Technical University of Denmark

31761 Renewables in electricity markets

Assignment 1

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Group 25

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Table of contributions

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Contribution to model development	25%	25%	25%	25%
Contribution to programming	25%	25%	25%	25%
Contribution to the analysis of numerical results	25%	25%	25%	25%
Contribution to writing the report	25%	25%	25%	25%

Nomenclature

Description of parameters:

- $\bar{P}^{G}_{g,t}$: Maximum power output in MW of generator g in hour t
- $\bar{P}^{D}_{d,t}$: Demand in MW at load d in hour t
- U_d: Bid price of each demand d
- C_g: offer price for generator g
- R^{U}_{g}: Maximum ramp up in MW/h for generator g
- R^{D}_{g}: Maximum ramp down in MW/h for generator g
- P^{ini}_{g}: Initial power output of generating unit g in MW
- SOC^{BS}_{t}: State of charge(SOC) for the battery storage(BS) in hour t
- Cap^{BS}: BS capacity
- Ch_rate^{BS}: Maximum charging of BS in one hour
- Dis_rate^{BS}: Maximum discharging of BS in one hour
- η_c: charging efficiency of BS
- η_d: discharging efficiency of BS
- α: initial SOC of BS
- $B_{n,m}$: Susceptance of the transmission line between node n and m in p.u.
- $F_{n,m}$: Power capacity of transmission line between node n and m in MW
- $ATC_{a,b}$: Available Transfer Capacity from zone a to b
- PS^{9}_{g}: production schedule in hour nine for generator g in MW
- DA^{9}: Day-ahead price in hour nine in \$
- $Curt$: Load curtailment cost in \$/MWh
- R^{+}_{g}: Maximum upregulation reserve capacity in MW of generator unit g
- R^{-}_{g}: Maximum downregulation reserve capacity in MW of generator unit g
- C^{U}_{g}: Upward reserve capacity cost of generating unit g
- C^{D}_{g}: Downward reserve capacity cost of generating unit g

Decision variables used throughout the assignment:

- $p_{g,t}^{G} \geq 0$: Power generation in MW from generator g in hour t
- $p_{d,t}^{D} \geq 0$: Power demand in MW in hour t from load d
- $Ch_{t}^{BS} \geq 0$: Charge of battery storage(BS) in MW in hour t
- $Dis_{t}^{BS} \geq 0$: Discharge of BS in MW in hour t
- $SOC_{t}^{BS} \geq 0$: State of charge of BS in MW in hour t
- $\theta_{n,t}$: Phase angle of the voltage phasor in radians at node n in hour t
- $f_{a,b,t}$: Power flow in MW between zone a and b in hour t
- $p_{g}^{Gu} \geq 0$: Upward balancing service from generator g in MW
- $p_{g}^{Gd} \geq 0$: Downward balancing service from generator g in MW
- $p^{Dc} \geq 0$: Load curtailment of system demand
- $r_{g,t}^{U}$: Upward reserve capacity in MW to be provided by generator g in hour t
- $r_{g,t}^{D}$: Downward reserve capacity in MW to be provided by generator g in hour t

Step 1

In this assignment the electric power network chosen was case study 1: *IEEE 24-bus reliability test system* [1]. Additionally six wind farms are added to the system as well as a battery storage system. The wind farm data was taken from [2] where data from one scenario in zone 1 to 6 in the first 24 hours was used to create the simulated power generation of 6 wind farms, see the appendix. In this section a market-clearing optimization model for a copper-plate version of the 24-bus system is developed to be cleared over a time period of 24 hours.

The mathematical model

Objective function

$$\max_{p^G_{g,t}, p^D_{d,t}} \sum_{d,t} U_d \cdot p^D_{d,t} - \sum_{g,t} C_g \cdot p^G_{g,t} \tag{1}$$

Subject to the following constraints:

$$0 \leq p^G_{g,t} \leq \bar{P}^G_{g,t} \qquad \forall g, \forall t \tag{2}$$

$$0 \leq p^D_{d,t} \leq \bar{P}^D_{d,t} \qquad \forall d, \forall t \tag{3}$$

$$\sum_d p^D_{d,t} + Ch^{BS}_t = \sum_g p^G_{g,t} + Dis^{BS}_t \qquad \forall t \tag{4}$$

$$p^G_{g,t} - p^G_{g,t-1} \leq R^U_g \qquad \forall g \in 1,..,12, \forall t \in 2,..,T \tag{5}$$

$$p^G_{g,t} - p^G_{g,t-1} \geq R^D_g \qquad \forall g \in 1,..,12, \forall t \in 2,..,T \tag{6}$$

$$p^G_{g,t} - P^{ini}_g \leq R^U_g \qquad \forall g \in 1,..,12, t = 1 \tag{7}$$

$$p^G_{g,t} - P^{ini}_g \geq R^D_g \qquad \forall g \in 1,..,12, t = 1 \tag{8}$$

$$0 \leq SOC^{BS}_t \leq Cap^{BS} \qquad \forall t \tag{9}$$

$$SOC^{BS}_t - SOC^{BS}_{t-1} = Ch^{BS}_t \cdot \eta_c - \frac{1}{\eta_d} Dis^{BS}_t \qquad \forall t \in 2,..,T \tag{10}$$

$$SOC^{BS}_t - \alpha = Ch^{BS}_t \cdot \eta_c - \frac{1}{\eta_d} Dis^{BS}_t \qquad t = 1 \tag{11}$$

$$Ch^{BS}_t \leq Ch\_rate^{BS} \qquad \forall t \tag{12}$$

$$Dis^{BS}_t \leq Dis\_rate^{BS} \qquad \forall t \tag{13}$$

The objective function (1) maximizes social welfare. Constraint 2 ensures that generation is within limits of 0MW and the maximum specified power output of generators. Likewise, constraint 3 ensures that demand at load d is being met within the limit of 0MW and total demand of that load in hour t. Constraint 4 ensures power balance between generation and demand. Constraints 5-8 ensures that ramp up and down limits are obeyed in all hours. This is done by ensuring that the difference in generation between two consecutive hours is below the ramping limit. The battery storage(BS) system is implemented using the five constraints 9-13. State of charge is in all hours between zero and the battery capacity. The two constraints 10-11 ensures the battery charge/discharge flow while considering charge and discharge efficiencies. Initially there is $\alpha = 0$MW in the BS. Lastly, constraint 12-13 ensures that charge and discharge in hour t is restricted to a maximum within each hour.

