

PEARSON NEW INTERNATIONAL EDITION

Statistical Reasoning for Everyday Life

Jeff Bennett William L. Briggs

Mario F. Triola

Fourth Edition



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PEARSON®

Pearson Education Limited
Edinburgh Gate
Harlow
Essex CM20 2JE
England and Associated Companies throughout the world

Visit us on the World Wide Web at: www.pearsoned.co.uk

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PEARSON®

ISBN 10: 1-292-04021-1
ISBN 13: 978-1-292-04021-9

British Library Cataloguing-in-Publication Data
A catalogue record for this book is available from the British Library

Printed in the United States of America

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Speaking of Statistics

Nước uống của bạn có an toàn không? Có bao nhiêu người tán thành kế hoạch ngân sách của Tổng thống? Chúng ta có nhận được giá trị xứng đáng cho đồng tiền chăm sóc sức khỏe của mình không? Những câu hỏi như vậy chỉ có thể được giải quyết thông qua các nghiên cứu thống kê. Trong chương này, chúng ta sẽ thảo luận về các nguyên tắc cơ bản của nghiên cứu thống kê và đặt nền tảng cho nghiên cứu chi tiết về thống kê. Trong quá trình thực hiện, chúng ta sẽ xem xét nhiều ví dụ đa dạng cho thấy các nghiên cứu thống kê được thiết kế tốt có thể cung cấp hướng dẫn cho chính sách xã hội và các quyết định cá nhân như thế nào, cũng như một số trường hợp mà số liệu thống kê có thể gây hiểu lầm.

Statistical thinking will one day be as necessary for efficient citizenship as the ability to read and write.

—H. G. Wells

LEARNING GOALS

1 What Is/Are Statistics?

Hiểu 2 nghĩa của thuật ngữ thống kê và ý tưởng cơ bản đằng sau bất kỳ nghiên cứu thống kê nào, bao gồm các mối quan hệ giữa tổng thể, mẫu, thống kê mẫu và thông số tổng thể.

2 Sampling

Hiểu được tầm quan trọng của việc chọn mẫu đại diện và làm quen với một số phương pháp lấy mẫu phổ biến.

3 Types of Statistical Studies

Hiểu sự khác biệt giữa các nghiên cứu quan sát và thí nghiệm; nhận dạng các vấn đề chính trong thí nghiệm, bao gồm lựa chọn điều trị và nhóm kiểm soát, hiệu ứng giả dược, và mờ mắt.

4 Should You Believe a Statistical Study?

Có thể đánh giá các nghiên cứu thống kê mà bạn thấy trên các phương tiện truyền thông, vì vậy bạn có thể quyết định liệu kết quả có ý nghĩa.

FOCUS TOPICS

Focus on Psychology: Are You Driving "Drunk" on Your Cell Phone?

Focus on Public Health: Lối sống của bạn có lành mạnh?

WHAT IS/ARE STATISTICS?

Chủ đề thống kê thường bị rập khuôn là khô khan hoặc kỹ thuật, nhưng nó chạm đến hầu hết mọi thứ trong xã hội hiện đại. Số liệu thống kê có thể cho ta biết liệu một loại thuốc mới có hiệu quả trong điều trị ung thư hay không, nó có thể giúp các thanh tra nông nghiệp đảm bảo rằng thực phẩm an toàn và nó là điều cần thiết để tiến hành và giải thích các cuộc thăm dò dư luận. Các doanh nghiệp sử dụng số liệu thống kê trong nghiên cứu thị trường và quảng cáo. Chúng ta thậm chí còn sử dụng số liệu thống kê trong thể thao, thường là cách xếp hạng các đội và vận động viên. Thật vậy, bạn sẽ khó có thể nghĩ ra bất kỳ chủ đề nào không liên quan đến số liệu thống kê theo một cách quan trọng nào đó.

Mục tiêu chính của văn bản này là giúp bạn tìm hiểu những ý tưởng cốt lõi đằng sau các phương pháp thống kê. Những ý tưởng cơ bản này không khó hiểu, mặc dù nắm vững các chi tiết và lý thuyết đằng sau chúng có thể cần nhiều năm nghiên cứu. Một trong những điều tuyệt vời về thống kê là ngay cả một lượng nhỏ lý thuyết được trình bày trong cuốn sách này cũng sẽ giúp bạn hiểu được số liệu thống kê bạn gặp trong tin tức, trong lớp học hoặc nơi làm việc và trong cuộc sống hàng ngày của bạn.

Một nơi tốt để bắt đầu là với chính thuật ngữ thống kê, có thể là số ít hoặc số nhiều và có ý nghĩa khác nhau trong hai trường hợp. Khi nó là số ít, thống kê là khoa học giúp chúng tôi hiểu cách thu thập, sắp xếp và diễn giải các con số hoặc thông tin khác về chủ đề nào đó; chúng ta coi các con số hoặc các phần thông tin khác là dữ liệu. Khi nó là số nhiều, số liệu thống kê là dữ liệu thực tế mô tả một số đặc điểm. Ví dụ, nếu có 30 sinh viên trong lớp của bạn và họ có độ tuổi từ 17 đến 64, các con số “30 sinh viên,” “17 năm,” và “64 năm” đều là số liệu thống kê mô tả lớp học của bạn theo một cách nào đó.

Two Definitions of Statistics

- Thống kê là khoa học về thu thập, tổ chức và giải thích dữ liệu.
- Thống kê là dữ liệu (số hoặc các phần thông tin khác) mô tả hoặc tóm tắt điều gì đó.

How Statistics Works

Theo các bản tin, 111,3 triệu người Mỹ đã xem New York Giants thắng Super Bowl XLVI, điều này giải thích tại sao các nhà mạng có thể yêu cầu các nhà quảng cáo trả hơn 3 triệu đô la cho một đoạn quảng cáo dài 30 giây. Nhưng bạn có thể thắc mắc: Ai đã đếm hết hàng triệu người này?

Câu trả lời là không ai cả. Tuyên bố rằng 111,3 triệu người đã xem Super Bowl đã đến từ các nghiên cứu thống kê được thực hiện bởi một công ty có tên Nielsen Media Research. Công ty này tổng hợp xếp hạng Nielsen nổi tiếng của mình bằng cách theo dõi thói quen xem truyền hình của người dân ở chỉ khoảng 5.000 căn nhà.

Nếu bạn chưa quen với nghiên cứu thống kê, kết luận của Nielsen có vẻ hơi quá.

Làm sao một người có thể rút ra kết luận về hàng triệu người chỉ bằng cách nghiên cứu vài nghìn người? Tuy nhiên, khoa học thống kê cho thấy kết luận này có thể khá chính xác, miễn là nghiên cứu thống kê được tiến hành đúng cách. Hãy coi xếp hạng của Nielsen về Super Bowl là một ví dụ và hỏi một số câu hỏi chính sẽ minh họa cách thống kê hoạt động nói chung.

What Is the Goal of the Research?

Mục tiêu của Nielsen là xác định tổng số người Mỹ đã xem Super Bowl, bên trong ngôn ngữ thống kê, chúng tôi nói rằng Nielsen quan tâm đến **tổng thể** của tất cả người Mỹ. Các con số mà Nielsen hy vọng xác định được—số người đã xem Super Bowl—là một đặc điểm riêng của tổng thể. Trong thống kê, đặc điểm của tổng thể được gọi là **thông số tổng thể**.

Mặc dù chúng ta thường nghĩ tổng thể(population) là một nhóm người, nhưng tổng thể thống kê có thể là bất kỳ loại nhóm nào—con người, động vật hoặc đồ vật. Ví dụ, trong một nghiên cứu về an toàn ô tô, tổng thể có thể là tất cả các xe ô tô trên đường. Tương tự, thuật ngữ thống số tổng thể có thể đề cập đến bất kỳ đặc điểm nào của tổng thể. Trong trường hợp an toàn ô tô, các thông số tổng thể có thể bao gồm tổng số ô tô trên đường trong một khoảng thời gian nhất định, tỷ lệ tai nạn giữa các ô tô trên đường hoặc phạm vi trọng lượng của ô tô trên đường.

BY THE WAY

Mặc dù đôi khi bạn sẽ thấy dữ liệu được sử dụng như một từ đồng nghĩa số ít với thông tin, về mặt kỹ thuật nó là số nhiều: Một mẩu thông tin được gọi là một mốc thời gian, và hai hoặc nhiều hơn phần được gọi là dữ liệu.

Ở đây, thuật ngữ statistics số ít gọi là thống kê, số nhiều gọi là số liệu thống kê.

BY THE WAY

Thống kê bắt nguồn từ việc thu thập dữ liệu điều tra dân số và thuế, đó là công việc của nhà nước. Đó là lý do tại sao từ state là từ gốc của từ thống kê.

Tổng thể trong một nghiên cứu thống kê là tập hợp đầy đủ những người hoặc vật đang được nghiên cứu.

Thông số tổng thể là những con số cụ thể mô tả đặc điểm của quần thể.

EXAMPLE 1 Populations and Population Parameters

Đối với mỗi tình huống sau đây, hãy mô tả tổng thể đang được nghiên cứu và xác định một số thông số tổng thể sẽ được quan tâm

- Bạn làm việc cho Bảo hiểm Nông dân và bạn được yêu cầu xác định số tiền trung bình trả cho nạn nhân tai nạn trong xe ô tô không có túi khí một bên.
- Bạn đã được McDonald's thuê để xác định trọng lượng của khoai tây được giao mỗi tuần cho khoai tây chiên.
- Bạn là phóng viên kinh doanh đưa tin về Genentech Corporation và bạn đang điều tra liệu phương pháp điều trị mới của nó có hiệu quả đối với bệnh bạch cầu ở trẻ em hay không.

SOLUTION

- Population bao gồm những người đã nhận được thanh toán bảo hiểm cho tai nạn trong những chiếc xe thiếu túi khí bên. Population parameter là average số tiền trả cho những người này. (Sau này, thuật ngữ "average" sẽ được thay thế bằng thuật ngữ chính xác hơn thuật ngữ "mean".)
- Population bao gồm tất cả khoai tây được giao hàng tuần cho món khoai tây chiên kiểu Pháp. Các population parameter bao gồm trọng lượng trung bình của khoai tây và sự thay đổi của trọng số (ví dụ: hầu hết chúng gần hay xa mức trung bình?).
- Population bao gồm tất cả trẻ em mắc bệnh bạch cầu. Các population parameter quan trọng là tỷ lệ trẻ em hồi phục mà không cần điều trị mới và tỷ lệ những đứa trẻ hồi phục với phương pháp điều trị mới.

What Actually Gets Studied?

Nếu các nhà nghiên cứu tại Nielsen là toàn năng, họ có thể xác định số lượng người xem Super Bowl bằng cách khảo sát từng cá nhân người Mỹ. Nhưng không ai có thể làm điều đó, vì vậy thay vào đó họ cố gắng ước tính số lượng người Mỹ xem bằng cách nghiên cứu một nhóm tương đối nhỏ mọi người. Nielsen cố gắng tìm hiểu về dân số của tất cả người Mỹ bằng cách theo dõi cẩn thận thói quen xem của một mẫu người Mỹ nhỏ hơn nhiều. Cụ thể hơn, Nielsen sử dụng các thiết bị ghi âm trong khoảng 5.000 ngôi nhà, vì vậy những người sống trong những ngôi nhà này là mẫu người Mỹ mà Nielsen nghiên cứu.

Các phép đo cá nhân mà Nielsen thu thập từ những người trong 5.000 ngôi nhà tạo thành **raw data**. Nielsen thu thập nhiều dữ liệu thô—ví dụ: khi nào và bao lâu mỗi TV trong gia đình được bật, chương trình nào được điều chỉnh và ai trong gia đình đang xem. Nielsen sau đó hợp nhất những dữ liệu thô này thành một tập hợp các số đặc trưng cho mẫu, chẳng hạn như tỷ lệ phần trăm người xem trong mẫu đã xem từng chương trình truyền hình riêng tổng số người trong mẫu đã xem Super Bowl. Những con số này được gọi là **sample statistics**.

BY THE WAY

Arthur C. Nielsen thành lập công ty và phát minh ra nghiên cứu thị trường vào năm 1923. Ông đã giới thiệu Nielsen Radio Index để đánh giá đài phát thanh các chương trình vào năm 1942 và mở rộng phương pháp lập chương trình truyền hình của mình vào những năm 1960. Công ty hiện cũng theo dõi các phương tiện truyền thông khác (Internet, điện thoại thông minh, v.v.) và phải liên tục điều chỉnh phương pháp khoa học của mình với các công nghệ truyền thông mới.



Definitions

Một sample là một tập hợp con của population mà dữ liệu thực sự được lấy từ đó.

Các phép đo hoặc quan sát thực tế được thu thập từ sample tạo thành raw data.

Sample statistics là những con số mô tả đặc điểm của sample được tìm thấy bằng cách hợp nhất hoặc tổng hợp raw data.

BY THE WAY

Theo định nghĩa của Bộ Lao động, một người không làm việc chưa chắc đã thất nghiệp. Ví dụ, các bà mẹ nội trợ và bố không được tính trong số những người thất nghiệp trừ khi họ tích cực cố gắng tìm việc làm, và những người đã cố gắng tìm việc nhưng bỏ cuộc trong thất vọng không được tính là thất nghiệp.

EXAMPLE 2 Unemployment Survey

Bộ Lao động Hoa Kỳ định nghĩa lực lượng lao động dân sự là tất cả những người hoặc có việc làm hoặc tích cực tìm kiếm việc làm. Hàng tháng, Bộ Lao động báo cáo tỷ lệ thất nghiệp, là tỷ lệ phần trăm những người tích cực tìm kiếm việc làm trong toàn bộ lực lượng lao động dân sự. Để xác định tỷ lệ thất nghiệp, Bộ Lao động khảo sát 60.000 hộ gia đình. Đối với các báo cáo thất nghiệp, hãy mô tả từng điều sau đây.

- a.** population
- b.** sample
- c.** raw data
- d.** sample statistics
- e.** population parameters

SOLUTION

- a.** Population là nhóm mà Bộ lao động muốn tìm hiểu, đó là tất cả những người tạo nên lực lượng lao động dân sự.
- b.** Sample bao gồm tất cả những người trong số 60.000 hộ gia đình được khảo sát.
- c.** Raw Data bao gồm tất cả các thông tin được thu thập trong cuộc khảo sát.
- d.** Sample Statistics tóm tắt Raw Data cho Sample. Trong trường hợp này, Sample Statistics là phần trăm số người trong mẫu đang tích cực tìm kiếm việc làm. (Bộ Lao động cũng tính toán số liệu thống kê mẫu tương tự cho các nhóm nhỏ trong dân số, chẳng hạn như tỷ lệ thanh thiếu niên, nam giới, phụ nữ và cựu chiến binh thất nghiệp.)
- e.** Các Population Parameter là các đặc điểm của toàn bộ Population tương ứng với Sample Statistics. Trong trường hợp này, Population Parameter là tỷ lệ thất nghiệp thực tế. Lưu ý rằng Bộ Lao động không thực sự đo lường tham số này, bởi vì dữ liệu chỉ được thu thập cho Sample và sau đó được sử dụng để ước tính Population Parameter.

...

How Do Sample Statistics Relate to Population Parameters?

Giả sử Nielsen nhận thấy rằng 31% số người trong 5.000 ngôi nhà trong mẫu của họ đã xem Siêu cúp. "31%" này là một Sample Statistics, bởi vì nó đặc trưng cho Sample. Nhưng cái Nielsen thực sự muốn biết là Population Parameter tương ứng, là phần trăm tất cả người Mỹ đã xem Super Bowl.

Không có cách nào để các nhà nghiên cứu của Nielsen biết giá trị chính xác của Population Parameter, bởi vì họ chỉ nghiên cứu một mẫu. Tuy nhiên, các nhà nghiên cứu của Nielsen hy vọng rằng họ đã làm đúng rằng Sample Statistics là một ước tính tốt về Population Parameter. Nói cách khác, họ muốn kết luận rằng vì 31% mẫu đã xem Super Bowl, khoảng 31% dân số cũng xem Super Bowl. Một trong những mục đích chính của thống kê là để giúp các nhà nghiên cứu đánh giá tính hợp lệ của loại kết luận này.

TIME OUT TO THINK

Giả sử Nielsen kết luận rằng 30% người Mỹ đã xem Super Bowl. Có bao nhiêu người xem? (Dân số của Hoa Kỳ là khoảng 310 triệu người.)

Khoa học thống kê cung cấp các phương pháp cho phép các nhà nghiên cứu xác định mức độ hiệu quả của một Sample Statistic ước tính một Population Parameter. Ví dụ: kết quả từ các cuộc khảo sát hoặc thăm dò ý kiến thường được trích dẫn cùng với một giá trị được gọi là **Margin of Error**. Bằng cách cộng và trừ biên sai số khỏi Sample Statistic, chúng ta tìm thấy một dải giá trị hoặc **khoảng tin cậy** có khả năng chứa Population Parameter. Trong hầu hết các trường hợp, Margin of Error được xác định để chúng ta có thể tin tưởng 95% rằng phạm vi này chứa Population Parameter; Hình 1 cho thấy lời giải thích về điều này từ New York Times. Trong trường hợp xếp hạng của Nielsen, biên độ sai số là khoảng 1%. Do đó, nếu 31% mẫu đang xem Super Bowl, thì chúng tôi có thể tin tưởng 95% rằng phạm vi từ 30% đến 32% chứa tỷ lệ phần trăm dân số thực tế xem Super Bowl.

How the Poll Was Conducted

Thời báo New York Times/CBS mới nhất
Cuộc thăm dò ý kiến của bang New York
dựa trên các cuộc phỏng vấn qua
điện thoại được thực hiện 23 tháng 10
đến 28 tháng 10 với 1.315 người lớn
trên toàn tiểu bang. Trong số đó, 1.026
cho biết họ đã đăng ký bỏ phiếu.
Các cuộc phỏng vấn được thực hiện trong
cả hai Tiếng Anh hoặc tiếng Tây Ban Nha.

Về lý thuyết, 19 trong số 20 trường hợp
kết quả dựa trên các mẫu như vậy sẽ khác nhau
không quá 3% theo một trong hai hướng
từ những gì đã thu được bằng cách tìm
kiếm tất cả cư dân trưởng thành của bang
New York. Vì các nhóm nhỏ hơn, nguy cơ
lỗi lấy mẫu lớn hơn

Figure 1 The margin of error in a survey or opinion poll usually describes a range that is likely (with 95% confidence, meaning in 19 out of 20 cases) to contain the population parameter. This excerpt from the *New York Times* explains a margin of error of 3 percentage points.

Một trong những phát hiện đáng chú ý nhất của khoa học thống kê là có thể lấy kết quả có ý nghĩa từ các mẫu nhỏ đáng ngạc nhiên. Tuy nhiên, kích thước mẫu lớn hơn là tốt hơn (khi chúng khả thi), bởi vì sai số nói chung nhỏ hơn đối với mẫu lớn hơn. Ví dụ: biên sai số cho khoảng tin cậy 95% trong một nghiên cứu thăm dò được tiến hành tốt thường là khoảng 5% cho cỡ mẫu là 400, nhưng giảm xuống còn 3% cho cỡ mẫu 1.000 và 1% cho mẫu 10.000.

Definition

Biên độ sai số trong một nghiên cứu thống kê được sử dụng để mô tả phạm vi giá trị hoặc khoảng tin cậy, có khả năng chứa tham số tổng thể. Chúng ta tìm thấy khoảng tin cậy này bằng cách cộng và trừ biên độ sai số từ thống kê mẫu thu được trong nghiên cứu. Đó là, phạm vi của các giá trị có khả năng chứa tham số tổng thể là

$$\begin{aligned} \text{from} & \quad (\text{sample statistic} - \text{margin of error}) \\ \text{to} & \quad (\text{sample statistic} + \text{margin of error}) \end{aligned}$$

Biên độ sai số thường được xác định để đưa ra khoảng tin cậy 95%, nghĩa là 95% kích thước mẫu được sử dụng trong nghiên cứu sẽ chứa tham số tổng thể thực tế (và 5% thì không).

EXAMPLE 3 Sex and Politics

Trung tâm Nghiên cứu Nhân dân và Báo chí Pew đã phỏng vấn 1.002 người Mỹ trưởng thành và hỏi về lý do gần đây gia tăng các vụ bê bối tình dục giữa các quan chức được bầu. 57% số người được hỏi cho rằng sự gia tăng này là do giới truyền thông giám sát chặt chẽ hơn, trong khi 19% cho rằng sự gia tăng này là do các tiêu chuẩn đạo đức đang giảm sút. Biên độ lỗi cho cuộc thăm dò là 3%. Mô tả dân số và mẫu cho cuộc khảo sát này, và giải thích ý nghĩa thống kê mẫu 57%. Chúng ta có thể kết luận gì về tỷ lệ phần trăm của dân số tin rằng sự gia tăng các vụ bê bối tình dục chính trị là do sự giám sát chặt chẽ hơn của giới truyền thông?

SOLUTION Dân số là tất cả người Mỹ trưởng thành và mẫu bao gồm 1.002 người được phỏng vấn. Thống kê mẫu 57% là tỷ lệ phần trăm thực tế của những người trong mẫu người đã trả lời rằng sự giám sát chặt chẽ hơn của giới truyền thông là nguyên nhân dẫn đến sự gia tăng các vụ bê bối tình dục chính trị. Các Thống kê mẫu 57% và sai số 3% cho chúng tôi biết rằng phạm vi giá trị

$$\begin{aligned} \text{from} & \quad 57\% - 3\% = 54\% \\ \text{to} & \quad 57\% + 3\% = 60\% \end{aligned}$$

có khả năng (với độ tin cậy 95%) chứa tham số tổng thể, trong trường hợp này là tham số tỷ lệ phần trăm thực sự của tất cả người Mỹ trưởng thành tin rằng sự giám sát chặt chẽ hơn của giới truyền thông là nguyên nhân vì sự gia tăng các vụ bê bối tình dục chính trị.

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TIME OUT TO THINK

Trong cuộc thăm dò được mô tả trong Ví dụ 3, những người trả lời được đưa ra hai cách giải thích khả thi, phương tiện truyền thông giám sát tốt hơn và tiêu chuẩn đạo đức thấp hơn. Bạn có nghĩ rằng kết quả có thể khác nếu những người được hỏi được yêu cầu đưa ra lời giải thích của riêng họ? Giải thích.

Putting It All Together: The Process of a Statistical Study

Quy trình được Nielsen Media Research sử dụng tương tự như quy trình được sử dụng trong nhiều nghiên cứu thống kê. Hình 2 và hộp bên dưới tóm tắt các bước cơ bản trong một nghiên cứu thống kê. Hãy nhớ rằng các bước này phần nào được lý tưởng hóa và các bước thực tế có thể khác nhau giữa các nghiên cứu này với nghiên cứu khác. Hơn nữa, các chi tiết ẩn trong các bước cơ bản là cực kỳ quan trọng. Ví dụ, một mẫu được chọn tồi ở Bước 2 có thể khiến toàn bộ nghiên cứu trở nên vô nghĩa và phải hết sức cẩn thận trong việc suy luận kết luận về tổng thể từ các kết quả với mẫu nhỏ hơn nhiều tổng thể đó.

BY THE WAY

Các nhà thống kê thường chia môn học thành hai nhánh lớn: thống kê mô tả, đề cập đến với việc mô tả dữ liệu thông qua dạng đồ họa và thống kê mẫu, và thống kê suy luận, trong đó liên quan đến suy luận (hoặc ước tính) các tham số tổng thể số từ dữ liệu mẫu.

Basic Steps in a Statistical Study

- Step 1. Nêu mục tiêu nghiên cứu của bạn một cách chính xác; nghĩa là xác định tổng thể bạn muốn nghiên cứu và chính xác những gì bạn muốn tìm hiểu về nó.
- Step 2. Chọn một mẫu đại diện từ tổng thể.
- Step 3. Thu thập dữ liệu từ mẫu và tóm tắt những dữ liệu này bằng cách tìm các thống kê mẫu được quan tâm.
- Step 4. Sử dụng số liệu thống kê mẫu để suy luận về tổng thể.
- Step 5. Đến kết luận; xác định những gì bạn đã học và liệu bạn có đạt được mục tiêu của mình hay không.

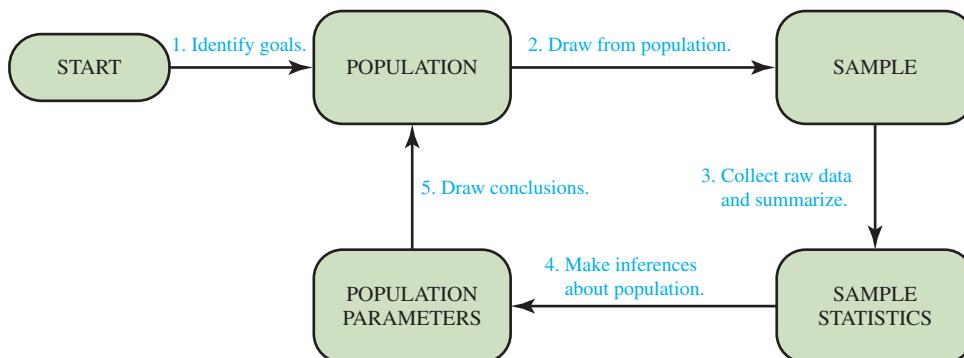


Figure 2 The process of a statistical study.

EXAMPLE 4 Identifying the Steps

Xác định cách các nhà nghiên cứu áp dụng năm bước cơ bản trong khảo sát từ Ví dụ 3.

SOLUTION Các bước áp dụng như sau

1. Các nhà nghiên cứu có mục tiêu tìm hiểu xem người Mỹ nghĩ gì về nguyên nhân của những bê bối chính trị. Họ chọn người Mỹ trưởng thành làm dân số, cố tình bỏ qua những đứa trẻ.
2. Họ đã chọn 1.002 người Mỹ trưởng thành để làm mẫu. Mặc dù chúng ta không được cho biết làm thế nào mẫu đã được rút ra, chúng tôi sẽ giả định rằng nó đã được rút ra sao cho 1.002 người Mỹ trưởng thành điển hình của toàn bộ dân số Mỹ trưởng thành.

3. Họ thu thập dữ liệu thô bằng cách hỏi những người trong mẫu một câu hỏi đơn giản. Dữ liệu thô là những câu trả lời cá nhân cho câu hỏi. Họ tóm tắt những dữ liệu này với thống kê mẫu, chẳng hạn như tỷ lệ phần trăm tổng thể của những người trong mẫu đã chọn từng câu trả lời.
4. Các kỹ thuật khoa học thống kê cho phép các nhà nghiên cứu suy ra các đặc điểm tổng thể. Trong trường hợp này, suy luận bao gồm ước tính tham số dân số có liên quan và tính toán sai số.
5. Bằng cách đảm bảo rằng nghiên cứu được tiến hành đúng cách và giải thích các ước tính của tham số tổng thể, các nhà nghiên cứu đã rút ra kết luận chung về thái độ của người Mỹ liên quan đến những vụ bê bối gần đây.

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Statistics: Decisions for an Uncertain World

Hầu hết các ví dụ mà chúng ta đã thảo luận cho đến nay đều liên quan đến khảo sát hoặc thăm dò ý kiến, nhưng chủ đề của thống kê bao gồm nhiều hơn thế, các thí nghiệm được thiết kế để thử nghiệm các phương pháp điều trị y tế mới, các phân tích về sự nguy hiểm của sự nóng lên toàn cầu và thậm chí cả những đánh giá về giá trị của một trường đại học giáo dục. Thật vậy, công bằng mà nói rằng mục đích chính của thống kê là giúp chúng ta ra quyết định tốt bất cứ khi nào ta phải đối mặt với một loạt các lựa chọn.

The Purpose of Statistics

Thống kê có nhiều công dụng, nhưng có lẽ mục đích quan trọng nhất của nó là giúp chúng ta đưa ra những quyết định đúng đắn về các vấn đề liên quan đến sự không chắc chắn.

Mục đích này sẽ rõ ràng trong hầu hết các nghiên cứu tình huống và ví dụ mà chúng tôi xem xét trong văn bản này, nhưng thỉnh thoảng trước hết chúng ta sẽ phải thảo luận về một chút lý thuyết có vẻ hơi trừu tượng. Nếu bạn ghi nhớ mục đích chung của số liệu thống kê, thì cuối cùng bạn sẽ được khen thưởng khi bạn thấy lý thuyết giúp chúng ta hiểu thế giới của chúng ta như thế nào. Case Study sau đây sẽ cung cấp cho bạn một hướng vị của những gì ở phía trước. Nó liên quan đến một số ý tưởng lý thuyết quan trọng dẫn đến một trong những thành tựu vĩ đại nhất của thế kỷ 20 về sức khỏe cộng đồng.

CASE STUDY The Salk Polio Vaccine

Nếu bạn đã làm cha mẹ vào những năm 1940 hoặc 1950, một trong những nỗi sợ hãi lớn nhất của bạn sẽ là là căn bệnh được gọi là bệnh bại liệt. Mỗi năm trong đợt dịch bại liệt kéo dài này, hàng ngàn trẻ nhỏ bị liệt vì căn bệnh này. Năm 1954, một thí nghiệm lớn đã được tiến hành để kiểm tra hiệu quả của một loại vắc-xin mới do Tiến sĩ Jonas Salk (1914–1995) tạo ra. Thí nghiệm bao gồm một mẫu gồm 400.000 trẻ em được chọn từ dân số của tất cả trẻ em ở Hoa Kỳ. Một nửa trong số 400.000 trẻ em này được tiêm vắc-xin Salk. Nửa còn lại được tiêm chỉ chứa nước muối. (Các tiêm nước muối là một giả dược; xem Phần 3.) Trong số những đứa trẻ nhận Salk vắc-xin, chỉ có 33 người mắc bệnh bại liệt. Ngược lại, có 115 trường hợp mắc bệnh bại liệt trong số trẻ em không tiêm vắc-xin Salk. Sử dụng các kỹ thuật của khoa học thống kê mà chúng ta sẽ nghiên cứu sau đó, các nhà nghiên cứu kết luận rằng vắc-xin có hiệu quả trong việc ngăn ngừa bệnh bại liệt. Do đó, họ quyết định thực hiện một nỗ lực lớn để cải thiện vắc-xin Salk và phân phối nó cho dân số của tất cả trẻ em. Nhờ vắc-xin này (và những vắc-xin cải tiến đã phát triển sau này), nỗi kinh hoàng của bệnh bại liệt giờ đây phần lớn chỉ còn là ký ức của quá khứ.

BY THE WAY

Bệnh bại liệt nhanh chóng trở nên hiếm gặp ở Hoa Kỳ sau quá trình phát triển của vắc-xin Salk, nhưng nó vẫn còn phổ biến ở các nước kém phát triển. Một nỗ lực toàn cầu để tiêm chủng trẻ em chống lại bệnh bại liệt bắt đầu từ 1998 và đã đạt được nhiều thành công tuy chưa đạt mục tiêu của nó là loại bỏ hoàn toàn bệnh tật.



The greatest reward for doing is the opportunity to do more.

—Jonas Salk

Section 1 Exercises

Statistical Literacy and Critical Thinking

- 1. Population and Sample.** What is a *population*, what is a *sample*, and what is the difference between them?
 - 2. Statistic and Statistics.** Suppose that, in a discussion, one person refers to baseball *statistics* and another refers to the use of *statistics* in showing that a particular drug is an effective treatment. Do both uses of the term *statistics* have the same meaning? If not, how do they differ?
 - 3. Statistics and Parameters.** What is a sample statistic, what is a population parameter, and what is the difference between them?
 - 4. Margin of Error.** What is the margin of error in a statistical study, and why is it important?
- Does It Make Sense?** For Exercises 5–10, decide whether the statement makes sense (or is clearly true) or does not make sense (or is clearly false). Explain clearly; not all of these have definitive answers, so your explanation is more important than your chosen answer.
- 5. Statistics and Parameters.** My professor conducted a statistical study in which he was unable to measure any sample statistics, but he succeeded in determining the population parameters with a very small margin of error.
 - 6. Poor Poll.** A poll conducted two weeks before the election found that Smith would get 70% of the vote, with a margin of error of 3%, but he ended up losing the election anyway.
 - 7. Poll Certainty.** There is no doubt that Johnson won the election, because an exit poll showed that she received 54% of the vote and the margin of error is only 3 percentage points.
 - 8. Beating Nielsen.** A new startup company intends to compete with Nielsen Media Research by providing data with a larger margin of error for the same price.
 - 9. Depression Sample.** The goal of my study is to learn about depression among people who have suffered through a family tragedy, so I plan to choose a sample from the population of patients in support groups for loss of a spouse.
 - 10. New Product.** Our market research department surveyed 1,000 consumers on their attitude toward our new product. Because the people in this sample were so enthusiastic in their desire to purchase the product, we have decided to roll out a nationwide advertising campaign.

Concepts and Applications

Population, Sample, Statistic, and Parameter. Exercises 11–14 each describe a statistical study. In each case, identify the sample, the population, the sample statistic, and the population parameter.

- 11. Smoking Poll.** In a Gallup poll of 1,018 adults in the United States, it was found that 22% smoked cigarettes in the past week.
 - 12. Birth Weights.** For 186 randomly selected babies, the average (mean) of their birth weights is 3,103 grams (based on data from “Cognitive Outcomes of Preschool Children with Prenatal Cocaine Exposure,” by Singer et al., *Journal of the American Medical Association*, Vol. 291, No. 20).
 - 13. Garlic and Cholesterol.** In a test of the effectiveness of garlic for lowering cholesterol, 47 adult subjects were treated with Garlicin, which is garlic in a processed tablet form. Cholesterol levels were measured before and after the treatment. The changes in their levels of LDL cholesterol (in mg/dL) have an average (mean) of 3.2 (based on data from “Effect of Raw Garlic vs Commercial Garlic Supplements on Plasma Lipid Concentrations in Adults With Moderate Hypercholesterolemia,” by Gardner et al., *Archives of Internal Medicine*, Vol. 167).
 - 14. Job Interview Mistakes.** In an Accountemps survey of 150 senior executives, 47% said that the most common job interview mistake is to have little or no knowledge of the company.
- Identifying the Range of Values.** In Exercises 15–18, use the given statistics and margin of error to identify the range of values (confidence interval) likely to contain the true value of the population parameter.
- 15. Wake Up.** A Braun Research poll asked 1,000 office workers how they wake up in time for work; 60% of them said that they use an alarm clock. The margin of error of was 3 percentage points.
 - 16. Wash Up.** *USA Today* reported that among 6,028 adults observed in restrooms, 85% washed their hands. The margin of error was 1 percentage point.
 - 17. Claim to Wash Up.** In a Harris Interactive survey of 1,006 adults, 96% say that they wash their hands when in a public restroom. The margin of error was 3 percentage points.
 - 18. Body Temperatures.** One hundred and six adults are randomly selected and tested for their body temperatures. Based on that sample, researchers estimated that the average (mean) body temperature is 98.2°F with a margin of error of 0.1°F .
 - 19. Global Warming Poll.** A Pew Research Center poll asked 1,708 randomly selected adults whether “global warming is a problem that requires immediate government action.” Results

showed that 55% of those surveyed said yes. The margin of error was 2 percentage points. Can a news reporter safely write that the majority (more than 50%) of people believe that immediate government action is required?

20. Nielsen Survey. In a survey of 25,047 Super Bowl viewers, 51% of them said that they enjoyed commercials more than the game. The margin of error is 0.6%. Can we conclude that the majority of Super Bowl viewers prefer commercials to the game? Why or why not?

21. Do People Lie About Voting? In a survey of 1,002 people, 701 (or 70%) said that they voted in the last presidential election (based on data from ICR Research Group). The margin of error was 3 percentage points. However, actual voting records show that only 61% of all eligible voters actually voted. Does this imply that people lied when they responded in the survey? Explain.

22. Why the Discrepancy? An Eagleton Institute poll asked men if they agreed with this statement: “Abortion is a private matter that should be left to women to decide without government intervention.” Among the men who were interviewed by women, 77% agreed with the statement. Among the men who were interviewed by men, 70% agreed with the statement. Assuming that the discrepancy is significant, how might that discrepancy be explained?

Interpreting Real Studies. For each of Exercises 23–26, do the following:

- Based on the given information, state what you think was the goal of the study. Identify a possible population and the population parameter of interest.
- Briefly describe the sample, raw data, and sample statistic for the study.
- Based on the sample statistic and the margin of error, identify the range of values (confidence interval) likely to contain the population parameter of interest.

23. Death Penalty. A Gallup poll asked 511 randomly selected adults if they favor the death penalty for a person convicted of murder, with 64% saying yes. The margin of error is 4 percentage points.

24. Prescription Drugs. A study of 3,005 adults ages 57 to 85 showed that 82% of them use at least one prescription drug. The margin of error is 2 percentage points (based on data from “Use of Prescription and Over-the-Counter Medications and Dietary Supplements Among Older Adults in the United States,” by Qato, et al., *Journal of the American Medical Association*, Vol. 300, No. 24).

25. Super Bowl. In a recent Super Bowl, a Nielsen report found that 45% of the 9,000 surveyed U.S. households had TV sets tuned to the game. The margin of error is 1 percentage point.

26. Piercings and Tattoos. A Harris Interactive survey of 514 human resources professionals showed that 46% of them say that piercings or tattoos make job applicants less likely to be hired. The margin of error for the survey was 4 percentage points.

Five Steps in a Study. Describe how you would apply the five basic steps in a statistical study (as listed in the box shown previously) to the issues in Exercises 27–30.

27. Cell Phones and Driving. You want to determine the percentage of drivers who use cell phones while they are driving.

28. Credit Scores. FICO (Fair Isaac Corporation) scores are routinely used to rate the quality of consumer credit. You want to determine the average (mean) FICO score of an adult in the United States.

29. Passenger Weight. Recognizing that overloading commercial aircraft would lead to unsafe flights, you want to determine the average (mean) weight of airline passengers.

30. Pacemaker Batteries. Because the batteries used in heart pacemakers are so critically important, you want to determine the average (mean) length of time that such batteries last before failure.

PROJECTS FOR THE INTERNET & BEYOND

31. Current Nielsen Ratings. Find the Nielsen ratings for the past week. What were the three most popular television shows? Explain the meaning of the “rating” and the “share” for each show.

32. Nielsen Methods. Nielsen Media Research supplies statistical data that is very important to advertisers and marketers. Visit the Nielsen Web site and read about one or more of their data products. Write a brief report on how they collect the data and how the data are used.

33. Comparing Airlines. The U.S. Department of Transportation routinely publishes on-time performance, lost baggage rates, and other statistics for different airline companies. Find a recent example of such statistics. Based on what you find, is it fair to say that any particular airline stands out as better or worse than others? Explain.

34. Labor Statistics. Use the Bureau of Labor Statistics Web site to find monthly unemployment rates over the past 12 months. If you assume that the monthly survey has a margin of error of about 0.2 percentage point, has there been a noticeable change in the unemployment rate over the past year? Explain.

35. Statistics and Safety. Identify a study that has been done (or should be done) to improve the safety of car drivers and passengers. Briefly describe the importance of statistics to the study.

36. Pew Research Center. The Pew Research Center for the People and the Press studies public attitudes toward the press, politics, and public policy issues. Go to its Web site and find the latest survey about attitudes. Select a particular recent survey, and write a summary of what was surveyed, how the survey was conducted, and what was found.

IN THE NEWS

- 37. Statistics in the News.** Identify three stories from the past week that involve statistics in some way. In each case, write a brief statement describing the role of statistics in the story.
- 38. Statistics in Your Major.** Write a brief description of some ways in which you think that the science of statistics can be used in your major field of study. (If you have not yet selected a major, answer the question for a major that you are considering.)
- 39. Statistics and Entertainment.** The Nielsen ratings are well known for their role in gauging television viewing. Identify another way that statistics are used in the entertainment industry. Briefly describe the role of statistics in this application.
- 40. Statistics in Sports.** Choose a sport and describe at least three different statistics commonly tracked by participants in or spectators of the sport. In each case, briefly describe the importance of the statistic to the sport.
- 41. Economic Statistics.** The government regularly publishes many different economic statistics, such as the unemployment rate, the inflation rate, and the surplus or deficit in the federal budget. Study recent newspapers and identify five important economic statistics. Briefly explain the purpose of each of these five statistics.

2

SAMPLING

Not everything that can be counted counts, and not everything that counts can be counted.

—Albert Einstein

Cách duy nhất để biết giá trị thực của tham số tổng thể là quan sát mọi thành phần của tổng thể. Ví dụ: để biết chính xác chiều cao trung bình của tất cả học sinh ở trường của bạn, bạn cần phải đo chiều cao của mỗi học sinh. Một tập hợp dữ liệu từ mọi thành viên của một tổng thể được gọi là **điều tra tổng thể**. Thật không may, việc tiến hành điều tra tổng thể thường không thực tế. Trong một số trường hợp, tổng thể quá lớn nên sẽ quá tốn kém hoặc tốn thời gian để thu thập dữ liệu từ mọi thành phần. Trong các trường hợp khác, một cuộc điều tra tổng thể có thể bị loại trừ vì nó sẽ cản trở mục tiêu tổng thể của một nghiên cứu. Ví dụ, một nghiên cứu được thiết kế để kiểm tra chất lượng của các thanh kẹo trước khi vận chuyển không thể liên quan đến điều tra tổng thể vì điều đó có nghĩa là thử nghiệm một phần của mỗi thanh kẹo, không để lại cái nào vnguyên vẹn để bán.

Definition

Điều tra tổng thể là việc thu thập dữ liệu từ mọi thành viên của tổng thể.

May mắn thay, hầu hết các nghiên cứu thống kê có thể được thực hiện mà không gặp rắc rối khi tiến hành điều tra tổng thể. Thay vì thu thập dữ liệu từ mọi thành viên, chúng ta thu thập dữ liệu từ một mẫu và sử dụng số liệu thống kê mẫu để suy luận về tổng thể. Tất nhiên, các suy luận sẽ chỉ hợp lý nếu các thành viên của mẫu đại diện cho tổng thể công bằng, ít nhất là về các đặc điểm được nghiên cứu. Đó là, chúng ta tìm kiếm một **mẫu đại diện** của tổng thể.

Definition

Một **mẫu đại diện** là một mẫu trong đó các đặc điểm liên quan của các thành phần mẫu nói chung giống với đặc điểm của tổng thể.

EXAMPLE 1 A Representative Sample for Heights

Giả sử bạn muốn xác định chiều cao trung bình của tất cả học sinh ở trường của bạn. Cái nào nhiều khả năng có thể là mẫu đại diện cho nghiên cứu này: đội bóng rổ nam hoặc học sinh trong lớp thống kê của bạn?

SOLUTION Đội bóng rổ nam không phải là mẫu đại diện cho nghiên cứu về chiều cao, vì nó chỉ bao gồm nam giới và cũng bởi các cầu thủ bóng rổ có xu hướng cao hơn so với trung bình. Chiều cao trung bình của các học sinh trong lớp thống kê của bạn có nhiều khả năng gần bằng với chiều cao trung bình của tất cả học sinh, vì vậy các thành viên trong lớp của bạn đại diện mẫu hơn các thành viên của đội bóng rổ nam.

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Bias

Hãy tưởng tượng rằng, đối với 5.000 ngôi nhà trong mẫu của mình, Nielsen chỉ chọn những ngôi nhà mà những người làm công nhân làm ca đêm. Vì người làm khuya không ở có nhà để xem truyền hình đêm khuya, Nielsen sẽ thấy các chương trình đêm khuya không được ưa chuộng trong các gia đình trong mẫu này. Rõ ràng, mẫu này sẽ không đại diện cho tất cả các hộ gia đình ở Mỹ, và nó sẽ là sai lầm khi kết luận rằng các buổi biểu diễn đêm khuya không được tất cả người Mỹ ưa chuộng. Chúng ta nói rằng một mẫu như vậy bị thiên vị vì những ngôi nhà trong mẫu khác nhau theo một cách cụ thể từ những ngôi nhà Mỹ "điển hình". (Trên thực tế, Nielsen rất cẩn thận để tránh những thiên vị trong việc lựa chọn mẫu.) Tổng quát hơn, thuật ngữ **thiên vị** đề cập đến bất kỳ vấn đề nào trong thiết kế hoặc tiến hành một nghiên cứu thống kê có xu hướng ủng hộ kết quả nhất định. Chúng ta không thể tin tưởng vào kết luận của một nghiên cứu thiên vị.

Definition

Một nghiên cứu thống kê bị **thiên vị** nếu thiết kế hoặc hành vi của nó có xu hướng ủng hộ một số kết quả nhất định.

Thiên vị có thể phát sinh theo nhiều cách. Ví dụ:

- Một mẫu bị thiên vị nếu các thành phần của mẫu khác với các thành phần chung của tổng thể theo một số cách cụ thể. Trong trường hợp đó, kết quả nghiên cứu sẽ phản ánh đặc điểm khác thường của mẫu chứ không phải là đặc điểm thực tế của tổng thể.
- Một nhà nghiên cứu là thiên vị nếu người đó có lợi ích cá nhân trong một kết quả cụ thể. Trong trường hợp đó, nhà nghiên cứu có thể cố ý hoặc vô ý bóp méo ý nghĩa thực sự của dữ liệu.
- Bản thân tập dữ liệu bị thiên vị nếu các giá trị của nó được thu thập một cách có chủ ý hoặc không chủ ý trong một cách làm cho dữ liệu không đại diện cho tổng thể.
- Ngay cả khi một nghiên cứu được thực hiện tốt, nó vẫn có thể được báo cáo một cách sai lệch. Ví dụ, một đồ thị đại diện cho dữ liệu có thể chỉ kể một phần của câu chuyện hoặc mô tả dữ liệu theo cách sai lệch.

Ngăn chặn thiên vị là một trong những thách thức lớn nhất trong nghiên cứu thống kê. Tìm kiếm sự thiên vị do đó là một trong những bước quan trọng nhất trong việc đánh giá một nghiên cứu thống kê hoặc báo cáo phương tiện truyền thông về một nghiên cứu thống kê.

EXAMPLE 2 Why Use Nielsen?

Nielsen Media Research kiểm tiền bằng cách tính phí các đài truyền hình và mạng cho dịch vụ của nó. Ví dụ: NBC trả tiền cho Nielsen để cung cấp xếp hạng cho các chương trình truyền hình của họ. Tại sao NBC không đơn giản thực hiện xếp hạng của riêng mình, thay vì trả tiền cho một công ty như Nielsen?

SOLUTION Chi phí quảng cáo trên một chương trình truyền hình phụ thuộc vào xếp hạng của chương trình. Xếp hạng càng cao, nhà mạng có thể tính phí quảng cáo càng cao—có nghĩa là NBC sẽ có một sự thiên vị rõ ràng nếu nó tiến hành xếp hạng của riêng mình. Do đó, các nhà quảng cáo sẽ không tin tưởng xếp hạng do NBC tự sản xuất. Bằng cách thuê một nguồn độc lập, chẳng hạn như Nielsen, NBC có thể cung cấp thông tin mà các nhà quảng cáo có nhiều khả năng tin tưởng hơn.

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BY THE WAY

Nhiều nghiên cứu y học là những thí nghiệm được thiết kế để kiểm tra xem liệu một thuốc mới có hiệu nghiệm. Trong một bài viết đăng trên Tạp chí Khoa học Hiệp hội y tế Hoa Kỳ, các tác giả nhận thấy rằng các nghiên cứu có kết quả khả quan (thuốc là hiệu quả) có nhiều khả năng được xuất bản hơn các nghiên cứu với kết quả âm tính (thuốc là không hiệu quả). "Sự xuất bản thiên vị" này có xu hướng tạo ra các loại thuốc mới, với tư cách là một nhóm, có vẻ hiệu quả hơn so với thực tế.



TIME OUT TO THINK

Việc NBC trả tiền cho Nielsen cho các dịch vụ của mình dường như mang lại cho Nielsen động cơ tài chính để khiến NBC có xếp hạng cao. Nếu bạn đã làm việc cho một công ty quảng cáo, những yếu tố nào khác có thể giúp bạn tin tưởng vào xếp hạng của Nielsen?

Sampling Methods

Một nghiên cứu thống kê tốt phải có một mẫu đại diện. Mặt khác, mẫu là thiên vị và kết luận từ nghiên cứu là không đáng tin cậy. Hãy xem xét một vài phương pháp lấy mẫu phổ biến, ít nhất về nguyên tắc, có thể cung cấp một mẫu đại diện.

Simple Random Samples

Trong hầu hết các trường hợp, cách tốt nhất để lấy mẫu đại diện là chọn ngẫu nhiên từ tổng thể. Mẫu ngẫu nhiên là mẫu trong đó mọi thành viên của tổng thể đều có một cơ hội được chọn vào mẫu. Ví dụ, bạn có thể nhận được một ngẫu nhiên lấy mẫu bằng cách cho mọi người trong một quần thể tung xúc xắc và chọn những người tung được mặt 6. Ngược lại, mẫu sẽ không ngẫu nhiên nếu bạn chọn tất cả những người cao hơn 6 feet, bởi vì mọi người sẽ không có cơ hội bình đẳng để được lựa chọn.

Trong thống kê, chúng ta thường quyết định trước cỡ mẫu cần thiết với lấy mẫu ngẫu nhiên đơn giản, mọi mẫu có thể có kích thước cụ thể đều có cơ hội được chọn như nhau. Ví dụ, để chọn một mẫu ngẫu nhiên đơn gồm 100 sinh viên từ tất cả các sinh viên trong trường của bạn, bạn có thể chỉ định một số cho mỗi học sinh trong trường của bạn và chọn mẫu bằng cách vẽ 100 số này từ một chiếc mũ. Miễn là số của mỗi học sinh có trong mũ chỉ một lần, mọi mẫu gồm 100 sinh viên đều có cơ hội được chọn như nhau. Như một cách nhanh hơn thay vì sử dụng mũ, bạn có thể chọn số học sinh với sự trợ giúp của máy tính hoặc máy tính có trình tạo số ngẫu nhiên tích hợp.

TIME OUT TO THINK

Tìm phím số ngẫu nhiên trên máy tính. (Gần như tất cả các máy tính khoa học đều có một.) Điều gì xảy ra khi bạn đẩy nó? Làm thế nào bạn có thể sử dụng phím số ngẫu nhiên để chọn một mẫu gồm 100 sinh viên?

Bởi vì lấy mẫu ngẫu nhiên đơn giản cho phép mọi mẫu cùng kích thước cụ thể có cơ hội được chọn như nhau, nó có khả năng cung cấp một mẫu đại diện, miễn là kích thước mẫu đủ lớn.

EXAMPLE 3 Local Resident Sampling

Bạn muốn tiến hành một cuộc thăm dò ý kiến trong đó tổng thể là tất cả cư dân trong một thị trấn. Bạn có thể chọn một mẫu ngẫu nhiên đơn giản bằng cách chọn ngẫu nhiên tên từ báo cáo thuế tài sản địa phương không?

SOLUTION Một mẫu được rút ra từ hồ sơ thuế bất động sản không phải là một mẫu ngẫu nhiên đơn giản của dân số thị trấn bởi vì những hồ sơ này sẽ chỉ liệt kê những người sở hữu tài sản trong thị trấn. Do đó, những hồ sơ này bị thiếu nhiều cư dân thị trấn và cũng có thể bao gồm những người những người sống ở nơi khác nhưng sở hữu tài sản trong thị trấn.

Systematic Sampling

Lấy mẫu ngẫu nhiên đơn giản có hiệu quả, nhưng trong nhiều trường hợp, chúng ta có thể nhận được kết quả tốt như nhau với một kỹ thuật đơn giản hơn. Giả sử bạn đang kiểm tra chất lượng của vi mạch do Intel sản xuất chip lắn ra khỏi dây chuyền lắp ráp, bạn có thể quyết định kiểm tra mọi chip thứ 50. Điều này nên cung cấp m

mẫu đại diện vì không có lý do gì để tin rằng mọi con chip thứ 50 đều có bất kỳ đặc điểm đặc biệt nào so với các chip khác. Loại lấy mẫu này, trong đó chúng tôi sử dụng một hệ thống chẳng hạn như chọn mọi thành viên thứ 50 của tổng thể, được gọi là **lấy mẫu có hệ thống**.

EXAMPLE 4 Museum Assessment

Khi Bảo tàng Không gian muốn thử nghiệm những ý tưởng khả thi cho một triển lãm Hệ mặt trời mới, một nhân viên đã phỏng vấn một mẫu khách tham quan được chọn bằng phương pháp lấy mẫu có hệ thống. Cô ấy đã phỏng vấn vị khách cứ sau 15 phút, chọn bất kỳ ai tình cờ bước vào triển lãm tại thời điểm đó. Bạn nghĩ tại sao cô ấy chọn lấy mẫu có hệ thống hơn là lấy mẫu ngẫu nhiên đơn giản? Lấy mẫu hệ thống có khả năng tạo ra mẫu đại diện trong trường hợp này không?

SOLUTION Lấy mẫu ngẫu nhiên đơn giản đôi khi có thể chọn hai khách tham quan đến ngay sau nhau mà nhân viên sẽ không có thời gian để phỏng vấn từng người trong số họ. Quá trình chọn khách sau 15 phút có hệ thống đã ngăn chặn vấn đề này. Bởi vì không có lý do gì để nghĩ rằng những người bước vào một thời điểm cụ thể là khác với những người vào sớm hơn hoặc muộn hơn vài phút, quá trình này có thể mang lại mẫu đại diện cho tổng thể du khách trong thời gian lấy mẫu.



EXAMPLE 5 When Systematic Sampling Fails

Bạn đang tiến hành một cuộc khảo sát về các sinh viên trong ký túc xá, trong đó nam giới được chỉ định phòng số lẻ, nữ được xếp vào phòng số chẵn. Bạn có thể có được một mẫu đại diện khi bạn chọn mỗi phòng thứ 10?

SOLUTION Không. Nếu bạn bắt đầu với một phòng lẻ, mọi phòng thứ 10 cũng sẽ là số lẻ (chẳng hạn như phòng số 3, 13, 23, p). Tương tự, nếu bạn bắt đầu với một số chẵn phòng, mỗi phòng thứ 10 cũng sẽ là chẵn. Do đó, bạn sẽ có được một mẫu bao gồm của tất cả nam hoặc tất cả nữ, không ai trong số họ là đại diện của dân số ký túc.



TIME OUT TO THINK

Giả sử bạn chọn mọi phòng thứ 5, thay vì mọi phòng thứ 10, trong Ví dụ 5. Liệu mẫu sau đó có đại diện?

BY THE WAY

Việc lấy mẫu được mô tả trong Ví dụ 4 đã được thực hiện trước để xây dựng các chuyến tham quan mô hình Hệ mặt trời, một mô hình trưng bày trải dọc theo National Mall từ Hàng không và Không gian Quốc gia Bảo tàng Smithsonian Lâu đài. Bức ảnh dưới đây cho thấy con trai của một trong những người sáng tạo triển lãm (cũng là tác giả của văn bản này) chạm vào mô hình Mặt trời.



Convenience Samples

Lấy mẫu hệ thống dễ hơn lấy mẫu ngẫu nhiên đơn giản nhưng cũng có thể không thực tế trong nhiều trường hợp. Ví dụ, giả sử bạn muốn biết tỷ lệ học sinh thuận tay trái ở trường của bạn. Sẽ mất nhiều công sức để chọn một mẫu ngẫu nhiên đơn giản hoặc một mẫu có hệ thống, vì cả hai đều yêu cầu rút ra từ tất cả học sinh trong trường. Ngược lại, nó sẽ dễ dàng sử dụng các sinh viên trong lớp thống kê của bạn làm mẫu — bạn chỉ cần hỏi học sinh thuận tay trái giơ tay. Loại mẫu này được gọi là **mẫu thuận tiện** vì nó được chọn để thuận tiện hơn là bởi một thủ tục phức tạp hơn. Để thử tìm tỷ lệ người thuận tay trái, mẫu tiện lợi của lớp thống kê của bạn có lẽ là tốt; không có lý do gì để nghĩ rằng sẽ có một tỷ lệ học sinh thuận tay trái trong một lớp thống kê khác với bất kỳ nơi nào khác. Nhưng nếu bạn đang cố xác định tỷ lệ sinh viên theo các chuyên ngành khác nhau, mẫu này sẽ bị sai lệch vì một số chuyên ngành yêu cầu một khóa học thống kê và những người khác thì không. Nhìn chung, lấy mẫu thuận tiện có xu hướng dễ bị sai lệch hơn so với hầu hết các hình thức lấy mẫu khác.

EXAMPLE 6 Salsa Taste Test

Một siêu thị muốn quyết định có nên bán một nhãn hiệu salsa mới hay không, vì vậy siêu thị cung cấp các món ăn miễn phí trong cửa hàng và hỏi mọi người xem họ nghĩ gì. Loại lấy mẫu nào đang được sử dụng? Là mẫu có thể đại diện cho tập hợp của tất cả những người mua sắm không?

SOLUTION Mẫu người mua sắm dừng lại để nếm thử món salsa là một mẫu thuận tiện.

bởi vì những người này tình cờ có mặt trong cửa hàng và sẵn sàng dùng thử sản phẩm mới. (đây là mẫu thuận tiện, trong đó mọi người chọn có tham gia vào mẫu hay không, còn được gọi là mẫu tự chọn. Chúng ta sẽ nghiên cứu thêm về các mẫu tự chọn trong Phần 4.) Mẫu này khó có thể đại diện cho tổng thể của tất cả những người mua sắm, bởi vì những kiểu người khác nhau có thể mua sắm vào những thời điểm khác nhau (ví dụ: cha mẹ ở nhà thường mua sắm vào buổi trưa hơn là cha mẹ đi làm) và chỉ những người thích salsa mới có khả năng tham gia. Tuy nhiên, dữ liệu có thể vẫn hữu ích vì ý kiến của những người thích salsa có lẽ là những thứ quan trọng nhất trong trường hợp này.

Cluster Samples

Cluster sampling liên quan đến việc chọn tất cả các thành viên trong các nhóm được lựa chọn, ngẫu nhiên hay cụm. Hãy tưởng tượng rằng bạn làm việc cho Bộ Nông nghiệp và muốn xác định tỷ lệ nông dân sử dụng kỹ thuật canh tác hữu cơ. Sẽ rất khó khăn và tốn kém nếu thu thập một mẫu ngẫu nhiên đơn giản hoặc một mẫu có hệ thống vì sẽ yêu cầu truy cập nhiều trang trại riêng lẻ nằm cách xa nhau. Một mẫu thuận tiện của nông dân trong một quận duy nhất sẽ bị sai lệch vì các hoạt động canh tác khác nhau giữa các vùng. Do đó có thể quyết định chọn ngẫu nhiên vài chục quận trên khắp Hoa Kỳ và khảo sát mọi nông dân ở mỗi quận đó. Ta nói rằng mỗi vùng chứa một cụm nông dân và mẫu bao gồm mọi nông dân trong các cụm được chọn ngẫu nhiên.

EXAMPLE 7 Gasoline Prices

Bạn muốn biết giá xăng trung bình tại các trạm xăng cách xe thuê tại các sân bay trong vòng 1 dặm. Giải thích cách bạn có thể sử dụng lấy mẫu cụm trong trường hợp này.

SOLUTION Bạn có thể chọn ngẫu nhiên một vài sân bay trên cả nước. Đối với những nơi này bạn kiểm tra giá xăng tại mọi trạm xăng trong vòng một dặm xung quanh vị trí thuê xe.

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BY THE WAY

As mandated by the U.S. Constitution, voting for the President is actually done by a small group of people called *electors*. Each state may select as many electors as it has members of Congress (counting both senators and representatives). When you cast a ballot for President, you actually cast a vote for your state's electors, each of whom has promised to vote for a particular presidential candidate. The electors cast their votes a few weeks after the general election.



Stratified Samples

Giả sử bạn đang tiến hành một cuộc thăm dò để dự đoán kết quả của nhiệm kỳ tổng thống tiếp theo của bầu cử Hoa Kỳ. Tổng thể được nghiên cứu đều có khả năng là cử tri, vì vậy bạn có thể chọn một mẫu ngẫu nhiên đơn giản từ tổng thể này. Tuy nhiên, vì cuộc bầu cử tổng thống được quyết định bởi phiếu đại cử tri được bầu trên cơ sở từng tiểu bang, bạn sẽ có dự đoán tốt hơn nếu bạn xác định sở thích của cử tri trong mỗi tiểu bang. Do đó, mẫu tổng thể của bạn phải bao gồm các mẫu ngẫu nhiên có tỷ lệ riêng biệt từ mỗi trong số 50 tiểu bang. Theo thuật ngữ thống kê, dân số trong số 50 tiểu bang đại diện cho các nhóm nhỏ, hoặc **tầng lớp**, của tổng dân số. Bởi vì **tổng thể** của bạn mẫu bao gồm các thành viên được chọn ngẫu nhiên từ mỗi tầng, bạn đã sử dụng **lấy mẫu phân tầng**.

EXAMPLE 8 Unemployment Data

Bộ Lao động Hoa Kỳ khảo sát 60.000 hộ gia đình mỗi tháng để tổng hợp báo cáo thất nghiệp (xem Ví dụ 2 trong Phần 1). Để chọn được những hộ này, trước hết sở nhóm các thành phố và quận thành khoảng 2.000 khu vực địa lý. Sau đó, nó chọn ngẫu nhiên các hộ gia đình để khảo sát trong các khu vực địa lý này. Làm thế nào đây là một ví dụ về lấy mẫu phân tầng? Các tầng là gì? Tại sao lấy mẫu phân tầng lại quan trọng trong trường hợp này?

SOLUTION Khảo sát thất nghiệp là một ví dụ về lấy mẫu phân tầng vì nó chia dân số thành các nhóm nhỏ. Các nhóm con, hay **tầng lớp**, là những người trong 2.000 vùng địa lý. Lấy mẫu phân tầng rất quan trọng trong trường hợp này vì tỷ lệ thất nghiệp có thể khác nhau ở các vùng địa lý khác nhau. Ví dụ, thất nghiệp tỷ lệ ở vùng nông thôn Kansas có thể rất khác so với ở Thung lũng Silicon. Bằng cách sử dụng lấy mẫu phân tầng, Bộ Lao động đảm bảo rằng mẫu của họ đại diện cho tất cả các khu vực địa lý.

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Summary of Sampling Methods

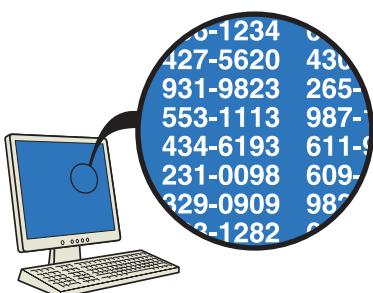
Hộp dưới đây và Hình 3 tóm tắt năm phương pháp lấy mẫu mà chúng ta đã thảo luận. Không có phương pháp đơn lẻ nào là “tốt nhất”, vì mỗi phương pháp đều có công dụng của nó. (Một số nghiên cứu thậm chí kết hợp hai hoặc nhiều loại lấy mẫu.)

Nhưng bất kể mẫu được chọn như thế nào, hãy ghi nhớ ba ý chính sau:

- Một nghiên cứu chỉ có thể thành công nếu mẫu đại diện cho dân số.
- Mẫu sai lệch không chắc là mẫu đại diện.
- Ngay cả một mẫu được chọn tốt vẫn có thể không mang tính đại diện chỉ vì không may mắn trong bản vẽ thực tế của mẫu.

Common Sampling Methods

- Lấy mẫu ngẫu nhiên đơn giản: Chúng tôi chọn một mẫu các mặt hàng sao cho mọi mẫu đều giống nhau size có cơ hội được chọn như nhau.
- Chọn mẫu có hệ thống: Chúng tôi sử dụng một hệ thống đơn giản để chọn mẫu, chẳng hạn như chọn cứ sau 10 hoặc mỗi thành viên thứ 50 của dân số.
- Lấy mẫu thuận tiện: Chúng tôi sử dụng một mẫu thuận tiện để chọn.
- Lấy mẫu theo cụm: Trước tiên, chúng tôi chia tổng thể thành các nhóm hoặc cụm và chọn một số trong số này cụm một cách ngẫu nhiên. Sau đó, chúng tôi lấy mẫu bằng cách chọn tất cả các thành viên trong mỗi các cụm đã chọn.
- Lấy mẫu phân tầng: Chúng tôi sử dụng phương pháp này khi lo ngại về sự khác biệt giữa các nhóm nhỏ hoặc tầng lớp trong một tổng thể. Đầu tiên chúng tôi xác định các tầng và sau đó rút một mẫu ngẫu nhiên bên trong mỗi tầng. Tổng mẫu bao gồm tất cả các mẫu từ các tầng riêng lẻ.



Simple Random Sampling:

Every sample of the same size has an equal chance of being selected. Computers are often used to generate random numbers.



Systematic Sampling:

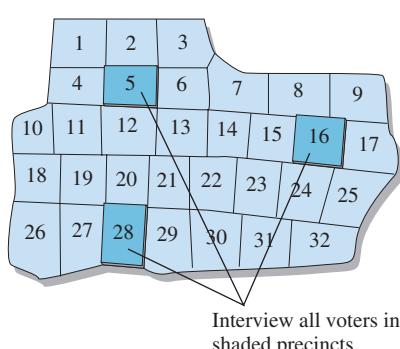
Select every k th member.



Convenience Sampling:

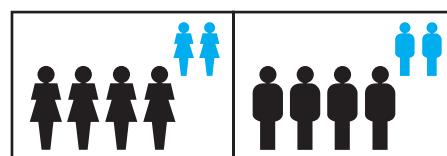
Use results that are readily available.

Election precincts in Carson County



Cluster Sampling:

Divide the population into clusters, randomly select some of those clusters, then choose all members of the selected clusters.



Stratified Sampling:

Partition the population into at least two strata, then draw a sample from each.

Figure 3 Common sampling methods.

EXAMPLE 9 Sampling Methods

Xác định loại lấy mẫu được sử dụng trong mỗi trường hợp sau.

- Thu hoạch táo từ một vườn cây ăn quả được thu thập trong 1.200 giờ. Một thanh tra nông nghiệp chọn ngẫu nhiên 25 giờ và sau đó kiểm tra từng quả táo trong mỗi giờ này để tìm giun.
- Một nhà nghiên cứu giáo dục muốn biết, tại một trường đại học cụ thể, nam hay nữ có xu hướng đặt nhiều câu hỏi hơn trong lớp. Trong số 10.000 sinh viên tại trường đại học, cô phỏng vấn 50 nam và 50 nữ được chọn ngẫu nhiên.
- Khi cố gắng tìm hiểu về các hệ hành tinh, các nhà thiên văn học tiến hành một cuộc khảo sát bằng cách tìm hành tinh trong số 100 ngôi sao lân cận.
- Để xác định ai sẽ giành được bóng đá có chữ ký, một chương trình máy tính sẽ chọn ngẫu nhiên số vé của 11 người trong một sân vận động chật kín người.

SOLUTION

- Việc kiểm tra quả táo là một ví dụ về lấy mẫu theo cụm vì người kiểm tra bắt đầu bằng một tập hợp các cụm (giờ) được chọn ngẫu nhiên và sau đó kiểm tra mọi quả táo trong cụm đã chọn.
- Các nhóm nam và nữ đại diện cho hai tầng lớp khác nhau trong nghiên cứu này, vì vậy đây là một ví dụ về lấy mẫu phân tầng.
- Các nhà thiên văn học có lẽ tập trung vào các ngôi sao gần đó vì chúng dễ nghiên cứu hơn trong điều kiện trường hợp, vì vậy đây là một ví dụ về lấy mẫu thuận tiện.
- Vì máy tính chọn 11 số vé một cách ngẫu nhiên nên mỗi số vé có một cơ hội được chọn như nhau. Đây là một ví dụ về lấy mẫu ngẫu nhiên đơn giản.

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Section 2 Exercises

Statistical Literacy and Critical Thinking

- Census and Sample.** What is a *census*, what is a sample, and what is the difference between them?
- Biased Sample.** In a survey before a general election (as opposed to a primary), a sample is drawn randomly from a list of registered Democrats. Is there anything wrong with this sampling method?
- Cluster and Stratified Sampling.** Cluster sampling and stratified sampling both involve selecting subjects in subgroups of the population. What is the difference between those two types of sampling?
- Sample of Students.** A college statistics teacher conducted a study by recording whether each student in his class was right-handed. The objective was to form a conclusion about the proportion of college students that is right-handed. What type of sample was obtained? Is this sample likely to be biased? Why or why not?

Does It Make Sense? For Exercises 5–8, decide whether the statement makes sense (or is clearly true) or does not make sense (or is clearly false). Explain clearly; not all of these have

definitive answers, so your explanation is more important than your chosen answer.

5. Graduation Age. For a statistics class project, I conducted a census to determine the mean age of students when they earn their bachelor's degrees.

6. Convenience Sample. For a statistics class project, I used a convenience sample, but the results may still be meaningful.

7. Biased Sample. The study must have been biased, because it concluded that 75% of Americans are more than 6 feet tall.

8. Death Row. There are currently 3,242 convicts on death row (based on 2010 data from the Bureau of Justice Statistics). We obtained a simple random sample of those convicts by compiling a numbered list, then using a computer to randomly generate 20 numbers between 1 and 3,242, then selecting the convicts that correspond to the generated numbers.

Concepts and Applications

Census. In Exercises 9–12, determine whether a census is practical in the situations described. Explain your reasoning.

9. Laker Heights. You want to determine the mean height of all basketball players on the LA Lakers team.

10. High School Heights. You want to determine the mean height of all high school basketball players in the United States.

11. IQ Scores. You want to determine the mean IQ score of all statistics instructors in the United States.

12. Instructor Ages. You want to determine the mean age of all statistics instructors at the University of Colorado.

Representative Samples? In Exercises 13–16, identify the sample, population, and sampling method. Then comment on whether you think it is likely that the sample is representative of the population.

13. Senate Terms. A political scientist randomly selects 4 of the 100 senators currently serving in Congress, then finds the lengths of time that they have served.

14. Super Bowl. During the Super Bowl game, Nielsen Media Research conducts a survey of 5,108 randomly selected households and finds that 44% of them have television sets tuned to the Super Bowl.

15. Gun Ownership. In a Gallup poll of 1,059 randomly selected adults, 39% answered “yes” when asked “Do you have a gun in your home?”

16. Mail Survey. A graduate student conducts a research project about how adult Americans send thank-you notes. She uses the U.S. Postal Service to mail a survey to 500 adults that she knows and asks them to mail back a response to this question: “Do you prefer to send thank you notes by e-mail or snail mail (the U.S. Postal Service)?” She gets back 65 responses, with 42 of them indicating a preference for snail mail.

Evaluate the Sample Choices. Exercises 17–18 each describe the goal of a study, then offer you four options for obtaining a sample. In each case, decide which sample is most likely to be a representative sample, and explain why. Then explain why each of the other choices is *not* likely to make a representative sample for the study.

17. Credit Card Debt. You want to determine the mean amount of credit card debt owed by adult consumers in Florida.

- Sample 1: The Florida drivers who own and have registered Land Rover vehicles
- Sample 2: The first 1,000 Florida residents listed in the Fort Lauderdale phone book
- Sample 3: The first 1,000 Florida residents in a complete list of all Florida telephone numbers
- Sample 4: The Florida residents who mail back a survey printed in the *Miami Herald*

18. California Voters. You want to conduct a survey to determine the proportion of eligible voters in California likely to vote for the Democratic presidential candidate in the next election.

- Sample 1: All eligible voters in San Diego County
- Sample 2: All eligible voters in the city of Sonoma
- Sample 3: All eligible voters who respond to a CNN Internet survey
- Sample 4: Every 1,000th person on a complete list of all eligible voters in California

Bias. Are there sources of bias in the situations described in Exercises 19–22? Explain.

19. Movie Critic. A film critic for ABC News gives her opinion of the latest movie from Disney, which also happens to own ABC.

20. Car Reviews. *Consumer Reports* magazine prints a review of new cars and does not accept free products or advertising from anyone.

21. GMO Soybeans. Monsanto hires independent university scientists to determine whether its new, genetically engineered soybean poses any threat to the environment.

22. Drug Study Funding. The *Journal of the American Medical Association* prints an article evaluating a drug, and some of the physicians who wrote the article received funding from the pharmaceutical company that produces the drug.

Sampling Methods. In Exercises 23–38, identify which of the following applies: simple random sample, systematic sample, convenience sample, stratified sample, or cluster sample. In each case, state whether you think the procedure is likely to yield a representative sample or a biased sample, and explain why.

23. Clinical Trial. In phase II testing of a new drug designed to increase the red blood cell count, a researcher obtains envelopes with the names and addresses of all treated subjects. She wants to increase the dosage in a sub-sample of 12 subjects, so she thoroughly mixes all of the envelopes in a bin, then pulls 12 of those envelopes to identify the subjects to be given the increased dosage.

24. Sobriety Checkpoint. Police set up a sobriety checkpoint at which every fifth driver is stopped and interviewed.

25. Exit Polls. On days of presidential elections, the news media organize an exit poll in which specific polling stations are randomly selected and all voters are surveyed as they leave the premises.

26. Education and Sports. A researcher for the Spaulding athletic equipment company is studying the relationship between level of education and participation sports. She conducts a survey of 40 randomly selected golfers, 40 randomly selected tennis players, and 40 randomly selected swimmers.

- 27. Ergonomics.** An engineering student measures the strength of fingers used to push buttons by testing family members.
- 28. Tax Cheating.** An Internal Revenue Service researcher investigates false reporting of tip income by waiters and waitresses by surveying all waiters and waitresses at 20 randomly selected restaurants.
- 29. MTV Survey.** A marketing expert for MTV is planning a survey in which 500 people will be randomly selected from each age group of 10–19, 20–29, and so on.
- 30. Credit Card Data.** A professor surveyed students in her class to obtain sample data consisting of the number of credit cards students possess.
- 31. Fundraising.** Fundraisers for the College of Newport test a new telemarketing campaign by obtaining an alphabetical list of all alumni and selecting every 100th name on that list.
- 32. Telephone Poll.** In a Gallup poll of 1,059 adults, the interview subjects were selected by using a computer to randomly generate telephone numbers that were then called.
- 33. Market Research.** A market researcher has partitioned all California residents into categories of unemployed, employed full time, and employed part time. She is surveying 50 people from each category.
- 34. Student Drinking.** Motivated by a student who died from binge drinking, the College of Newport conducts a study of student drinking by randomly selecting 10 different classes and interviewing all of the students in each of those classes.
- 35. Magazine Survey.** *People* magazine chooses its “best-dressed celebrities” by compiling responses from readers who mail in a survey printed in the magazine.
- 36. Heart Transplants.** A medical researcher at Johns Hopkins University obtains a numbered list of all patients waiting for a heart transplant, then uses a computer to select the patients corresponding to the 50 numbers randomly generated by computer.
- 37. Quality Control.** A sample of manufactured CDs is obtained by using a computer to randomly generate a number between 1 and 1,000 for each CD, and the CD is selected if the generated number is 1,000.
- 38. Seat Belts.** Every 500th seat belt is tested by stressing it until it fails.
- Choose a Sampling Method.** For each of Exercises 39–42, suggest a sampling method that is likely to produce a representative sample. Explain why you chose this method over other methods.
- 39. Student Election.** You want to predict the winner of an upcoming election for student body president.
- 40. Blood Type.** You want to determine the percentage of people in this country in each of the four major blood groups (A, B, AB, and O).
- 41. Heart Deaths.** You want to determine the percentage of deaths due to heart disease each year.
- 42. Mercury in Tuna.** You want to determine the average mercury content of the tuna fish consumed by U.S. residents.
-  **PROJECTS FOR THE INTERNET & BEYOND**
- 43. Public Opinion Poll.** Use information available on the Web site of a polling organization, such as Gallup, Harris, Pew, or Yankelovich, to answer the following questions.
- How exactly is a sample of subjects selected?
 - Based on what you have learned, do you think the poll results are reliable? If so, why? If not, why not?
- 44. Unemployment Sample.** Use the Bureau of Labor Statistics Web page to find details on how the bureau chooses the sample of households in its monthly survey. Write a short summary of the procedure and why it is likely to yield a representative sample.
- 45. Selective Voting.** The Academy Awards, the Heisman Trophy, and the *New York Times* “Bestseller List” are just three examples of selections that are determined by the votes of specially selected individuals. Pick one of these selection processes, and describe who votes and how those people are chosen. Discuss sources of bias in the process.

IN THE NEWS

- 46. Sampling in the News.** Find a recent news report about a statistical study that you find interesting. Write a short summary of how the sample for the study was chosen, and briefly discuss whether you think the sample was representative of the population under study.
- 47. Opinion Poll Sample.** Find a recent news report about an opinion poll carried out by a news organization (such as Gallup, Harris, *USA Today*, *New York Times*, or CNN). Briefly describe the sample and how it was chosen. Was the sample chosen in a way that was likely to introduce any bias? Explain.
- 48. Political Polls.** Find results from a recent poll conducted by a political organization (such as the Republican or Democratic party or an organization that seeks to influence Congress on some particular issue). Briefly describe the sample and how it was chosen. Was the sample chosen in a way that was likely to introduce any bias? Should you be more concerned about bias in such a poll than you would be in a poll conducted by a news organization? Explain.

3**TYPES OF STATISTICAL STUDIES**

Statistical studies are conducted in many different ways. In all cases, the people, animals (or other living things), or objects chosen for the sample are called the **subjects** of the study. If the subjects are people, it is common to refer to them as **participants** in the study.

Definition

The **subjects** of a study are the people, animals (or other living things), or objects chosen for the sample; if the subjects are people, they may be called the **participants** in the study.

There are two basic types of statistical study: observational studies and experiments. In an **observational study**, we observe or measure specific characteristics while trying to be careful to avoid influencing or modifying the characteristics we are observing. The Nielsen ratings are an example of an observational study, because Nielsen uses devices to *observe* what the subjects are watching on TV, but does not try to influence what they watch.

Note that an observational study may involve activities that go beyond the usual definition of *observing*. Measuring people's weights requires interacting with them, as in asking them to stand on a scale. But in statistics, we consider these measurements to be observations because the interactions do not change people's weights. Similarly, an opinion poll in which researchers conduct in-depth interviews is considered observational as long as the researchers attempt only to learn people's opinions, not to change them.

In contrast, consider a medical study designed to test whether large daily doses of vitamin C help prevent colds. To conduct this study, the researchers must ask some people in the sample to take large doses of vitamin C every day. This type of statistical study is called an **experiment**. The purpose of an experiment is to study the effects of some **treatment**—in this case, large daily doses of vitamin C.

You can observe a lot by just watching.

—Yogi Berra

Two Basic Types of Statistical Study

- In an **observational study**, researchers observe or measure characteristics of the subjects, but do not attempt to influence or modify these characteristics.
- In an **experiment**, researchers apply some **treatment** and observe its effects on the subjects of the experiment.

EXAMPLE 1 Type of Study

Identify the study as an observational study or an experiment.

- a. The Salk polio vaccine study (see the Case Study given previously)
- b. A poll in which college students are asked if they commute or live on campus

SOLUTION

- a. The Salk vaccine study was an *experiment* because researchers tested a treatment—in this case, the vaccine—to see whether it reduced the incidence of polio.
- b. The poll is an *observational study* because it attempts to determine how college students go to their classes, and it does not try to influence how they get there.

**Identifying the Variables**

Statistical studies—whether observations or experiments—generally are attempts to measure what we call **variables of interest**. The term *variable* refers to an item or quantity that can vary or take on different values, and variables of interest are those we seek to learn about. For

example, variables of interest in the Nielsen studies of television viewing habits include *show being watched* and *number of viewers*. The variable *show being watched* can take on different values such as “Super Bowl” or “60 Minutes.” The variable *number of viewers* depends on the popularity of a particular show. In essence, the raw data in any statistical study are the different values of the variables of interest.

In cases where we think cause and effect may be involved, we sometimes subdivide the variables of interest into two categories. For example, each person in the study of vitamin C and colds may take a different daily dose of vitamin C and may end up with a different number of colds over some period of time. Because we are trying to learn if vitamin C causes a lower number of colds, we say that *daily dose of vitamin C* is an **explanatory variable**—it may explain or cause a change in the number of colds. Similarly, we say that *number of colds* is a **response variable**, because we expect it to respond to changes in the explanatory variable (the dose of vitamin C).

Definitions

A variable is any item or quantity that can vary or take on different values.

The **variables of interest** in a statistical study are the items or quantities that the study seeks to measure.

When cause and effect may be involved, an **explanatory variable** is a variable that may explain or cause the effect, while a **response variable** is a variable that responds to changes in the explanatory variable.

EXAMPLE 2 Identify the Variables

Identify the variables of interest for each study.

- The Salk polio vaccine study
- A poll in which college students are asked if they commute or live on campus

SOLUTION

- The two variables of interest in the Salk vaccine study are *vaccine* and *polio*. They are variables because they can take on two different values: A child either did or did not get the vaccine and either did or did not contract polio. In this case, because the study seeks to determine whether the vaccine prevents polio, we say that *vaccine* is the explanatory variable (it may explain a change in the incidence of polio) and *polio* the response variable (it is supposed to change in response to the vaccine).
- The variables of interest are the responses to the question asking how students get to their classes, and the proportions of responses to the possible different choices (such as commute, live on campus, refuse to answer, don’t know). (There is no cause and effect involved in this study, so we do not need to decide whether the variable is explanatory or response.)



Observational Studies

The observational studies we have discussed up to this point, such as Nielsen ratings, opinion polls, and determining the mean height of students, are studies in which the data are all generally collected around the same time. Sometimes, however, observational studies look at past data or are designed to look at future data over a long period of time.

A **retrospective study** (also called a *case-control* study) is an observational study that uses data from the past—such as official records or past interviews—to learn about some issue of concern. Retrospective studies are especially valuable in cases where it may be impractical or unethical to perform an experiment. For example, suppose we want to learn how alcohol consumed during pregnancy affects newborn babies. Because it is already known that

consuming alcohol during pregnancy can be harmful, it would be highly unethical to ask pregnant mothers to test the “treatment” of consuming alcohol. However, because many mothers consumed alcohol in past pregnancies (either before the dangers were known or choosing to ignore the dangers), we can do a retrospective study in which we compare children born to those mothers to children born to mothers who did not consume alcohol.

Sometimes, the data we need to reach clear conclusions are not available in past records. In those cases, researchers may set up a **prospective study** (sometimes called a *longitudinal study*) designed to collect observations in the future from groups that share common factors. A classic example of a prospective study is the Harvard Nurses’ Health Study, which was started in 1976 in order to collect data about how different lifestyles affect women’s health (see the “Focus on Public Health” section at the end of this chapter). The study, still ongoing today, has followed thousands of nurses over more than three decades, collecting data about their lifestyles and health.

Variations on Observational Studies

The most familiar observational studies are those in which data are collected all at once (or as close to that as possible). Two variations on observational studies are also common:

- A **retrospective** (or **case-control**) **study** uses data from the past, such as official records or past interviews.
- A **prospective** (or **longitudinal**) **study** is set up to collect data in the future from groups that share common factors.

EXAMPLE 3 Observational Study

You want to know whether children born prematurely do as well in elementary school as children born at full term. What type of study should you do?

SOLUTION An observational, retrospective study is the only real option in this case. You would collect data on past births and compare the elementary school performance of those born prematurely to that of those born at full term.



Experiments

Because experiments require active intervention, such as applying a treatment, we must take special care to ensure that they are designed in ways that will provide the information we seek. Let’s examine a few of the issues that arise in the design of experiments.

The Need for Controls

Consider an experiment that gives some participants in a study vitamin C to determine its effect on colds. Suppose the people taking vitamin C daily get an average of 1.5 colds in a three-month period. How can the researchers know whether the subjects would have gotten more colds without the vitamin C? To answer this type of question, the researchers must conduct their experiment with two (or more) groups of subjects: One group takes large doses of vitamin C daily and another group does not. As we’ll discuss shortly, in most cases it is important that participants be *randomly* assigned to the two groups.

The group of people who are randomly assigned to take vitamin C is called the **treatment group** because its members receive the treatment being tested (vitamin C). The group of people who do *not* take vitamin C is called the **control group**. The researchers can be confident that vitamin C is an effective treatment only if the people in the treatment group get significantly fewer colds than the people in the control group.

BY THE WAY

The control group gets its name from the fact that it helps control the way we interpret experimental results.

Treatment and Control Groups

The **treatment group** in an experiment is the group of subjects who receive the treatment being tested.

The **control group** in an experiment is the group of subjects who do *not* receive the treatment being tested.

EXAMPLE 4 Treatment and Control

Look again at the Salk polio vaccine Case Study. What was the treatment? Which group of children constituted the treatment group? Which constituted the control group?

SOLUTION The treatment was the Salk vaccine. The treatment group consisted of the children who received the Salk vaccine. The control group consisted of the children who did not get the Salk vaccine and instead got an injection of salt water.

**BY THE WAY**

The so-called Mozart effect holds that listening to Mozart can make babies smarter. The supposed effect spawned an entire industry of Mozart products for children. The state of Georgia even began passing out Mozart CDs to new mothers. However, more recent studies of the Mozart effect have been unable to substantiate the claimed effect.

**EXAMPLE 5 Mozart Treatment**

A study divided college students into two groups. One group listened to Mozart or other classical music before being assigned a specific task and the other group simply was assigned the task without listening to the music. Researchers found that those listening to the classical music performed the task slightly better, but only if they did the task within a few minutes of listening to the music. (The two groups performed equally on tasks given later). Identify the treatment and the control and treatment groups.



SOLUTION The treatment was the classical music. The treatment group consisted of the students who listened to the music. The control group consisted of the students who did not listen to the music.

Confounding Variables

Using control groups helps to ensure that we account for known variables that could affect a study's results. However, researchers may be unaware of or be unable to account for other important variables. Consider an experiment in which a statistics teacher seeks to determine whether students who study collaboratively (in study groups with other students) earn higher grades than students who study independently. The teacher chooses five students who will study collaboratively (the treatment group) and five others who will study independently (the control group). To ensure that the students all have similar abilities and will study diligently, the teacher chooses only students with high grade-point averages. At the end of the semester, the teacher finds that the students who studied collaboratively earned higher grades.

The variables of interest for this study are *collaborative study* (whether they do so or not) and *final grade*. But suppose that, unbeknownst to the teacher, the collaborative students all lived in a dormitory where a curfew ensured that they got plenty of sleep. This fact introduces a new variable—which we might call *amount of sleep*—that might partially explain the results. In other words, the experiment's conclusion may *seem* to support the benefits of collaborative study, but this conclusion is not justified because the teacher did not account for how much students slept.

In statistical terminology, this study suffers from **confounding**. The higher grades may be due either to the variable of interest (*collaborative study*) or to the differing amounts of sleep or to a combination of both. Because the teacher did not account for differences in the amount of sleep, we say that *amount of sleep* is a **confounding variable** for this study. You can probably think of other potentially confounding variables that could affect a study like this one.

Definition

A study suffers from **confounding** if the effects of different variables are mixed so we cannot determine the specific effects of the variables of interest. The variables that lead to the confusion are called **confounding variables**.



CASE STUDY Confounding Drug Results

An advisory panel of the Federal Drug Administration (FDA) recently recommended revoking the approval of the advanced breast cancer drug Avastin, which at the time was the world's best-selling cancer drug. Early trials had found that, when used in combination with a certain chemotherapy drug, Avastin delayed the growth of tumors by five months compared to using the chemotherapy drug alone. However, additional trials found less significant delays and no improvement in the length or quality of the lives of those taking Avastin. The differing results in different trials suggest that confounding variables have not all been accounted for. One possible confounding variable may be the choice of chemotherapy drug. Indeed, studies in Europe found Avastin to be effective, but were based on using it in combination with a different chemotherapy drug than the one approved in the United States. As a result, European regulators were approving Avastin for expanded use at the same time that regulators in the United States were recommending against it. The lesson should be clear: Even when thousands of lives and millions of dollars are on the line, confounding variables may be very difficult to eliminate, making decisions far more uncertain than doctors or patients (or government regulators) would like.

Assigning Treatment and Control Groups

As the collaborative study experiment illustrates, results are almost sure to suffer from confounding if the treatment and control groups differ in important ways (other than receiving or not receiving the treatment). Researchers generally employ two strategies to prevent such differences and thereby ensure that the treatment and control groups can be compared fairly. First, they assign participants to the treatment and control groups *at random*, meaning that they use a technique designed to ensure that each participant has an equal chance of being assigned to either group. When the participants are randomly assigned, it is less likely that the people in the treatment and control groups will differ in some way that will affect the study results.

Second, researchers try to ensure that the treatment and control groups are sufficiently large. For example, in the collaborative study experiment, including 50 students in each group rather than five would have made it much less likely that all the students in one group would live in a special dormitory.

Strategies for Selecting Treatment and Control Groups

- **Select groups at random.** Make sure that the subjects of the experiment are assigned to the treatment or control group at random, meaning that each subject has an equal chance of being assigned to either group.
- **Use sufficiently large groups.** Make sure that the treatment and control groups are both sufficiently large that they are unlikely to differ in a significant way (aside from the fact that one group gets the treatment and the other does not).

EXAMPLE 6 Salk Study Groups

Briefly explain how the two strategies for selecting treatment and control groups were used in the Salk polio vaccine study.

SOLUTION A total of about 400,000 children participated in the study, with half receiving an injection of the Salk vaccine (the treatment group) and the other half receiving an injection of salt water (the control group). The first strategy was implemented by choosing children for the two groups randomly from among all the children. The second strategy was implemented by using a large number of participants (200,000 in each group) so that the two groups were unlikely to differ by chance.

The Placebo Effect

When an experiment involves people, effects can occur simply because people know they are part of the experiment. For example, suppose you are testing the effectiveness of a new anti-depression drug. You find 500 people who suffer from depression and randomly divide them into a treatment group that receives the new drug and a control group that does not. A few weeks later, interviews with the patients show that people in the treatment group tend to be feeling much better than people in the control group. Can you conclude that the new drug works?

Unfortunately, it's quite possible that the mood of people receiving the drug improved simply because they were happy to be getting some kind of treatment, which means you cannot be sure that the drug really helped. This type of effect, in which people improve because they believe that they are receiving a useful treatment, is called the **placebo effect**. (The word *placebo* comes from the Latin "to please.")

To distinguish between results caused by a placebo effect and results that are truly due to the treatment, researchers try to make sure that the participants do not know whether they are part of the treatment or control group. To accomplish this, the researchers give the people in the control group a **placebo**: something that looks or feels just like the treatment being tested, but lacks its active ingredients. For example, in a test of a drug that comes in pill form, the placebo might be a pill of the same shape and size that contains sugar instead of the real drug. In a test of an injected vaccine, the placebo might be an injection that contains only a saline solution (salt water) instead of the real vaccine. In a recent test of the effectiveness of acupuncture, the placebo consisted of treatment with needles as in real acupuncture, except the needles were not put in the special places that acupuncturists claim to be important.

As long as the participants do not know whether they received the real treatment or a placebo, the placebo effect ought to affect the treatment and control groups equally. If the results for the two groups are significantly different, it is reasonable to believe that the differences can be attributed to the treatment. For example, in the study of the anti-depression drug, we would conclude that the drug was effective only if the control group received a placebo and members of the treatment group improved much more than members of the control group. For even better control, some experiments use three groups: a treatment group, a placebo group, and a control group. The placebo group is given a placebo while the control group is given nothing.

There are men on whom the mere sight of medicine is operative.

—French philosopher Michel de Montaigne (1533–1592)

BY THE WAY

The placebo effect can be remarkably powerful. In some studies, up to 75% of the participants receiving the placebo actually improve. Nevertheless, different researchers disagree about the strength of the placebo effect, and some even question the reality of the effect.

Definitions

A placebo lacks the active ingredients of a treatment being tested in a study, but looks or feels like the treatment so that participants cannot distinguish whether they are receiving the placebo or the real treatment.

The **placebo effect** refers to the situation in which patients improve simply because they believe they are receiving a useful treatment.

Note: Although participants should not know whether they belong to the treatment or control group, for ethical reasons it is very important that participants be told that some of them will be given a placebo, rather than the real treatment.

BY THE WAY

A related effect, known as the *Hawthorne effect*, occurs when treated subjects somehow respond differently simply because they are part of an experiment—regardless of the particular way in which they are treated. The Hawthorne effect gets its name from the fact that it was first observed in a study of factory workers at Western Electric's Hawthorne plant.

EXAMPLE 7 Vaccine Placebo

What was the placebo in the Salk polio vaccine study? Why did researchers use a placebo in this experiment?

SOLUTION The placebo was the salt water injection given to the children in the control group. To understand why the researchers used a placebo for the control group, suppose that a placebo had *not* been used. When improvements were observed in the treatment group, it would have been impossible to know whether the improvements were due to the vaccine or to the placebo effect. In order to remove this confounding, all participants had to believe that they were being treated in the same way. This ensured that any placebo effect would occur in both groups equally, so that researchers could attribute any remaining differences to the vaccine.

TIME OUT TO THINK

Although participants should not know whether they belong to the treatment or control (placebo) group, for ethical reasons they should be told that some of them will receive a placebo rather than the real treatment. This was not always the case in decades past, when participants were sometimes told that they all received a treatment when in fact some received a placebo. Should researchers be allowed to use results of past studies that do not meet today's ethical criteria? Defend your opinion.

Experimenter Effects

Even if the study subjects don't know whether they received the real treatment or a placebo, the *experimenters* may still have an effect. In testing an anti-depression drug, for example, experimenters will probably interview patients to find out whether they are feeling better. But if the experimenters know who received the real drug and who received the placebo, they may inadvertently smile more at the people in the treatment group. Their smiles might improve those participants' moods, making it seem as if the treatment worked when in fact the improvement was caused by the experimenter. This type of confounding, in which the experimenter somehow influences the results, is called an **experimenter effect** (or a Rosenthal effect). The only way to avoid experimenter effects is to make sure that the experimenters don't know which subjects are in which group.

Definition

An **experimenter effect** occurs when a researcher or experimenter somehow influences subjects through such factors such as facial expression, tone of voice, or attitude.

EXAMPLE 8 Child Abuse?

In a famous case, two couples from Bakersfield, California, were convicted of molesting dozens of preschool-age children at their daycare center. The evidence for the abuse came primarily from interviews with the children. However, the conviction was overturned—after one man had served 14 years in prison—when a judge re-examined the interviews and concluded that the children had given answers that they thought the interviewers wanted to hear. If we think of the interviewers as experimenters, this is an example of an experimenter effect because the interviewers influenced the children's answers through the tone and style of their questioning. •••

Blinding

In statistical terminology, the practice of keeping people in the dark about who is in the treatment group and who is in the control group is called **blinding**. A **single-blind** experiment is one in which the participants don't know which group they belong to, but the experimenters do know. If neither the participants nor the experimenters know who belongs to each group, the study is said to be **double-blind**. Of course, *someone* has to keep track of the two groups in order to evaluate the results at the end. In a double-blind experiment, the researchers conducting the study typically hire experimenters to make any necessary contact with the participants. The researchers thereby avoid any contact with the participants, ensuring that they cannot influence them in any way. The Salk polio vaccine study was double-blind because neither the participants (the children) nor the experimenters (the doctors and nurses giving the injections and diagnosing polio) knew who got the real vaccine and who got the placebo.

BY THE WAY

Many similar cases of supposedly widespread child abuse at daycare centers and preschools are being re-examined to see if experimenter effects (by those who interviewed the children) may have led to wrongful convictions. Similar claims of experimenter effects have been made in cases involving repressed memory, in which counseling supposedly helped people retrieve lost memories of traumatic events.



Blinding in Experiments

An experiment is **single-blind** if the participants do not know whether they are members of the treatment group or members of the control group, but the experimenters do know.

An experiment is **double-blind** if neither the participants nor any experimenters know who belongs to the treatment group and who belongs to the control group.

EXAMPLE 9 What's Wrong with This Experiment?

For each of the experiments described below, identify any problems and explain how the problems could have been avoided.

- a. A new drug for attention deficit disorder (ADD) is supposed to make affected children more polite. Randomly selected children suffering from ADD are divided into treatment and control groups. The experiment is single-blind. Experimenters evaluate how polite the children are during one-on-one interviews.
- b. Researchers wonder whether drinking coffee before an exam improves performance. Fifty coffee-drinkers are told not to drink coffee in the four hours before an exam, and 50 non-coffee-drinkers are asked to drink at least two cups of coffee in the four hours before an exam. Both groups had the same average performance on the exam.
- c. Researchers wonder if the effects of a rare degenerative disease can be slowed by exercise. They identify six people suffering from the disease and randomly assign three to a treatment group that exercises every day and three to a control group that avoids exercise. After six months, they compare the amounts of degeneration in each group.
- d. A chiropractor performs adjustments on 25 patients with back pain. Afterward, 18 of the patients say they feel better. He concludes that the adjustments are an effective treatment.

SOLUTION

- a. The experimenters assess politeness in interviews, but because they know which children received the real drug, they may inadvertently speak differently to these children during the interviews. Or, they might interpret the children's behavior differently because they know which subjects received the real drug. These are experimenter effects that can confound the study results. The experiment should have been double-blind.
 - b. Both groups took the exam under conditions that were unnatural for them: Coffee-drinkers were deprived of coffee and non-coffee-drinkers were forced to drink it. This fact almost certainly introduced confounding variables that were not taken into account in interpreting the results. In fact, this is an experiment that is difficult to design flawlessly.
 - c. The results of this study will be difficult to interpret because the sample sizes are not sufficiently large. In addition, if those who exercise are asked not to (or vice versa), it may create other health problems that could confound the results.
 - d. The 25 patients who receive adjustments represent a treatment group, but this study lacks a control group. The patients may be feeling better because of a placebo effect rather than any real effect of the adjustments. The chiropractor might have improved his study by hiring an actor to do a fake adjustment (one that feels similar, but doesn't actually conform to chiropractic guidelines) on a control group. Then he could have compared the results in the two groups to see whether a placebo effect was involved.
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EXAMPLE 10 Identifying the Study Type

For each of the following questions, what type of statistical study is most likely to lead to an answer? Be as specific as possible.

- a. What is the average (mean) income of stock brokers?
- b. Do seat belts save lives?
- c. Can lifting weights improve runners' times in a 10-kilometer (10K) race?
- d. Does skin contact with a particular glue cause a rash?
- e. Can a new herbal remedy reduce the severity of colds?
- f. Do supplements of resveratrol (an extract from red grapes) increase life span?

With proper treatment, a cold can be cured in a week. Left to itself, it may linger for seven days.

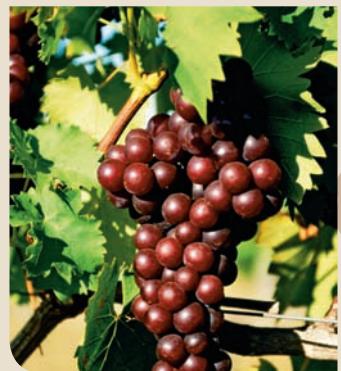
—Medical Folk Saying

SOLUTION

- a. An *observational study* can tell us the mean income of stock brokers. We need only survey the brokers, and the survey itself will not change their incomes.
- b. It would be unethical to do an experiment in which some people were told to wear seat belts and others were told *not* to wear them. A study to determine whether seat belts save lives must therefore be *observational*. Because some people choose to wear seat belts and others choose not to, we can conduct a *retrospective study*. By comparing the death rates in accidents between those who do and do not wear seat belts, we can learn whether seat belts save lives. (They do.)
- c. We need an *experiment* to determine whether lifting weights can improve runners' 10K times. We select randomly from a group of runners to create a treatment group of runners who are put on a weight-lifting program and a control group that is asked to stay away from weights. We must try to ensure that all other aspects of their training are similar. Then we can see whether the runners in the lifting group improve their times more than those in the control group. We cannot use blinding in this experiment because there is no way to prevent participants from knowing whether they are lifting weights.
- d. An *experiment* can help us determine whether skin contact with the glue causes a rash. In this case, it's best to use a *single-blind experiment* in which we apply the real glue to participants in one group and apply a placebo that looks the same, but lacks the active ingredient, to members of the control group. There is no need for a double-blind experiment because it seems unlikely that the experimenters could influence whether a person gets a rash. (However, if the question of whether the subject *has* a rash is subject to interpretation, the experimenter's knowledge of who got the real treatment could affect this interpretation.)
- e. We should use a *double-blind experiment* to determine whether a new herbal remedy can reduce the severity of colds. Some participants get the actual remedy, while others get a placebo. We need the double-blind conditions because the severity of a cold may be affected by mood or other factors that researchers might inadvertently influence. In the double-blind experiment, the researchers do not know which participants belong to which group and thus cannot treat the two groups differently.
- f. Resveratrol has been identified and made available in supplement form only recently; we will need many years of data to determine whether it has an effect on life span. We therefore should use a prospective study designed to monitor participants over many years. The participants could keep written records regarding whether and how much of the supplement they take, and eventually researchers could analyze the data to see if resveratrol has an effect on life span.

BY THE WAY

Experiments have found that mice live longer and have greater endurance when taking large doses of resveratrol, an extract from the skin of red grapes. However, it is not yet known whether high doses of resveratrol would have the same effects for humans, nor is it known whether such doses are safe for humans. Moreover, some scientists question whether the results from the mice studies have been properly interpreted.



Meta-Analysis

All individual statistical studies are either observational studies or experiments. In recent years, however, statisticians have found it useful to “mine” groups of past studies to see if we can learn something that we were unable to learn from the individual studies. For example, hundreds of studies have considered the possible effects of vitamin C on colds, so researchers might decide to review the data from many of these studies as a group. This type of study, in which researchers review many past studies as a group, is called a **meta-analysis**.

Definition

In a **meta-analysis**, researchers review many past studies. The meta-analysis considers these studies as a combined group, with the aim of finding trends that were not evident in the individual studies.



CASE STUDY Music and Autism: A Meta-Analysis

Numerous individual studies with different approaches have suggested that music is beneficial to children and adolescents with autism, but the results have varying levels of significance. Rather than conducting a separate new study, a researcher at Florida State University decided to conduct a meta-analysis that combined data from all the existing studies. This meta-analysis provided a stronger correlation between music therapy and behavioral improvement than the individual studies provided by themselves, and it helped lead to much greater use of music therapy for autistic children. Of course, a meta-analysis can only be as good as the data from the studies that go into it, and in this case some psychologists remain unconvinced of the benefits of music therapy. They argue that all the studies have ignored confounding variables, especially the possibility that the children are responding more to the attention and enthusiasm of the therapists than to the music itself. While meta-analysis can be a very useful technique, it is still subject to the “garbage in, garbage out” phenomenon, so we must be just as careful in interpreting the conclusions of a meta-analysis as we are with any other type of statistical study.

Section 3 Exercises

Statistical Literacy and Critical Thinking

- 1. Placebo.** What is a placebo, and why is it important in an experiment to test the effectiveness of a drug?
- 2. Blinding.** What is blinding, and why is it important in an experiment to test the effectiveness of a drug?
- 3. Confounding.** In testing the effectiveness of a new vaccine, suppose that researchers used males for the treatment group and females for the placebo group. What is confounding, and how would it affect such an experiment?
- 4. Ethics.** A clinical trial of a new drug designed to treat hypertension (high blood pressure) is designed to last for three years, but after the first year it becomes clear that the drug is highly successful. Is it ethical to continue the trial with the result that some hypertensive subjects continue to receive a placebo instead of the effective treatment?
- 5. Clothing Color.** A researcher plans to investigate the belief that people are more comfortable in the summer sun when they wear clothing with light colors instead of clothing with dark colors. Does it make sense to use a double-blind experiment in this case? Is it easy to implement blinding in this case? Explain.
- 6. Lawn Treatment.** A researcher plans to test the effectiveness of a new fertilizer on grass growth. Does it make sense to use a double-blind experiment in this case?
- 7. Treating Depression.** A psychologist has developed a procedure for modifying behavior so that subjects suffering from depression can greatly improve that condition. In formal tests of the effectiveness of the treatment, what is an experimenter effect, and how might it be avoided?

- 8. Improving IQ Scores.** A psychologist develops a procedure for improving IQ scores by training subjects to become better at taking tests. A standard IQ test is used for evaluating the effectiveness of the procedure. In this case, is it necessary to take precautions against an experimenter effect? Why or why not?

Concepts and Applications

Type of Study. For Exercises 9–20, state the type of study. Be as specific as possible.

- 9. Quality Control.** Apple selects a simple random sample of iPhone batteries. The voltage of each battery is measured.
- 10. Quality Control.** Apple selects a simple random sample of iPhone batteries. The voltage of each battery is measured after being heated to 43°C.
- 11. Magnetic Bracelets.** Some cruise ship passengers are given magnetic bracelets, which they agree to wear in an attempt to eliminate or diminish the effects of motion sickness. Others are given similar bracelets that have no magnetism.
- 12. Touch Therapy.** Nine-year-old Emily Rosa became an author of an article in the *Journal of the American Medical Association* after she tested professional touch therapists.

- Using a cardboard partition, she held her hand above the therapist's hand, and the therapist was asked to identify the hand that Emily chose.
- 13. Twins.** In a study of hundreds of Swedish twins, it was determined that the level of mental skills was more similar in identical twins (twins coming from a single egg) than in fraternal twins (twins coming from two separate eggs) (*Science*).
- 14. Texting and Driving.** A study of 2,500 fatal car crashes identified those that involved drivers who were texting and those who were not.
- 15. Gender Selection.** In a study of the YSORT gender selection method developed by the Genetics & IVF Institute, 152 couples using the method had 127 baby boys and 25 baby girls.
- 16. AOL Poll.** An America OnLine (AOL) poll resulted in 1,651 responses to the question that asked which of four organizations has the most unethical people in charge. Among the respondents, 36% chose government.
- 17. GMO Corn.** Researchers at New York University found that the genetically modified corn known as Bt corn releases an insecticide through its roots into the soil, while the insecticide was not released by corn that was not genetically modified (*Nature*).
- 18. Hygiene Survey.** In a Harris Interactive survey of 1,006 randomly selected subjects, 96% of adults said that they wash their hands in public restrooms.
- 19. Magnet Treatment.** In a study of the effects of magnets on back pain, some subjects were treated with magnets while others were given non-magnetic devices with a similar appearance. The magnets did not appear to be effective in treating back pain (*Journal of the American Medical Association*).
- 20. Power Lines and Cancer.** Hundreds of separate and individual scientific and statistical studies have been done to determine whether high-voltage overhead power lines increase the incidence of cancer among those living nearby. A summary study based on many previous studies concluded that there is no significant link between power lines and cancer (*Journal of the American Medical Association*).
- What's Wrong with This Experiment?** For each of the (hypothetical) studies described in Exercises 21–28, identify any problems that are likely to cause confounding and explain how the problems could be avoided. Discuss any other problems that might affect the results.
- 21. Poplar Tree Growth.** An experiment is designed to evaluate the effectiveness of irrigation and fertilizers on poplar tree growth. Fertilizer is used with one group of poplar trees in a moist region, and irrigation is used with poplar trees in a dry region.
- 22. Internet Shopping.** Two hundred volunteers are recruited for a study of how Internet shopping affects purchases. Each person is allowed to choose whether to be in the Internet user group or the group that agrees not to use the Internet for shopping. After one month, the purchases of the two groups are compared.
- 23. Octane Rating.** In a comparison of gasoline with different octane ratings, 24 vans are driven with 87 octane gasoline, while 28 sport utility vehicles are driven with 91 octane gasoline. After being driven for 250 miles, the amount of gasoline consumed is measured for each vehicle.
- 24. Aspirin Trial.** In Phase I of a clinical trial designed to test the effectiveness of aspirin in preventing heart attacks, aspirin is given to three people and a placebo to seven other people.
- 25. Treating Back Pain.** A physician conducts a clinical trial of the effectiveness of running as a treatment for back pain. One group undergoes the running treatment while a control group does not.
- 26. Athlete's Foot.** In a clinical trial of the effectiveness of a lotion used to treat tinea pedis (athlete's foot), the physicians who evaluate the results know which subjects were given the treatment and which were given a placebo.
- 27. Weight Lifting.** In a test of the effects of lifting heavy weights on blood pressure, one group undergoes a treatment consisting of a weight-lifting program while another group lifts tennis balls.
- 28. Paint Mixtures.** In durability tests of Benjamin Moore paint and Sherwin Williams paint, the researchers who evaluate the results know which samples are from each of the two different brands.
- Analyzing Experiments.** Exercises 29–32 present questions that might be addressed in an experiment. If you were to design the experiment, how would you choose the treatment and control groups? Should the experiment be single-blind, double-blind, or neither? Explain your reasoning.
- 29. Beethoven and Intelligence.** Does listening to Beethoven make infants more intelligent?
- 30. Lipitor and Cholesterol.** Does the drug Lipitor result in lower cholesterol levels?
- 31. Ethanol and Mileage.** Does an ethanol additive in gasoline cause reduced mileage?
- 32. Home Siding.** Does aluminum siding on a home last longer than wood siding?
-  **PROJECTS FOR THE INTERNET & BEYOND**
- 33. Experimenter Effects in Repressed Memory Cases.** Search the Internet for articles and information about the controversy regarding recovering repressed memories. Briefly summarize one or two of the most interesting cases and, based on what you read, express your own opinion as to whether the allegedly recovered memories are being influenced by experimenter effects.
- 34. Ethics in Experiments.** In an infamous study conducted in Tuskegee, Alabama, from 1932 to 1972, African American males were told that they were receiving treatment for

syphilis, but in fact they were not. The researchers' hidden goal was to study the long-term effects of the disease. Use the Internet to learn about the history of the Tuskegee syphilis study. Have a class discussion about the ethical issues involved in this case, or write a short essay summarizing the case and its ethical lessons.

