REDES NEURAIS ARTIFICIAIS

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zenilton@pucminas.br

Rede Neural Artificial (sem metáfora cognitiva)

(**Antes**) Função de predição: $f = Wx$

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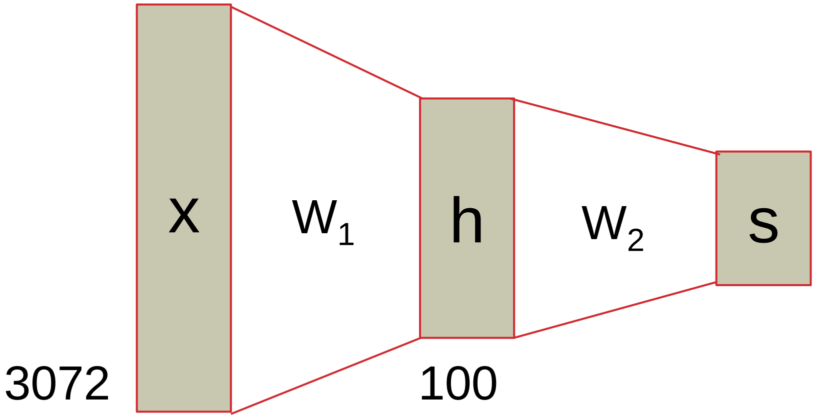


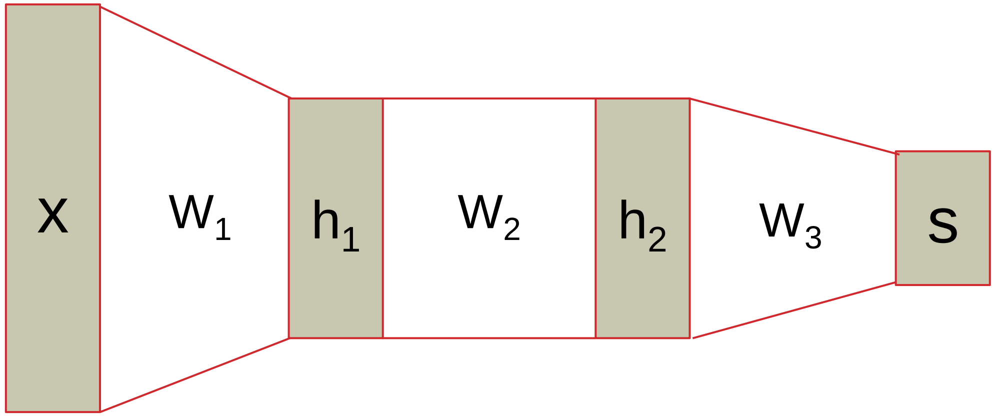
Rede Neural Artificial (sem metáfora cognitiva)

(**Antes**) Função de predição: $f = Wx$

(**Agora**) Rede neural de 2 camadas: $f = W_2 \max(0, W_1 x)$

ou Rede neural de 3 camadas:

$$f = W_3 \max(0, W_2 \max(0, W_1 x))$$



Código para Treino de Rede Neural de 2 Camadas

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import numpy as np
from numpy.random import randn

N, D_in, H, D_out = 64, 1000, 100, 10
x, y = randn(N, D_in), randn(N, D_out)
w1, w2 = randn(D_in, H), randn(H, D_out)

for t in range(2000):
  h = 1 / (1 + np.exp(-x.dot(w1)))
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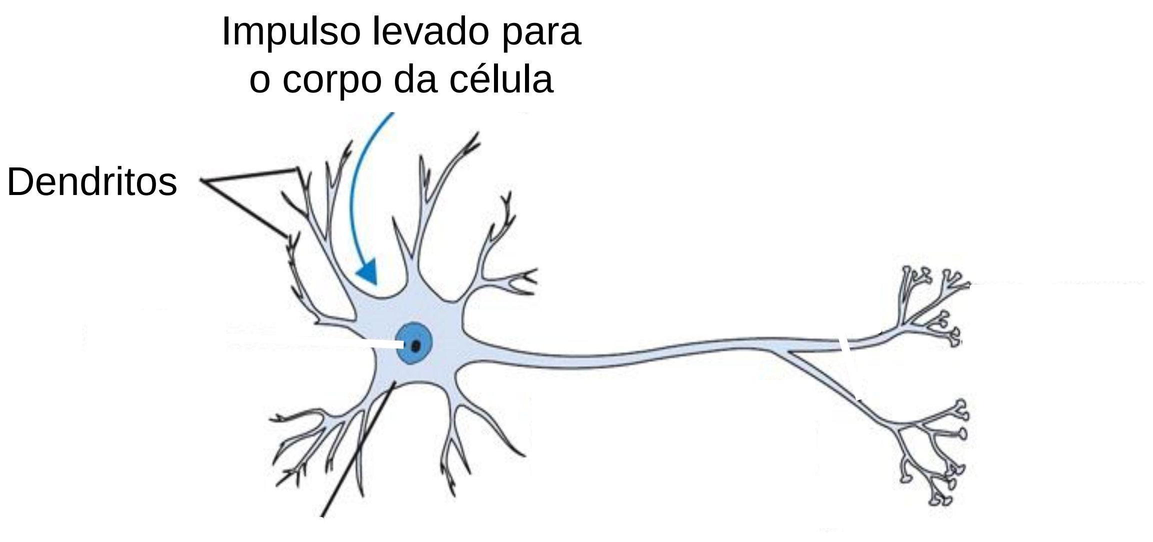
Apenas ~20 linhas!

Inspiração Biológica



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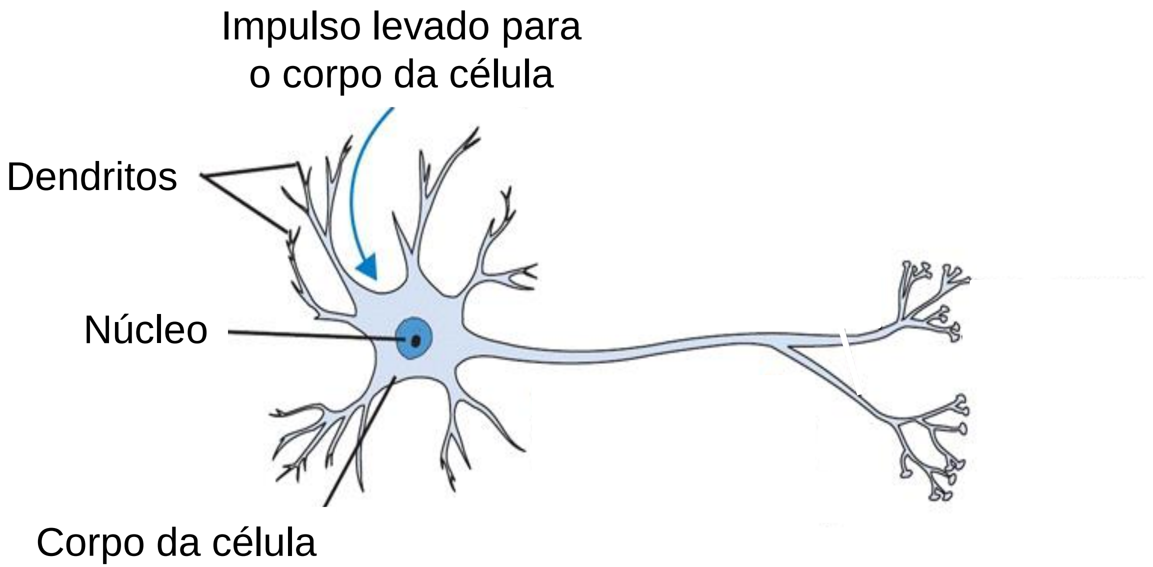

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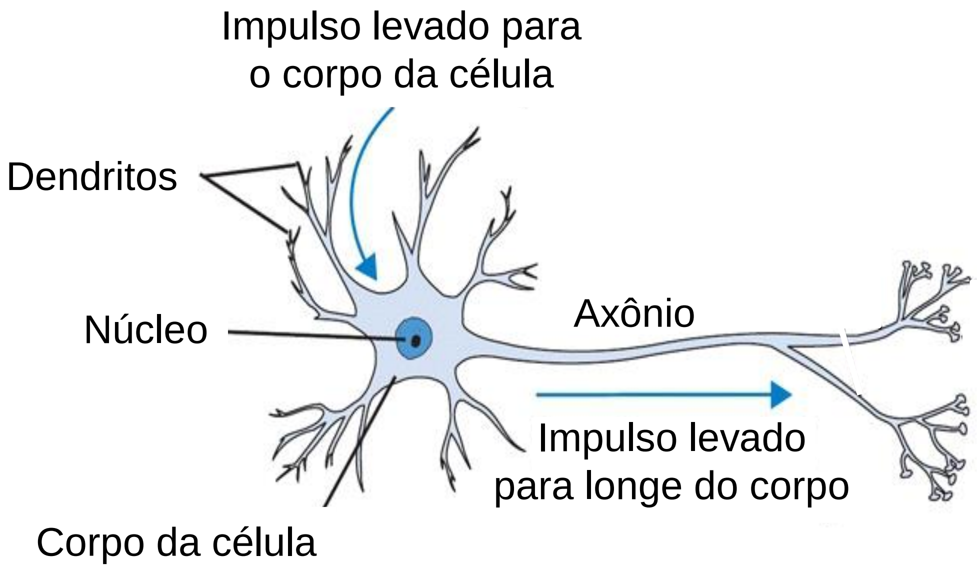

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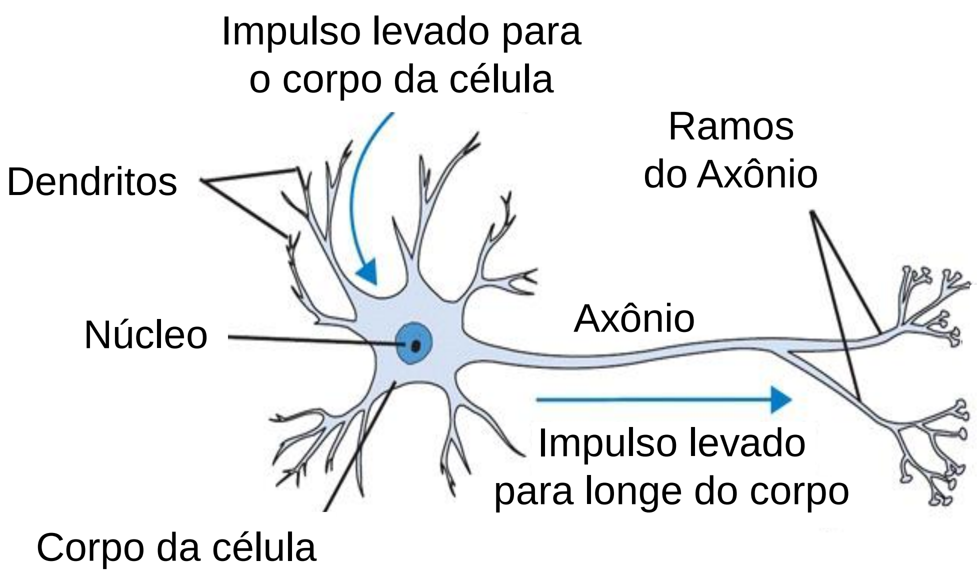

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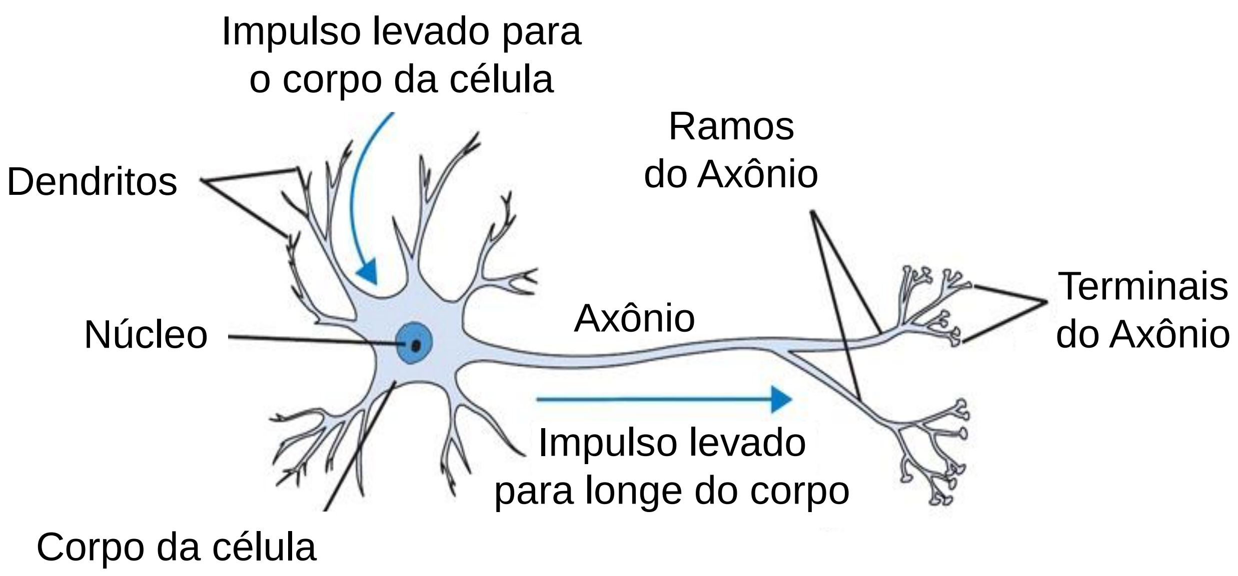

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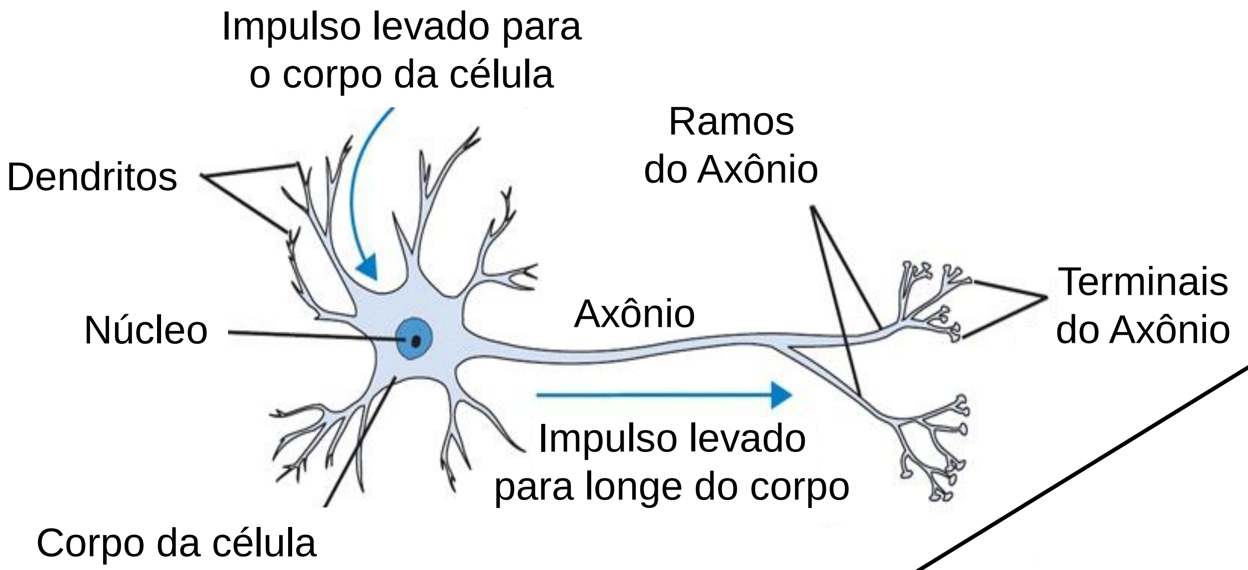

