

# Pool Strategy of a Producer With Endogenous Formation of Locational Marginal Prices

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**Abstract**—This paper considers a strategic power producer that trades electric energy in an electricity pool. It provides a procedure to derive the optimal offering strategy of this producer. A multiperiod network-constrained market-clearing algorithm is considered. Uncertainty on demand bids and offering strategies of rival producers is also modeled. The proposed procedure to derive strategic offers relies on a bilevel programming model whose upper-level problem represents the profit maximization of the strategic producer while the lower-level one represents the market clearing and the corresponding price formation. This bilevel model is reduced to a mixed-integer linear programming problem using the duality theory and the Karush–Kuhn–Tucker optimality conditions. Results from an illustrative example and a case study are reported and discussed. Finally, some relevant conclusions are duly drawn.

**Index Terms**—Electricity pool, endogenous price formation, LMP, offering strategy, power producer.

## NOTATION

The main notation used in the paper is stated below for quick reference. Other symbols are defined as needed throughout the text.

### Indices:

$t$	Index for time periods running from 1 to $T$ .
$i$	Index for the generating units of the strategic producer running from 1 to $I$ .
$j$	Index for the other generating units (owned by nonstrategic producers) running from 1 to $J$ .
$b$	Index for generation blocks running from 1 to $B$ .
$d$	Index for demands running from 1 to $D$ .
$k$	Index for demand blocks running from 1 to $K$ .
$n, m$	Indices for buses running from 1 to $N$ , and from 1 to $M$ , respectively.

### Constants:

$\lambda_{tib}^S$	Marginal cost of block $b$ of unit $i$ of the strategic producer in period $t$ .
$\lambda_{tjb}^O$	Marginal cost of block $b$ of unit $j$ of a nonstrategic producer in period $t$ .
$\lambda_{tdk}^D$	Marginal utility of block $k$ of demand $d$ in period $t$ .
$R_i^{\text{UP}}$	Ramp-up limit for generation unit $i$ of the strategic producer.

$R_i^{\text{LO}}$	Ramp-down limit for generation unit $i$ of the strategic producer.
$P_{tib}^{S^{\text{max}}}$	Upper limit of block $b$ of generation unit $i$ of the strategic producer in period $t$ .
$P_{tjb}^{O^{\text{max}}}$	Upper limit of block $b$ of generation unit $j$ of a nonstrategic producer in period $t$ .
$P_{tdk}^{D^{\text{max}}}$	Upper limit of block $k$ of demand $d$ in period $t$ .
$B_{nm}$	Susceptance of line $n - m$ .
$C_{nm}^{\text{max}}$	Transmission capacity of line $n - m$ .

These constants include subscript  $w$  if they refer to scenario  $w$ .

### Variables:

$\beta_{tn}$	Locational marginal price (LMP) in period $t$ at bus $n$ .
$\alpha_{tib}^S$	Price offer of block $b$ of unit $i$ of the strategic producer in period $t$ .
$P_{tib}^S$	Power produced by block $b$ of unit $i$ of the strategic producer in period $t$ .
$P_{tjb}^O$	Power produced by block $b$ of unit $j$ of a nonstrategic producer in period $t$ .
$P_{tdk}^D$	Power consumed by block $k$ of demand $d$ in period $t$ .
$\delta_{tn}$	Voltage angle of bus $n$ in period $t$ .

These variables include subscript  $w$  if they refer to scenario  $w$ .

### Dual Variables:

The dual variables below are associated with the following constraints:

$\lambda_{t,n}$	Generation-demand equilibrium in period $t$ at bus $n$ .
$\mu_{tib}^{S^{\text{max}}}$	Capacity of block $b$ of unit $i$ in period $t$ .
$\mu_{tib}^{S^{\text{min}}}$	Minimum production of block $b$ of unit $i$ in period $t$ .
$\mu_{tjb}^{O^{\text{max}}}$	Capacity of block $b$ of unit $j$ in period $t$ .
$\mu_{tjb}^{O^{\text{min}}}$	Minimum production of block $b$ of unit $j$ in period $t$ .
$\mu_{tdk}^{D^{\text{max}}}$	Capacity of block $k$ of demand $d$ in period $t$ .
$\mu_{tdk}^{D^{\text{min}}}$	Minimum power of block $k$ of demand $d$ in period $t$ .
$\nu_{tnm}^{\text{max}}$	Transmission capacity of line $n - m$ in period $t$ and direction $n - m$ .
$\nu_{tnm}^{\text{min}}$	Transmission capacity of line $n - m$ in period $t$ and direction $m - n$ .
$\xi_{tn}^{\text{max}}$	Upper bound of the voltage angle in period $t$ at bus $n$ .
$\xi_{tn}^{\text{min}}$	Lower bound of the voltage angle in period $t$ at bus $n$ .
$\xi_t^1$	Voltage angle in period $t$ at bus $n = 1$ .

These dual variables include subscript  $w$  if they refer to scenario  $w$ .

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## I. INTRODUCTION

### A. Background and Aim

WE consider a price-maker power producer that trades electric energy in a pool-based electricity market. This producer owns a sufficiently large number of generating units of diverse types to be able to alter the formation of the market clearing prices. These units are adequately distributed throughout the electric power network so that congestion-based strategies are possible.

The pool is cleared once a day, one day ahead, and on an hourly basis. The market clearing procedure embodies a dc representation of the network including both first and second Kirchhoff laws [1]. Its solution renders the hourly energy sold and bought, and the hourly locational marginal prices (LMPs).

Specifically, this paper provides a procedure to derive the best offering strategy for the considered strategic producer so that its benefit from trading in the pool is maximized. The offering strategy is derived modeling precisely the formation of the LMPs through constraints describing the actual market clearing. This is achieved using a bilevel programming approach that results in a mathematical program with equilibrium constraints (MPEC) [2].

### B. Approach

The proposed bilevel programming approach includes an upper-level problem and a lower-level one that constrains the upper-level problem.

The upper-level problem seeks obtaining the optimal offering strategy and represents the profit maximization of the considered strategic producer, whose revenues depend on the clearing LMPs obtained at the lower-level problem.

The lower-level problem represents the market clearing of the pool, and its solution provides, on an hourly basis, the energy bought, the energy sold, and the LMPs. The formulation of this lower-level market clearing problem requires one to hypothesize the behavior and data of both rival producers and consumers. Uncertainty can be incorporated into the model by considering multiple lower-level market clearing problems, each of them representing a possible realization of the uncertain parameters.

Since the lower-level problem is assumed to be continuous and convex (it is actually linear), it can be replaced by its Karush–Kuhn–Tucker (KKT) conditions, which yields an MPEC. This MPEC is nonlinear as it includes products of variables, but it can be efficiently linearized without approximations using the strong duality theorem [3] and the linearization procedure for the KKT conditions provided in [4].

The resulting mixed-integer linear programming problem can be solved using a conventional branch-and-cut solver [5].

### C. Literature Review

Bilevel models and complementarity theory techniques have been recently proposed to tackle electricity market problems.

A linear complementarity problem (LCP) is proposed in [6] to formulate an imperfect competition model for electricity producers. The market equilibrium is derived concatenating the optimality conditions of the producers, the grid owner, and the market clearing conditions.

A bilevel model is presented in [7] in which participants seek to maximize their corresponding profits under the constraints that their dispatches and the price are determined by an optimal power flow. Based on this model, [8] proposes an algorithm to find Nash equilibria in the market. A similar bilevel model is used in [9] to analyze the effectiveness of an independent system operator if producers have market power.

Reference [10] presents a strategic gaming model where the single-firm problem is formulated as an MPEC. The model includes a dc power flow representation, but the effect of network congestion is not analyzed. A single clearing period and linear offer curves are considered. To solve the resulting nonlinear programming problem, a penalty interior point algorithm is used.

Reference [11] presents a binary expansion solution approach for the problem of strategic offering under uncertainty in short-term electricity markets. This binary expansion is applied in [12] to address the problem of the optimal offering strategy by electricity producers in day-ahead energy auctions.

