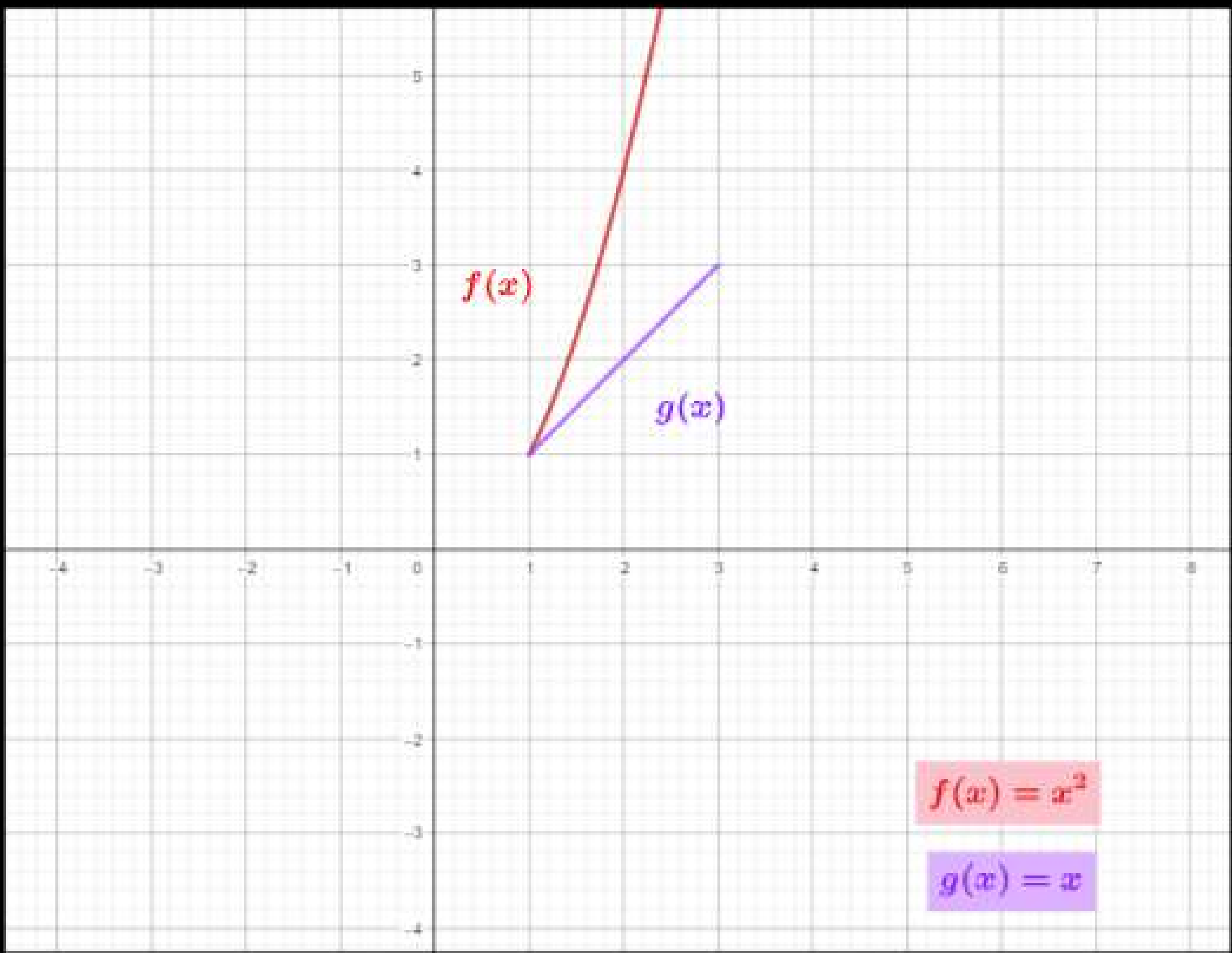
Resuelva la siguiente integral triple:

$$\int_0^{\frac{3}{2}} \int_0^{2z} \int_0^{y} (x + y + z)\,dx\,dy\,dz$$

Sean f y g funciones reales y derivables. Demostrar que la función $z = f(x, y) + g\left(\frac{x}{y}\right)$ satisface la ecuación diferencial:

$$x^2 \frac{\partial^2 z}{\partial x^2} - y^2 \frac{\partial^2 z}{\partial y^2} + x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y} = 0$$