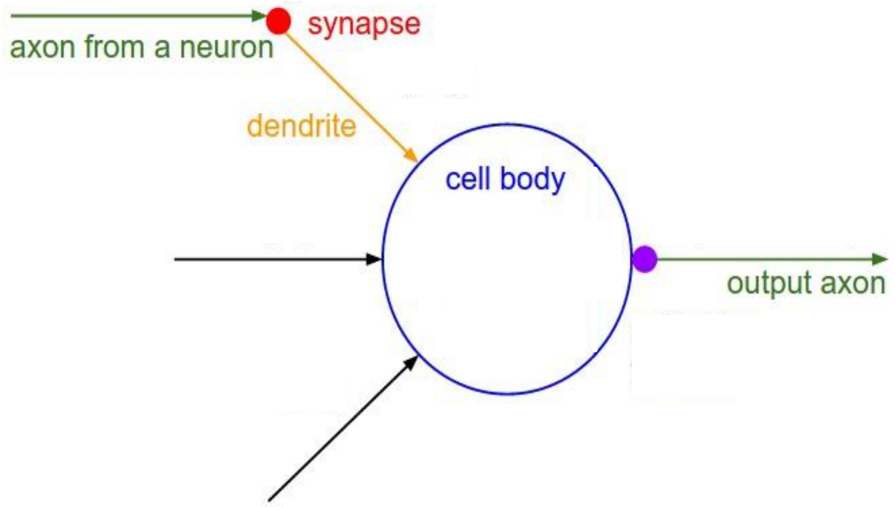

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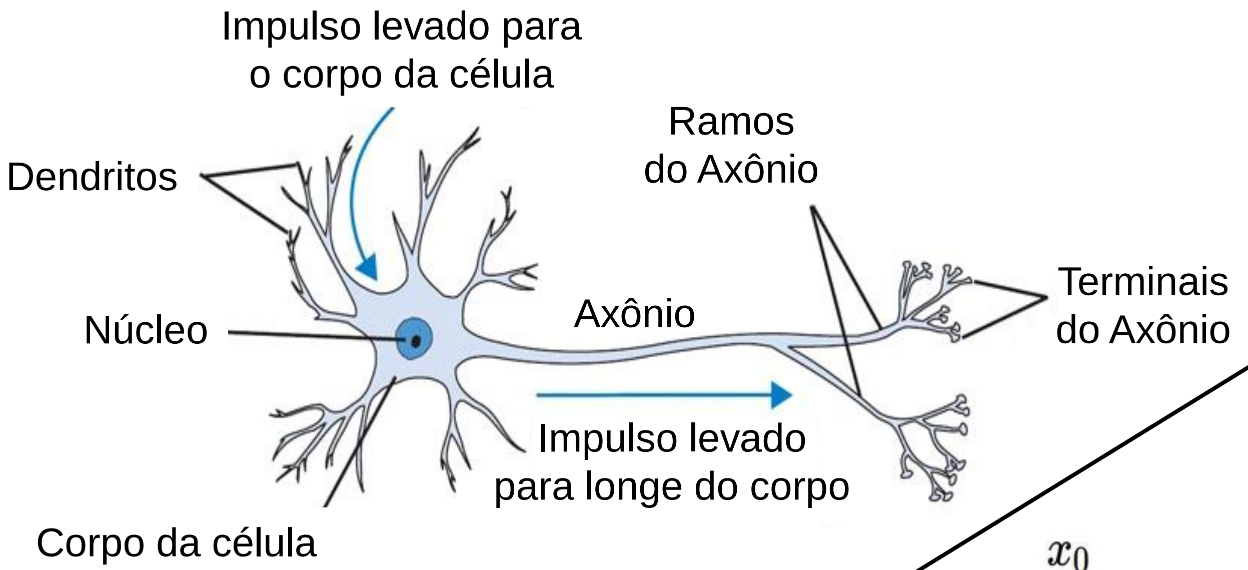

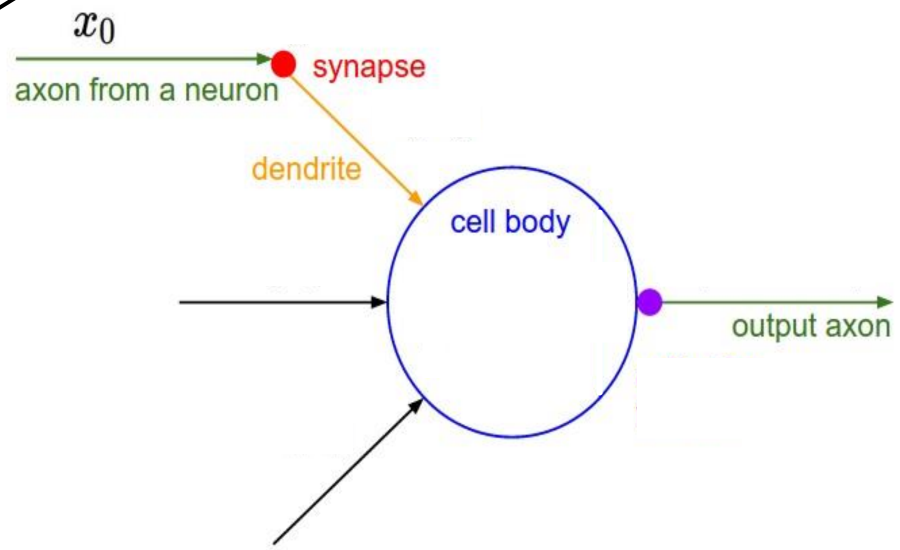

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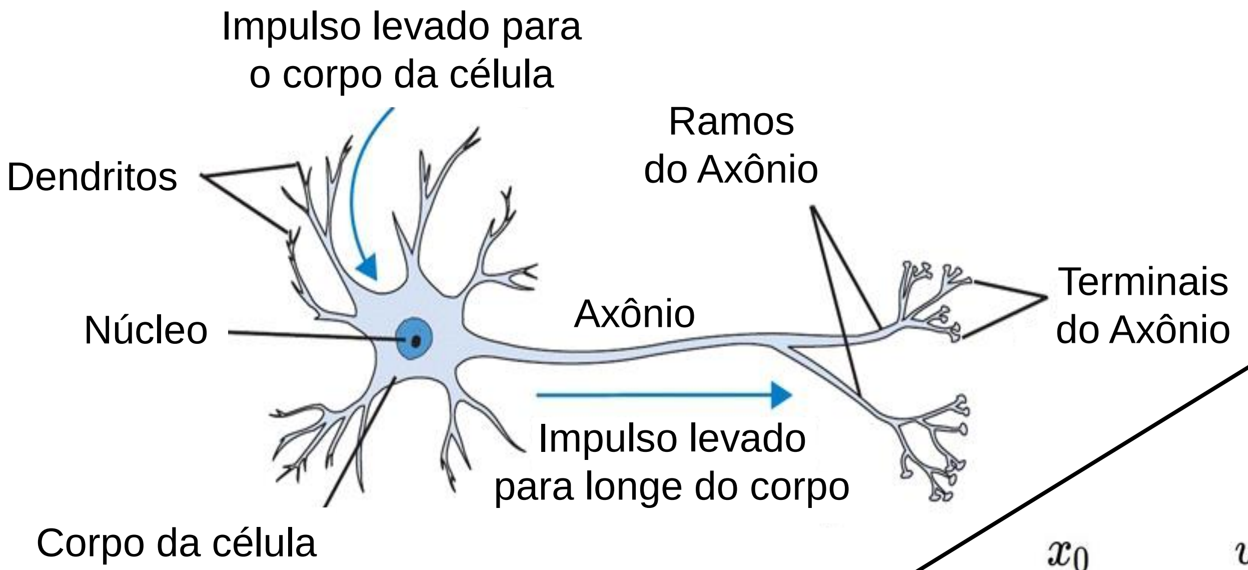

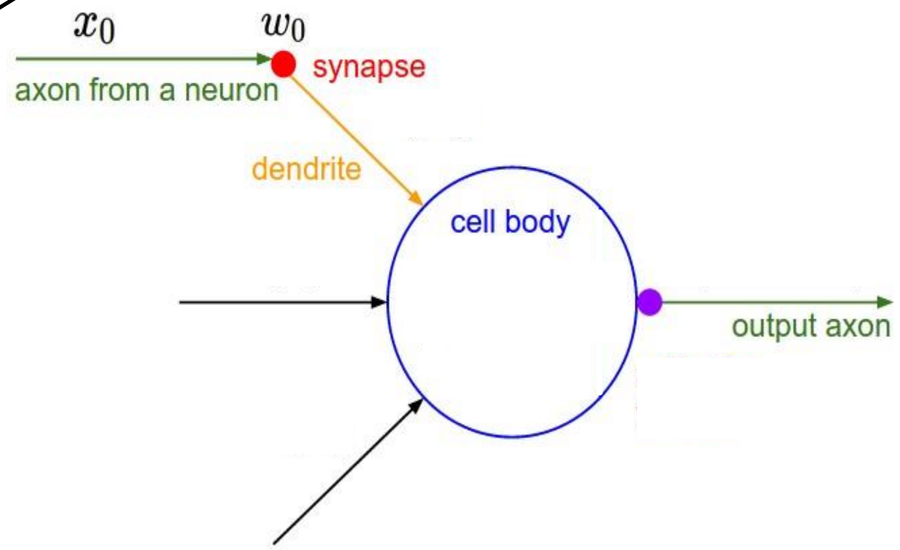

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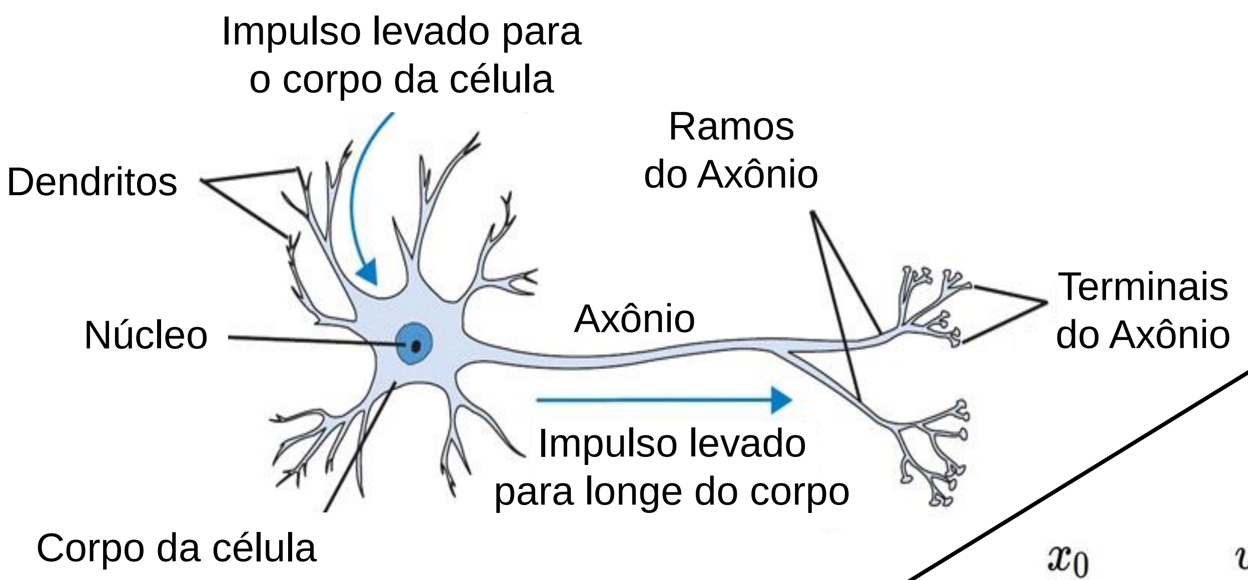

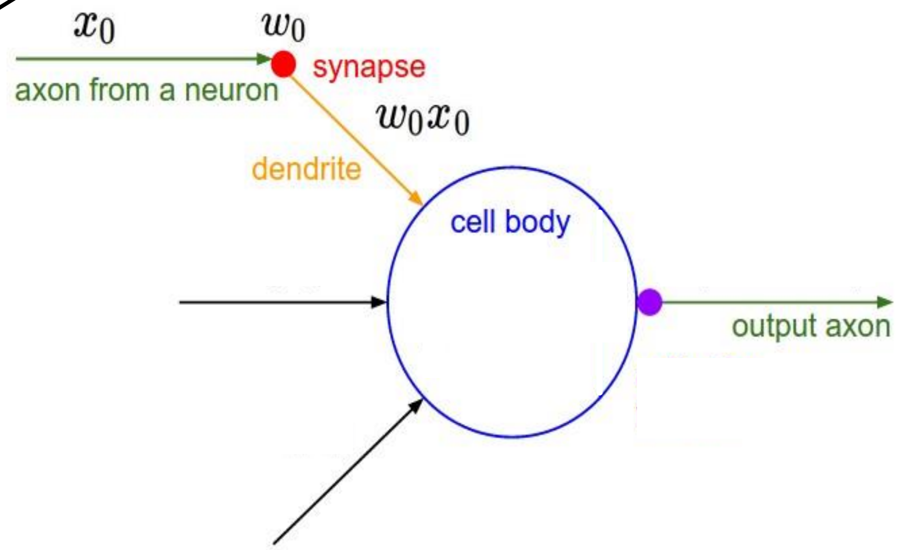

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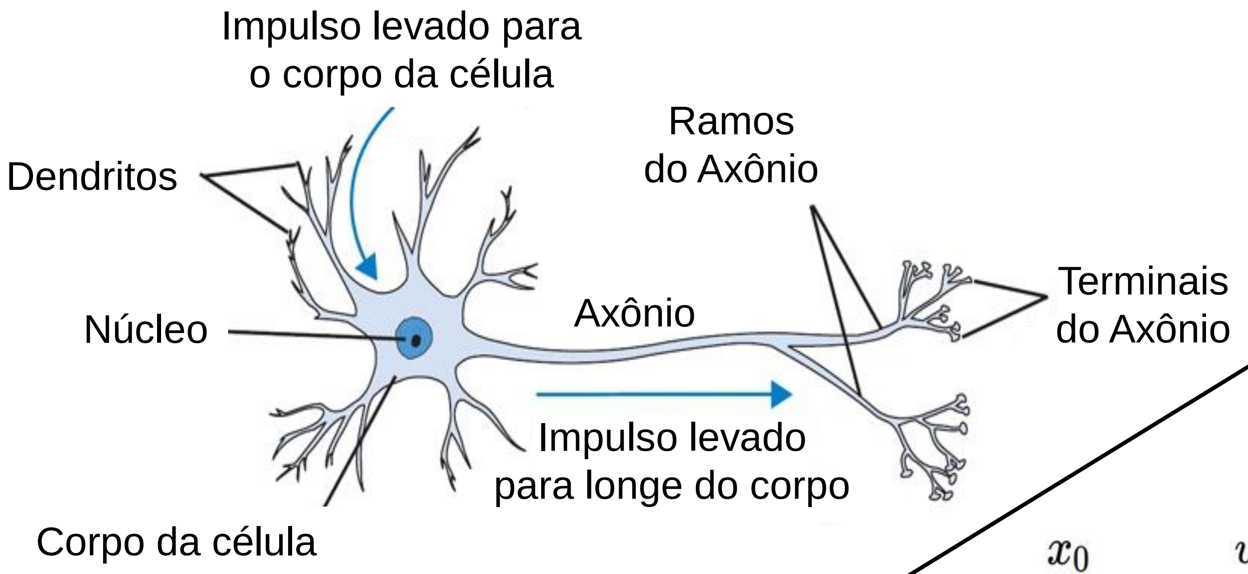