35. Debate: Should We Use Data from Unethical Experiments? Past research often did not conform to today's ethical standards. In extreme cases, such as research conducted by doctors in Nazi Germany, the researchers sometimes killed the subjects of their experiments. While this past unethical research clearly violated the human rights of the experimental subjects, in some cases it led to insights that could help people today. Is it ethical to use the results of unethical research?

36. Study Stopped Early. It sometimes happens that study is stopped early before its completion. Use the Internet to find an example of such a study. Why was the study stopped? Should it have been stopped, or would it be better to complete the study?

IN THE NEWS

- 37. Observational Studies.** Search through recent newspapers or journals and find an example of a statistical study that was observational. Briefly describe the study and summarize its conclusions.
- 38. Experimental Studies.** Search through recent newspapers or journals and find an example of a statistical study that involved an experiment. Briefly describe the study and summarize its conclusions.
- 39. Retrospective Studies.** Search through recent newspapers or journals and find an example of an observational, retrospective study. Briefly describe the study and summarize its conclusions.
- 40. Meta-Analysis.** Search through recent newspapers or journals and find an example of a meta-analysis. Briefly describe the study and summarize its conclusions.

4

SHOULD YOU BELIEVE A STATISTICAL STUDY?

If I have ever made any valuable discoveries, it has been owing more to patient attention, than to any other talent.

—Isaac Newton

Eight Guidelines for Critically Evaluating a Statistical Study

1. *Get a Big Picture View of the Study.* For example, you should understand the goal of the study, the population that was under study, and whether the study was observational or an experiment.
2. *Consider the Source.* In particular, look for any potential biases on the part of the researchers.
3. *Look for Bias in the Sample.* That is, decide whether the sampling method was likely to produce a representative sample.
4. *Look for Problems Defining or Measuring the Variables of Interest.* Ambiguity in the variables can make it difficult to interpret reported results.
5. *Beware of Confounding Variables.* If the study neglected potential confounding variables, its results may not be valid.
6. *Consider the Setting and Wording in Surveys.* In particular, look for anything that might tend to produce inaccurate or dishonest responses.
7. *Check That Results Are Presented Fairly.* For example, check whether the study really supports the conclusions that are presented in the media.
8. *Stand Back and Consider the Conclusions.* For example, evaluate whether study achieved its goals. If so, do the conclusions make sense and have practical significance?

News reports do not always provide enough information for you to apply all eight guidelines, but you can usually find additional information on the Web. Look for clues such as “reported by NASA” or “published in the *New England Journal of Medicine*” to help you track down original sources or other relevant information.

Guideline 1: Get a Big Picture View of the Study

The first thing you should do when you hear about a statistical study is figure out what it was all about; that is, get a big picture view of the study that will allow you to consider its results in an appropriate context. A good starting point for gaining a big picture view comes in trying to answer these basic questions:

- What was the goal of the study?
- What was the population under study? Was the population clearly and appropriately defined?
- Was the study an observational study, an experiment, or a meta-analysis? If it was an observational study, was it retrospective? If it was an experiment, was it single- or double-blind, and were the treatment and control groups properly randomized? Given the goal, was the type of study appropriate?

EXAMPLE 1 Appropriate Type of Study?

Imagine the following (hypothetical) newspaper report: “Researchers gave 100 participants their individual astrological horoscopes and asked whether the horoscopes were accurate. 85% of the participants said their horoscopes were accurate. The researchers concluded that horoscopes are valid most of the time.” Analyze this study according to Guideline 1.

SOLUTION The goal of the study was to determine the validity of horoscopes. Based on the news report, it appears that the study was *observational*: The researchers simply asked the participants about the accuracy of the horoscopes. However, because the accuracy of a horoscope is somewhat subjective, the study goal would have been better served by a controlled experiment in which some people were given their actual horoscope and others were given a fake horoscope (which would serve as a placebo). Then the researchers could have looked for differences between the two groups. Moreover, because researchers could easily influence the results by how they questioned the participants, the experiment should have been double-blind. In summary, the type of study was inappropriate to the goal and its results are meaningless.



BY THE WAY

Surveys show that nearly half of Americans believe their horoscopes. However, in controlled experiments, the predictions of horoscopes come true no more often than would be expected by chance.



TIME OUT TO THINK

Try your own test of horoscopes. Find yesterday’s horoscope for each of the 12 signs and put each one on a separate piece of paper, without anything identifying the sign. Shuffle the pieces of paper randomly, and ask a few people to guess which one was supposed to be their personal horoscope. How many people choose the right one? Discuss your results.

EXAMPLE 2 Does Aspirin Prevent Heart Attacks?

A study reported in the *New England Journal of Medicine* (Vol. 318, No. 4) sought to determine whether aspirin is effective in preventing heart attacks. It involved 22,000 male physicians considered to be at risk for heart attacks. The men were divided into a treatment group that took aspirin and a control group that did not. The results were so convincing in favor of the benefits of aspirin that the experiment was stopped for ethical reasons before it was completed, and the subjects were informed of the results. Many news reports led with the headline that taking aspirin can help prevent heart attacks. Analyze this headline according to Guideline 1.

SOLUTION The study was an experiment, which is appropriate, and its results appear convincing. However, the fact that the sample consisted only of men means that the results should be considered to apply only to the population of men. Because results of medical tests on men do not necessarily apply to women, the headlines misstated the results when they did not qualify the population.



BY THE WAY

Many recent studies show substantial differences in the ways men and women respond to the same medical treatments. For example, aspirin is more effective at thinning blood in men than in women (thinning is thought to help prevent heart attacks in some people). Morphine controls pain better in women, but ibuprofen is more effective for men. And women reject heart transplants more often than men. These differences may stem from interactions with hormones that differ in men and women or from differences in the rates at which men and women metabolize different drugs.

Guideline 2: Consider the Source

Statistical studies are supposed to be objective, but the people who carry them out and fund them may be biased. It is therefore important to consider the source of a study and evaluate the potential for biases that might invalidate the study's conclusions.

Bias may be obvious in cases where a statistical study is carried out for marketing, promotional, or other commercial purposes. For example, a toothpaste advertisement that claims "4 out of 5 dentists prefer our brand" appears to be statistically based, but we are given no details about how the survey was conducted. Because the advertisers obviously want to say good things about their brand, it's difficult to take the statistical claim seriously without much more information about how the result was obtained.

Other cases of bias may be more subtle. For example, suppose that a carefully conducted study concludes that a new drug helps cure cancer. On the surface, the study might seem quite believable. But what if the study was funded by a drug company that stands to gain billions of dollars in sales if the drug is proven effective? The researchers may well have carried out their work with great integrity despite the source of funding, but it might be worth a bit of extra investigation to be sure.

Major statistical studies are usually evaluated by experts who are supposed to be unbiased. The most common process by which scientists examine each other's research is called **peer review** (because the scientists who do the evaluation are *peers* of those who conducted the research). Reputable scientific journals require all research reports to be peer reviewed before the research is accepted for publication. Peer review does not guarantee that a study is valid, but it lends credibility because it implies that other experts agree that the study was carried out properly.

Definition

Peer review is a process in which several experts in a field evaluate a research report before the report is published.

BY THE WAY

After decades of arguing to the contrary, in 1999 the Philip Morris Company—the world's largest seller of tobacco products—publicly acknowledged that smoking causes lung cancer, heart disease, emphysema, and other serious diseases. Shortly thereafter, Philip Morris changed its name to Altria.

EXAMPLE 3 Is Smoking Healthy?

By 1963, research had so clearly shown the health dangers of smoking that the Surgeon General of the United States publicly announced that smoking is bad for health. Research done since that time built further support for this claim. However, while the vast majority of studies showed that smoking is unhealthy, a few studies found no dangers from smoking and perhaps even health *benefits*. These studies generally were carried out by the Tobacco Research Institute, funded by the tobacco companies. Analyze these studies according to Guideline 2.

SOLUTION Even in a case like this, it can be difficult to decide whom to believe. However, the studies showing smoking to be unhealthy came primarily from peer-reviewed research. In contrast, the studies carried out at the Tobacco Research Institute had a clear potential for bias. The *potential* for bias does not mean the research was biased, but the fact that it contradicts virtually all other research on the subject should be cause for concern.



EXAMPLE 4 Press Conference Science

Suppose the nightly TV news shows scientists at a press conference announcing that they've discovered evidence that a newly developed chemical can stop the aging process. The work has not yet gone through the peer review process. Analyze this study according to Guideline 2.

SOLUTION Scientists often announce the results of their research at a press conference so that the public may hear about their work as soon as possible. However, a great deal of expertise may be required to evaluate their study for possible biases or other errors—which is the goal of the peer review process. Until the work is peer reviewed and published in a reputable journal, any findings should be considered preliminary—especially about an astonishing claim such as being able to stop the aging process.



Guideline 3: Look for Bias in the Sample

A statistical study cannot be valid unless the sample is representative of the population under study. Poor sampling methods almost guarantee a biased sample that makes the study results useless.

Biased samples can arise in many ways, but two closely related problems are particularly common. The first problem, called **selection bias** (or a **selection effect**), occurs whenever researchers *select* their sample in a way that tends to make it unrepresentative of the population. For example, a pre-election poll that surveys only registered Republicans has selection bias because it is unlikely to reflect the opinions of non-Republican voters.

The second problem, called **participation bias**, can arise when people *choose* to be part of a study—that is, when the participants are volunteers. The most common form of participation bias occurs in **self-selected surveys** (or **voluntary response surveys**)—surveys or polls in which people decide for themselves whether to participate. In such cases, people who feel strongly about an issue are more likely to participate, and their opinions may not represent the opinions of the larger population that has less emotional attachment to the issue.

BY THE WAY

More than a third of all Americans routinely shut the door or hang up the phone when contacted for a survey, thereby making self-selection a problem for legitimate pollsters. One reason people hang up may be the proliferation of selling under the guise of market research (often called “sugging”), in which a telemarketer pretends you are part of a survey in order to try to get you to buy something.

Definitions

Selection bias (or a **selection effect**) occurs whenever researchers *select* their sample in a biased way.

Participation bias occurs any time participation in a study is voluntary.

A **self-selected survey** (or **voluntary response survey**) is one in which people decide for themselves whether to be included in the survey.



CASE STUDY The 1936 Literary Digest Poll

The *Literary Digest*, a popular magazine of the 1930s, successfully predicted the outcomes of several elections using large polls. In 1936, editors of the *Literary Digest* conducted a particularly large poll in advance of the presidential election. They randomly chose a sample of 10 million people from various lists, including names in telephone books and rosters of country clubs. They mailed a postcard “ballot” to each of these 10 million people. About 2.4 million people returned the postcard ballots. Based on the returned ballots, the editors of the *Literary Digest* predicted that Alf Landon would win the presidency by a margin of 57% to 43% over Franklin Roosevelt. Instead, Roosevelt won with 62% of the popular vote. How did such a large survey go so wrong?

The sample suffered from both selection bias and participation bias. The selection bias arose because the *Literary Digest* chose its 10 million names in ways that favored affluent people. For example, selecting names from telephone books meant choosing only from those who could afford telephones back in 1936. Similarly, country club members are usually quite wealthy. The selection bias favored the Republican Landon because affluent voters of the 1930s tended to vote for Republican candidates.

The participation bias arose because return of the postcard ballots was voluntary, so people who felt strongly about the election were more likely to be among those who returned their ballots. This bias also tended to favor Landon because he was the challenger—people who did not like President Roosevelt could express their desire for change by returning the postcards. Together, the two forms of bias made the sample results useless, despite the large number of people surveyed.

BY THE WAY

A young pollster named George Gallup conducted his own surveys prior to the 1936 election. Using interviews with only 3,000 randomly selected people at a time, he correctly predicted the outcome of the election, and even gained insights into how the *Literary Digest* poll went wrong. Gallup went on to establish a very successful polling organization.

EXAMPLE 5 Self-Selected Poll

The television show *Nightline* conducted a poll in which viewers were asked whether the United Nations headquarters should be kept in the United States. Viewers could respond to the poll by paying 50 cents to call a special phone number with their opinions. The poll drew

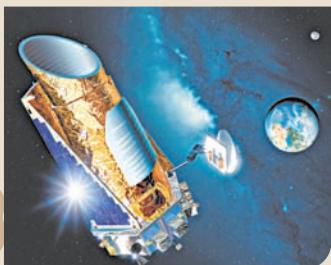
186,000 responses, of which 67% favored moving the United Nations out of the United States. Around the same time, a poll using simple random sampling of 500 people found that 72% wanted the United Nations to *stay* in the United States. Which poll is more likely to be representative of the general opinions of Americans?

SOLUTION The *Nightline* sample was severely biased. It had selection bias because its sample was drawn only from the show's viewers, rather than from all Americans. The poll itself was a self-selected survey in which viewers not only chose whether to respond, but also had to pay 50 cents to participate. This cost made it even more likely that respondents would be those who felt a need for change. Despite its large number of respondents, the *Nightline* survey was therefore unlikely to give meaningful results. In contrast, the simple random sample of 500 people is quite likely to be representative, so the finding of this small survey has a better chance of representing the true opinions of all Americans.



BY THE WAY

NASA's *Kepler* mission, launched in 2009, is an orbiting telescope that has made the first detections of planets as small as Earth around other stars. *Kepler* looks for slight dimming of a star's light each time an orbiting planet passes in front of it (a "transit"), which means it can detect planets only for the small fraction of stars that happen to have their planetary systems aligned with our line of sight.



EXAMPLE 6 Planets Around Other Stars

Until the mid-1990s, astronomers had never found conclusive evidence for planets outside our own solar system. But improving technology made it possible to begin finding such planets, and more than 2,000 had been discovered by mid-2012. The existing technology makes it easier to find large planets than small ones and easier to find planets that orbit close to their stars than planets that orbit far from their stars. According to the leading theory of solar system formation, large planets should form far from their stars, not close by. But large planets in close orbits are quite common among the planets discovered to date. Does this mean there is something wrong with the leading theory of solar system formation?

SOLUTION Even if large planets in close orbits are relatively rare, the fact that current technology makes these rare cases the easiest ones to find introduces a *selection effect* that biases the sample (of discovered planets) toward that type. In fact, most astronomers think the existing solar system formation theory is still correct and that large planets in close-in orbits are not as common as they appear. Those that do exist can be explained by physical interactions that can cause large planets to migrate inward as a planetary system forms.



Guideline 4: Look for Problems Defining or Measuring the Variables of Interest

Results of a statistical study may be difficult to interpret if the variables under study are difficult to define or measure. For example, imagine trying to conduct a study of how exercise affects resting heart rates. The variables of interest would be *amount of exercise* and *resting heart rate*. Both variables are difficult to define and measure. In the case of *amount of exercise*, it's not clear what the definition covers—does it include walking to class? Even if we specify the definition, how can we measure *amount of exercise* given that some forms of exercise are more vigorous than others?

TIME OUT TO THINK

What are the challenges of defining and measuring resting heart rate?

EXAMPLE 7 Can Money Buy Love?

A Roper poll reported in *USA Today* involved a survey of the wealthiest 1% of Americans. The survey found that these people would pay an average of \$487,000 for *true love*, \$407,000 for *great intellect*, \$285,000 for *talent*, and \$259,000 for *eternal youth*. Analyze this result according to Guideline 4.

SOLUTION The variables in this study are very difficult to define. How, for example, do you define *true love*? And does it mean true love for a day, a lifetime, or something else?

Similarly, does the ability to balance a spoon on your nose constitute *talent*? Because the variables are so poorly defined, it's likely that different people interpreted them differently, making the results very difficult to interpret.

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EXAMPLE 8 Illegal Drug Supply

A commonly quoted statistic is that law enforcement authorities succeed in stopping only about 10% to 20% of the illegal drugs entering the United States. Should you believe this statistic?

SOLUTION There are essentially two variables in a study of illegal drug interception: *quantity of illegal drugs intercepted* and *quantity of illegal drugs NOT intercepted*. It should be relatively easy to measure the quantity of illegal drugs that law enforcement officials intercept. However, because the drugs are illegal, it's unlikely that anyone is reporting the quantity of drugs that are *not* intercepted. How, then, can anyone know that the intercepted drugs are 10% to 20% of the total? In a *New York Times* analysis, a police officer was quoted as saying that his colleagues refer to this type of statistic as "PFA" for "pulled from the air."

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Guideline 5: Beware of Confounding Variables

Variables that are *not intended* to be part of a study can make it difficult to interpret results properly. As discussed in Section 3, it's not always easy to discover these *confounding variables*. Sometimes they are discovered only years after a study is completed, and other times they are not discovered at all, in which case a study's conclusion may be accepted even though it's not correct. Fortunately, confounding variables are sometimes more obvious and can be discovered simply by thinking hard about factors that may have influenced a study's results.

EXAMPLE 9 Radon and Lung Cancer

Radon is a radioactive gas produced by natural processes (the decay of uranium) in the ground. The gas can leach into buildings through the foundation and can accumulate to relatively high concentrations if doors and windows are closed. Imagine a (hypothetical) study that seeks to determine whether radon gas causes lung cancer by comparing the lung cancer rate in Colorado, where radon gas is fairly common, with the lung cancer rate in Hong Kong, where radon gas is less common. Suppose the study finds that the lung cancer rates are nearly the same. Would it be reasonable to conclude that radon is *not* a significant cause of lung cancer?

SOLUTION The variables of interest are *amount of radon* (an explanatory variable in this case) and *lung cancer rate* (a response variable). However, radon gas is not the only possible cause of lung cancer. For example, smoking can cause lung cancer, so *smoking rate* may be a confounding variable in this study—especially because the smoking rate in Hong Kong is much higher than the smoking rate in Colorado. As a result, we cannot draw any conclusions about radon and lung cancer without taking the smoking rate into account (and perhaps other variables as well). In fact, careful studies have shown that radon gas *can* cause lung cancer, and the U.S. Environmental Protection Agency (EPA) recommends taking steps to prevent radon from building up indoors.

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Guideline 6: Consider the Setting and Wording in Surveys

Even when a survey is conducted with proper sampling and with clearly defined terms and questions, you should watch for problems in the setting or wording that might produce inaccurate or dishonest responses. Dishonest responses are particularly likely when the survey concerns sensitive subjects, such as personal habits or income. For example, the question "Do you cheat on your income taxes?" is unlikely to elicit honest answers from those who cheat, unless they are assured of complete confidentiality (and perhaps not even then).

In other cases, even honest answers may not really be accurate if the wording of questions invites bias. Sometimes just the order of words in a question can affect the outcome. A poll conducted in Germany asked the following two questions.

- Would you say that traffic contributes more or less to air pollution than industry?
- Would you say that industry contributes more or less to air pollution than traffic?

BY THE WAY

Many hardware stores sell simple kits that you can use to test whether radon gas is accumulating in your home. If it is, the problem can be eliminated by installing an appropriate "radon mitigation" system, which usually consists of a fan that blows the radon out from under the house before it can get into the house.



BY THE WAY

People are more likely to choose the item that comes first in a survey because of what psychologists call the *availability error*—the tendency to make judgments based on what is *available* in the mind. Professional polling organizations take great care to avoid this problem; for example, they may pose a question with two choices in one order to half the people in the sample and in the opposite order to the other half.

The only difference is the order of the words *traffic* and *industry*, but this difference dramatically changed the results: With the first question, 45% answered traffic and 32% answered industry. With the second question, only 24% answered traffic while 57% answered industry.

EXAMPLE 10 Do You Want a Tax Cut?

The Republican National Committee commissioned a poll to find out whether Americans supported their proposed tax cuts. Asked “Do you favor a tax cut?,” a large majority of respondents answered *yes*. Should we conclude that Americans supported the proposal?

SOLUTION A question like “Do you favor a tax cut?” is biased because it does not give other options or discuss any consequences. In fact, other polls conducted at the same time showed a similarly large majority expressing great concern about federal deficits. Indeed, support for the tax cuts was far lower when the question was asked by independent organizations in the form “Would you favor a tax cut even if it increased the federal deficit?”

**EXAMPLE 11 Sensitive Survey**

Two surveys asked Catholics in the Boston area whether contraceptives should be made available to unmarried women. The first survey involved in-person interviews, and 44% of the respondents answered *yes*. The second survey was conducted by mail and telephone, and 75% of the respondents answered *yes*. Which survey was more likely to be accurate?

SOLUTION Contraceptives are a sensitive topic, particularly among Catholics (because the Catholic Church officially opposes contraceptives). The first survey, with in-person interviews, may have encouraged dishonest responses. The second survey made responses seem more private and therefore was more likely to reflect the respondents’ true opinions.

**Guideline 7: Check That Results Are Presented Fairly**

Even when a statistical study is done well, it may be misrepresented in graphics or concluding statements. Researchers occasionally misinterpret the results of their own studies or jump to conclusions that are not supported by the results, particularly when they have personal biases. News reporters may misinterpret a survey or jump to unwarranted conclusions that make a story seem more spectacular. Misleading graphics are especially common. You should always look for inconsistencies between the interpretation of a study (in pictures and in words) and any actual data given along with it.

EXAMPLE 12 Does the School Board Need a Statistics Lesson?

The school board in Boulder, Colorado, created a hubbub when it announced that 28% of Boulder school children were reading “below grade level” and hence concluded that methods of teaching reading needed to be changed. The announcement was based on reading tests on which 28% of Boulder school children scored below the national average for their grade. Do these data support the board’s conclusion?

SOLUTION The fact that 28% of Boulder children scored below the national average for their grade implies that 72% scored at or above the national average. Thus, the school board’s ominous statement about students reading “below grade level” makes sense only if “grade level” means the national average score for a particular grade. This interpretation of “grade level” is curious because it would imply that half the students in the nation are always below grade level—no matter how high the scores. It may still be the case that teaching methods needed to be improved, but these data did not justify that conclusion.



Guideline 8: Stand Back and Consider the Conclusions

Finally, even if a study seems reasonable according to all the previous guidelines, you should stand back and consider the conclusions. Ask yourself questions such as these:

- Did the study achieve its goals?
- Do the conclusions make sense?
- Can you rule out alternative explanations for the results?
- If the conclusions make sense, do they have any practical significance?

EXAMPLE 13 Extraordinary Claims

A recent study by a respected psychologist, Daryl J. Bem (Cornell University) claimed to find evidence for the existence of extrasensory perception, or ESP. Dr. Bem's results showed relatively small effects; for example, in one experiment in which subjects were asked to identify a picture hidden behind a screen, 53% gave the right answer versus the 50% expected by pure chance. Nevertheless, Dr. Bem claimed that the results were statistically significant, and his results were peer reviewed and published in a respected scholarly journal. Should you conclude that ESP really exists?

Extraordinary claims require extraordinary evidence.

—Carl Sagan

SOLUTION Although the study may well have been done carefully, the claim of ESP should be considered what scientists often call an “extraordinary claim,” because decades of study have never previously turned up any indisputable evidence for ESP and its existence would seem to violate some well-established laws of physics. Moreover, Dr. Bem’s claim that the results were statistically significant has been disputed by other scientists and statisticians, who argue that he used the wrong type of analysis and that a proper analysis shows the results were consistent with pure chance. Clearly, for a claim as extraordinary as one of ESP, we would need far stronger evidence before concluding that it actually exists.



EXAMPLE 14 Practical Significance

An experiment is conducted in which the weight losses of people who try a new “Fast Diet Supplement” are compared to the weight losses of a control group of people who try to lose weight in other ways. After eight weeks, the results show that the treatment group lost an average of one-half pound more than the control group. Assuming that it has no dangerous side effects, does this study suggest that the Fast Diet Supplement is a good treatment for people wanting to lose weight?

SOLUTION Compared to the average person’s body weight, a weight loss of one-half pound hardly matters at all. So while loss results may be interesting, they don’t seem to have much practical significance.



Section 4 Exercises

Statistical Literacy and Critical Thinking

1. **Peer Review.** What is peer review? How is it useful?
2. **Selection Bias and Participation Bias.** Describe and contrast selection bias and participation bias in sampling.
3. **Self-Selected Surveys.** Why are self-selected surveys almost always prone to participation bias?
4. **Confounding Variables.** What are confounding variables, and what problems can they cause?

Does It Make Sense? For Exercises 5–8, decide whether the statement makes sense (or is clearly true) or does not make sense (or is clearly false). Explain clearly; not all of these have

definitive answers, so your explanation is more important than your chosen answer.

5. **Large Survey.** A survey involving a larger sample of subjects is always better than one involving a smaller sample.
6. **Survey Location.** The survey of the use of credit among adult Americans suffered from selection bias because the questionnaires were handed out only on college campuses.
7. **Vitamin C and Colds.** My experiment proved that vitamin C can reduce the

severity of colds, because I controlled the experiment carefully for every possible confounding variable.

- 8. Diet Effectiveness.** The Simon diet is effective because it was used by a sample of 1,000 subjects and there was a mean weight loss of 1.7 pounds during a six-month study.

Concepts and Applications

Applying Guidelines. In Exercises 9–16, determine which of the eight guidelines appears to be most relevant. Explain your reasoning.

- 9. Hygiene.** The Winslow Supply Company manufactures deodorants and sponsored a survey showing that good personal hygiene is critically important for success in a job interview.
- 10. Smoking.** A clinical trial involved the use of a nicotine gum as an aid to help smokers stop smoking. The clinical trial involved 1,000 college students who were paid for their participation, and the results showed that the nicotine gum treatment was highly successful. The researchers concluded that the use of their nicotine gum would be a successful treatment for the population of all smokers.
- 11. Goodness.** In a study of 1,200 college students, each was asked whether he or she was a good person.
- 12. Agriculture.** Researchers conclude that an irrigation system used to grow tomatoes in California is more effective than a competing system used in Arizona.
- 13. New York City Subway Survey.** The New York City Transit Authority routinely conducts a survey of satisfaction by distributing surveys on subways. The passengers take the surveys home and return them by mail.
- 14. Election Poll.** Under the headline “Turner predicted to win in a landslide,” it was reported that 54% of voters in a pre-election poll prefer Turner, compared with 46% for her opponent.
- 15. Nuclear Energy Poll.** Randomly selected adults were asked: “Do you agree or disagree with increasing the production of nuclear energy that could potentially kill thousands of innocent people?”
- 16. Counterfeit Goods.** A consortium of manufacturers plans a study designed to compare the value of counterfeit goods produced in the United States in the year 2000 to the current year.
- Bias.** In each of Exercises 17–20, identify and explain at least one source of bias in the study described. Then suggest how the bias might have been avoided.
- 17. Chocolate.** An article in *Journal of Nutrition* (Vol. 130, No. 8) noted that chocolate is rich in flavonoids. The article reports that “regular consumption of foods rich in flavonoids may reduce the risk of coronary heart disease.” The study received funding from Mars, Inc., the candy company, and the Chocolate Manufacturers Association.
- 18. Famous Book.** When author Shere Hite wrote *Woman and Love: A Cultural Revolution in Progress*, she based conclusions about the general population of all women on 4,500

replies that she received after mailing 100,000 questionnaires to various women’s groups.

- 19. Political Polling.** You receive a call in which the caller claims to be conducting a national opinion research poll. You are asked if your opinion about congressional candidate John Sweeney would change if you knew that Sweeney once had a car crash while driving under the influence of alcohol.

- 20. Survey Method.** You conduct a survey to find the percentage of people in your state who can name the lieutenant governor, who plans to run for the United States Senate. You obtain addresses from a list of property owners in the state and you mail a survey to 850 randomly selected people from the list.

- 21. It's All in the Wording.** Princeton Survey Research Associates did a study for *Newsweek* magazine illustrating the effects of wording in a survey. Two questions were asked:

- Do you personally believe that abortion is wrong?
- Whatever your own personal view of abortion, do you favor or oppose a woman in this country having the choice to have an abortion with the advice of her doctor?

To the first question, 57% of the respondents replied yes, while 36% responded no. In response to the second question, 69% of the respondents favored the choice, while 24% opposed the choice. Discuss why the two questions produced seemingly contradictory results. How could the results of the questions be used selectively by various groups?

- 22. Tax or Spend?** A Gallup poll asked the following two questions:

- Do you favor a tax cut or “increased spending on other government programs”? *Result:* 75% for the tax cut.
- Do you favor a tax cut or “spending to fund new retirement savings accounts, as well as increased spending on education, defense, Medicare and other programs”? *Result:* 60% for the spending.

Discuss why the two questions produced seemingly contradictory results. How could the results of the questions be used selectively by various groups?

What Do You Want to Know? Exercises 23–26 pose two related questions that might form the basis of a statistical study. Briefly discuss how the two questions differ and how these differences would affect the goal of a study and the design of the study.

23. Internet Dating

First question: What percentage of Internet dates lead to marriage?

Second question: What percentage of marriages begin with Internet dates?

24. Full-Time Faculty

First question: What percentage of introductory classes on campus are taught by full-time faculty members?

Second question: What percentage of full-time faculty members teach introductory classes?

25. Binge Drinking

First question: How often do college students do binge drinking?

Second question: How often is binge drinking done by college students?

26. Statistics Courses

First question: What is the proportion of college graduates who have taken a statistics course?

Second question: What is the proportion of statistics courses taken by college students?

Accurate Headlines? Exercises 27 and 28 give a headline and a brief description of the statistical news story that accompanied the headline. In each case, discuss whether the headline accurately represents the story.

27. Headline: “Drugs shown in 98 percent of movies”

Story summary: A “government study” claims that drug use, drinking, or smoking was depicted in 98% of the top movie rentals (Associated Press).

28. Headline: “Sex more important than jobs”

Story summary: A survey found that 82% of 500 people interviewed by phone ranked a satisfying sex life as important or very important, while 79% ranked job satisfaction as important or very important (Associated Press).

Stat-Bytes. Politicians commonly believe that they must make their political statements (often called sound-bytes) very short because the attention span of listeners is so short. A similar effect occurs in reporting statistical news. Major statistical studies are often reduced to one or two sentences. The summaries of statistical reports in Exercises 29–32 are taken from various news sources. Describe what crucial information is missing in the given statement and what more you would want to know before you acted on the report.

29. Confidence in Military. *USA Today* reports on a Harris poll claiming that the percentage of adults with a “great deal of confidence” in military leaders stands at 54%.

30. Top Restaurants. CNN reports on a Zagat Survey of America’s Top Restaurants that found that “only nine restaurants achieved a rare 29 out of a possible 30 rating and none of those restaurants are in the Big Apple.”

31. Forecasting Weather. A *USA Today* headline reported that “More companies try to bet on forecasting weather.” The article gave examples of companies believing that long-range forecasts are reliable, and four companies were cited.

32. Births in China. A *USA Today* headline reported that “China thrown off balance as boys outnumber girls,” and an accompanying graph showed that for every 100 girls born in China, 116.9 boys are born.


PROJECTS FOR THE INTERNET & BEYOND

33. Analyzing a Statistical Study. Find a detailed report on some recent statistical study of interest to you. Write a short report applying each of the eight guidelines given in this section. (Some of the guidelines may not apply to the particular study you are analyzing; in that case, explain why the guideline is not applicable.)

34. Twin Studies. Researchers doing statistical studies in biology, psychology, and sociology are grateful for the existence of twins. Twins can be used to study whether certain traits are inherited from parents (nature) or acquired from one's surroundings during upbringing (nurture). Identical twins are formed from the same egg in the mother and have the same genetic material. Fraternal twins are formed from two separate eggs and share roughly half of the same genetic material. Find a published report of a twin study. Discuss how identical and fraternal twins are used to form case and control groups. Apply Guidelines 1–8 to the study and comment on whether you find the conclusions of the report convincing.

35. Professional Journals. Consult an issue of a professional journal. Select one specific article and use the ideas of this section to summarize and evaluate the study.

IN THE NEWS

36. Applying the Guidelines. Find a recent newspaper article or television report about a statistical study on a topic that you find interesting. Write a short report applying each of the eight guidelines given in this section. (Some of the guidelines may not apply to the particular study you are analyzing; in that case, explain why the guideline is not applicable.)

37. Believable Results. Find a recent news report about a statistical study whose results you believe are meaningful and important. In one page or less, summarize the study and explain why you find it believable.

38. Unbelievable Results. Find a recent news report about a statistical study whose results you *don't* believe are meaningful and important. In one page or less, summarize the study and explain why you don't believe its claims.



CHAPTER REVIEW EXERCISES

1. Tats. A Harris poll surveyed 2,320 adults in the United States, among which 14% said that they have at least one tattoo. The margin of error is 2 percentage points.

- a. Interpret the margin of error by identifying the range of values likely to contain the percentage of adults with tattoos.
- b. Identify the population.
- c. Is this study an experiment or an observational study? Explain. Identify the variable of interest.
- d. Is the reported value of 14% a population parameter or a sample statistic? Why?
- e. If you learned that survey subjects responded to a magazine article asking readers to phone in their responses, would you consider the survey results to be valid? Why or why not?
- f. Describe a procedure for selecting the survey subjects using a simple random sample.
- g. Describe a procedure for selecting similar survey subjects using stratified sampling.
- h. Describe a procedure for selecting similar survey subjects using cluster sampling.
- i. Describe a procedure for selecting similar survey subjects using systematic sampling.
- j. Describe a procedure for selecting similar survey subjects using a convenience sample.

2. Simple Random Sample. An important element of this chapter is the concept of a simple random sample.

- a. What is a simple random sample?
- b. When the Bureau of Labor Statistics conducts a survey, it begins by partitioning the United States adult population into 2,007 groups called *primary sampling units*. Assume that these primary sampling units all contain the same number of adults. If you randomly select one adult from each primary sampling unit, is the result a simple random sample? Why or why not?
- c. Refer to the primary sampling units described in part b and describe a sampling plan that results in a simple random sample.

3. Clinical Trial of Bystolic. In clinical trials of the drug Bystolic used to treat hypertension (high blood pressure), 677 Bystolic users were observed for adverse reactions. It was found that among those treated with Bystolic, 7% experienced headaches.

- a. Based on the given information, can you conclude that in some cases, Bystolic causes headaches? Why or why not?
- b. While 677 subjects were treated with Bystolic, another 205 subjects were given a placebo, and 6% of the placebo group experienced headaches. What does this additional information suggest about headaches as an adverse reaction to the use of Bystolic?
- c. Is this clinical trial an observational study or an experiment? Explain.
- d. In this clinical trial, what is blinding and why is it important in testing the effects of Bystolic?
- e. What is an experimenter effect, and how might this effect be minimized?

4. Wording of a Survey Question. In *The Superpollsters*, David W. Moore describes an experiment in which different subjects were asked if they agree with the following statements:

- i. Too little money is being spent on welfare.
 - ii. Too little money is being spent on assistance to the poor.
- Even though it is the poor who receive welfare, only 19% agreed when the word “welfare” was used, but 63% agreed with “assistance to the poor.”
- a. Which of the two questions should be used in a survey? Why?
 - b. If you are working on a campaign for a conservative candidate for Congress, and you want to emphasize opposition to the use of federal funds for assistance to the poor, which of the two questions would you use? Why?
 - c. Is it ethical to deliberately word a survey question so that it influences responses? Why or why not?

CHAPTER QUIZ

Choose the best answer to each of the following questions. Explain your reasoning with one or more complete sentences.

1. You conduct a poll in which you randomly select 1,200 college students in California and ask if they have taken an online course. The *population* for this study is: (a) All students who have taken an online course; (b) the 1200 college students that you interview; (c) all college students in California.
2. For the poll described in Exercise 1, which sampling plan would likely yield results that are *most* biased: (a) Mail the survey to college students in California and use the returned responses; (b) randomly select 20 colleges in California, then randomly select 60 students at each college; (c) obtain a numbered list of all college students in California and select every 100th name until a sample size of 1,200 is obtained.
3. When we say that a sample is *representative* of the population, we mean that: (a) the results found for the sample are similar to those we would find for the entire population; (b) the sample is very large; (c) the sample was chosen in the best possible way.
4. Consider an experiment designed to see whether cash incentives can improve school attendance. The researcher chooses two groups of 100 high school students: She offers one group \$10 for every week of perfect attendance. She tells the other group that they are part of an experiment but does not give them any incentive. The students who do not receive an incentive represent: (a) the treatment group; (b) the control group; (c) the observation group.
5. The experiment described in Exercise 4 is: (a) single-blind; (b) double-blind; (c) not blind.
6. The purpose of a *placebo* is: (a) to prevent participants from knowing whether they belong to the treatment group or the control group; (b) to distinguish between the cases and the controls in a case-control study; (c) to determine whether diseases can be cured without any treatment.
7. If we see a *placebo effect* in an experiment to test a new treatment designed to cure warts, it means: (a) the experiment was not properly double-blind; (b) the experimental groups were too small; (c) warts were cured among members of the control group.
8. An experiment is single-blind if: (a) it lacks a treatment group; (b) it lacks a control group; (c) the participants do not know whether they belong to the treatment or control group.
9. Poll X predicts that Powell will receive 49% of the vote, while Poll Y predicts that she will receive 53% of the vote. Both polls have a margin of error of 3 percentage points. What can you conclude? (a) one of the two polls must have been conducted poorly; (b) the two polls are consistent with one another; (c) Powell will receive 51% of the vote.
10. A survey reveals that 24% of adults believe that the most fun way to flirt is through instant messages. The margin of error is 3 percentage points. The confidence interval for this poll is: (a) from 18% to 30%; (b) from 24% to 27%; (c) from 21% to 27%.
11. A study conducted by the oil company Exxon Mobil shows that there was no lasting damage from a large oil spill in Alaska. This conclusion: (a) is definitely invalid, because the study was biased; (b) may be correct, but the potential for bias means you should look very closely at how the conclusion was reached; (c) could be correct if it falls within the confidence interval of the study.
12. The television show *American Idol* selects winners from votes cast by anyone who wants to vote. This means the winner: (a) is the person most Americans want to win; (b) may or may not be the person most Americans want to win, because the voting is subject to participation bias; (c) may or may not be the person most Americans want to win, because the voting should have been double-blind.
13. Consider an experiment in which you measure the weights of randomly selected cars. The variable of interest in this study is: (a) the size of the sample; (b) the weights of the cars; (c) the average (mean) weight of all cars.
14. Imagine a survey of randomly selected people in which it is found that people who use sunscreen were *more* likely to have been sunburned in the past year. Which explanation for this result seems most likely? (a) sunscreen is useless; (b) the people in this study all used sunscreen that had passed its expiration date; (c) people who use sunscreen are more likely to spend time in the sun.
15. If a statistical study is carefully conducted in every possible way, then: (a) its results must be correct; (b) we can have confidence in its results, but it is still possible that the results are not correct; (c) we say that the study is perfectly biased.



FOCUS ON PSYCHOLOGY

Are You Driving "Drunk" on Your Cell Phone?

One of the hottest topics in statistics deals with how cell phones and other types of distraction affect the abilities and reactions of drivers, and whether these distractions lead to more accidents and fatalities than would occur otherwise. These questions are studied by scientists in a variety of disciplines but are especially important to psychologists, because the answers depend on how the human brain reacts to different types of stimuli.

Many different types of statistical study have looked at the issue. Some researchers study accident reports to see what fraction of accidents involve talking on a cell phone, texting, or other distractions. Some have looked at cell phone records of people who have been in crashes, to see if the phone was in use at the time of the crash. These studies clearly indicate higher crash and fatality rates associated with distracted driving. Perhaps most notably, the University of Utah's Applied Cognition Lab has conducted a series of studies in which subjects use simulators to drive cars under a variety of distraction conditions, including use of a hand-held cell phone, use of a hands-free cell phone, and while texting. They compare these results with those in which the same subjects are not distracted and are either sober or intoxicated. Their astonishing conclusion: Even with hands-free devices, talking on a cell phone makes drivers as dangerous as drunk drivers, and texting and other distractions can make drivers even more likely to cause a crash.

Should we believe this claim that using your cell phone essentially makes you a drunk driver? As always, there are many ways to evaluate the claim, but for practice, let's use the eight guidelines given in Section 4.

Guideline 1: Get a Big Picture View of the Study. The goal of the Utah studies is to learn about the relative danger of different types of distraction, especially of cell phones, and the population under study is all drivers. The study is an experiment, because it has the same subjects drive simulators under different conditions of distraction, with the control being the results when they are undistracted and sober.

Guideline 2: Consider the Source. We can generally assume that a university lab operates independently, and we have no reason to suspect that any pressure is put on researchers to come up with a particular result. Indeed, if there were pressure, it would likely come from cell phone companies that would have a bias toward *not* finding danger in their products, which is the opposite of what the researchers concluded. We



similarly have no reason to suspect bias on the part of the researchers themselves. It's almost impossible to be sure that no bias is involved, but in this case the source seems likely to be trustworthy.

Guideline 3: Look for Bias in the Sample. For the results to be valid, the group of subjects must be a representative sample of all drivers. Media reports of the Utah studies rarely talk about the sample selection method, but in this case it does not seem like it should be difficult to get a representative sample, since it's unlikely that any particular group of people would be especially prone or immune to distractions. So unless we see evidence to the contrary, it seems safe to assume that the sample was well chosen.

Guideline 4: Look for Problems Defining or Measuring the Variables of Interest. The Utah researchers study variables such as the reaction times of drivers in surprising situations and how often they get into crashes in their simulated drives. These variables are straightforward to define and measure.

Guideline 5: Beware of Confounding Variables. It's almost impossible to eliminate all possible confounding variables, and we can easily think of some variables that might affect these studies. For example, the driver responses might depend on the order and time at which different tests are conducted, as a driver may be fatigued as tests progress or might not be feeling well on a particular day. However, the researchers can in principle avoid most of these problems by changing the order of the tests for different drivers on different days and by having a large enough test group so that things like minor illnesses should not affect the overall results.

Guideline 6: Consider the Setting and Wording in Surveys. The Utah studies are not survey based, so this guideline does not apply. However, studies such as those that look at real crash rates are essentially surveys based on crash reports compiled by police or insurance companies, so it's worth

considering potential biases in them. For example, could it be that the role of cell phones in crashes might be either overreported or underreported? A little thought shows the former is unlikely, because no one would be expected to tell police they were on a cell phone if they weren't. But the opposite seems quite likely, because denying cell phone use might be a way of deflecting potential blame for a crash. We conclude that if the surveys are biased at all, it is most likely toward *underestimating* the danger of distracted driving.

Guideline 7: *Check That Results Are Presented Fairly.* The Utah studies have found clear evidence of the danger of cell phone use, with or without hand-free devices. The fact that these results are consistent with actual data from accidents gives further reason to believe that the results have been presented fairly.

Guideline 8: *Stand Back and Consider the Conclusions.* This last step is essentially our summary. We've found that the Utah studies have achieved their goals and lead to clear conclusions about the danger of using a cell phone while driving. In fact, the studies also show that similar or greater danger arises from texting while driving, programming your GPS while driving, finding music on your iPod while driving, and a variety of other distractions. Do these results have practical significance? They should, as they make it very clear that if you talk on your cell phone or are otherwise distracted while driving, you are putting your own life and the lives of others at risk.

QUESTIONS FOR DISCUSSION

1. Do you ever drive while talking on your cell phone, texting, or programming your GPS? If so, have you ever noticed evidence of your distraction, such as missing a turn or an especially close call for a collision? Have you seen such evidence of distraction when being driven by friends or family or taxi drivers?
 2. Public safety advocates and many insurance companies are using the data about distracted driving to argue in favor of laws banning the use of cell phones while driving. Do you support or oppose such laws? Defend your opinion.
 3. The fact that the studies found essentially no difference in danger between hand-held and hands-free devices comes as a surprise to most people. Research studies on brain activity that have provided an explanation for this surprising fact, often called "inattention blindness." Discuss how and why talking on a hands-free cell phone turns out to increase the risk of crashes, while talking to a passenger does not.
 4. Find the latest statistics from the U.S. Department of Transportation concerning the number of crashes and fatalities estimated to be caused by distracted driving each year. Do you think the estimates are likely to be accurate? Why or why not?
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FOCUS ON PUBLIC HEALTH

Is Your Lifestyle Healthy?

Consider the following findings from statistical studies:

- Smoking increases the risk of heart disease.
- Eating margarine can increase the risk of heart disease.
- One glass of wine per day can protect against heart disease but increases the risk of breast cancer.
- Potato chips and sugary sodas are the foods most strongly associated with weight gain.

You are probably familiar with some of these findings, and perhaps you've even altered your lifestyle as a result of them. But where do they come from? Remarkably, these and hundreds of other important findings on public health come from huge prospective studies that have provided data for hundreds of smaller statistical studies. The longest-running of these is the Harvard Nurses' Health Study, which began in 1976 when Dr. Frank E. Speizer decided to study the long-term effects of oral contraceptives. He mailed questionnaires to approximately 370,000 registered nurses and received more than 120,000 responses. He chose to survey nurses because he believed that their medical training would make their responses more reliable than those of the general public.

As Dr. Speizer and his colleagues sifted through the data in the returned questionnaires, they realized that the study could be expanded to include more than just the effects of contraceptives. Today, this research team continues to follow many of the original 120,000 respondents.

Annual questionnaires are still a vital part of the study, allowing researchers to gather data about what the nurses eat; what medicines and vitamins they take; whether and how much they exercise, drink, and smoke; and what illnesses they have contracted. Some of the nurses also provide blood samples, which are used to measure such things as cholesterol level, hormone levels, genetic variations, and residues from pesticides and environmental pollutants. Dr. Speizer's faith in nurses has proven justified, as they reliably complete surveys and almost always provide properly drawn and labeled blood samples upon request.

After more than three decades of correspondence, both the researchers and the nurses say they feel a sense of closeness. Many of the nurses look forward to hearing from the researchers and say that the study has helped them to pay more attention to how they live their lives. Today, as the original nurses become elderly, the study is beginning to turn out results that should shed light on factors that influence longevity and health in old age.

The success of the Harvard Nurses Study has spurred its expansion and many similar studies of large groups. When you



see statistical reports based on these studies, remember the hundreds of thousands of people whose willingness to participate in these studies is making life better for everyone.

QUESTIONS FOR DISCUSSION

1. Consider some of the results that are likely to come from the Harvard Nurses' Health Study over the next 10 to 20 years. What types of results do you think will be most important? Do you think the findings will alter the way you live your life?
2. Explain why the Harvard Nurses' Health Study is an observational study. Critics sometimes say that the results would be more valid if obtained by experiments rather than observations. Discuss whether it would be possible to gather similar data by carrying out experiments in a practical and ethical way.
3. In principle, the Harvard Nurses' Health Study is subject to participation bias because only 120,000 of the original 370,000 questionnaires were returned. Should the researchers be concerned about this bias? Why or why not?
4. Another potential pitfall comes from the fact that the questionnaires often deal with sensitive issues of personal health, and researchers have no way to confirm that the nurses answer honestly. Do you think that dishonesty could be leading researchers to incorrect conclusions? Defend your opinion.
5. All of the participants in the Harvard Nurses' Health Study were women. Do you think that the results also are of use to men? Why or why not?
6. Do a Web search for news articles that discuss results from the Harvard Nurses' Health Study or other similar studies. Choose one recent result that interests you, and discuss what it means and how it may affect public health or your own health in the future.

ANSWERS

Section 1

1. A population is the complete set of people or things being studied, while a sample is a subset of the population. The difference is that the sample is only a part of the complete population.
3. A sample statistic is a numerical measure of a characteristic of a *sample* found by consolidating or summarizing raw data. A population parameter is a numerical measure of a characteristic of a *population*. Because it is usually impractical to directly measure population parameters for large populations, we usually infer likely values of the population parameters from the measured sample statistics.
5. Does not make sense. Population parameters are typically inferred from sample statistics. When population parameters are known, sample statistics are not needed and the concept of “margin of error” does not apply.
7. Does not make sense; in fact, the statement is clearly false. The confidence interval in this case runs from $54\% - 3\% = 51\%$ to $54\% + 3\% = 57\%$. Assuming the margin of error is defined as it is in most cases, this interval is a 95% confidence interval, meaning that we can be 95% confident that Johnson received between 51% and 57% of the vote. While this makes it quite likely that Johnson really did receive a majority of the vote (assuming the poll was conducted well), it is not a certainty, as there is still a 5% chance that her true percentage of the vote was above 57% or below 51% — and it is possible that it could be enough below so that she did not win a majority of the votes.
9. Does not make sense. The sample should be drawn from the population of all people who have suffered through a family tragedy, which is not the same as the population of patients in support groups for loss of a spouse. There are many types of family tragedies besides loss of a spouse, and not all of those people join support groups.
11. *Sample:* The 1,018 adults selected. *Population:* All adults in the United States. *Sample statistic:* 22%. The value of the population parameter is not known, but it is the percentage of all adults in the United States who smoked cigarettes in the past week.
13. *Sample:* The 47 subjects treated with Garlicin. *Population:* The complete set of all adults. *Sample statistic:* 3.2. The value of the population parameter is not known, but it is the average (mean) change in LDL cholesterol.
15. 57% to 63%
17. 93% to 99%
19. Yes. Although there is no guarantee, the results suggest that the majority of adults believe that immediate government action is required, because the percentage is most likely between 53% and 57%.
21. Yes. Based on the survey, the actual percentage of voters is expected to be between 67% and 73%, which does not include the 61% value based on actual voter results. If the survey was conducted well, then it is unlikely that it would be so different

from the actual results, implying either that respondents intentionally lied to appear favorable to the pollsters or that their memories may have been inaccurate.

23. **a.** *Goal:* Determine the percentage of all adults in favor of the death penalty for people convicted of murder. *Population:* The complete set of all adults. *Population parameter:* The percentage of all adults who are in favor of the death penalty for people convicted of murder. **b.** *Sample:* The 511 selected subjects. *Raw data:* Individual responses to the question. *Sample statistic:* 64%. **c.** 60% to 68%
25. **a.** *Goal:* Determine the percentage of households with a TV tuned to the Super Bowl. *Population:* The complete set of all U.S. households. *Population parameter:* The percentage of all U.S. households tuned to the Super Bowl. **b.** *Sample:* The sample of 9,000 households selected for the survey. *Raw data:* Individual indications of whether the household has a TV tuned to the Super Bowl. *Sample statistic:* 45%. **c.** 44% to 46%
27. *Step 1.* Goal: Identify the percentage of all drivers who use cell phones while they are driving. *Step 2.* Choose a sample of drivers. *Step 3.* Collect data from the sample of drivers to determine the percentage of them who use a cell phone while driving. *Step 4.* Use the techniques of the science of statistics to make an inference about the percentage of all people who use cell phones while they are driving. *Step 5.* Based on the likely value of the population parameter, form a conclusion about the percentage of people who use cell phones while they are driving.

Section 2

1. A census is the collection of data from every member of the population, but a sample is a collection of data from only part of the population.
3. With cluster sampling, we select all members of randomly selected subgroups (or clusters), but with stratified sampling, we select samples from each of the different subgroups (or strata).
5. Does not make sense because it is not possible or practical to survey every student, as would be required for a census.
7. Makes sense. Because it is obvious that the percentage of Americans more than 6 feet tall is well under 75%, the study must have somehow used a biased sample.
9. A census is practical because the population consists of the small number of players on the LA Lakers team, and it is easy to obtain their heights.
11. A census is not practical. The number of statistics instructors is large, and it would be extremely difficult to get them all to take an IQ test.
13. The sample is the service times of the 4 selected senators. The population consists of the service times of all 100 senators. This is an example of random sampling (or simple random sampling).

However, because the sample is so small, it is not likely to be representative of the population.

- 15.** The sample is the 1,059 selected adults, and the population is the complete set of all adults. This is an example of simple random sampling. Because the sample is fairly large and it was obtained by a reputable firm, the sample is likely to be representative of the population.
- 17.** Most representative: Sample 3, because the list is likely to represent the most people, and there is no reason to think that people with the first 1,000 numbers would differ in any particular way from other people. Sample 1 is biased because it involves owners of expensive vehicles. Sample 2 is biased because it involves people from only one geographic region. Sample 4 is biased because it involves a self-selected sample consisting of people who are more likely to have strong feelings about the issue of credit card debt.
- 19.** Yes. Because the film critic indirectly works for Disney, he or she may be more inclined to submit a more favorable review.
- 21.** Yes. The university scientists receive the funding from Monsanto, so they might be inclined to please the company in the hopes of getting further funding in the future. Thus, there may be an inclination to provide favorable results. To determine whether this bias is a problem, you would need to explore the methods and conclusions very carefully.
- 23.** The sample is a simple random sample that is likely to be representative because there is no bias in the selection process.
- 25.** The sample is a cluster sample. It is likely to be representative, although the exact method of selecting the polling stations could affect whether the sample is biased.
- 27.** The sample is a convenience sample. It is likely to be biased, because the sample consists of family members likely to have the same socio-economic backgrounds and interests.
- 29.** The sample is a stratified sample. It is likely to be biased because people from those age groups are not evenly distributed throughout the population. However, the results could be weighted to reflect the age distribution of the population.
- 31.** The sample is a systematic sample. The sample is likely to be representative, because there is nothing about the alphabetical ordering that is likely to result in a biased sample.
- 33.** The sample is a stratified sample. It is likely to be biased because the population does not have equal numbers of people in each of the three categories. However, the results could be weighted to reflect the actual distribution of the population.
- 35.** The sample is a convenience sample. It is likely to be biased because it is a self-selected sample and consists of those with strong feelings about the issue.
- 37.** The sampling plan results in a simple random sample, so it is likely to be representative.
- 39.** Simple random sampling of the student body. Be sure you use a large enough sample size to get meaningful results.
- 41.** Cluster sampling of death records should give good data for this case.

Section 3

- 1.** A placebo is physically similar to a treatment, but it lacks any active ingredients, so it should not by itself produce any effects. A placebo is important so that results from subjects given a real treatment can be compared with the results from subjects given a placebo.
- 3.** Confounding is the mixing of effects from different factors so that we cannot determine the effects from the specific factors being studied. If males are given the treatment and females are given placebos, we would not know whether effects are due to the treatment or the sex of the participant.
- 5.** It makes sense to use a double-blind experiment, but the experimental design is tricky because the subjects can clearly see the clothing that they are wearing, and the evaluators can also see those colors. Blinding might be used by not telling the subjects about the experiment, so their knowledge of their clothes colors does not affect their results. Because it is difficult to use blinding for the evaluators, it is important to obtain results based on objective measures not subject to the judgments of the evaluators.
- 7.** The experimenter effect occurs when a researcher or experimenter somehow influences subjects through such factors as facial expression, tone of voice, or attitude. The experimenter effect can be avoided by using blinding so that those who evaluate the results do not know which subjects are given an actual treatment and which subjects are given no treatment or a placebo.
- 9.** Observational because the batteries are measured, but they are not treated.
- 11.** Experiment. The treatment group consists of the subjects given the magnetic bracelets, and the control group consists of the subjects given the bracelets that have no magnetism. It is probably not possible to use blinding with this study, because the passengers could easily test their bracelets to see if they are magnetic (by holding them near metal).
- 13.** This is an observational, retrospective study examining how a characteristic determined before birth (identical or fraternal twins) affected mental skills later.
- 15.** Experiment because the subjects are given a treatment. The treatment group consists of the 152 couples given the YSORT treatment. The control group consists of others not given any treatment.
- 17.** This is an experiment. The treatment group consists of the genetically modified corn, and the control group consists of corn not genetically modified.
- 19.** This is an experiment. The treatment group consists of those treated with magnets. The control group consists of those given the non-magnetic devices.
- 21.** Confounding is likely to occur. If there are differences in effects from the two groups, there is no way to know if those differences are attributable to the treatment (fertilizer or irrigation) or the type of region (moist or dry). This confounding can be avoided by using blocks of fertilized trees in both the moist region and dry region and using blocks of irrigated trees in both the moist region and dry region.

- 23.** Confounding is likely. If there are differences in the amounts of gasoline consumed, there would be no way to know whether those differences are due to the octane rating of the gasoline or the type of vehicle. Confounding can be avoided by using 87 octane gasoline in half of the vans and half of the sport utility vehicles and by using 91 octane gasoline in the other vehicles. Even better, conduct an experiment in which identical vehicles are driven under the same conditions (speed, distance, etc.) with the different octane gasolines.
- 25.** Subjects clearly know whether they are treated with running, so confounding is possible from a placebo effect. Moreover, there is no objective way to measure back pain, so different subjects may report changes in back pain differently. Also, there could be an experimenter effect that can be avoided with blinding of those who evaluate results.
- 27.** In this case, the tennis balls play the role of placebos. Confounding can occur because of a placebo effect and/or an experimenter effect, because it will be obvious to both subjects and experimenters whether they are lifting heavy weights. It would be better to use the heavy weights and the tennis balls with the same subjects at different times, to see if the different regimens affect blood pressure.
- 29.** The control group consists of those who do not listen to Beethoven and the treatment group consists of those who do listen to Beethoven. By using subjects who are coded, blinding could be used so that those who measure intelligence are not influenced by their knowledge about the participants.
- 31.** The control group consists of cars using gasoline without the ethanol additive, and the treatment group consists of cars using gasoline with the ethanol additive. Blinding is not necessary for the cars, and it is probably unnecessary for the researchers because the mileage will likely be measured with objective tools.
- 11.** “Good” is not well defined and is difficult to measure. Guideline 4 is most relevant.
- 13.** The sample is self-selected, so participation bias is a serious issue. Guideline 3 is most relevant.
- 15.** The wording of the question is biased and tends to elicit negative responses. Guideline 6 is most relevant.
- 17.** Because much of the funding was provided by Mars and the Chocolate Manufacturers Association, the researchers may have been more inclined to provide favorable results. The bias could have been avoided if the researchers were not paid by the chocolate manufacturers. If that was the only way the research could be done, then the researchers should institute procedures to ensure that they publish all results, including negative ones.
- 19.** The wording of the question was biased to strengthen opposition against a particular candidate, and is likely to be a “push poll” financed by supporters of another candidate, rather than a legitimate poll. A better sampling method would involve questions devoid of such bias.
- 21.** The word “wrong” in the first question could be misleading. Some people might believe that abortion is wrong, but still favor choice. The second question could also be confusing, as some people might think that “advice of her doctor” means that the woman’s life is in danger, which could alter their opinion about abortion in this situation. Groups opposed to abortion would be likely to cite the results of the first question, while groups favoring abortion would be more likely to cite the results of the second question.
- 23.** The first question requires a study of Internet dates. The second question involves a study of married people to determine whether their first date was an Internet date. The two questions are likely to yield very different types of information. For example, the second question would tell you what percentage of current marriages began with Internet dates, but the first would only tell you how often Internet dating leads to marriage, which might not be any different from how often other forms of dating lead to marriage. The goals of the study need to be better defined and the questions framed to meet the goals.
- 25.** The first question involves a study of college students in general, and the second question involves a study of those who do binge drinking. The first question might be addressed by surveying college students. The second question would be addressed by surveying binge drinkers, and it would be much more difficult to survey this group.
- 27.** The headline refers to drugs whereas the story refers to “drug use, drinking, or smoking.” Because “drugs” is generally considered to consist of drugs other than cigarettes or alcohol, the headline is very misleading.
- 29.** No information is given about the meaning of “confidence.” The sample size and margin of error are not provided.
- 31.** No information is given to justify the statement that “more” companies try to bet on weather forecasting. If only the four cited companies are new, the increase is relatively insignificant.

Section 4

- 1.** Peer review is a process in which experts in a field evaluate a research report before the report is published. It is useful for lending credibility to the research because it implies that other experts agree that it was carried out properly.
- 3.** When participants select themselves for a survey, those with strong opinions about the topic being surveyed are more likely to participate, and this group is typically not representative of the general population.
- 5.** Does not make sense. A survey involving a large sample could be very poor if it involves a poor sampling method (such as a self-selected sample), while a smaller sample could produce much better results if it involves a sound sampling method (such as a simple random sample).
- 7.** Does not make sense. We often don’t even know if confounding variables are present, so we cannot be certain they’ve all been taken into account.
- 9.** The survey was funded by a source that can benefit through increased sales fostered by the survey results, so there is a potential for bias in the survey. Guideline 2 is most relevant.

Chapter Review Exercises

- 1.** **a.** 12% to 16% **b.** All adults in the United States **c.** It is an observational study because the subjects were not treated or modified in any way. The variable of interest is whether the subject has a tattoo. For this survey, that variable has two values: yes or no. **d.** The value is a sample statistic because it is based on the sample of 2,320 adults, not the population of all adults. **e.** No, because it would be a self-selected sample with a likely participation bias. **f.** Use a computer to randomly generate Social Security numbers, identify the people with those Social Security numbers, then select those people. **g.** Select a sample of subjects in each state. **h.** Select all of the adults in several randomly selected streets. **i.** Select every 10th adult, by address, on each street in a city. **j.** Select your classmates.
- 3.** **a.** No, because there is no information about the occurrence of headaches among people who do not use Bystolic. Based on the given information, it is possible that 7% of the population

experiences headaches whether or not Bystolic is used. **b.** It appears that Bystolic users have about the same rate of headaches as those given a placebo, so headaches do not appear to be an adverse reaction to Bystolic use. **c.** This is an experiment because subjects are given a treatment. **d.** With blinding, the trial participants do not know whether they are getting Bystolic or a placebo, and those who evaluate the results also do not know. **e.** An experimenter effect occurs if the experimenter somehow influences subjects through such factors as facial expression, tone of voice, or attitude. It can be avoided through the use of blinding.

Chapter Quiz

- 1.** c **3.** a **5.** c **7.** c **9.** b
11. b **13.** b **15.** b

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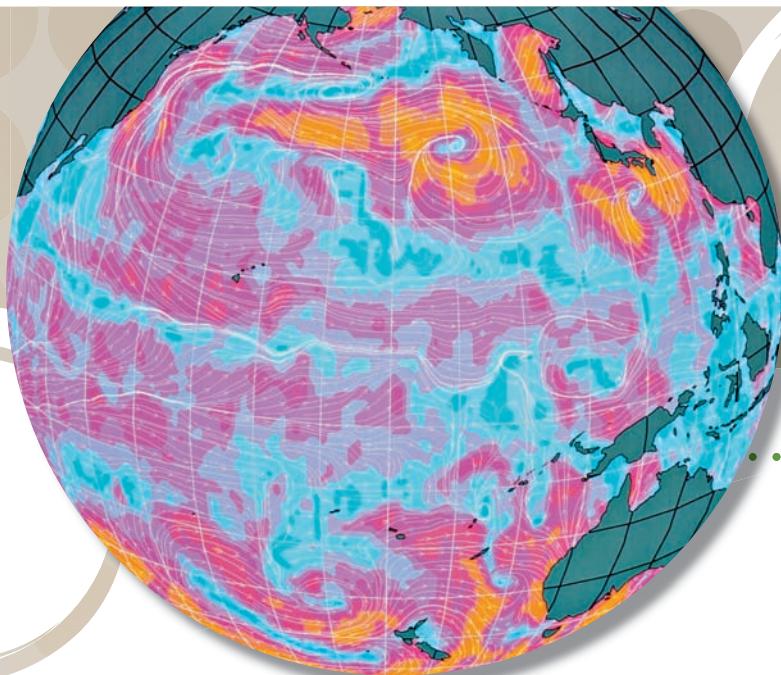
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Measurement in Statistics

We all know how to measure quantities such as height, weight, and temperature. However, in statistical studies there are many other kinds of measurements, and we must be sure that they are defined, obtained, and reported carefully. In this chapter, we will discuss a few important concepts associated with measurements and statistics. As you'll see, these concepts are very useful to understanding the statistical reports you encounter in your daily life.

Practically no one knows what they're talking about when it comes to numbers in the newspapers. And that's because we're always quoting other people who don't know what they're talking about, like politicians and stock-market analysts.

—Molly Ivins (1944–2007), newspaper columnist

LEARNING GOALS

1 Data Types and Levels of Measurement

Identify data as qualitative or quantitative, discrete or continuous, and by level of measurement (nominal, ordinal, interval, or ratio).

2 Dealing with Errors

Understand the difference between random and systematic errors, describe errors by their absolute and relative sizes, and know the difference between accuracy and precision in measurements.

3 Uses of Percentages in Statistics

Understand how percentages are used to report statistical results and recognize ways in which they are sometimes misused.

4 Index Numbers

Understand the concept of an index number and the use of the Consumer Price Index (CPI).

FOCUS TOPICS

Focus on Politics: Who Benefited Most from Lower Tax Rates?

Focus on Economics: Can a Redefined CPI Help Solve the Budget Crisis?

1

DATA TYPES AND LEVELS OF MEASUREMENT

One of the challenges in statistics is deciding how best to summarize and display data. Different types of data call for different types of summaries. In this section, we'll discuss how data are categorized, an idea that will help us when we consider data summaries and displays.

Data Types

Data come in two basic types: qualitative and quantitative. **Qualitative data** have values that can be placed into *nonnumerical categories*. (For this reason, qualitative data are sometimes called *categorical* data.) For example, eye color data are qualitative because they are categorized by colors such as blue, brown, and hazel. Other examples of qualitative data include flavors of ice cream, names of employers, genders of animals, and movie or restaurant ratings, such as bad, average, good, and excellent.

Quantitative data have numerical values representing counts or measurements. The times of runners in a race, the incomes of college graduates, and the numbers of students in different classes are all examples of quantitative data.

Data Types

Qualitative (or categorical) data consist of values that can be placed into nonnumerical categories.

Quantitative data consist of values representing counts or measurements.

EXAMPLE 1 Data Types

Classify each of the following sets of data as qualitative or quantitative.

- Brand names of shoes in a consumer survey
- Scores on a multiple-choice exam
- Letter grades on an essay assignment
- Numbers on uniforms that identify players on a basketball team

SOLUTION

- Brand names are categories and therefore represent qualitative data.
- Scores on a multiple-choice exam are quantitative because they are counts of the number of correct answers.
- Letter grades on an essay assignment are qualitative because they represent different categories of performance (failing through excellent).
- The players' uniform numbers are qualitative because they do not represent a count or measurement; they are used solely for identification. You can tell that these numbers are qualitative rather than quantitative because you could not use them for computations. For example, it would make no sense to add or subtract the uniform numbers of different players.

• •

Discrete versus Continuous Data

Quantitative data can be further classified as continuous or discrete. Data are **continuous** if they can take on *any* value in a given interval. For example, a person's weight can be anything between 0 and a few hundred pounds, so data that consist of weights are continuous. Data are **discrete** if they can take on only particular values and not other values in between. For

example, the number of students in your class is discrete because it must be a whole number, and shoe sizes are discrete because they take on only integer and half-integer values such as 7, $7\frac{1}{2}$, 8, and $8\frac{1}{2}$. (Actual foot lengths are continuous, but such shoe sizes are discrete.)

Discrete versus Continuous Data

Continuous data can take on *any* value in a given interval.

Discrete data can take on only particular, distinct values and not other values in between.

EXAMPLE 2 Discrete or Continuous?

For each data set, indicate whether the data are discrete or continuous.

- Measurements of the time it takes to walk a mile
- The numbers of calendar years (such as 2013, 2014, 2015)
- The numbers of dairy cows on different farms
- The amounts of milk produced by dairy cows on a farm

SOLUTION

- Time can take on any value, so measurements of time are continuous.
- The numbers of calendar years are discrete because they cannot have fractional values. For example, on New Year's Eve of 2016, the year will change from 2016 to 2017; we'll never say the year is $2016\frac{1}{2}$.
- Each farm has a whole number of cows that we can count, so these data are discrete. (You cannot have fractional cows, for example.)
- The amount of milk that a cow produces can take on any value in some range, so the milk production data are continuous.

• • •

Levels of Measurement

Another way to classify data is by their *level of measurement*. The simplest level of measurement applies to variables such as eye color, ice cream flavors, or gender of animals. These variables can be described solely by names, labels, or categories. We say that such data are at a **nominal level of measurement**. (The word *nominal* refers to *names* for categories.) The nominal level of measurement does not involve any ranking or ordering of the data. For example, we could not say that blue eyes come before brown eyes or that vanilla ranks higher than chocolate.

When we describe data with a ranking or ordering scheme, such as star ratings of movies or restaurants, we are using an **ordinal level of measurement**. (The word *ordinal* refers to *order*.) Such data generally cannot be used in any meaningful way for computations. For example, it doesn't make sense to add star ratings—watching three one-star movies is not equivalent to watching one three-star movie.

TIME OUT TO THINK

Consider a survey that asks "What's your favorite flavor of ice cream?" We've said that ice cream flavors represent data at the nominal level of measurement. But suppose that, for convenience, the researchers enter the survey data into a computer by assigning numbers to the different flavors. For example, they assign 1 = vanilla, 2 = chocolate, 3 = cookies and cream, 4 = cherry Garcia, and so on. Does this change the ice cream flavor data from nominal to ordinal? Why or why not?

The ordinal level of measurement provides a ranking system, but it does not allow us to determine precise differences between measurements. For example, there is no way to determine the exact difference between a three-star movie and a two-star movie. In contrast, a temperature of 81°F is hotter than 80°F by the same amount that 28°F is hotter than 27°F. Temperature data are at a higher level of measurement, because the *intervals* (differences) between units on a temperature scale always mean the same definite amount. However, while intervals (which involve subtraction) between Fahrenheit temperatures are meaningful, *ratios* (which involve division) are not. For example, it is *not true* that 20°F is twice as hot as 10°F or that -40°F is twice as cold as -20°F. The reason ratios are meaningless on the Fahrenheit scale is that its *zero point is arbitrary* and does not represent a state of “no heat.” If intervals are meaningful but ratios are not, as is the case with Fahrenheit temperatures, we say that the data are at the **interval level of measurement**.

When both intervals and ratios are meaningful, we say that data are at the **ratio level of measurement**. For example, data consisting of distances are at the ratio level of measurement because a distance of 10 kilometers really is twice as far as a distance of 5 kilometers. In general, the ratio level of measurement applies to any scale with a *true zero*, which is a value that means *none* of whatever is being measured. In the case of distances, a distance of zero means “no distance.” Other examples of data at the ratio level of measurement include weights, speeds, and incomes.

Note that data at the nominal or ordinal level of measurement are always qualitative, while data at the interval or ratio level are always quantitative (and can therefore be either continuous or discrete). Figure 1 summarizes the possible data types and levels of measurement.

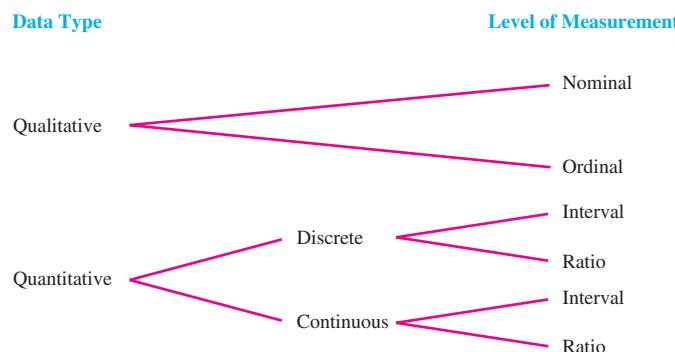


Figure 1 Data types and levels of measurement.

Levels of Measurement

The **nominal level of measurement** is characterized by data that consist of names, labels, or categories only. The data are qualitative and cannot be ranked or ordered.

The **ordinal level of measurement** applies to qualitative data that can be arranged in some order (such as low to high). It generally does not make sense to do computations with data at the ordinal level of measurement.

The **interval level of measurement** applies to quantitative data in which intervals are meaningful, but ratios are not. Data at this level have an arbitrary zero point.

The **ratio level of measurement** applies to quantitative data in which both intervals and ratios are meaningful. Data at this level have a true zero point.

EXAMPLE 3 Levels of Measurement

Identify the level of measurement (nominal, ordinal, interval, ratio) for each of the following sets of data.

- Numbers on uniforms that identify players on a basketball team
- Student rankings of cafeteria food as excellent, good, fair, or poor

- c. Calendar years of historic events, such as 1776, 1945, or 2001
- d. Temperatures on the Celsius scale
- e. Runners' times in the Boston Marathon

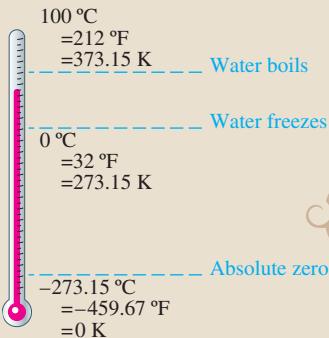
SOLUTION

- a. As discussed in Example 1, numbers on uniforms don't count or measure anything. They are at the nominal level of measurement because they are labels and do not imply any kind of ordering.
- b. A set of rankings represents data at the ordinal level of measurement because the categories (excellent, good, fair, or poor) have a definite order.
- c. An interval of one calendar year always has the same meaning. But ratios of calendar years do not make sense because the choice of the year 0 is arbitrary and does not mean "the beginning of time." Calendar years are therefore at the interval level of measurement.
- d. Like Fahrenheit temperatures, Celsius temperatures are at the interval level of measurement. An interval of 1°C always has the same meaning, but the zero point (1°C = freezing point of water) is arbitrary and does not mean "no heat."
- e. Marathon times have meaningful ratios—for example, a time of 6 hours really is twice as long as a time of 3 hours—because they have a true zero point at a time of 0 hours. •••

BY THE WAY

Scientists often measure temperatures on the Kelvin scale. Data on the Kelvin scale are at the ratio level of measurement, because the Kelvin scale has a true zero. A temperature of 0 Kelvin really is the coldest possible temperature. Called *absolute zero*, 0 K is equivalent to about -273.15°C or -459.67°F . (The degree symbol is not used for Kelvin temperatures.)

Temperature Scale



Section 1 Exercises

Statistical Literacy and Critical Thinking

- 1. Qualitative/Quantitative.** What is the difference between qualitative data and quantitative data?
- 2. Quantitative/Qualitative.** A football player is taking a statistics course and states that the names of the players on his team are qualitative, but they can be made quantitative by using the numbers on the jerseys of their uniforms. Is he correct? Why or why not?
- 3. Qualitative/Quantitative.** Is a researcher correct when she argues that all data are either qualitative or quantitative? Explain.
- 4. ZIP Codes.** A researcher argues that ZIP codes are quantitative data because they measure location, with low numbers in the east and high numbers in the west. Is she correct? Why or why not?
- 7. Reaction Times.** Braking reaction times (in seconds) are measured as part of a driver education program.
- 8. Physicians.** The specialties of physicians (cardiac surgeon, pediatrician, etc.)
- 9. Multiple Choice Test Questions.** The answers (a, b, c, d, e) to multiple choice test questions
- 10. Survey Responses.** The responses (yes, no, refuse to answer) from survey subjects when asked a question
- 11. Nielsen Survey.** The television shows being watched by households surveyed by Nielsen Media Research
- 12. Nielsen Ratings.** The number of households with a television in use when surveyed by Nielsen Media Research
- 13. Head Circumferences.** In studying different societies, an archeologist measures head circumferences of skulls
- 14. Shoe Sizes.** The shoe sizes (such as 8 or $10\frac{1}{2}$) of test subjects
- 15. GPA.** The grade point averages of randomly selected college students

Concepts and Applications

Qualitative vs. Quantitative Data. In Exercises 5–16, determine whether the data described are qualitative or quantitative and explain why.

- 5. Blood Groups.** The blood groups of A, B, AB, and O
- 6. White Blood Cells.** The white blood cell counts of different people, consisting of the numbers of white blood cells per microliter of blood

16. Area Codes. The area codes (such as 617) of the telephones of survey subjects

Discrete or Continuous. In Exercises 17–28, state whether the data described are discrete or continuous and explain why.

17. Aircraft Baggage. The numbers of checked bags on flights between San Francisco and Atlanta

18. Aircraft Baggage. The weights of checked bags on flights between San Francisco and Atlanta

19. Flights. The total numbers of flights by different airlines between San Francisco and Atlanta in the past month

20. Flights. The lengths in minutes of each of the flights between San Francisco and Atlanta in the past month

21. Chemistry. An experiment in chemistry is repeated, and the times it takes for a reaction to occur are recorded.

22. Test Times. The times required by students to complete a statistics test

23. Test Scores. The numerical scores on a statistics test

24. Traffic Count. Number of cars crossing the Golden Gate Bridge each hour

25. Car Speeds. The speeds of cars as they pass the center of the Golden Gate Bridge

26. Movie Ratings. The movie ratings by a critic, with 0 stars, 1/2 star, 1 star, and so on

27. Stars. Number of stars in each galaxy in the universe

28. Cola. The exact amounts of cola in different cans

Levels of Measurement. For the data described in Exercises 29–40, identify the level of measurement (nominal, ordinal, interval, or ratio).

29. Weights of Textbooks. Weights of college textbooks

30. Movie Ratings. A critic's movie recommendations of "must see," "good," "fair," "poor," or "avoid"

31. Movie Types. Types of movies (drama, comedy, etc.)

32. Temperatures. Body temperatures in Fahrenheit of all students in a statistics class

33. Cars. Classifications of cars by size as subcompact, compact, intermediate, full-size

34. Clinical Trial. Results from a clinical trial consisting of "true positive," "false positive," "true negative," or "false negative"

35. Grades. Final course grades of A, B, C, D, F

36. Distances. Distances traveled by college students as they drive from their homes to their colleges

37. SSN. Social Security numbers

38. Weights. Weights of the cola in cans of Diet Coke

39. Word Counts. Numbers of words spoken in a day by a sample of males

40. Car Safety Ratings. *Consumer Reports* safety ratings of cars: 0 = unsafe up to 3 = safest

Meaningful Ratios? In Exercises 41–48, determine whether the given statement represents a meaningful ratio, so that the ratio level of measurement applies. Explain.

41. Movie Rating. A movie with a 4-star rating is twice as good as one with a 2-star rating.

42. Wind Speed. Wind with a speed of 40 mi/h moves four times as fast as wind with a speed of 10 mi/h.

43. IQ Score as a Measure of Intelligence. One subject has an IQ score of 140 while another subject has an IQ score of 70, so the first subject is twice as intelligent as the second subject.

44. Temperatures. On August 6, it was 80°F in New York City, so it was twice as hot as on December 7, when it was 40°F.

45. Art Dating. Using carbon dating, one sculpture is found to be 1,000 years old while a second sculpture is found to be 500 years old, so the first sculpture is twice as old as the second.

46. Carbon Dating. Using carbon dating, one sample of wood is found to be twice as old as another, because the first sample is found to be 200 years old while the other sample is 100 years old.

47. Salary. An employee with a salary of \$150,000 earns twice as much as one with a \$75,000 salary.

48. SAT Scores. A person with an SAT score of 2200 is twice as qualified for college as a person with a score of 1100.

Complete Classification. In Exercises 49–56, determine whether the data described are qualitative or quantitative and give their level of measurement. If the data are quantitative, state whether they are continuous or discrete. Give a brief explanation.

49. Marathon Times. Finish times of the New York City Marathon

50. Marathon Runners. Home nations (such as U. S., France, Kenya) of runners in a marathon

51. Employee ID Numbers. The employees of the Telektronics Corporation have six-digit identification numbers that are randomly generated.

52. Employee Service Times. Seniority of each employee at the Telektronics Corporation is based on the length of time that has passed since the employee was first hired.

53. Employee Hiring Years. The years in which employees were hired (such as 2000, 1995, 2012) are used to determine their pension plan.

54. Political Survey. In a survey of voter preferences, the political parties of respondents are recorded as coded numbers 1, 2, 3, 4, or 5 (where 1 = Democrat, 2 = Republican, 3 = Liberal, 4 = Conservative, 5 = other).

55. Product Ratings. *Consumer Reports* magazine lists ratings of "best buy," "recommended," or "not recommended" for each of several different computers.

56. Quality Control. Apple tests each of its manufactured iPhones and labels each as acceptable or defective.

2 DEALING WITH ERRORS

We turn now to the issue of how to deal with errors in measurement. First we will consider the various types of errors that can occur, and then we'll discuss how to account for possible errors when we state results. Note that while we will phrase most of this discussion in terms of measurements, it applies equally well to estimates or projections, such as population estimates or projected revenues for a corporation.

Mistakes are the portals of discovery.

—James Joyce

Types of Error: Random and Systematic

Broadly speaking, measurement errors fall into two categories: random errors and systematic errors. An example will illustrate the difference.

Suppose you work in a pediatric office and use a digital scale to weigh babies. If you've ever worked with babies, you know that they usually aren't very happy about being put on a scale. Their thrashing and crying tends to shake the scale, making the readout jump around. For the case shown in Figure 2a, you could conceivably record the baby's weight as anything between about 14.5 and 15.0 pounds. We say that the shaking of the scale introduces a **random error** because any particular measurement may be either too high or too low.

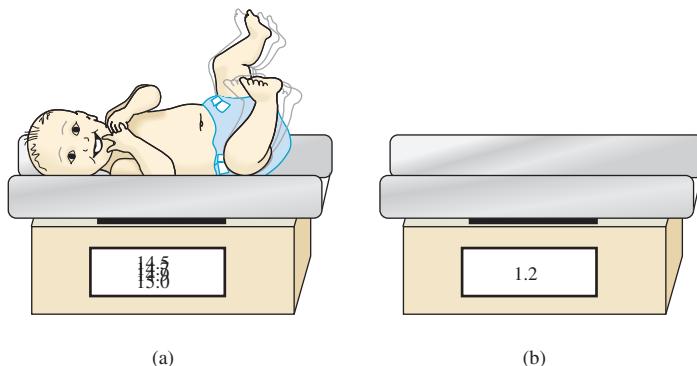


Figure 2 (a) The baby's motion introduces random errors. (b) The scale reads 1.2 pounds when empty, introducing a systematic error that makes all measurements 1.2 pounds too high.

Now suppose you have been measuring weights of babies all day long with the scale shown in Figure 2b. At the end of the day, you notice that the scale reads 1.2 pounds even when there is nothing on it. If you assume that this problem had been present all day, then every measurement you made was high by 1.2 pounds. This type of error is called a **systematic error** because it is caused by an error in the measurement *system*—an error that consistently (systematically) affects all measurements.

Two Types of Measurement Error

Random errors occur because of random and inherently unpredictable events in the measurement process.

Systematic errors occur when there is a problem in the measurement system that affects all measurements in the same way.

BY THE WAY

A systematic error in which a scale's measurements differ consistently from the true values is called a *calibration error*. You can test the calibration of a scale by putting known weights on it, such as weights that are 2, 5, 10, and 20 pounds, and making sure that the scale gives the expected readings.

A systematic error affects all measurements in the same way, such as making them all too high or all too low. If you discover a systematic error, you can go back and adjust the affected measurements. In contrast, the unpredictable nature of random errors makes it impossible to correct for them. However, you can minimize the effects of random errors by making many

measurements and averaging them. For example, if you measure the baby's weight 10 times, your measurements will probably be too high in some cases and too low in others. You can therefore get a better value by averaging the 10 individual measurements.

BY THE WAY

The fact that urban areas tend to be warmer than they would be in the absence of human activity is often called the *urban heat island effect*. Major causes of this effect include heat released by burning fuel in automobiles, homes, and industry and the fact that pavement and large masonry buildings tend to retain heat from sunlight.



A little inaccuracy sometimes saves a ton of explanation.

—H. H. Munro (Saki)

TIME OUT TO THINK

Go to a Web site (such as www.time.gov) that gives the current time. How far off is your clock or watch? Describe possible sources of random and systematic errors in your timekeeping.

EXAMPLE 1 Errors in Global Warming Data

Scientists studying global warming need to know how the average temperature of the entire Earth, or the *global average temperature*, has changed with time. Consider two difficulties in trying to interpret historical temperature data from the early 20th century: (1) Temperatures were measured with simple thermometers and the data were recorded by hand, and (2) most temperature measurements were recorded in or near urban areas, which tend to be warmer than surrounding rural areas because of heat released by human activity. Discuss whether each of these two difficulties produces random or systematic errors, and consider the implications of these errors.

SOLUTION The first difficulty involves *random errors* because people undoubtedly made occasional errors in reading the thermometers, in calibrating the thermometers, and in recording temperature readings. There is no way to predict whether any individual reading is correct, too high, or too low. However, if there are several readings for the same region on the same day, averaging these readings can minimize the effects of the random errors.

The second difficulty involves a *systematic error* because the excess heat in urban areas always causes the temperature reading to be higher than it would be otherwise. If the researchers can estimate how much this systematic error affected the temperature readings, they can correct the data for this error.

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CASE STUDY The Census

The Constitution of the United States mandates a census of the population every 10 years. The U.S. Census Bureau conducts the census (and also does many other demographic studies).

In attempting to count the population, the Census Bureau relies largely on a survey that is supposed to include every household in the United States. However, many *random errors* occur in this survey process. For example, some people fill out survey forms incorrectly and some responses are recorded incorrectly by Census Bureau employees.

The census also is subject to several types of *systematic error*. For example, the difficulty of counting the homeless and the fact that undocumented aliens may try to hide their presence both tend to lead to undercounts in the population. Other systematic errors tend to cause overcounts—such as college students being counted both at home and at their school residences, or children of divorced parents being counted in both households.

The 2010 census, for example, found a United States population of about 308.7 million people, but this reflects only the actual survey results. In principle, the Census Bureau could use statistical studies to correct for the systematic errors and make the count more accurate, but that is not allowed by current law.

TIME OUT TO THINK

The question of whether the Census Bureau should be allowed to adjust its “official” count on the basis of statistical surveys is very controversial. The Constitution calls for an “actual enumeration” of the population (Article 1, Section 2, Subsection 2). Do you believe that this wording precludes or allows the use of statistical surveys in the official count? Defend your opinion. Also discuss reasons why Democrats tend to favor the use of sampling methods while Republicans tend to oppose it.

Size of Errors: Absolute versus Relative

Besides wanting to know whether an error is random or systematic, we often want to know whether an error is big enough to be of concern or small enough to be unimportant. An example should clarify the idea.

Suppose you go to the store and buy what you think is 6 pounds of hamburger, but because the store’s scale is poorly calibrated, you actually get only 4 pounds. You’d probably be upset by this 2-pound error. Now suppose you are buying hamburger for a huge town barbecue and you order 3,000 pounds of hamburger, but you actually receive only 2,998 pounds. You are short by the same 2 pounds as before, but in this case the error probably doesn’t seem very important.

In more technical language, the 2-pound error in both cases is an **absolute error**—it describes how far the claimed or measured value lies from the true value. A **relative error** compares the size of the absolute error to the true value. The relative error for the first case is fairly large because the absolute error of 2 pounds is half the true weight of 4 pounds; we say that the relative error is $2/4$, or 50%. In contrast, the relative error for the second case is the absolute error of 2 pounds divided by the true hamburger weight of 2,998 pounds, which is only $2/2998 \approx 0.00067$, or 0.067%.

Absolute and Relative Error

The **absolute error** describes how far a claimed or measured value lies from the true value:

$$\text{absolute error} = \text{claimed or measured value} - \text{true value}$$

The **relative error** compares the size of the absolute error to the true value. It is often expressed as a percentage:

$$\text{relative error} = \frac{\text{absolute error}}{\text{true value}} \times 100\%$$

Note that the absolute and relative errors are *positive* when the claimed or measured value is greater than the true value, and *negative* when the claimed or measured value is less than the true value.

EXAMPLE 2 Absolute and Relative Error

Find the absolute and relative error in each case.

- Your true weight is 100 pounds, but a scale says you weigh 105 pounds.
- The government claims that a program costs \$99.0 billion, but an audit shows that the true cost is \$100.0 billion.

SOLUTION

- a. The measured value is the scale reading of 105 pounds and the true value is 100 pounds:

$$\begin{aligned}\text{absolute error} &= \text{measured value} - \text{true value} \\ &= 105 \text{ lb} - 100 \text{ lb} \\ &= 5 \text{ lb}\end{aligned}$$

$$\begin{aligned}\text{relative error} &= \frac{\text{absolute error}}{\text{true value}} \times 100\% \\ &= \frac{5 \text{ lb}}{100 \text{ lb}} \times 100\% \\ &= 5\%\end{aligned}$$

The measured weight is too high by 5 pounds, or by 5%.

- b. The claimed cost is \$99.0 billion and the true cost is \$100.0 billion:

$$\begin{aligned}\text{absolute error} &= \text{claimed value} - \text{true value} \\ &= \$99.0 \text{ billion} - \$100.0 \text{ billion} \\ &= -\$1.0 \text{ billion}\end{aligned}$$

$$\begin{aligned}\text{relative error} &= \frac{\text{absolute error}}{\text{true value}} \times 100\% \\ &= \frac{-\$1.0 \text{ billion}}{\$100.0 \text{ billion}} \times 100\% \\ &= -1\%\end{aligned}$$

The claimed cost is too low by \$1.0 billion, or by 1%.

• • •

*A billion here, a billion there;
soon you're talking real money.*

—attributed to Senator Everett Dirksen

BY THE WAY

In 1999, NASA lost the \$160 million Mars Climate Orbiter when engineers sent it very precise computer instructions in English units (pounds) but the spacecraft software interpreted them in metric units (kilograms). In other words, the loss occurred because the very precise instructions were actually quite inaccurate!

**Describing Results: Accuracy and Precision**

Two key ideas about any reported value are its *accuracy* and its *precision*. Although these terms are often used interchangeably in English, they are not quite the same thing.

The goal of any measurement is to obtain a value that is as close as possible to the *true value*. **Accuracy** describes how close the measured value lies to the true value. **Precision** describes the amount of detail in the measurement. For example, suppose a census says that the population of your hometown is 72,453 but the true population is 96,000. The census value of 72,453 is quite precise because it seems to tell us the exact count, but it is not very accurate because it is nearly 25% smaller than the actual population of 96,000. Note that accuracy is usually defined by relative error rather than absolute error. For example, if a company projects sales of \$7.30 billion and true sales turn out to be \$7.32 billion, we say the projection was quite accurate because it was off by less than 1%, even though the error of \$0.02 billion represents \$20 million.

Definitions

Accuracy describes how closely a measurement approximates a true value. An accurate measurement is close to the true value. (*Close* is generally defined as a small *relative error*, rather than a small *absolute error*.)

Precision describes the amount of detail in a measurement.

We generally assume that the precision with which a number is reported reflects what was actually measured. If you say that you weigh 132 pounds, we assume that you measured your weight only to the nearest pound. In that case, a more precise measurement might find

that you weighed, say, 132.3 pounds or 131.6 pounds (both of these would round to 132). In contrast, if you say that you weigh 132.0 pounds, we assume that you measured your weight to the nearest tenth of a pound. This assumption means that you should never report a measurement with more precision than is justified. For example, if you actually measured your weight only to the nearest pound, it would be wrong to say that you weighed 132.0 pounds, because that would imply you made a measurement to the nearest tenth of a pound.

EXAMPLE 3 Accuracy and Precision in Your Weight

Suppose that your true weight is 102.4 pounds. The scale at the doctor's office, which can be read only to the nearest quarter pound, says that you weigh $102\frac{1}{4}$ pounds. The scale at the gym, which gives a digital readout to the nearest 0.1 pound, says that you weigh 100.7 pounds. Which scale is more *precise*? Which is more *accurate*?

SOLUTION The scale at the gym is more *precise* because it gives your weight to the nearest tenth of a pound, whereas the doctor's scale gives your weight only to the nearest quarter pound. However, the scale at the doctor's office is more *accurate* because its value is closer to your true weight.

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TIME OUT TO THINK

In Example 3, we need to know your true weight to determine which scale is more accurate. But how would you know your true weight? Can you ever be sure that you know it? Explain.



CASE STUDY Understanding Census Results

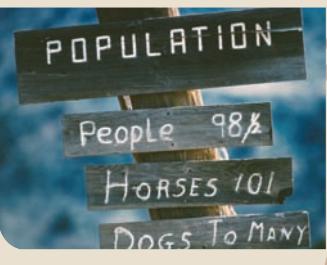
Upon completing the 2010 census, the U.S. Census Bureau reported a population of 308,745,538 (on April 1, 2010), thereby implying an exact count of everyone living in the United States. Unfortunately, such a precise count could not possibly be as accurate as it seems to imply.

Even in principle, the only way to get an exact count of the number of people living in the United States would be to count everyone *instantaneously*. Otherwise, the count would be off because, for example, an average of about eight births and four deaths occur every minute in the United States.

In fact, the Census is conducted over a period of many months, so the actual population on a given date could not really be known. Moreover, the Census results are affected by both random and systematic errors (see the previous Case Study). A more honest report of the Census result would use much less precision—for example, stating the population as “about 310 million.” In fairness to the Census Bureau, their detailed reports explained the uncertainties in the population count, but these uncertainties were rarely mentioned by the press.

BY THE WAY

The digits in a number that were actually measured are called *significant digits*. All the digits in a number are significant except for zeros that are included so that the decimal point can be properly located. For example, 0.001234 has four significant digits; the zeros are required for proper placement of the decimal point. The number 1,234,000,000 has four significant digits for the same reason. The number 132.0 has four significant digits; the zero is significant because it is not required for the proper placement of the decimal point and therefore implies an actual measurement of zero tenths.



CASE STUDY Trillions Disappear in Unjustified Budget Precision

In early 2001, politicians heralded a projected U.S. federal budget *surplus* of \$5.6 trillion over the next 10 years. When 2011 actually arrived, rather than having gained trillions in surplus money, the government had instead gone another \$9 trillion into debt. In other words, the projection proved to be in error by nearly \$15 trillion (going from *positive* \$5.6 trillion to *negative* \$9 trillion).

You've probably heard or read news reports about this multi-trillion-dollar turn in government fortunes, and people of different political persuasions may feel quite differently about its cause. For our purposes here, the most remarkable thing about it was the unjustified precision of the original projection. After all, the projection of \$5.6 trillion was quoted precise to the nearest *tenth* of a trillion dollars (which means to the nearest hundred billion dollars), implying that the actual value would be between about \$5.5 and \$5.7 trillion. Now that's what you might call an "Ooops!"

In fairness to the economists responsible for the projection, they were well aware of the associated uncertainties. Moreover, changes that occurred after the projection was made—including the terrorist attacks of 9/11, wars in Afghanistan and Iraq, substantial cuts in taxes, and a major recession—make it unsurprising that the projection turned out so far off base. But the lesson is clear: Next time you hear politicians or the media talking about future federal budgets with great precision, remember it is quite likely that their numbers will prove to be abysmally inaccurate.

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Summary: Dealing with Errors

The ideas we've covered in this section are a bit technical, but very important to understanding measurements and errors. Let's briefly summarize how the ideas relate to one another.

- Errors can occur in many ways, but generally can be classified into one of two basic types: random errors or systematic errors.
- Whatever the source of an error, its size can be described in two different ways: as an absolute error or as a relative error.
- Once a measurement is reported, we can evaluate it in terms of its accuracy and its precision.

Section 2 Exercises

Statistical Literacy and Critical Thinking

- Error Type.** When recording the weight of a watermelon, a supermarket clerk writes the wrong number. Is this type of error a random error or a systematic error? Explain.
- Standard Weight.** A standard weight defined to represent exactly 1 kg is kept by the National Institute of Standards and Technology. If you put this 1-kilogram true weight on a scale and the scale says it is 1.002 kg, what is the absolute error and the relative error of the measurement?
- Standard Weight.** Using the same standard weight from Exercise 2, assume that you put the weight on a scale and record the weight as 1.2034278 kg. Describe the accuracy and precision of the recorded weight. Explain.
- World Population.** At a particular moment, the U.S. Census Bureau population clock shows that the global population is 7,118,233,027 people. Describe the accuracy and precision of that population number.

Does It Make Sense? For Exercises 5–8, decide whether the statement makes sense (or is clearly true) or does not make sense (or is clearly false). Explain clearly; not all of these have definitive answers, so your explanation is more important than your chosen answer.

- Species of Fish.** There are 24,627 species of fish on Earth.

Relative Error. The relative error that a microbiologist makes in measuring a cell must be less than the relative error that an astronomer makes in measuring a galaxy, because cells are smaller than galaxies.

Scanner Error. The Jenkins supermarket manager claims that the scanning errors on purchased items are random, and about half of the errors are in favor of the supermarket.

Aircraft Baggage. An American Airlines agent tells you that you must pay a surcharge because your checked bag weighs 23.018 kg, which exceeds the limit of 23 kg, and that there's no doubt that the scale is correct because it measures to the thousandth of a kilogram.

Concepts and Applications

- Tax Audit.** A tax auditor reviewing a tax return looks for several kinds of problems, including these two: (1) mistakes made in entering or calculating numbers on the tax return

and (2) places where the taxpayer reported income dishonestly. Discuss whether each problem involves random or systematic errors.

10. Safe Air Travel. Before taking off, a pilot is supposed to set the aircraft altimeter to the elevation of the airport. A pilot leaves from Denver (altitude 5,280 feet) with her altimeter set to 2,500 feet. Explain how this affects the altimeter readings throughout the flight. What kind of error is this?

11. Technical Specifications. An iPod battery is supposed to provide 3.7 volts. An aftermarket supplier manufactures 5,000 replacement batteries and finds that they have a mean of 3.7 V, but about half of the batteries have less than 3.7 volts and half have more than 3.7 volts. Does the error appear to be random or systematic? Explain.

12. Drunk Driving Data. For data collected on car driving fatalities, a researcher claims that while many fatalities are recorded as involving alcohol, many others are missed because the deceased are not tested for alcohol consumption. If this is true, what kind of error is introduced and how does it affect the values of the data?

Sources of Errors. For each measurement described in Exercises 13–20, identify at least one likely source of random errors and also identify at least one likely source of systematic errors.

13. Contributions. A survey asks people for the amount of money they donated to charity in the past year.

14. Tax Returns. The annual incomes of 200 people are obtained from their tax returns.

15. Passenger Weights. For a flight on a small plane, the pilot asks passengers what they weigh.

16. M&Ms. The weights of individual M&M plain candies were obtained by placing each candy in a paper cup, then obtaining the weight without accounting for the weight of the cup.

17. Radar Speeds. Speeds of cars are recorded by a police officer who uses a radar gun.

18. Counterfeit Products. The police commissioner in New York City estimates the annual value of counterfeit goods sold in the city.

19. Cigarette Sales. The health commissioner of Los Angeles estimates the number of cigarettes smoked in her city from data for taxes collected on sales of cigarettes.

20. Measuring Length. A groundskeeper measures the length and width of a school's athletic field using a ruler that is 1 foot long.

Absolute and Relative Errors. In Exercises 21–24, find the values of the absolute and relative errors.

21. Credit Card Bill. You receive a Visa credit card bill for \$2,995, but it includes a charge of \$1,750 that was not valid. (That is, the true value is \$1,750 less than the bill claims.)

22. Steak Weight. A steak at a restaurant actually weighs 18 ounces (the true value), but the menu claims that it is a 20-ounce steak.

23. Wrong Change. When purchasing lunch in a cafeteria, the actual (true value) of change due is \$2.75, but the incorrect amount of \$1.75 is given instead.

24. Baker's Dozen. The bakery menu claims that there are 12 doughnuts in a bag, but the baker always puts 13 doughnuts (the true value) in each bag.

25. Minimizing Errors. Twenty-five people, including yourself, are to measure the length of a room to the nearest tenth of a millimeter. Assume that everyone uses the same well-calibrated measuring device, such as a tape measure.

a. All 25 measurements are not likely to be exactly the same; thus, the measurements will contain some sources of error. Are these errors systematic or random? Explain.

b. If you want to minimize the effect of random errors in determining the length of the room, which is the better choice: to report your own personal measurement as the length of the room or to report the average of all 25 measurements? Explain.

c. Describe any possible sources of systematic errors in the measurement of the room length.

d. Can the process of averaging all 25 measurements help reduce any systematic errors? Why or why not?

26. Minimizing Errors. When weighing a model 22F car battery, the measuring instrument is very precise, and the weight is obtained 10 consecutive times.

a. All 10 measurements are not likely to be exactly the same; thus, the measurements will contain some sources of error. Are these errors systematic or random? Explain.

b. If you want to minimize the effect of random errors in determining the true weight of the battery, which is the better choice: to choose one of the 10 measurements at random or to report the average of all 10 measurements? Explain.

c. Describe any possible sources of systematic errors in the 10 measurements.

d. Can the process of averaging all 10 weights help reduce any systematic errors? Why or why not?

27. Accuracy and Precision in Corvette Weight. A new Corvette weighs 3,273 lb. A manufacturer's scale that is accurate to the nearest 10 lb gives the weight as 3,250 lb, while the U.S. Department of Transportation uses a scale that is accurate to the nearest 0.1 lb and obtains a weight of 3,298.2 lb. Which measurement is more *precise*? Which is more *accurate*? Explain.

28. Accuracy and Precision in Height. Assume that your statistics professor has a height of exactly 175.2 cm. Assume that this height is measured with a tape measure that can be read to the nearest mm (or 1/10 cm) and results from two different measurements are reported as 175 cm and 175.5 cm. Which measurement is more *precise*? Which is more *accurate*? Explain.

- 29. Accuracy and Precision in Weight.** Suppose your weight is 52.55 kilograms. A scale at a health clinic that gives weight measurements to the nearest half kilogram gives your weight as 53 kilograms. A digital scale at the gym that gives readings to the nearest 0.01 kilogram gives your weight as 52.88 kilograms. Which measurement is more precise? Which is more *accurate*? Explain.
- 30. Accuracy and Precision in Weight.** Suppose your weight is 52.55 kilograms. A scale at a health clinic that gives weight measurements to the nearest half kilogram gives your weight as $52\frac{1}{2}$ kilograms. A digital scale at the gym that gives readings to the nearest 0.01 kilogram gives your weight as 51.48 kilograms. Which measurement is more precise? Which is more *accurate*? Explain.

Believable Facts? Exercises 31–38 give statements of “fact” coming from statistical measurements. For each statement, briefly discuss possible sources of error in the measurement. Then, considering the precision with which the measurement is given, discuss whether you think the fact is believable.

- 31. Population.** The population of the United States in 1860 was 31,443,321.
- 32. Motor Vehicle Deaths.** Last year there were 38,929 deaths in the United States due to motor vehicle crashes.
- 33. Population of China.** Last year, the population of China was 1,339,414,205 people.
- 34. Tallest Building.** The Burj Khalifa in Dubai is 2,717 feet tall, making it the world’s tallest building.
- 35. Gateway Arch.** The St. Louis Gateway Arch is 630.2377599694 feet tall.
- 36. Cell Phones.** The *Newport Chronicle* reported that there are now 5 billion cell phones in use.
- 37. College Students.** Wikipedia reports that there are currently 14,261,778 college students in the United States.
- 38. Threatened Species.** The U.S. government now lists 1,879 endangered or threatened species of animals and plants.



- 39. The Census.** Go to the Web site for the U.S. Census Bureau and learn about the census conducted every 10 years. How and when will data be collected for the next census? Are any significant changes in the collection process planned?

- 40. Census Controversies.** Use the Library of Congress’s “Thomas” Web site to find out about any pending legislation concerning the collection or use of census data. If you find more than one legislative bill pending, choose one to study in depth. Summarize the proposed legislation, and briefly discuss arguments both for and against it.
- 41. Wristwatch Errors.** Use a Web site that gives you the local time (such as www.time.gov) to set a watch to the nearest second. Then compare the time on your watch with the times on friends’ watches. Record the errors with positive signs for watches that are ahead of the true time and negative signs for those watches that are behind the true time. Use the concepts of this section to describe the accuracy of the wristwatches in your sample.

IN THE NEWS

- 42. Random and Systematic Errors.** Find a recent news report that gives a quantity that was measured statistically (for example, a report of population, average income, or the number of homeless people). Write a short description of how the quantity was measured, and briefly describe any likely sources of either random or systematic errors. Overall, do you think that the reported measurement was accurate? Why or why not?
- 43. Absolute and Relative Errors.** Find a recent news report that describes some mistake in a measured, estimated, or projected number (for example, a budget projection that turned out to be incorrect). In words, describe the size of the error in terms of both absolute error and relative error.
- 44. Accuracy and Precision.** Find a recent news article that causes you to question accuracy or precision. For example, the article might report a figure with more precision than you think is justified, or it might cite a figure that you know is inaccurate. Write a summary of the report, and explain why you question its accuracy or precision (or both).

3**USES OF PERCENTAGES IN STATISTICS**

Statistical results are often stated with percentages. A percentage is simply a way of expressing a fraction; the words *per cent* literally mean “divided by 100.” However, percentages are often used in subtle ways. Consider a statement that appeared in a front-page article in the *New York Times*:

The percentage of smokers among 8th graders is up 44 percent, to 10.4 percent.

Although the statement uses percentages in a valid way, it can be difficult to understand what it means by “up 44%, to 10.4%.” In this section, we will investigate some of the subtle uses and abuses of percentages. Before we begin, you should review the following basic rules regarding conversions between fractions and percentages.

Conversions Between Fractions and Percentages

To convert a percentage to a common fraction: Replace the % symbol with division by 100; simplify the fraction if necessary.

$$\text{Example: } 25\% = \frac{25}{100} = \frac{1}{4}$$

To convert a percentage to a decimal: Drop the % symbol and move the decimal point two places to the left (that is, divide by 100).

$$\text{Example: } 25\% = 0.25$$

To convert a decimal to a percentage: Move the decimal point two places to the right (that is, multiply by 100) and add the % symbol.

$$\text{Example: } 0.43 = 43\%$$

To convert a common fraction to a percentage: First convert the common fraction to a decimal; then convert the decimal to a percentage.

$$\text{Example: } \frac{1}{5} = 0.2 = 20\%$$

EXAMPLE 1 **Newspaper Survey**

A newspaper reports that 54% of 1,069 people surveyed said that the President is doing a good job. How many people said that the President is doing a good job?

SOLUTION The 54% represents the fraction of respondents who said the President is doing a good job. Because “of” usually indicates multiplication, we multiply:

$$54\% \times 1,069 = 0.54 \times 1,069 = 577.26 \approx 577$$

About 577 out of the 1,069 people said the President is doing a good job. Note that we round the answer to 577 to obtain a whole number of people, because the number of people must be a discrete (integer) value. (The symbol \approx means “approximately equal to.”)

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Using Percentages to Describe Change

Percentages are often used to describe how data change with time. For example, suppose the population of a town was 10,000 in 1970 and 15,000 in 2000. We can express this change in two basic ways:

- Because the population rose by 5,000 people (from 10,000 to 15,000), we say that the **absolute change** in the population was 5,000 people.
- Because the increase of 5,000 people was 50% of the starting population of 10,000, we say that the **relative change** in the population was 50%.

In general, calculating an absolute or relative change always involves two numbers: a starting number, or **reference value**, and a **new value**. Once we identify these two values, we can calculate the absolute and relative change with the following formulas. A change is positive if the new value is greater than the reference value and negative if the new value is less than the reference value.

Absolute and Relative Change

The **absolute change** is the actual increase or decrease from a reference value to a new value:

$$\text{absolute change} = \text{new value} - \text{reference value}$$

The **relative change** is the size of the absolute change in comparison to the reference value and can be expressed as a percentage:

$$\text{relative change} = \frac{\text{new value} - \text{reference value}}{\text{reference value}} \times 100\%$$

TIME OUT TO THINK

Compare the formulas for absolute and relative change to the formulas for absolute and relative error, given in Section 2. Why are they so similar?

EXAMPLE 2 World Population Growth

BY THE WAY

According to United Nations and U.S. Census Bureau estimates, world population passed 7 billion in early 2012—only 13 years after passing the 6 billion mark. The population is still growing by more than 75 million people each year, which means it takes only about four years for the world to add a population equivalent to that of the entire United States.



Estimated world population in 1950 was 2.6 billion. By the end of 2010, it had reached 6.9 billion. Describe the absolute and relative change in world population from 1950 to 2010.

SOLUTION The reference value is the 1950 population of 2.6 billion and the new value is the 2010 population of 6.9 billion.

$$\text{absolute change} = \text{new value} - \text{reference value}$$

$$= 6.9 \text{ billion} - 2.6 \text{ billion}$$

$$= 4.3 \text{ billion}$$

$$\text{relative change} = \frac{\text{new value} - \text{reference value}}{\text{reference value}} \times 100\%$$

$$= \frac{6.9 \text{ billion} - 2.6 \text{ billion}}{2.6 \text{ billion}} \times 100\%$$

$$= 165.4\%$$

World population increased by 4.3 billion people, or by about 165%, during the 60-year period from 1950 to 2010.



Using Percentages for Comparisons

Percentages are also commonly used to compare two numbers:

- The **reference value** is the number that we are using as the basis for a comparison.
- The **compared value** is the other number, which we compare to the reference value.

We can then express the absolute or relative difference between these two values with formulas very similar to those for absolute and relative change. The difference is positive if the compared value is greater than the reference value and negative if the compared value is less than the reference value.