1. Define una región a partir de las funciones que ves en el recuadro, dibújala.
2. De acuerdo a dicha región, construye las siguientes integrales:

   a. $\iint\limits_R x^2 \, dx \, dy$

   b. $\iint\limits_R x^2 \, dy \, dx$

3. Resuelve las integrales y verifica que el resultado es el mismo.

Cambia región

Cambia integral

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(1) $$\int_0^{3/2}\int_0^{2z}\int_0^{y} (x+y+z)\, dx\, dy\, dz$$

1) $$\int_0^{y} x+y+z\, dx = \int_0^{y} x\, dx + (y+z)\int 1\, dx =$$

$$= \frac{1}{2}x^2\Big|_0^{y} + (y+z).\,x\Big|_0^{y} = \frac{1}{2}\left[(y)^2-(0)^2\right] + (y+z).(y-0)$$

$$= \frac{1}{2}y^2 + y^2 + zy = \frac{3}{2}y^2 + zy$$

2) $$\int_0^{2z} \frac{3}{2}y^2 + zy\, dy = \frac{3}{2}\int_0^{2z} y^2\, dy + z\int_0^{2z} y\, dy =$$

$$= \frac{\cancel{3}}{2}\cdot\frac{1}{\cancel{3}}y^3\Big|_0^{2z} + z.\frac{1}{2}y^2\Big|_0^{2z} = \frac{1}{2}\left[(2z)^3-(0)^3\right] + \frac{1}{2}z\left[(2z)^2-(0)^2\right]$$

$$\frac{1}{\cancel{2}}.\cancel{8}z^3 + \frac{1}{\cancel{2}}z.\cancel{4}z^2 = 4z^3 + 2z^3 = 6z^3$$

3) $$\int_0^{3/2} 6z^3\, dz = 6\int_0^{3/2} z^3\, dz = \cancel{6}.\frac{1}{\cancel{4}}z^4\Big|_0^{3/2} = \frac{3}{2}\left[\left(\frac{3}{2}\right)^4 - (0)^4\right]$$

$$= \frac{3}{2}\cdot\frac{81}{16} = \boxed{\frac{243}{32}}$$

②

SACO LA PRIMERA Y SEGUNDA DERIVADA PARCIAL CON RESPECTO A X. COMO Z ES LA SUMA de f y g, DERIVO Y SUMO f y g

$$\frac{dz}{dx} = \frac{df}{dx} + \frac{dg}{dx} = \boxed{f'(xy).y + g'\left(\frac{x}{y}\right).\frac{1}{y}}$$

$$\frac{d^2z}{dx^2} = y.f''(xy).y + \frac{1}{y}.g''\left(\frac{x}{y}\right).\frac{1}{y} = \boxed{f''(xy).y^2 + g''\left(\frac{x}{y}\right).\frac{1}{y^2}}$$

HAGO LO MISMO CON Y

$$\frac{dz}{dy} = f'(xy).x + g'\left(\frac{x}{y}\right).\left(-\frac{x}{y^2}\right) = \boxed{f'(xy).x - g'\left(\frac{x}{y}\right)\frac{x}{y^2}}$$

$$\frac{d^2z}{dy^2} = f''(xy).x.x - \left[-g''\left(\frac{x}{y}\right).\frac{x^2}{y^4} - 2xy^{-3}.g'\left(\frac{x}{y}\right)\right]$$

$$= \boxed{x^2 f''(xy) + \frac{x^2}{y^4} g''\left(\frac{x}{y}\right) + 2xy^{-3}.g'\left(\frac{x}{y}\right)}$$

