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Graph Coloring Models for Production Line Scheduling Optimization

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I. INTRODUCTION

Production scheduling is a necessity for companies that want to remain competitive in increasingly dynamic markets [1]. As a result, it has become a field of extensive research in order to find effective approaches to this problem. In particular, production lines are the backbone of production and require precise time management. Any delays in the manufacture of a single component can trigger a chain reaction in every company that depends on it.

In recent years, the rise of industry 5.0 [2] has further broadened the scope of production scheduling, introducing not only economic and operational considerations, but also ecological and environmental ones. The environmental footprint caused by manufacturing has become a critical concern, with production lines accounting for 12% of emissions in the US alone [3] due to high energy consumption and inefficient resource allocation. As global industries expand, these inefficiencies compound, amplifying these environmental burdens.

Addressing these challenges requires the integration of optimization techniques that enable a simultaneous focus on environmental objectives, in addition to economic and operational goals. By incorporating resource allocation and scheduling strategies, we aim to significantly reduce the environmental footprint of manufacturing systems without compromising productivity. This optimization represents not only a technological opportunity but also an environmental necessity.

Thus, the central problem addressed in this paper concerns how to schedule tasks across production lines in a way that optimizes resource utilization while minimizing energy consumption and environmental impact. Traditional scheduling approaches often prioritize productivity metrics, often overlooking ecological concerns. Therefore, there is a need for methods that integrate sustainability directly into production planning.

Specifically, we will cover a graph-theoretic optimization method based on the graph coloring problem to reduce idle times, balance workloads, and minimize energy consumption, ultimately enhancing both operational and environmental performance. [4] [5] [6].

FIGURE 1: The environmental footprint caused by production lines has become a critical concern. Source: Wikimedia Commons

In summary, the main contributions of this paper are:

- ...
- ...
- ...

The rest of the paper will be structured as follows: it will start with Section II, where...

II. RELATED WORKS

Graph coloring has always been considered a fundamental problem in the combinatorial optimization field, often being applied to scheduling problems. The act of assigning colors to graph vertices in a way that no two connected vertices share a color naturally lends itself to many resource allocation and conflict avoidance scenarios. As such, many works have explored algorithms that try to solve the problem itself as well as how they can be applied to diverse scheduling tasks.

The idea of applying the graph coloring problem to these scheduling tasks is not unfounded. In [4] they applied various approaches, such as iterated local search algorithms to the well-known timetable scheduling problem and found that approaching the task as a graph coloring problem was the most efficient way to solve it, causing the least conflicts in allocation. On the same note, [7] and [8] also explored in depth which genetic operators offered the greatest results to solve the graph coloring problem.

The most common scheduling problem tackled by applying graph coloring is **resource allocation**. For example, in [5], by representing tasks as graph vertices and conflicts as edges, graph coloring is used to schedule resource projects efficiently while minimizing conflicts between tasks that require shared resources. This allows for conflict-free schedules with improved resource utilization.

Similarly, this resource allocation approach was applied to cloud computing. [6] investigates how graph coloring can be utilized to dynamically allocate virtual resources to tasks, reducing execution times for most tasks by optimizing task-to-resource assignments under capacity constraints.

In [9] they take it a step further and use graph coloring problem algorithms as an objective function in clustering of jobs with different priorities across multiple shifts, which were subject to setup and capacity constraints. The objective was to minimize the number of urgent clusters (clusters that contain at least one urgent job) and the total number of clusters.

A notable number of recent works focus on **mixed graph coloring problems**, an extension of graph coloring where some edges may be directed. This is due to their applicability to scheduling tasks where not only do some tasks conflict with one another, but some tasks do precede others – two very recent examples being in [1] and [10]. The former article applied mixed graphs to the job-shop scheduling problem (where the operations of each job are completely ordered), proposing an algorithm to minimize both execution and job completion time, while the latter addressed parallel execution environments in the context of multiprocessor task scheduling, where minimizing idle processor time is crucial.

The evolution and recent developments of the mixed graph coloring problem were extensively surveyed by the main author of the previously mentioned multiprocessor task scheduling article in [11], which provides a comprehensive overview of the development of mixed graph coloring techniques and their broad applicability to various resource scheduling contexts.

Lastly, more recent work has explored modern techniques as an approach to graph coloring. [12] introduces a neural network based method for finding minimal-cost colorings, demonstrating promising results on complex graphs, relevant to more contrived scheduling tasks.