Some other works model the interaction between electricity market agents as equilibrium problems with equilibrium constraints (EPECs). Particularly, [13] presents a mixed-integer linear programming solution approach for the EPEC of finding Nash equilibrium in short-term electricity markets and [14] studies a bilevel noncooperative game-theoretic model with LMPs. EPECs have also proven to be an effective tool for modeling two-settlement electricity markets (see, for instance, [15]).

A more detailed ac network representation of the system is proposed in [16], where oligopolistic competition in a centralized power market is characterized by a multileader single-follower game. Agents exercise strategic behavior in both active and reactive power.

Reference [17] identifies Nash equilibria if generating companies game through their incremental cost offers (supply functions). It provides a mixed-integer linear programming scheme to find equilibria without approximations or iterations.

### D. Contributions

Within the modeling framework reviewed in the previous subsection, the contributions of the paper are fivefold:

- 1) to describe a procedure to determine the optimal offering strategies of a power producer in a pool-based electricity market with endogenous formation of LMPs;
- 2) to consider the uncertainty associated with demand bids and generating offers in 1);
- 3) to formulate an MPEC to derive the optimal offering strategies in 1) considering a multiperiod network-constrained market clearing algorithm;
- 4) to convert the MPEC in 2) into an equivalent mixed-integer linear programming problem solvable using available branch-and-cut solvers;
- 5) to analyze and discuss a variety of illustrative examples and realistic case studies, including the effects of network congestion and uncertainty.

### E. Paper Organization

The remainder of this paper is organized as follows. Section II presents the proposed bilevel model, its corresponding MPEC

and its equivalent linear form. Section III provides results from an illustrative example; these results are discussed and analyzed in detail. Section IV gives some results and provides some discussions for a realistic case study. Computational issues are presented in Section V. Section VI provides some relevant conclusions derived from the study carried out. KKT conditions for the lower-level problem are provided in Appendix A, while the linearization carried out is explained in Appendix B.

## II. MODEL

### A. Model Assumptions

For clarity, the main assumptions of the proposed model are summarized below.

- 1) The model is to be used by a power producer to derive hourly offering curves (24) for the day-ahead market, which is cleared 10 to 15 h prior to actual power delivery.
- 2) Forward contracts (which may contribute to mitigate market power [18]) are not explicitly modeled since they are already settled when the day-ahead market takes place. We assume that producers and consumers take them into account for defining their capacity limits.
- 3) The network is explicitly modeled within the market clearing problem through a dc approximation.
- 4) The 24 hourly clearing prices are obtained as the 24 dual variables associated with the hourly market clearing constraints.
- 5) The producers explicitly anticipate the impact of their actual offers on the pool and the pool price. This is done modeling the actual price formation through the lower-level market clearing problem.
- 6) The description that we use of each power unit includes most of its features, i.e., piecewise linear operating cost (not necessarily convex), capacity limits, and ramping limits.
- 7) We model explicitly stepwise offer curves for producers and stepwise bidding curves for consumers.

### B. Bilevel Model

The problem to identify the best offering strategy for the strategic producer can be stated using the following bilevel model:

$$\underset{\alpha_{tib}^S, P_{tib}^S, \forall t, \forall i, \forall b}{\text{Minimize}} \sum_{tib} \lambda_{tib}^S P_{tib}^S - \sum_{t(i \in \Psi_n)b} \beta_{tn} P_{tib}^S \quad (1)$$

subject to

$$\sum_b P_{(t+1)ib}^S - \sum_b P_{tib}^S \leq R_i^{\text{UP}}, \quad \forall t < T, \forall i \quad (2)$$

$$\sum_b P_{tib}^S - \sum_b P_{(t+1)ib}^S \leq R_i^{\text{LO}}, \quad \forall t < T, \forall i \quad (3)$$

$$\beta_{tn} = \lambda_{tn}, \quad \forall t, \forall n \quad (4)$$

$$P_{tib}^S \in \arg \left\{ \begin{array}{l} \text{Minimize} \sum_{tib} \alpha_{tib}^S P_{tib}^S + \sum_{tjb} \lambda_{tjb}^O P_{tjb}^O \\ - \sum_{tdk} \lambda_{tdk}^D P_{tdk}^D \end{array} \right. \quad (5)$$

subject to

$$\sum_{(i \in \Psi_n)b} P_{tib}^S + \sum_{(j \in \Psi_n)b} P_{tjb}^O - \sum_{(d \in \Psi_n)k} P_{tdk}^D = \sum_{m \in \Theta_n} B_{nm} (\delta_{tn} - \delta_{tm}) : \lambda_{tn}, \quad \forall t, \forall n \quad (6)$$

$$0 \leq P_{tib}^S \leq P_{tib}^{S\max} : \mu_{tib}^{S\min}, \mu_{tib}^{S\max}, \quad \forall t, \forall i, \forall b \quad (7)$$

$$0 \leq P_{tjb}^O \leq P_{tjb}^{O\max} : \mu_{tjb}^{O\min}, \mu_{tjb}^{O\max}, \quad \forall t, \forall j, \forall b \quad (8)$$

$$0 \leq P_{tdk}^D \leq P_{tdk}^{D\max} : \mu_{tdk}^{D\min}, \mu_{tdk}^{D\max}, \quad \forall t, \forall d, \forall k \quad (9)$$

$$-C_{nm}^{\max} \leq B_{nm} (\delta_{tn} - \delta_{tm}) \leq C_{nm}^{\max} : \nu_{tnm}^{\min}, \nu_{tnm}^{\max} \quad \forall t, \forall n, \forall m \in \Theta_n \quad (10)$$

$$-\pi \leq \delta_{tn} \leq \pi : \xi_{tn}^{\min}, \xi_{tn}^{\max}, \quad \forall t, \forall n \quad (11)$$

$$\delta_{tn} = 0 : \xi_t^1, \quad \forall t, n = 1 \quad (12)$$

The upper-level problem (1)–(4) represents the profit maximization of the strategic producer. The objective function (1) is the minus profit (cost minus revenues) of the strategic producer. Constraints (2) and (3) are the ramping limits of the generation units. A more detailed ramping model can be easily incorporated in this problem as stated in [19]. Equation (4) states that LMPs are endogenously generated within the lower-level problem. The productions  $P_{tib}^S$  belong to the feasible region defined by the lower-level problem, as stated in (5). Note that  $i \in \Psi_n$  identifies the generating units of producer  $i$  located at bus  $n$ , while  $\lambda_{tn}$  is the dual variable of the equilibrium constraint at bus  $n$  and time  $t$ , obtained from the lower-level problem.

The lower-level problem (5)–(12) represents the market clearing with the target of maximizing the social welfare as expressed by (5). A dc linear approximation of the network is used to represent the power balance at each node as well as the line capacity limits. Equation (6) enforces the power balance at every bus. Equations (7)–(9) are power bounds for blocks of both generation and demand. Constraints (10) enforce the transmission capacity limits of each line. Constraints (11) fix angle bounds for each node, and constraints (12) impose  $n_1$  to be the slack bus.  $m \in \Theta_n$  identifies the buses  $m$  connected to bus  $n$  in all time periods. Dual variables are indicated at the corresponding equations following a colon.

The lower-level problem is linear as the market operator takes  $\alpha_{tip}$  as a parameter.

It should be noted that model (1)–(12) considers known all data pertaining to the offers of all competitors and the bids of all demands.

### C. MPEC

The lower-level problem (5)–(12) is continuous and linear, and thus, it can be replaced by its KKT conditions. The result of including these equilibrium constraints in the upper-level problem is an MPEC, whose formulation is

$$\text{Minimize} \sum_{tib} \lambda_{tib}^S P_{tib}^S - \sum_{t(i \in \Psi_n)b} \lambda_{tn} P_{tib}^S \quad (13)$$

subject to

$$\sum_b P_{(t+1)ib}^S - \sum_b P_{tib}^S \leq R_i^{\text{UP}}, \quad \forall t, \forall i \quad (14)$$

$$\sum_b P_{tib}^S - \sum_b P_{(t+1)ib}^S \leq R_i^{\text{LO}}, \quad \forall t, \forall i \quad (15)$$

$$(78) - (93). \quad (16)$$

For the sake of clarity and conciseness, KKT conditions (78)–(93) are stated in Appendix A.

