XINRUI ZHANG　　Project 4　　EECS 592　　zxinrui

1. R&N 13.7

a. # of atomic events $= \binom{52}{5} = \cancel{259880}\ \underline{\underline{2598960}}$

b. $P(\text{atomic events}) = \dfrac{1}{\underline{\underline{2598960}}} = \underline{\underline{0.00000038477}}$

c. Royal flush means 10, J, Q, K, A for 4 suits

$P(\text{Royal flush}) = \dfrac{1}{\underline{\underline{649740}}} = \underline{0.000001539}$

Four of a kind means A, A, A, A + X　　$13 \times (52-4) = 624$
⋮
K K K K + X

$P(\text{Four of a kind}) = \dfrac{624}{2598960} = \underline{\underline{\dfrac{1}{4165}}} = \underline{\underline{0.00024010}}$

2. R&N 13.18

a. H := pick a coin, get a head
F := pick a fake coin (two heads)

$P(F|H) = \dfrac{P(F,H)}{P(H)} = \dfrac{P(H|F)\,P(F)}{P(H|F)\,P(F) + P(H|\sim F)\cdot P(\sim F)}$

$= \dfrac{1\cdot\frac{1}{n}}{1\cdot\frac{1}{n} + \frac{1}{2}\cdot\frac{n-1}{n}} = \underline{\underline{\dfrac{2}{n+1}}}$

b. kH := continue filpping a coin, get k heads.

$P(F|kH) = \dfrac{P(kH|F)\cdot P(F)}{P(kH|F)\cdot P(F) + P(kH|\sim F)\cdot P(\sim F)}$

For a already picked coined, the get a head or tail events are independent.

$P(kH|F) = P(H|F)^k = 1$, $P(kH|\sim F) = P(H|F)^k = (\frac{1}{2})^k$

$P(F|kH) = \dfrac{\frac{1}{n}\cdot 1}{\frac{1}{n}\cdot 1 + \frac{n-1}{n}\cdot(\frac{1}{2})^k} = \dfrac{2^k}{2^k+n-1}$

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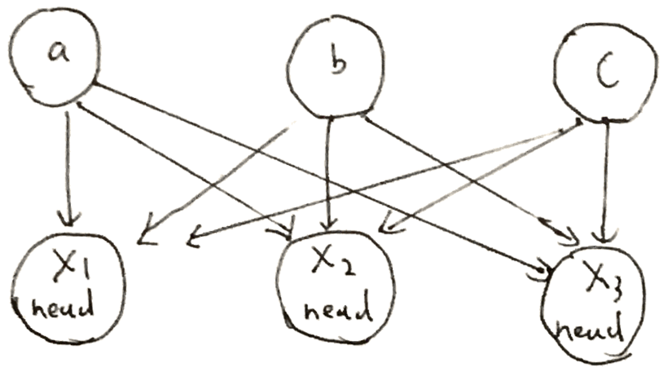
2.

c. $P(\text{Error}) = P(KH, \neg F) = P(KH \mid \neg F) \cdot P(\neg F) = \frac{1}{2^k} \cdot \frac{n-1}{n}$

$= \frac{n-1}{n \cdot 2^k}$

3. R&N Problem 14.1

a.



CPT Tables: for a, b, c denote as $P(X)$, $X \in \{a, b, c\}$

P(X)
1/3

for X_1, X_2, X_3 denote as $P(X_i)$, $i \in \{1, 2, 3\}$ means Event X_i shows head.

a	b	c	$P(X_i)$
T	F	F	0.2
F	T	F	0.6
F	F	T	0.8

b. ~~P(C | H,H,T)~~ H: denotes X_i shows head, T denotes ~~T denotes~~ X_i shows tail

X_1, X_2, X_3 are independent events. c denotes coin

$P(C \mid H,H,T) = \cancel{P(C} \frac{P(C, H, H, T)}{P(H,H,T)} = \frac{P(C) \cdot P(H|C) \cdot P(H|C) \cdot P(T|C)}{P(H,H,T)}$

$P(a \mid H,H,T) \propto \cancel{P(a|HHT)}\ P(H|a) \cdot P(H|a) \cdot P(T|a) \cdot P(a) = \frac{1}{5} \cdot \frac{1}{5} \cdot \frac{4}{5} \cdot \frac{1}{3} = \frac{4}{375}$

$P(b \mid H,H,T) \propto \ldots = \frac{3}{5} \cdot \frac{3}{5} \cdot \frac{2}{5} \cdot \frac{1}{3} = \frac{6}{125}$

$P(c \mid H,H,T) \propto \ldots = \frac{4}{5} \cdot \frac{4}{5} \cdot \frac{1}{5} \cdot \frac{1}{3} = \frac{16}{375}$

$P(b \mid HHT)$ has the maximum probability. b was most likely to be drawn.

