

BELJING NORMAL UNIVERSITY
SCHOOL OF MATHEMATICS

Template

appleDog

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1 hpp

1.1 heading

```

1  #include <bits/stdc++.h>
2
3  // using namespace std;
4
5  using LL = long long;
6  using i128 = __int128;
7  using PII = std::pair<int, int>;
8  /*
9  using UI = unsigned int;
10 using ULL = unsigned long long;
11 using ULL = unsigned long long;
12 using PIL = std::pair<int, LL>;
13 using PLI = std::pair<LL, int>;
14 using PLL = std::pair<LL, LL>;
15 using vi = std::vector<int>;
16 using vvi = std::vector<vi>;
17 using vl = std::vector<LL>;
18 using vvl = std::vector<vl>;
19 using vpi = std::vector<PII>;
20 */
21
22 #define ff first
23 #define ss second
24 #define all(v) v.begin(), v.end()
25 #define rall(v) v.rbegin(), v.rend()
26
27 #ifdef LOCAL
28 #include "debug.h"
29 #else
30 #define debug(...) \
31     do { \
32     } while (false)
33 #endif
34
35 constexpr int inf = 0x3f3f3f3f;
36 constexpr LL INF = 1e18;
37 constexpr int lowbit(int x) { return x & -x; }
38 /*
39 constexpr int add(int x, int y) { return x + y < mod ? x + y : x - mod + y; }
40 constexpr int sub(int x, int y) { return x < y ? mod + x - y : x - y; }
41 constexpr int mul(LL x, int y) { return x * y % mod; }
42 constexpr void Add(int& x, int y) { x = add(x, y); }
43 constexpr void Sub(int& x, int y) { x = sub(x, y); }
44 constexpr void Mul(int& x, int y) { x = mul(x, y); }
45 constexpr int pow(int x, int y, int z = 1) {
46     for (; y; y /= 2) {
47         if (y & 1) Mul(z, x);
48         Mul(x, x);
49     }
50     return z;
51 }
52 temps constexpr int add(Ts... x) {
53     int y = 0;
54     (... , Add(y, x));
55     return y;
56 }
57 temps constexpr int mul(Ts... x) {
58     int y = 1;
59     (... , Mul(y, x));
60     return y;
61 }
62 */
63 tandu bool Max(T& x, const U& y) { return x < y ? x = y, true : false; }
64 tandu bool Min(T& x, const U& y) { return x > y ? x = y, true : false; }
65
66 void solut() {
67     ;
68 }
69
70 int main() {
71     std::ios::sync_with_stdio(false);
72     std::cin.tie(0);
73     int t = 1;
74     std::cin >> t;
75     while (t--) {
76         solut();
77     }
78     return 0;
79 }

```

1.2 debug.h

```

1  template <typename T, typename U>
2  std::ostream& operator<<(std::ostream& os, const std::pair<T, U>& p) {
3      return os << '<' << p.first << ',' << p.second << '>';
4  }
5
6  template <
7      typename T, typename = decltype(std::begin(std::declval<T>()))
8      typename = std::enable_if_t<!std::is_same_v<T, std::string>>>
9  std::ostream& operator<<(std::ostream& os, const T& c) {
10     auto it = std::begin(c);
11     if (it == std::end(c)) return os << "{}";
12     for (os << '{' << *it; ++it != std::end(c); os << ',' << *it);
13     return os << '}';
14 }
15
16 #define debug(arg...) \
17     do { \
18         std::cerr << "[" #arg "]" :"; \
19         dbg(arg); \
20     } while (false)
21
22 template <typename... Ts>
23 void dbg(Ts... args) {
24     (... , (std::cerr << ' ' << args));
25     std::cerr << std::endl;
26 }

```

2 shell scripts

2.1 linux version

```

1  #!/bin/bash
2
3  cd "$1"
4
5  g++ -o main -O2 -std=c++17 -DLOCAL main.cpp -ftrapv -fsanitize=address,undefined
6
7  for input in *.in; do
8      output=${input%.*}.out
9      answer=${input%.*}.ans
10
11      ./main < $input > $ouput
12
13      echo "case ${input%.*}: "
14      echo "My: "
15      cat $output
16      echo "Answer: "
17      cat $answer
18
19  done

```

2.2 windows version

```

1  @echo off
2
3  cd %1
4
5  del .\main.exe
6
7  g++ -o main.exe main.cpp -DLOCAL -std=c++17 -ftrapv
8
9  for %%i in (*.in) do (
10     main.exe < %%i > %%~ni.out
11     echo case %%~ni:
12     echo My:
13     type %%~ni.out
14     echo Answer:
15     type %%~ni.ans
16 )
17
18 cd ../shell

```

3 data structure

3.1 stack

```

1 vi stk;
2 for (int i = 1; i <= n; i++){
3     while (!stk.empty() and stk.back() > a[i]) {
4         stk.pop_back();
5     }
6     stk.push_back(a[i]);
7 }

```

3.2 queue

```

1 std::deque<int> q;
2 for (int i = 1; i <= n; i++) {
3     while (!q.empty() and a[q.back()] >= a[i]) q.pop_back();
4     if (!q.empty() and i - q.front() >= k) q.pop_front();
5     q.push_back(i);
6 }

```

3.3 DSU

```

1 /* DSU */
2 vi fa(n + 1);
3 std::iota(all(fa), 0);
4 std::function<int(int)> find = [&] (int x) -> int{
5     return x == fa[x] ? x : fa[x] = find(fa[x]);
6 };
7 auto merge = [&] (int x, int y) -> void{
8     x = find(x), y = find(y);
9     if (x == y) return;
10    // operations //
11    fa[y] = x;
12 };

```

3.4 spare table

一维

```

1 /* spare table */
2 int B = 30;
3 vvi f(n + 1, vi(B));
4 vi Log2(n + 1);
5 auto init = [&]() -> void {
6     for (int i = 1; i <= n; i++) {
7         f[i][0] = a[i];
8         if (i > 1) Log2[i] = Log2[i / 2] + 1;
9     };
10    int t = Log2[n];
11    for (int j = 1; j <= t; j++) {
12        for (int i = 1; i <= n - (1 << j) + 1; i++) {
13            f[i][j] = std::max(f[i][j - 1], f[i + (1 << (j - 1))][j - 1]);
14        }
15    }
16 };
17 init();
18 auto query = [&](int l, int r) -> int {
19     int t = Log2[r - l + 1];
20     return std::max(f[l][t], f[r - (1 << t) + 1][t]);
21 };

```

二维


```

1  /* spare table */
2  intB = 30;
3  std::vector f(n + 1, std::vector<std::array<std::array<int, B>, B>>(m + 1));
4  vi Log2(n + 1);
5  auto init = [&]() -> void {
6      for (int i = 2; i <= std::max(n, m); i++) {
7          Log2[i] = Log2[i / 2] + 1;
8      }
9      for (int i = 2; i <= n; i++) {
10         for (int j = 2; j <= m; j++) {
11             f[i][j][0][0] = a[i][j];
12         }
13     }
14     for (int ki = 0; ki <= Log2[n]; ki++) {
15         for (int kj = 0; kj <= Log2[n]; kj++) {
16             if (!ki && !kj) continue;
17             for (int i = 1; i <= n - (1 << ki) + 1; i++) {
18                 for (int j = 1; j <= m - (1 << kj) + 1; j++) {
19                     if (ki) {
20                         f[i][j][ki][kj] =
21                             std::max(f[i][j][ki - 1][kj], f[i + (1 << (ki - 1))][j][ki - 1][kj]);
22                     } else {
23                         f[i][j][ki][kj] =
24                             std::max(f[i][j][ki][kj - 1], f[i][j + (1 << (kj - 1))][ki][kj - 1]);
25                     }
26                 }
27             }
28         }
29     }
30 };
31 init();
32 auto query = [&](int x1, int y1, int x2, int y2) -> int {
33     int ki = Log2[x2 - x1 + 1], kj = Log2[y2 - y1 + 1];
34     int t1 = f[x1][y1][ki][kj];
35     int t2 = f[x2 - (1 << ki) + 1][y1][ki][kj];
36     int t3 = f[x1][y2 - (1 << kj) + 1][ki][kj];
37     int t4 = f[x2 - (1 << ki) + 1][y2 - (1 << kj) + 1][ki][kj];
38     return std::max({t1, t2, t3, t4});
39 };

```

3.5 Cartesian tree

一种特殊的平衡树, 用元素的值作为平衡点节点的 *val*, 元素的下标作为 *key*.

```

1  /* cartesian tree */
2  vi ls(n + 1), rs(n + 1), stk(n + 1);
3  int top = 1;
4  for (int i = 1; i <= n; i++) {
5      int k = top;
6      while (k && a[stk[k]] > a[i]) k--;
7      if (k) rs[stk[k]] = i;
8      if (k < top) ls[i] = stk[k + 1];
9      stk[++k] = i;
10     top = k;
11 }

```

3.6 segment tree

TODO

3.7 persistent segment tree

单点修改, 版本拷贝

n 个数, m 次操作, 操作分别为

1. v_i 1 loc_i $value_i$: 将第 v_i 个版本的 $a[loc_i]$ 修改为 $value_i$,
2. v_i 2 loc_i : 拷贝第 v_i 个版本, 并查询该版本的 $a[loc_i]$.

```

1 // 洛谷 P3919 【模板】可持久化线段树 1 (可持久化数组)
2
3 struct node {
4     int l, r, key;
5 };
6
7 int main() {
8     std::ios::sync_with_stdio(false);
9     std::cin.tie(0);
10    std::cout.tie(0);
11
12    int n, m;
13    std::cin >> n >> m;
14    vi a(n + 1);
15    for (int i = 1; i <= n; i++) {
16        std::cin >> a[i];
17    }
18
19    /* hjt segment tree */
20    int idx = 0;
21    vi root(m + 1);
22    std::vector<node> tr(n * 25);
23
24    std::function<int(int, int)> build = [&](int l, int r) -> int {
25        int p = ++idx;
26        if (l == r) {
27            tr[p].key = a[l];
28            return p;
29        }
30        int mid = (l + r) >> 1;
31        tr[p].l = build(l, mid);
32        tr[p].r = build(mid + 1, r);
33        return p;
34    };
35
36    std::function<int(int, int, int, int, int)> modify = [&](int p, int l, int r, int k,
37                                                            int x) -> int {
38        int q = ++idx;
39        tr[q].l = tr[p].l, tr[q].r = tr[p].r;
40        if (tr[q].l == tr[q].r) {
41            tr[q].key = x;
42            return q;
43        }
44        int mid = (l + r) >> 1;
45        if (k <= mid) {
46            tr[q].l = modify(tr[q].l, l, mid, k, x);
47        } else {
48            tr[q].r = modify(tr[q].r, mid + 1, r, k, x);
49        }
50        return q;
51    };
52
53    std::function<int(int, int, int, int)> query = [&](int p, int l, int r, int k) -> int {
54        if (tr[p].l == tr[p].r) {
55            return tr[p].key;
56        }
57        int mid = (l + r) >> 1;
58        if (k <= mid) {
59            return query(tr[p].l, l, mid, k);
60        } else {
61            return query(tr[p].r, mid + 1, r, k);
62        }
63    };
64
65    root[0] = build(1, n);
66
67    for (int i = 1; i <= m; i++) {
68        int op, ver, k, x;
69        std::cin >> ver >> op;
70        if (op == 1) {
71            std::cin >> k >> x;
72            root[i] = modify(root[ver], 1, n, k, x);
73        } else {
74            std::cin >> k;
75            root[i] = root[ver];
76            std::cout << query(root[ver], 1, n, k) << '\n';
77        }
78    }
79
80    return 0;
81 }

```

区间第 k 小

长度为 n 的序列 a , m 次查询, 每次查询 $[l, r]$ 中的第 k 小值.

```

1 // 洛谷P3834 【模板】可持久化线段树 2
2
3 struct node {
4     int l, r, cnt;
5 };
6
7 int main() {
8     std::ios::sync_with_stdio(false);
9     std::cin.tie(0);
10    std::cout.tie(0);
11
12    int n, m;
13    std::cin >> n >> m;
14    vi a(n + 1), v;
15    for (int i = 1; i <= n; i++) {
16        std::cin >> a[i];
17        v.push_back(a[i]);
18    }
19    std::sort(all(v));
20    v.erase(unique(all(v)), v.end());
21    auto find = [&](int x) -> int { return std::lower_bound(all(v), x) - v.begin() + 1; };
22
23    /* hjt segment tree */
24    std::vector<node> tr(n * 25);
25    vi root(n + 1);
26    int idx = 0;
27
28    std::function<int(int, int)> build = [&](int l, int r) -> int {
29        int p = ++idx;
30        if (l == r) return p;
31        int mid = (l + r) >> 1;
32        tr[p].l = build(l, mid), tr[p].r = build(mid + 1, r);
33        return p;
34    };
35
36    std::function<int(int, int, int, int)> modify = [&](int p, int l, int r, int x) -> int {
37        int q = ++idx;
38        tr[q] = tr[p];
39        if (tr[q].l == tr[q].r) {
40            tr[q].cnt++;
41            return q;
42        }
43        int mid = (l + r) >> 1;
44        if (x <= mid) {
45            tr[q].l = modify(tr[q].l, l, mid, x);
46        } else {
47            tr[q].r = modify(tr[q].r, mid + 1, r, x);
48        }
49        tr[q].cnt = tr[tr[q].l].cnt + tr[tr[q].r].cnt;
50        return q;
51    };
52
53    std::function<int(int, int, int, int, int)> query = [&](int p, int q, int l, int r,
54                                                         int x) -> int {
55        if (l == r) return l;
56        int cnt = tr[tr[p].l].cnt - tr[tr[q].l].cnt;
57        int mid = (l + r) >> 1;
58        if (x <= cnt) {
59            return query(tr[p].l, tr[q].l, l, mid, x);
60        } else {
61            return query(tr[p].r, tr[q].r, mid + 1, r, x - cnt);
62        }
63    };
64
65    root[0] = build(1, v.size());
66
67    for (int i = 1; i <= n; i++) {
68        root[i] = modify(root[i - 1], 1, v.size(), find(a[i]));
69    }
70    for (int i = 1; i <= m; i++) {
71        int l, r, k;
72        std::cin >> l >> r >> k;
73        std::cout << v[query(root[r], root[l - 1], 1, v.size(), k) - 1] << '\n';
74    }
75
76    return 0;
77 }
78

```

3.8 treap

fhq treap

n 次操作, 操作分为如下 6 种:

1. 插入数 x ;
2. 删除数 x (若有多个相同的数, 只删除一个);
3. 查询数 x 的排名 (排名定义为小于 x 的数的个数 + 1);
4. 查询排名为 x 的数;
5. 求 x 的前驱 (前驱定义为小于 x 的最大数);
6. 求 x 的后继 (后继定义为大于 x 的最小数).

```

1 struct node {
2     node *ch[2];
3     int key, val;
4     int cnt, size;
5
6     node(int _key) : key(_key), cnt(1), size(1) {
7         ch[0] = ch[1] = nullptr;
8         val = rand();
9     }
10
11     // node(node * _node) {
12     //     key = _node->key, val = _node->val, cnt = _node->cnt, size = _node->size;
13     // }
14
15     inline void push_up() {
16         size = cnt;
17         if (ch[0] != nullptr) size += ch[0]->size;
18         if (ch[1] != nullptr) size += ch[1]->size;
19     }
20 };
21
22 struct treap {
23     #define _2 second.first
24     #define _3 second.second
25
26     node *root;
27
28     pair<node *, node *> split(node *p, int key) {
29         if (p == nullptr) return {nullptr, nullptr};
30         if (p->key <= key) {
31             auto temp = split(p->ch[1], key);
32             p->ch[1] = temp.first;
33             p->push_up();
34             return {p, temp.second};
35         } else {
36             auto temp = split(p->ch[0], key);
37             p->ch[0] = temp.second;
38             p->push_up();
39             return {temp.first, p};
40         }
41     }
42
43     pair<node *, pair<node *, node *> > split_by_rank(node *p, int rank) {
44         if (p == nullptr) return {nullptr, {nullptr, nullptr}};
45         int ls_size = p->ch[0] == nullptr ? 0 : p->ch[0]->size;
46         if (rank <= ls_size) {
47             auto temp = split_by_rank(p->ch[0], rank);
48             p->ch[0] = temp._3;
49             p->push_up();
50             return {temp.first, {temp._2, p}};
51         } else if (rank <= ls_size + p->cnt) {
52             node *lt = p->ch[0];
53             node *rt = p->ch[1];
54             p->ch[0] = p->ch[1] = nullptr;
55             return {lt, {p, rt}};
56         } else {
57             auto temp = split_by_rank(p->ch[1], rank - ls_size - p->cnt);
58             p->ch[1] = temp.first;
59             p->push_up();

```

```

60         return {p, {temp._2, temp._3}};
61     }
62 }
63
64 node *merge(node *u, node *v) {
65     if (u == nullptr && v == nullptr) return nullptr;
66     if (u != nullptr && v == nullptr) return u;
67     if (v != nullptr && u == nullptr) return v;
68     if (u->val < v->val) {
69         u->ch[1] = merge(u->ch[1], v);
70         u->push_up();
71         return u;
72     } else {
73         v->ch[0] = merge(u, v->ch[0]);
74         v->push_up();
75         return v;
76     }
77 }
78
79 void insert(int key) {
80     auto temp = split(root, key);
81     auto l_tr = split(temp.first, key - 1);
82     node *new_node;
83     if (l_tr.second == nullptr) {
84         new_node = new node(key);
85     } else {
86         l_tr.second->cnt++;
87         l_tr.second->push_up();
88     }
89     node *l_tr_combined = merge(l_tr.first, l_tr.second == nullptr ? new_node : l_tr.second);
90     root = merge(l_tr_combined, temp.second);
91 }
92
93 void remove(int key) {
94     auto temp = split(root, key);
95     auto l_tr = split(temp.first, key - 1);
96     if (l_tr.second->cnt > 1) {
97         l_tr.second->cnt--;
98         l_tr.second->push_up();
99         l_tr.first = merge(l_tr.first, l_tr.second);
100     } else {
101         if (temp.first == l_tr.second) temp.first = nullptr;
102         delete l_tr.second;
103         l_tr.second = nullptr;
104     }
105     root = merge(l_tr.first, temp.second);
106 }
107
108 int get_rank_by_key(node *p, int key) {
109     auto temp = split(p, key - 1);
110     int ret = (temp.first == nullptr ? 0 : temp.first->size) + 1;
111     root = merge(temp.first, temp.second);
112     return ret;
113 }
114
115 int get_key_by_rank(node *p, int rank) {
116     auto temp = split_by_rank(p, rank);
117     int ret = temp._2->key;
118     root = merge(temp.first, merge(temp._2, temp._3));
119     return ret;
120 }
121
122 int get_prev(int key) {
123     auto temp = split(root, key - 1);
124     int ret = get_key_by_rank(temp.first, temp.first->size);
125     root = merge(temp.first, temp.second);
126     return ret;
127 }
128
129 int get_nex(int key) {
130     auto temp = split(root, key);
131     int ret = get_key_by_rank(temp.second, 1);
132     root = merge(temp.first, temp.second);
133     return ret;
134 }
135 };
136
137 treap tr;
138
139 int main() {
140     ios::sync_with_stdio(false);
141     cin.tie(0);
142     cout.tie(0);
143
144     srand(time(0));
145
146     int n;

```

```

147     cin >> n;
148     while (n--> 0) {
149         int op, x;
150         cin >> op >> x;
151         if (op == 1) {
152             tr.insert(x);
153         } else if (op == 2) {
154             tr.remove(x);
155         } else if (op == 3) {
156             cout << tr.get_rank_by_key(tr.root, x) << '\n';
157         } else if (op == 4) {
158             cout << tr.get_key_by_rank(tr.root, x) << '\n';
159         } else if (op == 5) {
160             cout << tr.get_prev(x) << '\n';
161         } else {
162             cout << tr.get_nex(x) << '\n';
163         }
164     }
165     return 0;
166 }

```

用 01 trie 实现的一种方式

同样的题目, 注意使用 01 trie 只能存在非负数. 速度能快不少, 但只能单点操作, 而且有点费空间.

```

1 // 洛谷 P3369 【模板】普通平衡树
2
3 struct Treap {
4     int id = 1, maxlog = 25;
5     int ch[N * 25][2], siz[N * 25];
6
7     int newnode() {
8         id++;
9         ch[id][0] = ch[id][1] = siz[id] = 0;
10        return id;
11    }
12
13    void merge(int key, int cnt) {
14        int u = 1;
15        for (int i = maxlog - 1; i >= 0; i--) {
16            int v = (key >> i) & 1;
17            if (!ch[u][v]) ch[u][v] = newnode();
18            u = ch[u][v];
19            siz[u] += cnt;
20        }
21    }
22
23    int get_key_by_rank(int rank) {
24        int u = 1, key = 0;
25        for (int i = maxlog - 1; i >= 0; i--) {
26            if (siz[ch[u][0]] >= rank) {
27                u = ch[u][0];
28            } else {
29                key |= (1 << i);
30                rank -= siz[ch[u][0]];
31                u = ch[u][1];
32            }
33        }
34        return key;
35    }
36
37    int get_rank_by_key(int rank) {
38        int key = 0;
39        int u = 1;
40        for (int i = maxlog - 1; i >= 0; i--) {
41            if ((rank >> i) & 1) {
42                key += siz[ch[u][0]];
43                u = ch[u][1];
44            } else {
45                u = ch[u][0];
46            }
47            if (!u) break;
48        }
49        return key;
50    }
51
52    int get_prev(int x) { return get_key_by_rank(get_rank_by_key(x)); }
53    int get_next(int x) { return get_key_by_rank(get_rank_by_key(x + 1) + 1); }
54 } treap;
55
56 const int num = 1e7;
57 int n, op, x;

```

```

58
59 int main() {
60     std::ios::sync_with_stdio(false);
61     std::cin.tie(0);
62     std::cout.tie(0);
63
64     std::cin >> n;
65     for (int i = 1; i <= n; i++) {
66         std::cin >> op >> x;
67         if (op == 1) {
68             treap.merge(x + num, 1);
69         } else if (op == 2) {
70             treap.merge(x + num, -1);
71         } else if (op == 3) {
72             std::cout << treap.get_rank_by_key(x + num) + 1 << '\n';
73         } else if (op == 4) {
74             std::cout << treap.get_key_by_rank(x) - num << '\n';
75         } else if (op == 5) {
76             std::cout << treap.get_prev(x + num) - num << '\n';
77         } else if (op == 6) {
78             std::cout << treap.get_next(x + num) - num << '\n';
79         }
80     }
81     return 0;
82 }