Overall, these works illustrate the variety of applications of graph coloring to scheduling problems across multiple domains, reflecting the continued relevance of graph coloring as a tool for addressing scheduling challenges.

III. STATEMENT OF THE PROBLEM

The Graph Coloring Problem is a classical combinatorial optimization problem defined on an undirected graph $G = (V, E)$, where V denotes the set of vertices and $E \subseteq V \times V$ the set of edges connecting pairs of vertices. The objective of the Graph Coloring Problem is to assign a color $d(v) \in D$ from a finite color set D to each vertex $v \in V$ so that no two adjacent vertices share the same color.

$$d(u) \neq d(v), \quad \forall (u, v) \in E.$$

The goal is to minimize the total number of colors $|D|$ required to achieve a valid coloring. This problem is NP-Hard [9].

The Graph Scheduling Problem has been found to be equivalent to General Shop Scheduling Problems [10]. The Job Shop Scheduling problem consists of a set of jobs $J = \{J_1, J_2, \ldots, J_s\}$ that must be processed on different machines $M = \{M_1, M_2, \ldots, M_m\}$. Each Job $J_k \in J$ consists of a set of ordered operations $O = \{O_1, O_2, \ldots O_o\}$ that need to be performed on specific machines $M_t \in M$.

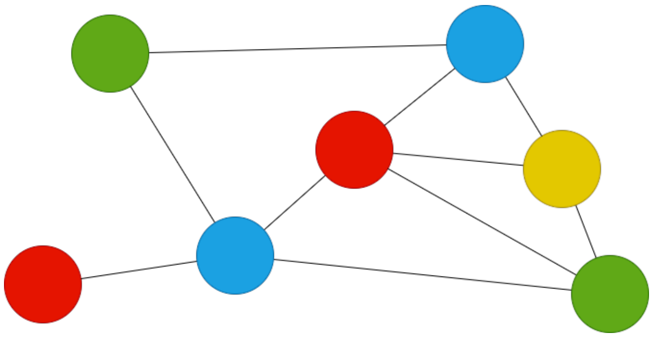

FIGURE 2: Example of a solved graph coloring problem G(7,10).

Two operations of the same job cannot be performed at the same time. Machines spend t_n time units (slots) on each operation. To solve this problem, it is necessary to find a schedule that minimizes the completion time $C_{\max} = \max\{C_1, C_2, \ldots, C_s\}$ of the problem while following the two restrictions without generating conflicts. [10]

$$J|t_n|C_{max}$$

To preserve analogy with the graph coloring problem, operations O are modeled as vertex V, task conflicts as edges E, and the minimization objective as colors D.

In this paper, we are going to solve a Job Scheduling problem modeled as a Graph Coloring Problem to minimize the ecological impact of a production line.

A. MAIN OBJECTIVE

To address the environmental concerns of our problem, we consider two possible strategies: minimizing the total operating time of machines and minimizing the number of machines operating simultaneously. Although the second objective may appear counterintuitive, it reduces overall energy consumption by maximizing the utilization of active machines [13]. In addition, this strategy mitigates energy consumption peaks, making it the preferred objective in our formulation. Therefore, we define our problem as follows.

$$\min F(x) = M(x) \tag{1}$$

$$M(x) = \sum_{t=0}^{T_{\max}} |M_t| \tag{2}$$

Where M_t is the set of machines active in slot t,$|M_t|$ is the number of machines that work simultaneously in slot t and $T_{\max}$ is the total number of slots in the scheduling horizon.

B. CONSTRAINTS

In (1) We are only minimizing the number of machines per time slot, however, we need to maintain the same restrictions as the original scheduling problem. This is formulated as constraints. Each constraint represents edges in the graph of the problem.

1) Machine conflict constraint

A single machine can execute only one operation at a time, analogous to a vertex in a graph being connected only with different color vertex. Consequently, our formulation must prevent multiple operations from being scheduled on the same machine within a single time slot. This constraint can be expressed as.

$$P_{\text{mach}} = \sum_{i,j}^{O} \sum_{\substack{i<j \\ M_i = M_j}} \text{overlap}(i,j) \tag{3}$$

Where P_{mach} is the total penalty due to overlapping operations assigned to the same machine, O is the total number of operations and M_i, M_j are the machines assigned to the operations i and j, respectively.