#### D. Equivalent Linear Formulation

Considering the linearization stated in Appendix B, the final mixed-integer linear programming problem, equivalent to problem (5)–(12), is as follows:

$$\text{Minimize } \sum_{tib} \lambda_{tib}^S P_{tib}^S - X \quad (17)$$

subject to

1) Ramping limits:

$$\sum_b P_{(t+1)ib}^S - \sum_b P_{tib}^S \leq R_i^{\text{UP}}, \quad \forall t, \forall i \quad (18)$$

$$\sum_b P_{tib}^S - \sum_b P_{(t+1)ib}^S \leq R_i^{\text{LO}}, \quad \forall t, \forall i \quad (19)$$

2) KKT equalities (see Appendix A):

$$(78) - (83)$$

3) Linearization of  $\sum \lambda_{t,n} P_{t,i,b}^S$  (see Appendix B):

$$\begin{aligned} X = & - \sum_{tjb} \lambda_{tjb}^O P_{tjb}^O + \sum_{tdk} \lambda_{tdk}^D P_{tdk}^D \\ & - \sum_{tjb} \mu_{tjb}^{O^{\max}} P_{tjb}^{O^{\max}} \\ & - \sum_{tdk} \mu_{tdk}^{D^{\max}} P_{tdk}^{D^{\max}} - \sum_{tn(m \in \Theta_n)} \nu_{tnm}^{\min} C_{nm}^{\max} \\ & - \sum_{tn(m \in \Theta_n)} \nu_{tnm}^{\max} C_{nm}^{\max} - \sum_{tn} \xi_{tn}^{\max} \pi \\ & - \sum_{tn} \xi_{tn}^{\min} \pi \end{aligned} \quad (20)$$

4) Linearization of (84), (85), and (86) (see [4]):

$$P_{tib}^S \geq 0, \quad \forall t, \forall i, \forall b \quad (21)$$

$$P_{tjb}^O \geq 0, \quad \forall t, \forall j, \forall b \quad (22)$$

$$P_{tdk}^D \geq 0, \quad \forall t, \forall d, \forall k \quad (23)$$

$$\mu_{tib}^{S^{\min}} \geq 0, \quad \forall t, \forall i, \forall b \quad (24)$$

$$\mu_{tjb}^{O^{\min}} \geq 0, \quad \forall t, \forall j, \forall b \quad (25)$$

$$\mu_{tdk}^{D^{\min}} \geq 0, \quad \forall t, \forall d, \forall k \quad (26)$$

$$P_{tib}^S \leq (1 - \omega_{tib}^{S^{\min}}) M^P, \quad \forall t, \forall i, \forall b \quad (27)$$

$$P_{tjb}^O \leq (1 - \omega_{tjb}^{O^{\min}}) M^P, \quad \forall t, \forall j, \forall b \quad (28)$$

$$P_{tdk}^D \leq (1 - \omega_{tdk}^{D^{\min}}) M^P, \quad \forall t, \forall d, \forall k \quad (29)$$

$$\mu_{tib}^{S^{\min}} \leq \omega_{tib}^{S^{\min}} M^{\mu P}, \quad \forall t, \forall i, \forall b \quad (30)$$

$$\mu_{tjb}^{O^{\min}} \leq \omega_{tjb}^{O^{\min}} M^{\mu P}, \quad \forall t, \forall j, \forall b \quad (31)$$

$$\mu_{tdk}^{D^{\min}} \leq \omega_{tdk}^{D^{\min}} M^{\mu P}, \quad \forall t, \forall d, \forall k \quad (32)$$

$$\omega_{tib}^{S^{\min}}, \omega_{tjb}^{O^{\min}}, \omega_{tdk}^{D^{\min}} \in \{0, 1\}$$

where  $M^P$  and  $M^{\mu P}$  are large enough constants.

5) Linearization of (87), (88), and (89) (see [4]): See equations (33)–(44) at the bottom of the page.

$$P_{tib}^{S^{\max}} - P_{tib}^S \geq 0, \quad \forall t, \forall i, \forall b \quad (33)$$

$$P_{tjb}^{O^{\max}} - P_{tjb}^O \geq 0, \quad \forall t, \forall j, \forall b \quad (34)$$

$$P_{tdk}^{D^{\max}} - P_{tdk}^D \geq 0, \quad \forall t, \forall d, \forall k \quad (35)$$

$$\mu_{tib}^{S^{\max}} \geq 0, \quad \forall t, \forall i, \forall b \quad (36)$$

$$\mu_{tjb}^{O^{\max}} \geq 0, \quad \forall t, \forall j, \forall b \quad (37)$$

$$\mu_{tdk}^{D^{\max}} \geq 0, \quad \forall t, \forall d, \forall k \quad (38)$$

$$P_{tib}^{S^{\max}} - P_{tib}^S \leq (1 - \omega_{tib}^{S^{\max}}) M^P, \quad \forall t, \forall i, \forall b \quad (39)$$

$$P_{tjb}^{O^{\max}} - P_{tjb}^O \leq (1 - \omega_{tjb}^{O^{\max}}) M^P, \quad \forall t, \forall j, \forall b \quad (40)$$

$$P_{tdk}^{D^{\max}} - P_{tdk}^D \leq (1 - \omega_{tdk}^{D^{\max}}) M^P, \quad \forall t, \forall d, \forall k \quad (41)$$

$$\mu_{tib}^{S^{\max}} \leq \omega_{tib}^{S^{\max}} M^{\mu P}, \quad \forall t, \forall i, \forall b \quad (42)$$

$$\mu_{tjb}^{O^{\max}} \leq \omega_{tjb}^{O^{\max}} M^{\mu P}, \quad \forall t, \forall j, \forall b \quad (43)$$

$$\mu_{tdk}^{D^{\max}} \leq \omega_{tdk}^{D^{\max}} M^{\mu P}, \quad \forall t, \forall d, \forall k \quad (44)$$

$$\omega_{tib}^{S^{\max}}, \omega_{tjb}^{O^{\max}}, \omega_{tdk}^{D^{\max}} \in \{0, 1\}$$

6) Linearization of (90) and (91) (see [4]):

$$C_{nm}^{\max} - B_{nm}(\delta_{tn} - \delta_{tm}) \geq 0, \quad \forall t, \forall n, \forall m \in \Theta_n \quad (45)$$

$$C_{nm}^{\max} + B_{nm}(\delta_{tn} - \delta_{tm}) \geq 0, \quad \forall t, \forall n, \forall m \in \Theta_n \quad (46)$$

$$\nu_{tnm}^{\max} \geq 0, \quad \forall t, \forall n, \forall m \in \Theta_n \quad (47)$$

$$\nu_{tnm}^{\min} \geq 0, \quad \forall t, \forall n, \forall m \in \Theta_n \quad (48)$$

$$C_{nm}^{\max} - B_{nm}(\delta_{tn} - \delta_{tm}) \leq (1 - \psi_{tnm}^{\max})M^C \quad \forall t, \forall n, \forall m \in \Theta_n \quad (49)$$

$$C_{nm}^{\max} + B_{nm}(\delta_{tn} - \delta_{tm}) \leq (1 - \psi_{tnm}^{\min})M^C \quad \forall t, \forall n, \forall m \in \Theta_n \quad (50)$$

$$\nu_{tnm}^{\max} \leq \psi_{tnm}^{\max}M^{\nu C}, \quad \forall t, \forall n, \forall m \in \Theta_n \quad (51)$$

$$\nu_{tnm}^{\min} \leq \psi_{tnm}^{\min}M^{\nu C}, \quad \forall t, \forall n, \forall m \in \Theta_n \quad (52)$$

$$\psi_{tnm}^{\max}, \psi_{tnm}^{\min} \in \{0, 1\}$$

where  $M^C$  and  $M^{\nu C}$  are large enough constants.

