

BELJING NORMAL UNIVERSITY
SCHOOL OF MATHEMATICS

Template

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目录

1	hpp	3
1.1	heading	3
1.2	debug.h	4
2	shell scripts	5
2.1	linux version	5
2.2	windows version	5
3	data structure	6
3.1	stack	6
3.2	queue	6
3.3	DSU	6
3.4	spare table	6
3.5	Cartesian tree	7
3.6	segment tree	7
3.7	segment tree split	9
3.8	persistent segment tree	10
3.9	sweep line	12
3.10	treap	13
3.11	splay	17
3.12	link cut tree	20
3.13	Lichao tree	20
3.14	ODT	21
4	string	21
4.1	kmp	21
4.2	z function	22
4.3	manacher	22
4.4	AC automaton	22
4.5	PAM	23
4.6	Suffix Array	24
4.7	Cantor expansion	25
4.8	trie	26
5	math - number theory	28

目录	3
5.1 mod int	28
5.2 Eculid	29
5.3 inverse	30
5.4 sieve	31
5.5 powerful number	33
5.6 block	34
5.7 CRT & exCRT	35
5.8 BSGS & exBSGS	36
5.9 Miller Rabin	36
5.10 Pollard Rho	37
5.11 quadratic residu	37
5.12 Lucas	38
5.13 Wilson	40
5.14 LTE	40
5.15 Mobius inversion	41
6 math - polynomial	43
6.1 FTT	43
6.2 FWT	44
6.3 class polynomial	46
6.4 wsy poly	50
7 math - game theory	57
7.1 nim game	57
7.2 anti - nim game	57
8 math - linear algebra	58
8.1 matrix	58
8.2 linear basis	59
8.3 linear programming	60
8.4 bm	60
9 complex number	62
10 graph	63
10.1 topology sort	63
10.2 shortest path	63
10.3 minimum spanning tree	66

10.4	SCC	66
10.5	DCC	67
10.6	2-sat	70
10.7	minimum ring	71
10.8	tree - center of gravity	71
10.9	tree - DSU on tree	71
10.10	tree - AHU	72
10.11	tree - LCA	73
10.12	tree - heavy light decomposition	73
10.13	tree - virtual tree	74
10.14	tree - pseudo tree	75
10.15	tree - divide and conquer on tree	76
10.16	tree - matrix tree	78
10.17	Prefür sequence	79
10.18	network flow - maximal flow	79
10.19	network flow - minimum cost flow	82
10.20	network flow - minimal cut	84
10.21	network flow - upper / lower bound	85
10.22	network flow - other versions	86
10.23	matching - matching on bipartite graph	88
10.24	matching - matching on general graph	90
11	geometry	91
11.1	two demention	91
11.2	convex	91
11.3	half plane union	93
11.4	rotate	93
11.5	Simpson	94
12	offline algorithm	95
12.1	discretization	95
12.2	Mo algorithm	95
12.3	回滚莫队	95
12.4	CDQ	96
12.5	segment tree devide and conquer	97
13	Print All Cases	99

目录	5
13.1 print all trees with n nodes	99
13.1.1 有根树	99
14 Magic	101
14.1 magic heap	101
14.2 operator queue	101
14.3 Fast GCD	102
14.4 $q \equiv \frac{a}{b} \bmod mod$	102

1 hpp

1.1 heading

```

1  #include <bits/stdc++.h>
2
3  // using namespace std;
4
5  using LL = long long;
6  using i128 = __int128;
7  using PII = std::pair<int, int>;
8  /*
9  using UI = unsigned int;
10 using ULL = unsigned long long;
11 using ULL = unsigned long long;
12 using PII = std::pair<int, LL>;
13 using PLI = std::pair<LL, int>;
14 using PLL = std::pair<LL, LL>;
15 using vi = std::vector<int>;
16 using vvi = std::vector<vi>;
17 using vl = std::vector<LL>;
18 using vvl = std::vector<vl>;
19 using vpi = std::vector<PII>;
20 */
21
22 #define ff first
23 #define ss second
24 #define all(v) v.begin(), v.end()
25 #define rall(v) v.rbegin(), v.rend()
26
27 #ifdef LOCAL
28 #include "debug.h"
29 #else
30 #define debug(...) \
31     do { \
32     } while (false)
33 #endif
34
35 constexpr int inf = 0x3f3f3f3f;
36 constexpr LL INF = 1e18;
37 constexpr int lowbit(int x) { return x & -x; }
38 /*
39 constexpr int add(int x, int y) { return x + y < mod ? x + y : x - mod + y; }
40 constexpr int sub(int x, int y) { return x < y ? mod + x - y : x - y; }
41 constexpr int mul(LL x, int y) { return x * y % mod; }
42 constexpr void Add(int& x, int y) { x = add(x, y); }
43 constexpr void Sub(int& x, int y) { x = sub(x, y); }
44 constexpr void Mul(int& x, int y) { x = mul(x, y); }
45 constexpr int pow(int x, int y, int z = 1) {
46     for (; y; y /= 2) {
47         if (y & 1) Mul(z, x);
48         Mul(x, x);
49     }
50     return z;
51 }
52 temps constexpr int add(Ts... x) {
53     int y = 0;
54     (... , Add(y, x));
55     return y;
56 }
57 temps constexpr int mul(Ts... x) {
58     int y = 1;
59     (... , Mul(y, x));
60     return y;
61 }
62 */
63 tandu bool Max(T& x, const U& y) { return x < y ? x = y, true : false; }
64 tandu bool Min(T& x, const U& y) { return x > y ? x = y, true : false; }
65
66 void solut() {
67     ;
68 }
69
70 int main() {
71     std::ios::sync_with_stdio(false);
72     std::cin.tie(0);
73     int t = 1;
74     std::cin >> t;
75     while (t--) {
76         solut();
77     }
78     return 0;
79 }

```

1.2 debug.h

```

1  template <typename T, typename U>
2  std::ostream& operator<<(std::ostream& os, const std::pair<T, U>& p) {
3      return os << '<' << p.first << ',' << p.second << '>';
4  }
5
6  template <
7      typename T, typename = decltype(std::begin(std::declval<T>())),
8      typename = std::enable_if_t<!std::is_same_v<T, std::string>>>
9  std::ostream& operator<<(std::ostream& os, const T& c) {
10     auto it = std::begin(c);
11     if (it == std::end(c)) return os << "{}";
12     for (os << '{' << *it; ++it != std::end(c); os << ',' << *it);
13     return os << '}';
14 }
15
16 #define debug(arg...) \
17     do { \
18         std::cerr << "[" #arg "]" :"; \
19         dbg(arg); \
20     } while (false)
21
22 template <typename... Ts>
23 void dbg(Ts... args) {
24     (... , (std::cerr << ' ' << args));
25     std::cerr << std::endl;
26 }

```

2 shell scripts

2.1 linux version

```
1  #!/bin/bash
2
3  cd "$1"
4
5  g++ -o main -O2 -std=c++17 -DLOCAL main.cpp -ftrapv -fsanitize=address,undefined
6
7  for input in *.in; do
8      output=${input%.*}.out
9      answer=${input%.*}.ans
10
11      ./main < $input > $output
12
13      echo "case ${input%.*}: "
14      echo "My: "
15      cat $output
16      echo "Answer: "
17      cat $answer
18
19  done
```

2.2 windows version

```
1  @echo off
2
3  cd %1
4
5  del .\main.exe
6
7  g++ -o main.exe main.cpp -DLOCAL -std=c++17 -ftrapv
8
9  for %%i in (*.in) do (
10     main.exe < %%i > %%~ni.out
11     echo case %%~ni:
12     echo My:
13     type %%~ni.out
14     echo Answer:
15     type %%~ni.ans
16 )
17
18 cd ../shell
```


3 data structure

3.1 stack

```

1 vi stk;
2 for (int i = 1; i <= n; i++){
3     while (!stk.empty() and stk.back() > a[i]) {
4         stk.pop_back();
5     }
6     stk.push_back(a[i]);
7 }

```

3.2 queue

```

1 std::deque<int> q;
2 for (int i = 1; i <= n; i++) {
3     while (!q.empty() and a[q.back()] >= a[i]) q.pop_back();
4     if (!q.empty() and i - q.front() >= k) q.pop_front();
5     q.push_back(i);
6 }

```

3.3 DSU

```

1 /* DSU */
2 vi fa(n + 1);
3 std::iota(all(fa), 0);
4 std::function<int(int)> find = [&] (int x) -> int{
5     return x == fa[x] ? x : fa[x] = find(fa[x]);
6 };
7 auto merge = [&] (int x, int y) -> void{
8     x = find(x), y = find(y);
9     if (x == y) return;
10    // operations //
11    fa[y] = x;
12 };

```

3.4 spare table

一维

```

1 /* spare table */
2 int B = 30;
3 vvi f(n + 1, vi(B));
4 vi Log2(n + 1);
5 auto init = [&]() -> void {
6     for (int i = 1; i <= n; i++) {
7         f[i][0] = a[i];
8         if (i > 1) Log2[i] = Log2[i / 2] + 1;
9     }
10    int t = Log2[n];
11    for (int j = 1; j <= t; j++) {
12        for (int i = 1; i <= n - (1 << j) + 1; i++) {
13            f[i][j] = std::max(f[i][j - 1], f[i + (1 << (j - 1))][j - 1]);
14        }
15    }
16 };
17 init();
18 auto query = [&](int l, int r) -> int {
19     int t = Log2[r - l + 1];
20     return std::max(f[l][t], f[r - (1 << t) + 1][t]);
21 };

```

二维

```

1  /* spare table */
2  intB = 30;
3  std::vector f(n + 1, std::vector<std::array<std::array<int, B>, B>>(m + 1));
4  vi Log2(n + 1);
5  auto init = [&]() -> void {
6      for (int i = 2; i <= std::max(n, m); i++) {
7          Log2[i] = Log2[i / 2] + 1;
8      }
9      for (int i = 2; i <= n; i++) {
10         for (int j = 2; j <= m; j++) {
11             f[i][j][0][0] = a[i][j];
12         }
13     }
14     for (int ki = 0; ki <= Log2[n]; ki++) {
15         for (int kj = 0; kj <= Log2[n]; kj++) {
16             if (!ki && !kj) continue;
17             for (int i = 1; i <= n - (1 << ki) + 1; i++) {
18                 for (int j = 1; j <= m - (1 << kj) + 1; j++) {
19                     if (ki) {
20                         f[i][j][ki][kj] =
21                             std::max(f[i][j][ki - 1][kj], f[i + (1 << (ki - 1))][j][ki - 1][kj]);
22                     } else {
23                         f[i][j][ki][kj] =
24                             std::max(f[i][j][ki][kj - 1], f[i][j + (1 << (kj - 1))][ki][kj - 1]);
25                     }
26                 }
27             }
28         }
29     }
30 };
31 init();
32 auto query = [&](int x1, int y1, int x2, int y2) -> int {
33     int ki = Log2[x2 - x1 + 1], kj = Log2[y2 - y1 + 1];
34     int t1 = f[x1][y1][ki][kj];
35     int t2 = f[x2 - (1 << ki) + 1][y1][ki][kj];
36     int t3 = f[x1][y2 - (1 << kj) + 1][ki][kj];
37     int t4 = f[x2 - (1 << ki) + 1][y2 - (1 << kj) + 1][ki][kj];
38     return std::max({t1, t2, t3, t4});
39 };

```

3.5 Cartesian tree

一种特殊的平衡树, 用元素的值作为平衡点节点的 *val*, 元素的下标作为 *key*.

```

1  /* cartesian tree */
2  vi ls(n + 1), rs(n + 1), stk(n + 1);
3  int top = 1;
4  for (int i = 1; i <= n; i++) {
5      int k = top;
6      while (k and a[stk[k]] > a[i]) k--;
7      if (k) rs[stk[k]] = i;
8      if (k < top) ls[i] = stk[k + 1];
9      stk[++k] = i;
10     top = k;
11 }

```

3.6 segment tree

```

1  /* segment tree @ czr */
2  const int N = 100010;
3  struct node {
4      int l, r;
5      ll sum, maxn, add, set;
6      bool addflag, setflag;
7  } tr[N << 2];
8
9  void push_up(int u) {
10     tr[u].sum = tr[u << 1].sum + tr[u << 1 | 1].sum;
11     tr[u].maxn = max(tr[u << 1].maxn, tr[u << 1 | 1].maxn);
12 }
13 // 0 4 0 0 0
14 // 2 6 2 2 2
15 // 2 6 2 4 4
16
17 void push_down(int u) {
18     auto& root = tr[u], &left = tr[u << 1], &right = tr[u << 1 | 1];
19     if (root.setflag) {

```

```

20     assert(!root.addflag);
21     left.add = 0, left.set = root.set, left.addflag = false, left.setflag = true;
22     right.add = 0, right.set = root.set, right.addflag = false, right.setflag = true;
23     left.sum = root.set * (left.r - left.l + 1);
24     right.sum = root.set * (right.r - right.l + 1);
25     left.maxn = root.set;
26     right.maxn = root.set;
27     root.set = 0, root.setflag = false;
28 }
29 if (root.addflag) {
30     assert(!root.setflag);
31     if (left.setflag) left.set += root.add;
32     else left.add += root.add, left.addflag = true;
33     if (right.setflag) right.set += root.add;
34     else right.add += root.add, right.addflag = true;
35
36     left.sum += root.add * (left.r - left.l + 1);
37     right.sum += root.add * (right.r - right.l + 1);
38     left.maxn += root.add;
39     right.maxn += root.add;
40     root.add = 0, root.addflag = false;
41 }
42 assert(root.add == 0);
43 }
44
45 void build(int u, int l, int r, vector<ll>& a) {
46     if (l == r) {
47         tr[u].l = tr[u].r = l, tr[u].sum = tr[u].maxn = a[l];
48         tr[u].add = 0, tr[u].set = 0;
49         tr[u].addflag = tr[u].setflag = false;
50     } else {
51         tr[u].l = l, tr[u].r = r, tr[u].add = 0, tr[u].set = 0;
52         tr[u].addflag = tr[u].setflag = false;
53         int mid = l + r >> 1;
54         build(u << 1, l, mid, a);
55         build(u << 1 | 1, mid + 1, r, a);
56         push_up(u);
57     }
58 }
59
60 // 区间加
61 void modify(int u, int l, int r, ll d) {
62     if (l > r) return;
63     if (tr[u].l >= l && tr[u].r <= r) {
64         if (tr[u].setflag) tr[u].set += d;
65         else tr[u].add += d, tr[u].addflag = true;
66         tr[u].sum += d * (tr[u].r - tr[u].l + 1);
67         tr[u].maxn += d;
68     } else {
69         push_down(u);
70         int mid = tr[u].l + tr[u].r >> 1;
71         if (l <= mid) modify(u << 1, l, r, d);
72         if (r > mid) modify(u << 1 | 1, l, r, d);
73         push_up(u);
74     }
75 }
76
77 // 区间赋值
78 void update(int u, int l, int r, ll x) {
79     if (l > r) return;
80     if (tr[u].l >= l && tr[u].r <= r) {
81         tr[u].set = x, tr[u].setflag = true;
82         tr[u].add = 0, tr[u].addflag = false;
83         tr[u].sum = x * (tr[u].r - tr[u].l + 1);
84         tr[u].maxn = x;
85     } else {
86         push_down(u);
87         int mid = tr[u].l + tr[u].r >> 1;
88         if (l <= mid) update(u << 1, l, r, x);
89         if (r > mid) update(u << 1 | 1, l, r, x);
90         push_up(u);
91     }
92 }
93
94 ll query_sum(int u, int l, int r) {
95     if (l > r) return 0;
96     if (tr[u].l >= l && tr[u].r <= r) return tr[u].sum;
97     else {
98         ll res = 0;
99         push_down(u);
100         int mid = tr[u].l + tr[u].r >> 1;
101         if (l <= mid) res += query_sum(u << 1, l, r);
102         if (r > mid) res += query_sum(u << 1 | 1, l, r);
103         return res;
104     }
105 }
106

```

```

107 ll query_maxn(int u, int l, int r) {
108     if (l > r) return -1e18;
109     if (tr[u].l >= l && tr[u].r <= r) return tr[u].maxn;
110     else {
111         ll res = -1e18;
112         push_down(u);
113         int mid = tr[u].l + tr[u].r >> 1;
114         if (l <= mid) res = max(res, query_maxn(u << 1, l, r));
115         if (r > mid) res = max(res, query_maxn(u << 1 | 1, l, r));
116         return res;
117     }
118 }
119
120 // 找到最小 i 使得 sum(l, i) >= k
121 ll find_presum_idx(int u, int l, int r, int x) {
122     if (tr[u].l == tr[u].r) return tr[u].l;
123     else {
124         push_down(u);
125         int mid = tr[u].l + tr[u].r >> 1;
126         if (r <= mid) {
127             return find_presum_idx(u << 1, l, r, x);
128         } else if (l > mid) {
129             return find_presum_idx(u << 1 | 1, l, r, x);
130         } else {
131             ll lsum = query_sum(u << 1, l, r);
132             if (lsum >= x) return find_presum_idx(u << 1, l, mid, x);
133             else return find_presum_idx(u << 1 | 1, mid + 1, r, x - lsum);
134         }
135     }
136 }

```

3.7 segment tree split

```

1  /* segment tree split @ wrb */
2  #include<bits/stdc++.h>
3  using namespace std;
4  namespace Acc{
5      using i64=int64_t;
6      enum{N=200009,M=10000000};
7      i64 v[M];
8      int lc[M],rc[M],tot,a[N],r[N];
9      auto up=[](int o){
10         v[o]=v[lc[o]]+v[rc[o]];
11     };
12     void bd(int&o,int l,int r){
13         if(o==++tot,l==r)return cin>>v[o],void();
14         int md=l+r>>1;
15         bd(lc[o],l,md),bd(rc[o],md+1,r),up(o);
16     }
17     void spl(int&o,int&x,int l,int r,int L,int R){
18         if(l<=L&&R<=r)return o=x,x=0,void();
19         int md=L+R>>1;
20         o=++tot;
21         if(l<=md)spl(lc[o],lc[x],l,r,L,md);
22         if(r>md)spl(rc[o],rc[x],l,r,md+1,R);
23         up(o),up(x);
24     }
25     void mg(int&o,int x,int l,int r){
26         if(!o||!x)return o|=x,void();
27         if(l==r)return v[o]+=v[x],void();
28         int md=l+r>>1;
29         mg(lc[o],lc[x],l,md);
30         mg(rc[o],rc[x],md+1,r);
31         up(o);
32     }
33     void ins(int&o,int l,int r,int x,int k){
34         if(!o)o=++tot;
35         if(v[o]==k,l==r)return;
36         int md=l+r>>1;
37         x<=md?ins(lc[o],l,md,x,k):ins(rc[o],md+1,r,x,k);
38     }
39     i64 qry(int o,int l,int r,int L,int R){
40         if(!o)return 0;
41         if(l<=L&&R<=r)return v[o];
42         int md=L+R>>1;i64 z=0;
43         if(l<=md)z=qry(lc[o],l,r,L,md);
44         if(r>md)z+=qry(rc[o],l,r,md+1,R);
45         return z;
46     }
47     int kth(int o,int l,int r,int k){
48         if(l==r)return l;
49         if(k>v[o])return -1;

```

```

50     int md=l+r>>1;
51     if(k<=v[lc[o]])return kth(lc[o],l,md,k);
52     else return kth(rc[o],md+1,r,k-v[lc[o]]);
53 }
54 auto work=[]() {
55     int n,m,i,x,y,o=1;
56     for(cin>>n>>m,bd(r[1],1,n);m--;)switch(cin>>i,i){
57         case 0:cin>>i>>x>>y,spl(r[++o],r[i],x,y,1,n);break;
58         case 1:cin>>x>>y,mg(r[x],r[y],1,n);break;
59         case 2:cin>>i>>x>>y,ins(r[i],1,n,y,x);break;
60         case 3:cin>>i>>x>>y,cout<<qry(r[i],x,y,1,n)<<'\\n';break;
61         case 4:cin>>i>>x,cout<<kth(r[i],1,n,x)<<'\\n';break;
62     }
63 };
64 }
65 int main(){
66     ios::sync_with_stdio(0);
67     cin.tie(0),Acc::work();
68 }

```

3.8 persistent segment tree

单点修改，版本拷贝

n 个数， m 次操作，操作分别为

1. v_i 1 loc_i $value_i$: 将第 v_i 个版本的 $a[loc_i]$ 修改为 $value_i$,
2. v_i 2 loc_i : 拷贝第 v_i 个版本，并查询该版本的 $a[loc_i]$.

```

1 // 洛谷 P3919 【模板】可持久化线段树 1 (可持久化数组)
2
3 struct node {
4     int l, r, key;
5 };
6
7 int main() {
8     std::ios::sync_with_stdio(false);
9     std::cin.tie(0);
10    std::cout.tie(0);
11
12    int n, m;
13    std::cin >> n >> m;
14    vi a(n + 1);
15    for (int i = 1; i <= n; i++) {
16        std::cin >> a[i];
17    }
18
19    /* hjt segment tree */
20    int idx = 0;
21    vi root(m + 1);
22    std::vector<node> tr(n * 25);
23
24    std::function<int(int, int)> build = [&](int l, int r) -> int {
25        int p = ++idx;
26        if (l == r) {
27            tr[p].key = a[l];
28            return p;
29        }
30        int mid = (l + r) >> 1;
31        tr[p].l = build(l, mid);
32        tr[p].r = build(mid + 1, r);
33        return p;
34    };
35
36    std::function<int(int, int, int, int, int)> modify = [&](int p, int l, int r, int k,
37                                                            int x) -> int {
38        int q = ++idx;
39        tr[q].l = tr[p].l, tr[q].r = tr[p].r;
40        if (tr[q].l == tr[q].r) {
41            tr[q].key = x;
42            return q;
43        }
44        int mid = (l + r) >> 1;
45        if (k <= mid) {
46            tr[q].l = modify(tr[q].l, l, mid, k, x);
47        } else {

```

```

48     tr[q].r = modify(tr[q].r, mid + 1, r, k, x);
49 }
50 return q;
51 };
52
53 std::function<int(int, int, int, int)> query = [&](int p, int l, int r, int k) -> int {
54     if (tr[p].l == tr[p].r) {
55         return tr[p].key;
56     }
57     int mid = (l + r) >> 1;
58     if (k <= mid) {
59         return query(tr[p].l, l, mid, k);
60     } else {
61         return query(tr[p].r, mid + 1, r, k);
62     }
63 };
64
65 root[0] = build(1, n);
66
67 for (int i = 1; i <= m; i++) {
68     int op, ver, k, x;
69     std::cin >> ver >> op;
70     if (op == 1) {
71         std::cin >> k >> x;
72         root[i] = modify(root[ver], 1, n, k, x);
73     } else {
74         std::cin >> k;
75         root[i] = root[ver];
76         std::cout << query(root[ver], 1, n, k) << '\n';
77     }
78 }
79
80 return 0;
81 }

```

区间第 k 小

长度为 n 的序列 a , m 次查询, 每次查询 $[l, r]$ 中的第 k 小值.

```

1 // 洛谷P3834 【模板】可持久化线段树 2
2
3 struct node {
4     int l, r, cnt;
5 };
6
7 int main() {
8     std::ios::sync_with_stdio(false);
9     std::cin.tie(0);
10    std::cout.tie(0);
11
12    int n, m;
13    std::cin >> n >> m;
14    vi a(n + 1), v;
15    for (int i = 1; i <= n; i++) {
16        std::cin >> a[i];
17        v.push_back(a[i]);
18    }
19    std::sort(all(v));
20    v.erase(unique(all(v)), v.end());
21    auto find = [&](int x) -> int { return std::lower_bound(all(v), x) - v.begin() + 1; };
22
23    /* hjt segment tree */
24    std::vector<node>(n * 25);
25    vi root(n + 1);
26    int idx = 0;
27
28    std::function<int(int, int)> build = [&](int l, int r) -> int {
29        int p = ++idx;
30        if (l == r) return p;
31        int mid = (l + r) >> 1;
32        tr[p].l = build(l, mid), tr[p].r = build(mid + 1, r);
33        return p;
34    };
35
36    std::function<int(int, int, int, int)> modify = [&](int p, int l, int r, int x) -> int {
37        int q = ++idx;
38        tr[q] = tr[p];
39        if (tr[q].l == tr[q].r) {
40            tr[q].cnt++;
41            return q;
42        }
43        int mid = (l + r) >> 1;

```

```

44     if (x <= mid) {
45         tr[q].l = modify(tr[q].l, l, mid, x);
46     } else {
47         tr[q].r = modify(tr[q].r, mid + 1, r, x);
48     }
49     tr[q].cnt = tr[tr[q].l].cnt + tr[tr[q].r].cnt;
50     return q;
51 };
52
53 std::function<int(int, int, int, int, int)> query = [&](int p, int q, int l, int r,
54                                                     int x) -> int {
55     if (l == r) return l;
56     int cnt = tr[tr[p].l].cnt - tr[tr[q].l].cnt;
57     int mid = (l + r) >> 1;
58     if (x <= cnt) {
59         return query(tr[p].l, tr[q].l, l, mid, x);
60     } else {
61         return query(tr[p].r, tr[q].r, mid + 1, r, x - cnt);
62     }
63 };
64
65 root[0] = build(1, v.size());
66
67
68 for (int i = 1; i <= n; i++) {
69     root[i] = modify(root[i - 1], 1, v.size(), find(a[i]));
70 }
71 for (int i = 1; i <= m; i++) {
72     int l, r, k;
73     std::cin >> l >> r >> k;
74     std::cout << v[query(root[r], root[l - 1], 1, v.size(), k) - 1] << '\n';
75 }
76
77 return 0;
78 }

```

3.9 sweep line

```

1  /* sweep line @ czr */
2  struct Node {
3      int l, r;
4      ll sum, length, res;
5  } tr[N << 2];
6
7  void push_up(int u) {
8      tr[u].res = (tr[u << 1].res + tr[u << 1 | 1].res) % Mod;
9  }
10
11 void update_length(int u) {
12     if (tr[u].sum) {
13         tr[u].length = tr[u].res;
14     } else {
15         if (tr[u].l == tr[u].r) tr[u].length = 0;
16         else tr[u].length = (tr[u << 1].length + tr[u << 1 | 1].length) % Mod;
17     }
18 }
19
20 void build(int u, int l, int r) {
21     if (l == r) tr[u] = {l, r, 0, 0, 0};
22     else {
23         tr[u] = {l, r, 0, 0, 0};
24         int mid = l + r >> 1;
25         build(u << 1, l, mid);
26         build(u << 1 | 1, mid + 1, r);
27         push_up(u);
28     }
29 }
30
31 void modify(int u, int l, int r, int op) {
32     if (tr[u].l >= l && tr[u].r <= r) {
33         tr[u].sum += op;
34         update_length(u);
35     } else {
36         int mid = tr[u].l + tr[u].r >> 1;
37         if (l <= mid) modify(u << 1, l, r, op);
38         if (r > mid) modify(u << 1 | 1, l, r, op);
39         push_up(u);
40         update_length(u);
41     }
42 }
43
44 void change(int u, int x, ll d) {

```

```

45 |     if (tr[u].l == tr[u].r) {
46 |         tr[u].res = (tr[u].res + d) % Mod;
47 |         update_length(u);
48 |     } else {
49 |         int mid = tr[u].l + tr[u].r >> 1;
50 |         if (x <= mid) change(u << 1, x, d);
51 |         else change(u << 1 | 1, x, d);
52 |         push_up(u);
53 |         update_length(u);
54 |     }
55 | }

```

```

1  /* sweep line @ wrb */
2  #define int long long
3  const int N = 2e5+10;
4  int b[N<<1], n, len, ans;
5  struct node{
6      int y1, y2, x, k;
7  } a[N<<1];
8  struct Seg{
9      #define lc (o<<1)
10     #define rc (o<<1|1)
11     static const int N = 5e6+10;
12     int sum[N], cnt[N], tag[N];
13     void push_up(int o, int l, int r){
14         if(sum[o]) cnt[o] = b[r+1] - b[l];
15         else cnt[o] = cnt[lc] + cnt[rc];
16     }
17     void add(int o, int l, int r, int L, int R, int k){
18         if(r < L || l > R) return;
19         if(l == L && r == R) return (void)(sum[o] += k, push_up(o, l, r));
20         int mid = L + R >> 1;
21         if(r <= mid) add(lc, l, r, L, mid, k);
22         else if(l > mid) add(rc, l, r, mid+1, R, k);
23         else add(lc, l, mid, L, mid, k), add(rc, mid+1, r, mid+1, R, k);
24         push_up(o, L, R);
25     }
26 #undef lc
27 #undef rc
28 } t;
29 void work(){
30     cin >> n;
31     for(int i=1, x1, y1, x2, y2; i<=n; i++){
32         cin >> x1 >> y1 >> x2 >> y2;
33         b[i*2-1] = y1, b[i*2] = y2, a[i*2-1] = {y1, y2, x1, 1}, a[i*2] = {y1, y2, x2, -1};
34     }
35     n <<= 1;
36     sort(b+1, b+n+1), len = unique(b+1, b+n+1) - b - 1;
37     for(int i=1; i<=n; i++) a[i].y1 = lower_bound(b+1, b+len+1, a[i].y1) - b, a[i].y2 = lower_bound(b+1, b+len+1, a[i].y2) - b;
38     sort(a+1, a+n+1, [](node a, node b) -> bool {return a.x < b.x;});
39     for(int i=1; i<=n; i++){
40         t.add(1, a[i].y1, a[i].y2-1, 1, len-1, a[i].k);
41         ans += t.cnt[1] * (a[i+1].x - a[i].x);
42     }
43     cout << ans;
44 }
45 #undef int

```

3.10 treap

fhq treap

n 次操作, 操作分为如下 6 种:

1. 插入数 x ;
2. 删除数 x (若有多个相同的数, 只删除一个);
3. 查询数 x 的排名 (排名定义为小于 x 的数的个数 + 1);
4. 查询排名为 x 的数;
5. 求 x 的前驱 (前驱定义为小于 x 的最大数);

