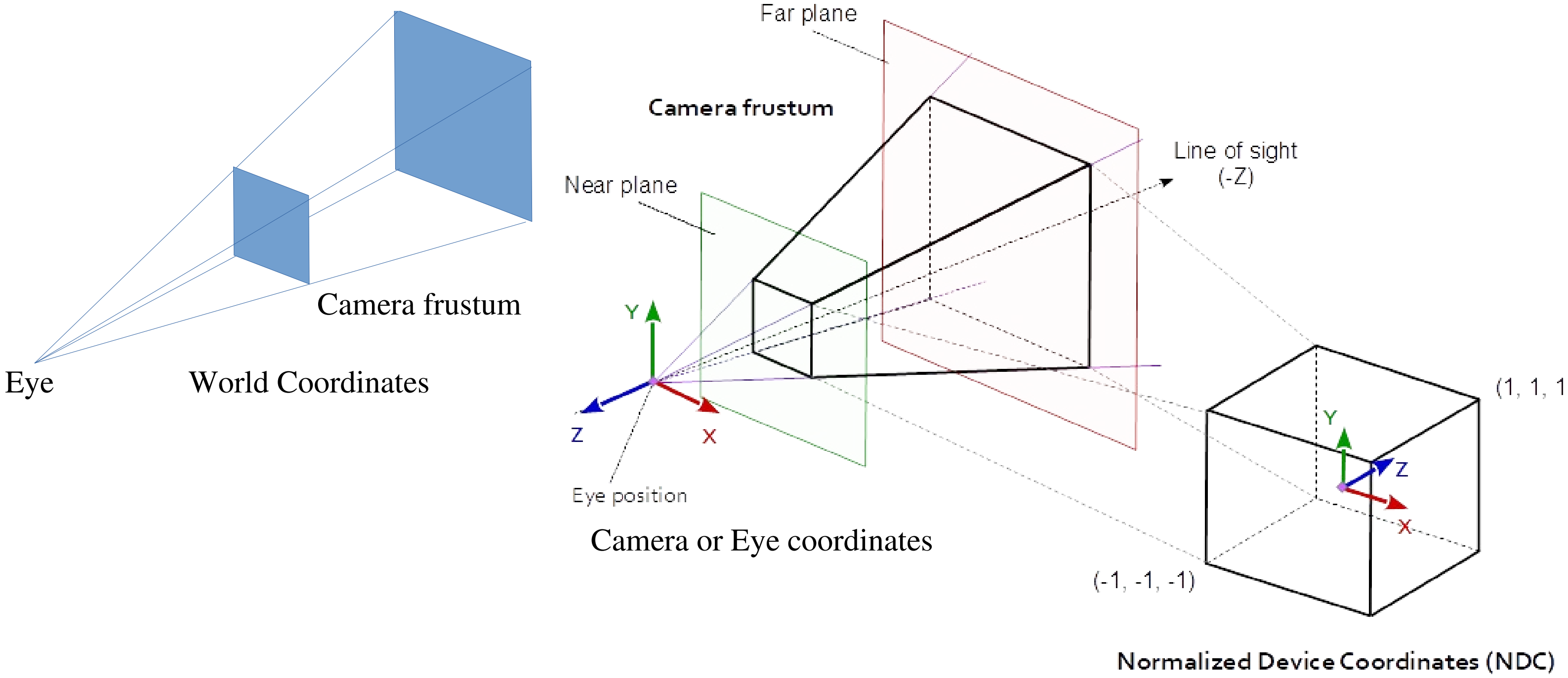
Perspective



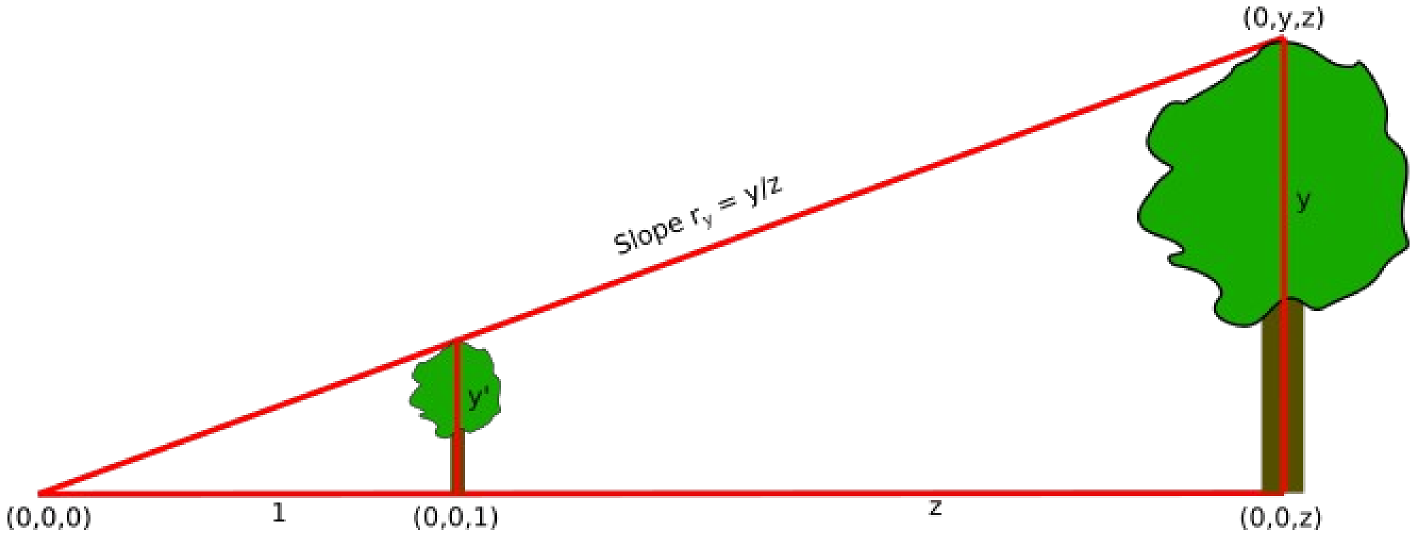
Perspective Transformation Objective

Map the region within the frustum to the NDC cube.

The frustum is the region of space projecting onto a screen.

The NDC cube is the space the GPU operates within.

This map requires both a **4x4 linear transformation** and a separate **division**.



Perspective requires a division

The perspective projection of a point (x,y,z) onto a view plane z=1

By similar triangles:

$\frac{y'}{1} = \frac{y}{z}$, and similarly for x :

$(x', y') = (x/z, y/z)$