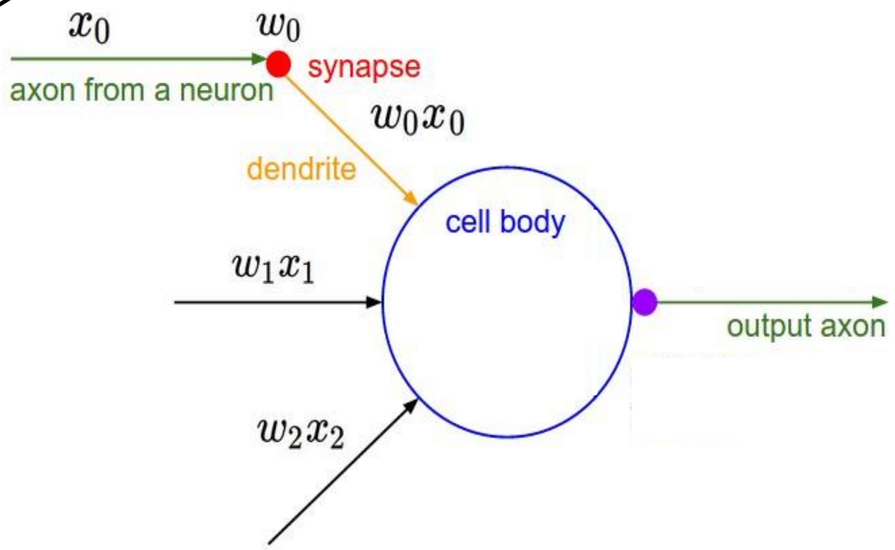

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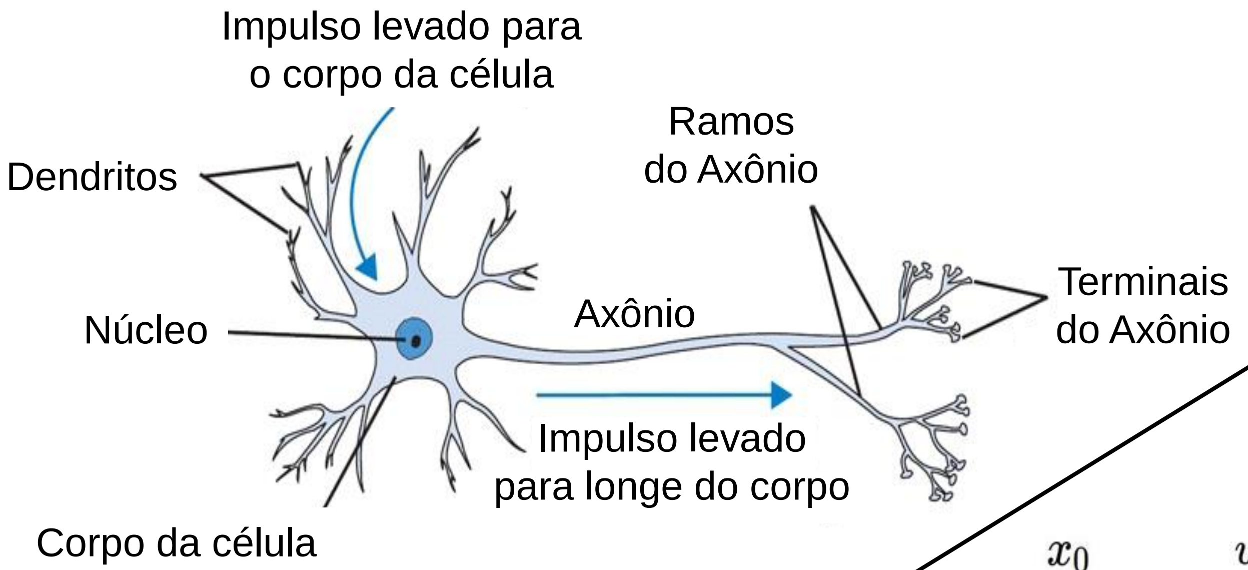

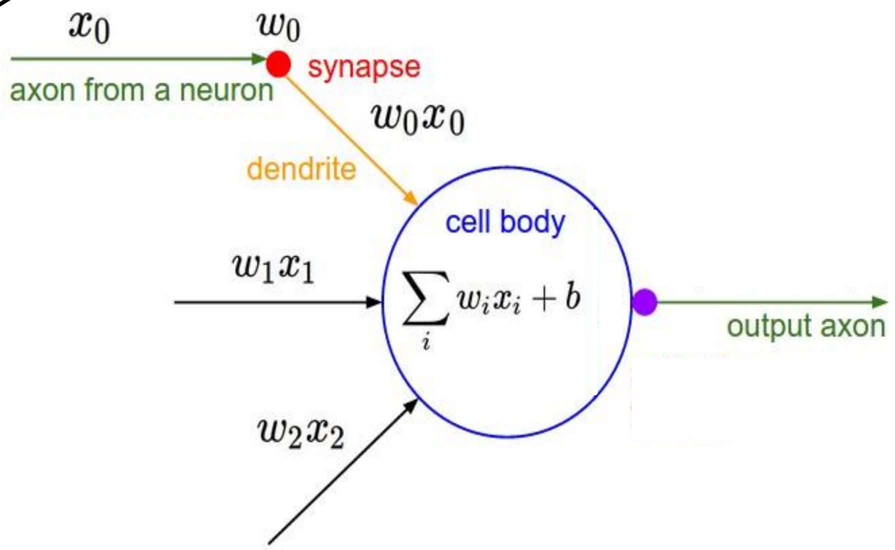

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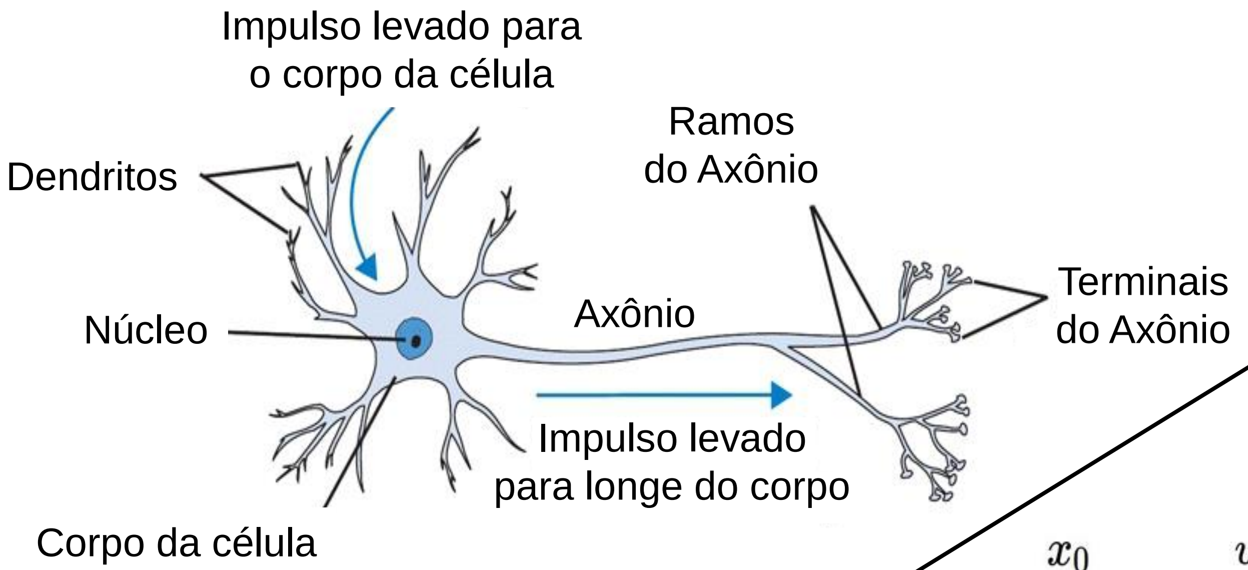

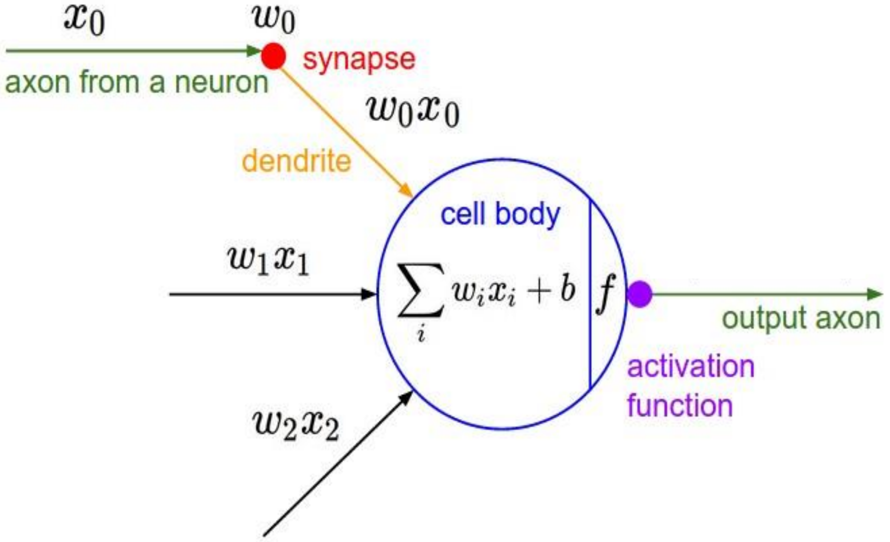

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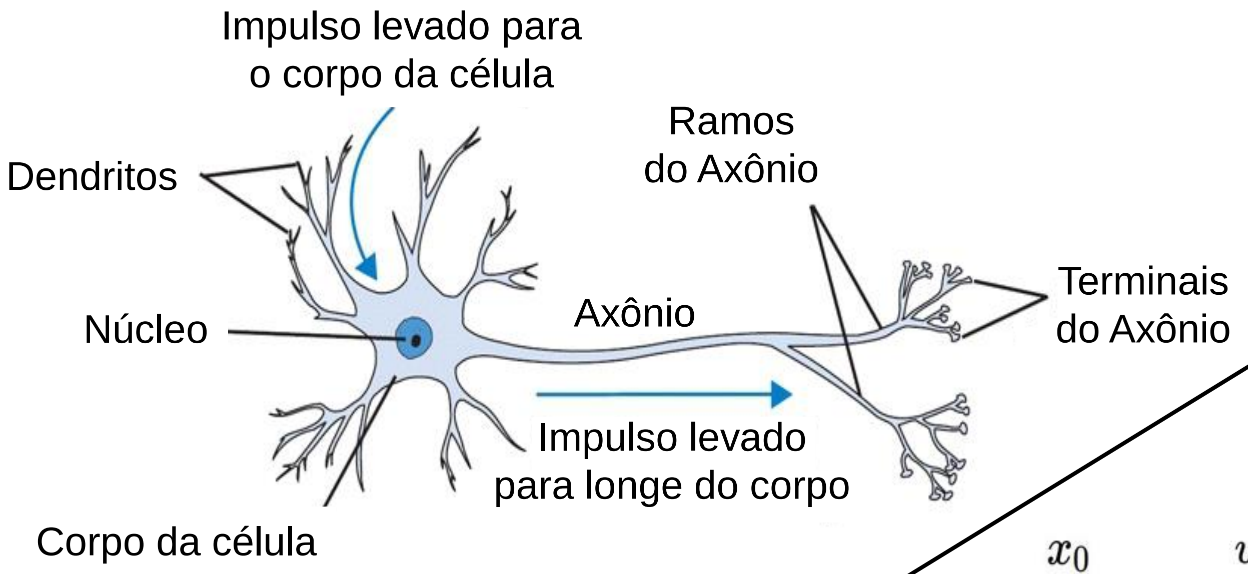

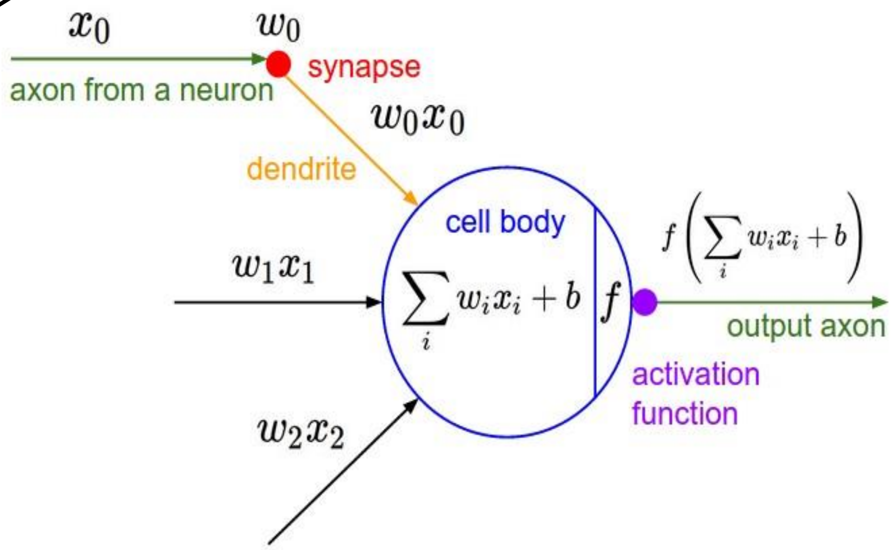

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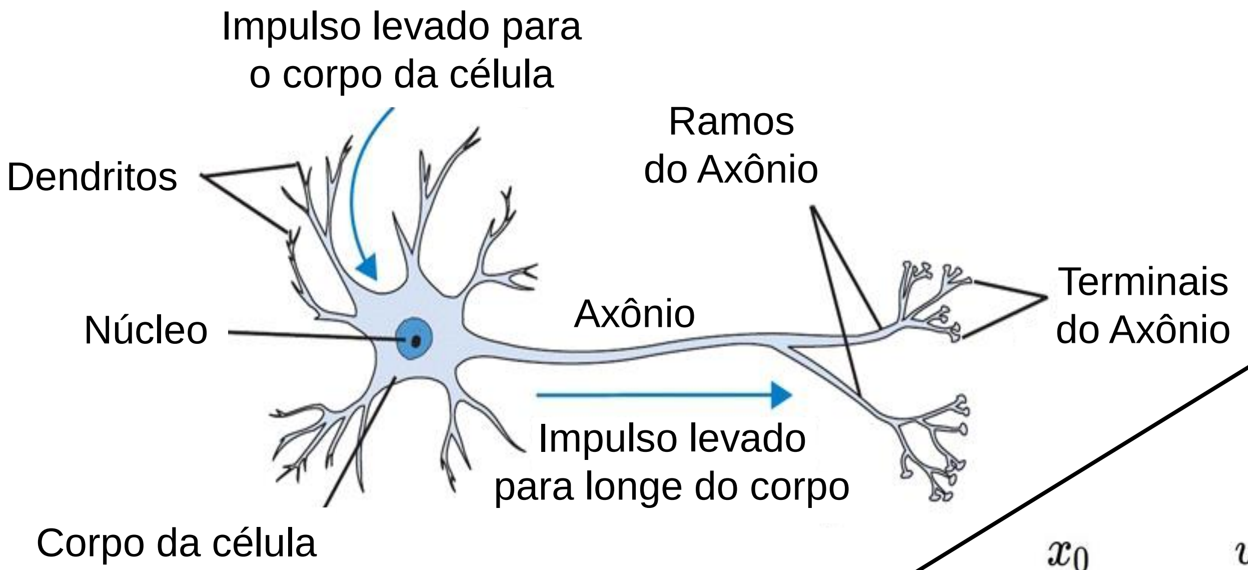
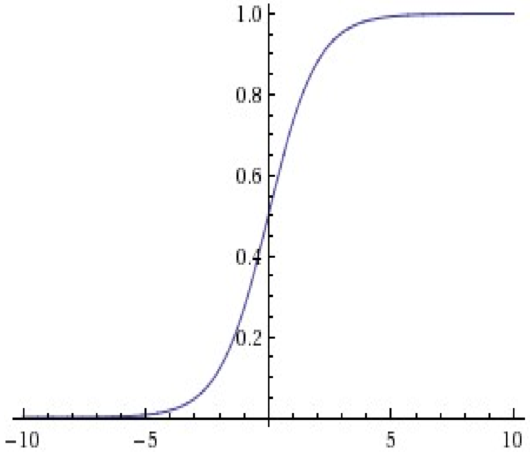
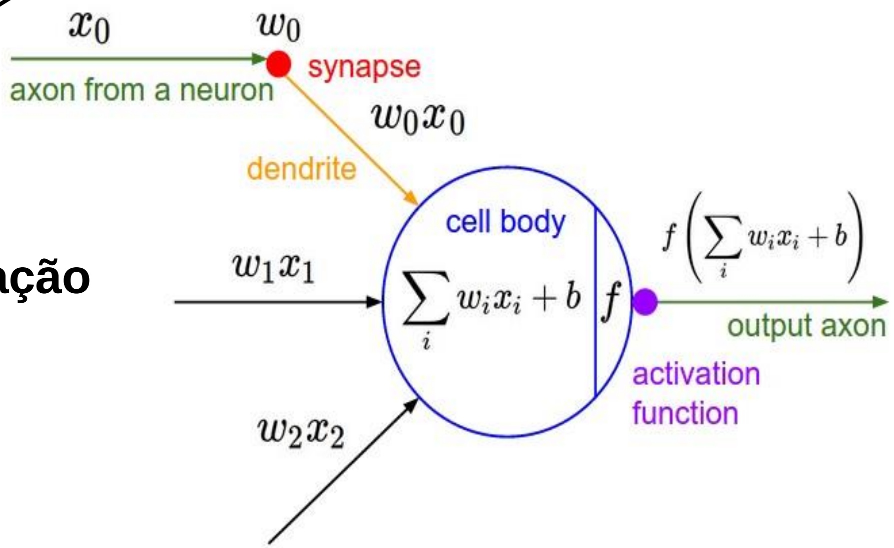