6. 求 x 的后继 (后继定义为大于 x 的最小数).

```

1 struct node {
2     node *ch[2];
3     int key, val;
4     int cnt, size;
5
6     node(int _key) : key(_key), cnt(1), size(1) {
7         ch[0] = ch[1] = nullptr;
8         val = rand();
9     }
10
11     // node(node *_node) {
12     //     key = _node->key, val = _node->val, cnt = _node->cnt, size = _node->size;
13     // }
14
15     inline void push_up() {
16         size = cnt;
17         if (ch[0] != nullptr) size += ch[0]->size;
18         if (ch[1] != nullptr) size += ch[1]->size;
19     }
20 };
21
22 struct treap {
23     #define _2 second.first
24     #define _3 second.second
25
26     node *root;
27
28     pair<node *, node *> split(node *p, int key) {
29         if (p == nullptr) return {nullptr, nullptr};
30         if (p->key <= key) {
31             auto temp = split(p->ch[1], key);
32             p->ch[1] = temp.first;
33             p->push_up();
34             return {p, temp.second};
35         } else {
36             auto temp = split(p->ch[0], key);
37             p->ch[0] = temp.second;
38             p->push_up();
39             return {temp.first, p};
40         }
41     }
42
43     pair<node *, pair<node *, node *> > split_by_rank(node *p, int rank) {
44         if (p == nullptr) return {nullptr, {nullptr, nullptr}};
45         int ls_size = p->ch[0] == nullptr ? 0 : p->ch[0]->size;
46         if (rank <= ls_size) {
47             auto temp = split_by_rank(p->ch[0], rank);
48             p->ch[0] = temp._3;
49             p->push_up();
50             return {temp.first, {temp._2, p}};
51         } else if (rank <= ls_size + p->cnt) {
52             node *lt = p->ch[0];
53             node *rt = p->ch[1];
54             p->ch[0] = p->ch[1] = nullptr;
55             return {lt, {p, rt}};
56         } else {
57             auto temp = split_by_rank(p->ch[1], rank - ls_size - p->cnt);
58             p->ch[1] = temp.first;
59             p->push_up();
60             return {p, {temp._2, temp._3}};
61         }
62     }
63
64     node *merge(node *u, node *v) {
65         if (u == nullptr && v == nullptr) return nullptr;
66         if (u != nullptr && v == nullptr) return u;
67         if (v != nullptr && u == nullptr) return v;
68         if (u->val < v->val) {
69             u->ch[1] = merge(u->ch[1], v);
70             u->push_up();
71             return u;
72         } else {
73             v->ch[0] = merge(u, v->ch[0]);
74             v->push_up();
75             return v;
76         }
77     }
78
79     void insert(int key) {
80         auto temp = split(root, key);
81         auto l_tr = split(temp.first, key - 1);
82         node *new_node;
83         if (l_tr.second == nullptr) {
84             new_node = new node(key);

```

```

85     } else {
86         l_tr.second->cnt++;
87         l_tr.second->push_up();
88     }
89     node *l_tr_combined = merge(l_tr.first, l_tr.second == nullptr ? new_node : l_tr.second);
90     root = merge(l_tr_combined, temp.second);
91 }
92
93 void remove(int key) {
94     auto temp = split(root, key);
95     auto l_tr = split(temp.first, key - 1);
96     if (l_tr.second->cnt > 1) {
97         l_tr.second->cnt--;
98         l_tr.second->push_up();
99         l_tr.first = merge(l_tr.first, l_tr.second);
100    } else {
101        if (temp.first == l_tr.second) temp.first = nullptr;
102        delete l_tr.second;
103        l_tr.second = nullptr;
104    }
105    root = merge(l_tr.first, temp.second);
106 }
107
108 int get_rank_by_key(node *p, int key) {
109     auto temp = split(p, key - 1);
110     int ret = (temp.first == nullptr ? 0 : temp.first->size) + 1;
111     root = merge(temp.first, temp.second);
112     return ret;
113 }
114
115 int get_key_by_rank(node *p, int rank) {
116     auto temp = split_by_rank(p, rank);
117     int ret = temp._2->key;
118     root = merge(temp.first, merge(temp._2, temp._3));
119     return ret;
120 }
121
122 int get_prev(int key) {
123     auto temp = split(root, key - 1);
124     int ret = get_key_by_rank(temp.first, temp.first->size);
125     root = merge(temp.first, temp.second);
126     return ret;
127 }
128
129 int get_nex(int key) {
130     auto temp = split(root, key);
131     int ret = get_key_by_rank(temp.second, 1);
132     root = merge(temp.first, temp.second);
133     return ret;
134 }
135 };
136
137 treap tr;
138
139 int main() {
140     ios::sync_with_stdio(false);
141     cin.tie(0);
142     cout.tie(0);
143
144     srand(time(0));
145
146     int n;
147     cin >> n;
148     while (n-- > 0) {
149         int op, x;
150         cin >> op >> x;
151         if (op == 1) {
152             tr.insert(x);
153         } else if (op == 2) {
154             tr.remove(x);
155         } else if (op == 3) {
156             cout << tr.get_rank_by_key(tr.root, x) << '\n';
157         } else if (op == 4) {
158             cout << tr.get_key_by_rank(tr.root, x) << '\n';
159         } else if (op == 5) {
160             cout << tr.get_prev(x) << '\n';
161         } else {
162             cout << tr.get_nex(x) << '\n';
163         }
164     }
165     return 0;
166 }

```

用 01 trie 实现的一种方式

同样的题目, 注意使用 01 trie 只能存在非负数. 速度能快不少, 但只能单点操作, 而且有点费空间.

```

1 // 洛谷 P3369 【模板】普通平衡树
2
3 struct Treap {
4     int id = 1, maxlog = 25;
5     int ch[N * 25][2], siz[N * 25];
6
7     int newnode() {
8         id++;
9         ch[id][0] = ch[id][1] = siz[id] = 0;
10        return id;
11    }
12
13    void merge(int key, int cnt) {
14        int u = 1;
15        for (int i = maxlog - 1; i >= 0; i--) {
16            int v = (key >> i) & 1;
17            if (!ch[u][v]) ch[u][v] = newnode();
18            u = ch[u][v];
19            siz[u] += cnt;
20        }
21    }
22
23    int get_key_by_rank(int rank) {
24        int u = 1, key = 0;
25        for (int i = maxlog - 1; i >= 0; i--) {
26            if (siz[ch[u][0]] >= rank) {
27                u = ch[u][0];
28            } else {
29                key |= (1 << i);
30                rank -= siz[ch[u][0]];
31                u = ch[u][1];
32            }
33        }
34        return key;
35    }
36
37    int get_rank_by_key(int rank) {
38        int key = 0;
39        int u = 1;
40        for (int i = maxlog - 1; i >= 0; i--) {
41            if ((rank >> i) & 1) {
42                key += siz[ch[u][0]];
43                u = ch[u][1];
44            } else {
45                u = ch[u][0];
46            }
47            if (!u) break;
48        }
49        return key;
50    }
51
52    int get_prev(int x) { return get_key_by_rank(get_rank_by_key(x)); }
53    int get_next(int x) { return get_key_by_rank(get_rank_by_key(x + 1) + 1); }
54 } treap;
55
56 const int num = 1e7;
57 int n, op, x;
58
59 int main() {
60     std::ios::sync_with_stdio(false);
61     std::cin.tie(0);
62     std::cout.tie(0);
63
64     std::cin >> n;
65     for (int i = 1; i <= n; i++) {
66         std::cin >> op >> x;
67         if (op == 1) {
68             treap.merge(x + num, 1);
69         } else if (op == 2) {
70             treap.merge(x + num, -1);
71         } else if (op == 3) {
72             std::cout << treap.get_rank_by_key(x + num) + 1 << '\n';
73         } else if (op == 4) {
74             std::cout << treap.get_key_by_rank(x) - num << '\n';
75         } else if (op == 5) {
76             std::cout << treap.get_prev(x + num) - num << '\n';
77         } else if (op == 6) {
78             std::cout << treap.get_next(x + num) - num << '\n';
79         }
80     }
81     return 0;

```

82 | }

3.11 splay

文艺平衡树

初始为 1 到 n 的序列, m 次操作, 每次将序列下标为 $[l \sim r]$ 的区间翻转.

```

1 // 洛谷 P3391 【模板】文艺平衡树
2
3 struct node {
4     int ch[2], fa, key;
5     int siz, flag;
6
7     void init(int _fa, int _key) { fa = _fa, key = _key, siz = 1; }
8 };
9
10 struct splay {
11     node tr[N];
12     int n, root, idx;
13
14     bool get(int u) { return u == tr[tr[u].fa].ch[1]; }
15
16     void pushup(int u) { tr[u].siz = tr[tr[u].ch[0]].siz + tr[tr[u].ch[1]].siz + 1; }
17
18     void pushdown(int u) {
19         if (tr[u].flag) {
20             std::swap(tr[u].ch[0], tr[u].ch[1]);
21             tr[tr[u].ch[0]].flag ^= 1, tr[tr[u].ch[1]].flag ^= 1;
22             tr[u].flag = 0;
23         }
24     }
25
26     void rotate(int x) {
27         int y = tr[x].fa, z = tr[y].fa;
28         int op = get(x);
29         tr[y].ch[op] = tr[x].ch[op ^ 1];
30         if (tr[x].ch[op ^ 1]) tr[tr[x].ch[op ^ 1]].fa = y;
31         tr[x].ch[op ^ 1] = y;
32         tr[y].fa = x, tr[x].fa = z;
33         if (z) tr[z].ch[y == tr[z].ch[1]] = x;
34         pushup(y), pushup(x);
35     }
36
37     void opt(int u, int k) {
38         for (int f = tr[u].fa; f = tr[u].fa, f != k; rotate(u)) {
39             if (tr[f].fa != k) rotate(get(u) == get(f) ? f : u);
40         }
41         if (k == 0) root = u;
42     }
43
44     void output(int u) {
45         pushdown(u);
46         if (tr[u].ch[0]) output(tr[u].ch[0]);
47         if (tr[u].key >= 1 && tr[u].key <= n) {
48             std::cout << tr[u].key << ' ';
49         }
50         if (tr[u].ch[1]) output(tr[u].ch[1]);
51     }
52
53     void insert(int key) {
54         idx++;
55         tr[idx].ch[0] = root;
56         tr[idx].init(0, key);
57         tr[root].fa = idx;
58         root = idx;
59         pushup(idx);
60     }
61
62     int kth(int k) {
63         int u = root;
64         while (1) {
65             pushdown(u);
66             if (tr[u].ch[0] && k <= tr[tr[u].ch[0]].siz) {
67                 u = tr[u].ch[0];
68             } else {
69                 k -= tr[tr[u].ch[0]].siz + 1;
70                 if (k <= 0) {
71                     opt(u, 0);
72                     return u;
73                 } else {

```

```

74         u = tr[u].ch[1];
75     }
76 }
77 }
78 }
79
80 } splay;
81
82 int n, m, l, r;
83
84 int main() {
85     std::ios::sync_with_stdio(false);
86     std::cin.tie(0);
87     std::cout.tie(0);
88
89     std::cin >> n >> m;
90     splay.n = n;
91     splay.insert(-inf);
92     rep(i, 1, n) splay.insert(i);
93     splay.insert(inf);
94     rep(i, 1, m) {
95         std::cin >> l >> r;
96         l = splay.kth(l), r = splay.kth(r + 2);
97         splay.opt(l, 0), splay.opt(r, 1);
98         splay.tr[splay.tr[r].ch[0]].flag ^= 1;
99     }
100     splay.output(splay.root);
101
102     return 0;
103 }

```

普通平衡树

```

1 // 洛谷 P3369 【模板】普通平衡树
2
3 struct node {
4     int ch[2], fa, key, siz, cnt;
5
6     void init(int _fa, int _key) { fa = _fa, key = _key, siz = cnt = 1; }
7
8     void clear() { ch[0] = ch[1] = fa = key = siz = cnt = 0; }
9 };
10
11 struct splay {
12     node tr[N];
13     int n, root, idx;
14
15     bool get(int u) { return u == tr[tr[u].fa].ch[1]; }
16
17     void pushup(int u) { tr[u].siz = tr[tr[u].ch[0]].siz + tr[tr[u].ch[1]].siz + tr[u].cnt; }
18
19     void rotate(int x) {
20         int y = tr[x].fa, z = tr[y].fa;
21         int op = get(x);
22         tr[y].ch[op] = tr[x].ch[op ^ 1];
23         if (tr[x].ch[op ^ 1]) tr[tr[x].ch[op ^ 1]].fa = y;
24         tr[x].ch[op ^ 1] = y;
25         tr[y].fa = x, tr[x].fa = z;
26         if (z) tr[z].ch[y == tr[z].ch[1]] = x;
27         pushup(y), pushup(x);
28     }
29
30     void opt(int u, int k) {
31         for (int f = tr[u].fa; f = tr[u].fa, f != k; rotate(u)) {
32             if (tr[f].fa != k) {
33                 rotate(get(u) == get(f) ? f : u);
34             }
35         }
36         if (k == 0) root = u;
37     }
38
39     void insert(int key) {
40         if (!root) {
41             idx++;
42             tr[idx].init(0, key);
43             root = idx;
44             return;
45         }
46         int u = root, f = 0;
47         while (1) {
48             if (tr[u].key == key) {
49                 tr[u].cnt++;
50                 pushup(u), pushup(f);

```

```

51         opt(u, 0);
52         break;
53     }
54     f = u, u = tr[u].ch[tr[u].key < key];
55     if (!u) {
56         idx++;
57         tr[idx].init(f, key);
58         tr[f].ch[tr[f].key < key] = idx;
59         pushup(idx), pushup(f);
60         opt(idx, 0);
61         break;
62     }
63 }
64 }
65
66 // 返回节点编号 //
67 int kth(int rank) {
68     int u = root;
69     while (1) {
70         if (tr[u].ch[0] && rank <= tr[tr[u].ch[0]].siz) {
71             u = tr[u].ch[0];
72         } else {
73             rank -= tr[tr[u].ch[0]].siz + tr[u].cnt;
74             if (rank <= 0) {
75                 opt(u, 0);
76                 return u;
77             } else {
78                 u = tr[u].ch[1];
79             }
80         }
81     }
82 }
83
84 // 返回排名 //
85 int nlt(int key) {
86     int rank = 0, u = root;
87     while (1) {
88         if (tr[u].key > key) {
89             u = tr[u].ch[0];
90         } else {
91             rank += tr[tr[u].ch[0]].siz;
92             if (tr[u].key == key) {
93                 opt(u, 0);
94                 return rank + 1;
95             }
96             rank += tr[u].cnt;
97             if (tr[u].ch[1]) {
98                 u = tr[u].ch[1];
99             } else {
100                 return rank + 1;
101             }
102         }
103     }
104 }
105
106 int get_prev(int key) { return kth(nlt(key) - 1); }
107
108 int get_next(int key) { return kth(nlt(key) + 1); }
109
110 void remove(int key) {
111     nlt(key);
112     if (tr[root].cnt > 1) {
113         tr[root].cnt--;
114         pushup(root);
115         return;
116     }
117     int u = root, l = get_prev(key);
118     tr[tr[u].ch[1]].fa = l;
119     tr[l].ch[1] = tr[u].ch[1];
120     tr[u].clear();
121     pushup(root);
122 }
123
124 void output(int u) {
125     if (tr[u].ch[0]) output(tr[u].ch[0]);
126     std::cout << tr[u].key << ' ';
127     if (tr[u].ch[1]) output(tr[u].ch[1]);
128 }
129
130 } splay;
131
132 int n, op, x;
133
134 int main() {
135     std::ios::sync_with_stdio(false);
136     std::cin.tie(0);
137     std::cout.tie(0);

```

```

138
139 splay.insert(-inf), splay.insert(inf);
140
141 std::cin >> n;
142 for (int i = 1; i <= n; i++) {
143     std::cin >> op >> x;
144     if (op == 1) {
145         splay.insert(x);
146     } else if (op == 2) {
147         splay.remove(x);
148     } else if (op == 3) {
149         std::cout << splay.nlt(x) - 1 << endl;
150     } else if (op == 4) {
151         std::cout << splay.tr[splay.kth(x + 1)].key << endl;
152     } else if (op == 5) {
153         std::cout << splay.tr[splay.get_prev(x)].key << endl;
154     } else if (op == 6) {
155         std::cout << splay.tr[splay.get_next(x)].key << endl;
156     }
157 }
158
159 return 0;
160 }

```

3.12 link cut tree

```

1  /* link cut tree @ wrb */
2  struct LCT{
3      int v[N], r[N], f[N], s[N][2], st[N], tp;
4      void pu(int x){v[x]=a[x]^v[s[x][0]]^v[s[x][1]};}
5      void flp(int x){r[x]^=1, std::swap(s[x][0], s[x][1]);}
6      void pd(int x){if(r[x])flp(s[x][0]), flp(s[x][1]), r[x]=0;}
7      bool isrt(int x){return s[f[x]][0]!=x&&s[f[x]][1]!=x;}
8      void rtt(int x){
9          int y=f[x], z=f[y], k=(s[y][1]==x); if(!isrt(y)) s[z][y==s[z][1]]=x;
10         f[x]=z, f[y]=x, f[s[x][k^1]]=y, s[y][k]=s[x][k^1], s[x][k^1]=y, pu(y), pu(x);}
11     void spl(int x){
12         st[tp++]=x; for(int i=x; !isrt(i); i=f[i]) st[tp++]=f[i];
13         while(tp)pd(st[--tp]);
14         while(!isrt(x)){
15             if(!isrt(f[x]))rtt((s[f[x]][0]==x)^(s[f[f[x]]][0]==f[x])?x:f[x]);
16             rtt(x);
17         }
18     }
19     void acc(int x){for(int y=0; x=y, x=f[x]) spl(x), s[x][1]=y, pu(x);}
20     void mkrt(int x){acc(x), spl(x), flp(x);}
21     int fdrt(int x){acc(x), spl(x); while(s[x][0])x=s[x][0]; spl(x); return x;}
22     void cut(int x, int y){mkrt(x); if(x==fdrt(y)&&f[y]==x&&!s[y][0])s[x][1]=f[y]=0, pu(x);}
23     void lk(int x, int y){mkrt(x); if(x!=fdrt(y))f[x]=y;}
24 }t;

```

3.13 Lichao tree

```

1  /* Lichao tree @ wrb */
2  #include<bits/stdc++.h>
3  using namespace std;
4  namespace Acc{
5      #define lc (o<<1)
6      #define rc (o<<1|1)
7      const int N = 4e5+10;
8      int v[N], n, l, r, z;
9      double k[N], b[N];
10     inline void r1(int&x){x=(x+z-1)%39989+1;}
11     inline void r2(int&x){x=(x+z-1)%1000000000+1;}
12     double f(int o, int x){
13         return k[o]*x+b[o];
14     }
15     int beat(int x, int a, int b){
16         double u=f(a, x), v=f(b, x);
17         return fabs(u-v)<=1e-8?a<b:u>v;
18     }
19     void add(int o, int L, int R, int x){
20         int md=L+R>>1;
21         if(L<=L&&R<=R){
22             if(!v[o])return (void)(v[o]=x);
23             if(beat(L, v[o], x) && beat(R, v[o], x))return;
24             if(beat(L, x, v[o]) && beat(R, x, v[o]))return (void)(v[o]=x);
25             if(beat(md, x, v[o]))swap(x, v[o]);

```

```

26         if(beat(L,x,v[o]))add(lc,L,md,x);
27         else add(rc,md+1,R,x);
28         return;
29     }
30     if(r>md)add(rc,md+1,R,x);
31     if(l<=md)add(lc,L,md,x);
32 }
33 int ask(int o,int L,int R){
34     if(L==R)return v[o];
35     int md=L+R>>1,h=l<=md?ask(lc,L,md):ask(rc,md+1,R);
36     return beat(l,h,v[o])?h:v[o];
37 }
38 void work(){
39     cin>>n;
40     for(int op,y1,y2,c=0;n--;){
41         cin>>op;
42         if(op){
43             cin>>l>>y1>>r>>y2,++c,r1(l),r2(y1),r1(r),r2(y2);
44             if(l==r)k[c]=0,b[c]=max(y1,y2);
45             else {
46                 if(l>r)swap(l,r),swap(y1,y2);
47                 k[c]=(y2-y1+0.)/(r-l),b[c]=y1-k[c]*l;
48             }
49             add(1,1,4e4+10,c);
50         }else cin>>l,r1(l),cout<<(z=ask(1,1,4e4+10))<<'\\n';
51     }
52 }
53 }
54 int main(){
55     return Acc::work(),0;
56 }

```

3.14 ODT

```

1  /* ODT @ wrb */
2  struct T{
3      int l,r,v;
4      T(int a,int b=-1,int c=-1):l(a),r(b),v(c){}
5      bool operator<(const T&_)const{return l<_.l;}
6  };
7  set<T>s;
8  auto spl(int p){
9      auto it=s.lower_bound(p);
10     if(it!=end(s) && it->l==p)return it;
11     --it;
12     int l=it->l,r=it->r,v=it->v;
13     s.erase(it),s.insert(T(l,p-1,v));
14     return s.insert(T(p,r,v)).first;
15 }
16 void asgn(int l,int r,int v){
17     auto ed=spl(r+1),bg=spl(l);
18     s.erase(bg,ed);
19     auto i=s.insert(T(l,r,v)).first,j=prev(i);
20     if(i!=begin(s)&&j->v==v)l=j->l,s.erase(j);
21     if((j=next(i))!=end(s)&&j->v==v)r=j->r,s.erase(j);
22     s.erase(i),s.insert(T(l,r,v));
23 }

```

4 string

4.1 kmp

```

1  /* kmp */
2  auto kmp = [&](const std::string& s) -> vi {
3      int n = s.length();
4      vi next(n);
5      for (int i = 1; i < n; i++) {
6          int j = next[i - 1];
7          while (j > 0 and s[i] != s[j]) j = next[j - 1];
8          if (s[i] == s[j]) j++;
9          next[i] = j;
10     }
11     return next;
12 };

```


4.2 z function

```

1  /* exkmp */
2  auto exkmp = [&](const std::string& s) -> vi {
3      int n = s.size();
4      vi z(n);
5      for (int i = 1, l = 0, r = 0; i < n; i++) {
6          if (i <= r and z[i - l] < r - i + 1) {
7              z[i] = z[i - l];
8          } else {
9              z[i] = std::max(0, r - i + 1);
10             while (z[i] + i < n and s[z[i]] == s[z[i] + i]) z[i]++;
11         }
12         if (z[i] + i - 1 > r) {
13             l = i;
14             r = z[i] + i - 1;
15         }
16     }
17     return z;
18 };

```

4.3 manacher

```

1  /* manacher @ wrb */
2  auto Manacher = [&](const std::string& t) {
3      std::string s = "#";
4      for (char c : t) s += c, s += '#';
5      int i, o = 0, r = 0, n = s.size();
6      std::vector<int> p(n, 1), q(n);
7      for (i = 0; i < n; ++i) {
8          if (i <= r) p[i] = std::min(r - i + 1, p[2 * o - i]);
9          for (; p[i] <= i && s[i + p[i]] == s[i - p[i]]; ++p[i]);
10         if (i + p[i] - 1 > r) r = i + p[i] - 1, o = i;
11     }
12     return p;
13 };

```

4.4 AC automaton

```

1  /* AC auto */
2  int cnt = 0;
3  const int N = 2e5 + 10;
4  static std::array<std::array<int, 26>, N> tr;
5  static std::array<int, N> exist, fail, ans, point;
6  vi order;
7
8  auto insert = [&](const auto& s) {
9      int p = 0;
10     for (const auto& ch : s) {
11         int c = ch - 'a';
12         if (!tr[p][c]) tr[p][c] = ++cnt;
13         p = tr[p][c];
14     }
15     exist[p]++;
16     return p;
17 };
18
19 auto build = [&]() {
20     std::queue<int> q;
21     for (int i = 0; i < 26; i++) {
22         if (tr[0][i]) q.push(tr[0][i]);
23     }
24     while (!q.empty()) {
25         auto u = q.front();
26         q.pop();
27         order.push_back(u);
28         for (int i = 0; i < 26; i++) {
29             if (tr[u][i]) {
30                 fail[tr[u][i]] = tr[fail[u]][i];
31                 q.push(tr[u][i]);
32             } else {
33                 tr[u][i] = tr[fail[u]][i];
34             }
35         }
36     }
37 };

```

```

38 |
39 | auto query = [&](const auto& s) {
40 |     int p = 0;
41 |     for (const auto ch : s) {
42 |         p = tr[p][ch - 'a'];
43 |         ans[p]++;
44 |     }
45 |     return;
46 | };
47 |
48 | void solve () {
49 |     for (int i = 0; i < n; i++) {
50 |         point[i] = insert(t);
51 |     }
52 |     build();
53 |     query(s);
54 |     /* fail 树上子树求和 */
55 |     reverse(all(order));
56 |     for (const auto& i : order) ans[fail[i]] += ans[i];
57 | }

```

4.5 PAM

```

1  | /* PAM @ ddl */
2  | std::vector<node> tr;
3  | std::vector<int> stk;
4  | auto newnode = [&](int len) {
5  |     tr.emplace_back();
6  |     tr.back().len = len;
7  |     return (int) tr.size() - 1;
8  | };
9  | auto PAMinit = [&]() {
10 |     newnode(0), tr.back().fail = 1;
11 |     newnode(-1), tr.back().fail = 0;
12 |     stk.push_back(-1);
13 | };
14 | PAMinit();
15 | auto getfail = [&](int v) {
16 |     while (stk.end()[-2 - tr[v].len] != stk.back()) {
17 |         v = tr[v].fail;
18 |     }
19 |     return v;
20 | };
21 | auto insert = [&](int last, int c, int cnt) {
22 |     stk.emplace_back(c);
23 |     int x = getfail(last);
24 |     if (!tr[x].ch[c]) {
25 |         int u = newnode(tr[x].len + 2);
26 |         tr[u].fail = tr[getfail(tr[x].fail)].ch[c];
27 |         tr[x].ch[c] = u;
28 |         /* tr[u].size = tr[tr[u].fail].size + 1; */
29 |         /* Can be used to count the number of types of palindromic strings ending at the current
30 |          * position */
31 |     }
32 |     tr[tr[x].ch[c]].size += cnt;
33 |     return tr[x].ch[c];
34 | };
35 | auto build = [&]() { /* DP on fail tree */
36 |     int ans = 0;
37 |     for (int i = (int) tr.size() - 1; i > 1; i--) {
38 |         tr[tr[i].fail].size += tr[i].size;
39 |         /* options */
40 |     }
41 |     return ans;
42 | };
43 | /* PAM */
44 | int ans = 0, last = 0;
45 | for (int i = 0; i < n; i++) {
46 |     last = insert(last, s[i] - 'a', 1);
47 | }

```

```

1  | /* PAM @ wrb */
2  | template<const int M = 26>
3  | struct PAM {
4  |     struct T {
5  |         int len, d, fa, ch[M];
6  |         T() : len(), d(), fa(), ch() {}
7  |     };
8  |     int las;
9  |     string s;
10 |     vector<T> t;

```

```

11     vector<int> bl;
12     size_t count() const {
13         return t.size() - 2;
14     }
15     const T& operator[] (const size_t& p) const {
16         return t[p];
17     }
18     const T& ask(const size_t& p) const {
19         return t[bl[p]];
20     }
21     int gf(int o, int p) {
22         while (p - t[o].len - 1 < 0 || s[p - t[o].len - 1] != s[p]) o = t[o].fa;
23         return o;
24     }
25     void append(int c) {
26         int p = s.size(), o;
27         s += c, o = gf(las, p);
28         if (t[o].ch[c] == 0) {
29             t.emplace_back();
30             t.back().len = t[o].len + 2;
31             t.back().fa = t[gf(t[o].fa, p)].ch[c];
32             t.back().d = t[t.back().fa].d + 1;
33             t[o].ch[c] = t.size() - 1;
34         }
35         bl.emplace_back(las = t[o].ch[c]);
36     }
37     PAM() : las(), s(), t(2) {
38         t[0].fa = t[1].fa = 1, t[1].len = -1;
39     }
40     PAM(const string& str, int h) : las(), s(), t(2) {
41         t[0].fa = t[1].fa = 1, t[1].len = -1;
42         for (char c : str) append(c - h);
43     }
44 };

```

4.6 Suffix Array

```

1  /* suffix array and ST table @ jiangly */
2  auto suffixArray = [&](const std::string& s) {
3      int n = s.length();
4      vi sa(n), rk(n);
5      std::iota(all(sa), 0);
6      std::sort(all(sa), [&](int a, int b) { return s[a] < s[b]; });
7      rk[sa[0]] = 0;
8      for (int i = 1; i < n; ++i) {
9          rk[sa[i]] = rk[sa[i - 1]] + (s[sa[i]] != s[sa[i - 1]]);
10     }
11     int k = 1;
12     vi tmp(n), cnt(n);
13     tmp.reserve(n);
14     while (rk[sa[n - 1]] < n - 1) {
15         tmp.clear();
16         for (int i = 0; i < k; ++i) tmp.push_back(n - k + i);
17         for (const auto& i : sa) {
18             if (i >= k) tmp.push_back(i - k);
19         }
20         std::fill(all(cnt), 0);
21         for (int i = 0; i < n; i++) cnt[rk[i]]++;
22         for (int i = 1; i < n; i++) cnt[i] += cnt[i - 1];
23         for (int i = n - 1; i >= 0; i--) sa[--cnt[rk[tmp[i]]]] = tmp[i];
24         std::swap(rk, tmp);
25         rk[sa[0]] = 0;
26         for (int i = 1; i < n; i++) {
27             rk[sa[i]] = rk[sa[i - 1]] + (tmp[sa[i - 1]] < tmp[sa[i]] or sa[i - 1] + k == n or
28                                     tmp[sa[i - 1] + k] < tmp[sa[i] + k]);
29         }
30         k *= 2;
31     }
32     vi height(n);
33     for (int i = 0, j = 0; i < n; ++i) {
34         if (rk[i] == 0) continue;
35         if (j) --j;
36         while (s[i + j] == s[sa[rk[i] - 1] + j]) ++j;
37         height[rk[i]] = j;
38     }
39     return std::make_tuple(sa, rk, height);
40 };
41 auto [sa, rk, height] = suffixArray(s);
42 vvi f(n, vi(30, inf));
43 vi Log2(n);
44 auto init = [&]() -> void {
45     for (int i = 0; i < n; i++) {

```

```

46     f[i][0] = height[i];
47     if (i > 1) Log2[i] = Log2[i / 2] + 1;
48 };
49 int t = Log2.back();
50 for (int j = 1; j <= t; j++) {
51     for (int i = 0; i <= n - (1 << j); i++) {
52         f[i][j] = std::min(f[i][j - 1], f[i + (1 << (j - 1))][j - 1]);
53     }
54 }
55 };
56 init();
57 auto query = [&](int l, int r) -> int {
58     int t = Log2[r - l + 1];
59     return std::min(f[l][t], f[r - (1 << t) + 1][t]);
60 };
61 auto lcp = [&](int i, int j) {
62     i = rk[i], j = rk[j];
63     if (i > j) std::swap(i, j);
64     return query(i + 1, j);
65 };

```

```

1  /* suffix array @ wrb */
2  auto SA = [](std::string s) {
3      int n = s.size(), m = 128, i, j, l;
4      std::vector<int> ct(m), sa(n), rk(n), h(n), a(n);
5      for (i = 0; i < n; ++i) ++ct[rk[i] = s[i]];
6      for (i = 1; i < m; ++i) ct[i] += ct[i - 1];
7      for (i = n - 1; ~i; --i) sa[--ct[rk[i]]] = i;
8      for (l = 1; l < n; l *= 2) {
9          for (j = 0, i = n - 1; i >= n - l; --i) a[j++] = i;
10         for (i = 0; i < n; ++i) if (sa[i] >= l) a[j++] = sa[i] - l;
11         ct = std::vector<int>(m);
12         for (i = 0; i < n; ++i) ++ct[rk[a[i]]];
13         for (i = 1; i < m; ++i) ct[i] += ct[i - 1];
14         for (i = n - 1; ~i; --i) sa[--ct[rk[a[i]]]] = a[i];
15         std::swap(rk, a), rk[sa[0]] = 0;
16         for (i = 1; i < n; ++i) {
17             rk[sa[i]] = rk[sa[i - 1]] + (a[sa[i]] != a[sa[i - 1]] || a[(sa[i] + l) % n] != a[(sa[i - 1] +
18                 l) % n]);
19         }
20         if ((m = rk[sa[n - 1]] + 1) == n) break;
21     }
22     for (i = j = 0; i + 1 < n; h[rk[i++]] = j) {
23         for (j ? --j : 0; s[i + j] == s[sa[rk[i] - 1] + j]; ++j);
24     }
25     sa.erase(sa.begin());
26     rk.erase(rk.begin());
27     h.erase(h.begin());
28     return make_tuple(sa, rk, h);
29 };
30 //h[i] : LCP(rk[i], rk[i - 1])

```

4.7 Cantor expansion

```

1  /* Cantor expression @ wrb */
2  std::cin >> n, fac[0] = 1;
3  for (int i = 1; i <= n; ++i) {
4      std::cin >> a[i];
5      fac[i] = 1ll * fac[i - 1] * i % P;
6  }
7  auto ins = [&](int x) {
8      for (; x <= n; x += x & -x) ++t[x];
9  };
10 auto ask = [&](int x) {
11     int z = 0;
12     for (; x; x ^= x & -x) z += t[x];
13     return z;
14 };
15 int z = 0;
16 for (int i = n; i; --i) {
17     z = (z + 1ll * fac[n - i] * ask(a[i])) % P;
18     ins(a[i]);
19 }
20 std::cout << ++z << '\n';

```

4.8 trie

普通字典树 (单词匹配)

```

1  /* trie */
2  int cnt;
3  std::vector<std::array<int, 26>> trie(n + 1);
4  vi exist(n + 1);
5  auto insert = [&](const std::string& s) -> void {
6      int p = 0;
7      for (const auto ch : s) {
8          int c = ch - 'a';
9          if (!trie[p][c]) trie[p][c] = ++cnt;
10         p = trie[p][c];
11     }
12     exist[p] = true;
13 };
14 auto find = [&](const string& s) -> bool {
15     int p = 0;
16     for (const auto ch : s) {
17         int c = ch - 'a';
18         if (!trie[p][c]) return false;
19         p = trie[p][c];
20     }
21     return exist[p];
22 };

```

01 字典树 (求最大异或值)

给定 n 个数, 取两个数进行异或运算, 求最大异或值.

```

1  /* trie */
2  int cnt = 0;
3  std::vector<std::array<int, 2>> trie(N);
4  auto insert = [&](int x) -> void {
5      int p = 0;
6      for (int i = 30; i >= 0; i--) {
7          int c = (x >> i) & 1;
8          if (!trie[p][c]) trie[p][c] = ++cnt;
9          p = trie[p][c];
10     }
11 };
12 auto find = [&](int x) -> int {
13     int sum = 0, p = 0;
14     for (int i = 30; i >= 0; i--) {
15         int c = (x >> i) & 1;
16         if (trie[p][c ^ 1]) {
17             p = trie[p][c ^ 1];
18             sum += (1 << i);
19         } else {
20             p = trie[p][c];
21         }
22     }
23     return sum;
24 };

```

字典树合并

来自浙大城市学院 2023 校赛 E 题.