The overlap function can be mathematically represented by.

$$\text{overlap}(i,j) = \begin{cases} s_i + d_i - s_j & \text{if } s_i < s_j < s_i + d_i \\ s_j + d_j - s_i & \text{if } s_j < s_i < s_j + d_j \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

Where s_i, s_j are the start times (slot indices) of operations i and j and d_i, d_j are the durations (in slots) of operations i and j.

2) Precedence conflict constraint

As exposed previously, operations on each job are ordered. To be able to express this in the Graph Coloring Problem, we need to create a new constraint. It is described by the following expression.

$$P_{\text{prec}} = \sum_{i,j}^{O} \max\left(0,\ (s_i + d_i) - s_j\right) \tag{5}$$

Where P_{prec} is the total penalty for violating job precedence constraints, O is the total number of operations, $(s_i + d_i)$ is the completion time of the operation i and s_j is the start time of its successor operation j.

3) Makespan penalty

If we consider only the number of machines operating per time slot, the total completion time (makespan) may increase uncontrollably, leading to counterproductive outcomes. To mitigate this risk, we introduce a makespan penalty that discourages excessively long schedules. This constraint is defined as follows:

$$C_{\max} = \max\left(s_s + d_s\right) \tag{6}$$

Where $C_{\max}$ is the makespan, the completion time of the last operation in the schedule.

C. FINAL PROBLEM DEFINITION

If we combine the function (1) with the constraints (3), (5) and (6) we obtain the following equation.

$$\begin{aligned} \min\ & F(x) = \gamma M(x) + \delta C_{max} \\ \text{s.t. } & P_{\text{mach}} = 0, \\ & P_{\text{prec}} = 0, \\ & C_{\text{max}} > 0 \\ & \gamma > \delta \end{aligned} \tag{7}$$

Where $F(x)$ is the total cost function to minimize, $M(x)$ represents the sum of machines used in each slot, C_{max} is the makespan, γ and δ are coefficients, P_{mach} is the amount of time the operations overlap and P_{prec} is the number of precedence conflicts .

D. ASSUMPTIONS

Assumption 1: In this problem, we assume that reducing energy consumption leads to a corresponding decrease in environmental pollution.

Assumption 2: We assume that all machines consume energy at the same rate. If this were not the case, the total number of machines operating in each time slot would need to be weighted by their respective levels of energy consumption.

$$M(x) = \sum_{t=0}^{T_{\text{max}}} |E_t \cdot M_t|$$

Where E_t is the energy consumption of machine t M_t.

Assumption 3: We assume that every operation takes an exact number of time slots.

Assumption 4: We assume time slots of 1 minute.

Assumption 5: We assume that all machines are available at any time and remain in working condition.

Assumption 6: We assume that no external factors alter the original scheduling problem.

E. DATA SET ACQUISITION

The data set used in this study was obtained from [14]. It contains a collection of Job Scheduling Problem instances compiled from key publications in the scheduling field. In total, it includes 90 problem instances, some of which provide the optimal makespan values. This data set enables an evaluation of our proposed approach in various scheduling scenarios.

IV. PROPOSED EVOLUTIONARY APPROACH

To solve the problem defined in Section III, we propose an evolutionary optimization approach based on a Genetic Algorithm (GA). Genetic Algorithms are stochastic search heuristics inspired by the process of natural selection [15], [16]. They are particularly suitable for combinatorial optimization problems such as scheduling and resource allocation, where the search space is discrete and highly non-linear [17]. Our proposed GA iteratively evolves a population of candidate schedules represented as graph colorings, with the goal of minimizing the total number of active machines (proxy for energy consumption) and the overall makespan.

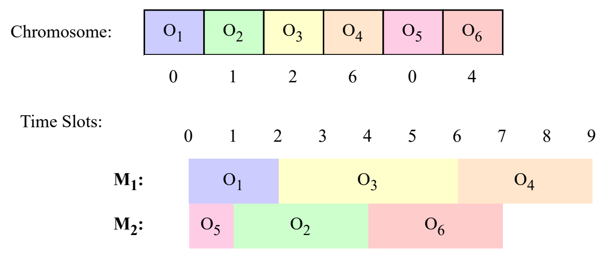

FIGURE 3: Chromosome representation and it's equivalent schedule.