7) Linearization of (92) and (93) (see [4]):

$$\pi - \delta_{tn} \geq 0, \quad \forall t, \forall n \quad (53)$$

$$\pi + \delta_{tn} \geq 0, \quad \forall t, \forall n \quad (54)$$

$$\xi_{tn}^{\max} \geq 0, \quad \forall t, \forall n \quad (55)$$

$$\xi_{tn}^{\min} \geq 0, \quad \forall t, \forall n \quad (56)$$

$$\pi - \delta_{tn} \leq (1 - \varphi_{tn}^{\max})M^\delta, \quad \forall t, \forall n \quad (57)$$

$$\pi + \delta_{tn} \leq (1 - \varphi_{tn}^{\min})M^\delta, \quad \forall t, \forall n \quad (58)$$

$$\xi_{tn}^{\max} \leq \varphi_{tn}^{\max}M^{\xi\delta}, \quad \forall t, \forall n \quad (59)$$

$$\xi_{tn}^{\min} \leq \varphi_{tn}^{\min}M^{\xi\delta}, \quad \forall t, \forall n \quad (60)$$

$$\varphi_{tn}^{\max}, \varphi_{tn}^{\min} \in \{0, 1\}$$

where  $M^\delta$  and  $M^{\xi\delta}$  are large enough constants.

Note that the optimization variables of the problem (17)–(60) are:  $\alpha_{tib}^S$ ,  $\lambda_{tn}$ ,  $P_{tib}^S$ ,  $P_{tjb}^O$ ,  $P_{tdk}^D$ ,  $\mu_{tib}^{\min}$ ,  $\mu_{tjb}^{\min}$ ,  $\mu_{tdk}^{\min}$ ,  $\mu_{tib}^{\max}$ ,  $\mu_{tjb}^{\max}$ ,  $\mu_{tdk}^{\max}$ ,  $\delta_{tn}$ ,  $\xi_{tn}^{\max}$ ,  $\xi_{tn}^{\min}$ ,  $\nu_{tnm}^{\max}$ ,  $\nu_{tnm}^{\min}$ ,  $\omega_{tib}^{\min}$ ,  $\omega_{tjb}^{\min}$ ,  $\omega_{tdk}^{\min}$ ,  $\omega_{tib}^{\max}$ ,  $\omega_{tjb}^{\max}$ ,  $\omega_{tdk}^{\max}$ ,  $\psi_{tnm}^{\max}$ ,  $\psi_{tnm}^{\min}$ ,  $\varphi_{tn}^{\max}$ ,  $\varphi_{tn}^{\min}$ .

### E. Offer Building

The result of model (17)–(60) provides the optimal offer prices  $\alpha_{tib}^S$ , which always coincide with LMPs. Then, a naive offer strategy is as follows: offer all production blocks of any unit at the resulting LMP of the node where the unit is located. However, this is not a convenient offer curve for two reasons:

- 1) The market clearing assumes increasing offer curves, which results in the production blocks being “filled” in order. A flat offer curve renders multiple solutions and degeneracy.
- 2) Most actual markets require increasing offer curves.

It is possible to modify the offer strategy in order to obtain an increasing offer curve that achieves the same results as model (17)–(60) provides. Among several building alternatives, we propose one based on the marginal cost curve. It consists of bidding all the energy blocks at their corresponding marginal costs except those blocks that actually set the prices (marginal blocks), which are offered at the corresponding LMPs ( $\alpha_{tib}^S$ ):

- 1) If  $P_{tib}^S = 0$  and  $\alpha_{tib}^S < \lambda_{tib}^S$ , then  $\alpha_{tib}^{fS} = \lambda_{tib}^S$ .

To ensure that this energy block remains non-accepted, it is offered at its marginal cost, which is above  $\alpha_{tib}^S$ .

- 2) If  $P_{tib}^S = 0$  and  $\alpha_{tib}^S > \lambda_{tib}^S$ , then  $\alpha_{tib}^{fS} = \alpha_{tib}^S$ .

This energy block is not accepted, but its marginal cost is smaller than  $\alpha_{tib}^S$ . Thus, this block is offered at  $\alpha_{tib}^S$  to guarantee that it remains non-accepted.

- 3) If  $0 < P_{tib}^S < P_{tib}^{S\max}$ , then  $\alpha_{tib}^{fS} = \alpha_{tib}^S - \varepsilon$ .

This energy block is offered at a value slightly smaller than  $\alpha_{tib}^S$  to guarantee that it is partially accepted.

- 4) If  $P_{tib}^S = P_{tib}^{S\max}$ , then  $\alpha_{tib}^{fS} = \lambda_{tib}^S$ .

This energy block is offered at its marginal cost, which is lower than the  $\alpha_{tib}^S$ , to guarantee that it is fully accepted.

Note that values  $\alpha_{tib}^{fS}$  allow building the final offering curves and  $\varepsilon$  is a small positive constant, e.g.,  $10^{-6}$ .

### F. Uncertainty Modeling

Relevant uncertainties affecting the day-ahead offering strategy of a producer include:

- 1) consumers' bids;
- 2) rival producers' offers.

These uncertainties can be incorporated into problem (1)–(12) by using a set of scenarios that considers all possible realizations of the uncertain parameters, replicating decisions variables, and considering multiple lower-level market clearing problems. This translates into the following formulation:

$$\begin{aligned} & \text{Minimize} \\ & \alpha_{tibw}^S, P_{tibw}^S \quad \forall t, \forall i, \forall b, \forall w \\ & \sum_w \Pi_w \left( \sum_{tib} \lambda_{tib}^S P_{tibw}^S - \sum_{t(i \in \Psi_n)b} \beta_{tnw} P_{tibw}^S \right) \end{aligned} \quad (61)$$

subject to

$$\sum_b P_{(t+1)ibw}^S - \sum_b P_{tibw}^S \leq R_i^{\text{UP}}, \quad \forall t < T, \forall i, \forall w \quad (62)$$

$$\sum_b P_{tibw}^S - \sum_b P_{(t+1)ibw}^S \leq R_i^{\text{LO}}, \quad \forall t < T, \forall i, \forall w \quad (63)$$

$$\beta_{tnw} = \lambda_{tnw}, \quad \forall t, \forall n, \forall w \quad (64)$$

$$P_{tibw}^S \in \arg \left\{ \begin{aligned} & \text{Minimize} \sum_{tib} \alpha_{tibw}^S P_{tibw}^S \\ & + \sum_{tjb} \lambda_{tjbw}^O P_{tjbw}^O - \sum_{tdk} \lambda_{tdkw}^D P_{tdkw}^D \end{aligned} \right. \quad (65)$$

subject to (66)–(77) at the bottom of the next page.

Note that all optimization variables depend on the scenario index  $w$ . Each scenario constitutes a realization of the rival producers' offers ( $\lambda_{tjbw}^O, P_{tjbw}^{O\max}$ ) and of the consumers bids ( $\lambda_{tdkw}^D, P_{tdkw}^{D\max}$ ). The objective function (61) is the minus expected profit and  $\Pi_w$  is the probability associated with scenario  $w$ . The multiple lower-level problems are represented by (65)–(72).

Model (61)–(72) provides a pair of production quantities ( $P_{tibw}^S$ ) and prices ( $\lambda_{tnw}$ ) per scenario  $w$ , which can be used to build an offering curve, i.e., the optimal production offer for each possible market price. Note that additional constraints are needed to ensure that the resulting offer curves are increasing in price. Equations (73)–(77) guarantee that higher productions correspond to higher prices while maintaining the linearity

of the model. Equations (73)–(77) require the definition of a new scenario index ( $w'$ ) and the use of binary variables ( $x_{tiww'}$ ,  $y_{tiww'}$ ,  $z_{tiww'}$ ) and large constants ( $M^x$ ,  $M^y$ ).

The linearization techniques for the single-scenario problem (1)–(12) are equally suitable for model (61)–(77). However, while the number of variables (primal and dual) increases linearly with the number of scenarios, the computational time increases exponentially due mainly to the binary variables needed to linearize the complementarity conditions of the multiple lower-level problems.

The use of an appropriate scenario reduction technique is thus needed because the initial number of scenarios may result in an intractable optimization problem. The final number of scenarios should be selected properly balancing accuracy and tractability. However, the generation of an appropriate set of scenarios and its subsequent reduction keeping as much as possible the stochastic information contained in the original set is beyond the scope of this paper; the interested reader can find appropriated information in [20].