COMPRUEBO SI LA IDENTIDAD $x^2\frac{d^2z}{dx^2} - y^2\frac{d^2z}{dy^2} + x\frac{dz}{dx} - y\frac{dz}{dy} = 0$ SE CUMPLE

$$x^2\left(f''(xy).y^2 + g''\left(\frac{x}{y}\right).\frac{1}{y^2}\right) - y^2\left(x^2 f''(xy) + \frac{x^2}{y^4}.g''\left(\frac{x}{y}\right) + 2xy^{-3}.g'\left(\frac{x}{y}\right)\right)$$
$$+ x\left(y f'(xy) + \frac{1}{y}g'\left(\frac{x}{y}\right)\right) - y\left(x f'(xy) - xy^{-2}.g'\left(\frac{x}{y}\right)\right) =$$

$$\cancel{x^2y^2f''(xy)} + \cancel{\frac{x^2}{y^2}g''\left(\frac{x}{y}\right)} - \cancel{x^2y^2f''(xy)} - \cancel{\frac{x^2}{y^2}g''\left(\frac{x}{y}\right)} - \cancel{2xy^{-1}g'\left(\frac{x}{y}\right)}$$
$$+ \cancel{xyf'(xy)} + \cancel{\frac{x}{y}g'\left(\frac{x}{y}\right)} - \cancel{xyf'(xy)} + \cancel{xy^{-1}g'\left(\frac{x}{y}\right)} = 0$$

SE CANCELAN TODOS LOS TERMINOS Y SE DEMUESTRA QUE LA ECUACIÓN SE CUMPLE

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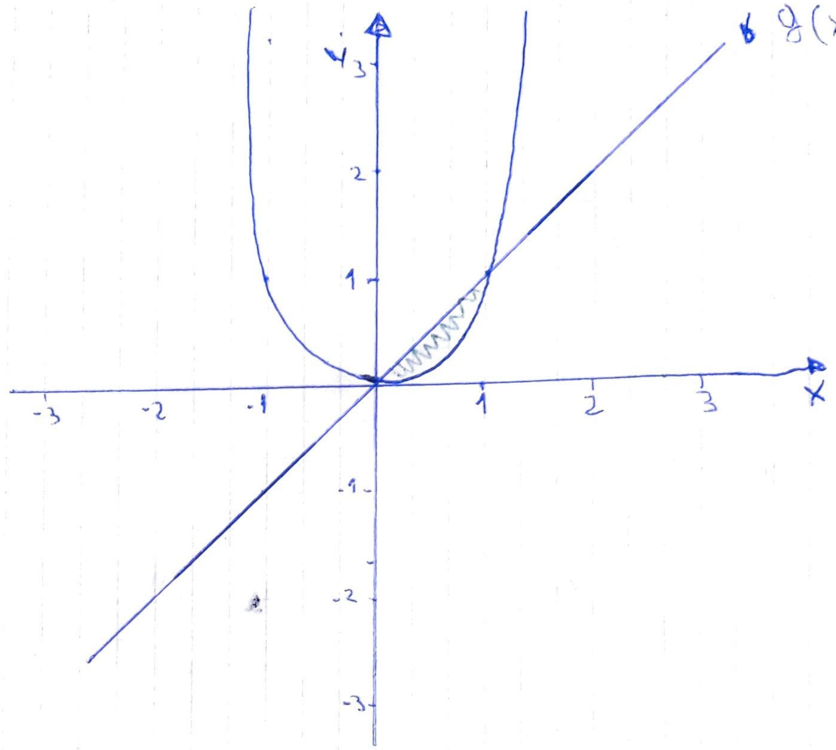
(2)

3)



