# 信息安全数学基础作业 #3

XXX：202XX80XXXXXXXX

2023 年 4 月 16 日

求证：

$$H(XY) = H(X) + H(Y) - I(X;Y)$$

证明：

$$\begin{aligned}
H(XY) + I(X;Y) &= -(\sum_{i=1}^{n}\sum_{j=1}^{m} P(x_i,y_j)log_2 P(x_i,y_j) + \sum_{i=1}^{n}\sum_{j=1}^{m} P(x_i,y_j)log_2 \frac{P(x_i)}{P(x_i|y_j)}) \\
&= -(\sum_{i=1}^{n}\sum_{j=1}^{m} P(x_i,y_j)log_2 (\frac{P(x_i,y_j)P(x_i)}{P(x_i|y_j)})) \\
&= -(\sum_{i=1}^{n}\sum_{j=1}^{m} P(x_i,y_j)log_2 \frac{P(y_j)P(x_i|y_j)P(x_i)}{P(x_i|y_j)}) \\
&= -(\sum_{i=1}^{n}\sum_{j=1}^{m} P(x_i,y_j)log_2 (P(x_i)P(y_j))) \\
&= -(\sum_{i=1}^{n}\sum_{j=1}^{m} P(x_i,y_j)log_2 P(x_i) + \sum_{i=1}^{n}\sum_{j=1}^{m} P(x_i,y_j)log_2 P(y_j)) \\
&= -(\sum_{i=1}^{n}(\sum_{j=1}^{m} P(x_i,y_j))log_2 P(x_i) + \sum_{j=1}^{m}(\sum_{i=1}^{n} P(x_i,y_j))P(y_j)) \\
&= -(\sum_{i=1}^{n} P(x_i)log_2 P(x_i) + \sum_{j=1}^{m} P(y_j)log_2 P(y_j)) \\
&= H(X) + H(Y)
\end{aligned}$$

故命题得证。