But we don't want $(x', y', z') = (x/z, y/z, z/z)$

because z/z loses all depth information

Homogeneous coordinates when $w \neq 0$

Use 4D points (x,y,z,w) to represent 3D points like this

$(x, y, z, w) \rightarrow (x/w, y/w, z/w)$

For non-zero scalars:

$(sx, sy, sz, sw) \rightarrow (sx/sw, sy/sw, sz/sw) = (x/w, y/w, z/w)$

We can interpret $w=0$ as:

points at infinity., or

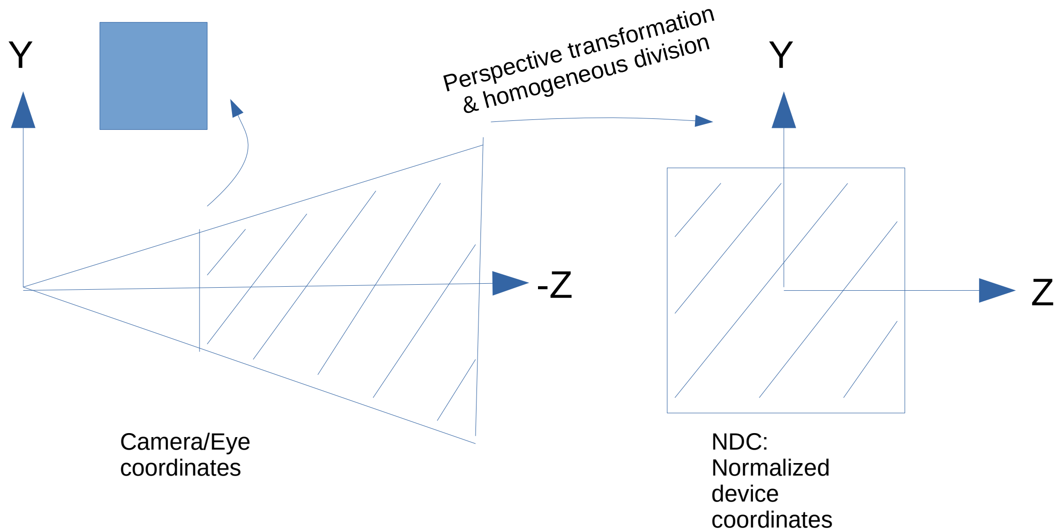
vectors (directions)

We can rig w to contain a useful quantity for perspective

Homogeneous coordinates when $w=0$

Consider this sequence as $w \rightarrow 0$.