Absolute and Relative Difference

The **absolute difference** is the difference between the compared value and the reference value:

$$\text{absolute difference} = \text{compared value} - \text{reference value}$$

The **relative difference** describes the size of the absolute difference in comparison to the reference value and can be expressed as a percentage:

$$\text{relative difference} = \frac{\text{compared value} - \text{reference value}}{\text{reference value}} \times 100\%$$

EXAMPLE 3 Russian and American Life Expectancy

According to United Nations data, life expectancy for American men is about 76 years, while life expectancy for Russian men is about 63 years. Using the life expectancy of Russian men as the reference value, compare the life expectancy of American men with that of Russian men in absolute and relative terms.

SOLUTION We want to compare the American male life expectancy with the Russian male life expectancy, so the Russian male life expectancy is the reference value and the American male life expectancy is the compared value:

$$\text{absolute difference} = \text{compared value} - \text{reference value}$$

$$= 76 \text{ years} - 63 \text{ years}$$

$$= 13 \text{ years}$$

$$\text{relative difference} = \frac{\text{compared value} - \text{reference value}}{\text{reference value}} \times 100\%$$

$$= \frac{76 \text{ years} - 63 \text{ years}}{63 \text{ years}} \times 100\%$$

$$\approx 21\%$$

The life expectancy of American men is 13 years greater in absolute terms and 21% greater in relative terms than the life expectancy of Russian men.

BY THE WAY

No one knows all the reasons for the low life expectancy of Russian men, but one contributing factor is alcoholism, which is much more common in Russia than in America.



Of versus More Than

Consider a population that *triples* in size from 200 to 600. There are two equivalent ways to state this change with percentages:

- Using *more than*: The new population is 200% *more than* the original population. Here, we are looking at the relative change in the population:

$$\begin{aligned}\text{relative change} &= \frac{\text{new value} - \text{reference value}}{\text{reference value}} \times 100\% \\ &= \frac{600 - 200}{200} \times 100\% \\ &= 200\%\end{aligned}$$

- Using *of*: The new population is 300% *of* the original population, which means it is three times the original population. Here, we are looking at the *ratio* of the new population to the original population:

$$\frac{\text{new population}}{\text{original population}} = \frac{600}{200} = 3.00 = 300\%$$

Notice that the percentages in the “more than” and “of” statements are related by $300\% = 100\% + 200\%$. This leads to the following general relationship.

Of versus More Than (or Less Than)

- If the new or compared value is $P\%$ *more than* the reference value, then it is $(100 + P)\%$ *of* the reference value.
- If the new or compared value is $P\%$ *less than* the reference value, then it is $(100 - P)\%$ *of* the reference value.

For example, 40% *more than* the reference value is 140% *of* the reference value, and 40% *less than* the reference value is 60% *of* the reference value. When you hear statistics quoted with percentages, it is very important to listen carefully for the key words *of* and *more than* (or *less than*)—and hope that the speaker knows the difference.

EXAMPLE 4 World Population

In Example 2, we found that world population in 2010 was about 165% more than world population in 1950. Express this change with an “*of*” statement.

SOLUTION World population in 2010 was 165% more than world population in 1950. Because $(100 + 165)\% = 265\%$, the 2010 population was 265% *of* the 1950 population. This means that the 2010 population was 2.65 times the 1950 population. • •

EXAMPLE 5 Sale!

A store is having a “25% off” sale. How does a sale price compare to an original price?

SOLUTION The “25% off” means that a sale price is 25% *less than* the original price, which means it is $(100 - 25)\% = 75\%$ *of* the original price. For example, if an item’s original price was \$100, its sale price is \$75. • •

TIME ⏱ OUT TO THINK

One store advertises “1/3 off everything!” Another store advertises “Sale prices just 1/3 of original prices!” Which store is having the bigger sale? Explain.

Percentages of Percentages

Percentage changes and percentage differences can be particularly confusing when the values *themselves* are percentages. Suppose your bank increases the interest rate on your savings account from 3% to 4%. It’s tempting to say that the interest rate increases by 1%, but that

statement is ambiguous at best. The interest rate increases by 1 *percentage point*, but the relative change in the interest rate is 33%:

$$\frac{4\% - 3\%}{3\%} \times 100\% = 0.33 \times 100\% = 33\%$$

You can therefore say that the bank raised your interest rate by 33%, even though the actual rate increased by only 1 percentage point (from 3% to 4%).

Percentage Points versus %

When you see a change or difference expressed in *percentage points*, you can assume it is an *absolute* change or difference. If it is expressed as a percentage, it probably is a *relative* change or difference.

EXAMPLE 6 Care in Wording

Assume that 40% of the registered voters in Carson City are Republicans. Read the following questions carefully and give the most appropriate answers.

- The percentage of voters registered as Republicans is 25% higher in Freetown than in Carson City. What percentage of the registered voters in Freetown are Republicans?
- The percentage of voters registered as Republicans is 25 percentage points higher in Freetown than in Carson City. What percentage of the registered voters in Freetown are Republicans?

*If you can't convince them,
confuse them.*

—Harry S. Truman

SOLUTION

- We interpret the “25%” as a relative difference, and 25% of 40% is 10% (because $0.25 \times 0.40 = 0.10$). Therefore, the percentage of registered Republicans in Freetown is $40\% + 10\% = 50\%$.
- In this case, we interpret the “25 percentage points” as an absolute difference, so we simply add this value to the percentage of Republicans in Carson City. Therefore, the percentage of registered Republicans in Freetown is $40\% + 25\% = 65\%$. • •

Section 3

Statistical Literacy and Critical Thinking

- Percentages.** Last year's budget for the legislative branch of the U.S. government was \$4919 million, and this year it is \$5333 million. Consider last year's budget of \$4919 million to be the reference value.
 - What is the absolute change in the budget from last year to this year?
 - What is the relative change in the budget from last year to this year?
 - Next year's budget is estimated to be \$5185 million. What is the percentage decrease from this year's budget of \$5333?
 - If next year's budget is changed so that it is 5% less than this year's budget of \$5333 million, what is the amount of next year's budget?
- Percentage** A *New York Times* editorial criticized a chart caption that described a dental rinse as one that “reduces plaque on teeth by over 300%.” If the dental rinse removes all of the plaque, what percentage is removed? Is it possible to reduce plaque by over 300%?
- Percentage Points.** A Ridgid survey of 1,023 high school students showed that 25% of them plan to enter the field of information technology, and the margin of error is 3 percentage points. Why is it misleading to state that the margin of error is 3% instead of 3 percentage points?
- Of and More Than.** In an Opinion Research poll, 1,072 adults were asked what they would

do with their old cell phones, and 44.0% of them said that they would donate them to charity. What is the actual number of respondents who plan to donate their old cell phones to charity? If another poll is to be conducted with a sample size that is 5% greater than the sample size of 1,072 adults, how many subjects will be included in this new poll?

Does It Make Sense? For Exercises 5–8, decide whether the statement makes sense (or is clearly true) or does not make sense (or is clearly false). Explain clearly; not all of these have definitive answers, so your explanation is more important than your chosen answer.

5. Cell Phones. The percentage of people with cell phones increased by 1.2 million people.

6. Salary Percentages. The CEO of the Brandon Marketing Group announces that all employees must take a 5% cut in pay this year, but they will all get a 5% raise next year, so the salaries will then be the same as they are now.

7. Interest Rate. The Jefferson Valley Bank increased its new-car loan rate by 100%.

8. Interest Rate. The Jefferson Valley Bank increased its new-car loan rate (annual) by 100 percentage points.

Concepts and Applications

9. Fractions, Decimals, Percentages. Express each of the following numbers in the three forms of a fraction, decimal, and percentage.

- a. 75% b. $\frac{3}{8}$ c. 0.4 d. 80%

10. Fractions, Decimals, Percentages. Express the following numbers in the three forms of fraction, decimal, and percentage.

- a. 350% b. 2.5 c. -0.44 d. -200%

11. Percentage Practice. A study was conducted of pleas made by 1,348 criminals. Among those criminals, 956 pleaded guilty and 392 of them were sentenced to prison. Among 72 other criminals who pleaded not guilty, 58 were sent to prison (based on data from “Does It Pay to Plead Guilty?” by Brereton and Casper, *Law and Society Review*, Vol. 16, No. 1).

- a. What percentage of the criminals pleaded guilty?
b. What percentage of the criminals were sent to prison?

c. Among those who pleaded guilty, what is the percentage who were sent to prison?

d. Among those who pleaded not guilty, what is the percentage who were sent to prison?

12. Percentage Practice. A study was conducted to determine whether flipping a penny or spinning a penny has an effect on the proportion of heads. Among 49,437 trials, 29,015 involved flipping pennies, and 14,709 of those pennies turned up heads. The other 20,422 trials involved spinning pennies, and 9,197 of those pennies turned up heads (based on data from Robin Lock as given in *Chance News*).

- a. What percentage of the trials involved flipping pennies?
b. What percentage of the trials involved spinning pennies?
c. Among the pennies that were flipped, what is the percentage that turned up heads?
d. Among the pennies that were spun, what is the percentage that turned up heads?

Relative Change. Exercises 13–20 each provide two values. For each pair of values, use a percentage to express their relative change or difference. Use the second given value as the reference value, and express results to the nearest percentage point. Also, write a statement describing the result.

13. Newspapers. The number of daily newspapers in the United States is now 1,387, and it was 2,226 in 1900.

14. Cars. There are now 143,781,202 registered passenger cars, and in 1980 there were 121,601,000.

15. Airline Flights. This January there were 751,183 scheduled passenger flights in the United States, and in January of 1996 there were 634,343.

16. Bankruptcies. There were 1,531,997 bankruptcy cases filed last year, and in the year 2000 there were 1,276,900 bankruptcy cases filed.

17. Newspapers. The daily circulation of the *Wall Street Journal* is currently 2.09 million (the largest in the country). The daily circulation of *USA Today* is currently 1.83 million (the second largest in the country).

18. Car Sales. In the current month, 18,830 Toyota Camry cars were sold, and there were 18,341 Honda Civic cars sold.

19. Airports. Chicago’s O’Hare Airport handled 67 million passengers last year. As the busiest airport in the world, Atlanta’s Hartsfield Airport handled 89 million passengers last year.

- 20. Tourists.** Last year, France ranked as the number one tourist destination with 78 million international arrivals. The United States ranked second with 58 million international arrivals.

Surveys. Some important analyses of survey results require that you know the actual number of subjects whose responses fall into a particular category. In Exercises 21–24, find the actual number of respondents corresponding to the given percentage.

- 21. Personal Calls.** In an At-A-Glance survey of 1,385 office workers, 4.8% said that they do not make personal phone calls.

- 22. Interview Mistakes.** In an Accountemps survey of 150 executives, 47% said that the most common interview mistake is to have little or no knowledge of the company.

- 23. Televisions.** In a Frank N. Magid Associates survey of 1,005 adults, 83% reported having more than one television at home.

- 24. Cell Phones.** In a Harris Interactive survey of 9,132 adults, 89% reported being cell phone users.

Of vs. More Than. Fill in the blanks in Exercises 25–28. Briefly explain your reasoning in each case.

- 25. Weights.** If a truck weighs 40% more than a car, then the truck's weight is ____% of the car's weight.

- 26. Areas.** If the area of Norway is 24% more than the area of Colorado, then Norway's area is ____% of Colorado's area.

- 27. Population.** If the population of Montana is 20% less than the population of New Hampshire, then Montana's population is ____% of New Hampshire's population.

- 28. Salary.** The Vice President's salary is currently 42% less than the President's salary, so the Vice President's salary is ____% of the President's salary.

- 29. Margin of Error.** A Gallup poll of 1,012 adults showed that 89% of Americans say that human cloning should not be allowed. The margin of error was 3 percentage points. Would it matter if a newspaper reported the margin of error as “3%”? Explain.

- 30. Margin of Error.** A Pew Research Center survey of 3,002 adults showed that the percentage who listen to National Public Radio is probably between 14% and 18%. How should a newspaper report the margin of error? Explain.

Percentages of Percentages. Exercises 31–34 describe changes in which the measurements themselves are percentages. Express each change in two ways: (1) as an absolute

difference in terms of percentage points and (2) as a relative difference in terms of percent.

- 31.** The percentage of high school seniors using alcohol decreased from 68.2% in 1975 to 52.7% now.

- 32.** The percentage of the world's population living in developed countries decreased from 27.1% in 1970 to 19.5% now.

- 33.** The five-year survival rate for Caucasians for all forms of cancer increased from 39% in the 1960s to 61% now.

- 34.** The five-year survival rate for Blacks for all forms of cancer increased from 27% in the 1960s to 48% now.

PROJECTS FOR THE INTERNET & BEYOND

- 35. World Population.** Find the current estimate of world population on the U.S. Census Bureau's world population clock. Describe the percentage change in population since the 6 billion mark was passed during 1999. Also find how the population clock estimates are made, and discuss the uncertainties in estimating world population.

- 36. Drug Use Statistics.** Go to the Web site for the National Center on Addiction and Substance Abuse (CASA) and find a recent report giving statistics on substance abuse. Write a summary of the new research, giving at least some of the conclusions in terms of percentages.

IN THE NEWS

- 37. Percentages.** Find three recent news reports in which percentages are used to describe statistical results. In each case, describe the meaning of the percentage.

- 38. Percentage Change.** Find a recent news report in which percentages are used to express the change in a statistical result from one time to another (such as an increase in population or in the number of children who smoke). Describe the meaning of the change. Be sure to watch for key words such as *of* or *more than*.

- 39. Quote Interpretation.** Consider this quote: “The rate [of smoking] among 10th graders jumped 45 percent, to 18.3 percent, and the rate for 8th graders is up 44 percent, to 10.4 percent.” Briefly explain the meaning of each of the percentages in this statement.

4

INDEX NUMBERS**BY THE WAY**

By showing only 10-year increments, Table 1 hides substantial price variations that occurred within each decade. For example, while the 1990 average gasoline price was nearly identical to the 1980 price, actual prices during the 1980s ranged from below \$0.90 to almost \$1.50 per gallon. Similarly, during the 2000s the price ranged from a low of just over \$1 (in late 2001) to a high of over \$4 (in mid-2008) per gallon.

**BY THE WAY**

The term *index* is commonly used for almost any kind of number that provides a useful comparison, even when the numbers are not standard index numbers. For example, body mass index (BMI) provides a way of comparing people by height and weight, but is defined without any reference value. Specifically, body mass index is defined as weight (in kilograms) divided by height (in meters) squared.

You've probably heard about **index numbers**, such as the Consumer Price Index, the Producer Price Index, or the Consumer Confidence Index. Index numbers are very common in statistics because they provide a simple way to compare measurements made at different times or in different places. In this section, we'll investigate the meaning and use of index numbers, focusing on the Consumer Price Index (CPI). Let's start with an example using gasoline prices.

Table 1 shows the average price of gasoline in the United States for selected years from 1960 to 2010. (These are real prices from those years; that is, they have *not* been adjusted for inflation.) Suppose that, instead of the prices themselves, we want to know how the price of gasoline in different years compares with the 1980 price. One way to compare the prices would

TABLE 1		Average Gasoline Prices (per gallon)	
Year	Price	Price as a percentage of 1980 price	Price index (1980 = 100)
1960	\$0.31	25.4%	25.4
1970	\$0.36	29.5%	29.5
1980	\$1.22	100.0%	100.0
1990	\$1.23	100.8%	100.8
2000	\$1.56	127.9%	127.9
2010	\$2.84	232.8%	232.8

Source: U.S. Department of Energy; prices are the year-long average for all grades of gasoline.

be to express each year's price as a percentage of the 1980 price. For example, by dividing the 1970 price by the 1980 price, we find that the 1970 price was 29.5% of the 1980 price:

$$\frac{1970 \text{ price}}{1980 \text{ price}} = \frac{\$0.36}{\$1.22} = 0.295 = 29.5\%$$

Proceeding similarly for each of the other years, we can calculate all the prices as percentages of the 1980 price. The third column of Table 1 shows the results. Note that the percentage for 1980 is 100%, because we chose the 1980 price as the reference value.

Now look at the last column of Table 1. It is identical to the third column, except we dropped the % signs. This simple change converts the numbers from percentages to a *price index*, which is one type of index number. The statement "1980 = 100" in the column heading shows that the reference value is the 1980 price. In this case, there's really no difference between stating the comparisons as percentages and as index numbers—it's a matter of choice and convenience. However, as we'll see shortly, it's traditional to use index numbers rather than percentages in cases where many factors are being considered simultaneously.

Index Numbers

An **index number** provides a simple way to compare measurements made at different times or in different places. The value at one particular time (or place) must be chosen as the *reference value* (or *base value*). The index number for any other time (or place) is

$$\text{index number} = \frac{\text{value}}{\text{reference value}} \times 100$$

EXAMPLE 1 Finding an Index Number

Suppose the cost of gasoline today is \$3.60 per gallon. Using the 1980 price as the reference value, find the price index number for gasoline today.

SOLUTION We use the 1980 price of \$1.22 per gallon (see Table 1) as the reference value to find the index number for the \$3.60 gasoline price today:

$$\text{index number} = \frac{\text{current price}}{1980 \text{ price}} \times 100 = \frac{\$3.60}{\$1.22} \times 100 = 295.1$$

This index number for the current price is 295.1, which means the current gasoline price is 295.1% of the 1980 price.



TIME OUT TO THINK

Find the actual price of gasoline today at a nearby gas station. What is the gasoline price index for today's price, with the 1980 price as the reference value?

Making Comparisons with Index Numbers

The primary purpose of index numbers is to facilitate comparisons. For example, suppose we want to know how much more expensive gas was in 2000 than in 1980. We can get the answer easily from Table 1, which uses the 1980 price as the reference value. This table shows that the price index for 2000 was 127.9, which means that the price of gasoline in 2000 was 127.9% of the 1980 price. Equivalently, we can say that the 2000 price was 1.279 times the 1980 price.

We can also do comparisons when neither value is the reference value. For example, suppose we want to know how much more expensive gas was in 1990 than in 1960. We find the answer by dividing the index numbers for the two years:

$$\frac{\text{index number for 1990}}{\text{index number for 1960}} = \frac{100.8}{25.4} = 3.97$$

The 1990 price was 3.97 times the 1960 price, or 397% of the 1960 price. In other words, the same amount of gas that cost \$1.00 in 1960 would have cost \$3.97 in 1990.

A study of economics usually reveals that the best time to buy anything is last year.

—Marty Allen

EXAMPLE 2 Using the Gas Price Index

Use Table 1 to answer the following questions.

- Suppose that it cost \$16.00 to fill your gas tank in 1980. How much did it cost to buy the same amount of gas in 2010?
- Suppose that it cost \$20.00 to fill your gas tank in 2000. How much did it cost to buy the same amount of gas in 1960?

SOLUTION

- Table 1 shows that the price index (1980 = 100) for 2010 was 232.8, which means that the price of gasoline in 2010 was 232.8% of the 1980 price. So the 2010 price of gas that cost \$16.00 in 1980 was

$$232.8\% \times \$16.00 = 2.328 \times \$16.00 = \$37.25$$

- Table 1 shows that the price index (1980 = 100) for 2000 was 127.9 and the index for 1960 was 25.4. Dividing the index numbers, we find that the price of gasoline in 1960 was a fraction

$$\frac{\text{index number for 1960}}{\text{index number for 2000}} = \frac{25.4}{127.9} = 0.1986$$

of the price in 2000. Therefore, gas that cost \$20.00 in 2000 cost $0.1986 \times \$20.00 = \3.97



The Consumer Price Index

We've seen that the price of gas has risen substantially with time. Most other prices and wages have also risen, a phenomenon we call **inflation**. (Prices and wages occasionally decline with time, which is *deflation*.) Changes in the actual gasoline price therefore are not very meaningful unless we compare them to the overall rate of inflation, which is measured by the **Consumer Price Index (CPI)**.

The Consumer Price Index is computed and reported monthly by the U.S. Bureau of Labor Statistics. It represents an average of prices for a sample of goods, services, and housing. The monthly sample consists of more than 60,000 items. The details of the data collection and index calculation are fairly complex, but the CPI itself is a simple index number. Table 2 shows the average annual CPI over a 35-year period. Currently, the reference value for the CPI is an average of prices during the period 1982–1984, which is why the table says “1982 – 1984 = 100.”

TECHNICAL NOTE

The government measures two consumer price indices: The CPI-U is based on products thought to reflect the purchasing habits of all urban consumers, whereas the CPI-W is based on the purchasing habits of only wage earners. (Table 2 shows the CPI-U.)

TABLE 2		Average Annual Consumer Price Index (1982 – 1984 = 100)					
Year	CPI	Year	CPI	Year	CPI	Year	CPI
1977	60.6	1987	113.6	1997	160.5	2007	207.3
1978	65.2	1988	118.3	1998	163.0	2008	215.3
1979	72.6	1989	124.0	1999	166.6	2009	214.5
1980	82.4	1990	130.7	2000	172.2	2010	218.1
1981	90.9	1991	136.2	2001	177.1	2011	224.9
1982	96.5	1992	140.3	2002	179.9		
1983	99.6	1993	144.5	2003	184.0		
1984	103.9	1994	148.2	2004	188.9		
1985	107.6	1995	152.4	2005	195.3		
1986	109.6	1996	156.9	2006	201.6		

The Consumer Price Index

The **Consumer Price Index (CPI)**, which is computed and reported monthly, is based on prices in a sample of more than 60,000 goods, services, and housing costs.

The CPI allows us to compare overall prices at different times. For example, to find out how much higher typical prices were in 2010 than in 1995, we divide the CPIs for the two years:

$$\frac{\text{CPI for 2010}}{\text{CPI for 1995}} = \frac{218.1}{152.4} = 1.43$$

Based on the CPI, typical prices in 2010 were 1.43 times those in 1995. For example, a typical item that cost \$1,000 in 1995 would, on average, have cost \$1,430 in 2010. Of course, individual items may have price changes that are different from the average. For example, computer prices for equivalent computing power *fell* significantly from 1995 to 2010, meaning you could buy a much more powerful computer in 2010 for the same or less money. In contrast, the average price of health insurance more than doubled during the same period, so health insurance prices rose much faster than the overall rate of inflation.

EXAMPLE 3 CPI Changes

Suppose you needed \$30,000 to maintain a particular standard of living in 2000. How much would you have needed in 2010 to maintain the same living standard? Assume that the average price of your typical purchases has risen at the same rate as the Consumer Price Index (CPI).

SOLUTION We compare the Consumer Price Indices for 2010 and 2000:

$$\frac{\text{CPI for 2010}}{\text{CPI for 2000}} = \frac{218.1}{172.2} = 1.27$$

That is, typical prices in 2010 were about 1.27 times those in 2000. So if you needed \$30,000 in 2000, you would have needed $1.27 \times \$30,000 = \$38,100$ to have the same standard of living in 2010.

• •

Adjusting Prices for Inflation

In mid-2000, gasoline prices suddenly jumped, reaching a peak of about \$1.87 per gallon. Although that might sound cheap by today's standards, at the time it caused consumer outrage as news reports trumpeted the "highest gasoline prices in history."

In terms of actual prices (as posted at the pump), the \$1.87 price per gallon did indeed shatter the previous record high of \$1.47 from 1981. But were gas prices *really* at a record high? If we want to compare prices fairly, we must take into account the effects of inflation. To do that, we must first see how typical prices changed between 1981 and 2000, which we can do by dividing the CPIs for those years, using data from Table 2:

$$\frac{\text{CPI for 2000}}{\text{CPI for 1981}} = \frac{172.2}{90.9} = 1.89$$

Because the CPI rose by a factor of 1.89, the 1981 gasoline price of \$1.47 was equivalent to a 2000 price of $\$1.47 \times 1.89 = \2.78 . In the language of economics, we say that \$1.47 in "1981 dollars" was equivalent to \$2.78 in "2000 dollars." Because the actual price of gasoline in 2000 was much less than \$2.78, the 2000 price was *not* as high as the 1981 price in "real terms," meaning prices adjusted for inflation. Figure 3 shows more than 50 years of annual price data for both actual gasoline prices and prices adjusted to 2011 dollars. Notice that, in real terms, the 1981 average annual price remained the record until quite recently.

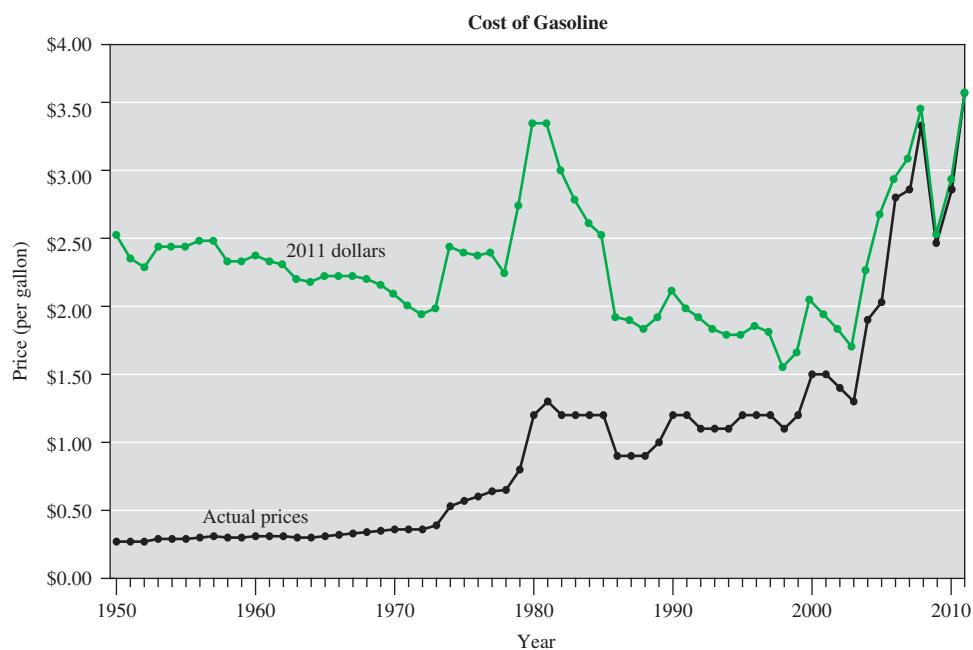


Figure 3 Gasoline prices (annual average), 1950–2011. Note that, because we use 2011 dollars for the inflation-adjusted prices, the actual and adjusted prices are the same for that year.

Source: U.S. Energy Information Administration.



USING TECHNOLOGY

INFLATION CALCULATOR

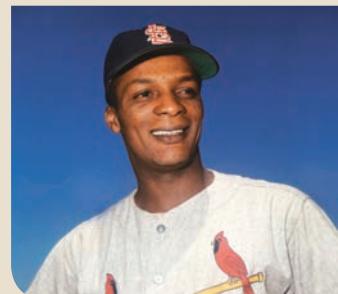
The U.S. Bureau of Labor Statistics (BLS) provides an online inflation calculator that allows you to adjust prices for any pair of years (based on the CPI). Search for "inflation calculator" on the BLS Web site.

CPI Inflation Calculator

\$ <input type="text" value="100.00"/>	
in <input type="button" value="1965"/>	
Has the same buying power as:	
\$ <input type="text" value="716.58"/>	
in <input type="button" value="2011"/>	
<input type="button" value="Calculate"/>	

BY THE WAY

Salaries of professional athletes were once kept low because the players were not allowed to offer their skills in the free market ("free agency"). That changed after star baseball player Curt Flood filed suit against Major League Baseball in 1970. Flood ultimately lost when the Supreme Court ruled in favor of baseball in 1972, but the process he set in motion (toward free agency) was unstoppable.



EXAMPLE 4 Baseball Salaries

In 1987, the mean salary for major league baseball players was \$412,000. In 2011, it was \$3,319,000. Compare the increase in mean baseball salaries to the overall rate of inflation as measured by the Consumer Price Index.

SOLUTION First, we compare the Consumer Price Indices for 2011 and 1987:

$$\frac{\text{CPI for 2011}}{\text{CPI for 1987}} = \frac{224.9}{113.6} = 1.98$$

Next, we compare the average baseball salaries for those two years:

$$\frac{\text{average baseball salary for 2011}}{\text{average baseball salary for 1987}} = \frac{\$3,319,000}{\$412,000} = 8.06$$

During the same period of time that average prices (as measured by the CPI) approximately doubled, the mean baseball salary rose by more than 800%. In other words, the mean salaries of major league baseball players rose more than four times as much as the overall rate of inflation.



BY THE WAY

Thinking of becoming a comedian? Then you'll probably want to check the Cost of Laughing Index (there really is one!), which tracks costs of items such as rubber chickens, Groucho Marx glasses, and admission to comedy clubs.

Section 4

Statistical Literacy and Critical Thinking

- Index Number.** A newspaper reports that the Gas Price Index in 2011 was \$3.92 per gallon. What is wrong with that statement?
- Index Number.** If computer costs in the year 2000 are set equal to 100 so they can be used as the basis for determining index numbers, and the index number for the year 2012 is 15, what do we know about computer costs in 2012 compared with computer costs in 2000?
- CPI.** If the prices of goods, services, and housing increase, must the Consumer Price Index increase? Explain.
- CPI.** If the Consumer Price Index increases, must wages also increase? Explain.

Concepts and Applications

Gasoline Price Index. In Exercises 5–8, use the gasoline price index from Table 1. Briefly explain your reasoning in each case.

- Current Data.** Suppose the cost of gasoline today is \$5.00 per gallon. What is the price index number for gasoline today, with the 1980 price as the reference value? (See the data in Table 1.)
- 2006 Index.** The average price of a gallon of gas was \$2.62 in 2006. What is the price index for gasoline in 2006, with the 1980 price as the reference value?
- 1998 Price.** Using the 1980 price as the reference value, the gasoline price index for 1998 is 0.9. What was the cost of a gallon of gasoline in 1998?

8. 2005 Price. Using the 1980 price as the reference value, the gasoline price index for 2005 is 189.3. What was the cost of a gallon of gasoline in 2005?

9. Reconstructing the Gasoline Price Index. Identify the six price indexes in Table 1 that result from using the price from 2000 as the reference value. (*Hint:* Create a column for price as a percentage of 2000 price and another column giving the price index with 2000 = 100.)

10. Reconstructing the Gasoline Price Index. Identify the six price indexes in Table 1 that result from using the price from 1970 as the reference value. (*Hint:* Create a column for price as a percentage of 1970 price and another column giving the price index with 1970 = 100.)

11. Using the Gas Price Index. If it cost \$19.52 to fill your gas tank in 1980, how much would it have cost to fill the same tank in 2010?

12. Using the Gas Price Index. If it cost \$23.40 to fill your gas tank in 2000, how much would it have cost to fill the same tank in 2010?

13. Private College Costs. The average annual cost (tuition, fees, and room and board) at four-year private universities rose from

\$5,600 in 1980 to \$37,000 in 2010. Calculate the percentage rise in cost from 1980 to 2010, and compare it with the overall rate of inflation as measured by the Consumer Price Index.

- 14. Public College Costs.** The average annual cost (tuition, fees, and room and board) at four-year public universities rose from \$2,550 in 1980 to \$16,100 in 2010. Calculate the percentage rise in cost from 1980 to 2010, and compare it with the overall rate of inflation as measured by the Consumer Price Index.
- 15. Home Prices—South.** The typical (median) price of a new single-family home in the South (United States) rose from \$75,300 in 1990 to \$122,000 in 2010. Calculate the percentage rise in cost of a home from 1990 to 2010, and compare it with the overall rate of inflation as measured by the Consumer Price Index.
- 16. Home Prices—West.** The typical (median) price of a new single-family home in the West (United States) rose from \$129,600 in 1990 to \$235,600 in 2010. Calculate the percentage rise in cost of a home from 1990 to 2010, and compare it with the overall rate of inflation as measured by the Consumer Price Index.

Housing Price Index. Realtors use an index to compare housing prices in major cities throughout the country. The index numbers for several cities are given in the table below. If you know the price of a particular home in your town, you can use the index to find the price of a comparable house in another town:

$$\text{price}_{(\text{other town})} = \frac{\text{price}_{(\text{your town})}}{\text{index in your town}} \times \text{index in other town}$$

Use the housing price index in Exercises 17–20.

City	Index	City	Index
Denver	100	Boston	358
Miami	194	Las Vegas	101
Phoenix	86	Dallas	81
Atlanta	90	Cheyenne	60
Baltimore	150	San Francisco	382

- 17. Housing Prices.** If you see a house valued at \$300,000 in Denver, find the price of a comparable house in Miami and Cheyenne.
- 18. Housing Prices.** If you see a house valued at \$500,000 in Boston, find the price of a comparable house in Baltimore and Phoenix.
- 19. Housing Prices** If you see a house valued at \$250,000 in Cheyenne, find the price of a comparable house in San Francisco and Boston.

- 20. Housing Prices.** If you see a house valued at \$1,000,000 in Boston, find the price of a comparable house in San Francisco and Cheyenne.

PROJECTS FOR THE INTERNET & BEYOND

- 21. Consumer Price Index.** Go to the Consumer Price Index home page and find the latest news release with updated figures for the CPI. Summarize the news release and any important trends in the CPI.
- 22. Producer Price Index.** Go to the Producer Price Index (PPI) home page. Read the overview and recent news releases. Write a short summary describing the purpose of the PPI and how it is different from the CPI. Also summarize any important recent trends in the PPI.
- 23. Consumer Confidence Index.** Use a search engine to find recent news about the Consumer Confidence Index. After studying the news, write a short summary of what the Consumer Confidence Index is trying to measure and describe any recent trends in the Consumer Confidence Index.
- 24. Human Development Index.** The United Nations Development Programme regularly releases its Human Development Report. A closely watched finding of this report is the Human Development Index (HDI), which measures the overall achievements in a country in three basic dimensions of human development: life expectancy, educational attainment, and adjusted income. Find the most recent copy of this report and investigate exactly how the HDI is defined and computed.
- 25. Convenience Store Index.** Go to a local supermarket and find the prices of a few staples, such as bread, milk, juice, and coffee. Compute the total cost of those items. Then go to a few smaller convenience stores and find the prices of the same items. Using the supermarket total as the reference value, compute the index numbers for the convenience stores.

IN THE NEWS

- 26. Consumer Price Index.** Find a recent news report that includes a reference to the Consumer Price Index. Briefly describe how the Consumer Price Index is important in the story.
- 27. Index Numbers.** Find a recent news report that includes an index number other than the Consumer Price Index. Describe the index number and its meaning, and discuss how the index is important in the story.

CHAPTER REVIEW EXERCISES

1. Astrology. In a Harris Interactive poll of 2,303 adults, 26% said that they believe in astrology.

- Find the true number of the poll respondents who believe in astrology.
- Is the number of respondents who believe in astrology from a discrete data set or a continuous data set? Explain.
- Of the 2,303 adults who were polled, 1,382 believe in the devil. What percentage of respondents believe in the devil?
- Among the respondents, 1,382 believe in the devil, and the number who believe in UFOs is 53% of that number. How many believe in UFOs?
- If we compile the ages of all respondents, what is the level of measurement (nominal, ordinal, interval, ratio) of those ages?
- If we classify the respondents according to gender, what is the level of measurement (nominal, ordinal, interval, ratio) of the gender data?

2. AOL Poll. In an America OnLine poll of 671 people, 36% answered yes when asked if today's NFL games are too long.

- What is the number of respondents who answered yes?
- Among the respondents, 40 answered by saying that they were not sure. What is the percentage of "not sure" responses?
- Given that the possible responses are "yes, no, not sure," what is the level of measurement of those responses? (nominal, ordinal, interval, ratio)
- Given that the poll was conducted by asking America OnLine users to respond to a question that was posted on the Web site, what do you conclude about the poll results? Is it likely that the results reflect the opinion of the general population?

3. Health Care Spending. Total spending on health care in the United States rose from \$80 billion in 1973 to \$2.5 trillion in 2010. The Consumer Price Index was 44.4 in 1973, and it was 218.1 in 2010 (with 1982–1984 = 100). Compare the change in health care spending from 1973 to 2010 to the overall rate of inflation as measured by the Consumer Price Index.

4. Minimum Wage. The accompanying table lists the federal hourly minimum in both actual dollars at the time and 1996 dollars. The table entries correspond to years in which the minimum wage changed (based on data from the Department of Labor).

Year	Actual Dollars	1996 Dollars
1938	0.25	2.78
1939	0.30	3.39
1945	0.40	3.49
1950	0.75	4.88
1956	1.00	5.77
1961	1.25	6.41
1967	1.40	6.58
1968	1.60	7.21
1974	2.00	6.37
1976	2.30	6.34
1978	2.65	6.38
1979	2.90	6.27
1981	3.35	5.78
1990	3.50	4.56
1991	4.25	4.90
1996	4.75	4.75
1997	5.15	5.03
2009	7.25	5.22

- According to the table, how much is \$0.25 in 1938 dollars worth in 1996 dollars?
- According to the table, how much is \$1.00 in 1956 dollars worth in 1996 dollars?
- Note that the minimum wage for 2009 in actual dollars is greater than the minimum wage for 2009 in 1996 dollars. What does that tell us about the minimum wage in 2009?

CHAPTER QUIZ

1. Sitting heights of 750 randomly selected licensed drivers are measured. Are those values continuous or discrete?
2. What is the level of measurement of the values described in the preceding exercise? (nominal, ordinal, interval, ratio)
3. A researcher measures the sitting height of a subject and records a value of 91.4 cm, but the subject's actual sitting height is 89.0 cm. What is the absolute error?
4. A researcher measures the sitting height of a subject and records a value of 91.4 cm, but the subject's actual sitting height is 89.0 cm. What is the relative error?
5. Recorded survey results include the states in which respondents reside. What is the level of measurement of those data? (nominal, ordinal, interval, ratio)
6. Sitting heights of 750 randomly selected licensed drivers are measured and 38 of those heights are greater than 97.2 cm. What is the percentage of sitting heights greater than 97.2 cm?
7. In a Gallup poll of 1,038 adults, 5% of the respondents said that second-hand smoke is not at all harmful. How many respondents said that second-hand smoke is not at all harmful?
8. Two different students measure the height of an instructor who is actually 178.44 cm tall. The first student obtains a measurement of 178 cm and the second student obtains a measurement of 179.18 cm. Which measurement is more accurate? Which measurement is more precise?
9. The Telektronics Company has been in business for five years, and the table lists the net profits in each of those years. Using the first year as a reference, find the index number for the net profit in the second year.

Year	Net Profit
1	\$12,335
2	\$15,257
3	\$23,444
4	\$31,898
5	\$47,296

10. Refer to the same table used for Exercise 9. If the net profit in the sixth year is projected to be 12% more than in the fifth year, what is that projected net profit for the sixth year?

FOCUS ON POLITICS

Who Benefited Most from Lower Tax Rates?

Politicians have a remarkable capacity to cast numbers in whatever light best supports their beliefs. There are many ways in which numbers can be made to support a particular position, but one of the most common occurs through selective use of relative (percentages) and absolute numbers.

Consider the two charts shown in Figure 4. Both purport to show effects of the tax rate cuts originally enacted under President Bush in 2001 and renewed under President Obama in 2010. (Under the 2001 law, the tax cuts would have expired at the end of 2010. The 2010 law extended them through 2012.) The chart in Figure 4a, created by supporters of the tax cuts, indicates that the rich ended up paying more under the tax cuts than they would have otherwise. Figure 4b, created by opponents of the tax cuts, shows that the rich received far more benefit from the tax cuts than lower-income taxpayers. The two charts therefore seem contradictory, because the first seems to indicate that the rich paid more while the second seems to indicate that they paid less.

Which story is right? In fact, both of the graphs are accurate and show data from reputable sources. (The Department of the Treasury, cited as the source of the data for Figure 4a, is an agency of the federal government; the Joint Committee on Taxation, cited in Figure 4b, is a nonpartisan committee of the



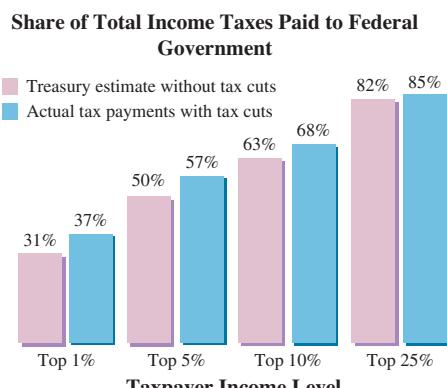
U.S. Congress.) The seemingly opposing claims arise from the way in which each group chose its data.

The tax cut supporters show the *percentage of total taxes* that the rich paid with the cuts and what calculations suggest they would have paid without them. The title stating that the “rich pay more” therefore means that the tax cuts led them to pay a higher percentage of total taxes. However, if total tax revenue also was lower than it would have been without the cuts (as it was), a higher percentage of total taxes could still mean lower absolute dollars. It is these lower absolute dollars that are shown by the opponents of the tax cut.

Politicians and government officials usually abuse numbers and logic in the most elementary ways. They simply cook figures to suit their purpose, use obscure measures of economic performance, and indulge in horrendous examples of chart abuse, all in the name of disguising unpalatable truths.

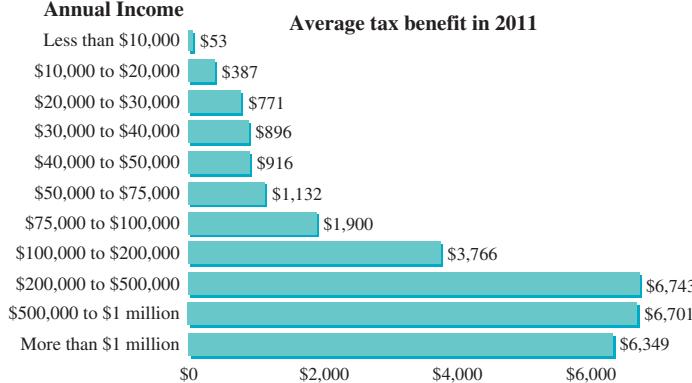
—A. K. Dewdney, *200% of Nothing*

Rich Pay More Under Tax Cuts



(a)

High-Income Households Receive Highest Dollar Benefit from Tax Cuts



(b)

Figure 4 Both charts purport to show effects on higher-income households of tax cuts enacted in 2001 and renewed in 2010, but they are designed to support opposing conclusions. (a) Adapted from a graph published in *The American—The Journal of the American Enterprise Institute*. (b) Adapted from a graph published by the Center on Budget and Policy Priorities (cbpp.org).

Which side was being more fair? Neither, really. The supporters have deliberately focused on a percentage in order to mask the absolute change, which would be less favorable to their position. The opponents focused on the absolute change, but neglected to mention the fact that the wealthy pay most of the taxes. Unfortunately, this type of “selective truth” is very common when it comes to numbers, especially those tied up in politics.

QUESTIONS FOR DISCUSSION

1. Discuss the chart in Figure 4a. Why does it only show taxpayer income levels from the top 1% to the top 25%? What does it tell us about the relative tax burden on each group shown? Do you think its claim that the “rich pay more” is an honest depiction or a distortion of the facts? Defend your opinion.
 2. Discuss the chart in Figure 4b. Why does it show actual incomes rather than incomes as a percentage of all taxpayers? What does it tell us about the effects of the tax cuts on each income level? Do you think it supports the claim in its title? Defend your opinion.
 3. Do you think that either of the charts in Figure 4 accurately portrays the overall “fairness” of the tax cuts? If so, which one and why? If not, how do you think the data could have been portrayed more fairly?
 4. Do a Web search on “benefit of tax cuts” and find an article arguing for or against a particular tax cut proposal or policy. Briefly summarize the article and discuss whether you think the article made its case well.
 5. Have any tax law changes been proposed by the U.S. Congress or the President this year? Discuss the fairness of current proposals.
- • • • • • • • • • •



FOCUS ON ECONOMICS

Can a Redefined CPI Help Solve the Budget Crisis?

We're all aware that the U.S. government has a sky-high debt of more than \$15 trillion, which amounts to approximately \$50,000 for every man, woman, and child in the nation. The only way to pay off the debt is through some combination of increased government revenues or decreased government spending. Unfortunately, a quick look at the news will show that neither option is politically popular, because some people are adamantly opposed to tax increases while others are adamantly opposed to cuts in government benefits. But what if part of the problem exists only because of a misunderstanding and misapplication of the Consumer Price Index (CPI)?

Benefits are popular. Paying for benefits is extremely unpopular.

—John Danforth, former U.S. Senator
(Republican–Missouri)

This question arises because most economists believe that the CPI overstates the true effects of inflation. The thrust of the economic argument is that the standard calculation of the CPI contains at least two systematic errors that overstate how inflation affects the cost of living. First, from one month to the next, the CPI is based on changes in the prices of particular items at particular stores. In reality, however, if the price of an item rises at one store, consumers often buy it more cheaply at another store, and if the price rises at all stores, consumers may substitute a similar but lower-priced item (such as a different brand of the same product). This “price substitution” effect means that consumers don’t find their actual costs rising as much as the CPI indicates. Second, the CPI tracks changes in the price of “typical” items purchased by consumers at any given time, but it does not account for the effects of changes or improvements in these items with time. For example, the data used in computing the CPI may show that a typical cell phone is 10% more expensive than a cell phone from a few years ago, but these data do not account for the fact that today’s cell phones have many more capabilities. The data therefore overstate the effect of the cell phone price rise, because you are getting so much more for your money today.

The accuracy of the CPI as a measure of the increase in the cost of living may seem like an academic debate, but it has real consequences for the government budget. The reason is that many budget items are tied to inflation, including one that is particularly important to government revenue and another that is particularly important to government spending. On the



revenue side, the government raises the income thresholds for different tax rates each year, with the goal of making sure that people’s tax rates go up only if they actually increase their standard of living, as opposed to going up just because inflation has made living more costly. For decades, the increases in the thresholds have been tied to the CPI. But if the CPI overstates the effects of inflation on living standards, then instead of holding rates steady, the threshold changes have effectively lowered tax rates. On the spending side, the government annually increases the checks provided to recipients of Social Security and other benefits, again with the goal of making sure that the benefits reflect changes in the cost of living. (The annual increase is called a “cost of living adjustment,” or COLA for short.) If the CPI is overstating the effects of inflation, then these cost of living adjustments have been larger than is really needed to maintain the same standard of living.

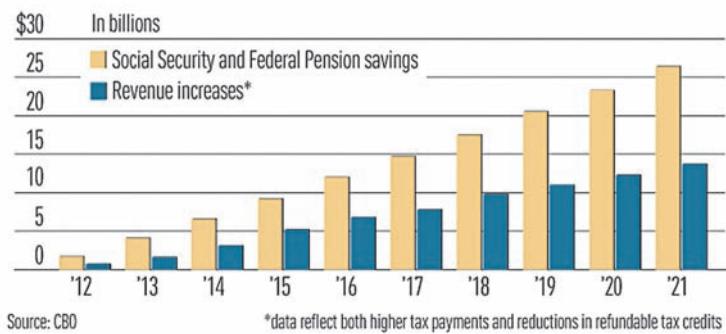
Clearly, the federal government’s budget picture could be dramatically improved by linking changes in tax rates and cost of living adjustments to a value that more accurately reflects inflation than the current CPI. In fact, the government already computes something called a “chained CPI” that was designed specifically to address this problem. Figure 5 shows how both revenue and spending would be different if the government tied changes to the chained CPI rather than the standard CPI. If you add all the revenue increases and spending savings that are shown in the chart, you’ll find that this simple change would reduce the government’s deficit by more than \$200 billion over the next decade. The savings would be even greater in the following decade, because each year’s savings builds on the previous year’s.

QUESTIONS FOR DISCUSSION

1. Find examples in your own spending or the spending of friends and family of substitution effects and the purchase of products that were unavailable or lower quality

Diet COLA

Using a lower measure of inflation would reduce cost-of-living adjustments and raise tax revenue



Source: CBO

*data reflect both higher tax payments and reductions in refundable tax credits

Figure 5 Revenue increases and spending savings that would arise from trying to account for the ways in which the CPI tends to overstate inflation. Adapted from “Obama Mulls COLA Switch, Debt Talks Will Continue,” by Jed Graham, from *Investor’s Business Daily*, July 7, 2011. Copyright © 2011 by *Investor’s Business Daily*. Reprinted with permission.

a few years ago. Overall, do you think these examples support the claim that the CPI overstates the effects of inflation? Defend your opinion.

2. Because adjusting the CPI would lead to a slower rise in the income thresholds for various tax rates, conservatives often claim that this change would mean “tax increases.” Similarly, because the change would reduce the annual increases in cost of living adjustments, liberals often claim that it would mean a “benefit cut” to Social Security recipients. Do you think either claim is accurate? Defend your opinion.
3. As this text goes to press, the government is considering making a change to the chained CPI. Has it happened

yet? Investigate the current status of the debate over whether the CPI should be adjusted for the purposes of setting tax rates and cost of living adjustments.

4. In Figure 5, notice that the revenue increases and spending savings get larger each year. Discuss why this occurs, focusing on how a change in the CPI compounds over time, much like compound interest in a bank account.
5. While most economists believe that the CPI overstates the effects of inflation, a few believe that it actually understates the effects, especially on lower-income people. Research why some economists come to this conclusion, then draw your own conclusion: Does the CPI sometimes underestimate the effects of inflation?

ANSWERS

Section 1

1. Qualitative data consist of values that can be placed into different nonnumerical categories, whereas quantitative data consist of values representing counts or measurements.
3. Yes. Data consist of either qualities or quantities (numbers), so all data are either qualitative or quantitative.
5. Blood groups are qualitative because they don't measure or count anything.
7. Braking reaction times are quantitative because they consist of measurements.
9. The answers to multiple choice test questions are qualitative because they don't measure or count anything.
11. The television shows are qualitative because they don't measure or count anything.
13. Head circumferences are quantitative because they consist of measurements.
15. The grade point averages are quantitative because they are measures of course grades.
17. The numbers are discrete because only the counting numbers are used, and no values between counting numbers are possible.
19. The numbers are discrete because they are counts. Only the counting numbers are used, and no values between counting numbers are possible.
21. The times are continuous data because they can be any value within some range of values.
23. The numerical test scores are discrete data because they can be counting numbers only.
25. The speeds are continuous data because they can be any value within some range of values.
27. The numbers are discrete data because they can be counting numbers only.
29. Ratio
31. Nominal
33. Ordinal
35. Ordinal
37. Nominal
39. Ratio
41. The ratio level does not apply. The ratio is not meaningful because the stars don't measure or count anything. Differences between star values are not meaningful.
43. The ratio level does not apply. IQ tests do not measure intelligence on the type of scale required for the ratio level. Someone with an IQ score of 140 is not necessarily twice as intelligent as someone with an IQ score of 70.

45. The ratio level applies. The age of 1,000 years is twice as long as the age of 500 years.
47. The ratio level applies. The \$150,000 salary amount is twice the salary of \$75,000, so the ratio of "twice" is meaningful.
49. The data are quantitative and are at the ratio level. The data are continuous. The times have a natural zero starting point and the times can be any values within a particular range.
51. The data are qualitative and are at the nominal level of measurement. The numbers are different ways to express the names, and they don't measure or count anything.
53. The data are quantitative and are at the interval level of measurement. The data are discrete because they consist of whole numbers only. The years are measured from an arbitrary reference (the year 0), not a natural zero starting point. Differences between the years are meaningful values, but ratios are not meaningful.
55. The data are qualitative at the ordinal level of measurement. The ratings consist of an ordering, but they do not represent counts or measurements.

Section 2

1. It is a random error, because there is no way to predict when a clerk will write a number incorrectly or whether the number will be too high or too low.
3. Because the recorded weight has so many decimal places, it is very precise. Because the recorded weight is not very close to the true weight, it is not very accurate.
5. Does not make sense. The number is too precise. Some species have not yet been identified, and new species are being found.
7. Makes sense. With random error, we expect that about half of the errors will be in favor of the supermarket and half will be in favor of the consumer.
9. Mistakes tend to result from random errors, but dishonesty tends to result in systematic errors that benefit the taxpayer.
11. Because about half of the batteries have more than 3.7 volts and half have less than 3.7 volts, it appears that we have a random error.
13. Random errors could occur when people make estimates because they don't recall the exact amounts. Systematic errors could occur when people tend to report that they donate more money than they actually donate so that they appear to be more benevolent.
15. Random errors could occur when people don't know their actual weight and they report an amount that is wrong. Systematic errors could occur when people intentionally lie about their weight by reporting a value that is considerably lower than the true weight.
17. Random errors could occur with an inaccurate radar gun or with honest mistakes made when the officer records the speeds. Systematic errors could occur with a radar gun that is incorrectly calibrated so that it consistently reads too high or too low.

- 19.** Random errors occur with errors in the calculations. Systematic errors may undercount the actual number if some cigarettes are sold illegally without the tax being collected.
- 21.** Absolute error: \$1,750. Relative error: 141% (using the absolute error of \$1,750 and the correct bill amount of \$1,245).
- 23.** Absolute error: $-\$1$. Relative error: -36.4% .
- 25.** **a.** These errors are random. If they were systematic, there would be a tendency for the measurements to be all too high or all too low. **b.** Use the average because it is likely to be in error by less than most of the individual measurements and it is more reliable than any single measurement. **c.** Systematic errors might result if the students have all been taught to use the tape measure in the same incorrect way. **d.** No. If there is a systematic error in the measurements, that same error will be present in the average.
- 27.** The Department of Transportation scale is more precise because its weight of 3,298.2 lb has more decimal places than the other weight of 3,250 lb. The manufacturer's scale is more accurate because its weight of 3,250 lb is closer to the true weight. (3,250 lb is in error by 23 lb, but the weight of 3,298.2 lb is in error by 25.2 lb.)
- 29.** The measurement of 52.88 kilograms is both more precise (because it has more decimal places) and more accurate (because it is closer to your weight).
- 31.** The given number is very precise, but it is not likely to be accurate. We cannot even measure the population this precisely today, and the uncertainties were certainly greater in 1860.
- 33.** The given number is very precise, but it is not likely to be accurate. There are many people in China who are not counted. The census of any nation is likely to be in error by considerable amounts due to the inherent difficulties in conducting a national census. The population of China likely changed during the course of the year.
- 35.** It is easy to accurately measure the height of a structure with a reasonable degree of precision, such as the nearest 1/10 foot, but the given number has far too much precision (it implies knowing the height to a size smaller than an atom!), so the claim is not believable.
- 37.** The number of college students is constantly changing with new enrollments and dropouts, so the given number must be an estimate. The precision of the given number is probably unjustified. The given number seems to be accurate, but we would need more information to know if it really is accurate.
- 7.** Makes sense. The 100% increase indicates that the loan rate doubled. For example, if the loan rate doubled from 5% to 10%, it increased the loan rate by 100%.
- 9.** **a.** $75/100$ or $3/4$, 0.75, 75% **b.** $3/8$, 0.375, 37.5% **c.** 4/10 or $2/5$, 0.4, 40% **d.** $80/100$ or $4/5$, 0.8, 80%
- 11.** **a.** 71% **b.** 33% **c.** 41% **d.** 81%
- 13.** -38% (38% decrease from 1990)
- 15.** 18% (18% increase from January 1996)
- 17.** 14%. The *Wall Street Journal* has 14% more circulation than *USA Today*.
- 19.** -25% . O'Hare handled 25% fewer passengers than Atlanta's Hartsfield Airport.
- 21.** 66
- 23.** 834
- 25.** 140%. The truck weighs 100% of the car's weight plus another 40%.
- 27.** 80%. The population of Montana is the population of New Hampshire minus 20%.
- 29.** Yes. Three percentage points corresponds to a range from 86% to 92%, which was intended. A margin of error of 3% corresponds to 3% of 89%, which is $0.03 \times 0.89 = 0.0267$, but this is not what was meant.
- 31.** -15.5 percentage points; -22.7%
- 33.** 22 percentage points; 56.4%

Section 4

Section 3

- 1.** **a.** \$414 million **b.** 8.4% **c.** 2.8% decrease
d. \$5066.35 million
- 3.** The statement that the margin of error is 3% incorrectly implies that the error can be up to 3% of 25%, which is 0.75%, but the actual error can be up to 3 percentage points away from 25% (from 22% to 28%).
- 5.** Does not make sense. The number of people with cell phones may have increased by 1.2 million, but 1.2 million people is not a percentage, so the statement is not sensible.

- 1.** An index number is a ratio without any units, such as dollars. The number appears to be the actual cost of gasoline in 2011, not an index number.
- 3.** Yes. The Consumer Price Index is based on the prices of goods, services, and housing, so increases in those prices will result in an increase in the Consumer Price Index.
- 5.** 409.8
- 7.** \$1.10
- 9.** 19.9, 23.1, 78.2, 78.8, 100, 182.1
- 11.** \$45.44
- 13.** The tuition in 2010 is about 660% of the amount in 1980, which means a rise of about 550%. The CPI in 2010 is 264.7% of the CPI in 1980, for a rise of 164.7%. So the cost of tuition rose much more than the cost of typical goods, services, and housing.
- 15.** The home prices in 2010 are about 162% of the amount in 1990, which means a rise of about 62%. The CPI in 2010 is 167% of the CPI in 1980, for a rise of 67%. So the cost of homes rose at a slightly lower rate than the CPI.
- 17.** \$582,000; \$180,000
- 19.** \$1,591,667; \$1,491,667

Chapter Review Exercises

- 1.** a. 599 b. Discrete, because only the counting numbers are used, and no values between counting numbers are possible.
 c. 60% d. 732 e. Ratio f. Nominal
- 3.** The health care spending in 2010 is 3125% of the amount in 1973, but the CPI in 2010 is 491% of the 1973 CPI, so health care spending grew at a much greater rate than the general rate for goods, services, and housing.

Chapter Quiz

- 1.** Continuous **3.** -2.4 cm **5.** Nominal
7. 52 **9.** 123.7

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Visual Displays of Data



LEARNING GOALS

1 Frequency Tables

Be able to create and interpret frequency tables.

2 Picturing Distributions of Data

Be able to create and interpret basic bar graphs, dotplots, pie charts, histograms, stemplots, line charts, and time-series graphs.

3 Graphics in the Media

Understand how to interpret the many types of more complex graphics that are commonly found in news media.

4 A Few Cautions About Graphics

Critically evaluate graphics and identify common ways in which graphics can be misleading.

FOCUS TOPICS

Focus on History: Can War Be Described with a Graph?

Focus on Environment: Are We Changing Earth's Atmosphere?

Visual Displays of Data

Whether you look at a newspaper, a corporate annual report, or a government study, you are almost sure to see tables and graphs of statistical data. Some of these tables and graphs are very simple; others can be quite complex. Some make it easy to understand the data; others may be confusing or misleading. In this chapter, we'll study some of the many ways in which statistical data are commonly displayed in tables and graphs. Because the ability to convey concepts through graphs is so valuable in today's data-driven society, the skills developed in this chapter are crucial for success in nearly every profession.

The greatest value of a picture is when it forces us to notice what we never expected to see.

—John Tukey

1

FREQUENCY TABLES

A teacher records the following list of the grades she gave to her 25 students on a set of essays:

A	C	C	B	C	D	C	C	F	D	C	C	C
B	B	A	B	D	B	A	A	B	F	C	B	

This list contains all of the grades, but it isn't easy to read. A better way to display these data is with a **frequency table** (Table 1)—a table showing the number of times, or **frequency**, that each grade appears. The five possible grades (A, B, C, D, F) are called the **categories** (or classes) for the table.

Definitions

A basic **frequency table** has two columns:

- The first column lists all the **categories** of data.
- The second column lists the **frequency** of each category, which is the number of data values in the category.

TABLE 1		Frequency Table for a Set of Essay Grades
Grade	Frequency	
A	4	
B	7	
C	9	
D	3	
F	2	
Total	25	

EXAMPLE 1 Taste Test

The Rocky Mountain Beverage Company wants feedback on its new product, Coral Cola, and sets up a taste test with 20 people. Each individual is asked to rate the taste of the cola on a 5-point scale:

(bad taste) 1 2 3 4 5 (excellent taste)

The 20 ratings are as follows:

1	3	3	2	3	3	4	3	2	4
2	3	5	3	4	5	3	4	3	1

Construct a frequency table for these data.

SOLUTION The variable of interest is *taste*, and this variable can take on five values: the taste categories 1 through 5. (Note that the data are qualitative and at the ordinal level of measurement.) We construct a table with these five categories in the left column and their frequencies in the right column, as shown in Table 2.

TABLE 2		Taste Test Ratings
Taste scale	Frequency	
1	2	
2	3	
3	9	
4	4	
5	2	
Total	20	

Relative and Cumulative Frequency

Look again at the essay grades listed in Table 1. In some cases, we might be more interested in the fraction (or percentage) of students, rather than the actual number of students, who received each grade. Such a fraction is called a **relative frequency**. For example, because 4 of the 25 students received A grades, the relative frequency of A grades is $4/25$, or 0.16, or 16%.

In other cases, we might want to know the answer to questions like How many students got a grade of C or better? The answer to this question is the sum of the frequencies of A, B, and C grades, which we call the **cumulative frequency**. Table 3 repeats the data from Table 1, but this time with added columns showing the calculations for the relative and cumulative frequencies.

TABLE 3				Essay Grade Table with Relative and Cumulative Frequency
Grade	Frequency	Relative frequency	Cumulative frequency	
A	4	$4/25 = 0.16$	4	
B	7	$7/25 = 0.28$	$7 + 4 = 11$	
C	9	$9/25 = 0.36$	$9 + 7 + 4 = 20$	
D	3	$3/25 = 0.12$	$3 + 9 + 7 + 4 = 23$	
F	2	$2/25 = 0.08$	$2 + 3 + 9 + 7 + 4 = 25$	
Total	25	1	25	

TECHNICAL NOTE

Most frequency tables start with the lowest category, but tables of grades commonly start with the highest category (A). Note that category can affect cumulative frequency. For example, if we reversed the order in Table 3, the cumulative frequency for C would be the number of grades of C or lower, rather than C or higher.

Note that the sum of the relative frequencies must equal 1 (or 100%), because each individual relative frequency is a fraction of the total frequency. (Rounding sometimes causes the total to be slightly different from 1.) The cumulative frequency for the last category must always equal the total number of data values, because it represents the total number of data values in that category *and* all the preceding categories.

Definitions

The **relative frequency** of any category is the fraction (or percentage) of the data values that fall in that category:

$$\text{relative frequency} = \frac{\text{frequency in category}}{\text{total frequency}}$$

The **cumulative frequency** of any category is the number of data values in that category *and all preceding* categories.

Keep in mind that cumulative frequencies make sense only for data categories that have a clear order. That is, we can use cumulative frequencies for data at the ordinal, interval, and ratio levels of measurement, but not for data at the nominal level of measurement.

EXAMPLE 2 Taste Test: Relative and Cumulative Frequency

Using the taste test data from Example 1, create a frequency table with columns for the relative and cumulative frequencies. What percentage of the respondents gave the cola the highest rating? What percentage gave the cola one of the three lowest ratings?

TECHNICAL NOTE

A cumulative frequency divided by the total frequency, such as the 70% of respondents in the taste test giving a rating of 3 or higher, is called a *relative cumulative frequency*.

SOLUTION We find the relative frequencies by dividing the frequency in each category by the total frequency of 20. We find the cumulative frequencies by adding the frequency in each category to the sum of the frequencies in all preceding categories. Table 4 shows the results. The relative frequency column shows that 0.10, or 10%, of the respondents gave the cola the highest rating. The cumulative frequency column shows that 14 out of 20 people, or 70%, gave the cola a rating of 3 or lower.

TABLE 4 Relative and Cumulative Frequencies			
Taste scale	Frequency	Relative frequency	Cumulative frequency
1	2	$2/20 = 0.10$	2
2	3	$3/20 = 0.15$	$3 + 2 = 5$
3	9	$9/20 = 0.45$	$9 + 3 + 2 = 14$
4	4	$4/20 = 0.20$	$4 + 9 + 3 + 2 = 18$
5	2	$2/20 = 0.10$	$2 + 4 + 9 + 3 + 2 = 20$
Total	20	1	20

• • •

Binning Data

When we deal with quantitative data categories, it's often useful to group, or **bin**, the data into categories that cover a range of possible values. For example, in a table of income levels, it might be useful to create bins of \$0 to \$20,000, \$20,001 to \$40,000, and so on. In this case, the frequency of each bin is simply the number of people with incomes in that bin.



USING TECHNOLOGY—FREQUENCY TABLES

Excel Excel is easy to use for statistical tables and calculations. The following steps work with the essay grade data from Table 1. The screenshot on the left shows the Excel table with the formulas, and the one on the right shows the results of the formulas.

1. Create columns for the grade and frequency data, which you must type in manually (columns B and C below). At the bottom of column C, use the SUM function to compute the total frequency, which is entered in cell C8.
2. Compute the relative frequency (Column D) by dividing each frequency in Column C by the total frequency from cell C8. You can enter the formula for the first row ($=C3/\$C\8) and then use the “fill down” editing option to put the correct formulas in the remaining rows. Note: When using the “fill down” option, you must include the dollar signs in front of C and 8 to make the reference to cell C8 an “absolute cell reference.” Without these dollar signs, the “fill down” option would make the cell reference shift down (becoming C9, C10, etc.) in each row, which would be incorrect in this case.
3. Cumulative frequency (Column E) is the total of all the frequencies up to a given category. The first row shows “=C3” because cell C3 contains the frequency for A grades. The next row ($=E3+C4$) starts with the value in the prior row (cell E3) and add the frequency for B grades (cell C4). The pattern continues for the remaining rows, which you can fill with the “fill down” option.

A	B	C	D	E
	Grade	Frequency	Relative Frequency	Cumulative Frequency
3	A	4	=C3/\$C\$8	=C3
4	B	7	=C4/\$C\$8	=E3+C4
5	C	9	=C5/\$C\$8	=E4+C5
6	D	3	=C6/\$C\$8	=E5+C6
7	F	2	=C7/\$C\$8	=E6+C7
8	Total	=SUM(C3:C7)=SUM(D3:D7)	=C8	

A	B	C	D	E
1	Grade	Frequency	Relative Frequency	Cumulative Frequency
2	A	4	16%	11
3	B	7	28%	20
4	C	9	36%	23
5	D	3	12%	25
6	F	2	8%	25
7	Total	25	100%	25

EXAMPLE 3 Binned Exam Scores

Consider the following set of 20 scores from a 100-point exam:

76 80 78 76 94 75 98 77 84 88 81 72 91 72 74 86 79 88 72 75

Determine appropriate bins and make a frequency table. Include columns for relative and cumulative frequency, and interpret the cumulative frequency for this case.

SOLUTION The scores range from 72 to 98. One way to group the data is with 5-point bins. The first bin represents scores from 95 to 99, the second bin represents scores from 90 to 94, and so on. Note that there is no overlap between bins. We then count the frequency (the number of scores) in each bin. For example, only 1 score is in bin 95 to 99 (the high score of 98) and 2 scores are in bin 90 to 94 (the scores of 91 and 94). Table 5 shows the complete frequency table. In this case, we interpret the cumulative frequency of any bin to be the total number of scores in *or above* that bin. For example, the cumulative frequency of 6 for the bin 85 to 89 means that there were 6 scores of 85 or higher.

TABLE 5 Frequency Table for Binned Exam Scores

Scores	Frequency	Relative frequency	Cumulative frequency
95 to 99	1	$1/20 = 0.05$	1
90 to 94	2	$2/20 = 0.10$	$2 + 1 = 3$
85 to 89	3	$3/20 = 0.15$	$3 + 2 + 1 = 6$
80 to 84	3	$3/20 = 0.15$	$3 + 3 + 2 + 1 = 9$
75 to 79	7	$7/20 = 0.35$	$7 + 3 + 3 + 2 + 1 = 16$
70 to 74	4	$4/20 = 0.20$	$4 + 7 + 3 + 3 + 2 + 1 = 20$
Total	20	1	20

“Data! Data! Data!” he cried impatiently. “I can’t make bricks without clay.”

— Sherlock Holmes in Sir Arthur Conan Doyle’s *The Adventure of the Copper Beeches*



Section 1 Exercises

Statistical Literacy and Critical Thinking

- Frequency Table.** What is a frequency table? Explain what we mean by the categories (or classes) and frequencies.
- Relative Frequency.** A frequency table of grades has five classes (A, B, C, D, F) with frequencies of 4, 12, 16, 6, and 2. What are the relative frequencies of the five classes?
- Cumulative Frequency.** A frequency table of grades has five classes with frequencies of 4, 12, 16, 6, and 2. What are the corresponding cumulative frequencies of the five classes?
- Frequency Table.** The first class in a frequency table of incomes shows a frequency of 24 corresponding to the range of values from \$0 to \$999. Using only this information about the frequency table, is it possible to identify the original 24 sample values that are summarized by this class? Explain.

Does It Make Sense? For Exercises 5–8, decide whether the statement makes sense (or is clearly true) or does not make sense (or is clearly false). Explain clearly. Not all of these statements have definitive answers, so your explanation is more important than your chosen answer.

- Frequency Table.** A friend tells you that her frequency table has two columns labeled *State* and *Median Income*.
- Relative Frequency.** The relative frequency of category A in a table is 0.27 or 27%.
- Cumulative Frequency.** The cumulative frequency of a category in a table is 25.5.
- Bins.** For a given data set of IQ scores, if you increase the width of the bins, the number of bins decreases.

Concepts and Applications

- Frequency Table Practice.** Professor Diaz records the following final grades in one of her courses:

A A A B B B B B B B C C
C C C C C C C D D F F F F

Construct a frequency table for these grades. Include columns for relative frequency and cumulative frequency.

- Frequency Table Practice.** A guidebook for San Francisco lists 2 one-star restaurants, 12 two-star restaurants, 18 three-star restaurants, 6 four-star restaurants, and 2 five-star restaurants. Make a frequency table for these ratings. Include columns for relative frequency and cumulative frequency.

- Weights of Coke.** Construct a frequency table for the weights (in pounds) given below of 36 cans of regular Coke.

Start the first bin at 0.7900 pound and use a bin width of 0.0050 pound. Discuss your findings.

0.8192	0.8194	0.8211	0.8176
0.8062	0.8143	0.8110	0.8152
0.7901	0.8152	0.8079	0.8161
0.8161	0.8163	0.8194	0.8247
0.8165	0.8172	0.8150	0.8264
0.8207	0.8073	0.8295	0.8170
0.8150	0.8189	0.8181	0.8284
0.8128	0.8229	0.8251	0.8244
0.8244	0.8126	0.8044	0.8192

- Weights of Diet Coke.** Construct a frequency table for the weights (in pounds) given below of 36 cans of Diet Coke. Start the first bin at 0.7750 pound and use a bin width of 0.0050 pound. Discuss your findings.

0.7773	0.7874	0.7868	0.7802
0.7806	0.7907	0.7879	0.7833
0.7923	0.7910	0.7813	0.7859
0.7822	0.7896	0.7839	0.7861
0.7874	0.7852	0.7870	0.7826
0.7837	0.7872	0.7923	0.7760
0.7758	0.7822	0.7844	0.7892
0.7830	0.7771	0.7881	0.7822
0.7852	0.7879	0.7885	0.7811

- Oscar-Winning Actors.** The following data show the ages of all Academy Award-winning male actors at the time when they won their award, through 2012. Make a frequency table for the data, using bins of 20–29, 30–39, and so on. Discuss your findings.

44	41	62	52	41	34	52	41	37	38	34	32	40	43	
56	41	39	49	57	41	38	42	52	51	35	30	39	41	44
49	35	47	31	47	37	57	42	45	42	44	62	43	42	48
49	56	38	60	30	40	42	36	76	39	53	45	36	62	43
51	32	42	54	52	37	38	32	45	60	46	40	36	47	29
43	37	38	45	50	48	60	50	39						

- Body Temperatures.** The following data show the body temperatures (°F) of randomly selected subjects. Construct a frequency table with seven classes: 96.9–97.2, 97.3–97.6, 97.7–98.0, and so on.

98.6	98.6	98.0	98.0	99.0	98.4	98.4	98.4
98.4	98.6	98.6	98.8	98.6	97.0	97.0	98.8
97.6	97.7	98.8	98.0	98.0	98.3	98.5	97.3
98.7	97.4	98.9	98.6	99.5	97.5	97.3	97.6
98.2	99.6	98.7	99.4	98.2	98.0	98.6	98.6
97.2	98.4	98.6	98.2	98.0	97.8	98.0	98.4
98.6	98.6						

- 15. Missing Information.** The following table shows grades for a term paper in an English class. The table is incomplete. Use the information given to fill in the missing entries and complete the table.

Category	Frequency	Relative frequency
A	?	?
B	?	18%
C	?	24%
D	11	?
F	6	?
Total	50	?

- 16. Missing Information.** The following table shows grades for performances in a drama class. The table is incomplete. Use the information given to fill in the missing entries and complete the table.

Category	Frequency	Cumulative frequency
A	?	1
B	6	?
C	7	?
D	?	23
F	?	25
Total	?	?

- 17. Loaded Die.** One of the authors drilled a hole in a die, filled it with a lead weight, and then proceeded to roll it. The results are given in the following frequency table.

- According to the data, how many times was the die rolled?
- How many times was the outcome greater than 2?
- What percentage of outcomes were 6?
- List the relative frequencies that correspond to the given frequencies.
- List the cumulative frequencies that correspond to the given frequencies.

Outcome	Frequency
1	27
2	31
3	42
4	40
5	28
6	32

- 18. Interpreting Family Data.** Consider the following frequency table for the number of children in American families.

- According to the data, how many families are there in America?
- How many families have two or fewer children?
- What percentage of American families have no children?
- What percentage of American families have three or more children?

Number of children	Number of families (millions)
0	35.54
1	14.32
2	13.28
3	5.13
4 or more	1.97

- 19. Computer Keyboards.** The traditional keyboard configuration is called a *Qwerty* keyboard because of the positioning of the letters QWERTY on the top row of letters. Developed in 1872, the Qwerty configuration supposedly forced people to type slower so that the early typewriters would not jam. Developed in 1936, the Dvorak keyboard supposedly provides a more efficient arrangement by positioning the most used keys on the middle row (or “home” row), where they are more accessible.

A *Discover* magazine article suggested that you can measure the ease of typing by using this point rating system: Count each letter on the home row as 0, count each letter on the top row as 1, and count each letter on the bottom row as 2. For example, the word *statistics* would result in a rating of 7 on the Qwerty keyboard and 1 on the Dvorak keyboard, as shown below.

	S	T	A	T	I	S	T	I	C	S	
Qwerty keyboard	0	1	0	1	1	0	1	1	2	0	(sum = 7)
Dvorak keyboard	0	0	0	0	0	0	0	0	1	0	(sum = 1)

Using this rating system with each of the 52 words in the Preamble to the Constitution, we get the rating values below.

Qwerty Keyboard Word Ratings:

2	2	5	1	2	6	3	3	4	2	4	0	5
7	7	5	6	6	8	10	7	2	2	10	5	8
2	5	4	2	6	2	6	1	7	2	7	2	3
8	1	5	2	5	2	14	2	2	6	3	1	7

Dvorak Keyboard Word Ratings:

2	0	3	1	0	0	0	0	2	0	4	0	3
4	0	3	3	1	3	5	4	2	0	5	1	4
0	3	5	0	2	0	4	1	5	0	4	0	1
3	0	1	0	3	0	1	2	0	0	0	1	4

- Create a frequency table for the Qwerty word ratings data. Use bins of 0–2, 3–5, 6–8, 9–11, and 12–14. Include a column for relative frequency.
- Create a frequency table for the Dvorak word ratings data, using the same bins as in part a. Include a column for relative frequency.
- Based on your results from parts a and b, which keyboard arrangement is easier for typing? Explain.

20. Double Binning. The students in a statistics class conduct a transportation survey of students in their high school. Among other data, they record the age and mode of transportation between home and school for each student. The following table gives some of the data that were collected. For age: 1 = 14 years, 2 = 15 years, 3 = 16 years, 4 = 17 years, 5 = 18 years. For transportation: 1 = walk, 2 = school bus, 3 = public bus, 4 = drive, 5 = other.

Student	Age	Transportation	Student	Age	Transportation
1	1	1	11	3	5
2	5	1	12	5	5
3	2	2	13	1	2
4	3	5	14	5	5
5	4	3	15	5	5
6	1	1	16	4	4
7	5	2	17	2	2
8	2	1	18	3	1
9	3	4	19	3	3
10	1	3	20	1	4

- a. Classify the two variables, *age* and *transportation*, as qualitative or quantitative, and give the level of measurement for each.
- b. In order to be analyzed or displayed, the data must be binned with respect to both variables. Count the number of students in each of the 25 age/transportation categories and fill in the blank cells in the following table.

Age	Transportation					
		1	2	3	4	5
1						
2						
3						
4						
5						

PROJECTS FOR THE INTERNET & BEYOND

- 21. Energy Table.** The U.S. Energy Information Administration (EIA) Web site offers dozens of tables relating to energy use, energy prices, and pollution. Explore the selection of tables. Find a table of raw data that is of interest to you and convert it to an appropriate frequency table. Briefly discuss what you can learn from the frequency table that is less obvious in the raw data table.
- 22. Endangered Species.** The Web site for the World Conservation Monitoring Centre in Great Britain provides data on extinct, endangered, and threatened animal species. Explore these data and summarize some of your more interesting findings with frequency tables.

23. Navel Data. The *navel ratio* is defined to be a person's height divided by the height (from the floor) of his or her navel. An old theory says that, on average, the navel ratio of humans is the golden ratio: $(1 + \sqrt{5})/2$. Measure the navel ratio of each person in your class. What percentage of students have a navel ratio within 5% of the golden ratio? What percentage of students have a navel ratio within 10% of the golden ratio? Does the old theory seem reliable?

24. Your Own Frequency Table (Unbinned). Collect your own frequency data for some set of categories that will *not* require binning. (For example, you might collect data by asking friends to do a taste test on some brand of cookie.) State how you collected your data, and make a list of all your raw data. Then summarize the data in a frequency table. Include a column for relative frequency, and also include a column for cumulative frequency if it is meaningful.

25. Your Own Frequency Table (Binned). Collect your own frequency data for some set of categories that *will* require binning (for example, weights of your friends or scores on a recent exam). State how you collected your data, and make a list of all your raw data. Then summarize the data in a frequency table. Include columns for relative frequency and cumulative frequency.

IN THE NEWS

- 26. Frequency Tables.** Find a recent news article that includes some type of frequency table. Briefly describe the table and how it is useful to the news report. Do you think the table was constructed in the best possible way for the article? If so, why? If not, what would you have done differently?
- 27. Relative Frequencies.** Find a recent news article that gives at least some data in the form of relative frequencies. Briefly describe the data, and discuss why relative frequencies were useful in this case.
- 28. Cumulative Frequencies.** Find a recent news article that gives at least some data in the form of cumulative frequencies. Briefly describe the data, and discuss why cumulative frequencies were useful in this case.
- 29. Temperature Data.** Look for a weather report that lists yesterday's high temperatures in many American cities. Choosing appropriate bins, make a frequency table for the high temperature data. Include columns for relative frequency and cumulative frequency. Briefly describe how and why you chose your bins.

2

PICTURING DISTRIBUTIONS OF DATA

A frequency table shows how a variable is distributed over chosen categories, so we say that it summarizes the **distribution** of data. We can often gain deeper insight into a distribution with a picture or graph. In this section, we'll study a few of the most common ways of visualizing data distributions.

Definition

The **distribution** of a variable refers to the way its values are spread over all possible values. We can summarize a distribution with a table or a graph.

Bar Graphs, Dotplots, and Pareto Charts

A **bar graph** uses a set of bars to represent the frequency (or relative frequency) of each category: the higher the frequency, the longer the bar. The bars can be either vertical or horizontal. Figure 1 shows a vertical bar graph based on the essay grade data in Table 1; there are five bars because there are five data categories (the grades A, B, C, D, F). Several key features help make the graph look good:

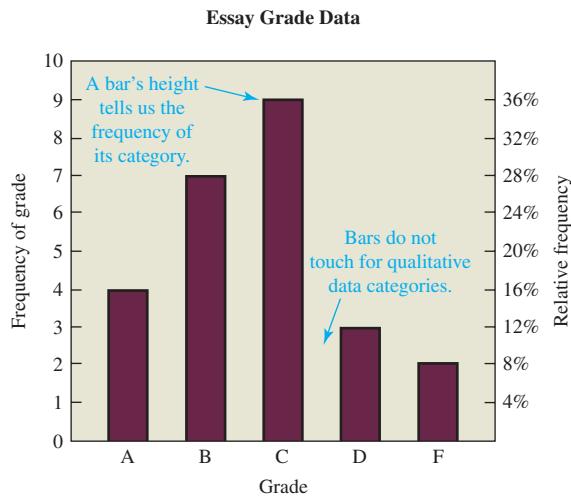


Figure 1 Bar graph for the essay grade data in Table 1.

- Because the highest frequency is 9 (for C grades), running the vertical scale from 0 to 10 allows all the bars to fit well.
- The height of each bar is proportional to its frequency. The graph is easy to read because we chose a total height of 5 centimeters, so that each centimeter of height corresponds to a frequency of 2.
- Because the data are qualitative, the bars do not need to touch. The widths of the bars have no special meaning. We therefore draw them with uniform widths.
- The graph is clearly labeled, in accord with the rules in the following summary.

Important Labels for Graphs

Title/caption: The graph should have a title or caption (or both) that explains what is being shown and, if applicable, lists the source of the data.

Vertical scale and label: Numbers along the vertical axis should clearly indicate the scale and line up with the *tick marks*—marks along the axis that precisely locate the numerical values. Include an axis title that describes the variable that the numbers represent.

Horizontal scale and label: The categories should be clearly indicated along the horizontal axis; tick marks are not necessary for qualitative data but should be used with quantitative data. Include an axis title that describes the variable that the categories represent.

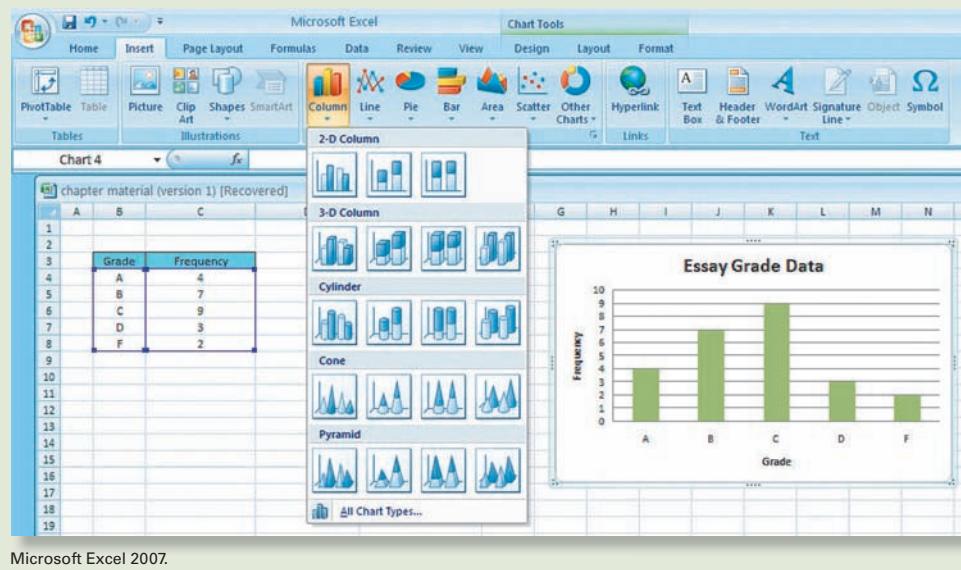
Legend: If multiple data sets are displayed on a single graph, include a legend or key to identify the individual data sets.



USING TECHNOLOGY—BAR GRAPHS

Excel Excel can make many types of statistical graphs. Let's start with a bar graph using the grade data in Table 1. The basic process is as follows, though the details will vary with different versions of Excel.

1. Starting with the frequency table created in the Using Technology box, select the letter grades (column B) and the frequencies (column C).
2. Choose the type of chart that you want from the Insert menu; here, we choose a 2-D “column” chart, which is Excel's name for a bar graph. The screen shot below shows the process and result.
3. You can customize the labels on the bar graph. The procedure differs for different versions of Excel, but in most versions, a right-click allows you to change the axes and other labels; some versions offer dialog boxes for changing the labels.



A **dotplot** is a variation on a bar graph in which we use dots rather than bars to represent the frequencies. Each dot represents one data value; for example, a stack of 4 dots means a frequency of 4. Figure 2 shows a dotplot for the essay data set. Dotplots are convenient when making graphs of raw data by hand, because you can tally the data by making a dot for each data value. You may then convert the graph to a bar chart for a formal report.

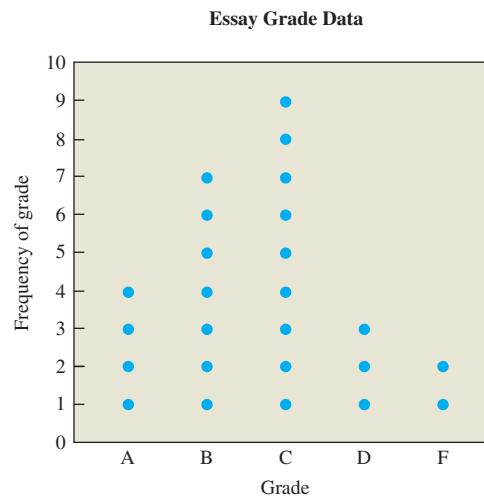


Figure 2 Dotplot for the essay grade data in Table 1.

A **Pareto chart** is a bar graph in which the bars are arranged in frequency order. This can be useful for data at the nominal level of measurement. For example, Figure 3 shows both a bar chart (with the cities in alphabetical order) and a Pareto chart for the populations of the five



Figure 3 (a) Bar graph showing populations for the five largest cities in the United States (2010 census). (b) Pareto chart for the same data.

Source: U.S. Census Bureau.

largest cities in the United States. Although the only difference is in the order of the bars, most people find the Pareto chart easier to study. Note that Pareto charts do not work for data at the ordinal level of measurement or higher; for example, it would not make sense to make a Pareto chart for the essay grade data, because the grades (A, B, C, D, F) already have a natural order.

Definitions

A **bar graph** consists of bars representing frequencies (or relative frequencies) for particular categories. The bar lengths are proportional to the frequencies.

A **dotplot** is similar to a bar graph, except each individual data value is represented with a dot.

A **Pareto chart** is a bar graph with the bars arranged in frequency order. Pareto charts make sense only for data at the nominal level of measurement.

BY THE WAY

Pareto charts were invented by Italian economist Vilfredo Pareto (1848–1923). Pareto is best known for developing methods of analyzing income distributions, but his most important contributions probably were in developing new ways of applying mathematics and statistics to economic analysis.

TIME OUT TO THINK

Would it be practical to make a dotplot for the population data in Figure 3? Would it make sense to make a Pareto chart for data concerning SAT scores? Explain.

EXAMPLE 1 Carbon Dioxide Emissions

Carbon dioxide (CO_2) is released into the atmosphere by the combustion of fossil fuels (oil, coal, natural gas). Table 6 lists the eight countries that emit the most carbon dioxide each year. Construct Pareto charts for the total emissions and for the average emissions per person. Why do the two charts appear to be so different?

TABLE 6 The World's Eight Leading Emitters of Carbon Dioxide

Country/region	Total CO_2 emissions (millions of metric tons)	Per person CO_2 emissions (metric tons)
United States	5,833	19.18
China	6,534	4.91
Russia	1,729	12.29
Japan	1,495	1.31
India	1,214	9.54
Germany	829	10.06
Canada	574	17.27
United Kingdom	572	9.38

Source: U.S. Department of Energy, Annual Energy Outlook 2011 (provides data for 2008 emissions).

SOLUTION The categories are the countries and the frequencies are the data values. The total emissions are given in units of “millions of metric tons” and the highest value in these units is 6,534; therefore, a range of 0 to 7,000 makes a good choice for the vertical scale. Table 6 already lists the total emissions in descending order, so this is the order we use for the Pareto chart in Figure 4a. The per person emissions are given in metric tons, and the highest value is 19.18 for the United States; therefore, a range of 0 to 20 works well. To make the Pareto chart in Figure 4b, we put the tallest bar (for the U.S.) at the left and continue in descending order.

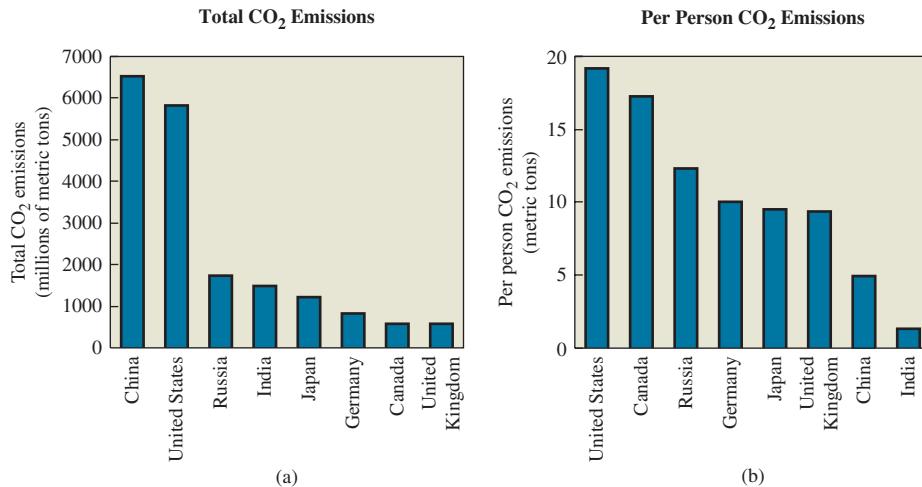


Figure 4 Pareto charts for (a) carbon dioxide emissions by country and (b) per person carbon dioxide emissions.

The fact that the two Pareto charts have the countries in different orders tells us that the biggest total emitters of carbon dioxide are not necessarily the biggest per person emitters. In particular, although China has the largest total emissions, the emissions per person are far higher in the United States and Canada.



TIME OUT TO THINK

Most people around the world aspire to a standard of living like that in the United States. Suppose that to achieve this, the rest of the world's per person carbon dioxide emissions rose to the same level as that in the United States. What consequences might this have for the world? Defend your opinions.

Pie Charts

Pie charts are commonly used to show relative frequency distributions. The entire pie represents the total relative frequency of 100%, so the sizes of the individual slices, or wedges, represent the relative frequencies of the various categories. As a simple example, Figure 5 shows a pie chart for the essay grade data. Note that the size of each wedge reflects the relative frequencies that we found in Table 3. To make comparisons easier, the relative frequencies (as percentages) are written on the wedges.

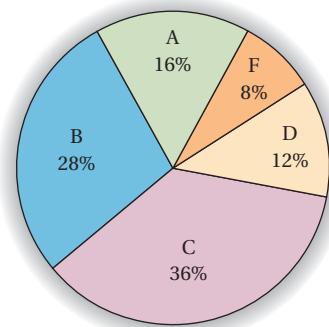


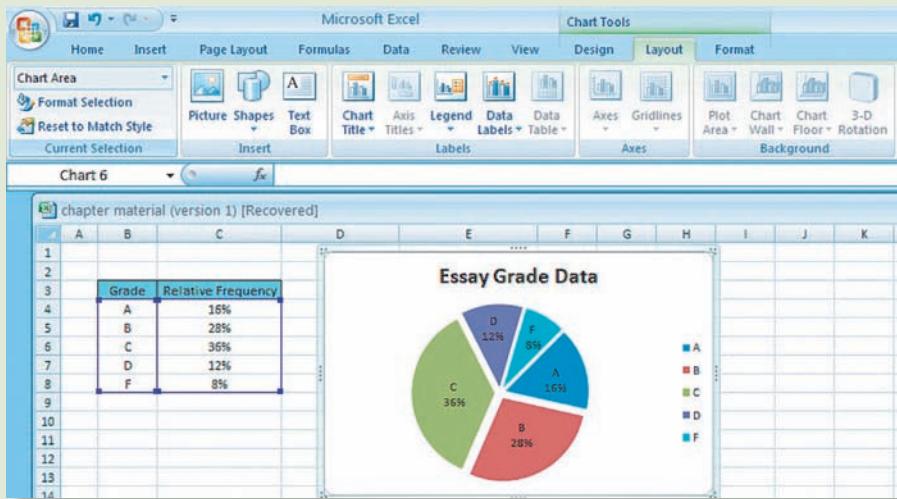
Figure 5 Pie chart for the essay grade data in Tables 1 and 3.



USING TECHNOLOGY—PIE CHARTS

Excel Making a pie chart in Excel is very similar to making a bar graph, except:

- For a pie chart, you will probably want to select the relative frequencies rather than the frequencies (though both will work); it may be helpful to cut and paste these data so they are next to the grade letters.
- Choose a pie chart rather than a column chart from the Insert menu.
- The labeling process is different from that for bar graphs, as are the options for colors and other decorative features. The accompanying screen display shows one set of options used in Excel for Windows; you should experiment with other options to learn about the possibilities.



Microsoft Excel 2007.

Definition

A **pie chart** is a circle divided so that each wedge represents the *relative frequency* of a particular category. The wedge size is proportional to the relative frequency. The entire pie represents the total relative frequency of 100%.

EXAMPLE 2 Simple Pie Chart

The registered voters of Rochester County are 25% Democrats, 25% Republicans, and 50% independents. Construct a pie chart to represent the party affiliations.

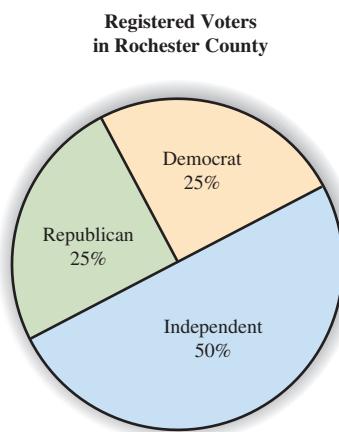


Figure 6 Pie chart for Example 2.

SOLUTION Because Democrats and Republicans each represent 25% of the voters, the wedges for Republicans and Democrats each occupy 25%, or one-fourth, of the pie. Independents represent half of the voters, so their wedge occupies the remaining half of the pie. Figure 6 shows the result. As always, note the importance of clear labeling.



TECHNICAL NOTE

Different books define the terms *histogram* and *bar graph* differently, and there are no universally accepted definitions. In this text, a bar graph is any graph that uses bars, and histograms are the types of bar graphs used for quantitative data categories.

Histograms and Line Charts

As we've seen, bar graphs and pie charts are used primarily for data in which the categories are qualitative, such as letter grades or countries. For quantitative data categories, the two most common types of graphics are *histograms* and *line charts*.

A **histogram** is essentially a bar graph in which the data categories are quantitative. The bars in a histogram must follow the natural order of the numerical categories. The widths of the bars must be equal, and they must have a specific meaning. For example, Figure 7a shows a histogram for the binned exam data of Table 5. Notice that the width of each bar represents 5 points on the exam. The bars in the histogram touch each other because there are no gaps between the categories.

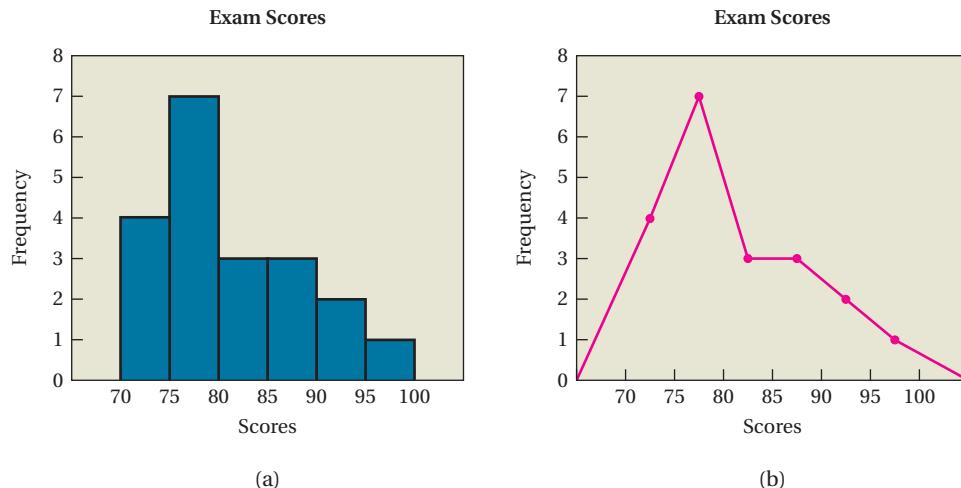


Figure 7 The numerical grade data from Table 5 shown as (a) a histogram and (b) a line chart.

TECHNICAL NOTE

A line chart created from a set of frequency data is often called a *frequency polygon* because it consists of many straight line segments that take the shape of a many-sided figure.

Definitions

A **histogram** is a bar graph showing a distribution for quantitative data (at the interval or ratio level of measurement); the bars have a natural order and the bar widths have specific meaning.

A **line chart** shows a distribution of quantitative data as a series of dots connected by lines. For each dot, the horizontal position is the *center* of the bin it represents and the vertical position is the frequency value for the bin.

EXAMPLE 3 Oscar-Winning Actresses

The following data show the ages (at the time when they won the award) of all Academy Award-winning actresses through 2012, sorted into age order. Display the data using 10-year bins. Discuss the results.

Ages of Actresses at Time of Academy Award (through 2012)

21	22	24	24	25	25	25	25	26	26	26
26	26	27	27	27	27	28	28	28	28	29
29	29	29	29	29	29	29	30	30	30	31
31	31	32	32	32	32	33	33	33	33	33
33	34	34	34	35	35	35	35	35	36	36
37	37	38	38	38	38	39	39	40	41	41
41	41	41	42	42	45	45	48	49	49	54
60	61	61	61	62	63	74	80			

Source: Academy of Motion Picture Arts and Sciences.

SOLUTION The first step is to bin the data. If you make a tally using 10-year bins of ages 20–29, 30–39, and so on, you'll find the frequencies shown in Table 7. Because the data are quantitative, either a histogram or line chart can be made to show the frequencies. Figure 8 shows both, with the line chart overlaying the histogram. Note that the vertical axis spans the range 0 to 40, which easily fits the largest frequency of 34 (for ages 30–39), and the dots for the line chart are placed at the center of each bin.

The data show that most actresses win the award at a fairly young age, which stands in contrast to the older ages of most male winners of Best Actor (see Exercise 13 in Section 1). Many actresses believe this difference arises because Hollywood producers rarely make movies that feature older women in strong character roles.

TABLE 7		Frequency table for Ages of Actresses at Time of Academy Award (through 2012)	
Age		Number of actresses	
20–29		29	
30–39		34	
40–49		13	
50–59		1	
60–69		6	
70–79		1	
80–89		1	

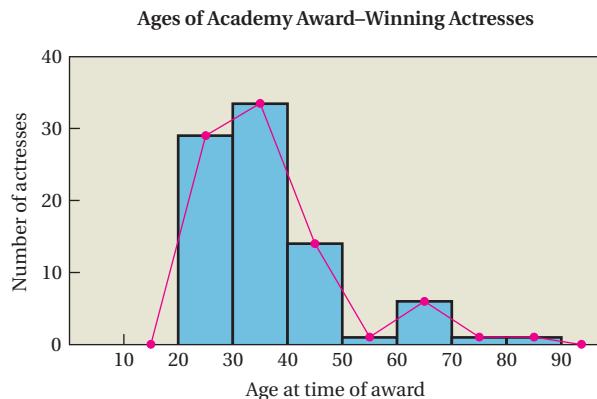


Figure 8 Histogram and line chart (overlaying the histogram) for the ages of Academy Award-winning actresses.

BY THE WAY

The oldest Oscar-winning actress was Jessica Tandy, who won in 1990 for the film *Driving Miss Daisy*.



Variations on the Theme

There are many variations of histograms and line charts, but we'll discuss just two here. A histogram or line chart for which the data categories on the horizontal axis are time intervals is called a **time-series graph**.

Another common variation, at least among statisticians if not the media, is called a **stemplot** (or stem-and-leaf plot). A stemplot looks somewhat like a histogram turned sideways, except in place of bars we see a listing of the raw data values. Stemplots can be made in several ways, but one common way is to separate the data values into two parts: the stem (such as the leftmost digit) and the leaf (such as the rightmost digit). The following example shows the process.

Definitions

A histogram or line chart in which the horizontal axis represents *time* is called a **time-series graph**.

A **stemplot** (or *stem-and-leaf plot*) is somewhat like a histogram turned sideways, except in place of bars we see a listing of data.

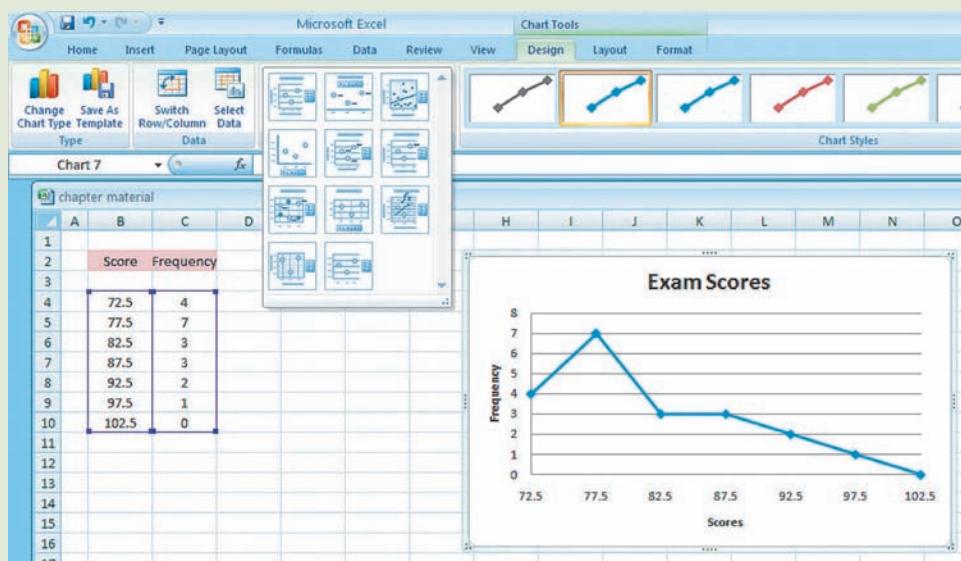


USING TECHNOLOGY—LINE CHARTS AND HISTOGRAMS

Excel Line charts are easy to create in Excel. The screen shot below shows the process for the binned data from Table 5. Follow these steps:

1. To get the dots in the centers of the bins for the scores, enter the center point of each bin in Column B. Then enter the frequencies in Column C.
2. Select the scores and frequencies, then choose the chart type “scatter” but with the option for connecting points with straight lines. You will get the graph as shown.
3. Use the chart options to improve design, labels, and more.

Note: You can also create a line chart with the “line” chart option in Excel. In that case, select *only* the frequencies when you begin the graphing process; then, in the “source data” dialog box, choose “series” and select the scores (Column B) as the “X values.” The resulting graph should look the same as that created with the “scatter” option, except the data points will not have dots.



Microsoft Excel 2007.

Excel can also generate histograms with the use of add-ins, including the Data Analysis add-in that can be installed with some versions of Excel.

STATDISK To create a histogram in STATDISK, enter or open data in the STATDISK Data Window, click **Data**, click **Histogram**, and then click on the **Plot** button. To use your own settings, click on the “User defined” button before clicking on Plot. Click on the **Turn labels on** button to see the frequency for each class.

TI-83/84 PLUS To create a histogram using a TI-83/84 Plus calculator, enter or a list of data in L1 or use a list of values assigned to a name. Select the **STAT PLOT** function by pressing **2ND** **Y=**. Press **ENTER** and use the arrow keys to turn Plot1 to “On” and select the graph with bars. The screen display should be as shown here.

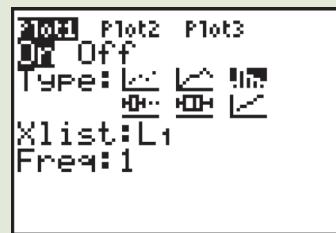


Image used with permission of Texas Instruments, Inc.

If you want to let the calculator determine the settings, press **ZOOM** **9** to get a histogram with default settings. (To use your own settings, press **WINDOW** and enter the maximum and minimum values. Press **GRAPH** to obtain the graph.)

EXAMPLE 4 Stemplot for Oscar-Winning Actresses

Make a stemplot for the data from Example 3. Discuss the pros and cons of the stemplot over a histogram.

SOLUTION The data are the ages of the actresses (at time of award), which range from 21 to 80. We therefore choose to use the tens column of the ages as the stems and the ones column as the leaves. Because the ages range from the 20s to the 80s, the stem values range from 2 to 8; we put these on the left side of the stemplot shown in Figure 9. On the right, we place all the ones values that go with each tens value. For example, the first row of data from Example 3 shows the following ages:

21 22 24 24 25 25 25 25 26 26 26

All of these ages are in the 20s, so they all have the same stem of “2” in Figure 9. We then place each of these ages as one of the leaves on the right, which is why the first set of the leaves begins 1, 2, 4, 4, 5, 5, 5, 5, 6, 6, 6. You can continue through the data from Example 3 to see how all the leaves were created. Notice, for example, that there is only one age in the 50s, and it is 54; that is why the stem 5 (which represents the 50s) has only a single leave with value 4.

The primary advantage of the stemplot over the histogram from Figure 8 is that the stemplot actually contains all of the original data. For example, the histogram shows only that there were 6 actresses with ages from 60–69; in contrast, by looking at the leaves that go with the stem “6” in the stemplot, you can see that the actual ages of these six actresses were 60, 61, 61, 61, 62, and 63. The primary drawback to the stemplot is that the extra information it contains makes it more visually complex than a histogram.

Stem (tens)	Leaves (ones)
2	1 2 4 4 5 5 5 6 6 6 6 6 7 7 7 7 8 8 8 8 9 9 9 9 9 9 9 9
3	0 0 0 1 1 1 2 2 2 2 3 3 3 3 3 4 4 4 5 5 5 5 6 6 7 7 8 8 8 8 9
4	0 1 1 1 1 1 2 2 5 5 8 9 9
5	4
6	0 1 1 1 2 3
7	7
8	0

Figure 9 Stemplot for the ages of Academy Award-winning actresses.

• •

EXAMPLE 5 Interpreting a Time-Series Graph

Figure 10 shows a time-series graph for the death rate (deaths per 1,000 people) in the United States since 1900. (For example, the 1905 death rate of 16 means that, for each 1,000 people living at the beginning of 1905, 16 people died during the year.) Discuss the general trend. Also consider the spike in 1918: If someone told you that this spike was due to battlefield deaths in World War I, would you believe it? Explain.

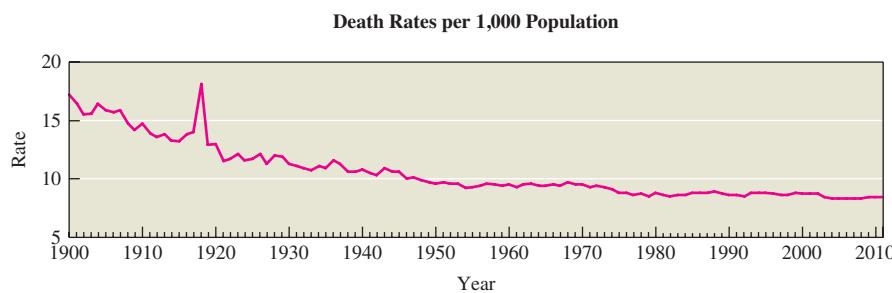


Figure 10 Historical U.S. death rates per 1,000 people.

Source: National Center for Health Statistics.

SOLUTION The general trend in death rates is clearly downward, presumably because of improvements in medical science. For example, bacterial diseases such as pneumonia were major killers in the early 1900s, but are largely curable with antibiotics today. Regarding the

spike in 1918: Although it coincides with the end of World War I, if the war were the cause of this spike, then we might expect to see a similar spike during World War II, but we don't. This suggests that there must have been some other reason for the spike. In fact, the spike reflects the effects of the deadly flu pandemic of 1918, which killed 850,000 people in the United States and an estimated 20 million people worldwide.

• • •

TIME OUT TO THINK

Do you get annual flu shots? Does knowing the tremendous impact of the 1918 flu epidemic affect your opinion as to the value of flu shots? Defend your opinion.

Section 2 Exercises

Statistical Literacy and Critical Thinking

- 1. Distribution.** What do we mean by the distribution of data?
- 2. Visualizing Data.** How is a histogram or line chart more helpful than a list of sample values for understanding a data distribution?
- 3. Pareto Chart and Pie Chart.** What is an important advantage of a Pareto chart over a pie chart?
- 4. Histogram and Stemplot.** Assume that a data set is used to construct a histogram and a stemplot. Using only the histogram, is it possible to re-create the original list of data values? Using only the stemplot, is it possible to re-create the original list of data values? What is an advantage of a stemplot over a histogram?

Does It Make Sense? For Exercises 5–8, decide whether the statement makes sense (or is clearly true) or does not make sense (or is clearly false). Explain clearly. Not all of these statements have definitive answers, so your explanation is more important than your chosen answer.