Choosing relevant bid price of elastic demand

The bid price of demand is not given, however comparatively high prices should be used such that the majority of demands will be supplied. In order to decide on relevant prices, a supply curve for the six wind farms and 12 conventional generators was made. This was done assuming an offer price of 0$/MWh for the wind farm units and the mean power supply of the 24 hour wind data used. Similarly, for the conventional generators the given day-ahead offer price and maximum power output of the units were used. The demand curve was built using hour 18 with a high system demand of 2650.5MW and the individual load's fractions of the system demand. Bid prices were chosen to begin in the same range as the most expensive generator and end a bit higher than the supply price (see figure 1). Thus the majority of demands will be supplied in all hours.

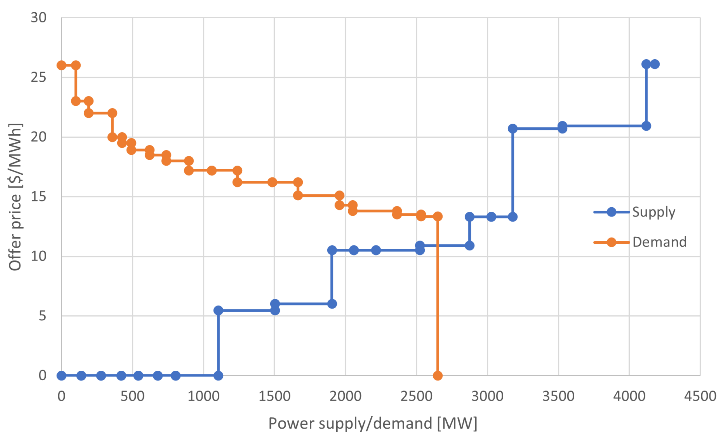


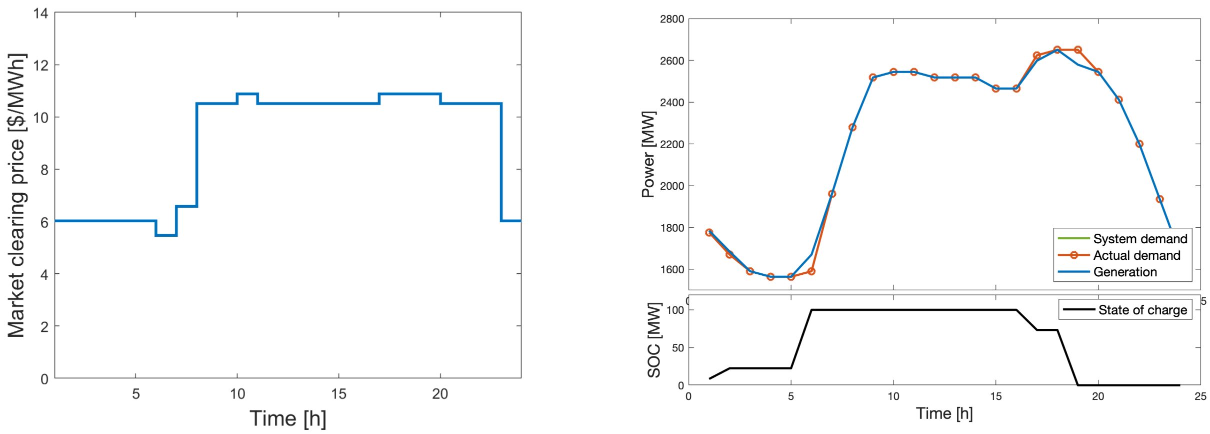
Figure 1: Supply and demand curves

Battery implementation

The battery used for investigation was chosen to have a capacity of 100MW based on the battery storage found at Hornsdale Power Reserve in Australia [3]. The battery was chosen in order to test the system, when the capacitance of the battery is similar in size to the maximum power output of the generators. Furthermore Hornsdale Power Reserve had a large amount of available data. The battery has a charge rate of 71.1MW and discharge rate of 79.9MW [4]. The efficiency is estimated to be 97% since the reserve consists of lithium ion batteries, that have an efficiency above 95% [5]. The battery is placed at node 21 because the transmission lines will not be constrained and there is a wind farm located at that node as well. For the model it is assumed that the battery does not make any offers or bids, but purely functions as a storage option. This also means that the battery does not have a direct influence on the objective function.

Results

When using a uniform pricing scheme and solving the copper plate model for the system with battery, the resulting hourly market clearing price can be seen figure 2a. This resulted in a total social welfare of **708,042$**. From figure 2a, where hour 1 is the hour between midnight and 1am, it can be seen that the market clearing price is lower during the night than during the day. The price peaks in hour 10 and from hour 17 to 19. This also where the demand is at its highest and thus more generators are needed. Figure 2b shows how generation correlates to the demand as well as the state of charge of the battery. Thus it can be seen from figure 2b that the battery is charged during the morning and discharged during the evening, when demand is at its highest.



(a) Market clearing price

(b) System demand and generation plus BS SOC

Figure 2: Relationship between market clearing price, demand, generation and battery SOC

Table 1: Total profit of every asset, where generator 1 to 12 are conventional generators and 13-18 are wind farms

Generator	1	2	3	4	5	6	7	8	9
Profit [$]	0	0	0	0	0	2290	2290	277,460	328,720
Generator	10	11	12	13	14	15	16	17	18
Profit [$]	640,380	1530	-1950	308,670	299,720	300,510	256,190	295,480	270,960

Table 1 shows the resulting profits of the 18 generators. Apart from that, the battery had a total profit of **914.0 $**. To calculate this, it was assumed that the cost of charging at a certain hour is equal to the market clearing price at that time. Likewise, the revenue is calculated by assuming that the power discharged is sold at the market clearing price of that hour. It is interesting to note that all generators using renewable sources have a high profit. This is seen for generator 10, that has the largest profit, as well as generators 13 to 18, which are all wind farms. This is also expected as generator 10 and the wind farms have a cost of 0, and generator 10 has a larger maximum power output than the wind farms (300 MW compared to 200 MW) and thus stand to have a higher profit. Only generator 8 and 9 are within the same profit range as the renewable generators. These are also the two generators with the lowest day-ahead offer price (6.02 $ and 5.47 $ respectively) and the highest maximum power output. The battery is making some profit, however not a lot in comparison to other generators. Comparing the battery to generator 11 there is only a difference of 600 $ and thus in order to get a better estimate of the potential profit of the battery an investigation for a longer period of time, for example a year or decade, should be done.

Generator 12 has a negative profit of -1950 $ since it cannot ramp down fast enough in the beginning of the day. Looking at the data provided it can be seen that the generator starts with an initial power output of 280 MW and can only ramp down with 240 MW/h. Thus it has to generate 40 MW at hour 2 (2 hours after midnight), where the market clearing price is lower than its production cost.