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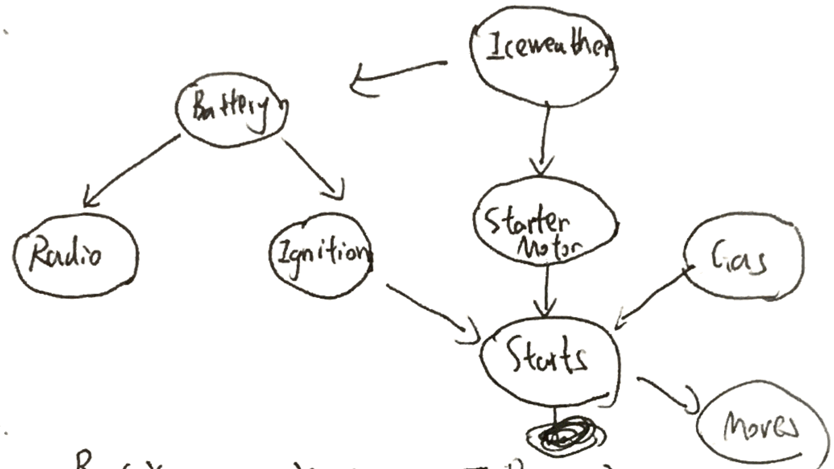
4.

a. The effect of ice weather

1. The battery could die
2. The starter could stick

a.



b.

$$P(X_1, \ldots X_n) = \prod P(X_i)$$

$n = 8$

$2n = 16$ There are 16 independent values

c. Ice: ice weather, Ba: Battery, R: Radio, I: Ignition, SM: Starter Motor

G: Gas   S: starts   M: Move

2 states T and F

$$P(Ice, Ba, R, I, SM, G, S, M) = P(Ice)\, P(Ba|Ice)\, P(R|Ba)\, P(I|Ba)\, P(SM|Ice)\, P(G)\, P(S|I, S, G)\, P(M|S)$$

$$2^1 + 2^2 + 2^4 + 2^2 + 2^2 + 2^1 + 2^4 + 2^2 = 40$$

d. Iceweather and Radio

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5.

a. $P(A=t, B=f, C=t, D=f) = P(A=t) \cdot P(B=f|A=t) \cdot P(C=t)\, P(D=f|B=f, C=t)$

$= 0.4 \cdot 0.7 \cdot 0.7 \cdot 0.7 = 0.1372$

b. $P(D=t|A=f, B=t, C=f) = \dfrac{P(D=t, A=f, B=t, C=f)}{P(A=f, B=t, C=f)} = \dfrac{P(D=t|B=t, C=f)\, P(A=f)\, P(B=t|A=f)\, P(C=f)}{\cancel{P(A=f)\, P(B=t|A=f)\, P(C=f) \cdot P(D=t|B=t, C=f)} + P(A=f)\, P(B=t|A=f) \cancel{P(D=f|B=t, C=f)} \cdot P(C)}$

$= \dfrac{0.5 \cdot 0.6 \cdot 0.3 \cdot 0.9}{0.6 \cdot 0.9 \cdot 0.3} = 0.5$

c. $P(A=t|B=f, C=t, D=f) = \dfrac{P(A=t, B=f, C=t, D=f)}{P(B=f, C=t, D=f)}$

$= \dfrac{P(A=t)\, P(B=f|A=t)\, P(C=t) \cdot P(D=f|B=f, C=t)}{\cancel{P(B=f|A=t)\ P(C}\ \big(P(A=t)\, P(B=f|A=t)\, P(C=t)\, P(D=f|B=f, C=t) + P(A=f)\, P(B=f|A=f)\big)}$

$= \dfrac{0.4 \times 0.7 \times 0.7 \times 0.7}{(0.4 \times 0.7 + 0.6 \times 0.1) \times 0.7 \times 0.7} = 0.8235$

d. $P(B=f|A=t, C=f) = \dfrac{P(B=f, A=t, C=f)}{P(A=t, C=f)} = \dfrac{P(B=f|A=t)\, P(A=t)\, P(C=f)}{P(A=t) \cdot P(C=f)}$

$= \dfrac{0.7 \times 0.4 \times 0.3}{0.4 \times 0.3} = 0.7$

e. $P(B=\text{false}) = P(A=f) \cdot P(B=f|A=f) + P(B=f|A=t)\, P(A=t)$

$= 0.1 \times 0.6 + 0.7 \times 0.4 = 0.34$

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6. Exam Bayes net

a. Given Ex ~~DP: Doesn't panic~~, ~~H: Hard working~~, Ps: Gains practical skill and S: success: are conditionally independent of intelligent (I).