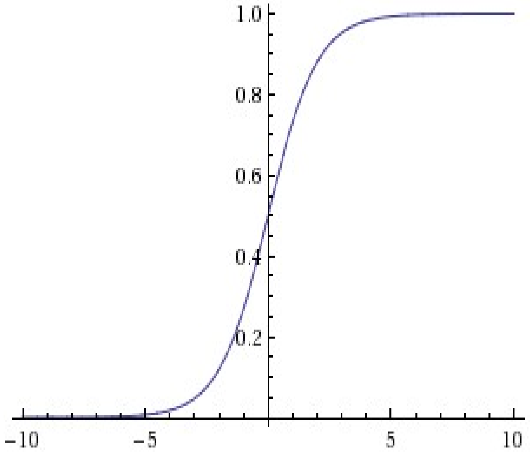
Função de ativação sigmoide

$$\frac{1}{1+e^{-x}}$$



Inspiração Biológica

```python
class Neuron:
  # ...
  def neuron_tick(inputs):
    """ assume inputs and weights are 1-D numpy arrays and bias is a number """
    cell_body_sum = np.sum(inputs * self.weights) + self.bias
    firing_rate = 1.0 / (1.0 + math.exp(-cell_body_sum)) # sigmoid activation function
    return firing_rate
```



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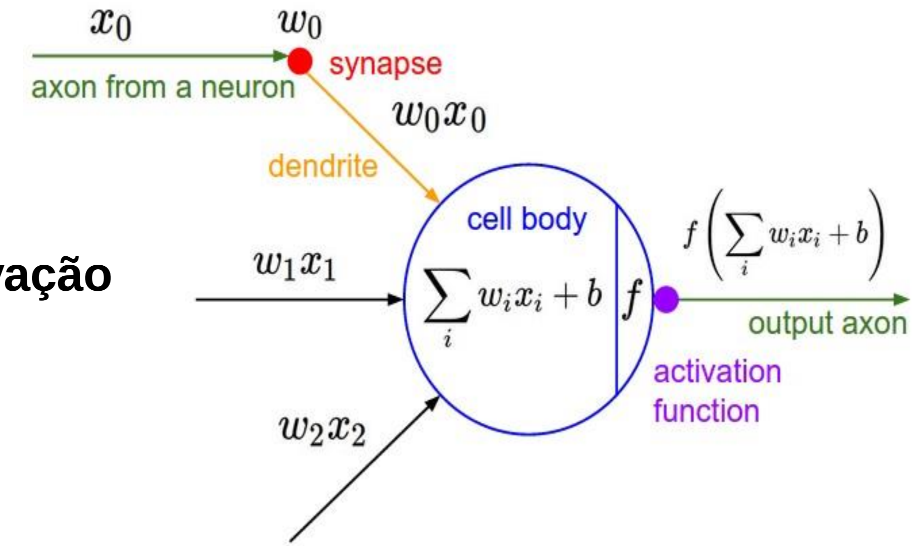
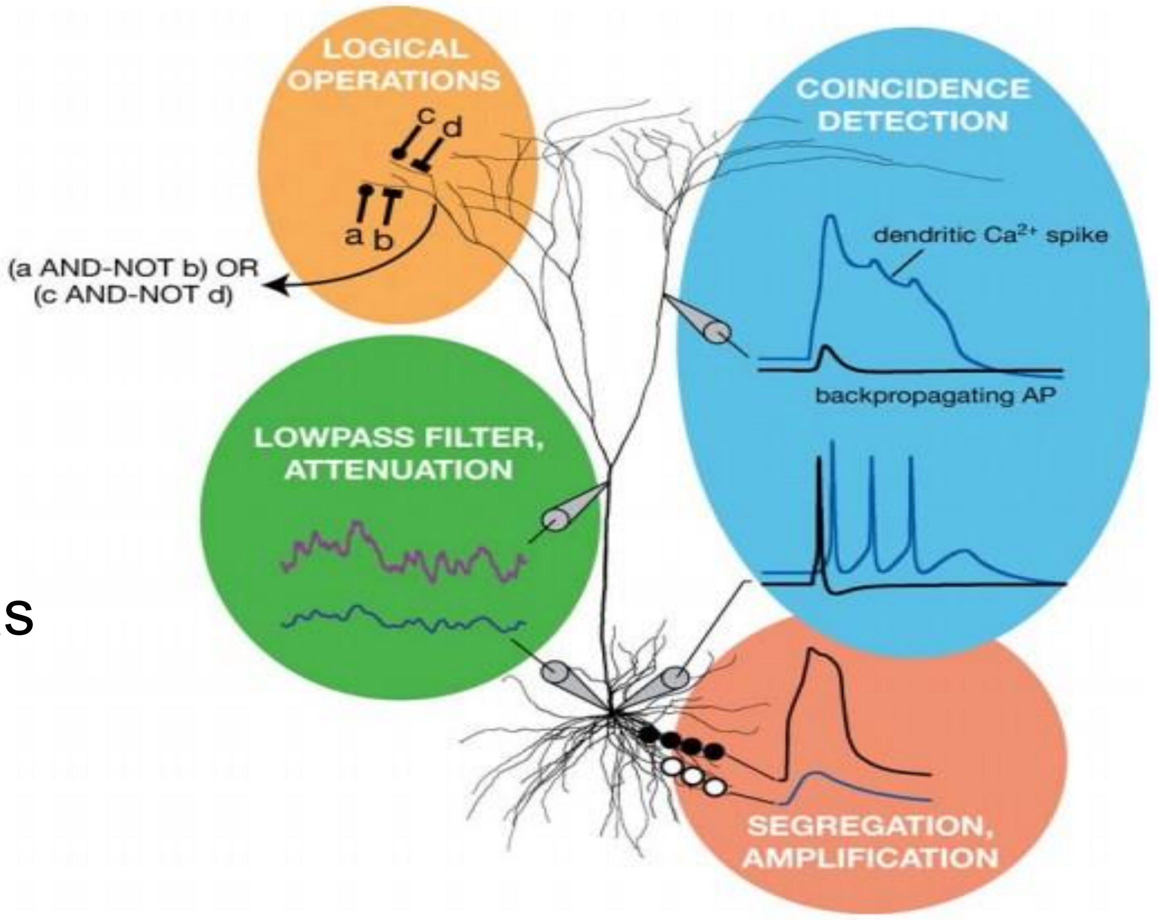


Cuidados com Analogias

Neurônio biológicos:

- Vários tipos diferentes
- Dendritos pode realizar computações não-lineares
- Sinapses não representam apenas um “simples peso” mas sim um complexo sistema dinâmico não-linear

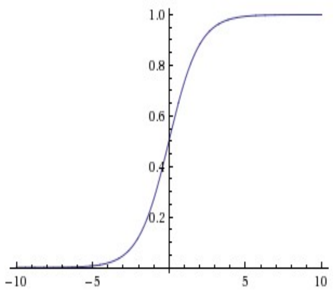


London, M., & Häusser, M. Dendritic computation. *Annual Review of Neuroscience*, *28*: 503-532, (2005).

Algumas Funções de Ativação

Sigmoid

$$\sigma(x) = 1/(1 + e^{-x})$$



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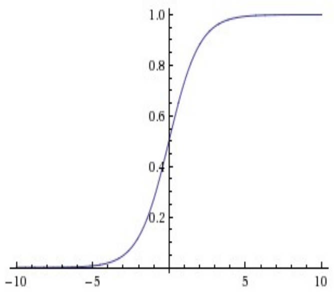
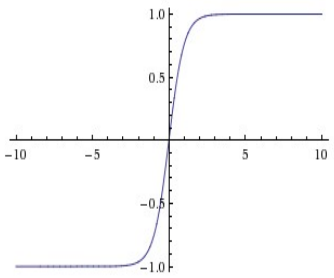


Tanh tanh(x)



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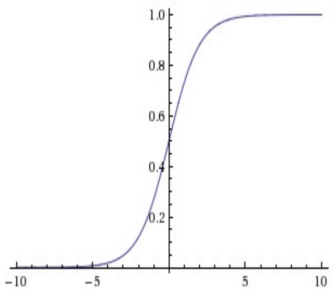

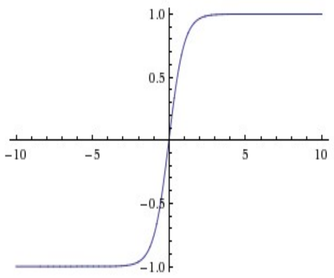

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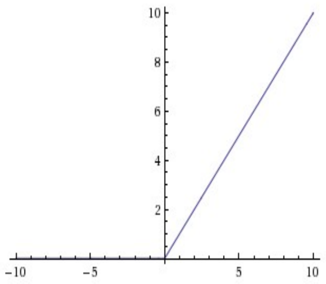

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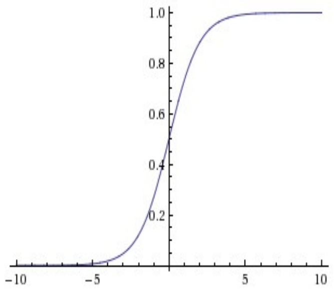

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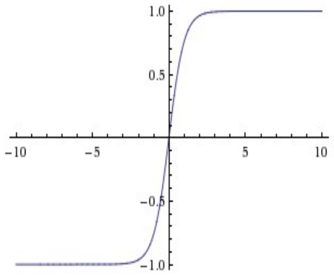

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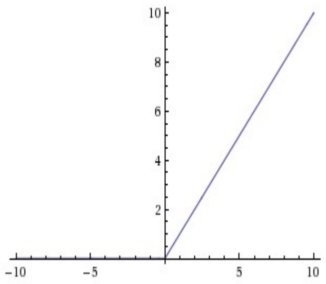
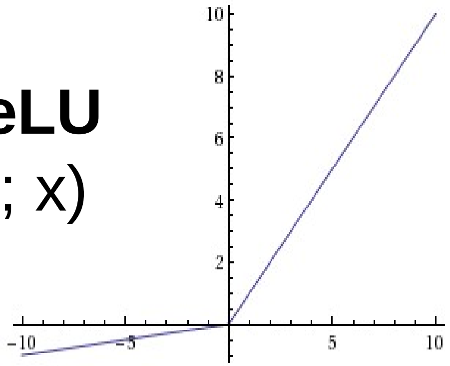


Leaky ReLU
max(0,1x; x)



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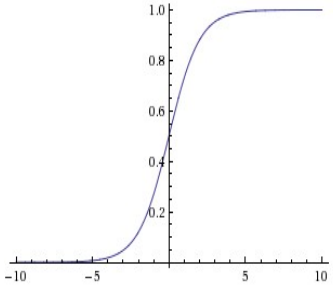

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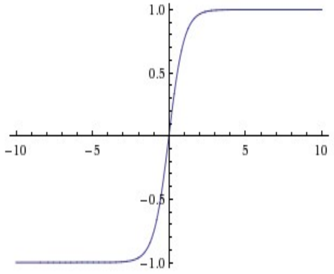

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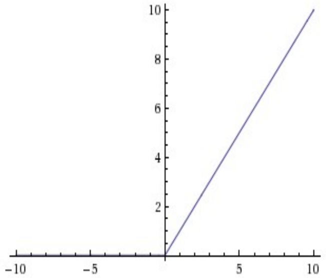

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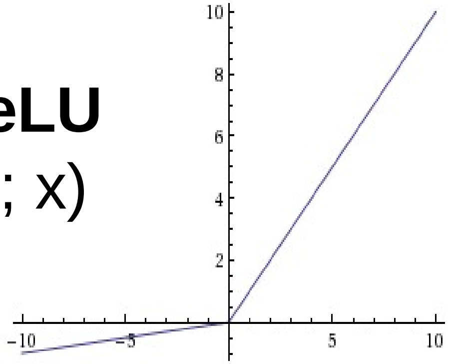
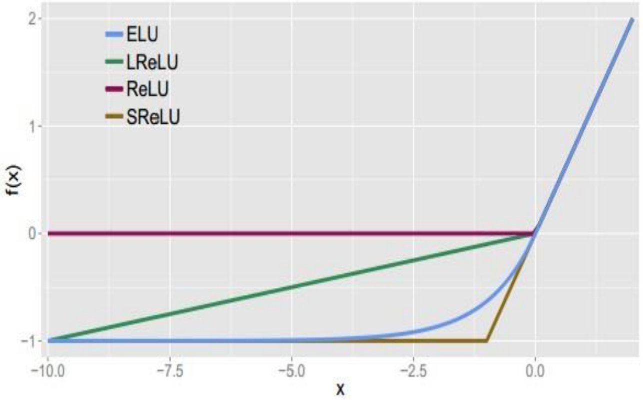


ELU

$$f(x) = \begin{cases} x & \text{if } x > 0 \\ \alpha\,(\exp(x) - 1) & \text{if } x \leq 0 \end{cases}$$



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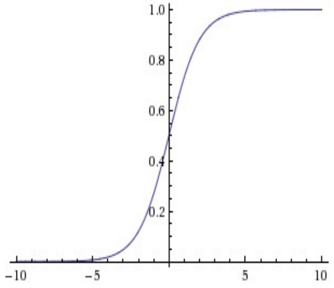
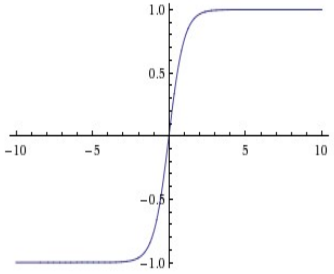
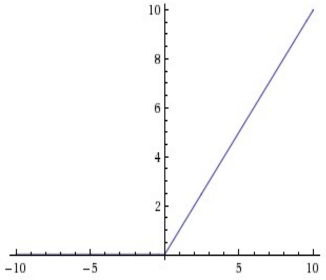