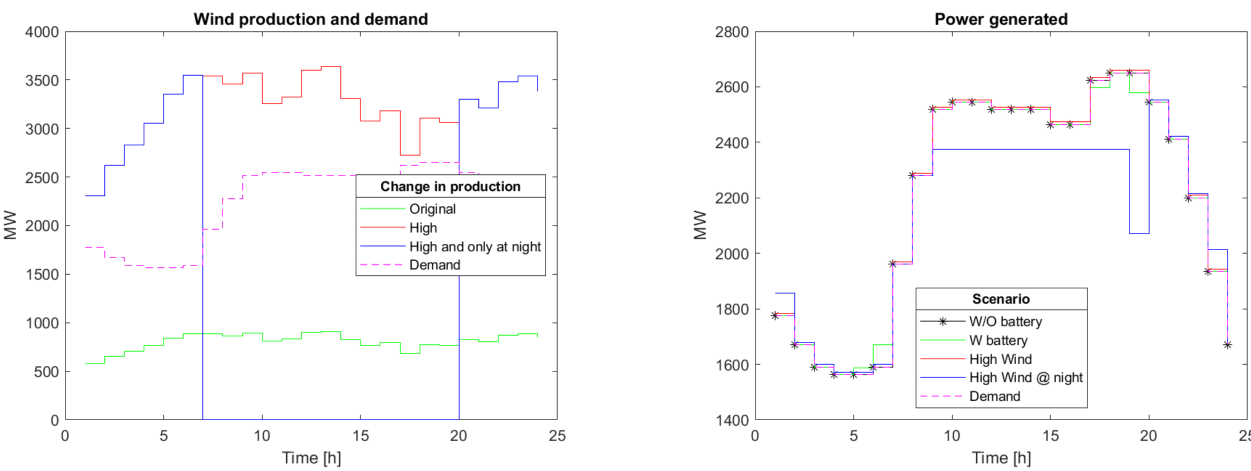
Table 2: Total utility of each load

Load	1	2	3	4	5	6	7	8	9
Profit [$]	33507	24597	42252	14693	13468	24340	21383	27575	25459
Load	10	11	12	13	14	15	16	17	-
Profit [$]	28381	33908	24793	34027	9251	27840	14215	9639	-

From table 2 it can be seen that node 1, 11 and 13 has the highest utility and node 14 and 17 has the lowest.

Sensitivity analysis of battery implementation

Four different scenarios are created in order to analyze the impact of the battery. The first scenario uses the originally assumed wind conditions that generates the power output as seen in the appendix and no battery (W/O battery), the second uses the battery mentioned above and originally assumed wind conditions (W battery). The third scenario uses the battery and assumes a high wind generation that is four times the generation under normal conditions (High Wind). The fourth scenario uses the battery and high wind generation during the night and no wind generation during the day(High Wind @ night). The resulting power generation from the wind farms can be seen in figure 3a and the total power generation, not including the contribution of the battery, can be seen in figure 3b.

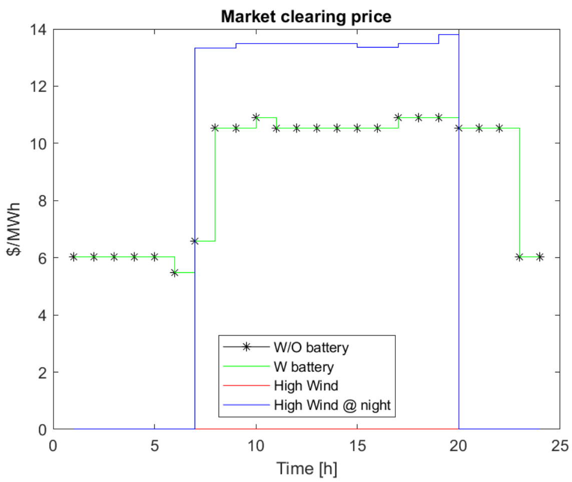


(a) Wind power generation of the different scenarios compared to demand

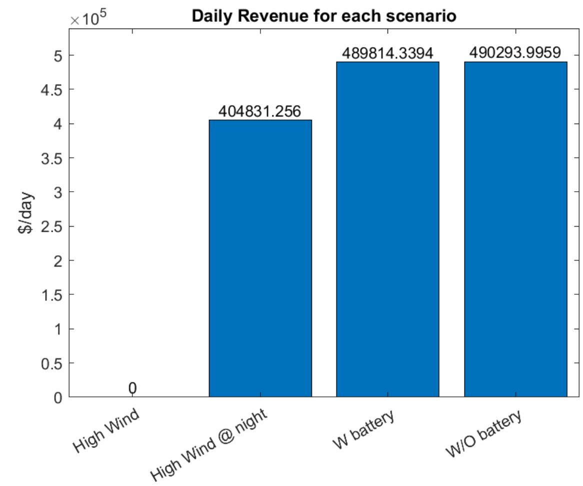
(b) Total power generation of the different scenarios compared to demand

Figure 3: Power generated throughout the day

From figure 4a and 3b it becomes evident that the two scenarios assuming the wind conditions given in the appendix have very similar graphs for the power production and market clearing. This indicates that under originally assumed wind conditions the battery does not have a large affect on the system. A possible explanation for this is, that the power generated from wind never exceeds the demand. Thus, the power that could be stored in the battery would come with a cost from a conventional generator. This is not more optimal for the system than using the conventional generators and their ramping capabilities on their own. From figure 3b it is also clear that in the High Wind @ night scenario, power generation from the generators is lower than demand, meaning that the remaining power is supplied from the battery, which was charged at night with cheap wind power.



(a) Market clearing price of the different scenarios



(b) Total power cost of the day for each scenario

Figure 4: Different scenarios effect on the cost of power

Figure 4a shows how the different scenarios affect the market clearing price and the total daily revenue of all generators. As expected the scenario with high wind has a market clearing price and daily revenue of zero, since the wind power generation is always higher than the demand. It becomes interesting when analyzing the scenario with high wind production during the night and no production during the day. Here the market clearing price becomes relatively high during the day, since all of the demand must be covered by conventional generators. As seen in figure 4b the sum of the revenue for all generators is lower in higher wind scenarios where wind power generation exceeds demand as the excess energy can be stored in a battery for later use. This lower revenue would seem undesirable for the generators. However, by lowering the revenue, the social welfare will increase. Thus, society could stand to benefit from the implementation of a battery in this case. In this model the cost of charging the battery plus the gain of discharging the battery has not been included, which could have affected the results. Also the battery could have been implemented as part of a generator facility where a potential gain in revenue for the generators could be investigated.
As can be seen in the scenarios assuming normal wind conditions, the battery has relatively small influence on the revenue. However, this might change if the battery was modeled more in detail and had the opportunity to bid different prices when it acts as a load and as a generator.
The battery can also be profitable in other ways. Today, the Hornsdale Power Reserve earns the majority of its revenue from offering frequency control ancillary services [6]. A battery power reserve can also reduce the cost of these services, as seen in Australia, where the cost has been reduced with 90% [7].

