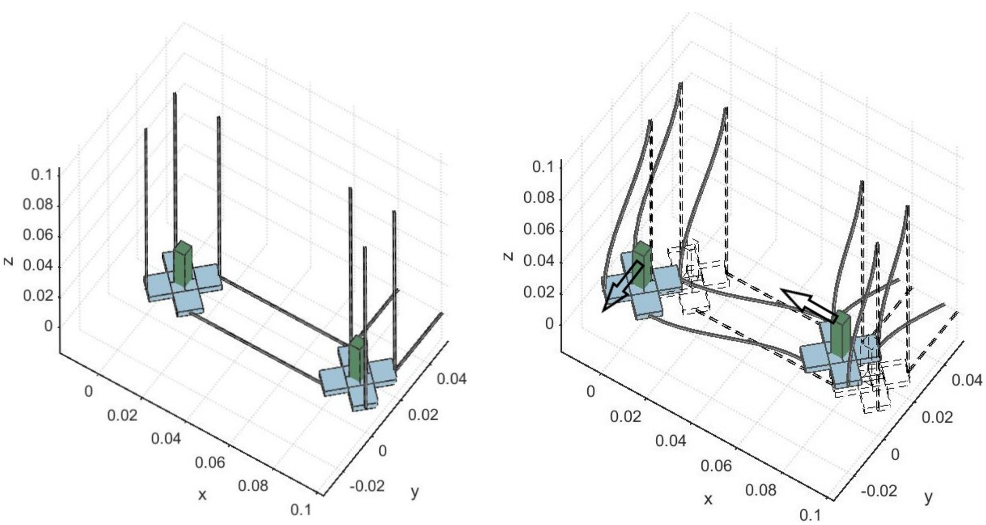

Figure 1: Design in a) static position and b) with forces in x and y applied

Design motivation

The rigid bodies are taken to be square plates in our design.The design complies with the maximum measures required and after some analysis some conclusions are explained in the following section regarding the other requirements as accuracy and the area our design could cover. It should be noted that in order to reduce unnecessary deformations in the design, a compliance center was taken as a point that is in the center of the rigid bodies and that is located at 1/5 the length of the wire flexures. In this way the force applied by the actuators in x and y direction can be directed to the compliance centers, thus reducing a very abrupt displacement especially in z direction upwards. As for the strengths of our design, we can indicate the ease of manufacturing it and also that the components are very light. As weak points of the design are the high frequency vibration of the same, due to long and slender components, it has an unwanted movement in z upwards so that the accuracy can be affected and it will be shown later big stresses will affect our design .

Wire flexures assumptions:

- length = 6 of 115 mm, 2 of 55 mm, 2 60 mm.
- radius = 1 mm
- material = Aluminium alloy (E = 73 GPa, G = 26 GPa, Ys = 340 Mpa)
- Density = 2700 kg/m^3

Rigid bodies assumptions:

- length = 20 mm
- width = 20 mm
- thickness = 3 mm
- Density = 7800 kg/m^3

Calculations

w = width , t = thickness

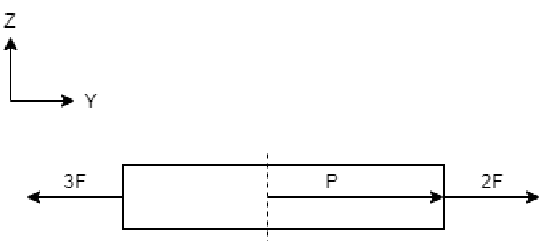

Figure 2: FBD for calculations

$$\sum Fx = P - 3F - 2F = 0 \tag{1}$$

$$\sum M0 = 2M - Nw + Pt = 0 \tag{2}$$

$$\Theta(L) = ML/EI + FL^2/2EI = 0 \tag{3}$$

$$v(L) = ML^2/2EI + FL^3/3EI \tag{4}$$

$$\sigma(x) = M(x)y/I \tag{5}$$

$$\sum M = M - M(x) - Fx = 0 \tag{6}$$

From eq. 3 its found that $M = -FL/2$ and from eq. 1 its found that $P = 5F$, then substitute this into eq. 2 that gives $N = -FL + 5Ft$. To find center of compliance N = 0, so its find that $t = l/5$. l = 115 mm which is the length of the wire flexures, so **t = 23 mm**.