A. CHROMOSOME REPRESENTATION

Each individual in the population represents a possible solution to our scheduling problem. The chromosome is encoded as an integer vector $X = [s_1, s_2, \ldots, s_n]$ (see Fig.3 for an illustrative example), where each gene $s_i \in \mathbb{Z}^+$ represents the starting slot of operation i, the length of the chromosome corresponding to the total number of operations in the instance.

This representation is particularly advantageous for us because it maintains a direct correspondence with the graph coloring analogy we employed: each operation is a vertex, each color corresponds to a time slot, and constraints ensure that adjacent vertices (i.e., conflicting operations) receive different colors. This encoding also facilitates the detection of precedence and machine conflicts, as both can be computed directly from the vector of start times.

B. POPULATION INITIALIZATION

Our initial population P_0 is designed to combine random and heuristic initialization. The individuals are constructed using a *greedy job-based heuristic* which, starting from a randomized individual, corrects the operations of their schedule sequentially according to their job order and assigns each operation to the earliest available time slot on its required machine. This heuristic ensures that the initial population is feasible or near-feasible.

C. FITNESS EVALUATION

The fitness function evaluates the quality of each individual based on the objective function and the constraints defined in Section III. It combines penalties for constraint violations with environmental and temporal objectives as follows:

$$f(X) = \frac{1}{1 + \alpha P_{\text{mach}} + \beta P_{\text{prec}} + \gamma M(X) + \delta C_{\text{max}}} \tag{8}$$

Where P_{mach} and P_{prec} are the penalty terms for machine and precedence conflicts, respectively, $M(X)$ is the total machine activation time (energy proxy) defined in (1), and C_{max} is the makespan penalty introduced in (6). The weighting parameters $\alpha, \beta, \gamma, \delta$ ensure a balance between the feasibility of an individual and the optimization pressure, with $\alpha, \beta \gg \gamma, \delta$.

This fitness formulation prioritizes feasibility, ensuring that the algorithm first eliminates scheduling conflicts before optimizing environmental efficiency and throughput.

D. GENETIC OPERATORS

At each generation, new individuals are created by applying crossover and mutation operators to selected parents (see Fig.4). We chose a tournament selection mechanism to ensure competitive pressure while maintaining diversity.

1) Crossover

A uniform crossover operator is applied to exchange genes (time slots) between two parent chromosomes with probability p_c. For each gene i, the offspring inherits the value of one of the parents with equal probability:

$$X_i^{(child)} = \begin{cases} X_i^{(parent1)} & \text{if } rand() < 0.5, \\ X_i^{(parent2)} & \text{otherwise.} \end{cases}$$

After crossover, a repair procedure will be executed to correct any precedence or machine conflicts. This is achieved by shifting conflicting operations to the nearest feasible slot, ensuring that the offspring remains valid or near-valid.

2) Mutation

Mutation introduces stochastic variations to preserve genetic diversity and improve exploration over exploitation of the search space. Two types of mutation operators are used, applied with probability p_m:

- **Shift mutation:** moves the starting slot of an operation forward or backward by a random offset $\Delta \in [-k, k]$.
- **Swap mutation:** exchanges the starting slots of two randomly selected operations, typically assigned to the same machine.

After mutation, the same repair mechanism used in the crossover phase ensures that feasibility constraints are not violated.

E. REPLACEMENT AND ELITISM

The population is updated using an elitist replacement strategy. The best individuals according to the fitness function are preserved across generations to guarantee that the best-found solution is never lost. The remaining population slots are filled with newly generated offspring, ensuring both convergence and exploration.

F. TERMINATION CRITERIA

The algorithm terminates when one of the following conditions is met:

- The maximum number of generations $G_{\max}$ is reached.
- The improvement in the best fitness value over a fixed number of consecutive generations falls below a threshold ϵ.

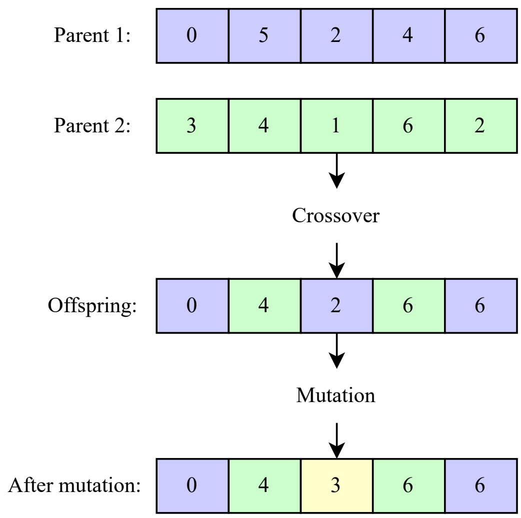

FIGURE 4: Illustration of crossover and mutation operations in the proposed GA. The offspring inherits genes from both parents and is subsequently modified by mutation.