Step 2

In this step, the copper plate model from step 1 is extended to include network limits. First, by modeling the system with a nodal structure, where every node represents a bus. Afterwards with a zonal structure, where every zone contains a number of buses.

Nodal system implementation

When the setup of the system is nodal, each node is connected to some of the other nodes by transmission lines. Therefore, power cannot flow freely between buses and new constraints are needed in the model, so that power flow limits are not exceeded. The power flow between two nodes n and m is modeled by $B_{n,m}\left(\theta_{n,t}-\theta_{m,t}\right)$ where $B_{n,m}$ is the susceptance of the transmission line between the nodes and $\theta_{n,t}$ the phase angle of the voltage phasor at node n in hour t.
In order to include a nodal structure, multiple changes are made to the optimization problem from step 1. Firstly, the phase angle of each node is included as a decision variable to the objective function and the battery storage system is now indexed by both s, for the nodal position, and t:

$$\max_{p^G_{g,t},p^D_{d,t},\theta_{n,t}} \sum_{d,t} U_d \cdot p^D_{d,t} - \sum_{g,t} C_g \cdot p^G_{g,t} \tag{14}$$

The balance constraint (4) from step 1 now accounts for all nodes in all hours. In the equation, Ψ_n refers to the set of loads, generators and batteries located at bus n, and Ω_n refers to the nodes m connected to n with transmission lines.

$$\sum_{d\in\Psi_n} p^D_{d,t} + \sum_{m\in\Omega_n} B_{n,m}\left(\theta_{n,t}-\theta_{m,t}\right) + \sum_{s\in\Psi_n} Ch^{BS}_{s,t} = \sum_{g\in\Psi_n} p^G_{g,t} + \sum_{s\in\Psi_n} Dis^{BS}_{s,t} \qquad \forall n, \forall t \tag{15}$$

Constraint 15 includes the term for modeling power flow between nodes, implying that if demand at one node is not supplied by a generator placed at the given node, power can be transported from neighboring nodes. Similarly, excess power can be transported away from node n to connected nodes. Furthermore, the following constraints are added to the optimization problem from step 1:

$$-F_{n,m} \leq B_{n,m}\left(\theta_{n,t}-\theta_{m,t}\right) \leq F_{n,m} \qquad \forall n, \forall m, \forall t \tag{16}$$

$$-\pi \leq \theta_{n,t} \leq \pi \qquad \forall n, \forall t \tag{17}$$

$$\theta_{1,t} = 0 \qquad \forall t \tag{18}$$

Equation 16 ensures that the power capacity of the transmission line between node n and m is not exceeded in every given hour. Equation 17 restricts phase angles to only acquire values between $-\pi$ and π and, lastly, equation 18 sets a reference phase angle of 0 at node 1 in all hours.

In order to implement the problem in Julia, data for transmission line capacities is taken from the 24-bus system [1]. The susceptance is set to 500 p.u. for all transmission lines. This scenario is denoted as a non-restricted scenario where no transmission lines should be congested. Subsequent to the non-restricted scenario, a sensitivity scenario was created where the capacity of transmission lines connecting the nodes (15,21), (14,16) and (13,23) is restricted to 400 MW, 250 MW and 250 MW, respectively, as suggested in C. Ordoudis et al. [1]. This scenario is denoted the restricted scenario.

Solving the non-restricted nodal optimization problem yields identical results as in the W-battery scenario from step 1 with a social welfare of $708,042$\$. The black line on figure 5 show the hourly nodal market clearing price in the non-restricted scenario. The nodal prices are equal for all nodes in each respective hour and the hourly nodal market price in one hour is equal to the hourly market price in step 1. The results thus indicate that no transmission lines are congested, as there would otherwise be different nodal prices.

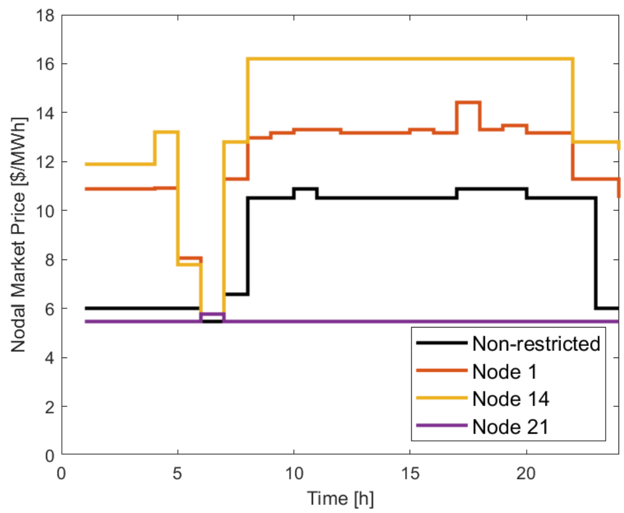
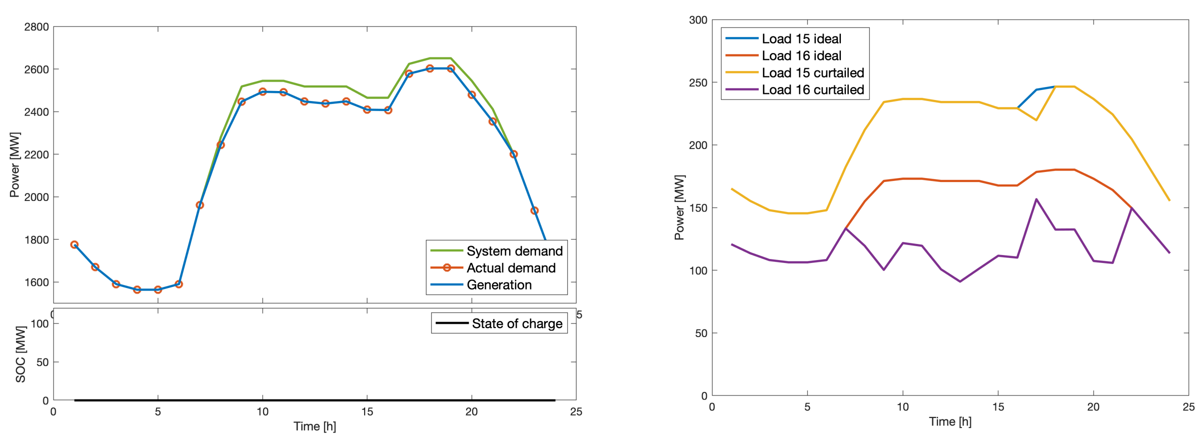