```

3.9 splay

文艺平衡树

初始为 1 到 n 的序列, m 次操作, 每次将序列下标为 $[l \sim r]$ 的区间翻转.

```

1 // 洛谷 P3391 【模板】文艺平衡树
2
3 struct node {
4     int ch[2], fa, key;
5     int siz, flag;
6
7     void init(int _fa, int _key) { fa = _fa, key = _key, siz = 1; }
8 };
9
10 struct splay {
11     node tr[N];
12     int n, root, idx;
13
14     bool get(int u) { return u == tr[tr[u].fa].ch[1]; }
15
16     void pushup(int u) { tr[u].siz = tr[tr[u].ch[0]].siz + tr[tr[u].ch[1]].siz + 1; }
17
18     void pushdown(int u) {
19         if (tr[u].flag) {
20             std::swap(tr[u].ch[0], tr[u].ch[1]);
21             tr[tr[u].ch[0]].flag ^= 1, tr[tr[u].ch[1]].flag ^= 1;
22             tr[u].flag = 0;
23         }
24     }
25
26     void rotate(int x) {
27         int y = tr[x].fa, z = tr[y].fa;
28         int op = get(x);
29         tr[y].ch[op] = tr[x].ch[op ^ 1];
30         if (tr[x].ch[op ^ 1]) tr[tr[x].ch[op ^ 1]].fa = y;
31         tr[x].ch[op ^ 1] = y;
32         tr[y].fa = x, tr[x].fa = z;
33         if (z) tr[z].ch[y == tr[z].ch[1]] = x;
34         pushup(y), pushup(x);
35     }
36
37     void opt(int u, int k) {
38         for (int f = tr[u].fa; f = tr[u].fa, f != k; rotate(u)) {
39             if (tr[f].fa != k) rotate(get(u) == get(f) ? f : u);
40         }
41         if (k == 0) root = u;
42     }
43
44     void output(int u) {
45         pushdown(u);
46         if (tr[u].ch[0]) output(tr[u].ch[0]);
47         if (tr[u].key >= 1 && tr[u].key <= n) {
48             std::cout << tr[u].key << ' ';
49         }
50     }
51 }

```

```

50     if (tr[u].ch[1]) output(tr[u].ch[1]);
51 }
52
53 void insert(int key) {
54     idx++;
55     tr[idx].ch[0] = root;
56     tr[idx].init(0, key);
57     tr[root].fa = idx;
58     root = idx;
59     pushup(idx);
60 }
61
62 int kth(int k) {
63     int u = root;
64     while (1) {
65         pushdown(u);
66         if (tr[u].ch[0] && k <= tr[tr[u].ch[0]].siz) {
67             u = tr[u].ch[0];
68         } else {
69             k -= tr[tr[u].ch[0]].siz + 1;
70             if (k <= 0) {
71                 opt(u, 0);
72                 return u;
73             } else {
74                 u = tr[u].ch[1];
75             }
76         }
77     }
78 }
79
80 } splay;
81
82 int n, m, l, r;
83
84 int main() {
85     std::ios::sync_with_stdio(false);
86     std::cin.tie(0);
87     std::cout.tie(0);
88
89     std::cin >> n >> m;
90     splay.n = n;
91     splay.insert(-inf);
92     rep(i, 1, n) splay.insert(i);
93     splay.insert(inf);
94     rep(i, 1, m) {
95         std::cin >> l >> r;
96         l = splay.kth(l), r = splay.kth(r + 2);
97         splay.opt(l, 0), splay.opt(r, 1);
98         splay.tr[splay.tr[r].ch[0]].flag ^= 1;
99     }
100     splay.output(splay.root);
101
102     return 0;
103 }

```

普通平衡树

```

1 // 洛谷 P3369 【模板】普通平衡树
2
3 struct node {
4     int ch[2], fa, key, siz, cnt;
5
6     void init(int _fa, int _key) { fa = _fa, key = _key, siz = cnt = 1; }
7
8     void clear() { ch[0] = ch[1] = fa = key = siz = cnt = 0; }
9 };
10
11 struct splay {
12     node tr[N];
13     int n, root, idx;
14
15     bool get(int u) { return u == tr[tr[u].fa].ch[1]; }
16
17     void pushup(int u) { tr[u].siz = tr[tr[u].ch[0]].siz + tr[tr[u].ch[1]].siz + tr[u].cnt; }
18
19     void rotate(int x) {
20         int y = tr[x].fa, z = tr[y].fa;
21         int op = get(x);
22         tr[y].ch[op] = tr[x].ch[op ^ 1];
23         if (tr[x].ch[op ^ 1]) tr[tr[x].ch[op ^ 1]].fa = y;
24         tr[x].ch[op ^ 1] = y;
25         tr[y].fa = x, tr[x].fa = z;
26         if (z) tr[z].ch[y == tr[z].ch[1]] = x;

```



```

27     pushup(y), pushup(x);
28 }
29
30 void opt(int u, int k) {
31     for (int f = tr[u].fa; f = tr[u].fa, f != k; rotate(u)) {
32         if (tr[f].fa != k) {
33             rotate(get(u) == get(f) ? f : u);
34         }
35     }
36     if (k == 0) root = u;
37 }
38
39 void insert(int key) {
40     if (!root) {
41         idx++;
42         tr[idx].init(0, key);
43         root = idx;
44         return;
45     }
46     int u = root, f = 0;
47     while (1) {
48         if (tr[u].key == key) {
49             tr[u].cnt++;
50             pushup(u), pushup(f);
51             opt(u, 0);
52             break;
53         }
54         f = u, u = tr[u].ch[tr[u].key < key];
55         if (!u) {
56             idx++;
57             tr[idx].init(f, key);
58             tr[f].ch[tr[f].key < key] = idx;
59             pushup(idx), pushup(f);
60             opt(idx, 0);
61             break;
62         }
63     }
64 }
65
66 // 返回节点编号 //
67 int kth(int rank) {
68     int u = root;
69     while (1) {
70         if (tr[u].ch[0] && rank <= tr[tr[u].ch[0]].siz) {
71             u = tr[u].ch[0];
72         } else {
73             rank -= tr[tr[u].ch[0]].siz + tr[u].cnt;
74             if (rank <= 0) {
75                 opt(u, 0);
76                 return u;
77             } else {
78                 u = tr[u].ch[1];
79             }
80         }
81     }
82 }
83
84 // 返回排名 //
85 int nlt(int key) {
86     int rank = 0, u = root;
87     while (1) {
88         if (tr[u].key > key) {
89             u = tr[u].ch[0];
90         } else {
91             rank += tr[tr[u].ch[0]].siz;
92             if (tr[u].key == key) {
93                 opt(u, 0);
94                 return rank + 1;
95             }
96             rank += tr[u].cnt;
97             if (tr[u].ch[1]) {
98                 u = tr[u].ch[1];
99             } else {
100                 return rank + 1;
101             }
102         }
103     }
104 }
105
106 int get_prev(int key) { return kth(nlt(key) - 1); }
107
108 int get_next(int key) { return kth(nlt(key) + 1); }
109
110 void remove(int key) {
111     nlt(key);
112     if (tr[root].cnt > 1) {
113         tr[root].cnt--;

```

```

114         pushup(root);
115         return;
116     }
117     int u = root, l = get_prev(key);
118     tr[tr[u].ch[1]].fa = l;
119     tr[l].ch[1] = tr[u].ch[1];
120     tr[u].clear();
121     pushup(root);
122 }
123
124 void output(int u) {
125     if (tr[u].ch[0]) output(tr[u].ch[0]);
126     std::cout << tr[u].key << ' ';
127     if (tr[u].ch[1]) output(tr[u].ch[1]);
128 }
129
130 } splay;
131
132 int n, op, x;
133
134 int main() {
135     std::ios::sync_with_stdio(false);
136     std::cin.tie(0);
137     std::cout.tie(0);
138
139     splay.insert(-inf), splay.insert(inf);
140
141     std::cin >> n;
142     for (int i = 1; i <= n; i++) {
143         std::cin >> op >> x;
144         if (op == 1) {
145             splay.insert(x);
146         } else if (op == 2) {
147             splay.remove(x);
148         } else if (op == 3) {
149             std::cout << splay.nlt(x) - 1 << endl;
150         } else if (op == 4) {
151             std::cout << splay.tr[splay.kth(x + 1)].key << endl;
152         } else if (op == 5) {
153             std::cout << splay.tr[splay.get_prev(x)].key << endl;
154         } else if (op == 6) {
155             std::cout << splay.tr[splay.get_next(x)].key << endl;
156         }
157     }
158
159     return 0;
160 }

```

4 string

4.1 kmp

```

1  /* kmp */
2  auto kmp = [&](const std::string& s) -> vi {
3      int n = s.length();
4      vi next(n);
5      for (int i = 1; i < n; i++) {
6          int j = next[i - 1];
7          while (j > 0 and s[i] != s[j]) j = next[j - 1];
8          if (s[i] == s[j]) j++;
9          next[i] = j;
10     }
11     return next;
12 };

```

4.2 z function

```

1  /* exkmp */
2  auto exkmp = [&](const std::string& s) -> vi {
3      int n = s.size();
4      vi z(n);
5      for (int i = 1, l = 0, r = 0; i < n; i++) {
6          if (i <= r and z[i - 1] < r - i + 1) {
7              z[i] = z[i - 1];
8          } else {

```

```

9         z[i] = std::max(0, r - i + 1);
10         while (z[i] + i < n and s[z[i]] == s[z[i] + i]) z[i]++;
11     }
12     if (z[i] + i - 1 > r) {
13         l = i;
14         r = z[i] + i - 1;
15     }
16 }
17 return z;
18 };

```

4.3 manacher

TODO

AC automaton

```

1  /* AC auto */
2  int cnt = 0;
3  const int N = 2e5 + 10;
4  static std::array<std::array<int, 26>, N> tr;
5  static std::array<int, N> exist, fail, ans, point;
6  vi order;
7
8  auto insert = [&](const auto& s) {
9      int p = 0;
10     for (const auto& ch : s) {
11         int c = ch - 'a';
12         if (!tr[p][c]) tr[p][c] = ++cnt;
13         p = tr[p][c];
14     }
15     exist[p]++;
16     return p;
17 };
18
19 auto build = [&]() {
20     std::queue<int> q;
21     for (int i = 0; i < 26; i++) {
22         if (tr[0][i]) q.push(tr[0][i]);
23     }
24     while (!q.empty()) {
25         auto u = q.front();
26         q.pop();
27         order.push_back(u);
28         for (int i = 0; i < 26; i++) {
29             if (tr[u][i]) {
30                 fail[tr[u][i]] = tr[fail[u]][i];
31                 q.push(tr[u][i]);
32             } else {
33                 tr[u][i] = tr[fail[u]][i];
34             }
35         }
36     }
37 };
38
39 auto query = [&](const auto& s) {
40     int p = 0;
41     for (const auto& ch : s) {
42         p = tr[p][ch - 'a'];
43         ans[p]++;
44     }
45     return;
46 };
47
48 void solve () {
49     for (int i = 0; i < n; i++) {
50         point[i] = insert(t);
51     }
52     build();
53     query(s);
54     /* fail 树上子树求和 */
55     reverse(all(order));
56     for (const auto& i : order) ans[fail[i]] += ans[i];
57 }

```

4.4 PAM

```

1  /* PAM @ ddl */
2  std::vector<node> tr;
3  std::vector<int> stk;
4  auto newnode = [&](int len) {
5      tr.emplace_back();
6      tr.back().len = len;
7      return (int) tr.size() - 1;
8  };
9  auto PAMinit = [&]() {
10     newnode(0), tr.back().fail = 1;
11     newnode(-1), tr.back().fail = 0;
12     stk.push_back(-1);
13 };
14 PAMinit();
15 auto getfail = [&](int v) {
16     while (stk.end()[-2 - tr[v].len] != stk.back()) {
17         v = tr[v].fail;
18     }
19     return v;
20 };
21 auto insert = [&](int last, int c, int cnt) {
22     stk.emplace_back(c);
23     int x = getfail(last);
24     if (!tr[x].ch[c]) {
25         int u = newnode(tr[x].len + 2);
26         tr[u].fail = tr[getfail(tr[x].fail)].ch[c];
27         tr[x].ch[c] = u;
28         /* tr[u].size = tr[tr[u].fail].size + 1; */
29         /* Can be used to count the number of types of palindromic strings ending at the current
30          * position */
31     }
32     tr[tr[x].ch[c]].size += cnt;
33     return tr[x].ch[c];
34 };
35 auto build = [&]() { /* DP on fail tree */
36     int ans = 0;
37     for (int i = (int) tr.size() - 1; i > 1; i--) {
38         tr[tr[i].fail].size += tr[i].size;
39         /* options */
40     }
41     return ans;
42 };
43 /* PAM */
44 int ans = 0, last = 0;
45 for (int i = 0; i < n; i++) {
46     last = insert(last, s[i] - 'a', 1);
47 }

```

4.5 Suffix Array

```

1  /* suffix array and ST table @ jiangly */
2  auto suffixArray = [&](const std::string& s) {
3      int n = s.length();
4      vi sa(n), rk(n);
5      std::iota(all(sa), 0);
6      std::sort(all(sa), [&](int a, int b) { return s[a] < s[b]; });
7      rk[sa[0]] = 0;
8      for (int i = 1; i < n; ++i) {
9          rk[sa[i]] = rk[sa[i - 1]] + (s[sa[i]] != s[sa[i - 1]]);
10     }
11     int k = 1;
12     vi tmp(n), cnt(n);
13     tmp.reserve(n);
14     while (rk[sa[n - 1]] < n - 1) {
15         tmp.clear();
16         for (int i = 0; i < k; ++i) tmp.push_back(n - k + i);
17         for (const auto& i : sa) {
18             if (i >= k) tmp.push_back(i - k);
19         }
20         std::fill(all(cnt), 0);
21         for (int i = 0; i < n; i++) cnt[rk[i]]++;
22         for (int i = 1; i < n; i++) cnt[i] += cnt[i - 1];
23         for (int i = n - 1; i >= 0; i--) sa[--cnt[rk[tmp[i]]]] = tmp[i];
24         std::swap(rk, tmp);
25         rk[sa[0]] = 0;
26         for (int i = 1; i < n; i++) {
27             rk[sa[i]] = rk[sa[i - 1]] + (tmp[sa[i - 1]] < tmp[sa[i]] or sa[i - 1] + k == n or
28                 tmp[sa[i - 1] + k] < tmp[sa[i] + k]);

```

```

29     }
30     k *= 2;
31 }
32 vi height(n);
33 for (int i = 0, j = 0; i < n; ++i) {
34     if (rk[i] == 0) continue;
35     if (j) --j;
36     while (s[i + j] == s[sa[rk[i] - 1] + j]) ++j;
37     height[rk[i]] = j;
38 }
39 return std::make_tuple(sa, rk, height);
40 };
41 auto [sa, rk, height] = suffixArray(s);
42 vvi f(n, vi(30, inf));
43 vi Log2(n);
44 auto init = [&]() -> void {
45     for (int i = 0; i < n; i++) {
46         f[i][0] = height[i];
47         if (i > 1) Log2[i] = Log2[i / 2] + 1;
48     };
49     int t = Log2.back();
50     for (int j = 1; j <= t; j++) {
51         for (int i = 0; i <= n - (1 << j); i++) {
52             f[i][j] = std::min(f[i][j - 1], f[i + (1 << (j - 1))][j - 1]);
53         }
54     }
55 };
56 init();
57 auto query = [&](int l, int r) -> int {
58     int t = Log2[r - l + 1];
59     return std::min(f[l][t], f[r - (1 << t) + 1][t]);
60 };
61 auto lcp = [&](int i, int j) {
62     i = rk[i], j = rk[j];
63     if (i > j) std::swap(i, j);
64     return query(i + 1, j);
65 };

```

4.6 trie

普通字典树 (单词匹配)

```

1  /* trie */
2  int cnt;
3  std::vector<std::array<int, 26>> trie(n + 1);
4  vi exist(n + 1);
5  auto insert = [&](const std::string& s) -> void {
6      int p = 0;
7      for (const auto ch : s) {
8          int c = ch - 'a';
9          if (!trie[p][c]) trie[p][c] = ++cnt;
10         p = trie[p][c];
11     }
12     exist[p] = true;
13 };
14 auto find = [&](const string& s) -> bool {
15     int p = 0;
16     for (const auto ch : s) {
17         int c = ch - 'a';
18         if (!trie[p][c]) return false;
19         p = trie[p][c];
20     }
21     return exist[p];
22 };

```

01 字典树 (求最大异或值)

给定 n 个数, 取两个数进行异或运算, 求最大异或值.

```

1  /* trie */
2  int cnt = 0;
3  std::vector<std::array<int, 2>> trie(N);
4  auto insert = [&](int x) -> void {
5      int p = 0;
6      for (int i = 30; i >= 0; i--) {
7          int c = (x >> i) & 1;

```

```

8         if (!trie[p][c]) trie[p][c] = ++cnt;
9         p = trie[p][c];
10    }
11};
12auto find = [&](int x) -> int {
13    int sum = 0, p = 0;
14    for (int i = 30; i >= 0; i--) {
15        int c = (x >> i) & 1;
16        if (trie[p][c ^ 1]) {
17            p = trie[p][c ^ 1];
18            sum += (1 << i);
19        } else {
20            p = trie[p][c];
21        }
22    }
23    return sum;
24};

```

字典树合并

来自浙大城市学院 2023 校赛 E 题.

给定一棵根为 1 的树, 每个点的点权为 w_i . 一共 q 次询问, 每次给出一对 u, v , 询问以 v 为根的子树上的点与 u 的权值最大异或值.

```

1  int main() {
2      std::ios::sync_with_stdio(false);
3      std::cin.tie(0);
4
5      int n, m;
6      std::cin >> n;
7      vi w(n + 1);
8      for (int i = 1; i <= n; i++) std::cin >> w[i];
9      vvi e(n + 1);
10     for (int i = 1, u, v; i < n; i++) {
11         std::cin >> u >> v;
12         e[u].push_back(v);
13         e[v].push_back(u);
14     }
15
16     // 离线询问 //
17     std::cin >> m;
18     std::vector<vpi> q(n + 1);
19     vi ans(m + 1);
20     for (int i = 1; i <= m; i++) {
21         int u, v;
22         std::cin >> u >> v;
23         q[v].emplace_back(u, i);
24     }
25
26     // 01 trie //
27     std::vector<std::array<int, 2>> tr(1);
28     auto new_node = [&]() -> int {
29         tr.emplace_back();
30         return tr.size() - 1;
31     };
32     vi id(n + 1);
33     auto insert = [&](int root, int x) {
34         int p = root;
35         for (int i = 29; i >= 0; i--) {
36             int c = x >> i & 1;
37             if (!tr[p][c]) tr[p][c] = new_node();
38             p = tr[p][c];
39         }
40     };
41     auto query = [&](int root, int x) -> int {
42         int ans = 0, p = root;
43         for (int i = 29; i >= 0; i--) {
44             int c = x >> i & 1;
45             if (tr[p][c ^ 1]) {
46                 p = tr[p][c ^ 1];
47                 ans += (1 << i);
48             } else {
49                 p = tr[p][c];
50             }
51         }
52         return ans;
53     };
54     std::function<int(int, int)> merge = [&](int a, int b) -> int {
55         // b 的信息挪到 a 上 //
56         if (!a) return b;

```

```
57         if (!b) return a;
58         tr[a][0] = merge(tr[a][0], tr[b][0]);
59         tr[a][1] = merge(tr[a][1], tr[b][1]);
60         return a;
61     };
62     std::function<void(int, int)> dfs = [&](int u, int fa) {
63         id[u] = new_node();
64         insert(id[u], w[u]);
65         for (auto v : e[u]) {
66             if (v == fa) continue;
67             dfs(v, u);
68             id[u] = merge(id[u], id[v]);
69         }
70         for (auto [v, i] : q[u]) {
71             ans[i] = query(id[u], w[v]);
72         }
73     };
74     dfs(1, 0);
75     for (int i = 1; i <= m; i++) std::cout << ans[i] << endl;
76     return 0;
77 }
```

5 math - number theory

5.1 mod int

```

1  template <int P>
2  struct Mint {
3      int v = 0;
4
5      // reflection //
6      template <typet = int>
7      constexpr operator T() const {
8          return v;
9      }
10
11     // constructor //
12     constexpr Mint() = default;
13     template <typet>
14     constexpr Mint(T x) : v(x % P) {}
15     constexpr int val() const { return v; }
16     constexpr int mod() { return P; }
17
18     // io //
19     friend std::istream& operator>>(std::istream& is, Mint& x) {
20         LL y;
21         is >> y;
22         x = y;
23         return is;
24     }
25     friend std::ostream& operator<<(std::ostream& os, Mint x) { return os << x.v; }
26
27     // comparison //
28     friend constexpr bool operator==(const Mint& lhs, const Mint& rhs) { return lhs.v == rhs.v; }
29     friend constexpr bool operator!=(const Mint& lhs, const Mint& rhs) { return lhs.v != rhs.v; }
30     friend constexpr bool operator<(const Mint& lhs, const Mint& rhs) { return lhs.v < rhs.v; }
31     friend constexpr bool operator<=(const Mint& lhs, const Mint& rhs) { return lhs.v <= rhs.v; }
32     friend constexpr bool operator>(const Mint& lhs, const Mint& rhs) { return lhs.v > rhs.v; }
33     friend constexpr bool operator>=(const Mint& lhs, const Mint& rhs) { return lhs.v >= rhs.v; }
34
35     // arithmetic //
36     template <typet>
37     friend constexpr Mint power(Mint a, T n) {
38         Mint ans = 1;
39         while (n) {
40             if (n & 1) ans *= a;
41             a *= a;
42             n >>= 1;
43         }
44         return ans;
45     }
46     friend constexpr Mint inv(const Mint& rhs) { return power(rhs, P - 2); }
47     friend constexpr Mint operator+(const Mint& lhs, const Mint& rhs) {
48         return lhs.val() + rhs.val() < P ? lhs.val() + rhs.val() : lhs.val() - P + rhs.val();
49     }
50     friend constexpr Mint operator-(const Mint& lhs, const Mint& rhs) {
51         return lhs.val() < rhs.val() ? lhs.val() + P - rhs.val() : lhs.val() - rhs.val();
52     }
53     friend constexpr Mint operator*(const Mint& lhs, const Mint& rhs) {
54         return static_cast<LL>(lhs.val() * rhs.val() % P);
55     }
56     friend constexpr Mint operator/(const Mint& lhs, const Mint& rhs) { return lhs * inv(rhs); }
57     Mint operator+() const { return *this; }
58     Mint operator-() const { return Mint() - *this; }
59     constexpr Mint& operator++() {
60         v++;
61         if (v == P) v = 0;
62         return *this;
63     }
64     constexpr Mint& operator--() {
65         if (v == 0) v = P;
66         v--;
67         return *this;
68     }
69     constexpr Mint& operator++(int) {
70         Mint ans = *this;
71         ++*this;
72         return ans;
73     }
74     constexpr Mint& operator--(int) {
75         Mint ans = *this;
76         --*this;
77         return ans;
78     }
79     constexpr Mint& operator+=(const Mint& rhs) {

