Decode Ways II

A message containing letters from `A-Z` is being encoded to numbers using the following mapping way:

```
'A' -> 1
'B' -> 2
...
'Z' -> 26
```

Beyond that, now the encoded string can also contain the character '*', which can be treated as one of the numbers from 1 to 9.

Given the encoded message containing digits and the character '*', return the total number of ways to decode it.

Also, since the answer may be very large, you should return the output mod $10^9 + 7$.

Example 1:

```
Input: "*"
Output: 9
Explanation: The encoded message can be decoded to the string: "A", "B", "C", "D", "E
", "F", "G", "H", "I".
```

Example 2:

```
Input: "1*"
Output: 9 + 9 = 18
```

Note:

1. The length of the input string will fit in range [1, 10^5].
2. The input string will only contain the character '*' and digits '0' - '9'.

Solution 1

```
def numDecodings(self, S):
    MOD = 10**9 + 7
    e0, e1, e2 = 1, 0, 0
    for c in S:
        if c == '*':
            f0 = 9*e0 + 9*e1 + 6*e2
            f1 = e0
            f2 = e0
        else:
            f0 = (c > '0') * e0 + e1 + (c <= '6') * e2
            f1 = (c == '1') * e0
            f2 = (c == '2') * e0
        e0, e1, e2 = f0 % MOD, f1, f2
    return e0
```

Let's keep track of:

- `e0 =` current number of ways we could decode, ending on any number;
- `e1 =` current number of ways we could decode, ending on an open 1;
- `e2 =` current number of ways we could decode, ending on an open 2;

(Here, an "open 1" means a 1 that may later be used as the first digit of a 2 digit number, because it has not been used in a previous 2 digit number.)

With the right idea of what to keep track of, our dp proceeds straightforwardly.

Say we see some character `c`. We want to calculate `f0, f1, f2`, the corresponding versions of `e0, e1, e2` after parsing character `c`.

If `c == '*'`, then the number of ways to finish in total is: we could put * as a single digit number (`9*e0`), or we could pair * as a 2 digit number `1*` in `9*e1` ways, or we could pair * as a 2 digit number `2*` in `6*e2` ways. The number of ways to finish with an open 1 (or 2) is just `e0`.

If `c != '*'`, then the number of ways to finish in total is: we could put `c` as a single digit if it is not zero (`(c>'0')*e0`), or we could pair `c` with our open 1, or we could pair `c` with our open 2 if it is 6 or less (`(c<='6')*e2`). The number of ways to finish with an open 1 (or 2) is `e0` iff `c == '1'` (or `c == '2'`).

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Solution 2

The idea is DP. One of the hints is that you need mod the answer with a huge prime number.

1. For any string s longer than 2, we can decode either the last 2 characters as a whole or the last 1 character. So dp[i] = dp[i-1]* f(s.substr(i,1)) + dp[i-2]* f(s.substr(i-1, 2)). f() is the number of ways to decode a string of length 1 or 2. f() could be 0, for example f("67").
2. There is a lot of cases and corner cases for f(string s). For example, * cannot be '0', so ** has 15 instead of 16 possibilities, because "20" is excluded. But the time complexity is still O(n).
   The code is as below.

```
class Solution {
public:
    int numDecodings(string s) {
        int n = s.size(), p = 1000000007;
        // f2 is the answer to sub string ending at position i; Initially i = 0.
        long f1 = 1, f2 = helper(s.substr(0,1));
        // DP to get f2 for sub string ending at position n-1;
        for (int i = 1; i < n; i++) {
            long f3 = (f2*helper(s.substr(i, 1)))+(f1*helper(s.substr(i-1, 2)));
            f1 = f2;
            f2 = f3%p;
        }
        return f2;
    }
private:
    int helper(string s) {
        if (s.size() == 1) {
            if (s[0] == '*') return 9;
            return s[0] == '0'? 0:1;
        }
        // 11-26, except 20 because '*' is 1-9
        if (s == "**")
            return 15;
        else if (s[1] =='*') {
            if (s[0] =='1') return 9;
            return s[0] == '2'? 6:0;
        }
        else if (s[0] == '*')
            return s[1] <= '6'? 2:1;
        else
            // if two digits, it has to be in [10 26]; no leading 0
            return stoi(s) >= 10 && stoi(s) <= 26? 1:0;
    }
};
```

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Solution 3

The idea for DP is simple when using two helper functions
ways(i) -> that gives the number of ways of decoding a single character
and
ways(i, j) -> that gives the number of ways of decoding the two character string formed by i and j.
The actual recursion then boils down to :

```
f(i) = (ways(i) * f(i+1)) + (ways(i, i+1) * f(i+2))
```

The solution to a string f(i), where i represents the starting index,

f(i) = no.of ways to decode the character at i, which is ways(i) + solve for remainder of the string using recursion f(i+1)
and
no.of ways to decode the characters at i and i+1, which is ways(i, i+1) + solve for remainder of the string using recursion f(i+2)

The base case is ,

```
return ways(s.charAt(i)) if(i == j)
```

The above recursion when implemented with a cache, is a viable DP solution, but it leads to stack overflow error, due to the depth of the recursion. So its better to convert to memoized version.

For the memoized version, the equation changes to

```
f(i) = ( f(i-1) * ways(i) ) + ( f(i-2) *ways(i-1, i) )
```

This is exactly the same as the previous recursive version in reverse,

The solution to a string f(i), where i represents the ending index of the string,

f(i) = solution to the prefix of the string f(i-1) + no.of ways to decode the character at i, which is ways(i)
and
solution to the prefix f(i-2) + no.of ways to decode the characters at i - 1 and i, which is ways(i-1, i)

```
public class Solution {
    public static int numDecodings(String s) {
        long[] res = new long[2];
        res[0] = ways(s.charAt(0));
        if(s.length() < 2) return (int)res[0];

        res[1] = res[0] * ways(s.charAt(1)) + ways(s.charAt(0), s.charAt(1));
        for(int j = 2; j < s.length(); j++) {
            long temp = res[1];
            res[1] = (res[1] * ways(s.charAt(j)) + res[0] * ways(s.charAt(j-1), s.c
harAt(j))) % 1000000007;
            res[0] = temp;
        }
        return  (int)res[1];
    }

    private static int ways(int ch) {
        if(ch == '*') return 9;
        if(ch == '0') return 0;
        return 1;
    }

    private static int ways(char ch1, char ch2) {
        String str = "" + ch1 + "" + ch2;
        if(ch1 != '*' && ch2 != '*') {
            if(Integer.parseInt(str) >= 10 && Integer.parseInt(str) <= 26)
                return 1;
        } else if(ch1 == '*' && ch2 == '*') {
            return 15;
        } else if(ch1 == '*') {
            if(Integer.parseInt(""+ch2) >= 0 && Integer.parseInt(""+ch2) <= 6)
                return 2;
            else
                return 1;
        } else {
            if(Integer.parseInt(""+ch1) == 1 ) {
                return 9;
            } else if(Integer.parseInt(""+ch1) == 2 ) {
                return 6;
            }
        }
        return 0;
    }
}
```

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