- 5. Histogram.** I made a histogram to depict frequency counts of answers to the question "What political party do you belong to?"
- 6. Pie Chart.** I used a pie chart to illustrate a data set showing how the cost of college has changed over time.
- 7. Pareto Chart.** A quality control engineer wants to draw attention to the most serious causes of defects, so she

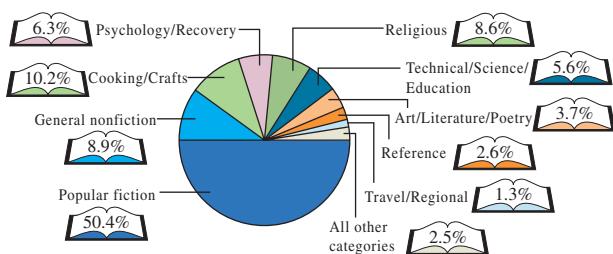


Figure 11 Source: Wall Street Journal Almanac, based on data from the Book Industry Study Group.

uses a Pareto chart to illustrate the frequencies of the different causes of defects.

- 8. Peak Values.** I made both a histogram and a line chart for a data set of the crime rate (number of crimes committed) each year from 1960 to 2010. The histogram showed that the crime rate peaked in 1982, while the line chart showed that it peaked in 1983.

Concepts and Applications

Most Appropriate Display. Exercises 9–12 describe data sets but do not give actual data. For each data set, state the type of graphic that you believe would be most appropriate for displaying the data, if they were available. Explain your choice.

- 9. Incomes.** Incomes of college graduates who took a statistics course
- 10. Political Party.** The political party affiliations of 1000 survey subjects
- 11. Movie Theaters.** Number of movie theatres for each year since 1960
- 12. Ages of Crash Victims.** Ages of people who died in car crashes last year
- 13. What People Are Reading.** The pie chart in Figure 11 shows the results of a survey about what people are reading.
 - a.** Summarize these data in a table of relative frequencies.
 - b.** Construct a Pareto chart for these data.
 - c.** Which do you think is a better representation of the data: the pie chart or the Pareto chart? Why?

14. Histogram. The histogram in Figure 12 depicts cotinine levels (in milligrams per milliliter) of a sample of subjects who smoke cigarettes. Cotinine is a metabolite of nicotine, which means that cotinine is produced by the body when nicotine is absorbed. The data are from the Third National Health and Nutrition Examination Survey.

- How many subjects are represented in the histogram?
- How many subjects have cotinine levels below 400?
- How many subjects have cotinine levels above 150?
- What is the highest possible cotinine level of a subject represented in this histogram?

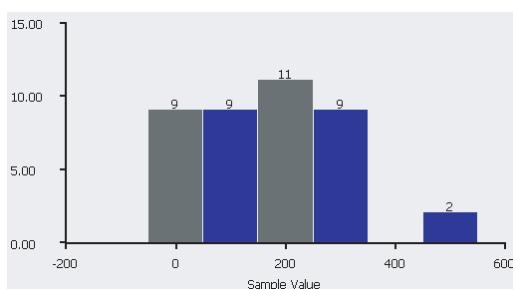


Figure 12

15. Weights of Coke. Exercise 11 in Section 1 required the construction of a frequency table from the weights (in pounds) of 36 cans of regular Coke. Use that frequency table to construct the corresponding histogram.

16. Weights of Diet Coke. Exercise 12 in Section 1 required the construction of a frequency table for the weights (in pounds) of 36 cans of Diet Coke. Use that frequency table to construct the corresponding histogram.

17. Oscar-Winning Actors. Exercise 13 in Section 1 required the construction of a frequency table for the ages of recent Academy Award-winning male actors at the time when they won their award. Use that frequency table to construct the corresponding histogram.

18. Body Temperatures. Exercise 14 in Section 1 required the construction of a frequency table for a list of body temperatures ($^{\circ}\text{F}$) of randomly selected subjects. Use that frequency table to construct the corresponding histogram.

19. Job Hunting. A survey was conducted to determine how employees found their jobs. The table below lists the successful methods identified by 400 randomly selected employees. The data are based on results from the National Center for Career Strategies. Construct a Pareto chart that corresponds to the given data. Based on these results, what appears to be the best strategy for someone seeking employment?

Job sources of survey respondents	Frequency
Help-wanted ads	56
Executive search firms	44
Networking	280
Mass mailing	20

20. Job Sources. Refer to the data given in Exercise 19, and construct a pie chart. Compare the pie chart to the Pareto chart. Can you determine which graph is more effective in showing the relative importance of job sources?

21. Job Application Mistakes Chief financial officers of U.S. companies were surveyed about areas in which job applicants make mistakes. Here are the areas and the frequency of responses: interview (452); resume (297); cover letter (141); reference checks (143); interview follow-up (113); screening call (85). These results are based on data from Robert Half Finance and Accounting. Construct a pie chart representing the given data.

22. Job Application Mistakes Construct a Pareto chart of the data given in Exercise 21. Compare the Pareto chart to the pie chart. Which graph is more effective in showing the relative importance of the mistakes made by job applicants?

23. Dotplot. Refer to the QWERTY data in Exercise 19 in Section 1 and construct a dotplot.

24. Dotplot. Refer to the Dvorak data in Exercise 19 in Section 1 and construct a dotplot. Compare the result to the dotplot in Exercise 23. Based on the results, does either keyboard configuration appear to be better? Explain.

25. Time-Series Graph for Cell Phone Subscriptions. The following table shows the numbers of cell phone subscriptions (in thousands) in the United States for various years. Construct a time-series graph of the data. “Linear” growth would result in a graph that is approximately a straight line. Does the time-series graph appear to show linear growth?

Year	Number	Year	Number
1986	682	2000	109478
1988	2069	2002	140767
1990	5283	2004	182140
1992	11033	2006	233000
1994	24134	2008	262700
1996	44043	2010	302900
1998	69209		

26. Time Series for Motor Vehicle Deaths. The following values are numbers of motor vehicle deaths in the United States for years beginning with 1980. The data are arranged in order by row. Construct a time-series graph and then determine whether there appears to be a trend. If so, provide a possible explanation.

51,091	49,301	43,945	42,589	44,257	43,825
46,087	46,390	47,087	45,582	44,599	41,508
39,250	40,150	40,716	41,817	42,065	42,013
41,501	41,717	41,945	42,196	43,005	42,884
42,836	43,443	42,642	41,059	37,261	33,808
			32,708		

27. Stemplot. Construct a stemplot of these test scores: 67, 72, 85, 75, 89, 89, 88, 90, 99, 100. How does the stemplot show the distribution of these data?

- 28. Stemplot.** Listed below are the lengths (in minutes) of animated children's movies. Construct a stemplot. Does the stemplot show the distribution of the data? If so, how?

83	88	120	64	69	71	76	74	75	76	75
75	79	80	78	78	83	77	71	83	80	73
72	82	74	84	90	89	81	81	90	79	92
82	89	82	74	86	76	81	75	75	77	70
75	64	73	74	71	94					



PROJECTS FOR THE INTERNET & BEYOND

- 29. CO₂ Emissions.** Look for updated data on international carbon dioxide emissions. Create a graph of the latest data and discuss any important features or trends that you notice.
- 30. Energy Table.** Explore the energy tables at the U.S. Energy Information Administration (EIA) Web site. Choose a table that you find interesting and make a graph of its data. You may choose any of the graph types discussed in this section. Explain how you made your graph, and briefly discuss what can be learned from it.
- 31. Statistical Abstract.** Go to the Web site for the *Statistical Abstract of the United States*. Explore the selection of “frequently requested tables.” Choose one table of interest to you and make a graph from its data. You may choose any of the graph types discussed in this section. Explain how you made your graph and briefly discuss what can be learned from it.
- 32. Navel Data.** Create an appropriate display of the navel data collected in Exercise 23 of Section 1. Discuss any special properties of this distribution.

IN THE NEWS

- 33. Bar Graphs.** Find a recent news article that includes a bar graph with qualitative data categories.
- Briefly explain what the bar graph shows, and discuss whether it helps make the point of the news article. Are the labels clear?
 - Briefly discuss whether the bar graph could be recast as a dotplot.
 - Is the bar graph already a Pareto chart? If so, explain why you think it was drawn this way. If not, do you think it would be clearer if the bars were rearranged to make a Pareto chart? Explain.
- 34. Pie Charts.** Find a recent news article that includes a pie chart. Briefly discuss the effectiveness of the pie chart. For example, would it be better if the data were displayed in a bar graph rather than a pie chart? Could the pie chart be improved in other ways?
- 35. Histograms.** Find a recent news article that includes a histogram. Briefly explain what the histogram shows, and discuss whether it helps make the point of the news article. Are the labels clear? Is the histogram a time-series diagram? Explain.
- 36. Line Charts.** Find a recent news article that includes a line chart. Briefly explain what the line chart shows, and discuss whether it helps make the point of the news article. Are the labels clear? Is the line chart a time-series diagram? Explain.

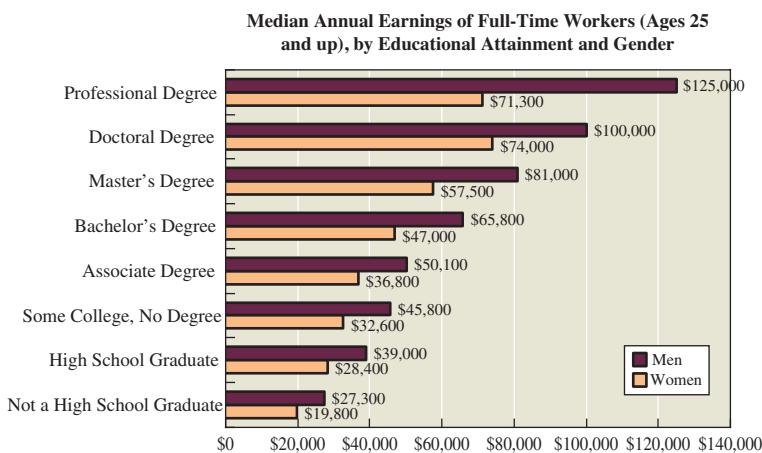
3 GRAPHICS IN THE MEDIA

The basic graphs we have studied so far are only the beginning of the many ways to depict data visually. In this section, we will explore some of the more complex types of graphics that are common in the media.

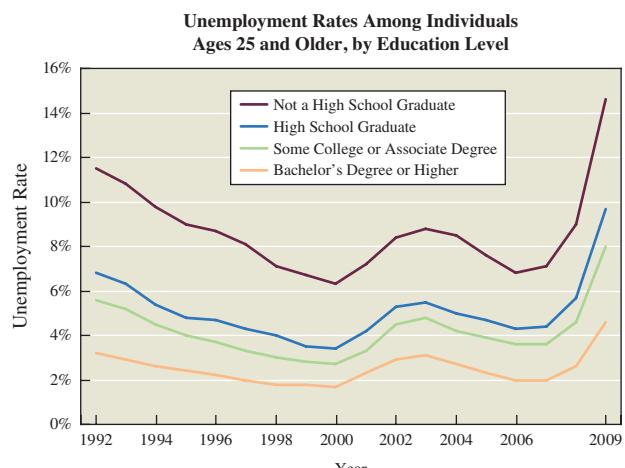
Multiple Bar Graphs and Line Charts

A **multiple bar graph** is a simple extension of a regular bar graph: It has two or more sets of bars that allow comparison between two or more data sets. All the data sets must have the same categories so that they can be displayed on the same graph. Figure 13 is a multiple bar graph with two sets of bars, one for men and one for women. (The median is a type of average.)

Notice that the data categories (the different levels of educational attainment by gender) in Figure 13 are qualitative, which makes a bar chart the best choice for display. In cases for which data categories are quantitative, a **multiple line chart** is often a better choice. Figure 14 shows time-series data using four different lines for four different data sets. The multiple line chart works because all four data sets represent the same pair of variables: time and unemployment rate. They differ only in the groups for which the data are plotted, with each line representing data for a different level of educational attainment.

**Figure 13** A multiple bar graph.

Source: The College Board, *Trends in Higher Education 2010*, based on 2008 data from the U.S. Census Bureau.

**Figure 14** A multiple line chart.

Source: The College Board, *Trends in Higher Education 2010*, based on data through 2009 from the U.S. Bureau of Labor Statistics.

EXAMPLE 1 Education Pays

Study Figures 13 and 14. What general messages do they reveal? Comment on how the use of the multiple bar and line graphics helps convey these messages.

SOLUTION Figure 13 conveys two clear messages. First, by looking at the bars across all the categories, we see clearly that people with greater education have significantly higher median incomes. For example, the median earnings for men with a bachelor's degree are more than \$25,000 higher than for men without a college education. This means that over a typical 40-year career (age 25 to 65), a college education is worth an extra *\$1 million* in income, at least on average. The second message conveyed by the graph is that for equivalent levels of educational attainment, women still earn much less than men.

Figure 14 shows another added value of education: no matter what the unemployment rate (at least over the time period shown), unemployment has always been significantly lower for more highly educated people.

The graphic choices work well because of the easy comparisons they allow. For example, if the bar graphs for men and women were shown separately, it would be much more difficult to see the fact that women earn less than men with the same education. Similarly, if the unemployment line charts were shown separately, our eyes would be drawn more to the trends with time than to the more important differences in the unemployment rates for people with different levels of education.



TIME OUT TO THINK

Together, Figures 13 and 14 make clear that education can be very valuable financially because it can easily raise your lifetime earnings by \$1 million while dramatically reducing your risk of unemployment. Discuss whether these facts should alter the way you approach your own education or public policies regarding higher education.

BY THE WAY

More detailed data show that median earnings are higher for college graduates than non-graduates in nearly every field of work. However, the variations are quite significant. For example, students majoring in technical fields such as science or engineering generally earn more than students with other majors, and for any particular major, students who study more and get better grades tend to earn more than students who study less.

Stack Plots

Another way to show two or more related data sets simultaneously is with a **stack plot**, which shows different data sets in a vertical stack. Data can be stacked in both bar charts and line charts.

Figure 15 shows a stack plot of federal government spending categories as a percentage of the total, with data projected through 2016. Note that government spending is broken into five broad categories, with each category represented by a wedge in the figure. The

BY THE WAY

The “all other” category in Figure 15 might sound unimportant, but it actually includes nearly every government program you’ve ever heard of, including all programs that support education, roads and transportation, air traffic control and safety, homeland security, science, the space program, the justice system, international affairs, consumer safety, environmental protection, and much more.

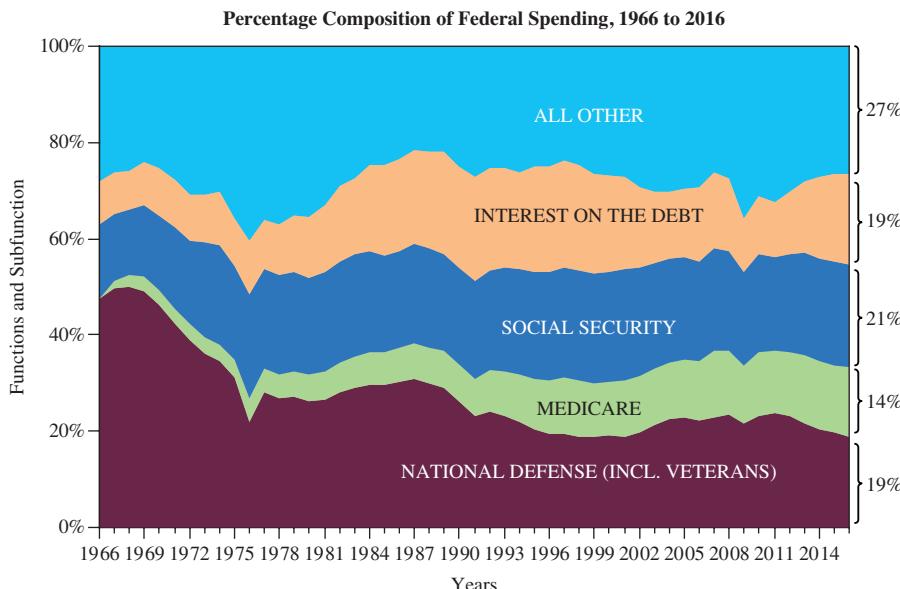


Figure 15 A stack plot for U.S. federal government spending by category as a percentage of total spending. Based on actual data through 2010, and projections made in 2010 for the years 2011 through 2016.

Source: U.S. Office of Management and Budget, Historical Tables published in 2011.

thickness of a wedge at a particular time tells you its value at that time. For example, the fact that the wedge for National Defense (which, in this chart, includes spending on Veterans) falls from about 48% in 1966 to about 19% in 2016 tells us that the percentage of the federal budget that goes to defense has declined dramatically in recent decades. The graph also makes clear that the trend over time is for Social Security, Medicare, and interest on the debt to make up a rapidly growing percentage of overall spending.

EXAMPLE 2 Interpreting the Stack Plot

As you are undoubtedly aware, the government currently spends much more money than it takes in; in fact, in recent years the revenue has been about one-third less than the spending. Therefore, balancing the budget requires either a dramatic reduction in spending, a substantial increase in tax revenue, or some combination of both. Suppose that you wanted to balance the budget with spending cuts only. How could you do it?

SOLUTION Achieving a balanced budget through spending cuts alone would require cutting total spending by about one-third, which essentially means finding a way to take 33% out of the 2016 values shown at the right. In principle, there are many ways we could do this, but in practice there are difficult constraints. Interest on the debt is an obligation that cannot be reduced without a government default, and a default could potentially cripple the economy. In addition, many conservatives feel strongly that defense should not be cut any further, while many liberals tend to believe that there should be no cuts in Social Security and Medicare. Those constraints put heavy pressure on finding a way to cut spending without touching interest on the debt, Social Security, Medicare, or defense. However, because the remaining “all other” adds up to only about 27% of the budget in 2016, it’s not possible to cut the budget by a third from this spending alone. The conclusion is that balancing the budget through spending cuts alone would require deep cuts not only to “all other” government programs but also to national defense, Social Security, and Medicare.

TIME OUT TO THINK

If you were a member of Congress, how would you propose to balance the budget? Do you think your proposal could pass Congress? Do you think you’d be able to be re-elected if it did? Defend your opinions.

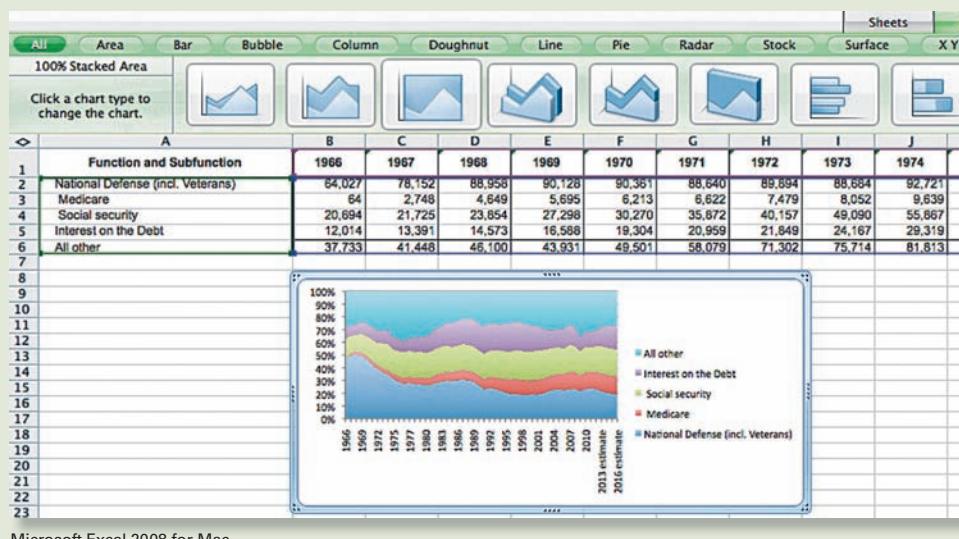


USING TECHNOLOGY—GRAPHS WITH MULTIPLE DATA SETS

Excel Excel provides a variety of options for making multiple bar graphs, multiple line graphs, stack plots, and more. The screen shot below shows some of the data that went into making Figure 15, along with a first attempt at a graph. Excel provides several ways of creating the desired graph, but here's a simple process: Notice that the years run along row 1, the different categories of spending are in column 1, and the data cells show the actual spending amounts in millions of dollars.

1. Enter all the data. Notice that the years run along row 1, the different categories of spending are in column 1, and the data cells show the actual spending amounts in millions of dollars.
2. Select all six rows, which means including both the data cells and the labels in row 1 and column 1. Then use the “Insert” menu to choose “Insert Chart...” That will bring up the chart selections that you see above row 1 in the screen shot below.
3. Choose your chart option. For this particular screen shot, we chose the third chart from the left; to the left of the chart options, you'll see that this chart type is identified as “100% stacked area,” which means it makes the stack plot as percentages of each year's total rather than from the actual dollar values.
4. From this point, getting to a final graphic like that in Figure 15 is just a matter of choosing labeling options and other decorative features. Most of these features can be added within Excel, though sometimes it is easier to add them by importing the Excel graph into an art or photo editing program.

You should try your own similar data set and experiment with the various chart types. If you don't get the result that you expect, use Excel's “help” function to get more detailed instructions about making various types of graphs.



Microsoft Excel 2008 for Mac.

Geographical Data

There are many cases in which we are interested in geographical patterns in data. Figure 16 shows one common way of depicting **geographical data**. In this case, the map shows trends in energy use per capita (per person) in different states. The actual data values are shown in small print with each state, while the color coding shows the binned categories listed in the legend.

The display in Figure 16 works well for the energy data set because each state is associated with a unique energy usage per person. For data that vary continuously across geographical areas, a **contour map** is more convenient. Figure 17 shows a contour map of temperature over the United States at a particular time. Each of the *contours* (curvy lines) connects locations with the same temperature. For example, the temperature is 50° F everywhere along the contour labeled 50° and 60° F everywhere along the contour labeled 60°. Between

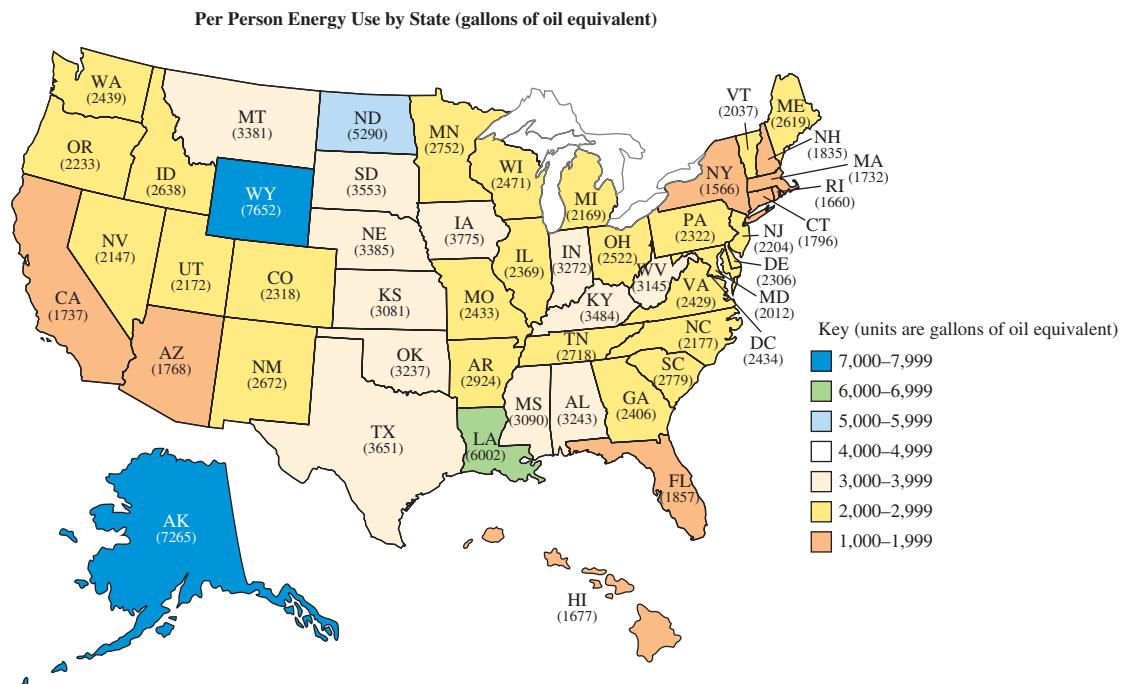


Figure 16 Geographical data can be displayed with a color-coded map. These data show per person energy usage by state, in units of “gallons of oil equivalent”; that is, the data represent the amount of oil that each person would use if all of the energy were generated by burning oil. In reality, about 40% of U.S. energy comes from oil, with most of the rest from coal, natural gas, nuclear, and hydroelectric power.

Source: U.S. Energy Information Administration, State Energy Data System (2009 data released 2011).

these two contours, the temperature is between 50°F and 60°F. Note that more closely spaced contours mean that temperature varies more greatly with distance. For example, the closely packed contours in the northeast indicate that the temperature varies substantially over small distances. To make the graph easier to read, the regions between adjacent contours are color-coded.

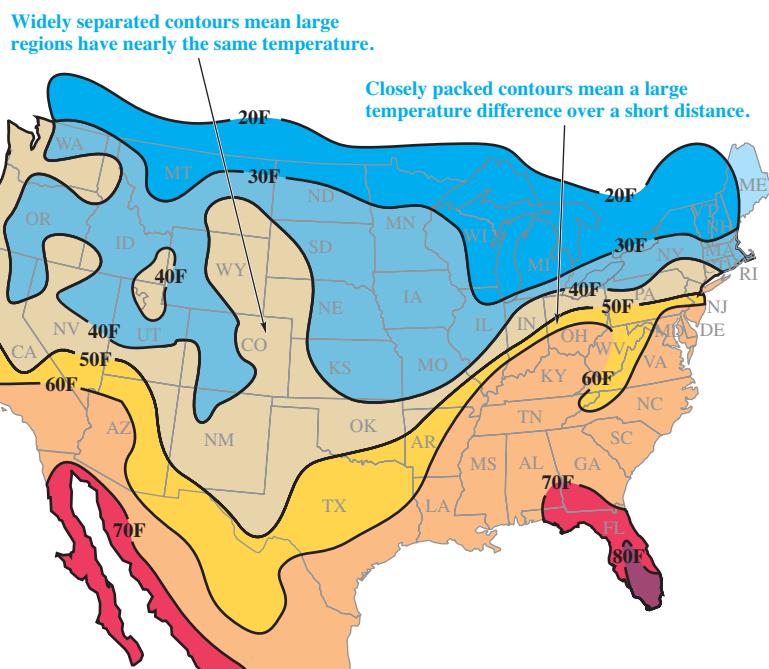


Figure 17 Geographical data that vary continuously, such as temperatures, can be displayed with a contour map.

EXAMPLE 3 Interpreting Geographical Data

Study Figures 16 and 17 and use them to answer the following questions.

- Do you see any geographical trends that might explain the states with the lowest energy usage per person?
- Were there any temperatures above 80°F in the United States on the date shown in Figure 17?

SOLUTION

- The color coding shows that the states in the lowest category of energy use per person are all either warm-weather states (CA, AZ, FL, and HI) or states in the more densely populated regions of the northeast (NY, NH, CT, MA, and RI).
- The 80°F contour passes through southern Florida, so the parts of Florida that lie south of this contour had temperatures above 80°F.

• •

Three-Dimensional Graphics

Today, computer software makes it easy to give almost any graph a three-dimensional appearance. For example, Figure 18 shows the same bar graph as Figure 1, but “dressed up” with a three-dimensional look. It may look nice, but the three-dimensional effects are purely cosmetic; they do not provide any information that wasn’t already shown in Figure 1.

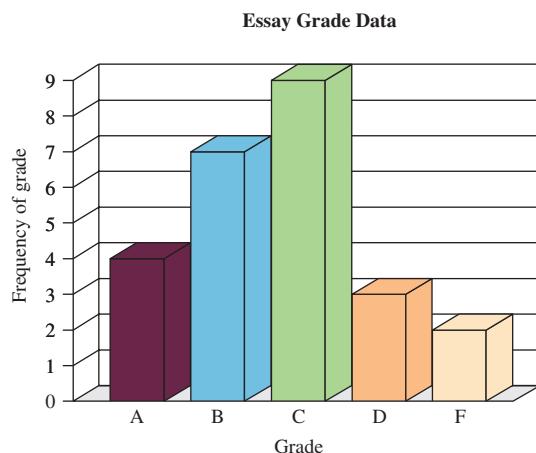


Figure 18 This graph has a three-dimensional appearance, but it shows only two-dimensional data.

In contrast, each of the three axes in Figure 18 carries distinct information, making the graph a true three-dimensional graph. Researchers studying migration patterns of a bird species (the *Bobolink*) counted the number of birds flying over seven New York cities throughout the night. As shown on the inset map, the cities were aligned east-west so that the researchers would learn what parts of the state the birds flew over, and at what times of night, as they headed south for the winter. Notice that the three axes measure *number of birds*, *time of night*, and *east-west location*.

EXAMPLE 5 Bird Migration

Based on Figure 19, at about what time was the largest number of birds flying over the east-west line marked by the seven cities? Over what part of New York did most of the birds fly? Approximately how many birds passed over Oneonta around 12:00 midnight?

SOLUTION The number of birds detected in all the cities peaked between 3 and 5 hours after 8:30 p.m., or between about 11:30 p.m. and 1:30 a.m. More birds flew over the two easternmost cities of Oneonta and Jefferson than over cities farther west, which means that most of the birds were flying over the eastern part of the state. To answer the specific question about

SONIC MAPPING TRACES BIRD MIGRATION

Sensors across New York State counted each occurrence of the nocturnal flight call of the bobolink to trace the fall migration on the night of Aug. 28–29, 1993. Computerized, the data showed the heaviest swath passing over the eastern part of the state.

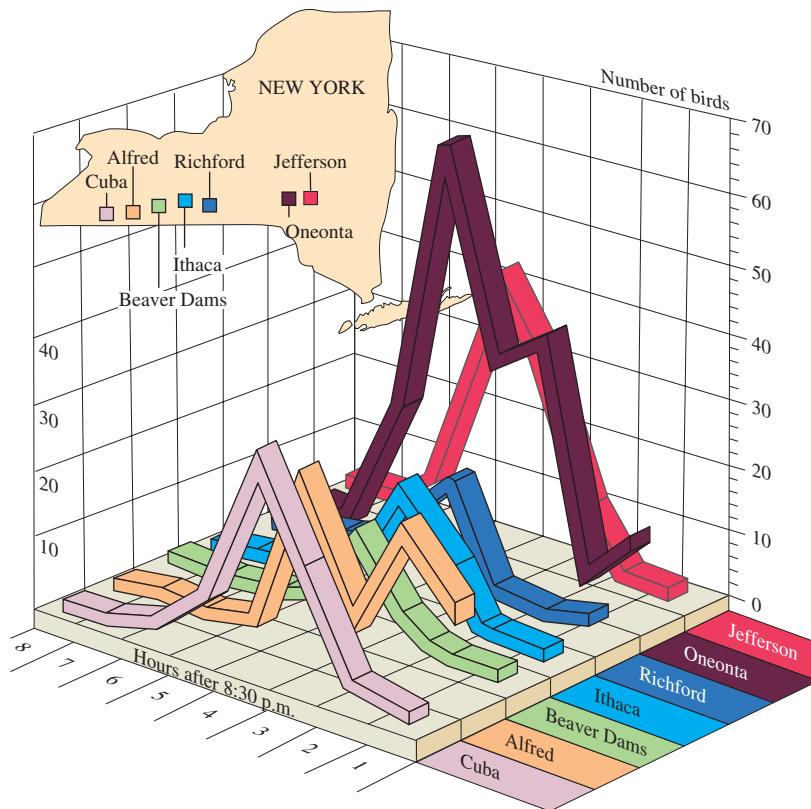


Figure 19 This graph shows true three-dimensional data.

Source: "Graph of Ornithology Data," by Bill Evans, from *New York Times*. © New York Times.

Oneonta, note that 12:00 midnight is the midpoint of time category 4. On the graph, this time aligns with the dip between peaks on the line at Oneonta. Looking across to the *number of birds* axis, we see that about 30 birds were flying over Oneonta at that time.



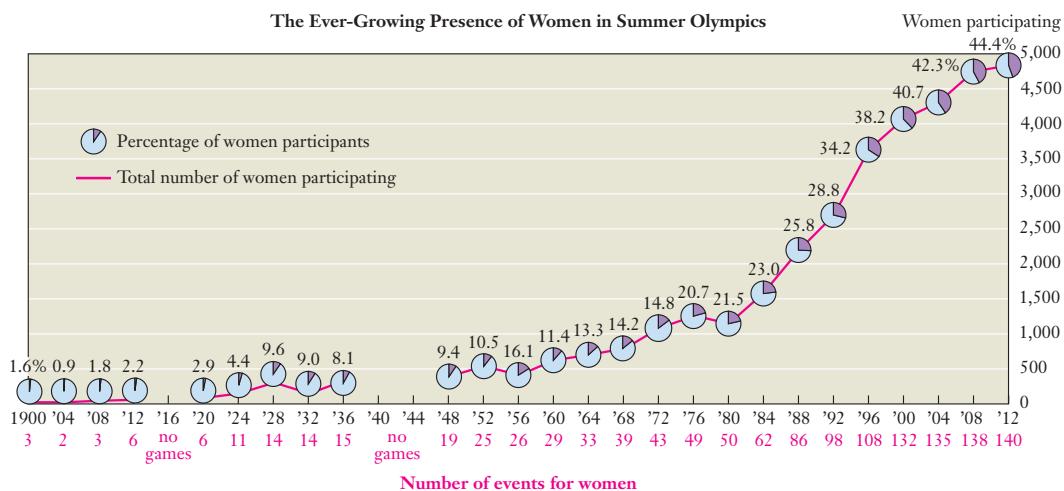
Combination Graphics

All of the graphic types we have studied so far are common and fairly easy to create. But the media today are often filled with many varieties of even more complex graphics. For example, Figure 20 shows a graphic concerning the participation of women in the summer Olympics. This single graphic combines a line chart, many pie charts, and numerical data. It is certainly a case of a picture being worth far more than a thousand words.

EXAMPLE 6 Olympic Women

Describe three trends shown in Figure 20.

SOLUTION The line chart shows that the total number of women competing in the summer Olympics has risen fairly steadily, especially since the 1960s, reaching nearly 5,000 in the 2012 games. The pie charts show that the percentage of women among all competitors has also increased, surpassing 44% in the 2012 games. The bold red numbers at the bottom show that the number of events in which women compete has also increased dramatically, reaching 140 in the 2012 games.

**Figure 20** Women in the Olympics.Source: Adapted from the *New York Times*, based on data from the International Olympic Committee.

TIME OUT TO THINK

Which of the trends shown in Figure 20 are likely to continue over the next few Olympic games? Which are not? Explain.

Section 3 Exercises

Statistical Literacy and Critical Thinking

1. Three-Dimensional Histograms. Can every histogram be converted to a three-dimensional graphic? Does the three-dimensional version of a histogram have more visual appeal? Does the three-dimensional version of a histogram provide any information not provided by the original histogram?

2. Multiple Bar Graph. What is a multiple bar graph, and how is it helpful?

3. Geographical Data. What are geographical data? Identify at least two ways to display geographical data.

4. Contour Map. What is a contour on a contour map? What does it mean when contours are close together? What does it mean when they are far apart?

Does It Make Sense? For Exercises 5–8, decide whether the statement makes sense (or is clearly true) or does not make sense (or is clearly false). Explain clearly. Not all of these statements have definitive answers, so your explanation is more important than your chosen answer.

5. Three-Dimensional Graph. A quality control engineer claims that because cars are three-dimensional objects, she needs a three-dimensional graph to display the production cost of a Corvette for each of the past 10 years.

6. Contour Map. A contour map could be used to display the ages of all full-time students at your college.

7. Geographic Data. A graphic artist for a magazine is depicting the populations of the 10 largest U.S. cities by using bars of different heights, with the bars positioned on the locations of the cities on a map of the United States.

8. Stack Plot. A multiple bar graph could be used to show the numbers of males, females, and total students at your college for each of the past 10 years.

Concepts and Applications

9. Genders of Students. The stack plot in Figure 21 shows the numbers of male and female higher education students for different years. Projections are from the U.S. National Center for Education Statistics.

- In words, discuss the trends revealed on this graphic.

- b.** Redraw the graph as a multiple line chart. Briefly discuss the advantages and disadvantages of the two different representations of this particular data set.

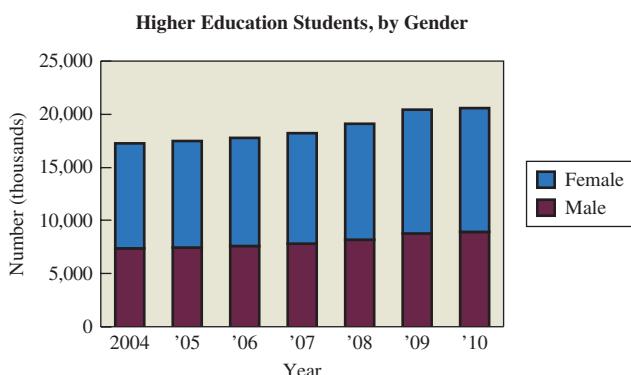


Figure 21 Source: National Center for Education Statistics.

- 10. Home Prices by Region.** The graph in Figure 22 shows home prices in different regions of the United States. Note that the data have *not* been adjusted for the effects of inflation.

- In words, describe the general trends that apply to the home price data for all regions.
- In words, describe any differences that you notice among the different regions.

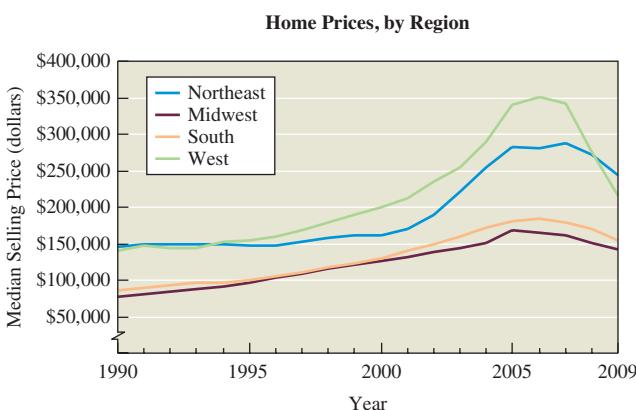


Figure 22 Source: National Association of Realtors.

- 11. Gender and Salary.** Consider the display in Figure 23 of median salaries of males and females in recent years.

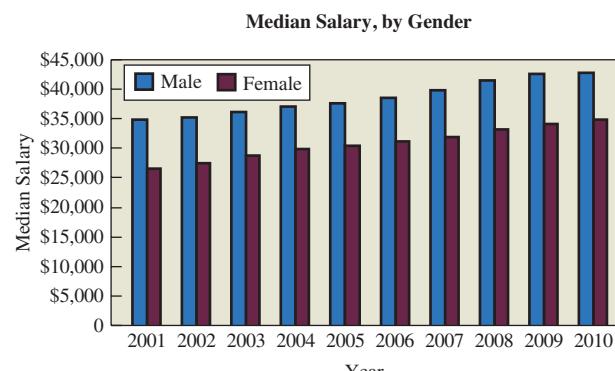


Figure 23 Source: U.S. Census Bureau.

- What story does the graph convey?

- Redraw the graph as a multiple (two) line chart. Briefly discuss the advantages and disadvantages of the two different representations of this particular data set.

- 12. Marriage and Divorce Rates.** The graph in Figure 24 depicts the U.S. marriage and divorce rates for selected years since 1900. The marriage rates are depicted by the blue bars and the divorce rates are depicted by the red bars. Both rates are given in units of marriages/divorces per 1,000 people in the population (Department of Health and Human Services).

- Why do these data consist of marriage and divorce *rates* rather than total numbers of marriages and divorces? Comment on any trends that you observe in these rates, and give plausible historical and sociological explanations for these trends.
- Construct a stack plot of the marriage and divorce rate data. For each bar, place the divorce rate above the marriage rate. Which graph makes the comparisons easier: the multiple bar graph shown here or the stack plot? Explain.



Figure 24 Source: National Center for Health Statistics.

- 13. Entitlement Spending.** The stack plot in Figure 25 shows Congressional Budget Office data for actual (through 2011) and projected spending on entitlement programs through 2085 as percentages of the gross domestic product (GDP). Interpret the graph and summarize its message.

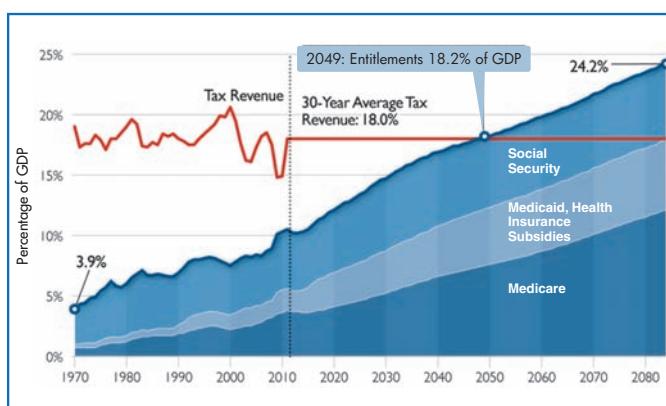


Figure 25 Source: "Tax Revenues Devoured By Medicare, Medicaid, and Social Security in 2045," from heritage.org, based on data from the Congressional Budget Office. Reprinted with permission.

- 14. College Degrees.** The stacked line chart in Figure 26 shows the numbers of bachelor's degrees awarded to men and women since 1970.

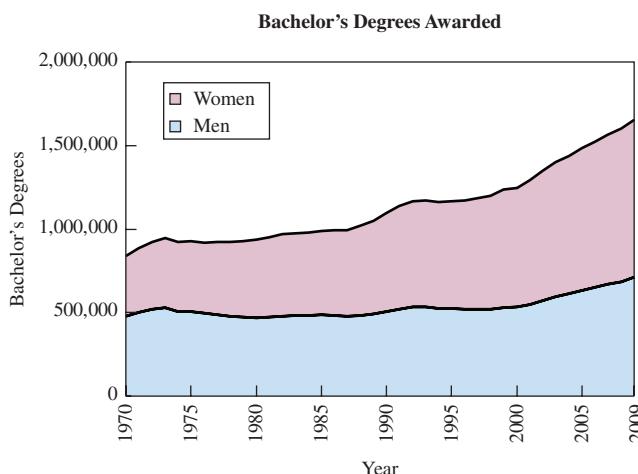


Figure 26 Source: National Center for Education Statistics.

- Estimate the numbers of bachelor's degrees awarded to men and to women (separately) in 1970 and in 2010.
- About when were the number of bachelor's degrees equal for males and females?
- Comment on the overall trend.
- Do you think the stacked line chart is an effective way to display these data? Briefly discuss other ways that might have been used instead.

- 15. Melanoma Mortality.** Figure 27 shows how the mortality rate from *melanoma* (a form of skin cancer) varies on a county-by-county basis across the United States. The legend shows that the darker the shading in a county, the higher the mortality rate. Discuss a few of the trends revealed in the figure. If you were researching skin cancer, which regions might warrant special study? Why?

- 16. School Segregation.** One way of measuring segregation is the likelihood that a black student will have classmates who are white. Figure 28 shows the probability that a black

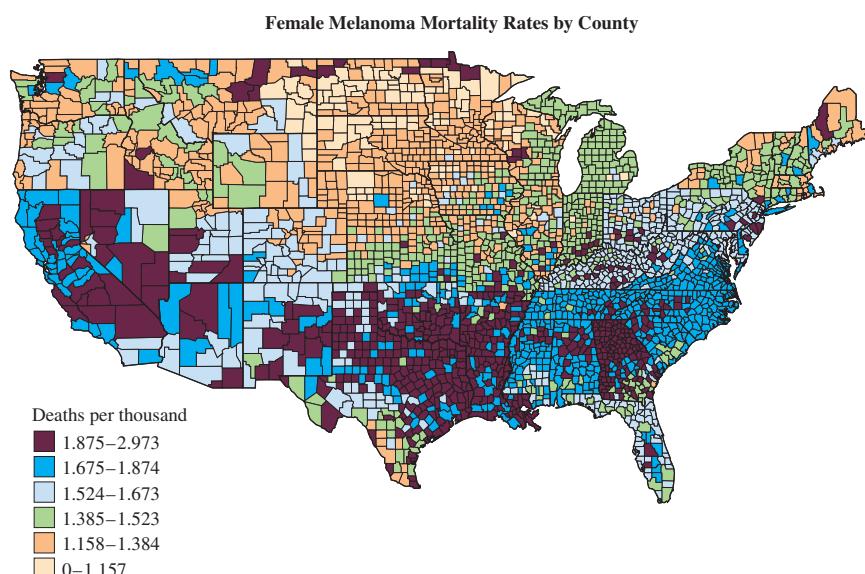


Figure 27 Source: "Female Melanoma Mortality Rates by County," by Professor Karen Kafadar, Mathematics Department, University of Colorado at Denver. Reprinted with permission.

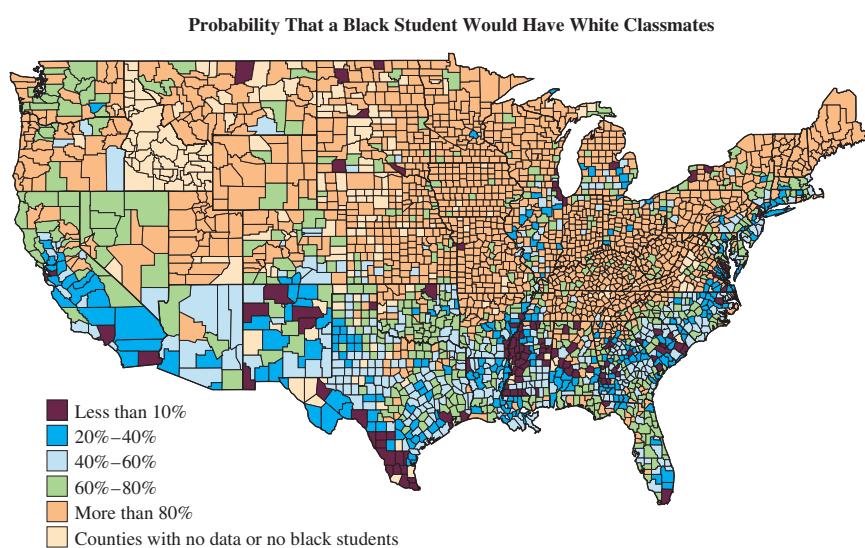


Figure 28 Source: New York Times.

student would have white classmates, by county, during a recent academic year. Do there appear to be any significant regional differences? Can you pick out any differences between urban and rural areas? Discuss possible explanations for a few of the trends that you see in the figure.

Creating Graphics. Exercises 17–20 give tables of real data. For each table, make a graphical display of the data. You may choose any graphic type that you feel is appropriate to the data set. In addition to making the display, write a few sentences explaining why you chose this type of display and a few sentences describing interesting patterns in the data.

- 17. Drinking and Driving.** The following table lists the numbers of persons killed in fatal car crashes for three different categories of blood alcohol content (BAC) of drivers. The data are from the U.S. Census Bureau.

Year	BAC of 0.0 (No alcohol)	BAC of 0.01 to 0.07	BAC of 0.08 and higher
1990	24,083	2,810	17,572
1995	25,968	2,300	13,423
2000	26,300	2,349	13,171
2005	27,542	2,350	13,532
2010 (est)	21,787	1,988	11,242

- 18. Daily Newspapers.** The following table gives the number of daily newspapers and their total circulation (in millions) for selected years since 1920 (*Editor & Publisher*).

Year	Number of daily newspapers	Circulation (millions)
1920	2,042	27.8
1930	1,942	39.6
1940	1,878	41.1
1950	1,772	53.9
1960	1,763	58.8
1970	1,748	62.1
1980	1,747	62.2
1990	1,611	62.3
2000	1,485	56.1
2010 (est)	1,402	50.0

- 19. Firearm Fatalities.** The following table summarizes deaths due to firearms in different nations in a recent year (Coalition to Stop Gun Violence).

Country	Total firearms deaths	Homicides by firearms	Suicides by firearms	Fatal accidents by firearms
United States	35,563	15,835	18,503	1,225
Germany	1,197	168	1,004	25
Canada	1,189	176	975	38
Australia	536	96	420	20
Spain	396	76	219	101
United Kingdom	277	72	193	12
Sweden	200	27	169	4
Vietnam	131	85	16	30
Japan	93	34	49	10

- 20. Working Mothers.** The following table lists labor force participation rates (as percentages) of mothers categorized according to the age of their youngest child (based on data from the Bureau of Labor Statistics).

Year	Youngest child aged 6 to 17 years	Youngest child under 6 years of age
1980	64.3	46.8
1985	69.9	53.5
1990	74.7	58.2
1995	76.4	62.3
2000	79.0	65.3
2005	76.9	62.6
2010 (est)	77.5	63.6



PROJECTS FOR THE INTERNET & BEYOND

- 21. Weather Maps.** Many Web sites offer contour maps with current weather data. Find at least two contour weather maps and discuss what they show.
- 22. The Federal Budget.** Go to the Web site for the U.S. Office of Management and Budget (OMB) and look for some of its charts related to the federal budget. Pick two charts of particular interest to you and discuss the data they show.

IN THE NEWS

- 23. Multiple Bar Graphs.** Find an example of a multiple bar graph or multiple line chart in a recent news report. Comment on the effectiveness of the display. Could another display have been used to depict the same data?
- 24. Stack Plots.** Find an example of a stack plot in a recent news report. Comment on the effectiveness of the display. Could another display have been used to depict the same data?
- 25. Geographical Data.** Find an example of a graph of geographical data in a recent news report. Comment on the effectiveness of the display. Could another display have been used to depict the same data?
- 26. Three-Dimensional Displays.** Find an example of a three-dimensional display in a recent news report. Are three dimensions needed, or are they included for cosmetic reasons? Comment on the effectiveness of the display. Could another display have been used to depict the same data?
- 27. Fancy News Graphics.** Find an example in the news of a graphic that combines two or more of the basic graphic types. Briefly explain what the graphic is showing, and discuss the effectiveness of the graphic.

4

FEW CAUTIONS ABOUT GRAPHICS

As we have seen, graphics can offer clear and meaningful summaries of statistical data. However, even well-made graphics can be misleading if we are not careful in interpreting them, and poorly made graphics are almost always misleading. Moreover, some people use graphics in deliberately misleading ways. In this section, we discuss a few of the more common ways in which graphics can lead us astray.

Perceptual Distortions

Many graphics are drawn in a way that distorts our perception of them. Figure 29 shows one of the most common types of distortion. Dollar-shaped bars are used to show the declining value of the dollar over time. The problem is that the values are represented by the *lengths* of the dollar bills; for example, a 2010 dollar was worth \$0.39 in 1980 dollars and therefore is drawn so that it is 39% as long as the 1980 dollar. However, our eyes tend to focus on the *areas* of the dollar bills, and the area of the 2010 dollar is only about 15% of the area of the 1980 dollar (because both the length and width of the 2010 dollar are reduced to 39% of the 1980 sizes, and $0.39^2 \approx 0.15$). This gives the perception that the value of the dollar shrank even more than it really did.

TIME ⏱ OUT TO THINK

Suppose the three dollars shown in Figure 29 were each represented by a three-dimensional stack of pennies, with the height of the stack proportional to the value of the dollar. Would that make the visual distortion of the data greater, less, or the same as the distortion in Figure 29? Explain.

Watch the Scales

Figure 30a shows the percentage of college students since 1910 who were women. At first glance, it appears that this percentage grew by a huge margin after about 1950. But the vertical axis scale does not begin at zero and does not end at 100%. The increase is still substantial but looks far less dramatic if we redraw the graph with the vertical axis covering the full range of 0 to 100% (Figure 30b). From a mathematical point of view, leaving out the zero point on a scale is perfectly honest and can make it easier to see small-scale trends in data. Nevertheless, as this example shows, it can be visually deceptive if you don't study the scale carefully.

Women as a Percentage of All College Students

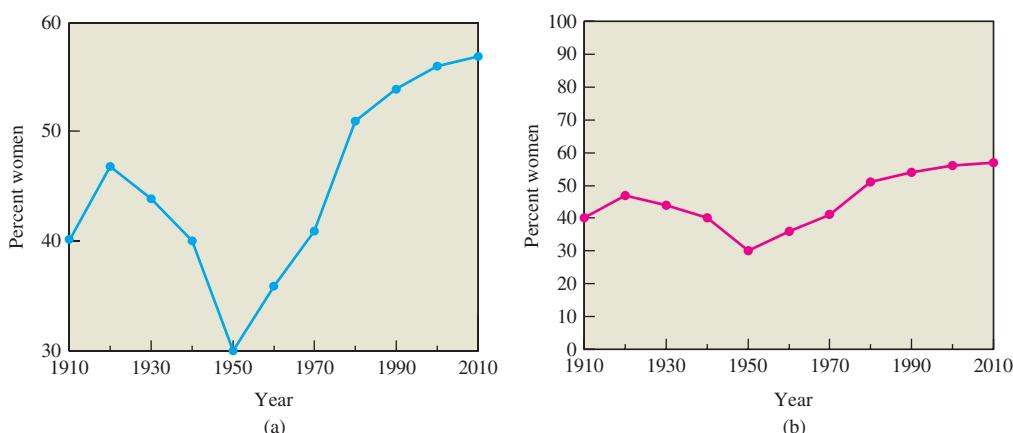


Figure 30 Both graphs show the same data, but they look very different because their vertical scales have different ranges.

Source: National Center for Education Statistics.

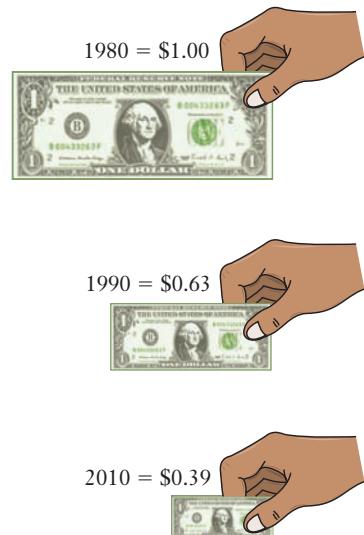


Figure 29 The lengths of the dollars are proportional to their spending power, but our eyes are drawn to the areas, which decline more than the lengths.

BY THE WAY

German researchers in the latter part of the 19th century studied many types of graphics. The type of distortion shown in Figure 29 was so common that they gave it its own name, which translates roughly as “the old goosing up the effect by squaring the eyeball trick.”



The easiest person to deceive is one's own self.

—Edward Bulwer-Lytton

Another issue can arise when graphs use nonlinear scales, meaning scales in which each increment does not always represent the same change in value. Consider Figure 31a, which shows how the speeds of the fastest computers have increased with time. At first glance, it appears that speeds have been increasing linearly. For example, it might look as if the speed increased by the same amount from 1990 to 2000 as it did from 1950 to 1960. However, if you look more closely, you'll see that each tick mark on the vertical scale represents a *tenfold* increase in speed. Now we see that computer speed grew from about 1 to 100 calculations per second from 1950 to 1960 and from about 100 million to 10 billion calculations per second between 1990 and 2000. This type of scale is called an **exponential scale** (or *logarithmic scale*) because it grows by powers of 10 and powers of 10 are *exponents*. (For example, 3 is the exponent in $10^3 = 1,000$.) It is always possible to convert an exponential scale back to an ordinary linear scale as shown in Figure 31b. However, comparing the two graphs should make clear why the exponential scale is so useful in this case: The exponential scale clearly shows the rapid gains in computer speeds, while the ordinary scale makes it impossible to see any detail in the early years shown on the graph. More generally, exponential scales are useful whenever data vary over a huge range of values.

BY THE WAY

In 1965, Intel founder Gordon E. Moore predicted that advances in technology would allow computer chips to double in power roughly every two years. This principle is now called *Moore's law*, and it has held fairly true ever since Moore first stated it.

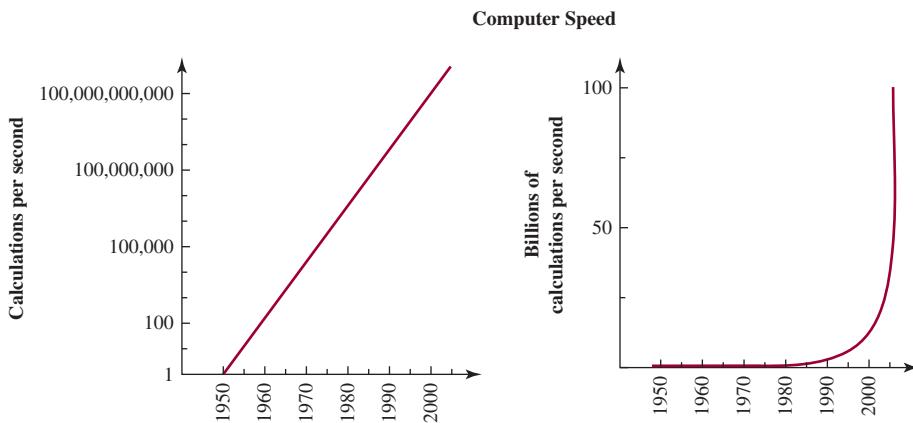


Figure 31 Both graphs show the same data, but the graph on the left uses an exponential scale.

TIME OUT TO THINK

Based on Figure 31a, can you predict the speed of the fastest computers in 2020? Could you make the same prediction with Figure 31b? Explain.



CASE STUDY Asteroid Threat

Asteroids and comets occasionally hit the Earth. Small ones tend to burn up in the atmosphere or create small craters on impact, but larger ones can cause substantial devastation. About 65 million years ago, an asteroid about 10 kilometers in diameter hit the Earth, leaving a 200-kilometer-wide crater on the coast of the Yucatan peninsula in Mexico. Scientists estimate that this impact caused the extinction of about three-quarters of all species living on Earth at the time, including all the dinosaurs.

Clearly, a similar impact would be bad news for our civilization. We might therefore want to understand the likelihood of such an event. Figure 32 shows a graph relating the size of impacting asteroids and comets to the frequency with which such objects hit the Earth. Because of the wide range of sizes and time scales involved, *both* axes on this graph are exponential. The horizontal axis shows impactor (asteroid or comet) sizes, with each tick representing a power of 10. The vertical axis shows the frequency of impact; moving up on the vertical axis corresponds to more frequent events. With this double exponential graph, we can see trends clearly. For example, small objects of about 1 meter

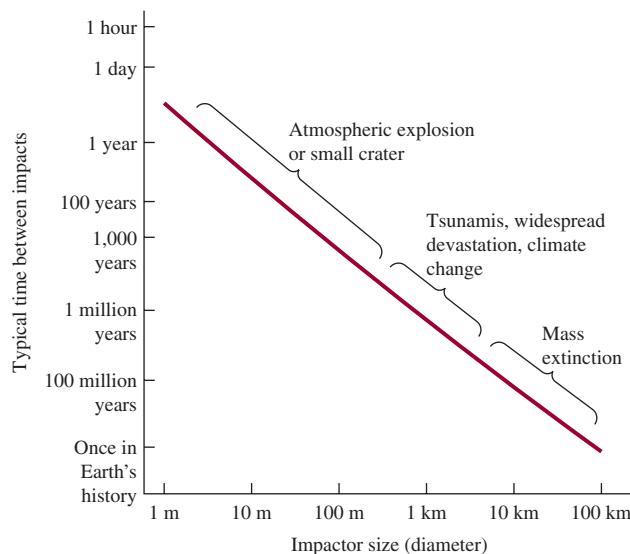


Figure 32 This graph shows how the frequency of impacts—and the magnitude of their effects—depends on the size of the impactor. Note that smaller impacts are much more frequent than larger ones.

in size strike the Earth every day, but cause little damage. At the other extreme, objects large enough to cause a mass extinction hit only about once every hundred million years.

The intermediate cases are probably the most worrisome. The graph indicates that objects that could cause “widespread devastation”—such as wiping out the population of a large city—can be expected as often as once every thousand years. This is often enough to warrant at least some preventive action. Currently, astronomers are trying to make more precise predictions about when an object might hit the Earth. If they discover an object that will hit the Earth, scientists will need to find a way to deflect it to prevent the impact.

BY THE WAY

More than 150 impact craters have been identified around the Earth. One of the best preserved is Meteor Crater in Arizona (photo below). It formed about 50,000 years ago when an asteroid about 50 meters across crashed to Earth, releasing energy equivalent to that of a 20-megaton hydrogen bomb and leaving a crater more than a kilometer across and 200 meters deep.



Percentage Change Graphs

Is college getting more or less expensive? If you didn’t look too carefully, Figure 33a might lead you to conclude that after peaking in the early 2000s, the cost of public colleges fell during the rest of that decade. But look more closely and you’ll see that the vertical axis on this graph represents the *percentage change* in costs. The drop-off therefore means only that costs rose by smaller amounts, not that they fell. Actual college costs are shown in Figure 33b, which makes it clear that they rose every year. Graphs that show percentage change are very common; you’ll find them in the financial news almost every day. But as you can see, they can be very misleading if you don’t realize that they are showing change rather than actual values.

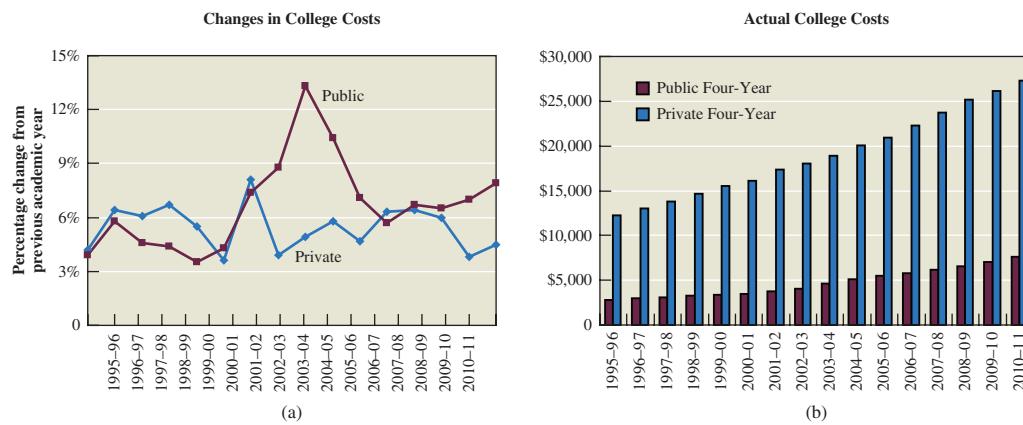


Figure 33 Trends in college costs: (a) annual percent change; (b) actual costs.

Source: The College Board.

Get your facts first, and then you can distort them as much as you please.

—Mark Twain

Pictographs

Pictographs are graphs embellished with additional artwork. The artwork may make the graph more appealing, but it can also distract or mislead. Figure 34 is a pictograph showing the rise in world population from 1804 to 2040 (numbers for future years are based on United Nations intermediate-case projections). The lengths of the bars correctly correspond to world population for the years listed. However, the artistic embellishments of this graph are deceptive in several ways. For example, your eye may be drawn to the figures of people lining the globe. Because this line of people rises from the left side of the pictograph to the center and then falls, it might give the impression that future world population will decline. In fact, the line of people is purely decorative and carries no information.

The more serious problem with this pictograph is that it makes it appear that world population has been rising linearly. However, notice that the time intervals on the horizontal axis are not the same in each case. For example, the interval between the bars for 1 billion and 2 billion people is 123 years (from 1804 to 1927), but the interval between the bars for 5 billion and 6 billion people is only 12 years (from 1987 to 1999).

Pictographs are very common. As this example shows, however, you have to study them carefully to extract the essential information and not be distracted by the cosmetic effects.

BY THE WAY

Demographers often characterize population growth by a doubling time—the time it takes the population to double. During the late 20th century, the doubling time for the human population was about 40 years. If the population continued to double at this rate, world population would reach more than 30 billion by 2100 and 190 billion by 2200. By about 2650, the human population would be so large that it would not fit on the Earth, even if everyone stood elbow to elbow everywhere.

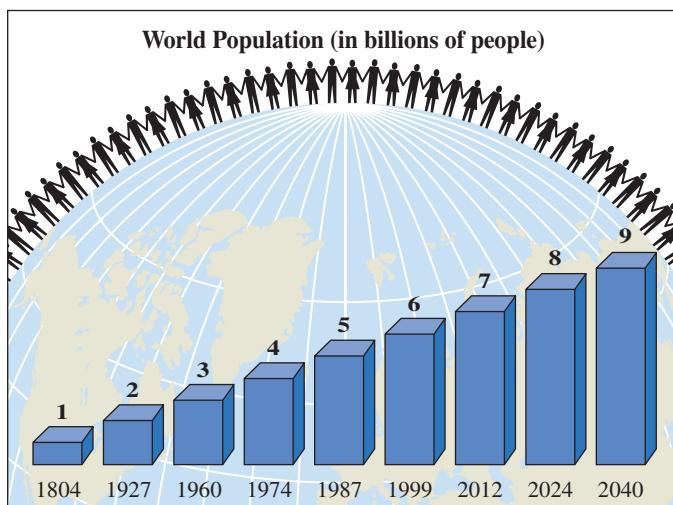


Figure 34 A pictograph of world population. The pictures add visual impact, but can also be misleading. Also notice that the horizontal scale (time) is not linear.

Source: United Nations Population Division, future projections based on intermediate case assumptions.

Section 4 Exercises

Statistical Literacy and Critical Thinking

1. Exaggerating a Difference. Under standard test conditions, the braking distance for a Honda Civic is 136 feet and the braking distance for a VW Jetta is 137 feet. Is that difference meaningful? How could a graph be constructed so that the difference is greatly exaggerated?

2. Graph of Populations. When constructing a graph showing the population of the United States and the population of Mexico, an illustrator draws two different people with heights proportional to the populations. Identify a way in which the graph might be misleading? What is the general name for such graphs that use drawings of people or objects?

3. Vertical Scale. A line chart has a vertical scale with values of 0, 1, 10, 100, 1000 and 10,000. What is the name of such a scale? What is an advantage of such a scale?

4. Sugar Cubes. To show how sugar production doubled from 1990 to now, an illustrator draws two sugar cubes. The first cube is drawn with a length of 1 cm on each side and the second cube is drawn with a length of 2 cm on each side. What are the volumes of the two sugar cubes? Is the illustration misleading? If so, how?

Concepts and Applications

- 5. Car Mileage.** Figure 35 shows the highway fuel consumption (mi/gal) for the Chevrolet Aveo and the Honda Civic. How is the graph misleading? How could it be drawn so that it is not misleading?

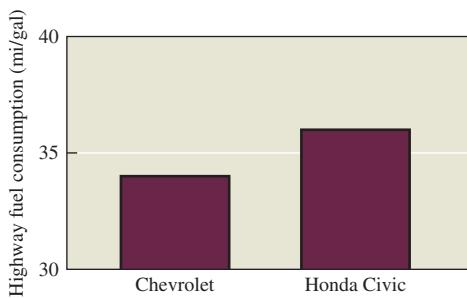


Figure 35 Car Mileage.

- 6. Comparing State Populations.** Figure 36 depicts 2010 populations for California, Texas, and New York (based on data from the 2010 Census). How is this graph misleading? How could it be drawn so that it is not misleading?

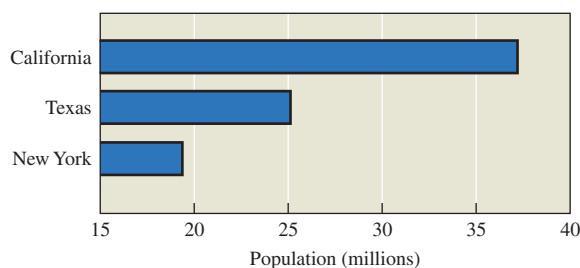


Figure 36 State Populations.

- 7. Pictograph.** Figure 37 depicts the amounts of daily oil consumption in the United States and Japan. Does the illustration accurately depict the data? Why or why not?

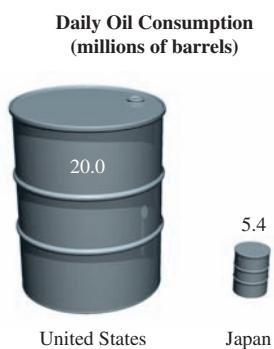


Figure 37

- 8. Pictograph.** Refer to Figure 37 used in Exercise 7 and construct a bar chart to depict the same data in a way that is fair and objective.

- 9. Three-Dimensional Pies.** The pie charts in Figure 38 give the percentage of Americans in three age categories in 1990 and 2050 (projected).

- Consider the 1990 age distribution. The actual percentages for the three categories for 1990 were 87.5% (others), 11.3% (60–84), and 1.2% (85+). Does the pie chart show these values accurately? Explain.
- Consider the 2050 age distribution. The actual percentages for the three categories for 2050 were 80.0% (others), 15.4% (60–84), and 4.6% (85+). Does the pie chart show these values accurately? Explain.
- Using the actual percentages given in parts a and b, draw flat (two-dimensional) pie charts to display these data. Explain why these pie charts give a more accurate picture than the three-dimensional pies.
- Comment on the general trends shown in the two pie charts.

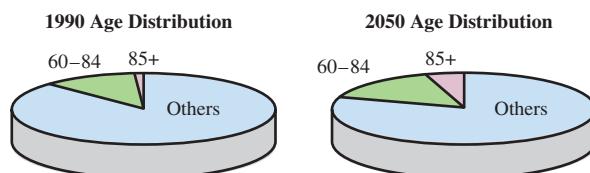


Figure 38 Source: U.S. Census Bureau.

- 10. Moore's Law.** In 1965, Intel cofounder Gordon Moore initiated what has since become known as *Moore's law*: the number of transistors per square inch on integrated circuits will double approximately every 18 months. In the table below, the first row lists different years and the second row lists the number of transistors (in thousands) for different years.

Year	Transistors	Year	Transistors
1971	23	1997	7500
1974	5	2000	42,000
1978	29	2002	220,000
1982	120	2003	410,000
1985	275	2007	789,000
1989	1180	2011	2,600,000
1993	3100		

- Construct a time-series graph of these data, using a uniform scale on both axes.
- Make an exponential graph of these data in which the subdivisions on the vertical axis are 0, 1, 10, 100, 1,000, 10,000, 100,000, 1,000,000, and 100,000,000.
- Compare the graphs in parts a and b.

- 11. Percentage Change in the CPI.** The graph in Figure 39 shows the percentage change in the CPI over recent years. In what year (of the years displayed) was the change in the CPI the greatest? What happened in 2009? How do actual prices in 2010 compare to those in 1990? Based on this graph, what can you conclude about changes in prices during the period shown?

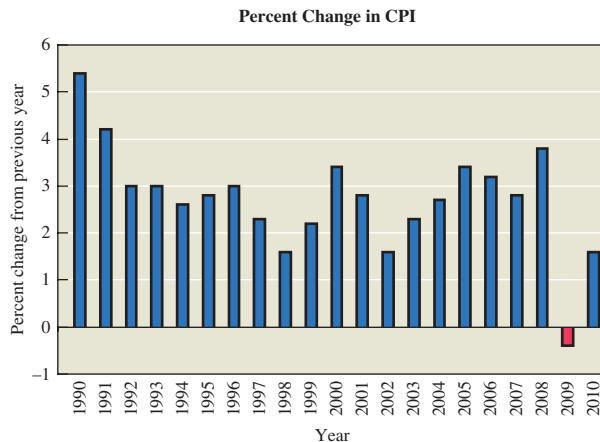


Figure 39 Source: U.S. Bureau of Labor Statistics.

- 12. Seasonal Effects on Schizophrenia?** The graph in Figure 40 shows data regarding the relative risk of schizophrenia among people born in different months.

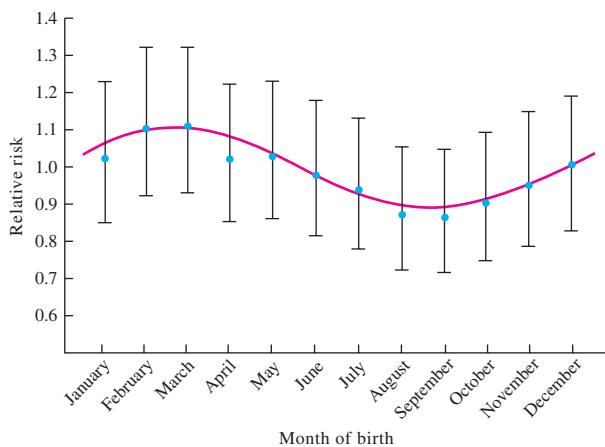


Figure 40 Source: New England Journal of Medicine.

- Note that the scale of the vertical axis does not include zero. Sketch the same risk curve using an axis that includes zero. Comment on the effect of this change.
- Each value of the relative risk is shown with a dot at its most likely value and with an “error bar” indicating the range in which the data value probably lies. The study concludes that “the risk was also significantly associated with the season of birth.” Given the size of the error bars, does this claim appear justified? (Is it possible to draw a flat line that passes through all of the error bars?)
- Constant Dollars.** The graph in Figure 41 shows the minimum wage in the United States, together with its

purchasing power, which is adjusted for inflation with 1996 used as the reference year. The graph represents the years from 1955 to 2011. Summarize what the graph shows.

Minimum Wage and Its Purchasing Power

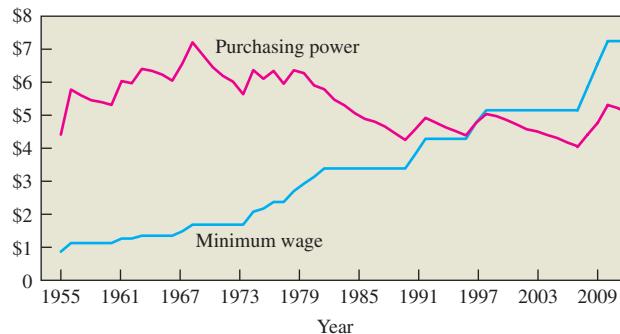


Figure 41 Source: U.S. Department of Labor.

- 14. Double Horizontal Scale.** The graph in Figure 42 shows simultaneously the number of births in this country during two time periods: 1946–1964 and 1977–1994. When did the first baby boom peak? When did the second baby boom peak? Why do you think the designer of this display chose to superimpose the two time intervals, rather than use a single time scale from 1946 through 1994?

Baby Boomers and Their Babies

The baby-boom generation, born between 1946 and 1964, produced their own smaller boom between 1977 and 1994.

Number of U.S. Births, 1946 through 1964 and 1977 through 1994 (in millions)

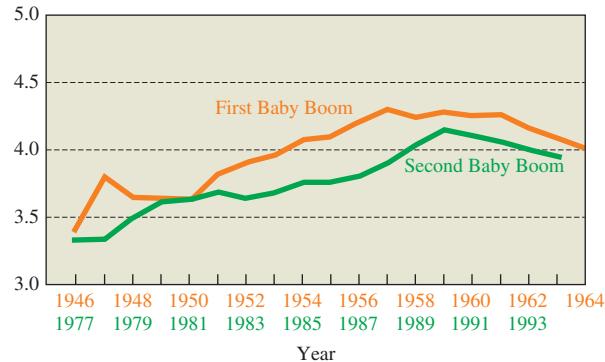


Figure 42 Source: Based on data from the National Center for Health Statistics.



- USA Snapshot.** *USA Today* offers a daily pictograph for its “Snapshot.” Find a snapshot from a recent issue of *USA Today*. Briefly discuss its purpose and effectiveness.
- Image Search.** Choose some topic that interests you for which you think that good statistical graphs should be available. Do an image search for the topic in Google, Bing, or other search engine. Does the search give you what you were looking for? Briefly discuss the value of the search results in terms of what you hoped to find.

IN THE NEWS

- 17. Distortions in the News.** Find an example in a recent news report of a graph that involves some type of perceptual distortion. Explain the effects of the distortion, and describe how the graph could have been drawn more honestly.
- 18. Scale Problems in the News.** Find an example in a recent news report of a graph in which the vertical scale does not start at zero. Suggest why the graph was drawn that way and also discuss any ways in which the graph might be misleading as a result.
- 19. Economic Graph in the News.** Find an example in a recent news report of a graph that shows economic data over time. Are the data adjusted for inflation? Discuss the meaning of the graph and any ways in which it might be deceptive.
- 20. Pictograph in the News.** Find an example of a pictograph in a recent news report. Discuss what the pictograph attempts to show, and discuss whether the artistic embellishments help or hinder this purpose.
- 21. Outstanding News Graph.** Find a graph from a recent news report that, in your opinion, is truly outstanding in displaying data visually. Discuss what the graph shows, and explain why you think it is so outstanding.
- 22. Not-So-Outstanding News Graph.** Find a graph from a recent news report that, in your opinion, fails in its attempt to display data visually in a meaningful way. Discuss what the graph was trying to show, explain why it failed, and explain how it could have been done better.

CHAPTER REVIEW EXERCISES

Listed below are measured weights (in pounds) of the contents in samples of cans of regular Pepsi and Diet Pepsi. Use these data for Exercises 1–3.

Regular Pepsi:

0.8258	0.8156	0.8211	0.8170	0.8216	0.8302
0.8192	0.8192	0.8271	0.8251	0.8227	0.8256
0.8139	0.8260	0.8227	0.8388	0.8260	0.8317
0.8247	0.8200	0.8172	0.8227	0.8244	0.8244
0.8319	0.8247	0.8214	0.8291	0.8227	0.8211
0.8401	0.8233	0.8291	0.8172	0.8233	0.8211

Diet Pepsi:

0.7925	0.7868	0.7846	0.7938	0.7861	0.7844
0.7795	0.7883	0.7879	0.7850	0.7899	0.7877
0.7852	0.7756	0.7837	0.7879	0.7839	0.7817
0.7822	0.7742	0.7833	0.7835	0.7855	0.7859
0.7775	0.7833	0.7835	0.7826	0.7815	0.7791
0.7866	0.7855	0.7848	0.7806	0.7773	0.7775

- 1. a.** Construct a frequency table for the weights of regular Pepsi. Use bins of

0.8130–0.8179
0.8180–0.8229
0.8230–0.8279
0.8280–0.8329
0.8330–0.8379
0.8380–0.8429

- b.** Construct a frequency table for the weights of diet Pepsi. Use bins of

0.7740–0.7779
0.7780–0.7819
0.7820–0.7859
0.7860–0.7899
0.7900–0.7939

- c.** Compare the frequency tables from parts a and b. What notable differences are there? How can those notable differences be explained?

- 2. a.** Construct a relative frequency table for the weights of regular Pepsi. Use bins of

0.8130–0.8179
0.8180–0.8229
0.8230–0.8279
0.8280–0.8329
0.8330–0.8379
0.8380–0.8429

- b.** Construct a cumulative frequency table for the weights of regular Pepsi.

- 3. a.** Use the result from Exercise 1a to construct a histogram for the weights of regular Pepsi.
b. Use the result from Exercise 1b to construct a histogram for the weights of Diet Pepsi.

- c. Compare the histograms from parts a and b. How are they similar and how are they different?
- 4. Pie Chart of Awful Sounds.** In a survey, 1,004 adults were asked to identify the most frustrating sound that they hear in a day. Two hundred seventy-nine chose jackhammers, 388 chose car alarms, 128 chose barking dogs, and 209 chose crying babies (based on data from Kelton Research). Construct a pie chart depicting these data.
- 5. Pareto Chart.** Construct a Pareto chart from the data given in Exercise 4. Compare the Pareto chart to the pie chart. Which graph is more effective in showing the numbers of frustrating sounds? Explain.
- 6. Bar Chart.** Figure 43 shows the numbers of U.S. adoptions from China in the years 2005 and 2010. What is wrong with

this graph? Draw a graph that depicts the data in a fair and objective way.

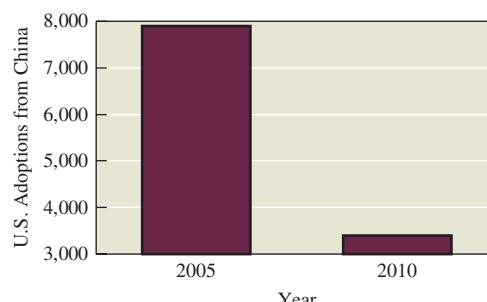


Figure 43

CHAPTER QUIZ

- The IQ scores of 500 college football players are randomly selected. Which graph would be most appropriate for these data: histogram, bar chart, pie chart, multiple bar graph, or stack plot?
- As a quality control manager at Sony, you find that defective CDs have various causes, including worn machinery, human error, bad supplies, and packaging mistreatment. Which of the following graphs would be best for describing the causes of defects: histogram, bar chart, Pareto chart, dotplot, or pie chart?
- A stemplot is created from the intervals (min) between eruptions of the Old Faithful geyser in Yellowstone National Park, and one row of that stemplot is 6|1222279. Identify the values represented by that row.
- The first class in a frequency table is 20–29 and the corresponding frequency is 15. What does the value of 15 indicate?
- The first class in a relative frequency table is 20–29 and the corresponding relative frequency is 0.25. What does the value of 0.25 indicate?
- The third class in a frequency table is 40–49 and the corresponding cumulative frequency is 80. What does the value of 80 indicate?
- The bar chart in Figure 44 depicts the number of twin births in the United States in the years 2000 and 2008. In what way is this graph misleading?

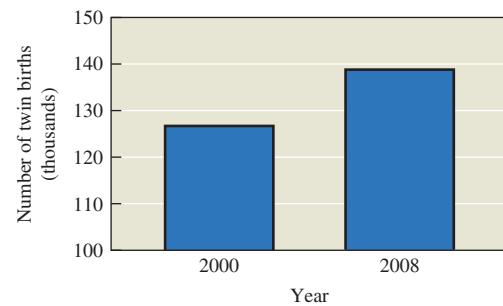


Figure 44

- Construct a dotplot representing these six IQ scores: 90, 95, 95, 100, 105, 110.
- Identify the values represented by the following stemplot:

0	0	1	1
1	0	1	2
3			
4	9	9	

- In one county, the cost of snow removal tripled from 1980 to the current year. Why is it misleading to represent the amounts with images of two rectangular snowplows if the second is three times as wide and three times as tall as the first snowplow?

FOCUS ON HISTORY

Can War Be Described with a Graph?

Can a war be described with a graph? Figure 45, created by Charles Joseph Minard in 1869, does so remarkably well. This graph tells the story of Napoleon's ill-fated Russian campaign of 1812, sometimes called Napoleon's death march.

The underlying map on Minard's graph shows a roughly 500-mile strip of land extending from the Niemen River on the Polish-Russian border to Moscow. The blue strip depicts the forward march of Napoleon's army. On Minard's original drawing, each millimeter of width represented 6,000 men; this reproduction is shown at a smaller size than the original. The march begins at the far left, where the strip is widest. Here, an army of 422,000 men triumphantly began a march toward Moscow on June 24, 1812. At the time, it was the largest army ever mobilized.

The narrowing of the strip as it approaches Moscow represents the unfolding decimation of the army. (The offshoots represent battalions that were sent off in other directions along the way.) Napoleon had brought only minimal food supplies, and hot summer weather accompanied by heavy rains brought rampant disease. Starvation, disease, and combat losses killed



thousands of men each day. By the time the army entered Moscow on September 14, it had shrunk to 100,000 men. The worst was yet to come.

To Napoleon's dismay, the Russians evacuated Moscow prior to the French army's arrival. Deprived of the opportunity to engage the Russian troops and feeling that his army's condition was too poor to continue on to the Russian capital of St. Petersburg, Napoleon took his troops southward out of Moscow. The lower part of the strip on the graph (shown in maroon) represents the retreat, and the dark blue line near the bottom of the figure shows the nighttime temperatures as winter approached. We see that freezing temperatures had already set in by October 18.

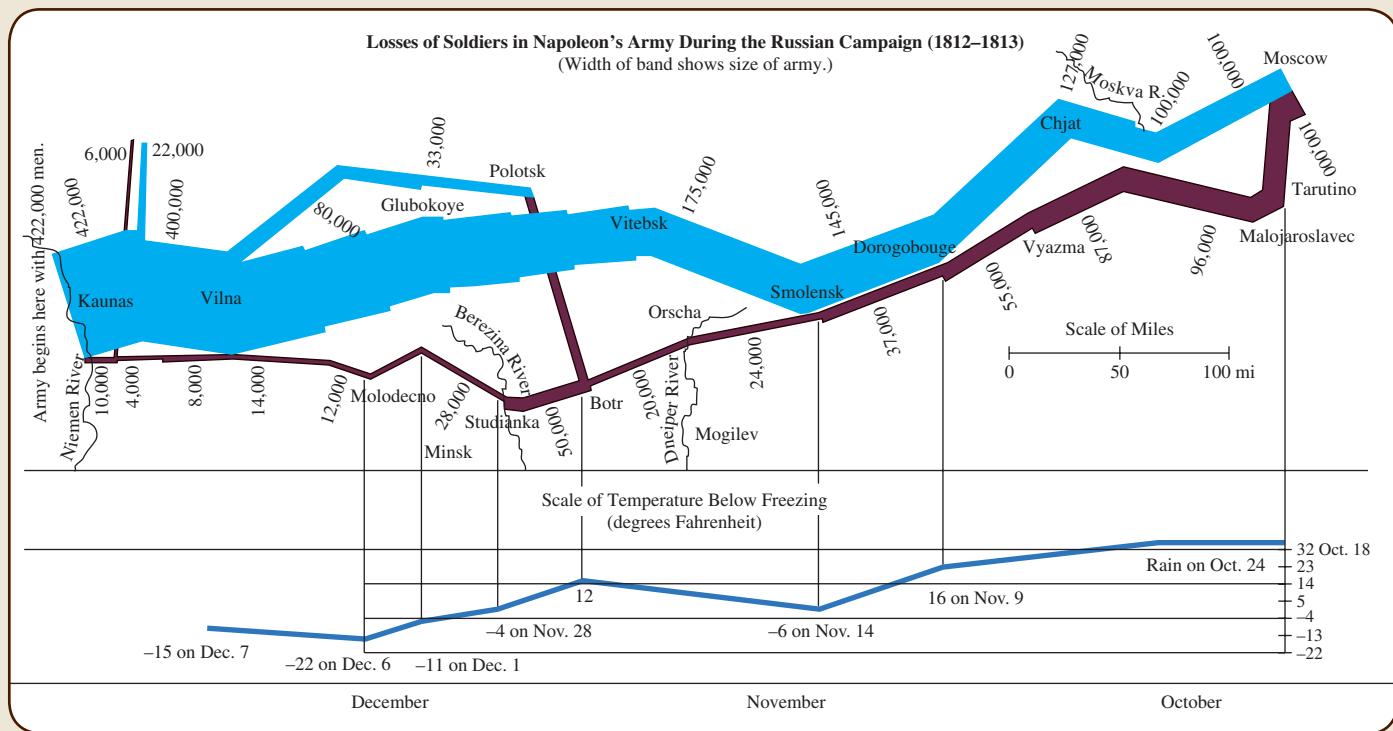


Figure 45 Source: Edward R. Tufte, *The Visual Display of Quantitative Information* (Cheshire, CT: Graphics Press, 1983, 2001). Reprinted with permission.

Temperatures plunged below 0°F in late November. The sudden narrowing of the lower strip around November 28 shows where 22,000 men perished on the banks of the Berezina River. Three-fourths of the survivors froze to death over the next few days, many on the bitter cold night of December 6. By the time the army reached Poland on December 14, only 10,000 of the original 422,000 remained.

In a famous analysis of graphical techniques, author Edward Tufte described Minard's graph as possibly "the best statistical graphic ever drawn." But a more dramatic statement came from a contemporary of Minard, E. J. Marey, who wrote that this graphic "brought tears to the eyes of all France."

QUESTIONS FOR DISCUSSION

1. Discuss how this graph helps to overcome the impersonal nature of the many deaths in a war. What kind of impact does it have on you personally?
 2. Note that this graph plots six variables: two variables of direction (north-south and east-west), the size of the army, the location of the army, the direction of the army's movement, and temperatures during the retreat. Do you think Minard could have gotten the point across with fewer variables? Why or why not?
 3. Discuss how you might make a similar graph for some other historical or political event.
- • • • • • • • • • •

FOCUS ON ENVIRONMENT

Are We Changing Earth's Atmosphere?

Sometimes a well-drawn figure can teach important lessons even before you understand everything about it. Figure 46 is a case in point.

As the label shows, the graph shows how the concentration of carbon dioxide in Earth's atmosphere varies with time. The units used for the concentration are *parts per million*, which means the number of carbon dioxide molecules among each one million molecules of air. For example, a concentration of 300 parts per million means that there are 300 molecules of carbon dioxide among each one million molecules of air, which is equivalent to a concentration of 3 in 10,000 or 0.03%.



Notice that the main graph shows how the carbon dioxide concentration has varied over the past 800,000 years. Scientists obtain these data by measuring the concentration of carbon dioxide trapped in bubbles in ancient ice, which they collect by drilling ice cores out of the ice sheets in Greenland.

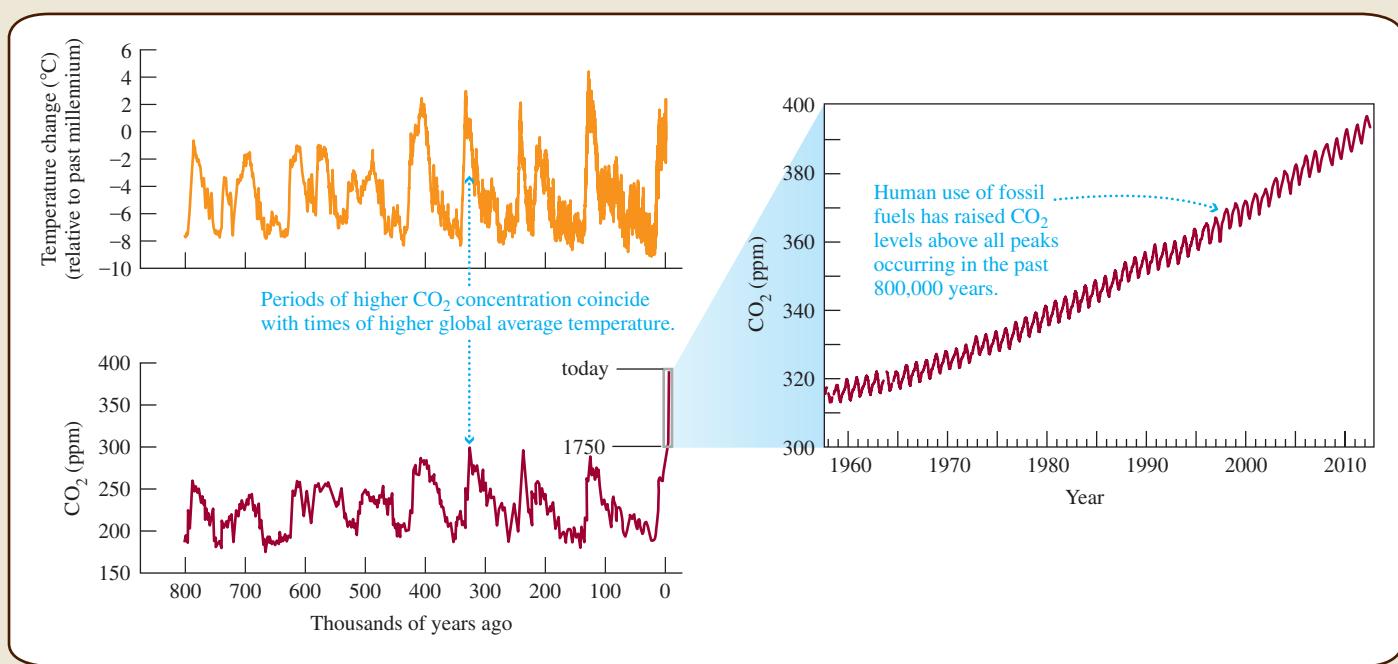


Figure 46 (Left) The atmospheric concentration of carbon dioxide and global average temperature reconstructed from ice core data for the past 800,000 years. Source: European Project for Ice Coring in Antarctica. (Right) The concentration has been directly measured in the atmosphere since the late 1950s. Note: The wiggles in the right graph represent seasonal fluctuations; the line running through the wiggles represents the general trend. Source: National Oceanic and Atmospheric Administration.

or Antarctica. The deepest ice cores drilled to date reach down to ice that was deposited more than 800,000 years ago, which is why the graph can show data going back that far. The zoom-in on the right shows the carbon dioxide concentration directly measured in Earth's atmosphere since the late 1950s.

Even without knowing anything about the role of carbon dioxide, the graph should certainly grab your attention. For one thing, the many ups and downs visible in the graph for the past 800,000 years show that the carbon dioxide concentration varies substantially through natural processes. But notice that in all of those ups and downs, the concentration never rose above about 300 parts per million until just a couple of centuries ago. Now, the zoom-out shows the concentration rising by some 2 to 3 parts per million per year, a rate at which it will pass 400 parts per million by 2015 and 500 parts per million by about 2060. Clearly, something dramatic is happening to the carbon dioxide concentration.

The dramatic change should lead you to ask other questions. First, you might wonder whether the carbon dioxide concentration is important, and the answer is revealed by the notes on the main graph. The same ice core data that allow reconstruction of the past carbon dioxide concentration also allow reconstruction of past temperatures on Earth, and comparison reveals that the carbon dioxide concentration and the temperature tend to rise and fall in tandem. That is, past ice ages were marked by low carbon dioxide concentrations, while past warm periods were marked by higher concentrations. This leads to the question of whether the change in carbon dioxide concentration *causes* the changes in temperature. Although this graph alone does not answer that question, other statistical studies provide strong evidence that there is indeed a cause and effect.

If the cause and effect are real, then the current dramatic rise in the carbon dioxide concentration should be cause for great concern. During many of the past warm periods, Earth's average temperature was several degrees warmer than it is today, so the fact that the carbon dioxide concentration is now skyrocketing suggests that the same might happen to our planet's temperature. Moreover, notice that the past data show that the changes in concentration (and temperature) often happen quite rapidly, suggesting that the process can feed back on

itself. In that case, there's a strong risk that we have already started a process that may cause Earth's temperature to rise rapidly and dramatically over the coming decades.

The remaining question is whether the current rise in the carbon dioxide concentration is a natural phenomenon like the past rises or something being caused by humans. On this point, there is no doubt. By carefully studying the isotopic makeup of the carbon dioxide in the atmosphere and comparing it to that of various carbon dioxide sources, scientists have found that added carbon dioxide is coming primarily from the burning of fossil fuels.

Perhaps you've heard debate about whether global warming is a real or imagined threat. With a bit of extra information, such as knowing that the added carbon dioxide comes from human activity, Figure 46 makes clear that there's nothing imaginary about it.

QUESTIONS FOR DISCUSSION

1. Study Figure 46 carefully. How does the carbon dioxide concentration today compare to that of 1750? How does that of 1750 compare to that during the past 800,000 years? What conclusions can you draw from your answers to these questions?
 2. In the past, a carbon dioxide concentration of 300 parts per million accompanied global average temperatures as much as about 9°F (5°C) higher than the global average temperature today. What do you think would happen if Earth's temperature rose that much over the next century? Is that a worst-case scenario? Explain.
 3. Discuss some of the factors that will affect the future concentration of carbon dioxide in the atmosphere. What do you think should be done to slow or stop the growth in the carbon dioxide concentration?
 4. Find the latest data for the atmospheric carbon dioxide concentration. Is the level still rising? How fast is it rising?
 5. The graphs in Figure 46 summarize a great deal of scientific data. How important are clear graphics to our understanding of these data? Defend your opinion.
- • • • •

ANSWERS

Section 1

- A frequency table has two columns, one for categories and one for frequencies. Categories are the different values that a variable may have (for example, different eye colors or different ranges of income). Frequencies are the counts of the numbers of data values in each category.
- 4, 16, 32, 38, 40
- The statement does not make sense, because a column in the frequency table must consist of frequency counts, but neither of the columns of *State* and *Median Income* consist of frequency counts.
- The statement does not make sense, because each individual frequency is a whole number, and any sum of frequencies must also be a whole number, so 25.5 is not a possible value of a cumulative frequency.

9.	Grade	Frequency	Relative frequency	Cumulative frequency
	A	3	10.0%	3
	B	10	33.3%	13
	C	11	36.7%	24
	D	2	6.7%	26
	F	4	13.3%	30
	Total	30	100%	30

11.	Weight (pounds)	Frequency	Relative frequency	Cumulative frequency
	0.7900–0.7949	1	1/36	1
	0.7950–0.7999	0	0	1
	0.8000–0.8049	1	1/36	2
	0.8050–0.8099	3	3/36	5
	0.8100–0.8149	4	4/36	9
	0.8150–0.8199	17	17/36	26
	0.8200–0.8249	6	6/36	32
	0.8250–0.8299	4	4/36	36
	Total	36	100%	36

- The age category of 40 to 49 includes the most actors.

Age	Number of Actors
20–29	1
30–39	27
40–49	35
50–59	14
60–69	6
70–79	1

15.	Category	Frequency	Relative frequency
	A	12	24%
	B	9	18%
	C	12	24%
	D	11	22%
	F	6	12%
	Total	50	100%

- a. 200 b. 142 c. 16% d. 0.135, 0.155, 0.210, 0.200, 0.140, 0.160 e. 27, 58, 100, 140, 168, 200

19. a.	Rating	Frequency	Relative frequency
	0–2	20	38.5%
	3–5	14	26.9%
	6–8	15	28.8%
	9–11	2	3.8%
	12–14	1	1.9%
	Total	52	100%

b.	Rating	Frequency	Relative frequency
	0–2	33	63.5%
	3–5	19	36.5%
	6–8	0	0%
	9–11	0	0%
	12–14	0	0%
	Total	52	100%

- c. The Dvorak keyboard appears more efficient because it has more lower ratings and fewer high ones.

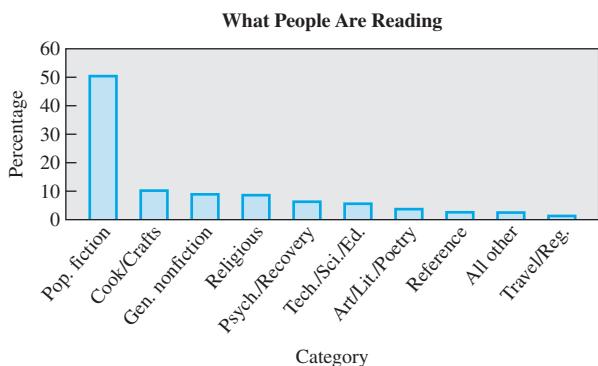
Section 2

- The distribution of data is the way data values are spread over all possible values.
- By arranging bars from highest to lowest, the Pareto chart draws attention to the most important categories. The pie chart does not do that.
- Does not make sense. Histograms require quantitative data, and names of political parties are qualitative.
- Makes sense.
- A histogram would work well to show the frequencies of the different categories of incomes.
- A time series graph would be effective in showing any trend in the number of movie theaters.
- a.

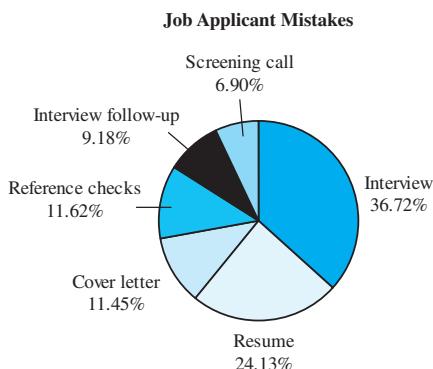
Reading Category	Relative Frequency
Popular fiction	50.4%
Cooking/Crafts	10.2%
General nonfiction	8.9%
Religious	8.6%
Psychology/Recovery	6.3%
Technical/Science/Education	5.6%
Art/Literature/Poetry	3.7%
Reference	2.6%
All other categories	2.5%
Travel/Regional	1.3%

Visual Displays of Data

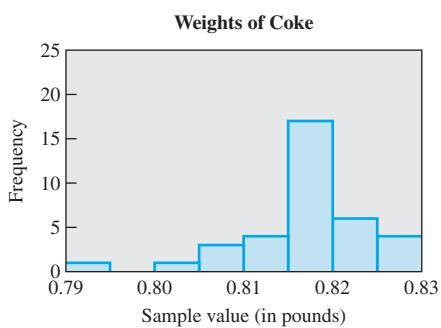
b.



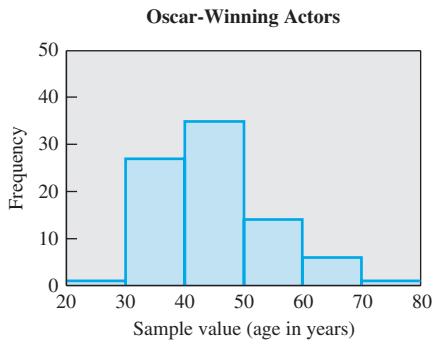
21.



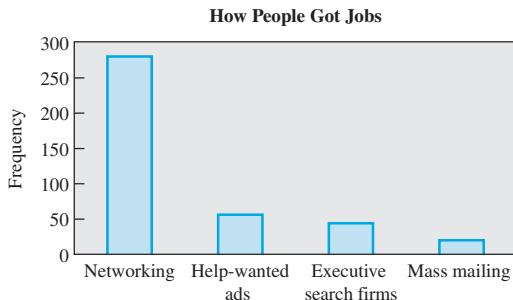
15.



17.

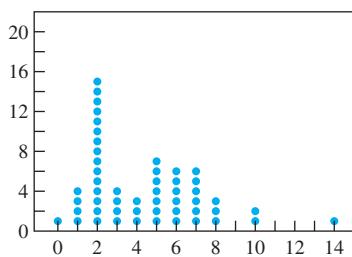


19.

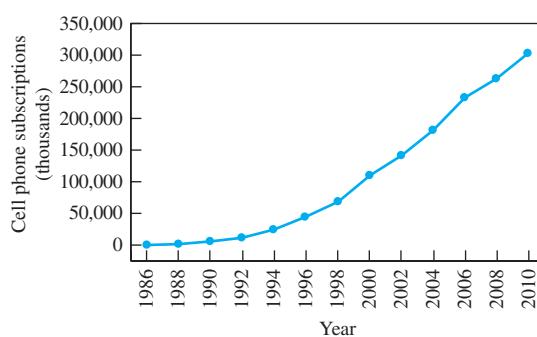


23.

QWERTY Keyboard Ratings



25.



The graph does not appear to show linear growth.

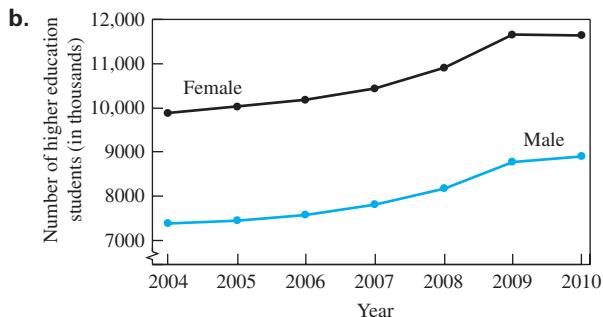
27.

Stem	Leaves
6	7
7	25
8	5899
9	09
10	0

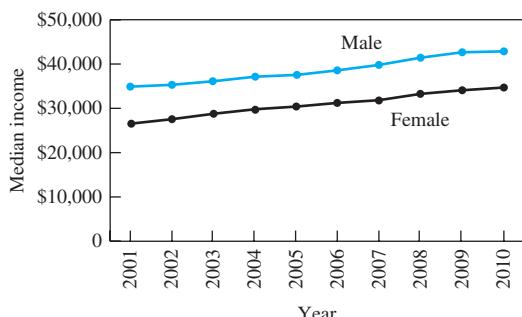
The lengths of the rows are similar to the height of bars in a histogram, so that longer rows of data correspond to higher frequencies.

Section 3

1. Yes, any histogram can be depicted as a three-dimensional histogram (as in Figure 3.18). The three-dimensional version of a histogram does provide more visual appeal, but it does not provide any additional information.
3. Geographical data are raw data corresponding to different geographic locations. Two examples of displays of geographical data are color-coded maps and contour maps.
5. Does not make sense.
7. Makes sense.
9. a. The numbers of females consistently outnumber the numbers of males. The numbers of both genders are increasing gradually over time.



11. a. Males consistently have much higher median salaries than females, and both males and females have steadily increasing salaries over time. b. The line graph makes it easier to examine the trend over time. Also, the line graph is less cluttered, so that the information is easier to understand and interpret.

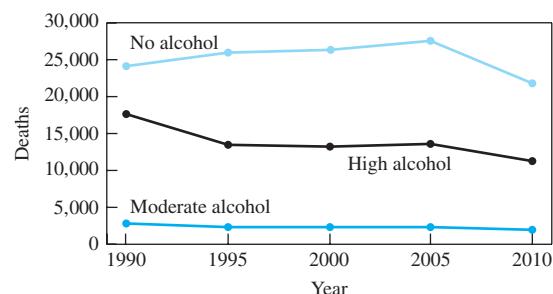


13. The stack plot shows that entitlement spending (Social Security, Medicare, Medicaid, and insurance subsidies) is projected to grow dramatically in coming decades. The graph shows that, without an increase in tax revenues, this entitlement spending would consume all government revenue by mid-century, which would leave no room for any other government programs, including the military. The clear message is that without changes, entitlement spending is on a track that would cause severe budgetary problems.
15. There appears to be a general trend of higher melanoma mortality in southern states and western states. This might be the result of people in these states spending more time outdoors, exposed to sunlight. As a researcher, you might be particularly interested in regions that deviate from general trends. For example, a county in eastern Washington state stands out

with a very high melanoma mortality. You might first want to verify that the data point is accurate, and not an error of some type. If it is accurate, you may want to find out why this one county has a higher melanoma mortality than surrounding counties.

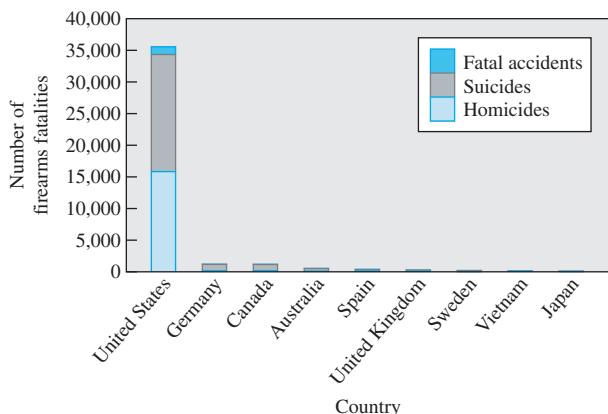
17.

Alcohol Involvement in Automobile Fatalities



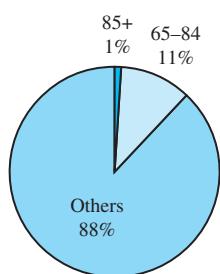
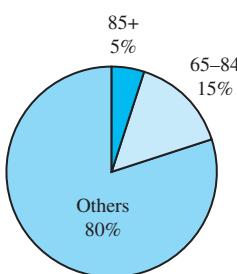
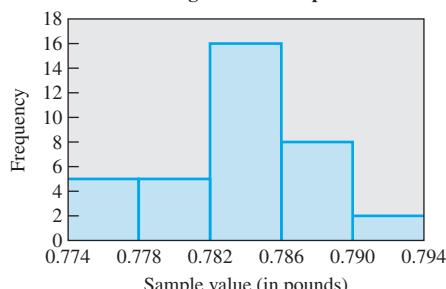
19.

Firearms Fatalities



Section 4

1. The difference of 1 foot is not meaningful. By drawing a bar graph with a vertical scale starting at a value such as 130 feet, the difference will be greatly exaggerated.
3. The scale is called an exponential scale. The advantage of such a scale is that it allows us to include values that vary over a very large range.
5. By starting the vertical scale at 30 instead of 0, the difference between the two amounts of fuel consumption is greatly exaggerated. The graph would not be misleading if it is changed so that the vertical scale begins at 0.
7. The amount of oil used by each country appears to be related to the volume of the barrels in the pictograph, when it is really related to the height of the barrels. The U.S. consumption is about four times that of Japan, not 64 times as suggested by the volumes of the barrels.
9. a, b. Because of the three-dimensional appearance of the pie charts, the sizes of the wedges on the page do not match the percentages. Instead they show how the wedges would look if the entire pie was tilted at an angle. This distortion makes it difficult to see the true relationships among the categories.

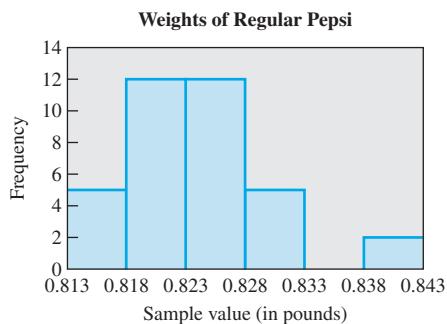
c. 1990 Age Distribution**2050 Age Distribution****b.****Weights of Diet Pepsi**

- d.** In 2050, there will be relatively more older people and fewer younger people in the U.S. population than in 1990. Note, however, that because of population growth, all age groups are expected to be larger in number in 2050 than in 1990.
- 11.** The percent change in the CPI is greatest in 1990. In 2009, the percent change in CPI is negative, so prices decreased from those in 2008. Prices have increased in every year (except for 2009), but the increases near the end of the time period are lower than those near the beginning.
- 13.** The actual minimum wage in unadjusted dollars has either remained constant or has risen steadily since 1955. The purchasing power increased from 1955 to 1968, then decreased from 1968 to 1989, and it has been relatively stable in recent years. The purchasing power in 2011 is just slightly higher than it was in 1955.