给定一棵根为 1 的树, 每个点的点权为 w_i . 一共 q 次询问, 每次给出一对 u, v , 询问以 v 为根的子树上的点与 u 的权值最大异或值.

```

1  int main() {
2      std::ios::sync_with_stdio(false);
3      std::cin.tie(0);
4
5      int n, m;
6      std::cin >> n;
7      vi w(n + 1);
8      for (int i = 1; i <= n; i++) std::cin >> w[i];
9      vvi e(n + 1);
10     for (int i = 1, u, v; i < n; i++) {

```

```

11     std::cin >> u >> v;
12     e[u].push_back(v);
13     e[v].push_back(u);
14 }
15
16 // 离线询问 //
17 std::cin >> m;
18 std::vector<vpi> q(n + 1);
19 vi ans(m + 1);
20 for (int i = 1; i <= m; i++) {
21     int u, v;
22     std::cin >> u >> v;
23     q[v].emplace_back(u, i);
24 }
25
26 // 01 trie //
27 std::vector<std::array<int, 2>> tr(1);
28 auto new_node = [&]() -> int {
29     tr.emplace_back();
30     return tr.size() - 1;
31 };
32 vi id(n + 1);
33 auto insert = [&](int root, int x) {
34     int p = root;
35     for (int i = 29; i >= 0; i--) {
36         int c = x >> i & 1;
37         if (!tr[p][c]) tr[p][c] = new_node();
38         p = tr[p][c];
39     }
40 };
41 auto query = [&](int root, int x) -> int {
42     int ans = 0, p = root;
43     for (int i = 29; i >= 0; i--) {
44         int c = x >> i & 1;
45         if (tr[p][c ^ 1]) {
46             p = tr[p][c ^ 1];
47             ans += (1 << i);
48         } else {
49             p = tr[p][c];
50         }
51     }
52     return ans;
53 };
54 std::function<int(int, int)> merge = [&](int a, int b) -> int {
55     // b 的信息挪到 a 上 //
56     if (!a) return b;
57     if (!b) return a;
58     tr[a][0] = merge(tr[a][0], tr[b][0]);
59     tr[a][1] = merge(tr[a][1], tr[b][1]);
60     return a;
61 };
62 std::function<void(int, int)> dfs = [&](int u, int fa) {
63     id[u] = new_node();
64     insert(id[u], w[u]);
65     for (auto v : e[u]) {
66         if (v == fa) continue;
67         dfs(v, u);
68         id[u] = merge(id[u], id[v]);
69     }
70     for (auto [v, i] : q[u]) {
71         ans[i] = query(id[u], w[v]);
72     }
73 };
74 dfs(1, 0);
75 for (int i = 1; i <= m; i++) std::cout << ans[i] << endl;
76 return 0;
77 }

```

5 math - number theory

5.1 mod int

```

1  template <int P>
2  struct Mint {
3      int v = 0;
4
5      // reflection //
6      template <typet = int>
7      constexpr operator T() const {
8          return v;
9      }
10
11     // constructor //
12     constexpr Mint() = default;
13     template <typet>
14     constexpr Mint(T x) : v(x % P) {}
15     constexpr int val() const { return v; }
16     constexpr int mod() { return P; }
17
18     // io //
19     friend std::istream& operator>>(std::istream& is, Mint& x) {
20         LL y;
21         is >> y;
22         x = y;
23         return is;
24     }
25     friend std::ostream& operator<<(std::ostream& os, Mint x) { return os << x.v; }
26
27     // comparision //
28     friend constexpr bool operator==(const Mint& lhs, const Mint& rhs) { return lhs.v == rhs.v; }
29     friend constexpr bool operator!=(const Mint& lhs, const Mint& rhs) { return lhs.v != rhs.v; }
30     friend constexpr bool operator<(const Mint& lhs, const Mint& rhs) { return lhs.v < rhs.v; }
31     friend constexpr bool operator<=(const Mint& lhs, const Mint& rhs) { return lhs.v <= rhs.v; }
32     friend constexpr bool operator>(const Mint& lhs, const Mint& rhs) { return lhs.v > rhs.v; }
33     friend constexpr bool operator>=(const Mint& lhs, const Mint& rhs) { return lhs.v >= rhs.v; }
34
35     // arithmetic //
36     template <typet>
37     friend constexpr Mint power(Mint a, T n) {
38         Mint ans = 1;
39         while (n) {
40             if (n & 1) ans *= a;
41             a *= a;
42             n >>= 1;
43         }
44         return ans;
45     }
46     friend constexpr Mint inv(const Mint& rhs) { return power(rhs, P - 2); }
47     friend constexpr Mint operator+(const Mint& lhs, const Mint& rhs) {
48         return lhs.val() + rhs.val() < P ? lhs.val() + rhs.val() : lhs.val() - P + rhs.val();
49     }
50     friend constexpr Mint operator-(const Mint& lhs, const Mint& rhs) {
51         return lhs.val() < rhs.val() ? lhs.val() + P - rhs.val() : lhs.val() - rhs.val();
52     }
53     friend constexpr Mint operator*(const Mint& lhs, const Mint& rhs) {
54         return static_cast<LL>(lhs.val()) * rhs.val() % P;
55     }
56     friend constexpr Mint operator/(const Mint& lhs, const Mint& rhs) { return lhs * inv(rhs); }
57     Mint operator+() const { return *this; }
58     Mint operator-() const { return Mint() - *this; }
59     constexpr Mint& operator++() {
60         v++;
61         if (v == P) v = 0;
62         return *this;
63     }
64     constexpr Mint& operator--() {
65         if (v == 0) v = P;
66         v--;
67         return *this;
68     }
69     constexpr Mint& operator++(int) {
70         Mint ans = *this;
71         ++*this;
72         return ans;
73     }
74     constexpr Mint& operator--(int) {
75         Mint ans = *this;
76         --*this;
77         return ans;
78     }
79     constexpr Mint& operator+=(const Mint& rhs) {

```

```

80     v = v + rhs;
81     return *this;
82 }
83 constexpr Mint& operator--(const Mint& rhs) {
84     v = v - rhs;
85     return *this;
86 }
87 constexpr Mint& operator*=(const Mint& rhs) {
88     v = v * rhs;
89     return *this;
90 }
91 constexpr Mint& operator/=(const Mint& rhs) {
92     v = v / rhs;
93     return *this;
94 }
95 };
96 using Z = Mint<998244353>;

```

5.2 Eculid

欧几里得算法

```
1 std::gcd(a, b)
```

扩展欧几里得算法

```

1  /* exgcd */
2  auto exgcd = [&](LL a, LL b, LL& x, LL& y) {
3      LL x1 = 1, x2 = 0, x3 = 0, x4 = 1;
4      while (b != 0) {
5          LL c = a / b;
6          std::tie(x1, x2, x3, x4, a, b) =
7              std::make_tuple(x3, x4, x1 - x3 * c, x2 - x4 * c, b, a - b * c);
8      }
9      x = x1, y = x2;
10 };
11 auto exgcd = [&](auto&& self, LL a, LL b, LL& x, LL& y) {
12     if (!b) {
13         x = 1, y = 0;
14         return a;
15     }
16     LL d = self(self, b, a % b, y, x);
17     y -= a / b * x;
18     return d;
19 };

```

```

1  auto exgcd = [&](auto&& self, LL a, LL b, LL& x, LL& y) {
2      if (!b) {
3          x = 1, y = 0;
4          return;
5      }
6      self(self, b, a % b, y, x);
7      y -= a / b * x;
8  };

```

```

1  auto exgcd = [&](auto&& self, LL a, LL b, LL& x, LL& y) {
2      if (!b) {
3          x = 1, y = 0;
4          return a;
5      }
6      LL d = self(self, b, a % b, y, x);
7      y -= a / b * x;
8      return d;
9  };

```

类欧几里得算法

一般形式: 求 $f(a, b, c, n) = \sum_{i=0}^n \lfloor \frac{ai+b}{c} \rfloor$

$$f(a, b, c, n) = nm - f(c, c - b - 1, a, m - 1)$$

```

1 LL f(LL a, LL b, LL c, LL n) {
2     if (a == 0) return ((b / c) * (n + 1));
3     if (a >= c || b >= c)
4         return f(a % c, b % c, c, n) + (a / c) * n * (n + 1) / 2 + (b / c) * (n + 1);
5     LL m = (a * n + b) / c;
6     LL v = f(c, c - b - 1, a, m - 1);
7     return n * m - v;
8 }

```

更进一步, 求: $g(a, b, c, n) = \sum_{i=0}^n i \lfloor \frac{ai+b}{c} \rfloor$ 以及 $h(a, b, c, n) = \sum_{i=0}^n \lfloor \frac{ai+b}{c} \rfloor^2$

$$g(a, b, c, n) = \lfloor \frac{mn(n+1) - f(c, c-b-1, a, m-1) - h(c, c-b-1, a, m-1)}{2} \rfloor$$

$$h(a, b, c, n) = nm(m+1) - 2f(c, c-b-1, a, m-1) - 2g(c, c-b-1, a, m-1) - f(a, b, c, n)$$

```

1 const int inv2 = 499122177;
2 const int inv6 = 166374059;
3
4 LL f(LL a, LL b, LL c, LL n);
5 LL g(LL a, LL b, LL c, LL n);
6 LL h(LL a, LL b, LL c, LL n);
7
8 struct data {
9     LL f, g, h;
10 };
11
12 data calc(LL a, LL b, LL c, LL n) {
13     LL ac = a / c, bc = b / c, m = (a * n + b) / c, n1 = n + 1, n21 = n * 2 + 1;
14     data d;
15     if (a == 0) {
16         d.f = bc * n1 % mod;
17         d.g = bc * n % mod * n1 % mod * inv2 % mod;
18         d.h = bc * bc % mod * n1 % mod;
19         return d;
20     }
21     if (a >= c || b >= c) {
22         d.f = n * n1 % mod * inv2 % mod * ac % mod + bc * n1 % mod;
23         d.g =
24             ac * n % mod * n1 % mod * n21 % mod * inv6 % mod + bc * n % mod * n1 % mod * inv2 % mod;
25         d.h = ac * ac % mod * n % mod * n1 % mod * n21 % mod * inv6 % mod +
26             bc * bc % mod * n1 % mod + ac * bc % mod * n % mod * n1 % mod;
27         d.f %= mod, d.g %= mod, d.h %= mod;
28         data e = calc(a % c, b % c, c, n);
29         d.h += e.h + 2 * bc % mod * e.f % mod + 2 * ac % mod * e.g % mod;
30         d.g += e.g, d.f += e.f;
31         d.f %= mod, d.g %= mod, d.h %= mod;
32         return d;
33     }
34     data e = calc(c, c - b - 1, a, m - 1);
35     d.f = n * m % mod - e.f, d.f = (d.f % mod + mod) % mod;
36     d.g = m * n % mod * n1 % mod - e.h - e.f, d.g = (d.g * inv2 % mod + mod) % mod;
37     d.h = n * m % mod * (m + 1) % mod - 2 * e.g - 2 * e.f - d.f;
38     d.h = (d.h % mod + mod) % mod;
39     return d;
40 }

```

5.3 inverse

线性递推

$$a^{-1} \equiv -\lfloor \frac{p}{a} \rfloor \times (p \% a)^{-1}$$

```

1 /* inverse */
2 vi inv(n + 1);
3 auto sieve_inv = [&](int n) {
4     inv[1] = 1;
5     for (int i = 2; i <= n; i++) {
6         inv[i] = 1ll * (p - p / i) * inv[p % i] % p;
7     }
8 };

```

求 n 个数的逆元

```

1  /* inverse */
2  auto inverse = [&](const vi& a) {
3      int n = a.size();
4      vi b(n), f(n), ivf(n);
5      f[0] = a[0];
6      for (int i = 1; i < n; i++) {
7          f[i] = 1ll * f[i - 1] * a[i] % p;
8      }
9      ivf.back() = quick_power(f.back(), p - 2, p);
10     for (int i = n - 1; i; i--) {
11         ivf[i - 1] = 1ll * ivf[i] * a[i] % p;
12     }
13     b[0] = ivf[0];
14     for (int i = 1; i < n; i++) {
15         b[i] = 1ll * ivf[i] * f[i - 1] % p;
16     }
17     return b;
18 };

```

5.4 sieve

素数

```

1  vi prime, is_prime(n + 1, 1);
2  auto Euler_sieve = [&](int n){
3      for (int i = 2; i <= n; i++) {
4          if (is_prime[i]) prime.push_back(i);
5          for (auto p : prime) {
6              if (i * p > n) break;
7              is_prime[i * p] = 0;
8              if (i % p == 0) break;
9          }
10     }
11 };

```

欧拉函数

```

1  vi phi(n + 1), prime;
2  vi is_prime(n + 1, 1);
3  auto get_phi = [&](int n) {
4      int cnt = 0;
5      phi[1] = 1;
6      for (int i = 2; i <= n; i++) {
7          if (is_prime[i]) {
8              prime.push_back(i);
9              phi[i] = i - 1;
10         }
11         for (auto p : prime) {
12             if (i * p > n) break;
13             is_prime[i * p] = 0;
14             if (i % p) {
15                 phi[i * p] = phi[i] * phi[p];
16             } else {
17                 phi[i * p] = phi[i] * p;
18                 break;
19             }
20         }
21     }
22 };

```

约数和

```

1  vi g(n + 1), d(n + 1), prime;
2  vi is_prime(n + 1, 1);
3  auto get_d = [&](int n) {
4      int tot = 0;
5      g[1] = d[1] = 1;
6      for (int i = 2; i <= n; i++) {
7          if (is_prime[i]) {

```

```

8         prime.push_back(i);
9         d[i] = g[i] = i + 1;
10    }
11    for (auto p : prime) {
12        if (i * p > n) break;
13        is_prime[i * p] = 0;
14        if (i % p == 0) {
15            g[i * p] = g[i] * p + 1;
16            d[i * p] = d[i] / g[i] * g[i * p];
17            break;
18        } else {
19            d[i * p] = d[i] * d[p];
20            g[i * p] = 1 + p;
21        }
22    }
23 }
24 };

```

莫比乌斯函数

```

1  vi mu(n + 1), prime;
2  vi is_prime(n + 1, 1);
3  auto get_mu = [&](int n) {
4      mu[1] = 1;
5      for (int i = 2; i <= n; i++) {
6          if (is_prime[i]) {
7              prime.push_back(i);
8              mu[i] = -1;
9          }
10         for (auto p : prime) {
11             if (i * p > n) break;
12             is_prime[i * p] = 0;
13             if (i % p == 0) {
14                 mu[i * p] = 0;
15                 break;
16             }
17             mu[i * p] = -mu[i];
18         }
19     }
20 };

```

杜教筛

```

1  const int N = 1e7;
2  vi mu(N + 1), phi(N + 1), prime;
3  vl sum_phi(N + 1), sum_mu(N + 1);
4  vi is_prime(N + 1, 1);
5  std::map<LL, LL> mp_mu;
6
7  /* 计算 1 ~ 10^7 的 mu */
8  auto get_mu = [&](int n) {
9      phi[1] = mu[1] = 1;
10     for (int i = 2; i <= n; i++) {
11         if (is_prime[i]) {
12             prime.push_back(i);
13             phi[i] = i - 1;
14             mu[i] = -1;
15         }
16         for (auto p : prime) {
17             if (i * p > n) break;
18             is_prime[i * p] = 0;
19             if (i % p == 0) {
20                 phi[i * p] = phi[i] * p;
21                 mu[i * p] = 0;
22                 break;
23             }
24             phi[i * p] = phi[i] * phi[p];
25             mu[i * p] = -mu[i];
26         }
27     }
28 };
29 get_mu(N);
30 for (int i = 1; i <= N; i++) {
31     sum_phi[i] = sum_phi[i - 1] + phi[i];
32     sum_mu[i] = sum_mu[i - 1] + mu[i];
33 }
34
35 /* 杜教筛：求 mu 的前缀和 */

```

```

36 std::function<LL(LL)> S_mu = [&](LL x) -> LL {
37     if (x <= N) return sum_mu[x];
38     auto it = mp_mu.find(x);
39     if (it != mp_mu.end()) return mp_mu[x];
40     LL ans = 1;
41     for (LL i = 2, j; i <= x; i = j + 1) {
42         j = x / (x / i);
43         ans -= S_mu(x / i) * (j - i + 1);
44     }
45     return mp_mu[x] = ans;
46 };
47
48 /* 杜教筛: 求 phi 的前缀和 */
49 auto S_phi = [&](LL x) -> LL {
50     if (x <= N) return sum_phi[x];
51     LL ans = 0;
52     for (LL i = 1, j; i <= x; i = j + 1) {
53         j = x / (x / i);
54         ans += 1ll * (S_mu(j) - S_mu(i - 1)) * (x / i) * (x / i);
55     }
56     return (ans - 1) / 2 + 1;
57 };

```

5.5 powerful number

目标: 求积性函数 $f(n)$ 的前缀和. 做法如下:

1. 构造积性函数 $g(n)$, 满足其易求前缀和且素数处函数值等于 f 的函数值.
2. 构造 $h = f/g$, 即 $f = h * g$ (狄利克雷卷积), 容易知道 $h(1) = 1$. 容易计算出 h 在非 powerful number 处函数值均为 0.
3. 根据

$$\begin{aligned}
 F(n) &= \sum_{i=1}^n f(i) \\
 &= \sum_{i=1}^n \sum_{d|i} g(i)h(i/d) \\
 &= \sum_{d=1}^n \sum_{i=1}^{\lfloor \frac{n}{d} \rfloor} h(d)g(i) \\
 &= \sum_{d=1}^n h(d)G\left(\left\lfloor \frac{n}{d} \right\rfloor\right) \\
 &= \sum_{d=1,2,\dots,n, d \text{ is powerful number}} h(d)G\left(\left\lfloor \frac{n}{d} \right\rfloor\right)
 \end{aligned}$$

发现只需要计算 h 在 powerful number 处的函数值即可, 可以边搜索边计算.

给定 $f(p^k) = p^k(p^k - 1)$ 为积性函数, 计算其前缀和.

```

1 auto powerfulNumber = [&](LL n) {
2     /* 1. construct g, and compute G */
3     /* int m = sqrt(n); // maybe TLE // */
4     int m = 2e6;
5     std::vector<Z> Gs(m + 1);
6     vi prime, is_prime(m + 1, 1);
7     Gs[1] = 1;
8     for (int i = 2; i <= m; i++) {
9         if (is_prime[i]) {
10             prime.push_back(i);
11             Gs[i] = i - 1;
12         }
13         for (auto p : prime) {
14             if (i * p > m) break;
15             is_prime[i * p] = 0;
16             if (i % p) {
17                 Gs[i * p] = Gs[i] * Gs[p];
18             } else {

```

```

19         Gs[i * p] = Gs[i] * p;
20         break;
21     }
22 }
23 }
24 for (int i = 2; i <= m; i++) {
25     Gs[i] = Gs[i - 1] + Z(i) * Gs[i];
26 }
27 std::map<LL, Z> mp;
28 auto G = [&](auto&& self, LL n) {
29     if (n <= m) return Gs[n];
30     if (mp.find(n) != mp.end()) return mp[n];
31     Z ans = Z(n) * Z(n + 1) * Z(n * 2 + 1) * inv6;
32     for (LL l = 2, r, k; l <= n; l = r + 1) {
33         k = n / l, r = n / (n / l);
34         ans -= (Z(r) * Z(r + 1) - Z(l - 1) * Z(l)) * inv2 * self(self, k);
35     }
36     return mp[n] = ans;
37 };
38 /* 2. compute h(p^c) */
39 vvl ps(prime.size());
40 std::vector<std::vector<Z>> hs(prime.size());
41 int len = 0;
42 for (int i = 0; i < prime.size(); i++) {
43     LL p = prime[i], now = p * p, c = 2;
44     ps[i] = {1, p}, hs[i] = {1, 0};
45     while (now <= n) {
46         ps[i].push_back(now);
47         Z ans = Z(ps[i][c]) * (Z(ps[i][c]) - 1);
48         for (int j = 1; j <= c; j++) {
49             ans -= Z(ps[i][j]) * Z(ps[i][j - 1]) * Z(p - 1) * hs[i][c - j];
50         }
51         hs[i].push_back(ans);
52         now *= p, c += 1;
53     }
54     len += ps[i].size();
55 }
56 debug(len);
57 /* 3. search powerful number */
58 Z ans = 0;
59 auto dfs = [&](auto&& self, int id, LL now, Z hd) -> void {
60     ans += hd * G(G, n / now);
61     for (int i = id; i < prime.size(); i++) {
62         int p = prime[i], c = 2;
63         if (now > n / p / p) break;
64         for (LL x = now * p * p; x <= n; x *= p, c++) {
65             if (hs[i][c]) self(self, i + 1, x, hd * hs[i][c]);
66         }
67     }
68 };
69 dfs(dfs, 0, 1, 1);
70 return ans;
71 };

```

5.6 block

分块的逻辑

下取整 $\lfloor \frac{n}{g} \rfloor = k$ 的分块 ($g \leq n$)

```

1 for(int l = 1, r, k; l <= n; l = r + 1){
2     k = n / l;
3     r = n / (n / l);
4     debug(l, r, k);
5 }

```

$k = \lfloor \frac{n}{g} \rfloor$ 从大到小遍历 $\lfloor \frac{n}{g} \rfloor$ 的所有取值, $[l, r]$ 对应的是 g 取值的区间.

```

1 n = 11
2 [l, r, k] : 1 1 11
3 [l, r, k] : 2 2 5
4 [l, r, k] : 3 3 3
5 [l, r, k] : 4 5 2
6 [l, r, k] : 6 11 1

```

上取整 $\lceil \frac{n}{g} \rceil = k$ 的分块 ($g < n$)

```

1 for(int l = 1, r, k; l < n; l = r + 1){
2     k = (n + l - 1) / l;
3     r = (n + k - 2) / (k - 1) - 1;
4     debug(l, r, k);
5 }

```

$k = \lceil \frac{n}{g} \rceil$ 从大到小遍历 $\lceil \frac{n}{g} \rceil$ 的所有取值, $[l, r]$ 对应的是 g 取值的区间.

```

1 n = 11
2 [l, r, k] : 1 1 11
3 [l, r, k] : 2 2 6
4 [l, r, k] : 3 3 4
5 [l, r, k] : 4 5 3
6 [l, r, k] : 6 10 2

```

一般形式

计算 $\sum_{i=1}^n f(i) \lfloor \frac{n}{i} \rfloor$, 设 $s(i)$ 为 $f(i)$ 的前缀和。

```

1 for (int l = 1, r; l <= n; l = r + 1) {
2     r = n / (n / l);
3     ans += (s[r] - s[l - 1]) * (n / l);
4 }

```

$$\sum_{i=1}^n f(i) \lfloor \frac{a}{i} \rfloor \lfloor \frac{b}{i} \rfloor$$

```

1 for (int l = 1, r, r1, r2; l <= n; l = r + 1) {
2     if (a / l) {
3         r1 = a / (a / l);
4     } else {
5         r1 = n;
6     }
7     if (b / l) {
8         r2 = b / (b / l);
9     } else {
10        r2 = n;
11    }
12    r = min(min(r1, r2), n);
13    ans += (s[r] - s[l - 1]) * (a / l) * (b / l);
14 }

```

5.7 CRT & exCRT

求解

$$\begin{cases} N \equiv a_1 \pmod{m_1} \\ N \equiv a_2 \pmod{m_2} \\ \dots \\ N \equiv a_n \pmod{m_n} \end{cases}$$

$$\text{有 } N \equiv \sum_{i=1}^k a_i \times \text{inv}\left(\frac{M}{m_i}, m_i\right) \times \left(\frac{M}{m_i}\right) \pmod{M}$$

```

1 /* CRT */
2 auto crt = [&](int n, const vi& a, const vi& m) -> LL{
3     LL ans = 0, M = 1;
4     for(int i = 1; i <= n; i++) M *= m[i];
5     for(int i = 1; i <= n; i++){
6         ans = (ans + a[i] * inv(M / m[i], m[i]) * (M / m[i])) % M;
7     }
8     return (ans % M + M) % M;
9 };

```

扩展中国剩余定理

```

1 /* exCRT */
2 auto excrt = [&](int n, const vi& a, const vi& m) -> LL{
3     LL A = a[1], M = m[1];

```

```

4   for (int i = 2; i <= n; i++) {
5       LL x, y, d = std::gcd(M, m[i]);
6       exgcd(M, m[i], x, y);
7       LL mod = M / d * m[i];
8       x = x * (a[i] - A) / d % (m[i] / d);
9       A = ((M * x + A) % mod + mod) % mod;
10      M = mod;
11  }
12  return A;
13 };

```

5.8 BSGS & exBSGS

求解满足 $a^x \equiv b \pmod p$ 的 x

```

1  /* BSGS */
2  /* return value = -1e18 means no solution */
3  auto BSGS = [&](LL a, LL b, LL p) {
4      if (1 % p == b % p) return 0ll;
5      LL k = std::sqrt(p) + 1;
6      std::unordered_map<LL, LL> hash;
7      for (LL i = 0, j = b % p; i < k; i++) {
8          hash[j] = i;
9          j = j * a % p;
10     }
11     LL ak = 1;
12     for (int i = 1; i <= k; i++) ak = ak * a % p;
13     for (int i = 1, j = ak; i <= k; i++) {
14         if (hash.count(j)) return 1ll * i * k - hash[j];
15         j = 1ll * j * ak % p;
16     }
17     return -INF;
18 };

```

$(a, p) \neq 1$ 的情形

```

1  /* exBSGS */
2  /* return value < 0 means no solution */
3  auto exBSGS = [&](auto&& self, LL a, LL b, LL p) {
4      b = (b % p + p) % p;
5      if (1ll % p == b % p) return 0ll;
6      LL x, y, d = std::gcd(a, p);
7      exgcd(exgcd, a, p, x, y);
8      if (d > 1) {
9          if (b % d != 0) return -INF;
10         exgcd(exgcd, a / d, p / d, x, y);
11         return self(self, a, b / d * x % (p / d), p / d) + 1;
12     }
13     return BSGS(a, b, p);
14 };

```

5.9 Miller Rabin

```

1  /* Miller Rabin */
2  vl vv = {2, 325, 9375, 28178, 450775, 9780504, 1795265022};
3  auto quick_power = [&](LL a, LL n, LL mod) {
4      LL ans = 1;
5      while (n) {
6          if (n & 1) ans = (i128) ans * a % mod;
7          a = (i128) a * a % mod;
8          n >>= 1;
9      }
10     return ans;
11 };
12
13 auto millerRabin = [&](LL n) {
14     if (n < 2 || n % 6 % 4 != 1) return (n | 1) == 3;
15     int s = __builtin_ctzll(n - 1);
16     LL d = n >> s;
17     for (auto a : vv) {
18         LL p = quick_power(a % n, d, n);
19         int i = s;
20         while (p != 1 and p != n - 1 and a % n and i--) p = (i128) p * p % n;
21         if (p != n - 1 and i != s) return false;
22     }
23     return true;

```

```
24 };
```

5.10 Pollard Rho

能在 $O(n^{\frac{1}{4}})$ 的时间复杂度随机出一个 n 的非平凡因数.

```
1  /* pollard rho */
2  auto pollard_rho = [&](LL x) -> LL{
3      LL s = 0, t = 0, val = 1;
4      LL c = rand() % (x - 1) + 1;
5      for(int goal = 1;; goal <= 1, s = t, val = 1){
6          for(int step = 1; step <= goal; step++){
7              t = ((i128) t * t + c) % x;
8              val = (i128) val * abs(t - s) % x;
9              if(step % 127 == 0){
10                 LL d = std::gcd(val, x);
11                 if(d > 1) return d;
12             }
13         }
14         LL d = std::gcd(val, x);
15         if(d > 1) return d;
16     }
17 };
```

利用 Miller Rabin 和 Pollard Rho 进行素因数分解

```
1  auto factorize = [&](LL a) -> vl{
2      vl ans, stk;
3      for (auto p : prime) {
4          if (p > 1000) break;
5          while (a % p == 0) {
6              ans.push_back(p);
7              a /= p;
8          }
9          if (a == 1) return ans;
10     }
11     stk.push_back(a);
12     while (!stk.empty()) {
13         LL b = stk.back();
14         stk.pop_back();
15         if (miller_rabin(b)) {
16             ans.push_back(b);
17             continue;
18         }
19         LL c = b;
20         while (c >= b) c = pollard_rho(b);
21         stk.push_back(c);
22         stk.push_back(b / c);
23     }
24     return ans;
25 };
```

5.11 quadratic residu

```
1  /* cipolla */
2  auto cipolla = [&](int x) {
3      std::srand(time(0));
4      auto check = [&](int x) -> bool { return pow(x, (mod - 1) / 2) == 1; };
5      if (!x) return 0;
6      if (!check(x)) return -1;
7      int a, b;
8      while (1) {
9          a = rand() % mod;
10         b = sub(mul(a, a), x);
11         if (!check(b)) break;
12     }
13     PII t = {a, 1};
14     PII ans = {1, 0};
15     auto mulp = [&](PII x, PII y) -> PII {
16         auto [x1, x2] = x;
17         auto [y1, y2] = y;
18         int c = add(mul(x1, y1), mul(x2, y2, b));
19         int d = add(mul(x1, y2), mul(x2, y1));
20         return {c, d};
21     };
```



```

22     for (int i = (mod + 1) / 2; i; i >>= 1) {
23         if (i & 1) ans = mulp(ans, t);
24         t = mulp(t, t);
25     }
26     return std::min(ans.ff, mod - ans.ff);
27 }

```

5.12 Lucas

卢卡斯定理

用于求大组合数，并且模数是一个不大的素数。

$$\binom{n}{m} \bmod p = \binom{\lfloor n/p \rfloor}{\lfloor m/p \rfloor} \cdot \binom{n \bmod p}{m \bmod p} \bmod p$$

$\binom{n \bmod p}{m \bmod p}$ 可以直接计算， $\binom{\lfloor n/p \rfloor}{\lfloor m/p \rfloor}$ 可以继续使用卢卡斯计算。

递归至 $m = 0$ 的时候，返回 1。

p 不太大，一般在 10^5 左右。

```

1  auto C = [&](LL n, LL m, LL p) -> LL {
2      if (n < m) return 0;
3      if (m == 0) return 1;
4      return fac[n] * inv_fac[m] % p * inv_fac[n - m] % p;
5  };
6  /* lucas */
7  auto lucas = [&](auto&& self, LL n, LL m, LL p) -> LL {
8      if (n < m) return 0;
9      if (m == 0) return 1;
10     return C(n % p, m % p, p) * self(self, n / p, m / p, p) % p;
11 }

```

素数在组合数中的次数

Legengre 给出一种 $n!$ 中素数 p 的幂次的计算方式为：

$$\sum_{1 \leq j} \left\lfloor \frac{n}{p^j} \right\rfloor.$$

另一种计算方式利用 p 进制下各位数字和：

$$v_p(n!) = \frac{n - S_p(n)}{p - 1}.$$

则有

$$v_p(C_m^n) = \frac{S_p(n) + S_p(m - n) - S_p(m)}{p - 1}.$$

扩展卢卡斯定理

计算

$$\binom{n}{m} \bmod p,$$

p 可能为合数。

第一部分：CRT.

