Solution of Homework

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Question 1.
Find the area under the following curve and demonstrate it on a figure.

$f(x) = x^2$, evaluate the $\int_0^2 f(x)dx$.

$$\int_0^2 f(x)dx = \int_0^2 x^2 dx$$

As a general rule, we can say that the antiderivative of x^n is, for values of n other than -1, $\frac{x^{n+1}}{n+1}$ plus a constant that we can safely ignore when doing definite integrals of this type. Then t this integral evaluates as

$$\frac{1}{3}x^3|_2^0 = \frac{1}{3}(2^3 - 0^3) = \frac{8}{3}$$

Question 2.
Evaluate the following integrals or explain why it doesn't exist.

1. $$\int_0^1 x^{\frac{3}{7}} dx$$

   As a general rule, we can say that the antiderivative of x^n is, for values of n other than -1, $\frac{x^{n+1}}{n+1}$ plus a constant that we can safely ignore when doing definite integrals of this type. Then this integral evaluates as

   $$\frac{x^{\frac{10}{7}}}{\frac{10}{7}}|_0^1 = \frac{1^{\frac{10}{7}}}{\frac{10}{7}} - \frac{0^{\frac{10}{7}}}{\frac{10}{7}} = \frac{1}{\frac{10}{7}} = \frac{7}{10}$$

2. $$\int_8^9 e^x dx$$

As a general rule, we can write the antiderivative of e^x is e^x plus a constant. Then in this case, the integral evaluates at

$$e^x|_9^8 = e^9 - e^8$$

3.

$$\int_3^3 \sqrt{x^5}dx$$

We don't actually have to find the integral of this function. Notice that we are evaluating the integral from 3 to 3. Whatever the actual antiderivative function F actually is, $F(3) - F(3) = 0$, so 0 is the solution. Intuitively, we are taking the area under curve at a single point, which has no area.

Question 3.

A group of three students in the first-year political science decided to grab a drink at downtown Champaign after the second day of math camp because they are worn out after the discussion of calculus. After having five shots of tequila each, three pitchers of beer, they had enough fun and decided to start the trip back home.

Student A got on a bike and started peddling at a velocity of $v_A(t) = 2t^4 + t$, where t represents minutes. However, student A crashed into a garbage bin and ended the peddling after 2 minutes.

Student B started to run at a velocity of $v_B(t) = 4\sqrt{t}$. After 4 minutes, he didn't feel good and decided to stop and waited for his friend to pick him up.

Student C couldn't even stand up, so he slowly crawled at a velocity of $v_c(t) = 2e^{-t}$ for 20 minutes. Then he felt asleep on the sidewalk.

Generally, if an object moves along a straight line with position function $s(t)$, then its velocity $v(t) = s^{'}(t)$. The Fundamental Theorem of Calculus tells us that Total Distance Traveled:

$$s(t_2) - s(t_1) = \int_{t_1}^{t_2} v(t)dt$$

Without using the calculator, assuming all three students traveled in a straight line, use this formula to calculate the distance traveled by student A, B and C. Who travelled the farthest? The least far?

Solution:

For student A:

$$\int_0^2 2t^4+t = (\frac{2}{5}t^5+\frac{1}{2}t^2)|_0^2 = \frac{2}{5}(2^5)+\frac{2}{5}(2^2)-[\frac{2}{5}(0^5)+\frac{2}{5}(0^2)] = \frac{2(32)}{5}+\frac{4}{2}-[0+0] = \frac{64}{5}+2 = \frac{74}{5}$$

For student B:

$$\int_0^4 4\sqrt{t} = 4(\frac{2}{3})t^{\frac{2}{3}}|_4^0 = \frac{3}{8}4^{\frac{2}{3}} - \frac{3}{8}0^{\frac{2}{3}} = \frac{3}{8}(\sqrt{4^3} - 0 = \frac{3}{8}(\sqrt{64} = \frac{3}{8}(8) = \frac{3}{64}$$

For student C:

$$\int_0^{20} 2e^{-t} = -2e^{-t}|_{20}^0 = -2e^{-20} - [-2e^0] = -2e^{-20} + 2(1) = -\frac{2}{e^20} + 2 \approx 0 + 2 = 2$$

Clearly, Student C had the shortest trip, and Student B went the farthest.

Question 4.
An individual benefits from an action whenever X > 10. If X is a random variable distributed uniformly on [0, 25], what is the probability that the individual will benefit?
Hint:

$$Pr(X \leq x) = F(x) = \int_{-\infty}^{x} f(t)dt$$

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$$Pr(Y > y) = 1 - Pr(Y \leq y)$$

Solution:

$$Pr(Y > 10) = 1 - Pr(Y \leq 10) = 1 - \int_0^{10} f(t)dt = 1 - F(10) = 1 - \frac{10}{25} = 0.6$$