Chapter Review Exercises

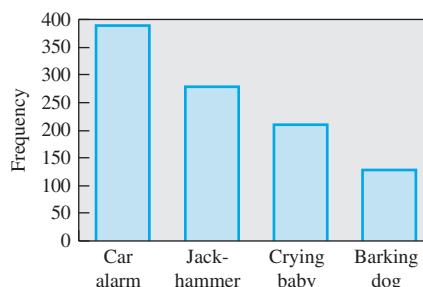
- 1. a.** The frequencies are 5, 12, 12, 5, 0, 2. **b.** The frequencies are 5, 5, 16, 8, 2. **c.** The weights of regular Pepsi are consistently larger than those of Diet Pepsi. The weights of regular Pepsi are larger because of the sugar content that is not included in the cans of Diet Pepsi.

- 3. a.**



c. The shapes of the histograms are not dramatically different, indicating that the distributions of the weights are similar. However, the range of values is very different, indicating that the weights of regular Pepsi are considerably higher than those of Diet Pepsi.

- 5.** By drawing attention to the most frustrating sounds, the Pareto chart is more effective. By using a meaningful scale, the Pareto chart also does a better job of showing the relative importance of the different frustrating sounds.



Chapter Quiz

- Histogram
- 61 min, 62 min, 62 min, 62 min, 62 min, 67 min, 69 min
- 25% of the values are between 20 and 29 inclusive.
- Using a vertical scale that does not start at zero causes the difference between the two frequencies to be exaggerated.
- 0, 1, 1, 10, 11, 12, 49, 49

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Describing Data

This chapter discusses some of the common methods for describing the center, shape, and variation of a collection of data. These methods are central to data analysis and, as you'll see, have applications to nearly every statistical study you encounter in the news. We'll conclude the chapter by studying a few surprises that occasionally turn up even when we look at data carefully.

It's no use trying to sum
people up. One must follow
hints, not exactly what is said,
nor yet entirely what is done.

—Virginia Woolf

LEARNING GOALS

1 What Is Average?

Understand the difference between a mean, median, and mode; how each is affected by outliers; and when it is appropriate to use a weighted mean.

2 Shapes of Distributions

Describe the general shape of a distribution in terms of its number of modes, skewness, and variation.

3 Measures of Variation

Understand and interpret these common measures of variation: range, the five-number summary, and standard deviation.

4 Statistical Paradoxes

Investigate a few common paradoxes that arise in statistics, such as how it is possible that most people who fail a "90% accurate" polygraph test may actually be telling the truth.

FOCUS TOPICS

Focus on the Stock Market: What's Average About the Dow?

Focus on Economics: Are the Rich Getting Richer?

1

WHAT IS AVERAGE?

The term *average* comes up frequently in the news and other reports, but it does not always have the same meaning. As you will see in this section, the most appropriate definition of *average* depends on the situation.

Mean, Median, and Mode

TABLE 1		Five Science Fiction Movie Series
Series	Number of movies (as of 2012)	
Alien	4	
Planet of the Apes	7	
Star Trek	11	
Star Wars	6	
Terminator	4	

Note: Counts only major releases in each series; does not include *Prometheus* in the *Alien* series.

Table 1 shows the number of movies (original and sequels or prequels) in each of five popular science fiction series. What is the average number of films in these series? One way to answer this question is to compute the **mean**. (The formal term of *arithmetic mean* is commonly referred to simply as the *mean*.) We find the mean by dividing the total number of movies by five (because there are five series listed in the data set):

$$\text{mean} = \frac{4 + 7 + 11 + 6 + 4}{5} = \frac{32}{5} = 6.4$$

In other words, these five series have a mean of 6.4 movies. More generally, we find the mean of any data set by dividing the sum of all the data values by the number of data values. The mean is what most people think of as the average. In essence, it represents the balance point for a quantitative data distribution, as shown in Figure 1.

We could also describe the average number of films by computing the **median**, or middle value, of the data set. To find a median, we arrange the data values in ascending (or descending) order, repeating data values that appear more than once. If the number of values is odd, there is exactly one value in the middle of the list, and this value is the median. If the number of values is even, there are two values in the middle of the list, and the median is the number that lies halfway between them. Putting the data in Table 1 in ascending order gives the list 4, 4, 6, 7, 11. The median number of movies is 6 because 6 is the middle number in the list.

The **mode** is the most common value or group of values in a data set. In the case of the movies, the mode is 4 because this value occurs twice in the data set, while the other values occur only once. A data set may have one mode, more than one mode, or no mode. Sometimes the mode refers to a group of closely spaced values rather than a single value. The mode is used more commonly for qualitative data than for quantitative data, as neither the mean nor the median can be used with qualitative data.

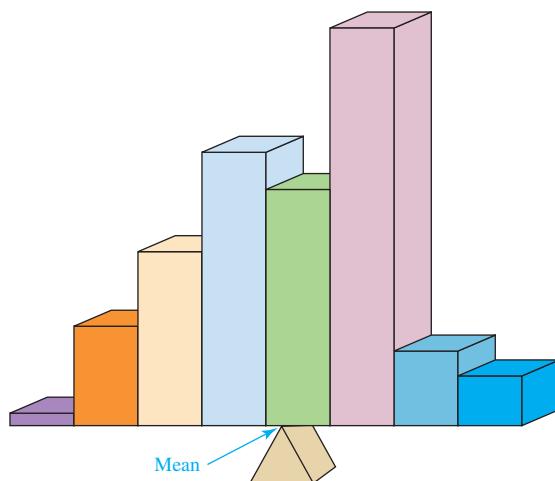


Figure 1 A histogram made from blocks would balance at the position of its mean.



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Definitions—Measures of Center in a Distribution

The **mean** is what we most commonly call the average value. It is found as follows:

$$\text{mean} = \frac{\text{sum of all values}}{\text{total number of values}}$$

The **median** is the middle value in the sorted data set (or halfway between the two middle values if the number of values is even).

The **mode** is the most common value (or group of values) in a data set.

When rounding, we will use the following rule for all the calculations discussed in this chapter.

Rounding Rule for Statistical Calculations

In general, you should express answers with *one more* decimal place of precision than is found in the raw data. For example, if the data are given as whole numbers, you should round their mean to the nearest tenth; if the data are given to the nearest tenth, you should round their mean to the nearest hundredth; and so on. *As always, round only the final answer and not any intermediate values used in your calculations.*

Notice how we applied this rule in the movie example. The data in Table 1 consist of whole numbers, so we stated the mean as 6.4. If the calculation happens to end up with the same number of significant digits as the raw data, then keeping an extra decimal place is optional.

EXAMPLE 1 Price Data

Eight grocery stores sell the PR energy bar for the following prices:

\$1.09 \$1.29 \$1.29 \$1.35 \$1.39 \$1.49 \$1.59 \$1.79

Find the mean, median, and mode for these prices.

SOLUTION The *mean* price is \$1.41:

$$\begin{aligned}\text{mean} &= \frac{\$1.09 + \$1.29 + \$1.29 + \$1.35 + \$1.39 + \$1.49 + \$1.59 + \$1.79}{8} \\ &= \$1.41\end{aligned}$$

To find the *median*, we first sort the data in ascending order:

$$\begin{array}{ccccccc} \$1.09, \$1.29, \$1.29, & \underbrace{\$1.35, \$1.39,}_{2 \text{ middle values}} & \underbrace{\$1.49, \$1.59, \$1.79}_{3 \text{ values above}} \\ 3 \text{ values below} & & & & & & \end{array}$$

**USING TECHNOLOGY—MEAN, MEDIAN, AND MODE**

Excel Excel provides the built-in function AVERAGE for calculating a mean and separate functions for MEDIAN and MODE. The screen shot below shows the use of these functions for the data from Example 1. Column B shows the functions and Column C shows the results.

	A	B	C
1	Data	1.09	
2		1.29	
3		1.29	
4		1.35	
5		1.39	
6		1.49	
7		1.59	
8		1.79	
9	Mean	=AVERAGE(B1:B8)	1.41
10	Median	=MEDIAN(B1:B8)	1.37
11	Mode	=MODE(B1:B8)	1.29
12			

Note: If you have a Windows version of Excel, you can get even more information by installing the Data Analysis Toolpak; to find it, use the Help feature and search for "Data Analysis." Once installed, click on **Data**, select **Data Analysis**, then select **Descriptive Statistics** in the pop-up window, and click **OK**. In the dialog box, enter the input range (such as A1:A8 for 8 values in column A), click on **Summary Statistics**, then click **OK**. Results will include the mean and median, as well as other statistics to be discussed in the following sections.

Alternatively, the XLSTAT add-in that is a supplement to this book can be used with both Windows and Mac computers. Enter the data as described above, click on **XLSTAT**, click on **Visualizing Data**, then select Descriptive Statistics. Enter the range of the data, such as A1:A8. (If the first row is the name of the data, be sure to click on the box next to "Sample labels."

STATDISK Enter the data in the Data Window or open an existing data set. Click on **Data** and select **Descriptive Statistics**. Now click on **Evaluate** to get the various descriptive statistics, including the mean and median, as well as other statistics to be discussed in the following sections.

TI-83/84 PLUS First enter the data in list L1 by pressing **STAT**, then selecting **Edit** and pressing the **ENTER** key. After the data values have been entered, press **STAT** and select **CALC**, then select **1-Var Stats** and press the **ENTER** key twice. The display will include the mean and median, as well as other statistics to be discussed in the following sections. Use the down-arrow key **▼** to view the results that don't fit on the initial display.

Because there are eight prices (an even number), there are two values in the middle of the list: \$1.35 and \$1.39. Therefore the median lies halfway between these two values, which we calculate by adding them and dividing by 2:

$$\text{median} = \frac{\$1.35 + \$1.39}{2} = \$1.37$$

Using the rounding rule, we could express the mean and median as \$1.410 and \$1.370, respectively, though the extra zeros are optional in this case.

The *mode* is \$1.29 because this price occurs more times than any other price. •••

Effects of Outliers

To explore the differences among the mean, median, and mode, imagine that the five graduating seniors on a college basketball team receive the following first-year contract offers to play in the National Basketball Association (zero indicates that the player did not receive a contract offer):

0 0 0 0 \$10,000,000

The mean contract offer is

$$\text{mean} = \frac{0 + 0 + 0 + 0 + \$10,000,000}{5} = \$2,000,000$$

Is it therefore fair to say that the *average* senior on this basketball team received a \$2 million contract offer?

TABLE 2

Comparison of Mean, Median, and Mode

Measure	Definition	How common?	Existence	Takes every value into account?	Affected by outliers?	Advantages
Mean	$\frac{\text{sum of all values}}{\text{total number of values}}$	most familiar “average”	always exists	yes	yes	commonly understood; works well with many statistical methods
Median	middle value	common	always exists	no (aside from counting the total number of values)	no	when there are outliers, may be more representative of an “average” than the mean
Mode	most frequent value	sometimes used	may be no mode, one mode, or more than one mode	no	no	most appropriate for qualitative data

Not really. The problem is that the single player receiving the large offer makes the mean much larger than it would be otherwise. If we ignore this one player and look only at the other four, the mean contract offer is zero. Because this one value of \$10,000,000 is so extreme compared with the others, we say that it is an **outlier** (or *outlying value*). As our example shows, an outlier can pull the mean significantly upward (or downward), thereby making the mean unrepresentative of the data set as a whole.

Definition

An **outlier** in a data set is a value that is much higher or much lower than almost all other values.

While the outlier pulls the mean contract offer upward, it has no effect on the median contract offer, which remains zero for the five players. In general, the value of an outlier has no effect on the median, because outliers don’t lie in the middle of a data set. Outliers do not affect the mode either. Table 2 summarizes the characteristics of the mean, median, and mode, including the effects of outliers on each measure.

TIME OUT TO THINK

Is it fair to use the median as the average contract offer for the five players? Why or why not?

Deciding how to deal with outliers is one of the more important issues in statistics. Sometimes, as in our basketball example, an outlier is a legitimate value that must be understood in order to interpret the mean and median properly. Other times, outliers may indicate mistakes in a data set. Deciding when outliers are important and when they may simply be mistakes can be very difficult.

EXAMPLE 2 Mistake?

A track coach wants to determine an appropriate heart rate for her athletes during their workouts. She chooses five of her best runners and asks them to wear heart monitors during a workout. In the middle of the workout, she reads the following heart rates for the five athletes: 130, 135, 140, 145, 325. Which is a better measure of the average in this case—the mean or the median? Why?

SOLUTION Four of the five values are fairly close together and seem reasonable for mid-workout heart rates. The high value of 325 is an outlier. This outlier seems likely to be a mistake (perhaps caused by a faulty heart monitor), because anyone with such a high heart rate should be in cardiac arrest. If the coach uses the mean as the average, she will be including this

BY THE WAY

A survey once found that geography majors from the University of North Carolina had a far higher mean starting salary than geography majors from other schools. The reason for the high mean turned out to be a single outlier—the basketball superstar and geography major named Michael Jordan.





outlier—which means she will be including any mistake made when it was recorded. If she uses the median as the average, she'll have a more reasonable value, because the median won't be affected by the outlier.



“Average” Confusion

The different meanings of *average* can lead to confusion. Sometimes this confusion arises because we are not told whether the average is the mean or the median, and other times because we are not given enough information about how the average was computed. The following examples illustrate two such situations.

EXAMPLE 3 Wage Dispute

A newspaper surveys wages for workers in regional high-tech companies and reports an average of \$22 per hour. The workers at one large firm immediately request a pay raise, claiming that they work as hard as employees at other companies but their average wage is only \$19. The management rejects their request, telling them that they are *overpaid* because their average wage, in fact, is \$23. Can both sides be right? Explain.



SOLUTION Both sides can be right if they are using different definitions of *average*. In this case, the workers may be using the median while the management uses the mean. For example, imagine that there are only five workers at the company and their wages are \$19, \$19, \$19, \$19, and \$39. The median of these five wages is \$19 (as the workers claimed), but the mean is \$23 (as management claimed).



EXAMPLE 4 Which Mean?

All 100 first-year students at a small college take three courses in the Core Studies program. Two courses are taught in large lectures, with all 100 students in a single class. The third course is taught in 10 classes of 10 students each. Students and administrators get into an argument about whether classes are too large. The students claim that the mean size of their Core Studies classes is 70. The administrators claim that the mean class size is only 25. Can both sides be right? Explain

Figures won’t lie, but liars will figure.

—Charles H. Grosvenor

SOLUTION The students calculated the mean size of the classes in which each student is personally enrolled. Each student is taking two classes with enrollments of 100 and one class with an enrollment of 10, so the mean size of each student's classes is

$$\frac{\text{total enrollment in student's classes}}{\text{number of classes student is taking}} = \frac{100 + 100 + 10}{3} = 70$$

The administrators calculated the mean enrollment in all classes. There are two classes with 100 students and 10 classes with 10 students, making a total enrollment of 300 students in 12 classes. The mean enrollment per class is

$$\frac{\text{total enrollment}}{\text{number of classes}} = \frac{300}{12} = 25$$

The two claims about the mean are both correct, but the two sides are talking about different means. The students calculated the mean *class size per student*, while the administrators calculated the mean number of *students per class*.



TIME OUT TO THINK

In Example 4, could the administrators redistribute faculty assignments so that all classes have 25 students each? How? Discuss the advantages and disadvantages of such a change.

Weighted Mean

Suppose your course grade is based on four quizzes and one final exam. Each quiz counts as 15% of your final grade, and the final counts as 40%. Your quiz scores are 75, 80, 84, and 88, and your final exam score is 96. What is your overall score?

Because the final exam counts more than the quizzes, a simple mean of the five scores does not give your final score. Instead, we must assign a *weight* (indicating the relative importance) to each score. In this case, we assign weights of 15 (for the 15%) to each of the quizzes and 40 (for the 40%) to the final. We then find the **weighted mean** by adding the products of each score and its weight and then dividing by the sum of the weights:

$$\begin{aligned}\text{weighted mean} &= \frac{(75 \times 15) + (80 \times 15) + (84 \times 15) + (88 \times 15) + (96 \times 40)}{15 + 15 + 15 + 15 + 40} \\ &= \frac{8745}{100} = 87.45\end{aligned}$$

The weighted mean of 87.45 properly accounts for the different weights of the quizzes and the exam. Following the rounding rule, we round this score to 87.5.

Weighted means are appropriate whenever the data values vary in their degree of importance. You can always find a weighted mean using the following formula.

BY THE WAY

Sports statistics that rate players or teams according to their performance in many different categories are usually weighted means. Examples include the earned run average (ERA) and slugging percentage in baseball, the quarterback rating in football, and computerized rankings of college teams.

Definitions

A **weighted mean** accounts for variations in the relative importance of data values. Each data value is assigned a weight and the weighted mean is

$$\text{weighted mean} = \frac{\text{sum of (each data value} \times \text{its weight)}}{\text{sum of all weights}}$$

TIME OUT TO THINK

Because the weights are percentages in the course grade example, we could think of the weights as 0.15 and 0.40 rather than 15 and 40. Calculate the weighted mean by using the weights of 0.15 and 0.40. Do you still find the same answer? Why or why not?

EXAMPLE 5 GPA

Randall has 38 credits with a grade of A, 22 credits with a grade of B, and 7 credits with a grade of C. What is his grade point average (GPA)? Base the GPA on values of 4.0 points for an A, 3.0 points for a B, and 2.0 points for a C.

SOLUTION The grades of A, B, and C represent data values of 4.0, 3.0, and 2.0, respectively. The numbers of credits are the weights. The As represent a data value of 4 with a weight of 38, the Bs represent a data value of 3 with a weight of 22, and the Cs represent a data value of 2 with a weight of 7. The weighted mean is

$$\text{weighted mean} = \frac{(4 \times 38) + (3 \times 22) + (2 \times 7)}{38 + 22 + 7} = \frac{232}{67} = 3.46$$

Following our rounding rule, we round Randall's GPA from 3.46 to 3.5.

EXAMPLE 6 Stock Voting

Voting in corporate elections is usually weighted by the amount of stock owned by each voter. Suppose a company has five stockholders who vote on whether the company should embark on a new advertising campaign. The votes (Y = yes, N = no) are as follows:

Stockholder	Shares owned	Vote
A	225	Y
B	170	Y
C	275	Y
D	500	N
E	90	N

According to the company's bylaws, the measure needs 60% of the vote to pass. Does it pass?

SOLUTION We can regard a yes vote as a value of 1 and a no vote as a value of 0. The number of shares is the weight for the vote of each stockholder, so Stockholder A's vote represents a value of 1 with a weight of 225, stockholder B's vote represents a value of 1 with a weight of 170, and so on. The weighted mean vote is

$$\begin{aligned} \text{weighted mean} &= \frac{(1 \times 225) + (1 \times 170) + (1 \times 275) + (0 \times 500) + (0 \times 90)}{225 + 170 + 275 + 500 + 90} \\ &= \frac{670}{1260} \approx 0.53 \end{aligned}$$

The weighted vote is 53% (or 0.53) in favor, which is short of the required 60%, so the measure does not pass.

**Means with Summation Notation (Optional Section)**

Many statistical formulas, including the formula for the mean, can be written compactly with a mathematical notation called *summation notation*. The symbol Σ (the Greek capital letter *sigma*) is called the *summation sign* and indicates that a set of numbers should be added. We use the symbol x to represent *each* value in a data set, so we write the sum of all the data values as

$$\text{sum of all values} = \Sigma x$$

TECHNICAL NOTE

Summations are often written with the use of an *index* that specifies how to step through the sum. For example, the symbol x_i indicates the *i*th data value in the set; the letter *i* is the index. We then write the sum of all values as

$$\sum_{i=1}^n x_i$$

We read this expression as "the sum of the x_i values, starting with $i = 1$ and continuing to $i = n$, where n is the total number of data values in the set." With this notation, the mean is written

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$$

For example, if a sample consists of 25 exam scores, Σx represents the sum of all 25 scores. Similarly, if a sample consists of the incomes of 10,000 families, Σx represents the total dollar value of all 10,000 incomes.

We use n to represent the total number of values in the sample. Thus, the general formula for the mean is

$$\bar{x} = \text{sample mean} = \frac{\text{sum of all values}}{\text{total number of values}} = \frac{\Sigma x}{n}$$

The symbol \bar{x} is the standard symbol for the mean of a sample. When dealing with the mean of a population rather than a sample, statisticians instead use the Greek letter μ (mu).

Summation notation also makes it easy to express a general formula for the weighted mean. Again we use the symbol x to represent each data value, and we let w represent the weight of each data value. The sum of the products of each data value and its corresponding weight is $\Sigma(x \times w)$. The sum of the weights is Σw . Therefore, the formula for the weighted mean is

$$\text{weighted mean} = \frac{\Sigma(x \times w)}{\Sigma w}$$

Means and Medians with Binned Data (Optional Section)

The ideas of this section can be extended to binned data simply by assuming that the middle value in the bin represents all the data values in the bin. For example, consider the following table of 50 binned integer data values:

Bin	Frequency
0–6	10
7–13	10
14–20	10
21–27	20

The middle value of the first bin is 3, so we assume that the value of 3 occurs 10 times. Continuing this way, the sum of the 50 values in the table is

$$(3 \times 10) + (10 \times 10) + (17 \times 10) + (24 \times 20) = 780$$

The mean is therefore $780/50 = 15.6$. With 50 values, the median is between the 25th and 26th values. These values fall within the bin 14–20, so we call this bin the **median class** for the data. The mode is the bin with the highest frequency—the bin 21–27 in this case.

Section 1 Exercises

Statistical Literacy and Critical Thinking

- Average and Mean.** Do the terms “average” and “mean” have the same meaning? Explain.
- Mean and Median.** A statistics class consists of 24 students, all but one of whom are unemployed or are employed in low-paying part-time jobs. One student works as an executive secretary earning \$75,000 per year. Which does a better job of describing the income of a typical student in the class: the mean or the median? Why?
- Outlier.** For the same statistics class described in Exercise 2, is the executive secretary’s salary an outlier? Why or why not? In general, is an outlier defined in an exact way so that it can be clearly and objectively identified?
- Mean Income.** An economist wants to find the mean annual income for all adults in the United States. She knows that it is not practical to survey each member of the adult population, so she refers to an almanac and finds the mean income listed for each of the 50 states. She adds the 50 state means and divides by 50. Is the result the mean income for the United States? Why or why not?
- Does It Make Sense?** For Exercises 5–8, decide whether the statement makes sense (or is clearly true) or does not make sense (or is clearly false). Explain clearly; not all of these statements have definitive answers, so your explanation is more important than your chosen answer.
 - Mean.** The number on the jersey of each New York Giants football player is recorded, then the mean of those numbers is computed.
 - Mode.** A data set of incomes has modes of \$50,000 and \$80,000.
- Mean, Median, and Mode.** A researcher studying an income distribution obtains the same value of \$75,000 for the mean, median, and mode.
- Weighted Mean.** A professor calculates final grades using a weighted mean in which the final exam counts twice as much as the midterm.
- Appropriate Average.** Exercises 9–12 list “averages” that someone might want to know. In each case, state whether the mean or median would give a better description of the “average.” Explain your reasoning.
- Height.** The average height of all active professional basketball players
- Salary.** The average salary of all active professional basketball players
- Ages.** The following ages (years) of survey respondents: 22, 19, 21, 27, “over 65,” “over 80”
- Pulse Rates.** The average resting pulse rate of 500 randomly selected female statistics students

Concepts and Applications

- Mean, Median, and Mode.** Exercises 13–20 each list a set of numbers. In each case, find the mean, median, and mode of the listed numbers.

- 13. Number of Words.** Pages from *Merriam-Webster's Collegiate Dictionary*, 11th edition, were randomly selected. Here are the numbers of words defined on those pages:

51 63 36 43 34 62 73 39 53 79

- 14. Space Shuttle Flights.** Listed below are the durations (in hours) of a sample of all flights of NASA's Space Transport System (space shuttle):

73 95 235 192 165 262 191 376
259 235 381 331 221 244 0

- 15. Perception of Time.** Actual times (in seconds) recorded when statistics students participated in an experiment to test their ability to determine when one minute (60 seconds) had passed:

53 52 75 62 68 58 49 49

- 16. Body Temperatures.** Body temperatures (in degrees Fahrenheit) of randomly selected normal and healthy adults:

98.6 98.6 98.0 98.0 99.0
98.4 98.4 98.4 98.4 98.6

- 17. Blood Alcohol.** Blood alcohol concentrations of drivers involved in fatal crashes and then given jail sentences (based on data from the U.S. Department of Justice):

0.27 0.17 0.17 0.16 0.13 0.24
0.29 0.24 0.14 0.16 0.12 0.16

- 18. Old Faithful Geyser.** Time intervals (in minutes) between eruptions of Old Faithful geyser in Yellowstone National Park:

98 92 95 87 96 90
65 92 95 93 98 94

- 19. Weights of M&Ms.** Weights (in grams) of randomly selected M&M plain candies:

0.957 0.912 0.842 0.925 0.939 0.886
0.914 0.913 0.958 0.947 0.920

- 20. Quarters.** Weights (in grams) of quarters in circulation:

5.60 5.63 5.58 5.56 5.66 5.58 5.57 5.59
5.67 5.61 5.84 5.73 5.53 5.58 5.52 5.65
5.57 5.71 5.59 5.53 5.63 5.68

- 21. Alphabetic States.** The following table gives the total area in square miles (land and water) of the seven states with names beginning with the letters A through C.

State	Area
Alabama	52,200
Alaska	615,200
Arizona	114,000
Arkansas	53,200
California	158,900
Colorado	104,100
Connecticut	5,500

- a. Find the mean area and median area for these states.
b. Which state is an outlier on the high end? If you eliminate this state, what are the new mean and median areas for this data set?

- c. Which state is an outlier on the low end? If you eliminate this state, what are the new mean and median areas for this data set?

- 22. Outlier Coke.** Cans of regular Coca-Cola vary slightly in weight. Here are the measured weights of seven cans, in pounds:

0.8161 0.8194 0.8165 0.8176
0.7901 0.8143 0.8126

- a. Find the mean and median of these weights.
b. Which, if any, of these weights would you consider to be an outlier? Explain.
c. What are the mean and median weights if the outlier is excluded?

- 23. Raising Your Grade.** Suppose you have scores of 80, 84, 87, and 89 on quizzes in a mathematics class.

- a. What is the mean of these scores?
b. What score would you need on the next quiz to have an overall mean of 88?
c. If the maximum score on a quiz is 100, is it possible to have a mean of 90 after the fifth quiz? Explain.

- 24. Raising Your Grade.** Suppose you have scores of 60, 70, 65, 85, and 85 on exams in a sociology class.

- a. What is the mean of these scores?
b. What score would you need on the next exam to have an overall mean of 75?
c. If the maximum score on an exam is 100, what is the maximum mean score that you could possibly have after the next exam? Explain.

- 25. Comparing Averages.** Suppose that school district officials claim that the average reading score for fourth-graders in the district is 73 (out of a possible 100). As a principal, you know that your fourth-graders had the following scores: 55, 60, 68, 70, 87, 88, 95. Would you be justified in claiming that your students scored above the district average? Explain.

- 26. Comparing Averages.** Suppose the National Basketball Association (NBA) reports that the average height of basketball players is 6'8". As a coach, you know that the players on your starting lineup have heights of 6'5", 6'6", 6'6", 7'0", and 7'2". Would you be justified in claiming that your starting lineup has above average height for the NBA? Explain.

- 27. Average Peaches.** A grocer has three baskets of peaches. One holds 50 peaches and weighs 18 pounds, one holds 55 peaches and weighs 22 pounds, and the third holds 60 peaches and weighs 24 pounds. What is the mean weight of all of the peaches combined? Explain.

- 28. Average Confusion.** An instructor has a first-period class with 25 students, and they had a mean score of 86% on the midterm exam. The second-period class has 30 students, and they had a mean score of 84% on the same exam. Does it follow that the mean score for both classes combined is 85%? Explain.

- 29. Different Means?** Each of the 300 students at a high school takes the same four courses. Three of the courses are each taught in 15 classes of 20 students each. The fourth course is taught in 3 classes of 100 students each. Find the mean class size experienced by each student. Find the mean class size in the 48 courses. Are the two means the same?
- 30. Final Grade.** Your course grade is based on one midterm that counts as 15% of your final grade, one class project that counts as 20% of your final grade, a set of homework assignments that counts as 40% of your final grade, and a final exam that counts as 25% of your final grade. Your midterm score is 75, your project score is 90, your homework score is 85, and your final exam score is 72. What is your overall final score?
- 31. Batting Average.** A batting average in baseball is determined by dividing the total number of hits by the total number of at-bats (neglecting walks, sacrifices, and a few other special cases). A player goes 2 for 4 (2 hits in 4 at-bats) in the first game, 0 for 3 in the second game, and 3 for 5 in the third game. What is his batting average? In what way is this number an “average”?
- 32. Averaging Averages.** Suppose a player has a batting average over many games of 0.200 (he’s not very good). In his next game, he goes 2 for 4, which is a batting average of 0.500 for the game. Does it follow that his new batting average is $(0.200 + 0.500)/2 = 0.350$? Explain.
- 33. Batteries.** A manufacturer uses two different production sites to make batteries for cell phones. There is a defect rate of 2% at one of the sites, and the defect rate at the other site is 4%. Does it follow that when the batteries from the two sites are combined, the overall rate of defects is 3%? Explain.
- 34. Slugging Average.** In addition to the batting average, another measure of hitting performance in baseball is called the slugging average. In finding a slugging average, a single is worth 1 point, a double is worth 2 points, a triple is worth 3 points, and a home run is worth 4 points. A player’s slugging average is the total number of points divided by the total number of
- at-bats (neglecting walks, sacrifices, and a few other special cases). A player has three singles in five at-bats in the first game, a triple and a single in four at-bats in the second game, and a double and a home run in five at-bats in the third game.
- What is his batting average?
 - What is his slugging average?
 - Is it possible for a slugging average to be more than 1? Explain.
- 35. Stockholder Voting.** A small company has four stockholders. One stockholder has 400 shares, a second stockholder has 300 shares, a third stockholder has 200 shares, and the fourth stockholder has 100 shares. In a vote on a new advertising campaign, the first stockholder votes yes, and the other three stockholders vote no. Explain how the outcome of the vote can be expressed as a weighted mean. What is the outcome of the vote?
- 36. GPA.** One common system for computing a grade point average (GPA) assigns 4 points to an A, 3 points to a B, 2 points to a C, 1 point to a D, and 0 points to an F. What is the GPA of a student who gets an A in a 4-credit course, a B in each of two 3-credit courses, and a C in a 1-credit course?
- 37. Phenotypes of Peas.** An experiment was conducted to determine whether a deficiency of carbon dioxide in soil affects the phenotypes of peas. Listed below are the phenotype codes, where 1 = smooth yellow, 2 = smooth green, 3 = wrinkled yellow, and 4 = wrinkled green. Can the measures of center be obtained for these values? Do the results make sense?
- | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 1 | 1 | 1 | 1 | 1 | 1 | 4 | 1 | 2 | 2 | 1 | 2 |
| 3 | 3 | 2 | 3 | 1 | 3 | 1 | 3 | 1 | 3 | 2 | 2 | |
- 38. U.S. Population Center.** Imagine taking a huge flat map of the United States and placing weights on it to represent where people live. The point at which the map would balance is called the mean center of population. Figure 2

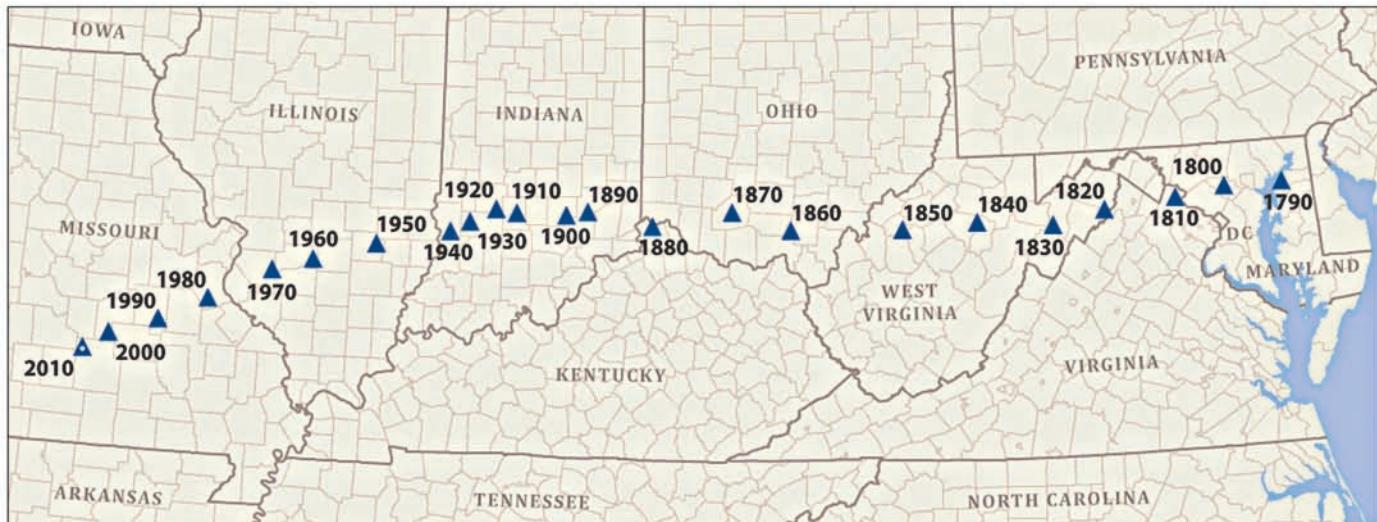


Figure 2 Mean center of population.

Source: Statistical Abstract of the United States.

shows how the location of the mean center of population has shifted from 1790 to 2010. Briefly explain the pattern shown on this map.

PROJECTS FOR THE INTERNET & BEYOND

39. Salary Data. Many Web sites offer data on salaries in different careers. Find salary data for a career you are considering. What are the mean and median salaries for this career? How do these salaries compare with those of other careers that interest you?

40. Is the Median the Message? Read the article “The Median Isn’t the Message,” by Stephen Jay Gould, which is posted on the Web. Write a few paragraphs in which you describe the message that Gould was trying to get across. How is this message important to other patients diagnosed with cancer?

41. Navel Data. The *navel ratio* is defined to be a person’s height divided by the height (from the floor) of his or her navel. Measure the navel ratio of each person in your class and bin the data. Then make a frequency table, and draw

a histogram of the distribution. What is the mean of the distribution? What is the median of the distribution? An old theory says that, on average, the navel ratio of humans is the golden ratio: $(1 + \sqrt{5})/2$. Does this theory seem accurate based on your observations?

IN THE NEWS

42. Daily Averages. Cite three examples of averages that you deal with in your own life (such as grade point average or batting average). In each case, explain whether the average is a mean, a median, or some other type of average. Briefly describe how the average is useful to you.

43. Averages in the News. Find three recent news articles that refer to some type of average. In each case, explain whether the average is a mean, a median, or some other type of average.

2

SHAPES OF DISTRIBUTIONS

In the previous section we discussed how to describe the center of a quantitative data distribution with measures such as the mean and median. We now turn our attention to the overall *shape* of a distribution, which we often describe with three characteristics: its number of modes, its symmetry or skewness, and its variation. Although these three characteristics carry less information than a complete graph of the distribution, they are still useful.

Note that, because we are interested in the *general* shapes of distributions, it’s often easier to examine graphs that show smooth curves to fit the original data sets. Figure 3 shows three examples of this idea, two in which the distributions are shown as histograms and one in which the distribution is shown as a line chart. In each case, the smooth curves make good approximations to the original distributions.

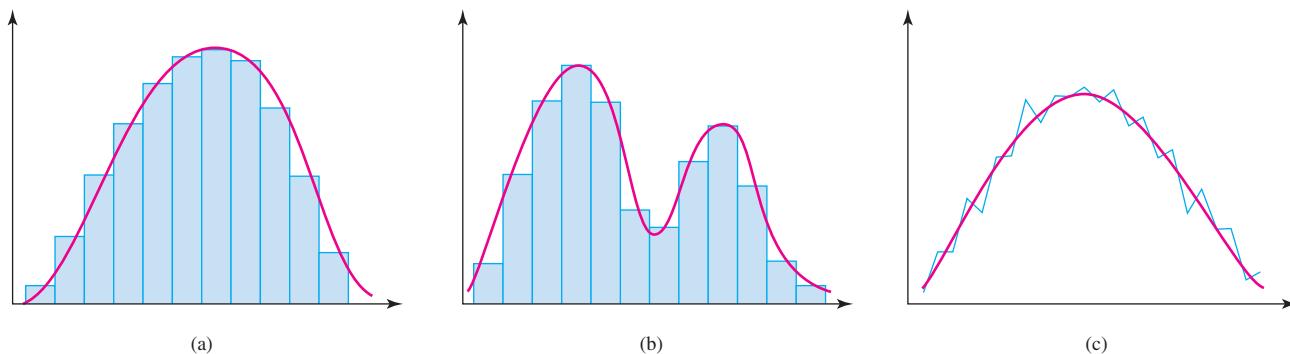


Figure 3 The smooth curves approximate the shapes of the distributions.

Number of Modes

One simple way to describe the shape of a distribution is by its number of peaks, or modes. Figure 4a shows a distribution, called a **uniform distribution**, that has no mode because all data values have the same frequency. Figure 4b shows a distribution with a single peak as its mode. It is called a **single-peaked**, or **unimodal**, distribution. By convention, any peak in a distribution is considered a mode, even if not all peaks have the same height. For example,

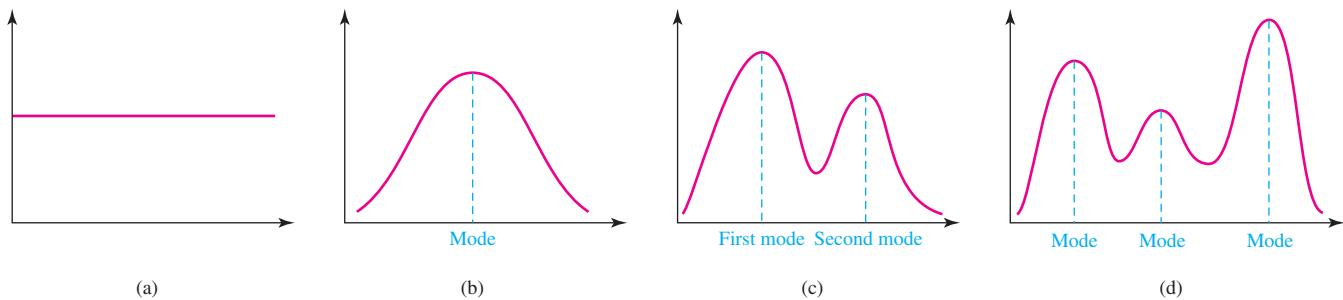


Figure 4 (a) A uniform distribution has no mode. (b) A single-peaked distribution has one mode. (c) A bimodal distribution has two modes. (d) A trimodal distribution has three modes.

the distribution in Figure 4c is said to have two modes, even though the second peak is lower than the first; it is a *bimodal* distribution. Similarly, the distribution in Figure 4d is said to have three modes; it is a *trimodal* distribution.

EXAMPLE 1 Number of Modes

How many modes would you expect for each of the following distributions? Why? Make a rough sketch for each distribution, with clearly labeled axes.

- Heights of 1,000 randomly selected adult women
- Hours spent watching football on TV in January for 1,000 randomly selected adult Americans
- Weekly sales throughout the year at a retail clothing store for children
- The number of people with particular last digits (0 through 9) in their Social Security numbers

SOLUTION Figure 5 shows sketches of the distributions.

- The distribution of heights of women is single-peaked because many women are at or near the mean height, with fewer and fewer women at heights much greater or less than the mean.
- The distribution of times spent watching football on TV for 1,000 randomly selected adult Americans is likely to be bimodal (two modes). One mode represents the mean watching time of men, and the other represents the mean watching time of women.
- The distribution of weekly sales throughout the year at a retail clothing store for children is likely to have several modes. For example, it will probably have a mode in spring for sales of summer clothing, a mode in late summer for back-to-school sales, and another mode in winter for holiday sales.
- The last digits of Social Security numbers are essentially random, so the number of people with each different last digit (0 through 9) should be about the same. That is, about 10% of all Social Security numbers end in 0, 10% end in 1, and so on. It is therefore a uniform distribution with no mode.

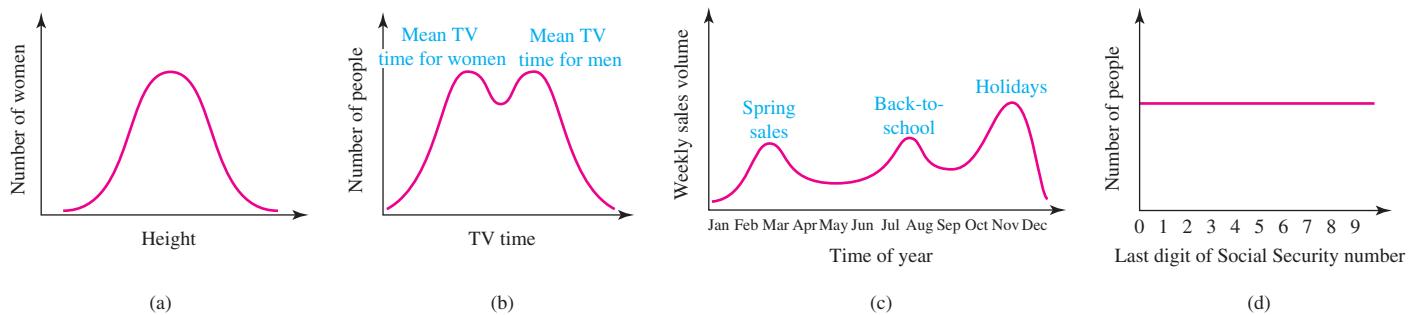


Figure 5 Sketches for Example 1.



Symmetry or Skewness

A second simple way to describe the shape of a distribution is in terms of its symmetry or skewness. A distribution is **symmetric** if its left half is a mirror image of its right half. The distributions in Figure 6 are all symmetric. The symmetric distribution in Figure 6a, with a single peak and a characteristic bell shape, is known in statistics as a *normal distribution*.

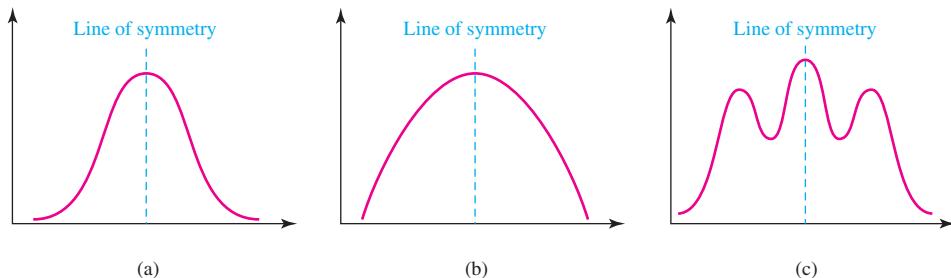


Figure 6 These distributions are all symmetric because their left halves are mirror images of their right halves. Note that (a) and (b) are single-peaked (unimodal), whereas (c) is triple-peaked (trimodal).

A distribution that is not symmetric must have values that tend to be more spread out on one side than on the other. In this case, we say that the distribution is **skewed**. Figure 7a shows a distribution in which the values are more spread out on the left, meaning that some values are outliers at low values. We say that such a distribution is **left-skewed** (or *negatively skewed*), because it looks as if it has a tail that has been pulled toward the left. Figure 7b shows a distribution in which the values are more spread out on a tail extending to the right, making it **right-skewed** (or *positively skewed*).

Figure 7 also shows how skewness affects the relative positions of the mean, median, and mode. By definition, the mode is at the peak in a single-peaked distribution. A left-skewed distribution pulls both the mean and median to the left of the mode, meaning to values less than the mode. In addition, outliers at the low end of the data set make the mean less than the median (see Table 2). Similarly, a right-skewed distribution pulls the mean and median to the right of the mode, and the outliers at the high end of the data set make the mean greater than the median. When the distribution is symmetric and single-peaked, both the mean and the median are equal to the mode.

TECHNICAL NOTE

A left-skewed distribution is also called *negatively skewed*, and a right-skewed distribution is also called *positively skewed*. A symmetric distribution has *zero skewness*.

Definitions

A distribution is **symmetric** if its left half is a mirror image of its right half.

A distribution is **left-skewed** if its values are more spread out on the left side.

A distribution is **right-skewed** if its values are more spread out on the right side.

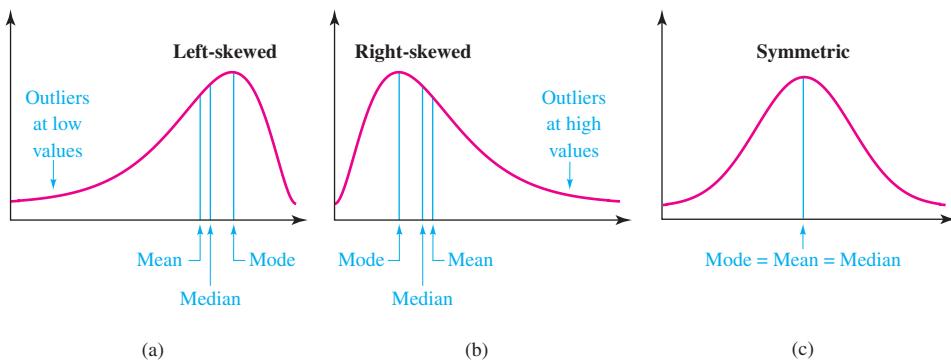


Figure 7 (a) Skewed to the left (left-skewed): The mean and median are less than the mode. (b) Skewed to the right (right-skewed): The mean and median are greater than the mode. (c) Symmetric distribution: The mean, median, and mode are the same.

TIME OUT TO THINK

Which is a better measure of “average” (or of the *center* of the distribution) for a skewed distribution: the median or the mean? Why?

EXAMPLE 2 Skewness

For each of the following situations, state whether you expect the distribution to be symmetric, left-skewed, or right-skewed. Explain.

- Heights of a sample of 100 women
- Family income in the United States
- Speeds of cars on a road where a visible patrol car is using radar to detect speeders

SOLUTION

- The distribution of heights of women is symmetric, because roughly equal numbers of women are shorter and taller than the mean and extremes of height are rare on either side of the mean.
- The distribution of family incomes is right-skewed. Most families are middle-class, so the mode of this distribution is a middle-class income. But a few very high-income families pull the mean to a considerably higher value, stretching the distribution to the right (high-income) side.
- Drivers usually slow down when they are aware of a patrol car looking for speeders. Few if any drivers will be exceeding the speed limit, but some drivers tend to slow to well below the speed limit. Thus, the distribution of speeds is therefore left-skewed, with a mode near the speed limit but a few cars going well below the speed limit.

BY THE WAY

Median family income in the United States is about \$50,000, which is substantially lower than the *mean* family income of about \$68,000 (Data for 2011).

TIME OUT TO THINK

In ordinary English, the term *skewed* is often used to mean something that is distorted or depicted in an unfair way. How is this use of *skew* related to its meaning in statistics?

BY THE WAY

Speed kills. On average, in the United States, someone is killed in an auto accident about every 12 minutes. About one-third of these fatalities involve a speeding driver.

Variation

A third way to describe a distribution is by its **variation**, which is a measure of how much the data values are spread out. A distribution in which most data are clustered together has a low variation. As shown in Figure 8a, such a distribution has a fairly sharp peak. The variation is higher when the data are distributed more widely around the center, which makes the peak broader. Figure 8b shows a distribution with a moderate variation and Figure 8c shows a distribution with a high variation. We’ll discuss methods for describing the variation quantitatively in the next section.

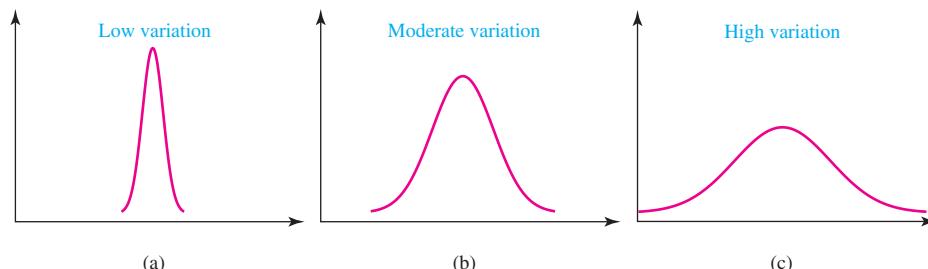


Figure 8 From left to right, these three distributions have increasing variation.

Definitions

Variation describes how widely data are spread out about the center of a data set.

EXAMPLE 3 Variation in Marathon Times

How would you expect the variation to differ between times in the Olympic marathon and times in the New York City marathon? Explain.

SOLUTION The Olympic marathon invites only elite runners, whose times are likely to be clustered relatively near world-record times. The New York City marathon allows runners of all abilities, whose times are spread over a very wide range (from near the world record of just over two hours to many hours). Therefore, the variation among the times should be greater in the New York City marathon than in the Olympic marathon.



Section 2 Exercises

Statistical Literacy and Critical Thinking

1. Symmetry. In the United States, there are many people with little or no accumulated wealth, and there are a few people with very large amounts of wealth. What does this suggest about the symmetry of the distribution of wealth?

2. Distribution. When the digits 0 through 9 are selected for a state lottery, the digits are selected in a way that they are all equally likely. Which term best describes the distribution of selected digits: skewed, bimodal, uniform, or unimodal?

3. IQ Scores. Consider the IQ scores of professors who teach statistics courses compared with the IQ scores of adults randomly selected from the general population. Which of these two sets of IQ scores has less variation? What effect does the lower variation have on a graph of the distribution of those IQ scores?

4. Skewness. What is skewness in a graph?

Does It Make Sense? For Exercises 5–8, decide whether the statement makes sense (or is clearly true) or does not make sense (or is clearly false). Explain clearly; not all of these statements have definitive answers, so your explanation is more important than your chosen answer.

5. Symmetry. Because a data set has three modes, it must have a skewed distribution.

6. Symmetry. Examination of the data set reveals that it is symmetric with a mean of 98.2 and a median of 98.2.

7. Distribution. Examination of a data set reveals that its distribution is left-skewed and unimodal.

8. Uniform Distribution. Examination of a data set reveals that the mean and median are both equal to 98.2, so the distribution must be uniform.

Concepts and Applications

9. Old Faithful. The histogram in Figure 9 shows the times between eruptions of Old Faithful geyser in Yellowstone National Park for a sample of 300 eruptions (with 299 times between eruptions). Over the histogram, draw a smooth curve that captures its general features. Then classify the distribution according to its number of modes and its symmetry or skewness. In words, summarize the meaning of your results.

Times Between Eruptions of Old Faithful

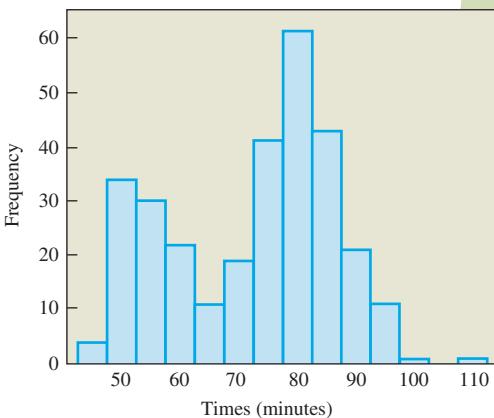


Figure 9 Source: Hand et al., *Handbook of Small Data Sets*.

10. Chip Failures. The histogram in Figure 10 shows the time until failure for a sample of 108 computer chips. Over the histogram, draw a smooth curve that captures its general features. Then classify the distribution according to its number of modes and its symmetry or skewness. In words, summarize the meaning of your results.

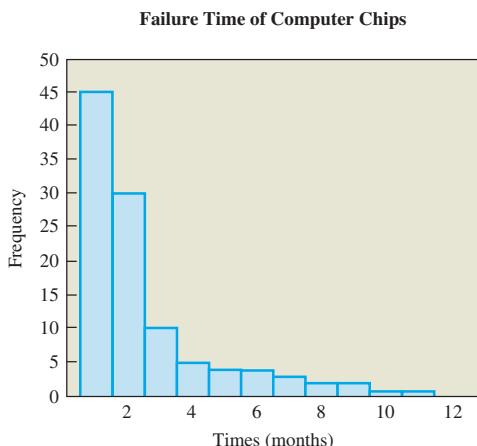


Figure 10 Source: Hand et al., *Handbook of Small Data Sets*.

- 11. Rugby Weights.** The histogram in Figure 11 shows the weights of a sample of 391 rugby players. Over the histogram, draw a smooth curve that captures its general features. Then classify the distribution according to its number of modes and its symmetry or skewness. In words, summarize the meaning of your results.

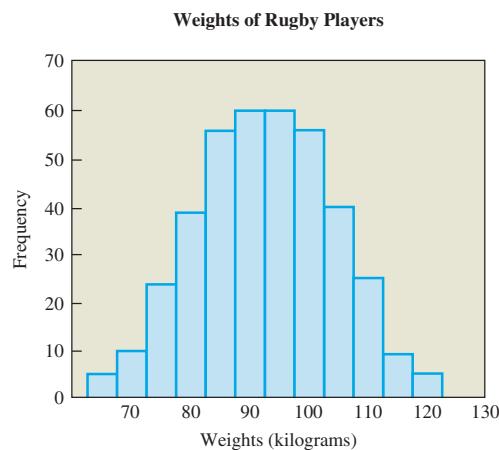


Figure 11 Source: Hand et al., *Handbook of Small Data Sets*.

- 12. Penny Weights.** The histogram in Figure 12 shows the weights (in grams) of 72 pennies. Over the histogram, draw a smooth curve that captures its general features. Then classify the distribution according to its number of modes and its symmetry or skewness. What feature of the graph reflects the fact that 35 of the pennies were made before 1983 and consist of 95% copper and 5% zinc whereas the other 37 pennies were made after 1983 and are 2.5% copper and 97.5% zinc?

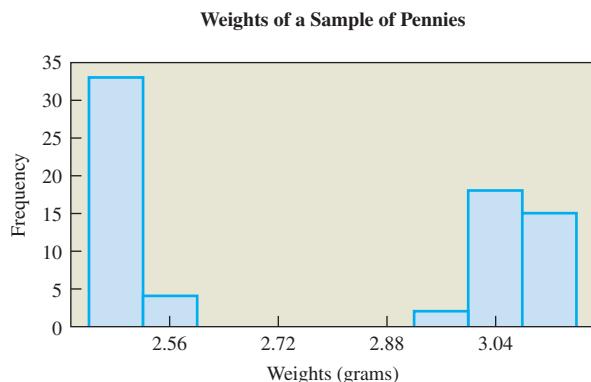


Figure 12 Source: Measurements by Mario F.Triola.

- 13. Baseball Salaries.** In a recent year, the 817 professional baseball players had salaries with the following characteristics:

- The mean was \$3,250,178.
 - The median was \$1,152,000.
 - The salaries ranged from a low of \$400,000 to a high of \$33,000,000.
- Describe the shape of the distribution of salaries. Is the distribution symmetric? Is it left-skewed? Is it right-skewed?
 - About how many players had salaries of \$1,152,000 or higher?

- 14. Boston Rainfall.** The daily rainfall amounts (in inches) for Boston in a recent year consist of 365 values with these properties:

- The mean daily rainfall amount is 0.083 inch.
 - The median of the daily rainfall amounts is 0 inches.
 - The minimum daily rainfall amount is 0 inches and the maximum is 1.48 inches.
- How is it possible that the minimum of the 365 values is 0 inches and the median is also 0 inches?
 - Describe the distribution as symmetric, left-skewed, or right-skewed.
 - Can you determine the exact number of days that it rained? Can you conclude anything about the number of days that it rained? Explain.

Describing Distributions. For each distribution described in Exercises 15–26, answer the following questions:

- How many modes would you expect for the distribution?
 - Would you expect the distribution to be symmetric, left-skewed, or right-skewed?
- 15. Incomes.** The annual incomes of all those in a statistics class, including the instructor
- 16. Reaction Times.** The reaction times of 500 randomly selected drivers, measured under standard conditions
- 17. Heights.** The heights of 250 randomly selected male attorneys
- 18. Heights.** The heights of 500 male students, half of whom are adults while the other half are eight years of age
- 19. Weights of Cola.** The weights of the cola in 1000 randomly selected cans of Coke
- 20. Vehicle Weights.** The weights of cars in a fleet consisting of 50 compact cars and 50 delivery trucks
- 21. Patients.** The ages of 1,000 randomly selected patients being treated for dementia
- 22. Speeds.** The speeds of drivers on a highway in Montana
- 23. Patron Ages.** The ages of adults who visit the National Air and Space Museum

- 24. Patron Ages.** The ages of people who visit Disneyworld
- 25. Incomes.** The incomes of people sitting in luxury boxes at the Super Bowl
- 26. Times.** The amounts of time that 5000 randomly selected individual taxpayers used to prepare their federal tax returns

PROJECTS FOR THE INTERNET & BEYOND



- 27. New York Marathon.** The Web site for the New York City marathon gives frequency data for finish times in the most recent marathon. Study the data, make a rough sketch of the distribution, and describe the shape of the distribution in words.
- 28. Tax Stats.** The IRS Web site provides statistics collected from tax returns on income, refunds, and much more. Choose a set of statistics from this Web site and study the distribution. Describe the distribution in words, and discuss anything you learn that is relevant to national tax policies.
- 29. Social Security Data.** Survey a sample of fellow students, asking each to indicate the last digit of her or his Social Security number. Also ask each participant to indicate the fifth digit. Draw one graph showing the distribution of the

last digits and another graph showing the distribution of the fifth digits. Compare the two graphs. What notable difference becomes apparent?

IN THE NEWS

- 30. Distributions in the News.** Find three recent examples in the news of distributions shown as histograms or line charts. Over each distribution, draw a smooth curve that captures its general features. Then classify the distribution according to its number of modes, symmetry or skewness, and variation.
- 31. Trimodal Distribution.** Give an example of a real distribution that you expect to have *three* modes. Make a rough sketch of the distribution; be sure to label the axes on your sketch.
- 32. Skewed Distribution.** Give an example of a real distribution that you would expect to be either right- or left-skewed. Make a rough sketch of the distribution; be sure to label the axes on your sketch.

3

MEASURES OF VARIATION

In Section 2, we saw how to describe variation qualitatively. In this section we describe quantitative measures of variation.

Why Variation Matters

We mortals cross the ocean of
this world,
Each in his average cabin of a
life.

—Robert Browning

Imagine customers waiting in line for tellers at two different banks. Customers at Big Bank can enter any one of three different lines leading to three different tellers. Best Bank also has three tellers, but all customers wait in a single line and are called to the next available teller. The following values are waiting times, in minutes, for 11 customers at each bank. The times are arranged in ascending order.

Big Bank (three lines): 4.1 5.2 5.6 6.2 6.7 7.2 7.7 7.7 8.5 9.3 11.0

Best Bank (one lines): 6.6 6.7 6.7 6.9 7.1 7.2 7.3 7.4 7.7 7.8 7.8

You'll probably find more unhappy customers at Big Bank than at Best Bank, but this is *not* because the average wait is any longer. In fact, you should verify for yourself that the mean and median waiting times are 7.2 minutes at both banks. The difference in customer satisfaction comes from the *variation* at the two banks. The waiting times at Big Bank vary over a fairly wide range, so a few customers have long waits and are likely to become annoyed. In contrast, the variation of the waiting times at Best Bank is small, so all customers feel they are being treated roughly equally. Figure 13 shows the difference in the two variations with histograms in which the data values are binned to the nearest minute.

TIME OUT TO THINK

Explain *why* Big Bank, with three separate lines, should have a greater variation in waiting times than Best Bank. Then consider several places where you commonly wait in lines, such as a grocery store, a bank, a theme park ride, or a fast food restaurant. Do these places use a single customer line that feeds multiple clerks or multiple lines? If a place uses multiple lines, do you think a single line would be better? Explain.



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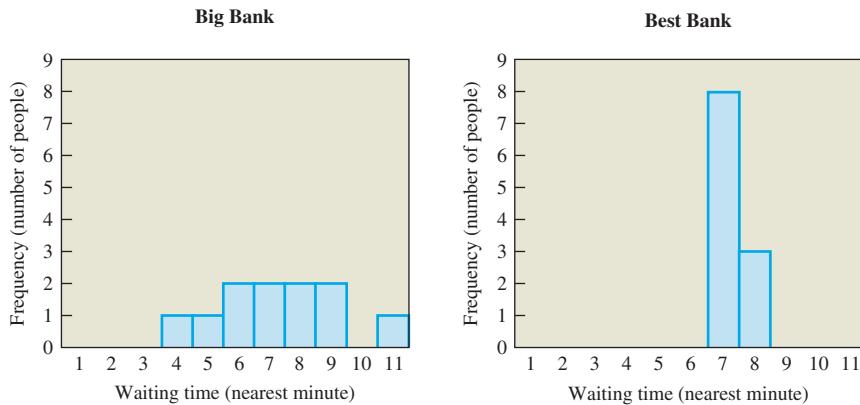


Figure 13 Histograms for the waiting times at Big Bank and Best Bank, shown with data binned to the nearest minute.

Range

The simplest way to describe the variation of a data set is to compute its **range**, defined as the difference between the highest (maximum) and lowest (minimum) values. For the example of the two banks, the waiting times for Big Bank vary from 4.1 to 11.0 minutes, so the range is $11.0 - 4.1 = 6.9$ minutes. The waiting times for Best Bank vary from 6.6 to 7.8 minutes, so the range is $7.8 - 6.6 = 1.2$ minutes. The range for Big Bank is much larger, reflecting its greater variation.

Definitions

The **range** of a set of data values is the difference between its highest and lowest data values:

$$\text{range} = \text{highest value (max)} - \text{lowest value (min)}$$

Although the range is easy to compute and can be useful, it occasionally can be misleading, as the next example shows.

BY THE WAY

The idea of waiting in line (or *queuing*) is important not only for people but also for data, particularly for data streaming through the Internet. Major corporations often employ statisticians to help them make sure that data move smoothly and without bottlenecks through their servers and Web pages.



EXAMPLE 1 Misleading Range

Consider the following two sets of quiz scores for nine students. Which set has the greater range? Would you also say that this set has the greater variation?

<i>Quiz 1:</i>	1	10	10	10	10	10	10	10	10
<i>Quiz 2:</i>	2	3	4	5	6	7	8	9	10

SOLUTION The range for Quiz 1 is $10 - 1 = 9$ points, which is greater than the range for Quiz 2 of $10 - 2 = 8$ points. However, aside from a single low score (an outlier), Quiz 1 has no variation at all because every other student got a 10. In contrast, no two students got the same score on Quiz 2, and the scores are spread throughout the list of possible scores. Quiz 2 therefore has greater variation even though Quiz 1 has greater range.

**Quartiles and the Five-Number Summary**

A better way to describe variation is to consider a few intermediate data values in addition to the high and low values. A common way involves looking at the **quartiles**, or values that divide the data distribution into quarters. The following list repeats the waiting times at the two banks, with the quartiles shown in bold. Note that the middle quartile, which divides the data set in half, is simply the median.

	Lower quartile (Q_1)	Median (Q_2)	Upper quartile (Q_3)	
<i>Big Bank:</i>	4.1 5.2 5.6 6.2 6.7 7.2 7.7 7.7 8.5 9.3 11.0	↓	↓	↓
<i>Best Bank:</i>	6.6 6.7 6.7 6.9 7.1 7.2 7.3 7.4 7.7 7.8 7.8			

TECHNICAL NOTE

Statisticians do not universally agree on the procedure for calculating quartiles, and different procedures and different technologies can result in slightly different values.

Definitions

The **lower quartile** (or **first quartile** or Q_1) divides the lowest fourth of a data set from the upper three-fourths. It is the median of the data values in the *lower half* of a data set. (Exclude the middle value in the data set if the number of data points is odd.)

The **middle quartile** (or **second quartile** or Q_2) is the overall median.

The **upper quartile** (or **third quartile** or Q_3) divides the lowest three-fourths of a data set from the upper fourth. It is the median of the data values in the *upper half* of a data set. (Exclude the middle value in the data set if the number of data points is odd.)

Once we know the quartiles, we can describe a distribution with a **five-number summary**, consisting of the low value, the lower quartile, the median, the upper quartile, and the high value. For the waiting times at the two banks, the five-number summaries are as follows:

<i>Big Bank:</i>	low	= 4.1	<i>Best Bank:</i>	low	= 6.6
	lower quartile	= 5.6		lower quartile	= 6.7
	median	= 7.2		median	= 7.2
	upper quartile	= 8.5		upper quartile	= 7.7
	high	= 11.0		high	= 7.8

The Five-Number Summary

The **five-number summary** for a data distribution consists of the following five numbers:

low value	lower quartile	median	upper quartile	high value
-----------	----------------	--------	----------------	------------

We can display the five-number summary with a graph called a **boxplot** (or *box-and-whisker plot*). Using a number line for reference, we enclose the values from the lower to the upper quartiles in a box. We then draw a line through the box at the median and add two “whiskers,” extending from the box to the low and high values. Figure 14 shows boxplots for the bank waiting times. Both the box and the whiskers for Big Bank are broader than those for Best Bank, indicating that the waiting times have greater variation at Big Bank.

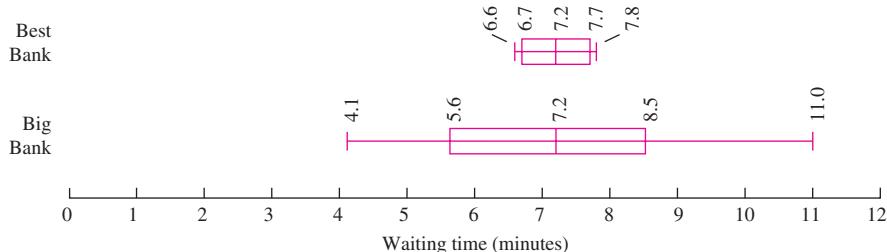


Figure 14 Boxplots show that the variation of the waiting times is greater at Big Bank than at Best Bank.

Drawing a Boxplot

- Step 1. Draw a number line that spans all the values in the data set.
- Step 2. Enclose the values from the lower to the upper quartile in a box. (The thickness of the box has no meaning.)
- Step 3. Draw a line through the box at the median.
- Step 4. Add “whiskers” extending to the low and high values.

TECHNICAL NOTE

The boxplots shown in this text are called *skeletal boxplots*. Some boxplots are drawn with outliers marked by an asterisk (*) or a dot and the whiskers extending only to the smallest and largest *nonoutliers*; these types of boxplots are called *modified boxplots*.



USING TECHNOLOGY—BOXPLOTS

Excel Although Excel itself is not designed to generate a boxplot, it can be generated using XLSTAT that is a supplement to this book. Load XLSTAT (the Excel add-in), then enter or copy the data into a column of the spreadsheet. Click on **XLSTAT**, click on **Visualizing Data**, then select **Univariate plots**. Enter the range of cells containing the data, such as A1:A11. (If the first cell includes the name of the data, click on the box next to “Sample labels.”) Click **OK** to continue. The result will be a boxplot with two features: (1) The boxplot will be vertical, and (2) the exact values of the quartiles are likely to be somewhat different from those found using the procedure described above.

STATDISK Enter the data in the Data Window, then click on **Data**, then **Boxplot**. Click on the columns that you want to include, then click on **Plot**.

TI-83/84 PLUS Enter the sample data in list L1 or enter the data and assign them to a list name. Now select **STAT PLOT** by pressing **2ND Y=**. Press **ENTER**, then select the option of **ON**. For a simple boxplot as described in Part 1 of this section, select the boxplot type that is positioned in the middle of the second row; for a modified boxplot as described in Part 2 of this section, select the boxplot that is positioned at the far left of the second row. The Xlist should indicate L1 and the Freq value should be 1. Now press **ZOOM** and select option 9 for **ZoomStat**. Press **ENTER** and the boxplot should be displayed. You can use the arrow keys to move right or left so that values can be read from the horizontal scale.

EXAMPLE 2 Passive and Active Smoke

One way to study exposure to cigarette smoke is by measuring blood levels of *serum cotinine*, a metabolic product of nicotine that the body absorbs from cigarette smoke. Table 3 lists serum cotinine levels from samples of 50 smokers (“active smoke”) and 50 nonsmokers who are exposed to cigarette smoke at home or at work (“passive smoke”). Compare the two data sets (smokers and nonsmokers) with five-number summaries and boxplots, and discuss your results.

BY THE WAY

Passive smoke is particularly harmful to young children. Apparently, the toxins in cigarette smoke have a greater effect on developing bodies than on full-grown adults. A similar effect is found for most other toxins, which is why it is especially important to limit children's exposure to toxic chemicals.

**TABLE 3**

Serum Cotinine Levels (nanograms per milliliter of blood) in Samples of 50 Smokers and 50 Nonsmokers Exposed to Passive Smoke, with Data Values Listed in Ascending Order

Order number	Smokers	Nonsmokers	Order number	Smokers	Nonsmokers
1	0.08	0.03	26	34.21	0.82
2	0.14	0.07	27	36.73	0.97
3	0.27	0.08	28	37.73	1.12
4	0.44	0.08	29	39.48	1.23
5	0.51	0.09	30	48.58	1.37
6	1.78	0.09	31	51.21	1.40
7	2.55	0.10	32	56.74	1.67
8	3.03	0.11	33	58.69	1.98
9	3.44	0.12	34	72.37	2.33
10	4.98	0.12	35	104.54	2.42
11	6.87	0.14	36	114.49	2.66
12	11.12	0.17	37	145.43	2.87
13	12.58	0.20	38	187.34	3.13
14	13.73	0.23	39	226.82	3.54
15	14.42	0.27	40	267.83	3.76
16	18.22	0.28	41	328.46	4.58
17	19.28	0.30	42	388.74	5.31
18	20.16	0.33	43	405.28	6.20
19	23.67	0.37	44	415.38	7.14
20	25.00	0.38	45	417.82	7.25
21	25.39	0.44	46	539.62	10.23
22	29.41	0.49	47	592.79	10.83
23	30.71	0.51	48	688.36	17.11
24	32.54	0.51	49	692.51	37.44
25	32.56	0.68	50	983.41	61.33

Note: The column "Order number" is included to make it easier to read the table.

Source: National Health and Nutrition Examination Survey, National Institutes of Health.

SOLUTION The two data sets are already in ascending order, making it easy to construct the five-number summary. Each has 50 data points, so the median lies halfway between the 25th and 26th values. For the smokers, the 25th and 26th values are 32.56 and 34.21, respectively, so the median is

$$\frac{32.56 + 34.21}{2} = 33.385$$

For the nonsmokers, the 25th and 26th values are 0.68 and 0.82, respectively, so the median is

$$\frac{0.68 + 0.82}{2} = 0.75$$

The lower quartile is the median of the *lower half* of the values, which is the 13th value in each set. The upper quartile is the median of the *upper half* of the values, which is the 38th value in each set. The five-number summaries for the two data sets are as follows:

Active smoke:

low = 0.08 ng/ml
lower quartile = 12.58 ng/ml
median = 33.385 ng/ml
upper quartile = 187.34 ng/ml
high = 983.41 ng/ml

Passive smoke:

low = 0.03 ng/ml
lower quartile = 0.20 ng/ml
median = 0.75 ng/ml
upper quartile = 3.13 ng/ml
high = 61.33 ng/ml

Figure 15 shows boxplots for the two data sets. The boxplots make it easy to see some key features of the data sets. For example, it is immediately clear that the active smokers have a higher median level of serum cotinine, as well as a greater variation in levels. We conclude that smokers absorb considerably more nicotine than do nonsmokers exposed to passive smoke. Nevertheless, the levels in the passive smokers are much higher than those found in people who had no exposure to cigarette smoke (as demonstrated by other data, not shown here). Indeed, the nonsmoker with the high value for passive smoke has absorbed more nicotine than

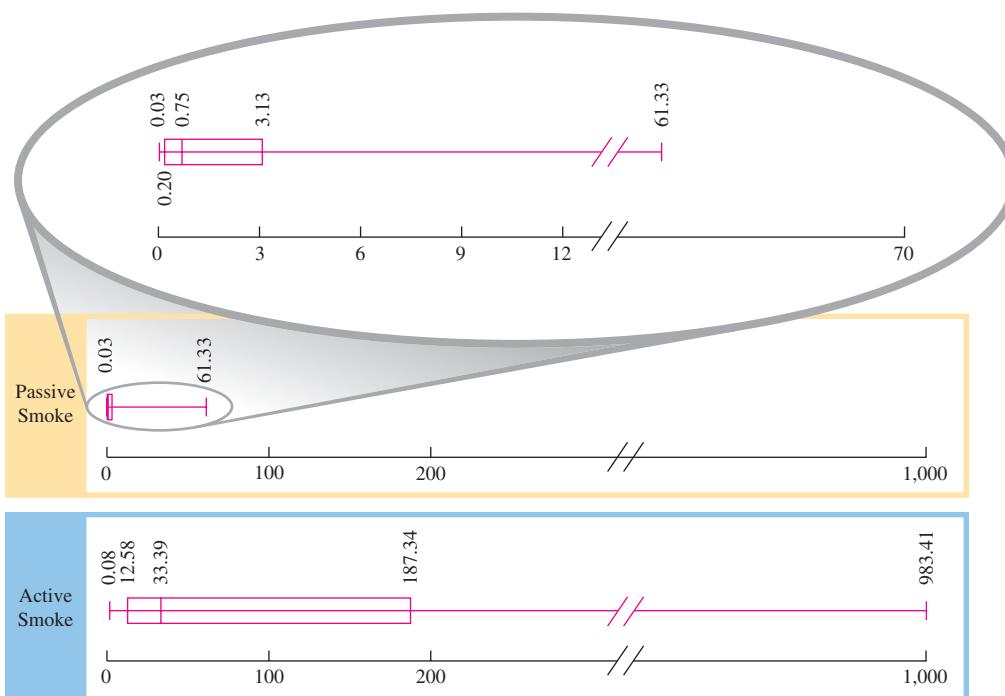


Figure 15 Boxplots for the data in Table 3.

the median smoker. We conclude that passive smoke can expose nonsmokers to significant amounts of nicotine. Given the known dangers of cigarette smoke, these results give us reason to be concerned about possible health effects from passive smoke.



Percentiles

Quartiles divide a data set into 4 segments. It is possible to divide a data set even more. For example, *quintiles* divide a data set into 5 segments, and *deciles* divide a data set into 10 segments. It is particularly common to divide data sets into 100 segments using **percentiles**. Roughly speaking, the 35th percentile, for example, is a value that separates the bottom 35% of the data values from the top 65%. (More precisely, the 35th percentile is greater than or equal to at least 35% of the data values and less than or equal to at least 65% of the data values.)

If a data value lies between two percentiles, it is common to say that the data value lies *in* the lower percentile. For example, if you score higher than 84.7% of all people taking a college entrance examination, we say that your score is in the 84th percentile.

TECHNICAL NOTE

As with quartiles, statisticians and various statistics software packages may use slightly different procedures to calculate percentiles, resulting in slightly different values.

Definitions

The ***n*th percentile** of a data set divides the bottom $n\%$ of data values from the top $(100 - n)\%$. A data value that lies between two percentiles is often said to lie *in* the lower percentile. You can approximate the percentile of any data value with the following formula:

$$\text{percentile of data value} = \frac{\text{number of values less than this data value}}{\text{total number of values in data set}} \times 100$$

There are different procedures for finding a data value corresponding to a given percentile, but one approximate approach is to find the L th value, where L is the product of the percentile (in decimal form) and the sample size. For example, with 50 sample values, the 12th percentile is around the $0.12 \times 50 = 6$ th value.

EXAMPLE 3 Smoke Exposure Percentiles

Answer the following questions concerning the data in Table 3.

- What is the percentile for the data value of 104.54 ng/ml for smokers?
- What is the percentile for the data value of 61.33 ng/ml for nonsmokers?
- What data value marks the 36th percentile for the smokers? For the nonsmokers?

SOLUTION The following results are approximate.

- a. The data value of 104.54 ng/ml for smokers is the 35th data value in the set, which means that 34 data values lie below it. Thus, its percentile is

$$\frac{\text{number of values less than } 104.54 \text{ ng/ml}}{\text{total number of values in data set}} = \times 100 = \frac{34}{50} \times 100 = 68$$

In other words, the 35th data value marks the 68th percentile.

- b. The data value of 61.33 ng/ml for nonsmokers is the 50th and highest data value in the set, which means that 49 data values lie below it. Thus, its percentile is

$$\frac{\text{number of values less than } 61.33 \text{ ng/ml}}{\text{total number of values in data set}} \times 100 = \frac{49}{50} \times 100 = 98$$

In other words, the highest data value in this set lies in the 98th percentile.

- c. Because there are 50 data values in the set, the 36th percentile is around the $0.36 \times 50 = 18$ th value. For smokers this value is 20.16 ng/ml, and for nonsmokers it is 0.33 ng/ml. . . .

TECHNICAL NOTE

The standard deviation formula given here, in which we divide the sum of the squared deviations by the total number of data values *minus 1*, is technically valid only for data from *samples*. When dealing with *populations*, we do not subtract the 1. In this text, we use only the sample formula.

We divide by $n - 1$ because there are only $n - 1$ independent sample values. With a given mean, only $n - 1$ values can be freely assigned any number before the last value is determined. Here, we say that we have $n - 1$ *degrees of freedom*.

Standard Deviation

The five-number summary characterizes variation well, but statisticians often prefer to describe variation with a single number. The single number most commonly used to describe variation is called the **standard deviation**.

The standard deviation is a measure of how widely data values are spread around the mean of a data set. To calculate a standard deviation, we first find the mean and then find how much each data value “deviates” from the mean. Consider our bank data sets, in which the mean waiting time was 7.2 minutes for both Big Bank and Best Bank. For a waiting time of 8.2 minutes, the **deviation** from the mean is equal to $8.2 \text{ minutes} - 7.2 \text{ minutes} = 1.0$ minute, meaning that it is 1.0 minute greater than the mean. For a waiting time of 5.2 minutes, the deviation from the mean is equal to $5.2 \text{ minutes} - 7.2 \text{ minutes} = -2 \text{ minutes}$ (*negative* 2 minutes), because it is 2.0 minutes *less* than the mean.

In essence, the standard deviation is a measure of the average of all the deviations from the mean. However, because the mean of the deviations is always zero (because the positive deviations exactly balance the negative deviations), we calculate the standard deviation by first finding a mean of the *squares* of the deviations (because squares are always positive) and taking a square root in the end. For technical reasons, we divide the sum of the squares by the total number of data values *minus 1*.

Calculating the Standard Deviation

To calculate the standard deviation for any data set:

- Step 1. Compute the mean of the data set. Then find the deviation from the mean for every data value by subtracting the mean from the data value. That is, for every data value,

$$\text{deviation from mean} = \text{data value} - \text{mean}$$

- Step 2. Find the squares (second power) of all the deviations from the mean.

- Step 3. Add all the squares of the deviations from the mean.

- Step 4. Divide this sum by the total number of data values *minus 1*.

- Step 5. The standard deviation is the square root of this quotient. Overall, these steps produce the standard deviation formula:

$$\text{standard deviation} = \sqrt{\frac{\text{sum of (deviations from the mean)}^2}{\text{total number of data values} - 1}}$$

TECHNICAL NOTE

The result of Step 4 is called the **variance** of the distribution. In other words, the standard deviation is the square root of the variance. Although the variance is used in many advanced statistical computations, we will not use it in this text.

Note that, because we square the deviations in Step 3 and then take the square root in Step 5, the units of the standard deviation are the same as the units of the data values. For example, if the data values have units of minutes, the standard deviation also has units of minutes.

The standard deviation formula is easy to use in principle, but the calculations become tedious for all but the smallest data sets. As a result, it is usually calculated with the aid of a calculator or computer. Nevertheless, you'll find the standard deviation formula easier to understand if you try a few examples in which you work through the calculations in detail.

EXAMPLE 4 Calculating Standard Deviation

Calculate the standard deviations for the waiting times at Big Bank and Best Bank.

SOLUTION We follow the five steps to calculate the standard deviations. Table 4 shows how to organize the work in the first three steps. The first column for each bank lists the waiting times (in minutes). The second column lists the deviations from the mean (Step 1), which we already know to be 7.2 minutes for both banks. The third column lists the squares of the deviations (Step 2). We add all the squared deviations to find the sum at the bottom of the third column (Step 3). For Step 4, we divide the sums from Step 3 by the total number of data values *minus 1*. Because there are 11 data values, we divide by 10:

$$\text{Big Bank: } \frac{38.46}{10} = 3.846$$

$$\text{Best Bank: } \frac{1.98}{10} = 0.198$$

Finally, Step 5 tells us that the standard deviations are the square roots of the numbers from Step 4:

$$\text{Big Bank: standard deviation} = \sqrt{3.846} \approx 1.96 \text{ minutes}$$

$$\text{Best Bank: standard deviation} = \sqrt{0.198} \approx 0.44 \text{ minutes}$$

We conclude that the standard deviation of the waiting times is about 1.96 minutes at Big Bank and 0.44 minute at Best Bank. As we expected, the waiting times showed greater variation at Big Bank, which is why the lines at Big Bank annoyed more customers than did those at Best Bank.

TABLE 4 Calculating Standard Deviation

Big Bank			Best Bank		
Time	Deviation (Time – Mean)	(Deviation) ²	Time	Deviation (Time – Mean)	(Deviation) ²
4.1	4.1 – 7.2 = -3.1	(-3.1) ² = 9.61	6.6	6.6 – 7.2 = -0.6	(-0.6) ² = 0.36
5.2	5.2 – 7.2 = -2.0	(-2.0) ² = 4.00	6.7	6.7 – 7.2 = -0.5	(-0.5) ² = 0.25
5.6	5.6 – 7.2 = -1.6	(-1.6) ² = 2.56	6.7	6.7 – 7.2 = -0.5	(-0.5) ² = 0.25
6.2	6.2 – 7.2 = -1.0	(-1.0) ² = 1.00	6.9	6.9 – 7.2 = -0.3	(-0.3) ² = 0.09
6.7	6.7 – 7.2 = -0.5	(-0.5) ² = 0.25	7.1	7.1 – 7.2 = -0.1	(-0.1) ² = 0.01
7.2	7.2 – 7.2 = 0.0	(0.0) ² = 0.0	7.2	7.2 – 7.2 = 0.0	(0.0) ² = 0.0
7.7	7.7 – 7.2 = 0.5	(0.5) ² = 0.25	7.3	7.3 – 7.2 = 0.1	(0.1) ² = 0.01
7.7	7.7 – 7.2 = 0.5	(0.5) ² = 0.25	7.4	7.4 – 7.2 = 0.2	(0.2) ² = 0.04
8.5	8.5 – 7.2 = 1.3	(1.3) ² = 1.69	7.7	7.7 – 7.2 = 0.5	(0.5) ² = 0.25
9.3	9.3 – 7.2 = 2.1	(2.1) ² = 4.41	7.8	7.8 – 7.2 = 0.6	(0.6) ² = 0.36
11.0	11.0 – 7.2 = 3.8	(3.8) ² = 14.44	7.8	7.8 – 7.2 = 0.6	(0.6) ² = 0.36
		Sum = 38.46			Sum = 1.98

• • •

**USING TECHNOLOGY—STANDARD DEVIATION**

Excel The built-in Excel function STDEV automates the calculation of standard deviation, so that all you have to do is enter the data and then use the function. The screen shot below shows the process for the Big Bank data, with the functions shown in Column B and the results in Column C.

	A	B	C
1		Big Bank	
2	Data	4.1	
3		5.2	
4		5.6	
5		6.2	
6		6.7	
7		7.2	
8		7.7	
9		7.7	
10		8.5	
11		9.3	
12		11	
13	Mean	=AVERAGE(B2:B12)	7.2
14	St. Dev.	=STDEV(B2:B12)	1.96
15			

Note: Alternatively, the procedures given in Section 1 using the Analysis Tookit or XLSTAT will give results that include the value of the standard deviation

Statdisk or **TI 83/84**: Use the same procedures given in Section 1, and the results will include the value of the standard deviation.

TIME OUT TO THINK

Look closely at the individual deviations in Table 4 in Example 4. Do the standard deviations for the two data sets seem like reasonable “averages” for the deviations? Explain.

Interpreting the Standard Deviation

A good way to develop a deeper understanding of the standard deviation is to consider an approximation called the **range rule of thumb**, summarized in the following box.

TECHNICAL NOTE

Another way of interpreting the standard deviation uses a mathematical rule called *Chebyshev's Theorem*. It states that, for any data distribution, at least 75% of all data values lie within two standard deviations of the mean, and at least 89% of all data values lie within three deviations of the mean.

The Range Rule of Thumb

The standard deviation is *approximately* related to the range of a distribution by the **range rule of thumb**:

$$\text{standard deviation} \approx \frac{\text{range}}{4}$$

If we know the range of a distribution ($\text{range} = \text{high} - \text{low}$), we can use this rule to estimate the standard deviation. Alternatively, if we know the standard deviation, we can use this rule to estimate the low and high values as follows:

$$\text{low value} \approx \text{mean} - (2 \times \text{standard deviation})$$

$$\text{high value} \approx \text{mean} + (2 \times \text{standard deviation})$$

The range rule of thumb does not work well when the high or low values are outliers.

The range rule of thumb works reasonably well for data sets in which values are distributed fairly evenly. It does not work well when the high or low values are extreme outliers. You must therefore use judgment in deciding whether the range rule of thumb is applicable in a particular case, and in all cases remember that the range rule of thumb yields rough approximations, not exact results.

EXAMPLE 5 Using the Range Rule of Thumb

Use the range rule of thumb to estimate the standard deviations for the waiting times at Big Bank and Best Bank. Compare the estimates to the actual values found in Example 4.

SOLUTION The waiting times for Big Bank vary from 4.1 to 11.0 minutes, which means a range of $11.0 - 4.1 = 6.9$ minutes. The waiting times for Best Bank vary from 6.6 to 7.8 minutes, for a range of $7.8 - 6.6 = 1.2$ minutes. Thus, the range rule of thumb gives the following estimates for the standard deviations:

$$\text{Big Bank: standard derivation} \approx \frac{6.9}{4} = 1.7$$

$$\text{Best Bank: standard derivation} \approx \frac{1.2}{4} = 0.3$$

The actual standard deviations calculated in Example 4 are 1.96 and 0.44, respectively. For these two cases, the estimates from the range rule of thumb slightly underestimate the actual standard deviations. Nevertheless, the estimates put us in the right ballpark, showing that the rule is useful.



EXAMPLE 6 Estimating a Range

Studies of the gas mileage of a Prius under varying driving conditions show that it gets a mean of 45 miles per gallon with a standard deviation of 4 miles per gallon. Estimate the minimum and maximum typical gas mileage amounts that you can expect under ordinary driving conditions.

SOLUTION From the range rule of thumb, the low and high values for gas mileage are approximately

$$\text{low value} \approx \text{mean} - (2 \times \text{standard deviation}) = 45 - (2 \times 4) = 37$$

$$\text{high value} \approx \text{mean} + (2 \times \text{standard deviation}) = 45 + (2 \times 4) = 53$$

The range of gas mileage for the car is roughly from a minimum of 37 miles per gallon to a maximum of 53 miles per gallon.



BY THE WAY

Technologies such as catalytic converters have helped reduce the amounts of many pollutants emitted by cars (per mile driven), but burning less gasoline is the only way to reduce carbon dioxide emissions that cause global warming. This is a major reason why auto manufacturers are developing high-mileage hybrid vehicles and zero-emission vehicles that run on electricity or fuel cells.



Standard Deviation with Summation Notation (Optional Section)

The summation notation introduced earlier makes it easy to write the standard deviation formula in a compact form. Recall that x represents the individual values in a data set and \bar{x} represents the mean of the data set. We can therefore write the deviation from the mean for any data value as

$$\text{deviation} = \text{data value} - \text{mean} = x - \bar{x}$$

We can now write the sum of all squared deviations as

$$\text{sum of all squared deviations} = \sum(x - \bar{x})^2$$

The remaining steps in the calculation of the standard deviation are to divide this sum by $n - 1$ and then take the square root. You should confirm that the following formula summarizes the five steps in the earlier box:

$$s = \text{standard deviation} = \sqrt{\frac{\sum(x - \bar{x})^2}{n - 1}}$$

The symbol s is the conventional symbol for the standard deviation of a sample. For the standard deviation of a population, statisticians use the Greek letter σ (sigma), and the term $n - 1$ in the formula is replaced by N (the population size). Consequently, you will get slightly different results for the standard deviation depending on whether you assume the data represent a sample or a population.

TECHNICAL NOTE

The formula for the variance is

$$s^2 = \frac{\sum(x - \bar{x})^2}{n - 1}$$

The standard symbol for the variance, s^2 , reflects the fact that it is the square of the standard deviation.

Section 3 Exercises

Statistical Literacy and Critical Thinking

1. **Range.** How is the range computed for a set of sample data? Is the range a measure of variation? What is a major disadvantage of the range?
2. **Standard Deviation.** Assume that you are manufacturing aspirin tablets that are supposed to contain 325 mg of aspirin. If the standard deviation of the amounts of aspirin is calculated, which value would you prefer: a standard deviation of 20 mg or a standard deviation of 10 mg? Why?
3. **Correct Statement?** In the book *How to Lie with Charts*, the author writes that “the standard deviation is usually shown as plus or minus the difference between the high and the mean, and the low and the mean. For example, if the mean is 1, the high is 3, and the low is –1, the standard deviation is ± 2 . ” Is that statement correct? Why or why not?
4. **Quartiles.** For the salaries paid to the 817 professional baseball players in a recent year, the first quartile is \$4,355,101. What do we mean when we say that \$4,355,101 is the first quartile?

Does It Make Sense? For Exercises 5–8, decide whether the statement makes sense (or is clearly true) or does not make sense (or is clearly false). Explain clearly; not all of these statements have definitive answers, so your explanation is more important than your chosen answer.

5. **SAT Score.** Jennifer received an SAT score that was equal to the first quartile and the 35th percentile.
6. **Lengths.** The house key lengths of 15 statistics students are measured and rounded to the nearest centimeter, and all 15 values are the same, so the standard deviation is 0 cm.
7. **Baseball Salaries.** For a recent year, the 817 salaries paid to professional baseball players have a median of \$1,152,000 and a second quartile with the same value of \$1,152,000.
8. **Baseball Salaries.** If the range of salaries paid to baseball players in the American League is less than the range of salaries paid to baseball players in the National League, then the American League salaries must have a smaller standard deviation than the National League salaries.

Concepts and Applications

Range and Standard Deviation. Exercises 9–16 each list a set of numbers. In each case, find the range and standard deviation. (The same sets of numbers were used in Exercises 13–20 in Section 1.)

9. **Number of Words.** Pages from *Merriam-Webster's Collegiate Dictionary*, 11th edition, were randomly selected. Here are the numbers of words defined on those pages:

51 63 36 43 34 62 73 39 53 79

Describing Data

10. **Space Shuttle Flights.** Listed below are the durations (in hours) of a sample of all flights of NASA's Space Transport System (space shuttle):

73 95 235 192 165 262 191 376
259 235 381 331 221 244 0

11. **Perception of Time.** Actual times (in seconds) recorded when statistics students participated in an experiment to test their ability to determine when one minute (60 seconds) had passed:

53 52 75 62 68 58 49 49

12. **Body Temperatures.** Body temperatures (in degrees Fahrenheit) of randomly selected normal and healthy adults:

98.6 98.6 98.0 98.0 99.0
98.4 98.4 98.4 98.4 98.6

13. **Blood Alcohol.** Blood alcohol concentrations of drivers involved in fatal crashes and then given jail sentences (based on data from the U.S. Department of Justice):

0.27 0.17 0.17 0.16 0.13 0.24
0.29 0.24 0.14 0.16 0.12 0.16

14. **Old Faithful Geyser.** Time intervals (in minutes) between eruptions of Old Faithful geyser in Yellowstone National Park:

98 92 95 87 96 90
65 92 95 93 98 94

15. **Weights of M&Ms.** Weights (in grams) of randomly selected M&M plain candies:

0.957 0.912 0.842 0.925 0.939 0.886
0.914 0.913 0.958 0.947 0.920

16. **Quarters.** Weights (in grams) of quarters in circulation:

5.60 5.63 5.58 5.56 5.66 5.58 5.57 5.59
5.67 5.61 5.84 5.73 5.53 5.58 5.52 5.65
5.57 5.71 5.59 5.53 5.63 5.68

Comparing Variation. In Exercises 17–20, find the range and standard deviation for each of the two samples and then compare the two sets of results.

17. **It's Raining Cats.** Statistics are sometimes used to compare or identify authors of different works. The lengths of the first 20 words in the foreword by Tennessee Williams in *Cat on a Hot Tin Roof* are listed along with the lengths of the first 20 words in *The Cat in the Hat* by Dr. Seuss. Does there appear to be a difference in variation?

Cat on a Hot Tin Roof:

2	6	2	2	1	4	4	2	4	2
3	8	4	2	2	7	7	2	3	11

The Cat in the Hat:

3	3	3	3	5	2	3	3	3	2
4	2	2	3	2	3	5	3	4	4

- 18. BMI for Miss America.** The trend of thinner Miss America winners has generated charges that the contest encourages unhealthy diet habits among young women. Listed below are body mass indexes (BMI) for Miss America winners from two different time periods. Does there appear to be a difference in variation?

BMI (from the 1920s and 1930s):

20.4 21.9 22.1 22.3 20.3 18.8 18.9 19.4 18.4 19.1

BMI (from recent winners):

19.5 20.3 19.6 20.2 17.8 17.9 19.1 18.8 17.6 16.8

- 19. Weather Forecast Accuracy.** In an analysis of the accuracy of weather forecasts, the actual high temperatures are compared with the high temperatures predicted one day earlier and the high temperatures predicted five days earlier. Listed below are the errors between the predicted temperatures and the actual high temperatures for consecutive days in Dutchess County, New York. Do the standard deviations suggest that the temperatures predicted one day in advance are more accurate than those predicted five days in advance, as we might expect?

(actual high) – (high predicted one day earlier):

2	2	0	0	-3	-2	1
-2	8	1	0	-1	0	1

(actual high) – (high predicted five days earlier):

0	-3	2	5	-6	-9	4
-1	6	-2	-2	-1	6	-4

- 20. Treatment Effect.** Researchers at Pennsylvania State University conducted experiments with poplar trees. Listed below are weights (in kilograms) of poplar trees given no treatment and poplar trees treated with fertilizer and irrigation. Does there appear to be a difference between the two standard deviations?

No treatment:

0.15 0.02 0.16 0.37 0.22

Fertilizer and irrigation:

2.03 0.27 0.92 1.07 2.38

- 21. Calculating Percentiles.** A statistics professor with too much time on his hands weighed each M&M candy in a bag of 465 plain M&M candies.

- a. One of the M&Ms weighed 0.776 gram and it was heavier than 25 of the other M&Ms. What is the percentile of this particular value?

- b. One of the M&Ms weighed 0.876 gram and it was heavier than 322 of the other M&Ms. What is the percentile of this particular value?

- c. One of the M&Ms weighed 0.856 gram and it was heavier than 224 of the other M&Ms. What is the percentile of this particular value?

- 22. Calculating Percentiles.** A data set consists of the 85 ages of women at the time that they won an Oscar in the category of best actress.

- a. One of the actresses was 40 years of age, and she was older than 63 of the other actresses at the time that they won Oscars. What is the percentile of the age of 40?

- b. One of the actresses was 54 years of age, and she was older than 76 of the other actresses at the time that they won Oscars. What is the percentile of the age of 54?

- c. One of the actresses was 60 years of age, and she was older than 77 of the other actresses at the time that they won Oscars. What is the percentile of the age of 60?

- 23. Understanding Standard Deviation.** The following four sets of 7 numbers all have a mean of 9.

{9, 9, 9, 9, 9, 9, } {8, 8, 9, 9, 9, 10, 10}

{8, 8, 8, 9, 10, 10, 10} {6, 6, 6, 9, 12, 12, 12}

- a. Make a histogram for each set.

- b. Give the five-number summary and draw a boxplot for each set.

- c. Compute the standard deviation for each set.

- d. Based on your results, briefly explain how the standard deviation provides a useful single-number summary of the variation in these data sets.

- 24. Understanding Standard Deviation.** The following four sets of 7 numbers all have a mean of 6.

{6, 6, 6, 6, 6, 6, } {5, 5, 6, 6, 7, 7}

{5, 5, 5, 6, 7, 7, 7}, {3, 3, 3, 6, 9, 9, 9}

- a. Make a histogram for each set.

- b. Give the five-number summary and draw a boxplot for each set.

- c. Compute the standard deviation for each set.

- d. Based on your results, briefly explain how the standard deviation provides a useful single-number summary of the variation in these data sets.

- Comparing Variations.** For each of Exercises 25–28, do the following:

- a. Find the mean, median, and range for each of the two data sets.
- b. Give the five-number summary and draw a boxplot for each of the two data sets.
- c. Find the standard deviation for each of the two data sets.
- d. Apply the range rule of thumb to estimate the standard deviation of each of the two data sets. How well does the rule work in each case? Briefly discuss why it does or does not work well.

- e. Based on all your results, compare and discuss the two data sets in terms of their center and variation.

25. The following data sets give the ages in years of a sample of cars in a faculty parking lot and a student parking lot at the College of Portland.

Faculty:

2 3 1 0 1 2 4 3 3 2 1

Student:

5 6 8 2 7 10 1 4 6 10 9

26. The following data sets give the driving speeds in miles per hour of the first nine cars to pass through a school zone and the first nine cars to pass through a downtown intersection.

School:

20 18 23 21 19 18 17 24 25

Downtown:

29 31 35 24 31 26 36 31 28

27. The following data sets show the ages of the first seven U.S. Presidents (Washington through Jackson) and seven recent U.S. Presidents (Ford through Obama) at the time of inauguration.

First 7:

57 61 57 57 58 57 61

Last 7:

61 52 69 64 46 54 47

28. The following data sets give the approximate lengths of Beethoven's nine symphonies and Mahler's nine symphonies (in minutes).

Beethoven:

28 36 50 33 30 40 38 26 68

Mahler:

52 85 94 50 72 72 80 90 80

29. **Manufacturing.** You are in charge of a manufacturing process that produces car batteries that are supposed to provide 12 volts of power. Manufacturing occurs at two different sites. The first site produces batteries with a mean of 12.1 volts and a standard deviation of 0.5 volt, while the second site produces batteries with a mean of 12.2 volts and a standard deviation of 0.1 volt. Which site has better quality? Why?

30. **Managing Complaints.** You manage a small ice cream shop in which your employees scoop the ice cream by hand. Each night, you total your sales and the total volume of ice cream sold. You find that on nights when an employee named Ben is working, the mean price of the ice cream sold is \$1.75 per pint with a standard deviation of \$0.05. On nights when an employee named Jerry is working, the mean price of the ice cream sold is \$1.70 per pint with a standard deviation of \$0.35. Which employee is more likely to be generating complaints of "too small" servings? Explain.

31. **Portfolio Standard Deviation.** The book *Investments*, by Zvi Bodie, Alex Kane, and Alan Marcus, claims that the annual percentage returns for investment portfolios with a single stock have a standard deviation of 0.55, while the annual percentage returns for portfolios with 32 stocks have a standard deviation of 0.325. Explain how the standard deviation measures the risk in these two types of portfolios.

32. **Batting Standard Deviation.** For the past 100 years, the mean batting average in the major leagues has remained fairly constant at about 0.260. However, the standard deviation of batting averages has decreased from about 0.049 in the 1870s to 0.031 in the present. What does this tell us about the batting averages of players? Based on these facts, would you expect batting averages above 0.350 to be more or less common today than in the past? Explain.



PROJECTS FOR THE INTERNET & BEYOND

33. **Secondhand Smoke.** At the Web sites of the American Lung Association and the U.S. Environmental Protection Agency, find statistical data concerning the health effects of secondhand (passive) smoke. Write a short summary of your findings and your opinions about whether and how this health issue should be addressed by government.

34. **Kids and the Media.** A recent study by the Kaiser Family Foundation looked at the role of media (for example, television, books, computers) in the lives of children. The report, which is on the Kaiser Family Foundation Web site, gives many data distributions concerning, for example, how much time children spend daily with each medium. Study at least three of the distributions in the report that you find particularly interesting. Summarize each distribution in words, and discuss your opinions of the social consequences of the findings.

35. **Measuring Variation.** The range and standard deviation use different approaches to measure variation in a data set. Construct two different data sets configured so that the range of the first set is *greater than* the range of the second set (suggesting that the first set has more variation) but the standard deviation of the first set is *less than* the standard deviation of the second set (suggesting that the first set has less variation).

IN THE NEWS

36. **Ranges in the News.** Find two examples of data distributions in recent news reports; they may be given either as tables or as graphs. In each case, state the range of the distribution and explain its meaning in the context of the news report. Estimate the standard deviation by applying the range rule of thumb.

37. **Summarizing a News Data Set.** Find an example of a data distribution given in the form of a table in a recent news report. Make a five-number summary and a boxplot for the distribution.

4

STATISTICAL PARADOXES

The government administers polygraph tests (“lie detectors”) to new applicants for sensitive security jobs. The polygraph tests are reputed to be 90% accurate; that is, they catch 90% of the people who are lying and validate 90% of the people who are truthful. Most people therefore guess that only 10% of the people who fail a polygraph test have been falsely identified as lying. In fact, the actual percentage of false accusations can be *much* higher — more than 90% in some cases. How can this be?

We’ll discuss the answer soon, but the moral of this story should already be clear: Even when we describe data carefully according to the principles discussed in the first three sections of this chapter, we may still be led to very surprising conclusions. Before we get to the polygraph issue, let’s start with a couple of other statistical surprises.

Better in Each Case, but Worse Overall

Suppose a pharmaceutical company creates a new treatment for acne. To decide whether the new treatment is better than an older treatment, the company gives the old treatment to 90 patients and gives the new treatment to 110 patients. Some patients had mild acne and others had severe acne. Table 5 summarizes the results after four weeks of treatment, broken down by which treatment was given and whether the patient’s acne was mild or severe. If you study the table carefully, you will notice these key facts:

- Among patients with *mild* acne:
 - 10 received the old treatment and 2 were cured, for a 20% cure rate.
 - 90 received the new treatment and 30 were cured, for a 33% cure rate.
- Among patients with *severe* acne:
 - 80 received the old treatment and 40 were cured, for a 50% cure rate.
 - 20 received the new treatment and 12 were cured, for a 60% cure rate.

		Results of Acne Treatments			
		Mild acne		Severe acne	
		Cured	Not cured	Cured	Not cured
Old treatment		2	8	40	40
New treatment		30	60	12	8

Notice that the new treatment had a higher cure rate *both* for patients with mild acne (33% for the new treatment vs. 20% for the old) and for patients with severe acne (60% for the new treatment vs. 50% for the old). Is it therefore fair for the company to claim that their new treatment is better than the old treatment?

At first, this might seem to make sense. But instead of looking at the data for the mild and severe acne patients separately, let’s look at the *overall* results:

- A total of 90 patients received the old treatment and 42 were cured (2 out of 10 with mild acne and 40 out of 80 with severe acne), for an overall cure rate of $42/90 = 46.7\%$.
- A total of 110 patients received the new treatment and 42 were cured (30 out of 90 with mild acne and 12 out of 20 with severe acne), for an overall cure rate of $42/110 = 38.2\%$.

Overall, the *old* treatment had the higher cure rate, despite the fact that the new treatment had a higher rate for both mild and severe acne cases.

This example illustrates that it is possible for something to appear better in each of two or more group comparisons but actually be worse overall. If you look carefully, you’ll see that this occurs because of the way in which the overall results are divided into unequally sized groups (in this case, mild acne patients and severe acne patients).

BY THE WAY

The general case in which a set of data gives different results for each of several group comparisons than it does when the groups are taken together is known as *Simpson’s paradox*, so named because it was described by Edward Simpson in 1951. However, the same idea was actually described around 1900 by Scottish statistician George Yule.



EXAMPLE 1 Who Played Better?

Table 6 gives the shooting performance of two players in each half of a basketball game. Shaq had a higher shooting percentage in both the first half (40% to 25%) and the second half (75% to 70%). Can Shaq claim that he had the better game?

TABLE 6 Basketball Shots						
Player	First half			Second half		
	Baskets	Attempts	Percent	Baskets	Attempts	Percent
Shaq	4	10	40%	3	4	75%
Vince	1	4	25%	7	10	70%

SOLUTION No, and we can see why by looking at the overall game statistics. Shaq made a total of 7 baskets (4 in the first half and 3 in the second half) on 14 shots (10 in the first half and 4 in the second half), for an overall shooting percentage of $7/14 = 50\%$. Vince made a total of 8 baskets on 14 shots, for an overall shooting percentage of $8/14 = 57.1\%$. Surprisingly, even though Shaq had a higher shooting percentage in both halves, Vince had a better overall shooting percentage for the game.



BY THE WAY

This mammogram example and the polygraph example that follows it illustrate cases in which conditional probabilities lead to confusion. The proper way of handling conditional probabilities was discovered by the Reverend Thomas Bayes (1702–1761) and is often called *Bayes rule*.

Does a Positive Mammogram Mean Cancer?

We often associate tumors with cancers, but most tumors are not cancers. Medically, any kind of abnormal swelling or tissue growth is considered a tumor. A tumor caused by cancer is said to be *malignant* (or *cancerous*); all others are said to be *benign*.

Imagine you are a doctor or nurse treating a patient who has a breast tumor. The patient will be understandably nervous, but you can give her some comfort by telling her that only about 1 in 100 breast tumors turns out to be malignant. But, just to be safe, you order a mammogram to determine whether her tumor is one of the 1% that are malignant.

Now, suppose the mammogram comes back positive, suggesting that the tumor is malignant. Mammograms are not perfect, so the positive result does not necessarily mean that your patient has breast cancer. More specifically, let's assume that the mammogram screening is 85% accurate: It will correctly identify 85% of malignant tumors as malignant and 85% of benign tumors as benign. When you tell your patient that her mammogram was positive, what should you tell her about the chance that she actually has cancer?

Because the mammogram screening is 85% accurate, most people guess that the positive result means that the patient probably has cancer. Studies have shown that most doctors also believe this to be the case and would tell the patient to be prepared for cancer treatment. But a more careful analysis shows otherwise. In fact, the chance that the patient has cancer is still quite small—about 5%. We can see why by analyzing some numbers.

Consider a study in which mammograms are given to 10,000 women with breast tumors. Assuming that 1% of tumors are malignant, $1\% \times 10,000 = 100$ of the women actually have cancer; the remaining 9,900 women have benign tumors. Table 7 summarizes the mammogram results. Notice the following:

- The mammogram screening correctly identifies 85% of the 100 malignant tumors as malignant. Thus, it gives positive (malignant) results for 85 of the malignant tumors; these cases are called **true positives**. In the other 15 malignant cases, the result is negative, even though the women actually have cancer; these cases are **false negatives**.

TABLE 7 Summary of Results for 10,000 Mammograms (when in fact 100 tumors are malignant and 9,900 are benign)			
	Tumor is malignant	Tumor is benign	Total
Positive mammogram	85 true positives	1,485 false positives	1,570
Negative mammogram	15 false negatives	8,415 true negatives	8,430
Total	100	9,900	10,000

- The mammogram screening correctly identifies 85% of the 9,900 benign tumors as benign. Thus, it gives negative (benign) results for $85\% \times 9,900 = 8,415$ of the benign tumors; these cases are **true negatives**. The remaining $9,900 - 8,485 = 1,485$ women get positive results in which the mammogram incorrectly identifies their tumors as malignant; these cases are **false positives**.

Overall, the mammogram screening gives positive results to 85 women who actually have cancer and to 1,485 women who do *not* have cancer. The total number of positive results is $85 + 1,485 = 1,570$. Because only 85 of these are true positives (the rest are false positives), the chance that a positive result really means cancer is only $85/1,570 = 0.054$, or 5.4%. Therefore, when your patient's mammogram comes back positive, you should reassure her that there's still only a small chance that she has cancer.

EXAMPLE 2 False Negatives

Suppose you are a doctor seeing a patient with a breast tumor. Her mammogram comes back negative. Based on the numbers in Table 7, what is the chance that she has cancer?