```



```

80     v = v + rhs;
81     return *this;
82 }
83 constexpr Mint& operator--(const Mint& rhs) {
84     v = v - rhs;
85     return *this;
86 }
87 constexpr Mint& operator*=(const Mint& rhs) {
88     v = v * rhs;
89     return *this;
90 }
91 constexpr Mint& operator/=(const Mint& rhs) {
92     v = v / rhs;
93     return *this;
94 }
95 };
96 using Z = Mint<998244353>;

```

5.2 Eculid

欧几里得算法

```

1 std::gcd(a, b)

```

扩展欧几里得算法

```

1 /* exgcd */
2 auto exgcd = [&](LL a, LL b, LL& x, LL& y) {
3     LL x1 = 1, x2 = 0, x3 = 0, x4 = 1;
4     while (b != 0) {
5         LL c = a / b;
6         std::tie(x1, x2, x3, x4, a, b) =
7             std::make_tuple(x3, x4, x1 - x3 * c, x2 - x4 * c, b, a - b * c);
8     }
9     x = x1, y = x2;
10 };
11 auto exgcd = [&](auto&& self, LL a, LL b, LL& x, LL& y) {
12     if (!b) {
13         x = 1, y = 0;
14         return a;
15     }
16     LL d = self(self, b, a % b, y, x);
17     y -= a / b * x;
18     return d;
19 };

```

```

1 auto exgcd = [&](auto&& self, LL a, LL b, LL& x, LL& y) {
2     if (!b) {
3         x = 1, y = 0;
4         return;
5     }
6     self(self, b, a % b, y, x);
7     y -= a / b * x;
8 };

```

```

1 auto exgcd = [&](auto&& self, LL a, LL b, LL& x, LL& y) {
2     if (!b) {
3         x = 1, y = 0;
4         return a;
5     }
6     LL d = self(self, b, a % b, y, x);
7     y -= a / b * x;
8     return d;
9 };

```

类欧几里得算法

一般形式: 求 $f(a, b, c, n) = \sum_{i=0}^n \lfloor \frac{ai+b}{c} \rfloor$

$$f(a, b, c, n) = nm - f(c, c - b - 1, a, m - 1)$$

```

1 LL f(LL a, LL b, LL c, LL n) {
2   if (a == 0) return ((b / c) * (n + 1));
3   if (a >= c || b >= c)
4     return f(a % c, b % c, c, n) + (a / c) * n * (n + 1) / 2 + (b / c) * (n + 1);
5   LL m = (a * n + b) / c;
6   LL v = f(c, c - b - 1, a, m - 1);
7   return n * m - v;
8 }

```

更进一步, 求: $g(a, b, c, n) = \sum_{i=0}^n i \lfloor \frac{ai+b}{c} \rfloor$ 以及 $h(a, b, c, n) = \sum_{i=0}^n \lfloor \frac{ai+b}{c} \rfloor^2$

$$g(a, b, c, n) = \lfloor \frac{mn(n+1) - f(c, c-b-1, a, m-1) - h(c, c-b-1, a, m-1)}{2} \rfloor$$

$$h(a, b, c, n) = nm(m+1) - 2f(c, c-b-1, a, m-1) - 2g(c, c-b-1, a, m-1) - f(a, b, c, n)$$

```

1 const int inv2 = 499122177;
2 const int inv6 = 166374059;
3
4 LL f(LL a, LL b, LL c, LL n);
5 LL g(LL a, LL b, LL c, LL n);
6 LL h(LL a, LL b, LL c, LL n);
7
8 struct data {
9   LL f, g, h;
10 };
11
12 data calc(LL a, LL b, LL c, LL n) {
13   LL ac = a / c, bc = b / c, m = (a * n + b) / c, n1 = n + 1, n21 = n * 2 + 1;
14   data d;
15   if (a == 0) {
16     d.f = bc * n1 % mod;
17     d.g = bc * n % mod * n1 % mod * inv2 % mod;
18     d.h = bc * bc % mod * n1 % mod;
19     return d;
20   }
21   if (a >= c || b >= c) {
22     d.f = n * n1 % mod * inv2 % mod * ac % mod + bc * n1 % mod;
23     d.g =
24       ac * n % mod * n1 % mod * n21 % mod * inv6 % mod + bc * n % mod * n1 % mod * inv2 % mod;
25     d.h = ac * ac % mod * n % mod * n1 % mod * n21 % mod * inv6 % mod +
26       bc * bc % mod * n1 % mod + ac * bc % mod * n % mod * n1 % mod;
27     d.f %= mod, d.g %= mod, d.h %= mod;
28     data e = calc(a % c, b % c, c, n);
29     d.h += e.h + 2 * bc % mod * e.f % mod + 2 * ac % mod * e.g % mod;
30     d.g += e.g, d.f += e.f;
31     d.f %= mod, d.g %= mod, d.h %= mod;
32     return d;
33   }
34   data e = calc(c, c - b - 1, a, m - 1);
35   d.f = n * m % mod - e.f, d.f = (d.f % mod + mod) % mod;
36   d.g = m * n % mod * n1 % mod - e.h - e.f, d.g = (d.g * inv2 % mod + mod) % mod;
37   d.h = n * m % mod * (m + 1) % mod - 2 * e.g - 2 * e.f - d.f;
38   d.h = (d.h % mod + mod) % mod;
39   return d;
40 }

```

5.3 inverse

线性递推

$$a^{-1} \equiv -\lfloor \frac{p}{a} \rfloor \times (p \% a)^{-1}$$

```

1 /* inverse */
2 vi inv(n + 1);
3 auto sieve_inv = [&](int n) {
4   inv[1] = 1;
5   for (int i = 2; i <= n; i++) {
6     inv[i] = 1ll * (p - p / i) * inv[p % i] % p;
7   }
8 };

```

求 n 个数的逆元

```

1  /* inverse */
2  auto inverse = [&](const vi& a) {
3      int n = a.size();
4      vi b(n), f(n), ivf(n);
5      f[0] = a[0];
6      for (int i = 1; i < n; i++) {
7          f[i] = 1ll * f[i - 1] * a[i] % p;
8      }
9      ivf.back() = quick_power(f.back(), p - 2, p);
10     for (int i = n - 1; i; i--) {
11         ivf[i - 1] = 1ll * ivf[i] * a[i] % p;
12     }
13     b[0] = ivf[0];
14     for (int i = 1; i < n; i++) {
15         b[i] = 1ll * ivf[i] * f[i - 1] % p;
16     }
17     return b;
18 };

```

5.4 sieve

素数

```

1  vi prime, is_prime(n + 1, 1);
2  auto Euler_sieve = [&](int n){
3      for (int i = 2; i <= n; i++) {
4          if (is_prime[i]) prime.push_back(i);
5          for (auto p : prime) {
6              if (i * p > n) break;
7              is_prime[i * p] = 0;
8              if (i % p == 0) break;
9          }
10     }
11 };

```

欧拉函数

```

1  vi phi(n + 1), prime;
2  vi is_prime(n + 1, 1);
3  auto get_phi = [&](int n) {
4      int cnt = 0;
5      phi[1] = 1;
6      for (int i = 2; i <= n; i++) {
7          if (is_prime[i]) {
8              prime.push_back(i);
9              phi[i] = i - 1;
10         }
11         for (auto p : prime) {
12             if (i * p > n) break;
13             is_prime[i * p] = 0;
14             if (i % p) {
15                 phi[i * p] = phi[i] * phi[p];
16             } else {
17                 phi[i * p] = phi[i] * p;
18                 break;
19             }
20         }
21     }
22 };

```

约数和

```

1  vi g(n + 1), d(n + 1), prime;
2  vi is_prime(n + 1, 1);
3  auto get_d = [&](int n) {
4      int tot = 0;
5      g[1] = d[1] = 1;
6      for (int i = 2; i <= n; i++) {
7          if (is_prime[i]) {

```

```

8         prime.push_back(i);
9         d[i] = g[i] = i + 1;
10    }
11    for (auto p : prime) {
12        if (i * p > n) break;
13        is_prime[i * p] = 0;
14        if (i % p == 0) {
15            g[i * p] = g[i] * p + 1;
16            d[i * p] = d[i] / g[i] * g[i * p];
17            break;
18        } else {
19            d[i * p] = d[i] * d[p];
20            g[i * p] = 1 + p;
21        }
22    }
23 }
24 };

```

莫比乌斯函数

```

1  vi mu(n + 1), prime;
2  vi is_prime(n + 1, 1);
3  auto get_mu = [&](int n) {
4      mu[1] = 1;
5      for (int i = 2; i <= n; i++) {
6          if (is_prime[i]) {
7              prime.push_back(i);
8              mu[i] = -1;
9          }
10         for (auto p : prime) {
11             if (i * p > n) break;
12             is_prime[i * p] = 0;
13             if (i % p == 0) {
14                 mu[i * p] = 0;
15                 break;
16             }
17             mu[i * p] = -mu[i];
18         }
19     }
20 };

```

杜教筛

```

1  const int N = 1e7;
2  vi mu(N + 1), phi(N + 1), prime;
3  vl sum_phi(N + 1), sum_mu(N + 1);
4  vi is_prime(N + 1, 1);
5  std::map<LL, LL> mp_mu;
6
7  /* 计算 1 ~ 10^7 的 mu */
8  auto get_mu = [&](int n) {
9      phi[1] = mu[1] = 1;
10     for (int i = 2; i <= n; i++) {
11         if (is_prime[i]) {
12             prime.push_back(i);
13             phi[i] = i - 1;
14             mu[i] = -1;
15         }
16         for (auto p : prime) {
17             if (i * p > n) break;
18             is_prime[i * p] = 0;
19             if (i % p == 0) {
20                 phi[i * p] = phi[i] * p;
21                 mu[i * p] = 0;
22                 break;
23             }
24             phi[i * p] = phi[i] * phi[p];
25             mu[i * p] = -mu[i];
26         }
27     }
28 };
29 get_mu(N);
30 for (int i = 1; i <= N; i++) {
31     sum_phi[i] = sum_phi[i - 1] + phi[i];
32     sum_mu[i] = sum_mu[i - 1] + mu[i];
33 }
34
35 /* 杜教筛：求 mu 的前缀和 */

```

```

36 std::function<LL(LL)> S_mu = [&](LL x) -> LL {
37     if (x <= N) return sum_mu[x];
38     auto it = mp_mu.find(x);
39     if (it != mp_mu.end()) return mp_mu[x];
40     LL ans = 1;
41     for (LL i = 2, j; i <= x; i = j + 1) {
42         j = x / (x / i);
43         ans -= S_mu(x / i) * (j - i + 1);
44     }
45     return mp_mu[x] = ans;
46 };
47
48 /* 杜教筛: 求 phi 的前缀和 */
49 auto S_phi = [&](LL x) -> LL {
50     if (x <= N) return sum_phi[x];
51     LL ans = 0;
52     for (LL i = 1, j; i <= x; i = j + 1) {
53         j = x / (x / i);
54         ans += 1ll * (S_mu(j) - S_mu(i - 1)) * (x / i) * (x / i);
55     }
56     return (ans - 1) / 2 + 1;
57 };

```

5.5 block

分块的逻辑

下取整 $\lfloor \frac{n}{g} \rfloor = k$ 的分块 ($g \leq n$)

```

1 for(int l = 1, r, k; l <= n; l = r + 1){
2     k = n / l;
3     r = n / (n / l);
4     debug(l, r, k);
5 }

```

$k = \lfloor \frac{n}{g} \rfloor$ 从大到小遍历 $\lfloor \frac{n}{g} \rfloor$ 的所有取值, $[l, r]$ 对应的是 g 取值的区间.

```

1 n = 11
2 [l, r, k] : 1 1 11
3 [l, r, k] : 2 2 5
4 [l, r, k] : 3 3 3
5 [l, r, k] : 4 5 2
6 [l, r, k] : 6 11 1

```

上取整 $\lceil \frac{n}{g} \rceil = k$ 的分块 ($g < n$)

```

1 for(int l = 1, r, k; l < n; l = r + 1){
2     k = (n + l - 1) / l;
3     r = (n + k - 2) / (k - 1) - 1;
4     debug(l, r, k);
5 }

```

$k = \lceil \frac{n}{g} \rceil$ 从大到小遍历 $\lceil \frac{n}{g} \rceil$ 的所有取值, $[l, r]$ 对应的是 g 取值的区间.

```

1 n = 11
2 [l, r, k] : 1 1 11
3 [l, r, k] : 2 2 6
4 [l, r, k] : 3 3 4
5 [l, r, k] : 4 5 3
6 [l, r, k] : 6 10 2

```

一般形式

计算 $\sum_{i=1}^n f(i) \lfloor \frac{n}{i} \rfloor$, 设 $s(i)$ 为 $f(i)$ 的前缀和。

```

1 for (int l = 1, r; l <= n; l = r + 1) {
2     r = n / (n / l);
3     ans += (s[r] - s[l - 1]) * (n / l);
4 }

```

$$\sum_{i=1}^n f(i) \lfloor \frac{a}{i} \rfloor \lfloor \frac{b}{i} \rfloor$$

```

1 for (int l = 1, r, r1, r2; l <= n; l = r + 1) {
2     if (a / l) {
3         r1 = a / (a / l);
4     } else {
5         r1 = n;
6     }
7     if (b / l) {
8         r2 = b / (b / l);
9     } else {
10        r2 = n;
11    }
12    r = min(min(r1, r2), n);
13    ans += (s[r] - s[l - 1]) * (a / l) * (b / l);
14 }

```

5.6 CRT & exCRT

求解

$$\begin{cases} N \equiv a_1 \pmod{m_1} \\ N \equiv a_2 \pmod{m_2} \\ \dots \\ N \equiv a_n \pmod{m_n} \end{cases}$$

$$\text{有 } N \equiv \sum_{i=1}^k a_i \times \text{inv}\left(\frac{M}{m_i}, m_i\right) \times \left(\frac{M}{m_i}\right) \pmod{M}$$

```

1 /* CRT */
2 auto crt = [&](int n, const vi& a, const vi& m) -> LL{
3     LL ans = 0, M = 1;
4     for(int i = 1; i <= n; i++) M *= m[i];
5     for(int i = 1; i <= n; i++){
6         ans = (ans + a[i] * inv(M / m[i], m[i]) * (M / m[i])) % M;
7     }
8     return (ans % M + M) % M;
9 };

```

扩展中国剩余定理

```

1 /* exCRT */
2 auto exCRT = [&](int n, const vi& a, const vi& m) -> LL{
3     LL A = a[1], M = m[1];
4     for (int i = 2; i <= n; i++) {
5         LL x, y, d = std::gcd(M, m[i]);
6         exgcd(M, m[i], x, y);
7         LL mod = M / d * m[i];
8         x = x * (a[i] - A) / d % (m[i] / d);
9         A = ((M * x + A) % mod + mod) % mod;
10        M = mod;
11    }
12    return A;
13 };

```

5.7 BSGS & exBSGS

求解满足 $a^x \equiv b \pmod{p}$ 的 x

```

1 /* BSGS */
2 /* return value = -1e18 means no solution */
3 auto BSGS = [&](LL a, LL b, LL p) {
4     if (1 % p == b % p) return 0ll;
5     LL k = std::sqrt(p) + 1;
6     std::unordered_map<LL, LL> hash;
7     for (LL i = 0, j = b % p; i < k; i++) {
8         hash[j] = i;
9         j = j * a % p;
10    }
11    LL ak = 1;
12    for (int i = 1; i <= k; i++) ak = ak * a % p;

```

```

13     for (int i = 1, j = ak; i <= k; i++) {
14         if (hash.count(j)) return 1ll * i * k - hash[j];
15         j = 1ll * j * ak % p;
16     }
17     return -INF;
18 };

```

$(a, p) \neq 1$ 的情形

```

1  /* exBSGS */
2  /* return value < 0 means no solution */
3  auto exBSGS = [&](auto&& self, LL a, LL b, LL p) {
4      b = (b % p + p) % p;
5      if (1ll % p == b % p) return 0ll;
6      LL x, y, d = std::gcd(a, p);
7      exgcd(exgcd, a, p, x, y);
8      if (d > 1) {
9          if (b % d != 0) return -INF;
10         exgcd(exgcd, a / d, p / d, x, y);
11         return self(self, a, b / d * x % (p / d), p / d) + 1;
12     }
13     return BSGS(a, b, p);
14 };

```

5.8 Miller Rabin

```

1  /* Miller Rabin */
2  vl vv = {2, 325, 9375, 28178, 450775, 9780504, 1795265022};
3  auto quick_power = [&](LL a, LL n, LL mod) {
4      LL ans = 1;
5      while (n) {
6          if (n & 1) ans = (i128) ans * a % mod;
7          a = (i128) a * a % mod;
8          n >>= 1;
9      }
10     return ans;
11 };
12
13 auto millerRabin = [&](LL n) {
14     if (n < 2 || n % 6 % 4 != 1) return (n | 1) == 3;
15     int s = __builtin_ctzll(n - 1);
16     LL d = n >> s;
17     for (auto a : vv) {
18         LL p = quick_power(a % n, d, n);
19         int i = s;
20         while (p != 1 and p != n - 1 and a % n and i--) p = (i128) p * p % n;
21         if (p != n - 1 and i != s) return false;
22     }
23     return true;
24 };

```

5.9 Pollard Rho

能在 $O(n^{\frac{1}{4}})$ 的时间复杂度随机出一个 n 的非平凡因数.

```

1  /* pollard rho */
2  auto pollard_rho = [&](LL x) -> LL{
3      LL s = 0, t = 0, val = 1;
4      LL c = rand() % (x - 1) + 1;
5      for(int goal = 1;; goal <= 1, s = t, val = 1){
6          for(int step = 1; step <= goal; step++){
7              t = ((i128) t * t + c) % x;
8              val = (i128) val * abs(t - s) % x;
9              if(step % 127 == 0){
10                 LL d = std::gcd(val, x);
11                 if(d > 1) return d;
12             }
13         }
14         LL d = std::gcd(val, x);
15         if(d > 1) return d;
16     }
17 };

```

利用 Miller Rabin 和 Pollard Rho 进行素因数分解

```

1 auto factorize = [&](LL a) -> vl{
2     vl ans, stk;
3     for (auto p : prime) {
4         if (p > 1000) break;
5         while (a % p == 0) {
6             ans.push_back(p);
7             a /= p;
8         }
9         if (a == 1) return ans;
10    }
11    stk.push_back(a);
12    while (!stk.empty()) {
13        LL b = stk.back();
14        stk.pop_back();
15        if (miller_rabin(b)) {
16            ans.push_back(b);
17            continue;
18        }
19        LL c = b;
20        while (c >= b) c = pollard_rho(b);
21        stk.push_back(c);
22        stk.push_back(b / c);
23    }
24    return ans;
25 };

```

5.10 quadratic residu

```

1 /* cipolla */
2 auto cipolla = [&](int x) {
3     std::srand(time(0));
4     auto check = [&](int x) -> bool { return pow(x, (mod - 1) / 2) == 1; };
5     if (!x) return 0;
6     if (!check(x)) return -1;
7     int a, b;
8     while (1) {
9         a = rand() % mod;
10        b = sub(mul(a, a), x);
11        if (!check(b)) break;
12    }
13    PII t = {a, 1};
14    PII ans = {1, 0};
15    auto mulp = [&](PII x, PII y) -> PII {
16        auto [x1, x2] = x;
17        auto [y1, y2] = y;
18        int c = add(mul(x1, y1), mul(x2, y2, b));
19        int d = add(mul(x1, y2), mul(x2, y1));
20        return {c, d};
21    };
22    for (int i = (mod + 1) / 2; i; i >>= 1) {
23        if (i & 1) ans = mulp(ans, t);
24        t = mulp(t, t);
25    }
26    return std::min(ans.ff, mod - ans.ff);
27 }

```

5.11 Lucas

卢卡斯定理

用于求大组合数, 并且模数是一个不大的素数.

$$\binom{n}{m} \bmod p = \binom{\lfloor n/p \rfloor}{\lfloor m/p \rfloor} \cdot \binom{n \bmod p}{m \bmod p} \bmod p$$

$\binom{n \bmod p}{m \bmod p}$ 可以直接计算, $\binom{\lfloor n/p \rfloor}{\lfloor m/p \rfloor}$ 可以继续使用卢卡斯计算.

递归至 $m = 0$ 的时候, 返回 1.

p 不太大, 一般在 10^5 左右.


```

1 auto C = [&](LL n, LL m, LL p) -> LL {
2     if (n < m) return 0;
3     if (m == 0) return 1;
4     return fac[n] * inv_fac[m] % p * inv_fac[n - m] % p;
5 };
6 /* lucas */
7 auto lucas = [&](auto&& self, LL n, LL m, LL p) -> LL {
8     if (n < m) return 0;
9     if (m == 0) return 1;
10    return C(n % p, m % p, p) * self(self, n / p, m / p, p) % p;
11 }

```

素数在组合数中的次数

Legengre 给出一种 $n!$ 中素数 p 的幂次的计算方式为:

$$\sum_{1 \leq j} \left\lfloor \frac{n}{p^j} \right\rfloor.$$

另一种计算方式利用 p 进制下各位数字和:

$$v_p(n!) = \frac{n - S_p(n)}{p - 1}.$$

则有

$$v_p(C_m^n) = \frac{S_p(n) + S_p(m - n) - S_p(m)}{p - 1}.$$

扩展卢卡斯定理

计算

$$\binom{n}{m} \bmod p,$$

p 可能为合数.

第一部分: CRT.

原问题变成求

$$\left\{ \begin{array}{l} \binom{n}{m} \equiv a_1 \bmod p_1^{\alpha_1} \\ \binom{n}{m} \equiv a_2 \bmod p_2^{\alpha_2} \\ \dots \\ \binom{n}{m} \equiv a_k \bmod p_k^{\alpha_k} \end{array} \right.$$

在求出 a_i 之后就可以利用 CRT 求出答案.

第二部分: 移除分子分母中的素数

问题转换成求解

$$\binom{n}{m} \bmod q^k.$$

等价于

$$\frac{\frac{n!}{q^x}}{\frac{m!}{q^y} \frac{(n-m)!}{q^z}} q^{x-y-z} \bmod q^k,$$

其中 x 表示 $n!$ 中 q 的次数, y, z 同理.