原问题变成求

$$\left\{ \begin{array}{l} \binom{n}{m} \equiv a_1 \pmod{p_1^{\alpha_1}} \\ \binom{n}{m} \equiv a_2 \pmod{p_2^{\alpha_2}} \\ \dots \\ \binom{n}{m} \equiv a_k \pmod{p_k^{\alpha_k}} \end{array} \right.$$

在求出 a_i 之后就可以利用 CRT 求出答案.

第二部分: 移除分子分母中的素数

问题转换成求解

$$\binom{n}{m} \pmod{q^k}.$$

等价于

$$\frac{\frac{n!}{q^x}}{\frac{m!}{q^y} \frac{(n-m)!}{q^z}} q^{x-y-z} \pmod{q^k},$$

其中 x 表示 $n!$ 中 q 的次数, y, z 同理.

第三部分: 威尔逊定理的推论

问题转换为求

$$\frac{n!}{q^x} \pmod{q^k}.$$

可以利用威尔逊定理的推论.

```

1  /* exlucas */
2  auto exLucas = [&](LL n, LL m, LL p) {
3      auto inv = [&](LL a, LL p) {
4          LL x, y;
5          exgcd(a, p, x, y);
6          return (x % p + p) % p;
7      };
8
9      auto func = [&](auto&& self, LL n, LL pi, LL pk) {
10         if (!n) return 1ll;
11         LL ans = 1;
12         for (LL i = 2; i <= pk; i++) {
13             if (i % pi) ans = ans * i % p;
14         }
15         ans = quick_power(ans, n / pk, pk);
16         for (LL i = 2; i <= n % pk; i++) {
17             if (i % pi) ans = ans * i % pk;
18         }
19         ans = ans * self(self, n / pi, pi, pk) % pk;
20         return ans;
21     };
22
23     auto multiLucas = [&](LL n, LL m, LL pi, LL pk) {
24         LL cnt = 0;
25         for (LL i = n; i; i /= pi) cnt += i / pi;
26         for (LL i = m; i; i /= pi) cnt -= i / pi;
27         for (LL i = n - m; i; i /= pi) cnt -= i / pi;
28         LL ans = quick_power(pi, cnt, pk) * func(func, n, pi, pk) % pk;
29         ans = ans * inv(func(func, m, pi, pk), pk) % pk;
30         ans = ans * inv(func(func, n - m, pi, pk), pk) % pk;
31         return ans;
32     };
33
34     auto crt = [&](const vl& a, const vl& m, int k) {
35         LL ans = 0;
36
37         for (int i = 0; i < k; i++) {
38             ans = (ans + a[i] * inv(p / m[i], m[i]) * (p / m[i])) % p;
39         }

```

```

40     return (ans % p + p) % p;
41 };
42
43 vl a, prime;
44 LL pp = p;
45 for (int i = 2; i * i <= pp; i++) {
46     if (pp % i) continue;
47     prime.push_back(1);
48     while (pp % i == 0) {
49         prime.back() *= i;
50         pp /= i;
51     }
52     a.push_back(multiLucas(n, m, i, prime.back()));
53 }
54 if (pp > 1) {
55     prime.push_back(pp);
56     a.push_back(multiLucas(n, m, pp, pp));
57 }
58 return crt(a, prime, a.size());
59 };

```

5.13 Wilson

简单结论

对于素数 p 有

$$(p-1)! \equiv -1 \pmod{p}.$$

推论

令 $(n!)_p$ 表示不大于 n 且不被 p 整除的正整数的乘积.

特殊情形: n 为素数 p 时即为上述结论.

一般结论: 对素数 p 和正整数 q 有

$$((p^q)!)_p \equiv \pm 1 \pmod{p^q}.$$

详细定义:

$$((p^q)!)_p = \begin{cases} 1 & \text{if } p = 2 \text{ and } q \geq 3, \\ -1 & \text{other wise.} \end{cases}$$

5.14 LTE

将素数 p 在整数 n 中的个数记为 $v_p(n)$.

$$(n, p) = 1$$

对所有素数 p 和满足 $(n, p) = 1$ 的整数 n , 有

1. 若 $p \mid x - y$, 则有

$$v_p(x^n - y^n) = v_p(x - y).$$

2. 若 $p \mid x - y$, 则对奇数 n 有

$$v_p(x^n + y^n) = v_p(x + y).$$

p 是奇素数

对所有奇素数 p 有

1. 若 $p \mid x - y$, 则有

$$v_p(x^n - y^n) = v_p(x - y) + v_p(n).$$

2. 若 $p \mid x - y$, 则对奇数 n 有

$$v_p(x^n + y^n) = v_p(x + y) + v_p(n).$$

$p = 2$

对 $p = 2$ 且 $p \mid x - y$ 有

1. 对奇数 n 有

$$v_2(x^n - y^n) = v_2(x - y).$$

2. 对偶数 n 有

$$v_2(x^n - y^n) = v_2(x - y) + v_2(x + y) + v_2(n) - 1.$$

除此之外, 对上述 x, y, n , 若 $4 \mid x - y$, 有

1. $v_2(x + y) = 1$.
2. $v_2(x^n - y^n) = v_2(x - y) + v_2(n)$.

5.15 Mobius inversion

莫比乌斯函数

$$\mu(n) = \begin{cases} 1 & n = 1, \\ 0 & n \text{ 含有平方因子}, \\ (-1)^k & k \text{ 为 } n \text{ 的本质不同素因子个数}. \end{cases}$$

性质

$$\sum_{d \mid n} \mu(d) = \begin{cases} 1 & n = 1 \\ 0 & n \neq 1 \end{cases}.$$

$$\varphi(n) = \sum_{d \mid n} d \cdot \mu\left(\frac{n}{d}\right).$$

反演结论

$$[gcd(i, j) = 1] = \sum_{d \mid gcd(i, j)} \mu(d).$$

$O(n \log n)$ 求莫比乌斯函数

```

1 mu[1] = 1;
2 for (int i = 1; i <= n; i++){
3     for (int j = i + i; j <= n; j += i){

```

```

4 |      mu[j] -= mu[i];
5 |    }
6 | }

```

莫比乌斯变换

设 $f(n), F(n)$.

1. $F(n) = \sum_{d|n} f(d)$, 则 $f(n) = \sum_{d|n} \mu(d) F\left(\frac{n}{d}\right)$.
2. $F(n) = \sum_{n|d} f(d)$, 则 $f(n) = \sum_{n|d} \mu\left(\frac{d}{n}\right) F(d)$.

6 math - polynomial

6.1 FTT

FFT 与拆系数 FFT

```

1  const int sz = 1 << 23;
2  int rev[sz];
3  int rev_n;
4  void set_rev(int n) {
5      if (n == rev_n) return;
6      for (int i = 0; i < n; i++) rev[i] = (rev[i / 2] | (i & 1) * n) / 2;
7      rev_n = n;
8  }
9  template void butterfly(T* a, int n) {
10     set_rev(n);
11     for (int i = 0; i < n; i++) {
12         if (i < rev[i]) std::swap(a[i], a[rev[i]]);
13     }
14 }
15
16 namespace Comp {
17
18 long double pi = 3.141592653589793238;
19
20 template struct complex {
21     T x, y;
22     complex(T x = 0, T y = 0) : x(x), y(y) {}
23     complex operator+(const complex& b) const { return complex(x + b.x, y + b.y); }
24
25     complex operator-(const complex& b) const { return complex(x - b.x, y - b.y); }
26
27     complex operator*(const complex& b) const {
28         return complex<T>(x * b.x - y * b.y, x * b.y + y * b.x);
29     }
30     complex operator~() const { return complex(x, -y); }
31     static complex unit(long double rad) { return complex(std::cos(rad), std::sin(rad)); }
32 };
33
34 } // namespace Comp
35
36 struct fft_t {
37     typedef Comp::complex<double> complex;
38     complex wn[sz];
39
40     fft_t() {
41         for (int i = 0; i < sz / 2; i++) {
42             wn[sz / 2 + i] = complex::unit(2 * Comp::pi * i / sz);
43         }
44         for (int i = sz / 2 - 1; i; i--) wn[i] = wn[i * 2];
45     }
46
47     void operator()(complex* a, int n, int type) {
48         if (type == -1) std::reverse(a + 1, a + n);
49         butterfly(a, n);
50         for (int i = 1; i < n; i *= 2) {
51             const complex* w = wn + i;
52             for (complex* b = a, t; b != a + n; b += i + 1) {
53                 t = b[i];
54                 b[i] = *b - t;
55                 *b = *b + t;
56                 for (int j = 1; j < i; j++) {
57                     t = (++b)[i] * w[j];
58                     b[i] = *b - t;
59                     *b = *b + t;
60                 }
61             }
62         }
63         if (type == 1) return;
64         for (int i = 0; i < n * 2; i++) ((double*) a)[i] /= n;
65     }
66 } FFT;
67
68 typedef decltype(FFT)::complex complex;
69
70 vi fft(const vi& f, const vi& g) {
71     static complex ff[sz];
72     int n = f.size(), m = g.size();
73     vi h(n + m - 1);
74     if (std::min(n, m) <= 50) {
75         for (int i = 0; i < n; i++) {

```

```

76         for (int j = 0; j < m; ++j) {
77             h[i + j] += f[i] * g[j];
78         }
79     }
80     return h;
81 }
82 int c = 1;
83 while (c + 1 < n + m) c *= 2;
84 std::memset(ff, 0, sizeof(decltype(*(ff))) * (c));
85 for (int i = 0; i < n; i++) ff[i].x = f[i];
86 for (int i = 0; i < m; i++) ff[i].y = g[i];
87 FFT(ff, c, 1);
88 for (int i = 0; i < c; i++) ff[i] = ff[i] * ff[i];
89 FFT(ff, c, -1);
90 for (int i = 0; i + 1 < n + m; i++) h[i] = std::llround(ff[i].y / 2);
91 return h;
92 }
93
94 vi mtt(const vi& f, const vi& g) {
95     static complex ff[3][sz], gg[2][sz];
96     static int s[3] = {1, 31623, 31623 * 31623};
97     int n = f.size(), m = g.size();
98     vi h(n + m - 1);
99     if (std::min(n, m) <= 50) {
100         for (int i = 0; i < n; ++i) {
101             for (int j = 0; j < m; ++j) {
102                 Add(h[i + j], mul(f[i], g[j]));
103             }
104         }
105         return h;
106     }
107     int c = 1;
108     while (c + 1 < n + m) c *= 2;
109     for (int i = 0; i < 2; ++i) {
110         std::memset(ff[i], 0, sizeof(decltype(*(ff[i]))) * (c));
111         std::memset(gg[i], 0, sizeof(decltype(*(ff[i]))) * (c));
112         for (int j = 0; j < n; ++j) ff[i][j].x = f[j] / s[i] % s[1];
113         for (int j = 0; j < m; ++j) gg[i][j].x = g[j] / s[i] % s[1];
114         FFT(ff[i], c, 1);
115         FFT(gg[i], c, 1);
116     }
117     for (int i = 0; i < c; ++i) {
118         ff[2][i] = ff[1][i] * gg[1][i];
119         ff[1][i] = ff[1][i] * gg[0][i];
120         gg[1][i] = ff[0][i] * gg[1][i];
121         ff[0][i] = ff[0][i] * gg[0][i];
122     }
123     for (int i = 0; i < 3; ++i) {
124         FFT(ff[i], c, -1);
125         for (int j = 0; j + 1 < n + m; ++j) {
126             Add(h[j], mul(std::llround(ff[i][j].x) % mod, s[i]));
127         }
128     }
129     FFT(gg[1], c, -1);
130     for (int i = 0; i + 1 < n + m; ++i) {
131         Add(h[i], mul(std::llround(gg[1][i].x) % mod, s[1]));
132     }
133     return h;
134 }

```

6.2 FWT

各种分治过程: **and**:

$$\begin{aligned} \text{FWT}[A] &= \text{merge}(\text{FWT}[A_0] + \text{FWT}[A_1], \text{FWT}[A_1]), \\ \text{UFWT}[A'] &= \text{merge}(\text{UFWT}[A'_0] - \text{UFWT}[A'_1], \text{UFWT}[A'_1]). \end{aligned}$$

or:

$$\begin{aligned} \text{FWT}[A] &= \text{merge}(\text{FWT}[A_0], \text{FWT}[A_0] + \text{FWT}[A_1]), \\ \text{UFWT}[A'] &= \text{merge}(\text{UFWT}[A'_0], -\text{UFWT}[A'_0] + \text{UFWT}[A'_1]). \end{aligned}$$

xor:

$$\text{FWT}[A] = \text{merge}(\text{FWT}[A_0] + \text{FWT}[A_1], \text{FWT}[A_0] - \text{FWT}[A_1]),$$

$$\text{UFWT}[A'] = \text{merge} \left(\frac{\text{UFWT}[A'_0] + \text{UFWT}[A'_1]}{2}, \frac{\text{UFWT}[A'_0] - \text{UFWT}[A'_1]}{2} \right).$$

```

1  /* FWT */
2  auto FWT_and = [&](vi v, int type) -> vi {
3      int n = v.size();
4      for (int mid = 1; mid < n; mid <= 1) {
5          for (int block = mid < 1, j = 0; j < n; j += block) {
6              for (int i = j; i < j + mid; i++) {
7                  LL x = v[i], y = v[i + mid];
8                  if (type == 1) {
9                      v[i] = add(x, y);
10                 } else {
11                     v[i] = sub(x, y);
12                 }
13             }
14         }
15     }
16     return v;
17 };
18
19 auto FWT_or = [&](vi v, int type) -> vi {
20     int n = v.size();
21     for (int mid = 1; mid < n; mid <= 1) {
22         for (int block = mid < 1, j = 0; j < n; j += block) {
23             for (int i = j; i < j + mid; i++) {
24                 LL x = v[i], y = v[i + mid];
25                 if (type == 1) {
26                     v[i + mid] = add(x, y);
27                 } else {
28                     v[i + mid] = sub(y, x);
29                 }
30             }
31         }
32     }
33     return v;
34 };
35
36 auto FWT_xor = [&](vi v, int type) -> vi {
37     int n = v.size();
38     for (int mid = 1; mid < n; mid <= 1) {
39         for (int block = mid < 1, j = 0; j < n; j += block) {
40             for (int i = j; i < j + mid; i++) {
41                 LL x = v[i], y = v[i + mid];
42                 v[i] = add(x, y);
43                 v[i + mid] = sub(x, y);
44                 if (type == -1) {
45                     Mul(v[i], inv2);
46                     Mul(v[i + mid], inv2);
47                 }
48             }
49         }
50     }
51     return v;
52 };
53
54 a = FWT(a, 1), b = FWT(b, 1);
55 for (int i = 0; i < (1 << n); i++) {
56     c[i] = mul(a[i], b[i]);
57 }
58 c = FWT(c, -1);

```

```

1  /* FWT @ wrb */
2  void FMTor(int f[]) {
3      for (int i = 0; i < n; ++i)
4          for (int j = 0; j < m; ++j)
5              if (j >> i & 1) f[j] = (f[j] + f[j ^ 1 << i]) % P;
6  }
7  void FMToriv(int f[]) {
8      for (int i = 0; i < n; ++i)
9          for (int j = 0; j < m; ++j)
10             if (j >> i & 1) f[j] = (f[j] - f[j ^ 1 << i] + P) % P;
11 }
12 void FMTand(int f[]) {
13     for (int i = 0; i < n; ++i)
14         for (int j = 0; j < m; ++j)
15             if (~j >> i & 1) f[j] = (f[j] + f[j ^ 1 << i]) % P;
16 }
17 void FMTandiv(int f[]) {
18     for (int i = 0; i < n; ++i)
19         for (int j = 0; j < m; ++j)
20             if (~j >> i & 1) f[j] = (f[j] - f[j ^ 1 << i] + P) % P;
21 }
22 void FWT(int f[]) {

```



```

23     for (int len = 1; len < m; len *= 2) {
24         for (int i = 0; i < m; i += len * 2) {
25             for (int j = i; j < i + len; ++j) {
26                 int x = f[j], y = f[j + len];
27                 f[j] = (x + y) % P;
28                 f[j + len] = (x - y + P) % P;
29             }
30         }
31     }
32 }
33 void FWTiv(int f[]) {
34     for (int len = 1; len < m; len *= 2) {
35         for (int i = 0; i < m; i += len * 2) {
36             for (int j = i; j < i + len; ++j) {
37                 int x = f[j], y = f[j + len];
38                 f[j] = 111 * (x + y) * iv2 % P;
39                 f[j + len] = 111 * (x - y + P) * iv2 % P;
40             }
41         }
42     }
43 }

```

6.3 class polynomial

```

1  class polynomial : public vi {
2      public:
3          polynomial() = default;
4          polynomial(const vi& v) : vi(v) {}
5          polynomial(vi&& v) : vi(std::move(v)) {}
6
7          int degree() { return size() - 1; }
8
9          void clearzero() {
10             while (size() && !back()) pop_back();
11         }
12 };
13
14
15 polynomial& operator+=(polynomial& a, const polynomial& b) {
16     a.resize(std::max(a.size(), b.size()), 0);
17     for (int i = 0; i < b.size(); i++) {
18         Add(a[i], b[i]);
19     }
20     a.clearzero();
21     return a;
22 }
23
24 polynomial operator+(const polynomial& a, const polynomial& b) {
25     polynomial ans = a;
26     return ans += b;
27 }
28
29 polynomial& operator-=(polynomial& a, const polynomial& b) {
30     a.resize(std::max(a.size(), b.size()), 0);
31     for (int i = 0; i < b.size(); i++) {
32         Sub(a[i], b[i]);
33     }
34     a.clearzero();
35     return a;
36 }
37
38 polynomial operator-(const polynomial& a, const polynomial& b) {
39     polynomial ans = a;
40     return ans -= b;
41 }
42
43 class ntt_t {
44     public:
45         static const int maxbit = 22;
46         static const int sz = 1 << maxbit;
47         static const int mod = 998244353;
48         static const int g = 3;
49
50         std::array<int, sz + 10> w;
51         std::array<int, maxbit + 10> len_inv;
52
53         ntt_t() {
54             int wn = pow(g, (mod - 1) >> maxbit);
55             w[0] = 1;
56             for (int i = 1; i <= sz; i++) {
57                 w[i] = mul(w[i - 1], wn);
58             }
59             len_inv[maxbit] = pow(sz, mod - 2);

```

```

60     for (int i = maxbit - 1; ~i; i--) {
61         len_inv[i] = add(len_inv[i + 1], len_inv[i + 1]);
62     }
63 }
64
65 void operator()(vi& v, int& n, int type) {
66     int bit = 0;
67     while ((1 << bit) < n) bit++;
68     int tot = (1 << bit);
69     v.resize(tot, 0);
70     vi rev(tot);
71     n = tot;
72     for (int i = 0; i < tot; i++) {
73         rev[i] = rev[i >> 1] >> 1;
74         if (i & 1) {
75             rev[i] |= tot >> 1;
76         }
77     }
78     for (int i = 0; i < tot; i++) {
79         if (i < rev[i]) {
80             std::swap(v[i], v[rev[i]]);
81         }
82     }
83     for (int midd = 0; (1 << midd) < tot; midd++) {
84         int mid = 1 << midd;
85         int len = mid << 1;
86         for (int i = 0; i < tot; i += len) {
87             for (int j = 0; j < mid; j++) {
88                 int w0 = v[i + j];
89                 int w1 = mul(
90                     w[type == 1 ? (j << maxbit - midd - 1) : (len - j << maxbit - midd - 1)],
91                     v[i + j + mid]);
92                 v[i + j] = add(w0, w1);
93                 v[i + j + mid] = sub(w0, w1);
94             }
95         }
96     }
97     if (type == -1) {
98         for (int i = 0; i < tot; i++) {
99             v[i] = mul(v[i], len_inv[bit]);
100         }
101     }
102 }
103 } NTT;

```

乘法

```

1  polynomial& operator*=(polynomial& a, const polynomial& b) {
2      if (!a.size() || !b.size()) {
3          a.resize(0);
4          return a;
5      }
6      polynomial tmp = b;
7      int deg = a.size() + b.size() - 1;
8      int temp = deg;
9
10     // 项数较小直接硬算
11
12     if ((LL) a.size() * (LL) b.size() <= (LL) deg * 50LL) {
13         tmp.resize(0);
14         tmp.resize(deg, 0);
15         for (int i = 0; i < a.size(); i++) {
16             for (int j = 0; j < b.size(); j++) {
17                 tmp[i + j] = add(tmp[i + j], mul(a[i], b[j]));
18             }
19         }
20         a = tmp;
21         return a;
22     }
23
24     // 项数较多跑 NTT
25
26     NTT(a, deg, 1);
27     NTT(tmp, deg, 1);
28     for (int i = 0; i < deg; i++) {
29         Mul(a[i], tmp[i]);
30     }
31     NTT(a, deg, -1);
32     a.resize(temp);
33     return a;
34 }
35

```

```

36 polynomial operator*(const polynomial& a, const polynomial& b) {
37     polynomial ans = a;
38     return ans *= b;
39 }

```

逆

```

1  polynomial inverse(const polynomial& a) {
2      polynomial ans({pow(a[0], mod - 2)});
3      polynomial temp;
4      polynomial tempa;
5      int deg = a.size();
6      for (int i = 0; (1 << i) < deg; i++) {
7          tempa.resize(0);
8          tempa.resize(1 << i << 1, 0);
9          for (int j = 0; j != tempa.size() and j != deg; j++) {
10             tempa[j] = a[j];
11         }
12         temp = ans * (polynomial({2}) - tempa * ans);
13         if (temp.size() > (1 << i << 1)) {
14             temp.resize(1 << i << 1, 0);
15         }
16         temp.clearzero();
17         std::swap(temp, ans);
18     }
19     ans.resize(deg);
20     return ans;
21 }

```

对数

```

1  polynomial differential(const polynomial& a) {
2      if (!a.size()) {
3          return a;
4      }
5      polynomial ans(vi(a.size() - 1));
6      for (int i = 1; i < a.size(); i++) {
7          ans[i - 1] = mul(a[i], i);
8      }
9      return ans;
10 }
11
12 polynomial integral(const polynomial& a) {
13     polynomial ans(vi(a.size() + 1));
14     for (int i = 0; i < a.size(); i++) {
15         ans[i + 1] = mul(a[i], pow(i + 1, mod - 2));
16     }
17     return ans;
18 }
19
20 polynomial ln(const polynomial& a) {
21     int deg = a.size();
22     polynomial da = differential(a);
23     polynomial inva = inverse(a);
24     polynomial ans = integral(da * inva);
25     ans.resize(deg);
26     return ans;
27 }

```

指数

```

1  polynomial exp(const polynomial& a) {
2      polynomial ans({1});
3      polynomial temp;
4      polynomial tempa;
5      polynomial tempaa;
6      int deg = a.size();
7      for (int i = 0; (1 << i) < deg; i++) {
8          tempa.resize(0);
9          tempa.resize(1 << i << 1, 0);
10         for (int j = 0; j != tempa.size() and j != deg; j++) {
11             tempa[j] = a[j];
12         }
13         tempaa = ans;

```

```

14     tempaa.resize(1 << i << 1);
15     temp = ans * (tempa + polynomial({1}) - ln(tempaa));
16     if (temp.size() > (1 << i << 1)) {
17         temp.resize(1 << i << 1, 0);
18     }
19     temp.clearzero();
20     std::swap(temp, ans);
21 }
22 ans.resize(deg);
23 return ans;
24 }

```

根号

```

1 polynomial sqrt(polynomial& a) {
2     polynomial ans({cipolla(a[0])});
3     if (ans[0] == -1) return ans;
4     polynomial temp;
5     polynomial tempa;
6     polynomial tempaa;
7     int deg = a.size();
8     for (int i = 0; (1 << i) < deg; i++) {
9         tempa.resize(0);
10        tempa.resize(1 << i << 1, 0);
11        for (int j = 0; j != tempa.size() and j != deg; j++) {
12            tempa[j] = a[j];
13        }
14        tempaa = ans;
15        tempaa.resize(1 << i << 1);
16        temp = (tempa * inverse(tempaa) + ans) * inv2;
17        if (temp.size() > (1 << i << 1)) {
18            temp.resize(1 << i << 1, 0);
19        }
20        temp.clearzero();
21        std::swap(temp, ans);
22    }
23    ans.resize(deg);
24    return ans;
25 }
26
27 // 特判 //
28
29 int cnt = 0;
30 for (int i = 0; i < a.size(); i++) {
31     if (a[i] == 0) {
32         cnt++;
33     } else {
34         break;
35     }
36 }
37 if (cnt) {
38     if (cnt == n) {
39         for (int i = 0; i < n; i++) {
40             std::cout << "0 ";
41         }
42         std::cout << endl;
43         return 0;
44     }
45     if (cnt & 1) {
46         std::cout << "-1" << endl;
47         return 0;
48     }
49     polynomial b(vi(a.size() - cnt));
50     for (int i = cnt; i < a.size(); i++) {
51         b[i - cnt] = a[i];
52     }
53     a = b;
54 }
55 a.resize(n - cnt / 2);
56 a = sqrt(a);
57 if (a[0] == -1) {
58     std::cout << "-1" << endl;
59     return 0;
60 }
61 reverse(all(a));
62 a.resize(n);
63 reverse(all(a));

```

6.4 wsy poly

```

1  #include <bits/stdc++.h>
2
3  using ul = std::uint32_t;
4  using li = std::int32_t;
5  using ll = std::int64_t;
6  using ull = std::uint64_t;
7  using llf = long double;
8  using lf = double;
9  using vul = std::vector<ul>;
10 using vvul = std::vector<vul>;
11 using pulb = std::pair<ul, bool>;
12 using vpulb = std::vector<pulb>;
13 using vvpulb = std::vector<vpulb>;
14 using vb = std::vector<bool>;
15
16 const ul base = 998244353;
17
18 std::mt19937 rnd;
19
20 ul plus(ul a, ul b) { return a + b < base ? a + b : a + b - base; }
21
22 ul minus(ul a, ul b) { return a < b ? a + base - b : a - b; }
23
24 ul mul(ul a, ul b) { return ull(a) * ull(b) % base; }
25
26 void exgcd(li a, li b, li& x, li& y) {
27     if (b) {
28         exgcd(b, a % b, y, x);
29         y -= x * (a / b);
30     } else {
31         x = 1;
32         y = 0;
33     }
34 }
35
36 ul inverse(ul a) {
37     li x, y;
38     exgcd(a, base, x, y);
39     return x < 0 ? x + li(base) : x;
40 }
41
42 ul pow(ul a, ul b) {
43     ul ret = 1;
44     ul temp = a;
45     while (b) {
46         if (b & 1) {
47             ret = mul(ret, temp);
48         }
49         temp = mul(temp, temp);
50         b >>= 1;
51     }
52     return ret;
53 }
54
55
56 ul sqrt(ul x) {
57     ul a;
58     ul w2;
59     while (true) {
60         a = rnd() % base;
61         w2 = minus(mul(a, a), x);
62         if (pow(w2, base - 1 >> 1) == base - 1) {
63             break;
64         }
65     }
66     ul b = base + 1 >> 1;
67     ul rs = 1, rt = 0;
68     ul as = a, at = 1;
69     ul qs, qt;
70     while (b) {
71         if (b & 1) {
72             qs = plus(mul(rs, as), mul(mul(rt, at), w2));
73             qt = plus(mul(rs, at), mul(rt, as));
74             rs = qs;
75             rt = qt;
76         }
77         b >>= 1;
78         qs = plus(mul(as, as), mul(mul(at, at), w2));
79         qt = plus(mul(as, at), mul(as, at));
80         as = qs;
81         at = qt;
82     }
83     return rs + rs < base ? rs : base - rs;
84 }