SOLUTION For the 10,000 cases summarized in Table 7, the mammograms are negative for 15 women with cancer and for 8,415 women with benign tumors. The total number of negative results is $15 + 8,415 = 8,430$. Thus, the fraction of women with cancer who have false negatives is $15/8,430 = 0.0018$, or slightly less than 2 in 1,000. In other words, the chance that a woman with a negative mammogram has cancer is only about 2 in 1,000. •••

TIME OUT TO THINK

While the chance of cancer with a negative mammogram is small, it is not zero. Therefore, it might seem like a good idea to biopsy all tumors, just to be sure. However, biopsies involve surgery, which means they can be painful and expensive, among other things. Given these facts, do you think that biopsies should be routine for all tumors? Should they be routine for cases of positive mammograms? Defend your opinion.

Polygraphs and Drug Tests

We're now ready to return to the question asked at the beginning of this section, about how a 90% accurate polygraph test can lead to a surprising number of false accusations. The explanation is very similar to that used in the case of the mammograms.

Suppose the government gives the polygraph test to 1,000 applicants for sensitive security jobs. Further suppose that 990 of these 1,000 people tell the truth on their polygraph test, while only 10 people lie. For a test that is 90% accurate, we find the following results:

- Of the 10 people who lie, the polygraph correctly identifies 90%, meaning that 9 fail the test (they are identified as liars) and 1 passes.
- Of the 990 people who tell the truth, the polygraph correctly identifies 90%, meaning that $90\% \times 990 = 891$ truthful people pass the test and the other $10\% \times 990 = 99$ truthful people fail the test.

Figure 16 summarizes these results. The total number of people who fail the test is $9 + 99 = 108$. Of these, only 9 were actually liars; the other 99 were falsely accused of lying. That is, 99 out of 108, or $99/108 = 91.7\%$, of the people who fail the test were actually telling the truth.

The percentage of people who are falsely accused in any real situation depends on both the accuracy of the test and the proportion of people who are lying. Nevertheless, for the numbers given here, we have an astounding result: Assuming the government rejects applicants who fail the polygraph test, then almost 92% of the rejected applicants were actually being truthful and may have been highly qualified for the jobs.

BY THE WAY

The accuracy of breast cancer screening is rapidly improving; newer technologies, including digital mammograms and ultrasounds, appear to achieve accuracies near 98%. The most definitive test for cancer is a biopsy, though even biopsies can miss cancers if they are not taken with sufficient care. If you have negative tests but are still concerned about an abnormality, ask for a second opinion. It may save your life.

BY THE WAY

A polygraph, often called a "lie detector," measures a variety of bodily functions including heart rate, skin temperature, and blood pressure. Polygraph operators look for subtle changes in these functions that typically occur when people lie. However, polygraph results have never been allowed as evidence in criminal proceedings. First of all, 90% accuracy is far too low for justice. In addition, studies show that polygraphs are easily fooled by people who train to beat them.



Describing Data

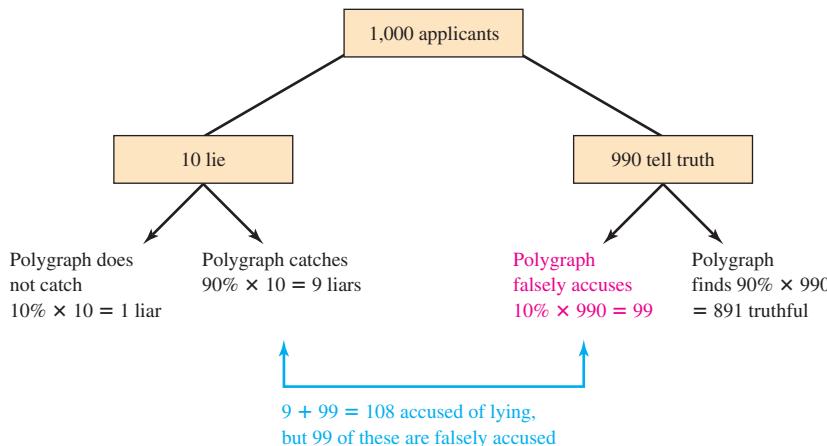


Figure 16 A tree diagram summarizes results of a 90% accurate polygraph test for 1,000 people, of whom only 10 are lying.

TIME OUT TO THINK

Imagine that you are falsely accused of a crime. The police suggest that, if you are truly innocent, you should agree to take a polygraph test. Would you do it? Why or why not?

EXAMPLE 3 High School Drug Testing

All athletes participating in a regional high school track and field championship must provide a urine sample for a drug test. Those who fail are eliminated from the meet and suspended from competition for the following year. Studies show that, at the laboratory selected, the drug tests are 95% accurate. Assume that 4% of the athletes actually use drugs. What fraction of the athletes who fail the test are falsely accused and therefore suspended without cause?

SOLUTION The easiest way to answer this question is by using some sample numbers. Suppose there are 1,000 athletes in the meet. Then 4%, or 40 athletes, actually use drugs; the remaining 960 athletes do not use drugs. In that case, the 95% accurate drug test should return the following results:

- 95% of the 40 athletes who use drugs, or $0.95 \times 40 = 38$ athletes, fail the test. The other 2 athletes who use drugs pass the test.
 - 95% of the 960 athletes who do not use drugs pass the test, but 5% of these 960, or $0.05 \times 960 = 48$ athletes, fail.

The total number of athletes who fail the test is $38 + 48 = 86$. But 48 of these athletes who fail the test, or $48/86 = 56\%$, are actually nonusers. Despite the 95% accuracy of the drug test, more than half of the suspended students are innocent of drug use.

Section 4 Exercises

Statistical Literacy and Critical Thinking

- 1. False Positive and False Negative.** A baseball player is given a test for banned substances. For this test, what is a false positive? What is a false negative? What is a true positive? What is a true negative?
 - 2. Positive Test Result.** Jennifer is given a pregnancy test. What does it mean when she is told that the result is positive?

- 3. Test Result.** If you apply a test for the presence of alcohol, what is the result called if the test correctly indicates that the subject has not consumed alcohol?

- 4. Better in Each Half, Worse Overall.** When the Giants and Patriots football teams play each other, can one of the quarterbacks have a higher passing

percentage in each half while having a lower passing percentage for the entire game?

Does It Make Sense? For Exercises 5–8, decide whether the statement makes sense (or is clearly true) or does not make sense (or is clearly false). Explain clearly; not all of these statements have definitive answers, so your explanation is more important than your chosen answer.

5. Course Average. Ann and Bret are taking the same statistics course, in which the final grade is determined by assignments and exams. Ann's mean score on the assignments is higher than Bret's, and Ann's mean score on the exams is higher than Bret's. It follows that Ann's overall mean score in the course is higher than Bret's.

6. Batting Average. Ann's batting average for the first half of the softball season is higher than Bret's, and Ann's batting average for the second half of the season is higher than Bret's. It follows that Ann's batting average for the entire season is higher than Bret's.

7. Test Results. After taking a test for the presence of a disease, a patient is happy because the physician announces that the test results are positive.

8. Test Accuracy. If a drug test is 90% accurate, it follows that 90% of those who test positive are actual drug users.

Concepts and Applications

9. Batting Percentages. The table below shows the batting records of two baseball players in the first half (first 81 games) and last half of a season.

Player	First half		
	Hits	At-bats	Batting average
Josh	50	150	0.333
Jude	10	50	0.200
Second half			
Player	Hits	At-bats	Batting average
Josh	35	70	0.500
Jude	70	150	0.467

Who had the higher batting average in the first half of the season? Who had the higher batting average in the second half? Who had the higher overall batting average? Explain how these results illustrate Simpson's paradox.

10. Passing Percentages. The table below shows the passing records of two rival quarterbacks in the first half and second half of a football game.

Player	First half		
	Completions	Attempts	Percent
Allan	8	20	40%
Abner	2	6	33%
Second half			
Player	Completions	Attempts	Percent
Allan	3	6	50%
Abner	12	25	48%

Who had the higher completion percentage in the first half? Who had the higher completion percentage in the second half? Who had the higher overall completion percentage? Explain how these results illustrate Simpson's paradox.

11. Test Scores. The table below shows eighth-grade mathematics test scores in Nebraska and New Jersey. The scores are separated according to the race of the student. Also shown are the state averages for all races.

	White	Nonwhite	Average for all races
Nebraska	281	250	277
New Jersey	283	252	272

Source: National Assessment of Educational Progress, from *Chance* magazine.

- a. Which state had the higher scores in both racial categories? Which state had the higher overall average across both racial categories?
- b. Explain how a state could score lower in both categories and still have a higher overall average.
- c. Now consider the table below, which gives the percentages of whites and nonwhites in each state. Use these percentages to verify that the overall average test score in Nebraska is 277, as claimed in the first table.

	White	Nonwhite
Nebraska	87%	13%
New Jersey	66%	34%

- d. Use the racial percentages to verify that the overall average test score in New Jersey is 272, as claimed in the first table.
- e. Explain briefly, in your own words, how Simpson's paradox appeared in this case.

12. Test Scores. Consider the following table comparing the grade point averages (GPAs) and mathematics SAT scores of high school students in 1988 and 1998 (before the SAT test format was revised).

GPA	% students		SAT score		Change
	1988	1998	1988	1998	
A +	4	7	632	629	-3
A	11	15	586	582	-4
A -	13	16	556	554	-2
B	53	48	490	487	-3
C	19	14	431	428	-3
Overall average			504	514	+10

Source: Cited in *Chance*, Vol. 12, No. 2, 1999, from data in *New York Times*, September 2, 1999.

- a. In general terms, how did the SAT scores of the students in the five grade categories change between 1988 and 1998?
- b. How did the overall average SAT score change between 1988 and 1998?
- c. How is this an example of Simpson's paradox?

- 13. Tuberculosis Deaths.** The following table shows deaths due to tuberculosis (TB) in New York City and Richmond, Virginia, in 1910.

Race	New York	
	Population	TB deaths
White	4,675,000	8400
Nonwhite	92,000	500
Total	4,767,000	8900
Race	Richmond	
	Population	TB deaths
White	81,000	130
Nonwhite	47,000	160
Total	128,000	290

Source: Cohen and Nagel, An Introduction to Logic and Scientific Method, Harcourt, Brace and World, 1934.

- Compute the death rates for whites, nonwhites, and all residents in New York City.
- Compute the death rates for whites, nonwhites, and all residents in Richmond.
- Explain why this is an example of Simpson's paradox and explain how the paradox arises.

- 14. Weight Training.** Two cross-country running teams participated in a (hypothetical) study in which a fraction of each team used weight training to supplement a running workout. The remaining runners did not use weight training. At the end of the season, the mean improvement in race times (in seconds) was recorded in the table below.

	Mean improvement (seconds)		
	Weight training	No weight training	Team average
Gazelles	10	2	6.0
Cheetahs	9	1	6.2

Describe how Simpson's paradox arises in this table. Resolve the paradox by finding the percentage of each team that used weight training.

- 15. Basketball Records.** Consider the following (hypothetical) basketball records for Spelman and Morehouse Colleges.

	Spelman College	Morehouse College
Home games	10 wins, 19 losses	9 wins, 19 losses
Away games	12 wins, 4 losses	56 wins, 20 losses

- Give numerical evidence to support the claim that Spelman College has a better team than Morehouse College.
 - Give numerical evidence to support the claim that Morehouse College has a better team than Spelman College.
 - Which claim do you think makes more sense? Why?
- 16. Better Drug.** Two drugs, A and B, were tested on a total of 2,000 patients, half of whom were women and half of

whom were men. Drug A was given to 900 patients and Drug B to 1,100 patients. The results appear in the table below.

	Women	Men
Drug A	5 of 100 cured	400 of 800 cured
Drug B	101 of 900 cured	196 of 200 cured

- Give numerical evidence to support the claim that Drug B is more effective than Drug A.
- Give numerical evidence to support the claim that Drug A is more effective than Drug B.
- Which claim do you think makes more sense? Why?

- 17. Polygraph Test.** The results in the table below are from experiments conducted by researchers Charles R. Honts (Boise State University) and Gordon H. Barland (Department of Defense Polygraph Institute). In each case, it was known whether the subject lied, so the table indicates when the polygraph test was correct.

	Did the Subject Actually Lie?	
	No	Yes
Polygraph test indicated that the subject <i>lied</i> .	15	42
Polygraph test indicated that the subject did <i>not lie</i> .	32	9

- Based on the test results, how many subjects appeared to be lying? Of these, how many were actually lying and how many were telling the truth? What percentage of those who appear to be lying were not actually lying?
- Based on the test results, how many subjects appeared to be telling the truth? Of those, how many were actually telling the truth? What percentage of those who appeared to be telling the truth are actually truthful?

- 18. Disease Test.** Suppose a test for a disease is 80% accurate for those who have the disease (true positives) and 80% accurate for those who do not have the disease (true negatives). Within a sample of 4,000 patients, the incidence rate of the disease matches the national average, which is 1.5%.

	Disease	No disease	Total
Test positive	48	788	836
Test negative	12	3,152	3,164
Total	60	3,940	4,000

- Of those with the disease, what percentage test positive?
- Of those who test positive, what percentage have the disease? Compare this result to the one in part a and explain why they are different.
- Suppose a patient tests positive for the disease. As a doctor using this table, how would you describe the patient's chance of actually having the disease? Compare this figure to the overall incidence rate of the disease.

Further Applications

- 19. Hiring Statistics.** (This problem is based on an example in “Ask Marilyn” column in *Parade Magazine*.) A company decided to expand, so it opened a factory, generating 455 jobs. For the 70 white-collar positions, 200 males and 200 females applied. Of the females who applied, 20% were hired, while only 15% of the males were hired. Of the 400 males applying for the blue-collar positions, 75% were hired, while 85% of the 100 females who applied were hired. How does looking at the white-collar and blue-collar positions separately suggest a hiring preference for women? Do the overall data support the idea that the company hires women preferentially? Explain why this is an example of Simpson’s paradox and how the paradox can be resolved.
- 20. Drug Trials.** (This problem is based on an example from the “Ask Marilyn” column in *Parade Magazine*.) A company runs two trials of two treatments for an illness. In the first trial, Treatment A cures 20% of the cases (40 out of 200) and Treatment B cures 15% of the cases (30 out of 200). In the second trial, Treatment A cures 85% of the cases (85 out of 100) and Treatment B cures 75% of the cases (300 out of 400). Which treatment had the better cure rate in the two trials individually? Which treatment had the better overall cure rate? Explain why this is an example of Simpson’s paradox and how the paradox can be resolved.
- 21. HIV Risks.** The New York State Department of Health estimates a 10% rate of HIV for the at-risk population and a 0.3% rate for the general population. Tests for HIV are 95% accurate in detecting both true negatives and true positives. Random selection and testing of 5,000 at-risk people and 20,000 people from the general population results in the following table.
- | | | At-risk population | |
|---------------------|-----|--------------------|---------------|
| | | Test positive | Test negative |
| Infected | 475 | 25 | |
| Not infected | 225 | 4,275 | |
| | | General population | |
| | | Test positive | Test negative |
| Infected | 57 | 3 | |
| Not infected | 997 | 18,943 | |
- a. Verify that incidence rates for the general and at-risk populations are 0.3% and 10%, respectively. Also verify that detection rates for the general and at-risk populations are 95%.
- b. Consider the at-risk population. Of those with HIV, what percentage test positive? Of those who test positive,
- what percentage have HIV? Explain why these two percentages are different.
- c. Suppose a patient in the at-risk category tests positive for the disease. As a doctor using this table, how would you describe the patient’s chance of actually having the disease? Compare this figure with the overall incidence rate of the disease.
- d. Consider the general population. Of those with HIV, what percentage test positive? Of those who test positive, what percentage have HIV? Explain why these two percentages are different.
- e. Suppose a patient in the general population tests positive for the disease. As a doctor using this table, how would you describe the patient’s chance of actually having the disease? Compare this figure to the overall incidence rate of the disease.



PROJECTS FOR THE INTERNET & BEYOND

- 22. Polygraph Arguments.** Visit Web sites devoted to either opposing or supporting the use of polygraph tests. Summarize the arguments on both sides, specifically noting the role that false negative rates play in the discussion.
- 23. Drug Testing.** Explore the issue of drug testing either in the workplace or in athletic competitions. Discuss the legality of drug testing in these settings and the accuracy of the tests that are commonly conducted.
- 24. Cancer Screening.** Investigate recommendations concerning routine screening for some type of cancer (for example, breast cancer, prostate cancer, or colon cancer). Explain how the accuracy of the screening test is measured. How is the test useful? How can its results be misleading?

IN THE NEWS

- 25. Polygraphs.** Find a recent article in which someone or some group proposes a polygraph test to determine whether a person is being truthful. In light of what you know about polygraph tests, do you think the results will be meaningful? Why or why not?
- 26. Drug Testing and Athletes.** Find a news report concerning drug testing of athletes. Summarize how the testing is being used, and discuss whether the testing is reliable.

CHAPTER REVIEW EXERCISES

- 1. Nicotine in Cigarettes.** Listed below are the nicotine amounts (in mg per cigarette) for samples of filtered and non-filtered cigarettes. Do filters appear to be effective in reducing the amount of nicotine?

Non-filtered: 1.1 1.7 1.7 1.1 1.1 1.4 1.1 1.4 1.0 1.2

Filtered: 0.4 1.0 1.2 0.8 0.8 1.0 1.1 1.1 1.1 0.78

- a. Find the mean and median for each of the two data sets.
- b. Find the range and standard deviation for each of the two data sets.
- c. Give the five-number summary and construct a boxplot for each of the two data sets.
- d. Apply the range rule of thumb to estimate the standard deviation of each of the two data sets. How well does the rule work in each case? Briefly discuss why it does or does not work well.
- e. Based on all your results, compare and discuss the two data sets in terms of their center and variation. Does there appear to be a difference between the amounts of nicotine in non-filtered cigarettes and filtered cigarettes?

- 2.** Combine the two samples from Review Exercise 1 and find the following:

- a. The percentile for the amount of 1.4 mg
- b. The mode

- a. What is the standard deviation for a sample of 50 values, all of which are the same?
- b. Which of the following two car batteries would you prefer to buy, and why?
 - One taken from a population with a mean life of 48 months and a standard deviation of 2 months
 - One taken from a population with a mean life of 48 months and a standard deviation of 6 months
- c. If an outlier is included with a sample of 50 values, what is the effect of the outlier on the mean?
- d. If an outlier is included with a sample of 50 values, what is the effect of the outlier on the median?
- e. If an outlier is included with a sample of 50 values, what is the effect of the outlier on the range?
- f. If an outlier is included with a sample of 50 values, what is the effect of the outlier on the standard deviation?

CHAPTER QUIZ

1. When you add the pulse rates of 65, 74, 88, 77, and 92, then divide by the number of values, the result is 79.2. Which term best describes this value: average, mean, median, mode, or standard deviation?
2. Find the median of the pulse rates given in Exercise 1.
3. What is the range of the pulse rates given in Exercise 1?
4. The standard deviation of the pulse rates given in Exercise 1 is 10.9. What characteristic does that value measure?
5. A histogram is constructed for a large set of pulse rates of adult males, and it is found that the distribution is symmetric and unimodal. What does this imply about the values of the mean and median?
6. Indicate whether the given statement could apply to a data set consisting of 1,000 values that are all different.
 - a. The 20th percentile is greater than the 30th percentile.
 - b. The median is greater than the first quartile.
7. A standard test for braking reaction time of drivers is designed so that the mean is 2.00 sec and the standard deviation is 0.25 sec. Based on the range rule of thumb, what are the likely low and high values?
8. Use the range rule of thumb to estimate the standard deviation of the pulse rates given in Exercise 1. How does the result compare to the actual standard deviation of 10.9?
9. Find the standard deviation of these body temperatures ($^{\circ}\text{F}$): 98.2, 98.2, 98.2, 98.2, and 98.2.
10. Identify the components that constitute the five-number summary for a data set.



FOCUS ON

THE STOCK MARKET

What's Average About the Dow?

As “averages” go, this one is extraordinary. You can’t watch the news without hearing what happened to it, and many people spend hours tracking it each day. It is by far the most famous indicator of stock market performance. We are talking, of course, about the Dow Jones Industrial Average, or DJIA for short. But what exactly is it?

The easiest way to understand the DJIA is by looking at its history. As the modern industrial era got under way in the late 19th century, most people considered stocks to be dangerous and highly speculative investments. One reason was a lack of regulation that made it easy for wealthy speculators, unscrupulous managers, and corporate raiders to manipulate stock prices. But another reason was that, given the complexities of daily stock trading, even Wall Street professionals had a hard time figuring out whether stocks in general were going up (a “bull market”) or down (a “bear market”). Charles H. Dow, the founder (along with Edward D. Jones) and first editor of the *Wall Street Journal*, believed he could rectify this problem by creating an “average” for the stock market as a whole. If the average was up, the market was up, and if the average was down, the market was down.

To keep the average simple, Dow chose 12 large corporations to include in his average. On May 26, 1896, he added the stock prices of these 12 companies and divided by 12, finding a mean stock price of \$40.94. This was the first value for the DJIA. As Dow had hoped, it suddenly became easy for the public to follow the market’s direction just by comparing his average from day to day, month to month, or year to year.

The basic idea behind the DJIA is still the same, although the list now includes 30 stocks rather than 12; the list is selected by the editors of the *Wall Street Journal*, who occasionally change the stocks on the list. However, the DJIA is no longer the mean price of its 30 stocks. Instead, it is calculated by adding the prices of its 30 stocks and dividing by a special divisor. Because of this divisor, we now think of the DJIA as an index that helps us keep track of stock values, rather than as an actual average of stock prices.

The divisor is designed to preserve continuity in the underlying value represented by the DJIA, and it therefore must change whenever the list of 30 stocks changes or when a company on the list has a stock split. A simple example shows why the divisor must change when the list changes. Suppose the DJIA consisted of only 2 stocks (rather than 30): Stock



A with a price of \$100 and Stock B with a price of \$50. The mean price of these two stocks is $(\$100 + \$50)/2 = \$75$. Now, suppose that we change the list by replacing Stock B with Stock C and that Stock C’s price is \$200. The new mean is $(\$100 + \$200)/2 = \$150$, so merely replacing one stock on the list would raise the mean price from \$75 to \$150. Therefore, to keep the “value” of the DJIA constant when we change this list, we must divide the new mean of \$150 by 2. In this way, the DJIA remains 75 both before and after the list change, but we can no longer think of this 75 as a mean price in dollars.

To see why a stock split changes the divisor, again suppose the index consists of just two stocks: Stock X at \$100 and Stock Y at \$50, for a mean price of \$75. Now, suppose Stock X undergoes a 2-for-1 stock split, so that its new price is \$50. With both stocks now priced at \$50, the mean price after the stock split would also be \$50. In other words, even though a stock split does not affect a company’s total value (it only changes the number and prices of its shares), we’d find a drop in the mean price from \$75 to \$50. In this case, we can preserve continuity by dividing the new mean of 50 by 2/3 (which is equivalent to multiplying by 3/2) so that the DJIA holds at 75 both before and after the stock split.

Just as in these simple examples, the real divisor changes with every list change or stock split, so it has changed many times since Charles Dow first calculated the DJIA as an actual mean. The current value of the divisor is published daily in the *Wall Street Journal*.

Given that there are now well over 10,000 actively traded stocks, it might seem remarkable that a sample of only 30 could reflect overall market activity. But today, when computers make it easy to calculate stock market “averages” in many other ways, we can look at historical data and see that the DJIA has indeed been a reliable indicator of overall market performance. Figure 17 shows the historical performance of the DJIA.

If you study Figure 17 carefully, you may be tempted to think that you can see patterns that would allow you to forecast precise values of the market in the future. Unfortunately, no



Figure 17 Historical values of the Dow Jones Industrial Average, 1900 through 2011 Note that the vertical axis uses an exponential scale in which the value doubles with each equivalent increment in height; this makes it easier to see the changes that occurred when the DJIA was low compared with its value today. Chart courtesy of StockCharts.com.

one has ever found a way to make reliable forecasts, and most economists now believe that such forecasts are impossible.

The futility of trying to forecast the market is illustrated by the story of the esteemed Professor Benjamin Graham, often called the father of “value investing.” In the spring of 1951, one of his students came to him for some investment advice. Professor Graham noted that the DJIA then stood at 250, but that it had fallen below 200 at least once during every year since its inception in 1896. Because it had not yet fallen below 200 in 1951, Professor Graham advised his student to hold off on buying until it did. Professor Graham presumably followed his own advice, but the student did not. Instead, the student invested his “about 10 thousand bucks” in the market right away. As it turned out, the market never did fall below 200 in 1951 or any time thereafter. And the student, named Warren Buffet, became a billionaire many times over.

QUESTIONS FOR DISCUSSION

1. The stock market is still considered a riskier investment than, say, bank savings accounts or bonds. Nevertheless, financial advisors almost universally recommend holding at least some stocks, which is quite different from the situation that prevailed a century ago. What role do you think the DJIA played in building investors’ confidence in the stock market?

2. The DJIA is only one of many different stock market indices in wide use today. Briefly look up a few other indices, such as the S&P 500, the Russell 2000, and the NASDAQ. How do these indices differ from the DJIA? Do you think that any of them should be considered more reliable indicators of the overall market than the DJIA? Why or why not?

3. The 30 stocks in the DJIA represent a sample of the more than 10,000 actively traded stocks, but it is not a *random* sample because it is chosen by particular editors for particular reasons that may include personal biases. Suppose that you chose a random sample of 30 stocks and tracked their prices. Do you think that such a random sample would track the market as well as the stocks in the DJIA? Why or why not?

4. Create your own “portfolio” of 10 stocks that you’d like to own, and assume you own 100 shares of each. Calculate the total value of your portfolio today, and track price changes over the next month. At the end of the month, calculate the percent change in the value of your portfolio. How did the performance of your portfolio compare to the performance of the DJIA during the month? If you really owned these stocks, would you continue to hold them or would you sell? Explain.



FOCUS ON ECONOMICS

Are the Rich Getting Richer?

The media love to report on the lavish spending of the super-rich, making it seem that the rich keep getting richer while the rest of us are left behind. But is it true?

If we wish to draw general conclusions about how the average person is faring compared to the rich, we must look at the overall income distribution. Economists have developed a number, called the **Gini Index**, that is used to describe the level of equality or inequality in the income distribution. The Gini Index is defined so that it can range only between 0 and 1. A Gini Index of 0 indicates perfect income equality, in which every person has precisely the same income. A Gini Index of 1 indicates perfect inequality, in which a single person has all the income and no one else has anything. Figure 18 shows the Gini Index in the United States since 1947. Note that the Gini Index fell from 1947 to 1968, indicating that the income distribution became more uniform during this period. The Gini Index has generally risen ever since, indicating that the rich are, indeed, getting richer.

Although the Gini Index provides a simple single-number summary of income inequality, the number itself is fairly difficult to interpret (and to calculate). An alternative way to look at the income distribution is to study income quintiles, which divide the population into fifths by income. Often, the highest quintile is further broken down to show how the top 5% of income earners compares with others.



Figure 19 shows the share of total income received by each quintile and the top 5% in the United States in different decades. The height of each bar (the number on top of it) represents the share of total income. For example, the 3.3 on the bar for the lowest quintile in 2010 means that the poorest 20% of the population received only 3.3% of the total income in the United States. Similarly, the 50.2 on the bar for the top quintile in 2010 means that the richest 20% of the population received 50.2% of the income. Note also that the richest 5% received 21.3% of the income—nearly double the total income of the poorest 40% of the population. If you study this graph carefully, you'll see that the share of income earned by the first four quintiles—which means all but the richest 20% of the population—dropped since 1970. Meanwhile, the share earned by the richest 20% rose substantially, as did the share of the top 5%. In other words, this graph also confirms that the rich have been getting richer compared to most of the population.

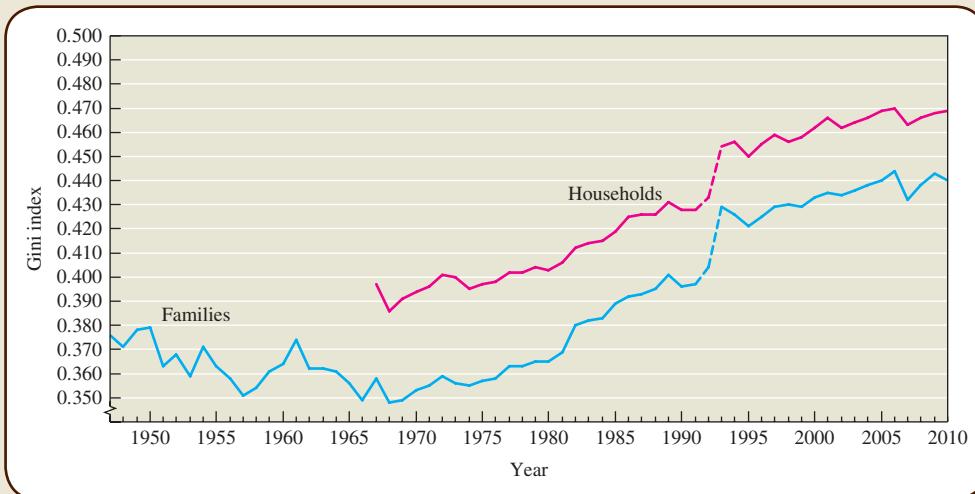


Figure 18 Gini Index for families and households, 1947–2010. Household data, which include single people and households in which the members are not part of the same family, have been taken only since 1967. The dashed segments in 1993 indicate a change in the methodology for data collection, so the corresponding rise in the Gini Index may be partially or wholly due to this change rather than a real change in income inequality. *Source:* Adapted from data by the U.S. Census Bureau.

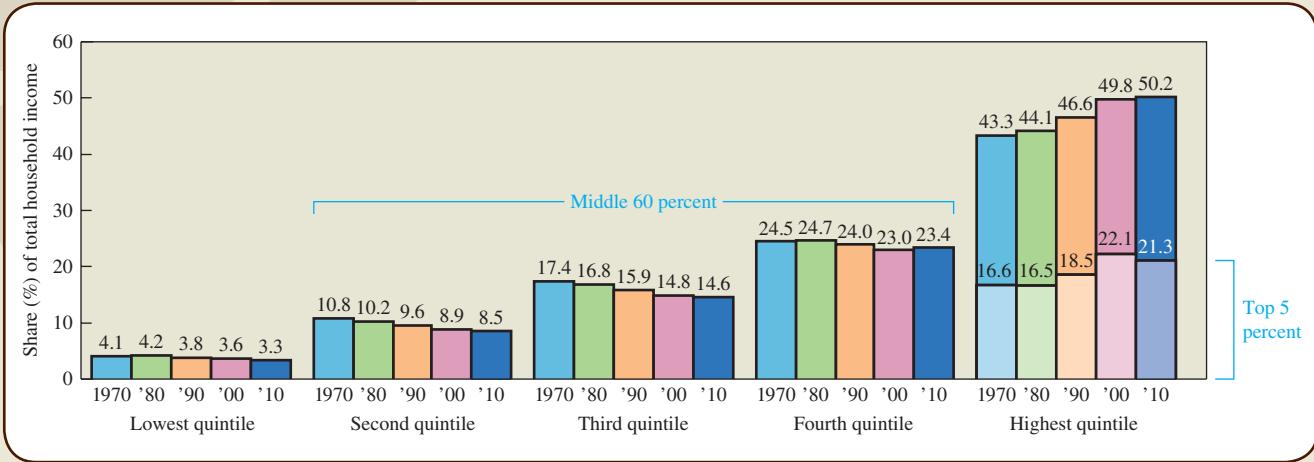


Figure 19 Share of total household income by quintile (and top 5%) at 10-year intervals. Source: U.S. Census Bureau.

Now that we've established that the rich are getting richer, the next question is whether it matters. Most people, including most economists, have traditionally assumed that rising income inequality is bad for democracies. But a few economists from both the left and the right of the political spectrum argue that the change in recent decades is different. For one thing, the change meets a widely accepted ethical condition called the *Pareto criterion*, after the Italian economist Vilfredo Pareto (for whom Pareto charts are also named): Any change is good if it makes someone better off without making anyone else worse off. The Pareto criterion appears to be satisfied because overall growth in the U.S. economy has been helping nearly everyone. In other words, most people may have a smaller percentage of total income than they had in the past, but they still have more absolute income and therefore are living better than they did in the past.

Secondly, today's rich differ from the rich in the past. For example, as recently as 1980, 60% of the "Forbes 400" (the richest 400 people) had inherited most of their wealth. Today, less than 20% of the Forbes 400 represents old money. The implication is that while you had to be born rich in the past, today you can *become* rich by getting educated and working hard. Surely, it is a good thing to encourage education and hard work.

Finally, while overall income inequality has increased, the income inequality among different races and between men and women has decreased. In other words, it is now easier than it was in the past for African Americans, Hispanics, and

women to earn as much as white males. Again, this is surely a good thing for our democratic values, even if we still have a long way to go before the inequalities are completely eliminated.

QUESTIONS FOR DISCUSSION

1. Compare several different ways of looking at the data shown in Figure 18 and Figure 19. For example, does one seem to indicate a larger change in income inequality than the other? Can you think of other possible ways to display income data that might give a different picture than those shown here?
 2. Do you agree that the Pareto criterion is a good way to evaluate the ethics of economic change? Why or why not?
 3. Overall, do you think the increase in income inequality has been a good or bad thing for the United States? Will it be good if the trend continues? Defend your opinion.
 4. Although economic data suggest that the vast majority of Americans are better off today than they were a few decades ago, the poorest Americans still live in difficult economic conditions. What do you think can or should be done to help improve the lives of the poor? Can your suggestion be implemented without harming the overall economy? Explain.
- • • • • • • • • •

ANSWERS

Section 1

1. No. The mean refers specifically to the sum of all data values divided by the number of data values. The term “average” could refer to any one of several different statistics, including the mean, median, or mode.
3. Yes. The executive secretary’s income is much higher than the incomes of the other 23 students, so it is an outlier. In general, the definition of an outlier is somewhat vague, so an outlier cannot be clearly and objectively identified. (However, statisticians have created rules that can be applied to identify outliers in some cases.)
5. Does not make sense.
7. Makes sense.
9. Mean. For the population of basketball players, there are only a few heights that are extreme, and they are not extreme by very large amounts. In this case, the mean would be better because it takes every height into account.
11. Median. The exact values of “over” ages are not known, so the mean cannot be computed.
13. Mean: 53.3; median: 52.0; mode: none
15. Mean: 58.3 sec; median: 55.5 sec; mode: 49 sec
17. Mean: 0.188; median: 0.165; mode: 0.16
19. Mean: 0.9194 g; median: 0.9200 g; there is no mode
21. a. Mean: 157,586; median: 104,100 b. Alaska is an outlier on the high end. Without Alaska the mean is 81,317 and the median is 78,650. c. Connecticut is an outlier on the low end. Without Connecticut (but with Alaska) the mean is 182,933 and the median is 109,050.
23. a. 85.0 b. 100 c. No; a mean of 90 for five quizzes requires a total of 450 points, which you cannot reach even with 100 on the fifth quiz.
25. Mean score of your students: 74.7; median score: 70.0. So it depends on the meaning of “average.” If it is mean, then your students are above average in their scores. If it is median, then your students are below average.
27. 0.39 pound
29. Each student is taking three classes with enrollments of 20 each and one class with an enrollment of 100, so the mean class size for each student is $(20 + 20 + 20 + 100)/4 = 40$. There are three classes with 100 students each and 45 classes with 20 students each, so the 48 courses have a mean class size of $(300 + 900)/48 = 25$. The two means are very different.
31. 0.417; it is the average number of hits per at-bat.
33. Probably not. The overall rate of defects would be 3% only if both sites produce the same number of batteries.
35. The outcome is no, by 600 votes to 400 votes.
37. Mean: 1.9; median: 2; mode: 1. The mode of 1 correctly indicates that the smooth yellow peas occur more than any other phenotype, but the mean and median don’t make sense with these data at the nominal level of measurement.

Section 2

1. The distribution is not symmetric.
 3. Because the professors have successfully completed a rigorous program of education, their IQ scores are likely to all be higher than average and closer together, so they probably have less variation than IQ scores of randomly selected adults. The lower variation of IQ scores of the professors results in a graph that is narrower with less spread than the graph of IQ scores from the randomly selected adults.
 5. Does not make sense.
 7. Makes sense.
 9. Two modes, left-skewed, wide variation.
 11. Single-peaked, nearly symmetric, moderate variation.
 13. a. The distribution is right-skewed. It is not symmetric.
b. 409 (half of 817)
 15. a. One mode of \$0 b. Right-skewed, because there will be many students making little or nothing.
 17. a. One mode b. Symmetric
 19. a. One mode b. Symmetric
 21. a. One mode b. Left-skewed
 23. a. One mode b. Right-skewed
 25. a. One mode b. Right-skewed
-
1. The range is found by subtracting the lowest sample value from the highest. It is a measure of variation. A major disadvantage of the range is that its value depends on only the lowest and highest sample values and it does not take every value into account.
 3. The statement is incorrect because it defines the standard deviation as a value that depends on the minimum and maximum values, but the standard deviation uses every data value.
 5. Does not make sense.
 7. Makes sense.
 9. Range: 45.0 words; standard deviation: 15.7 words
 11. Range: 26.0 sec; standard deviation: 9.5 sec
 13. Range: 0.170; standard deviation: 0.057
 15. Range: 0.1160 g; standard deviation: 0.0336 g
 17. *Cat on a Hot Tin Roof*: range = 10.0 and standard deviation = 2.6. *The Cat in the Hat*: range = 3.0 and standard deviation = 0.9. There is much less variation among the word lengths in *The Cat in the Hat*.
 19. One day: range = 11.0 degrees and standard deviation = 2.6 degrees. Five day: range = 15.0 degrees and standard deviation = 4.5 degrees. As expected, there appears to be greater variation in errors from the five-day forecast.

- 21.** a. 5th percentile b. 69th percentile c. 48th percentile
- 23.** Answers for data set (1) only: a. Histogram has frequency of 7 for data value of 9; no other values. b. low value = 9; lower quartile = 9; median = 9; upper quartile = 9; high value = 9 c. 0
- 25.** a. Faculty: mean = 2, median = 2, range = 4; student: mean = 6.2, median = 6, range = 9. b. Faculty: low = 0, lower quartile = 1, median = 2, upper quartile = 3, high = 4; student: low = 1, lower quartile = 4, median = 6, upper quartile = 9, high = 10. c. Faculty: st. dev. = 1.2; student: st. dev. = 3.0. d. Faculty: range/4 = 1.0; student: range/4 = 2.3.
- 27.** a. First 7: mean = 58.3, median = 57, range = 4; Last 7: mean = 56.1, median = 54, range = 23. b. First 7: low = 57, lower quartile = 57, median = 57, upper quartile = 61, high = 61; Last 7: low = 46, lower quartile = 47, median = 54, upper quartile = 64, high = 69. c. First 7: st. dev. = 1.9; Last 7: st. dev. = 8.7. d. First 7: range/4 = 1; Last 7: range/4 = 5.75.
- 29.** The second site has better quality. Although the second site has a mean that is a little farther from the target of 12 volts, the standard deviation is much less, so the voltages will be more consistent.
- 31.** Having more stocks in the portfolio reduces the variation in the returns, because some stocks are likely to go up and others go down. The reduced variation means reduced risk, though also a lower likelihood of large gains.

Section 4

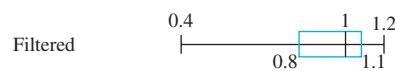
- 1.** A false positive occurs when the test indicates use of banned substances for someone who does not actually use them. A false negative occurs when the test indicates that banned substances are not used by someone who actually does use them. A true positive occurs when the player tests positive and he actually does use banned substances. A true negative occurs when the player tests negative and he does not use banned substances.
- 3.** True negative
- 5.** Makes sense.
- 7.** Does not make sense.
- 9.** Josh; Josh; Jude
- 11.** a. New Jersey; Nebraska
- 13.** a. Whites: 0.18%; nonwhites: 0.54%; total 0.19%. b. Whites: 0.16%; nonwhites: 0.34%; total 0.23%. c. The rate for both whites and nonwhites was higher in New York than in Richmond, yet the overall rate was higher in Richmond than in New York. The percentage of nonwhites was significantly lower in New York than in Richmond.
- 15.** a. Spelman has a better record for home games (34.5% vs. 32.1%) and away games (75.0% vs. 73.3%), individually. b. Morehouse has a better overall average. c. Morehouse has a better team, as teams are generally rated on overall records.
- 17.** a. 57 appeared to be lying. 15 of them were telling the truth, and 42 were actually lying. 26% of those who appear to be lying were not actually lying. b. 41 appeared to be telling the truth. 32 of them were actually telling the truth, and 9 were

lying. 78% of those who appeared to be telling the truth were actually truthful.

- 19.** A higher percentage of women than men were hired in both the white-collar and blue-collar positions, suggesting a hiring preference for women. Overall, 20% of the 200 females (40) who applied for white-collar positions were hired, and 85% of the 100 females (85) who applied for blue-collar positions were hired. Thus $40 + 85 = 125$ of the $200 + 100 = 300$ females who applied were hired, a percentage of 41.7. Overall, 15% of the 200 males (30) who applied for white-collar positions were hired, and 75% of the 400 males (300) who applied for blue-collar positions were hired. Thus $30 + 300 = 330$ of the $200 + 400 = 600$ males who applied were hired, a percentage of 55.0.
- 21.** a. In the general population, $57 + 3 = 60$ of the 20,000 in the sample are infected. This is an incidence rate of $60/20000 = 0.003$ or 0.3%. In the “at-risk” population, $475 + 25 = 500$ of the 5,000 in the sample are infected. This is an incidence rate of $500/5000 = 10.0\%$. b. In the “at-risk” category, 475 out 500 infected with HIV test positive, or 95%. Of those who test positive, 475 out of $475 + 225 = 700$ have HIV, a percentage of $475/700 = 0.679$ or 67.9%. These two figures are different because they measure different things. The 700 who test positive include 225 who were false positives. While the test correctly identifies 95% of those who have HIV, it also incorrectly identifies some who do not have HIV. Thus only 67.9 % of those who test positive actually have HIV. c. In the “at-risk” population, a patient who tests positive for the disease has about a 68% chance of actually having the disease. This is nearly 7 times as great as the incidence rate (10%) of the disease in the at-risk category. Thus the test is very valuable in identifying those with HIV. d. In the general population, patients with HIV test positive 57 times out of 60, or 95% of the time. Of those who test positive, 57 out of $57 + 997 = 1054$ actually have HIV, a percentage of $57/1054 = 0.054$ or 5.4%. These two figures are different because they measure different things. The 1054 who test positive include 997 who were false positives. While the test correctly identifies 95% of those who have HIV, it also incorrectly identifies some who do not have HIV. Thus only 5.4 % of those who test positive actually have HIV. e. In the general population, a patient who tests positive for the disease has about a 5.4% chance of actually having the disease. This is 18 times as great as the incidence rate (0.3%) of the disease in the general population. Thus the test is very valuable in identifying those with HIV.

Chapter Review Exercises

- 1.** a. Non-filtered: Mean is 1.28 mg and median is 1.15 mg. Filtered: Mean is 0.93 mg and median is 1.00 mg. b. Non-filtered: Range is 0.70 mg and standard deviation is 0.26 mg. Filtered: Range is 0.80 mg and standard deviation is 0.24 mg.



- d. Non-filtered: Standard deviation is estimated to be 0.18 mg and it is actually 0.26 mg. Filtered: Standard deviation is

estimated to be 0.20 and it is actually 0.24 mg. Both estimates are in the general ballpark of the actual values. **e.** There does appear to be a difference. The nicotine in filtered cigarettes appears to be generally lower, so the filters appear to be at least somewhat effective.

- 3. a. 0 b.** Although both batteries have the same mean lifetime, the batteries with the smaller standard deviation are better because their lifetimes will be closer to the mean, and fewer of them will strand drivers by failing sooner than expected. **c.** The outlier pulls the mean either up or down depending on whether it is above

or below the mean, respectively. **d.** The outlier has no effect on the median. **e.** The outlier increases the range. **f.** The outlier increases the standard deviation.

Chapter Quiz

1. Mean 3. 27.0
5. The mean and median are equal.
7. 1.50 sec and 2.50 sec 9. Zero

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A Normal World

When you walk into a store, how do you know if a sale price is really a good price? When you exercise and your heart rate rises, how do you know if it has risen enough, but not too much, for a good workout? If your 12-year-old daughter runs a mile in 5 minutes, is she a future Olympic hopeful? These questions seem very different, but from a statistical standpoint they are very similar: Each one asks whether a particular number (price, heart rate, running time) is somehow unusual. In this chapter, we will discuss how we can answer such questions with the aid of the bell-shaped *normal distribution*.

Nothing in life is to be feared.
It is only to be understood.

—Marie Curie

LEARNING GOALS

1 What Is Normal?

Understand the normal distribution and identify situations in which a normal distribution is likely to arise.

2 Properties of the Normal Distribution

Know how to interpret the normal distribution in terms of the 68-95-99.7 rule, standard scores, and percentiles.

3 The Central Limit Theorem

Understand the basic idea behind the Central Limit Theorem and its important role in statistics.

FOCUS TOPICS

Focus on Education: What Can We Learn from SAT Trends?

Focus on Psychology: Are We Smarter than Our Parents?

1

WHAT IS NORMAL?**BY THE WAY**

The Scottish politician John Sinclair (1754–1835) was one of the first collectors of economic, demographic, and agricultural data. He is credited with introducing the words *statistics* and *statistical* into the English language, having heard them used in Germany to refer to matters of state.

Suppose a friend is pregnant and due to give birth on June 30. Would you advise her to schedule an important business meeting for June 16, two weeks before the due date? Answering this question requires knowing whether the baby is likely to arrive more than 14 days before the due date. For that, we need to examine data concerning due dates and actual birth dates.

Figure 1 is a histogram for a distribution of 300 natural births at Providence Memorial Hospital; the data are hypothetical, but based on how births would be distributed without medical intervention. The horizontal axis shows how many days before or after the due date a baby was born. Negative numbers represent births *prior* to the due date, zero represents a birth on the due date, and positive numbers represent births *after* the due date. The left vertical axis shows the number of births for each 4-day bin. For example, the frequency of 35 for the highest bar corresponds to the bin from -2 days to 2 days; it shows that out of the 300 total births in the sample, 35 births occurred within 2 days of the due date.

To answer our question about whether a birth is likely to occur more than 14 days early, it is more useful to look at the *relative frequencies*. Recall that the relative frequency of any data value is its frequency divided by the total number of data values. Figure 1 shows relative frequencies on the right vertical axis. For example, the bin for -14 days to -10 days (shaded dark blue) has a relative frequency of about 0.07, or 7%. That is, about 7% of the 300 births occurred between 14 days and 10 days before the due date.

We can find the proportion of births that occurred more than 14 days before the due date simply by adding the relative frequencies for the bins to the left of -14 ; you can measure the graph to confirm that these bins have a total relative frequency of about 0.21, which means that about 21% of the births in this data set occurred more than 14 days before the due date. Based on these data, your friend has about a 1 in 5 chance of her baby being born on or before the date of the business meeting. If the meeting is important, it might be good to schedule it earlier.

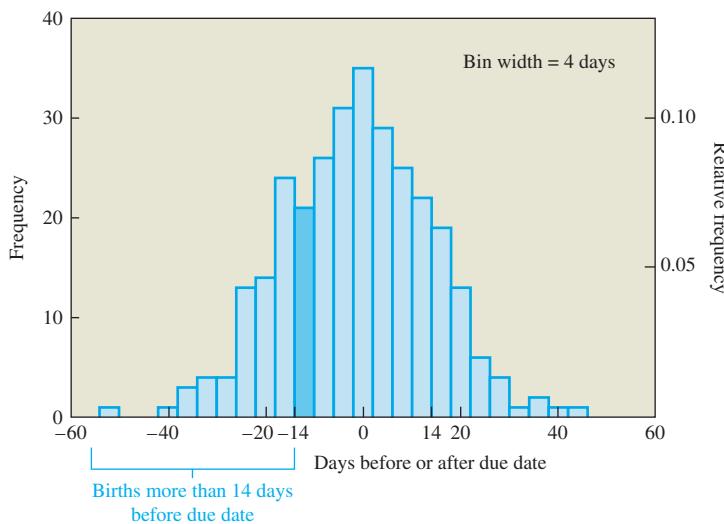


Figure 1 Histogram of frequencies (left vertical axis) and relative frequencies (right vertical axis) for birth dates relative to due date. (These data are hypothetical.) Negative numbers refer to births before the due date; positive numbers refer to births after the due date. The width of each bin is 4 days. For example, the bin shaded dark blue represents births occurring between 10 and 14 days early.

TIME OUT TO THINK

Suppose the friend plans to take a three-month maternity leave after the birth. Based on the data in Figure 1 and assuming a due date of June 30, should she promise to be at work on October 10?

The Normal Distribution

The distribution of the birth data has a fairly distinctive shape, which is easier to see if we overlay the histogram with a smooth curve (Figure 2). For our present purposes, the shape of this smooth distribution has three very important characteristics:

- The distribution is *single-peaked*. Its mode, or most common birth date, is the due date.
- The distribution is *symmetric* around its single peak; therefore, its median and mean are the same as its mode. The median is the due date because equal numbers of births occur before and after this date. The mean is also the due date because, for every birth before the due date, there is a birth the same number of days after the due date.
- The distribution is spread out in a way that makes it resemble the shape of a bell, so we call it a “bell-shaped” distribution.

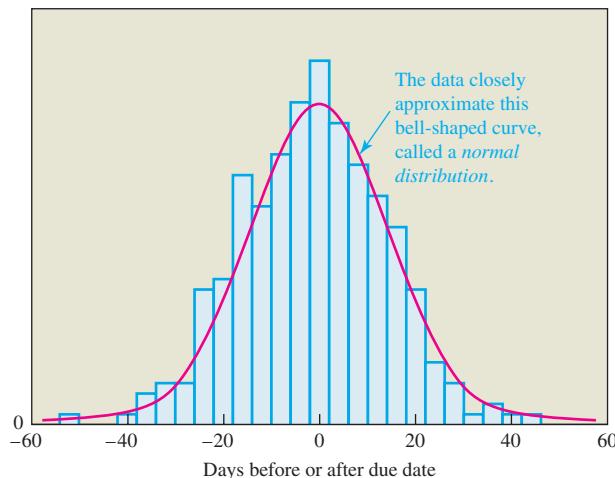


Figure 2 A smooth normal distribution curve is drawn over the histogram of Figure 1.

TIME OUT TO THINK

The histogram in Figure 2, which is based on natural births, is fairly symmetric. Today, doctors usually induce birth if a woman goes too far past her due date. How would the shape of the histogram change if it included induced births?

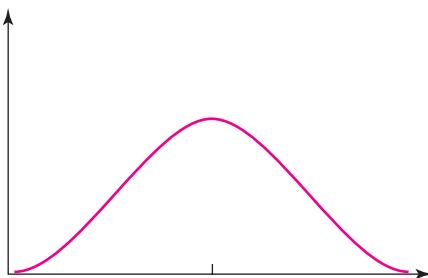
The smooth distribution in Figure 2, with these three characteristics, is called a **normal distribution**. All normal distributions have the same characteristic bell shape, but they can differ in their mean and in their variation. Figure 3 shows two different normal distributions. Both have the same mean, but distribution (a) has greater variation. As we'll discuss in the next section, knowing the *standard deviation* of a normal distribution tells us everything we need to know about its variation. Therefore, a normal distribution can be fully described with just two numbers: its mean and its standard deviation.

TECHNICAL NOTE

Although we will not use it in this text, the following algebraic function describes a normal distribution with mean μ and standard deviation σ :

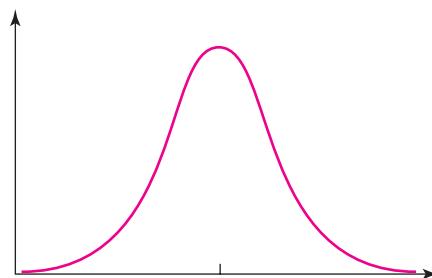
$$y = \frac{e^{-\frac{1}{2}((x-\mu)/\sigma)^2}}{\sigma\sqrt{2\pi}}$$

Larger standard deviation



(a)

Smaller standard deviation



(b)

Figure 3 Both distributions are normal and have the same mean of 75, but the distribution on the left has a larger standard deviation.

Definition

The **normal distribution** is a symmetric, bell-shaped distribution with a single peak. Its peak corresponds to the mean, median, and mode of the distribution. Its variation can be characterized by the standard deviation of the distribution.

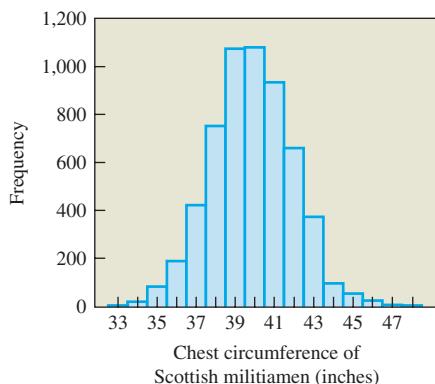
BY THE WAY

Using data taken from French and Scottish soldiers, the Belgian social scientist Adolphe Quetelet realized in the 1830s that human characteristics such as height and chest circumference are normally distributed. This observation led him to coin the term “the average man.” Quetelet was the first foreign member of the American Statistical Association.

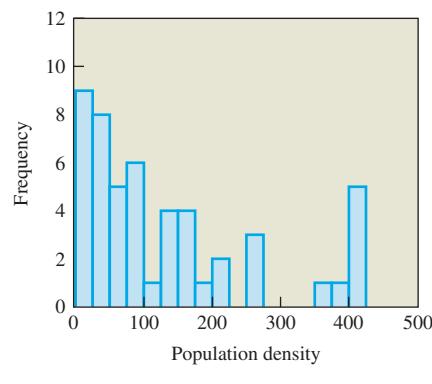


EXAMPLE 1 Normal Distributions?

Figure 4 shows two distributions: (a) a famous data set of the chest sizes of 5,738 Scottish militiamen collected in about 1846 and (b) the distribution of the population densities of the 50 states. Is either distribution a normal distribution? Explain.



(a)



(b)

Figure 4 Source of (a): Adolphe Quetelet, *Lettres à S. A. R. le Duc Régnant de Saxe-Cobourg et Gotha*, 1846.

SOLUTION The distribution in Figure 4a is nearly symmetric, with a mean between 39 and 40 inches. Values far from the mean are less common, giving it the bell shape of a normal distribution. The distribution in Figure 4b shows that most states have low population densities, but a few have much higher densities. This fact makes the distribution right-skewed, so it is not a normal distribution.

• • •

The Normal Distribution and Relative Frequencies

Recall that the total relative frequency for any data set must be 1. Now consider the smooth curve for the normal distribution in Figure 2, which is repeated in Figure 5. Although we no longer show individual bars, we can still associate the height of the normal curve with the relative frequency. The fact that the relative frequencies must sum to 1 becomes the condition that the area under the normal curve must be 1.

The key idea is this: *The relative frequency for any range of data values is the area under the curve covering that range of values.* For example, a precise calculation shows that the shaded region to the left of -14 days in Figure 5 represents about 18% of the total area under the curve. We therefore conclude that the relative frequency is about 0.18 for data values less than -14 days, which means that about 18% of births are more than 14 days early. Similarly, the shaded region to the right of 18 days represents about 12% of the total area under the curve. We therefore conclude that the relative frequency is about 0.12 for data values greater than 18 days, which means that about 12% of births are more than 18 days late. Altogether, we see that $18\% + 12\% = 30\%$ of all births are either more than 14 days early or more than 18 days late.

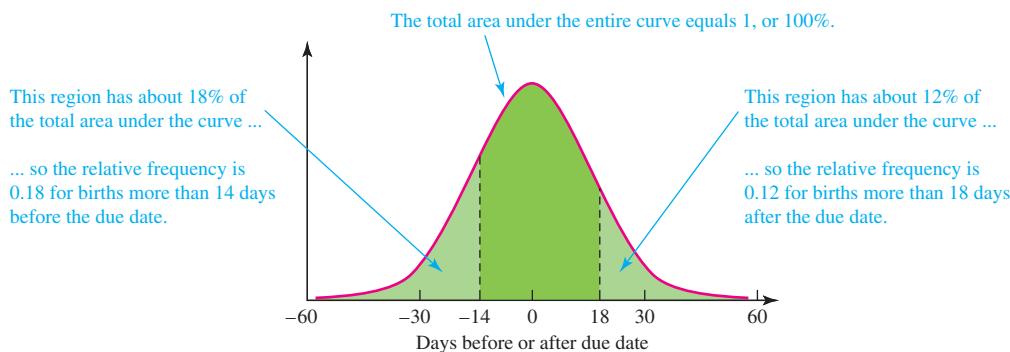


Figure 5 The percentage of the total area in any region under the normal curve tells us the relative frequency of data values in that region.

Relative Frequencies and the Normal Distribution

- The area that lies under the normal distribution curve corresponding to a range of values on the horizontal axis is the relative frequency of those values.
- Because the total relative frequency must be 1, the total area under the normal distribution curve must equal 1, or 100%.

EXAMPLE 2 Estimating Areas

Look again at the normal distribution in Figure 5.

- Estimate the percentage of births occurring between 0 and 60 days after the due date.
- Estimate the percentage of births occurring between 14 days before and 14 days after the due date.

SOLUTION

- About half of the total area under the curve lies in the region between 0 days and 60 days. This means that about 50% of the births in the sample occur between 0 and 60 days after the due date.
- Figure 5 shows that about 18% of the births occur more than 14 days before the due date. Because the distribution is symmetric, about 18% must also occur more than 14 days after the due date. Therefore, a total of about $18\% + 18\% = 36\%$ of births occur either more than 14 days before or more than 14 days after the due date. The question asked about the remaining region, which means *between* 14 days before and 14 days after the due date, so this region must represent $100\% - 36\% = 64\%$ of the births.



TIME OUT TO THINK

About what percentage of births in Figure 5 occur between 14 days early and 18 days late? Explain.
(Hint: Remember that the total area under the curve is 100%).

BY THE WAY

The normal distribution curve is often called a *Gaussian curve* in honor of the 19th-century German mathematician Carl Friedrich Gauss. The American logician Charles Peirce introduced the term *normal distribution* in about 1870.

When Can We Expect a Normal Distribution?

We can better appreciate the importance of the normal distribution if we understand why it is so common. Consider a human characteristic such as height, which closely approximates a normal distribution. Most men or women have heights clustered near the mean height (for their sex), so a data set of heights has a peak at the mean height. But as we consider heights increasingly far from the mean on either side, we find fewer and fewer people. This “tailing off” of heights far from the mean produces the two tails of the normal distribution.

On a deeper level, any quantity that is influenced by many factors is likely to follow a normal distribution. Adult heights are the result of many genetic and environmental factors. Scores on SAT tests or IQ tests tend to be normally distributed because each test score is determined from many individual test questions. Sports statistics, such as batting averages, tend to be normally distributed because they involve many people with many different levels of skill. More generally, we can expect a data set to have a nearly normal distribution if it meets the following conditions.

Conditions for a Normal Distribution

A data set that satisfies the following four criteria is likely to have a nearly normal distribution:

1. Most data values are clustered near the mean, giving the distribution a well-defined single peak.
2. Data values are spread evenly around the mean, making the distribution symmetric.
3. Larger deviations from the mean become increasingly rare, producing the tapering tails of the distribution.
4. Individual data values result from a combination of many different factors, such as genetic and environmental factors.

EXAMPLE 3 Is It a Normal Distribution?

Which of the following variables would you expect to have a normal or nearly normal distribution?

- a. Scores on a very easy test
- b. Shoe sizes of a random sample of adult women
- c. The number of apples in each of 100 full bushel baskets

SOLUTION

- a. Tests have a maximum possible score (100%) that limits the size of data values. If the test is very easy, the mean will be high and many scores will be near the maximum. The fewer lower scores can be spread out well below the mean. We therefore expect the distribution to be left-skewed and non-normal.
- b. Foot length is a human trait determined by many genetic and environmental factors. We therefore expect lengths of women’s feet to cluster near a mean and become less common farther from the mean, giving the distribution a bell shape, so the lengths have a nearly normal distribution.
- c. The number of apples in a bushel basket varies with the size of the apples. We expect that in the distribution there will be a single mode that should be close to the mean number of apples per basket. The number of baskets with more than the mean number of apples should be close to the number of baskets with fewer than the mean number of apples. We therefore expect the number of apples per basket to have a nearly normal distribution.

TIME OUT TO THINK

Would you expect scores on a moderately difficult exam to have a normal distribution? Suggest two more quantities that you would expect to be normally distributed.

Section 1 Exercises

Statistical Literacy and Critical Thinking

1. Normal Distribution. When we refer to a “normal” distribution, does the word *normal* have the same meaning as in ordinary language, or does it have a special meaning in statistics? What exactly is a normal distribution?

2. Normal Distribution. A normal distribution is informally and loosely described as a probability distribution that is “bell-shaped” when graphed. Draw a rough sketch of the bell shape that characterizes normal distributions.

3. Random Digits. Many states have lotteries that involve the random selection of digits 0, 1, 2, ..., 9. Is the distribution of those digits a normal distribution? Why or why not?

4. Areas. Birth weights in the United States are normally distributed with a mean (in grams) of 3420 g and a standard deviation of 495 g. If you graph this normal distribution, the area to the right of 4000 g is 0.12. What is the area to the left of 4000 g?

Does It Make Sense? For Exercises 5–8, decide whether the statement makes sense (or is clearly true) or does not make sense (or is clearly false). Explain clearly; not all of these have definitive answers, so your explanation is more important than your chosen answer.

5. Heights of Women. A sample of 2,000 women is randomly selected, and it is found that the heights of the women are normally distributed with a mean of 63.6 in.

6. Brains. As part of a study of the relationship between brain size and IQ, a random sample of 250 adult males is obtained and their brain volumes are measured and found to be normally distributed.

7. IQ Scores. The mean of a normally distributed set of IQ scores is 100, and 60% of the scores are over 105.

8. Salaries. An economist plans to obtain the current salaries of all professional football players, and she predicts that those salaries will have a normal distribution.

Concepts and Applications

9. What Is Normal? Identify the distribution in Figure 6 that is not normal. Of the two normal distributions, which has the larger standard deviation?

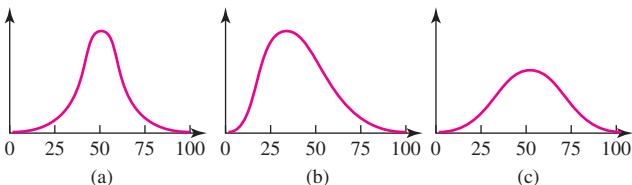


Figure 6

10. What Is Normal? Identify the distribution in Figure 7 that is not normal. Of the two normal distributions, which has the larger standard deviation?

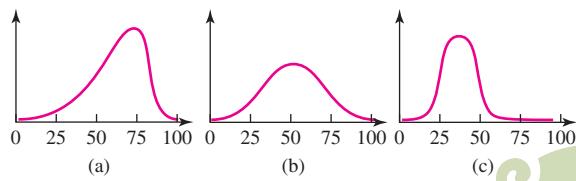


Figure 7

Normal Variables. For each of the data sets in Exercises 11–18, state whether you would expect it to be normally distributed. Explain your reasoning.

11. Weights of Quarters. The exact weights of a random sample of quarters manufactured in 2012 by the U.S. Mint.

12. Incomes. The incomes of randomly selected adults in the United States.

13. Lottery. The numbers selected in the Pennsylvania “Match 6” lottery, in which players attempt to match six randomly selected numbers between 1 and 49.

14. SAT Scores. All of the SAT scores from last year.

15. White Blood Cell Counts. The measured white blood cell counts of 500 randomly selected adult women.

16. Flight Delays. The lengths of time that commercial aircraft are delayed before departing.

17. Waiting Times. The waiting times at a bus stop if the bus comes once every 10 minutes and you arrive at random times.

18. Parking Ticket Fines. The amounts of the fines from parking tickets found on a random sample of 1,000 parked cars.

19. Movie Lengths. Figure 8 shows a histogram for the lengths of 60 movies. The mean movie length is 110.5 minutes. Is this distribution close to normal? Should this variable have a normal distribution? Why or why not?

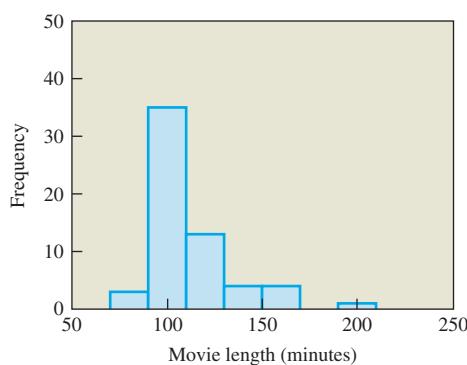


Figure 8

- 20. Pulse Rates.** Figure 9 shows a histogram for the pulse rates of 98 students. The mean pulse rate is 71.2 beats per minute. Is this distribution close to normal? Should this variable have a normal distribution? Why or why not?

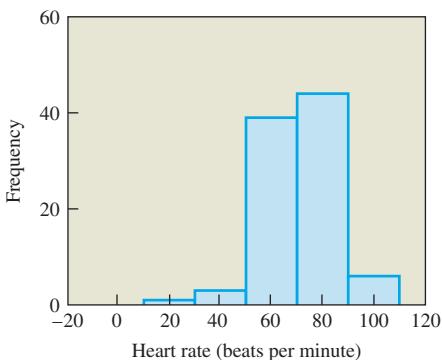


Figure 9

- 21. Quarter Weights.** Figure 10 shows a histogram for the weights of 50 randomly selected quarters. The mean weight is 5.62 grams. Is this distribution close to normal? Should this variable have a normal distribution? Why or why not?

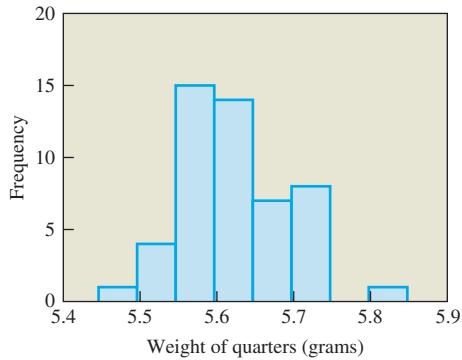


Figure 10

- 22. Aspirin Weights.** Figure 11 shows a histogram for the weights of 30 randomly selected aspirin tablets. The mean weight is 665.4 milligrams. Is this distribution close to normal? Should this variable have a normal distribution? Why or why not?

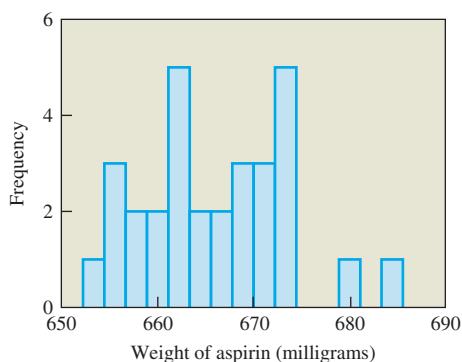


Figure 11

- 23. Areas and Relative Frequencies.** Consider the graph of the normal distribution in Figure 12, which gives relative frequencies in a distribution of men's heights. The distribution has a mean of 69.6 inches and a standard deviation of 2.8 inches.

- What is the total area under the curve?
- Estimate (using area) the relative frequency of values less than 67.
- Estimate the relative frequency of values greater than 67.
- Estimate the relative frequency of values between 67 and 70.
- Estimate the relative frequency of values greater than 70.

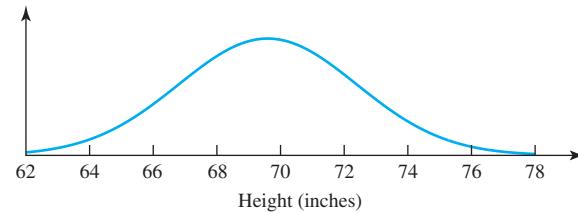


Figure 12

- 24. Areas and Relative Frequencies.** Consider the graph of the normal distribution in Figure 13, which shows the relative frequencies in a distribution of IQ scores. The distribution has a mean of 100 and a standard deviation of 16.

- What is the total area under the curve?
- Estimate (using area) the relative frequency of values less than 100.
- Estimate the relative frequency of values greater than 110.
- Estimate the relative frequency of values less than 110.
- Estimate the relative frequency of values between 100 and 110.

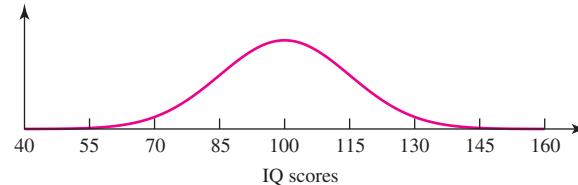


Figure 13

- 25. Estimating Areas.** Consider the graph of the normal distribution in Figure 14, which illustrates the relative frequencies in a distribution of systolic blood pressures for a sample of female students. The distribution has a standard deviation of 14.

- What is the mean of the distribution?
- Estimate (using area) the percentage of students whose blood pressure is less than 100.
- Estimate the percentage of students whose blood pressure is between 110 and 130.
- Estimate the percentage of students whose blood pressure is greater than 130.

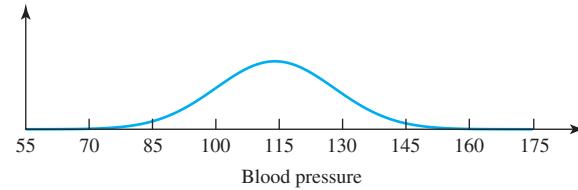


Figure 14

26. Estimating Areas. Consider the graph of the normal distribution in Figure 15, which gives the relative frequencies in a distribution of body weights for a sample of male students.

- What is the mean of the distribution?
- Estimate (using area) the percentage of students whose weight is less than 140.
- Estimate the percentage of students whose weight is greater than 170.
- Estimate the percentage of students whose weight is between 140 and 160.

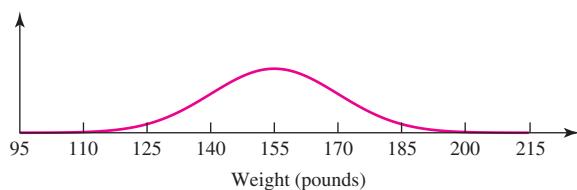


Figure 15

PROJECTS FOR THE INTERNET & BEYOND

27. SAT Score Distributions. The College Board Web site gives the distribution of SAT scores (usually in 50-point bins). Collect these data and construct a histogram for each part of the test. Discuss the validity of the claim that SAT scores are normally distributed.

28. Finding Normal Distributions. Using the guidelines given in the text, choose a variable that you think should be nearly normally distributed. Collect at least 30 data values for the variable and make a histogram. Comment on how closely the distribution fits a normal distribution. In what ways does it differ from a normal distribution? Try to explain the differences.

29. Movie Lengths. Collect data to support or refute the claim that movies have gotten shorter over the decades. Specifically, make a histogram of movie lengths for each decade from the 1940s through the present, find the mean movie length for each sample, and comment on whether these distributions are normal. Discuss your results and give plausible reasons for any trends that you observe.

IN THE NEWS

30. Normal Distributions. Rarely does a news article refer to the actual distribution of a variable or state that a variable is normally distributed. Nevertheless, variables mentioned in news reports must have some distribution. Find two variables in news reports that you suspect have nearly normal distributions. Explain your reasoning.

31. Non-normal Distributions. Find two variables in news reports that you suspect *do not* have nearly normal distributions. Explain your reasoning.

2

PROPERTIES OF THE NORMAL DISTRIBUTION

Consider a *Consumer Reports* survey in which participants were asked how long they owned their last TV set before they replaced it. The variable of interest in this survey is *replacement time for television sets*. Based on the survey, the distribution of replacement times has a mean of about 8.2 years, which we denote as μ (the Greek letter *mu*). The standard deviation of the distribution is about 1.1 years, which we denote as σ (the Greek letter *sigma*). Making the reasonable assumption that the distribution of TV replacement times is approximately normal, we can picture it as shown in Figure 16.

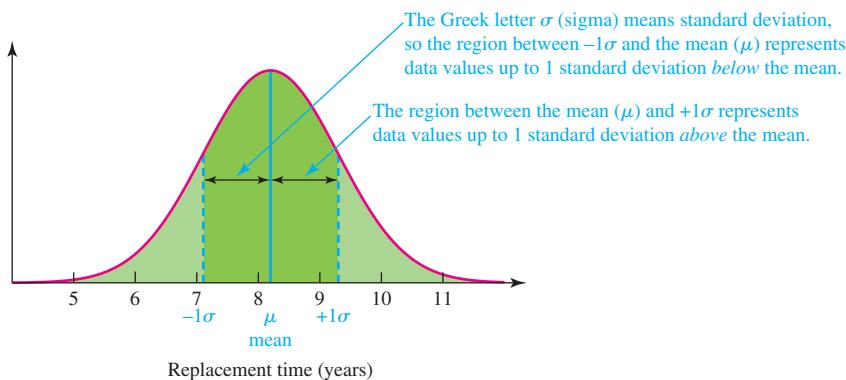


Figure 16 Normal distribution for replacement times for TV sets with a mean of $\mu = 8.2$ years and a standard deviation of $\sigma = 1.1$ years.

TIME OUT TO THINK

Apply the four criteria for a normal distribution to explain why the distribution of TV replacement times should be approximately normal.

TECHNICAL NOTE

A normal distribution can have any value for the mean and any positive value for the standard deviation. The term *standard normal distribution* specifically refers to a normal distribution with a mean of 0 and a standard deviation of 1.

Because all normal distributions have the same bell shape, knowing the mean and standard deviation of a distribution allows us to know much about where the data values lie. For example, if we measure areas under the curve in Figure 16, we find that about two-thirds of the area lies within 1 standard deviation of the mean, which in this case is between $8.2 - 1.1 = 7.1$ years and $8.2 + 1.1 = 9.3$ years. Therefore, the TV replacement time is between 7.1 and 9.3 years for about two-thirds of the people surveyed. Similarly, about 95% of the area lies within 2 standard deviations of the mean, which in this case is between $8.2 - 2.2 = 6.0$ years and $8.2 + 2.2 = 10.4$ years. We conclude that the TV replacement time is between 6.0 and 10.4 years for about 95% of the people surveyed.

A simple rule, called the **68-95-99.7 rule**, gives precise guidelines for the percentage of data values that lie within 1, 2, and 3 standard deviations of the mean for any normal distribution. The following box states the rule in words, and Figure 17 shows it visually.

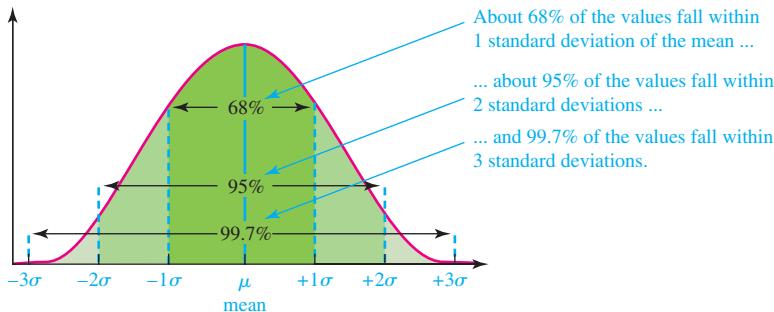


Figure 17 Normal distribution illustrating the 68-95-99.7 rule.

The 68-95-99.7 Rule for a Normal Distribution

- About 68% (more precisely, 68.3%), or just over two-thirds, of the data values fall within 1 standard deviation of the mean.
- About 95% (more precisely, 95.4%) of the data values fall within 2 standard deviations of the mean.
- About 99.7% of the data values fall within 3 standard deviations of the mean.

EXAMPLE 1 SAT Scores

The tests that make up the verbal (critical reading) and mathematics parts of the SAT (and the GRE, LSAT, and GMAT) are designed so that their scores are normally distributed with a mean of $\mu = 500$ and a standard deviation of $\sigma = 100$. Interpret this statement.

TECHNICAL NOTE

As discussed in the “Focus on Psychology” section at the end of the chapter, the mean score on a particular component of the SAT may differ from 500 depending on the test and the year it is given.

SOLUTION From the 68-95-99.7 rule, about 68% of students have scores within 1 standard deviation (100 points) of the mean of 500 points; that is, about 68% of students score between 400 and 600. About 95% of students score within 2 standard deviations (200 points) of the mean, or between 300 and 700. And about 99.7% of students score within 3 standard deviations (300 points) of the mean, or between 200 and 800. Figure 18 shows this interpretation graphically; note that the horizontal axis shows both actual scores and distance from the mean in standard deviations.

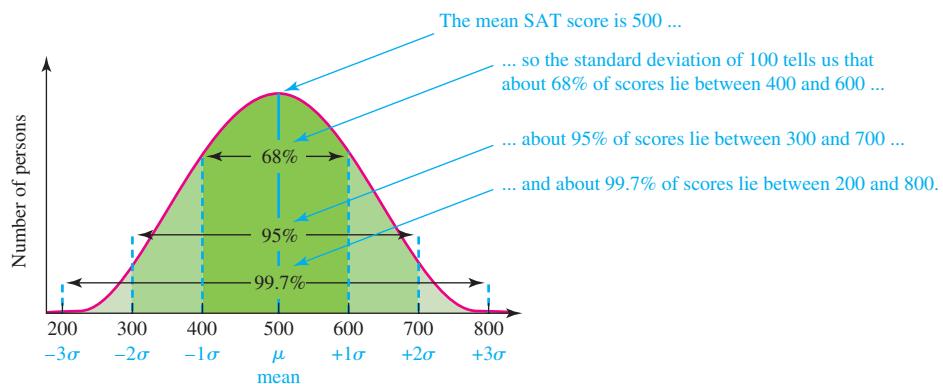


Figure 18 Normal distribution for scores on components of the SAT, showing the percentages associated with 1, 2, and 3 standard deviations.

• •

EXAMPLE 2 Detecting Counterfeits

Vending machines can be adjusted to reject coins above and below certain weights. The weights of legal U.S. quarters have a normal distribution with a mean of 5.67 grams and a standard deviation of 0.0700 gram. If a vending machine is adjusted to reject quarters that weigh more than 5.81 grams and less than 5.53 grams, what percentage of legal quarters will be rejected by the machine?

SOLUTION A weight of 5.81 is 0.14 gram, or 2 standard deviations, above the mean. A weight of 5.53 is 0.14 gram, or 2 standard deviations, below the mean. Therefore, by accepting only quarters within the weight range 5.53 to 5.81 grams, the machine accepts quarters that are within 2 standard deviations of the mean and rejects those that are more than 2 standard deviations from the mean. By the 68-95-99.7 rule, 95% of legal quarters will be accepted and 5% of legal quarters will be rejected.

• •

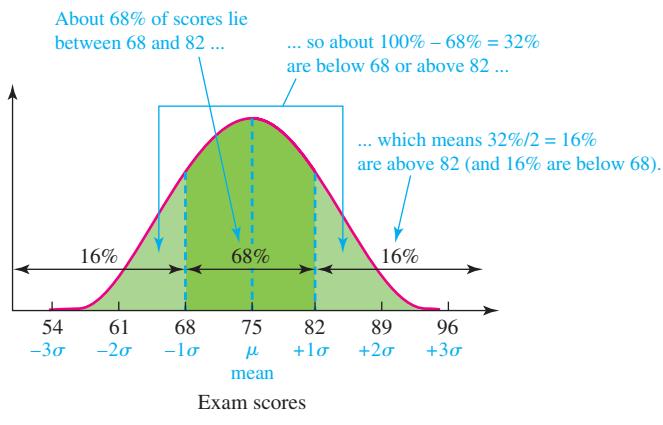
Applying the 68-95-99.7 Rule

We can apply the 68-95-99.7 rule to answer many questions about the frequencies (or relative frequencies) of data values in a normal distribution. Consider an exam taken by 1,000 students for which the scores are normally distributed with a mean of $\mu = 75$ and a standard deviation of $\sigma = 7$. How many students scored above 82?

A score of 82 is 7 points, or 1 standard deviation, above the mean of 75. The 68-95-99.7 rule tells us that about 68% of the scores are *within* 1 standard deviation of the mean. Therefore, about $100\% - 68\% = 32\%$ of the scores are *more than* 1 standard deviation from the mean. Half of this 32%, or 16%, of the scores are more than 1 standard deviation *below* the mean; the other 16% of the scores are more than 1 standard deviation *above* the mean (Figure 19a). We conclude that about 16% of 1,000 students, or 160 students, scored above 82.

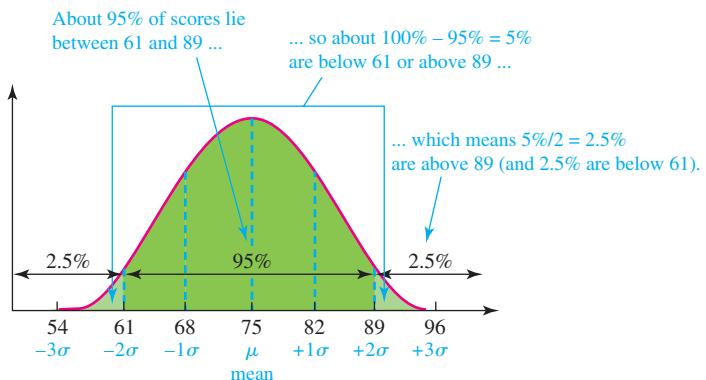
BY THE WAY

The circulation rate for U.S. pennies is estimated to be about 34% (compared with 88% for quarters). This means that about two-thirds of the more than 300 million pennies produced each month by the U.S. Mint make a one-way trip to piggy banks or penny jars.



Exam scores

(a)



Exam scores

(b)

Figure 19 A normal distribution of test scores with a mean of 75 and a standard deviation of 7. (a) 68% of the scores lie within 1 standard deviation of the mean. (b) 95% of the scores lie within 2 standard deviations of the mean.

Similarly, suppose we want to know how many students scored below 61. A score of 61 is 14 points, or 2 standard deviations, below the mean of 75. The 68-95-99.7 rule tells us that about 95% of the scores are *within* 2 standard deviations of the mean, so about 5% of the scores are *more than* 2 standard deviations from the mean. Half of this 5%, or 2.5%, of the scores are more than 2 standard deviations *below* the mean (Figure 19b), so we conclude that about 2.5% of 1,000 students, or 25 students, scored below 61.

Because 95% of the scores fall between 61 and 89, we sometimes say the scores outside this range are *unusual* because they are relatively rare.

Identifying Unusual Results

In statistics, we often need to distinguish values that are typical, or “*usual*,” from values that are “*unusual*.” By applying the 68-95-99.7 rule, we find that about 95% of all values from a normal distribution lie within 2 standard deviations of the mean. This implies that, among all values, 5% lie more than 2 standard deviations away from the mean. We can use this property to identify values that are relatively “*unusual*”: **Unusual values** are values that are more than 2 standard deviations away from the mean.

EXAMPLE 3 Traveling and Pregnancy

Consider again the question of whether you should advise a pregnant friend to schedule an important business meeting 2 weeks before her due date. Actual data suggest that the number of days between the birth date and the due date is normally distributed with a mean of $\mu = 0$ days and a standard deviation of $\sigma = 15$ days. How would you help your friend make the decision? Would a birth 2 weeks before the due date be considered “*unusual*”?

SOLUTION Your friend is assuming that she will *not* have given birth 14 days, or roughly 1 standard deviation, before her due date. But because this outcome is well within 2 standard deviations of the mean, it is *not* unusual. By the 68-95-99.7 rule, the day of birth for 68% of pregnancies is within 1 standard deviation of the mean, or between -15 days and 15 days from the due date. This means that about $100\% - 68\% = 32\%$ of births occur either more than 15 days early or more than 15 days late. Therefore, half of this 32%, or 16%, of all births are more than 15 days early (Figure 20). You should tell your friend that about 16% of all births occur more than 15 days before the due date. If your friend likes to think in terms of probability, you could say that there is a 0.16 (about 1 in 6) chance that she will give birth on or before her meeting date.

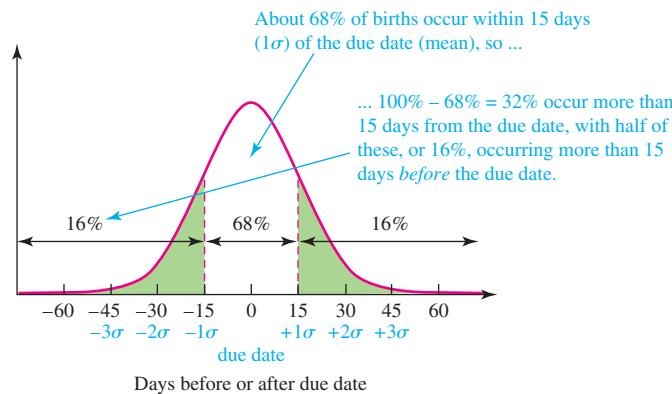


Figure 20 About 16% of births occur more than 15 days before the due date.

EXAMPLE 4 **Normal Heart Rate**

You measure your resting heart rate at noon every day for a year and record the data. You discover that the data have a normal distribution with a mean of 66 and a standard deviation of 4. On how many days was your heart rate below 58 beats per minute?

SOLUTION A heart rate of 58 is 8 (or 2 standard deviations) below the mean. According to the 68-95-99.7 rule, about 95% of the data values are within 2 standard deviations of the mean. Therefore, 2.5% of the data values are more than 2 standard deviations *below* the mean, and 2.5% of the data values are more than 2 standard deviations *above* the mean. On 2.5% of 365 days, or about 9 days, your measured heart rate was below 58 beats per minute.

**TIME OUT TO THINK**

As Example 4 suggests, multiple measurements of the resting heart rate of a *single* individual are normally distributed. Would you expect the average resting heart rates of *many* individuals to be normally distributed? Which distribution would you expect to have the larger standard deviation? Why?

Standard Scores

The 68-95-99.7 rule applies only to data values that are exactly 1, 2, or 3 standard deviations from the mean. For other cases, we can generalize this rule if we know precisely how many standard deviations from the mean a particular data value lies. The number of standard deviations a data value lies above or below the mean is called its **standard score** (or *z-score*), often abbreviated by the letter *z*. For example:

- The standard score of the mean is $z = 0$, because it is 0 standard deviations from the mean.
- The standard score of a data value 1.5 standard deviations *above* the mean is $z = 1.5$.
- The standard score of a data value 2.4 standard deviations *below* the mean is $z = -2.4$.

The following box summarizes the computation of standard scores.

Computing Standard Scores

The number of standard deviations a data value lies above or below the mean is called its **standard score** (or *z-score*), defined by

$$z = \text{standard score} = \frac{\text{data value} - \text{mean}}{\text{standard deviation}}$$

The standard score is positive for data values above the mean and negative for data values below the mean.

EXAMPLE 5 **Standard IQ Scores**

The Stanford-Binet IQ test is scaled so that scores have a mean of 100 and a standard deviation of 16. Find the standard scores for IQs of 85, 100, and 125.

SOLUTION We calculate the standard scores for these IQs by using the standard score formula with a mean of 100 and standard deviation of 16.

$$\text{standard score for } 85: z = \frac{85 - 100}{16} = -0.94$$

$$\text{standard score for } 100: z = \frac{100 - 100}{16} = 0.00$$

$$\text{standard score for } 125: z = \frac{125 - 100}{16} = 1.56$$

We can interpret these standard scores as follows: 85 is 0.94 standard deviation *below* the mean, 100 is equal to the mean, and 125 is 1.56 standard deviations *above* the mean. Figure 21 shows these values on the distribution of IQ scores.

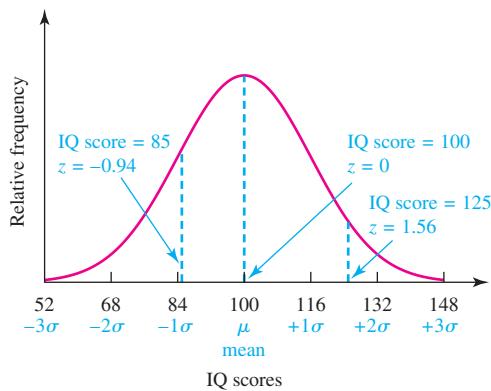


Figure 21 Standard scores for IQ scores of 85, 100, and 125.



Standard Scores and Percentiles

Once we know the standard score of a data value, the properties of the normal distribution allow us to find its **percentile** in the distribution. You are probably familiar with the idea of percentiles. For example, if you scored in the 45th percentile on the SAT, 45% of the SAT scores were lower than yours.

Percentiles

The n th percentile of a data set is the smallest value in the set with the property that $n\%$ of the data values are less than or equal to it. A data value that lies between two percentiles is said to lie in the lower percentile.

We can convert standard scores to percentiles with a *standard score table*, such as Table 1, or with computer software. For each of many standard scores in a normal distribution, the table gives the percentage of values in the distribution *less than or equal to* that value. For example, the table shows that 55.96% of the values in a normal distribution have a standard score less than or equal to 0.15. In other words, a data value with a standard score of 0.15 lies in the 55th percentile.



USING TECHNOLOGY—STANDARD SCORES IN EXCEL

Excel's built-in function STANDARDIZE returns the standard score for any data value, given the mean and standard deviation of the distribution. Enter the data in the form "`=STANDARDIZE(data value, mean, standard deviation)`". The screen shot below shows this function with a table for the data value, mean, and standard deviation from Example 5; the function references the cells (B1, B2, B3) containing those data. Column C shows the results.

	A	B	C
1	data value	95	
2	mean	100	
3	standard deviation	16	
4	standard score	<code>=STANDARDIZE(B1,B2,B3)</code>	-0.3125
5			

TABLE 1		Standard Scores and Percentiles for a Normal Distribution (cumulative values from the left)					
Standard score	%	Standard score	%	Standard score	%	Standard score	%
-3.5	0.02	-1.0	15.87	0.0	50.00	1.1	86.43
-3.0	0.13	-0.95	17.11	0.05	51.99	1.2	88.49
-2.9	0.19	-0.90	18.41	0.10	53.98	1.3	90.32
-2.8	0.26	-0.85	19.77	0.15	55.96	1.4	91.92
-2.7	0.35	-0.80	21.19	0.20	57.93	1.5	93.32
-2.6	0.47	-0.75	22.66	0.25	59.87	1.6	94.52
-2.5	0.62	-0.70	24.20	0.30	61.79	1.7	95.54
-2.4	0.82	-0.65	25.78	0.35	63.68	1.8	96.41
-2.3	1.07	-0.60	27.43	0.40	65.54	1.9	97.13
-2.2	1.39	-0.55	29.12	0.45	67.36	2.0	97.72
-2.1	1.79	-0.50	30.85	0.50	69.15	2.1	98.21
-2.0	2.28	-0.45	32.64	0.55	70.88	2.2	98.61
-1.9	2.87	-0.40	34.46	0.60	72.57	2.3	98.93
-1.8	3.59	-0.35	36.32	0.65	74.22	2.4	99.18
-1.7	4.46	-0.30	38.21	0.70	75.80	2.5	99.38
-1.6	5.48	-0.25	40.13	0.75	77.34	2.6	99.53
-1.5	6.68	-0.20	42.07	0.80	78.81	2.7	99.65
-1.4	8.08	-0.15	44.04	0.85	80.23	2.8	99.74
-1.3	9.68	-0.10	46.02	0.90	81.59	2.9	99.81
-1.2	11.51	-0.05	48.01	0.95	82.89	3.0	99.87
-1.1	13.57	0.0	50.00	1.0	84.13	3.5	99.98

Note: The table shows percentiles for standard scores between -3.5 and +3.5, though much lower and higher standard scores are possible. The % column gives the percentage of values in the distribution less than the corresponding standard score.

EXAMPLE 6 Cholesterol Levels

Cholesterol levels in men 18 to 24 years of age are normally distributed with a mean of 178 and a standard deviation of 41.

- a. What is the percentile for a 20-year-old man with a cholesterol level of 190?
- b. What cholesterol level corresponds to the 90th percentile, the level at which treatment may be necessary?

SOLUTION

- a. The *standard score* for a cholesterol level of 190 is

$$z = \text{standard score} = \frac{\text{data value} - \text{mean}}{\text{standard deviation}} = \frac{190 - 178}{41} \approx 0.29$$

Table 1 shows that a standard score of 0.29 corresponds to about the 61st percentile.

- b. Table 1 shows that 90.32% of all data values have a standard score less than 1.3. That is, the 90th percentile is about 1.3 standard deviations above the mean. Given the mean cholesterol level of 178 and the standard deviation of 41, a cholesterol level 1.3 standard deviations above the mean is

$$\underbrace{178}_{\text{mean}} + (\underbrace{1.3 \times 41}_{\text{1.3 standard deviations}}) = 231.3$$

The 90th percentile begins at a cholesterol level of about 231, so anyone with this level or higher may be in need of treatment.



**USING TECHNOLOGY—STANDARD SCORES AND PERCENTILES**

Excel The built-in function NORMDIST saves you the work of using a table like Table 1. Find a data value's percentile in a normal distribution by entering “=NORMDIST (data value, mean, standard deviation, TRUE)”; the fourth input (TRUE) is necessary to get a percentile result. The screen shot below shows the calculation of the standard score and percentile for the data in Example 6a. Column C shows the results; note that the percentile is displayed as 0.615, so you need to multiply by 100 to express it as a percentage (61.5%).

	A	B	C
1	data value	190	
2	mean	178	
3	standard deviation	41	
4	standard score	=STANDARDIZE(B1,B2,B3)	0.29
5	percentile	=NORMDIST(B1,B2,B3,TRUE)	0.615
6			

STATDISK STATDISK can be used in place of Table 1. Select **Analysis, Probability Distributions, Normal Distribution**. Either enter the z score to find corresponding areas, or enter the cumulative area from the left to find the z score. After entering a value, click on the **Evaluate** button. The display will include the area to the left of the z score as well as the area to the right of the z score.

TI-83/84 Plus

- **Finding Area:** To find the area between two values, press **2ND**, then press **VARS** to get to the **DISTR** (distribution) menu. Select **normalcdf**. Enter the two values, the mean, and the standard deviation, all separated by commas, as in this format: (left value, right value, mean, standard deviation). *Hint:* If there is no left value, enter the left value as **-999999**, and if there is no right value, enter the right value as **999999**. For example, **normalcdf(80, 85, 100, 15)** returns a value of 0.0674 (rounded), indicating that for a normally distributed population with mean 100 and standard deviation 15, 6.74% of the values are between 80 and 85.
- **Finding x Value:** To find a value corresponding to a known area, press **2ND**, then press **VARS** to get to the **DISTR** (distribution) menu. Select **invNorm**, and proceed to enter the total area to the left of the value, the mean, and the standard deviation in this format with the commas included: (total area to the left, mean, standard deviation). For example, **invNorm(0.4, 100, 15)** returns a value of 96.2 (rounded), indicating that for a normally distributed population with mean 100 and standard deviation 15, the value of 96.2 has an area of 0.4 (or 40%) to its left.

EXAMPLE 7 IQ Scores

IQ scores are normally distributed with a mean of 100 and a standard deviation of 16 (see Example 5). What are the IQ scores for people in the 75th and 40th percentiles on IQ tests?

SOLUTION Table 1 shows that the 75th percentile falls *between* standard scores of 0.65 and 0.70; we can estimate that it has a standard score of about 0.67. This corresponds to an IQ that is 0.67 standard deviation, or about $0.67 \times 16 = 11$ points, above the mean of 100. Therefore, a person in the 75th percentile has an IQ of 111. The 40th percentile corresponds to a standard score of approximately -0.25 , or a score that is $0.25 \times 16 = 4$ points *below* the mean of 100. So a person in the 40th percentile has an IQ of 96. • • •

EXAMPLE 8 Women in the Army

The heights of American women ages 18 to 24 are normally distributed with a mean of 65 inches and a standard deviation of 2.5 inches. In order to serve in the U.S. Army, women must be between 58 inches and 80 inches tall. What percentage of women are ineligible to serve based on their height?

SOLUTION The standard scores for the army's minimum and maximum heights of 58 inches and 80 inches are

$$\text{For 58 inches: } z = \frac{58 - 65}{2.5} = -2.8$$

$$\text{For 80 inches: } z = \frac{80 - 65}{2.5} = 6.0$$

Table 1 shows that a standard score of -2.8 corresponds to the 0.26 percentile. A standard score of 6.0 does not appear in Table 1, which means it is above the 99.98th percentile.



(the highest percentile shown in the table). We conclude that 0.26% of all women are too short to serve in the army and fewer than 0.02% of all women are too tall to serve in the army. Altogether, fewer than about 0.28% of all women, or about 1 out of every 400 women, are ineligible to serve in the army based on their height.



Toward Probability

Suppose you pick a baby at random and ask whether the baby was born more than 15 days prior to his or her due date. Because births are normally distributed around the due date with a standard deviation of 15 days, we know that 16% of all births occur more than 15 days prior to the due date (see Example 3). For an individual baby chosen at random, we can therefore say that there's a 0.16 chance (about 1 in 6) that the baby was born more than 15 days early. In other words, the properties of the normal distribution allow us to make a *probability statement* about an individual. In this case, our statement is that the probability of a birth occurring more than 15 days early is 0.16.

This example shows that the properties of the normal distribution can be restated in terms of ideas of probability. In the next section, we will use the basic ideas we have discussed so far to introduce one of the most important concepts in statistics.

Section 2 Exercises

Statistical Literacy and Critical Thinking

- 1. Standard Score.** Men's heights are normally distributed with a mean of 69.0 in. and a standard deviation of 2.8 in. What is the standard z -score for a man with a height of 69.0 in.?
- 2. Standard Score.** The standard score for the height of a male is $z = -2$. Is the height of this male above or below the mean height of all males? How many standard deviations away from the mean is this?
- 3. Distributions.** For rolling a die, the mean outcome is 3.5. Can we apply the 68-95-99.7 rule and conclude that 95% of all outcomes fall within 2 standard deviations of 3.5? Why or why not?
- 4. z-Scores and Percentages.** Table 1 includes standard scores and percentiles. Can a z -score be a negative value? Can a percentile be a negative value? Explain.

Does It Make Sense? For Exercises 5–8, decide whether the statement makes sense (or is clearly true) or does not make sense (or is clearly false). Explain clearly; not all of these have definitive answers, so your explanation is more important than your chosen answer.

- 5. Test Scores.** Scores on a statistics test are normally distributed with a mean of 75 and a standard deviation of 75.
- 6. Birth Weights.** Birth weights (in grams) in Lichtenstein are normally distributed with a mean of 3,420 g and a standard deviation of 0 g.
- 7. Depth Perception Scores.** Scores on a standard test of depth perception are normally distributed with two different modes.
- 8. SAT Scores.** SAT scores are normally distributed with a mean of 1518 and a standard deviation of 325.

Concepts and Applications

- 9. Using the 68-95-99.7 Rule.** A test of depth perception is designed so that scores are normally distributed with a mean of 50 and a standard deviation of 10. Use the 68-95-99.7 rule to find the following values.
 - a. Percentage of scores less than 50
 - b. Percentage of scores less than 60
 - c. Percentage of scores greater than 70
 - d. Percentage of scores greater than 40
 - e. Percentage of scores between 40 and 70
- 10. Using the 68-95-99.7 Rule.** Assume the resting pulse rates for a sample of individuals are normally distributed with a mean of 70 and a standard deviation of 15. Use the 68-95-99.7 rule to find the following quantities.
 - a. Percentage of pulse rates less than 70
 - b. Percentage of pulse rates greater than 55
 - c. Percentage of pulse rates between 55 and 70
 - d. Percentage of pulse rates between 55 and 100
 - e. Percentage of pulse rates between 70 and 100
- 11. Applying the 68-95-99.7 Rule.** In a study of facial behavior, people in a control group are timed for eye contact in a 5-minute period. Their times are normally distributed with a mean of 184.0 seconds and a standard deviation of 55.0 seconds (based on data from "Ethological Study of Facial Behavior

in Nonparanoid and Paranoid Schizophrenic Patients," by Pittman, Olk, Orr, and Singh, *Psychiatry*, Vol. 144, No. 1). Use the 68-95-99.7 rule to find the indicated quantity.

- Find the percentage of times within 55.0 seconds of the mean of 184.0 seconds.
- Find the percentage of times within 110.0 seconds of the mean of 184.0 seconds.
- Find the percentage of times within 165.0 seconds of the mean of 184.0 seconds.
- Find the percentage of times between 184.0 seconds and 294 seconds.

12. Applying the 68-95-99.7 Rule. When designing the placement of a CD player in a new model car, engineers must consider the forward grip reach of the driver. Women have forward grip reaches that are normally distributed with a mean of 27.0 inches and a standard deviation of 1.3 inches (based on anthropometric survey data from Gordon, Churchill et al.). Use the 68-95-99.7 rule to find the indicated quantity.

- Find the percentage of women with forward grip reaches between 24.4 inches and 29.6 inches.
- Find the percentage of women with forward grip reaches less than 30.9 inches.
- Find the percentage of women with forward grip reaches between 27.0 inches and 28.3 inches.

IQ Scores. For Exercises 13–24, use the normal distribution of IQ scores, which has a mean of 100 and a standard deviation of 16. Use Table 1 to find the indicated quantities. Note: Table 1 shows standard scores from -3.5 to $+3.5$. In these problems, for standard scores above 3.5 , use a percentile of 99.99%; for all standard scores below -3.5 , use a percentile of 0.01%.

13. Percentage of scores greater than 100

14. Percentage of scores less than 116

15. Percentage of scores less than 68

16. Percentage of scores greater than 108

17. Percentage of scores less than 132

18. Percentage of scores less than 92

19. Percentage of scores greater than 76

20. Percentage of scores greater than 148

21. Percentage of scores between 84 and 116

22. Percentage of scores between 68 and 132

23. Percentage of scores between 52 and 116

24. Percentage of scores between 76 and 108

Heights of Women. For Exercises 25–36, use the normal distribution of heights of adult women, which has a mean of 162 centimeters and a standard deviation of 6 centimeters. Use

Table 1 to find the indicated quantities. Note: Table 1 shows standard scores from -3.5 to $+3.5$. In these problems, for standard scores above 3.5 , use a percentile of 99.99%; for all standard scores below -3.5 , use a percentile of 0.01%.

- The percentage of heights greater than 162 centimeters
- The percentage of heights less than 168 centimeters
- The percentage of heights greater than 156 centimeters
- The percentage of heights greater than 171 centimeters
- The percentage of heights less than 177 centimeters
- The percentage of heights less than 147 centimeters
- The percentage of heights less than 144 centimeters
- The percentage of heights greater than 179 centimeters
- The percentage of heights between 156 centimeters and 168 centimeters
- The percentage of heights between 159 centimeters and 165 centimeters
- The percentage of heights between 148 centimeters and 170 centimeters
- The percentage of heights between 146 centimeters and 156 centimeters
- Coin Weights.** Consider the following table, showing the official mean weight and estimated standard deviation for five U.S. coins. Suppose a vending machine is designed to reject all coins with weights more than 2 standard deviations above or below the mean. For each coin, find the range of weights that is acceptable to the vending machine. In each case, what percentage of legal coins is rejected by the machine?

Coin	Weight (grams)	Estimated standard deviation (grams)
Cent	2.500	0.03
Nickel	5.000	0.06
Dime	2.268	0.03
Quarter	5.670	0.07
Half dollar	11.340	0.14

38. Pregnancy Lengths. Lengths of pregnancies are normally distributed with a mean of 268 days and a standard deviation of 15 days.

- What is the percentage of pregnancies that last less than 250 days?
- What is the percentage of pregnancies that last more than 300 days?
- If a birth is considered premature if the pregnancy lasts less than 238 days, what is the percentage of premature births?

39. SAT Scores. Based on data from the College Board, SAT scores are normally distributed with a mean of 1518 and a standard deviation of 325.

- Find the percentage of SAT scores greater than 2000.

- b. Find the percentage of SAT scores less than 1500.
- c. Find the percentage of SAT scores between 1600 and 2100.
- 40. GRE Scores.** Assume that the scores on the Graduate Record Exam (GRE) are normally distributed with a mean of 497 and a standard deviation of 115.
- A graduate school requires a GRE score of 650 for admission. To what percentile does this correspond?
 - A graduate school requires a GRE score in the 95th percentile for admission. To what actual score does this correspond?
- 41. Calibrating Barometers.** Researchers for a manufacturer of barometers (devices to measure atmospheric pressure) read each of 50 barometers at the same time of day. The mean of the readings is 30.4 (inches of mercury) with a standard deviation of 0.23 inch, and the readings appear to be normally distributed.
- What percentage of the barometers read over 31?
 - What percentage of the barometers read less than 30?
 - The company decides to reject barometers that read more than 1.5 standard deviations above or below the mean. What is the critical reading below which barometers will be rejected? What is the critical reading above which barometers will be rejected?
 - What would you take as the actual atmospheric pressure at the time the barometers were read? Explain.
- 42. Spelling Bee Scores.** At the district spelling bee, the 60 girls have a mean score of 71 points with a standard deviation of 6, while the 50 boys have a mean score of 66 points with a standard deviation of 5 points. Those students with a score greater than 75 are eligible to go to the state spelling bee. What percentage of those going to the state bee will be girls?
- 43. Being a Marine.** According to data from the National Health Survey, the heights of adult men are normally distributed with a mean of 69.0 inches and a standard deviation of 2.8 inches. The U.S. Marine Corps requires that men have heights between 64 inches and 78 inches. What percentage of American men are eligible for the Marines based on height?
- 44. Movie Lengths.** Based on a random sample of movie lengths, the mean length is 110.5 minutes with a standard deviation of 22.4 minutes. Assume that movie lengths are normally distributed.
- What fraction of movies are more than 2 hours long?
 - What fraction of movies are less than $1\frac{1}{2}$ hours long?
 - What is the probability that a randomly selected movie will be less than $2\frac{1}{2}$ hours long?



- 45. Normal Distribution Demonstrations on the Web.** Do an Internet search on the keywords “normal distribution” and find an animated demonstration of the normal distribution. Describe how the demonstration works and the useful features that you observed.
- 46. Estimating a Minute.** Ask survey subjects to estimate 1 minute without looking at a watch or clock. Each subject should say “go” at the beginning of the minute and then “stop” when he or she thinks that 1 minute has passed. (Alternatively, you could repeatedly time yourself by looking away from any watch or clock and then noting the correct time when you think that 1 minute has passed.) Use a watch to record the actual times. Construct a graph of the estimates. Is the graph approximately normal? Does the mean appear to be close to 1 minute?

3 THE CENTRAL LIMIT THEOREM

A high school English teacher has 100 seniors taking a college placement test. The test is designed to have a mean score of 500 and a standard deviation of 100. Using the methods of this chapter, she can determine the percentage of individual scores that are likely to be above, say, 600. But can she predict anything about the performance of her *group* of 100 students? For example, what is the likelihood that the mean score of the group will be above 600? This type of question, in which we ask about the mean score for a group or sample drawn from a much larger population, can be answered with the *Central Limit Theorem*.

Before we get to the theorem itself, we can develop some insight by thinking about dice rolling. Suppose we roll *one* die 1,000 times and record the outcome of each roll, which can be the number 1, 2, 3, 4, 5, or 6. Figure 22 shows a histogram of outcomes. All six outcomes have roughly the same relative frequency, because the die is equally likely to land in each of the six possible ways. That is, the histogram shows a (nearly) *uniform distribution*. We can compute that the mean is 3.41 and the standard deviation is 1.73 for this distribution.

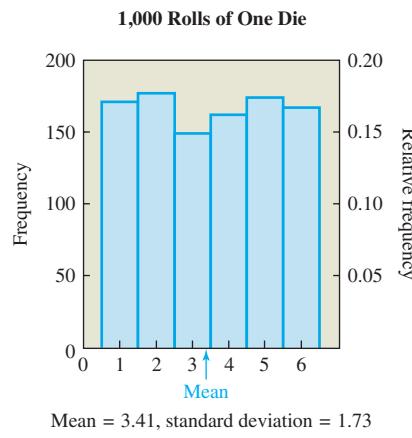


Figure 22 Frequency and relative frequency distribution of outcomes from rolling one die 1,000 times.

Now suppose we roll *two* dice 1,000 times and record the *mean* of the two numbers that appear on each roll (Figure 23). To find the mean for a single roll, we add the two numbers and divide by 2. For example, if the two dice come up 3 and 5, the mean for the roll is $(3 + 5)/2 = 4$. The possible values of the mean on a roll of two dice are 1.0, 1.5, 2.0, ..., 5.0, 5.5, 6.0.

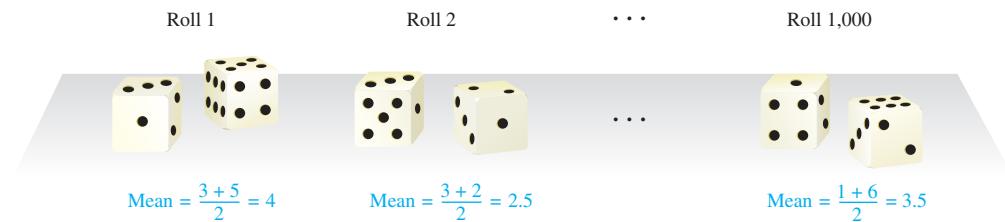


Figure 23 This diagram represents the idea of rolling two dice 1,000 times and recording the *mean* on each roll. The mean of the values on two dice is their sum divided by 2. This mean can range from $(1 + 1)/2 = 1$ to $(6 + 6)/2 = 6$.