### III. ILLUSTRATIVE EXAMPLE

#### A. Data

The considered test system is depicted in Fig. 1. There are two separated areas interconnected by two tie-lines. At the left-hand area, generation prevails, while at the right-hand one, the consumption does. “S” identifies units belonging to the strategic producers and “O” units of nonstrategic producers.

Table I provides data for the units considered in this example. Each column refers to a particular type of generation unit. The second row contains the power capacity of each unit, which is divided in four blocks (rows 3 to 6) with their associated production cost (rows 7 to 10). The last two rows contain the up and

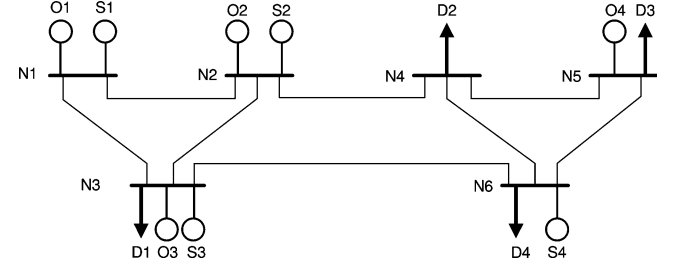


Fig. 1. Six-bus test system.

TABLE I  
TYPE AND DATA FOR THE GENERATING UNITS

Unit Type	oil	oil	hydro	coal	oil	coal	oil	coal	nuclear
$P_1$ [MW]	12	20	50	76	100	155	197	350	400
$P_1^{\max}$ [MWh]	2.4	15.8	15	15.2	25	54.25	68.95	140	100
$P_2^{\max}$ [MWh]	3.4	0.2	15	22.8	25	38.75	49.25	97.5	100
$P_3^{\max}$ [MWh]	3.6	3.8	10	22.8	20	31	39.4	52.5	120
$P_4^{\max}$ [MWh]	2.4	0.2	10	15.2	20	31	39.4	70	80
$\lambda_1^{S/O}$ [€/MWh]	23.41	11.09	0	11.46	18.60	9.92	10.08	19.20	5.31
$\lambda_2^{S/O}$ [€/MWh]	23.78	11.42	0	11.96	20.03	10.25	10.66	20.32	5.38
$\lambda_3^{S/O}$ [€/MWh]	26.84	16.06	0	13.89	21.67	10.68	11.09	21.22	5.53
$\lambda_4^{S/O}$ [€/MWh]	30.40	16.24	0	15.97	22.72	11.26	11.72	22.13	5.66
$R^{UP}$ [MW]	30	90	-	60	210	90	90	120	600
$R^{LO}$ [MW]	30	90	-	60	210	90	90	120	600

down ramping limits. Table II provides demand bids (energy and price) for each period of time. Each column corresponds to a time period (1 to 24), each row corresponds to a different bidding price (€/MWh), and the entries of this table identify the actual values of the energy bids (GWh). Note that the first demand block in each column is the largest one. Table III indicates how the total demand is shared among the buses. Table IV pro-

$$\sum_{(i \in \Psi_n) b} P_{tibw}^S + \sum_{(j \in \Psi_n) b} P_{tjbw}^O - \sum_{(d \in \Psi_n) k} P_{tdkw}^D = \sum_{m \in \Theta_n} B_{nm}(\delta_{tnw} - \delta_{tmw}) : \lambda_{tnw}, \quad \forall t, \forall n \quad (66)$$

$$0 \leq P_{tibw}^S \leq P_{tibw}^{S\max} : \mu_{tibw}^{S\min}, \mu_{tibw}^{S\max}, \quad \forall t, \forall i, \forall b \quad (67)$$

$$0 \leq P_{tjbw}^O \leq P_{tjbw}^{O\max} : \mu_{tjbw}^{O\min}, \mu_{tjbw}^{O\max}, \quad \forall t, \forall j, \forall b \quad (68)$$

$$0 \leq P_{tdkw}^D \leq P_{tdkw}^{D\max} : \mu_{tdkw}^{D\min}, \mu_{tdkw}^{D\max}, \quad \forall t, \forall d, \forall k \quad (69)$$

$$-C_{nm}^{\max} \leq B_{nm}(\delta_{tnw} - \delta_{tmw}) \leq C_{nm}^{\max} : \nu_{tnmw}^{\min}, \nu_{tnmw}^{\max}, \quad \forall t, \forall n, \forall m \in \Theta_n \quad (70)$$

$$-\pi \leq \delta_{tnw} \leq \pi : \xi_{tnw}^{\min}, \xi_{tnw}^{\max}, \quad \forall t, \forall n \quad (71)$$

$$\delta_{tnw} = 0 : \xi_{tnw}^1, \quad \forall t, n = 1 \quad \left. \vphantom{\delta_{tnw} = 0} \right\} \forall w \quad (72)$$

$$\lambda_{tnw} - \lambda_{tnw'} \leq x_{tiww'} M^x, \quad \forall t, \forall i \in \Psi_n, \forall w, \forall w' > w \quad (73)$$

$$\lambda_{tnw} - \lambda_{tnw'} \geq (x_{tiww'} - 1) M^x, \quad \forall t, \forall i \in \Psi_n, \forall w, \forall w' > w \quad (74)$$

$$\sum_b P_{tibw}^S - \sum_b P_{tibw'}^S \leq y_{tiww'} M^y, \quad \forall t, \forall i, \forall w, \forall w' > w \quad (75)$$

$$\sum_b P_{tibw}^S - \sum_b P_{tibw'}^S \geq (y_{tiww'} - 1) M^y, \quad \forall t, \forall i, \forall w, \forall w' > w \quad (76)$$

$$x_{tiww'} + y_{tiww'} = 2z_{tiww'} \quad \forall t, \forall i, \forall w, \forall w' > w \quad (77)$$

$$x_{tiww'}, y_{tiww'}, z_{tiww'} \in \{0, 1\}.$$

TABLE II  
DEMAND BLOCKS [GWh] FOR EACH PERIOD OF TIME

[€/MWh]	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
25.000												0.9	0.9					0.9	0.9	0.9	0.9			
24.968											0.9	0.025	0.025	0.9				0.025	0.025	0.025	0.025	0.9		
22.628											0.025	0.025	0.025	0.025				0.025	0.025	0.025	0.025	0.025		
20.876											0.025	0.025	0.025	0.025			0.9	0.025	0.025	0.025	0.025	0.025		
20.606											0.025	0.025	0.025	0.025			0.025	0.025	0.025	0.025	0.025	0.025		
20.378										0.9	0.025			0.025	0.9	0.9	0.025					0.025		
19.922										0.025					0.025	0.025	0.025							
19.532										0.025					0.025	0.025	0.025						0.9	
19.232									0.9	0.025					0.025	0.025							0.025	
18.932									0.025	0.025					0.025	0.025							0.025	
18.806									0.025														0.025	
18.344									0.025														0.025	
18.152									0.025															
17.940								0.9																0.9
17.612								0.025																0.025
17.430	0.9							0.025																0.025
17.250	0.025	0.9					0.9	0.025																0.025
17.216	0.025	0.025	0.9	0.9			0.025	0.025																0.025
16.886	0.025	0.025	0.025	0.025	0.9	0.9	0.025																	
16.790	0.025	0.025	0.025	0.025	0.025	0.025	0.025																	
16.380		0.025	0.025	0.025	0.025	0.025	0.025																	
16.320			0.025	0.025	0.025	0.025																		
16.130					0.025	0.025																		

TABLE III  
LOCATION AND DISTRIBUTION OF THE DEMAND

$d$	Bus	Factor (%)
1	3	19
2	4	27
3	5	27
4	6	27

TABLE IV  
LOCATION AND TYPE OF UNITS

Strategic units			Other units		
$i$	Type	Bus	$j$	Type	Bus
1	155	1	1	350	1
2	100	2	2	197	2
3	155	3	3	197	3
4	197	6	4	155	5

vides the location of the units throughout the network. Finally, note that we consider that all the lines have the same susceptance,  $B_{nm} = 9.412$  p.u.