Figure 5: Extract of hourly nodal market prices for the non-restricted (black line) and restricted scenario (yellow, orange, purple line)

In the restricted scenario the social welfare decreases to 674, 579$ and nodal market prices differ significantly from each other, though, generally higher compared to the non-restricted scenario. This is evident in figure 5 which depicts hourly nodal prices for nodes 1, 14 and 21. The market price at bus 21 is only around 5.5 $/MWh for all hours which is much cheaper relative the non-restricted scenario. This can be explained by the fact that generator 9 is positioned at node 21 and offers large power generation at a cheap price of 5.47. Figure 6a shows how the restricted scenario has a decreased system generation and load during daylight hours and the battery is not utilized. This is a non-ideal scenario where load shedding is imposed for load 15 and 16, shown in figure 6b. For load 15 there are only two hours where the curtailed load is smaller than the ideal load whereas load 16 undergoes load shedding in all daytime hours. Given that load 15 and 16 are located at bus 13 and 14 where two transmission lines were restricted, it is logical that the loads at these nodes are curtailed.



(a) System demand and generation for the restricted scenario

(b) Hourly curtailed demand profile for load 15 and 16 together with the ideal demand for this load

Figure 6: Demand and generation in the restricted scenario

Zonal system implementation

The zonal network optimization problem is similar to the nodal problem. In the zonal setup, the physical parameters of the power network are not considered. Instead the given network is divided into multiple zones, each with a set of nodes. The objective function of the zonal market clearing optimization is:

$$\max_{p^G_{g,t}, p^D_{d,t}, f_{a,b,t}} \sum_{d,t} U_d \cdot p^D_{d,t} - \sum_{g,t} C_g \cdot p^G_{g,t} \tag{19}$$

Instead of modeling power flow using physical parameters, the flow between zone a and b at hour t is now modeled by the decision variable $f_{a,b,t}$. The balance constraint from the original problem is now:

$$\sum_{d\in\Psi_a} p^D_{d,t} + \sum_{b\in\Omega_a} f_{a,b,t} + \sum_{s\in\Psi_a} Ch^{BS}_{s,t} = \sum_{g\in\Psi_a} p^G_{g,t} + \sum_{s\in\Psi_a} Dis^{BS}_{s,t} \qquad \forall a, \forall t \tag{20}$$

Additionally, the following constraints are added to the optimization problem:

$$-ATC_{b,a} \leq f_{a,b,t} \leq ATC_{a,b} \qquad \forall a, \forall b \in \Omega_a, \forall t \tag{21}$$

$$f_{a,b,t} = -f_{b,a,t} \qquad \forall a, \forall b, \forall t \tag{22}$$

Equation 21 restricts the power trade between two zones to be between the ATCs given by the different zones. Here ATCs can be asymmetrical meaning that the ATC given by zone a to zone b is not necessarily the same as the ATC given by zone b to a. Equation 22 ensures that the power trade between two zones is identical in magnitude and opposite in sign.

The system was divided in three zones as shown in figure 7. As it can be seen from the figure there are two conventional generators in zone 1 while there are four conventional generators in the other zones. Furthermore there are two wind farms in each zone.

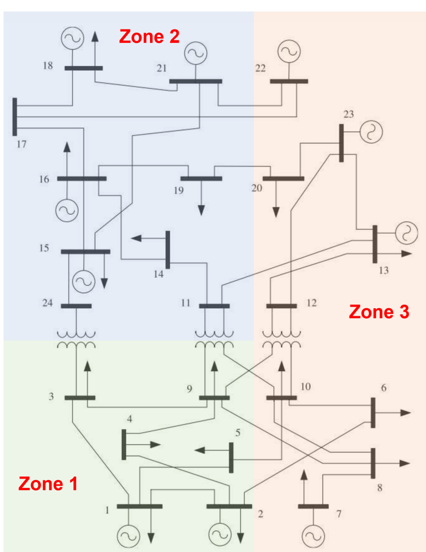


Figure 7: Zonal system setup

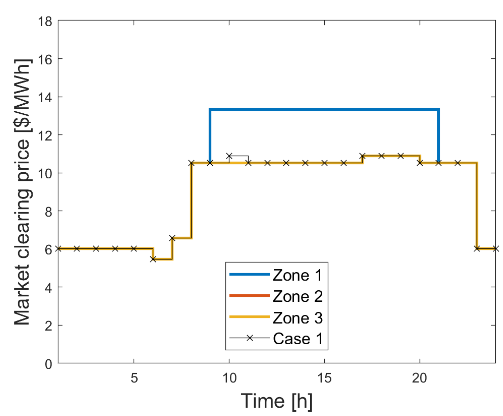


Figure 8: Case 2 - ATCs scaled to 15%

To do a sensitivity analysis for different values of ATCs, four case scenarios where created. Case 1 is when the ATCs between the three zones are set to the maximum transmission capacity. This leads to no congestions and thereby equal market prices in all three zones identical to the price of the copper plate model in step 1. In the visualization of the following three case studies, case 1 is therefore plotted as a single line for comparison. To create the next case study, the ATCs were scaled down symmetrically by a factor. When scaling down by 50% the results are equal to case 1. This suggest that the self sufficiency within each zone is good, since it does not change any of the individual zonal market prices. Case 2 was created by scaling the ATCs to 15% of the transmission capacity, as this gives new results. As shown in figure 8 the market price of zone 1 increases in quite a few hours. It increases because zone 1 has the fewest generators. Thus when transmission from the other zones are restricted a lot, more expensive generation in the zone 1 is needed to meet demand. Case 3 was created by scaling all ATCs to just 1% of the transmission capacity. The results are shown in figure 9 where it is seen that the price in zone 1 is now significantly larger in all hours. However both zone 2 and 3 experience hours with lower market prices than case 1. This is because almost no power is exported to zone 1, and the self sufficiency within zone 2 and 3 are good and have cheap power generation. In zone 3 there is even an hour with a market price of zero, because the renewable generation is large enough to meet demand.