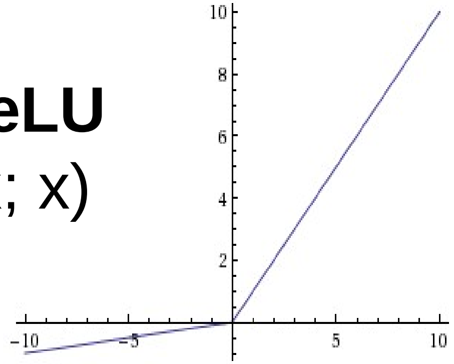
Tanh tanh(x)



ReLU max(0,x)

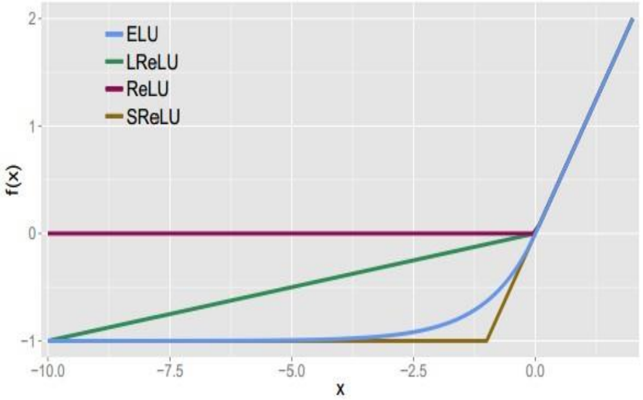


Leaky ReLU
max(0,1x; x)



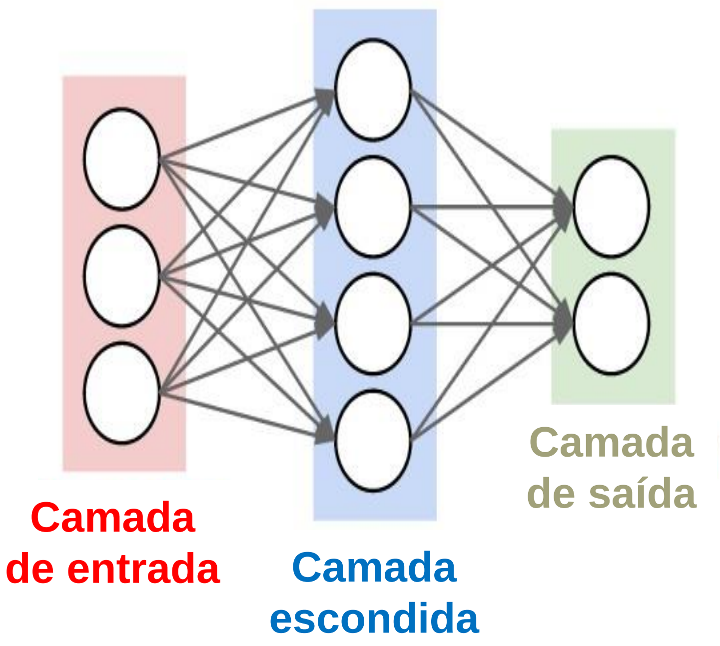
ELU

$$f(x) = \begin{cases} x & \text{if } x > 0 \\ \alpha\,(\exp(x) - 1) & \text{if } x \le 0 \end{cases}$$



Maxout $\max(w_1^T x + b_1, w_2^T x + b_2)$

Arquitetura de Rede Neural *Feed-Forward*



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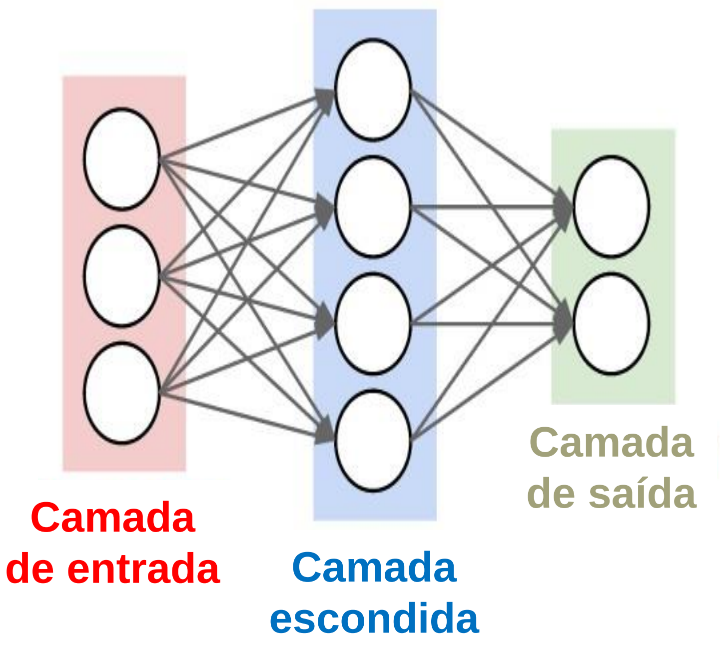
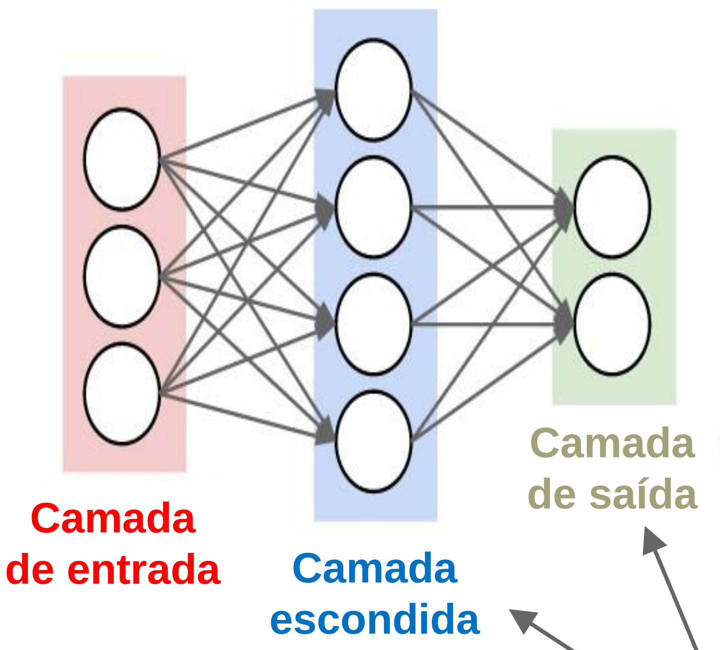


"Rede Neural de 2 camadas" ou
"Rede Neural com 1 camada escondida"

Arquitetura de Rede Neural *Feed-Forward*



Camadas “completamente conectadas”

“Rede Neural de 2 camadas” ou
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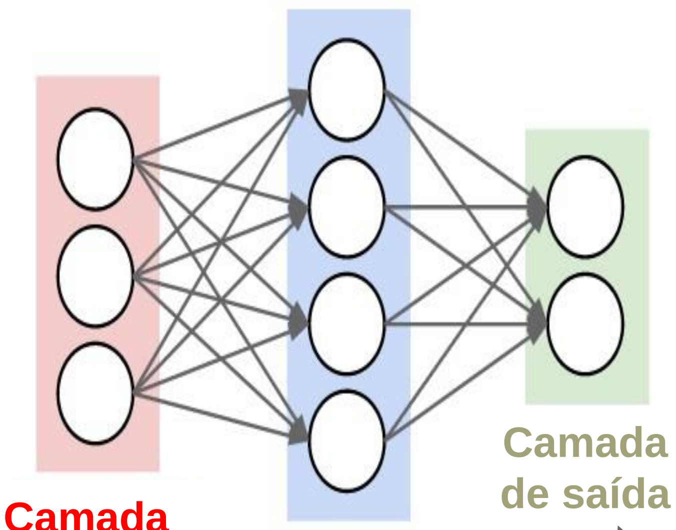
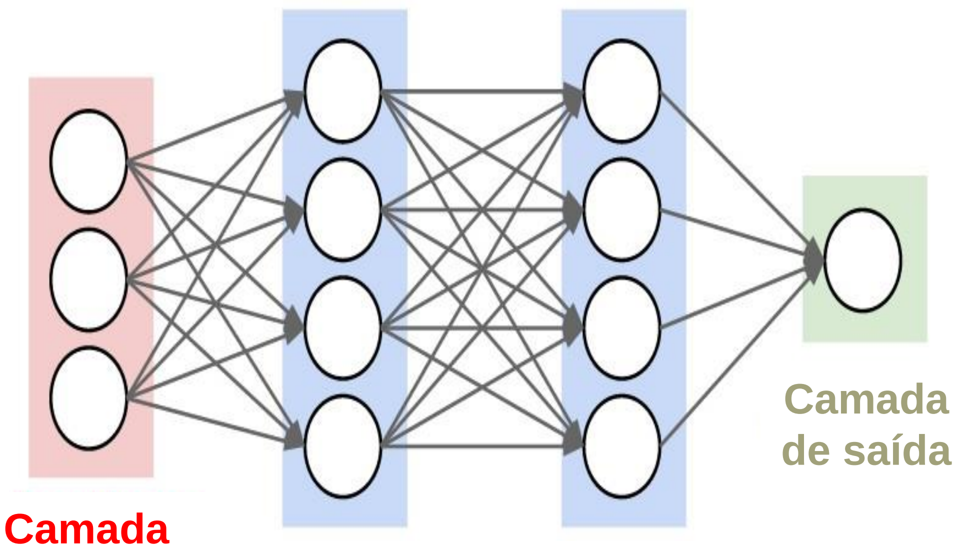
Arquitetura de Rede Neural *Feed-Forward*





Camadas "completamente conectadas"

"Rede Neural de 2 camadas" ou
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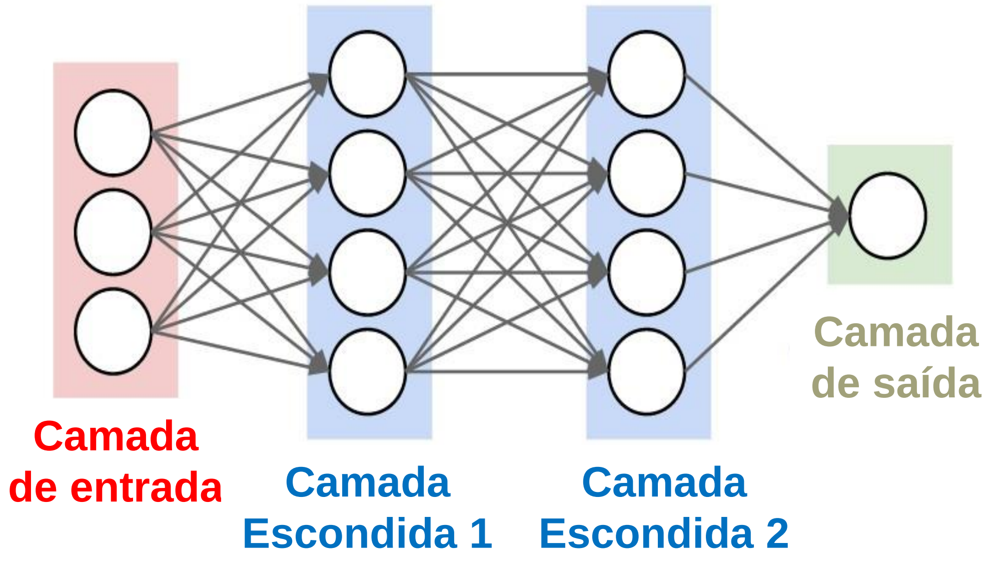
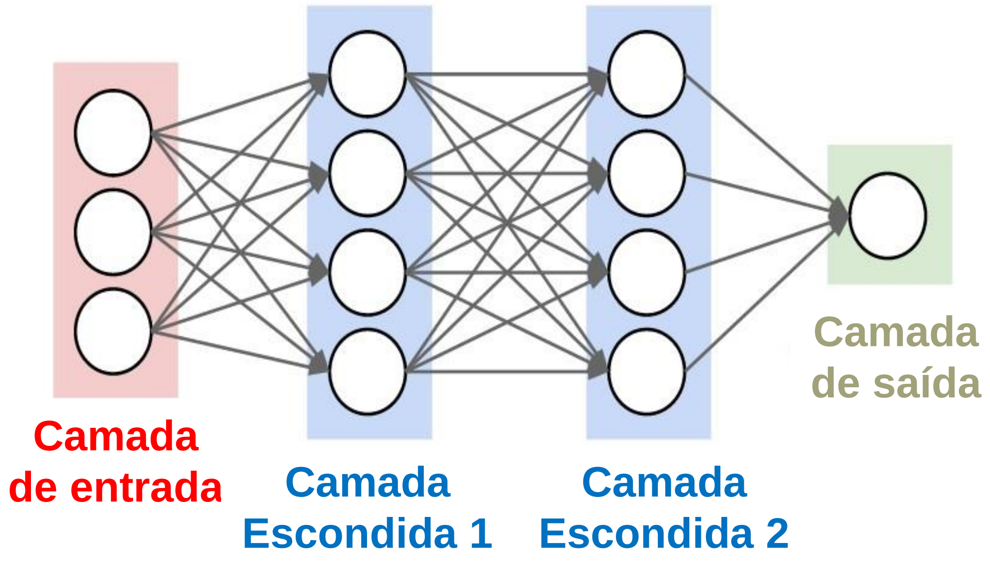
"Rede Neural de 3 camadas" ou
"Rede Neural com 2 camadas escondidas"

Exemplo de Avaliação de Rede *Feed-Forward*



Exemplo de Avaliação de Rede *Feed-Forward*



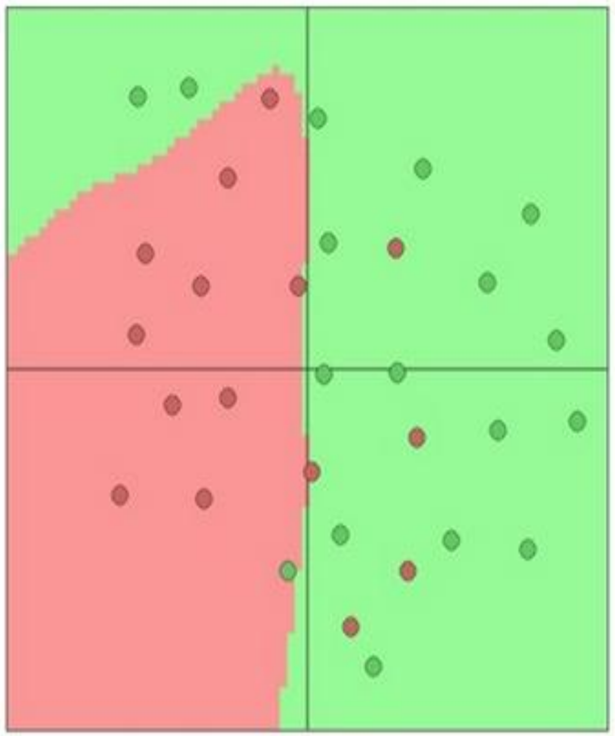
Pode-se avaliar eficientemente uma camada inteira de neurônios

```
f = lambda x: 1.0/(1.0 + np.exp(-x)) # activation function (use sigmoid)
x = np.random.randn(3, 1) # random input vector of three numbers (3x1)
h1 = f(np.dot(W1, x) + b1) # calculate first hidden layer activations (4x1)
h2 = f(np.dot(W2, h1) + b2) # calculate second hidden layer activations (4x1)
out = np.dot(W3, h2) + b3 # output neuron (1x1)
```