第三部分: 威尔逊定理的推论

问题转换为求

$$\frac{n!}{q^x} \bmod q^k.$$

可以利用威尔逊定理的推论.

```

1  /* exlucas */
2  auto exLucas = [&](LL n, LL m, LL p) {
3      auto inv = [&](LL a, LL p) {
4          LL x, y;
5          exgcd(a, p, x, y);
6          return (x % p + p) % p;
7      };
8
9      auto func = [&](auto&& self, LL n, LL pi, LL pk) {
10         if (!n) return 1ll;
11         LL ans = 1;
12         for (LL i = 2; i <= pk; i++) {
13             if (i % pi) ans = ans * i % p;
14         }
15         ans = quick_power(ans, n / pk, pk);
16         for (LL i = 2; i <= n % pk; i++) {
17             if (i % pi) ans = ans * i % pk;
18         }
19         ans = ans * self(self, n / pi, pi, pk) % pk;
20         return ans;
21     };
22
23     auto multiLucas = [&](LL n, LL m, LL pi, LL pk) {
24         LL cnt = 0;
25         for (LL i = n; i; i /= pi) cnt += i / pi;
26         for (LL i = m; i; i /= pi) cnt -= i / pi;
27         for (LL i = n - m; i; i /= pi) cnt -= i / pi;
28         LL ans = quick_power(pi, cnt, pk) * func(func, n, pi, pk) % pk;
29         ans = ans * inv(func(func, m, pi, pk), pk) % pk;
30         ans = ans * inv(func(func, n - m, pi, pk), pk) % pk;
31         return ans;
32     };
33
34     auto crt = [&](const vl& a, const vl& m, int k) {
35         LL ans = 0;
36
37         for (int i = 0; i < k; i++) {
38             ans = (ans + a[i] * inv(p / m[i], m[i]) * (p / m[i])) % p;
39         }
40         return (ans % p + p) % p;
41     };
42
43     vl a, prime;
44     LL pp = p;
45     for (int i = 2; i * i <= pp; i++) {
46         if (pp % i) continue;
47         prime.push_back(1);
48         while (pp % i == 0) {
49             prime.back() *= i;
50             pp /= i;
51         }
52         a.push_back(multiLucas(n, m, i, prime.back()));
53     }
54     if (pp > 1) {
55         prime.push_back(pp);
56         a.push_back(multiLucas(n, m, pp, pp));
57     }
58     return crt(a, prime, a.size());
59 };

```

5.12 Wilson

简单结论

对于素数 p 有

$$(p-1)! \equiv -1 \bmod p.$$

推论

令 $(n!)_p$ 表示不大于 n 且不被 p 整除的正整数的乘积.

特殊情形: n 为素数 p 时即为上述结论.

一般结论: 对素数 p 和正整数 q 有

$$((p^q)!)_p \equiv \pm 1 \pmod{p^q}.$$

详细定义:

$$((p^q)!)_p = \begin{cases} 1 & \text{if } p = 2 \text{ and } q \geq 3, \\ -1 & \text{other wise.} \end{cases}$$

5.13 LTE

将素数 p 在整数 n 中的个数记为 $v_p(n)$.

$$(n, p) = 1$$

对所有素数 p 和满足 $(n, p) = 1$ 的整数 n , 有

1. 若 $p \mid x - y$, 则有

$$v_p(x^n - y^n) = v_p(x - y).$$

2. 若 $p \mid x - y$, 则对奇数 n 有

$$v_p(x^n + y^n) = v_p(x + y).$$

p 是奇素数

对所有奇素数 p 有

1. 若 $p \mid x - y$, 则有

$$v_p(x^n - y^n) = v_p(x - y) + v_p(n).$$

2. 若 $p \mid x - y$, 则对奇数 n 有

$$v_p(x^n + y^n) = v_p(x + y) + v_p(n).$$

$$p = 2$$

对 $p = 2$ 且 $p \mid x - y$ 有

1. 对奇数 n 有

$$v_2(x^n - y^n) = v_2(x - y).$$

2. 对偶数 n 有

$$v_2(x^n - y^n) = v_2(x - y) + v_2(x + y) + v_2(n) - 1.$$

除此之外, 对上述 x, y, n , 若 $4 \mid x - y$, 有

1. $v_2(x + y) = 1$.

2. $v_2(x^n - y^n) = v_2(x - y) + v_2(n)$.

5.14 Mobius inversion

莫比乌斯函数

$$\mu(n) = \begin{cases} 1 & n = 1, \\ 0 & n \text{ 含有平方因子}, \\ (-1)^k & k \text{ 为 } n \text{ 的本质不同素因子个数}. \end{cases}$$

性质

$$\sum_{d|n} \mu(d) = \begin{cases} 1 & n = 1 \\ 0 & n \neq 1 \end{cases}.$$

$$\varphi(n) = \sum_{d|n} d \cdot \mu\left(\frac{n}{d}\right).$$

反演结论

$$[\gcd(i, j) = 1] = \sum_{d|\gcd(i, j)} \mu(d).$$

$O(n \log n)$ 求莫比乌斯函数

```

1 mu[1] = 1;
2 for (int i = 1; i <= n; i++){
3     for (int j = i + i; j <= n; j += i){
4         mu[j] -= mu[i];
5     }
6 }
```

莫比乌斯变换

设 $f(n), F(n)$.

1. $F(n) = \sum_{d|n} f(d)$, 则 $f(n) = \sum_{d|n} \mu(d) F\left(\frac{n}{d}\right)$.
2. $F(n) = \sum_{n|d} f(d)$, 则 $f(n) = \sum_{n|d} \mu\left(\frac{d}{n}\right) F(d)$.

6 math - polynomial

6.1 FTT

FFT 与拆系数 FFT

```

1  const int sz = 1 << 23;
2  int rev[sz];
3  int rev_n;
4  void set_rev(int n) {
5      if (n == rev_n) return;
6      for (int i = 0; i < n; i++) rev[i] = (rev[i / 2] | (i & 1) * n) / 2;
7      rev_n = n;
8  }
9  template void butterfly(T* a, int n) {
10     set_rev(n);
11     for (int i = 0; i < n; i++) {
12         if (i < rev[i]) std::swap(a[i], a[rev[i]]);
13     }
14 }
15
16 namespace Comp {
17
18     long double pi = 3.141592653589793238;
19
20     template struct complex {
21         T x, y;
22         complex(T x = 0, T y = 0) : x(x), y(y) {}
23         complex operator+(const complex& b) const { return complex(x + b.x, y + b.y); }
24
25         complex operator-(const complex& b) const { return complex(x - b.x, y - b.y); }
26
27         complex operator*(const complex& b) const {
28             return complex<T>(x * b.x - y * b.y, x * b.y + y * b.x);
29         }
30         complex operator~() const { return complex(x, -y); }
31         static complex unit(long double rad) { return complex(std::cos(rad), std::sin(rad)); }
32     };
33
34 } // namespace Comp
35
36 struct fft_t {
37     typedef Comp::complex<double> complex;
38     complex wn[sz];
39
40     fft_t() {
41         for (int i = 0; i < sz / 2; i++) {
42             wn[sz / 2 + i] = complex::unit(2 * Comp::pi * i / sz);
43         }
44         for (int i = sz / 2 - 1; i; i--) wn[i] = wn[i * 2];
45     }
46
47     void operator()(complex* a, int n, int type) {
48         if (type == -1) std::reverse(a + 1, a + n);
49         butterfly(a, n);
50         for (int i = 1; i < n; i *= 2) {
51             const complex* w = wn + i;
52             for (complex* b = a, t; b != a + n; b += i + 1) {
53                 t = b[i];
54                 b[i] = *b - t;
55                 *b = *b + t;
56                 for (int j = 1; j < i; j++) {
57                     t = (++b)[i] * w[j];
58                     b[i] = *b - t;
59                     *b = *b + t;
60                 }
61             }
62         }
63         if (type == 1) return;
64         for (int i = 0; i < n * 2; i++) ((double*) a)[i] /= n;
65     }
66 } FFT;
67
68 typedef decltype(FFT)::complex complex;
69
70 vi fft(const vi& f, const vi& g) {
71     static complex ff[sz];
72     int n = f.size(), m = g.size();
73     vi h(n + m - 1);
74     if (std::min(n, m) <= 50) {
75         for (int i = 0; i < n; i++) {

```

```

76         for (int j = 0; j < m; ++j) {
77             h[i + j] += f[i] * g[j];
78         }
79     }
80     return h;
81 }
82 int c = 1;
83 while (c + 1 < n + m) c *= 2;
84 std::memset(ff, 0, sizeof(decltype(*(ff))) * (c));
85 for (int i = 0; i < n; i++) ff[i].x = f[i];
86 for (int i = 0; i < m; i++) ff[i].y = g[i];
87 FFT(ff, c, 1);
88 for (int i = 0; i < c; i++) ff[i] = ff[i] * ff[i];
89 FFT(ff, c, -1);
90 for (int i = 0; i + 1 < n + m; i++) h[i] = std::llround(ff[i].y / 2);
91 return h;
92 }
93
94 vi mtt(const vi& f, const vi& g) {
95     static complex ff[3][sz], gg[2][sz];
96     static int s[3] = {1, 31623, 31623 * 31623};
97     int n = f.size(), m = g.size();
98     vi h(n + m - 1);
99     if (std::min(n, m) <= 50) {
100         for (int i = 0; i < n; ++i) {
101             for (int j = 0; j < m; ++j) {
102                 Add(h[i + j], mul(f[i], g[j]));
103             }
104         }
105         return h;
106     }
107     int c = 1;
108     while (c + 1 < n + m) c *= 2;
109     for (int i = 0; i < 2; ++i) {
110         std::memset(ff[i], 0, sizeof(decltype(*(ff[i]))) * (c));
111         std::memset(gg[i], 0, sizeof(decltype(*(ff[i]))) * (c));
112         for (int j = 0; j < n; ++j) ff[i][j].x = f[j] / s[i] % s[1];
113         for (int j = 0; j < m; ++j) gg[i][j].x = g[j] / s[i] % s[1];
114         FFT(ff[i], c, 1);
115         FFT(gg[i], c, 1);
116     }
117     for (int i = 0; i < c; ++i) {
118         ff[2][i] = ff[1][i] * gg[1][i];
119         ff[1][i] = ff[1][i] * gg[0][i];
120         gg[1][i] = ff[0][i] * gg[1][i];
121         ff[0][i] = ff[0][i] * gg[0][i];
122     }
123     for (int i = 0; i < 3; ++i) {
124         FFT(ff[i], c, -1);
125         for (int j = 0; j + 1 < n + m; ++j) {
126             Add(h[j], mul(std::llround(ff[i][j].x) % mod, s[i]));
127         }
128     }
129     FFT(gg[1], c, -1);
130     for (int i = 0; i + 1 < n + m; ++i) {
131         Add(h[i], mul(std::llround(gg[1][i].x) % mod, s[1]));
132     }
133     return h;
134 }

```

6.2 FWT

and

$$C_i = \sum_{j \& k} A_j B_k$$

分治过程

$$\text{FWT}[A] = \text{merge}(\text{FWT}[A_0] + \text{FWT}[A_1], \text{FWT}[A_1]),$$

$$\text{UFWT}[A'] = \text{merge}(\text{UFWT}[A'_0] - \text{UFWT}[A'_1], \text{UFWT}[A'_1]).$$

```

1  /* mod 998244353 */
2  int n = v.size();
3  for (int mid = 1; mid < n; mid <= 1) {
4      for (int block = mid < 1, j = 0; j < n; j += block) {
5          for (int i = j; i < j + mid; i++) {

```

```

6         LL x = v[i], y = v[i + mid];
7         if (type == 1) {
8             v[i] = add(x, y);
9         } else {
10            v[i] = sub(x, y);
11        }
12    }
13    }
14 }
15 return v;
16 };

```

or

$$C_i = \sum_{i=j|k} A_j B_k$$

分治过程

$$\text{FWT}[A] = \text{merge}(\text{FWT}[A_0], \text{FWT}[A_0] + \text{FWT}[A_1]),$$

$$\text{UFWT}[A'] = \text{merge}(\text{UFWT}[A'_0], -\text{UFWT}[A'_0] + \text{UFWT}[A'_1]).$$

```

1  /* mod 998244353 */
2  auto FWT_or = [&](vi v, int type) -> vi {
3      int n = v.size();
4      for (int mid = 1; mid < n; mid <= 1) {
5          for (int block = mid < 1, j = 0; j < n; j += block) {
6              for (int i = j; i < j + mid; i++) {
7                  LL x = v[i], y = v[i + mid];
8                  if (type == 1) {
9                      v[i + mid] = add(x, y);
10                 } else {
11                     v[i + mid] = sub(y, x);
12                 }
13             }
14         }
15     }
16     return v;
17 };

```

xor

$$C_i = \sum_{i=j \oplus k} A_j B_k$$

分治过程

$$\text{FWT}[A] = \text{merge}(\text{FWT}[A_0] + \text{FWT}[A_1], \text{FWT}[A_0] - \text{FWT}[A_1]),$$

$$\text{UFWT}[A'] = \text{merge}\left(\frac{\text{UFWT}[A'_0] + \text{UFWT}[A'_1]}{2}, \frac{\text{UFWT}[A'_0] - \text{UFWT}[A'_1]}{2}\right).$$

```

1  /* mod 998244353 */
2  auto FWT_xor = [&](vi v, int type) -> vi {
3      int n = v.size();
4      for (int mid = 1; mid < n; mid <= 1) {
5          for (int block = mid < 1, j = 0; j < n; j += block) {
6              for (int i = j; i < j + mid; i++) {
7                  LL x = v[i], y = v[i + mid];
8                  v[i] = add(x, y);
9                  v[i + mid] = sub(x, y);
10                 if (type == -1) {
11                     Mul(v[i], inv2);
12                     Mul(v[i + mid], inv2);
13                 }
14             }
15         }
16     }
17     return v;
18 };

```

统一地,

```

1 a = FWT(a, 1), b = FWT(b, 1);
2 for (int i = 0; i < (1 << n); i++) {
3     c[i] = mul(a[i], b[i]);
4 }
5 c = FWT(c, -1);

```

6.3 class polynomial

```

1 class polynomial : public vi {
2     public:
3         polynomial() = default;
4         polynomial(const vi& v) : vi(v) {}
5         polynomial(vi&& v) : vi(std::move(v)) {}
6
7         int degree() { return size() - 1; }
8
9         void clearzero() {
10             while (size() && !back()) pop_back();
11         }
12 };
13
14 polynomial& operator+=(polynomial& a, const polynomial& b) {
15     a.resize(std::max(a.size(), b.size()), 0);
16     for (int i = 0; i < b.size(); i++) {
17         Add(a[i], b[i]);
18     }
19     a.clearzero();
20     return a;
21 }
22
23 polynomial operator+(const polynomial& a, const polynomial& b) {
24     polynomial ans = a;
25     return ans += b;
26 }
27
28 polynomial& operator-=(polynomial& a, const polynomial& b) {
29     a.resize(std::max(a.size(), b.size()), 0);
30     for (int i = 0; i < b.size(); i++) {
31         Sub(a[i], b[i]);
32     }
33     a.clearzero();
34     return a;
35 }
36
37 polynomial operator-(const polynomial& a, const polynomial& b) {
38     polynomial ans = a;
39     return ans -= b;
40 }
41
42 class ntt_t {
43     public:
44         static const int maxbit = 22;
45         static const int sz = 1 << maxbit;
46         static const int mod = 998244353;
47         static const int g = 3;
48
49         std::array<int, sz + 10> w;
50         std::array<int, maxbit + 10> len_inv;
51
52         ntt_t() {
53             int wn = pow(g, (mod - 1) >> maxbit);
54             w[0] = 1;
55             for (int i = 1; i <= sz; i++) {
56                 w[i] = mul(w[i - 1], wn);
57             }
58             len_inv[maxbit] = pow(sz, mod - 2);
59             for (int i = maxbit - 1; ~i; i--) {
60                 len_inv[i] = add(len_inv[i + 1], len_inv[i + 1]);
61             }
62         }
63
64         void operator()(vi& v, int& n, int type) {
65             int bit = 0;
66             while ((1 << bit) < n) bit++;
67             int tot = (1 << bit);
68             v.resize(tot, 0);
69             vi rev(tot);
70             n = tot;
71             for (int i = 0; i < tot; i++) {

```



```

73         rev[i] = rev[i >> 1] >> 1;
74         if (i & 1) {
75             rev[i] |= tot >> 1;
76         }
77     }
78     for (int i = 0; i < tot; i++) {
79         if (i < rev[i]) {
80             std::swap(v[i], v[rev[i]]);
81         }
82     }
83     for (int midd = 0; (1 << midd) < tot; midd++) {
84         int mid = 1 << midd;
85         int len = mid << 1;
86         for (int i = 0; i < tot; i += len) {
87             for (int j = 0; j < mid; j++) {
88                 int w0 = v[i + j];
89                 int w1 = mul(
90                     w[type == 1 ? (j << maxbit - midd - 1) : (len - j << maxbit - midd - 1)],
91                     v[i + j + mid]);
92                 v[i + j] = add(w0, w1);
93                 v[i + j + mid] = sub(w0, w1);
94             }
95         }
96     }
97     if (type == -1) {
98         for (int i = 0; i < tot; i++) {
99             v[i] = mul(v[i], len_inv[bit]);
100         }
101     }
102 }
103 } NTT;

```

乘法

```

1 polynomial& operator*=(polynomial& a, const polynomial& b) {
2     if (!a.size() || !b.size()) {
3         a.resize(0);
4         return a;
5     }
6     polynomial tmp = b;
7     int deg = a.size() + b.size() - 1;
8     int temp = deg;
9
10    // 项数较小直接硬算
11
12    if ((LL) a.size() * (LL) b.size() <= (LL) deg * 50LL) {
13        tmp.resize(0);
14        tmp.resize(deg, 0);
15        for (int i = 0; i < a.size(); i++) {
16            for (int j = 0; j < b.size(); j++) {
17                tmp[i + j] = add(tmp[i + j], mul(a[i], b[j]));
18            }
19        }
20        a = tmp;
21        return a;
22    }
23
24    // 项数较多跑 NTT
25
26    NTT(a, deg, 1);
27    NTT(tmp, deg, 1);
28    for (int i = 0; i < deg; i++) {
29        Mul(a[i], tmp[i]);
30    }
31    NTT(a, deg, -1);
32    a.resize(temp);
33    return a;
34 }
35
36 polynomial operator*(const polynomial& a, const polynomial& b) {
37     polynomial ans = a;
38     return ans *= b;
39 }

```

逆

```

1 polynomial inverse(const polynomial& a) {
2     polynomial ans({pow(a[0], mod - 2)});

```

```

3   polynomial temp;
4   polynomial tempa;
5   int deg = a.size();
6   for (int i = 0; (1 << i) < deg; i++) {
7       tempa.resize(0);
8       tempa.resize(1 << i << 1, 0);
9       for (int j = 0; j != tempa.size() and j != deg; j++) {
10          tempa[j] = a[j];
11      }
12      temp = ans * (polynomial({2}) - tempa * ans);
13      if (temp.size() > (1 << i << 1)) {
14          temp.resize(1 << i << 1, 0);
15      }
16      temp.clearzero();
17      std::swap(temp, ans);
18  }
19  ans.resize(deg);
20  return ans;
21 }

```

对数

```

1   polynomial diffrential(const polynomial& a) {
2       if (!a.size()) {
3           return a;
4       }
5       polynomial ans(vi(a.size() - 1));
6       for (int i = 1; i < a.size(); i++) {
7           ans[i - 1] = mul(a[i], i);
8       }
9       return ans;
10  }
11
12  polynomial integral(const polynomial& a) {
13      polynomial ans(vi(a.size() + 1));
14      for (int i = 0; i < a.size(); i++) {
15          ans[i + 1] = mul(a[i], pow(i + 1, mod - 2));
16      }
17      return ans;
18  }
19
20  polynomial ln(const polynomial& a) {
21      int deg = a.size();
22      polynomial da = diffrential(a);
23      polynomial inva = inverse(a);
24      polynomial ans = integral(da * inva);
25      ans.resize(deg);
26      return ans;
27  }

```

指数

```

1   polynomial exp(const polynomial& a) {
2       polynomial ans({1});
3       polynomial temp;
4       polynomial tempa;
5       polynomial tempaa;
6       int deg = a.size();
7       for (int i = 0; (1 << i) < deg; i++) {
8           tempa.resize(0);
9           tempa.resize(1 << i << 1, 0);
10          for (int j = 0; j != tempa.size() and j != deg; j++) {
11              tempa[j] = a[j];
12          }
13          tempaa = ans;
14          tempaa.resize(1 << i << 1);
15          temp = ans * (tempa + polynomial({1}) - ln(tempaa));
16          if (temp.size() > (1 << i << 1)) {
17              temp.resize(1 << i << 1, 0);
18          }
19          temp.clearzero();
20          std::swap(temp, ans);
21      }
22      ans.resize(deg);
23      return ans;
24  }

```

根号

```

1 polynomial sqrt(polynomial& a) {
2     polynomial ans({cipolla(a[0])});
3     if (ans[0] == -1) return ans;
4     polynomial temp;
5     polynomial tempa;
6     polynomial tempaa;
7     int deg = a.size();
8     for (int i = 0; (1 << i) < deg; i++) {
9         tempa.resize(0);
10        tempa.resize(1 << i << 1, 0);
11        for (int j = 0; j != tempa.size() and j != deg; j++) {
12            tempa[j] = a[j];
13        }
14        tempaa = ans;
15        tempaa.resize(1 << i << 1);
16        temp = (tempa * inverse(tempaa) + ans) * inv2;
17        if (temp.size() > (1 << i << 1)) {
18            temp.resize(1 << i << 1, 0);
19        }
20        temp.clearzero();
21        std::swap(temp, ans);
22    }
23    ans.resize(deg);
24    return ans;
25 }
26
27 // 特判 //
28
29 int cnt = 0;
30 for (int i = 0; i < a.size(); i++) {
31     if (a[i] == 0) {
32         cnt++;
33     } else {
34         break;
35     }
36 }
37 if (cnt) {
38     if (cnt == n) {
39         for (int i = 0; i < n; i++) {
40             std::cout << "0 ";
41         }
42         std::cout << endl;
43         return 0;
44     }
45     if (cnt & 1) {
46         std::cout << "-1" << endl;
47         return 0;
48     }
49     polynomial b(vi(a.size() - cnt));
50     for (int i = cnt; i < a.size(); i++) {
51         b[i - cnt] = a[i];
52     }
53     a = b;
54 }
55 a.resize(n - cnt / 2);
56 a = sqrt(a);
57 if (a[0] == -1) {
58     std::cout << "-1" << endl;
59     return 0;
60 }
61 reverse(all(a));
62 a.resize(n);
63 reverse(all(a));

```

6.4 wsy poly

```

1 #include <bits/stdc++.h>
2
3 using ul = std::uint32_t;
4 using li = std::int32_t;
5 using ll = std::int64_t;
6 using ull = std::uint64_t;
7 using llf = long double;
8 using lf = double;
9 using vul = std::vector<ul>;
10 using vvul = std::vector<vul>;
11 using pulb = std::pair<ul, bool>;
12 using vpulb = std::vector<pulb>;
13 using vvpulb = std::vector<vpulb>;