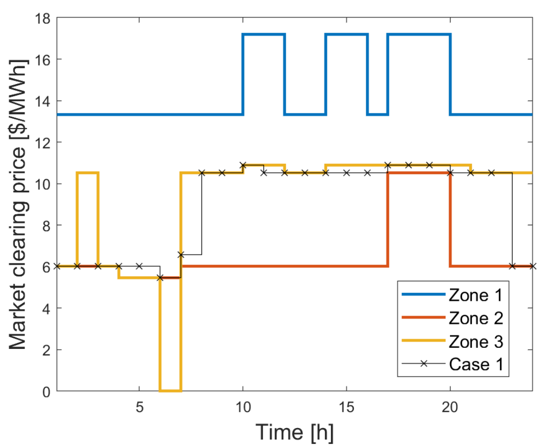


Figure 9: Case 3 - All ATCs scaled to 1%

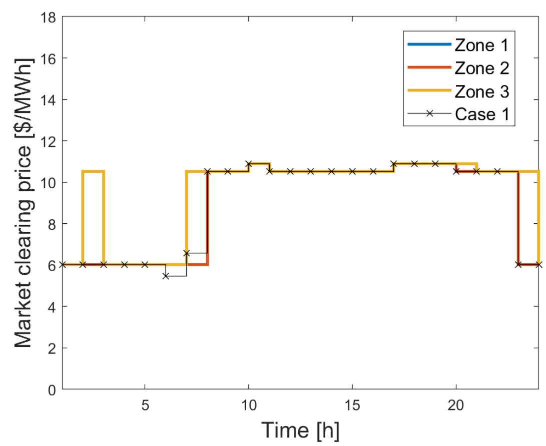


Figure 10: Case 4 - ATCs scaled to 1% except full ATC capacity from zone 2 and 3 to zone 1

Case 4 was made by restricting the ATCs to 1% of its max capacity except the ATCs from zone 2 and 3 to zone 1. Instead, these two allow maximum transmission capacity to zone 1. Thus the ATCs are asymmetric in case 4. The result is shown in figure 10 where the blue line representing market price of zone 1 is now identical to zone 2 and very similar to results of case 1. Thus in order to maximize the social welfare, it is more optimal for the system to have enough power transported to zone 1 compared to having hours with cheaper prices in zone 2 and 3 as it was seen in case 3. The results of social welfare for all four cases are shown in table 3, where the percentages refer to the relative change compared to case 1. As expected the social welfare is most optimal in case 1, and becomes worse in case 2 and 3. With the relaxation on two transmission lines in case 4, the resulting social welfare is a clear improvement of case 3.

Table 3: Resulting social welfare in the four cases of zonal system setup

	Case 1	Case 2	Case 3	Case 4
Social welfare [$]	708,042	706,449 (-0.08%)	680,649 (-3.9%)	707,471 (-0.2%)

Implication of nodal vs zonal framework

To look at the implications of nodal vs zonal frameworks to the market participants, the results of the market prices in the different scenarios can be compared. The nodal prices are much more volatile, as these were seen to change a lot with a restriction on just three transmission lines. For the zonal setup, the ATCs could all be restricted to 50% without causing any changes in the zonal prices. And even in case 2, where the ATCs were set 15% of the transmission capacity, it was just one zone that had an increase in market price for some hours. Thus for the demands there is a larger security of supply at cheaper costs in a zonal setup compared to the nodal structure. These conclusion are consistent with the fact that social welfare was larger in case 2-4 of the zonal setup compared to the restricted nodal setup. However from the perspective of generators, there will often be more generators competing on the market in the nodal setup, compared to zonal. Many of the generators with expensive bid prices, will simply not be scheduled for any generation in the zonal setup. As the market prices were found to be large at many nodes, when three lines were restricted, it will therefore lead to higher profits for the generators.

Step 3

In this step, the balancing market will be cleared in one specific hour. When modeling the balancing market, network constraints and intra-day market are discarded. Hour number nine was chosen to be modeled, as this has a high system demand of 2518 MW. An unexpected outage of generator 8, scheduled to produce at max capacity of 400MW on the day-ahead market, is assumed. Furthermore, it is modeled such that three of the wind farms produce 10% lower than their day-ahead forecast, while three other wind farms produce 15% more than their day-ahead forecast. The remaining conventional generators are potential balancing service providers.

Mathematical model

Apart from the input parameters used for step 1, the production schedule, PS_g^9, and market clearing price, DA^9, of hour nine is needed. This was simply found by running the code from step 1 and extracting the data from hour nine.

Objective function:

$$\min_{p_g^{Gu}, p_g^{Gd}, p^{Dc}} \sum_{g=1}^{12} (DA^9 + 0.1 \cdot C_g) p_g^{Gu} + Curt \cdot p^{Dc} - (DA^9 - 0.12 \cdot C_g) p_g^{Gd} \tag{23}$$

Subject to the following constraints:

$$\sum_{g=1}^{12} (p^{Dc} + p_g^{Gu} - p_g^{Gd}) = PS_8^9 + \sum_{g=13}^{15} PS_g^9 \cdot 0.1 - \sum_{g=16}^{18} PS_g^9 \cdot 0.15 \tag{24}$$

$$p_g^{Gu} \leq P_g^{max} - PS_g^9 \qquad \forall g \in 1, .., 12 \tag{25}$$

$$p_g^{Gd} \leq PS_g^9 \qquad \forall g \tag{26}$$

$$p^{Dc} \leq P_9^{total} \tag{27}$$

The objective function 23 minimizes balancing cost. It is seen that the conventional generators offer upward balancing service equal to the day-ahead price plus 10% of their production cost, while downward balancing service is at the day-ahead price minus 12% of their production cost. Load curtailment is set to a cost of \$500/MWh. Constraint 24 is the balance constraint where the total balancing need is found by production schedule of generator 8 plus the 10% lower production of three wind farms minus 15% larger production of the last three wind farms. This total imbalance in the system should then be adjusted by the generators and and the load that are capable of providing the balancing service. Constraint 25 ensures that the maximum possible up-regulation from the conventional generators is equal to the difference in max capacity and their day-ahead schedule in hour nine. Constraint 26 ensures that the maximum downward regulation from all generators is equal to their day-ahead production schedule of hour nine. Lastly constraint 27 ensures that the maximum load curtailment is equal to the total system demand of hour nine.