Definindo Tamanho das Camadas

Número de Neurônios na Camada Escondida

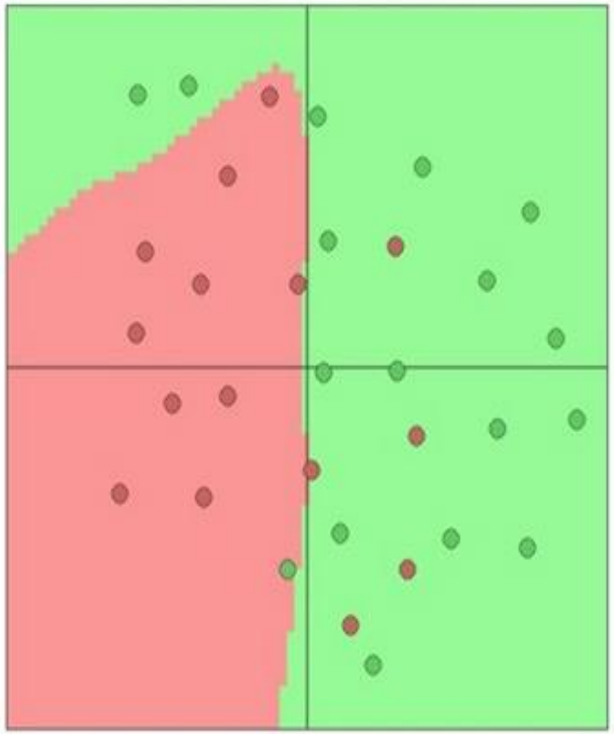
03 neurônios



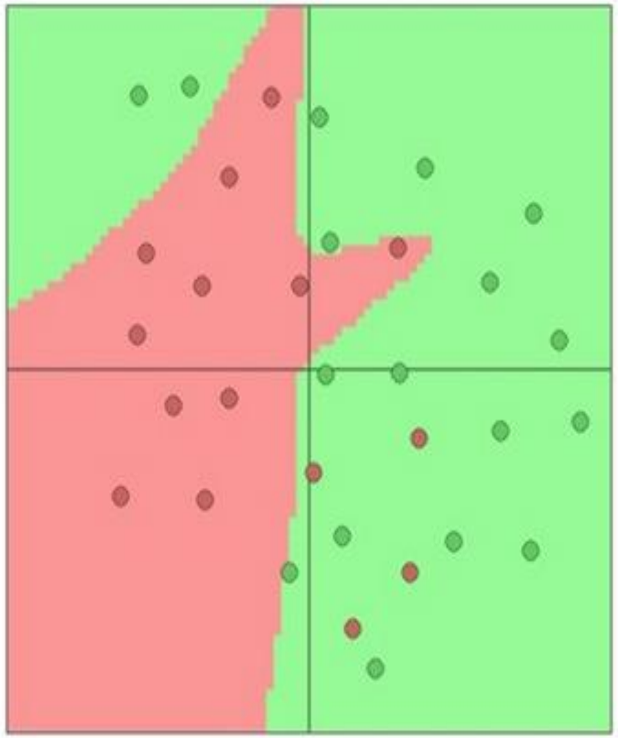
Definindo Tamanho das Camadas

Número de Neurônios na Camada Escondida

03 neurônios

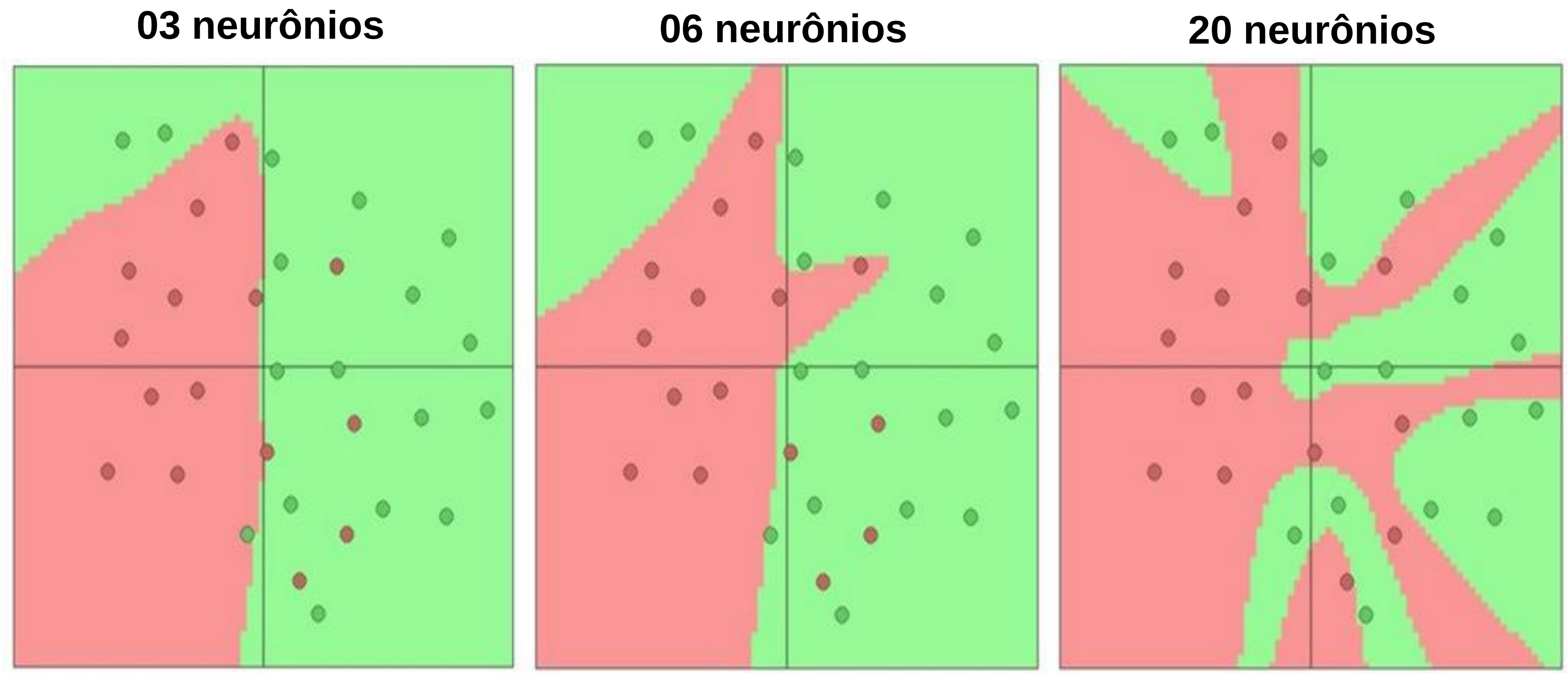


06 neurônios



Definindo Tamanho das Camadas

Número de Neurônios na Camada Escondida



Definindo Tamanho das Camadas

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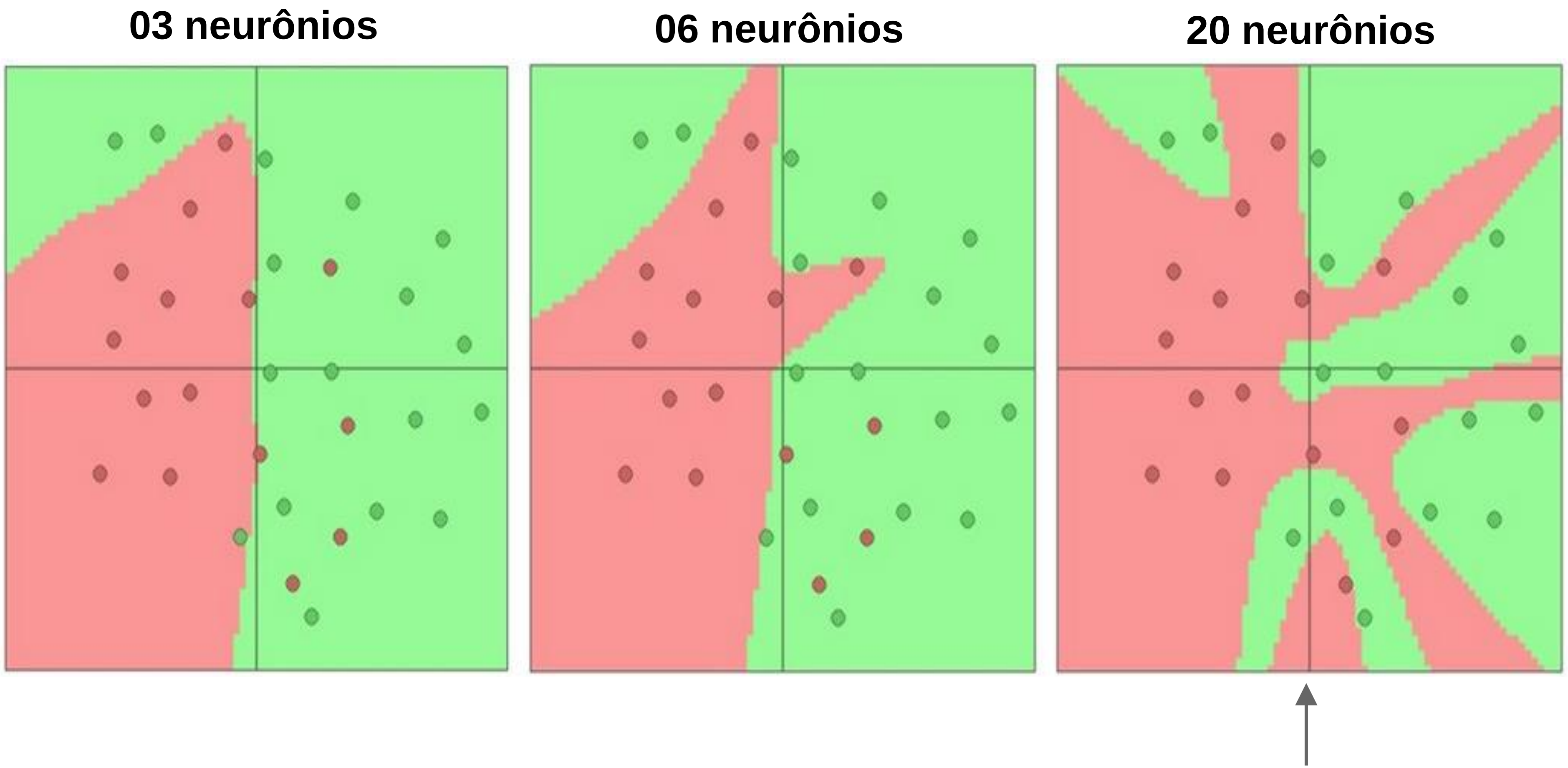
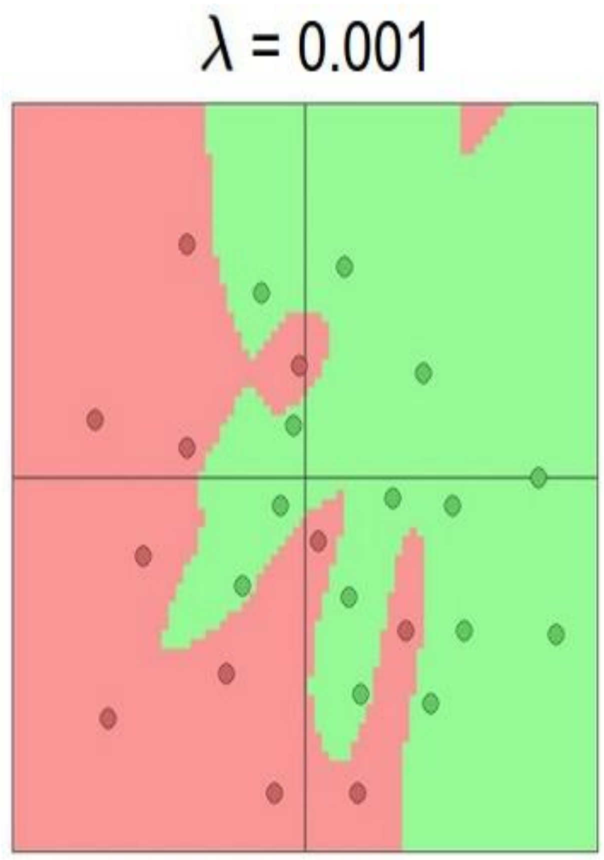


mais neurônios ≡ maior capacidade

Regularização

Não se deve usar o tamanho de uma rede para regularização

Deve-se aumentar a “força” da regularização



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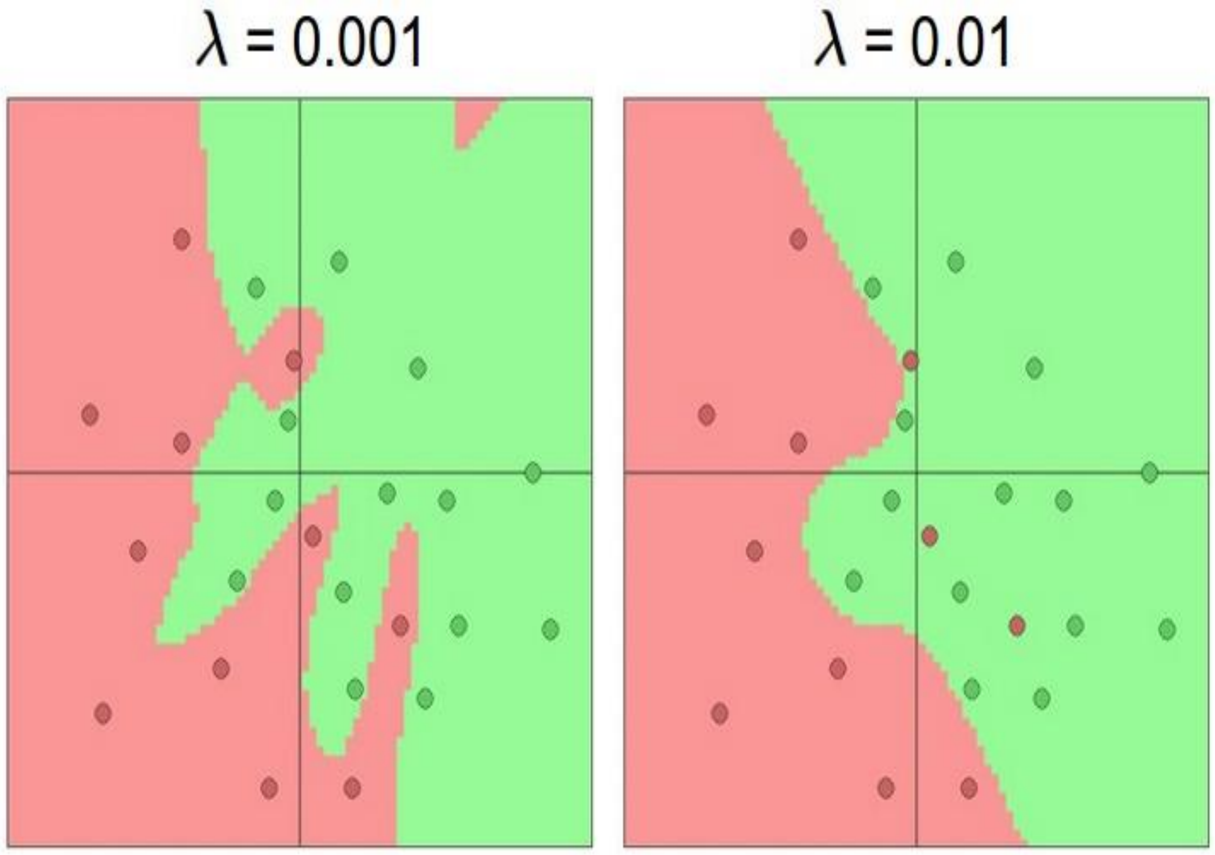