Figure 24a shows a typical result of rolling two dice 1,000 times. The most common values in this distribution are the central values 3.0, 3.5, and 4.0. These values are common because they can occur in several ways. For example, a mean of 3.5 can occur if the two dice land as 1 and 6, 2 and 5, 3 and 4, 4 and 3, 5 and 2, or 6 and 1. High and low values occur less frequently because they can occur in fewer ways. For example, a roll can have a mean of 1.0 only if both dice land showing a 1. Again, it is possible to compute the mean and standard deviation for this distribution, and they turn out to be 3.43 and 1.21, respectively.

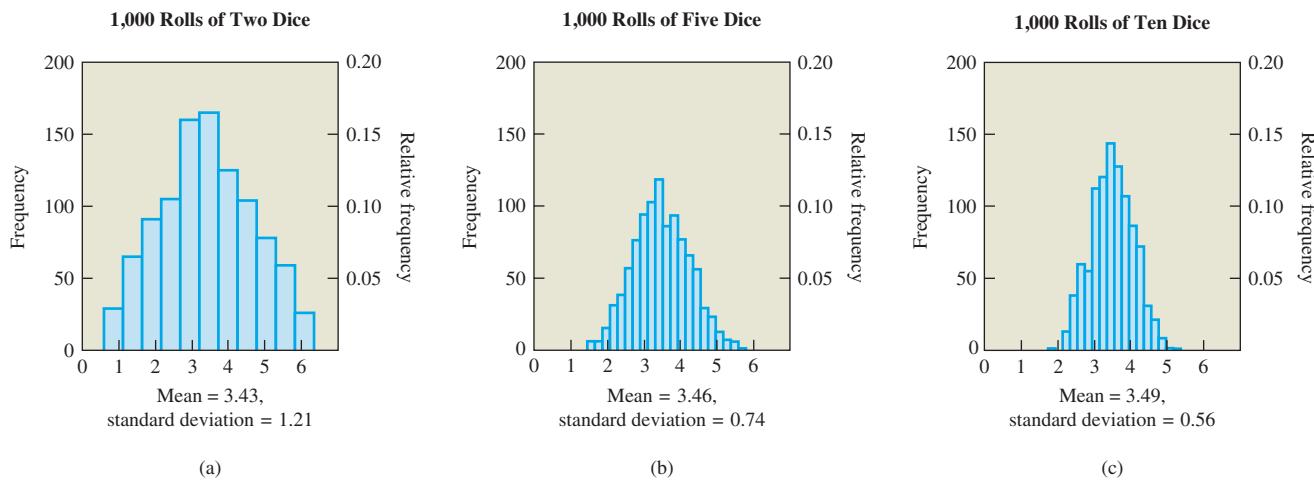


Figure 24 Frequency and relative frequency distributions of sample means from rolling (a) two dice 1,000 times, (b) five dice 1,000 times, and (c) ten dice 1,000 times.

What happens if we increase the number of dice we roll? Suppose we roll five dice 1,000 times and record the mean of the five numbers on each roll. A histogram for this experiment is shown in Figure 24b. Once again we see that the central values around 3.5 occur most frequently, but the spread of the distribution is narrower than in the two previous cases. Computing the mean and standard deviation of this distribution, we get values of 3.46 and 0.74, respectively.

If we further increase the number of dice to 10 on each of 1,000 rolls, we find the histogram in Figure 24c, which is even narrower. In this case, the mean is 3.49 and standard deviation is 0.56.

Table 2 summarizes the four experiments we've described. Columns 2 and 3 of the table refer to a *distribution of means* because, in each of the dice rolling experiments, we recorded the mean of each of 1,000 rolls (that is, the mean of one die on each roll, of two dice on each roll, of five dice on each roll, or of 10 dice on each roll). In other words, the mean for all 1,000 rolls in an experiment is *a mean of the distribution of means* (Column 2). Similarly, the standard deviation for all 1,000 rolls in an experiment is *a standard deviation of the distribution of means* (Column 3).

TABLE 2**Summary of Dice Rolling Experiments**

Number of dice rolled each time	Mean of the distribution of means	Standard deviation of the distribution of means
1	3.41	1.73
2	3.43	1.21
5	3.46	0.74
10	3.49	0.56

A remarkable insight emerges from these four experiments. Rolling $n = 1$ die 1,000 times can be regarded as taking 1,000 samples of size $n = 1$ from the population of all possible dice rolls. Rolling $n = 2$ dice 1,000 times can be viewed as taking 1,000 samples of size $n = 2$. Likewise, rolling $n = 5$ and $n = 10$ dice 1,000 times is like taking 1,000 samples of size $n = 5$ and $n = 10$, respectively. Table 2 shows that as the sample size increases, the mean of the distribution of means approaches the value 3.5 and the standard deviation becomes smaller (making the distribution narrower). More important, the distribution looks more and more like a normal distribution as the sample size increases. This latter fact may seem surprising because we have taken samples from a *uniform* distribution (the outcomes of rolling a single die shown in Figure 22), *not* from a normal distribution. Nevertheless, the distribution of means clearly approaches a normal distribution for large sample sizes. This fact is a consequence of the **Central Limit Theorem**.

The Central Limit Theorem

Suppose we take many random samples of size n for a variable with any distribution (not necessarily a normal distribution) and record the distribution of the *means* of each sample. Then,

1. The distribution of means will be approximately a normal distribution for large sample sizes.
2. The mean of the distribution of means approaches the population mean, μ , for large sample sizes.
3. The standard deviation of the distribution of means approaches σ/\sqrt{n} for large sample sizes, where σ is the standard deviation of the population.

TECHNICAL NOTE

- (1) For practical purposes, the distribution of means will be nearly normal if the sample size is larger than 30. (2) If the original population is normally distributed, then the sample means will be normally distributed for *any* sample size n . (3) In the ideal case, where the distribution of means is formed from *all* possible samples, the mean of the distribution of means equals μ and the standard deviation of the distribution of means equals σ/\sqrt{n} .

Be sure to note the third point above: The standard deviation of the distribution of sample means is not the standard deviation of the population, σ , but rather σ/\sqrt{n} , where n is the size of the samples.

TIME OUT TO THINK

Confirm that the standard deviations of the distributions of means given in Table 2 for $n = 2, 5, 10$ agree with the prediction of the Central Limit Theorem, given that $\sigma = 1.73$ (the population standard deviation found in Figure 22). For example, with $n = 2$, $\sigma/\sqrt{2} = 1.22 \approx 1.21$.

Let's summarize the ingredients of the Central Limit Theorem. We always start with a particular variable, such as the outcomes of rolling a die or the weights of people, that varies randomly over a population. The variable has a certain mean, μ , and standard deviation, σ , which we may or may not know. This variable can have a distribution of *any* shape, not necessarily the shape of a normal distribution. Now we take many samples of that variable, with n items in each sample, and find the mean of each sample (such as the mean value of n dice or the mean weight of a sample of n people). If we then make a histogram of the means from the many samples, we will see a distribution that is close to a normal distribution. The larger the sample size, n , the more closely the distribution of means approximates a normal distribution. Careful study of Figure 25 should help solidify these important ideas.

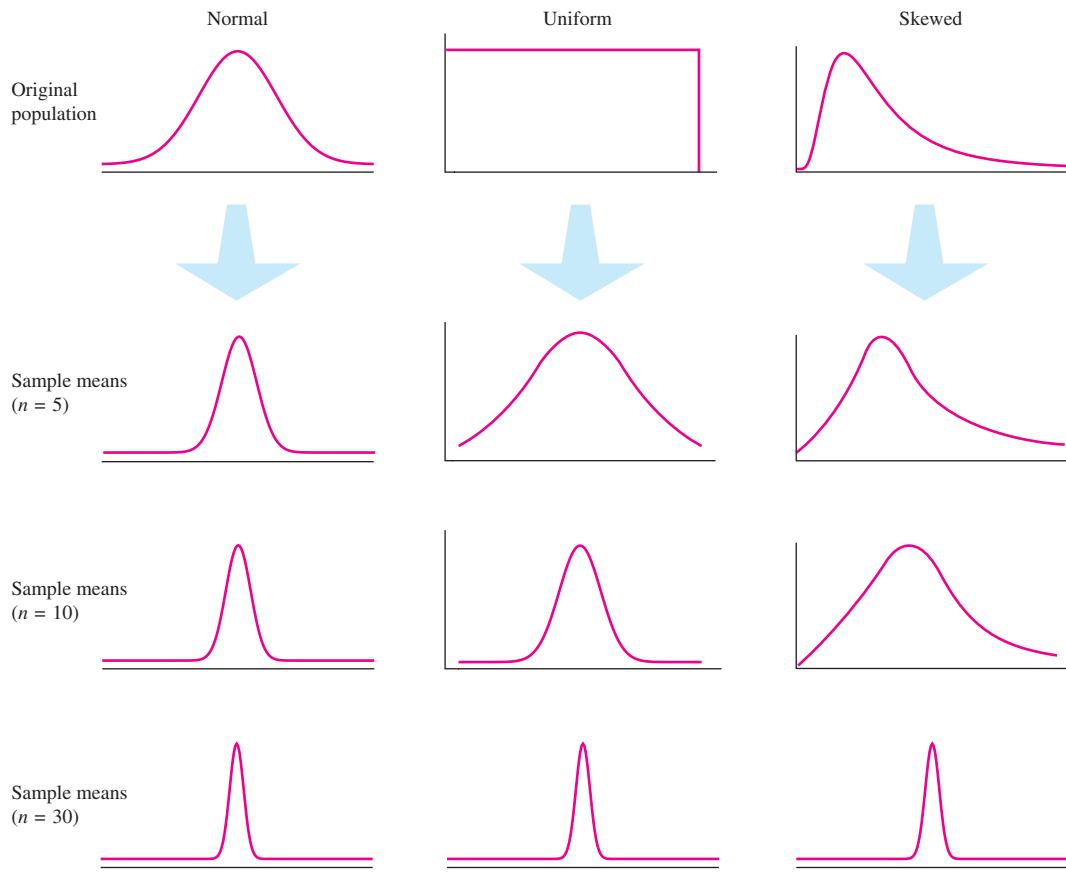


Figure 25 As the sample size increases ($n = 5, 10, 30$), the distribution of sample means approaches a normal distribution, regardless of the shape of the original distribution. The larger the sample size, the smaller is the standard deviation of the distribution of sample means.

EXAMPLE 1 Predicting Test Scores

You are a middle school principal and your 100 eighth-graders are about to take a national standardized test. The test is designed so that the mean score is $\mu = 400$ with a standard deviation of $\sigma = 70$. Assume the scores are normally distributed.

- What is the likelihood that *one* of your eighth-graders, selected at random, will score below 375 on the exam?
- Your performance as a principal depends on how well your entire *group* of eighth-graders scores on the exam. What is the likelihood that your group of 100 eighth-graders will have a *mean* score below 375?

SOLUTION

- a. In dealing with an individual score, we use the method of standard scores discussed in Section 2. Given the mean of 400 and standard deviation of 70, a score of 375 has a standard score of

$$z = \frac{\text{data value} - \text{mean}}{\text{standard deviation}} = \frac{375 - 400}{70} = -0.36$$

According to Table 1, a standard score of -0.36 corresponds to about the 36th percentile—that is, 36% of all students can be expected to score below 375. In other words, there is about a 0.36 chance that a randomly selected student will score below 375.

- b. The question about the mean of a *group* of students must be handled with the Central Limit Theorem. According to this theorem, if we take random samples of size $n = 100$ students and compute the mean test score of each group, the distribution of means is approximately normal. Moreover, the mean of this distribution is $\mu = 400$ and its standard deviation is $\sigma/\sqrt{n} = 70/\sqrt{100} = 7$. With these values for the mean and standard deviation, the standard score for a mean test score of 375 is

$$z = \frac{\text{data value} - \text{mean}}{\text{standard deviation}} = \frac{375 - 400}{7} = -3.57$$

Table 1 shows that a standard score of -3.57 corresponds to the 0.02th percentile, and the standard score in this case is even lower. In other words, fewer than 0.02% of all random samples of 100 students will have a mean score of less than 375. Therefore, the chance that a randomly selected group of 100 students will have a mean score below 375 is less than 0.0002, or about 1 in 5,000.

This example has an important lesson. The likelihood of an *individual* scoring below 375 is more than 1 in 3 (36%), but the likelihood of a *group* of 100 students having a mean score below 375 is less than 1 in 5,000 (0.02%). In other words, there is much more variation in the scores of individuals than in the means of groups of individuals. •••

When you are listening to corn pop, are you hearing the Central Limit Theorem?

—William A. Massey

EXAMPLE 2 Salary Equity

The mean salary of the 9,000 employees at Holley Inc. is $\mu = \$26,400$ with a standard deviation of $\sigma = \$2,420$. A pollster samples 400 randomly selected employees and finds that the mean salary of the sample is \$26,650. Is it likely that the pollster would get these results by chance, or does the discrepancy suggest that the pollster's results are suspect?

SOLUTION The question deals with the mean of a *group* of 400 individuals, which is a case for the Central Limit Theorem. The theorem tells us that if we select many groups of 400 individuals and compute the mean of each group, the distribution of means will be close to normal with a mean of $\mu = \$26,400$ and a standard deviation of $\sigma/\sqrt{n} = \$2,420/\sqrt{400} = \121 . Within the distribution of means, a mean salary of \$26,650 has a *standard score* of

$$z = \frac{\text{data value} - \text{mean}}{\text{standard deviation}} = \frac{\$26,650 - \$26,400}{\$121} = 2.07$$

In other words, if we assume that the sample is randomly selected, its mean salary is more than 2 standard deviations above the mean salary of the entire company. According to Table 1, a standard score of 2.07 lies near the 98th percentile. Thus, the mean salary of *this sample* is greater than the mean salary we would find in 98% of the possible samples of 400 workers. That is, the likelihood of selecting a group of 400 workers with a mean salary above \$26,650 is about 2%, or 0.02. The mean salary of the sample is surprisingly high; perhaps the survey was flawed. •••

TIME OUT TO THINK

Would a salary of \$26,650 for an *individual* worker lie above or below the 98th percentile? Explain.

The Value of the Central Limit Theorem

The Central Limit Theorem allows us to say something about the mean of a group if we know the mean, μ , and the standard deviation, σ , of the entire population. This can be useful, but it turns out that the opposite application is far more important.

Two major activities of statistics are making estimates of population means and testing claims about population means. Suppose we do *not* know the mean of a variable for the entire population. Is it possible to make a good estimate of the population mean (such as the mean income of all Internet users) knowing only the mean of a much smaller sample? As you can probably guess, being able to answer this type of question lies at the heart of statistical sampling, especially in polls and surveys. The Central Limit Theorem provides the key to answering such questions.

Section 3 Exercises

Statistical Literacy and Critical Thinking

- 1. Texas Lottery.** In each drawing for the Texas Pick 3 lottery, 3 digits between 0 and 9 (inclusive) are randomly selected. What is the distribution of the selected digits? If the mean is calculated for each drawing, can the distribution of the sample means be treated as a normal distribution?
- 2. Notation.** In this section, it was noted that the standard deviation of sample means is σ/\sqrt{n} . In that expression, what does σ represent and what does n represent?
- 3. Central Limit Theorem.** A process consists of repeating this operation: Randomly select two values from a normally distributed population and then find the mean of the two values. Will the sample means be normally distributed, even though each sample has only two values?
- 4. Central Limit Theorem.** The population of unemployed adults has ages with mean μ and standard deviation σ . Samples of unemployed adults are randomly selected so that there are exactly 100 in each sample. For each sample, the mean age is computed. What does the Central Limit Theorem tell us about the distribution of those mean ages?

Concepts and Applications

- 5. IQ Scores and the Central Limit Theorem.** IQ scores are normally distributed with a mean of 100 and a standard deviation of 16. Assume that many samples of size n are taken from a large population of people and the mean IQ score is computed for each sample.
 - a. If the sample size is $n = 64$, find the mean and standard deviation of the distribution of sample means.
 - b. If the sample size is $n = 100$, find the mean and standard deviation of the distribution of sample means.
 - c. Why is the standard deviation in part a different from the standard deviation in part b?

6. SAT Scores and the Central Limit Theorem

Theorem. Based on data from the College Board, assume that SAT scores are normally distributed with a mean of 1518 and a standard deviation of 325. Assume that many samples of size n are taken from a large population of students and the mean SAT score is computed for each sample.

- a. If the sample size is $n = 100$, find the mean and standard deviation of the distribution of sample means.
- b. If the sample size is $n = 2,500$, find the mean and standard deviation of the distribution of sample means.
- c. Why is the standard deviation in part a different from the standard deviation in part b?

7. Twelve-Sided Dice and the Central Limit Theorem

Theorem. Rolling a fair 12-sided die produces a uniformly distributed set of numbers between 1 and 12 with a mean of 6.5 and a standard deviation of 3.452. Assume that n 12-sided dice are rolled many times and the mean of the n outcomes is computed each time.

- a. Find the mean and standard deviation of the resulting distribution of sample means for $n = 81$.
- b. Find the mean and standard deviation of the resulting distribution of sample means for $n = 100$.
- c. Why is the standard deviation in part a different from the standard deviation in part b?

- 8. Ten-Sided Dice and the Central Limit Theorem.** Rolling a fair 10-sided die produces a uniformly distributed set of numbers between 1 and 10 with a mean of 5.5 and a standard deviation of 2.872. Assume that n 10-sided dice are rolled many times and the mean of the n outcomes is computed each time.
- Find the mean and standard deviation of the resulting distribution of sample means for $n = 49$.
 - Find the mean and standard deviation of the resulting distribution of sample means for $n = 400$.
 - Why is the standard deviation in part a different from the standard deviation in part b?
- Male Weights.** In Exercises 9–12, assume that weights of men are normally distributed with a mean of 170 lb and a standard deviation of 30 lb (approximate values based on data from the National Health and Nutrition Examination Survey).
- What percentage of individual men have weights less than 185 lb? If samples of 36 men are randomly selected and the mean weight is computed for each sample, what percentage of the sample means are less than 185 lb?
 - What percentage of individual men have weights greater than 167 lb? If samples of 100 men are randomly selected and the mean weight is computed for each sample, what percentage of the sample means are greater than 167 lb?
 - What percentage of individual men have weights between 164 lb and 176 lb? If samples of 100 men are randomly selected and the mean weight is computed for each sample, what percentage of sample means is between 164 lb and 176 lb?
 - What percentage of individual men have weights between 176 lb and 185 lb? If samples of 25 men are randomly selected and the mean weight is computed for each sample, what percentage of sample means is between 176 lb and 185 lb?
- 13. Amounts of Cola.** Assume that cans of cola are filled so that the actual amounts are normally distributed with a mean of 12.00 ounces and a standard deviation of 0.11 ounce.
- What is the likelihood that a sample of 36 cans will have a mean amount of at least 12.05 ounces?
 - Given the result in part a, is it reasonable to believe that the cans are actually filled with a mean of 12.00 ounces? If the mean is not 12.00 ounces, are consumers being cheated?
- 14. Designing Strobe Lights.** An aircraft strobe light is designed so that the times between flashes are normally distributed with a mean of 3.00 seconds and a standard deviation of 0.40 second.
- What is the likelihood that an individual time is greater than 4.00 seconds?
 - What is the likelihood that the mean for 60 randomly selected times is greater than 4.00 seconds?
- c.** Given that the strobe light is intended to help other pilots see an aircraft, which result is more relevant for assessing the safety of the strobe light: the result in part a or the result in part b? Why?
- 15. Designing Motorcycle Helmets.** Engineers must consider the breadths of male heads when designing motorcycle helmets for men. Men have head breadths that are normally distributed with a mean of 6.0 inches and a standard deviation of 1.0 inch (based on anthropometric survey data from Gordon, Churchill et al.).
- If one male is randomly selected, what is the likelihood that his head breadth is less than 6.2 inches?
 - The Safeguard Helmet company plans an initial production run of 100 helmets. How likely is it that 100 randomly selected men have a mean head breadth of less than 6.2 inches?
 - The production manager sees the result in part b and reasons that all helmets should be made for men with head breadths of less than 6.2 inches, because they would fit all but a few men. What is wrong with that reasoning?
- 16. Staying Out of Hot Water.** In planning for hot water requirements, the manager of the Luxurion Hotel finds that guests spend a mean of 11.4 minutes each day in the shower (based on data from the Opinion Research Corporation). Assume that the shower times are normally distributed with a standard deviation of 2.7 minutes.
- Find the percentage of guests who shower for more than 12 minutes.
 - The hotel has installed a system that can provide enough hot water provided that the mean shower time for 84 guests is less than 12 minutes. If the hotel currently has 84 guests, how likely is it that there will not be enough hot water? Does the current system appear to be effective?
- 17. Redesign of Ejection Seats.** When women were allowed to become pilots of fighter jets, engineers needed to redesign the ejection seats because they had been designed for men only. The ACES-II ejection seats were designed for men weighing between 140 pounds and 211 pounds. The population of women has normally distributed weights with a mean of 143 pounds and a standard deviation of 29 pounds (based on data from the National Health Survey).
- What percentage of women have weights between 140 pounds and 211 pounds?
 - If 36 women are randomly selected, how likely is it that their mean weight is between 140 pounds and 211 pounds?
 - For redesigning the fighter jet ejection seats to better accommodate women, which probability is more relevant: the result in part a or the result in part b? Why?
- 18. Labeling of M&M Packages.** M&M plain candies have weights that are normally distributed with a mean weight

of 0.8565 gram and a standard deviation of 0.0518 gram (based on measurements from one of the authors). A random sample of 100 M&M candies is obtained from a package containing 465 candies; the package label states that the net weight is 396.9 grams. (If every package has 465 candies, the mean weight of the candies must exceed $396.9/465 = 0.8535$ for the net contents to weigh at least 396.9 grams.)

- a. If 1 M&M plain candy is randomly selected, how likely is it that it weighs more than 0.8535 gram?
 - b. If 465 M&M plain candies are randomly selected, how likely is it that their mean weight is at least 0.8535 gram?
 - c. Given these results, does it seem that the Mars Company is providing M&M consumers with the amount claimed on the label?
- 19. Vending Machines.** Currently, quarters have weights that are normally distributed with a mean of 5.670 grams and a standard deviation of 0.062 gram. A vending machine is configured to accept only those quarters with weights between 5.550 grams and 5.790 grams.
- a. If 280 different quarters are inserted into the vending machine, what is the expected number of rejected quarters?
 - b. If 280 different quarters are inserted into the vending machine, how likely is it that the mean falls between the limits of 5.550 grams and 5.790 grams?
 - c. If you owned the vending machine, which result would concern you more: the result in part a or the result in part b? Why?

20. Aircraft Safety Standards. Federal Aviation Administration rules require airlines to estimate the weight of a passenger as 195 pounds, including carry-on baggage. Men have weights (without baggage) that are normally distributed with a mean of 172 pounds and a standard deviation of 29 pounds.

- a. If one adult male is randomly selected and is assumed to have 20 pounds of carry-on baggage, how likely is it that his total weight is greater than 195 pounds?
- b. If a Boeing 767-300 aircraft is full of 213 adult male passengers and each is assumed to have 20 pounds of carry-on baggage, how likely is it that the mean passenger weight (including carry-on baggage) is greater than 195 pounds? Does a pilot have to be concerned about exceeding this weight limit?

21. Kindergarten Desks. In designing new desks for an incoming class of 25 kindergarten girls, an important characteristic of the desks is that they must accommodate the sitting heights of those students. (The sitting height is the height

of a seated student measured from the bottom of the feet to the top of the knee.) Kindergarten girls have normally distributed sitting heights with a mean of 61.2 cm and a standard deviation of 3.1 cm.

- a. What percentage of individual kindergarten girls have sitting heights greater than 63 cm?
- b. For randomly selected groups of 25 kindergarten girls, what percentage of the group means is greater than 63 cm?
- c. Which result is more relevant for the design of the desks: the result from part a or part b? Why?

22. Blood Pressure in Women. Systolic blood pressure for women between the ages of 18 and 24 is normally distributed with a mean of 114.8 (millimeters of mercury) and a standard deviation of 13.1.

- a. What is the likelihood that an individual woman has a blood pressure above 125?
- b. Suppose a random sample of $n = 300$ women is selected and the mean blood pressure for the sample is computed. What is the likelihood that the mean blood pressure for the sample will be above 125?
- c. Suppose a random sample of $n = 300$ women is selected and the mean blood pressure for the sample is computed. What is the likelihood that the mean blood pressure for the sample will be below 114?

PROJECTS FOR THE INTERNET & BEYOND



23. Central Limit Theorem on the Internet. Doing an Internet search on “central limit theorem” will uncover many sites devoted to this subject. Find a site that has animated demonstrations of the Central Limit Theorem. Describe in your own words what you observed and how it illustrates the Central Limit Theorem.

24. The Quincunx on the Internet. Do an Internet search on “central limit theorem” or “quincunx” and find a site that has an animated demonstration of the quincunx (or Galton’s board). Describe the quincunx and explain how it illustrates the Central Limit Theorem.

25. Dice Rolling. Demonstrate the Central Limit Theorem using dice, as discussed in this section. Give each person in your class as many dice as possible. Begin by rolling one die and making a histogram of the outcomes. Then let every person roll two dice and make a histogram of the mean for each roll. Increase the number of dice in each roll as long as dice and time allow. Comment on the appearance of the histogram at each stage.

CHAPTER REVIEW EXERCISES

1. For each of the following situations, state whether the distribution of values is likely to be a normal distribution. Give a brief explanation justifying your choice.
 - a. Numbers resulting from spins of a roulette wheel. (There are 38 equally likely slots with numbers 0, 00, 1, 2, 3, ..., 36.)
 - b. Weights of 12-year old girls
 - c. Scores on a test designed to measure knowledge of current events
2. The College of Portland uses a diagnostics test to correctly place students in math courses. Scores on the test are normally distributed with a mean of 400 and a standard deviation of 50.
 - a. Using the 68-95-99.7 rule, find the percentage of scores within 50 of the mean of 400.
 - b. Using the 68-95-99.7 rule, find the percentage of scores between 300 and 500.
 - c. Is a score of 550 unusual? Why or why not?
3. Assume that body temperatures of healthy adults are normally distributed with a mean of 98.20°F and a standard deviation of 0.62°F (based on data from University of Maryland researchers).
 - a. If you have a body temperature of 99.00°F , what is your percentile score?
 - b. Convert 99.00°F to a standard score (or z -score).
 - c. Is a body temperature of 99.00°F “unusual”? Why or why not?
 - d. Fifty adults are randomly selected. What is the likelihood that the mean of their body temperatures is 97.98°F or lower?
 - e. A person’s body temperature is found to be 101.00°F . Is this result “unusual”? Why or why not? What should you conclude?
 - f. What body temperature is the 95th percentile?
 - g. What body temperature is the 5th percentile?
 - h. Bellevue Hospital in New York City uses 100.6°F as the lowest temperature considered to indicate a fever. What percentage of normal and healthy adults would be considered to have a fever? Does this percentage suggest that a cutoff of 100.6°F is appropriate?
 - i. If, instead of assuming that the mean body temperature is 98.20°F , we assume that the mean is 98.60°F (as many people believe), what is the chance of randomly selecting 106 people and getting a mean of 98.20°F or lower? (Continue to assume that the standard deviation is 0.62°F .) University of Maryland researchers did get such a result. What should we conclude?



CHAPTER QUIZ

1. Which of the following statements are correct?
 - a. A normal distribution is any distribution that typically occurs.
 - b. The graph of a normal distribution is bell-shaped.
 - c. The graph of a normal distribution has one mode.
 - d. In a normal distribution, the mean and median are equal.
 - e. In a normal distribution, the standard deviation is 0.
2. Car seat belts are tested for strength, and it is found that the results are normally distributed with a mean of 2,400 lb and a standard deviation of 400 lb. Using the 68-95-99.7 rule, find the percentage of scores between 1,600 lb and 3,200 lb.
3. Car seat belts are tested for strength, and it is found that the results are normally distributed with a mean of 2,400 lb and a standard deviation of 400 lb. Use Table 1 to find the percentage of scores between 1,600 lb and 3,200 lb.
4. A population is normally distributed with a mean of 850 and a standard deviation of 49, and samples of size 100 are randomly selected. What is the mean of the sample means?
5. A population is normally distributed with a mean of 850 and a standard deviation of 49, and samples of size 100 are randomly selected. What is the standard deviation of the sample means?
6. A population is normally distributed with a mean of 850 and a standard deviation of 49. What is the standard z -score corresponding to 801?
7. A population is normally distributed with a mean of 850 and a standard deviation of 49. What percentage of scores are greater than 801?
8. A population is normally distributed with a mean of 850 and a standard deviation of 49. If 2.28% of the scores are less than 752, what percentage of scores are greater than 752?
9. A population is normally distributed with a mean of 850 and a standard deviation of 49. Find the percentage of scores between 801 and 948.
10. Which of the following is likely to have a distribution that is closest to a normal distribution?
 - a. The outcomes that occur when a single die is rolled many times
 - b. The outcomes that occur when two dice are rolled many times and the mean is computed each time
 - c. The outcomes that occur when five dice are rolled many times and the mean is computed each time

FOCUS ON EDUCATION

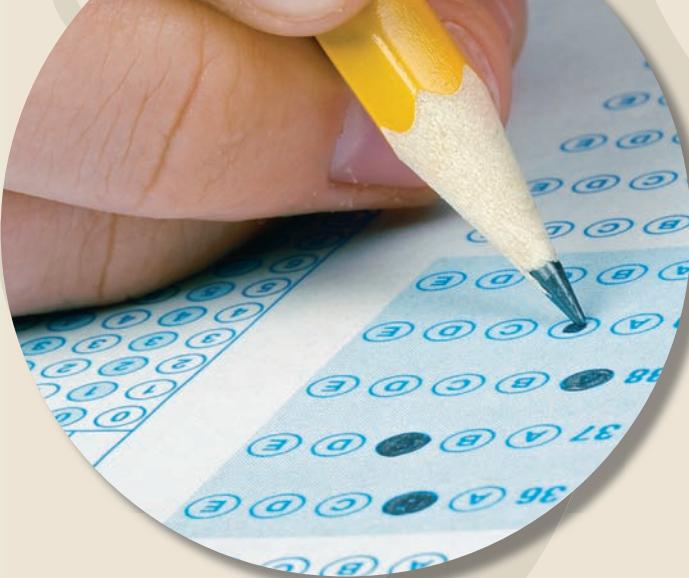
What Can We Learn from SAT Trends?

The Scholastic Aptitude Test (SAT) has been taken by college-bound high school students since 1941, when 11,000 students took the first test. Today it is taken by more than 2 million students each year. Until 2006, there were only two parts to the SAT: a “verbal” test and a mathematics test. In 2006, the verbal test was changed to become the “critical reading” test and writing was added as a third test.

The scores on each part of the SAT were originally scaled to have a mean of 500 and a standard deviation of 100. The maximum and minimum scores on each part are 800 and 200, respectively, which are 3 standard deviations above and below the mean. (Scores more than 3 standard deviations above or below the mean are assigned the maximum 800 or minimum 200.)

The mean does not stay at 500 from year to year, however. Each year’s test shares some common questions with prior years’ tests, and the College Board (which designs the test) uses these common questions to compare each year’s test results to those from prior years. The intent is for a score of 500 to represent the same level of achievement at all times, so scores can be compared from one year to the next. For example, if the mean score in a particular year is higher than 500, it implies students performed better on average than did students at the time that the average was set to 500.

Trends in SAT scores are widely used to assess the general state of American education. Figure 26 shows the average



scores on the verbal/critical reading (meaning the verbal test prior to 2006 and the critical reading test since) and math parts of the SAT between 1972 and 2010; results for the writing test are not shown, because there are not yet enough data to look for trends over time.

The trends show that if the scores from year to year are truly comparable, then students taking the SAT in recent years have significantly poorer verbal skills than students of a few decades ago, though somewhat better mathematical skills. But before we accept this conclusion, we must answer two important questions:

1. Are trends among the *sample* of high school students who take the SAT representative of trends for the *population* of all high school students?
2. Aside from the small number of common questions that link one year to the next, the test changes every year. Can test scores from one year legitimately be compared to scores in other years?

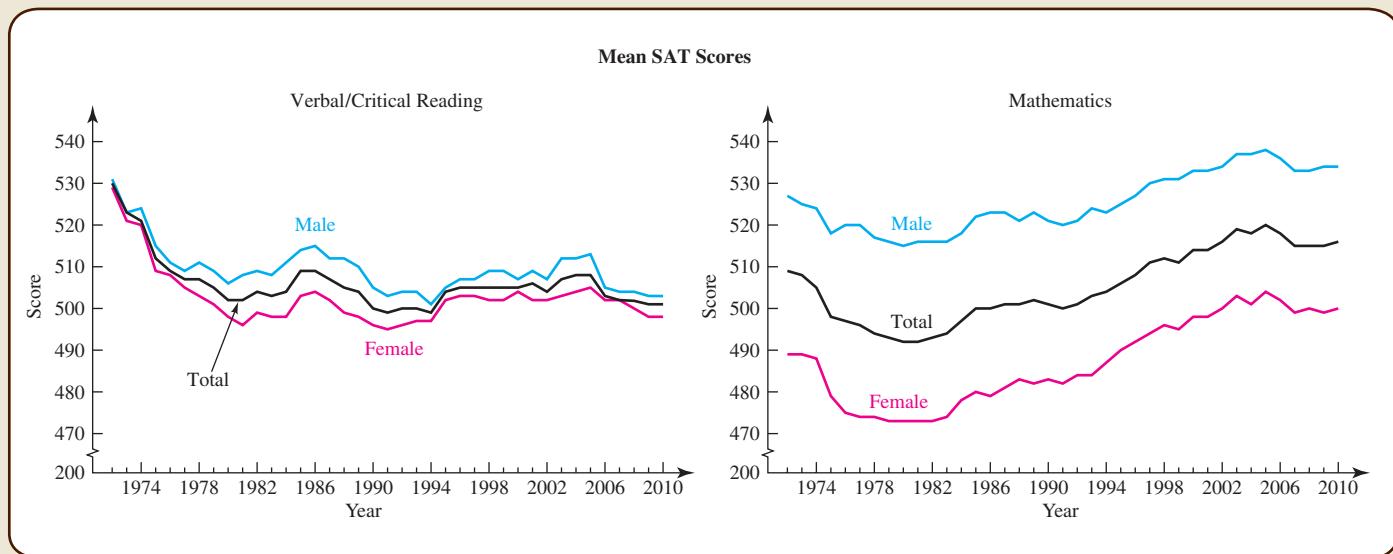


Figure 26 Verbal/critical reading and mathematics SAT scores, 1972–2010. Scores for years before 1996 are shown at their “recentered” values, rather than their original values. *Source:* The College Board.

Unfortunately, neither question can be answered with a clear yes. For example, only about a third of high school graduates took the SAT in the 1970s, while more than 50% of high school graduates take it today. If these samples represent the top tier of high school students, the decline in verbal scores might simply reflect the fact that a greater range of student abilities is represented in later samples.

The second question is even more difficult. Even if some test questions stay the same from one year to the next, different material may be emphasized in high schools, thereby changing the likelihood that students will answer the same questions correctly from one year to the next.

Further complications arise when the test undergoes major changes. For example, in 1994 the use of calculators was allowed for the first time on the mathematics section, and additional time was allotted for the entire test (the verbal test also underwent significant changes that year). Is it a coincidence that mathematics scores began an upward trend starting that year, or could it be that the test became “easier” as a result of the changes? Similarly, in 2006 the combined critical reading and mathematics scores suffered their largest decline in more than 30 years. Critics attributed the decline to the fact that the SAT became longer and harder as a result of adding the writing test, while the College Board attributed the decline to a decrease in the number of repeat test-takers. (Repeat test-takers tend to raise the average score, because individual students make significant gains in their scores when they take the test a second or third time.) Clearly, changes in the structure of the test make it difficult to determine whether changes in average scores reflect real changes in education.

Perhaps even more significantly, the calibration of the SAT test underwent a major change in 1996. By that time, scores had declined significantly from the originally planned mean of 500: The mean verbal score had fallen to about 420, while the mean mathematics score had fallen to about 470. Because the range of possible scores goes only from 200 to 800, these declines meant that the minimum and maximum scores no longer represented the same number of standard deviations from the mean, which created difficulties with statistical analysis. The College Board therefore decided to “recenter” all scores to a mean of 500 in 1996. The recentering affected different percentiles differently, but it effectively added about 80 points to the mean of the verbal scores and 30 points to the

mean of the mathematics scores. The scores shown in Figure 26 are the “recentered” scores. For example, Figure 26 shows a mean verbal score of almost exactly 500 in 1994, but if you look back at news reports from that time, you’ll see that the actual mean verbal score was about 420 in 1994.

The College Board argues that its statistical analysis has been done with great care, and that despite recentering and changes to the test, trends in the SAT reflect real changes in education. Critics generally agree that the statistical analysis has been done well, but argue that other factors, such as the change in the test-taking population and changes in our education system, make the comparisons invalid. For students, however, the key fact is that many colleges continue to use SAT scores as a part of their decision on whether to admit applicants. As long as that remains the case, the SAT will remain one of the nation’s most important tests—and the debate over its merits will surely continue.

QUESTIONS FOR DISCUSSION

- 1.** Do you think that comparisons of SAT scores over two years are meaningful? Over 10 years? Do you agree that the long-term trends indicate that students today have poorer verbal skills but better mathematical skills than those of a few decades ago? Defend your opinions.
 - 2.** The downward trend in verbal skills was a major reason why the College Board added the writing test in 2006. The hope was that adding the new test would cause students to focus more on writing in high school and therefore improve both in writing and in overall verbal abilities. What do you think of this rationale? Do you think it will be successful?
 - 3.** Notice that scores for males have been consistently higher than scores for females. Why do you think this is the case? Do you think that changes in our education system could eliminate this gap? Defend your opinions.
 - 4.** Discuss personal experiences with the SAT among your classmates. Based on these personal experiences, what do you think the SAT is measuring? Do you think the test is a reasonable way to predict students’ performance in college? Why or why not?
- •



FOCUS ON PSYCHOLOGY

Are We Smarter than Our Parents?

Most kids tend to think that they're smarter than their parents, but is it possible that they're right? If you believe the results of IQ tests, not only are we smarter than our parents, on average, but our parents are smarter than our grandparents. In fact, almost all of us would have ranked as geniuses if we'd lived a hundred years ago. Of course, before any of us start touting our Einstein-like abilities, it would be good to investigate what lies behind this startling claim.

The idea of an IQ, which stands for *intelligence quotient*, was invented by French psychologist Alfred Binet (1857–1911). Binet created a test that he hoped would identify children in need of special help in school.* He gave his test to many children and then calculated each child's IQ by dividing the child's "mental age" by his or her physical age (and multiplying by 100). For example, a 5-year-old child who scored as well as an average 6-year-old was said to have a mental age of 6, and therefore an IQ of $(6 \div 5) \times 100$, or 120. Note that, by this definition, IQ tests make sense only for children. However, later researchers, especially psychologists for the U.S. Army, extended the idea of IQ so that it could be applied to adults as well.

Today, IQ is defined by a normal distribution with a mean of 100 and a standard deviation of 16. Traditionally, psychologists have classified people with an IQ below 70 (about 2 standard deviations below the mean of 100) as "intellectually deficient," and people who score above 130 (about 2 standard deviations above the mean) as "intellectually superior."

You're probably aware of the controversy that surrounds IQ tests, which boils down to two key issues:

- Do IQ tests measure intelligence or something else?
- If they do measure intelligence, is it something that is innate and determined by heredity or something that can be molded by environment and education?

A full discussion of these issues is too involved to cover here, but a surprising trend in IQ scores sheds light on these issues. As we'll see shortly, the trend is quite pronounced, but it was long hidden because of the way IQ tests are scored. There are several different, competing versions of IQ tests, and most of them are regularly changed and updated. But in all cases, the scores are adjusted to fit a normal distribution with a mean of 100 and standard deviation of 16. In other words, the scoring of



an IQ test is essentially done in the same way that an instructor might grade an exam "on a curve." Because of this adjustment, the mean on IQ tests is *always* 100, which makes it impossible for measured IQ scores to rise and fall with time.

Nevertheless, a few IQ tests have not been changed and updated substantially over time, and even tests that have changed considerably often still repeat some old questions. In the early 1980s, a political science professor named Dr. James Flynn began to look at the raw, unadjusted scores on unchanged tests and questions. The results were astounding.

Dr. Flynn found that raw scores have been steadily rising, although the precise amount of the rise varies somewhat with the type of IQ test. The highest rates of increase are found on tests that purport to measure abstract reasoning abilities (such as the "Raven's" tests). For these tests, Dr. Flynn found that the unadjusted IQ scores of people in industrialized countries have been rising at a rate of about 6 points per decade. In other words, a person who scored 100 on a test given in 2010 would have scored about 106 on a test in 2000, 112 on a test in 1990, and so on. Over a hundred years, this would imply a rise of some 60 points, suggesting that someone who scores an "intellectually deficient" IQ of 70 today would have rated an "intellectually superior" IQ of 130 a century ago.

This long-term trend toward rising scores on IQ tests is now called the *Flynn effect*. It is present for all types of IQ tests, though not always to the same degree as with abstract reasoning tests. For example, Figure 27 shows how results changed on one of the most widely used IQ tests (the Stanford-Binet test) between 1932 and 1997. Note that, in terms of unadjusted scores, the mean rose by 20 points during that time period. In other words, if people who scored an IQ of 100 on a 1997 test were instead scored on a 1932 test, they would rate an IQ of 120. As the figure shows, about one-fourth of the 1997 test-takers would have rated "intellectually superior" on the 1932 test. There is some evidence that the rise in scores may have begun to slow or halt in more recent years, though the data are still subject to debate.

*Binet himself assumed that intelligence could be molded and warned against taking his tests as a measure of any innate or inherited abilities. However, many later psychologists concluded that IQ tests could measure innate intelligence, which led to their being used for separating school children, military recruits, and many other groups of people according to supposed intellectual ability.

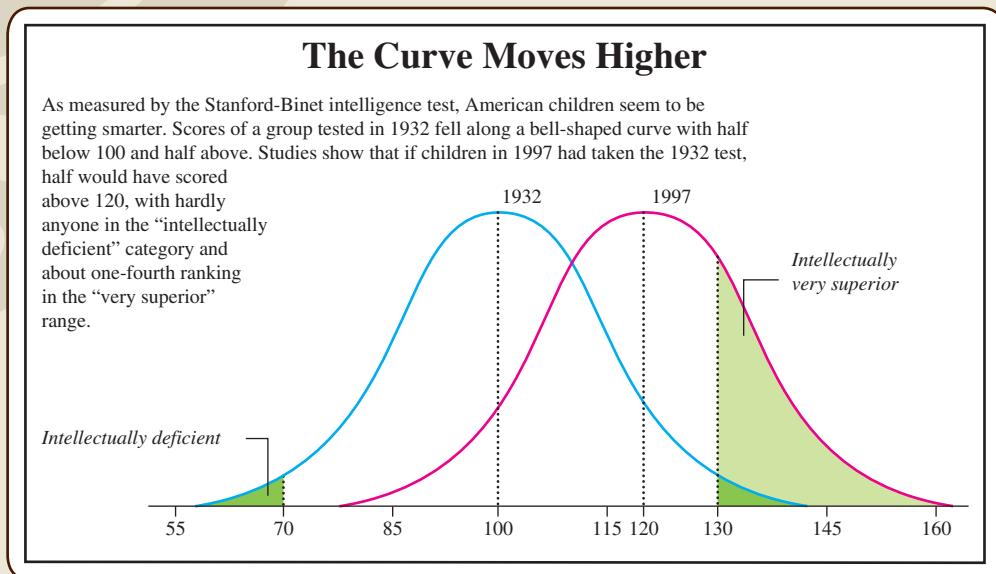


Figure 27 Source: Adapted from the *New York Times*, based on data from Neisser, Ulric (ed.). *The Rising Curve: Long-Term Gains in IQ and Related Measures*. American Psychological Association, 1998.

Many other scientists have investigated the Flynn effect, and there is general agreement that the long-term trend is real. The implication is clear: Whatever IQ tests measure, people today really *do* have more of it than people just a few decades ago. If IQ tests measure intelligence, then it means we really are smarter than our parents (on average), who in turn are smarter than our grandparents (on average).

Of course, if IQ tests don't measure “intelligence” but only measure some type of skill, then the rise in scores may indicate only that today's children have more practice at that skill than past children. The fact that the greatest rise is seen on tests of abstract thinking lends some support to this idea. These tests often involve such problems as solving puzzles and looking for patterns among sets of shapes, and these types of problems are now much more common in games than they were in the past.

While the Flynn effect does not answer the question of whether IQ tests measure intelligence, it may tell us one important thing: If IQs really have been rising as the Flynn effect suggests, then IQ cannot be an entirely inherited trait, because inherited traits cannot change that much in just a few decades. That is, if IQ tests are measuring intelligence, then intelligence can be molded by environmental as well as hereditary factors.

Dr. Flynn's discovery has already changed the way psychologists look at IQ tests, and it is sure to remain an active topic of research. Moreover, given the many uses to which modern society has put IQ tests, the Flynn effect is likely to have profound social and political consequences as well. So

back to our starting question: Are we smarter than our parents? We really can't say, but we can certainly hope so, because it will take a lot of brainpower to solve the problems of the future.

QUESTIONS FOR DISCUSSION

1. Which explanation do you favor for the Flynn effect: that people are getting smarter or that people are getting more practice at the skills measured on IQ tests? Defend your opinion.
2. The rise in performance on IQ tests contrasts sharply with a steady decline in performance over the past few decades on many tests that measure factual knowledge, such as the SAT. Think of several possible ways to explain these contrasting results, and form an opinion as to the most likely explanation.
3. Results on IQ tests tend to differ among different ethnic groups. Some people have used this fact to argue that some ethnic groups tend to be intellectually superior to others. Can such an argument still be supported in light of the Flynn effect? Defend your opinion.
4. Discuss some of the common uses of IQ tests. Do you think that IQ tests *should* be used for these purposes? Does the Flynn effect alter your thoughts about the uses of IQ tests? Explain.



ANSWERS

Section 1

1. The word *normal* has a special meaning in statistics. It refers to a specific category of distributions, all of them being bell-shaped.
 3. No. The ten different possible digits are all equally likely, so the graph of the distribution will tend to be flat, not bell-shaped.
 5. Makes sense.
 7. Does not make sense.
 9. Distribution b is not normal. Distribution c has the larger standard deviation.
 11. Normal. It is common for a manufactured product, such as quarters, to have a distribution that is normal. The weights typically vary above and below the mean weight by about the same amounts, so the distribution has one peak and is symmetric.
 13. Not normal. The outcomes of the numbers between 1 and 49 are all equally likely, so the distribution tends to be uniform, not normal. A graph of the distribution will tend to be very flat, not bell-shaped.
 15. Normal. Such physical measurements generally tend to be normally distributed. A small number of females will have extremely low counts, a small number will have extremely high counts, and the distribution will tend to peak around the value of the mean.
 17. Not normal. The waiting times will tend to be uniformly distributed.
 19. Not normal. Movie lengths have a minimum length (zero) and no maximum length.
 21. Nearly normal. The deviations from the mean are evenly distributed around the mean.
- 23.** a. 1 b. 0.20 c. 0.80 d. 0.35 e. 0.45
- 25.** a. 115 b. 15% c. 45% d. 15%

Section 2

1. 0
3. No. The rule applies to normal distributions, but the outcomes from a die roll have a uniform distribution, not a normal distribution.
5. Does not make sense. 7. Does not make sense.
9. a. 50% b. 84% c. 2.5% d. 84% e. 81.5%
11. a. 68% b. 95% c. 99.7% d. 47.5%
13. 50% 15. 2.28%
17. 97.72% 19. 93.32%
21. 68.26% 23. 84.00%
25. 50.00% 27. 84.13%
29. 99.38% 31. 0.13%
33. 68.26% 35. 89.25%

- 37.** In all cases, 5% of coins are rejected. Cents: 2.44 gm to 2.56 gm; Nickels: 4.88 gm to 5.12 gm; Dimes: 2.208 gm to 2.328 gm; Quarters: 5.530 gm to 5.810 gm; Half dollars: 11.060 gm to 11.620 gm.
- 39.** a. 6.68% b. 48.01% c. 36.54%
- 41.** a. 0.47% b. Approximately 4% c. 30.055 to 30.745 inches d. The best estimate would be the mean of the readings, or 30.4 inches.
- 43.** Approximately 96.33%

Section 3

1. Because the digits are equally likely, they have a uniform distribution. Because the sample means are based on samples of size 3 drawn from a population that does not have a normal distribution, we should not treat the sample means as having a normal distribution.
3. Yes. If the original population is normally distributed, the sample means will be normally distributed for any sample size, not just large sample sizes.
5. a. The mean is 100 and the standard deviation is 2.
b. The mean is 100 and the standard deviation is 1.6.
c. With larger sample sizes (as in part b), the means tend to be closer together, so they have less variation, which results in a smaller standard deviation.
7. a. The mean is 6.5 and the standard deviation is 0.384.
b. The mean is 6.5 and the standard deviation is 0.345.
c. With larger sample sizes (as in part b), the means tend to be closer together, so they have less variation, which results in a smaller standard deviation.
9. 69.2%; 99.9% 11. 15.9%; 95.4%
13. a. 0.35% b. No, it appears that the mean is greater than 12.00 oz, but consumers are not being cheated because the cans are being overfilled, not under filled.
15. a. 57.93% chance b. 97.72% chance c. Although the mean head breadth of 100 men is very likely to be less than 6.2 in., there could be many individual men that could not use the helmets because they have head breadths greater than 6.2 in. Based on the result from part a, these helmets would not fit about 42% of men.
17. a. 52.91% b. Approximately a 73% chance. c. Part a, because the seats will be occupied by individual women, not groups of women.
19. a. 16 (5.74% of 280) b. The standard *z* scores are beyond the range of those included in Table 5.1, so we conclude that it is very likely (more than a 99.96% chance) that the mean falls between the limits of 5.550 g and 5.790 g. c. Part a, because the individual rejected quarters could result in lost sales and lower profits.
21. a. 27.4% b. 0.2% c. Part a more relevant, because the individual desks will be occupied by individual girls, not groups of girls.

Chapter Review Exercises

- 1. a.** Because the 38 outcomes are all equally likely, the distribution is uniform, not normal. **b.** Weights of members in a homogeneous population, such as twelve-year old girls, typically have a normal distribution. **c.** Approximately normal. Such test scores tend to be normally distributed.
- 3. a.** 90 percentile **b.** 1.29 **c.** No, the data value lies less than 2 standard deviations from the mean. **d.** 0.0065
e. The temperature is unusual; it lies more than 2 standard deviations above the mean. **f.** 99.22 **g.** 97.18 **h.** Fewer

than 0.01%; yes, this temperature seems appropriate for claiming that a patient has a fever. **i.** The sample mean is 6.6 standard deviations below the mean; the chance of selecting such a sample is extremely small. The assumed mean (98.60) may be incorrect.

Chapter Quiz

- 1.** b, c, and d are correct. **3.** 95.44%
5. 4.9 **7.** 84% **9.** 81.85%

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Probability in Statistics

Most statistical studies seek to learn something about a *population* from a much smaller *sample*. Therefore, a key question in any statistical study is whether it is valid to generalize from a sample to the population. To answer this question, we must understand the likelihood, or probability, that what we've learned about the sample also applies to the population. In this chapter, we will focus on a few basic ideas of probability that are commonly used in statistics. As you will see, these ideas of probability also have many applications in their own right.

Probability is the very
guide of life.

—Cicero (106–43 BC)

LEARNING GOALS

- 1 **The Role of Probability in Statistics: Statistical Significance**
Understand the concept of statistical significance and the essential role that probability plays in defining it.
- 2 **Basics of Probability**
Find probabilities and construct probability distributions using theoretical and relative frequency methods.
- 3 **The Law of Large Numbers**
Understand the law of large numbers and related applications including expected values and the gambler's fallacy.
- 4 **Ideas of Risk and Life Expectancy**
Compute and interpret various measures of risk as they apply to travel, disease, and life expectancy.
- 5 **Combining Probabilities (Supplementary Section)**
Distinguish between independent and dependent events and between overlapping and non-overlapping events, and be able to calculate *and* and *either/or* probabilities.

FOCUS TOPICS

Focus on Social Science: Do Lotteries Harm the Poor?

Focus on Law: Is DNA Fingerprinting Reliable?

1

THE ROLE OF PROBABILITY IN STATISTICS: STATISTICAL SIGNIFICANCE

To see why probability is so important in statistics, imagine that you are trying to test whether a coin is fair—that is, whether it is equally likely to land on heads or tails. If you toss the coin 100 times and get 52 heads and 48 tails, should you conclude that the coin is unfair? No. While we should expect to see *roughly* 50 heads and 50 tails in every 100 tosses of a fair coin, we also expect some variation from one sample of 100 tosses to another. We’re not surprised to observe a small deviation from a perfect 50-50 split between heads and tails, because we expect small deviations to occur *by chance*.

In contrast, suppose you toss a coin 100 times and the results are 20 heads and 80 tails. This is such a substantial deviation from a 50-50 split that you’d probably conclude that the coin is unfair. In other words, while it’s *possible* that you observed a rare set of 100 tosses with a fair coin, it’s more likely that the coin is unfair. When the difference between what is observed and what is expected seems unlikely to be explained by chance alone, we say the difference is **statistically significant**.

Definition

A set of measurements or observations in a statistical study is said to be **statistically significant** if it is unlikely to have occurred by chance.

EXAMPLE 1 Likely Stories?

- a. A detective in Detroit finds that 25 of the 62 guns used in crimes during the past week were sold by the same gun shop. This finding is statistically significant. Because there are many gun shops in the Detroit area, having 25 out of 62 guns come from the same shop seems unlikely to have occurred by chance.
- b. In terms of the global average temperature, the 10 years from 2001–2010 were 10 of the 11 hottest years on record to that time, using a record going back to 1880. Having 10 of the 11 hottest years on record occur in a single decade is statistically significant. Indeed, having such a streak of hot years is very unlikely to have occurred by chance alone, and therefore provides significant evidence of a warming world.
- c. The team with the worst win-loss record in basketball wins one game against the defending league champions. This one win is *not* statistically significant because although we expect a team with a poor win-loss record to lose most of its games, we also expect it to win occasionally, even against the defending league champions.

• • •

From Sample to Population

Let’s look at the idea of statistical significance in an opinion poll. Suppose that in a poll of 1,000 randomly selected people, 51% support the President. A week later, in another poll with a different randomly selected sample of 1,000 people, only 49% support the President. Should you conclude that the opinions of Americans changed during the one week between the polls?

You can probably guess that the answer is *no*. The poll results are *sample statistics*: 51% of the people in the first sample support the President. We can use this result to estimate the *population parameter*, which is the percentage of *all* Americans who support the President. At best, if the first poll were conducted well, it would say that the percentage of Americans who support the President is *close* to 51%. Similarly, the 49% result in the second poll means that the percentage of Americans supporting the President is *close* to 49%. Because the two sample statistics differed only slightly (51% versus 49%), it’s quite possible that the

TECHNICAL NOTE

The difference between 49% and 51% is not statistically significant for typical polls, but it can be for a poll involving a very large sample size. In general, any difference can be significant if the sample size is large enough.

real percentage of Americans supporting the President did not change at all. Instead, the two polls reflect expected and reasonable differences between the two samples.

In contrast, suppose the first poll found that 75% of the sample supported the President, and the second poll found that only 30% supported the President. Assuming the polls were carefully conducted, it's highly unlikely that two groups of 1,000 randomly chosen people could differ so much by chance alone. In this case, we would look for another explanation. Perhaps Americans' opinions about the President really did change in the week between the polls.

In terms of statistical significance, the change from 51% to 49% in the first set of polls is *not* statistically significant, because we can reasonably attribute this change to chance variations between the two samples. However, in the second set of polls, the change from 75% to 30% is statistically significant, because it is unlikely to have occurred by chance.

EXAMPLE 2 Statistical Significance in Experiments

A researcher conducts a double-blind experiment that tests whether a new herbal formula is effective in preventing colds. During a 3-month period, the 100 randomly selected people in a treatment group take the herbal formula while the 100 randomly selected people in a control group take a placebo. The results show that 30 people in the treatment group get colds, compared with 32 people in the control group. Can we conclude that the herbal formula is effective in preventing colds?

SOLUTION Whether a person gets a cold during any 3-month period depends on many unpredictable factors. Therefore, we should not expect the number of people with colds in any two groups of 100 people to be exactly the same. In this case, the difference between 30 people getting colds in the treatment group and 32 people getting colds in the control group is small enough to be explainable by chance. That is, the difference is not statistically significant, and we should not conclude that the treatment is effective.



Quantifying Statistical Significance

In Example 2, we said that the difference between 30 colds in the treatment group and 32 colds in the control group was not statistically significant. This conclusion was fairly obvious because the difference was so small. But suppose that 24 people in the treatment group had colds, compared with the 32 in the control group. Would the difference between 24 and 32 be large enough to be considered statistically significant? The definition of statistical significance that we've been using so far is too vague to answer this question. We need a way to quantify the idea of statistical significance.

In general, we determine statistical significance by using probability to quantify the likelihood that a result may have occurred by chance. We ask a question like this one: *Is the probability that the observed difference occurred by chance less than or equal to 0.05 (or 1 in 20)?* If the answer is *yes* (the probability is less than or equal to 0.05), then we say that the difference is *statistically significant at the 0.05 level*. If the answer is *no*, the observed difference is reasonably likely to have occurred by chance, so we say that it is not statistically significant.

The choice of 0.05 is somewhat arbitrary, but it's a figure that statisticians frequently use. Nevertheless, other probabilities are sometimes used, such as 0.1 or 0.01. Statistical significance at the 0.01 level is stronger than significance at the 0.05 level, which is stronger than significance at the 0.1 level.

He that leaves nothing to chance will do few things ill, but will do very few things.

—George Savile Halifax

Quantifying Statistical Significance

- If the probability of an observed difference occurring by chance is 0.05 (or 1 in 20) or less, the difference is statistically significant at the 0.05 level.
- If the probability of an observed difference occurring by chance is 0.01 (or 1 in 100) or less, the difference is statistically significant at the 0.01 level.

You can probably see that caution is in order when working with statistical significance. We would expect roughly 1 in 20 trials to give results that are statistically significant at the 0.05 level even when the results actually occurred by chance. In other words, statistical significance at the 0.05 level—or at almost any level, for that matter—is *no guarantee* that an important effect or difference is present.

TIME OUT TO THINK

Suppose an experiment finds that people taking a new herbal remedy get fewer colds than people taking a placebo, and the results are statistically significant at the 0.01 level. Has the experiment *proven* that the herbal remedy works? Explain.

EXAMPLE 3 Polio Vaccine Significance

In the test of the Salk polio vaccine, 33 of the 200,000 children in the treatment group got paralytic polio, while 115 of the 200,000 in the control group got paralytic polio. Calculations show that the probability of this difference between the groups occurring by chance is less than 0.01. Describe the implications of this result.

SOLUTION The results of the polio vaccine test are statistically significant at the 0.01 level, meaning that there is a 0.01 chance (or less) that the difference between the control and treatment groups occurred by chance. Therefore, we can be fairly confident that the vaccine really was responsible for the fewer cases of polio in the treatment group. (In fact, the probability of the Salk results occurring by chance is *much* less than 0.01, so researchers were quite convinced that the vaccine worked; this probability is called a “*P*-value.”)

Section 1 Exercises

Statistical Literacy and Critical Thinking

- Statistical Significance.** In an experiment testing a method of gender selection intended to increase the likelihood that a baby is a girl, 1,000 couples give birth to 501 girls and 499 boys. A company representative argues that this is evidence that the method is effective, because the probability of getting 501 girls in 1,000 births by chance is only 0.025, which is less than 0.05. Do you agree with this argument? Why or why not?
- Statistical Significance.** Does the term *statistical significance* refer to results that are significant in the sense that they have great importance? Explain.
- Statistical Significance.** If a particular result is statistically significant at the 0.05 level, must it also be statistically significant at the 0.01 level? Why or why not?
- Statistical Significance.** If a particular result is statistically significant at the 0.01 level, must it also be statistically significant at the 0.05 level? Why or why not?

Does It Make Sense? For Exercises 5–8, decide whether the statement makes sense (or is clearly true) or does not make sense (or is clearly false). Explain clearly; not all of these have definitive answers, so your explanation is more important than your chosen answer.

- Unemployment.** The rate of unemployment has statistical significance because it has such a strong effect on the economy.
- Statistical Significance.** In an experiment testing a method of gender selection, 1,000 couples give birth to 550 girls and 450 boys. Because the probability of getting such extreme results by chance is only 0.0009, the results are statistically significant.
- Gender Selection.** In a test of a technique of gender selection, the 100 babies born consist of at least 80 girls. Because there is about 1 chance in a billion of getting at least 80 girls among 100 babies, the results are statistically significant.
- Clinical Trial.** In a clinical trial of a treatment for reducing back pain, the difference between the treatment group and the control group (with no treatment) was found to be statistically significant. This means that the treatment will definitely ease back pain for everyone.

Concepts and Applications

Subjective Significance. For each event in Exercises 9–16, state whether the difference between what occurred and what you would have expected by chance is statistically significant. Discuss any implications of the statistical significance.

9. **Statistics Class Survey.** In a survey, 50 students are randomly selected among all college students currently taking a statistics course, and it is found that they are all females.
10. **Voter Survey.** In a pre-election survey, 100 likely voters are randomly selected from adults in the United States, and it is found that 6% of them are Democrats.
11. **Rolls of a Die.** In 6 rolls of a six-sided die, the outcome of 6 never occurs.
12. **Lottery.** A common lottery game is to select four digits, each between 0 and 9 (inclusive). Winning requires that you get the same four digits that are drawn, and they must be in the same order. You select four digits for one ticket, and none of your selections matches the numbers drawn.
13. **Survey.** In conducting a survey of adults in the United States, a pollster claims that he randomly selected 20 subjects and all of them were women.
14. **Subway Riders.** A commuter enters a New York City subway car near Times Square and finds that it is occupied by 50 men, all of whom are bald.
15. **Jury Composition.** For a trial on a charge of failure to pay child support, the jury consists of exactly 6 men and 6 women.
16. **Clinical Trial.** In a clinical trial of a new drug intended to treat allergies, 5 of the 80 subjects in the treatment group experienced headaches, and 8 of the 160 subjects in the control group experienced headaches.
17. **Fuel Tests.** Thirty identical cars are selected for a fuel test. Half of the cars are filled with regular gasoline, and the other half are filled with a new experimental fuel. The cars in the first group average 29.3 miles per gallon, while the cars in the second group average 35.5 miles per gallon. Discuss whether this difference seems statistically significant.
18. **Carpal Tunnel Syndrome Treatments.** An experiment was conducted to determine whether there is a difference between the success rates from treating carpal tunnel syndrome with surgery and with splinting. The success rate for 73 patients treated with surgery was 92%, and the success rate for 83 patients treated with splints was 72% (based on data from “Splinting vs. Surgery in the Treatment of Carpal Tunnel Syndrome” by Gerritsen et al., *Journal of the American Medical Association*, Vol. 288, No. 10). Discuss whether this difference appears to be statistically significant.
19. **Gender Selection.** The Genetics and IVF Institute conducted a clinical trial of its method for gender selection. The latest actual results showed that among 945 babies born to couples using the XSORT method of gender selection, 879 were girls.
20. **Bednets and Malaria.** In a randomized controlled trial in Kenya, insecticide-treated bednets were tested as a way to reduce malaria. Among 343 infants who used the bednets, 15 developed malaria. Among 294 infants not using bednets, 27 developed malaria (based on data from “Sustainability of Reductions in Malaria Transmission and Infant Mortality in Western Kenya with Use of Insecticide-Treated Bednets” by Lindblade et al., *Journal of the American Medical Association*, Vol. 291, No. 21). Assuming that the bednets have no effect, there is a probability of 0.015 of getting these results by chance. Do the results appear to have statistical significance? Do the bednets appear to be effective?
21. **Human Body Temperature.** In a study by researchers at the University of Maryland, the body temperatures of 106 individuals were measured; the mean for the sample was 98.20°F. It is commonly believed that the mean body temperature is 98.6°F. The difference between the sample mean and the accepted value is significant at the 0.05 level.
 - a. Discuss the meaning of the significance level in this case.
 - b. If we assume that the mean body temperature is actually 98.6°F, the probability of getting a sample with a mean of 98.20°F or less is 0.000000001. Interpret this probability value.
22. **Seat Belts and Children.** In a study of children injured in automobile crashes (*American Journal of Public Health*, Vol. 82, No. 3), those wearing seat belts had a mean stay of 0.83 day in an intensive care unit. Those not wearing seat belts had a mean stay of 1.39 days. The difference in means between the two groups is significant at the 0.0001 level. Interpret this result.
23. **SAT Preparation.** A study of 75 students who took an SAT preparation course (*American Education Research Journal*, Vol. 19, No. 3) concluded that the mean improvement on the SAT was 0.6 point. If we assume that the preparation course has no effect, the probability of getting a mean improvement of 0.6 point by chance is 0.08. Discuss whether this preparation course results in statistically significant improvement.
24. **Weight by Age.** A National Health Survey determined that the mean weight of a sample of 804 men ages 25 to 34 years was 176 pounds, while the mean weight of a sample of 1,657 men ages 65 to 74 years was 164 pounds. The difference is significant at the 0.01 level. Interpret this result.



25. Significance in Vital Statistics. Visit a Web site that has vital statistics (for example, the U.S. Census Bureau or the National Center for Health Statistics). Choose a question such as one of the following:

- Are there significant differences in numbers of births among months?
- Are there significant differences in numbers of natural deaths among days of the week?
- Are there significant differences in infant mortality rates among selected states?
- Are there significant differences in incidences of a particular disease among selected states?
- Are there significant differences among the marriage rates in various states?

Collect the relevant data and determine subjectively whether you think the observed differences are significant. Explain why, and provide possible explanations for the significance or lack of significance.

26. Lengths of Rivers. Using an almanac or the Internet, find the lengths of the principal rivers of the world. Construct a list of the leading digits only. Does any particular digit occur more often than the others? Does that digit occur significantly more often? Explain.

IN THE NEWS

27. Statistical Significance. Find a recent newspaper article on a statistical study in which the idea of statistical significance is used. Write a one-page summary of the study and the result that is considered to be statistically significant. Also include a brief discussion of whether you believe the result, given its statistical significance.

28. Significant Experiment? Find a recent news story about a statistical study that used an experiment to determine whether some new treatment was effective. Based on the available information, briefly discuss what you can conclude about the statistical significance of the results. Given this significance (or lack thereof), do you think the new treatment is useful? Explain.

29. Personal Statistical Significance. Describe an incident in your own life that did not meet your expectation, defied the odds, or seemed unlikely to have occurred by chance. Would you call this incident statistically significant? To what did you attribute the event?

2

BASICS OF PROBABILITY

In this world, nothing is certain but death and taxes.

—Benjamin Franklin

Ideas of probability are fundamental to statistics, in part through the concept of statistical significance. We need to explore a few essential ideas of probability and their applications in everyday life.

Let's begin by considering a toss of two coins. Figure 1 shows that there are four different ways the coins could fall. We say that each of these four ways is a different **outcome** of the coin toss. The outcomes are the most basic possible results of the coin toss. But suppose we are interested only in the number of heads. Because the two middle outcomes in Figure 1 each have 1 head, we say that these two outcomes represent the same **event**. That is, an event describes one or more possible outcomes that all have the same property of interest—in this case, the same number of heads. Figure 1 shows that there are four possible outcomes for the two-coin toss, but only three possible events: 0 heads, 1 head, and 2 heads.

Definition

Outcomes are the most basic possible results of observations or experiments.

An **event** is a collection of one or more outcomes that share a property of interest.

Outcome								
Event	0 heads		1 head		1 head		2 heads	

Figure 1 The four possible outcomes for a toss of two coins. The two middle outcomes both represent the same event of 1 head.

Mathematically, we express probabilities as numbers between 0 and 1. For example, the probability of a coin landing on heads is one-half, or 0.5. If an event is impossible, such as the event of being in two places at the same time, we assign it a probability of 0. At the other extreme, an event that is certain to occur, such as the event of the Sun being above the horizon in the daytime, is given a probability of 1. Figure 2 shows the scale of probability values, along with common expressions of likelihood.

It's helpful to have some special notation for probability. We write $P(\text{event})$ to mean the probability of an event, and often denote events by letters or symbols. For example, if we use H to represent the event of a head on a coin toss, then we write the probability of heads as $P(H) = 0.5$.

Expressing Probability

The probability of an event, expressed as $P(\text{event})$, is always between 0 and 1 (inclusive). A probability of 0 means that the event is impossible and a probability of 1 means that the event is certain.

Theoretical Probabilities

There are three basic techniques for finding probabilities, called the *theoretical method*, the *relative frequency method*, and the *subjective method*. We'll begin with the theoretical method.

When we say that the probability of heads on a coin toss is $1/2$, we are assuming that the coin is fair and is equally likely to land on heads or tails. In essence, the probability is based on a theory of how the coin behaves, so we say that the probability of $1/2$ comes from the theoretical method. As another example, consider rolling a single die. Because there are six equally likely outcomes (Figure 3), the theoretical probability for each outcome is $1/6$.

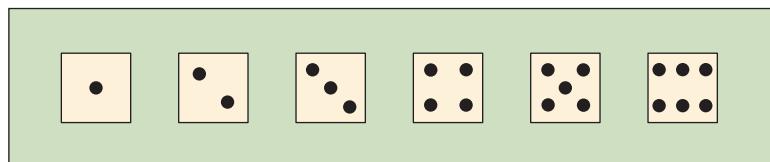


Figure 3 The six possible outcomes for a roll of one six-sided die.

As long as all outcomes are equally likely, we can use the following procedure to calculate theoretical probabilities.

Theoretical Method for Equally Likely Outcomes

Step 1. Count the total number of possible outcomes.

Step 2. Among all the possible outcomes, count the number of ways the event of interest, A , can occur.

Step 3. Determine the probability, $P(A)$, from

$$P(A) = \frac{\text{number of ways } A \text{ can occur}}{\text{total number of outcomes}}$$

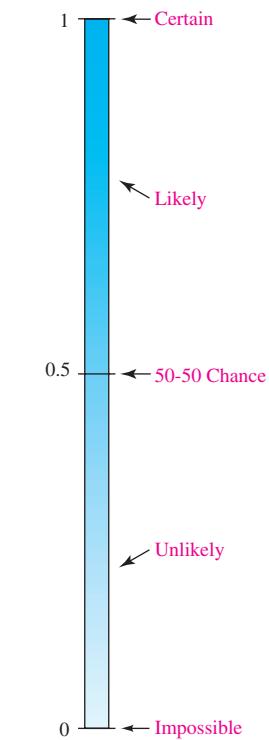


Figure 2 The scale shows various degrees of certainty as expressed by probabilities.

BY THE WAY

Mathematician Persi Diaconis has shown that seven shuffles of a new deck of cards are needed to give it an ordering in which all card arrangements are equally likely.

EXAMPLE 1 Guessing Birthdays

You select a person at random from a large group at a conference. What is the probability that the person selected has a birthday in July? Assume 365 days in a year.

SOLUTION If we assume that all birthdays are equally likely, we can use the three-step theoretical method.

- Step 1. Each possible birthday represents an outcome, so there are 365 possible outcomes.
- Step 2. July has 31 days, so 31 of the 365 possible outcomes represent the event of a July birthday.

BY THE WAY

Birthdays are not randomly distributed throughout the year. After accounting for the different lengths of months, January has the lowest birth rate and June and July have the highest birth rates.



Step 3. The probability that a randomly selected person has a birthday in July is

$$P(\text{July birthday}) = \frac{31}{365} \approx 0.0849$$

which is slightly more than 1 in 12. • • •

Counting Outcomes

Suppose we toss two coins and want to count the total number of outcomes. The toss of the first coin has two possible outcomes: heads (H) or tails (T). The toss of the second coin also has two possible outcomes. The two outcomes for the first coin can each occur with either of the two outcomes for the second coin, making a total of $2 \times 2 = 4$ possible outcomes for the two coins together. The tree diagram of Figure 4a shows an easy way to identify the four outcomes: HH, HT, TH, and TT.

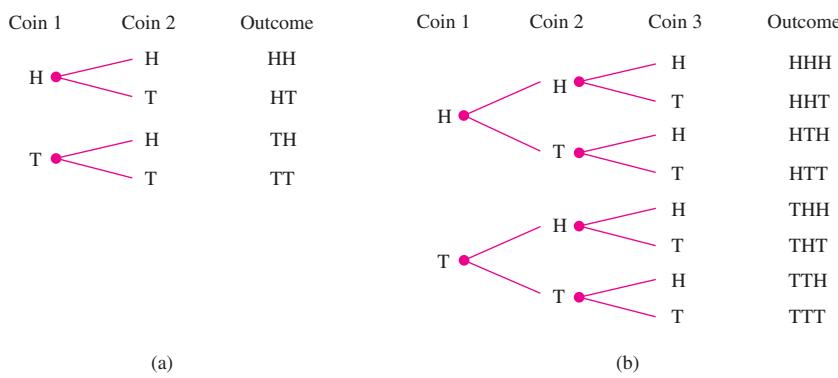


Figure 4 Tree diagrams showing the outcomes of tossing (a) two and (b) three coins.

TIME OUT TO THINK

Are the outcomes for tossing one coin twice in a row the same as those for tossing two coins at the same time? Explain.

We can now extend this thinking. If we toss three coins, there are $2 \times 2 \times 2 = 8$ possible outcomes, all shown in Figure 4b. This idea is the basis for the following counting rule.

Counting Outcomes

Suppose process A has a possible outcomes and process B has b possible outcomes. Assuming the outcomes of the processes do not affect each other, the number of different outcomes for the two processes combined is $a \times b$. This idea extends to any number of processes. For example, if a third process C has c possible outcomes, the number of possible outcomes for the three processes combined is $a \times b \times c$.

EXAMPLE 2 Dice Counting

- How many outcomes are there if you roll two fair dice?
- What is the probability of rolling two 1's (snake eyes) when two fair dice are rolled?

SOLUTION

- Rolling a single die has six equally likely outcomes (see Figure 3). Therefore, when two fair dice are rolled, there are $6 \times 6 = 36$ different outcomes.

- b. Of the 36 possible outcomes for rolling two fair dice, only one is the event of interest (two 1's). Therefore, the probability of rolling two 1's is

$$P(\text{two 1's}) = \frac{\text{number of ways two 1's can occur}}{\text{total number of outcomes}} = \frac{1}{36} = 0.0278$$
•••

EXAMPLE 3 Counting Children

What is the probability that a randomly selected family with three children has two girls and one boy? Assume that births of boys and girls are equally likely.

SOLUTION We apply the three-step theoretical method.

- Step 1. There are two possible outcomes for each birth: boy (B) or girl (G). For a family with three children, the total number of possible outcomes (birth orders) is $2 \times 2 \times 2 = 8$: BBB, BBG, BGB, BGG, GBB, GBG, GGB, GGG.
- Step 2. Of these eight possible outcomes, three of them have two girls and one boy: BGG, GBG, and GGB.
- Step 3. Therefore, the probability that a family with three children has two girls and one boy is

$$P(\text{two girls}) = \frac{\text{number of outcomes with two girls}}{\text{total number of outcomes}} = \frac{3}{8} = 0.375$$
•••

TIME OUT TO THINK

How many different four-child families are possible if birth order is taken into account? What is the probability of a couple having a four-child family with four girls?

BY THE WAY

Births of boys and girls are *not* equally likely. Naturally, there are approximately 105 male births for every 100 female births. However, male death rates are higher than female death rates, so female adults outnumber male adults.



Relative Frequency Probabilities

A second way to determine probabilities is to *approximate* the probability of an event A by making many observations and counting the number of times event A occurs. This approach is called the **relative frequency method** (or *empirical* method). For example, if we observe that it rains an average of 100 days per year, we might say that the probability of rain on a randomly selected day is 100/365. We apply this method as follows.

Relative Frequency Method

- Step 1. Repeat or observe a process many times and count the number of times the event of interest, A , occurs.
- Step 2. Estimate $P(A)$ by

$$P(A) = \frac{\text{number of times } A \text{ occurred}}{\text{total number of observations}}$$

EXAMPLE 4 500-Year Flood

Geological records indicate that a river has crested above a particular high flood level four times in the past 2,000 years. What is the relative frequency probability that the river will crest above the high flood level next year?

SOLUTION Based on the data, the probability of the river cresting above this flood level in any single year is

$$\frac{\text{number of years with flood}}{\text{total number of years}} = \frac{4}{2,000} = \frac{1}{500}$$

Because a flood of this magnitude occurs on average once every 500 years, it is called a “500-year flood.” The probability of having a flood of this magnitude in any given year is 1/500, or 0.002.



Subjective Probabilities

The third method for determining probabilities is to estimate a **subjective probability** using experience or intuition. For example, you could make a subjective estimate of the probability that a friend will be married in the next year or of the probability that a good grade in statistics will help you get the job you want.

BY THE WAY

Another approach to finding probabilities, called the *Monte Carlo method*, uses computer simulations. This technique essentially finds relative frequency probabilities; in this case, observations are made by causing a computer to behave in a way that is essentially the same as the actual event.

Three Approaches to Finding Probability

A **theoretical probability** is based on assuming that all outcomes are equally likely. It is determined by dividing the number of ways an event can occur by the total number of possible outcomes.

A **relative frequency probability** is based on observations or experiments. It is the relative frequency of the event of interest.

A **subjective probability** is an estimate based on experience or intuition.

EXAMPLE 5 Which Method?

Identify the method that resulted in the following statements.

- The chance that you'll get married in the next year is zero.
- Based on government data, the chance of dying in an automobile accident is about 1 in 8,000 (per year).
- The chance of rolling a 7 with a 12-sided die is 1/12.

SOLUTION

- This is a subjective probability because it is based on a feeling at the current moment.
- This is a relative frequency probability because it is based on observed data on past automobile accidents.
- This is a theoretical probability because it is based on assuming that a fair twelve-sided die is equally likely to land on any of its 12 sides.



BY THE WAY

Theoretical methods are also called *a priori* methods. The words *a priori* are Latin for “before the fact” or “before experience.”

Probability of an Event Not Occurring

Suppose we are interested in the probability that a particular event or outcome does *not* occur. For example, consider the probability of a wrong answer on a multiple-choice question with five possible answers. The probability of answering correctly with a random guess is 1/5, so the probability of *not* answering correctly is 4/5. Notice that the sum of the two probabilities must be 1, because the answer must be either right or wrong. We can generalize this idea.

Probability of an Event Not Occurring

Suppose the probability of an event A is $P(A)$. Then the probability that event A does *not* occur is $P(\text{not } A) = 1 - P(A)$. Note: The event *not A* is called the **complement** of the event A ; the “not” is often designated by a bar, so \bar{A} means *not A*.

EXAMPLE 6 Is Scanner Accuracy the Same for Specials?

In a study of checkout scanning systems, samples of purchases were used to compare the scanned prices to the posted prices. Table 1 summarizes results for a sample of 819 items. Based on these data, what is the probability that a regular-priced item has a scanning error? What is the probability that an advertised-special item has a scanning error?

TABLE 1 Scanner Accuracy		
	Regular-priced items	Advertised-special items
Undercharge	20	7
Overcharge	15	29
Correct price	384	364

Source: Ronald Goodstein, "UPC Scanner Pricing Systems: Are They Accurate?" *Journal of Marketing*, Vol. 58.

SOLUTION We can let R represent a regular-priced item being scanned correctly. Because 384 of the 419 regular-priced items are correctly scanned,

$$P(R) = \frac{384}{419} = 0.916$$

The event of a scanning error is the complement of the event of a correct scan, so the probability of a regular-priced item being subject to a scanning error (either undercharged or overcharged) is

$$P(\text{not } R) = 1 - 0.916 = 0.084$$

Now let A represent an advertised-special item being scanned correctly. Of the 400 advertised-special items in the sample, 364 are scanned correctly. Therefore,

$$P(A) = \frac{364}{400} = 0.910$$

The probability of an advertised-special item being scanned incorrectly is

$$P(\text{not } A) = 1 - 0.910 = 0.090$$

The error rates are nearly equal. However, Table 1 also shows that the errors differed in their effects on customers: Most of the errors made with advertised-special items were overcharges, while most of the errors with regular-priced items were undercharges.

BY THE WAY

A *lot* is a person's portion or allocation. The term came to be used for any object or marker that identified a person in a selection by chance. The casting of lots was used in the Trojan Wars to determine who would throw the first spear and in the Old Testament to divide conquered lands.

Probability Distributions

One of the most fundamental ideas in probability and statistics is that of a *probability distribution*. As the name suggests, a probability distribution is a distribution in which the variable of interest is associated with a probability.

Suppose you toss two coins simultaneously. As we found earlier (see Figure 4a), there are a total of four possible outcomes for the two coins: HH, HT, TH, and TT. Notice, however, that these four outcomes represent only three different events: one outcome represents 2 heads (HH), one outcome represents 2 tails (TT), and two outcomes (HT, TH) represent 1 head and 1 tail. Therefore, the probability of two heads is $P(\text{HH}) = 1/4 = 0.25$; the probability of two tails is $P(\text{TT}) = 1/4 = 0.25$; and the probability of one head and one tail is $P(\text{H and T}) = 2/4 = 0.50$. We can display this **probability distribution** as a table (Table 2) or a histogram (Figure 5). Note that the sum of all the probabilities must be 1 (because exactly one of the possible results must occur).

TABLE 2 Tossing Two Coins	
Event	Probability
2 heads, 0 tails	0.25
1 head, 1 tail	0.50
0 heads, 2 tails	0.25
Total	1

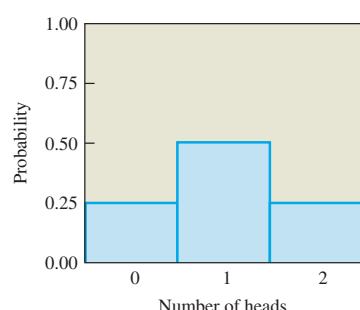


Figure 5 Histogram showing the probability distribution for the results of tossing two coins.

BY THE WAY

Another common way to express likelihood is to use odds. The odds *against* an event are the ratio of the probability that the event does not occur to the probability that it does occur. For example, the odds against rolling a 6 with a fair die are $(5/6)/(1/6)$, or 5 to 1. The odds used in gambling are called *payoff odds*; they express your net gain on a winning bet. For example, suppose that the payoff odds on a particular horse at a horse race are 3 to 1. This means that for each \$1 you bet on this horse, you will gain \$3 if the horse wins (and get your original \$1 back).

**Making a Probability Distribution**

A **probability distribution** represents the probabilities of all possible events. Do the following to make a display of a probability distribution:

- Step 1. List all possible *outcomes*. Use a table or figure if it is helpful.
- Step 2. Identify outcomes that represent the same *event*. Find the probability of each event.
- Step 3. Make a table in which one column lists each event and another column lists each probability. The sum of all the probabilities must be 1.

EXAMPLE 7 Tossing Three Coins

Make a probability distribution for the number of heads that occurs when three coins are tossed simultaneously.

SOLUTION We apply the three-step process.

- Step 1. The number of different outcomes when three coins are tossed is $2 \times 2 \times 2 = 8$ (see Figure 4b): HHH, HHT, HTH, HTT, THH, THT, TTH, and TTT.
- Step 2. The eight outcomes represent four possible events: 0 heads, 1 head, 2 heads, and 3 heads. Notice that only one outcome represents the event of 0 heads, so its probability is $1/8$; the same is true for the event of three heads (0 tails). The remaining events of 1 head (and 2 tails) or 2 heads (and 1 tail) each occur three times, so their probabilities are each $3/8$.
- Step 3. Table 3 shows the probability distribution, with the four events listed in the left column and their probabilities in the right column.

TABLE 3 Tossing Three Coins

Result	Probability
3 heads (0 tails)	$1/8$
2 heads (1 tail)	$3/8$
1 head (2 tails)	$3/8$
0 heads (3 tails)	$1/8$
Total	1

TIME OUT TO THINK

How many different *outcomes* are possible when you toss four coins? If you are interested in the number of heads, how many different *events* are possible?

EXAMPLE 8 Two Dice Distribution

Make a probability distribution for the sum of the dice when two dice are rolled.

SOLUTION There are six ways for each die to land (see Figure 3), so there are $6 \times 6 = 36$ possible outcomes for a roll of two dice. Table 4 shows all 36 outcomes by listing one die along the rows and the other along the columns, with each cell showing for the two dice.

TABLE 4**Outcomes and Sums for the Roll of Two Dice**

	1	2	3	4	5	6
1	$1 + 1 = 2$	$1 + 2 = 3$	$1 + 3 = 4$	$1 + 4 = 5$	$1 + 5 = 6$	$1 + 6 = 7$
2	$2 + 1 = 3$	$2 + 2 = 4$	$2 + 3 = 5$	$2 + 4 = 6$	$2 + 5 = 7$	$2 + 6 = 8$
3	$3 + 1 = 4$	$3 + 2 = 5$	$3 + 3 = 6$	$3 + 4 = 7$	$3 + 5 = 8$	$3 + 6 = 9$
4	$4 + 1 = 5$	$4 + 2 = 6$	$4 + 3 = 7$	$4 + 4 = 8$	$4 + 5 = 9$	$4 + 6 = 10$
5	$5 + 1 = 6$	$5 + 2 = 7$	$5 + 3 = 8$	$5 + 4 = 9$	$5 + 5 = 10$	$5 + 6 = 11$
6	$6 + 1 = 7$	$6 + 2 = 8$	$6 + 3 = 9$	$6 + 4 = 10$	$6 + 5 = 11$	$6 + 6 = 12$

Notice that the possible sums, which are the *events* in this case, are 2 through 12. We find the probability of each sum by counting the number of times it occurs and dividing by the total of 36 possible outcomes. For example, the five highlighted outcomes in the table have a sum of 8, so the probability of a sum of 8 is $5/36$. Table 5 shows the complete probability distribution, which is also shown as a histogram in Figure 6.

TABLE 5**Probability Distribution for the Sum of Two Dice**

Event (sum)	2	3	4	5	6	7	8	9	10	11	12	Total
Probability	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$	1

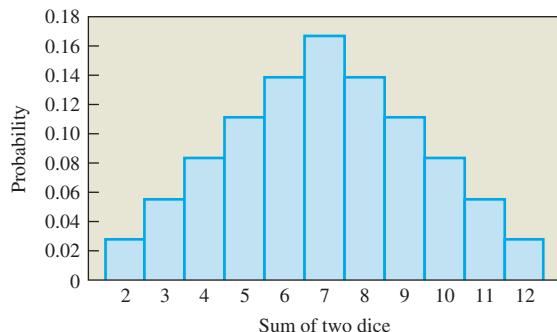


Figure 6 Histogram showing the probability distribution for the sum of two dice.

BY THE WAY

As early as 3600 BC, rounded bones called *astragali* were used much like dice for games of chance in the Middle East. Our familiar cubical dice appeared about 2000 BC in Egypt and in China. Playing cards were invented in China in the 10th century and brought to Europe in the 14th century.



Section 2 Exercises

Statistical Literacy and Critical Thinking

- Notation.** If A denotes the event that you answer a particular true/false test question correctly, what do each of the following represent: $P(A)$, $P(\text{not } A)$, $P(\bar{A})$, and what are their values?
- Probability of Life.** A student reasons that there is a probability of $1/2$ that there is life on Neptune, because there are two possible events: There is life or there is not life. Is this reasoning correct? Why or why not?
- Interpreting Probability.** What do we mean when we say that “the probability of getting 20 babies of the same gender when 20 random babies are born is $1/524,288$? Is such an event *unusual*? Why or why not?
- Subjective Probability.** Use subjective judgment to estimate the probability that the next time you ride an elevator, it gets stuck between floors.

Does It Make Sense? For Exercises 5–10, decide whether the statement makes sense (or is clearly true) or does not make sense (or is clearly false). Explain clearly; not all of these have definitive answers, so your explanation is more important than your chosen answer.

- Certain Event.** When randomly selecting a day of the week, it is certain that you will select a day containing the letter y , so $P(y) = 1$.
- Impossible Event.** Because it is impossible for Thanksgiving to fall on Tuesday, the probability of Thanksgiving falling on Tuesday is 0.
- Complementary Events.** If there is a 0.9 probability that it will rain

sometime today, then there is a probability of 0.1 that it will not rain sometime today.

8. Car Crash. An insurance company states that the probability that a particular car will be involved in a car crash this year is 0.6 and the probability that the car will not be involved in a car crash this year is 0.3.

9. Lightning. Jack estimates that the subjective probability of his being struck by lightning sometime next year is $1/2$.

10. Lightning. Jill estimates that the subjective probability of her being struck by lightning sometime next year is $1/1,000,000$.

Concepts and Applications

Theoretical Probabilities. For Exercises 11–18, use the theoretical method to determine the probability of the given outcome or event. State any assumptions that you need to make.

11. Die. Rolling a die and getting an outcome that is greater than 2.

12. Test Question. Making a correct random guess for an answer to a particular multiple-choice question with possible answers of a, b, c, d, e, one of which is correct.

13. Roulette. Getting an outcome of a red slot when a roulette wheel is spun (A roulette wheel has slots of 0, 00, 1, 2, 3, ..., 36, and 18 of those slots are red.).

14. Birthday. Finding that the next President of the United States was born on Saturday.

15. Birthday. Finding that the next person you meet has the same birthday as yours (Ignore leap years.).

16. Births. Finding that the next baby born in Alaska is a girl.

17. Births. Finding that the next baby born to a couple is a girl, given that the couple already has two children and they are both boys.

18. Dice. Rolling a pair of dice and getting an outcome (sum) of 12.

Complementary Events. Exercises 19–26 involve complementary events. In each exercise, find the probability of the given event. State any assumptions that you use.

19. Die. What is the probability of rolling a fair die and not getting an outcome less than 7?

20. Die. What is the probability of rolling a fair die and not getting an outcome that is greater than 6?

21. Week Days. What is the probability of randomly selecting a day of the week and not getting Monday?

22. Birthday. What is the probability of finding that the next President of the United States was not born on Saturday?

23. Basketball. What is the probability that a 55% free-throw shooter will miss her next free throw?

24. Testing. What is the probability of guessing incorrectly when making a random guess on a multiple-choice test

question with possible answers of a, b, c, d, and e, one of which is correct?

25. Baseball. What is the probability that a 0.280 hitter in baseball will not get a hit on his next at-bat?

26. Defects. What is the probability of not getting a defective fuse when one fuse is randomly selected from an assembly line and 1% of the fuses are defective?

Theoretical Probabilities. For Exercises 27–30, use the theoretical method to determine the probability of the given outcome or event. State any assumptions that you need to make.

27. M&Ms. A bag contains 10 red M&Ms, 15 blue M&Ms, and 20 yellow M&Ms. What is the probability of drawing a red M&M? A blue M&M? A yellow M&M? Something besides a yellow M&M?

28. Test Questions. The New England College of Medicine uses an admissions test with multiple-choice questions, each with five possible answers, only one of which is correct. If you guess randomly on every question, what score might you expect to get? (Express the answer as a percentage.)

29. Three-Child Family. Suppose you randomly select a family with three children. Assume that births of boys and girls are equally likely. What is the probability that the family has each of the following?

- a. Three girls
- b. Two boys and a girl
- c. A girl, a boy, and a boy, in that order
- d. At least one girl
- e. At least two boys

30. Four-Child Family. Suppose you randomly select a family with four children. Assume that births of boys and girls are equally likely.

- a. How many birth orders are possible? List all of them.
- b. What is the probability that the family has four boys? Four girls?
- c. What is the probability that the family has a boy, a girl, a boy, and a girl, in that order?
- d. What is the probability that the family has two girls and two boys in any order?

Relative Frequency Probabilities. Use the relative frequency method to estimate the probabilities in Exercises 31–34.

31. Weather Forecast. After recording the forecasts of your local weatherman for 30 days, you conclude that he gave a correct forecast 18 times. What is the probability that his next forecast will be correct?

32. Flood. What is the probability of a 100-year flood this year?

33. Basketball. Halfway through the season, a basketball player has hit 86% of her free throws. What is the probability that her next free throw will be successful?

34. Surgery. In a clinical trial of 73 carpal tunnel syndrome patients treated with surgery, 67 had successful treatments (based on data from “Splinting vs. Surgery in the Treatment of Carpal Tunnel Syndrome” by Gerritsen et al., *Journal of the American Medical Association*, Vol. 288, No. 10). What is the probability that the next surgery treatment will be successful?

35. Senior Citizen Probabilities. In the year 2000, there were 34.7 million people over 65 years of age out of a U.S. population of 281 million. In the year 2050, it is estimated that there will be 78.9 million people over 65 years of age out of a U.S. population of 394 million. Would your chances of meeting a person over 65 at random be greater in 2000 or in 2050? Explain.

36. Age at First Marriage. The following table gives percentages of women and men married for the first time in several age categories (U.S. Census Bureau).

	Under 20	20–24	25–29	30–34	35–44	45–64	Over 65
Women	16.6	40.8	27.2	10.1	4.5	0.7	0.1
Men	6.6	36.0	34.3	14.8	7.1	1.1	0.1

- a. What is the probability that a randomly encountered married woman was married, for the first time, between the ages of 35 and 44?
- b. What is the probability that a randomly encountered married man was married, for the first time, before he was 20 years old?
- c. Construct a bar chart consisting of side-by-side bars representing men and women.

37. Four-Coin Probability Distribution.

- a. Construct a table similar to Table 2, showing all possible outcomes of tossing four coins at once.
- b. Construct a table similar to Table 3, showing the probability distribution for the events 4 heads, 3 heads, 2 heads, 1 head, and 0 heads when you toss four coins at once.
- c. What is the probability of getting 2 heads and 2 tails when you toss four coins at once?
- d. What is the probability of tossing anything except 4 heads when you toss four coins at once?
- e. Which of the five possible events (4 heads, 3 heads, 2 heads, 1 head, 0 heads) is most likely to occur?

38. Colorado Lottery Distribution. The histogram in Figure 7 shows the distribution of 5,964 Colorado lottery numbers (possible values range from 1 to 42).

- a. Assuming the lottery drawings are random, what would you expect the probability of any number to be?
- b. Based on the histogram, what is the relative frequency probability of the most frequently appearing number?
- c. Based on the histogram, what is the relative frequency probability of the least frequently appearing number?

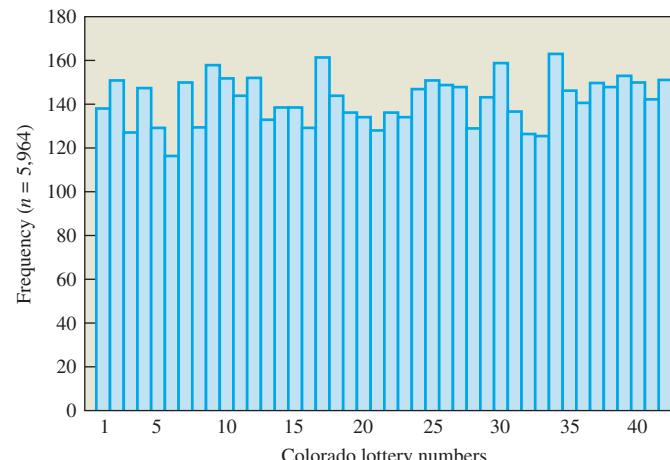


Figure 7 Colorado lottery distribution.

- d. Comment on the deviations of the empirical probabilities from the expected probabilities. Would you say these deviations are significant?

PROJECTS FOR THE INTERNET & BEYOND

39. Blood Groups. The four major blood groups are designated A, B, AB, and O. Within each group there are two Rh types: positive and negative. Using library resources or the Internet, find data on the relative frequency of blood groups, including the Rh types. Construct a table showing the probability of meeting someone in each of the eight combinations of blood group and Rh type.

40. Age and Gender. The proportions of men and women in the population change with age. Using current data from a Web site, construct a table showing the probability of meeting a male or a female in each of these age categories: 0–5, 6–10, 11–20, 21–30, 31–40, 41–50, 51–60, 61–70, 71–80, over 80.

41. Thumb Tack Probabilities. Find a standard thumb tack and practice tossing it onto a flat surface. Notice that there are two different outcomes: The tack can land point down or point up.

- a. Toss the tack 50 times and record the outcomes.
- b. Give the relative frequency probabilities of the two outcomes based on these results.
- c. If possible, ask several other people to repeat the process. How well do your probabilities agree?

42. Three-Coin Experiment. Toss three coins at once 50 times and record the outcomes in terms of the number of heads. Based on your observations, give the relative frequency probabilities of the outcomes. Do they agree with the theoretical probabilities? Explain and discuss your results.

43. Randomizing a Survey. Suppose you want to conduct a survey involving a sensitive question that not all participants may choose to answer honestly (for example,

question involving cheating on taxes or drug use). Here is a way to conduct the survey and protect the identity of respondents. We will assume that the sensitive question requires a *yes* or *no* answer. First, ask all respondents to toss a fair coin. Then give the following instructions:

- If you toss a head, then answer the decoy question (*yes/no*): Were you born on an even day of the month?
- If you toss a tail, then answer the real survey question (*yes/no*).

After all participants have answered *yes* or *no* to the question they were assigned, count the total numbers of *yes* and *no* responses.

- a. Choose a question that may not produce totally honest responses and conduct a survey in your class using this technique.
- b. Given only the total numbers of *yes* and *no* responses to both questions, explain how you can estimate the number of people who answered *yes* and *no* to the real question.
- c. Will the results computed in part b be exact? Explain.

- d. Suppose the decoy question was replaced by these instructions: If you toss a head, then answer *yes*. Can you still determine the number of people who answered *yes* and *no* to the real question?

IN THE NEWS

- 44. Theoretical Probabilities.** Find a news article or research report that cites a theoretical probability. Provide a one-paragraph discussion.
- 45. Relative Frequency Probabilities.** Find a news article or research report that makes use of a relative frequency (or empirical) probability. Provide a one-paragraph discussion.
- 46. Subjective Probabilities.** Find a news article or research report that refers to a subjective probability. Provide a one-paragraph discussion.
- 47. Probability Distributions.** Find a news article or research report that cites or makes use of a probability distribution. Provide a one-paragraph discussion.

3

THE LAW OF LARGE NUMBERS

The false ideas prevalent among all classes of the community, cultured as well as uncultured, respecting chance and luck, illustrate the truth that common consent argues almost of necessity [from] error.

—Richard Proctor,
Chance and Luck (1887 textbook)

If you toss a coin once, you cannot predict exactly how it will land; you can state only that the probability of a head is 0.5. If you toss the coin 100 times, you still cannot predict precisely how many heads will occur. However, you can reasonably expect to get heads *close to 50%* of the time. If you toss the coin 1,000 times, you can expect the proportion of heads to be even closer to 50%. In general, the more times you toss the coin, the closer the percentage of heads will be to exactly 50%. The idea that large numbers of events may show some pattern even while individual events are unpredictable is called the **law of large numbers** (sometimes called the *law of averages*). This law plays a very important role in statistics.

The Law of Large Numbers

The **law of large numbers** applies to a process for which the probability of an event *A* is $P(A)$ and the results of repeated trials do not depend on results of earlier trials (they are *independent*). It states: *If the process is repeated through many trials, the proportion of the trials in which event A occurs will be close to the probability $P(A)$. The larger the number of trials, the closer the proportion should be to $P(A)$.*

We can illustrate the law of large numbers with a die-rolling experiment. The probability of a 1 on a single roll is $P(1) = 1/6 = 0.167$. To avoid the tedium of rolling the die many times, we can let a computer *simulate* random rolls of the die. Figure 8 shows the results of a computer simulation of rolling a single die 5,000 times. The horizontal axis gives the number of rolls, and the height of the curve gives the proportion of 1's. Although the curve bounces around when the number of rolls is small, for larger numbers of rolls the proportion of 1's approaches the probability of 0.167—just as predicted by the law of large numbers.

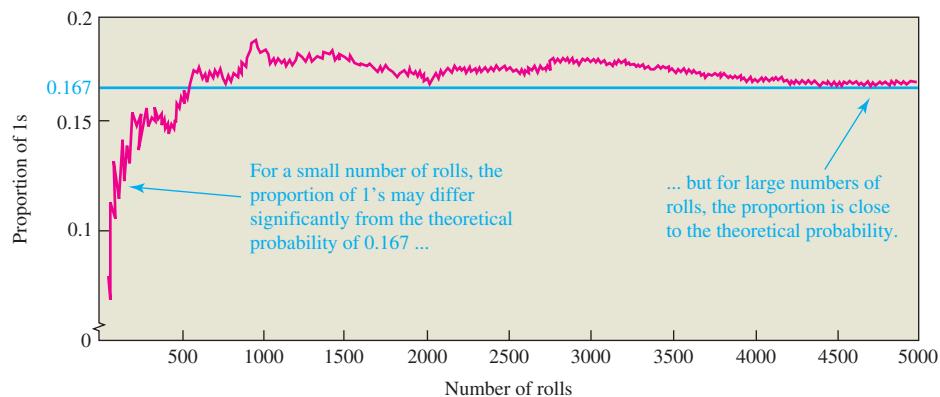


Figure 8 Results of a computer simulation of rolling a die. As the number of rolls grows large, the proportion of 1's gets close to the theoretical probability of a 1 on a single roll, which is 0.167.

EXAMPLE 1 Roulette

A roulette wheel has 38 numbers: 18 black numbers, 18 red numbers, and the numbers 0 and 00 in green. (Assume that all outcomes—the 38 numbers—have equal probability.)

- What is the probability of getting a red number on any spin?
- If patrons in a casino spin the wheel 100,000 times, how many times should you expect a red number?

SOLUTION

- The theoretical probability of getting a red number on any spin is

$$P(A) = \frac{\text{number of ways red can occur}}{\text{total number of outcomes}} = \frac{18}{38} = 0.474$$

- The law of large numbers tells us that as the game is played more and more times, the proportion of times that a red number appears should get closer to 0.474. In 100,000 tries, the wheel should come up red close to 47.4% of the time, or about 47,400 times. • • •

Expected Value

Suppose the InsureAll Company sells a special type of insurance in which it promises to pay you \$100,000 in the event that you must quit your job because of serious illness. Based on data from past claims, the probability that a policyholder will be paid for loss of job is 1 in 500. Should the insurance company expect to earn a profit if it sells the policies for \$250 each?

If InsureAll sells only a few policies, the profit or loss is unpredictable. For example, selling 100 policies for \$250 each would generate revenue of $100 \times \$250 = \$25,000$. If none of the 100 policyholders files a claim, the company will make a tidy profit. On the other hand, if InsureAll must pay a \$100,000 claim to even one policyholder, it will face a huge loss.

In contrast, if InsureAll sells a large number of policies, the law of large numbers tells us that the proportion of policies for which claims will have to be paid should be very close to the 1 in 500 probability for a single policy. For example, if the company sells 1 million policies, it should expect that the number of policyholders collecting on a \$100,000 claim will be close to

$$\underbrace{1,000,000}_{\text{number of policies}} \times \underbrace{\frac{1}{500}}_{\text{probability of } \$100,000 \text{ claim}} = 2,000 \text{ claims}$$

Paying these 2,000 claims will cost

$$2,000 \times \$100,000 = \$200 \text{ million}$$

BY THE WAY

Hurricane Katrina (in 2005) was the costliest natural catastrophe in U.S. history, responsible for nearly 2,000 deaths and damages estimated at more than \$80 billion.



This cost is an *average* of \$200 for each of the 1 million policies, which means that if the policies sell for \$250 each, the company should expect to earn an average of $\$250 - \$200 = \$50$ per policy. We call this average the **expected value** for each policy; note that it is “expected” only if the company sells a large number of policies.

Definition

The **expected value** of a variable is the weighted average of all its possible values. Because it is an average, we should expect to find the “expected value” only when there are a large number of events, so that the law of large numbers comes into play.

We can find the same expected value with a more formal procedure. The insurance example involves two distinct events, each with a particular *probability* and *value* for the company:

1. In the event that a person buys a policy, the value to the company is the \$250 price of the policy. The probability of this event is 1 because everyone who buys a policy pays \$250.
2. In the event that a person is paid for a claim, the value to the company is $-\$100,000$; it is negative because the company loses \$100,000 in this case. The probability of this event is $1/500$.

We now multiply the value of each event by its probability and add the results to find the expected value of each insurance policy:

$$\begin{aligned} \text{expected value} &= \$250 \times \underbrace{1}_{\substack{\text{value of} \\ \text{policy sale}}} + (-\$100,000) \times \underbrace{\frac{1}{500}}_{\substack{\text{probability of} \\ \text{paying claim}}} \\ &= \$250 - \$200 = \$50 \end{aligned}$$

This expected profit of \$50 per policy is the same answer we found earlier. Note that it amounts to a profit of \$50 million on sales of 1 million policies.

Calculating Expected Value

Consider two events, each with its own value and probability. The **expected value** is

$$\text{expected value} = (\text{value of event 1}) \times (\text{probability of event 1}) + (\text{value of event 2}) \times (\text{probability of event 2})$$

This formula can be extended to any number of events by including more terms in the sum.

TIME OUT TO THINK

Should the insurance company expect to see a profit of \$50 on each individual policy? Should it expect a profit of \$50,000 on 1,000 policies? Explain.

EXAMPLE 2 Lottery Expectations

Suppose that \$1 lottery tickets have the following probabilities: 1 in 5 to win a free ticket (worth \$1), 1 in 100 to win \$5, 1 in 100,000 to win \$1,000, and 1 in 10 million to win \$1 million. What is the expected value of a lottery ticket? Discuss the implications. (Note: Winners do *not* get back the \$1 they spend on the ticket.)

SOLUTION The easiest way to proceed is to make a table (below) of all the relevant events with their values and probabilities. We are calculating the expected value of a lottery ticket to *you*; thus, the ticket price has a negative value because it costs you money, while the values of the winnings are positive.

Event	Value	Probability	Value × probability
Ticket purchase	-\$1	1	$(-\$1) \times 1 = -\1.00
Win free ticket	\$1	$\frac{1}{5}$	$\$1 \times \frac{1}{5} = \0.20
Win \$5	\$5	$\frac{1}{100}$	$\$5 \times \frac{1}{100} = \0.05
Win \$1,000	\$1,000	$\frac{1}{100,000}$	$\$1,000 \times \frac{1}{100,000} = \0.01
Win \$1 million	\$1,000,000	$\frac{1}{10,000,000}$	$\$1,000,000 \times \frac{1}{10,000,000} = \0.10
			Sum of last column: - \$0.64

The expected value is the sum of all the products *value* × *probability*, which the final column of the table shows to be - \$0.64. In other words, averaged over many tickets, you should expect to lose 64¢ for each lottery ticket that you buy. If you buy, say, 1,000 tickets, you should expect to *lose* about $1,000 \times \$0.64 = \640 .

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DILBERT® by Scott Adams



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The Gambler's Fallacy

Consider a simple game involving a coin toss: You win \$1 if the coin lands heads, and you lose \$1 if it lands tails. Suppose you toss the coin 100 times and get 45 heads and 55 tails, putting you \$10 in the hole. Are you “due” for a streak of better luck?

You probably recognize that the answer is *no*: Your past bad luck has no bearing on your future chances. However, many gamblers—especially compulsive gamblers—guess just the opposite. They believe that when their luck has been bad, it’s due for a change. This mistaken belief is often called the **gambler’s fallacy** (or the *gambler’s ruin*).

Definition

The **gambler’s fallacy** is the mistaken belief that a streak of bad luck makes a person “due” for a streak of good luck.

One reason people succumb to the gambler’s fallacy is a misunderstanding of the law of large numbers. In the coin-toss game, the law of large numbers tells us that the proportion of heads tends to be closer to 50% for larger numbers of tosses. But this does *not* mean that you are likely to recover early losses. To see why, study Table 6, which shows results from computer simulations of large numbers of coin tosses. Note that as the number of tosses increases, the percentage of heads gets closer to exactly 50%, just as the law of large numbers predicts. However, the last column shows that the *difference* between the number of heads and the number of tails continues to grow—meaning that the losses (the difference between the numbers of heads and tails) grow larger even as the proportion of heads approaches 50%.

Everyone who bets any part of his fortune, however small, on a mathematically unfair game of chance acts irrationally... The imprudence of a gambler will be the greater the larger part of his fortune which he exposes to a game of chance.

—Daniel Bernoulli,
18th-century mathematician

TABLE 6		Outcomes of Coin Tossing Trials	
Number of tosses	Number of heads	Percentage of heads	Difference between numbers of heads and tails
100	45	45%	10
1,000	470	47%	60
10,000	4,950	49.5%	100
100,000	49,900	49.9%	200

EXAMPLE 3 Continued Losses

You are playing the coin-toss game in which you win \$1 for heads and lose \$1 for tails. After 100 tosses, you are \$10 in the hole because you have 45 heads and 55 tails. You continue playing until you've tossed the coin 1,000 times, at which point you've gotten 480 heads and 520 tails. Is this result consistent with what we expect from the law of large numbers? Have you gained back any of your losses? Explain.

