CS541 Homework 1

Homogeneous coordinates: Homogeneous coordinates are called such because that word means "all of the same or similar kind or nature". This problem should make sense of that designation by showing that any point, transformation matrix, or plane equation has a multitude of representations, all scalar multiples of each other.

Definitions: Let

- $P = (x,\ y,\ z,\ w)$ be a homogeneous point,
- $(x/w,\ y/w,\ z/w)$ be the "real" point represented by P,
- M be any 4x4 transformation matrix
  with elements $\begin{bmatrix} m_{11} & m_{12} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix}$, and
- $Q = [A,\ B,\ C,\ D]$ be the coefficients of any plane equation.
  Note that the usual plane equation formula $Ax + By + Cz + D = 0$
  becomes $QP = 0$ in the (usual) case of $P = (x, y, z, 1)^T$.

In problems 2-6, use the results of 1 whenever possible instead of breaking out the individual coordinates of points, vectors, and plane equations.
In particular do not write kM as $\begin{bmatrix} km_{11} & km_{12} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix}$

Hint: *Yes, most of these problems really are as easy as showing that factors of k in both numerators and denominators cancel out. Nevertheless, all these facts are worth noticing and understanding.*

Show the following:

1. Any scalar multiple of P, say kP, represents the same real point as P.

2. Matrix M will transform P and any scalar multiple kP to (different) homogeneous points that represent the same real point.

3. Matrix M and any scalar multiple of M will transform P and any scalar multiple of P to a multitude of homogeneous points, all of which represent the same real point.

4. If $Q = [A, B, C, D]$ represents the plane of all points (x,y,z) which satisfy $Ax + By + Cz + D = 0$ then any non-zero scalar multiple of Q represents the same plane. That is, a point is on Q if and only if it is on kQ.

5. **Why is this statement FALSE when it is so similar to the preceding statements:** If P is in the solution set of that plane equation, (i.e., P is on the plane), then any non-zero scalar multiple of P satisfies the plane equation.

6. So... In light the failure in (5), change the definition of the plane represented by Q to be the solution set of this new equation $Ax + By + Cz + Dw = 0$. Now, show that both (4) and (5) hold true with this new definition of a plane equation.