```

```

14 using vb = std::vector<bool>;
15
16 const ul base = 998244353;
17
18 std::mt19937 rnd;
19
20 ul plus(ul a, ul b) { return a + b < base ? a + b : a + b - base; }
21
22 ul minus(ul a, ul b) { return a < b ? a + base - b : a - b; }
23
24 ul mul(ul a, ul b) { return ull(a) * ull(b) % base; }
25
26 void exgcd(li a, li b, li& x, li& y) {
27     if (b) {
28         exgcd(b, a % b, y, x);
29         y -= x * (a / b);
30     } else {
31         x = 1;
32         y = 0;
33     }
34 }
35
36 ul inverse(ul a) {
37     li x, y;
38     exgcd(a, base, x, y);
39     return x < 0 ? x + li(base) : x;
40 }
41
42 ul pow(ul a, ul b) {
43     ul ret = 1;
44     ul temp = a;
45     while (b) {
46         if (b & 1) {
47             ret = mul(ret, temp);
48         }
49         temp = mul(temp, temp);
50         b >>= 1;
51     }
52     return ret;
53 }
54
55
56 ul sqrt(ul x) {
57     ul a;
58     ul w2;
59     while (true) {
60         a = rnd() % base;
61         w2 = minus(mul(a, a), x);
62         if (pow(w2, base - 1 >> 1) == base - 1) {
63             break;
64         }
65     }
66     ul b = base + 1 >> 1;
67     ul rs = 1, rt = 0;
68     ul as = a, at = 1;
69     ul qs, qt;
70     while (b) {
71         if (b & 1) {
72             qs = plus(mul(rs, as), mul(mul(rt, at), w2));
73             qt = plus(mul(rs, at), mul(rt, as));
74             rs = qs;
75             rt = qt;
76         }
77         b >>= 1;
78         qs = plus(mul(as, as), mul(mul(at, at), w2));
79         qt = plus(mul(as, at), mul(as, at));
80         as = qs;
81         at = qt;
82     }
83     return rs + rs < base ? rs : base - rs;
84 }
85
86 ul log(ul x, ul y, bool initd = false) {
87     static std::map<ul, ul> bs;
88     const ul d = std::round(std::sqrt(1l(base - 1)));
89     if (!initd) {
90         bs.clear();
91         for (ul i = 0, j = 1; i != d; ++i, j = mul(j, x)) {
92             bs[j] = i;
93         }
94     }
95     ul temp = inverse(pow(x, d));
96     for (ul i = 0, j = 1; i += d, j = mul(j, temp)) {
97         auto it = bs.find(mul(y, j));
98         if (it != bs.end()) {
99             return it->second + i;
100         }

```

```

101     }
102 }
103
104 ul powroot(ul x, ul y, bool initd = false) {
105     const ul g = 3;
106     ul lgx = log(g, x, initd);
107     li s, t;
108     exgcd(y, base - 1, s, t);
109     if (s < 0) {
110         s += base - 1;
111     }
112     return pow(g, ull(s) * ull(lgx) % (base - 1));
113 }
114
115 class polynomial : public vul {
116 public:
117     void clearzero() {
118         while (size() && !back()) {
119             pop_back();
120         }
121     }
122     polynomial() = default;
123     polynomial(const vul& a) : vul(a) {}
124     polynomial(vul&& a) : vul(std::move(a)) {}
125     ul degree() const { return size() - 1; }
126     ul operator()(ul x) const {
127         ul ret = 0;
128         for (ul i = size() - 1; ~i; --i) {
129             ret = mul(ret, x);
130             ret = plus(ret, vul::operator[](i));
131         }
132         return ret;
133     }
134 };
135
136 polynomial& operator+=(polynomial& a, const polynomial& b) {
137     a.resize(std::max(a.size(), b.size()), 0);
138     for (ul i = 0; i != b.size(); ++i) {
139         a[i] = plus(a[i], b[i]);
140     }
141     a.clearzero();
142     return a;
143 }
144
145 polynomial operator+(const polynomial& a, const polynomial& b) {
146     polynomial ret = a;
147     return ret += b;
148 }
149
150 polynomial& operator-=(polynomial& a, const polynomial& b) {
151     a.resize(std::max(a.size(), b.size()), 0);
152     for (ul i = 0; i != b.size(); ++i) {
153         a[i] = minus(a[i], b[i]);
154     }
155     a.clearzero();
156     return a;
157 }
158
159 polynomial operator-(const polynomial& a, const polynomial& b) {
160     polynomial ret = a;
161     return ret -= b;
162 }
163
164 class ntt_t {
165 public:
166     static const ul lgysz = 20;
167     static const ul sz = 1 << lgysz;
168     static const ul g = 3;
169     ul w[sz + 1];
170     ul leninv[lgysz + 1];
171     ntt_t() {
172         ul wn = pow(g, (base - 1) >> lgysz);
173         w[0] = 1;
174         for (ul i = 1; i <= sz; ++i) {
175             w[i] = mul(w[i - 1], wn);
176         }
177         leninv[lgysz] = inverse(sz);
178         for (ul i = lgysz - 1; ~i; --i) {
179             leninv[i] = plus(leninv[i + 1], leninv[i + 1]);
180         }
181     }
182     void operator()(vul& v, ul& n, bool inv) {
183         ul lgn = 0;
184         while ((1 << lgn) < n) {
185             ++lgn;
186         }
187         n = 1 << lgn;

```

```

188     v.resize(n, 0);
189     for (ul i = 0, j = 0; i != n; ++i) {
190         if (i < j) {
191             std::swap(v[i], v[j]);
192         }
193         ul k = n >> 1;
194         while (k & j) {
195             j &= ~k;
196             k >>= 1;
197         }
198         j |= k;
199     }
200     for (ul lgmid = 0; (1 << lgmid) != n; ++lgmid) {
201         ul mid = 1 << lgmid;
202         ul len = mid << 1;
203         for (ul i = 0; i != n; i += len) {
204             for (ul j = 0; j != mid; ++j) {
205                 ul t0 = v[i + j];
206                 ul t1 =
207                     mul(w[inv ? (len - j << lgsz - lgmid - 1) : (j << lgsz - lgmid - 1)],
208                        v[i + j + mid]);
209                 v[i + j] = plus(t0, t1);
210                 v[i + j + mid] = minus(t0, t1);
211             }
212         }
213     }
214     if (inv) {
215         for (ul i = 0; i != n; ++i) {
216             v[i] = mul(v[i], leninv[lgn]);
217         }
218     }
219 }
220 } ntt;
221
222 polynomial& operator*=(polynomial& a, const polynomial& b) {
223     if (!b.size() || !a.size()) {
224         a.resize(0);
225         return a;
226     }
227     polynomial temp = b;
228     ul npmp1 = a.size() + b.size() - 1;
229     if (ull(a.size()) * ull(b.size()) <= ull(npmp1) * ull(50)) {
230         temp.resize(0);
231         temp.resize(npmp1, 0);
232         for (ul i = 0; i != a.size(); ++i) {
233             for (ul j = 0; j != b.size(); ++j) {
234                 temp[i + j] = plus(temp[i + j], mul(a[i], b[j]));
235             }
236         }
237         a = temp;
238         a.clearzero();
239         return a;
240     }
241     ntt(a, npmp1, false);
242     ntt(temp, npmp1, false);
243     for (ul i = 0; i != npmp1; ++i) {
244         a[i] = mul(a[i], temp[i]);
245     }
246     ntt(a, npmp1, true);
247     a.clearzero();
248     return a;
249 }
250
251 polynomial operator*(const polynomial& a, const polynomial& b) {
252     polynomial ret = a;
253     return ret *= b;
254 }
255
256 polynomial& operator*=(polynomial& a, ul b) {
257     if (!b) {
258         a.resize(0);
259         return a;
260     }
261     for (ul i = 0; i != a.size(); ++i) {
262         a[i] = mul(a[i], b);
263     }
264     return a;
265 }
266
267 polynomial operator*(const polynomial& a, ul b) {
268     polynomial ret = a;
269     return ret *= b;
270 }
271
272 polynomial inverse(const polynomial& a, ul lgdeg) {
273     polynomial ret({inverse(a[0])});
274     polynomial temp;

```

```

275     polynomial tempa;
276     for (ul i = 0; i != lgdeg; ++i) {
277         tempa.resize(0);
278         tempa.resize(1 << i << 1, 0);
279         for (ul j = 0; j != tempa.size() && j != a.size(); ++j) {
280             tempa[j] = a[j];
281         }
282         temp = ret * (polynomial({2}) - tempa * ret);
283         if (temp.size() > (1 << i << 1)) {
284             temp.resize(1 << i << 1, 0);
285         }
286         temp.clearzero();
287         std::swap(temp, ret);
288     }
289     return ret;
290 }
291
292 void quotientremain(const polynomial& a, polynomial b, polynomial& q, polynomial& r) {
293     if (a.size() < b.size()) {
294         q = polynomial();
295         r = std::move(a);
296         return;
297     }
298     std::reverse(b.begin(), b.end());
299     auto ta = a;
300     std::reverse(ta.begin(), ta.end());
301     ul n = a.size() - 1;
302     ul m = b.size() - 1;
303     ta.resize(n - m + 1);
304     ul lgnmmp1 = 0;
305     while ((1 << lgnmmp1) < n - m + 1) {
306         ++lgnmmp1;
307     }
308     q = ta * inverse(b, lgnmmp1);
309     q.resize(n - m + 1);
310     std::reverse(b.begin(), b.end());
311     std::reverse(q.begin(), q.end());
312     r = a - b * q;
313 }
314
315 polynomial mod(const polynomial& a, const polynomial& b) {
316     polynomial q, r;
317     quotientremain(a, b, q, r);
318     return r;
319 }
320
321 polynomial quotient(const polynomial& a, const polynomial& b) {
322     polynomial q, r;
323     quotientremain(a, b, q, r);
324     return q;
325 }
326
327 polynomial sqrt(const polynomial& a, ul lgdeg) {
328     polynomial ret({sqrt(a[0])});
329     polynomial temp;
330     polynomial tempa;
331     for (ul i = 0; i != lgdeg; ++i) {
332         tempa.resize(0);
333         tempa.resize(1 << i << 1, 0);
334         for (ul j = 0; j != tempa.size() && j != a.size(); ++j) {
335             tempa[j] = a[j];
336         }
337         temp = (tempa * inverse(ret, i + 1) + ret) * (base + 1 >> 1);
338         if (temp.size() > (1 << i << 1)) {
339             temp.resize(1 << i << 1, 0);
340         }
341         temp.clearzero();
342         std::swap(temp, ret);
343     }
344     return ret;
345 }
346
347 polynomial differential(const polynomial& a) {
348     if (!a.size()) {
349         return a;
350     }
351     polynomial ret(vul(a.size() - 1, 0));
352     for (ul i = 1; i != a.size(); ++i) {
353         ret[i - 1] = mul(a[i], i);
354     }
355     return ret;
356 }
357
358 polynomial integral(const polynomial& a) {
359     polynomial ret(vul(a.size() + 1, 0));
360     for (ul i = 0; i != a.size(); ++i) {
361         ret[i + 1] = mul(a[i], inverse(i + 1));

```

```

362     }
363     return ret;
364 }
365
366 polynomial ln(const polynomial& a, ul lgdeg) {
367     polynomial da = differential(a);
368     polynomial inva = inverse(a, lgdeg);
369     polynomial ret = integral(da * inva);
370     if (ret.size() > (1 << lgdeg)) {
371         ret.resize(1 << lgdeg);
372         ret.clearzero();
373     }
374     return ret;
375 }
376
377 polynomial exp(const polynomial& a, ul lgdeg) {
378     polynomial ret({1});
379     polynomial temp;
380     polynomial tempa;
381     for (ul i = 0; i != lgdeg; ++i) {
382         tempa.resize(0);
383         tempa.resize(1 << i << 1, 0);
384         for (ul j = 0; j != tempa.size() && j != a.size(); ++j) {
385             tempa[j] = a[j];
386         }
387         temp = ret * (polynomial({1}) - ln(ret, i + 1) + tempa);
388         if (temp.size() > (1 << i << 1)) {
389             temp.resize(1 << i << 1, 0);
390         }
391         temp.clearzero();
392         std::swap(temp, ret);
393     }
394     return ret;
395 }
396
397 polynomial pow(const polynomial& a, ul k, ul lgdeg) { return exp(ln(a, lgdeg) * k, lgdeg); }
398
399 polynomial alpi[1 << 16][17];
400
401 polynomial getalpi(const ul x[], ul l, ul lgrml) {
402     if (lgrml == 0) {
403         return alpi[l][lgrml] = vul({minus(0, x[l]), 1});
404     }
405     return alpi[l][lgrml] = getalpi(x, l, lgrml - 1) * getalpi(x, l + (1 << lgrml - 1), lgrml - 1);
406 }
407
408 void multians(const polynomial& f, const ul x[], ul y[], ul l, ul lgrml) {
409     if (f.size() <= 700) {
410         for (ul i = l; i != l + (1 << lgrml); ++i) {
411             y[i] = f(x[i]);
412         }
413         return;
414     }
415     if (lgrml == 0) {
416         y[l] = f(x[l]);
417         return;
418     }
419     multians(mod(f, alpi[l][lgrml - 1]), x, y, l, lgrml - 1);
420     multians(mod(f, alpi[l + (1 << lgrml - 1)][lgrml - 1]), x, y, l + (1 << lgrml - 1), lgrml - 1);
421 }
422
423 ul sqrt(ul x) {
424     ul a;
425     ul w2;
426     while (true) {
427         a = rnd() % base;
428         w2 = minus(mul(a, a), x);
429         if (pow(w2, base - 1 >> 1) == base - 1) {
430             break;
431         }
432     }
433     ul b = base + 1 >> 1;
434     ul rs = 1, rt = 0;
435     ul as = a, at = 1;
436     ul qs, qt;
437     while (b) {
438         if (b & 1) {
439             qs = plus(mul(rs, as), mul(mul(rt, at), w2));
440             qt = plus(mul(rs, at), mul(rt, as));
441             rs = qs;
442             rt = qt;
443         }
444         b >>= 1;
445         qs = plus(mul(as, as), mul(mul(at, at), w2));
446         qt = plus(mul(as, at), mul(as, at));
447         as = qs;
448         at = qt;

```



```

449     }
450     return rs + rs < base ? rs : base - rs;
451 }
452
453 ul log(ul x, ul y, bool initied = false) {
454     static std::map<ul, ul> bs;
455     const ul d = std::round(std::sqrt(1f(base - 1)));
456     if (!initied) {
457         bs.clear();
458         for (ul i = 0, j = 1; i != d; ++i, j = mul(j, x)) {
459             bs[j] = i;
460         }
461     }
462     ul temp = inverse(pow(x, d));
463     for (ul i = 0, j = 1; i += d, j = mul(j, temp)) {
464         auto it = bs.find(mul(y, j));
465         if (it != bs.end()) {
466             return it->second + i;
467         }
468     }
469 }
470
471 ul powroot(ul x, ul y, bool initied = false) {
472     const ul g = 3;
473     ul lgx = log(g, x, initied);
474     li s, t;
475     exgcd(y, base - 1, s, t);
476     if (s < 0) {
477         s += base - 1;
478     }
479     return pow(g, ull(s) * ull(lgx) % (base - 1));
480 }
481
482 ul n;
483
484 int main() {
485     std::scanf("%u", &n);
486     polynomial f;
487     for (ul i = 0; i <= n; ++i) {
488         ul t;
489         std::scanf("%u", &t);
490         f.push_back(t % base);
491     }
492     polynomial g = exp(ln(f * inverse(f[0]), 17) * inverse(3), 17) * powroot(f[0], 3);
493     while (g.size() <= n) {
494         g.push_back(0);
495     }
496     for (ul i = 0; i <= n; ++i) {
497         if (i) {
498             std::putchar(' ');
499         }
500         std::printf("%u", g[i]);
501     }
502     std::putchar('\n');
503     return 0;
504 }

```

Lagrange interpolation

一般的插值

给出一个多项式 $f(x)$ 上的 n 个点 (x_i, y_i) , 求 $f(k)$.

插值的结果是

$$f(x) = \sum_{i=1}^n y_i \prod_{j \neq i} \frac{x - x_j}{x_i - x_j},$$

直接带入 k 并且取模即可, 时间复杂度 $O(n^2)$.

```

1 auto lagrange = (const vi& x, const vi& y, int n, int k) {
2     for (int i = 1; i <= n; i++) {
3         LL s1 = y[i] % mod, s2 = 1ll;
4         for (int j = 1; j <= n; j++) {
5             if (i != j) {
6                 s1 = s1 * (k - x[j]) % mod;
7                 s2 = s2 * (x[i] - x[j]) % mod;
8             }
9         }

```

```

10 |         Add(ans, mul(s1, quick_power(s2, mod - 2, mod)));
11 |     }
12 |     return ans;
13 | };

```

坐标连续的插值

给出的点是 (i, y_i) .

$$\begin{aligned}
 f(x) &= \sum_{i=1}^n y_i \prod_{j \neq i} \frac{x - x_j}{x_i - x_j} \\
 &= \sum_{i=1}^n y_i \prod_{j \neq i} \frac{x - j}{i - j} \\
 &= \sum_{i=1}^n y_i \cdot \frac{\prod_{j=1}^n (x - j)}{(x - i)(-1)^{n+1-i}(i - 1)!(n + 1 - i)!} \\
 &= \left(\prod_{j=1}^n (x - j) \right) \left(\sum_{i=1}^n \frac{(-1)^{n+1-i} y_i}{(x - i)(i - 1)!(n + 1 - i)!} \right),
 \end{aligned}$$

时间复杂度为 $O(n)$.

7 math - game theory

7.1 nim game

若 nim 和为 0, 则先手必败.

暴力打表.

```

1 vi SG(21, -1); /* 记忆化 */
2 std::function<int(int, int)> sg = [&](int x) -> int {
3     if (/* 为最终态 */) return SG[x] = 0;
4     if (SG[x] != -1) return SG[x];
5     vi st;
6     for (/* 枚举所有可到达的状态 y */) {
7         st.push_back(sg(y));
8     }
9     std::sort(all(st));
10    for (int i = 0; i < st.size(); i++) {
11        if (st[i] != i) return SG[x] = i;
12    }
13    return SG[x] = st.size();
14 };

```

7.2 anti - nim game

若

1. 所有堆的石子均为一个, 且 nim 和不为 0,
2. 至少有一堆石子超过一个, 且 nim 和为 0,

则先手必败.

8 math - linear algebra

8.1 matrix

determinant mod 998244353

```

1  /* determinant */
2  auto det = [&](int n, vvi e) -> int {
3      int ans = 1;
4      for (int i = 1; i <= n; i++) {
5          if (a[i][i] == 0) {
6              for (int j = i + 1; j <= n; j++) {
7                  if (a[j][i] != 0) {
8                      for (int k = i; k <= n; k++) {
9                          std::swap(a[i][k], a[j][k]);
10                     }
11                     ans = sub(mod, ans);
12                     break;
13                 }
14             }
15         }
16         if (a[i][i] == 0) return 0;
17         Mul(ans, a[i][i]);
18         int x = pow(a[i][i], mod - 2);
19         for (int k = i; k <= n; k++) {
20             Mul(a[i][k], x);
21         }
22         for (int j = i + 1; j <= n; j++) {
23             int x = a[j][i];
24             for (int k = i; k <= n; k++) {
25                 Sub(a[j][k], mul(a[i][k], x));
26             }
27         }
28     }
29     return ans;
30 };

```

matrix multiplication

$A_{n \times m}$ 与 $B_{m \times k}$ 相乘并模 998244353.

```

1  /* matrix multiplication */
2  auto matmul = [&](int n, int m, int k, const vvi& a, const vvi& b) -> vvi {
3      vvi c(n + 1, vi(k + 1));
4      for (int i = 1; i <= n; i++) {
5          for (int l = 1; l <= m; l++) {
6              int x = a[i][l];
7              for (int j = 1; j <= k; j++) {
8                  Add(c[i][j], mul(x, b[l][j]));
9              }
10         }
11     }
12     return c;
13 };

```

8.2 linear basis

```

1  /* linear basis */
2  vl p(63), s(63); /* basis and case */
3  auto insert = [&](LL x, int id) {
4      LL ans = 0;
5      for (int i = 62; i >= 0; i--) {
6          if (~(x >> i) & 1) continue;
7          if (!p[i]) {
8              p[i] = x;
9              s[i] = ans ^ (1ll << id);
10             break;
11         }
12         x ^= p[i], ans ^= s[i];
13     }
14     return x;
15 };

```

```
16 auto query = [&](LL x) {  
17     LL ans = 0;  
18     for (int i = 62; i >= 0; i--) {  
19         if (~(x >> i) & 1) continue;  
20         x ^= p[i], ans ^= s[i];  
21     }  
22     return (x ? -1 : ans);  
23 };  
24 auto queryMax = [&]() {  
25     LL ans = 0;  
26     for (int i = 62; i >= 0; i--)  
27         if ((ans ^ p[i]) > ans) ans ^= p[i];  
28     return ans;  
29 };
```

8.3 linear programming

9 complex number

```

1  tandu struct Comp {
2      T a, b;
3      Comp(T _a = 0, T _b = 0) { a = _a, b = _b; }
4      Comp operator+(const Comp& x) const { return Comp(a + x.a, b + x.b); }
5      Comp operator-(const Comp& x) const { return Comp(a - x.a, b - x.b); }
6      Comp operator*(const Comp& x) const { return Comp(a * x.a - b * x.b, a * x.b + b * x.a); }
7      bool operator==(const Comp& x) const { return a == x.a and b == x.b; }
8      T real() { return a; }
9      T imag() { return b; }
10     U norm() { return (U) a * a + (U) b * b; }
11     Comp conj() { return Comp(a, -b); }
12     Comp operator/(const Comp& x) const {
13         Comp y = x;
14         Comp c = Comp(a, b) * y.conj();
15         T d = y.norm();
16         return Comp(c.a / d, c.b / d);
17     }
18 };
19 typedef Comp<LL, LL> complex;
20 complex gcd(complex a, complex b) {
21     LL d = b.norm();
22     if (d == 0) return a;
23     std::vector<complex> v(4);
24     complex c = a * b.conj();
25     auto fdiv = [&](LL a, LL b) -> LL { return a / b - ((a ^ b) < 0 && (a % b)); };
26     v[0] = complex(fdiv(c.real(), d), fdiv(c.imag(), d));
27     v[1] = v[0] + complex(1, 0);
28     v[2] = v[0] + complex(0, 1);
29     v[3] = v[0] + complex(1, 1);
30     for (auto& x : v) {
31         x = a - x * b;
32     }
33     std::sort(all(v), [&](complex a, complex b) { return a.norm() < b.norm(); });
34     return gcd(b, v[0]);
35 };

```

10 graph

10.1 topology sort

```

1  /* topology sort */
2  vi top;
3  auto topsort = [&]() -> bool {
4      vi d(n + 1);
5      std::queue<int> q;
6      for (int i = 1; i <= n; i++) {
7          d[i] = e[i].size();
8          if (!d[i]) q.push(i);
9      }
10     while (!q.empty()) {
11         int u = q.front();
12         q.pop();
13         top.push_back(u);
14         for (auto v : e[u]) {
15             d[v]--;
16             if (!d[v]) q.push(v);
17         }
18     }
19     if (top.size() != n) return false;
20     return true;
21 };

```

10.2 shortest path

Floyd

```

1  /* floyd */
2  auto floyd = [&]() -> vvi {
3      vvi dist(n + 1, vi(n + 1, inf));
4      for (int i = 1; i <= n; i++) {
5          for (int j = 1; j <= n; j++) {
6              Min(dist[i][j], e[i][j]);
7          }
8          dist[i][i] = 0;
9      }
10     for (int k = 1; k <= n; k++) {
11         for (int i = 1; i <= n; i++) {
12             for (int j = 1; j <= n; j++) {
13                 Min(dist[i][j], dist[i][k] + dist[k][j]);
14             }
15         }
16     }
17     return dist;
18 };