To find the force needed to achieve a deflection of 12.5 mm in the flexures, the force should be solved from the deflection formula. From $P = 5F$ gives $F = P/5$ and substituting in $M = -FL/2$ gives $M = -PL/10$, then substituting into eq. 4 gives $PL^3/60EI$. Then, the force is solved giving $P = \frac{60EIv}{l1^3}$ But first to find the moment of area $I = \frac{5\pi}{4}r^4$ that gives **I = 2.4544e-13** m^4, then using values of assumptions gives **P = 8.83 N**

In order to find the maximum stress on the flex, it is cut, then the sum of moments is applied giving eq. 6, then gives $M(x) = M - Fx = -FL/2 - Fx$, the maximum stresses will be located at x = 0 and x = L, this give us $\sigma(0) = -FLy/2I$ and $\sigma(L) = -FLy/2I$. Then using values give us a result of $\sigma(0) = -206MPa$ and $\sigma(L) = 206MPa$

Numerical Analysis (Spacar)

The model was built with six wire flexures and two rigid bodies with the dimensions stated in Page 3. In order to model in spacar the rigid bodies had to be changed, 2 cross beams were used instead of a unique rigid body as in Fig 1. A total of 20 nodes and 20 elements were used in the model, with 8 fixed nodes.In order to see if it is possible for the mechanism to reach an area of 25mm*25mm, the center of the left rigid body was located at the center of this area and some actuator forces (assumptions) were used at the end points of the center of compliance of each rigid body shown in 3b. With the analysis in Spacar the mechanism could cover the area needed. For a single motion in Y direction, the most accurate result was achieved applying a force along the y axis of (Y = 6.18 N) at the center of compliance (top of green body on the left rigid body), reaching 12.5 mm either in positive or negative direction. The same analysis could be done for another cases of motion as for example, a single motion along the x direction with a force of (X = 8.9 N) or a combination of forces in x (right rigid body) and y (left rigid body) in order to reach the corner points located furthest from the center

The most important node to consider is node 5 (centering node). Also, it can be noticed that the stress resulting for a displacement to the corners of the area are the maximum, namely 1400 Mpa. this is clearly higher than the yield strength of the material selected, this means that our design will fail. This is because wire flexures used are not long enough in comparison to the deflection needed and in order to keep measurements within the requirements those members can not be longer.

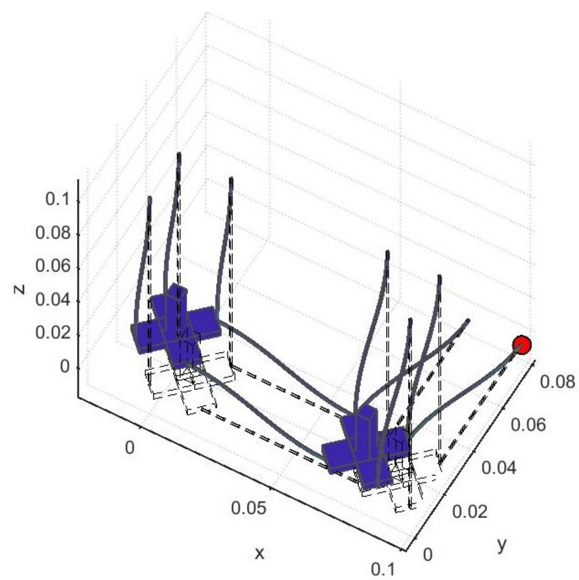

Figure 3: Maximum stress point at step(10)

Comparison with manual calculations

The manual calculations for a deflection of 12.5 mm in Y direction need 8.83 N of force, instead spacar makes the same displacement with a force of 6.18 N. This difference is mainly produced due to the fact that for manual calculations the moments of area were taking from the long wire flexures, assuming that all the flexures acting on the rigid body would be of the same length.

For the max stress, manual calculations give a result of $\sigma(0) = 206MPa$ and for spacar a result of 1400 MPa, this difference is caused because in manual calculations just was taken into account one flexure (one of the long ones) but because spacar take all of them into account, it find the biggest stress in one of the shortest flexures.

Ideal Physical Model (IPM) and Free body diagram (FBD)

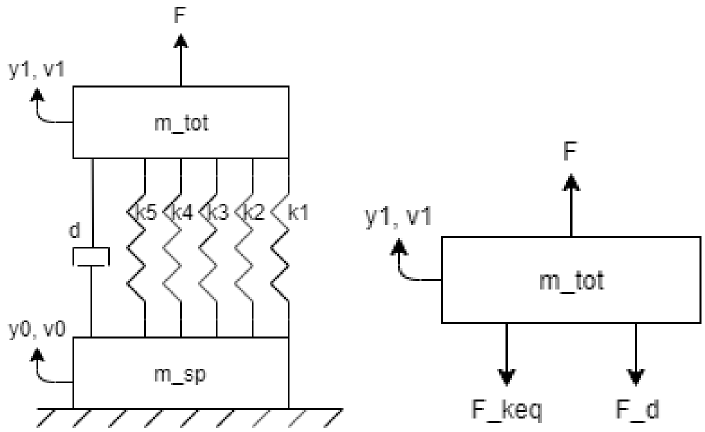

Figure 4: Ideal Physical Model (IPM) and Free body diagram (FBD)

- First, the masses are defined where the total mass is equal to

$$m_{tot} = m_{rb} + 5m_{eq} + 0.045 \tag{7}$$

  To find the m_{eq} of one of the flexures the following formula is used

$$m_{eq} = \frac{33}{40} m_{fl} \tag{8}$$

  where m_{fl} is the mass of one flexure, m_{rb} is the mas of the rigid body and 0.045 is the mass of the vacuum device. This gives a total mass of **0.9815 kg**.

