On-line Appendix to: Price-region bids in electricity markets

January 23, 2019

1 Case study: district heating utility model

As shown on Figure 1, the district heating system is composed of multiple elements. This section characterises the district heating utility's physical characteristics as a set of linear constraints. It then describes the utility's bidding strategy on day-ahead market.

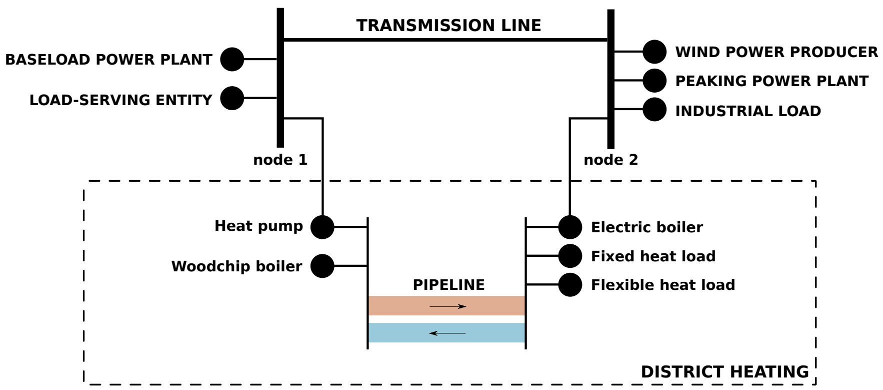

Figure 1: Overview of the case study power and heat system.

Let n_1, n_2 denote the two price areas in the power system. Let q_{sn_1k}, q_{sn_2k} respectively denote the utility's injection of electrical energy in n_1 and n_2. In this section, the index s refers to the district heating utility as an electricity market participant.

Let h_k^{wc}, h_k^{el} and h_k^{hp} denote the heat production in time period k of the woodchip boiler, the electrical boiler and the heat pump, respectively. Their unit is GJ/h. Let $H^{\mathrm{wc}} = 37.5$ GJ/h, $H^{\mathrm{el}} = 5$ GJ/h and $H^{\mathrm{hp}} = 30$ GJ/h denote the maximum heat production of these units. Let $\alpha^{\mathrm{el}} = 3$ GJ/MWh and $\alpha^{\mathrm{hp}} = 5$ GJ/MWh denote the power-to-heat conversion ratio of the electrical boiler and the heat pump, respectively. Equations (1a)-(1b) relate the withdrawal of electrical energy from n_1 and n_2 to the heat production by the heat pump and the electrical boiler. Equations (1c)-(1e) set bounds to the heat production of the different units.

$$h_k^{\text{el}} = -\alpha^{\text{hp}} q_{sn_1 k} \qquad \forall\, k \in \{1...K\}, \tag{1a}$$
$$h_k^{\text{el}} = -\alpha^{\text{el}} q_{sn_2 k} \qquad \forall\, k \in \{1...K\}, \tag{1b}$$
$$0 \leq h_k^{\text{hp}} \leq H^{\text{hp}} \qquad \forall\, k \in \{1...K\}, \tag{1c}$$
$$0 \leq h_k^{\text{el}} \leq H^{\text{el}} \qquad \forall\, k \in \{1...K\}, \tag{1d}$$
$$0 \leq h_k^{\text{wc}} \leq H^{\text{wc}} \qquad \forall\, k \in \{1...K\}. \tag{1e}$$

Let D_k^{fix} denote the heat demand of the inflexible load in time period k, in GJ/h. The inflexible load follows the demand profile displayed in Figure 2. Let d_k^{flex} denote the heat energy supplied to the flexible load in time period k, in GJ/h. Let $E^{\text{flex}} = 150$ GJ denote the total energy the flexible load requires over the horizon, and $R^{\text{flex}} = 10$ GJ/h the maximum rate at which it can be supplied. Equations (1f)-(1g) describe the flexible load constraints.