### B. Solution: Uncongested Network

First, we solve problem (17)–(60) without imposing capacity limits on the lines. The results are given in Table V. This table provides results for the strategic producer in two cases, exercising (strategic offer) and not exercising its market power (marginal-cost offer). Note that exercising market power results in higher profit and lower production. In the nonstrategic case, the price is the same for all buses and periods and equals 11.72 €/MWh. For the strategic case prices vary as shown in Fig. 2. The maximum power flow (strategic case) through lines 2–4, 3–6, and 6–4 are 269.62, 229.44, and 39.69 MW, respectively.

Table VI illustrates two examples of the offer building process described in Section II-F. The first one corresponds with the

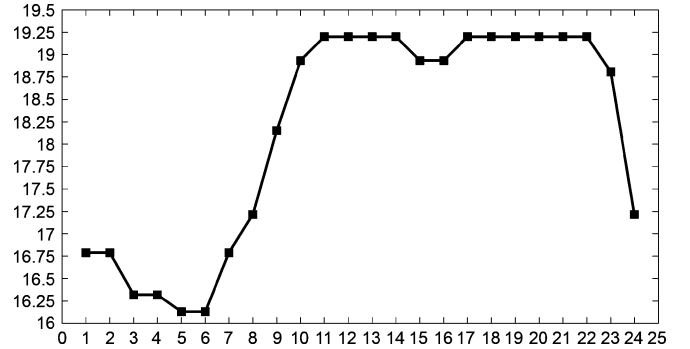


Fig. 2. Clearing prices in the uncongested network case.

TABLE V  
SOLUTION FOR THE UNCONGESTED NETWORK CASE

Strategic Offer					
	S1	S2	S3	S4	Total
Production [MWh]	3501.8	0	3464.8	3782.4	10749
Profit [€]	27202	0	27038	28861	83101
Marginal-Cost Offer					
Production [MWh]	3720	0	3720	3805.2	11245.2
Profit [€]	4826.7	0	4826.7	4562.5	14216

offer made by strategic unit 1 (S1) in the first hour. As Table VI shows, the first three energy blocks are fully dispatched and offered at marginal cost while the fourth one satisfies the condition  $0 < P_{1,1,4}^S < P_{1,1,4}^{S^{\max}}$  and is offered at  $\alpha_{1,1,4}^{fS} = \alpha_{1,1,4}^S - \epsilon$ . The second example is the offer made by strategic unit 2 (S2) in hour 12. In this case, the first energy block satisfies the conditions  $P_{12,2,1}^S = 0$  and  $\alpha_{12,2,1}^S > \lambda_{12,2,1}^S$  so it is offered at  $\alpha_{12,2,1}^{fS} = \alpha_{12,2,1}^S$ . The rest of the blocks are offered at  $\lambda_{12,2,b}^S$ ,

TABLE VI  
OFFER BUILDING FOR THE UNCONGESTED NETWORK CASE

Strategic Unit 1 (S1) in t=1				
$b$	$\lambda_{1,1,b}^S$ [€/MWh]	$P_{1,1,b}^S$ [MWh]	$\alpha_{1,1,b}^S$ [€/MWh]	$\alpha_{1,1,b}^{fS}$ [€/MWh]
1	9.92	54.25	16.79	9.92
2	10.25	38.75	16.79	10.25
3	10.68	31	16.79	10.68
4	11.26	14.40	16.79	16.79- $\epsilon$

Strategic Unit 2 (S2) in t=12				
$b$	$\lambda_{12,2,b}^S$ [€/MWh]	$P_{12,2,b}^S$ [MWh]	$\alpha_{12,2,b}^S$ [€/MWh]	$\alpha_{12,2,b}^{fS}$ [€/MWh]
1	18.60	0	19.20	19.20
2	20.03	0	19.20	20.03
3	21.67	0	19.20	21.67
4	22.72	0	19.20	22.72

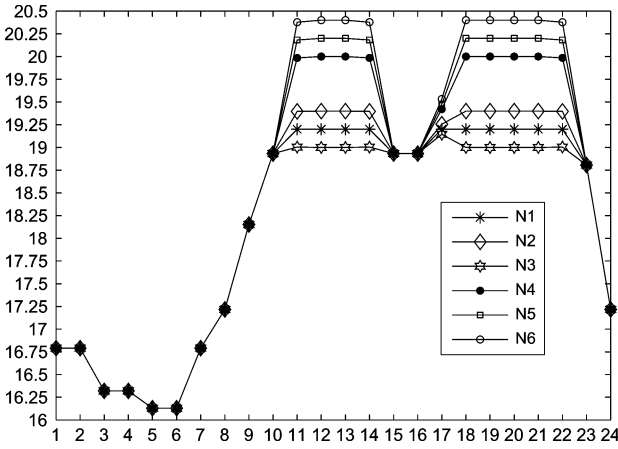


Fig. 3. LMPs with line 3–6 limited.

TABLE VII  
CAPACITY OF LINE 3–6 LIMITED TO 230 MW

	S1	S2	S3	S4	Total
Production [MWh]	3477.9	0	3498	3773.1	10749
Profit [€]	2691	0	27068	30519	84574

because they are not scheduled and  $\alpha_{12,2,b}^S$  is below their marginal costs.

### C. Solution: Congested Network

We fix the transmission capacity limit of line 3–6 to 230 MW, a value slightly above the maximum power flow through this line in the uncongested network case. For this new case, the solution of problem (17)–(60) shows that the strategic units offer to congest line 3–6 at some time periods so that unit S4 can sell its energy at higher prices. As it can be seen in Table VII, the profit of the strategic producer increases compared to the uncongested case while the production is the same. The prices for each bus and period for this case are depicted in Fig. 3. Due to congestion, bus 6 exhibits the highest prices at some periods, which allows unit S4 to increase its profit.

Similar results are obtained by fixing the maximum transmission capacity limit of line 2–4 to 270 MW.

It is also relevant to fix the maximum transmission capacity of line 4–6 to 39 MW, which, as opposed to the previous case,

TABLE VIII  
CAPACITY OF LINE 6–4 LIMITED TO 39 MW

	S1	S2	S3	S4	Total
Production [MWh]	3610.3	0	3356.3	3782.4	10749
Profit [€]	28027	0	26214	28861	83101

is a value slightly below the power flow obtained in the uncongested network case. In this case, the strategic offers derived from the solution of problem (17)–(60) are modified to avoid the congestion of this line, which would result in a distribution of prices that is not beneficial for the strategic producer. The resulting profit and productions are shown in Table VIII. The productions and profits of individual units are modified but not the total production as compared with the uncongested case. The resulting prices throughout time periods are equal to those depicted in Fig. 2.

From the strategic producer point of view, transmission lines can be classified into two types. The first one are the “congestable lines” (3–6, 2–4): if the capacity of these lines is not sufficiently large, the strategic units offer to congest them so its profit increases. The second type of lines are “noncongestable lines” (4–6): the strategic units offer to reduce its production in order not to congest these lines because congestion results in reduced profits.

### D. Solution: Stochastic Model

The uncongested network case from Section III-B but using model (61)–(77) is studied in this subsection. We consider eight equiprobable scenarios that differ on the producers offers,  $\lambda_{t,jbw}^O$ , and on the consumers bids,  $\lambda_{t,dkw}^D$ . They are generated by multiplying the offer/bid curves used in Section III-B by the entries of vector [1.5 1.2 1.0 0.9 0.7 0.5 0.3 0.2]. The use of this simple scenario generation technique allows us to obtain a wide range of market prices and production quantities to illustrate the building of the offer curves. Table IX provides results pertaining to the strategic producer for the cases of offering strategically and offering at marginal cost. Note once more that exercising market power results in higher expected profit and lower expected production. Selected offer curves obtained for different time periods are depicted in Fig. 4. Note that these curves are strictly increasing.

## IV. CASE STUDY

This section presents results from a case study based on the IEEE One Area Reliability Test System (RTS) described in [21]. A demand pattern similar to the one in Table II is considered with five demand blocks of 2.35, 0.1, 0.1, 0.1, and 0.1 GWh. This demand is shared among the buses as indicated in [21]. Table I provides data for the generating units and Table X provides their location in the network. We use the network described in [21] with the exception that the double-circuit lines are replaced by equivalent single-circuit ones.