Results

The balancing need in hour nine is an upregulation by 381.6MW. The resulting objective value, which represents the minimum balancing cost, is: **4426.5\$**.
25% of the upregulation is provided by generator 11 and 75% by generator 12. The balancing price(BP), found from the dual variable of constraint 24, is: **11.6\$/MWh**. This is the BP for all upward balancing service providers in one-price balancing settlement. In two-price balancing settlement the wind farms providing more than forecasted will receive the day-ahead price of hour nine (10.52\$/MWh), and those producing less than forecasted will be punished at the BP. The equations for profit calculation of the different generators are shown in (28)-(31) and the results are seen in table 4.

$$Profit_{conv} = DA^9 \cdot PS_g^9 + BP \cdot (p_g^{Gu}) - C_g \cdot (PS_g^9 + p_g^{Gu}) \tag{28}$$

$$Profit_{g8} = DA^9 \cdot 400 - BP \cdot 400 \tag{29}$$

$$Profit_{wind\_less\_gen} = DA^9 \cdot PS^9_{wind\_farm} - BP \cdot 0.1 \cdot PS^9_{wind\_farm} \tag{30}$$

$$Profit_{wind\_more\_gen} = DA^9 \cdot PS^9_{wind\_farm} + DA^9 \cdot 0.15 \cdot PS^9_{wind\_farm} \tag{31}$$

Equation 28 shows how to calculate the profit of the conventional generators as revenue from day-ahead and upward balancing market minus the costs of total production. Equation 29 explains what generator 8 should pay, as it was the primary source of imbalance with 400MW. Lastly equation 30 and 31 represent the profit calculations of the wind power plants producing respectively less and more than forecasted. The results for generators 1-5 are not shown in table 4, as these are not scheduled for any production in both day-ahead and balancing market and thus have zero profit. As expected, the only generator with a negative profit is number 8, where the unexpected outage happened. Generator 11 and 12 are the only conventional generators providing the balancing service, which allows them to earn a profit. Generator 11 was also scheduled for the DA-market, but with a production cost equal to the day-ahead market clearing price which would have resulted in zero profit. However in the balancing market, the BP is larger than production costs of the generator, and thus it yields a positive profit. Generator 12 was not scheduled for any production in the DA-market, but it benefits from the balancing market due to the BP price being larger than its production cost.

Table 4: Total profit of conventional generators and wind farms in a two-price balancing settlement

Generator	D-A schedule [MW]	Balancing schedule [MW]	Production cost [$]	Total profit [$]
G6	155	0	10.52	0
G7	155	0	10.52	0
G8	400	-400	6.02	-432
G9	400	0	5.47	2020
G10	300	0	0	3156
G11	215.9	94.1	10.52	102.5
G12	0	287.5	10.89	206.7
W1	166.4	-10% of D-A	0	1557.2
W2	141.5	-10% of D-A	0	1324.4
W3	153.8	-10% of D-A	0	1439.4
W4	130.9	+15% of D-A	0	1583.7
W5	147.6	+15% of D-A	0	1785.5
W6	151.9	+15% of D-A	0	1838.2

Step 4

In this step, the reserve and day-ahead markets should be cleared. Initially this is done in a sequential manner, following the practice in Europe, thereafter it is cleared in a joint manner following the US practice. The network constraints are disregarded in this step.

European practice

Reserve market

When the reserve and day-ahead markets are cleared sequentially, the reserve market is cleared first and its model consists of the following:

Objective function:

$$\min_{r^U_{g,t}, r^D_{g,t}} \sum_{g=1}^{12} \sum_{t} C^U_g r^U_{g,t} + C^D_g r^D_{g,t} \tag{32}$$

Subject to:

$$\sum_{g=1}^{12} r^U_{g,t} = \sum_{d} (\bar{P}^D_{d,t}) \cdot 0.15 \qquad \forall t \tag{33}$$

$$\sum_{g=1}^{12} r^D_{g,t} = \sum_{d} (\bar{P}^D_{d,t}) \cdot 0.10 \qquad \forall t \tag{34}$$

$$r^U_{g,t} \leq R^+_g \qquad \forall g \in 1,..12, t \tag{35}$$

$$r^D_{g,t} \leq R^-_g \qquad \forall g \in 1,..12, t \tag{36}$$

$$r^U_{g,t} + r^D_{g,t} \leq P^{max}_g \qquad \forall g \in 1,..12, t \tag{37}$$

The objective function in 32 minimizes the cost of up and down regulation reserves and is thus an expression for the total reserve cost. Equation 33 and 34 ensures that the requirements for the size of the reserves are

met in each hour. The requirements are that up and down regulation reserves are 15% and 10% of the system demand, respectively. Equation 35 and 36 make sure that in every hour, the up and down regulation reserve for each generator is not higher than the generator's maximum capacity for this. It is also important that the sum of a generator's up and down regulation capacity, in a specific hour, does not exceed the maximum power output of the generator. This is enforced in equation 37. It should also be noted that the wind farms are not participating in the given reserve market. Therefore it is only generator 1 to 12 that are included in the model.
The results from this model will be noted $r^{U*}_{g,t}$ and $r^{D*}_{g,t}$.

Day-ahead market
When the reserve market is cleared, its results should be included in the day-ahead market. The DA model in this step is almost identical to step 1. The objective function and all constraints from step 1 except equation (2), about power generation, will be included in the new DA model. Equation (2) is replaced by the following:

$$0 \leq p^G_{g,t} \leq \bar{P}^G_{g,t} \qquad \forall g \in 13,..,18,t \tag{38}$$
$$0 \leq p^G_{g,t} \leq \bar{P}^G_{g,t} - r^{U*}_{g,t} \qquad \forall g \in 1,..,12,t \tag{39}$$
$$p^G_{g,t} \geq r^{D*}_{g,t} \qquad \forall g \in 1,..,12,t \tag{40}$$

Equation 38 models the limits of the wind generation. Equation 39 ensures that the up regulation capacity that a generator has sold in the reserve market cannot be sold in the day-ahead market. Likewise equation 40 ensures that a generator will at least produce the amount of power that it has sold as down regulation reserve.

US practice

In the US the reserve and day-ahead markets are one joint market. Therefore, the US model of the markets includes all the constraints from the European model in this step, but where the fixed parameters $r^{U*}_{g,t}$ and $r^{D*}_{g,t}$ are changed to the decision variables $r^U_{g,t}$ and $r^D_{g,t}$ in equation 39 and 40. The objective function is changed to the following:

$$\max_{p^G_{g,t},p^D_{d,t},r^U_{g,t},r^D_{g,t}} \sum_{d,t}(U_d \cdot p^D_{d,t}) - \sum_{g,t}(C_g \cdot p^G_{g,t}) - \sum_{g=1}^{12}\sum_t (C^U_g r^U_{g,t} + C^D_g r^D_{g,t}) \tag{41}$$

This objectives combines the objectives of the reserve and day-ahead market. Since this is a maximization problem, the terms from the DA model are not changed. The reserve model was a minimization problem, and therefore its term in the objective function is now subtracted, in stead of added.

Results

Some of the results from the two models above will be presented in this section.
In figure 11 both the reserve cost and the social welfare are illustrated in every hour for both the European(EU) and the US model. The reserve cost and the social welfare are the elements being optimized in the EU model's first and second part, respectively. From figure 11 it is seen that the reserve cost is lowest in every hour of the EU model, compared to the US model. This is expected, since the minimization of these costs is performed initially in the EU model, while the US model also takes other elements into account when finding the optimal reserve schedule. Figure 11 also illustrates that there is a higher social welfare in the US model

in every hour, compared to the EU model. This means that it is a benefit for the US to model reserve and day-ahead market in one, as it increases the social welfare. In figure 12 the social welfare minus the reserve costs are plotted for every hour. This is what is being maximized in the US model. Therefore it is also expected that the US model has higher values in this graph, which is also the case.