```

Dijkstra

```

1  /* dijkstra */
2  auto dijkstra = [&](int s) -> vl {
3      vl dist(n + 1, INF);
4      vi vis(n + 1, 0);
5      dist[s] = 0;
6      std::priority_queue<PLI, std::vector<PLI>, std::greater<PLI>> q;
7      q.emplace(OLL, s);
8      while (!q.empty()) {
9          auto [dis, u] = q.top();
10         q.pop();
11         if (vis[u]) continue;
12         vis[u] = 1;
13         for (const auto& [v, w] : e[u]) {
14             if (dist[v] > dis + w) {
15                 dist[v] = dis + w;
16                 q.emplace(dist[v], v);
17             }
18         }
19     }
20     return dist;
21 };

```

SPFA

```

1  /* SPFA */
2  int n, m, s;
3  vl dist(n + 1, INF);
4  std::vector<bool> vis(n + 1);
5  std::vector<PLI> e(n + 1);
6  void spfa(int s){
7      for (int i = 1; i <= n; i++) dist[i] = INF;
8      dist[s] = 0;
9      std::queue<int> q;
10     q.push(s);
11     vis[s] = true;
12     while(q.size()){
13         auto u = q.front();
14         q.pop();
15         vis[u] = false;
16         for(const auto& [v, w] : e[u]){
17             if(dist[v] > dist[u] + w){
18                 dist[v] = dist[u] + w;
19                 if(!vis[v]){
20                     q.push(v);
21                     vis[v] = true;
22                 }
23             }
24         }
25     }
26 }

```

Johnson

```

1  /* johnson */
2  auto johnson = [&]() -> vl {
3      /* 负环 */
4      vl dist1(n + 1);
5      vi vis(n + 1), cnt(n + 1);
6      auto spfa = [&]() -> bool {
7          std::queue<int> q;
8          for (int u = 1; u <= n; u++) {
9              q.push(u);
10             vis[u] = false;
11         }
12         while (!q.empty()) {
13             auto u = q.front();
14             q.pop();
15             vis[u] = false;
16             for (auto [v, w] : e[u]) {
17                 if (dist1[v] > dist1[u] + w) {
18                     dist1[v] = dist1[u] + w;
19                     Max(cnt[v], cnt[u] + 1);
20                     if (cnt[v] >= n) return true;
21                     if (!vis[v]) {
22                         q.push(v);
23                         vis[v] = true;
24                     }
25                 }
26             }
27         }
28         return false;
29     };
30     /* dijkstra */
31     vl dist2(n + 1);
32     auto dijkstra = [&](int s) {
33         for (int u = 1; u <= n; u++) {
34             dist2[u] = 1e9;
35             vis[u] = false;
36         }
37         dist2[s] = 0;
38         std::priority_queue<PLI, std::vector<PLI>, std::greater<PLI>> q;
39         q.emplace(0, s);
40         while (!q.empty()) {
41             auto [d, u] = q.top();
42             q.pop();
43             if (vis[u]) continue;
44             vis[u] = true;
45             for (const auto& [v, w] : e[u]) {
46                 if (dist2[v] > d + w) {
47                     dist2[v] = d + w;
48                     q.emplace(dist2[v], v);
49                 }
50             }
51         }
52     };
53 }

```



```

51     }
52 };
53 if (spfa()) return vvl{};
54 for (int u = 1; u <= n; u++) {
55     for (auto& [v, w] : e[u]) {
56         w += dist1[u] - dist1[v];
57     }
58 }
59 vvl dist(n + 1, vl(n + 1));
60 for (int u; u <= n; u++) {
61     dijkstra(u);
62     for (int v = 1; v <= n; v++) {
63         if (dist2[v] == 1e9) {
64             dist[u][v] = INF;
65         } else {
66             dist[u][v] = dist2[v] + dist1[v] - dist1[u];
67         }
68     }
69 }
70 return dist;
71 };

```

最短路计数 - Dijkstra

```

1  /* dijkstra */
2  auto dijkstra = [&](int s) -> std::pair<vl, vi> {
3      vl dist(n + 1, INF);
4      vi cnt(n + 1), vis(n + 1);
5      dist[s] = 0;
6      cnt[s] = 1;
7      std::priority_queue<PLI, std::vector<PLI>, std::greater<PLI>> q;
8      q.emplace(0LL, s);
9      while (!q.empty()) {
10         auto [dis, u] = q.top();
11         q.pop();
12         if (vis[u]) continue;
13         vis[u] = 1;
14         for (const auto& [v, w] : e[u]) {
15             if (dist[v] > dis + w) {
16                 dist[v] = dis + w;
17                 cnt[v] = cnt[u];
18                 q.push({dist[v], v});
19             } else if (dist[v] == dis + w) {
20                 // cnt[v] += cnt[u];
21                 cnt[v] += cnt[u];
22                 cnt[v] %= mod;
23             }
24         }
25     }
26     return {dist, cnt};
27 };

```

最短路计数 - Floyd

```

1  /* floyd */
2  auto floyd() = [&] -> std::pair<vvi, vvi> {
3      vvi dist(n + 1, vi(n + 1, inf));
4      vvi cnt(n + 1, vi(n + 1));
5      for (int i = 1; i <= n; i++) {
6          for (int j = 1; j <= n; j++) {
7              Min(dist[i][j], e[i][j]);
8          }
9          dist[i][i] = 0;
10     }
11     for (int k = 1; k <= n; k++) {
12         for (int i = 1; i <= n; i++) {
13             for (int j = 1; j <= n; j++) {
14                 if (dist[i][j] == dist[i][k] + dist[k][j]) {
15                     cnt[i][j] += cnt[i][k] * cnt[k][j];
16                 } else if (dist[i][j] > dist[i][k] + dist[k][j]) {
17                     cnt[i][j] = cnt[i][k] * cnt[k][j];
18                     dist[i][j] = dist[i][k] + dist[k][j];
19                 }
20             }
21         }
22     }
23     return {dist, cnt};
24 };

```

负环

判断的是最短路长度.

```

1  /* SPFA */
2  auto spfa = [&]() -> bool {
3      std::queue<int> q;
4      vi vis(n + 1), cnt(n + 1);
5      for (int i = 1; i <= n; i++) {
6          q.push(i);
7          vis[i] = true;
8      }
9      while (!q.empty()) {
10         auto u = q.front();
11         q.pop();
12         vis[u] = false;
13         for (const auto& [v, w] : e[u]) {
14             if (dist[v] > dist[u] + w) {
15                 dist[v] = dist[u] + w;
16                 cnt[v] = cnt[u] + 1;
17                 if (cnt[v] >= n) return true;
18                 if (!vis[v]) {
19                     q.push(v);
20                     vis[v] = true;
21                 }
22             }
23         }
24     }
25     return false;
26 }

```

10.3 minimum spanning tree

Kruskal

```

1  /* kruskal */
2  std::vector<std::tuple<int, int, int>> edge;
3  auto kruskal = [&]() -> int {
4      std::sort(all(edge), [&](std::tuple<int, int, int> a, std::tuple<int, int, int> b) {
5          auto [x1, y1, w1] = a;
6          auto [x2, y2, w2] = b;
7          return w1 < w2;
8      });
9      int res = 0, cnt = 0;
10     for (int i = 0; i < m; i++) {
11         auto [a, b, w] = edge[i];
12         a = find(a), b = find(b);
13         if (a != b) {
14             fa[a] = b;
15             res += w;
16             /* res = std::max(res, w); */
17             cnt++;
18         }
19     }
20     if (cnt < n - 1) return -1;
21     return res;
22 }

```

10.4 SCC

Tarjan

```

1  /* tarjan */
2  vi dfn(n + 1), low(n + 1), stk(n + 1), belong(n + 1);
3  int timestamp = 0, top = 0, scc_cnt = 0;
4  std::vector<bool> in_stk(n + 1);
5  auto tarjan = [&](auto&& self, int u) -> void {
6      dfn[u] = low[u] = ++timestamp;
7      stk[++top] = u;
8      in_stk[u] = true;
9      for (const auto& v : e[u]) {
10         if (!dfn[v]) {
11             self(self, v);

```

```

12     Min(low[u], low[v]);
13 } else if (in_stk[v]) {
14     Min(low[u], dfn[v]);
15 }
16 }
17 if (dfn[u] == low[u]) {
18     scc_cnt++;
19     int v;
20     do {
21         v = stk[top--];
22         in_stk[v] = false;
23         belong[v] = scc_cnt;
24     } while (v != u);
25 }
26 };

```

10.5 DCC

点双连通分量

求点双连通分量.

```

1 vi dfn(n + 1), low(n + 1), is_bcc(n + 1), stk;
2 int timestamp = 0, bcc_cnt = 0, root = 0;
3 vvi bcc(2 * n + 1);
4 std::function<void(int, int)> tarjan = [&](int u, int fa) {
5     dfn[u] = low[u] = ++timestamp;
6     int child = 0;
7     stk.push_back(u);
8     if (u == root and e[u].empty()) {
9         bcc_cnt++;
10        bcc[bcc_cnt].push_back(u);
11        return;
12    }
13    for (auto v : e[u]) {
14        if (!dfn[v]) {
15            tarjan(v, u);
16            low[u] = std::min(low[u], low[v]);
17            if (low[v] >= dfn[u]) {
18                child++;
19                if (u != root or child > 1) {
20                    is_bcc[u] = 1;
21                }
22                bcc_cnt++;
23                int z;
24                do {
25                    z = stk.back();
26                    stk.pop_back();
27                    bcc[bcc_cnt].push_back(z);
28                } while (z != v);
29                bcc[bcc_cnt].push_back(u);
30            }
31        } else if (v != fa) {
32            low[u] = std::min(low[u], dfn[v]);
33        }
34    }
35 };
36 for (int i = 1; i <= n; i++) {
37     if (!dfn[i]) {
38         root = i;
39         tarjan(i, i);
40     }
41 }

```

求割点.

```

1 vi dfn(n + 1), low(n + 1), is_bcc(n + 1);
2 int timestamp = 0, bcc = 0, root = 0;
3 std::function<void(int, int)> tarjan = [&](int u, int fa) {
4     dfn[u] = low[u] = ++timestamp;
5     int child = 0;
6     for (auto v : e[u]) {
7         if (!dfn[v]) {
8             tarjan(v, u);
9             low[u] = std::min(low[u], low[v]);
10            if (low[v] >= dfn[u]) {
11                child++;
12                if ((u != root or child > 1) and !is_bcc[u]) {
13                    bcc++;

```

```

14         is_bcc[u] = 1;
15     }
16 }
17 } else if (v != fa) {
18     low[u] = std::min(low[u], dfn[v]);
19 }
20 }
21 };
22 for (int i = 1; i <= n; i++) {
23     if (!dfn[i]) {
24         root = i;
25         tarjan(i, i);
26     }
27 }

```

边双连通分量

求边双连通分量.

```

1  std::vector<vpi> e(n + 1);
2  for (int i = 1; i <= m; i++) {
3      int u, v;
4      std::cin >> u >> v;
5      e[u].emplace_back(v, i);
6      e[v].emplace_back(u, i);
7  }
8  vi dfn(n + 1), low(n + 1), is_ecc(n + 1), fa(n + 1), stk;
9  int timestamp = 0, ecc_cnt = 0;
10 vvi ecc(2 * n + 1);
11 std::function<void(int, int)> tarjan = [&](int u, int id) {
12     low[u] = dfn[u] = ++timestamp;
13     stk.push_back(u);
14     for (auto [v, idx] : e[u]) {
15         if (!dfn[v]) {
16             tarjan(v, idx);
17             low[u] = std::min(low[u], low[v]);
18         } else if (idx != id) {
19             low[u] = std::min(low[u], dfn[v]);
20         }
21     }
22     if (dfn[u] == low[u]) {
23         ecc_cnt++;
24         int v;
25         do {
26             v = stk.back();
27             stk.pop_back();
28             ecc[ecc_cnt].push_back(v);
29         } while (v != u);
30     }
31 };
32 for (int i = 1; i <= n; i++) {
33     if (!dfn[i]) {
34         tarjan(i, 0);
35     }
36 }

```

10.6 2-sat

给出 n 个集合, 每个集合有 2 个元素, 已知若个数对 (a, b) , 表示 a 与 b 矛盾. 要从每个集合各选择一个元素, 判断能否一共选 n 个两两不矛盾的元素.

```

1  /* two sat */
2  auto 2sat = [&](int n, const vpi& v) -> vi {
3      /* tarjan */
4      vvi e(2 * n);
5      vi dfn(2 * n), low(2 * n), stk(2 * n), belong(2 * n);
6      int timestamp = 0, top = 0, scc_cnt = 0;
7      std::vector<bool> in_stk(2 * n);
8      auto tarjan = [&](auto&& self, int u) -> void {
9          dfn[u] = low[u] = ++timestamp;
10         stk[++top] = u;
11         in_stk[u] = true;
12         for (const auto& v : e[u]) {
13             if (!dfn[v]) {
14                 self(self, v);
15                 Min(low[u], low[v]);

```

```

16         } else if (in_stk[v]) {
17             Min(low[u], dfn[v]);
18         }
19     }
20     if (dfn[u] == low[u]) {
21         scc_cnt++;
22         int v;
23         do {
24             v = stk[top--];
25             in_stk[v] = false;
26             belong[v] = scc_cnt;
27         } while (v != u);
28     }
29 };
30 for (const auto& [a, b] : v) {
31     e[a].push_back(b ^ 1);
32     e[b].push_back(a ^ 1);
33 }
34 for (int i = 0; i < 2 * n; i++) {
35     if (!dfn[i]) tarjan(tarjan, i);
36 }
37 vi ans;
38 for (int i = 0; i < 2 * n; i += 2) {
39     if (belong[i] == belong[i + 1]) return vi{};
40     ans.push_back(belong[i] > belong[i + 1] ? i + 1 : i);
41 }
42 return ans;
43 };

```

上述将 i 与 $i + 1$ 作为一个集合里的元素, 编号为 0 至 $2n - 1$.

10.7 minimum ring

Floyd

```

1  /* minimum ring */
2  auto min_circle = [&]() -> int {
3      vvi dist(n + 1, vi(n + 1, inf));
4      for (int i = 1; i <= n; i++) {
5          for (int j = 1; j <= n; j++) {
6              Min(dist[i][j], g[i][j]);
7          }
8          dist[i][i] = 0;
9      }
10     for (int k = 1; k <= n; k++) {
11         for (int i = 1; i < k; i++) {
12             for (int j = 1; j < i; j++) {
13                 Min(ans, dist[i][j] + g[i][k] + g[k][j]);
14             }
15         }
16         for (int i = 1; i <= n; i++) {
17             for (int j = 1; j <= n; j++) {
18                 Min(dist[i][j], dist[i][k] + dist[k][j]);
19             }
20         }
21     }
22     return ans;
23 };

```

tree - diameter

10.8 tree - center of gravity

```

1  /* center of gravity */
2  int sum; /* 点权和 */
3  vi size(n + 1), weight(n + 1), w(n + 1), depth(n + 1);
4  std::array<int, 2> centroid = {0, 0};
5  auto get_centroid = [&](auto&& self, int u, int fa) -> void {
6      size[u] = w[u];
7      weight[u] = 0;
8      for (auto v : e[u]) {
9          if (v == fa) continue;
10         self(self, v, u);
11         size[u] += size[v];

```

```

12     Max(weight[u], size[v]);
13 }
14 Max(weight[u], sum - size[u]);
15 if (weight[u] <= sum / 2) {
16     centroid[centroid[0] != 0] = u;
17 }
18 };

```

10.9 tree - DSU on tree

给出一棵 n 个节点以 1 为根的树, 每个节点染上一种颜色, 询问以 u 为节点的子树中有多少种颜色.

```

1 // Problem: U41492 树上数颜色
2
3 int main() {
4     std::ios::sync_with_stdio(false);
5     std::cin.tie(0);
6     std::cout.tie(0);
7
8     int n, m, dfn = 0, cnttot = 0;
9     std::cin >> n;
10    vvi e(n + 1);
11    vi siz(n + 1), col(n + 1), son(n + 1), dfnl(n + 1), dfnr(n + 1), rank(n + 1);
12    vi ans(n + 1), cnt(n + 1);
13
14    for (int i = 1; i < n; i++) {
15        int u, v;
16        std::cin >> u >> v;
17        e[u].push_back(v);
18        e[v].push_back(u);
19    }
20    for (int i = 1; i <= n; i++) {
21        std::cin >> col[i];
22    }
23    auto add = [&](int u) -> void {
24        if (cnt[col[u]] == 0) cnttot++;
25        cnt[col[u]]++;
26    };
27    auto del = [&](int u) -> void {
28        cnt[col[u]]--;
29        if (cnt[col[u]] == 0) cnttot--;
30    };
31    auto dfs1 = [&](auto&& self, int u, int fa) -> void {
32        dfnl[u] = ++dfn;
33        rank[dfn] = u;
34        siz[u] = 1;
35        for (auto v : e[u]) {
36            if (v == fa) continue;
37            self(self, v, u);
38            siz[u] += siz[v];
39            if (!son[u] or siz[son[u]] < siz[v]) son[u] = v;
40        }
41        dfnr[u] = dfn;
42    };
43    auto dfs2 = [&](auto&& self, int u, int fa, bool op) -> void {
44        for (auto v : e[u]) {
45            if (v == fa or v == son[u]) continue;
46            self(self, v, u, false);
47        }
48        if (son[u]) self(self, son[u], u, true);
49        for (auto v : e[u]) {
50            if (v == fa or v == son[u]) continue;
51            rep(i, dfnl[v], dfnr[v]) { add(rank[i]); }
52        }
53        add(u);
54        ans[u] = cnttot;
55        if (op == false) {
56            rep(i, dfnl[u], dfnr[u]) { del(rank[i]); }
57        }
58    };
59    dfs1(dfs1, 1, 0);
60    dfs2(dfs2, 1, 0, false);
61    std::cin >> m;
62    for (int i = 1; i <= m; i++) {
63        int u;
64        std::cin >> u;
65        std::cout << ans[u] << endl;
66    }
67    return 0;
68 }

```

10.10 tree - AHU

```

1  /* AHU */
2  std::map<vi, int> mapple;
3  std::function<int(vvi&, int, int)> tree_hash = [&](vvi& e, int u, int fa) -> int {
4      vi code;
5      if (u == 0) code.push_back(-1);
6      for (auto v : e[u]) {
7          if (v == fa) continue;
8          code.push_back(tree_hash(e, v, u));
9      }
10     std::sort(all(code));
11     int id = mapple.size();
12     auto it = mapple.find(code);
13     if (it == mapple.end()) {
14         mapple[code] = id;
15     } else {
16         id = it->ss;
17     }
18     return id;
19 };

```

10.11 tree - LCA

```

1  /* LCA */
2  int B = 30;
3  vvi e(n + 1), fa(n + 1, vi(B));
4  vi dep(n + 1);
5  auto dfs = [&](auto&& self, int u) -> void {
6      for (auto v : e[u]) {
7          if (v == fa[u][0]) continue;
8          dep[v] = dep[u] + 1;
9          fa[v][0] = u;
10         self(self, v);
11     }
12 };
13 auto init = [&]() -> void {
14     dep[root] = 1;
15     dfs(dfs, root);
16     for (int j = 1; j < B; j++) {
17         for (int i = 1; i <= n; i++) {
18             fa[i][j] = fa[fa[i][j - 1]][j - 1];
19         }
20     }
21 };
22 init();
23 auto LCA = [&](int a, int b) -> int {
24     if (dep[a] > dep[b]) std::swap(a, b);
25     int d = dep[b] - dep[a];
26     for (int i = 0; (1 << i) <= d; i++) {
27         if (d & (1 << i)) b = fa[b][i];
28     }
29     if (a == b) return a;
30     for (int i = B - 1; i >= 0 and a != b; i--) {
31         if (fa[a][i] == fa[b][i]) continue;
32         a = fa[a][i];
33         b = fa[b][i];
34     }
35     return fa[a][0];
36 };
37 auto dist = [&](int a, int b) -> int { return dep[a] + dep[b] - dep[LCA(a, b)] * 2; };

```

10.12 tree - heavy light decomposition

对一棵有根树进行如下 4 种操作:

1. 1 $x\ y\ z$: 将节点 x 到节点 y 的最短路径上所有节点的值加上 z .
2. 2 $x\ y$: 查询节点 x 到节点 y 的最短路径上所有节点的值的和.
3. 3 $x\ z$: 将以节点 x 为根的子树上所有节点的值加上 z .
4. 4 x : 查询以节点 x 为根的子树上所有节点的值的和.

```

1  /* heavy light decomposition */
2  int cnt = 0;
3  vi son(n + 1), fa(n + 1), siz(n + 1), depth(n + 1);
4  vi dfn(n + 1), rank(n + 1), top(n + 1), botton(n + 1);
5  auto dfs1 = [&](auto&& self, int u) -> void {
6      son[u] = -1, siz[u] = 1;
7      for (auto v : e[u]) {
8          if (depth[v] != 0) continue;
9          depth[v] = depth[u] + 1;
10         fa[v] = u;
11         self(self, v);
12         siz[u] += siz[v];
13         if (son[u] == -1 or siz[v] > siz[son[u]]) son[u] = v;
14     }
15 };
16 auto dfs2 = [&](auto&& self, int u, int t) -> void {
17     top[u] = t;
18     dfn[u] = ++cnt;
19     rank[cnt] = u;
20     botton[u] = dfn[u];
21     if (son[u] == -1) return;
22     self(self, son[u], t);
23     Max(botton[u], botton[son[u]]);
24     for (auto v : e[u]) {
25         if (v != son[u] and v != fa[u]) {
26             self(self, v, v);
27             Max(botton[u], botton[v]);
28         }
29     }
30 };
31 depth[root] = 1;
32 dfs1(dfs1, root);
33 dfs2(dfs2, root, root);
34
35 /* 求 LCA */
36 auto LCA = [&](int a, int b) -> int {
37     while (top[a] != top[b]) {
38         if (depth[top[a]] < depth[top[b]]) std::swap(a, b);
39         a = fa[top[a]];
40     }
41     return (depth[a] > depth[b] ? b : a);
42 };
43
44 /* 维护 u 到 v 的路径 */
45 while (top[u] != top[v]) {
46     if (depth[top[u]] < depth[top[v]]) std::swap(u, v);
47     opt(dfn[top[u]], dfn[u]);
48     u = fa[top[u]];
49 }
50 if (dfn[u] > dfn[v]) std::swap(u, v);
51 opt(dfn[u], dfn[v]);
52
53 /* 维护 u 为根的子树 */
54 opt(dfn[u], botton[u]);
55
56 /*
57 线段树的 build() 函数中
58 if(l == r) tree[u] = {l, l, w[rank[l]], 0};
59 */
60
61 build(1, 1, n);
62 for (int i = 1; i <= m; i++) {
63     int op, u, v;
64     LL k;
65     std::cin >> op;
66     if (op == 1) {
67         std::cin >> u >> v >> k;
68         while (top[u] != top[v]) {
69             if (depth[top[u]] < depth[top[v]]) std::swap(u, v);
70             modify(1, dfn[top[u]], dfn[u], k);
71             u = fa[top[u]];
72         }
73         if (dfn[u] > dfn[v]) std::swap(u, v);
74         modify(1, dfn[u], dfn[v], k);
75     } else if (op == 2) {
76         std::cin >> u >> v;
77         LL ans = 0;
78         while (top[u] != top[v]) {
79             if (depth[top[u]] < depth[top[v]]) std::swap(u, v);
80             ans = (ans + query(1, dfn[top[u]], dfn[u])) % p;
81             u = fa[top[u]];
82         }
83         if (dfn[u] > dfn[v]) std::swap(u, v);
84         ans = (ans + query(1, dfn[u], dfn[v])) % p;
85         std::cout << ans << endl;
86     } else if (op == 3) {