The best individual at termination represents the optimized production schedule, balancing minimal energy consumption and reduced makespan.

G. PARAMETER CONFIGURATION

The parameter values presented in Table 1 correspond to typical configurations reported in the literature for Genetic Algorithms applied to scheduling and combinatorial optimization [15], [18].

TABLE 1: Genetic Algorithm parameters

Parameter	Value
Population size	100
Crossover probability p_c	0.8
Mutation probability p_m	0.2
Tournament size	3
Generations $G_{\max}$	500
Penalty coefficients $(\alpha, \beta, \gamma, \delta)$	(1000, 1000, 1, 0.1)

These values will serve as a baseline for our implementation. Further tuning will be performed during the said implementation to evaluate their impact on convergence speed and solution quality.

H. ALGORITHM SUMMARY

The general procedure of the proposed GA is summarized in Algorithm 1.

The proposed GA thus provides a flexible and robust framework for solving the scheduling problem formulated as a graph coloring task. It effectively integrates constraint satisfaction, environmental optimization, and adaptive search

Algorithm 1 Proposed Genetic Algorithm for Graph Coloring-based Scheduling

Initialize population P_0
for $g \leftarrow 1$ **to** $G_{\max}$ **do**
    Evaluate fitness of each individual using Eq. (8)
    Select parents using tournament selection
    Apply crossover and mutation to generate offspring
    Repair infeasible offspring
    Form new population with elitism
    if improvement in best fitness $< \epsilon$ **then**
        break
    end if
end for
return best individual found

mechanisms to generate feasible and energy-efficient production schedules.

V. GUIDELINES FOR GRAPHICS PREPARATION AND SUBMISSION

A. TYPES OF GRAPHICS

The following list outlines the different types of graphics published in IEEE journals. They are categorized based on their construction, and use of color/shades of gray:

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Figures that are meant to appear in color, or shades of black/gray. Such figures may include photographs, illustrations, multicolor graphs, and flowcharts.

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Figures that are composed of only black lines and shapes. These figures should have no shades or half-tones of gray, only black and white.

3) Author photos

Head and shoulders shots of authors that appear at the end of our papers.

4) Tables

Data charts which are typically black and white, but sometimes include color.

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TABLE 2: Units for Magnetic Properties

Symbol	Quantity	Conversion from Gaussian and CGS EMU to SI [a]
Φ	magnetic flux	1 Mx $\rightarrow 10^{-8}$ Wb $= 10^{-8}$ V·s
B	magnetic flux density, magnetic induction	1 G $\rightarrow 10^{-4}$ T $= 10^{-4}$ Wb/m^2
H	magnetic field strength	1 Oe $\rightarrow 10^3/(4\pi)$ A/m
m	magnetic moment	1 erg/G $=$ 1 emu $\rightarrow 10^{-3}$ A·m^2 $= 10^{-3}$ J/T
M	magnetization	1 erg/(G·cm^3) $=$ 1 emu/cm^3 $\rightarrow 10^3$ A/m
$4\pi M$	magnetization	1 G $\rightarrow 10^3/(4\pi)$ A/m
σ	specific magnetization	1 erg/(G·g) $=$ 1 emu/g $\rightarrow$ 1 A·m^2/kg
j	magnetic dipole moment	1 erg/G $=$ 1 emu $\rightarrow 4\pi \times 10^{-10}$ Wb·m
J	magnetic polarization	1 erg/(G·cm^3) $=$ 1 emu/cm^3 $\rightarrow 4\pi \times 10^{-4}$ T
χ, κ	susceptibility	$1 \rightarrow 4\pi$
χ_ρ	mass susceptibility	1 cm^3/g $\rightarrow 4\pi \times 10^{-3}$ m^3/kg
μ	permeability	$1 \rightarrow 4\pi \times 10^{-7}$ H/m $= 4\pi \times 10^{-7}$ Wb/(A·m)
μ_r	relative permeability	$\mu \rightarrow \mu_r$
w, W	energy density	1 erg/cm^3 $\rightarrow 10^{-1}$ J/m^3
N, D	demagnetizing factor	$1 \rightarrow 1/(4\pi)$

Vertical lines are optional in tables. Statements that serve as captions for the entire table do not need footnote letters.