b. Given KM knows material I: Intelligent and H: Hard working are conditionally independent of success (S)

c. Given success (S) There is not a node ~~Ps: Gains practical skill~~ conditionally independent of high exam score (Ex).

d. $P(KM) = P(KM \mid I, H) P(I \mid H) P(H) + \cdots + P(KM \mid \sim I, \sim H) P(\sim I \mid \sim H) P(\sim H)$

H, and I are independent $P(I \mid H) = P(I)$

$\Rightarrow \quad = P(KM \mid I, H) P(I) P(H) + P(KM \mid \sim H, \sim I) P(\sim I) P(\sim H) + P(KM \mid \sim I, H) P(\sim I) P(H) + P(KM \mid I, \sim H) P(I) P(\sim H)$

$= 1 \times 0.7 \times 0.6 + 0.05 \times 0.3 \times 0.4 + 0.4 \times 0.7 \times 0.4 + 0.6 \times 0.3 \times 0.6$

$= \underline{0.646}$

e. $P(S \mid KM) = \dfrac{P(S, KM)}{P(KM)} = \dfrac{\cancel{P(S \mid PS, Ex) P(KM)}\ P(S, KM, PS, Ex) + \cdots}{\cancel{P(KM)}\ \cancel{P(S \mid PS, Ex)\ P(KM)\ P(PS \mid Ex)}\ P(KM)}$ $\cancel{P(EX)}$

Ps and Ex are independent

$= ~~P(S) +~~$ ~~Ps and Ex are independent~~ $\Rightarrow$

$= \cancel{P(S \mid PS, Ex) P(PS) P(Ex \mid KM, DP) + P(S \mid PS, \sim Ex) P(PS) P(\sim Ex \mid KM, DP)}$

$= \dfrac{\cancel{P(S, KM, PS)}\ P(S \mid \hat{PS}, \hat{EX}) P(\hat{PS}) P(\hat{EX} \mid \hat{DP}, KM) \cdot P(\hat{DP}) \cdot \cancel{P(KM)}}{\cancel{P(KM)}}$

$= P(S \mid PS, \sim EX) \cdot P(PS) \cdot P(\sim EX \mid DP, KM) P(DP)$
$+ P(S \mid PS, \sim EX) \cdot P(PS) \cdot P(\sim EX \mid DP, KM) P(DP)$
$\cancel{+ P(S)} + \cdots$
$+ P(S \mid \sim PS, \sim EX) P(\sim PS) P(\sim EX \mid \sim DP, KM) P(\sim DP)$

$= 0.238 + 0.08925 + 0.03675 + 0.00625 + 0.196 + 0.0375 + 0.0135 + 0.0135$

$= 0.72725$

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f. $P(PS|S) = \dfrac{P(PS,S)}{P(S)} = \dfrac{P(S|PS,EX)\,P(PS)\,P(EX) + P(S|PS,\neg EX)\,P(PS)\,P(\neg EX)}{P(S)}$

$P(EX) = P(EX|OP,KM)\cdot P(OP)\,P(KM) + \dots + P(EX|\neg OP,\neg KM)\,P(\neg OP)\,P(\neg KM)$

for OP and KM are independent

$\Rightarrow P(EX) = 0.85\times0.5\times0.646 + 0.7\times0.5\times0.646 + 0.2\times0.5\times0.354 + 0.1\times0.5\times0.354 = \cancel{0.5012765}\ 0.55375$

$P(\neg EX) = \cancel{0.4987235}\ 0.44625$

~~P(PS|S) =~~

EX and PS are independent.

$P(S) = P(S|EX,PS)\,P(PS)\cdot P(EX) + \dots\ P(S|\neg EX,\neg PS)\,P(\neg EX,\neg PS)$

$= (0.8\times0.7 + 0.7\times0.3)\times0.55375 + (0.7\times0.7 + 0.3\times0.3)\times0.44625$

$= 0.4263875 + 0.258825 = 0.6852125$

$0.8\times0.7\times0.55375 + 0.7\times0.7\times0.44625$

$P(PS|S) = \dfrac{\cancel{P(EX)\,P(S|PS,EX)\,P(PS) + P(S|PS,\neg EX)\,P(\neg EX) + \dots}}{0.6852125} = \dfrac{0.3101 + 0.2186625}{0.6852125}$

≈ 0.771775235

$\approx \underline{0.7722}$

g. $P(KM|S) = \dfrac{P(S\,KM)}{P(S)} = \dfrac{P(S|KM)\cdot P(KM)}{P(S)} = \dfrac{0.72725\times0.646}{0.6852125}$

≈ 0.6856318275

$\approx \underline{0.6856}$