```



```

87     std::cin >> u >> k;
88     modify(1, dfn[u], botton[u], k);
89 } else {
90     std::cin >> u;
91     std::cout << query(1, dfn[u], botton[u]) % p << endl;
92 }
93 }

```

10.13 tree - virtual tree

```

1  /* virtual tree */
2  auto build_vtree = [&](vi ver) -> void {
3      std::sort(all(ver), [&](int x, int y) { return dfn[x] < dfn[y]; });
4      vi stk = {1};
5      for (auto v : ver) {
6          int u = stk.back();
7          int lca = LCA(v, u);
8          if (lca != u) {
9              while (dfn[lca] < dfn[stk.end()[-2]]) {
10                 g[stk.end()[-2]].push_back(stk.back());
11                 stk.pop_back();
12             }
13             u = stk.back();
14             if (dfn[lca] != dfn[stk.end()[-2]]) {
15                 g[lca].push_back(u);
16                 stk.pop_back();
17                 stk.push_back(lca);
18             } else {
19                 g[lca].push_back(u);
20                 stk.pop_back();
21             }
22         }
23         stk.push_back(v);
24     }
25     while (stk.size() > 1) {
26         int u = stk.end()[-2];
27         int v = stk.back();
28         g[u].push_back(v);
29         stk.pop_back();
30     }
31 };

```

10.14 tree - pseudo tree

```

1  /* ring detection (directed) */
2  vi vis(n + 1), fa(n + 1), ring;
3  auto dfs = [&](auto&& self, int u) -> bool {
4      vis[u] = 1;
5      for (const auto& v : e[u]) {
6          if (!vis[v]) {
7              fa[v] = u;
8              if (self(self, v)) {
9                  return true;
10             }
11         } else if (vis[v] == 1) {
12             ring.push_back(v);
13             for (auto x = u; x != v; x = fa[x]) {
14                 ring.push_back(x);
15             }
16             reverse(all(ring));
17             return true;
18         }
19     }
20     vis[u] = 2;
21     return false;
22 };
23 for (int i = 1; i <= n; i++) {
24     if (!vis[i]) {
25         if (dfs(dfs, i)) {
26             // operations //
27         }
28     }
29 }
30
31 /* cycle detection (undirected) */
32 vi vis(n + 1), ring;
33 vpi fa(n + 1);

```

```

34 auto dfs = [&](auto&& self, int u, int from) -> bool {
35     vis[u] = 1;
36     for (const auto& [v, id] : e[u]) {
37         if (id == from) continue;
38         if (!vis[v]) {
39             fa[v] = {u, id};
40             if (self(self, v, id)) {
41                 return true;
42             }
43         } else if (vis[v] == 1) {
44             ring.push_back(v);
45             for (auto x = u; x != v; x = fa[x].ff) {
46                 ring.push_back(x);
47             }
48             return true;
49         }
50     }
51     vis[u] = 2;
52     return false;
53 };
54 for (int i = 1; i <= n; i++) {
55     if (!vis[i]) {
56         if (dfs(dfs, i, 0)) {
57             // operations //
58         }
59     }
60 }

```

10.15 tree - divide and conquer on tree

点分治

第一个题

一棵 $n \leq 10^4$ 个点的树，边权 $w \leq 10^4$ 。 $m \leq 100$ 次询问树上是否存在长度为 $k \leq 10^7$ 的路径。

```

1 // 洛谷 P3806 【模板】点分治1
2
3 int main() {
4     std::ios::sync_with_stdio(false);
5     std::cin.tie(0);
6     std::cout.tie(0);
7
8     int n, m, k;
9     std::cin >> n >> m;
10
11     std::vector<vpi> e(n + 1);
12     std::map<int, PII> mp;
13
14     for (int i = 1; i < n; i++) {
15         int u, v, w;
16         std::cin >> u >> v >> w;
17         e[u].emplace_back(v, w);
18         e[v].emplace_back(u, w);
19     }
20     for (int i = 1; i <= m; i++) {
21         std::cin >> k;
22         mp[i] = {k, 0};
23     }
24
25     /* centroid decomposition */
26     int top1 = 0, top2 = 0, root;
27     vi len1(n + 1), len2(n + 1), vis(n + 1);
28     static std::array<int, 20000010> cnt;
29
30     std::function<int(int, int)> get_size = [&](int u, int fa) -> int {
31         if (vis[u]) return 0;
32         int ans = 1;
33         for (auto [v, w] : e[u]) {
34             if (v == fa) continue;
35             ans += get_size(v, u);
36         }
37         return ans;
38     };
39
40     std::function<int(int, int, int, int)> get_root = [&](int u, int fa, int tot,
41                                                         int& root) -> int {
42         if (vis[u]) return 0;
43         int sum = 1, maxx = 0;
44         for (auto [v, w] : e[u]) {

```

```

45         if (v == fa) continue;
46         int tmp = get_root(v, u, tot, root);
47         Max(maxx, tmp);
48         sum += tmp;
49     }
50     Max(maxx, tot - sum);
51     if (2 * maxx <= tot) root = u;
52     return sum;
53 };
54
55 std::function<void(int, int, int)> get_dist = [&](int u, int fa, int dist) -> void {
56     if (dist <= 10000000) len1[++top1] = dist;
57     for (auto [v, w] : e[u]) {
58         if (v == fa or vis[v]) continue;
59         get_dist(v, u, dist + w);
60     }
61 };
62
63 auto solve = [&](int u, int dist) -> void {
64     top2 = 0;
65     for (auto [v, w] : e[u]) {
66         if (vis[v]) continue;
67         top1 = 0;
68         get_dist(v, u, w);
69         for (int i = 1; i <= top1; i++) {
70             for (int tt = 1; tt <= m; tt++) {
71                 int k = mp[tt].ff;
72                 if (k >= len1[i]) mp[tt].ss |= cnt[k - len1[i]];
73             }
74         }
75         for (int i = 1; i <= top1; i++) {
76             len2[++top2] = len1[i];
77             cnt[len1[i]] = 1;
78         }
79     }
80     for (int i = 1; i <= top2; i++) cnt[len2[i]] = 0;
81 };
82
83 std::function<void(int)> divide = [&](int u) -> void {
84     vis[u] = cnt[0] = 1;
85     solve(u, 0);
86     for (auto [v, w] : e[u]) {
87         if (vis[v]) continue;
88         get_root(v, u, get_size(v, u), root);
89         divide(root);
90     }
91 };
92
93 get_root(1, 0, get_size(1, 0), root);
94 divide(root);
95
96 for (int i = 1; i <= m; i++) {
97     if (mp[i].ss == 0) {
98         std::cout << "NAY" << endl;
99     } else {
100         std::cout << "AYE" << endl;
101     }
102 }
103
104 return 0;
105 }

```

第二个题

一棵 $n \leq 4 \times 10^4$ 个点的树, 边权 $w \leq 10^3$. 询问树上长度不超过 $k \leq 2 \times 10^4$ 的路径的数量.

```

1 // 洛谷 P4178 Tree
2
3 int main() {
4     std::ios::sync_with_stdio(false);
5     std::cin.tie(0);
6     std::cout.tie(0);
7
8     int n, k;
9     std::cin >> n;
10    std::vector<vpi> e(n + 1);
11    for (int i = 1; i < n; i++) {
12        int u, v, w;
13        std::cin >> u >> v >> w;
14        e[u].emplace_back(v, w);
15        e[v].emplace_back(u, w);
16    }
17    std::cin >> k;
18
19    /* centroid decomposition */

```

```

20 int root;
21 vi len, vis(n + 1);
22
23 std::function<int(int, int)> get_size = [&](int u, int fa) -> int {
24     if (vis[u]) return 0;
25     int ans = 1;
26     for (auto [v, w] : e[u]) {
27         if (v == fa) continue;
28         ans += get_size(v, u);
29     }
30     return ans;
31 };
32
33 std::function<int(int, int, int, int&)> get_root = [&](int u, int fa, int tot,
34                                                     int& root) -> int {
35     if (vis[u]) return 0;
36     int sum = 1, maxx = 0;
37     for (auto [v, w] : e[u]) {
38         if (v == fa) continue;
39         int tmp = get_root(v, u, tot, root);
40         maxx = std::max(maxx, tmp);
41         sum += tmp;
42     }
43     maxx = std::max(maxx, tot - sum);
44     if (2 * maxx <= tot) root = u;
45     return sum;
46 };
47
48 std::function<void(int, int, int)> get_dist = [&](int u, int fa, int dist) -> void {
49     len.push_back(dist);
50     for (auto [v, w] : e[u]) {
51         if (v == fa || vis[v]) continue;
52         get_dist(v, u, dist + w);
53     }
54 };
55
56 auto solve = [&](int u, int dist) -> int {
57     len.clear();
58     get_dist(u, 0, dist);
59     std::sort(all(len));
60     int ans = 0;
61     for (int l = 0, r = len.size() - 1; l < r;) {
62         if (len[l] + len[r] <= k) {
63             ans += r - l++;
64         } else {
65             r--;
66         }
67     }
68     return ans;
69 };
70
71 std::function<int(int)> divide = [&](int u) -> int {
72     vis[u] = true;
73     int ans = solve(u, 0);
74     for (auto [v, w] : e[u]) {
75         if (vis[v]) continue;
76         ans -= solve(v, w);
77         get_root(v, u, get_size(v, u), root);
78         ans += divide(root);
79     }
80     return ans;
81 };
82
83 get_root(1, 0, get_size(1, 0), root);
84 std::cout << divide(root) << endl;
85
86 return 0;
87 }

```

10.16 network flow - maximal flow

Dinic

```

1  /* dinic */
2  struct edge {
3      int from, to;
4      LL cap, flow;
5
6      edge(int u, int v, LL c, LL f) : from(u), to(v), cap(c), flow(f) {}
7  };
8

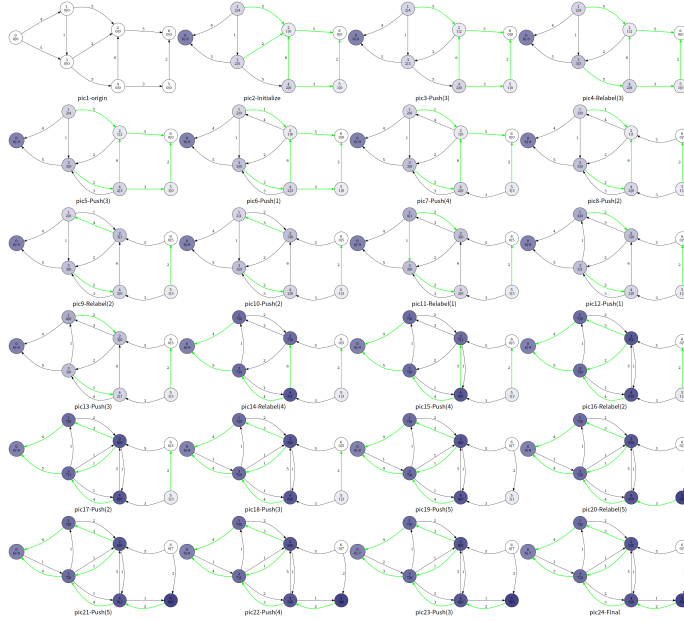
```

```

9 struct Dinic {
10     int n, m = 0, s, t;
11     std::vector<edge> e;
12     vi g[N];
13     int d[N], cur[N], vis[N];
14
15     void init(int n) {
16         for (int i = 0; i < n; i++) g[i].clear();
17         e.clear();
18         m = 0;
19     }
20
21     void add(int from, int to, LL cap) {
22         e.push_back(edge(from, to, cap, 0));
23         e.push_back(edge(to, from, 0, 0));
24         g[from].push_back(m++);
25         g[to].push_back(m++);
26     }
27
28     bool bfs() {
29         for (int i = 1; i <= n; i++) {
30             vis[i] = 0;
31         }
32         std::queue<int> q;
33         q.push(s), d[s] = 0, vis[s] = 1;
34         while (!q.empty()) {
35             int u = q.front();
36             q.pop();
37             for (int i = 0; i < g[u].size(); i++) {
38                 edge& ee = e[g[u][i]];
39                 if (!vis[ee.to] and ee.cap > ee.flow) {
40                     vis[ee.to] = 1;
41                     d[ee.to] = d[u] + 1;
42                     q.push(ee.to);
43                 }
44             }
45         }
46         return vis[t];
47     }
48
49     LL dfs(int u, LL now) {
50         if (u == t || now == 0) return now;
51         LL flow = 0, f;
52         for (int& i = cur[u]; i < g[u].size(); i++) {
53             edge& ee = e[g[u][i]];
54             edge& er = e[g[u][i] ^ 1];
55             if (d[u] + 1 == d[ee.to] and (f = dfs(ee.to, std::min(now, ee.cap - ee.flow))) > 0) {
56                 ee.flow += f, er.flow -= f;
57                 flow += f, now -= f;
58                 if (now == 0) break;
59             }
60         }
61         return flow;
62     }
63
64     LL dinic() {
65         LL ans = 0;
66         while (bfs()) {
67             for (int i = 1; i <= n; i++) cur[i] = 0;
68             ans += dfs(s, INF);
69         }
70         return ans;
71     }
72 } maxf;

```

HLPP



```

1  /* hlpp */
2  struct HLPP {
3      int n, m = 0, s, t;
4      std::vector<edge> e;      /* 边 */
5      std::vector<node> nd;     /* 点 */
6      std::vector<int> g[N];    /* 点的连边编号 */
7      std::priority_queue<node> q;
8      std::queue<int> qq;
9      bool vis[N];
10     int cnt[N];
11
12     void init() {
13         e.clear();
14         nd.clear();
15         for (int i = 0; i <= n + 1; i++) {
16             nd.pushback(node(inf, i, 0));
17             g[i].clear();
18             vis[i] = false;
19         }
20     }
21
22     void add(int u, int v, LL w) {
23         e.pushback(edge(u, v, w));
24         e.pushback(edge(v, u, 0));
25         g[u].pushback(m++);
26         g[v].pushback(m++);
27     }
28
29     void bfs() {
30         nd[t].hight = 0;
31         qq.push(t);
32         while (!qq.empty()) {
33             int u = qq.front();
34             qq.pop();
35             vis[u] = false;
36             for (auto j : g[u]) {
37                 int v = e[j].to;
38                 if (e[j].cap == 0 && nd[v].hight > nd[u].hight + 1) {
39                     nd[v].hight = nd[u].hight + 1;
40                     if (vis[v] == false) {
41                         qq.push(v);
42                         vis[v] = true;
43                     }
44                 }
45             }
46         }
47         return;
48     }
49
50     void _push(int u) {
51         for (auto j : g[u]) {
52             edge &ee = e[j], &er = e[j ^ 1];
53             int v = ee.to;
54             node &nu = nd[u], &nv = nd[v];
55             if (ee.cap && nv.hight + 1 == nu.hight) {

```

```

56         LL flow = std::min(ee.cap, nu.flow);
57         ee.cap -= flow, er.cap += flow;
58         nu.flow -= flow, nv.flow += flow;
59         if (vis[v] == false && v != t && v != s) {
60             q.push(nv);
61             vis[v] = true;
62         }
63         if (nu.flow == 0) break;
64     }
65 }
66 }
67
68 void relabel(int u) {
69     nd[u].hight = inf;
70     for (auto j : g[u]) {
71         int v = e[j].to;
72         if (e[j].cap && nd[v].hight + 1 < nd[u].hight) {
73             nd[u].hight = nd[v].hight + 1;
74         }
75     }
76 }
77
78 LL hlpp() {
79     bfs();
80     if (nd[s].hight == inf) return 0;
81     nd[s].hight = n;
82     for (int i = 1; i <= n; i++) {
83         if (nd[i].hight < inf) cnt[nd[i].hight]++;
84     }
85     for (auto j : g[s]) {
86         int v = e[j].to;
87         int flow = e[j].cap;
88         if (flow) {
89             e[j].cap -= flow, e[j ^ 1].cap += flow;
90             nd[s].flow -= flow, nd[v].flow += flow;
91             if (vis[v] == false && v != s && v != t) {
92                 q.push(nd[v]);
93                 vis[v] = true;
94             }
95         }
96     }
97     while (!q.empty()) {
98         int u = q.top().id;
99         q.pop();
100         vis[u] = false;
101         _push(u);
102         if (nd[u].flow) {
103             cnt[nd[u].hight]--;
104             if (cnt[nd[u].hight] == 0) {
105                 for (int i = 1; i <= n; i++) {
106                     if (i != s && i != t && nd[i].hight > nd[u].hight && nd[i].hight < n + 1) {
107                         nd[i].hight = n + 1;
108                     }
109                 }
110             }
111             relabel(u);
112             cnt[nd[u].hight]++;
113             q.push(nd[u]);
114             vis[u] = true;
115         }
116     }
117     return nd[t].flow;
118 }
119 } maxf;

```

10.17 network flow - minimum cost flow

Dinic + SPFA

```

1 struct edge {
2     int from, to;
3     LL cap, cost;
4
5     edge(int u, int v, LL c, LL w) : from(u), to(v), cap(c), cost(w) {}
6 };
7
8 struct MCMF {
9     int n, m = 0, s, t;
10    std::vector<edge> e;
11    vi g[N];
12    int cur[N], vis[N];

```

```

13 LL dist[N], minc;
14
15 void init(int n) {
16     for (int i = 0; i < n; i++) g[i].clear();
17     e.clear();
18     minc = m = 0;
19 }
20
21 void add(int from, int to, LL cap, LL cost) {
22     e.push_back(edge(from, to, cap, cost));
23     e.push_back(edge(to, from, 0, -cost));
24     g[from].push_back(m++);
25     g[to].push_back(m++);
26 }
27
28 bool spfa() {
29     rep(i, 1, n) { dist[i] = INF, cur[i] = 0; }
30     std::queue<int> q;
31     q.push(s), dist[s] = 0, vis[s] = 1;
32     while (!q.empty()) {
33         int u = q.front();
34         q.pop();
35         vis[u] = 0;
36         for (int j = cur[u]; j < g[u].size(); j++) {
37             edge& ee = e[g[u][j]];
38             int v = ee.to;
39             if (ee.cap && dist[v] > dist[u] + ee.cost) {
40                 dist[v] = dist[u] + ee.cost;
41                 if (!vis[v]) {
42                     q.push(v);
43                     vis[v] = 1;
44                 }
45             }
46         }
47         cur[u] = j;
48     }
49     return dist[t] != INF;
50 }
51
52 LL dfs(int u, LL now) {
53     if (u == t) return now;
54     vis[u] = 1;
55     LL ans = 0;
56     for (int& i = cur[u]; i < g[u].size() && ans < now; i++) {
57         edge &ee = e[g[u][i]], &er = e[g[u][i] ^ 1];
58         int v = ee.to;
59         if (!vis[v] && ee.cap && dist[v] == dist[u] + ee.cost) {
60             LL f = dfs(v, std::min(ee.cap, now - ans));
61             if (f) {
62                 minc += f * ee.cost, ans += f;
63                 ee.cap -= f;
64                 er.cap += f;
65             }
66         }
67     }
68     vis[u] = 0;
69     return ans;
70 }
71
72 PLL mcmf() {
73     LL maxf = 0;
74     while (spfa()) {
75         LL tmp;
76         while ((tmp = dfs(s, INF))) maxf += tmp;
77     }
78     return std::makepair(maxf, minc);
79 } minc_maxf;

```

Primal-Dual 原始对偶算法

```

1  /* primal dual */
2  struct edge {
3      int from, to;
4      LL cap, cost;
5  };
6  edge(int u, int v, LL c, LL w) : from(u), to(v), cap(c), cost(w) {}
7
8
9  struct node {
10     int v, e;
11 };
12 node(int _v = 0, int _e = 0) : v(_v), e(_e) {}
13

```



```

14
15 const int maxn = 5000 + 10;
16
17 struct MCMF {
18     int n, m = 0, s, t;
19     std::vector<edge> e;
20     vi g[maxn];
21     int dis[maxn], vis[maxn], h[maxn];
22     node p[maxn * 2];
23
24     void add(int from, int to, LL cap, LL cost) {
25         e.push_back(edge(from, to, cap, cost));
26         e.push_back(edge(to, from, 0, -cost));
27         g[from].push_back(m++);
28         g[to].push_back(m++);
29     }
30
31     bool dijkstra() {
32         std::priority_queue<PII, std::vector<PII>, std::greater<PII>> q;
33         for (int i = 1; i <= n; i++) {
34             dis[i] = inf;
35             vis[i] = 0;
36         }
37         dis[s] = 0;
38         q.push({0, s});
39         while (!q.empty()) {
40             int u = q.top().ss;
41             q.pop();
42             if (vis[u]) continue;
43             vis[u] = 1;
44             for (auto i : g[u]) {
45                 edge ee = e[i];
46                 int v = ee.to, nc = ee.cost + h[u] - h[v];
47                 if (ee.cap and dis[v] > dis[u] + nc) {
48                     dis[v] = dis[u] + nc;
49                     p[v] = node(u, i);
50                     if (!vis[v]) q.push({dis[v], v});
51                 }
52             }
53         }
54         return dis[t] != inf;
55     }
56
57     void spfa() {
58         std::queue<int> q;
59         for (int i = 1; i <= n; i++) h[i] = inf;
60         h[s] = 0, vis[s] = 1;
61         q.push(s);
62         while (!q.empty()) {
63             int u = q.front();
64             q.pop();
65             vis[u] = 0;
66             for (auto i : g[u]) {
67                 edge ee = e[i];
68                 int v = ee.to;
69                 if (ee.cap and h[v] > h[u] + ee.cost) {
70                     h[v] = h[u] + ee.cost;
71                     if (!vis[v]) {
72                         vis[v] = 1;
73                         q.push(v);
74                     }
75                 }
76             }
77         }
78     }
79
80     PLL mcmf() {
81         LL maxf = 0, minc = 0;
82         spfa();
83         while (dijkstra()) {
84             LL minf = INF;
85             for (int i = 1; i <= n; i++) h[i] += dis[i];
86             for (int i = t; i != s; i = p[i].v) minf = std::min(minf, e[p[i].e].cap);
87             for (int i = t; i != s; i = p[i].v) {
88                 e[p[i].e].cap -= minf;
89                 e[p[i].e ^ 1].cap += minf;
90             }
91             maxf += minf;
92             minc += minf * h[t];
93         }
94         return std::make_pair(maxf, minc);
95     }
96 } minc_maxf;

```

存在负环的网络

流满后推流, 转化为上下界网络流.