B) $\int_0^1 \int_{x^2}^{x} x^2 \, dy\, dx = \frac{1}{20}$

SACO LOS PUNTOS DE CORTE

$x^2 = x$

$x^2 - x = 0$

$x(x-1)=0 \; ; \; x=0 \; , \; x-1=0$

$x = 1$

$\int_0^1 \int_{x^2}^{x} x^2 \, dy\, dx = x^2 \int_{x^2}^{x} 1 \cdot dy =$

1) $x^2 \cdot y \Big|_{x^2}^{x} = x^2 \cdot [x - x^2] = x^3 - x^4$

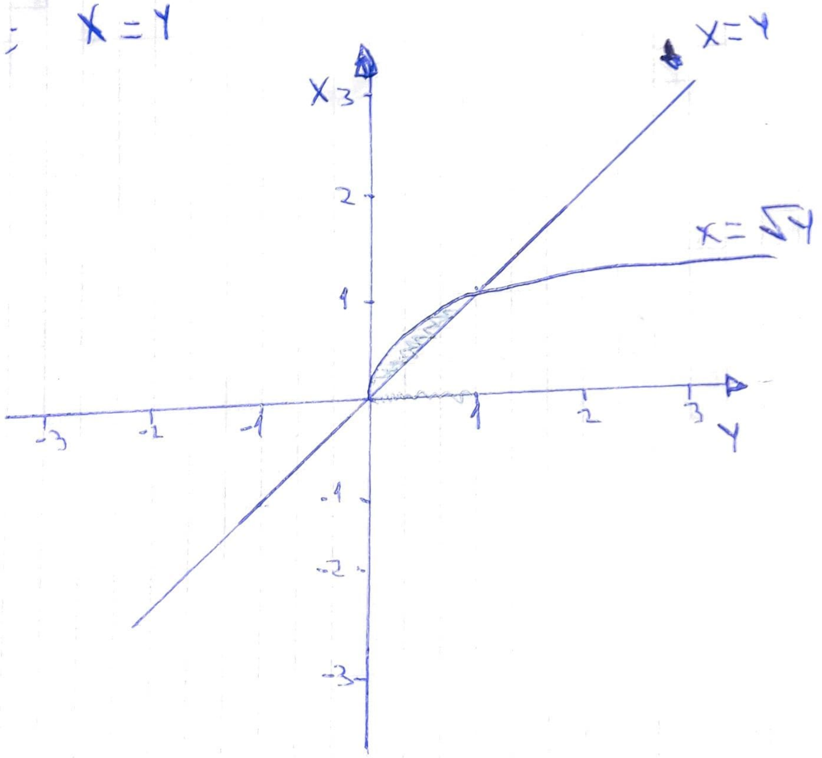
2) $\int_0^1 x^3 - x^4 \, dx = \frac{1}{4}x^4 \Big|_0^1 - \frac{1}{5}x^5 = \frac{1}{4}\cdot 1 - \frac{1}{5}\cdot 1 = \boxed{\frac{1}{20}}$

A) $\int_0^1 \int_Y^{\sqrt{Y}} x^2 \, dx \, dY = \boxed{\frac{1}{20}}$

Inverso

$Y = x^2 = X = \sqrt{X} ; -\sqrt{X}$

$Y = X \quad ; \quad X = Y$



$\int_0^1 \int_Y^{\sqrt{Y}} x^2 \, dx \, dY = 1) \int_Y^{\sqrt{Y}} x^2 \, dx = \frac{1}{3}x^3 \Big|_Y^{\sqrt{Y}} = \frac{1}{3}\left[(\sqrt{Y})^3 - (Y)^3\right]$

$= \frac{1}{3}\left[Y\sqrt{Y} - Y^3\right] = \frac{1}{3}Y\sqrt{Y} - \frac{1}{3}Y^3$

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$$\int_0^1 \frac{1}{3} Y\sqrt{Y} - \frac{1}{3} Y^3 \, dY = \int_0^1 \frac{1}{3} Y . Y^{1/2} - \frac{1}{3} Y^3 = \frac{1}{3}\int_0^1 Y^{3/2} \, dY - \frac{1}{3}\int Y^3 dY$$

$$\frac{1}{3} \cdot \frac{2}{5} Y^{5/2} \Big|_0^1 - \frac{1}{3} \cdot \frac{1}{4} Y^4 \Big|_0^1 = \frac{2}{15} \cdot 1 - \frac{1}{12} \cdot 1 = \boxed{\frac{1}{20}}$$