$$\sum_{k=1}^{K} d_k^{\text{flex}} = E^{\text{flex}}, \tag{1f}$$
$$0 \leq d_k^{\text{flex}} \leq R^{\text{flex}} \qquad \forall\, k \in \{1...K\}. \tag{1g}$$

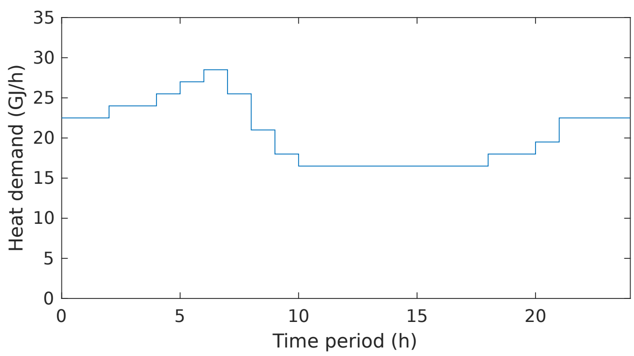

Figure 2: Demand profile of inflexible heat load.

The hot water pipelines are operated using constant mass flows, i.e. the speed at which water mass travels in the pipelines is fixed. Figure 3 provides a schematic representation of the supply-and-return pipelines system. Energy is injected from the left-hand side by increasing the temperature of the water flowing from the return pipe outlet to the supply pipe inlet at a fixed flow rate. Energy is withdrawn from the right-hand side by cooling down the temperature of the water flowing from the supply pipe outlet to the return pipe inlet at the same fixed flow rate. In this case study, it is assumed that it takes exactly one time period for the water mass to travel from one end to the other end of a pipe. This means that, in a given time period, the temperature of the water at the outlet of a pipe is equal to the temperature at the inlet of the pipe in the previous time period. Let t_k^{sup} denote the water temperature in the inlet of the supply pipe at time period k, and t_k^{ret} that of the return pipe, in °C.

The initial temperatures are set to $t_0^{\mathrm{sup}} = 70\,°\mathrm{C}$, $t_0^{\mathrm{ret}} = 50\,°\mathrm{C}$, and temperatures in the end of the horizon are enforced to reach the same value (see Equations (1h)-(1i)) so that the next day's operations are not compromised. Equations (1j)-(1k) enforce lower and upper bounds for temperatures, resp. $T^{\min} = 30\,°\mathrm{C}$ and $T^{\max} = 90\,°\mathrm{C}$. Equations (1l)-(1m) enforce that the temperatures in the return pipe are never higher than in the supply pipe.

$$t_K^{\mathrm{sup}} = t_0^{\mathrm{sup}}, \tag{1h}$$
$$t_K^{\mathrm{ret}} = t_0^{\mathrm{ret}}, \tag{1i}$$
$$T^{\min} \le t_k^{\mathrm{ret}} \qquad \forall\, k \in \{1...K\}, \tag{1j}$$
$$t_k^{\mathrm{sup}} \le T^{\max} \qquad \forall\, k \in \{1...K\}, \tag{1k}$$
$$t_{k-1}^{\mathrm{ret}} \le t_k^{\mathrm{sup}} \qquad \forall\, k \in \{1...K\}, \tag{1l}$$
$$t_k^{\mathrm{ret}} \le t_{k-1}^{\mathrm{sup}} \qquad \forall\, k \in \{1...K\}. \tag{1m}$$

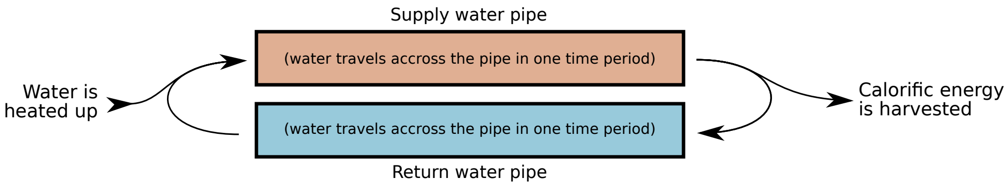

Figure 3: Supply and return district heating pipelines.