```

```

85 |
86 | ul log(ul x, ul y, bool initd = false) {
87 |     static std::map<ul, ul> bs;
88 |     const ul d = std::round(std::sqrt(1f(base - 1)));
89 |     if (!initd) {
90 |         bs.clear();
91 |         for (ul i = 0, j = 1; i != d; ++i, j = mul(j, x)) {
92 |             bs[j] = i;
93 |         }
94 |     }
95 |     ul temp = inverse(pow(x, d));
96 |     for (ul i = 0, j = 1;; i += d, j = mul(j, temp)) {
97 |         auto it = bs.find(mul(y, j));
98 |         if (it != bs.end()) {
99 |             return it->second + i;
100 |         }
101 |     }
102 | }
103 |
104 | ul powroot(ul x, ul y, bool initd = false) {
105 |     const ul g = 3;
106 |     ul lgx = log(g, x, initd);
107 |     li s, t;
108 |     exgcd(y, base - 1, s, t);
109 |     if (s < 0) {
110 |         s += base - 1;
111 |     }
112 |     return pow(g, ull(s) * ull(lgx) % (base - 1));
113 | }
114 |
115 | class polynomial : public vul {
116 | public:
117 |     void clearzero() {
118 |         while (size() && !back()) {
119 |             pop_back();
120 |         }
121 |     }
122 |     polynomial() = default;
123 |     polynomial(const vul& a) : vul(a) {}
124 |     polynomial(vul&& a) : vul(std::move(a)) {}
125 |     ul degree() const { return size() - 1; }
126 |     ul operator()(ul x) const {
127 |         ul ret = 0;
128 |         for (ul i = size() - 1; ~i; --i) {
129 |             ret = mul(ret, x);
130 |             ret = plus(ret, vul::operator[](i));
131 |         }
132 |         return ret;
133 |     }
134 | };
135 |
136 | polynomial& operator+=(polynomial& a, const polynomial& b) {
137 |     a.resize(std::max(a.size(), b.size()), 0);
138 |     for (ul i = 0; i != b.size(); ++i) {
139 |         a[i] = plus(a[i], b[i]);
140 |     }
141 |     a.clearzero();
142 |     return a;
143 | }
144 |
145 | polynomial operator+(const polynomial& a, const polynomial& b) {
146 |     polynomial ret = a;
147 |     return ret += b;
148 | }
149 |
150 | polynomial& operator-=(polynomial& a, const polynomial& b) {
151 |     a.resize(std::max(a.size(), b.size()), 0);
152 |     for (ul i = 0; i != b.size(); ++i) {
153 |         a[i] = minus(a[i], b[i]);
154 |     }
155 |     a.clearzero();
156 |     return a;
157 | }
158 |
159 | polynomial operator-(const polynomial& a, const polynomial& b) {
160 |     polynomial ret = a;
161 |     return ret -= b;
162 | }
163 |
164 | class ntt_t {
165 | public:
166 |     static const ul lgysz = 20;
167 |     static const ul sz = 1 << lgysz;
168 |     static const ul g = 3;
169 |     ul w[sz + 1];
170 |     ul leninv[lgysz + 1];
171 |     ntt_t() {

```

```

172     ul wn = pow(g, (base - 1) >> lgysz);
173     w[0] = 1;
174     for (ul i = 1; i <= sz; ++i) {
175         w[i] = mul(w[i - 1], wn);
176     }
177     leninv[lgysz] = inverse(sz);
178     for (ul i = lgysz - 1; ~i; --i) {
179         leninv[i] = plus(leninv[i + 1], leninv[i + 1]);
180     }
181 }
182 void operator()(vul& v, ul& n, bool inv) {
183     ul lgn = 0;
184     while ((1 << lgn) < n) {
185         ++lgn;
186     }
187     n = 1 << lgn;
188     v.resize(n, 0);
189     for (ul i = 0, j = 0; i != n; ++i) {
190         if (i < j) {
191             std::swap(v[i], v[j]);
192         }
193         ul k = n >> 1;
194         while (k & j) {
195             j ^= ~k;
196             k >>= 1;
197         }
198         j |= k;
199     }
200     for (ul lgmid = 0; (1 << lgmid) != n; ++lgmid) {
201         ul mid = 1 << lgmid;
202         ul len = mid << 1;
203         for (ul i = 0; i != n; i += len) {
204             for (ul j = 0; j != mid; ++j) {
205                 ul t0 = v[i + j];
206                 ul t1 =
207                     mul(w[inv ? (len - j << lgysz - lgmid - 1) : (j << lgysz - lgmid - 1)],
208                         v[i + j + mid]);
209                 v[i + j] = plus(t0, t1);
210                 v[i + j + mid] = minus(t0, t1);
211             }
212         }
213     }
214     if (inv) {
215         for (ul i = 0; i != n; ++i) {
216             v[i] = mul(v[i], leninv[lgn]);
217         }
218     }
219 }
220 } ntt;
221
222 polynomial& operator*=(polynomial& a, const polynomial& b) {
223     if (!b.size() || !a.size()) {
224         a.resize(0);
225         return a;
226     }
227     polynomial temp = b;
228     ul npmp1 = a.size() + b.size() - 1;
229     if (ull(a.size()) * ull(b.size()) <= ull(npmp1) * ull(50)) {
230         temp.resize(0);
231         temp.resize(npmp1, 0);
232         for (ul i = 0; i != a.size(); ++i) {
233             for (ul j = 0; j != b.size(); ++j) {
234                 temp[i + j] = plus(temp[i + j], mul(a[i], b[j]));
235             }
236         }
237         a = temp;
238         a.clearzero();
239         return a;
240     }
241     ntt(a, npmp1, false);
242     ntt(temp, npmp1, false);
243     for (ul i = 0; i != npmp1; ++i) {
244         a[i] = mul(a[i], temp[i]);
245     }
246     ntt(a, npmp1, true);
247     a.clearzero();
248     return a;
249 }
250
251 polynomial operator*(const polynomial& a, const polynomial& b) {
252     polynomial ret = a;
253     return ret *= b;
254 }
255
256 polynomial& operator*=(polynomial& a, ul b) {
257     if (!b) {
258         a.resize(0);

```

```

259     return a;
260 }
261 for (ul i = 0; i != a.size(); ++i) {
262     a[i] = mul(a[i], b);
263 }
264 return a;
265 }
266
267 polynomial operator*(const polynomial& a, ul b) {
268     polynomial ret = a;
269     return ret *= b;
270 }
271
272 polynomial inverse(const polynomial& a, ul lgdeg) {
273     polynomial ret({inverse(a[0])});
274     polynomial temp;
275     polynomial tempa;
276     for (ul i = 0; i != lgdeg; ++i) {
277         tempa.resize(0);
278         tempa.resize(1 << i << 1, 0);
279         for (ul j = 0; j != tempa.size() && j != a.size(); ++j) {
280             tempa[j] = a[j];
281         }
282         temp = ret * (polynomial({2}) - tempa * ret);
283         if (temp.size() > (1 << i << 1)) {
284             temp.resize(1 << i << 1, 0);
285         }
286         temp.clearzero();
287         std::swap(temp, ret);
288     }
289     return ret;
290 }
291
292 void quotientremain(const polynomial& a, polynomial b, polynomial& q, polynomial& r) {
293     if (a.size() < b.size()) {
294         q = polynomial();
295         r = std::move(a);
296         return;
297     }
298     std::reverse(b.begin(), b.end());
299     auto ta = a;
300     std::reverse(ta.begin(), ta.end());
301     ul n = a.size() - 1;
302     ul m = b.size() - 1;
303     ta.resize(n - m + 1);
304     ul lgnmmp1 = 0;
305     while ((1 << lgnmmp1) < n - m + 1) {
306         ++lgnmmp1;
307     }
308     q = ta * inverse(b, lgnmmp1);
309     q.resize(n - m + 1);
310     std::reverse(b.begin(), b.end());
311     std::reverse(q.begin(), q.end());
312     r = a - b * q;
313 }
314
315 polynomial mod(const polynomial& a, const polynomial& b) {
316     polynomial q, r;
317     quotientremain(a, b, q, r);
318     return r;
319 }
320
321 polynomial quotient(const polynomial& a, const polynomial& b) {
322     polynomial q, r;
323     quotientremain(a, b, q, r);
324     return q;
325 }
326
327 polynomial sqrt(const polynomial& a, ul lgdeg) {
328     polynomial ret({sqrt(a[0])});
329     polynomial temp;
330     polynomial tempa;
331     for (ul i = 0; i != lgdeg; ++i) {
332         tempa.resize(0);
333         tempa.resize(1 << i << 1, 0);
334         for (ul j = 0; j != tempa.size() && j != a.size(); ++j) {
335             tempa[j] = a[j];
336         }
337         temp = (tempa * inverse(ret, i + 1) + ret) * (base + 1 >> 1);
338         if (temp.size() > (1 << i << 1)) {
339             temp.resize(1 << i << 1, 0);
340         }
341         temp.clearzero();
342         std::swap(temp, ret);
343     }
344     return ret;
345 }

```



```

346 polynomial differential(const polynomial& a) {
347     if (!a.size()) {
348         return a;
349     }
350     polynomial ret(vul(a.size() - 1, 0));
351     for (ul i = 1; i != a.size(); ++i) {
352         ret[i - 1] = mul(a[i], i);
353     }
354     return ret;
355 }
356
357 polynomial integral(const polynomial& a) {
358     polynomial ret(vul(a.size() + 1, 0));
359     for (ul i = 0; i != a.size(); ++i) {
360         ret[i + 1] = mul(a[i], inverse(i + 1));
361     }
362     return ret;
363 }
364
365 polynomial ln(const polynomial& a, ul lgdeg) {
366     polynomial da = differential(a);
367     polynomial inva = inverse(a, lgdeg);
368     polynomial ret = integral(da * inva);
369     if (ret.size() > (1 << lgdeg)) {
370         ret.resize(1 << lgdeg);
371         ret.clearzero();
372     }
373     return ret;
374 }
375
376 polynomial exp(const polynomial& a, ul lgdeg) {
377     polynomial ret({1});
378     polynomial temp;
379     polynomial tempa;
380     for (ul i = 0; i != lgdeg; ++i) {
381         tempa.resize(0);
382         tempa.resize(1 << i << 1, 0);
383         for (ul j = 0; j != tempa.size() && j != a.size(); ++j) {
384             tempa[j] = a[j];
385         }
386         temp = ret * (polynomial({1}) - ln(ret, i + 1) + tempa);
387         if (temp.size() > (1 << i << 1)) {
388             temp.resize(1 << i << 1, 0);
389         }
390         temp.clearzero();
391         std::swap(temp, ret);
392     }
393     return ret;
394 }
395
396 polynomial pow(const polynomial& a, ul k, ul lgdeg) { return exp(ln(a, lgdeg) * k, lgdeg); }
397
398 polynomial alpi[1 << 16][17];
399
400 polynomial getalpi(const ul x[], ul l, ul lgrml) {
401     if (lgrml == 0) {
402         return alpi[l][lgrml] = vul({minus(0, x[l]), 1});
403     }
404     return alpi[l][lgrml] = getalpi(x, l, lgrml - 1) * getalpi(x, l + (1 << lgrml - 1), lgrml - 1);
405 }
406
407 void multians(const polynomial& f, const ul x[], ul y[], ul l, ul lgrml) {
408     if (f.size() <= 700) {
409         for (ul i = l; i != l + (1 << lgrml); ++i) {
410             y[i] = f(x[i]);
411         }
412         return;
413     }
414     if (lgrml == 0) {
415         y[l] = f(x[l]);
416         return;
417     }
418     multians(mod(f, alpi[l][lgrml - 1]), x, y, l, lgrml - 1);
419     multians(mod(f, alpi[l + (1 << lgrml - 1)][lgrml - 1]), x, y, l + (1 << lgrml - 1), lgrml - 1);
420 }
421
422 ul sqrt(ul x) {
423     ul a;
424     ul w2;
425     while (true) {
426         a = rnd() % base;
427         w2 = minus(mul(a, a), x);
428         if (pow(w2, base - 1 >> 1) == base - 1) {
429             break;
430         }
431     }
432 }

```

```

433     ul b = base + 1 >> 1;
434     ul rs = 1, rt = 0;
435     ul as = a, at = 1;
436     ul qs, qt;
437     while (b) {
438         if (b & 1) {
439             qs = plus(mul(rs, as), mul(mul(rt, at), w2));
440             qt = plus(mul(rs, at), mul(rt, as));
441             rs = qs;
442             rt = qt;
443         }
444         b >>= 1;
445         qs = plus(mul(as, as), mul(mul(at, at), w2));
446         qt = plus(mul(as, at), mul(as, at));
447         as = qs;
448         at = qt;
449     }
450     return rs + rs < base ? rs : base - rs;
451 }
452
453 ul log(ul x, ul y, bool initied = false) {
454     static std::map<ul, ul> bs;
455     const ul d = std::round(std::sqrt(1f(base - 1)));
456     if (!initied) {
457         bs.clear();
458         for (ul i = 0, j = 1; i != d; ++i, j = mul(j, x)) {
459             bs[j] = i;
460         }
461     }
462     ul temp = inverse(pow(x, d));
463     for (ul i = 0, j = 1;; i += d, j = mul(j, temp)) {
464         auto it = bs.find(mul(y, j));
465         if (it != bs.end()) {
466             return it->second + i;
467         }
468     }
469 }
470
471 ul powroot(ul x, ul y, bool initied = false) {
472     const ul g = 3;
473     ul lgx = log(g, x, initied);
474     li s, t;
475     exgcd(y, base - 1, s, t);
476     if (s < 0) {
477         s += base - 1;
478     }
479     return pow(g, ull(s) * ull(lgx) % (base - 1));
480 }
481
482 ul n;
483
484 int main() {
485     std::scanf("%u", &n);
486     polynomial f;
487     for (ul i = 0; i <= n; ++i) {
488         ul t;
489         std::scanf("%u", &t);
490         f.push_back(t % base);
491     }
492     polynomial g = exp(ln(f * inverse(f[0]), 17) * inverse(3), 17) * powroot(f[0], 3);
493     while (g.size() <= n) {
494         g.push_back(0);
495     }
496     for (ul i = 0; i <= n; ++i) {
497         if (i) {
498             std::putchar(' ');
499         }
500         std::printf("%u", g[i]);
501     }
502     std::putchar('\n');
503     return 0;
504 }

```

Lagrange interpolation

一般的插值

给出一个多项式 $f(x)$ 上的 n 个点 (x_i, y_i) , 求 $f(k)$.

插值的结果是

$$f(x) = \sum_{i=1}^n y_i \prod_{j \neq i} \frac{x - x_j}{x_i - x_j},$$

直接带入 k 并且取模即可, 时间复杂度 $O(n^2)$.

```

1  /* interpolation */
2  auto lagrange = (const vi& x, const vi& y, int n, int k) {
3      for (int i = 1; i <= n; i++) {
4          LL s1 = y[i] % mod, s2 = 1ll;
5          for (int j = 1; j <= n; j++) {
6              if (i != j) {
7                  s1 = s1 * (k - x[j]) % mod;
8                  s2 = s2 * (x[i] - x[j]) % mod;
9              }
10         }
11         Add(ans, mul(s1, quick_power(s2, mod - 2, mod)));
12     }
13     return ans;
14 };

```

坐标连续的插值

给出的点是 $(0, y_0), \dots, (n, y_n)$.

```

1  /* interpolation */
2  auto lagrange(int n, std::vector<Z> y, int x) -> Z {
3      if (x <= n) return y[x];
4      Z s1 = 1, s2 = 0;
5      for (int i = 1; i <= n; i++) s1 *= (x - i);
6      for (int i = 1; i <= n; i++) {
7          Z res = ((n - i) & 1 ? -y[i] : y[i]);
8          res /= (x - i);
9          res *= fiv[i - 1];
10         res *= fiv[n - i];
11         s2 += res;
12     }
13     return s1 * s2;
14 }

```

7 math - game theory

7.1 nim game

若 nim 和为 0, 则先手必败.

暴力打表.

```

1 vi SG(21, -1); /* 记忆化 */
2 std::function<int(int, int)> sg = [&](int x) -> int {
3     if (/* 为最终态 */) return SG[x] = 0;
4     if (SG[x] != -1) return SG[x];
5     vi st;
6     for (/* 枚举所有可到达的状态 y */) {
7         st.push_back(sg(y));
8     }
9     std::sort(all(st));
10    for (int i = 0; i < st.size(); i++) {
11        if (st[i] != i) return SG[x] = i;
12    }
13    return SG[x] = st.size();
14 };

```

7.2 anti - nim game

若

1. 所有堆的石子均为一个, 且 nim 和不为 0,
2. 至少有一堆石子超过一个, 且 nim 和为 0,

则先手必败.

8 math - linear algebra

8.1 matrix

determinant mod 998244353

```

1  /* determinant */
2  auto det = [&](int n, vvi e) -> int {
3      int ans = 1;
4      for (int i = 1; i <= n; i++) {
5          if (a[i][i] == 0) {
6              for (int j = i + 1; j <= n; j++) {
7                  if (a[j][i] != 0) {
8                      for (int k = i; k <= n; k++) {
9                          std::swap(a[i][k], a[j][k]);
10                     }
11                     ans = sub(mod, ans);
12                     break;
13                 }
14             }
15         }
16         if (a[i][i] == 0) return 0;
17         Mul(ans, a[i][i]);
18         int x = pow(a[i][i], mod - 2);
19         for (int k = i; k <= n; k++) {
20             Mul(a[i][k], x);
21         }
22         for (int j = i + 1; j <= n; j++) {
23             int x = a[j][i];
24             for (int k = i; k <= n; k++) {
25                 Sub(a[j][k], mul(a[i][k], x));
26             }
27         }
28     }
29     return ans;
30 };

```

determinant mod non-prime

```

1  /* determinant @ wrb */
2  int a[609][609];
3  int main() {
4      int n, P, z = 1;
5      std::cin >> n >> P;
6      for (int i = 1; i <= n; ++i) {
7          for (int j = 1; j <= n; ++j) std::cin >> a[i][j];
8      }
9      for (int i = 1; i <= n; ++i) {
10         for (int j = i + 1; j <= n; ++j) {
11             while (a[j][i]) {
12                 z = -z;
13                 int d = a[i][i] / a[j][i];
14                 for (int k = i; k <= n; ++k) {
15                     int x = (a[i][k] - 111 * d * a[j][k]) % P;
16                     a[i][k] = a[j][k], a[j][k] = x;
17                 }
18             }
19         }
20         z = 111 * z * a[i][i] % P;
21     }
22     std::cout << (z + P) % P;
23 }

```

matrix multiplication

$A_{n \times m}$ 与 $B_{m \times k}$ 相乘并模 998244353.

```

1  /* matrix multiplication */
2  auto matmul = [&](int n, int m, int k, const vvi& a, const vvi& b) -> vvi {
3      vvi c(n + 1, vi(k + 1));
4      for (int i = 1; i <= n; i++) {
5          for (int l = 1; l <= m; l++) {

```



```

41 struct Liner_Basis {
42     using u64 = unsigned long long;
43     static const size_t M = 60;
44     u64 a[M + 1];
45     size_t sz;
46     size_t size() {
47         return sz;
48     }
49     Liner_Basis& operator+=(u64 x) {
50         for (size_t i = M; ~i && x; --i)
51             if (x >> i & 1)
52                 if (a[i]) x ^= a[i];
53             else return a[i] = x, ++sz, *this;
54         return *this;
55     }
56     Liner_Basis& operator+=(const Liner_Basis&) {
57         for (u64 x : _a) if (x) *this += x;
58         return *this;
59     }
60     Liner_Basis operator+(u64 x) {
61         Liner_Basis z = *this;
62         return z += x;
63     }
64     Liner_Basis operator+(const Liner_Basis&) {
65         Liner_Basis z = *this;
66         return z += _;
67     }
68     u64 qry(u64 x = 0) {
69         for (size_t i = M; ~i; --i)
70             if ((x ^ a[i]) > x) x ^= a[i];
71         return x;
72     }
73     u64 rank(u64 x) {
74         u64 h = 1, z = 0;
75         for (size_t i = 0; i <= M; ++i)
76             if (a[i]) {
77                 if (x >> i & 1) z += h;
78                 h <<= 1;
79             }
80         return z;
81     }
82     u64 kth(u64 x) {
83         u64 z = 0;
84         for (size_t i = M; ~i; --i)
85             if (x >> i & 1) z ^= a[i];
86         return z;
87     }
88 }v;
89 using LB = Liner_Basis;

```

8.3 linear programming

8.4 bm

```

1  /* bm @ wrb */
2  const int p = 998244353;
3  auto power = [](int a, int b = p - 2) {
4      int z = 1;
5      while (b) {
6          if (b & 1) z = 1ll * z * a % p;
7          a = 1ll * a * a % p, b >>= 1;
8      }
9      return z;
10 };
11 vector<int> berlekamp_massey(const vector<int> &a) {
12     vector<int> v, last; // v is the answer, 0-based, p is the module
13     int k = -1, delta = 0;
14
15     for (int i = 0; i < (int)a.size(); i++) {
16         int tmp = 0;
17         for (int j = 0; j < (int)v.size(); j++)
18             tmp = (tmp + (long long)a[i - j - 1] * v[j]) % p;
19
20         if (a[i] == tmp) continue;
21
22         if (k < 0) {
23             k = i;
24             delta = (a[i] - tmp + p) % p;
25             v = vector<int>(i + 1);
26
27             continue;

```

```

28     }
29
30     vector<int> u = v;
31     int val = (long long)(a[i] - tmp + p) * power(delta, p - 2) % p;
32
33     if (v.size() < last.size() + i - k) v.resize(last.size() + i - k);
34
35     (v[i - k - 1] += val) %= p;
36
37     for (int j = 0; j < (int)last.size(); j++) {
38         v[i - k + j] = (v[i - k + j] - (long long)val * last[j]) % p;
39         if (v[i - k + j] < 0) v[i - k + j] += p;
40     }
41
42     if ((int)u.size() - i < (int)last.size() - k) {
43         last = u;
44         k = i;
45         delta = a[i] - tmp;
46         if (delta < 0) delta += p;
47     }
48 }
49
50 for (auto &x : v) x = (p - x) % p;
51 v.insert(v.begin(), 1);
52
53 return v;
54 // $\forall i, \sum_{j=0}^m a_{i-j} v_j = 0$
55 }

```


9 complex number

```

1  tandu struct Comp {
2      T a, b;
3      Comp(T _a = 0, T _b = 0) { a = _a, b = _b; }
4      Comp operator+(const Comp& x) const { return Comp(a + x.a, b + x.b); }
5      Comp operator-(const Comp& x) const { return Comp(a - x.a, b - x.b); }
6      Comp operator*(const Comp& x) const { return Comp(a * x.a - b * x.b, a * x.b + b * x.a); }
7      bool operator==(const Comp& x) const { return a == x.a and b == x.b; }
8      T real() { return a; }
9      T imag() { return b; }
10     U norm() { return (U) a * a + (U) b * b; }
11     Comp conj() { return Comp(a, -b); }
12     Comp operator/(const Comp& x) const {
13         Comp y = x;
14         Comp c = Comp(a, b) * y.conj();
15         T d = y.norm();
16         return Comp(c.a / d, c.b / d);
17     }
18 };
19 typedef Comp<LL, LL> complex;
20 complex gcd(complex a, complex b) {
21     LL d = b.norm();
22     if (d == 0) return a;
23     std::vector<complex> v(4);
24     complex c = a * b.conj();
25     auto fdiv = [&](LL a, LL b) -> LL { return a / b - ((a ^ b) < 0 && (a % b)); };
26     v[0] = complex(fdiv(c.real(), d), fdiv(c.imag(), d));
27     v[1] = v[0] + complex(1, 0);
28     v[2] = v[0] + complex(0, 1);
29     v[3] = v[0] + complex(1, 1);
30     for (auto& x : v) {
31         x = a - x * b;
32     }
33     std::sort(all(v), [&](complex a, complex b) { return a.norm() < b.norm(); });
34     return gcd(b, v[0]);
35 };

```

10 graph

10.1 topology sort

```

1  /* topology sort */
2  vi top;
3  auto topsort = [&]() -> bool {
4      vi d(n + 1);
5      std::queue<int> q;
6      for (int i = 1; i <= n; i++) {
7          d[i] = e[i].size();
8          if (!d[i]) q.push(i);
9      }
10     while (!q.empty()) {
11         int u = q.front();
12         q.pop();
13         top.push_back(u);
14         for (auto v : e[u]) {
15             d[v]--;
16             if (!d[v]) q.push(v);
17         }
18     }
19     if (top.size() != n) return false;
20     return true;
21 };

```

10.2 shortest path

Floyd

```

1  /* floyd */
2  auto floyd = [&]() -> vvi {
3      vvi dist(n + 1, vi(n + 1, inf));
4      for (int i = 1; i <= n; i++) {
5          for (int j = 1; j <= n; j++) {
6              Min(dist[i][j], e[i][j]);
7          }
8          dist[i][i] = 0;
9      }
10     for (int k = 1; k <= n; k++) {
11         for (int i = 1; i <= n; i++) {
12             for (int j = 1; j <= n; j++) {
13                 Min(dist[i][j], dist[i][k] + dist[k][j]);
14             }
15         }
16     }
17     return dist;
18 };

```

Dijkstra

```

1  /* dijkstra */
2  auto dijkstra = [&](int s) -> vl {
3      vl dist(n + 1, INF);
4      vi vis(n + 1, 0);
5      dist[s] = 0;
6      std::priority_queue<PLI, std::vector<PLI>, std::greater<PLI>> q;
7      q.emplace(0LL, s);
8      while (!q.empty()) {
9          auto [dis, u] = q.top();
10         q.pop();
11         if (vis[u]) continue;
12         vis[u] = 1;
13         for (const auto& [v, w] : e[u]) {
14             if (dist[v] > dis + w) {
15                 dist[v] = dis + w;
16                 q.emplace(dist[v], v);
17             }
18         }
19     }
20     return dist;
21 };

```

SPFA

```

1  /* SPFA */
2  int n, m, s;
3  vl dist(n + 1, INF);
4  std::vector<bool> vis(n + 1);
5  std::vector<PLI> e(n + 1);
6  void spfa(int s){
7      for (int i = 1; i <= n; i++) dist[i] = INF;
8      dist[s] = 0;
9      std::queue<int> q;
10     q.push(s);
11     vis[s] = true;
12     while(q.size()){
13         auto u = q.front();
14         q.pop();
15         vis[u] = false;
16         for(const auto& [v, w] : e[u]){
17             if(dist[v] > dist[u] + w){
18                 dist[v] = dist[u] + w;
19                 if(!vis[v]){
20                     q.push(v);
21                     vis[v] = true;
22                 }
23             }
24         }
25     }
26 }

```

Johnson

```

1  /* johnson */
2  auto johnson = [&]() -> vvl {
3      /* 负环 */
4      vl dist1(n + 1);
5      vi vis(n + 1), cnt(n + 1);
6      auto spfa = [&]() -> bool {
7          std::queue<int> q;
8          for (int u = 1; u <= n; u++) {
9              q.push(u);
10             vis[u] = false;
11         }
12         while (!q.empty()) {
13             auto u = q.front();
14             q.pop();
15             vis[u] = false;
16             for (auto [v, w] : e[u]) {
17                 if (dist1[v] > dist1[u] + w) {
18                     dist1[v] = dist1[u] + w;
19                     Max(cnt[v], cnt[u] + 1);
20                     if (cnt[v] >= n) return true;
21                     if (!vis[v]) {
22                         q.push(v);
23                         vis[v] = true;
24                     }
25                 }
26             }
27         }
28         return false;
29     };
30     /* dijkstra */
31     vl dist2(n + 1);
32     auto dijkstra = [&](int s) {
33         for (int u = 1; u <= n; u++) {
34             dist2[u] = 1e9;
35             vis[u] = false;
36         }
37         dist2[s] = 0;
38         std::priority_queue<PLI, std::vector<PLI>, std::greater<PLI>> q;
39         q.emplace(0, s);
40         while (!q.empty()) {
41             auto [d, u] = q.top();
42             q.pop();
43             if (vis[u]) continue;
44             vis[u] = true;
45             for (const auto& [v, w] : e[u]) {
46                 if (dist2[v] > d + w) {
47                     dist2[v] = d + w;
48                     q.emplace(dist2[v], v);
49                 }
50             }
51         }
52     };
53 }

```

```

51     }
52 };
53 if (spfa()) return vvl{};
54 for (int u = 1; u <= n; u++) {
55     for (auto& [v, w] : e[u]) {
56         w += dist1[u] - dist1[v];
57     }
58 }
59 vvl dist(n + 1, vl(n + 1));
60 for (int u; u <= n; u++) {
61     dijkstra(u);
62     for (int v = 1; v <= n; v++) {
63         if (dist2[v] == 1e9) {
64             dist[u][v] = INF;
65         } else {
66             dist[u][v] = dist2[v] + dist1[v] - dist1[u];
67         }
68     }
69 }
70 return dist;
71 };

```

最短路计数 - Dijkstra

```

1  /* dijkstra */
2  auto dijkstra = [&](int s) -> std::pair<vl, vi> {
3      vl dist(n + 1, INF);
4      vi cnt(n + 1, vis(n + 1));
5      dist[s] = 0;
6      cnt[s] = 1;
7      std::priority_queue<PLI, std::vector<PLI>, std::greater<PLI>> q;
8      q.emplace(OLL, s);
9      while (!q.empty()) {
10         auto [dis, u] = q.top();
11         q.pop();
12         if (vis[u]) continue;
13         vis[u] = 1;
14         for (const auto& [v, w] : e[u]) {
15             if (dist[v] > dis + w) {
16                 dist[v] = dis + w;
17                 cnt[v] = cnt[u];
18                 q.push({dist[v], v});
19             } else if (dist[v] == dis + w) {
20                 // cnt[v] += cnt[u];
21                 cnt[v] += cnt[u];
22                 cnt[v] %= mod;
23             }
24         }
25     }
26     return {dist, cnt};
27 };

```

最短路计数 - Floyd

```

1  /* floyd */
2  auto floyd() = [&] -> std::pair<vvi, vvi> {
3      vvi dist(n + 1, vi(n + 1, inf));
4      vvi cnt(n + 1, vi(n + 1));
5      for (int i = 1; i <= n; i++) {
6          for (int j = 1; j <= n; j++) {
7              Min(dist[i][j], e[i][j]);
8          }
9          dist[i][i] = 0;
10     }
11     for (int k = 1; k <= n; k++) {
12         for (int i = 1; i <= n; i++) {
13             for (int j = 1; j <= n; j++) {
14                 if (dist[i][j] == dist[i][k] + dist[k][j]) {
15                     cnt[i][j] += cnt[i][k] * cnt[k][j];
16                 } else if (dist[i][j] > dist[i][k] + dist[k][j]) {
17                     cnt[i][j] = cnt[i][k] * cnt[k][j];
18                     dist[i][j] = dist[i][k] + dist[k][j];
19                 }
20             }
21         }
22     }
23     return {dist, cnt};
24 };