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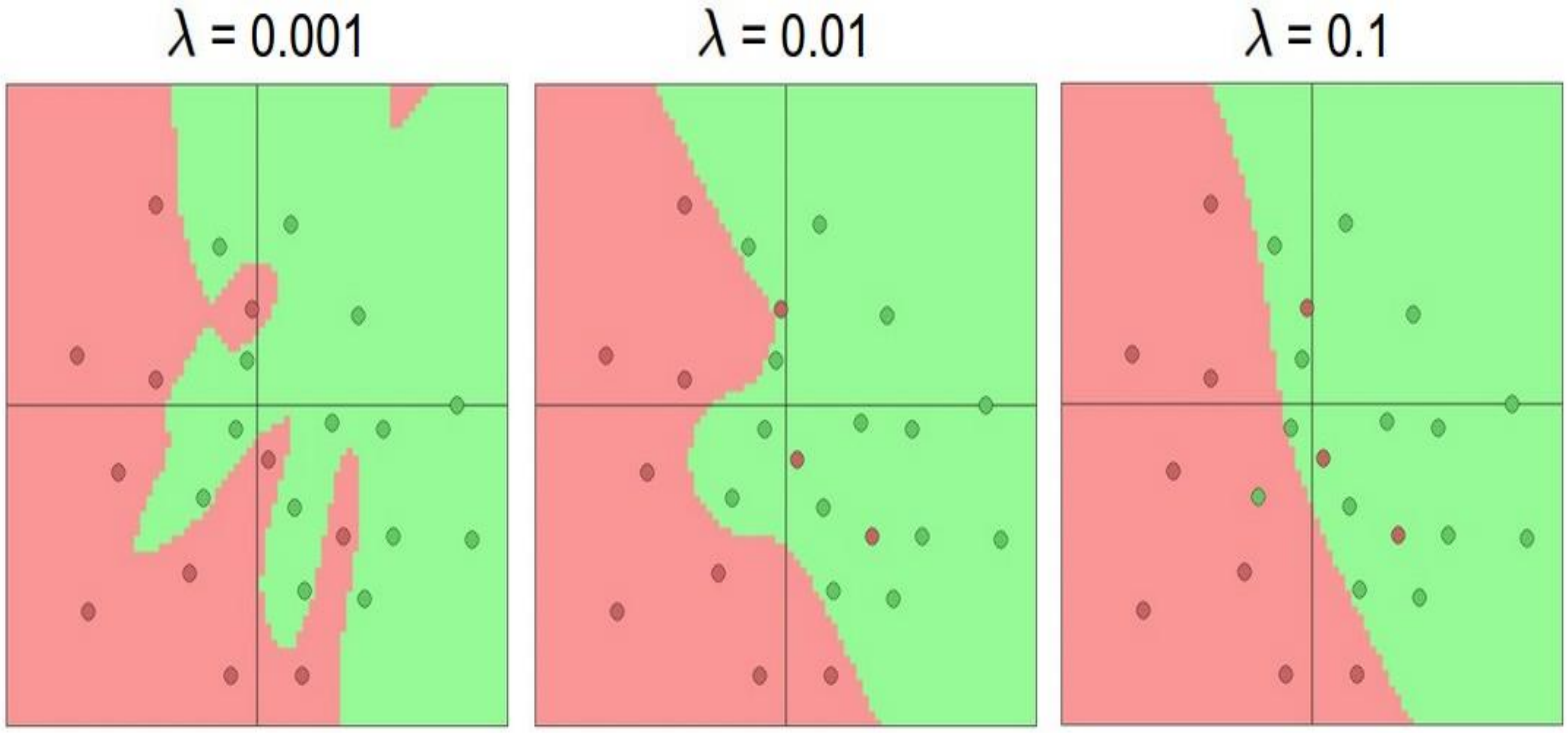


Um Pouco de História

A máquina **Mark I Perceptron** foi a primeira implementação do algoritmo perceptron

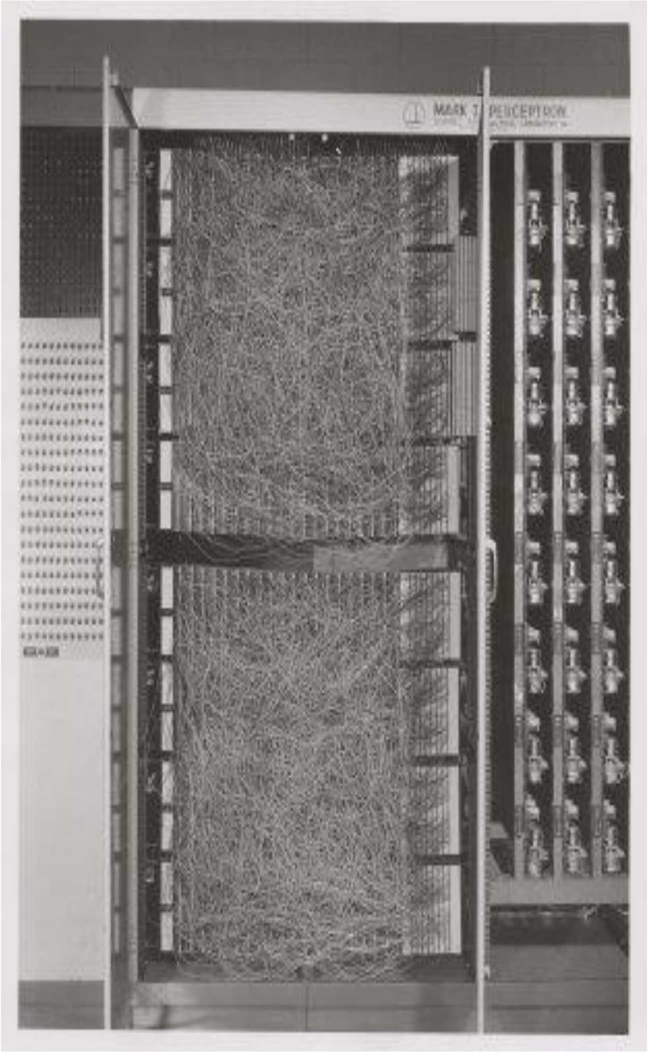
Essa máquina foi conectada a uma câmera capaz de produzir uma imagem de 400 pixels

Seu objetivo básico era o reconhecimento de imagens

$$f(x) = \begin{cases} 1 & \text{if } w \cdot x + b > 0 \\ 0 & \text{otherwise} \end{cases}$$

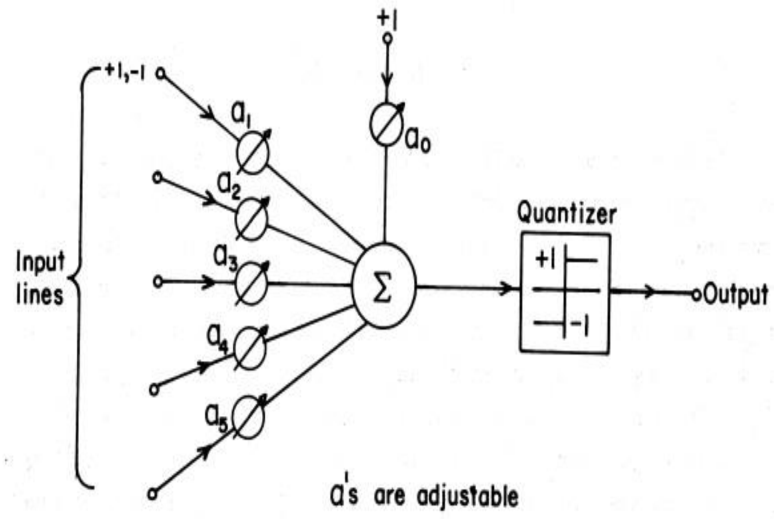
Regra de atualização :

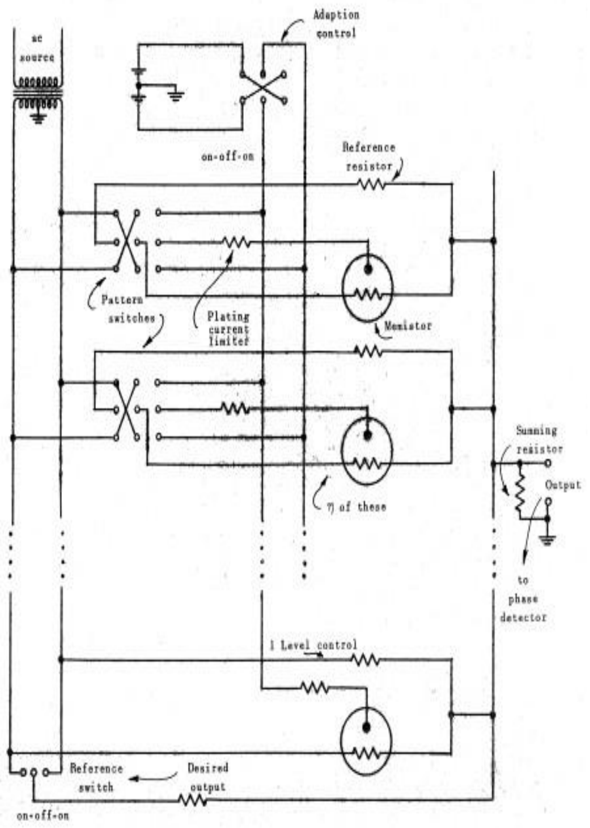
$$w_i(t+1) = w_i(t) + \alpha(d_j - y_j(t))x_{j,i},$$



Frank Rosenblatt, ~1957: Perceptron

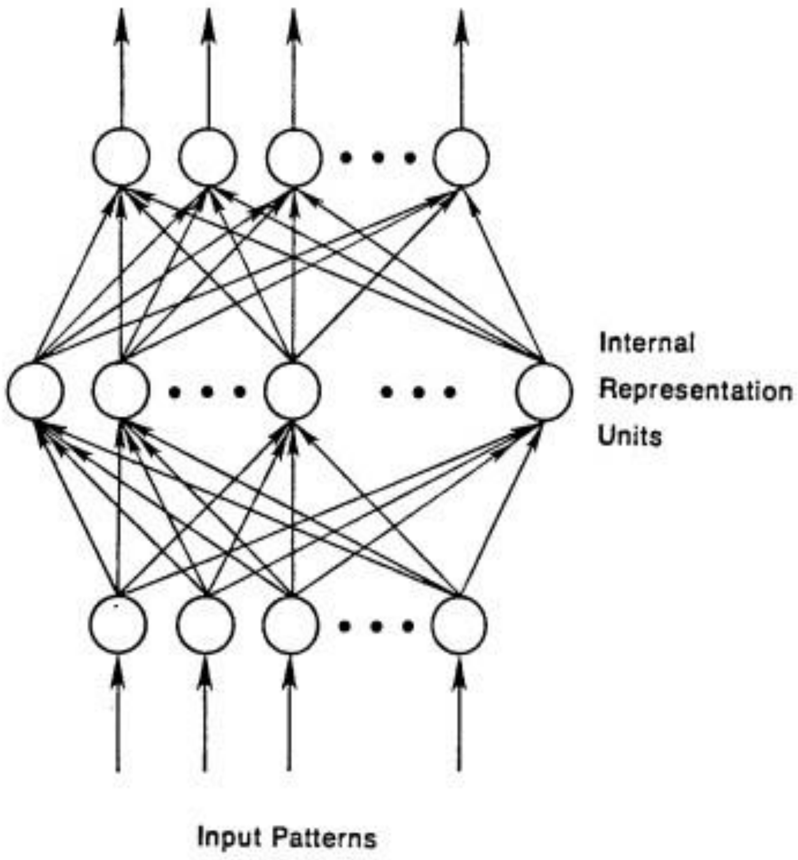
Um Pouco de História





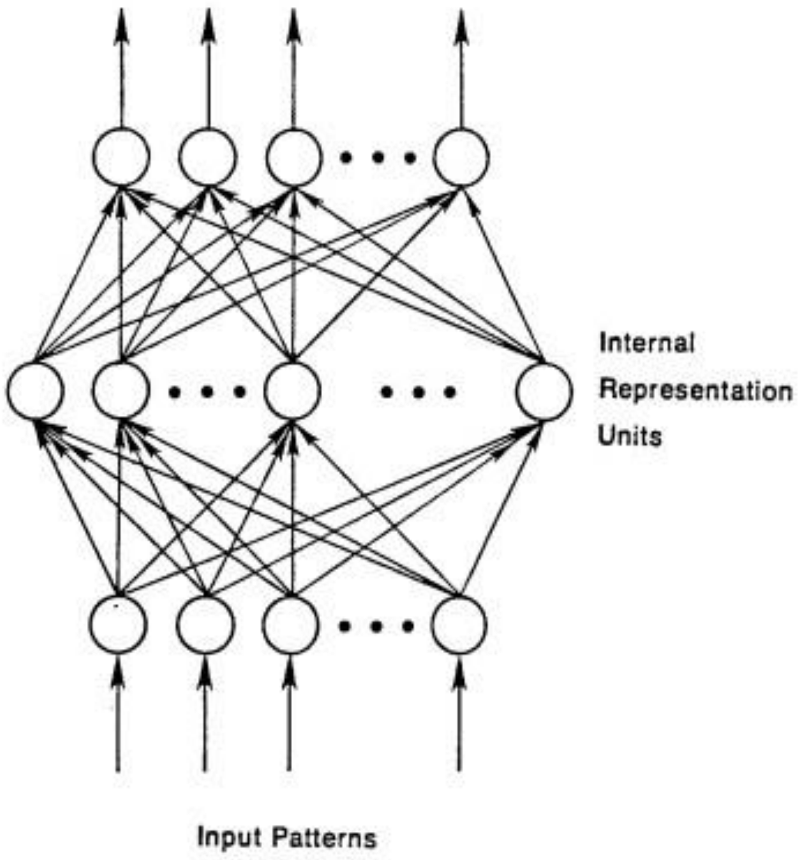
Widrow and Hoff, ~1960: Adaline/Madaline

Um Pouco de História



Rumelhart et al. 1986: Primeira vez em que a propagação retrógada se torna popular

Um Pouco de História



To be more specific, then, let

$$E_p = \frac{1}{2}\sum_j (t_{pj} - o_{pj})^2$$

be our measure of the error on input/output pattern p
overall measure of the error. We wish to show that the
dient descent in E when the units are linear. We wil
that

$$-\frac{\partial E_p}{\partial w_{ji}} = \delta_{pj}\, i_{pi},$$

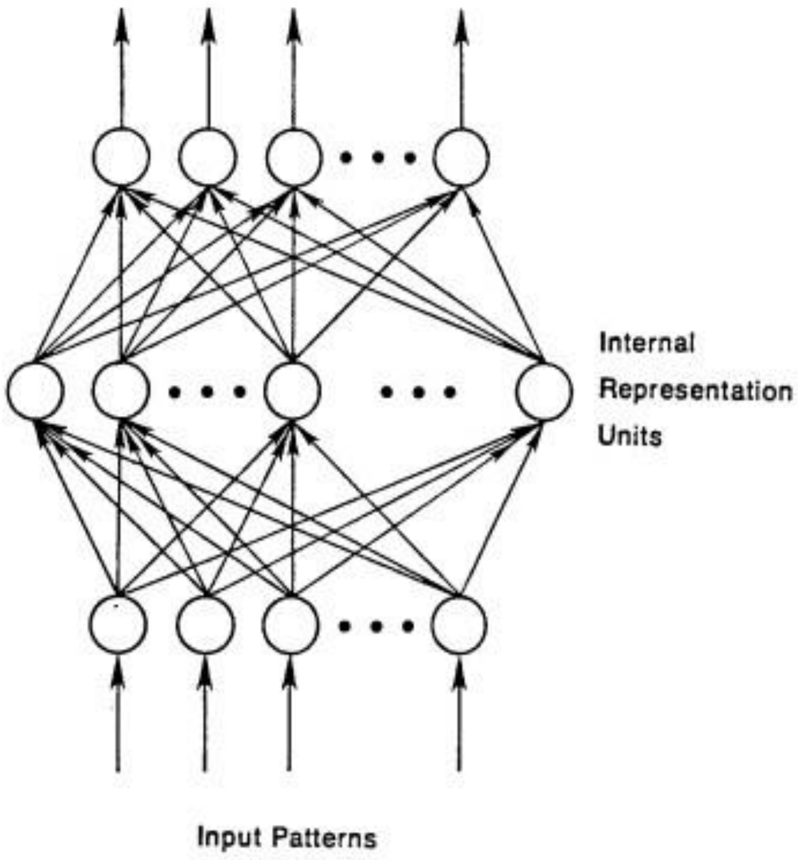
which is proportional to $\Delta_p w_{ji}$ *as prescribed by the del*
hidden units it is straightforward to compute the relevant
we use the chain rule to write the derivative as the prod
tive of the error with respect to the output of the unit tim
put with respect to the weight.