$(x, y, z, 1.0)$	$\rightarrow (x, y, z)$
$(x, y, z, 0.1)$	$\rightarrow 10(x, y, z)$
$(x, y, z, 0.001)$	$\rightarrow 1000(x, y, z)$
…	…
$(x, y, z, 0.0)$	$\rightarrow$ infinity in direction (x, y, z) or just direction (x, y, z)



Perspective projection transformation

Eye at origin, looking along the -Z axis

Perspective transformation maps frustum to $[\pm 1, \pm 1, \pm 1]$

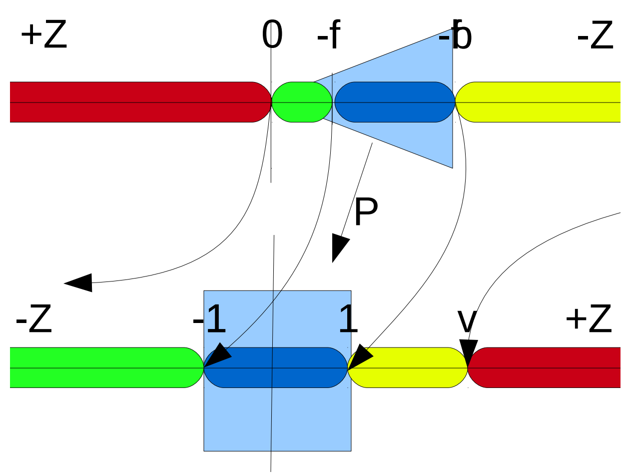
Frustum specified by 4 parameters:

- r_x : half width to viewing distance ratio
- r_y : half height to viewing distance ratio
- f : distance to view plane
- b : distance to far plane

$$P(r_x, r_y, f, b) = \begin{bmatrix} \frac{1}{r_x} & 0 & 0 & 0 \\ 0 & \frac{1}{r_y} & 0 & 0 \\ 0 & 0 & -\frac{b+f}{b-f} & -\frac{2fb}{b-f} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

Effect on depth values (note the Z-axis reversal)

$$\begin{array}{ccccccccccc} (& \infty & : & 0 &) & \rightarrow & (& v & : & \infty &) \\ (& 0 & : & -f &) & \rightarrow & (& -\infty & : & -1 &) \\ (& -f & : & -b &) & \rightarrow & (& -1 & : & +1 &) \\ (& -b & : & -\infty &) & \rightarrow & (& +1 & : & v &) \end{array}$$



Homogeneous coordinate facts

(x, y, z, w) when $w \neq 1$ is a "finite" point

Homogeneous division gives the associated 3D point: $(x, y, z, w) \rightarrow (x/w, y/w, z/w)$

$(x, y, z, 0)$ has multiple related interpretations:

vector (x, y, z)

point at ∞ in direction (x, y, z)

intersection of parallel lines in direction (x, y, z)

vanishing point in direction (x, y, z)

These facts can be seen by trying:

Translate (x, y, z, w) for both $w=0$ and $w \neq 0$

Consider $\lim_{w \to 0} (x, y, z, w)$

Some notes about Projection coordinate systems

In truth, this C.S. is most useful **before** the homogeneous division.

That is given a point (x,y,z,w) we have stated the bounds of Projection space as

$$-1 \leq \frac{x}{w} \leq +1$$

$$-1 \leq \frac{y}{w} \leq +1$$

$$0 \leq \frac{z}{w} \leq +1$$

but consider that (-x,-y,-z,-w) also satisfies these bounds.

The division by w loses (sign) info! We can't allow that!

This works:

$$-w \leq x \leq +w$$

$$-w \leq y \leq +w$$

$$0 \leq z \leq +w$$

Stereo projections

Require a shear before projection:

x_e is the eye's offset in the x direction

x_s is the screen center's offset in the x direction

$$\begin{bmatrix} 1 & 0 & \frac{x_e - x_s}{d} & -x_e \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Ways to present stereo images:

- Side-by-side
  - cross-eyed
  - wall-eyed aka parallel-eyed
  - end-on-mirror
  - double mirror system
  - head mounted display
  - stereo viewer aka transparency viewer

- 3D glasses
  - prismatic & self-masking crossview glasses
  - red-blue glasses (actually red-cyan) aka anaglyph
  - linearly polarized glasses and a silvered screen
  - circularly polarized glasses
  - flicker with synchronized glasses aka alternate frame sequencing
  - interference filter (infitec) glasses
  - ColorCode 3D

- other
  - lenticular sheet
  - parallax barrier
    - varifocal lens