```

负环

判断的是最短路长度.

```

1  /* SPFA */
2  auto spfa = [&]() -> bool {
3      std::queue<int> q;
4      vi vis(n + 1), cnt(n + 1);
5      for (int i = 1; i <= n; i++) {
6          q.push(i);
7          vis[i] = true;
8      }
9      while (!q.empty()) {
10         auto u = q.front();
11         q.pop();
12         vis[u] = false;
13         for (const auto& [v, w] : e[u]) {
14             if (dist[v] > dist[u] + w) {
15                 dist[v] = dist[u] + w;
16                 cnt[v] = cnt[u] + 1;
17                 if (cnt[v] >= n) return true;
18                 if (!vis[v]) {
19                     q.push(v);
20                     vis[v] = true;
21                 }
22             }
23         }
24     }
25     return false;
26 }

```

10.3 minimum spanning tree

Kruskal

```

1  /* kruskal */
2  std::vector<std::tuple<int, int, int>> edge;
3  auto kruskal = [&]() -> int {
4      std::sort(all(edge), [&](std::tuple<int, int, int> a, std::tuple<int, int, int> b) {
5          auto [x1, y1, w1] = a;
6          auto [x2, y2, w2] = b;
7          return w1 < w2;
8      });
9      int res = 0, cnt = 0;
10     for (int i = 0; i < m; i++) {
11         auto [a, b, w] = edge[i];
12         a = find(a), b = find(b);
13         if (a != b) {
14             fa[a] = b;
15             res += w;
16             /* res = std::max(res, w); */
17             cnt++;
18         }
19     }
20     if (cnt < n - 1) return -1;
21     return res;
22 }

```

10.4 SCC

Tarjan

```

1  /* tarjan */
2  vi dfn(n + 1), low(n + 1), stk(n + 1), belong(n + 1);
3  int timestamp = 0, top = 0, scc_cnt = 0;
4  std::vector<bool> in_stk(n + 1);
5  auto tarjan = [&](auto&& self, int u) -> void {
6      dfn[u] = low[u] = ++timestamp;
7      stk[++top] = u;
8      in_stk[u] = true;
9      for (const auto& v : e[u]) {
10         if (!dfn[v]) {
11             self(self, v);

```

```

12         Min(low[u], low[v]);
13     } else if (in_stk[v]) {
14         Min(low[u], dfn[v]);
15     }
16 }
17 if (dfn[u] == low[u]) {
18     scc_cnt++;
19     int v;
20     do {
21         v = stk[top--];
22         in_stk[v] = false;
23         belong[v] = scc_cnt;
24     } while (v != u);
25 }
26 };

```

10.5 DCC

点双连通分量

求点双连通分量.

```

1 vi dfn(n + 1), low(n + 1), is_bcc(n + 1), stk;
2 int timestamp = 0, bcc_cnt = 0, root = 0;
3 vvi bcc(2 * n + 1);
4 std::function<void(int, int)> tarjan = [&](int u, int fa) {
5     dfn[u] = low[u] = ++timestamp;
6     int child = 0;
7     stk.push_back(u);
8     if (u == root and e[u].empty()) {
9         bcc_cnt++;
10        bcc[bcc_cnt].push_back(u);
11        return;
12    }
13    for (auto v : e[u]) {
14        if (!dfn[v]) {
15            tarjan(v, u);
16            low[u] = std::min(low[u], low[v]);
17            if (low[v] >= dfn[u]) {
18                child++;
19                if (u != root or child > 1) {
20                    is_bcc[u] = 1;
21                }
22                bcc_cnt++;
23                int z;
24                do {
25                    z = stk.back();
26                    stk.pop_back();
27                    bcc[bcc_cnt].push_back(z);
28                } while (z != v);
29                bcc[bcc_cnt].push_back(u);
30            }
31        } else if (v != fa) {
32            low[u] = std::min(low[u], dfn[v]);
33        }
34    }
35 };
36 for (int i = 1; i <= n; i++) {
37     if (!dfn[i]) {
38         root = i;
39         tarjan(i, i);
40     }
41 }

```

求割点.

```

1 vi dfn(n + 1), low(n + 1), is_bcc(n + 1);
2 int timestamp = 0, bcc = 0, root = 0;
3 std::function<void(int, int)> tarjan = [&](int u, int fa) {
4     dfn[u] = low[u] = ++timestamp;
5     int child = 0;
6     for (auto v : e[u]) {
7         if (!dfn[v]) {
8             tarjan(v, u);
9             low[u] = std::min(low[u], low[v]);
10            if (low[v] >= dfn[u]) {
11                child++;
12                if ((u != root or child > 1) and !is_bcc[u]) {
13                    bcc++;

```

```

14         is_bcc[u] = 1;
15     }
16 } else if (v != fa) {
17     low[u] = std::min(low[u], dfn[v]);
18 }
19 }
20 }
21 };
22 for (int i = 1; i <= n; i++) {
23     if (!dfn[i]) {
24         root = i;
25         tarjan(i, i);
26     }
27 }

```

边双连通分量

求边双连通分量.

```

1  std::vector<vpi> e(n + 1);
2  for (int i = 1; i <= m; i++) {
3      int u, v;
4      std::cin >> u >> v;
5      e[u].emplace_back(v, i);
6      e[v].emplace_back(u, i);
7  }
8  vi dfn(n + 1), low(n + 1), is_ecc(n + 1), fa(n + 1), stk;
9  int timestamp = 0, ecc_cnt = 0;
10 vvi ecc(2 * n + 1);
11 std::function<void(int, int)> tarjan = [&](int u, int id) {
12     low[u] = dfn[u] = ++timestamp;
13     stk.push_back(u);
14     for (auto [v, idx] : e[u]) {
15         if (!dfn[v]) {
16             tarjan(v, idx);
17             low[u] = std::min(low[u], low[v]);
18         } else if (idx != id) {
19             low[u] = std::min(low[u], dfn[v]);
20         }
21     }
22     if (dfn[u] == low[u]) {
23         ecc_cnt++;
24         int v;
25         do {
26             v = stk.back();
27             stk.pop_back();
28             ecc[ecc_cnt].push_back(v);
29         } while (v != u);
30     }
31 };
32 for (int i = 1; i <= n; i++) {
33     if (!dfn[i]) {
34         tarjan(i, 0);
35     }
36 }

```

另一个版本

```

1  /* DCC @ wrb */
2  // 割点
3  std::vector<int> G[N];
4  int dfn[N], low[N], is_cut[N], tm, rt;
5  void tar(int u) {
6      int c = 0;
7      dfn[u] = low[u] = ++tm;
8      for (int v : G[u]) {
9          if (!dfn[v]) {
10             ++c, tar(v);
11             low[u] = std::min(low[u], low[v]);
12             if (low[v] == dfn[u]) is_cut[u] = 1;
13         } else low[u] = std::min(low[u], dfn[v]);
14     }
15     if (u == rt) is_cut[u] = c > 1;
16 }
17 int main() {
18     int n, m;
19     std::cin >> n >> m;
20     for (int x, y; m--; ) {

```

```

21     std::cin >> x >> y;
22     G[x].emplace_back(y);
23     G[y].emplace_back(x);
24 }
25 for (rt = 1; rt <= n; ++rt) {
26     if (!dfn[rt]) tar(rt);
27 }
28 std::vector<int> cut_v;
29 for (int i = 1; i <= n; ++i) {
30     if (is_cut[i]) cut_v.emplace_back(i);
31 }
32 std::cout << cut_v.size() << '\n';
33 for (int x : cut_v) std::cout << x << ' ';
34 }
35
36
37 // 桥
38 std::vector<std::pair<int, int>> G[N], brg;
39 int dfn[N], low[N], rt, tm;
40 void tar(int u, int fa, int fr) {
41     dfn[u] = low[u] = ++tm;
42     for (auto [v, i] : G[u]) {
43         if (!dfn[v]) {
44             tar(v, u, i), low[u] = std::min(low[u], low[v]);
45         } else if (i != fr) {
46             low[u] = std::min(low[u], dfn[v]);
47         }
48     }
49     if (u != rt && dfn[u] == low[u]) {
50         brg.emplace_back(std::minmax(u, fa));
51     }
52 }
53 int main() {
54     int n, m;
55     std::cin >> n >> m;
56     for (int i = 1, x, y; i <= m; ++i) {
57         std::cin >> x >> y;
58         G[x].emplace_back(y, i);
59         G[y].emplace_back(x, i);
60     }
61     for (rt = 1; rt <= n; ++rt) {
62         if (!dfn[rt]) tar(rt, -1, -1);
63     }
64     std::sort(begin(brg), end(brg));
65     for (auto [u, v] : brg) {
66         std::cout << u << ' ' << v << '\n';
67     }
68 }
69
70
71 // 点双
72 std::vector<int> G[N];
73 std::vector<std::vector<int>> bcc;
74 int dfn[N], low[N], tm, st[N], tp, rt;
75 void tar(int u) {
76     dfn[u] = low[u] = ++tm, st[++tp] = u;
77     if (G[u].empty()) bcc.push_back({u});
78     for (int v : G[u]) {
79         if (!dfn[v]) {
80             tar(v), low[u] = std::min(low[u], low[v]);
81             if (low[v] == dfn[u]) {
82                 bcc.push_back({u});
83                 do {
84                     bcc.back().emplace_back(st[tp]);
85                 } while (st[tp--] != v);
86             }
87         } else low[u] = std::min(low[u], dfn[v]);
88     }
89 }
90 int main() {
91     std::ios::sync_with_stdio(0);
92     std::cin.tie(0);
93     int n, m;
94     std::cin >> n >> m;
95     for (int x, y; m--;) {
96         std::cin >> x >> y;
97         G[x].emplace_back(y);
98         G[y].emplace_back(x);
99     }
100     for (int i = 0; i < n; ++i) tar(i);
101     std::cout << bcc.size() << '\n';
102     for (auto v : bcc) {
103         std::cout << v.size() << ' ';
104         for (int x : v) std::cout << x << ' ';
105         std::cout << '\n';
106     }
107 }

```



```

108
109
110 // 边双
111 std::vector<std::pair<int, int>> G[N];
112 std::vector<std::vector<int>> becc;
113 int dfn[N], low[N], tm, st[N], tp;
114 void tar(int u, int fr) {
115     dfn[u] = low[u] = ++tm, st[++tp] = u;
116     for (auto [v, i] : G[u]) {
117         if (!dfn[v]) {
118             tar(v, i), low[u] = std::min(low[u], low[v]);
119         } else if (i != fr) {
120             low[u] = std::min(low[u], dfn[v]);
121         }
122     }
123     if (dfn[u] == low[u]) {
124         becc.emplace_back();
125         do {
126             becc.back().emplace_back(st[tp]);
127         } while (st[tp--] != u);
128     }
129 }
130 int main() {
131     int n, m;
132     std::cin >> n >> m;
133     for (int i = 1, x, y; i <= m; ++i) {
134         std::cin >> x >> y;
135         G[x].emplace_back(y, i);
136         G[y].emplace_back(x, i);
137     }
138     for (int i = 0; i < n; ++i) {
139         if (!dfn[i]) tar(i, -1);
140     }
141     std::cout << becc.size() << '\n';
142     for (auto& v : becc) {
143         std::cout << v.size() << ' ';
144         for (int x : v) std::cout << x << ' ';
145         std::cout << '\n';
146     }
147 }

```

10.6 2-sat

给出 n 个集合, 每个集合有 2 个元素, 已知若个数对 (a, b) , 表示 a 与 b 矛盾. 要从每个集合各选择一个元素, 判断能否一共选 n 个两两不矛盾的元素.

```

1 /* two sat */
2 auto 2sat = [&](int n, const vpi& v) -> vi {
3     /* tarjan */
4     vvi e(2 * n);
5     vi dfn(2 * n), low(2 * n), stk(2 * n), belong(2 * n);
6     int timestamp = 0, top = 0, scc_cnt = 0;
7     std::vector<bool> in_stk(2 * n);
8     auto tarjan = [&](auto& self, int u) -> void {
9         dfn[u] = low[u] = ++timestamp;
10        stk[++top] = u;
11        in_stk[u] = true;
12        for (const auto& v : e[u]) {
13            if (!dfn[v]) {
14                self(self, v);
15                Min(low[u], low[v]);
16            } else if (in_stk[v]) {
17                Min(low[u], dfn[v]);
18            }
19        }
20        if (dfn[u] == low[u]) {
21            scc_cnt++;
22            int v;
23            do {
24                v = stk[top--];
25                in_stk[v] = false;
26                belong[v] = scc_cnt;
27            } while (v != u);
28        }
29    };
30    for (const auto& [a, b] : v) {
31        e[a].push_back(b ^ 1);
32        e[b].push_back(a ^ 1);
33    }
34    for (int i = 0; i < 2 * n; i++) {
35        if (!dfn[i]) tarjan(tarjan, i);
36    }
37 }

```

```

36     }
37     vi ans;
38     for (int i = 0; i < 2 * n; i += 2) {
39         if (belong[i] == belong[i + 1]) return vi{};
40         ans.push_back(belong[i] > belong[i + 1] ? i + 1 : i);
41     }
42     return ans;
43 };

```

上述将 i 与 $i + 1$ 作为一个集合里的元素, 编号为 0 至 $2n - 1$.

10.7 minimum ring

Floyd

```

1  /* minimum ring */
2  auto min_circle = [&]() -> int {
3      vvi dist(n + 1, vi(n + 1, inf));
4      for (int i = 1; i <= n; i++) {
5          for (int j = 1; j <= n; j++) {
6              Min(dist[i][j], g[i][j]);
7          }
8          dist[i][i] = 0;
9      }
10     for (int k = 1; k <= n; k++) {
11         for (int i = 1; i < k; i++) {
12             for (int j = 1; j < i; j++) {
13                 Min(ans, dist[i][j] + g[i][k] + g[k][j]);
14             }
15         }
16         for (int i = 1; i <= n; i++) {
17             for (int j = 1; j <= n; j++) {
18                 Min(dist[i][j], dist[i][k] + dist[k][j]);
19             }
20         }
21     }
22     return ans;
23 };

```

tree - diameter

10.8 tree - center of gravity

```

1  /* center of gravity */
2  int sum; /* 点权和 */
3  vi size(n + 1), weight(n + 1), w(n + 1), depth(n + 1);
4  std::array<int, 2> centroid = {0, 0};
5  auto get_centroid = [&](auto&& self, int u, int fa) -> void {
6      size[u] = w[u];
7      weight[u] = 0;
8      for (auto v : e[u]) {
9          if (v == fa) continue;
10         self(self, v, u);
11         size[u] += size[v];
12         Max(weight[u], size[v]);
13     }
14     Max(weight[u], sum - size[u]);
15     if (weight[u] <= sum / 2) {
16         centroid[centroid[0] != 0] = u;
17     }
18 };

```

10.9 tree - DSU on tree

给出一棵 n 个节点以 1 为根的树, 每个节点染上一种颜色, 询问以 u 为节点的子树中有多少种颜色.

```

1  // Problem: U41492 树上数颜色
2
3  int main() {

```

```

4   std::ios::sync_with_stdio(false);
5   std::cin.tie(0);
6   std::cout.tie(0);
7
8   int n, m, dfn = 0, cnttot = 0;
9   std::cin >> n;
10  vvi e(n + 1);
11  vi siz(n + 1), col(n + 1), son(n + 1), dfnl(n + 1), dfnr(n + 1), rank(n + 1);
12  vi ans(n + 1), cnt(n + 1);
13
14  for (int i = 1; i < n; i++) {
15      int u, v;
16      std::cin >> u >> v;
17      e[u].push_back(v);
18      e[v].push_back(u);
19  }
20  for (int i = 1; i <= n; i++) {
21      std::cin >> col[i];
22  }
23  auto add = [&](int u) -> void {
24      if (cnt[col[u]] == 0) cnttot++;
25      cnt[col[u]]++;
26  };
27  auto del = [&](int u) -> void {
28      cnt[col[u]]--;
29      if (cnt[col[u]] == 0) cnttot--;
30  };
31  auto dfs1 = [&](auto&& self, int u, int fa) -> void {
32      dfnl[u] = ++dfn;
33      rank[dfn] = u;
34      siz[u] = 1;
35      for (auto v : e[u]) {
36          if (v == fa) continue;
37          self(self, v, u);
38          siz[u] += siz[v];
39          if (!son[u] or siz[son[u]] < siz[v]) son[u] = v;
40      }
41      dfnr[u] = dfn;
42  };
43  auto dfs2 = [&](auto&& self, int u, int fa, bool op) -> void {
44      for (auto v : e[u]) {
45          if (v == fa or v == son[u]) continue;
46          self(self, v, u, false);
47      }
48      if (son[u]) self(self, son[u], u, true);
49      for (auto v : e[u]) {
50          if (v == fa or v == son[u]) continue;
51          rep(i, dfnl[v], dfnr[v]) { add(rank[i]); }
52      }
53      add(u);
54      ans[u] = cnttot;
55      if (op == false) {
56          rep(i, dfnl[u], dfnr[u]) { del(rank[i]); }
57      }
58  };
59  dfs1(dfs1, 1, 0);
60  dfs2(dfs2, 1, 0, false);
61  std::cin >> m;
62  for (int i = 1; i <= m; i++) {
63      int u;
64      std::cin >> u;
65      std::cout << ans[u] << endl;
66  }
67  return 0;
68 }

```

10.10 tree - AHU

```

1  /* AHU */
2  std::map<vi, int> mapple;
3  std::function<int(vi&, int, int)> tree_hash = [&](vvi& e, int u, int fa) -> int {
4      vi code;
5      if (u == 0) code.push_back(-1);
6      for (auto v : e[u]) {
7          if (v == fa) continue;
8          code.push_back(tree_hash(e, v, u));
9      }
10     std::sort(all(code));
11     int id = mapple.size();
12     auto it = mapple.find(code);
13     if (it == mapple.end()) {
14         mapple[code] = id;

```

```

15     } else {
16         id = it->ss;
17     }
18     return id;
19 };

```

10.11 tree - LCA

```

1  /* LCA */
2  int B = 30;
3  vvi e(n + 1), fa(n + 1, vi(B));
4  vi dep(n + 1);
5  auto dfs = [&](auto&& self, int u) -> void {
6      for (auto v : e[u]) {
7          if (v == fa[u][0]) continue;
8          dep[v] = dep[u] + 1;
9          fa[v][0] = u;
10         self(self, v);
11     }
12 };
13 auto init = [&]() -> void {
14     dep[root] = 1;
15     dfs(dfs, root);
16     for (int j = 1; j < B; j++) {
17         for (int i = 1; i <= n; i++) {
18             fa[i][j] = fa[fa[i][j - 1]][j - 1];
19         }
20     }
21 };
22 init();
23 auto LCA = [&](int a, int b) -> int {
24     if (dep[a] > dep[b]) std::swap(a, b);
25     int d = dep[b] - dep[a];
26     for (int i = 0; (1 << i) <= d; i++) {
27         if (d & (1 << i)) b = fa[b][i];
28     }
29     if (a == b) return a;
30     for (int i = B - 1; i >= 0 and a != b; i--) {
31         if (fa[a][i] == fa[b][i]) continue;
32         a = fa[a][i];
33         b = fa[b][i];
34     }
35     return fa[a][0];
36 };
37 auto dist = [&](int a, int b) -> int { return dep[a] + dep[b] - dep[LCA(a, b)] * 2; };

```

10.12 tree - heavy light decomposition

对一棵有根树进行如下 4 种操作:

1. 1 $x y z$: 将节点 x 到节点 y 的最短路径上所有节点的值加上 z .
2. 2 $x y$: 查询节点 x 到节点 y 的最短路径上所有节点的值的和.
3. 3 $x z$: 将以节点 x 为根的子树上所有节点的值加上 z .
4. 4 x : 查询以节点 x 为根的子树上所有节点的值的和.

```

1  /* heavy light decomposition */
2  int cnt = 0;
3  vi son(n + 1), fa(n + 1), siz(n + 1), depth(n + 1);
4  vi dfn(n + 1), rank(n + 1), top(n + 1), botton(n + 1);
5  auto dfs1 = [&](auto&& self, int u) -> void {
6      son[u] = -1, siz[u] = 1;
7      for (auto v : e[u]) {
8          if (depth[v] != 0) continue;
9          depth[v] = depth[u] + 1;
10         fa[v] = u;
11         self(self, v);
12         siz[u] += siz[v];
13         if (son[u] == -1 or siz[v] > siz[son[u]]) son[u] = v;
14     }
15 };

```

```

16 auto dfs2 = [&](auto&& self, int u, int t) -> void {
17     top[u] = t;
18     dfn[u] = ++cnt;
19     rank[cnt] = u;
20     botton[u] = dfn[u];
21     if (son[u] == -1) return;
22     self(self, son[u], t);
23     Max(botton[u], botton[son[u]]);
24     for (auto v : e[u]) {
25         if (v != son[u] and v != fa[u]) {
26             self(self, v, v);
27             Max(botton[u], botton[v]);
28         }
29     }
30 };
31 depth[root] = 1;
32 dfs1(dfs1, root);
33 dfs2(dfs2, root, root);
34
35 /* 求 LCA */
36 auto LCA = [&](int a, int b) -> int {
37     while (top[a] != top[b]) {
38         if (depth[top[a]] < depth[top[b]]) std::swap(a, b);
39         a = fa[top[a]];
40     }
41     return (depth[a] > depth[b] ? b : a);
42 };
43
44 /* 维护 u 到 v 的路径 */
45 while (top[u] != top[v]) {
46     if (depth[top[u]] < depth[top[v]]) std::swap(u, v);
47     opt(dfn[top[u]], dfn[u]);
48     u = fa[top[u]];
49 }
50 if (dfn[u] > dfn[v]) std::swap(u, v);
51 opt(dfn[u], dfn[v]);
52
53 /* 维护 u 为根的子树 */
54 opt(dfn[u], botton[u]);
55
56 /*
57 线段树的 build() 函数中
58 if(l == r) tree[u] = {l, l, w[rank[l]], 0};
59 */
60
61 build(1, 1, n);
62 for (int i = 1; i <= m; i++) {
63     int op, u, v;
64     LL k;
65     std::cin >> op;
66     if (op == 1) {
67         std::cin >> u >> v >> k;
68         while (top[u] != top[v]) {
69             if (depth[top[u]] < depth[top[v]]) std::swap(u, v);
70             modify(1, dfn[top[u]], dfn[u], k);
71             u = fa[top[u]];
72         }
73         if (dfn[u] > dfn[v]) std::swap(u, v);
74         modify(1, dfn[u], dfn[v], k);
75     } else if (op == 2) {
76         std::cin >> u >> v;
77         LL ans = 0;
78         while (top[u] != top[v]) {
79             if (depth[top[u]] < depth[top[v]]) std::swap(u, v);
80             ans = (ans + query(1, dfn[top[u]], dfn[u])) % p;
81             u = fa[top[u]];
82         }
83         if (dfn[u] > dfn[v]) std::swap(u, v);
84         ans = (ans + query(1, dfn[u], dfn[v])) % p;
85         std::cout << ans << endl;
86     } else if (op == 3) {
87         std::cin >> u >> k;
88         modify(1, dfn[u], botton[u], k);
89     } else {
90         std::cin >> u;
91         std::cout << query(1, dfn[u], botton[u]) % p << endl;
92     }
93 }

```

10.13 tree - virtual tree

```
1 /* virtual tree */
```

```

2 auto build_vtree = [&](vi ver) -> void {
3     std::sort(all(ver), [&](int x, int y) { return dfn[x] < dfn[y]; });
4     vi stk = {1};
5     for (auto v : ver) {
6         int u = stk.back();
7         int lca = LCA(v, u);
8         if (lca != u) {
9             while (dfn[lca] < dfn[stk.end()[-2]]) {
10                 g[stk.end()[-2]].push_back(stk.back());
11                 stk.pop_back();
12             }
13             u = stk.back();
14             if (dfn[lca] != dfn[stk.end()[-2]]) {
15                 g[lca].push_back(u);
16                 stk.pop_back();
17                 stk.push_back(lca);
18             } else {
19                 g[lca].push_back(u);
20                 stk.pop_back();
21             }
22         }
23         stk.push_back(v);
24     }
25     while (stk.size() > 1) {
26         int u = stk.end()[-2];
27         int v = stk.back();
28         g[u].push_back(v);
29         stk.pop_back();
30     }
31 };

```

10.14 tree - pseudo tree

```

1 /* ring detection (directed) */
2 vi vis(n + 1), fa(n + 1), ring;
3 auto dfs = [&](auto&& self, int u) -> bool {
4     vis[u] = 1;
5     for (const auto& v : e[u]) {
6         if (!vis[v]) {
7             fa[v] = u;
8             if (self(self, v)) {
9                 return true;
10            }
11        } else if (vis[v] == 1) {
12            ring.push_back(v);
13            for (auto x = u; x != v; x = fa[x]) {
14                ring.push_back(x);
15            }
16            reverse(all(ring));
17            return true;
18        }
19    }
20    vis[u] = 2;
21    return false;
22 };
23 for (int i = 1; i <= n; i++) {
24     if (!vis[i]) {
25         if (dfs(dfs, i)) {
26             // operations //
27         }
28     }
29 }
30
31 /* cycle detection (undirected) */
32 vi vis(n + 1), ring;
33 vpi fa(n + 1);
34 auto dfs = [&](auto&& self, int u, int from) -> bool {
35     vis[u] = 1;
36     for (const auto& [v, id] : e[u]) {
37         if (id == from) continue;
38         if (!vis[v]) {
39             fa[v] = {u, id};
40             if (self(self, v, id)) {
41                 return true;
42             }
43         } else if (vis[v] == 1) {
44             ring.push_back(v);
45             for (auto x = u; x != v; x = fa[x].ff) {
46                 ring.push_back(x);
47             }
48             return true;
49         }
50     }
51 }

```

```

50     }
51     vis[u] = 2;
52     return false;
53 };
54 for (int i = 1; i <= n; i++) {
55     if (!vis[i]) {
56         if (dfs(dfs, i, 0)) {
57             // operations //
58         }
59     }
60 }

```

10.15 tree - divide and conquer on tree

点分治

第一个题

一棵 $n \leq 10^4$ 个点的树，边权 $w \leq 10^4$ 。 $m \leq 100$ 次询问树上是否存在长度为 $k \leq 10^7$ 的路径。

```

1 // 洛谷 P3806 【模板】点分治1
2
3 int main() {
4     std::ios::sync_with_stdio(false);
5     std::cin.tie(0);
6     std::cout.tie(0);
7
8     int n, m, k;
9     std::cin >> n >> m;
10
11     std::vector<vpi> e(n + 1);
12     std::map<int, PII> mp;
13
14     for (int i = 1; i < n; i++) {
15         int u, v, w;
16         std::cin >> u >> v >> w;
17         e[u].emplace_back(v, w);
18         e[v].emplace_back(u, w);
19     }
20     for (int i = 1; i <= m; i++) {
21         std::cin >> k;
22         mp[i] = {k, 0};
23     }
24
25     /* centroid decomposition */
26     int top1 = 0, top2 = 0, root;
27     vi len1(n + 1), len2(n + 1), vis(n + 1);
28     static std::array<int, 20000010> cnt;
29
30     std::function<int(int, int)> get_size = [&](int u, int fa) -> int {
31         if (vis[u]) return 0;
32         int ans = 1;
33         for (auto [v, w] : e[u]) {
34             if (v == fa) continue;
35             ans += get_size(v, u);
36         }
37         return ans;
38     };
39
40     std::function<int(int, int, int, int)> get_root = [&](int u, int fa, int tot,
41                                                         int& root) -> int {
42         if (vis[u]) return 0;
43         int sum = 1, maxx = 0;
44         for (auto [v, w] : e[u]) {
45             if (v == fa) continue;
46             int tmp = get_root(v, u, tot, root);
47             Max(maxx, tmp);
48             sum += tmp;
49         }
50         Max(maxx, tot - sum);
51         if (2 * maxx <= tot) root = u;
52         return sum;
53     };
54
55     std::function<void(int, int, int)> get_dist = [&](int u, int fa, int dist) -> void {
56         if (dist <= 10000000) len1[++top1] = dist;
57         for (auto [v, w] : e[u]) {
58             if (v == fa or vis[v]) continue;
59             get_dist(v, u, dist + w);
60         }

```

```

61     };
62
63     auto solve = [&](int u, int dist) -> void {
64         top2 = 0;
65         for (auto [v, w] : e[u]) {
66             if (vis[v]) continue;
67             top1 = 0;
68             get_dist(v, u, w);
69             for (int i = 1; i <= top1; i++) {
70                 for (int tt = 1; tt <= m; tt++) {
71                     int k = mp[tt].ff;
72                     if (k >= len1[i]) mp[tt].ss |= cnt[k - len1[i]];
73                 }
74             }
75             for (int i = 1; i <= top1; i++) {
76                 len2[++top2] = len1[i];
77                 cnt[len1[i]] = 1;
78             }
79         }
80         for (int i = 1; i <= top2; i++) cnt[len2[i]] = 0;
81     };
82
83     std::function<void(int)> divide = [&](int u) -> void {
84         vis[u] = cnt[0] = 1;
85         solve(u, 0);
86         for (auto [v, w] : e[u]) {
87             if (vis[v]) continue;
88             get_root(v, u, get_size(v, u), root);
89             divide(root);
90         }
91     };
92
93     get_root(1, 0, get_size(1, 0), root);
94     divide(root);
95
96     for (int i = 1; i <= m; i++) {
97         if (mp[i].ss == 0) {
98             std::cout << "NAY" << endl;
99         } else {
100             std::cout << "AYE" << endl;
101         }
102     }
103
104     return 0;
105 }

```

第二个题

一棵 $n \leq 4 \times 10^4$ 个点的树, 边权 $w \leq 10^3$. 询问树上长度不超过 $k \leq 2 \times 10^4$ 的路径的数量.

```

1 // 洛谷 P4178 Tree
2
3 int main() {
4     std::ios::sync_with_stdio(false);
5     std::cin.tie(0);
6     std::cout.tie(0);
7
8     int n, k;
9     std::cin >> n;
10    std::vector<vpi> e(n + 1);
11    for (int i = 1; i < n; i++) {
12        int u, v, w;
13        std::cin >> u >> v >> w;
14        e[u].emplace_back(v, w);
15        e[v].emplace_back(u, w);
16    }
17    std::cin >> k;
18
19    /* centroid decomposition */
20    int root;
21    vi len, vis(n + 1);
22
23    std::function<int(int, int)> get_size = [&](int u, int fa) -> int {
24        if (vis[u]) return 0;
25        int ans = 1;
26        for (auto [v, w] : e[u]) {
27            if (v == fa) continue;
28            ans += get_size(v, u);
29        }
30        return ans;
31    };
32
33    std::function<int(int, int, int, int)> get_root = [&](int u, int fa, int tot,
34        int& root) -> int {
35        if (vis[u]) return 0;