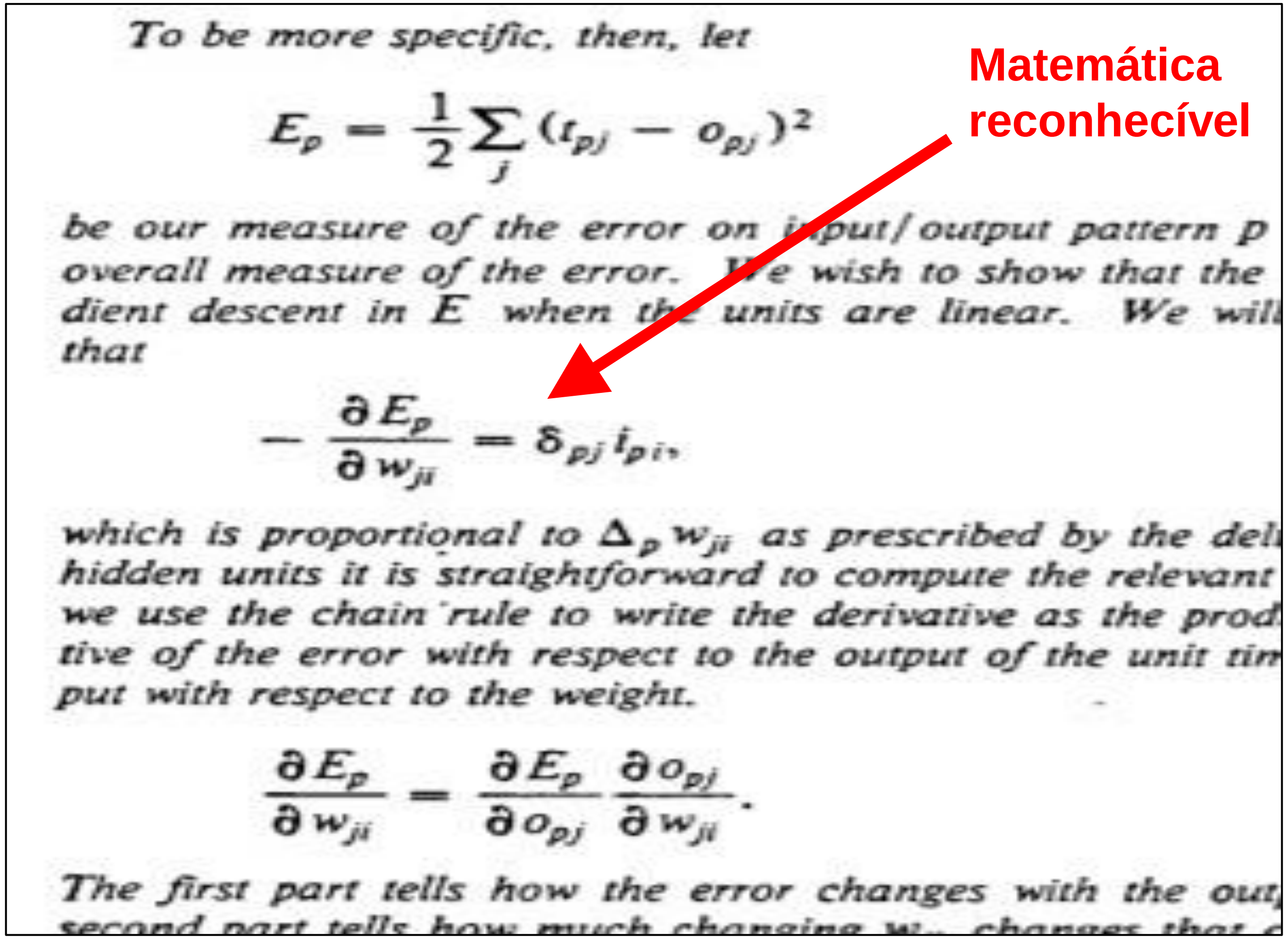
$$\frac{\partial E_p}{\partial w_{ji}} = \frac{\partial E_p}{\partial o_{pj}} \frac{\partial o_{pj}}{\partial w_{ji}}.$$

The first part tells how the error changes with the out
second part tells how much changing

Rumelhart et al. 1986: Primeira vez em que a propagação retrógada se torna popular

Um Pouco de História





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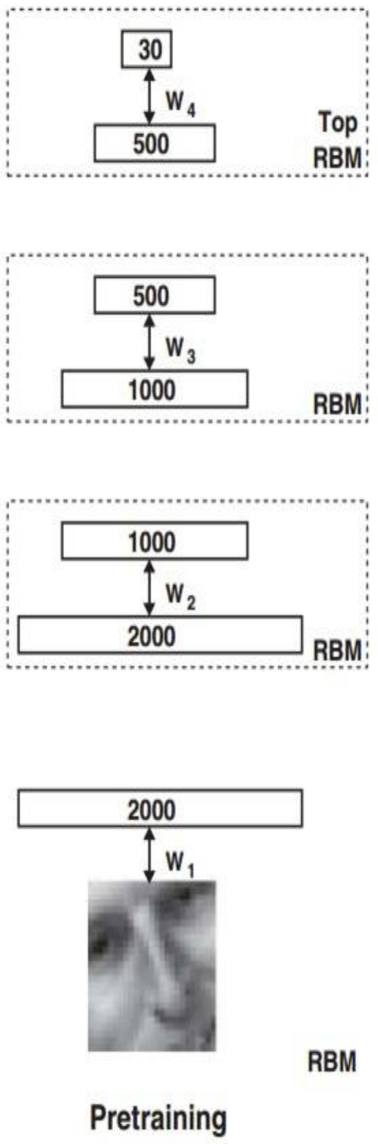
Matemática reconhecível

Rumelhart et al. 1986: Primeira vez em que a propagação retrógada se torna popular

Um Pouco de História

Hinton and Salakhutdinov 2006

Pesquisa revigorada em *Deep Learning*



Um Pouco de História

Hinton and Salakhutdinov 2006

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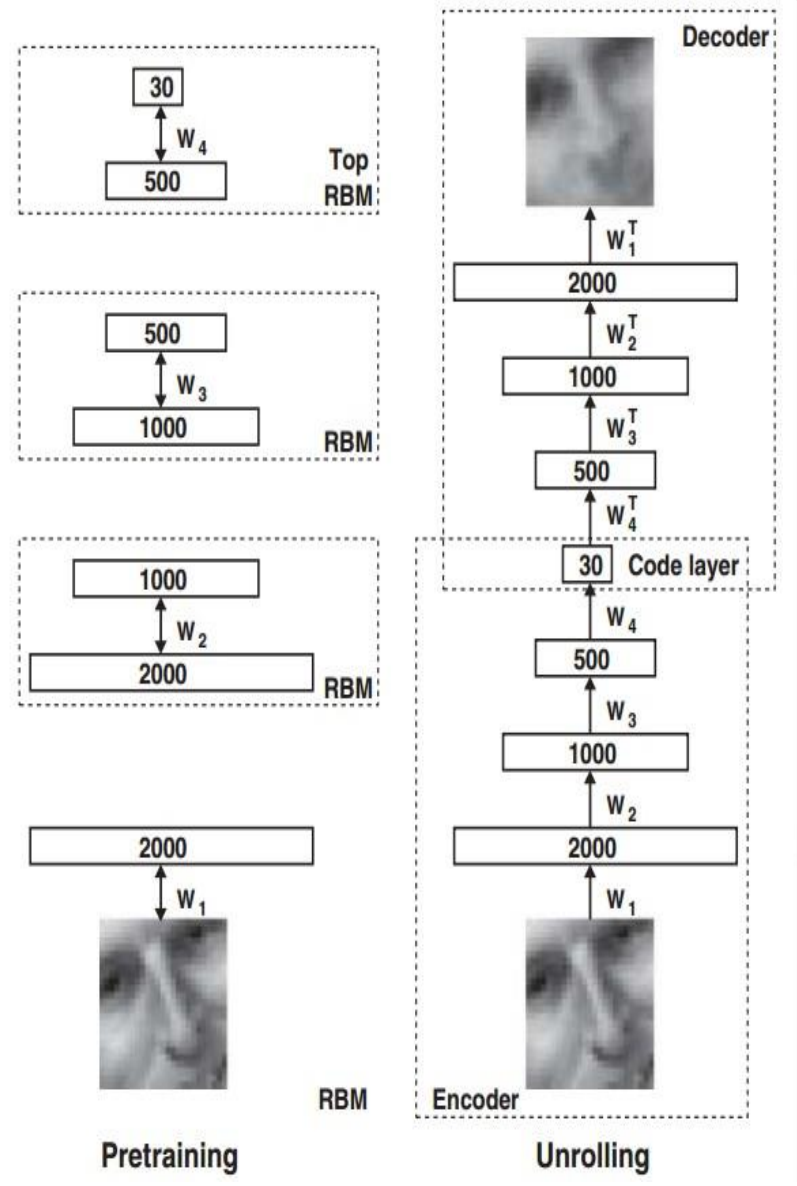

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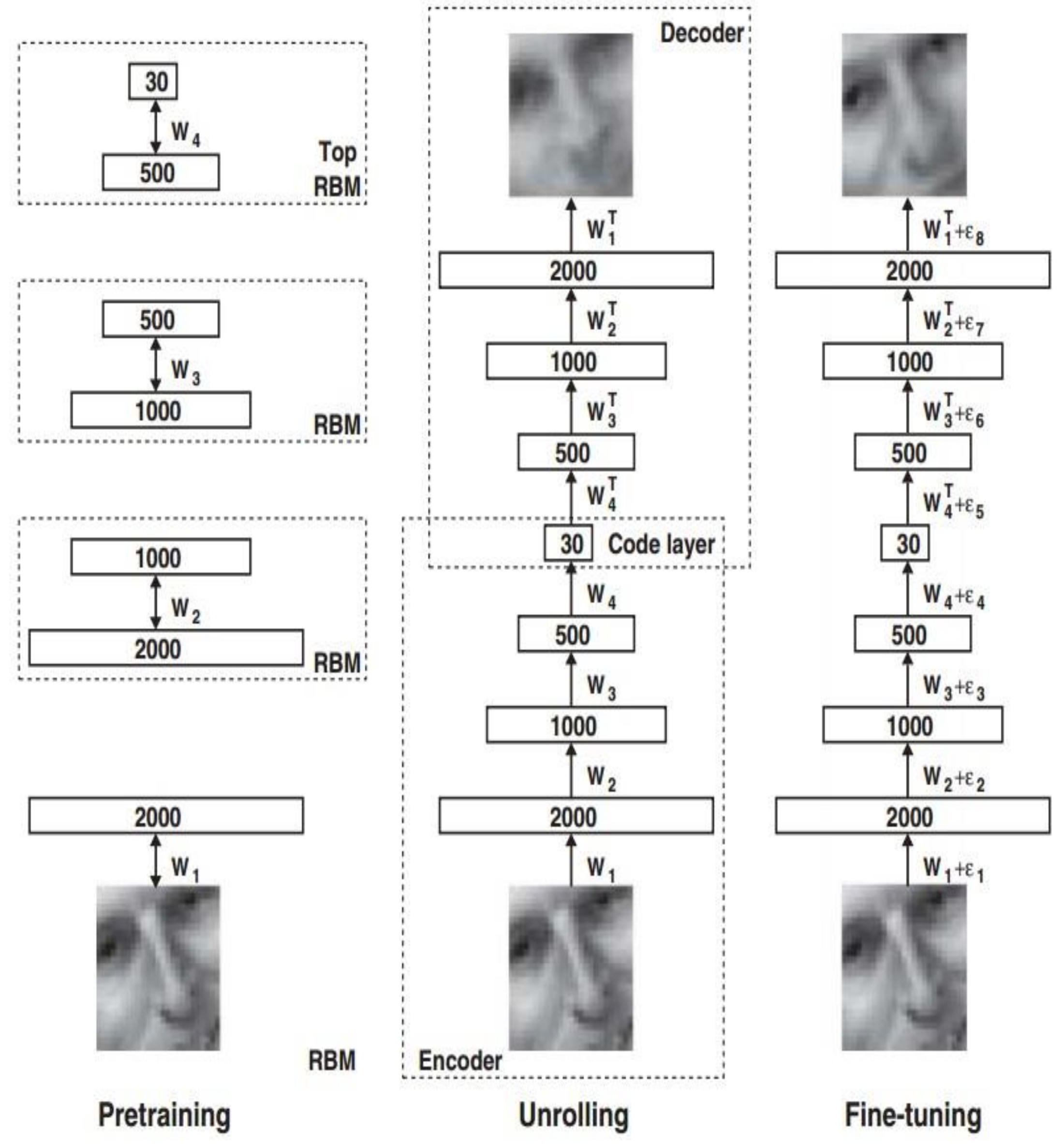
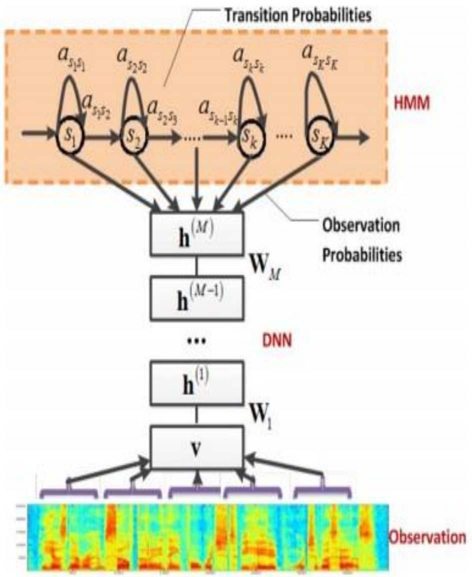


Um Pouco de História – 1os Resultados “Fortes”

Context-Dependent Pre-trained Deep Neural Networks for Large Vocabulary Speech Recognition
George Dahl, Dong Yu, Li Deng, Alex Acero, 2010



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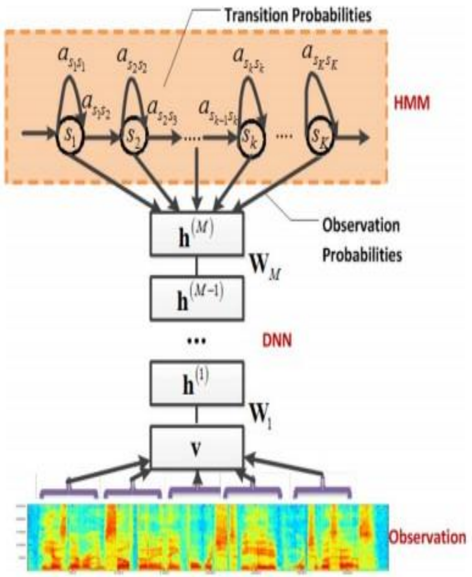

Imagenet classification with deep convolutional neural networks
Alex Krizhevsky, Ilya Sutskever, Geoffrey E Hinton, 2012