Let $\alpha^{\mathrm{pipe}} = 1.5$ GJ/°C denote the specific energy of the mass of water that travels the pipelines in one time period. The energy injected from area n_1 into the pipelines in a time period k writes $\alpha^{\mathrm{pipe}}(t_k^{\mathrm{sup}} - t_{k-1}^{\mathrm{ret}})$. The energy withdrawn from the pipelines in area n_2 in a time period k write $\alpha^{\mathrm{pipe}}(t_{k-1}^{\mathrm{sup}} - t_k^{\mathrm{ret}})$. Equations (1n)-(1o) respectively describe the conservation of energy in areas n_1 and n_2.

$$h_k^{\mathrm{wc}} + h_k^{hp} = \alpha^{\mathrm{pipe}}(t_k^{\mathrm{sup}} - t_{k-1}^{\mathrm{ret}}) \qquad \forall\, k \in \{1...K\}, \tag{1n}$$
$$h_k^{\mathrm{el}} + \alpha^{\mathrm{pipe}}(t_{k-1}^{\mathrm{sup}} - t_k^{\mathrm{ret}}) = D_k^{\mathrm{fix}} + d_k^{\mathrm{flex}} \qquad \forall\, k \in \{1...K\}. \tag{1o}$$

Let $\mathbf{q}_s$ denote the utility's injection profile, and let a vector $\mathbf{x}_s$ include all variables $\{h_k^{\mathrm{wc}}, h_k^{\mathrm{el}}, h_k^{\mathrm{hp}}, d_k^{\mathrm{flex}}, t_k^{\mathrm{sup}}, t_k^{\mathrm{ret}}, k = 1...K\}$. The district heating utility's feasible region of electricity injection $\mathcal{F}_s$ writes as:

$$\mathcal{F}_s = \left\{ \mathbf{q}_s \;\middle|\; \exists\, \mathbf{x}_s \in \mathbb{R}^{L_s} \text{ so that } (\mathbf{q}_s, \mathbf{x}_s) \text{ satisfies (1a)-(1o)} \right\}$$

The district heating system in this case study has only one variable cost component: the fuel cost of the woodchip boiler, $C^{\mathrm{wc}} = 40$ €/GJ. The district heating utility's cost function $\mathcal{C}_s(\mathbf{q}_s)$ thus writes as:

$$\mathcal{C}_s(\mathbf{q}_s) = \min_{\mathbf{x}_s} \left\{ \sum_{k=1}^{K} (C^{\mathrm{wc}} h_k^{\mathrm{wc}}), \text{ s.t. (1a)-(1o)} \right\}$$
$$- \min_{\mathbf{x}_s} \left\{ \sum_{k=1}^{K} (C^{\mathrm{wc}} h_k^{\mathrm{wc}}), \text{ so that } (\mathbf{0}, \mathbf{x}_s) \text{ satisfies (1a)-(1o)} \right\}$$

2 Case study: characteristics and bids of other market participants

This section describes the case study's other actors and their participation on the day-ahead market.

The transmission line is a system asset, modelled by the market operator based on physical characteristics communicated by the transmission system operator. It is modelled as a lossless transmission line, with a limited transmission capacity of 5 MW.

The wind power producer has a variable electricity supply capacity in a range between 0 and 25 MWh/h. The available power is assumed to be well-known by the wind power producer a day ahead of delivery. It offers energy in the market by placing price-quantity bids with bidding prices set to zero.

The load-serving entity has a variable electricity demand in a range between 5 and 15 MWh/h. The demand profile is assumed to be well-known by the load-serving entity a day ahead of delivery. It bids for energy in the market by placing price-quantity bids with bidding prices set to a value of lost load of 1000 e/MWh. This is a relatively high value so that the demand can be met even when electricity supply is scare.