```



```

36     int sum = 1, maxx = 0;
37     for (auto [v, w] : e[u]) {
38         if (v == fa) continue;
39         int tmp = get_root(v, u, tot, root);
40         maxx = std::max(maxx, tmp);
41         sum += tmp;
42     }
43     maxx = std::max(maxx, tot - sum);
44     if (2 * maxx <= tot) root = u;
45     return sum;
46 };
47
48 std::function<void(int, int, int)> get_dist = [&](int u, int fa, int dist) -> void {
49     len.push_back(dist);
50     for (auto [v, w] : e[u]) {
51         if (v == fa || vis[v]) continue;
52         get_dist(v, u, dist + w);
53     }
54 };
55
56 auto solve = [&](int u, int dist) -> int {
57     len.clear();
58     get_dist(u, 0, dist);
59     std::sort(all(len));
60     int ans = 0;
61     for (int l = 0, r = len.size() - 1; l < r;) {
62         if (len[l] + len[r] <= k) {
63             ans += r - l++;
64         } else {
65             r--;
66         }
67     }
68     return ans;
69 };
70
71 std::function<int(int)> divide = [&](int u) -> int {
72     vis[u] = true;
73     int ans = solve(u, 0);
74     for (auto [v, w] : e[u]) {
75         if (vis[v]) continue;
76         ans -= solve(v, w);
77         get_root(v, u, get_size(v, u), root);
78         ans += divide(root);
79     }
80     return ans;
81 };
82
83 get_root(1, 0, get_size(1, 0), root);
84 std::cout << divide(root) << endl;
85
86 return 0;
87 }

```

10.16 tree - matrix tree

```

1  const int N=33,M=152599,P=998244353;
2  int qpow(int a,int b=P-2){
3      int r=1;for(;b;b>>=1,a=1ll*a*a%P)if(b&1)r=1ll*r*a%P;return r;
4  }
5  struct T{int x,y,z;T(int a=0,int b=0,int c=0):x(a),y(b),z(c){}}e[N*N];
6  struct F{
7      int a,b;
8      F():a(),b(){}
9      F(int x,int y):a(x),b(y){}
10     F operator+(const F&_)const{return F((a+_a)%P,(b+_b)%P);}
11     F operator+=(const F&_)const{return *this=*this+_;}
12     F operator-(const F&_)const{return F((a-_a+P)%P,(b-_b+P)%P);}
13     F operator-=(const F&_)const{return *this=*this-_;}
14     F operator*(const F&_)const{return F((1ll*a*_b+1ll*b*_a)%P,1ll*b*_b%P);}
15     F operator*=(const F&_)const{return *this=*this*_;}
16     int operator&()const{return b?2:(a?1:0);}
17     bool operator!()const{return !a&&!b;}
18     F operator~()const{
19         int d=qpow(b);
20         return F((P-1ll*a*d%P*d%P)%P,d);
21     }
22 };
23 int fa[N],phi[M],n,m;
24 int gf(int x){return x==fa[x]?x:fa[x]=gf(fa[x]);}
25 int cal(int p){
26     F a[N][N],d,iv,z=F(0,1);
27     int i,j,k,l,x=0;iota(fa,fa+n+1,0);

```

```

28     for(i=1;i<=m;++i)if(e[i].z%p==0){
29         if((j=gf(e[i].x))!=(k=gf(e[i].y)))fa[j]=k;
30         j=e[i].x,k=e[i].y,l=e[i].z,++x;
31         a[j][k]-=F(1,1),a[k][j]-=F(1,1);
32         a[j][j]+=F(1,1),a[k][k]+=F(1,1);
33     }
34     for(j=0,i=1;i<=n;++i)if(fa[i]==i)++j;
35     if(j>1 || x<n-1)return 0;
36     for(i=1;i<n;++i){
37         for(k=i,j=i+1;j<n;++j)if(&a[j][i]>&a[k][i])k=j;
38         if(k!=i)swap(a[i],a[k]),z*=F(0,P-1);
39         if(!a[i][i])return 0;
40         for(z*=a[i][i][i],iv=-a[i][i],j=i;j<n;++j)a[i][j]*=iv;
41         for(j=i+1;j<n;++j)for(d=a[j][i],k=i;k<n;++k)a[j][k]-=a[i][k]*d;
42     }
43     return z.a;
44 }
45 void work(){
46     int h=0,i,j,x,y,z;
47     for(cin>>n>>m,i=1;i<=m;++i)cin>>x>>y>>z,e[i]=T(x,y,z),h=max(h,z);
48     iota(phi+1,phi+h+1,1);
49     for(i=1;i<=h;++i)for(j=i<=1;j<=h;j+=i)phi[j]=(phi[j]-phi[i]+P)%P;
50     for(z=0,i=1;i<=h;++i)z=(z+1ll*phi[i]*cal(i)%P)%P;
51     cout<<z<<'n';
52 }

```

10.17 Prefür sequence

```

1  \* prefür @ wrb *\
2  for(int i=1;i<n;i++)cin>>fa[i],d[fa[i]]++;
3  for(int i=1,j=1;i<n-1;i++,j++){
4      while(d[j])j++;
5      p[i]=fa[j];
6      while(i<n-1&&!--d[p[i]]&&j>p[i])p[i+1]=fa[p[i]],i++;
7  }
8
9  //////////////////////////////////////
10
11 for(int i=1;i<n-1;i++)cin>>p[i],d[p[i]]++;
12 p[n-1]=n;
13 for(int i=1,j=1;i<n;i++,j++){
14     while(d[j])j++;
15     fa[j]=p[i];
16     while(i<n&&!--d[p[i]]&&j>p[i])fa[p[i]]=p[i+1],i++;
17 }

```

10.18 network flow - maximal flow

Dinic

```

1  /* dinic */
2  struct edge {
3      int from, to;
4      LL cap, flow;
5  };
6  edge(int u, int v, LL c, LL f) : from(u), to(v), cap(c), flow(f) {}
7  };
8
9  struct Dinic {
10     int n, m = 0, s, t;
11     std::vector<edge> e;
12     vi g[N];
13     int d[N], cur[N], vis[N];
14
15     void init(int n) {
16         for (int i = 0; i < n; i++) g[i].clear();
17         e.clear();
18         m = 0;
19     }
20
21     void add(int from, int to, LL cap) {
22         e.push_back(edge(from, to, cap, 0));
23         e.push_back(edge(to, from, 0, 0));
24         g[from].push_back(m++);
25         g[to].push_back(m++);
26     }

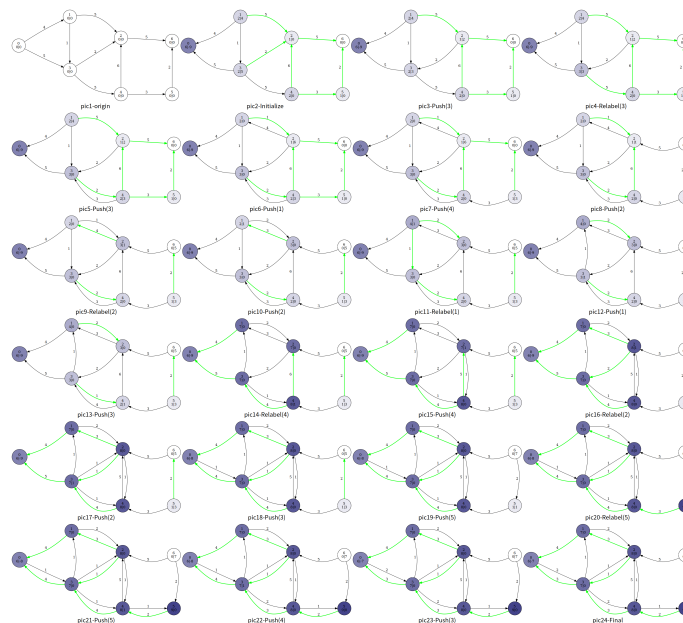
```

```

27
28 bool bfs() {
29     for (int i = 1; i <= n; i++) {
30         vis[i] = 0;
31     }
32     std::queue<int> q;
33     q.push(s), d[s] = 0, vis[s] = 1;
34     while (!q.empty()) {
35         int u = q.front();
36         q.pop();
37         for (int i = 0; i < g[u].size(); i++) {
38             edge& ee = e[g[u][i]];
39             if (!vis[ee.to] and ee.cap > ee.flow) {
40                 vis[ee.to] = 1;
41                 d[ee.to] = d[u] + 1;
42                 q.push(ee.to);
43             }
44         }
45     }
46     return vis[t];
47 }
48
49 LL dfs(int u, LL now) {
50     if (u == t || now == 0) return now;
51     LL flow = 0, f;
52     for (int& i = cur[u]; i < g[u].size(); i++) {
53         edge& ee = e[g[u][i]];
54         edge& er = e[g[u][i] ^ 1];
55         if (d[u] + 1 == d[ee.to] and (f = dfs(ee.to, std::min(now, ee.cap - ee.flow))) > 0) {
56             ee.flow += f, er.flow -= f;
57             flow += f, now -= f;
58             if (now == 0) break;
59         }
60     }
61     return flow;
62 }
63
64 LL dinic() {
65     LL ans = 0;
66     while (bfs()) {
67         for (int i = 1; i <= n; i++) cur[i] = 0;
68         ans += dfs(s, INF);
69     }
70     return ans;
71 }
72 } maxf;

```

HLPP



```

1  /* hlpp */
2  struct HLPP {
3      int n, m = 0, s, t;
4      std::vector<edge> e;    /* 边 */
5      std::vector<node> nd;   /* 点 */

```

```

6  std::vector<int> g[N];    /* 点的连边编号 */
7  std::priority_queue<node> q;
8  std::queue<int> qq;
9  bool vis[N];
10 int cnt[N];
11
12 void init() {
13     e.clear();
14     nd.clear();
15     for (int i = 0; i <= n + 1; i++) {
16         nd.pushback(node(inf, i, 0));
17         g[i].clear();
18         vis[i] = false;
19     }
20 }
21
22 void add(int u, int v, LL w) {
23     e.pushback(edge(u, v, w));
24     e.pushback(edge(v, u, 0));
25     g[u].pushback(m++);
26     g[v].pushback(m++);
27 }
28
29 void bfs() {
30     nd[t].hight = 0;
31     qq.push(t);
32     while (!qq.empty()) {
33         int u = qq.front();
34         qq.pop();
35         vis[u] = false;
36         for (auto j : g[u]) {
37             int v = e[j].to;
38             if (e[j].cap == 0 && nd[v].hight > nd[u].hight + 1) {
39                 nd[v].hight = nd[u].hight + 1;
40                 if (vis[v] == false) {
41                     qq.push(v);
42                     vis[v] = true;
43                 }
44             }
45         }
46     }
47     return;
48 }
49
50 void _push(int u) {
51     for (auto j : g[u]) {
52         edge &ee = e[j], &er = e[j ^ 1];
53         int v = ee.to;
54         node &nu = nd[u], &nv = nd[v];
55         if (ee.cap && nv.hight + 1 == nu.hight) {
56             LL flow = std::min(ee.cap, nu.flow);
57             ee.cap -= flow, er.cap += flow;
58             nu.flow -= flow, nv.flow += flow;
59             if (vis[v] == false && v != t && v != s) {
60                 q.push(nv);
61                 vis[v] = true;
62             }
63             if (nu.flow == 0) break;
64         }
65     }
66 }
67
68 void relabel(int u) {
69     nd[u].hight = inf;
70     for (auto j : g[u]) {
71         int v = e[j].to;
72         if (e[j].cap && nd[v].hight + 1 < nd[u].hight) {
73             nd[u].hight = nd[v].hight + 1;
74         }
75     }
76 }
77
78 LL hlpp() {
79     bfs();
80     if (nd[s].hight == inf) return 0;
81     nd[s].hight = n;
82     for (int i = 1; i <= n; i++) {
83         if (nd[i].hight < inf) cnt[nd[i].hight]++;
84     }
85     for (auto j : g[s]) {
86         int v = e[j].to;
87         int flow = e[j].cap;
88         if (flow) {
89             e[j].cap -= flow, e[j ^ 1].cap += flow;
90             nd[s].flow -= flow, nd[v].flow += flow;
91             if (vis[v] == false && v != s && v != t) {
92                 q.push(nd[v]);

```

```

93         vis[v] = true;
94     }
95 }
96 }
97 while (!q.empty()) {
98     int u = q.top().id;
99     q.pop();
100     vis[u] = false;
101     _push(u);
102     if (nd[u].flow) {
103         cnt[nd[u].hight]--;
104         if (cnt[nd[u].hight] == 0) {
105             for (int i = 1; i <= n; i++) {
106                 if (i != s && i != t && nd[i].hight > nd[u].hight && nd[i].hight < n + 1) {
107                     nd[i].hight = n + 1;
108                 }
109             }
110         }
111         relabel(u);
112         cnt[nd[u].hight]++;
113         q.push(nd[u]);
114         vis[u] = true;
115     }
116 }
117 return nd[t].flow;
118 }
119 } maxf;

```

10.19 network flow - minimum cost flow

Dinic + SPFA

```

1  /* Dinic + SPFA */
2  struct edge {
3      int from, to;
4      LL cap, cost;
5  };
6      edge(int u, int v, LL c, LL w) : from(u), to(v), cap(c), cost(w) {}
7  };
8
9  const int N = 2000;
10
11  struct MCMF {
12      int n, m = 0, s, t;
13      std::vector<edge> e;
14      vi g[N];
15      int cur[N], vis[N];
16      LL dist[N], minc;
17
18      void init(int n) {
19          for (int i = 0; i < n; i++) g[i].clear();
20          e.clear();
21          minc = m = 0;
22      }
23
24      void add(int from, int to, LL cap, LL cost) {
25          e.push_back(edge(from, to, cap, cost));
26          e.push_back(edge(to, from, 0, -cost));
27          g[from].push_back(m++);
28          g[to].push_back(m++);
29      }
30
31      bool spfa() {
32          for (int i = 1; i <= n; i++) {
33              dist[i] = INF, cur[i] = 0;
34          }
35          std::queue<int> q;
36          q.push(s), dist[s] = 0, vis[s] = 1;
37          while (!q.empty()) {
38              int u = q.front();
39              q.pop();
40              vis[u] = 0;
41              for (int j = cur[u]; j < g[u].size(); j++) {
42                  edge& ee = e[g[u][j]];
43                  int v = ee.to;
44                  if (ee.cap && dist[v] > dist[u] + ee.cost) {
45                      dist[v] = dist[u] + ee.cost;
46                      if (!vis[v]) {
47                          q.push(v);
48                          vis[v] = 1;
49                      }

```

```

50     }
51 }
52 }
53 return dist[t] != INF;
54 }
55
56 LL dfs(int u, LL now) {
57     if (u == t) return now;
58     vis[u] = 1;
59     LL ans = 0;
60     for (int& i = cur[u]; i < g[u].size() && ans < now; i++) {
61         edge &ee = e[g[u][i]], &er = e[g[u][i] ^ 1];
62         int v = ee.to;
63         if (!vis[v] && ee.cap && dist[v] == dist[u] + ee.cost) {
64             LL f = dfs(v, std::min(ee.cap, now - ans));
65             if (f) {
66                 minc += f * ee.cost, ans += f;
67                 ee.cap -= f;
68                 er.cap += f;
69             }
70         }
71     }
72     vis[u] = 0;
73     return ans;
74 }
75
76 PLL mcmf() {
77     LL maxf = 0;
78     while (spfa()) {
79         LL tmp;
80         while ((tmp = dfs(s, INF))) maxf += tmp;
81     }
82     return std::make_pair(maxf, minc);
83 }
84 } minc_maxf;

```

Primal-Dual 原始对偶算法

```

1  /* primal dual */
2  struct edge {
3      int from, to;
4      LL cap, cost;
5  };
6  edge(int u, int v, LL c, LL w) : from(u), to(v), cap(c), cost(w) {}
7  };
8
9  struct node {
10     int v, e;
11 };
12 node(int _v = 0, int _e = 0) : v(_v), e(_e) {}
13 };
14
15 const int maxn = 5000 + 10;
16
17 struct MCMF {
18     int n, m = 0, s, t;
19     std::vector<edge> e;
20     vi g[maxn];
21     int vis[maxn];
22     LL dis[maxn], h[maxn];
23     node p[maxn * 2];
24
25     void add(int from, int to, LL cap, LL cost) {
26         e.push_back(edge(from, to, cap, cost));
27         e.push_back(edge(to, from, 0, -cost));
28         g[from].push_back(m++);
29         g[to].push_back(m++);
30     }
31
32     bool dijkstra() {
33         std::priority_queue<PIL, std::vector<PIL>, std::greater<PIL>> q;
34         for (int i = 1; i <= n; i++) {
35             dis[i] = INF;
36             vis[i] = 0;
37         }
38         dis[s] = 0;
39         q.push({0, s});
40         while (!q.empty()) {
41             auto u = q.top().ss;
42             q.pop();
43             if (vis[u]) continue;
44             vis[u] = 1;
45             for (auto i : g[u]) {

```

```

46         edge ee = e[i];
47         int v = ee.to;
48         LL nc = ee.cost + h[u] - h[v];
49         if (ee.cap and dis[v] > dis[u] + nc) {
50             dis[v] = dis[u] + nc;
51             p[v] = node(u, i);
52             if (!vis[v]) q.push({dis[v], v});
53         }
54     }
55 }
56 return dis[t] != INF;
57 }
58
59 void spfa() {
60     std::queue<int> q;
61     for (int i = 1; i <= n; i++) h[i] = INF;
62     h[s] = 0, vis[s] = 1;
63     q.push(s);
64     while (!q.empty()) {
65         int u = q.front();
66         q.pop();
67         vis[u] = 0;
68         for (auto i : g[u]) {
69             edge ee = e[i];
70             int v = ee.to;
71             if (ee.cap and h[v] > h[u] + ee.cost) {
72                 h[v] = h[u] + ee.cost;
73                 if (!vis[v]) {
74                     vis[v] = 1;
75                     q.push(v);
76                 }
77             }
78         }
79     }
80 }
81
82 PLL mcmf() {
83     LL maxf = 0, minc = 0;
84     spfa();
85     while (dijkstra()) {
86         LL minf = INF;
87         for (int i = 1; i <= n; i++) h[i] += dis[i];
88         for (int i = t; i != s; i = p[i].v) minf = std::min(minf, e[p[i].e].cap);
89         for (int i = t; i != s; i = p[i].v) {
90             e[p[i].e].cap -= minf;
91             e[p[i].e ^ 1].cap += minf;
92         }
93         maxf += minf;
94         minc += minf * h[t];
95     }
96     return std::make_pair(maxf, minc);
97 }
98 } minc_maxf;

```

存在负环的网络

流满后推流, 转化为上下界网络流.

10.20 network flow - minimal cut

最小割解决的问题是将图中的点集 V 划分成 S 与 T , 使得 S 与 T 之间的连边的容量总和最小.

最大流最小割定理

网络中 s 到 t 的最大流流量的值等于所要求的最小割的值, 所以求最小割只需要跑 Dinic 即可.

获得 S 中的所有点

在 Dinic 的 bfs 函数中, 每次将所有点的 d 数组值改为无穷大, 最后跑完最大流之后 d 数组不为无穷大的就是和源点一起在 S 集合中的点.

例子

最小割的本质是对图中点集进行 2-划分, 网络流只是求解答案的手段.

1. 在图中花费最小的代价断开一些边使得源点 s 无法流到汇点 t .

直接跑最大流就得到了答案.

2. 在图中删除最少的点使得源点 s 无法流到汇点 t .

对每个点进行拆点, 在 i 与 i' 之间建立容量为 1 的有向边.

10.21 network flow - upper / lower bound

无源汇上下界可行流

每条有向边有流量的上下界限制, 但整张图并未确定源点与汇点. 如果存在满足每个点的流入量等于流出量, 且每条边的流量满足其上下界限制的流, 称之为可行流.

1. 将每条边先给予大小为下界的流量,
2. 对每个点计算总流入量 in_u 与总流出量 out_u 的值,
3. 建立超级源点到每个点, 容量大小为 $\max\{0, \text{in}_u - \text{out}_u\}$ 的边; 建立每个点到超级汇点, 容量大小为 $\max\{0, \text{out}_u - \text{in}_u\}$,
4. 跑从超级源点到超级汇点的最大流, 如果超级源点每条边都流满意味着存在可行流. 将每条边的流量加上预先给每条边设置的下界流量即为可行流方案.

有源汇上下界可行流

1. 建立汇点 t 到源点 s 的, 容量为 ∞ 的有向边, 将其转化为无源汇的问题.

有源汇上下界最大流

1. 建立汇点 t 到源点 s 的, 容量为 ∞ 的有向边, 将其转化为无源汇的问题,
2. 跑上下界可行流, 可行流流量为边 $t \xrightarrow{\infty} s$ 的流量.
3. 删除 $t \xrightarrow{\infty} s$ 的边, 再残量网络上跑 s 到 t 的最大流,
4. 答案等于可行流流量 + 最大流流量.

有源汇上下界最小流

1. 建立汇点 t 到源点 s 的, 容量为 ∞ 的有向边, 将其转化为无源汇的问题,
2. 跑上下界可行流, 可行流流量为边 $t \xrightarrow{\infty} s$ 的流量.
3. 删除 $t \xrightarrow{\infty} s$ 的边, 再残量网络上跑 t 到 s 的最大流,
4. 答案等于可行流流量 - 最大流流量.

有源汇上下界最小费用可行流

1. 按下界流满并计算费用,
2. 类似有源汇上下界最大流建图, 跑超级源点到超级汇点的费用流,
3. 答案等于按下界的费用加上后续残量网络.

10.22 network flow - other versions

```

1  /* dinic @ wrb */
2  template<typename T, T inf = numeric_limits<T>::max()>
3  struct Max_Flow {
4      vector<int> he, cur, d, ne, to;
5      vector<T> c;
6      int s, t;
7      Max_Flow(int m) : he(m, -1), s(-1), t(-1) {}
8      void add(int x, int y, T z = inf, T w = 0) {
9          // cerr << x << ' ' << y << ' ';
10         // if (z == inf) cerr << "inf\n";
11         // else cerr << z << '\n';
12         ne.emplace_back(he[x]);
13         he[x] = ne.size() - 1;
14         to.emplace_back(y);
15         c.emplace_back(z);
16         ne.emplace_back(he[y]);
17         he[y] = ne.size() - 1;
18         to.emplace_back(x);
19         c.emplace_back(w);
20     }
21     int bfs() {
22         queue<int> q;
23         d.assign(he.size(), -1);
24         q.emplace(s), d[s] = 0;
25         for (; q.size(); q.pop()) {
26             int u = q.front(), v;
27             for (int i = he[u]; ~i; i = ne[i]) {
28                 if (c[i] && d[v = to[i]] == -1) {
29                     d[v] = d[u] + 1;
30                     if (v == t) return 1;
31                     q.emplace(v);
32                 }
33             }
34         }
35         return 0;
36     };
37     T dfs(int u, T fl) {
38         if (u == t) return fl;
39         T z = 0, r;
40         for (int& i = cur[u], v; ~i; i = ne[i]) {
41             if (c[i] && d[v = to[i]] == d[u] + 1) {
42                 r = dfs(v, min(fl, c[i]));
43                 if (r == 0) d[v] = -1;
44                 else {
45                     fl -= r, z += r, c[i] -= r, c[i ^ 1] += r;
46                     if (fl == 0) return z;
47                 }
48             }
49         }
50         return z;
51     };
52     T dinic(int _s, int _t) {
53         T z = 0;
54         for (s = _s, t = _t; bfs();) {
55             cur = he, z += dfs(s, inf);
56         }
57         return z;
58     };
59 };

```

```

1  /* bounded flow @ lys */
2  #include <iostream>
3  #include <cstdio>
4  #include <algorithm>
5  #include <queue>
6  #include <vector>
7  #define int long long
8  using namespace std;
9

```

```

10 const int maxn = 50020;
11 const int inf = 1e18;
12
13 struct Dinic_limit
14 {
15     int st, sgn; // st = 1 表示有源汇; sgn 表示最大(1)最小(-1)流
16     struct edge
17     {
18         int x, y, cap, flow, cost;
19     };
20     int deg[maxn]; // rd - cd
21     vector<int> e[maxn];
22     vector<edge> edges;
23     int mx;
24     int mcmf;
25     void add(int x, int y, int cap, int cost)
26     {
27         edges.push_back({x, y, cap, 0, cost});
28         edges.push_back({y, x, 0, 0, -cost});
29         mx = max({mx, x, y});
30         int m = edges.size();
31         e[x].push_back(m - 2), e[y].push_back(m - 1);
32     }
33     void add(int x, int y, int l, int r, int cost)
34     {
35         if (cost >= 0)
36             add(x, y, r - l, cost), deg[y] += l, deg[x] -= l, mcmf += l * cost;
37         else
38             add(y, x, r - l, -cost), deg[y] += r, deg[x] -= r, mcmf += r * cost;
39     }
40     int s, t;
41     int vis[maxn], dis[maxn];
42     bool spfa()
43     {
44         queue<int> q;
45         fill(vis, vis + mx + 1, 0), fill(dis, dis + mx + 1, inf);
46         dis[s] = 0, q.push(s), vis[s] = 1;
47         while (!q.empty())
48         {
49             int x = q.front();
50             q.pop(), vis[x] = 0;
51             for (int i : e[x])
52             {
53                 auto k = edges[i];
54                 if (k.cap - k.flow > 0 && k.cost + dis[x] < dis[k.y])
55                 {
56                     dis[k.y] = dis[x] + k.cost;
57                     if (!vis[k.y])
58                         q.push(k.y), vis[k.y] = 1;
59                 }
60             }
61         }
62         return dis[t] != inf;
63     }
64     int cur[maxn];
65     int dfs(int x, int lim)
66     {
67         if (x == t || lim == 0)
68             return lim;
69         vis[x] = 1;
70         int res = 0, f;
71         for (int &i = cur[x]; i < (int)e[x].size(); i++)
72         {
73             auto &k = edges[e[x][i]];
74             if (!vis[k.y] && k.cost + dis[x] == dis[k.y] && (f = dfs(k.y, min(lim, k.cap - k.flow))))
75                 res += f, lim -= f, k.flow += f, edges[e[x][i] ^ 1].flow -= f, mcmf += f * k.cost;
76             if (lim == 0)
77                 break;
78         }
79         vis[x] = 0;
80         return res;
81     }
82     int dinic(int s_, int t_)
83     {
84         int ss = mx + 1, tt = ss + 1;
85         int tot = 0;
86         for (int i = 1; i <= mx; i++)
87             if (deg[i] > 0)
88                 add(ss, i, deg[i], 0), tot += deg[i];
89             else if (deg[i] < 0)
90                 add(i, tt, -deg[i], 0);
91         if (st)
92             add(t_, s_, 0, inf, 0);
93         s = ss, t = tt;
94         int res = 0;
95         while (spfa())
96             fill(cur, cur + mx + 1, 0), res += dfs(s, inf);

```

```

97     // cerr << res << " " << tot << endl;
98     if (res != tot)
99         return -1;
100    if (st == 0)
101        return 1;
102    res = -edges.back().flow;
103    edges.back().cap = edges.back().flow = 0;
104    edges[edges.size() - 2].cap = edges[edges.size() - 2].flow = 0;
105    s = s_, t = t_;
106    if (sgn == -1)
107        swap(s, t);
108    while (spfa())
109        fill(cur, cur + mx + 1, 0), res += sgn * dfs(s, inf);
110    return res;
111 }
112 void clear()
113 {
114     for (int i = 0; i <= mx; i++)
115         e[i].clear(), deg[i] = 0;
116     edges.clear();
117     mx = 0, mcmf = 0;
118 }
119 Dinic_limit(int st_ = 1, int sgn_ = 1) { st = st_, sgn = sgn_; }
120 // st = 1 表示有源汇; sgn 表示最大(1)最小(-1)流
121 // 使用时调用 dinic 函数, 返回-1表示无解, 否则返回最大/最小流
122 };
123
124 Dinic_limit G(1, 1);
125
126 signed main()
127 {
128     ios::sync_with_stdio(false), cin.tie(0);
129     int n, m, S, T;
130     cin >> n >> m >> S >> T;
131     for (int i = 0; i < m; i++)
132     {
133         int s, t, l, r, c;
134         cin >> s >> t >> l >> r >> c;
135         G.add(s, t, l, r, c);
136     }
137     int res = G.dinic(S, T);
138     if (res == -1)
139         cout << -1 << endl;
140     else
141     {
142         cout << res << " " << G.mcmf << endl;
143     }
144 }

```

10.23 matching - matching on bipartite graph

二分图最大匹配

Kuhn-Munkres

时间复杂度: $O(n^3)$.

```

1  /* Kuhn-Munkres */
2  auto KM = [&](int n1, int n2, vvi e) -> std::pair<vi, vi> {
3      vi vis(n2 + 1);
4      vi l(n1 + 1, -1), r(n2 + 1, -1);
5      std::function<bool(int)> dfs = [&](int u) -> bool {
6          for (auto v : e[u]) {
7              if (!vis[v]) {
8                  vis[v] = 1;
9                  if (r[v] == -1 or dfs(r[v])) {
10                     r[v] = u;
11                     return true;
12                 }
13             }
14         }
15         return false;
16     };
17     for (int i = 1; i <= n1; i++) {
18         std::fill(all(vis), 0);
19         dfs(i);
20     }
21     for (int i = 1; i <= n2; i++) {
22         if (r[i] == -1) continue;

```

```

23     l[r[i]] = i;
24 }
25 return {l, r};
26 };
27 auto [mchl, mchr] = KM(n1, n2, e);
28 std::cout << mchl.size() - std::count(all(mchl), -1) << endl;

```

Hopcroft-Karp

据说时间复杂度是 $O(m\sqrt{n})$ 的, 但是快的飞起.

```

1  /* Hopcroft-Karp */
2  vpi e(m);
3  auto hopcroft_karp = [&](int n, int m, vpi& e) -> std::pair<vi, vi> {
4      vi g(e.size()), l(n + 1, -1), r(m + 1, -1), d(n + 2);
5      for (auto [u, v] : e) d[u]++;
6      std::partial_sum(all(d), d.begin());
7      for (auto [u, v] : e) g[--d[u]] = v;
8      for (vi a, p, q(n + 1);) {
9          a.assign(n + 1, -1);
10         p.assign(n + 1, -1);
11         int t = 1;
12         for (int i = 1; i <= n; i++) {
13             if (l[i] == -1) {
14                 q[t++] = a[i] = p[i] = i;
15             }
16         }
17         bool match = false;
18         for (int i = 1; i < t; i++) {
19             int u = q[i];
20             if (l[a[u]] != -1) continue;
21             for (int j = d[u]; j < d[u + 1]; j++) {
22                 int v = g[j];
23                 if (r[v] == -1) {
24                     while (v != -1) {
25                         r[v] = u;
26                         std::swap(l[u], v);
27                         u = p[u];
28                     }
29                     match = true;
30                     break;
31                 }
32                 if (p[r[v]] == -1) {
33                     q[t++] = v = r[v];
34                     p[v] = u;
35                     a[v] = a[u];
36                 }
37             }
38         }
39         if (!match) break;
40     }
41     return {l, r};
42 };

```

二分图最大权匹配

Kuhn-Munkres

注意是否为完美匹配, 非完美选 0, 完美选 $-INF$. (存疑)

```

1  /* Kuhn-Munkres */
2  auto KM = [&](int n, vvl e) -> std::tuple<LL, vi, vi> {
3      vl la(n + 1), lb(n + 1), pp(n + 1), vx(n + 1);
4      vi l(n + 1, -1), r(n + 1, -1);
5      vi va(n + 1), vb(n + 1);
6      LL delta;
7      auto bfs = [&](int x) -> void {
8          int a, y = 0, y1 = 0;
9          std::fill(all(pp), 0);
10         std::fill(all(vx), INF);
11         r[y] = x;
12         do {
13             a = r[y], delta = INF, vb[y] = 1;
14             for (int b = 1; b <= n; b++) {
15                 if (!vb[b]) {
16                     if (vx[b] > la[a] + lb[b] - e[a][b]) {