The results obtained solving problem (17)–(60) are shown in Table XI. Note that, as expected, exercising market power results in higher profit and lower production than these profit and productions obtained with the marginal-cost offer. The time evolution of some representative LMPs for the marginal-cost



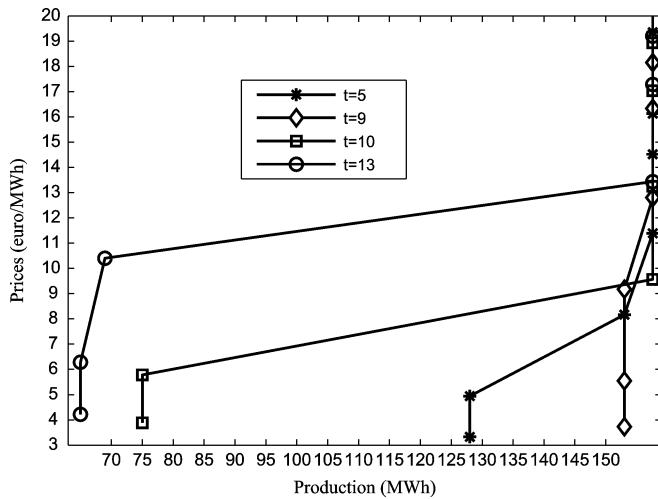


Fig. 4. Offer curves pertaining to strategic generator 1 for the stochastic case.

TABLE IX  
SOLUTION FOR THE STOCHASTIC CASE

Strategic Offer					
	S1	S2	S3	S4	Total
E. Production [MWh]	3088.3	0	3008.6	3326.7	9423.6
E. Profit [€]	16354	0	15657	16615	48626
Marginal-Cost Offer					
	S1	S2	S3	S4	Total
E. Production [MWh]	2331.4	0	2437.6	2715.2	7484.2
E. Profit [€]	6430.4	0	6430.4	7281.1	20141.9

TABLE X  
LOCATION AND TYPE OF UNITS FOR THE ONE AREA RTS CASE

Strategic units			Other units		
<i>i</i>	Type	Bus	<i>j</i>	Type	Bus
1,2	76	1	1,2	20	1
3	76	2	3,4	20	2
4	100	7	5	76	2
5	197	13	6,7	100	7
6	155	15	8,9	197	13
7	400	21	10-14	12	15
8	155	23	15	155	16
			16	400	18
			17-22	50	22
			23	155	23
			24	350	23

offer and the strategic offer cases are depicted in Figs. 5 and 6, respectively.

In this particular case study, congestion is not beneficial for the strategic producer that makes its offers to avoid it.

## V. COMPUTATIONAL ISSUES

Model (17)–(60) has been solved using CPLEX 11.0.1 [5] under GAMS [22] on a Sun Fire X4600 M2 with four processors at 2.60 GHz and 32 GB of RAM.

Table XII shows the computational time required for solving the problem pertaining to each case study. The required time increases with the size of the problem and with the congestion of any line. Note also that the computational time increases dramatically if scenarios are considered (case Section III-D), which is the main drawback of model (61)–(77). Moreover, these times

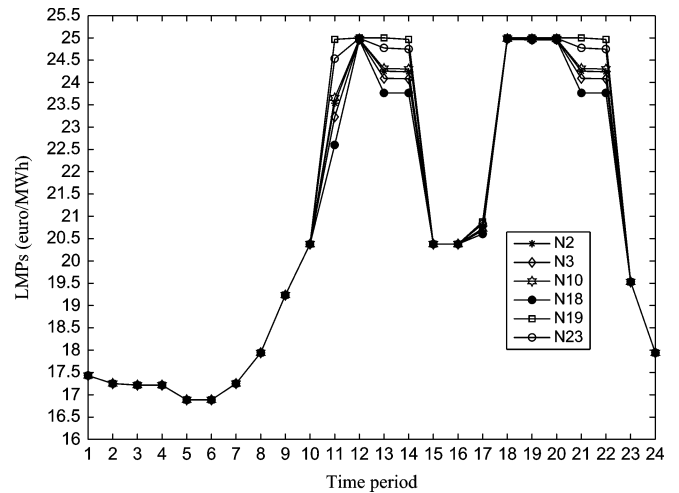


Fig. 5. Some LMPs offering at marginal cost for the RTS case.

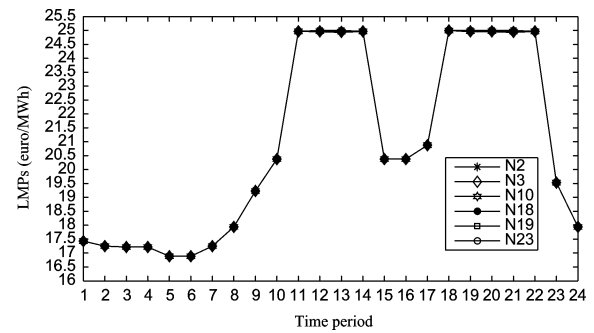


Fig. 6. Some LMPs with strategic offer for the RTS case.

TABLE XI  
SOLUTION FOR THE ONE AREA RTS CASE

Strategic Offer								
	S1	S2	S3	S4	S5	S6	S7	Total
Production [MWh]	36.48	18.24	8.9416	47.28	31.01	96	37.2	281.13
Profit [€]	27929	13965	3287.3	47959	38689	148120	38983	318932.3
Marginal-Cost Offer								
	S1	S2	S3	S4	S5	S6	S7	Total
Production [MWh]	36.48	18.24	10.6	47.28	37.2	96	37.2	283
Profit [€]	27245	13625	3449.3	47296	37815	145170	38773	313373

TABLE XII  
CPU TIME

	III-B	III-C	III-D	IV
CPU time [s]	2.91	5.82	204.77	449.33

are significantly dependent on the values of the linearization constants  $M$ , whose appropriate selection is important and requires generally a nontrivial trial-and-error process (see [23] for an alternative branch-and-bound method that avoids the use of constants  $M$ ).

It should be noted that for the particular problem addressed, the tune-up of these constants was achieved without particular numerical trouble. Note that constants  $M^P$ ,  $M^C$ ,  $M^\delta$ ,  $M^{\mu P}$ ,  $M^{\nu C}$ , and  $M^{\delta\delta}$  are related to power demands/generations, power flows, angles, demand/generation power bounds, line capacities, and angle bounds, respectively.

The values for constants related to primal variables are selected easily based on the physical nature of these variables.

However, since the selection of appropriate values for the constants related to dual variables may pose higher difficulty, the heuristic below has been developed and successfully implemented:

- 1) Solve a (single-level) market clearing problem considering that all the producers offer at marginal cost.
- 2) Obtain the marginal value of each relevant constraint, i.e., its relevant *dual variable value*.
- 3) Calculate the value of the each relevant constant as  $M = (\text{dual variable value} + 1) \times 100$ .

## VI. CONCLUSIONS

This paper provides a procedure to derive strategic offers for a power producer selling electric energy in a pool. LMPs are not input data but endogenously generated. Uncertainty on demand bids and offering strategies of rival producers is taken into account. The resulting problem is mixed-integer and linear and can be solved using available branch and cut solvers. Numerical simulations show the ability of the proposed technique to identify the strategic offers resulting in maximum profit. Moreover, network congestion can be used as a strategic mechanism to further increase profit.

Further research is needed to model strategic competition within the lower level problem, i.e., within the market clearing mechanism.

## APPENDIX A KKT CONDITIONS

The KKT conditions for the lower-level problem (5)–(12) are in (78)–(93) at the bottom of the page.

## APPENDIX B LINEARIZATION

The MPEC model (13)–(16) includes the following nonlinearities:

- 1) the term  $\lambda_{tn} P_{tib}^S$  in the objective function;
- 2) the complementarity conditions (84)–(93).

Nonlinearities in 2) can be linearized using the well-known linear expressions proposed in [4].