[a]Gaussian units are the same as cg emu for magnetostatics; Mx = maxwell, G = gauss, Oe = oersted; Wb = weber, V = volt, s = second, T = tesla, m = meter, A = ampere, J = joule, kg = kilogram, H = henry.

(.MPS), sizes them, and adjusts the resolution settings. When submitting your final paper, your graphics should all be submitted individually in one of these formats along with the manuscript.

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Most charts, graphs, and tables are one column wide (3.5 inches/88 millimeters/21 picas) or page wide (7.16 inches/181 millimeters/43 picas). The maximum depth a graphic can be is 8.5 inches (216 millimeters/54 picas). When choosing the depth of a graphic, please allow space for a caption. Figures can be sized between column and page widths if the author chooses, however it is recommended that figures are not sized less than column width unless when necessary.

There is currently one publication with column measurements that do not coincide with those listed above. Proceedings of the IEEE has a column measurement of 3.25 inches (82.5 millimeters/19.5 picas).

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G. COLOR SPACE

The term color space refers to the entire sum of colors that can be represented within the said medium. For our purposes, the three main color spaces are Grayscale, RGB (red/green/blue) and CMYK (cyan/magenta/yellow/black). RGB is generally used with on-screen graphics, whereas CMYK is used for printing purposes.

All color figures should be generated in RGB or CMYK color space. Grayscale images should be submitted in Grayscale color space. Line art may be provided in grayscale OR bitmap colorspace. Note that "bitmap colorspace" and "bitmap file format" are not the same thing. When bitmap color space is selected, .TIF/.TIFF/.PNG are the recommended file formats.

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1) Figure Axis labels

Figure axis labels are often a source of confusion. Use words rather than symbols. As an example, write the quantity "Magnetization," or "Magnetization M," not just "M." Put units in parentheses. Do not label axes only with units. As in Fig. 1, for example, write "Magnetization (A/m)" or "Magnetization ($A{\cdot}m^{-1}$)," not just "A/m." Do not label axes with a ratio of quantities and units. For example, write "Temperature (K)," not "Temperature/K."

Multipliers can be especially confusing. Write "Magnetization (kA/m)" or "Magnetization (10^3 A/m)." Do not write "Magnetization (A/m) $\times$ 1000" because the reader would not know whether the top axis label in Fig. 1 meant 16000 A/m or 0.016 A/m. Figure labels should be legible, approximately 8 to 10 point type.

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Multipart figures should be combined and labeled before final submission. Labels should appear centered below each subfigure in 8 point Times New Roman font in the format of (a) (b) (c).

J. FILE NAMING

Figures (line artwork or photographs) should be named starting with the first 5 letters of the author's last name. The next characters in the filename should be the number that represents the sequential location of this image in your article. For example, in author "Anderson's" paper, the first three figures would be named ander1.tif, ander2.tif, and ander3.ps.

Tables should contain only the body of the table (not the caption) and should be named similarly to figures, except that '.t' is inserted in-between the author's name and the table number. For example, author Anderson's first three tables would be named ander.t1.tif, ander.t2.ps, ander.t3.eps.

Author photographs should be named using the first five characters of the pictured author's last name. For example, four author photographs for a paper may be named: oppen.ps, moshc.tif, chen.eps, and duran.pdf.

If two authors or more have the same last name, their first initial(s) can be substituted for the fifth, fourth, third. . . letters of their surname until the degree where there is differentiation. For example, two authors Michael and Monica Oppenheimer's photos would be named oppmi.tif, and oppmo.eps.

K. REFERENCING A FIGURE OR TABLE WITHIN YOUR PAPER

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VI. CONCLUSION

[?] A conclusion section is not required. Although a conclusion may review the main points of the paper, do not replicate the abstract as the conclusion. A conclusion might elaborate on the importance of the work or suggest applications and extensions.

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Appendixes, if needed, appear before the acknowledgment.

ACKNOWLEDGMENT

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  See [?], [?].
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- Example when using et al.:
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