The peaking power plant has a flexible power output. Its variable cost of operation is positive and increases as the power output increases, following a piecewise linear function. This actor offers energy in the market by placing price-quantity bids.

The baseload power plant has a constant, predictable supply capacity of 10 MWh/h with no variable costs of operation. It offers this energy in the market by placing price-quantity bids with bidding prices set to zero.

The industrial load has a predictable inflexible demand for electrical energy following the profile from Figure 4. It bids for this energy in the market by placing price-quantity bids with bidding prices set to a value of loss load of 1000 €/MWh.

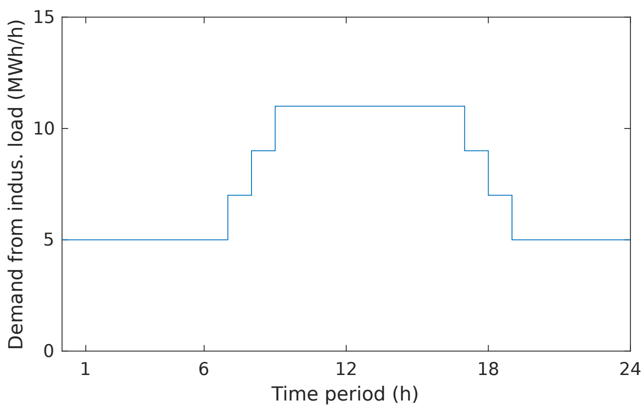

Figure 4: Industrial load's demand for electrical energy.

3 Case study: generation of price scenarios

Cases 1 and 2 in the article's case study rely on a set of electricity price forecast scenarios. These are generated by simulating different outcomes of the market-clearing program, using slighty different wind power and load profiles. The intention is to obtain representative price series for the system under study, accounting for a certain degree of uncertainty in wind power and load. Figures 5 and 6 below display the set of wind power and load profiles that are used to generate different scenarios. Combining the different profiles leads to 27 different scenarios. In Figures 5 and 6, the profiles that are highlighted correspond to the realised profiles in the day-ahead market.

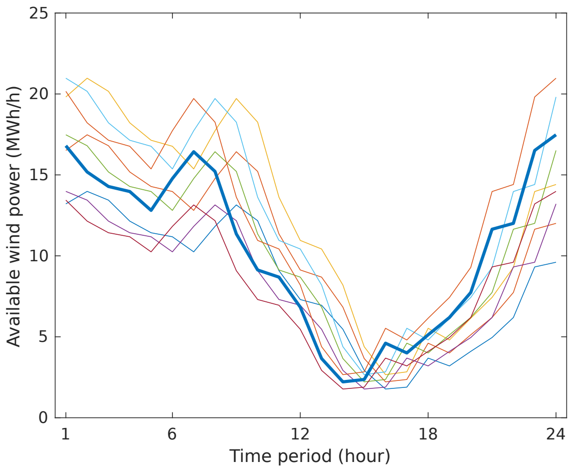

Figure 5: Set of likely available wind power profiles.

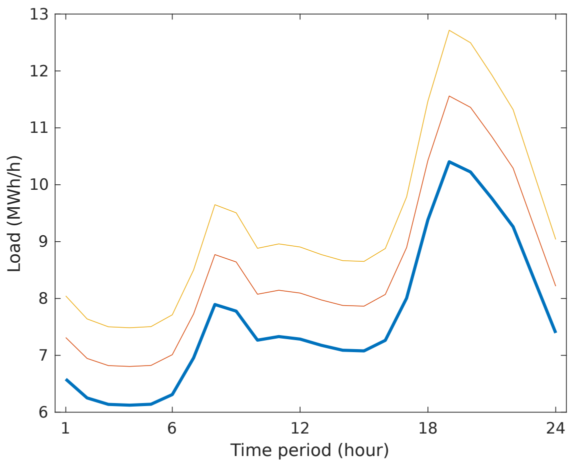

Figure 6: Set of likely load profiles (load serving entity).