```

```

17         vx[b] = la[a] + lb[b] - e[a][b];
18         pp[b] = y;
19     }
20     if (vx[b] < delta) {
21         delta = vx[b];
22         y1 = b;
23     }
24 }
25 }
26 for (int b = 0; b <= n; b++) {
27     if (vb[b]) {
28         la[r[b]] -= delta;
29         lb[b] += delta;
30     } else
31         vx[b] -= delta;
32 }
33 y = y1;
34 } while (r[y] != -1);
35 while (y) {
36     r[y] = r[pp[y]];
37     y = pp[y];
38 }
39 };
40 for (int i = 1; i <= n; i++) {
41     std::fill(all(vb), 0);
42     bfs(i);
43 }
44 LL ans = 0;
45 for (int i = 1; i <= n; i++) {
46     if (r[i] == -1) continue;
47     l[r[i]] = i;
48     ans += e[r[i]][i];
49 }
50 return {ans, l, r};
51 };
52
53 auto [ans, mchl, mchr] = KM(n, e);

```

10.24 matching - matching on general graph

11 geometry

11.1 two demention

点与向量

```

1 struct Point {
2     LL x = 0, y = 0;
3     Point() = default;
4     Point(long long x, long long y) : x(x), y(y) {}
5     operator bool() { return *this != Point{}; }
6     friend bool operator==(Point p, Point q) { return p.x == q.x and p.y == q.y; }
7     friend bool operator!=(Point p, Point q) { return !(p == q); }
8     friend Point operator+(Point p, Point q) { return {p.x + q.x, p.y + q.y}; }
9     friend Point operator-(Point p, Point q) { return {p.x - q.x, p.y - q.y}; }
10    friend LL dot(Point p, Point q) { return p.x * q.x + p.y * q.y; }
11    friend LL det(Point p, Point q) { return p.x * q.y - q.x * p.y; }
12    friend bool operator<(Point p, Point q) {
13        return std::pair{p.quad(), det(q, p)} < std::pair{q.quad(), 0ll};
14        return (p.x == q.x ? p.y < q.y : p.x < q.x);
15    }
16    int quad() const {
17        if (x > 0 && y >= 0) return 1;
18        if (x <= 0 and y > 0) return 2;
19        if (x < 0 and y <= 0) return 3;
20        if (x >= 0 and y < 0) return 4;
21        return 0;
22    }
23    friend LL dist(Point p, Point q) {
24        return (p.x - q.x) * (p.x - q.x) + (p.y - q.y) * (p.y - q.y);
25    }
26 };
27 std::istream& operator>>(std::istream& is, Point& p) { return is >> p.x >> p.y; }
28 std::ostream& operator<<(std::ostream& os, Point p) {
29     return os << '(' << p.x << ',' << p.y << ')';
30 }

```

线段

```

1 struct line {
2     point a, b;
3
4     line(point _a = {}, point _b = {}) { a = _a, b = _b; }
5
6     /* 交点类型为 double */
7     friend point iPoint(line p, line q) {
8         point v1 = p.b - p.a;
9         point v2 = q.b - q.a;
10        point u = q.a - p.a;
11        return q.a + (q.b - q.a) * ((u ^ v1) * 1. / (v1 ^ v2));
12    }
13
14    /* 极角排序 */
15    bool operator<(const line& p) const {
16        double t1 = std::atan2((b - a).y, (b - a).x);
17        double t2 = std::atan2((p.b - p.a).y, (p.b - p.a).x);
18        if (fabs(t1 - t2) > eps) {
19            return t1 < t2;
20        }
21        return ((p.a - a) ^ (p.b - a)) > eps;
22    }
23 };

```

11.2 convex

2D

```

1 /* andrew */
2 auto andrew = [&](std::vector<point>& v) -> std::vector<point> {
3     std::sort(all(v));

```

```

4      std::vector<point> stk;
5      for (int i = 0; i < n; i++) {
6          point x = v[i];
7          while (stk.size() > 1 and ((stk.end()[-1] - stk.end()[-2]) ^ (x - stk.end()[-2])) <= 0) {
8              stk.pop_back();
9          }
10         stk.push_back(x);
11     }
12     int tmp = stk.size();
13     for (int i = n - 2; i >= 0; i--) {
14         point x = v[i];
15         while (stk.size() > tmp and ((stk.end()[-1] - stk.end()[-2]) ^ (x - stk.end()[-2])) <= 0) {
16             stk.pop_back();
17         }
18         stk.push_back(x);
19     }
20     return stk;
21 };

```

求凸包 and 判断点与凸包关系

```

1  #include<bits/stdc++.h>
2  #ifdef LOCAL
3  #include "debug.h"
4  #else
5  #define debug(...) 0
6  #endif
7  #define all(v) begin(v), end(v)
8  using namespace std;
9  using pii = pair<int, int>;
10
11 template<typename T, typename P, T inf = numeric_limits<T>::max()>
12 struct Convex_Hull {
13     using ptt = pair<T, T>;
14     // using i128 = __int128;
15     vector<ptt> a, b;
16     T lox, hix;
17     P crs(const ptt& a, const ptt& b) {
18         return (P)a.first * b.second - (P)a.second * b.first;
19     }
20     ptt mns(const ptt& a, const ptt& b) {
21         return ptt{a.first - b.first, a.second - b.second};
22     };
23     Convex_Hull(vector<ptt> c) {
24         assert(c.size() > 0);
25         sort(begin(c), end(c));
26         vector<int> st = {0};
27         int n = c.size(), tp = 0;
28         for (int i = 1; i < n; ++i) {
29             while (tp > 0 &&
30                 crs(mns(c[st[tp]], c[st[tp - 1]]), mns(c[i], c[st[tp]])) <= 0) {
31                 --tp, st.pop_back();
32             }
33             st.emplace_back(i), ++tp;
34         }
35         int tmp = tp;
36         for (int i = n - 1; ~i; --i) {
37             while (tp > tmp &&
38                 crs(mns(c[st[tp]], c[st[tp - 1]]), mns(c[i], c[st[tp]])) <= 0) {
39                 --tp, st.pop_back();
40             }
41             st.emplace_back(i), ++tp;
42         }
43         for (int i = 0; i <= tmp; ++i) {
44             a.emplace_back(c[st[i]]);
45         }
46         for (int i = tmp; i <= tp; ++i) {
47             b.emplace_back(c[st[i]]);
48         }
49     }
50     // n >= 3
51     pair<int, vector<ptt>> insd(T x, T y) { // 0: outside, 1: invertex, 2:{u,v}: inedged, 3: inside
52         ptt o = {x, y};
53         if (x < a[0].first || x > b[0].first) return {0, {}};
54         int li = lower_bound(begin(a), end(a), ptt{x, -inf}) - begin(a);
55         if (o == a[li]) return {1, {}};
56         int hi = lower_bound(begin(b), end(b), ptt{x, inf}, greater{}) - begin(b);
57         if (o == b[hi]) return {1, {}};
58         if (li == 0) {
59             if (hi + 1 == b.size()) return {0, {}};
60             assert(b.end()[-1].first == b.end()[-2].first);
61             if (y < b.end()[-1].second || y > b.end()[-2].second) return {0, {}};
62             return {2, {b.end()[-1], b.end()[-2]}};

```

```

63     }
64     if (hi == 0) {
65         if (li + 1 == a.size()) return {0, {}};
66         assert(a.end()[-2].first == a.end()[-1].first);
67         if (y < a.end()[-2].second || y > a.end()[-1].second) return {0, {}};
68         return {2, {a.end()[-2], a.end()[-1]}};
69     }
70     P v1 = crs(mns(o, a[li - 1]), mns(a[li], a[li - 1]));
71     if (v1 == 0) return {2, {a[li - 1], a[li]}};
72     P v2 = crs(mns(o, b[hi - 1]), mns(b[hi], b[hi - 1]));
73     if (v2 == 0) return {2, {b[hi - 1], b[hi]}};
74     debug(v1, v2);
75     return {v1 > 0 && v2 > 0 || v1 < 0 && v2 < 0 ? 3 : 0, {}};
76 }
77 };
78
79 namespace Acc {
80     auto work = []() {
81         string ans[] = {"OUT", "ON", "ON", "IN"};
82         int n, q;
83         cin >> n;
84         vector<pair<int, int>> a(n);
85         for (auto& [x, y] : a) cin >> x >> y;
86         Convex_Hull<int, long long> ch(a);
87         debug(ch.a, ch.b);
88         cin >> q;
89         for (int x, y; q--;) {
90             cin >> x >> y;
91             cout << ans[ch.insd(x, y).first] << '\n';
92         }
93     };
94 }
95
96 int main() {
97     std::ios::sync_with_stdio(0);
98     std::cin.tie(0);
99     int T = 1;
100     // std::cin >> T;
101     while (T--) Acc::work();
102 }

```

11.3 half plane union

```

1  /* half plane union */
2  auto half_plane = [&](std::vector<line>& ln) -> std::vector<point> {
3      std::sort(all(ln));
4      ln.erase(
5          unique(
6              all(ln),
7              [](line& p, line& q) {
8                  double t1 = std::atan2((p.b - p.a).y, (p.b - p.a).x);
9                  double t2 = std::atan2((q.b - q.a).y, (q.b - q.a).x);
10                 return fabs((t1 - t2)) < eps;
11             }),
12          ln.end());
13      auto check = [&](line p, line q, line r) -> bool {
14          point a = iPoint(p, q);
15          return ((r.b - r.a) ^ (a - r.a)) < -eps;
16      };
17      line q[ln.size() + 2];
18      int hh = 1, tt = 0;
19      q[+tt] = ln[0];
20      q[+tt] = ln[1];
21      for (int i = 2; i < (int) ln.size(); i++) {
22          while (hh < tt and check(q[tt - 1], q[tt], ln[i])) tt--;
23          while (hh < tt and check(q[hh + 1], q[hh], ln[i])) hh++;
24          q[+tt] = ln[i];
25      }
26      while (hh < tt and check(q[tt - 1], q[tt], q[hh])) tt--;
27      while (hh < tt and check(q[hh + 1], q[hh], q[tt])) hh++;
28      q[tt + 1] = q[hh];
29      std::vector<point> ans;
30      for (int i = hh; i <= tt; i++) {
31          ans.push_back(iPoint(q[i], q[i + 1]));
32      }
33      return ans;
34 };

```

11.4 rotate


```

1  /* rotate @ wrb */
2  #include<cstdio>
3  #include<algorithm>
4  #define db double
5  namespace Acc{
6      const int N = 5e4+10;
7      struct node{
8          int x,y;
9      }a[N],stk[N];
10     db cmp(node a,node b,node c){return 1.*(c.x-a.x)*(c.y-b.y)-1.*(c.x-b.x)*(c.y-a.y);}
11     int dis(node a,node b){return ((a.x-b.x)*(a.x-b.x)+(a.y-b.y)*(a.y-b.y));}
12     int n,tp,ans;
13     void work(){
14         scanf("%d",&n);
15         for(int i=1;i<=n;i++)scanf("%d%d",&a[i].x,&a[i].y);
16         std::sort(a+1,a+n+1,[=](node a,node b)->bool{return a.x<b.x || (a.x==b.x && a.y<b.y);});
17         stk[1]=a[1],tp=1;
18         for(int i=2;i<=n;i++){
19             while(tp>1 && cmp(stk[tp-1],stk[tp],a[i])<=0)tp--;
20             stk[++tp]=a[i];
21         }
22         int tmp=tp;
23         for(int i=n-1;i>=1;i--){
24             while(tp>tmp && cmp(stk[tp-1],stk[tp],a[i])<=0)tp--;
25             stk[++tp]=a[i];
26         }
27         for(int i=1,j=3;i<tp;i++){
28             while(cmp(stk[i],stk[i+1],stk[j])<cmp(stk[i],stk[i+1],stk[j+1]))j=j%(tp-1)+1;
29             ans=std::max(ans,std::max(dis(stk[i],stk[j]),dis(stk[i+1],stk[j]))));
30         }
31         printf("%d",ans);
32     }
33 }
34 int main(){
35     return Acc::work(),0;
36 }

```

11.5 Simpson

```

1  #include <bits/stdc++.h>
2  using namespace std;
3  double a;
4  double f(double x) {
5      return pow(x, a / x - x);
6  }
7  double simpson(double l, double r) {
8      double mid = (l + r) / 2;
9      return (r - l) * (f(l) + 4 * f(mid) + f(r)) / 6;
10 }
11 double asr(double l, double r, double eps, double ans, int d) {
12     double mid = (l + r) / 2;
13     double Fl = simpson(l, mid), Fr = simpson(mid, r);
14     if (abs(Fl + Fr - ans) <= 15 * eps && d < 0) {
15         return Fl + Fr + (Fl + Fr - ans) / 15;
16     }
17     return asr(l, mid, eps / 2, Fl, d - 1) + asr(mid, r, eps / 2, Fr, d - 1);
18 }
19 double calc(double l, double r, double eps) {
20     return asr(l, r, eps, simpson(l, r), 12);
21 }
22 int main() {
23     cin >> a;
24     if (a < 0) {
25         cout << "orz\n";
26     } else {
27         cout << fixed << setprecision(5) << calc(1e-8, 15, 1e-8) << '\n';
28     }
29 }

```

12 offline algorithm

12.1 discretization

```

1 std::sort(all(a));
2 a.erase(unique(all(a)), a.end());
3 auto get_id = [&](const int& x) -> int { return lower_bound(all(a), x) - a.begin() + 1; };

```

12.2 Mo algorithm

普通莫队

```

1 int block = n / sqrt(2 * m / 3);
2 std::sort(all(q), [&](node a, node b) {
3     return a.l / block == b.l / block ? (a.r == b.r ? 0 : ((a.l / block) & 1) ^ (a.r < b.r))
4         : a.l < b.l;
5 });
6 auto move = [&](int x, int op) -> void {
7     if (op == 1) {
8         /* operations */
9     } else {
10        /* operations */
11    }
12 };
13 for (int k = 1, l = 1, r = 0; k <= m; k++) {
14     node Q = q[k];
15     while (l > Q.l) {
16         move(--l, 1);
17     }
18     while (r < Q.r) {
19         move(++r, 1);
20     }
21     while (l < Q.l) {
22         move(l++, -1);
23     }
24     while (r > Q.r) {
25         move(r--, -1);
26     }
27 }

```

12.3 回滚莫队

```

1 /* rollback Mo */
2 #include<bits/stdc++.h>
3 namespace Acc {
4     const int N = 200009;
5     int a[N], b[N], id[N], f[N], g[N], p[N], z[N];
6     std::pair<int, int> st[N];
7     struct T {
8         int l, r, o;
9     } q[N];
10    auto work = []() {
11        int n, m;
12        std::cin >> n;
13        for (int i = 1; i <= n; ++i) {
14            std::cin >> a[i], b[i] = a[i];
15        }
16        std::sort(b + 1, b + n + 1);
17        int ct = std::unique(b + 1, b + n + 1) - b - 1;
18        for (int i = 1; i <= n; ++i) {
19            a[i] = std::lower_bound(b + 1, b + ct + 1, a[i]) - b;
20        }
21        std::cin >> m;
22        for (int i = 1; i <= m; ++i) {
23            auto&[l, r, o] = q[i];
24            std::cin >> l >> r, o = i;
25        }
26        int B = ceil(n / sqrt(m));
27        for (int i = 1; i <= n; ++i) {
28            id[i] = (i - 1) / B + 1;
29        }
30        std::sort(q + 1, q + m + 1, [](T a, T b) {

```

```

31     return id[a.l] == id[b.l] ? a.r < b.r : a.l < b.l;
32 };
33 int ans = 0, L = 1, R = 0;
34 for (int i = 1; i <= m; ++i) {
35     auto[l, r, o] = q[i];
36     if (id[l] != id[q[i - 1].l]) {
37         ans = 0;
38         R = std::min(n, id[l] * B), L = R + 1;
39         memset(f + 1, 0, ct << 2);
40         memset(g + 1, 0, ct << 2);
41     }
42     if (id[l] == id[r]) {
43         for (int j = 1; j <= r; ++j) {
44             if (p[a[j]] == 0) p[a[j]] = j;
45             else ans = std::max(ans, j - p[a[j]]);
46         }
47         for (int j = 1; j <= r; ++j) p[a[j]] = 0;
48         z[o] = ans, ans = 0;
49     } else {
50         while (R < r) {
51             ++R, g[a[R]] = R;
52             if (f[a[R]] == 0) f[a[R]] = R;
53             else ans = std::max(ans, R - f[a[R]]);
54         }
55         int las = ans, t = L;
56         while (l < L) {
57             --L;
58             int x = f[a[L]], y = g[a[L]];
59             st[L] = std::make_pair(x, y);
60             f[a[L]] = L;
61             if (g[a[L]] == 0) g[a[L]] = L;
62             else ans = std::max(ans, g[a[L]] - L);
63         }
64         z[o] = ans;
65         for (int j = 1; j < t; ++j) {
66             auto[x, y] = st[j];
67             f[a[j]] = x, g[a[j]] = y;
68         }
69         ans = las, L = t;
70     }
71 }
72 for (int i = 1; i <= m; ++i) {
73     std::cout << z[i] << '\n';
74 }
75 };
76 }
77 int main() {
78     std::ios::sync_with_stdio(0);
79     std::cin.tie(0), Acc::work();
80 }

```

12.4 CDQ

n 个三维数对 (a_i, b_i, c_i) , 设 $f(i)$ 表示 $a_j \leq a_i, b_j \leq b_i, c_j \leq c_i (i \neq j)$ 的个数. 输出 $f(i) (0 \leq i \leq n-1)$ 的值.

```

1 // 洛谷 P3810 【模板】三维偏序 (陌上花开)
2
3 struct data {
4     int a, b, c, cnt, ans;
5
6     data(int _a = 0, int _b = 0, int _c = 0, int _cnt = 0, int _ans = 0) {
7         a = _a, b = _b, c = _c, cnt = _cnt, ans = _ans;
8     }
9
10    bool operator!=(data x) {
11        if (a != x.a) return true;
12        if (b != x.b) return true;
13        if (c != x.c) return true;
14        return false;
15    }
16 };
17
18 int main() {
19     std::ios::sync_with_stdio(false);
20     std::cin.tie(0);
21
22     int n, k;
23     std::cin >> n >> k;
24     static data v1[N], v2[N];
25     for (int i = 1; i <= n; i++) {

```

```

26     std::cin >> v1[i].a >> v1[i].b >> v1[i].c;
27 }
28 std::sort(v1 + 1, v1 + n + 1, [&](data x, data y) {
29     if (x.a != y.a) return x.a < y.a;
30     if (x.b != y.b) return x.b < y.b;
31     return x.c < y.c;
32 });
33 int t = 0, top = 0;
34 for (int i = 1; i <= n; i++) {
35     t++;
36     if (v1[i] != v1[i + 1]) {
37         v2[++top] = v1[i];
38         v2[top].cnt = t;
39         t = 0;
40     }
41 }
42 vi tr(N);
43 auto add = [&](int pos, int val) -> void {
44     while (pos <= k) {
45         tr[pos] += val;
46         pos += lowbit(pos);
47     }
48 };
49 auto query = [&](int pos) -> int {
50     int ans = 0;
51     while (pos > 0) {
52         ans += tr[pos];
53         pos -= lowbit(pos);
54     }
55     return ans;
56 };
57 std::function<void(int, int)> CDQ = [&](int l, int r) -> void {
58     if (l == r) return;
59     int mid = (l + r) >> 1;
60     CDQ(l, mid), CDQ(mid + 1, r);
61     std::sort(v2 + 1, v2 + mid + 1, [&](data x, data y) {
62         if (x.b != y.b) return x.b < y.b;
63         return x.c < y.c;
64     });
65     std::sort(v2 + mid + 1, v2 + r + 1, [&](data x, data y) {
66         if (x.b != y.b) return x.b < y.b;
67         return x.c < y.c;
68     });
69     int i = l, j = mid + 1;
70     while (j <= r) {
71         while (i <= mid && v2[i].b <= v2[j].b) {
72             add(v2[i].c, v2[i].cnt);
73             i++;
74         }
75         v2[j].ans += query(v2[j].c);
76         j++;
77     }
78     for (int ii = l; ii < i; ii++) {
79         add(v2[ii].c, -v2[ii].cnt);
80     }
81     return;
82 };
83 CDQ(1, top);
84 vi ans(n + 1);
85 for (int i = 1; i <= top; i++) {
86     ans[v2[i].ans + v2[i].cnt] += v2[i].cnt;
87 }
88 for (int i = 1; i <= n; i++) {
89     std::cout << ans[i] << endl;
90 }
91 return 0;
92 }

```

12.5 segment tree divide and conquer

```

1  /* seg div @ wrb */
2  #include<bits/stdc++.h>
3  using namespace std;
4  namespace Acc {
5      const int N = 1e5;
6      pair<int, int> q[N * 2];
7      vector<int> v[N * 4];
8      int n, l, r, p;
9      int fa[N * 2], sz[N * 2];
10     pair<int, int> st[N * 2];
11     int tp;
12     void ins(int o, int L, int R) {

```

```

13     if (r < L || l > R) return ;
14     if (l <= L && R <= r) {
15         v[o].emplace_back(p);
16         return ;
17     }
18     int md = L + R >> 1;
19     ins(o << 1, L, md);
20     ins(o << 1 | 1, md + 1, R);
21 }
22 auto gf = [](int x) {
23     while (x != fa[x]) x = fa[x];
24     return x;
25 };
26 auto mg = [](int x, int y) {
27     x = gf(x), y = gf(y);
28     if (x != y) {
29         if (sz[x] < sz[y]) swap(x, y);
30         fa[y] = x, sz[x] += sz[y];
31         st[++tp] = {x, y};
32     }
33 };
34 void dfs(int o, int L, int R) {
35     int lastp = tp;
36     for (int i : v[o]) {
37         auto[x, y] = q[i];
38         mg(x, y + n), mg(x + n, y);
39         if (gf(x) == gf(x + n)) {
40             for (int i = L; i <= R; ++i) {
41                 cout << "No\n";
42             }
43             goto _;
44         }
45     }
46     if (L == R) {
47         cout << "Yes\n";
48     } else {
49         int md = L + R >> 1;
50         dfs(o << 1, L, md);
51         dfs(o << 1 | 1, md + 1, R);
52     }
53 _:
54     for (; tp > lastp; --tp) {
55         auto[x, y] = st[tp];
56         fa[y] = y, sz[x] -= sz[y];
57     }
58 }
59 auto work = []() {
60     int m, k;
61     cin >> n >> m >> k;
62     for (int i = 1; i <= m; ++i) {
63         int x, y;
64         cin >> x >> y >> l >> r;
65         if (++l <= r) {
66             q[p = i] = {x, y}, ins(1, 1, k);
67         }
68     }
69     iota(fa + 1, fa + n * 2 + 1, 1);
70     fill(sz + 1, sz + n * 2 + 1, 1);
71     dfs(1, 1, k);
72 };
73 }
74 int main() {
75     ios::sync_with_stdio(0);
76     cin.tie(0), Acc::work();
77 }

```

13 Print All Cases

13.1 print all trees with n nodes

构造所有 n 个节点的树.

13.1.1 有根树

表示其数量的数列在 oeis 上编号为 A000081. $n = 1, 2, 3 \dots, 20$ 的项分别为:

1, 1, 2, 4, 9,
20, 48, 115, 286, 719,
1842, 4766, 12486, 32973, 87811,
235381, 634847, 1721159, 4688676, 12826228.

构造所有 $n \leq 20$ 的有根树的 (平均) 运行时间为 15.7054s.

```

1  /* integer partition */
2  int n = 5;
3  std::vector<vvi> part(n + 1);
4  auto integerPartition = [&](int n) {
5      // part[i] = {{i}};
6      for (int i = 1; i <= n; i++) {
7          part[i].push_back({i});
8          for (int j = 1; j < i; j++) {
9              for (const auto& v : part[i - j]) {
10                 vi tmp = v;
11                 tmp.push_back(j);
12                 std::sort(all(tmp));
13                 part[i].push_back(tmp);
14             }
15         }
16         std::sort(all(part[i]));
17         part[i].erase(unique(all(part[i])), part[i].end());
18     }
19 };
20 integerPartition(n);
21 /* find all trees */
22 std::vector<std::vector<std::string>> trees(n + 1);
23 auto allTrees = [&](int n) {
24     std::string s;
25     for (int i = 1; i < n; i++) s += '(';
26     for (int i = 1; i < n; i++) s += ')';
27     trees[n].push_back(s);
28     for (const auto& v : part[n - 1]) {
29         std::vector<std::string> now;
30         auto dfs = [&](auto&& self, int i) {
31             if (i == v.size()) {
32                 std::string s = "";
33                 auto tmp = now;
34                 std::sort(all(tmp));
35                 for (const auto& ss : tmp) s += '(' + ss + ')';
36                 trees[n].push_back(s);
37                 return;
38             }
39             for (const auto& s : trees[v[i]]) {
40                 now.push_back(s);
41                 self(self, i + 1);
42                 now.pop_back();
43             }
44         };
45         dfs(dfs, 0);
46     }
47     std::sort(all(trees[n]));
48     trees[n].erase(unique(all(trees[n])), trees[n].end());
49 };
50 for (int i = 1; i <= n; i++) {
51     allTrees(i);
52     debug(i, trees[i].size());
53     std::cout << '\n';
54 }
55 for (const auto& s : trees[n]) {
56     vvi e(n + 1);
57     vi fa(n + 1);
58     int cnt = 1, now = 1;

```

```
59     for (const auto& c : s) {
60         if (c == '(') {
61             cnt += 1;
62             e[now].push_back(cnt);
63             e[cnt].push_back(now);
64             fa[cnt] = now;
65             now = cnt;
66         } else {
67             now = fa[now];
68         }
69     }
70     debug(e);
71     /* do the things you need */
72 }
```

14 Magic

14.1 magic heap

对顶堆维护中位数.

```

1  /* magic heap */
2  struct MagicHeap {
3      LL suml = 0, sumr = 0;
4      std::priority_queue<int> ql;
5      std::priority_queue<int, std::vector<int>, std::greater<int>> qr;
6      void le2ri() {
7          auto x = ql.top();
8          suml -= x, ql.pop();
9          sumr += x, qr.push(x);
10     };
11     void ri2le() {
12         auto x = qr.top();
13         sumr -= x, qr.pop();
14         suml += x, ql.push(x);
15     };
16     void pushL(int x) { suml += x, ql.push(x); }
17     void pushR(int x) { sumr += x, qr.push(x); }
18     void push(int x) {
19         if (ql.empty()) {
20             pushL(x);
21         } else if (qr.empty()) {
22             (x <= ql.top() ? le2ri(), pushL(x) : pushR(x));
23         } else {
24             int le = ql.top(), ri = qr.top();
25             if (le <= x and x <= ri) {
26                 (ql.size() == qr.size() ? pushL(x) : pushR(x));
27             } else if (x < le) {
28                 if (ql.size() != qr.size()) le2ri();
29                 pushL(x);
30             } else {
31                 if (ql.size() <= qr.size()) ri2le();
32                 pushR(x);
33             }
34         }
35     }
36     int size() { return ql.size() + qr.size(); }
37     bool empty() { return ql.empty() and qr.empty(); }
38     LL val() { return suml + sumr; }
39     LL mid() { return ql.top(); }
40     LL dist() { return sumr - suml + ql.top() * (ql.size() - qr.size()); }
41 };

```

14.2 operator queue

双栈维护队列半群.

```

1  template <typename T, typename Op>
2  struct OpQueue {
3      static_assert(std::is_convertible_v<std::invoke_result_t<Op, T, T>, T>);
4      const T e;
5      const Op op;
6      std::vector<T> l, r, a;
7      OpQueue(T e, Op op) : e(e), op(op), l{e}, r{e} {}
8      T val() const { return op(l.back(), r.back()); }
9      void push(T x) {
10         r.push_back(op(r.back(), x));
11         a.push_back(x);
12     }
13     void pop() {
14         if (l.size() == 1) {
15             for (; !a.empty(); a.pop_back()) {
16                 l.push_back(op(a.back(), l.back()));
17             }
18             r.resize(1);
19         }
20         assert(l.size() > 1);
21         l.pop_back();
22     }
23     int size() const { return l.size() + r.size() - 2; }
24     bool empty() const { return l.size() + r.size() == 2; }
25 };

```



```

26
27 /* When using this, remember to replace "T" with correct type and "e" with identity in the half group. */
28 auto op = [] (T a, T b) -> T {
29     /* You operations */
30 };
31 OpQueue<T, decltype(op)> a(e, op);

```

14.3 Fast GCD

$O(V)$ 预处理, $O(1)$ 查询 GCD.

```

1  /* fast GCD @ luogu shit */
2  const int N = 5005, M = 1e6 + 5, S = 1000, P = 998244353;
3
4  int _a[M], _b[M], _c[M];
5  int v[M], p[M], r;
6  int f[S + 1][S + 1];
7  int a[N], b[N];
8
9  void Init() {
10     _a[1] = _b[1] = _c[1] = 1;
11     for (int i = 2; i <= M - 5; ++i) {
12         if (!v[i]) {
13             p[++r] = i;
14             _a[i] = _b[i] = 1, _c[i] = i;
15         }
16         int tp;
17         for (int j = 1; j <= r && (tp = i * p[j]) <= M - 5; ++j) {
18             v[tp] = 1;
19             _a[tp] = _a[i] * p[j];
20             _b[tp] = _b[i];
21             _c[tp] = _c[i];
22             if (_a[tp] > _b[tp]) {
23                 swap(_a[tp], _b[tp]);
24                 if (_b[tp] > _c[tp]) {
25                     swap(_b[tp], _c[tp]);
26                 }
27             }
28             if (!(i % p[j])) {
29                 break;
30             }
31         }
32     }
33
34     for (int i = 1; i <= S; ++i) {
35         f[0][i] = f[i][0] = i;
36         for (int j = 1; j <= S; ++j) {
37             f[i][j] = f[j % i][i];
38         }
39     }
40     return;
41 }
42
43 int gcd(int x, int y) {
44     int A = 1, tp = f[_a[x]][y % _a[x]];
45     A *= tp;
46     y /= tp;
47     tp = f[_b[x]][y % _b[x]];
48     A *= tp;
49     y /= tp;
50     tp = (v[_c[x]] ? f[_c[x]][y % _c[x]] : (y % _c[x] ? 1 : _c[x]));
51     A *= tp;
52     y /= tp;
53     return A;
54 }

```

14.4 $q \equiv \frac{a}{b} \pmod{mod}$

```

1  /* find q = a / b @ luogu shit */
2  pair<int, int> approx(int mod, int q, int bound) {
3      int x = q, y = mod, a = 1, b = 0;
4      while (x > bound) {
5          swap(x, y), swap(a, b);
6          a -= x / y * b;
7          x %= y;
8      }
9      return make_pair(x, a);

```

10 | }