10.18 network flow - minimal cut

最小割解决的问题是将图中的点集 V 划分成 S 与 T , 使得 S 与 T 之间的连边的容量总和最小.

最大流最小割定理

网络中 s 到 t 的最大流流量的值等于所要求的最小割的值, 所以求最小割只需要跑 Dinic 即可.

获得 S 中的所有点

在 Dinic 的 bfs 函数中, 每次将所有点的 d 数组值改为无穷大, 最后跑完最大流之后 d 数组不为无穷大的就是和源点一起在 S 集合中的点.

例子

最小割的本质是对图中点集进行 2-划分, 网络流只是求解答案的手段.

1. 在图中花费最小的代价断开一些边使得源点 s 无法流到汇点 t .

直接跑最大流就得到了答案.

2. 在图中删除最少的点使得源点 s 无法流到汇点 t .

对每个点进行拆点, 在 i 与 i' 之间建立容量为 1 的有向边.

10.19 network flow with upper / lower bound

10.19.1 无源汇上下界可行流

每条有向边有流量的上下界限制, 但整张图并未确定源点与汇点. 如果存在满足每个点的流入量等于流出量, 且每条边的流量满足其上下界限制的流, 称之为可行流.

1. 将每条边先给予大小为下界的流量,
2. 对每个点计算总流入量 in_u 与总流出量 out_u 的值,
3. 建立超级源点到每个点, 容量大小为 $\max\{0, \text{in}_u - \text{out}_u\}$ 的边; 建立每个点到超级汇点, 容量大小为 $\max\{0, \text{out}_u - \text{in}_u\}$,
4. 跑从超级源点到超级汇点的最大流, 如果超级源点每条边都流满意味着存在可行流. 将每条边的流量加上预先给每条边设置的下界流量即为可行流方案.

10.19.2 有源汇上下界可行流

1. 建立汇点 t 到源点 s 的, 容量为 ∞ 的有向边, 将其转化为无源汇的问题.

10.19.3 有源汇上下界最大流

1. 建立汇点 t 到源点 s 的, 容量为 ∞ 的有向边, 将其转化为无源汇的问题,
2. 跑上下界可行流, 可行流流量为边 $t \xrightarrow{\infty} s$ 的流量.
3. 删除 $t \xrightarrow{\infty} s$ 的边, 再残量网络上跑 s 到 t 的最大流,
4. 答案等于可行流流量 + 最大流流量.

10.19.4 有源汇上下界最小流

1. 建立汇点 t 到源点 s 的, 容量为 ∞ 的有向边, 将其转化为无源汇的问题,
2. 跑上下界可行流, 可行流流量为边 $t \xrightarrow{\infty} s$ 的流量.
3. 删除 $t \xrightarrow{\infty} s$ 的边, 再残量网络上跑 t 到 s 的最大流,
4. 答案等于可行流流量 - 最大流流量.

10.19.5 有源汇上下界最小费用可行流

1. 按下界流满并计算费用,
2. 类似有源汇上下界最大流建图, 跑超级源点到超级汇点的费用流,
3. 答案等于按下界的费用加上后续残量网络.

10.20 matching - matching on bipartite graph

二分图最大匹配

Kuhn-Munkres

时间复杂度: $O(n^3)$.

```

1  /* Kuhn-Munkres */
2  auto KM = [&](int n1, int n2, vvi e) -> std::pair<vi, vi> {
3      vi vis(n2 + 1);
4      vi l(n1 + 1, -1), r(n2 + 1, -1);
5      std::function<bool(int)> dfs = [&](int u) -> bool {
6          for (auto v : e[u]) {
7              if (!vis[v]) {
8                  vis[v] = 1;
9                  if (r[v] == -1 or dfs(r[v])) {
10                     r[v] = u;
11                     return true;
12                 }
13             }
14         }
15         return false;
16     };
17     for (int i = 1; i <= n1; i++) {
18         std::fill(all(vis), 0);
19         dfs(i);
20     }
21     for (int i = 1; i <= n2; i++) {
22         if (r[i] == -1) continue;
23         l[r[i]] = i;
24     }
25     return {l, r};
26 };
27 auto [mchl, mchr] = KM(n1, n2, e);
28 std::cout << mchl.size() - std::count(all(mchl), -1) << endl;

```

Hopcroft-Karp

据说时间复杂度是 $O(m\sqrt{n})$ 的, 但是快的飞起.

```

1  /* Hopcroft-Karp */
2  vpi e(m);
3  auto hopcroft_karp = [&](int n, int m, vpi& e) -> std::pair<vi, vi> {
4      vi g(e.size()), l(n + 1, -1), r(m + 1, -1), d(n + 2);
5      for (auto [u, v] : e) d[u]++;
6      std::partial_sum(all(d), d.begin());
7      for (auto [u, v] : e) g[--d[u]] = v;
8      for (vi a, p, q(n + 1);) {
9          a.assign(n + 1, -1);
10         p.assign(n + 1, -1);
11         int t = 1;
12         for (int i = 1; i <= n; i++) {
13             if (l[i] == -1) {
14                 q[t++] = a[i] = p[i] = i;
15             }
16         }
17         bool match = false;
18         for (int i = 1; i < t; i++) {
19             int u = q[i];
20             if (l[a[u]] != -1) continue;
21             for (int j = d[u]; j < d[u + 1]; j++) {
22                 int v = g[j];
23                 if (r[v] == -1) {
24                     while (v != -1) {
25                         r[v] = u;
26                         std::swap(l[u], v);
27                         u = p[u];
28                     }
29                     match = true;
30                     break;
31                 }
32                 if (p[r[v]] == -1) {
33                     q[t++] = v = r[v];
34                     p[v] = u;
35                     a[v] = a[u];
36                 }
37             }
38         }
39         if (!match) break;
40     }
41     return {l, r};
42 };

```

二分图最大权匹配

Kuhn-Munkres

注意是否为完美匹配, 非完美选 0, 完美选 $-INF$. (存疑)

```

1  /* Kuhn-Munkres */
2  auto KM = [&](int n, vvl e) -> std::tuple<LL, vi, vi> {
3      vl la(n + 1), lb(n + 1), pp(n + 1), vx(n + 1);
4      vi l(n + 1, -1), r(n + 1, -1);
5      vi va(n + 1), vb(n + 1);
6      LL delta;
7      auto bfs = [&](int x) -> void {
8          int a, y = 0, y1 = 0;
9          std::fill(all(pp), 0);
10         std::fill(all(vx), INF);
11         r[y] = x;
12         do {
13             a = r[y], delta = INF, vb[y] = 1;
14             for (int b = 1; b <= n; b++) {
15                 if (!vb[b]) {
16                     if (vx[b] > la[a] + lb[b] - e[a][b]) {
17                         vx[b] = la[a] + lb[b] - e[a][b];
18                         pp[b] = y;
19                     }
20                     if (vx[b] < delta) {
21                         delta = vx[b];
22                         y1 = b;
23                     }
24                 }
25             }
26             for (int b = 0; b <= n; b++) {

```

```
27         if (vb[b]) {
28             la[r[b]] -= delta;
29             lb[b] += delta;
30         } else
31             vx[b] -= delta;
32     }
33     y = y1;
34 } while (r[y] != -1);
35 while (y) {
36     r[y] = r[pp[y]];
37     y = pp[y];
38 }
39 };
40 for (int i = 1; i <= n; i++) {
41     std::fill(all(vb), 0);
42     bfs(i);
43 }
44 LL ans = 0;
45 for (int i = 1; i <= n; i++) {
46     if (r[i] == -1) continue;
47     l[r[i]] = i;
48     ans += e[r[i]][i];
49 }
50 return {ans, l, r};
51 };
52
53 auto [ans, mchl, mchr] = KM(n, e);
```

10.21 matching - matching on general graph

11 geometry

11.1 two demention

点与向量

```

1 struct Point {
2     LL x = 0, y = 0;
3     Point() = default;
4     Point(long long x, long long y) : x(x), y(y) {}
5     operator bool() { return *this != Point{}; }
6     friend bool operator==(Point p, Point q) { return p.x == q.x and p.y == q.y; }
7     friend bool operator!=(Point p, Point q) { return !(p == q); }
8     friend Point operator+(Point p, Point q) { return {p.x + q.x, p.y + q.y}; }
9     friend Point operator-(Point p, Point q) { return {p.x - q.x, p.y - q.y}; }
10    friend LL dot(Point p, Point q) { return p.x * q.x + p.y * q.y; }
11    friend LL det(Point p, Point q) { return p.x * q.y - q.x * p.y; }
12    friend bool operator<(Point p, Point q) {
13        return std::pair{p.quad(), det(q, p)} < std::pair{q.quad(), 0ll};
14        return (p.x == q.x ? p.y < q.y : p.x < q.x);
15    }
16    int quad() const {
17        if (x > 0 && y >= 0) return 1;
18        if (x <= 0 and y > 0) return 2;
19        if (x < 0 and y <= 0) return 3;
20        if (x >= 0 and y < 0) return 4;
21        return 0;
22    }
23    friend LL dist(Point p, Point q) {
24        return (p.x - q.x) * (p.x - q.x) + (p.y - q.y) * (p.y - q.y);
25    }
26 };
27 std::istream& operator>>(std::istream& is, Point& p) { return is >> p.x >> p.y; }
28 std::ostream& operator<<(std::ostream& os, Point p) {
29     return os << '(' << p.x << ',' << p.y << ')';
30 }

```

线段

```

1 struct line {
2     point a, b;
3
4     line(point _a = {}, point _b = {}) { a = _a, b = _b; }
5
6     /* 交点类型为 double */
7     friend point iPoint(line p, line q) {
8         point v1 = p.b - p.a;
9         point v2 = q.b - q.a;
10        point u = q.a - p.a;
11        return q.a + (q.b - q.a) * ((u ^ v1) * 1. / (v1 ^ v2));
12    }
13
14    /* 极角排序 */
15    bool operator<(const line& p) const {
16        double t1 = std::atan2((b - a).y, (b - a).x);
17        double t2 = std::atan2((p.b - p.a).y, (p.b - p.a).x);
18        if (fabs(t1 - t2) > eps) {
19            return t1 < t2;
20        }
21        return ((p.a - a) ^ (p.b - a)) > eps;
22    }
23 };

```

11.2 convex

2D

```

1 /* andrew */
2 auto andrew = [&](std::vector<point>& v) -> std::vector<point> {
3     std::sort(all(v));

```

```

4      std::vector<point> stk;
5      for (int i = 0; i < n; i++) {
6          point x = v[i];
7          while (stk.size() > 1 and ((stk.end()[-1] - stk.end()[-2]) ^ (x - stk.end()[-2])) <= 0) {
8              stk.pop_back();
9          }
10         stk.push_back(x);
11     }
12     int tmp = stk.size();
13     for (int i = n - 2; i >= 0; i--) {
14         point x = v[i];
15         while (stk.size() > tmp and ((stk.end()[-1] - stk.end()[-2]) ^ (x - stk.end()[-2])) <= 0) {
16             stk.pop_back();
17         }
18         stk.push_back(x);
19     }
20     return stk;
21 };

```

half plane

```

1  /* half plane */
2  auto half_plane = [&](std::vector<line>& ln) -> std::vector<point> {
3      std::sort(all(ln));
4      ln.erase(
5          unique(
6              all(ln),
7              [](line& p, line& q) {
8                  double t1 = atan2((p.b - p.a).y, (p.b - p.a).x);
9                  double t2 = atan2((q.b - q.a).y, (q.b - q.a).x);
10                 return fabs((t1 - t2)) < eps;
11             }),
12          ln.end());
13     auto check = [&](line p, line q, line r) -> bool {
14         point a = iPoint(p, q);
15         return ((r.b - r.a) ^ (a - r.a)) < -eps;
16     };
17     line q[ln.size() + 2];
18     int hh = 1, tt = 0;
19     q[++tt] = ln[0];
20     q[++tt] = ln[1];
21     for (int i = 2; i < (int) ln.size(); i++) {
22         while (hh < tt and check(q[tt - 1], q[tt], ln[i])) tt--;
23         while (hh < tt and check(q[hh + 1], q[hh], ln[i])) hh++;
24         q[++tt] = ln[i];
25     }
26     while (hh < tt and check(q[tt - 1], q[tt], q[hh])) tt--;
27     while (hh < tt and check(q[hh + 1], q[hh], q[tt])) hh++;
28     q[tt + 1] = q[hh];
29     std::vector<point> ans;
30     for (int i = hh; i <= tt; i++) {
31         ans.push_back(iPoint(q[i], q[i + 1]));
32     }
33     return ans;
34 };

```

12 offline algorithm

12.1 discretization

```

1 std::sort(all(a));
2 a.erase(unique(all(a)), a.end());
3 auto get_id = [&](const int& x) -> int { return lower_bound(all(a), x) - a.begin() + 1; };

```

12.2 Mo algorithm

普通莫队

```

1 int block = n / sqrt(2 * m / 3);
2 std::sort(all(q), [&](node a, node b) {
3     return a.l / block == b.l / block ? (a.r == b.r ? 0 : ((a.l / block) & 1) ^ (a.r < b.r))
4         : a.l < b.l;
5 });
6 auto move = [&](int x, int op) -> void {
7     if (op == 1) {
8         /* operations */
9     } else {
10        /* operations */
11    }
12 };
13 for (int k = 1, l = 1, r = 0; k <= m; k++) {
14     node Q = q[k];
15     while (l > Q.l) {
16         move(--l, 1);
17     }
18     while (r < Q.r) {
19         move(++r, 1);
20     }
21     while (l < Q.l) {
22         move(l++, -1);
23     }
24     while (r > Q.r) {
25         move(r--, -1);
26     }
27 }

```

12.3 CDQ

n 个三维数对 (a_i, b_i, c_i) , 设 $f(i)$ 表示 $a_j \leq a_i, b_j \leq b_i, c_j \leq c_i (i \neq j)$ 的个数. 输出 $f(i)$ ($0 \leq i \leq n-1$) 的值.

```

1 // 洛谷 P3810 【模板】三维偏序（陌上花开）
2
3 struct data {
4     int a, b, c, cnt, ans;
5
6     data(int _a = 0, int _b = 0, int _c = 0, int _cnt = 0, int _ans = 0) {
7         a = _a, b = _b, c = _c, cnt = _cnt, ans = _ans;
8     }
9
10    bool operator!=(data x) {
11        if (a != x.a) return true;
12        if (b != x.b) return true;
13        if (c != x.c) return true;
14        return false;
15    }
16 };
17
18 int main() {
19     std::ios::sync_with_stdio(false);
20     std::cin.tie(0);
21
22     int n, k;
23     std::cin >> n >> k;
24     static data v1[N], v2[N];
25     for (int i = 1; i <= n; i++) {

```



```

26     std::cin >> v1[i].a >> v1[i].b >> v1[i].c;
27 }
28 std::sort(v1 + 1, v1 + n + 1, [&](data x, data y) {
29     if (x.a != y.a) return x.a < y.a;
30     if (x.b != y.b) return x.b < y.b;
31     return x.c < y.c;
32 });
33 int t = 0, top = 0;
34 for (int i = 1; i <= n; i++) {
35     t++;
36     if (v1[i] != v1[i + 1]) {
37         v2[++top] = v1[i];
38         v2[top].cnt = t;
39         t = 0;
40     }
41 }
42 vi tr(N);
43 auto add = [&](int pos, int val) -> void {
44     while (pos <= k) {
45         tr[pos] += val;
46         pos += lowbit(pos);
47     }
48 };
49 auto query = [&](int pos) -> int {
50     int ans = 0;
51     while (pos > 0) {
52         ans += tr[pos];
53         pos -= lowbit(pos);
54     }
55     return ans;
56 };
57 std::function<void(int, int)> CDQ = [&](int l, int r) -> void {
58     if (l == r) return;
59     int mid = (l + r) >> 1;
60     CDQ(l, mid), CDQ(mid + 1, r);
61     std::sort(v2 + 1, v2 + mid + 1, [&](data x, data y) {
62         if (x.b != y.b) return x.b < y.b;
63         return x.c < y.c;
64     });
65     std::sort(v2 + mid + 1, v2 + r + 1, [&](data x, data y) {
66         if (x.b != y.b) return x.b < y.b;
67         return x.c < y.c;
68     });
69     int i = 1, j = mid + 1;
70     while (j <= r) {
71         while (i <= mid && v2[i].b <= v2[j].b) {
72             add(v2[i].c, v2[i].cnt);
73             i++;
74         }
75         v2[j].ans += query(v2[j].c);
76         j++;
77     }
78     for (int ii = 1; ii < i; ii++) {
79         add(v2[ii].c, -v2[ii].cnt);
80     }
81     return;
82 };
83 CDQ(1, top);
84 vi ans(n + 1);
85 for (int i = 1; i <= top; i++) {
86     ans[v2[i].ans + v2[i].cnt] += v2[i].cnt;
87 }
88 for (int i = 1; i <= n; i++) {
89     std::cout << ans[i] << endl;
90 }
91 return 0;
92 }

```

13 Print All Cases

13.1 print all trees with n nodes

构造所有 n 个节点的树.

13.1.1 有根树

表示其数量的数列在 oeis 上编号为 A000081. $n = 1, 2, 3 \dots, 20$ 的项分别为:

1, 1, 2, 4, 9,
20, 48, 115, 286, 719,
1842, 4766, 12486, 32973, 87811,
235381, 634847, 1721159, 4688676, 12826228.

构造所有 $n \leq 20$ 的有根树的 (平均) 运行时间为 15.7054s.

```

1  /* integer partition */
2  int n = 5;
3  std::vector<vvi> part(n + 1);
4  auto integerPartition = [&](int n) {
5      // part[i] = {{i}};
6      for (int i = 1; i <= n; i++) {
7          part[i].push_back({i});
8          for (int j = 1; j < i; j++) {
9              for (const auto& v : part[i - j]) {
10                 vvi tmp = v;
11                 tmp.push_back(j);
12                 std::sort(all(tmp));
13                 part[i].push_back(tmp);
14             }
15         }
16         std::sort(all(part[i]));
17         part[i].erase(unique(all(part[i])), part[i].end());
18     }
19 };
20 integerPartition(n);
21 /* find all trees */
22 std::vector<std::vector<std::string>> trees(n + 1);
23 auto allTrees = [&](int n) {
24     std::string s;
25     for (int i = 1; i < n; i++) s += '(';
26     for (int i = 1; i < n; i++) s += ')';
27     trees[n].push_back(s);
28     for (const auto& v : part[n - 1]) {
29         std::vector<std::string> now;
30         auto dfs = [&](auto&& self, int i) {
31             if (i == v.size()) {
32                 std::string s = "";
33                 auto tmp = now;
34                 std::sort(all(tmp));
35                 for (const auto& ss : tmp) s += '(' + ss + ')';
36                 trees[n].push_back(s);
37                 return;
38             }
39             for (const auto& s : trees[v[i]]) {
40                 now.push_back(s);
41                 self(self, i + 1);
42                 now.pop_back();
43             }
44         };
45         dfs(dfs, 0);
46     }
47     std::sort(all(trees[n]));
48     trees[n].erase(unique(all(trees[n])), trees[n].end());
49 };
50 for (int i = 1; i <= n; i++) {
51     allTrees(i);
52     debug(i, trees[i].size());
53     std::cout << '\n';
54 }
55 for (const auto& s : trees[n]) {
56     vvi e(n + 1);
57     vi fa(n + 1);
58     int cnt = 1, now = 1;

```

```
59     for (const auto& c : s) {
60         if (c == '(') {
61             cnt += 1;
62             e[now].push_back(cnt);
63             e[cnt].push_back(now);
64             fa[cnt] = now;
65             now = cnt;
66         } else {
67             now = fa[now];
68         }
69     }
70     debug(e);
71     /* do the things you need */
72 }
```

14 Magic

14.1 magic heap

对顶堆维护中位数.

```

1  /* magic heap */
2  struct MagicHeap {
3      LL suml = 0, sumr = 0;
4      std::priority_queue<int> ql;
5      std::priority_queue<int, std::vector<int>, std::greater<int>> qr;
6      void le2ri() {
7          auto x = ql.top();
8          suml -= x, ql.pop();
9          sumr += x, qr.push(x);
10     };
11     void ri2le() {
12         auto x = qr.top();
13         sumr -= x, qr.pop();
14         suml += x, ql.push(x);
15     };
16     void pushL(int x) { suml += x, ql.push(x); }
17     void pushR(int x) { sumr += x, qr.push(x); }
18     void push(int x) {
19         if (ql.empty()) {
20             pushL(x);
21         } else if (qr.empty()) {
22             (x <= ql.top() ? le2ri(), pushL(x) : pushR(x));
23         } else {
24             int le = ql.top(), ri = qr.top();
25             if (le <= x and x <= ri) {
26                 (ql.size() == qr.size() ? pushL(x) : pushR(x));
27             } else if (x < le) {
28                 if (ql.size() != qr.size()) le2ri();
29                 pushL(x);
30             } else {
31                 if (ql.size() <= qr.size()) ri2le();
32                 pushR(x);
33             }
34         }
35     }
36     int size() { return ql.size() + qr.size(); }
37     bool empty() { return ql.empty() and qr.empty(); }
38     LL val() { return suml + sumr; }
39     LL mid() { return ql.top(); }
40     LL dist() { return sumr - suml + ql.top() * (ql.size() - qr.size()); }
41 };

```

14.2 operator queue

双栈维护队列半群.

```

1  template <typename T, typename Op>
2  struct OpQueue {
3      static_assert(std::is_convertible_v<std::invoke_result_t<Op, T, T>, T>);
4      const T e;
5      const Op op;
6      std::vector<T> l, r, a;
7      OpQueue(T e, Op op) : e(e), op(op), l{e}, r{e} {}
8      T val() const { return op(l.back(), r.back()); }
9      void push(T x) {
10         r.push_back(op(r.back(), x));
11         a.push_back(x);
12     }
13     void pop() {
14         if (l.size() == 1) {
15             for (; !a.empty(); a.pop_back()) {
16                 l.push_back(op(a.back(), l.back()));
17             }
18             r.resize(1);
19         }
20         assert(l.size() > 1);
21         l.pop_back();
22     }
23     int size() const { return l.size() + r.size() - 2; }
24     bool empty() const { return l.size() + r.size() == 2; }
25 };

```

```
26 |  
27 | /* When using this, remember to replace "T" with correct type and "e" with identity in the half group. */  
28 | auto op = [](T a, T b) -> T {  
29 |     /* You operations */  
30 | };  
31 | OpQueue<T, decltype(op)> a(e, op);
```