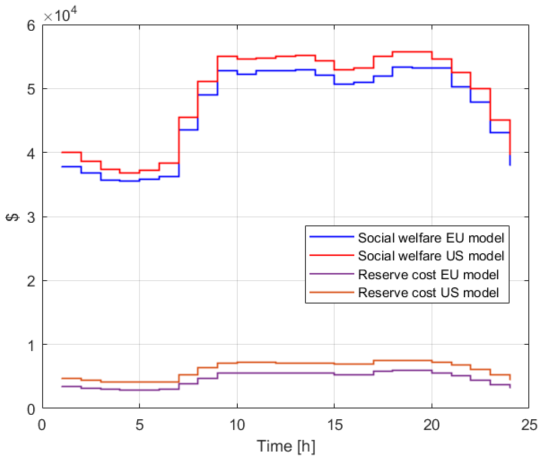


Figure 11: Social welfare and reserve cost of the two practices. Relates to the two objectives of the EU model.

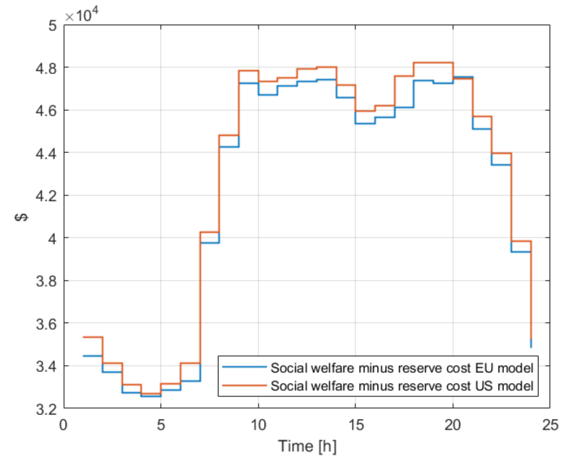


Figure 12: Social welfare minus reserve cost of the two practices. Relates to the objective of the US model.

Based on these results, one could argue that the US model is more favorable, as it performs better in its own objective and it also has the highest social welfare. However, this kind of investigation cannot stand alone when evaluating the two practices. This investigation has for example not considered if the European actor has the opportunity to make bilateral contracts, long time before the day-ahead market, and this could have an effect on the results.

In figure 13 the down regulation capacity of generator 4(g4) and 6(g6) is shown for both the EU and the US model. From this it is seen that the EU model chooses to use the full down regulation capacity of g4 (180MW) in almost every hour, but it has not chosen to buy down regulation capacity from g6. While the US model only uses a fraction of the g4 down regulation capacity in the peak hours but the full down regulation capacity of g6 (30MW) in every hour. The reason for the difference between these two schedules is that the EU model simply aims to have the lowest down regulation costs, and this is 7$/MWh for g4 and 14$/MWh for g6. While the US model also takes into account that when a generator is scheduled for down regulation it is forced to sell this energy at the DA market. So if the generation cost is high for a specific generator it might not be optimal to use it as down regulation capacity. Since the generation cost of g4 is 20.93$/MWh and the one for g6 is 10.52$/MWh, the US model finds it more feasible to invest in g6 than g4 down regulation capacity.

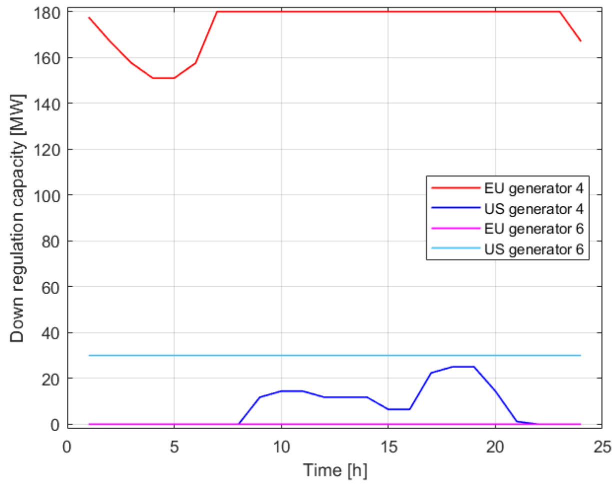


Figure 13: Down regulation capacity for two generators in the EU and US models

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Appendix

Hour/farm	farm 1	farm 2	farm 3	farm 4	farm 5	farm 6
0	74.46982	94.67942	102.1451	75.19539	114.2462	115.705
1	63.12577	125.1245	132.737	87.85176	128.3032	118.8781
2	75.31804	127.6919	139.0214	100.4989	133.0348	131.1821
3	73.4121	127.2677	145.3015	135.9442	135.287	145.6814
4	124.6061	133.1092	148.4107	144.3043	136.6002	151.3037
5	153.9935	137.9089	161.9446	144.8233	138.4989	148.9617
6	155.6187	142.3008	166.255	137.7931	137.893	144.7498
7	160.947	137.331	153.3728	137.2984	137.3808	139.0699
8	166.3675	141.4973	153.7776	130.9088	147.5918	151.9417
9	156.262	138.5449	129.5351	112.6748	146.5665	129.9621
10	158.9541	133.374	128.9511	114.2235	150.5	144.0347
11	165.481	155.1686	140.88	129.9039	152.0051	157.0579
12	163.1983	150.7746	144.188	141.0784	147.2883	162.3783
13	158.4072	139.9119	135.9548	130.306	129.0392	133.954
14	146.3653	129.5469	108.6523	107.4617	145.3945	130.8619
15	147.5313	140.8723	131.3121	108.3475	144.4942	122.8209
16	108.6399	135.0766	127.1314	96.67182	123.5225	89.87809
17	140.6819	141.1066	140.5166	124.117	131.6161	98.15602
18	145.935	143.7455	128.8967	121.6459	123.0537	102.9494
19	158.8208	153.8313	158.789	131.3894	127.0095	95.59066
20	158.8821	154.6901	149.5562	101.5658	137.7156	99.85586
21	173.1581	156.9537	163.2362	112.8652	151.351	113.0953
22	169.6514	152.1602	159.5835	129.8978	150.8511	122.6533
23	167.1041	142.4065	143.1139	125.6642	141.2701	125.8223

Figure 14: Power generated from each windfarm in MWh. The wind data is taken from [2] where data for zone 1 to 6 was downloaded. A specific scenario was chosen from each zone instead of using the mean across scenarios.