To find a linear expression for  $\lambda_{tn} P_{tib}^S$ , we use the strong duality condition and some of the KKT equalities.

The strong duality theorem says that if a problem is convex, the objective functions of the primal and dual problems have the same value at the optimum. Thus

$$\begin{aligned}
 & \sum_{tib} \alpha_{tib}^S P_{tib}^S \\
 &= - \sum_{tjb} \lambda_{tjb}^O P_{tjb}^O + \sum_{tdk} \lambda_{tdk}^D P_{tdk}^D \\
 & \quad - \sum_{tib} \mu_{tib}^{S\max} P_{tib}^{S\max} - \sum_{tjb} \mu_{tjb}^{O\max} P_{tjb}^{O\max} \\
 & \quad - \sum_{tdk} \mu_{tdk}^{D\max} P_{tdk}^{D\max} - \sum_{tn(m \in \Theta_n)} \nu_{tnm}^{\min} C_{nm}^{\max} \\
 & \quad - \sum_{tn(m \in \Theta_n)} \nu_{tnm}^{\max} C_{nm}^{\max} \\
 & \quad - \sum_{tn} \xi_{tn}^{\max} \pi - \sum_{tn} \xi_{tn}^{\min} \pi. \tag{94}
 \end{aligned}$$

$$\alpha_{tib}^S - \lambda_{tn} + \mu_{tib}^{S\max} - \mu_{tib}^{S\min} = 0, \quad \forall t, \forall i \in \Psi_n, \forall b \tag{78}$$

$$\lambda_{tjb}^O - \lambda_{tn} + \mu_{tjb}^{O\max} - \mu_{tjb}^{O\min} = 0, \quad \forall t, \forall j \in \Psi_n, \forall b \tag{79}$$

$$-\lambda_{tdk}^D + \lambda_{tn} + \mu_{tdk}^{D\max} - \mu_{tdk}^{D\min} = 0, \quad \forall t, \forall d \in \Psi_n, \forall k \tag{80}$$

$$\begin{aligned}
 & \sum_{m \in \Theta_n} B_{nm}(\lambda_{tn} - \lambda_{tm}) + \sum_{m \in \Theta_n} B_{nm}(\nu_{tnm}^{\max} - \nu_{tmn}^{\max}) \\
 & \quad + \sum_{m \in \Theta_n} B_{nm}(\nu_{tmn}^{\min} - \nu_{tnm}^{\min}) + \xi_{tn}^{\max} - \xi_{tn}^{\min} + (\xi_t^1)_{n=1} = 0, \quad \forall t, \forall n \tag{81}
 \end{aligned}$$

$$\sum_{ib} P_{t(i \in \Psi_n),b}^S + \sum_{jb} P_{t(j \in \Psi_n),b}^O - \sum_{dk} P_{t(d \in \Psi_n),k}^D = \sum_{m \in \Theta_n} B_{nm}(\delta_{tn} - \delta_{tm}), \quad \forall t, \forall n \tag{82}$$

$$\delta_{tn} = 0, \quad \forall t, n = 1 \tag{83}$$

$$0 \leq P_{tib}^S \perp \mu_{tib}^{S\min} \geq 0, \quad \forall t, \forall i, \forall b \tag{84}$$

$$0 \leq P_{tjb}^O \perp \mu_{tjb}^{O\min} \geq 0, \quad \forall t, \forall j, \forall b \tag{85}$$

$$0 \leq P_{tdk}^D \perp \mu_{tdk}^{D\min} \geq 0, \quad \forall t, \forall d, \forall k \tag{86}$$

$$0 \leq P_{tib}^{S\max} - P_{tib}^S \perp \mu_{tib}^{S\max} \geq 0, \quad \forall t, \forall i, \forall b \tag{87}$$

$$0 \leq P_{tjb}^{O\max} - P_{tjb}^O \perp \mu_{tjb}^{O\max} \geq 0, \quad \forall t, \forall j, \forall b \tag{88}$$

$$0 \leq P_{tdk}^{D\max} - P_{tdk}^D \perp \mu_{tdk}^{D\max} \geq 0, \quad \forall t, \forall d, \forall k \tag{89}$$

$$0 \leq C_{nm}^{\max} + B_{nm}(\delta_{tn} - \delta_{tm}) \perp \nu_{tnm}^{\min} \geq 0, \quad \forall t, \forall n, \forall m \in \Theta_n \tag{90}$$

$$0 \leq C_{nm}^{\max} - B_{nm}(\delta_{tn} - \delta_{tm}) \perp \nu_{tnm}^{\max} \geq 0, \quad \forall t, \forall n, \forall m \in \Theta_n \tag{91}$$

$$0 \leq \pi - \delta_{tn} \perp \xi_{tn}^{\max} \geq 0, \quad \forall t, \forall n \tag{92}$$

$$0 \leq \pi + \delta_{tn} \perp \xi_{tn}^{\min} \geq 0, \quad \forall t, \forall n \tag{93}$$

We can use some of the KKT equalities to obtain a relation between  $\alpha_{tib}^S P_{tib}^S$  and  $\lambda_{tn} P_{tib}^S$ . From (78)

$$\alpha_{tib}^S = \lambda_{tn} - \mu_{tib}^{S_{\max}} + \mu_{tib}^{S_{\min}}, \quad \forall t, \forall i \in \Psi_n, \forall b$$

thus

$$\sum_{tib} \alpha_{tib}^S P_{tib}^S = \sum_{t(i \in \Psi_n)b} (\lambda_{tn} - \mu_{tib}^{S_{\max}} + \mu_{tib}^{S_{\min}}) P_{tib}^S$$

and

$$\sum_{tib} \alpha_{tib}^S P_{tib}^S = \sum_{t(i \in \Psi_n)b} \lambda_{tn} P_{tib}^S - \sum_{tib} \mu_{tib}^{S_{\max}} P_{tib}^S + \sum_{tib} \mu_{tib}^{S_{\min}} P_{tib}^S. \quad (95)$$

From (84)

$$\mu_{tib}^{S_{\min}} P_{tib}^S = 0, \quad \forall t, \forall i, \forall b \Rightarrow \sum_{tib} \mu_{tib}^{S_{\min}} P_{tib}^S = 0. \quad (96)$$

From (87)

$$\mu_{tib}^{S_{\max}} P_{tib}^S = \mu_{tib}^{S_{\max}} P_{tib}^{S_{\max}}, \quad \forall t, \forall i, \forall b$$

thus

$$\sum_{tib} \mu_{tib}^{S_{\max}} P_{tib}^S = \sum_{tib} \mu_{tib}^{S_{\max}} P_{tib}^{S_{\max}}. \quad (97)$$

Substituting (96) and (97) in (95) renders

$$\sum_{tib} \alpha_{tib}^S P_{tib}^S = \sum_{t(i \in \Psi_n)b} \lambda_{tn} P_{tib}^S - \sum_{tib} \mu_{tib}^{S_{\max}} P_{tib}^{S_{\max}}. \quad (98)$$

Substituting (98) in (94) results in

$$\begin{aligned} X &= \sum_{t(i \in \Psi_n)b} \lambda_{tn} P_{tib}^S \\ &= - \sum_{tjb} \lambda_{tjb}^O P_{tjb}^O + \sum_{tdh} \lambda_{tdh}^D P_{tdh}^D \\ &\quad - \sum_{tjb} \mu_{tjb}^{O_{\max}} P_{tjb}^{O_{\max}} - \sum_{tdk} \mu_{tdk}^{D_{\max}} P_{tdk}^{D_{\max}} \\ &\quad - \sum_{tn(m \in \Theta_n)} \nu_{tnm}^{\min} C_{nm}^{\max} \\ &\quad - \sum_{tn(m \in \Theta_n)} \nu_{tnm}^{\max} C_{nm}^{\max} - \sum_{tn} \xi_{tn}^{\max} \pi \\ &\quad - \sum_{tn} \xi_{tn}^{\min} \pi. \end{aligned} \quad (99)$$

The last equation allows us calculating  $\sum \lambda_{tn} P_{tib}^S$  as a linear expression, which renders linear the objective function of the upper-level problem (1)–(4).

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