- Second, the spring constant equivalent for one flexure is defined as $k_{eq} = \frac{3Em_{fl}R^2}{2l^3}$, the formula already include the moment of inertia of a cylinder. In this case we have 5 flexures so a total constant was defined as $k_{tot} = 5k_{eq}$, giving as a result **603.7174 Ns/m**.

  After making a sum of forces in y direction the following differential equations was found:

$$m_{tot} * \ddot{y}(t) + Fk_{tot} * y(t) + Fd * \dot{y}(t) = F(t) \tag{9}$$

  From the last equation the eigenfrequency can be calculated, it gives the following equation:

$$w_n = \sqrt{\frac{k_{tot}}{m_{tot}}} \tag{10}$$

  introducing values in the equation give us a eigenfrequency of **24.80 rad/s**.

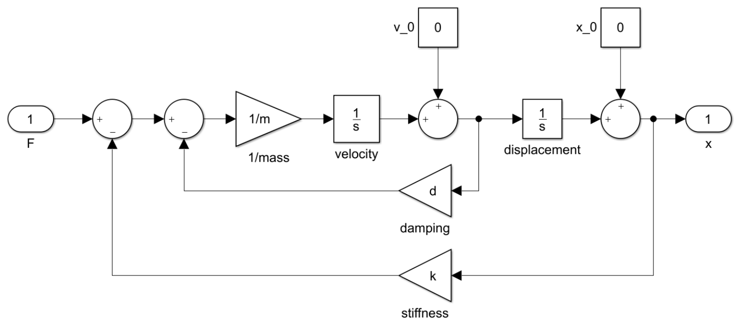

Figure 5: Block diagram

Transfer functions, poles, zeros and eigenfrequency

In order to determine the transfer function, eq.9 is necessary. First, it is necessary to say that all initial values are declared as zero and therefore these are not going to be taken into account. So, we substitute s^n depending the order of the variable y, this gives

$$m_{tot} * s^2 y(s) + Fk_{tot} * y(s) + Fd * sy(s) = F(s) \tag{11}$$

Then we carry the output y(s) to the left-hand side and the other terms to the right-hand side, which give us

$$y(s) = \frac{1}{ms^2 + k_{tot} + ds} F(s) \tag{12}$$

giving the transfer function

$$G(s) = \frac{1}{ms^2 + ds + k_{tot}} = \frac{1}{0.9815s^2 + 17.92s + 90838} \tag{13}$$

Now to find the poles and zeros, we have to equalize the numerator and denominator of the transfer function to zero. The numerator is 1 solo it is not possible to have zeros. To find the poles the equation is given as $ms^2+ds+k_{tot} = 0$, after solving the quadratic equation two poles are found $-0.7441 \pm 24.7906i$. The eigenfrequency is equal to the imaginary part of the poles so **24.79 rad/s**.

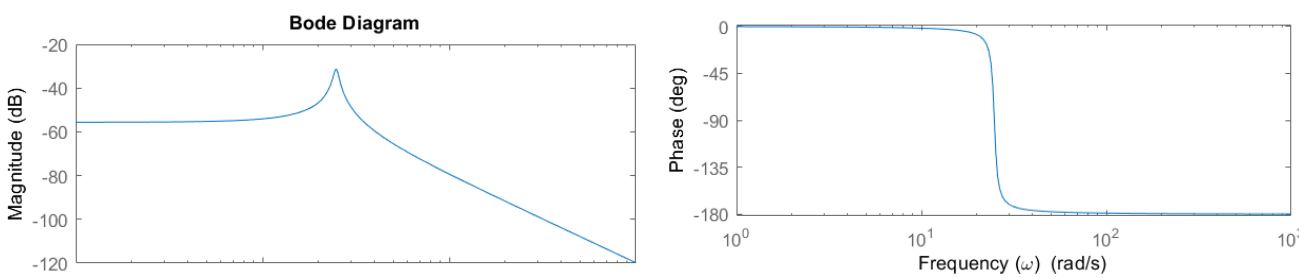

Figure 6: Bode diagram