SOLUTION The proportion of heads in your first 100 tosses was 45%. After 1,000 tosses, the proportion of heads has increased to 480 out of 1,000, or 48%. Because the proportion of heads moved closer to 50%, the results are consistent with what we expect from the law of large numbers. However, you've now won \$480 (for the 480 heads) and lost \$520 (for the 520 tails), for a net loss of \$40. Thus, your losses *increased*, despite the fact that the proportion of heads grew closer to 50%.



Streaks

BY THE WAY

The same kind of thinking about streaks applies to selecting lottery numbers. There are no special combinations of lottery numbers that are more likely to be drawn than other combinations. For example, the probability that the six winning numbers will be 1, 2, 3, 4, 5, 6 is exactly the same as the probability of any other combination of six numbers.

Another common misunderstanding that contributes to the gambler's fallacy involves expectations about streaks. Suppose you toss a coin six times and see the outcome HHHHHH (all heads). Then you toss it six more times and see the outcome HTTHTH. Most people would say that the latter outcome is "natural" while the streak of all heads is surprising. But, in fact, both outcomes are equally likely. The total number of possible outcomes for six coins is $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$, and every individual outcome has the same probability of 1/64.

Moreover, suppose you just tossed six heads and had to bet on the outcome of the next toss. You might think that, given the run of heads, a tail is "due" on the next toss. But the probability of a head or a tail on the next toss is still 0.50; the coin has no memory of previous tosses.

TIME OUT TO THINK

Is a family with six boys more or less likely to have a boy for the next child? Is a basketball player who has hit 20 consecutive free throws more or less likely to hit her next free throw? Is the weather on one day independent of the weather on the next (as assumed in the next example)? Explain.

EXAMPLE 4 Planning for Rain

A farmer knows that at this time of year in his part of the country, the probability of rain on a given day is 0.5. It hasn't rained in 10 days, and he needs to decide whether to start irrigating. Is he justified in postponing irrigation because he is due for a rainy day?

SOLUTION The 10-day dry spell is unexpected, and, like a gambler, the farmer is having a "losing streak." However, if we assume that weather events are independent from one day to the next, then the probability of rain is still 0.5 on any given day, so he is not "due" for rain.



Section 3 Exercises

Statistical Literacy and Critical Thinking

1. Law of Large Numbers. In your own words, describe the law of large numbers.

2. Expected Value. A geneticist computes the expected number of girls in 5 births and obtains the result of 2.5 girls. He rounds the result to 3 girls, reasoning that it is impossible to get 2.5 girls in 5 births. Is that reasoning correct? Why or why not?

3. Gambling Strategy. A professional gambler playing blackjack in the Venetian casino has lost each of his first 10 bets. He begins to place larger bets, reasoning that his current proportion of wins (which is 0) will increase to get closer to the average number of wins. Is his betting strategy sound? Is his reasoning correct? Explain.

4. Gambler's Fallacy. In your own words, describe the gambler's fallacy.

Does It Make Sense? For Exercises 5–8, decide whether the statement makes sense (or is clearly true) or does not make sense (or is clearly false). Explain clearly; not all of these have definitive answers, so your explanation is more important than your chosen answer.

5. Gambling Strategy. Steve learns that for a \$5 bet on a number in roulette, the expected return is \$4.74, but for a \$5 bet on the pass line in the game of craps, the expected return is \$4.93, so it is better to play this particular craps bet than the roulette bet.

6. Lottery. Kelly studies the Wisconsin Pick 3 lottery and finds that for a \$1 bet, the expected return is 50¢, so she reasons that this is bad bet and she does not play.

7. Lottery. When considering the chances of winning a lottery, Kim reasons that the number combination of 1, 2, 3, 4, 5, and 6 is less likely to occur because of the obvious pattern.

8. Lottery. Jennifer purchases a state lottery ticket and she avoids the combination of 1, 2, 3, 4, 5, and 6. She reasons that this combination has the same chance as any other combination, but the jackpot is divided among the winners, and many people are likely to select 1, 2, 3, 4, 5, and 6, so the jackpot will be much smaller, so it is better to select some other combination of numbers that will not be chosen by many other people.

Concepts and Applications

9. Gender Selection. In planning an experiment designed to test the effectiveness of a gender selection method, assume that boys and girls are equally likely to be born. Among 500 births, should we expect to get exactly 250 boys and 250 girls? As the number of births increases, what does the law of large numbers tell us about the proportion of girls?

10. Speedy Driver. A person who has a habit of driving fast has never had an accident or traffic citation. What does it mean to say that “the law of averages will catch up with him”? Is it true? Explain.

11. Should You Play? Suppose someone offers you this opportunity: You can place a bet of \$10 and roll a single die once. You win twice the outcome of the die. For example, if you roll a 6, you win \$12 for a net profit of \$2. What is the expected value of this game? Should you play?

12. Kentucky's Pick 4 Lottery. If you bet \$1 in Kentucky's Pick 4 lottery game, you either lose \$1 or gain \$4,999. (The winning prize is \$5,000, but your \$1 bet is not returned, so the net gain is \$4,999.) The game is played by selecting a four-digit number between 0000 and 9999. What is the probability of winning? If you bet \$1 on 1234, what is the expected value of your gain or loss?

13. Extra Points in Football. Football teams have the option of trying to score either 1 or 2 extra points after a touchdown. They get 1 point by kicking the ball through the goal posts or 2 points by running or passing the ball across the goal line. For a recent year in the NFL, 1-point kicks were successful 94% of the time, while 2-point attempts were successful only 37% of the time. In either case, failure means 0 points. Calculate the expected values of the 1-point and 2-point attempts. Based on these expected values, which option makes more sense in most cases? Can you think of any circumstances in which a team should make a decision different from what the expected values suggest? Explain.

14. Insurance Claims. An actuary at an insurance company estimates from existing data that on a \$1,000 policy, an average of 1 in 100 policyholders will file a \$20,000 claim, an average of 1 in 200 policyholders will file a \$50,000 claim, and an average of 1 in 500 policyholders will file a \$100,000 claim.

- What is the expected value to the company for each policy sold?
- If the company sells 100,000 policies, can it expect a profit? Explain the assumptions of this calculation.

15. Expected Waiting Time. Suppose that you arrive at a bus stop randomly, so all arrival times are equally likely. The bus arrives regularly every 30 minutes without delay (say, on the hour and on the half hour). What is the expected value of your waiting time? Explain how you got your answer.

- 16. Powerball Lottery.** The multi-state Powerball lottery advertises the following prizes and probabilities of winning for a single \$1 ticket. Assume the jackpot has a value of \$30 million one week. Note that there is more than one way to win some of the monetary prizes (for example, two ways to win \$100), so the table gives the probability for each way. What is the expected value of the winnings for a single lottery ticket? If you spend \$365 per year on the lottery, how much can you expect to win or lose?

Prize	Probability
Jackpot	1 in 80,089,128
\$100,000	1 in 1,953,393
\$5,000	1 in 364,042
\$100	1 in 8,879
\$100	1 in 8,466
\$7	1 in 207
\$7	1 in 605
\$4	1 in 188
\$3	1 in 74

- 17. Big Game.** The Multi-State Big Game lottery advertises the following prizes and probabilities of winning for a single \$1 ticket. The jackpot is variable, but assume it has an average value of \$3 million. Note that the same prize can be given to two outcomes with different probabilities. What is the expected value of a single lottery ticket to you? If you spend \$365 per year on the lottery, how much can you expect to win or lose?

Prize	Probability
Jackpot	1 in 76,275,360
\$150,000	1 in 2,179,296
\$5,000	1 in 339,002
\$150	1 in 9,686
\$100	1 in 7,705
\$5	1 in 220
\$5	1 in 538
\$2	1 in 102
\$1	1 in 62

- 18. Expected Value in Roulette.** When you give the Venetian casino in Las Vegas \$5 for a bet on the number 7 in roulette, you have a 37/38 probability of losing \$5 and you have a 1/38 probability of making a net gain of \$175. (The prize is \$180, but your \$5 bet is not returned, so the net gain is \$175.) If you bet \$5 that the outcome is an odd number, the probability of losing \$5 is 20/38 and the probability of making a net gain of \$5 is 18/38. (If you bet \$5 on an odd number and win, you are given \$10 that includes your bet, so the net gain is \$5.)

- If you bet \$5 on the number 7, what is your expected value?
- If you bet \$5 that the outcome is an odd number, what is your expected value?
- Which of these options is best: bet on 7, bet on odd, or don't bet? Why?

- 19. Expected Value in Casino Dice.** When you give a casino \$5 for a bet on the “pass line” in a casino game of dice,

there is a 251/495 probability that you will lose \$5 and there is a 244/495 probability that you will make a net gain of \$5. (If you win, the casino gives you \$5 and you get to keep your \$5 bet, so the net gain is \$5.) What is your expected value? In the long run, how much do you lose for each dollar bet?

- 20. New Jersey Pick 4.** In New Jersey’s Pick 4 lottery game, you pay 50¢ to select a sequence of four digits, such as 2273, from the 10,000 different possible four-digit sequences. If you select the same sequence of four digits that are drawn, you win and collect \$2788. What is your expected value? In the long run, how much do you lose for each 50¢ bet?

- 21. Psychology of Expected Values.** In 1953, a French economist named Maurice Allais conducted a survey of how people assess risk. Here are two scenarios that he used, each of which required people to choose between two options.

Decision 1:

Option A: 100% chance of gaining \$1,000,000

Option B: 10% chance of gaining \$2,500,000; 89% chance of gaining \$1,000,000; and 1% chance of gaining nothing

Decision 2:

Option A: 11% chance of gaining \$1,000,000 and 89% chance of gaining nothing

Option B: 10% chance of gaining \$2,500,000 and 90% chance of gaining nothing

Allais discovered that for decision 1, most people chose option A, while for decision 2, most people chose option B.

- For each decision, find the expected value of each option.
- Are the responses given in the survey consistent with the expected values?
- Give a possible explanation for the responses in Allais’s survey.

- 22. Expected Value for a Magazine Sweepstakes.** *Reader’s Digest* ran a sweepstakes in which prizes were listed along with the chances of winning: \$1,000,000 (1 chance in 90,000,000), \$100,000 (1 chance in 110,000,000), \$25,000 (1 chance in 110,000,000), \$5,000 (1 chance in 36,667,000), and \$2,500 (1 chance in 27,500,000).

- Assuming that there is no cost to enter the sweepstakes, find the expected value of the amount won for one entry.
- Find the expected value if the cost of entering this sweepstakes is the cost of a postage stamp. Is it worth entering this contest?

- 23. Gambler’s Fallacy and Dice.** Suppose you roll a die with a friend, with the following rules: For every even number you roll, you win \$1 from your friend; for every odd number you roll, you pay \$1 to your friend.

- a. What are the chances of rolling an even number on one roll of a fair die? An odd number?
- b. Suppose that on the first 100 rolls, you roll 45 even numbers. How much money have you won or lost?
- c. Suppose that on the second 100 rolls, your luck improves and you roll 47 even numbers. How much money have you won or lost over 200 rolls?
- d. Suppose that over the next 300 rolls, your luck again improves and you roll 148 even numbers. How much money have you won or lost over 500 rolls?
- e. What was the percentage of even numbers after 100, 200, and 500 rolls? Explain why this game illustrates the gambler's fallacy.
- f. How many even numbers would you have to roll in the next 100 rolls to break even?

- 24. Behind in Coin Tossing: Can You Catch Up?** Suppose that you toss a fair coin 100 times, getting 38 heads and 62 tails, which is 24 more tails than heads.

- a. Explain why, on your next toss, the *difference* in the numbers of heads and tails is as likely to grow to 25 as it is to shrink to 23.
- b. Extend your explanation from part a to explain why, if you toss the coin 1,000 more times, the final difference in the numbers of heads and tails is as likely to be larger than 24 as it is to be smaller than 24.
- c. Suppose that you continue tossing the coin. Explain why the following statement is true: If you stop at any random time, you always are more likely to have fewer heads than tails, in total.
- d. Suppose that you are betting on heads with each coin toss. After the first 100 tosses, you are well on the losing side (having lost the bet 62 times while winning only 38 times). Explain why, if you continue to bet, you will most likely remain on the losing side. How is this answer related to the gambler's fallacy?



PROJECTS FOR THE INTERNET & BEYOND

- 25. Analyzing Lotteries on the Web.** Go to the Web site for all U.S. lotteries and study the summary of state and multi-state lottery odds and prizes. Pick five lotteries and determine the expected value for winnings in each case. Discuss your results.
- 26. Law of Large Numbers.** Use a coin to simulate 100 births: Flip the coin 100 times, recording the results, and then convert the outcomes to genders of babies (tail = boy and head = girl). Use the results to fill in the following table. What happens to the proportion of girls as the sample size increases? How does this illustrate the law of large numbers?

Number of births	10	20	30	40	50	60	70	80	90	100
Proportion of girls										

IN THE NEWS

- 27. Personal Law of Large Numbers.** Describe a situation in which you personally have made use of the law of large numbers, either correctly or incorrectly. Why did you use the law of large numbers in this situation? Was it helpful?
- 28. Gambler's Fallacy in Life.** Describe a situation in which you or someone you know has fallen victim to the gambler's fallacy. How could the situation have been dealt with correctly?
- 29. Gambler's Ruin.** Describe a situation you know of in which someone lost nearly everything through gambling. Did his or her strategy appear to be rational, or did it appear to be the result of a destructive addiction?

4

IDEAS OF RISK AND LIFE EXPECTANCY

A smooth-talking but honest salesman comes to you, offering a new product:

I can't reveal the details yet, but you will love this product! It will improve your life in more ways than you can count. Its only downside is that it will eventually kill everyone who uses it. Will you buy one?

Not likely. After all, could any product be so great that you would die for it? A few weeks later, the salesman shows up again:

No one was buying, so we've made some improvements. Your chance of being killed by the product is now only 1 in 10. Ready to buy?

BY THE WAY

Roughly 90 people die in automobile accidents per day, or 1 every 16 minutes. Automobile accidents are the leading cause of death among people of ages 6 to 27 years. Vehicle occupants account for about 85% of automobile fatalities; pedestrians and cyclists account for the other 15%.



Despite the improvement, most people still would send the salesman home and wait for his inevitable return:

Okay, this time we've really perfected it. We've made it so safe that it would take 25 years for it to kill as many people as live in San Francisco. And it can be yours for a mere \$30,000.

You may be surprised to realize that, if you are like most Americans, you'll jump at this offer. The product is, after all, the automobile. It does indeed improve our lives in many ways, and \$30,000 is a typical price. And, given that more than 30,000 Americans are killed in auto accidents each year, it kills about the equivalent of the population of San Francisco (roughly 800,000) in about 25 years.

As this example shows, we frequently make tradeoffs between benefits and risks. In this section, we'll see how ideas of probability can help us quantify risk, thereby allowing us to make informed decisions about such tradeoffs.

Risk and Travel

Are you safer in a small car or in a sport utility vehicle? Are cars today safer than those 30 years ago? If you need to travel across country, are you safer flying or driving? To answer these and many similar questions, we must quantify the risk involved in travel. We can then make decisions appropriate for our own personal circumstances.

Travel risk is often expressed in terms of an **accident rate** or **death rate**. For example, suppose an annual accident rate is 750 accidents per 100,000 people. This means that, within a group of 100,000 people, on average 750 will have an accident over the period of a year. The statement is in essence an expected value, which means it also represents a probability: It tells us that the probability of a person being involved in an accident (in one year) is 750 in 100,000, or 0.0075.

This concept of travel risk is straightforward, but we must still interpret the numbers with care. For example, travel risks are sometimes stated *per 100,000 people*, as above, but other times they are stated *per trip* or *per mile*. If we use death rates *per trip* to compare the risks of flying and driving, we neglect the fact that airplane trips are typically much longer than automobile trips. Similarly, if we use accident rates *per person*, we neglect the fact that most automobile accidents involve only minor injuries.

EXAMPLE 1 Is Driving Getting Safer?

Figure 9 shows the number of automobile fatalities and the total number of miles driven (among all Americans) for each year over a period of more than four decades. In terms of death rate per mile driven, how has the risk of driving changed?

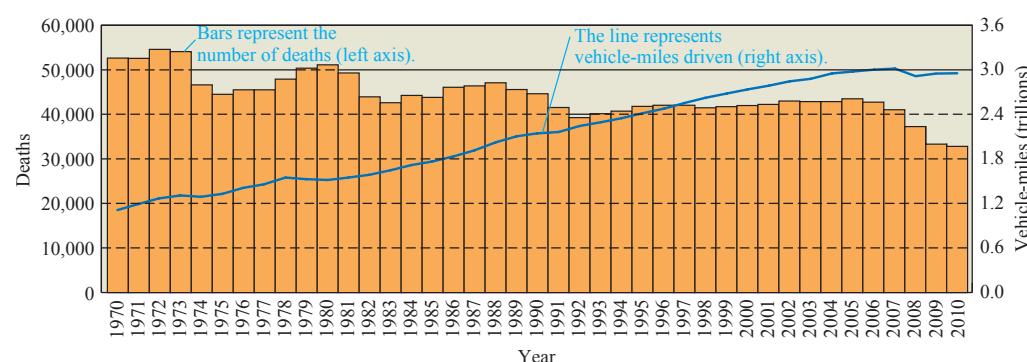


Figure 9 This graph shows the annual number of deaths (bars) and the annual number of vehicle-miles driven (line) in the United States from 1970 to 2010. Source: National Transportation Safety Board.

SOLUTION We start by comparing the approximate death rates per mile for 1970 and 2010.

$$1970: \frac{52,000 \text{ deaths}}{1 \times 10^{12} \text{ miles}} \approx 5.2 \times 10^{-8} \text{ death per mile}$$

$$2010: \frac{33,000 \text{ deaths}}{3 \times 10^{12} \text{ miles}} \approx 1.1 \times 10^{-8} \text{ death per mile}$$

Note that 5.2×10^{-8} death per mile is equivalent to 5.2 deaths per 100 million miles. We conclude that, over the approximately four decade period, the death rate per 100 million miles dropped from about 5.2 to about 1.1. By this measure, driving became much safer over the period. Most researchers believe the improvements resulted from better automobile design and from safety features, such as shoulder belts and air bags.



EXAMPLE 2 Which Is Safer: Flying or Driving?

For the period 1990 through 2010, the average (mean) number of deaths in commercial airplane accidents in the United States was roughly 60 per year. (The actual number varies significantly from year to year.) As of 2010, airplane passengers in the United States travel a total of about 8 billion miles per year. Use these numbers to calculate the death rate per mile of air travel. Compare the risk of flying to the risk of driving.

SOLUTION Assuming 60 deaths and 8 billion miles in an average year, the risk of air travel is

$$\frac{60 \text{ deaths}}{8 \times 10^9 \text{ miles}} \approx 0.75 \times 10^{-8} \text{ death per mile}$$

This risk of about 0.75 deaths per 100 million miles is about two thirds of the 2010 risk of 1.1 deaths per 100 million miles for driving (see Example 1). Note that, because the average air trip covers a considerably longer distance than the average driving trip, the risk *per trip* is higher for air travel, even though the risk *per mile* is higher for car travel.



*The cost of living is going up
and the chance of living is going down.*

—Flip Wilson,
comedian

TIME OUT TO THINK

Suppose you need to make the 800-mile trip from Atlanta to Houston. Do you think it is safer to fly or to drive? Defend your opinion.

Vital Statistics

Data concerning births and deaths of citizens, often called *vital statistics*, are very important to understanding risk-benefit tradeoffs. For example, insurance companies use vital statistics to assess risks and set rates. Health professionals study vital statistics to assess medical progress and decide where research resources should be concentrated. Demographers use birth and death rates to predict future population trends.

One important set of vital statistics, shown in Table 7, concerns causes of death. These data are extremely general; a more complete table would categorize data by age, sex, and race. Vital statistics are often expressed in terms of deaths per 100,000 people, which makes it easier to compare the rates for different years and for different states or countries.

Only those who risk going too far can possibly find out how far one can go.

—T. S. Eliot

TABLE 7 Leading Causes of Death in the United States (in a single recent year)

Cause	Deaths	Cause	Deaths
Heart disease	684,462	Diabetes	73,249
Cancer	554,643	Pneumonia/Influenza	65,681
Stroke	157,803	Alzheimer's disease	63,343
Pulmonary disease	126,128	Kidney disease	42,536
Accidents	105,695	Septicemia (blood poisoning)	34,243

Source: Centers for Disease Control and Prevention.

BY THE WAY

Among college-age students, alcohol consumption presents one of the most serious health risks. The National Institutes of Health estimates that alcohol contributes to 1,400 deaths and 500,000 injuries among college students each year, as well as to 70,000 cases of sexual assault.

EXAMPLE 3 Interpreting Vital Statistics

Assuming an approximate U.S. population of 300 million, find and compare risks per person and per 100,000 people for pneumonia (and influenza) and cancer.

SOLUTION We find the risk per person by dividing the number of deaths by the total population of 300 million:

$$\text{Pneumonia/influenza: } \frac{65,681 \text{ deaths}}{300,000,000 \text{ people}} \approx 0.000022 \text{ risk of death per person}$$

$$\text{Cancer: } \frac{554,643 \text{ deaths}}{300,000,000 \text{ people}} \approx 0.0018 \text{ risk of death per person}$$

To put these values in terms of deaths per 100,000 people, we simply multiply the per person rates by 100,000. We find that the pneumonia/influenza death rate is about 22 deaths per 100,000 people, which is much lower than the cancer death rate of 180 deaths per 100,000 people.

**TIME OUT TO THINK**

Table 7 suggests that the probability of death by stroke is about 50% higher than the probability of death by accident, but these data include all age groups. How do you think the risks of stroke and accident would differ between young people and older people? Explain.

BY THE WAY

Life expectancies vary widely around the world, from lows below 40 years in some sub-Saharan countries in Africa to highs above 80 years in several European and Asian countries. As of 2010, the United States ranked 30th in life expectancy.

Life Expectancy

One of the most commonly cited vital statistics is *life expectancy*, which is often used to compare overall health at different times or in different countries. The idea will be clearer if we start by looking at death rates. Figure 10a shows the overall U.S. death rate (or mortality rate), in deaths per 1,000 people, for different age groups. Note that there is an elevated risk of death near birth, after which the death rate drops to very low levels. At about 15 years of age, the death rate begins a gradual rise.

Figure 10b shows the **life expectancy** of Americans of different ages, defined as the number of *additional* years a person of a given age can expect to live on average. As we would expect, life expectancy is higher for younger people because, on average, they have longer left to live. At birth, the life expectancy of Americans today is about 78 years (75 years for men and 80 years for women).

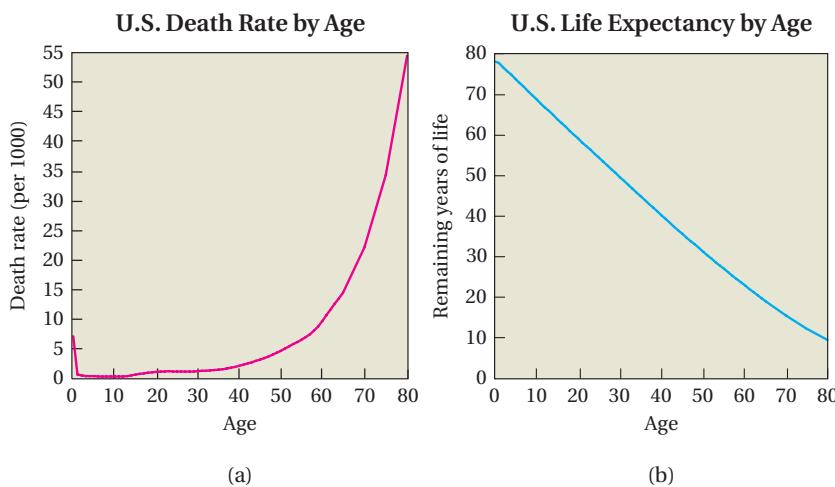


Figure 10 (a) The overall U.S. death rate (deaths per 1,000 people) for different ages. (b) Life expectancy (the number of additional years of life that can be expected on average) for different ages.

Source: U.S. National Center for Health Statistics.

Definition

Life expectancy is the number of additional years a person with a given age today can expect to live on average.

The subtlety in interpreting life expectancy comes from changes in medical science and public health. Life expectancies are calculated by studying *current* death rates. For example, when we say that the life expectancy of infants born today is 78 years, we mean that the average baby will live to age 78 *if there are no future changes* in medical science or public health. That is, while life expectancy provides a useful measure of current overall health, it should not be considered a *prediction* of future life spans.

In fact, because of advances in both medical science and public health, life expectancies increased dramatically during the 20th century, rising by about 60% (Figure 11). If this trend continues, today's infants are likely to live much longer than 78 years on average.

The reports of my death are greatly exaggerated.

—Mark Twain from London,
in a cable to the Associated Press

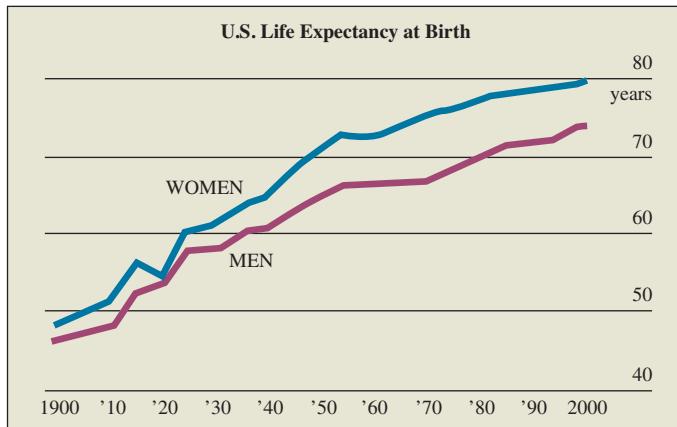


Figure 11 Changes in U.S. life expectancy during the 20th century.

Source: New York Times and National Center for Health Science Statistics.

TIME OUT TO THINK

Using Figure 11, compare the life expectancies of men and women. Briefly discuss these differences. Do they have any implications for social policy? For insurance rates? Explain.

EXAMPLE 4 Life Expectancies

Using Figure 10b, find the life expectancy of a 20-year-old person and of a 60-year-old person. Are the numbers consistent? Explain.

SOLUTION The graph shows that the life expectancy at age 20 is about 59 years and at age 60 is about 23 years. This means that an average 20-year-old can expect to live about 59 more years, to age 79. An average 60-year-old can expect to live about 23 more years, to age 83.

It might at first seem strange to calculate a longer average life span for 60-year-olds than 20-year-olds (83 years versus 79 years). But remember that life expectancies are based on *current* data. If there were no changes in medicine or public health, a 60-year-old would have a greater probability of reaching age 83 than a 20-year-old simply because he or she has already made it to age 60. However, if medicine and public health continue to improve, today's 20-year-olds may live to older ages than today's 60-year-olds.





CASE STUDY Life Expectancy and Social Security

Because of the changing age makeup of the U.S. population, the number of retirees qualifying for Social Security benefits is expected to become much larger in the future, while the number of wage earners paying Social Security taxes is expected to grow much more slowly. As a result, one of the biggest challenges to the future of Social Security is finding a way to make sure there is enough money to pay benefits for future retirees.

Current projections show that, without significant changes, the Social Security program will be bankrupt and unable to pay full benefits after about 2037. Social Security officials have proposed several different ways to solve this problem, including changes in the amounts of benefits paid, increases in the Social Security tax rate, changes in the retirement age, and partially or fully privatizing the Social Security program. Each of these proposals faces political obstacles. But, in addition, all of these proposals are based on assumptions about future life expectancy. Without corresponding changes in retirement age, longer lives mean more years of Social Security benefits.

More specifically, recent proposals by Social Security officials have assumed that life expectancy at birth will rise only slightly during this century. For example, the Social Security projections assume that American women will not reach a life expectancy of 82 years until about 2030—but women in many European and Asian nations have *already* achieved this life expectancy. In fact, during the 20th century, U.S. life expectancies rose an average of about three years per decade. If that trend continues, life expectancy for women will be nearly 86 by 2030, which means that the budgetary problems facing Social Security (and Medicare) will be far worse than current projections might otherwise suggest.

TIME OUT TO THINK

Suppose that life expectancy increases the same amount in this century as it did in the 20th century. What will life expectancy be by 2100? How would that affect programs like Social Security? What other effects would you expect it to have on society? Overall, do you think large increases in life expectancy would be good or bad for society? Defend your opinion.

Section 4 Exercises

Statistical Literacy and Critical Thinking

- Birth Rate.** The current U.S. birth rate is given as 13.5 per 1,000 population. When comparing population growth in different countries, why is it better to use birth *rates* instead of the actual numbers of births?
- Vital Statistics.** What are vital statistics?
- Life Expectancy.** What is life expectancy? Does a 30-year-old person have the same life expectancy as a 20-year-old person? Why or why not?
- Life Expectancy.** Based on recent data, a 20-year-old person in the United States has a life expectancy of 58.8 years. What does that mean?

Does It Make Sense? For Exercises 5–8, decide whether the statement makes sense (or is clearly true) or does not make sense (or is clearly false). Explain clearly. Not all of these have

definitive answers, so your explanation is more important than your chosen answer.

- Life Expectancy.** Because people are not expected to live longer than 90 years, a 100-year-old person has a negative life expectancy.
- Life Expectancy.** Your life expectancy increases as your age increases.
- Expected Age at Death.** As you become older, your expected age at death increases.
- Risk of Death.** In a recent year, the total numbers of deaths in the United States due to either accidents or pneumonia were approximately

equal. Therefore, the risks of death by accident and pneumonia per 100,000 people are approximately equal.

Commercial Aviation Fatality Rates. For Exercises 9–12, use the following table, which summarizes data on commercial aviation flights in the United States for three separate years.

Year	Departures (thousands)	Fatalities	Passenger miles (billions)	Passengers (millions)
2000	9035	92	692.8	666.2
2004	11,182	14	731.9	697.8
2008	10,437	3	722.8	690.2

9. For each of the three years, find the fatality rate in deaths per 1,000 departures. On the basis of those rates, which year was the safest? Why?
10. For each of the three years, find the fatality rate in deaths per billion passenger miles. On the basis of those rates, which year was the safest? Why?
11. For each of the three years, find the fatality rate in deaths per million passengers. On the basis of those rates, which year was the safest? Why?
12. For the year 2008, find the fatality rate in deaths per passenger mile. Why don't we report the fatality rate in units of deaths per passenger mile?

Life Table. For Exercises 13–16, use the data in the following table for people in the United States between the ages of 16 and 21 years.

Age interval	Probability of dying during the interval	Number surviving to the beginning of the interval	Number of deaths during the interval	Expected remaining lifetime (from the beginning of the interval)
16–17	0.000607	98,943	60	61.7
17–18	0.000706	98,883	70	60.7
18–19	0.000780	98,814	77	59.7
19–20	0.000833	98,736	82	58.8
20–21	0.000888	98,654	88	57.8

13. **Expected Lifetime.** How many years is a randomly selected 19-year-old expected to live beyond his or her 19th birthday?
14. **Expected Lifetime.** How many years is a randomly selected 17-year-old expected to live beyond his or her 17th birthday?
15. **Death Rate.** Life insurance companies must carefully monitor death rates. Before issuing a life insurance policy for a 19-year-old, the company needs to know the death rate for that age group. Find the death rate per 10,000 for people during their 20th year (age 19–20).
16. **Death Rate.** Find the death rate per 10,000 for people during their 17th year (age 16–17).
17. **High/Low U.S. Birth Rates.** The highest and lowest birth *rates* in the United States in 2008 were in Utah and Maine, respectively. Utah reported 55,633 births with a

population of 2,736,424 people. Maine reported 13,610 births with a population of 1,316,456 people. Use these data to find the birth rate in births per 1,000 people for Utah and Maine.

18. **High/Low U.S. Death Numbers.** In a recent year, there were 235,000 deaths in California, the highest number in the United States. The state with the lowest number of deaths was Alaska, with 3,000 deaths. The populations of California and Alaska were approximately 35,463,000 and 648,000, respectively.
 - a. Compute the death *rates* for California and Alaska in deaths per 1,000.
 - b. Based on the fact that California and Alaska had the highest and lowest death numbers, respectively, does it follow that California and Alaska had the highest and lowest death *rates*? Why or why not?
19. **U.S. Birth and Death Rates.** In 2011, the estimated U.S. population reached 313 million. The overall birth rate was estimated to be 13.8 births per 1,000, and the overall death rate was estimated to be 8.4 deaths per 1,000.
 - a. Approximately how many births were there in the United States?
 - b. About how many deaths were there in the United States?
 - c. Based on births and deaths alone (i.e., not counting immigration and emigration), about how much did the U.S. population rise during 2011?
 - d. Ignoring immigration and emigration, what is the 2011 rate of population growth of the United States? What is the population growth rate expressed as a percentage?
20. **China Birth and Death Rates.** These estimated 2011 values are for China: population = 1,336,718,015; birth rate = 12.3 per 1,000; death rate = 7.0 per 1,000.
 - a. Approximately how many births were there in China?
 - b. About how many deaths were there in China?
 - c. Based on births and deaths alone (i.e., not counting immigration and emigration), about how much did the population of China rise during 2011?
 - d. Ignoring immigration and emigration, what is the 2011 rate of population growth of China? What is the population growth rate expressed as a percentage?



21. **U.S. vs. World Life Expectancy.** You can find a great deal of data on the Web about life expectancies around the world. How does U.S. life expectancy compare to life expectancy in other developed countries? What might explain the differences you see? Based on your findings, discuss potential implications for social or government policy in the United States.

- 22. Male and Female Life Expectancies.** Find data about how and why male and female life expectancies are changing with time. Why do women have longer life expectancies than men? Should we expect male life expectancies to catch up with female life expectancies in the future? Summarize your findings with a short report.
- 23. Uganda Case Study.** Find data regarding life expectancies in Uganda over the past few decades. You will see that Ugandan life expectancy has risen dramatically in the past decade. What explains this rise, and what lessons might it have for improving life expectancy in other African nations?
- 24. Life Expectancy Calculations.** You will find many life expectancy calculators available on the Internet; try a few of them. Do they seem to give accurate or realistic results? Explore the statistical techniques that are used to make life expectancy tables.
- 25. Richter Scale for Risk.** The Royal Statistical Society has proposed a system of risk magnitudes and risk factors analogous to the Richter scale for measuring earthquakes. Go to the Internet to learn how these measures of risk are defined and computed. Using these measures, discuss the risks of various activities and events.
- 26. Understanding Risk.** The book *Against the Gods: The Remarkable Story of Risk* by Peter Bernstein (John Wiley, 1996) is an award-winning account of the history of probability and risk assessment. Find the book in a library or bookstore (it's a worthwhile purchase) and identify a particular event that changed our understanding of risk. Write a two-page essay on both the history and the consequences of this particular event.

IN THE NEWS

- 27. Travel Safety.** Find a recent news article discussing some aspect of travel safety (such as risk of accidents in automobiles or airplanes, the efficacy of child car seats, or the effects of driving while talking on a cell phone). Summarize any given statistics about risks, and give your overall opinion regarding the safety issue under discussion.
- 28. Vital Statistics.** Find a recent news report that gives current data about vital statistics or life expectancy. Summarize the report and the statistics, and discuss any personal or social implications of the new data.

5

COMBINING PROBABILITIES (SUPPLEMENTARY SECTION)

The ideas of probability that we have discussed to this point in the chapter will be sufficient for a great deal. However, probability has many more applications, both in statistics and in other areas of life. In this section, we investigate a few more ideas of probability and a few of their many applications.

And Probabilities

Chance favors only the prepared mind.

—Louis Pasteur,
19th-century scientist

Suppose you toss two fair dice and want to know the probability that *both* will come up 4. One way to find the probability is to consider the two tossed dice as a *single* toss of two dice. Then we can find the probability using the *theoretical* method (see Section 2). Because “double 4’s” is 1 of 36 possible outcomes, its probability is 1/36.

Alternatively, we can consider the two dice individually. For each die, the probability of a 4 is 1/6. We find the probability that both dice show a 4 by multiplying the individual probabilities:

$$P(\text{double 4's}) = P(4) \times P(4) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

By either method, the probability of rolling double 4’s is 1/36. In general, we call the probability of event A *and* event B occurring an **and probability** (or *joint probability*).

The advantage of the multiplication technique is that it can easily be extended to situations involving more than two events. For example, we might want to find the probability of getting 10 heads on 10 coin tosses or of having a baby *and* getting a pay raise in the same year. However, there is an important distinction that must be made when working with *and* probabilities. We must distinguish between events that are *independent* and events that are *dependent*. Let’s investigate each case.

Independent Events

The repeated roll of a single die produces **independent events** because the outcome of one roll does not affect the probabilities of the other rolls. Similarly, coin tosses are independent because one coin toss does not affect others. For independent events, we calculate the *and* probability by multiplying.

And Probability for Independent Events

Two events are **independent** if the outcome of one event does not affect the probability of the other event. Consider two independent events A and B with probabilities $P(A)$ and $P(B)$. The probability that A and B occur together is

$$P(A \text{ and } B) = P(A) \times P(B)$$

This principle can be extended to any number of independent events. For example, the probability of A , B , and a third independent event C is

$$P(A \text{ and } B \text{ and } C) = P(A) \times P(B) \times P(C)$$

EXAMPLE 1 Three Coins

Suppose you toss three fair coins. What is the probability of getting three tails?

SOLUTION Because coin tosses are independent, we multiply the probability of tails on each individual coin:

$$P(3 \text{ tails}) = \underbrace{P(\text{tails})}_{\text{coin 1}} \times \underbrace{P(\text{tails})}_{\text{coin 2}} \times \underbrace{P(\text{tails})}_{\text{coin 3}} = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

The probability that three tossed coins all land on tails is $1/8$ (which we also determined in Example 7 of Section 2 with much more work). • •

Dependent Events

A batch of 15 memory chips contains 5 defective chips. If you select a chip at random from the batch, the probability of getting a defect is $5/15$. Now, suppose that you select a defect on the first selection and put it in your pocket. What is the probability of getting a defect on the second selection?

Because you've removed one defective chip from the batch, the batch now contains only 14 chips, of which 4 are defective. Thus, the probability of getting a defective chip on the second draw is $4/14$. This probability is less than the $5/15$ probability on the first selection because the first selection changed the contents of the batch. Because the outcome of the first event affects the probability of the second event, these are **dependent events**.

Calculating the probability for dependent events still involves multiplying the individual probabilities, but we must take into account how prior events affect subsequent events. In the case of the batch of memory chips, we find the probability of getting two defective chips in a row by multiplying the $5/15$ probability for the first selection by the $4/14$ probability for the second selection.

$$P(2 \text{ defectives}) = \underbrace{P(\text{defective})}_{\text{first selection}} \times \underbrace{P(\text{defective})}_{\text{second selection}} \text{ if first selection defective} = \frac{5}{15} \times \frac{4}{14} = 0.0952$$

The probability of drawing two defective chips in a row is 0.0952, which is slightly less than $(5/15) \times (5/15) = 0.111$, the probability we get if we replace the first chip before the second selection.

BY THE WAY

Whether events are independent is not always obvious. Consider a set of basketball free throws by a single player. Some people argue that the probability of a basket is the same for each free throw. Others suggest that the psychological impact of success or failure on one free throw affects the player's chances of making the next one. Despite many statistical studies of basketball free throws, no one has clearly established which view is correct.



TECHNICAL NOTE

$P(B \text{ given } A)$ is called a *conditional probability*. In some texts, it is denoted $P(B|A)$.

And Probability for Dependent Events

Two events are **dependent** if the outcome of one event affects the probability of the other event. The probability that dependent events A and B occur together is

$$P(A \text{ and } B) = P(A) \times P(B \text{ given } A)$$

where $P(B \text{ given } A)$ means the probability of event B given the occurrence of event A .

This principle can be extended to any number of individual events. For example, the probability of dependent events A , B , and C is

$$P(A \text{ and } B \text{ and } C) = P(A) \times P(B \text{ given } A) \times P(C \text{ given } A \text{ and } B)$$

EXAMPLE 2 Bingo

The game of bingo involves drawing labeled buttons from a bin at random, without replacement. There are 75 buttons, 15 for each of the letters B, I, N, G, and O. What is the probability of drawing two B buttons in the first two selections?

SOLUTION Bingo involves dependent events because selected buttons are not replaced, so removing a button changes the contents of the bin. The probability of drawing a B on the first draw is $15/75$. If this occurs, 74 buttons remain in the bin, of which 14 are Bs. Therefore, the probability of drawing a B button on the second draw is $14/74$. The probability of drawing two B buttons in the first two selections is

$$P(\text{B and B}) = \underbrace{P(\text{B})}_{\text{first draw}} \times \underbrace{P(\text{B})}_{\substack{\text{second draw} \\ \text{given B on} \\ \text{first draw}}} = \frac{15}{75} \times \frac{14}{74} = 0.0378$$

**TIME OUT TO THINK**

Suppose you have a standard deck of cards (which has 13 hearts among the 52 cards). For your first experiment, you draw three cards from the deck without replacing any of the selected cards. For your second experiment, you draw a card, look at it, and then replace it and reshuffle the deck before drawing again. Without doing any calculations, is the probability of drawing 3 hearts in a row larger in the first experiment or the second? Explain.

EXAMPLE 3 Polling Probability

A polling organization has a list of 1,000 people for a telephone survey. The pollsters know that 433 people out of the 1,000 are members of the Democratic Party. Assuming that a person cannot be called more than once, what is the probability that the first two people called will be members of the Democratic Party?

The importance of probability can only be derived from the judgment that it is rational to be guided by it in action.

—John Maynard Keynes

SOLUTION This problem involves an *and* probability for dependent events: Once a person is called, that person cannot be called again. The probability of calling a member of the Democratic Party on the first call is $433/1,000$. With that person removed from the calling pool, the probability of calling a member of the Democratic Party on the second call is $432/999$. Therefore, the probability of calling two members of the Democratic Party on the first two calls is

$$\frac{433}{1,000} \times \frac{432}{999} = 0.1872$$

In this case, it's worth noting what would happen if we were to treat the two calls as independent events (because a selected person is not removed from the list). In that case, the

probability of calling a member of the Democratic Party would be 433/1,000 on both calls, and the probability of calling two members of the Democratic Party would be

$$\frac{433}{1,000} \times \frac{433}{1,000} = 0.1875$$

Notice that this result is nearly identical to the result we found when correctly doing the calculation with dependent events. In general, if relatively few items or people are selected from a large pool (in this case, 2 people out of 1,000), then dependent events can be treated as independent events with very little error. A common guideline is that we can treat the events as being independent when the sample size is less than 5% of the population size. This practice is commonly used by polling organizations.



Either/Or Probabilities

Suppose we want to know the probability that *either* of two events occurs, rather than the probability that both of two events occurs. In that case, we are looking for an **either/or probability**, such as the probability of having either a blue-eyed *or* a green-eyed baby or the probability of losing your home either to a fire *or* to a hurricane. As with *and* probabilities, there are two cases to consider; we call them *overlapping* and *non-overlapping* events.

Non-Overlapping Events

A coin can land on either heads *or* tails, but it can't land on both heads *and* tails at the same time. When two events cannot possibly occur at the same time, they are said to be **non-overlapping** (or *mutually exclusive*). We can represent non-overlapping events with a Venn diagram in which each circle represents an event. If the circles do not overlap, it means that the corresponding events cannot occur together. For example, we show the possibilities of heads and tails in a coin toss as two non-overlapping circles because a coin cannot land on both heads and tails at the same time (Figure 12). We can calculate either/or probabilities for non-overlapping events with the following rule.

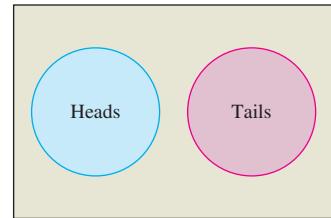


Figure 12 Venn diagram for non-overlapping events.

Either/Or Probability for Non-Overlapping Events

Two events are **non-overlapping** if they cannot occur at the same time. If A and B are non-overlapping events, the probability that either A or B occurs is

$$P(A \text{ or } B) = P(A) + P(B)$$

This principle can be extended to any number of non-overlapping events. For example, the probability that either event A , event B , or event C occurs is

$$P(A \text{ or } B \text{ or } C) = P(A) + P(B) + P(C)$$

provided that A , B , and C are all non-overlapping events.

EXAMPLE 4 Either/Or Dice

Suppose you roll a single die. What is the probability of rolling either a 2 or a 3?

SOLUTION The outcomes of 2 and 3 are non-overlapping because a single die can yield only one result. Each probability is $1/6$ (because there are 6 ways for the die to land), so the combined probability is

$$P(2 \text{ or } 3) = P(2) + P(3) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$$

The probability of rolling a 2 or a 3 is $1/3$.



TABLE 8		Tourism Committee	
	Men	Women	
American	2	6	
French	4	8	

Overlapping Events

To improve tourism between France and the United States, the two governments form a committee consisting of 20 people: 2 American men, 4 French men, 6 American women, and 8 French women (Table 8). If you meet one of these people at random, what is the probability that the person will be *either* a woman *or* a French person?

Twelve of the 20 people are French, so the probability of meeting a French person is $\frac{12}{20}$. Similarly, 14 of the 20 people are women, so the probability of meeting a woman is $\frac{14}{20}$. The sum of these two probabilities is

$$\frac{12}{20} + \frac{14}{20} = \frac{26}{20}$$

This cannot be the correct probability of meeting either a woman or a French person, because probabilities cannot be greater than 1. The Venn diagram in Figure 13 shows why simple addition was wrong in this situation. The left circle contains the 12 French people, the right circle contains the 14 women, and the American men are in neither circle. We see that there are 18 people who are either French or women (or both). Because the total number of people in the room is 20, the probability of meeting a person who is *either* French *or* a woman is $\frac{18}{20} = \frac{9}{10}$.

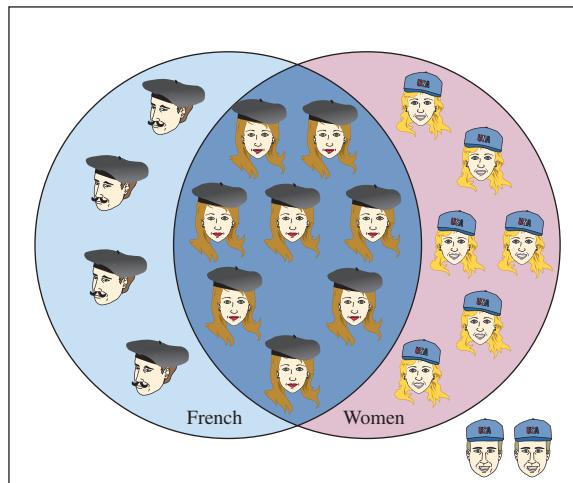


Figure 13 Venn diagram for overlapping events.

As the Venn diagram shows, simple addition was incorrect because the region in which the circles overlap contains 8 people who are *both* French *and* women. If we add the two individual probabilities, these 8 people get counted twice: once as women and once as French people. The probability of meeting one of these French women is $\frac{8}{20}$. We can correct the double counting error by subtracting out this probability. Thus, the probability of meeting a person who is either French or a woman is

$$P(\text{woman or French}) = \underbrace{\frac{14}{20}}_{\text{probability of a woman}} + \underbrace{\frac{12}{20}}_{\text{probability of a French person}} - \underbrace{\frac{8}{20}}_{\text{probability of a French woman}} = \frac{18}{20} = \frac{9}{10}$$

which agrees with the result found by counting.

We say that meeting a woman and meeting a French person are **overlapping** (or *non-mutually exclusive*) events because both can occur at the same time. Generalizing the procedure we used in this example, we find the following rule.

Either/Or Probability for Overlapping Events

Two events A and B are **overlapping** if they can occur together. For overlapping events, the probability that either A or B occurs is

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

The last term, $P(A \text{ and } B)$, corrects for the double counting of events in which A and B both occur together. Note that it is not necessary to use this formula. The correct probability can always be found by counting carefully and avoiding double counting.

TIME OUT TO THINK

Are the events of being born on a Wednesday or being born in Las Vegas overlapping? Are the events of being born on a Wednesday or being born in March overlapping? Are the events of being born on a Wednesday or being born on a Friday overlapping? Explain.

EXAMPLE 5 Minorities and Poverty

Suppose that the town of Pine Creek has 2,350 citizens, of which 1,950 are white and 400 are nonwhite. Further suppose that 11% of the white citizens, or 215 people, live below the poverty level, while 28% of the nonwhite citizens, or 112 people, live below the poverty level. If you visit Pine Creek, what is the probability of meeting (at random) a person who is *either* nonwhite *or* living below the poverty level?

SOLUTION Meeting a nonwhite citizen and meeting a person living in poverty are overlapping events. It's useful to make a small table such as Table 9, showing how many citizens are in each of the four categories.

You should check that the figures in the table are consistent with the given data and that the total in all four categories is 2,350. Because there are 400 nonwhite citizens, the probability of (randomly) meeting a nonwhite citizen is $400/2,350 = 0.170$. Because there are $215 + 112 = 327$ people living in poverty, the probability of meeting a citizen in poverty is $327/2,350 = 0.139$. The probability of meeting a person who is *both* nonwhite and living in poverty is $112/2,350 = 0.0477$. According to the rule for overlapping events, the probability of meeting *either* a nonwhite citizen *or* a person living in poverty is

$$P(\text{non white or poverty}) = 0.170 + 0.139 - 0.0477 = 0.261$$

The probability of meeting a citizen who is *either* nonwhite or living below the poverty level is about 1 in 4. Notice the importance of subtracting out the term that corresponds to meeting a person who is *both* nonwhite and living in poverty.

TABLE 9		Citizens in Pine Creek
	In poverty	Not in poverty
White	215	1,735
Nonwhite	112	288

Summary

Table 10 provides a summary of the formulas we've used in combining probabilities.

TABLE 10 Summary of Combining Probabilities

And probability: independent events	And probability: dependent events	Either/or probability: non-overlapping events	Either/or probability: overlapping events
$P(A \text{ and } B) = P(A) \times P(B)$	$P(A \text{ and } B) = P(A) \times P(B \text{ given } A)$	$P(A \text{ or } B) = P(A) + P(B)$	$P(A \text{ or } B) = P(A) \times P(B) - P(A \text{ and } B)$

Section 5 Exercises

Statistical Literacy and Critical Thinking

1. Independence. Let A denote the event of turning on your cell phone and finding that it works, and let B denote the event of turning on your car radio and finding that it works. Are events A and B independent or are they dependent?

2. Non-overlapping Events. In your own words, state what it means for two events to be non-overlapping.

3. Sampling with Replacement? The professor in a class of 25 students randomly selects a student and then randomly selects a second student. If all 25 students are available for the second selection, is this sampling with replacement or sampling without replacement? Is the second outcome independent of the first?

4. Complementary Events. Let A denote some event. Are events A and \bar{A} non-overlapping? Why or why not?

Does It Make Sense? For Exercises 5–8, decide whether the statement makes sense (or is clearly true) or does not make sense (or is clearly false). Explain clearly; not all of these have definitive answers, so your explanation is more important than your chosen answer.

5. Lottery. The numbers 5, 17, 18, 27, 36, and 41 were drawn in the last lottery; they should not be bet on in the next lottery because they are now less likely to occur.

6. Combining Probabilities. The probability of flipping a coin and getting heads is 0.5. The probability of selecting a red card when one card is drawn from a shuffled deck is also 0.5. When flipping a coin and drawing a card, the probability of getting heads or a red card is $0.5 + 0.5 = 1$.

7. Either/Or Probability. $P(A) = 0.5$ and $P(A \text{ or } B) = 0.8$.

8. Lottery. The probability of your winning the state lottery this week is not affected by whether you won that same lottery last week.

Concepts and Applications

9. Births. Assume that boys and girls are equally likely and that the gender of a child is independent of the gender of any brothers or sisters. If a couple already has three girls, find the probability of getting a girl when their fourth baby is born.

10. Births. A couple plans to have four children. Find the probability that the first two children are girls and the last two children are boys.

11. Password. A new computer owner creates a password consisting of five characters. She randomly selects a letter

of the alphabet for the first character and a digit (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) for each of the other four characters; the digits may be reused, so there are 10 possibilities for each of the four characters.

What is the probability that her password is A1234?

12. Wearing Hunter Orange. A study of hunting injuries and the wearing of hunter orange clothing showed that among 123 hunters injured when mistaken for game, 6 were wearing orange (based on data from the Centers for Disease Control and Prevention). If a follow-up study begins with the random selection of hunters from this sample of 123, find the probability that the first two hunters selected were both wearing orange.

- Assume that the first hunter is replaced before the next one is selected.
- Assume that the first hunter is not replaced before the second one is selected.
- Which makes more sense in this situation: selecting with replacement or selecting without replacement? Why?

13. Radio Tunes. An MP3 player is loaded with 60 musical selections: 30 rock selections, 15 jazz selections, and 15 blues selections. The player is set on “random play,” so selections are played randomly and can be repeated. What is the probability of each of the following events?

- The first four selections are all jazz.
- The first five selections are all blues.
- The first selection is jazz and the second is rock.
- Among the first four selections, none is rock.
- The second selection is the same song as the first.

14. Polling Calls. A telephone pollster has names and telephone numbers for 45 voters, 20 of whom are registered Democrats and 25 of whom are registered Republicans. Calls are made in random order. Suppose you want to find the probability that the first two calls are to Republicans.

- Are these independent or dependent events? Explain.
- If you treat them as *dependent* events, what is the probability that the first two calls are to Republicans?
- If you treat them as *independent* events, what is the probability that the first two calls are to Republicans?
- Compare the results of parts b and c.

Probability and Court Decisions. The data in the following table show the outcomes of guilty and not-guilty pleas in 1,028 criminal court cases. Use the data to answer Exercises 15–20.

	Guilty plea	Not-guilty plea
Sent to prison	392	58
Not sent to prison	564	14

Source: Brereton and Casper, "Does It Pay to Plead Guilty? Differential Sentencing and the Functioning of the Criminal Courts," *Law and Society Review*, Vol. 16, No. 1.

15. What is the probability that a randomly selected defendant either pled guilty or was sent to prison?
16. What is the probability that a randomly selected defendant either pled not guilty or was not sent to prison?
17. If two different defendants are randomly selected, what is the probability that they both entered guilty pleas?
18. If two different defendants are randomly selected, what is the probability that they both were sentenced to prison?
19. If a defendant is randomly selected, what is the probability that the defendant entered a guilty plea and was sent to prison?
20. If a defendant is randomly selected, what is the probability that the defendant entered a guilty plea and was not sent to prison?

Pedestrian Deaths. For Exercises 21–26, use the following table, which summarizes data on 985 pedestrian deaths that were caused by accidents (based on data from the National Highway Traffic Administration).

	Pedestrian intoxicated?	
	Yes	No
Driver intoxicated?	Yes	59
	No	266
		581

21. If one of the pedestrian deaths is randomly selected, find the probability that the pedestrian was intoxicated or the driver was intoxicated.
22. If one of the pedestrian deaths is randomly selected, find the probability that the pedestrian was not intoxicated or the driver was not intoxicated.
23. If one of the pedestrian deaths is randomly selected, find the probability that the pedestrian was intoxicated or the driver was not intoxicated.
24. If one of the pedestrian deaths is randomly selected, find the probability that the driver was intoxicated or the pedestrian was not intoxicated.
25. If two different pedestrian deaths are randomly selected, find the probability that they both involved intoxicated drivers.
26. If two different pedestrian deaths are randomly selected, find the probability that in both cases the pedestrians were intoxicated.

27. Drug Tests. An allergy drug is tested by giving 120 people the drug and 100 people a placebo. A control group consists of 80 people who were given no treatment. The number of people in each group who showed improvement appears in the table below.

	Allergy drug	Placebo	Control	Total
Improvement	65	42	31	138
No improvement	55	58	49	162
Total	120	100	80	300

- a. What is the probability that a randomly selected person in the study was given either the drug or the placebo?
 - b. What is the probability that a randomly selected person either improved or did not improve?
 - c. What is the probability that a randomly selected person either was given the drug or improved?
 - d. What is the probability that a randomly selected person was given the drug and improved?
- 28. Survey Refusals.** Refer to the following table summarizing results from a study of people who refused to answer survey questions (based on data from "I Hear You Knocking but You Can't Come In," by Fitzgerald and Fuller, *Sociological Methods and Research*, Vol. 11, No. 1). In each case, assume that one of the subjects is randomly selected.

	Age					
	18–21	22–29	30–39	40–49	50–59	60 and over
Responded	73	255	245	136	138	202
Refused	11	20	33	16	27	49

- a. What is the probability that the selected person refused to answer? Does that probability value suggest that refusals are a problem for pollsters? Why or why not?
- b. A pharmaceutical company is interested in opinions of the elderly, because they are either receiving Medicare or will receive it soon. What is the probability that the selected subject is someone 60 and over who responded?
- c. What is the probability that the selected person responded or is in the 18–21 age bracket?
- d. What is the probability that the selected person refused to respond or is over 59 years of age?

- 29. Probability Distributions and Genetics.** Many traits are controlled by a dominant gene, denoted by **A**, and a recessive gene, denoted by **a**. Suppose that two parents carry these genes in the proportion 3:1; that is, the probability of either parent giving the **A** gene is 0.75, and the probability of either parent giving the **a** gene is 0.25. Assume that the genes are selected from each parent randomly. To answer the following questions, imagine 100 trial "births."
- a. What is the probability that a child receives an **A** gene from both parents?

- b.** What is the probability that a child receives an **A** gene from one parent and an **a** gene from the other parent? Note that this can occur in two ways.
- c.** What is the probability that a child receives an **a** gene from both parents?
- d.** Make a table showing the probability distribution for all events.
- e.** If the combinations **AA** and **Aa** both result in the same dominant trait (say, brown hair) and **aa** results in the recessive trait (say, blond hair), what is the probability that a child will have the dominant trait?
- 30. BINGO.** The game of BINGO involves drawing numbered and lettered buttons at random from a barrel. The B numbers are 1–15, the I numbers are 16–30, the N numbers are 31–45, the G numbers are 46–60, and the O numbers are 61–75. Buttons are not replaced after they have been selected. What is the probability of each of the following events on the initial selections?
- Drawing a B button
 - Drawing two B buttons in a row
 - Drawing a B or an O
 - Drawing a B, then a G, then an N, in that order
 - Drawing anything but a B on each of the first five draws

At Least Once Problems. A common problem asks for the probability that an event occurs at least once in a given number of trials. Suppose the probability of a particular event is p (for example, the probability of drawing a heart from a deck of cards is 0.25). Then the probability that the event occurs at least once in N trials is

$$1 - (1 - p)^N$$

For example, the probability of drawing at least one heart in 10 draws (with replacement) is

$$1 - (1 - 0.25)^{10} = 0.944$$

Use this rule to solve Exercises 31 and 32.

31. The Bets of the Chevalier de Mère. It is said that probability theory was invented in the 17th century to explain the gambling of a nobleman named the Chevalier de Mère.

- In his first game, the Chevalier bet on rolling at least one 6 with four rolls of a fair die. If played repeatedly, is this a game he should expect to win?

- b.** In his second game, the Chevalier bet on rolling at least one double-6 with 24 rolls of two fair dice. If played repeatedly, is this a game he should expect to win?

32. HIV among College Students. Suppose that 3% of the students at a particular college are known to carry HIV.

- If a student has 6 sexual partners during the course of a year, what is the probability that at least one of them carries HIV?
- If a student has 12 sexual partners during the course of a year, what is the probability that at least one of them carries HIV?
- How many partners would a student need to have before the probability of an HIV encounter exceeded 50%?



PROJECTS FOR THE INTERNET & BEYOND

33. Simulation. A classic probability problem involves a king who wants to increase the proportion of women in his kingdom. He decrees that after a mother gives birth to a son, she is prohibited from having any more children. The king reasons that some families will have just one boy whereas other families will have a few girls and one boy, so the proportion of girls will be increased. Use coin tossing to simulate a kingdom that abides by this decree: “After a mother gives birth to a son, she will not have any other children.” If this decree is followed, does the proportion of girls increase?

IN THE NEWS

- 34.** A columnist for the *New York Daily News* (Stephen Allensworth) provided tips for selecting numbers in New York State’s lottery. He advocated a system based on the use of “cold digits,” which are digits that hit once or not at all in a seven-day period. He made this statement: “That [system] produces the combos 5–8–9, 7–8–9, 6–8–9, 0–8–9, and 3–8–9. These five combos have an excellent chance of being drawn this week. Good luck to all.” Can this system work? Why or why not?

CHAPTER REVIEW EXERCISES

For Exercises 1–7, use the data in the accompanying table (based on data from “Helmet Use and Risk of Head Injuries in Alpine Skiers and Snowboarders” by Sullheim et al., *Journal of the American Medical Association*, Vol. 295, No. 8).

	Head injured	Head not injured
Wore helmet	96	656
No helmet	480	2330

1. If one of the subjects is randomly selected, find the probability of selecting someone with a head injury.
2. If one of the subjects is randomly selected, find the probability of selecting someone who had a head injury or wore a helmet.
3. If one of the subjects is randomly selected, find the probability of selecting someone who did not wear a helmet or was not injured.
4. If one of the subjects is randomly selected, find the probability of selecting someone who wore a helmet and was injured.
5. If one of the subjects is randomly selected, find the probability of selecting someone who did not wear a helmet and was not injured.
6. If two different study subjects are randomly selected, find the probability that they both wore helmets.
7. If one of the subjects is randomly selected, find the probability of selecting someone who did not wear a helmet, given that the subject had head injuries.

8. Use subjective probability to estimate the probability of randomly selecting a car and selecting one that is black.
9. The Binary Computer Company manufactures computer chips used in DVD players. Those chips are made with a 27% yield, meaning that 27% of them are good and the others are defective.
 - a. If one chip is randomly selected, find the probability that it is *not* good.
 - b. If two chips are randomly selected, find the probability that they are both good.
 - c. If five chips are randomly selected, what is the *expected number* of good chips?
 - d. If five chips are randomly selected, find the probability that they are all good. If you did get five good chips among the five selected, would you continue to believe that the yield was 27%? Why or why not?
10. For a recent year, the fatality rate from motor vehicle crashes was reported as 15.2 per 100,000 population.
 - a. What is the probability that a randomly selected person will die this year as a result of a motor vehicle crash?
 - b. If two people are randomly selected, find the probability that they both die this year as the result of motor vehicle crashes, and express the result using three significant digits.
 - c. If two people are randomly selected, find the probability that neither of them dies this year as the result of motor vehicle crashes, and express the result using six decimal places.

CHAPTER QUIZ

1. A Las Vegas handicapper can correctly predict the winning professional football team 70% of the time. What is the probability that she is wrong in her next prediction?
2. For the same handicapper described in Exercise 1, find the probability that she is correct in each of her next two predictions.
3. Estimate the probability that a randomly selected prime-time television show will be interrupted with a news bulletin.
4. When conducting a clinical trial of the effectiveness of a gender selection method, it is found that there is a 0.342 probability that the results could have occurred by chance. Does the method appear to be effective?
5. If $P(A) = 0.4$, what is the value of $P(\bar{A})$?

In Exercises 6–10, use the following results:

In the judicial case of *United States v. City of Chicago*, discrimination was charged in a qualifying exam for the position

of fire captain. In the table below, Group A is a minority group and Group B is a majority group.

	Passed	Failed
Group A	10	14
Group B	417	145

6. If one of the test subjects is randomly selected, find the probability of getting someone who passed the exam.
7. Find the probability of randomly selecting one of the test subjects and getting someone who is in Group B or passed.
8. Find the probability of randomly selecting two different test subjects and finding that they are both in Group A.
9. Find the probability of randomly selecting one of the test subjects and getting someone who is in Group A and passed the exam.
10. Find the probability of getting someone who passed, given that the selected person is in Group A.

FOCUS ON SOCIAL SCIENCE

Do Lotteries Harm the Poor?

State-sponsored lotteries are a big business in the United States, generating more than \$50 billion in annual sales, of which about one-third (\$17 billion) ends up as state revenue. (The rest goes to prizes and expenses.) But are lotteries good social policy?

Lottery proponents point to several positive aspects. For example, lottery revenue helps states to fund education and recreation, while also allowing states to keep tax rates lower than they would be otherwise. Proponents also point out that lottery participation is voluntary and that polls show a large majority of Americans to be in favor of state-sponsored lotteries.

This favorable picture is part of the marketing and public relations of state lotteries. For example, Colorado state lottery officials offer statistics on the age, income, and education of lottery players compared to the general population (Figure 14). Within a few percentage points, the age of lottery players parallels that of the population as a whole. Similarly, the histogram for the income of lottery players gives the impression that lottery players as a whole are typical citizens—with the exception of the bars for incomes of \$15,000–\$25,000 and \$25,000–\$35,000, which show that the poor tend to play more than we would expect for their proportion of the population.

Despite the apparent benefits of lotteries, critics have long argued that lotteries are merely an unfair form of taxation. To investigate the reality, the *New York Times* conducted a study



of data from 48,875 people who had won at least \$600 in New Jersey lottery games. (In an ingenious bit of sampling, these winners were taken to be a random sample of all lottery players; after all, lottery winners are determined randomly. However, the sample is not really representative of all lottery players because winners tend to buy more than an average number of tickets.) By identifying the home zip codes of the lottery players, researchers were able to determine whether players came from areas with high or low income, high or low average education, and various other demographic characteristics. The overwhelming conclusion of the *New York Times* study was that lottery spending has a much greater impact *in relative terms* on those players with lower incomes and lower educational backgrounds. For example, the following were among the specific findings:

- People in the state's lowest income areas spend five times as much of their income on lotteries as those in the state's highest income areas (more than \$25 per \$10,000 of annual

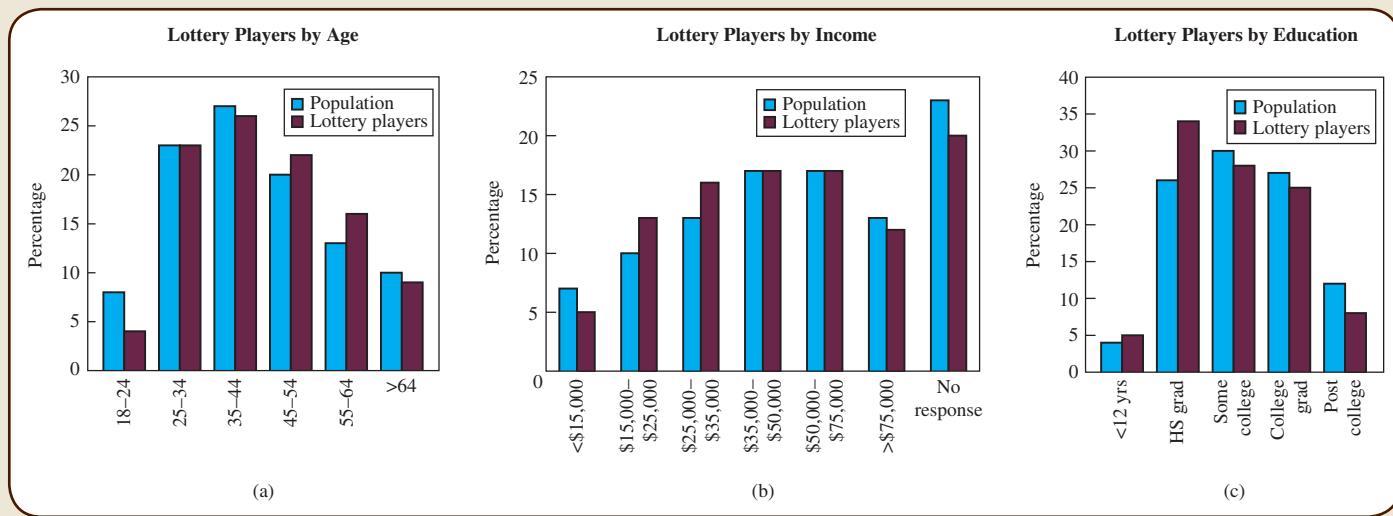


Figure 14 Three figures showing (a) age, (b) income, and (c) education of Colorado lottery players compared to population.

income in the lowest income areas, compared to less than \$5 per \$10,000 of annual income in the highest income areas).

- The number of lottery sales outlets (where lottery tickets can be purchased) is nearly twice as high per 10,000 people in low-income areas as in high-income areas.
- People in areas with the lowest percentage of college education spent over five times as much per \$10,000 of annual income as those in areas with the highest percentage of college education.
- Advertising and promotion of lotteries is focused in low-income areas.

Some of the results of the *New York Times* study are summarized in Figure 15. It suggests that while New Jersey has a progressive tax system (higher-income people pay a greater percentage of their income in taxes), the “lottery tax” is regressive. Moreover, the study found that the areas that

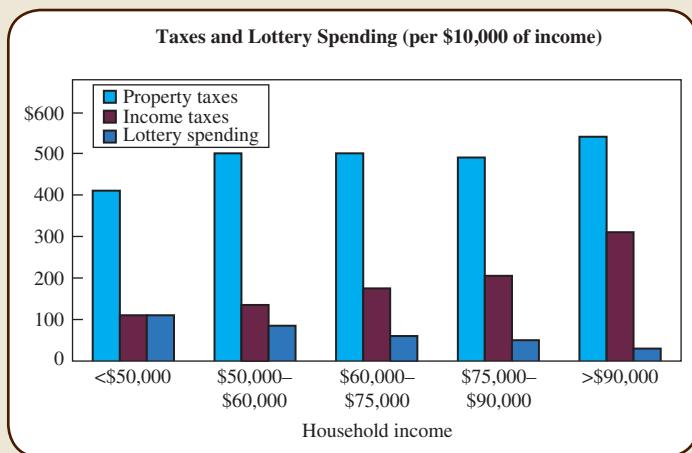


Figure 15 Taxes and lottery spending for New Jersey lottery winners (all figures are per \$10,000 of income).

Source: *New York Times*.

generate the largest percentage of lottery revenues do *not* receive a proportional share of state funding.

Other studies have found similar patterns in other states. The overall conclusions are inescapable: While lotteries provide many benefits to state governments, the revenue they produce comes disproportionately from poorer and less educated individuals.

QUESTIONS FOR DISCUSSION

1. Study Figure 14. Do lottery players appear to be a typical cross-section of American society based on age? Based on income? Based on level of education? Explain. How does the “no response” category affect these conclusions?
 2. Based on the more than \$50 billion in total annual lottery spending and the current U.S. population, about how much does the average person spend on the lottery each year? Now, using the statistics in Figure 14 to estimate the percentage of the population that plays the lottery, about how much does the average lottery player spend each year?
 3. Find and study a particular lottery advertisement, and determine whether it is misleading in any way.
 4. Considering all factors presented in this section and other facts that you can find, do you think lotteries are fair to poor or uneducated people? Should they remain legal? Should they be restricted in any way?
 5. An anonymous quote circulated on the Internet read “Lotteries are a tax on people who are bad at math.” Comment on the meaning and accuracy of this quote.
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FOCUS ON LAW

Is DNA Fingerprinting Reliable?

DNA fingerprinting (also called DNA profiling or DNA identification) is a major tool of law enforcement, used in criminal cases, in paternity cases, and even in the identification of human remains.

The scientific foundation for DNA identification has been in place for many decades. However, these ideas were not used in the courtroom until 1986. The case involved a 17-year-old boy accused of the rape and murder of two schoolgirls in Narborough, in the Midlands of England. During his interrogation, the suspect asked for a blood test, which was sent to the laboratory of a noted geneticist, Alec Jeffreys, at the nearby University of Leicester. Using methods that he had developed for paternity testing, Jeffreys compared the suspect's DNA to that found in samples from the victims. The tests showed that although both rapes and murders were committed by the same person, the person was *not* the suspect in custody. The following year, after more than 4,500 blood samples were collected, researchers made a positive identification of the murderer using Jeffreys's methods. Word of the British case and Jeffreys's methods spread rapidly. The techniques were swiftly tested, commercialized, and promoted.

To explore the essential roles that probability and statistics play in DNA identification, consider a simple eyewitness analogy. Suppose you are looking for a person who helped you out during a moment of need and you remember only three things about this person:

- The person was female.
- She had green eyes.
- She had long red hair.

If you find someone who matches this profile, can you conclude that this person is the woman who helped you? To answer, you need data telling you the probabilities that randomly selected individuals in the population have these characteristics. The probability that a person is female is about 1/2. Let's say that the probability of green eyes is about 0.06 (6% of the population has green eyes) and the probability of long red hair is 0.0075. If we assume that these characteristics occur independently of one another, then the probability that a randomly selected person has all three characteristics is

$$0.5 \times 0.06 \times 0.0075 = 0.000225$$

or about 2 in 10,000. This may seem relatively low, but it probably is not low enough to draw a definitive conclusion. For



example, a profile matched by only 2 in 10,000 people will still be matched by some 200 people in a city with a population of 1 million.

DNA identification is based on a similar idea, but it is designed so that the probability of a profile match is much lower. The DNA of every individual is unique and is the same throughout the individual. A single physical trait is determined by a small piece of DNA called a *gene* at a specific *locus* (location) on a *chromosome* (Figure 16); humans have 23 chromosomes and roughly 30,000 genes. A gene can take two or more (often hundreds) of different forms, called *alleles* (pronounced a-leels). Different alleles give rise to variations of a trait (for example, different hair colors or different blood types). Not only can different alleles appear at a locus, but the corresponding piece of DNA can have different lengths in different people (called *variable number of tandem repeats* or VNTRs). The genetic evidence that is collected and analyzed in the lab consists of the allele lengths or allele types at five to eight different loci.

Collecting genetic evidence (from samples of blood, tissues, hair, semen, or even saliva on a postage stamp) and analyzing it are straightforward, at least in theory. Nevertheless, the process is subject to both controversy and sources of error. Suppose, in our analogy, a person is found with "reddish brown" hair instead of "red" hair. Because many characteristics are continuous (not discrete) variables, should you rule this

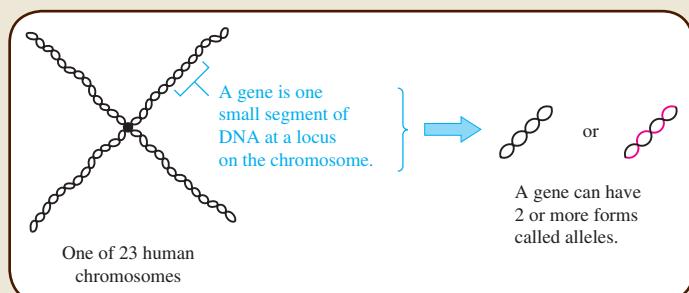


Figure 16 Diagram showing the relationship among chromosomes, alleles, genes, and loci.

person out or assume that reddish brown is close enough? For this reason, the issue of *binning* becomes extremely important. You might choose to include all people with hair that is any shade of red, or you could choose a narrower bin—say, bright red hair only—which would give a more discriminating test.

The same issue arises in genetic tests. When allele types or lengths are measured in the lab, there is enough variability or error in the measurements that these variables are continuous. Bin widths need to be chosen, and the choice is the source of debate. Bins with a small width give a more refined test, exclude more suspects, and ultimately provide stronger evidence against a defendant.

Other sources of scientific and statistical controversy come from assumptions about the independence of different genetic traits and in the populations chosen for measuring the frequency of different traits. For the former, DNA fingerprinting usually assumes that each genetic trait is independent of other genetic traits—which means that probabilities can be calculated with the multiplication rule for independent events—but some scientists suspect that the presence of a particular trait may affect the probability of other traits. For the latter, consider Figure 17, which shows allele data for four different Asian subpopulations. The horizontal axis shows 30 bins for the different allele measurements, and the vertical axis shows

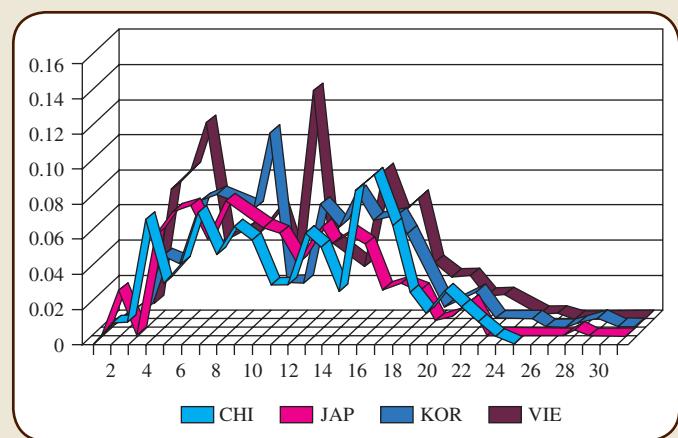


Figure 17 Binned frequency data for one allele, showing variability among four ethnic groups.

Source: Kathryn Roeder, "DNA Fingerprinting: A Review of the Controversy," *Statistical Science*, Vol. 9, No. 2, pp. 222–247.

the frequency for each bin. Because of the significant variation in the curves for the different subpopulations, calculations will yield different probabilities for the guilt of a suspect depending on whether the suspect is compared to his own Asian subpopulation, to the population of all Asian Americans, or to the population of all Americans. Fortunately, the calculations today yield small enough probabilities that DNA fingerprinting is considered very reliable, as long as no errors are made in examining the evidence.

QUESTIONS FOR DISCUSSION

1. The result of a DNA test is considered physical (as opposed to circumstantial) evidence. Yet it is much more sophisticated and difficult to understand than a typical piece of physical evidence, such as a weapon or a piece of clothing. Some people have therefore argued that DNA evidence should not be used in a criminal trial in which jury members may not fully understand how the evidence is collected and analyzed. What do you think of this argument? Defend your opinion.
2. Suppose that an allele has a *greater* frequency in a suspect's subpopulation than in the full population, but that the suspect is compared to the full population. How would this affect the probability of a match between the suspect and a DNA sample from the crime scene?
3. Evidence from blood tests can identify a suspect with a probability of about 1 in 200. Evidence from DNA tests often provides probabilities claimed to be on the order of 1 in 10 million. If you were a juror, would you accept such a probability as positive identification of a suspect?
4. The Innocence Project uses DNA to try to clear suspects wrongfully convicted of crimes. Is innocence easier to establish than guilt through DNA testing? Explain.
5. Discuss a few other ways DNA tests can be useful, such as in settling issues of paternity. Overall, how much do you think DNA evidence affects our society?



ANSWERS

Section 1

1. No, the argument is not valid. With 1000 births, any specific number of girls will have a very low probability, but 501 girls is very close to the 500 girls that is expected, so the result is not evidence that the method is effective.
3. No. Statistical significance at the 0.05 level means that there is less than a 0.05 probability that the result occurred by chance, but a probability less than 0.05 does not necessarily mean that there is less than a 0.01 chance.
5. Does not make sense.
7. Makes sense.
9. Statistically significant. The results are very unlikely to occur by chance.
11. Not statistically significant. We expect to get about one 6 in every six rolls, so not getting any 6s could easily occur by chance.
13. Statistically significant. It is very unlikely that when 20 adults are randomly selected, they are all women.
15. Not statistically significant. The result could easily occur by chance.
17. Even though the sample size is relatively small, such a large (21%) improvement in mileage is significant.
19. With 945 babies, the number of girls would usually be around 472.5, so the result of 879 girls is a substantial departure from the results expected by chance. The results appear to be statistically significant.
21. a. If 100 samples were selected, the mean temperature would be 98.20 or less in 5 or fewer of the samples. b. Selecting a sample with a mean this small is extremely unlikely and would not be expected by chance.
23. This result is not significant at the 0.05 level because the probability of it occurring by chance when there is no real improvement is greater than 0.05.

Section 2

1. $P(A)$ represents the probability that you answer the question correctly, $P(\text{not } A)$ is the probability that you do not answer the question correctly, and $P(\bar{A})$ also represents the probability that you do not answer the question correctly. Each of the three values is 0.5.
3. There is one chance out of 524,288 that all 20 babies are of the same gender. Such an event is unusual because its probability is so small.
5. Makes sense. 7. Makes sense.
9. Does not make sense.
11. $2/3$, assuming that the die is fair and the outcomes are equally likely.
13. $18/38$ or $9/19$, assuming that the 38 slots all have the same chance of being selected.
15. $1/365$, assuming that births on the 365 days are equally likely.

17. $1/2$ or 0.5

19. 0 **21.** $6/7$

23. 0.45 **25.** 0.720

27. 0.22; 0.33; 0.44; 0.56

29. a. $1/8 = 0.125$ (GGG) b. $3/8 = 0.375$ (BBG, BGB, GBB) c. $1/8 = 0.125$ (GBB) d. $7/8 = 0.875$ (GGG, BGG, GBG, GGB, BBG, BGB, GBB) e. $4/8 = 0.5$ (BBG, BGB, GBB, BBB)

31. 0.60 **33.** $P(\text{success}) = 0.86$

35. The probability that a person you meet at random is over 65 will be $34.7/281 = 0.123$ in 2000 and will be $78.9/394 = 0.200$ in 2050. Thus your chances will be greater in 2050.

Outcomes for Rolling Four Fair Coins					
Coin 1	Coin 2	Coin 3	Coin 4	Outcome	Probability
H	H	H	H	HHHH	1/16
H	H	H	T	HHHT	1/16
H	H	T	H	HHTH	1/16
H	H	T	T	HTTT	1/16
H	T	H	H	HTHH	1/16
H	T	H	T	HTHT	1/16
H	T	T	H	HTTH	1/16
H	T	T	T	HTTT	1/16
T	H	H	H	THHH	1/16
T	H	H	T	THHT	1/16
T	H	T	H	THTH	1/16
T	H	T	T	THTT	1/16
T	T	H	H	TTHH	1/16
T	T	H	T	TTHT	1/16
T	T	T	H	TTTH	1/16
T	T	T	T	TTTT	1/16

Probability Distribution for the Number of Heads in Rolling Four Fair Coins	
Result	Probability
4 Heads (0 Tails)	1/16
3 Heads (1 Tail)	4/16
2 Heads (2 Tails)	6/16
1 Head (3 Tails)	4/16
0 Heads (4 Tails)	1/16
Total	1

c. $6/16 = 0.375$ **d.** $15/16 = 0.9375$
(Probability is 0.375) **e.** 2 heads

Section 3

1. If some process is repeated over many trials, the proportion of trials in which some event occurs will be close to the probability of the event. As the number of trials increases, the proportion of

trials in which the event occurs gets closer to the probability of the event.

3. His betting strategy is unwise because the casino continues to have an advantage. His past performance does not affect future events. His reasoning is not correct. Although his proportion of winning bets is likely to increase, it could increase while he continues to lose more money. This flawed reasoning has caused many gamblers to lose large amounts of money.

5. Makes sense. **7.** Does not make sense.

9. No. You should not expect to get exactly 250 boys and 250 girls, because the probability of that particular outcome is very small. As the number of births increases, the proportion of girls should get closer to 0.5.

11. Expected value is $-\$3$. Because the expected value is negative, the game will produce a loss in the long run, so you should not play.

13. 0.94; 0.74

15. 15 minutes **17.** $-\$0.78$; $-\$285$

19. -7.07% ; -1.4%

21. a. **Decision 1**—Option A: Expected value = $\$1,000,000$; Option B: Expected value = $\$1,140,000$. **Decision 2**—Option A: Expected value = $\$110,000$; Option B: Expected value = $\$250,000$ **b.** Responses are not consistent with expected values in Decision 1, but they are consistent in Decision 2. **c.** It appears that people choose the certain outcome ($\$1,000,000$) in Decision 1.

23. a. 0.5; 0.5 **b.** Loss of $\$10$ **c.** Loss of $\$16$ **d.** Loss of $\$20$ **e.** 45%; 46%; 48%. The percentage of even numbers approaches 50%, but the difference between the numbers of even and odd numbers increases. **f.** 60 even numbers

Section 4

1. Because they are expressed as the numbers of births for each 1000 people in the population, the birth rates can be compared directly. The actual numbers of births should be considered in the context of the population sizes of the countries, so a comparison would require the numbers of births along with population sizes, and even then the comparison would not be easy because of the numbers involved.

3. Life expectancy is the number of additional years a person with a given age can expect to live on average. A 30-year-old person will have a shorter life expectancy than a 20-year-old person because the 30-year-old person is not expected to live as many additional years as the 20-year-old.

5. Does not make sense. **7.** Makes sense.

9. 2000: 0.0102; 2004: 0.0013; 2008: 0.0003. The year 2008 was the safest because it had the lowest number of fatalities per 1000 departures.

11. 2000: 0.1381; 2004: 0.0201; 2008: 0.0043. The year 2008 was the safest because it had the lowest number of fatalities per million passengers.

13. 58.8 years

15. 8.3 deaths per 10,000 people

17. Utah: 20.3; Maine: 10.3

19. a. 4,319,400 **b.** 2,629,200 **c.** 1,690,200
d. 0.0054; 0.54%

Section 5

1. The occurrence of event *A* does not affect the probability of event *B*, so the two events are independent.

3. Sampling with replacement. The second outcome is independent of the first.

5. Does not make sense.

7. Makes sense.

9. 1/2 or 0.5 **11.** 1/260,000

13. a. 0.0039 **b.** 0.00098 **c.** 0.125 **d.** 0.0625
e. $1/60 = 0.0167$. There are 60 equally likely songs available for each selection. No matter which song is played first, the probability that the next one is the same is $1/60 = 0.0167$.

15. $P(\text{guilty plea or sent to prison}) = 1014/1028 = 0.986$

17. 0.865 **19.** 0.381 **21.** 0.410

23. 0.920 **25.** 0.0195

27. a. 0.733 **b.** 1 **c.** 0.643 **d.** 0.22

29. a. 0.5625 **b.** 0.375 **c.** 0.0625
d.

Event	Probability
AA	0.5625
Aa	0.1875
aA	0.1875
aa	0.0625

31. a. 0.518 **b.** 0.491

Chapter Review Exercises

1. $576/3562 = 0.162$

3. $3466/3562 = 0.973$

5. $2330/3562 = 0.654$

7. $480/576 = 0.833$

9. a. 0.73 **b.** 0.073 **c.** 1.35 **d.** 0.0014; you should doubt the stated yield.

Chapter Quiz

1. 0.30

3. Answer varies, but an answer such as 0.01 or lower is reasonable.

5. 0.6 **7.** $572/586 = 0.976$

9. $10/586 = 0.0171$

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Correlation and Causality

Does smoking cause lung cancer? Are drivers more dangerous when on their cell phones? Is human activity causing global warming? A major goal of many statistical studies is to search for relationships among different variables so that researchers can then determine whether one factor *causes* another. Once a relationship is discovered, we can try to determine whether there is an underlying cause. In this chapter, we will study relationships known as correlations and explore how they are important to the more difficult task of searching for causality.

The person who knows "how"
will always have a job.

The person who knows "why"
will always be his boss.

—Diane Ravitch

LEARNING GOALS

1 Seeking Correlation

Define correlation, explore correlations with scatterplots, and understand the correlation coefficient as a measure of the strength of a correlation.

2 Interpreting Correlations

Be aware of important cautions concerning the interpretation of correlations, especially the effects of outliers, the effects of grouping data, and the crucial fact that correlation does not necessarily imply causality.

3 Best-Fit Lines and Prediction

Become familiar with the concept of a best-fit line, recognize when such lines have predictive value and when they do not, and understand the general concept of multiple regression.

4 The Search for Causality

Understand the difficulty of establishing causality from correlation, and investigate guidelines that can be used to help establish confidence in causality.

FOCUS TOPICS

Focus on Education: What Helps Children Learn to Read?

Focus on Environment: What Is Causing Global Warming?

1

SEEKING CORRELATION

Smoking is one of the leading causes of statistics.

—Fletcher Knebel

What does it mean when we say that smoking *causes* lung cancer? It certainly does *not* mean that you'll get lung cancer if you smoke a single cigarette. It does not even mean that you'll definitely get lung cancer if you smoke heavily for many years, as some heavy smokers do not get lung cancer. Rather, it is a *statistical* statement meaning that you are *much more likely* to get lung cancer if you smoke than if you don't smoke.

How did researchers learn that smoking causes lung cancer? The process began with informal observations, as doctors noticed that a surprisingly high proportion of their patients with lung cancer were smokers. These observations led to carefully conducted studies in which researchers compared lung cancer rates among smokers and nonsmokers. These studies showed clearly that heavier smokers were more likely to get lung cancer. In more formal terms, we say that there is a **correlation** between the variables *amount of smoking* and *likelihood of lung cancer*. A correlation is a special type of relationship between variables, in which a rise or fall in one goes along with a corresponding rise or fall in the other.

Definition

A **correlation** exists between two variables when higher values of one variable consistently go with higher values of another variable or when higher values of one variable consistently go with lower values of another variable.

Here are a few other examples of correlations:

- There is a correlation between the variables *height* and *weight* for people; that is, taller people tend to weigh more than shorter people.
- There is a correlation between the variables *demand for apples* and *price of apples*; that is, demand tends to decrease as price increases.
- There is a correlation between *practice time* and *skill* among piano players; that is, those who practice more tend to be more skilled.

BY THE WAY

Smoking is linked to many serious diseases besides lung cancer, including heart disease and emphysema. Smoking is also linked with many less lethal health conditions, such as premature skin wrinkling and sexual impotence.

It's important to realize that establishing a correlation between two variables does *not* mean that a change in one variable *causes* a change in the other. The correlation between smoking and lung cancer did not by itself prove that smoking causes lung cancer. We could imagine, for example, that some gene predisposes a person both to smoking and to lung cancer. Nevertheless, identifying the correlation was the crucial first step in learning that smoking causes lung cancer. We will discuss the difficult task of establishing causality later in this chapter. For now, we concentrate on how we look for, identify, and interpret correlations.

TIME OUT TO THINK

Suppose there really were a gene that made people prone to both smoking and lung cancer. Explain why we would still find a strong correlation between smoking and lung cancer in that case, but would not be able to say that smoking causes lung cancer.

Scatterplots

Table 1 lists data for a sample of gem-store diamonds—their prices and several common measures that help determine their value. Because advertisements for diamonds often quote only their weights (in carats), we might suspect a correlation between the weights and the prices. We can look for such a correlation by making a **scatterplot** (or *scatter diagram*) showing the relationship between the variables *weight* and *price*.

TABLE 1**Prices and Characteristics of a Sample of 23 Diamonds from Gem Dealers**

Diamond	Price	Weight (carats)	Depth	Table	Color	Clarity
1	\$6,958	1.00	60.5	65	3	4
2	\$5,885	1.00	59.2	65	5	4
3	\$6,333	1.01	62.3	55	4	4
4	\$4,299	1.01	64.4	62	5	5
5	\$9,589	1.02	63.9	58	2	3
6	\$6,921	1.04	60.0	61	4	4
7	\$4,426	1.04	62.0	62	5	5
8	\$6,885	1.07	63.6	61	4	3
9	\$5,826	1.07	61.6	62	5	5
10	\$3,670	1.11	60.4	60	9	4
11	\$7,176	1.12	60.2	65	2	3
12	\$7,497	1.16	59.5	60	5	3
13	\$5,170	1.20	62.6	61	6	4
14	\$5,547	1.23	59.2	65	7	4
15	\$7,521	1.29	59.6	59	6	2
16	\$7,260	1.50	61.1	65	6	4
17	\$8,139	1.51	63.0	60	6	4
18	\$12,196	1.67	58.7	64	3	5
19	\$14,998	1.72	58.5	61	4	3
20	\$9,736	1.76	57.9	62	8	2
21	\$9,859	1.80	59.6	63	5	5
22	\$12,398	1.88	62.9	62	6	2
23	\$11,008	2.03	62.0	63	8	3

Notes: Weight is measured in carats (1 carat = 0.2 gram). Depth is defined as 100 times the ratio of height to diameter.

Table is the size of the upper flat surface. (Depth and table determine "cut.") Color and clarity are each measured on standard scales, where 1 is best. For color, 1 = colorless, and increasing numbers indicate more yellow. For clarity, 1 = flawless, and 6 indicates that defects can be seen by eye.

BY THE WAY

The word *karats* (with a *k*) used to describe gold does not have the same meaning as the term *carats* (with a *c*) for diamonds and other gems. A carat is a measure of weight equal to 0.2 gram. Karats are a measure of the purity of gold: 24-karat gold is 100% pure gold; 18-karat gold is 75% pure (and 25% other metals); 12-karat gold is 50% pure (and 50% other metals); and so on.



Definition

A **scatterplot** (or *scatter diagram*) is a graph in which each point represents the values of two variables.

Figure 1 shows the scatterplot, which can be constructed with the following procedure.

- We assign one variable to each axis and label the axis with values that comfortably fit all the data. Sometimes the axis selection is arbitrary, but if we suspect that one variable depends on the other then we plot the *explanatory variable* on the horizontal axis and the *response variable* on the vertical axis. In this case, we expect the diamond price to depend at least in part on its weight; we therefore say that *weight* is the explanatory variable (because it helps

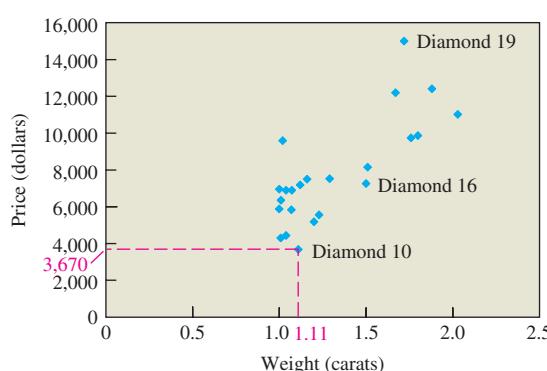


Figure 1 Scatterplot showing the relationship between the variables *price* and *weight* for the diamonds in Table 1. The dashed lines show how we find the position of the point for Diamond 10.

*explain the price) and *price* is the response variable (because it *responds* to changes in the explanatory variable). We choose a range of 0 to 2.5 carats for the *weight* axis and \$0 to \$16,000 for the *price* axis.*

2. For each diamond in Table 1, we plot a *single point* at the horizontal position corresponding to its weight and the vertical position corresponding to its price. For example, the point for Diamond 10 goes at a position of 1.11 carats on the horizontal axis and \$3,670 on the vertical axis. The dashed lines on Figure 1 show how we locate this point.
3. (Optional) We can label some (or all) of the data points, as is done for Diamonds 10, 16, and 19 in Figure 1.

Scatterplots get their name because the way in which the points are scattered may reveal a relationship between the variables. In Figure 1, we see a general upward trend indicating that diamonds with greater weight tend to be more expensive. The correlation is not perfect. For example, the heaviest diamond is not the most expensive. But the overall trend seems fairly clear.

TIME OUT TO THINK

Identify the points in Figure 1 that represent Diamonds 3, 7, and 23.

EXAMPLE 1 Color and Price

Using the data in Table 1, create a scatterplot to look for a correlation between a diamond's *color* and *price*. Comment on the correlation.

SOLUTION We expect price to depend on color, so we plot the explanatory variable *color* on the horizontal axis and the response variable *price* on the vertical axis in Figure 2. (You should check a few of the points against the data in Table 1.) The points appear much more scattered than in Figure 1. Nevertheless, you may notice a weak trend diagonally downward from the upper left toward the lower right. This trend represents a weak correlation in which diamonds with more yellow color (higher numbers for color) are less expensive. This trend is consistent with what we would expect, because colorless diamonds appear to sparkle more and are generally considered more desirable.

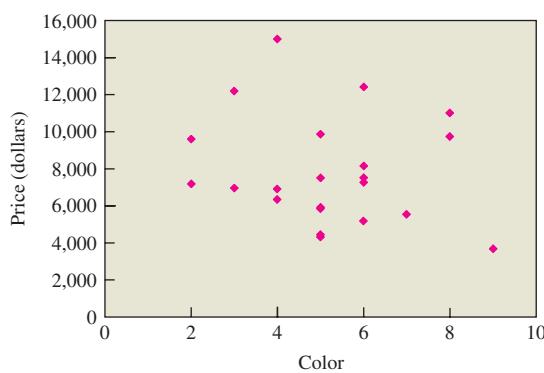


Figure 2 Scatterplot for the color and price data in Table 1.



TIME OUT TO THINK

Thanks to a large bonus at work, you have a budget of \$6,000 for a diamond ring. A dealer offers you the following two choices for that price. One diamond weighs 1.20 carats and has color = 4. The other weighs 1.18 carats and has color = 3. Assuming all other characteristics of the diamonds are equal, which would you choose? Why?

Types of Correlation

We have seen two examples of correlation. Figure 1 shows a fairly strong correlation between weight and price, while Figure 2 shows a weak correlation between color and price. We are now ready to generalize about types of correlation. Figure 3 shows eight scatterplots for variables called x and y . Note the following key features of these diagrams:

- Parts a to c show **positive correlations**: The values of y tend to increase with increasing values of x . The correlation becomes stronger as we proceed from a to c. In fact, c shows a perfect positive correlation, in which all the points fall along a straight line.
- Parts d to f show **negative correlations**: The values of y tend to decrease with increasing values of x . The negative correlation becomes stronger as we proceed from d to f. In fact, f shows a perfect negative correlation, in which all the points fall along a straight line.
- Part g shows **no correlation** between x and y : Values of x do not appear to be linked to values of y in any way.
- Part h shows a **nonlinear relationship**: x and y appear to be related but the relationship does not correspond to a straight line. (*Linear* means along a straight line, and *nonlinear* means *not* along a straight line.)

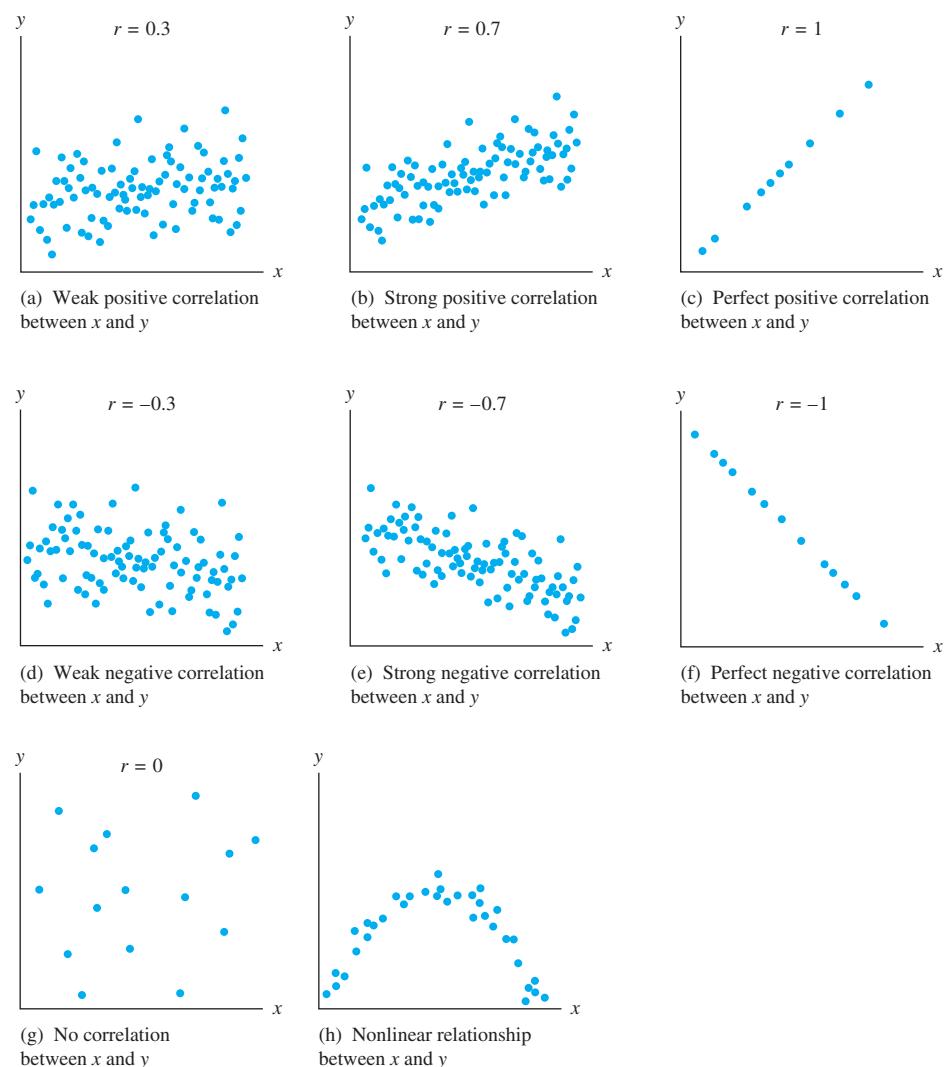


Figure 3 Types of correlation seen on scatterplots.

Types of Correlation

Positive correlation: Both variables tend to increase (or decrease) together.

Negative correlation: The two variables tend to change in opposite directions, with one increasing while the other decreases.

No correlation: There is no apparent (linear) relationship between the two variables.

Nonlinear relationship: The two variables are related, but the relationship results in a scatterplot that does not follow a straight-line pattern.

TECHNICAL NOTE

In this text we use the term *correlation* only for *linear* relationships. Some statisticians refer to nonlinear relationships as “*nonlinear correlations*.” There are techniques for working with nonlinear relationships that are similar to those described in this text for linear relationships.

EXAMPLE 2 Life Expectancy and Infant Mortality

Figure 4 shows a scatterplot for the variables *life expectancy* and *infant mortality* in 16 countries. What type of correlation does it show? Does this correlation make sense? Does it imply causality? Explain.

Life Expectancy and Infant Deaths

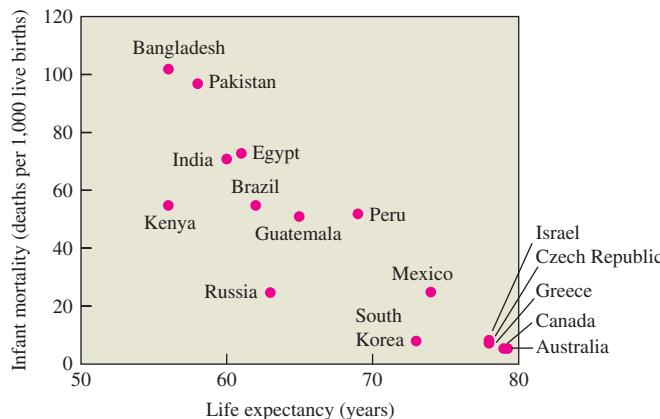


Figure 4 Scatterplot for life expectancy and infant mortality data.

Source: United Nations.

SOLUTION The diagram shows a moderate negative correlation in which countries with *lower* infant mortality tend to have *higher* life expectancy. It is a *negative* correlation because the two variables vary in opposite directions. The correlation makes sense because we would expect that countries with better health care would have both lower infant mortality and higher life expectancy. However, it does *not* imply causality between infant mortality and life expectancy: We would not expect that a concerted effort to reduce infant mortality would increase life expectancy significantly unless it was part of an overall effort to improve health care. (Reducing infant mortality will *slightly* increase life expectancy because having fewer infant deaths tends to raise the mean age of death for the population.)

• •

Measuring the Strength of a Correlation

For most purposes, it is enough to state whether a correlation is strong, weak, or nonexistent. However, sometimes it is useful to describe the strength of a correlation in more precise terms. Statisticians measure the strength of a correlation with a number called the **correlation coefficient**, represented by the letter r . The correlation coefficient is easy to calculate in principle, but the actual work is tedious unless you use a calculator or computer.

We can explore the interpretation of correlation coefficients by studying Figure 3, which shows the value of the correlation coefficient r for each scatterplot. Notice that the correlation coefficient is always between -1 and 1 . When points in a scatterplot lie close to an ascending straight line, the correlation coefficient is positive and close to 1 . When all the points lie close to a descending straight line, the correlation coefficient is negative with a value close to -1 . Points that do not fit any type of straight-line pattern or that lie close to a *horizontal* straight line (indicating that the y values have no dependence on the x values) result in a correlation coefficient close to 0 .

TECHNICAL NOTE

For the methods of this section, there is a requirement that the two variables result in data having a “bivariate normal distribution.” This basically means that for any fixed value of one variable, the corresponding values of the other variable have a normal distribution. This requirement is usually very difficult to check, so the check is often reduced to verifying that both variables result in data that are normally distributed.

Properties of the Correlation Coefficient, r

- The correlation coefficient, r , is a measure of the strength of a correlation. Its value can range only from -1 to 1 .
- If there is no correlation, the points do not follow any ascending or descending straight-line pattern, and the value of r is close to 0 .
- If there is a positive correlation, the correlation coefficient is positive ($0 < r \leq 1$): Both variables increase together. A perfect positive correlation (in which all the points on a scatterplot lie on an ascending straight line) has a correlation coefficient $r = 1$. Values of r close to 1 indicate a strong positive correlation and positive values closer to 0 indicate a weak positive correlation.
- If there is a negative correlation, the correlation coefficient is negative ($-1 \leq r < 0$): When one variable increases, the other decreases. A perfect negative correlation (in which all the points lie on a descending straight line) has a correlation coefficient $r = -1$. Values of r close to -1 indicate a strong negative correlation and negative values closer to 0 indicate a weak negative correlation.

EXAMPLE 3 U.S. Farm Size

Figure 5 shows a scatterplot for the variables *number of farms* and *mean farm size* in the United States. Each dot represents data from a single year between 1950 and 2000; on this diagram, the earlier years generally are on the right and the later years on the left. Estimate the correlation coefficient by comparing this diagram to those in Figure 3 and discuss the underlying reasons for the correlation.

BY THE WAY

In 1900, more than 40% of the U.S. population worked on farms; by 2000, less than 2% of the population worked on farms.

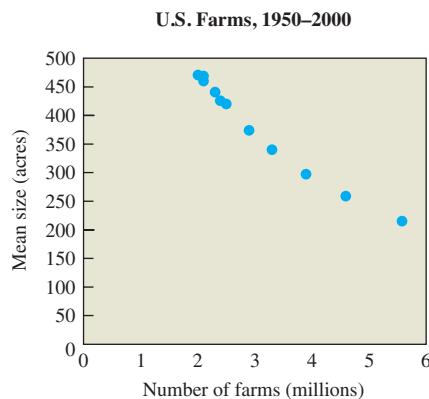


Figure 5 Scatterplot for farm size data.

Source: U.S. Department of Agriculture.

SOLUTION The scatterplot shows a strong negative correlation that most closely resembles the scatterplot in Figure 3f, suggesting a correlation coefficient around $r = -0.9$. The correlation shows that when there were fewer farms, they tended to have a larger mean size, and when there were more farms, they tended to have a smaller mean size. This trend reflects a basic change in the nature of farming: Prior to 1950, most farms were small family farms. Over time, these small farms were replaced by large farms owned by agribusiness corporations. •••

EXAMPLE 4 Accuracy of Weather Forecasts

The scatterplots in Figure 6 show two weeks of data comparing the actual high temperature for the day with the same-day forecast (part a) and the three-day forecast (part b). Estimate the correlation coefficient for each data set and discuss what these coefficients imply about weather forecasts.

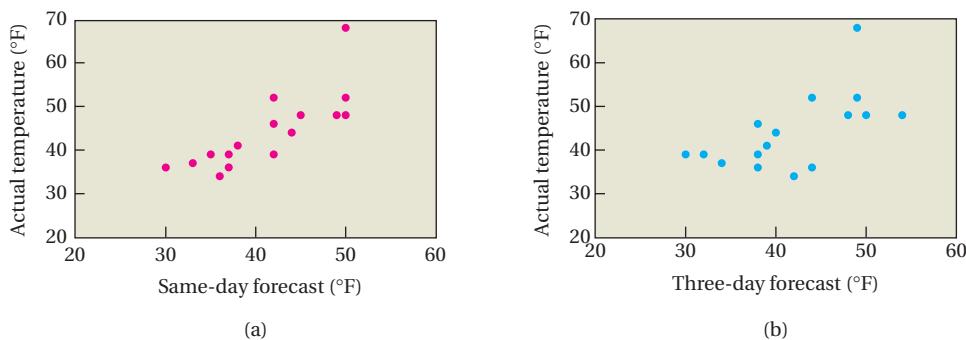


Figure 6 Comparison of actual high temperatures with (a) same-day and (b) three-day forecasts.

SOLUTION If every forecast were perfect, each actual temperature would equal the corresponding forecasted temperature. This would result in all points lying on a straight line and a correlation coefficient of $r = 1$. In Figure 6a, in which the forecasts were made at the beginning of the same day, the points lie fairly close to a straight line, meaning that same-day forecasts are closely related to actual temperatures. By comparing this scatterplot to the diagrams in Figure 3, we can reasonably estimate this correlation coefficient to be about $r = 0.8$. The correlation is weaker in Figure 6b, indicating that forecasts made three days in advance aren't as close to actual temperatures as same-day forecasts. This correlation coefficient is about $r = 0.6$. These results are unsurprising because we expect longer-term forecasts to be less accurate. •••

TIME OUT TO THINK

For further practice, visually estimate the correlation coefficients for the data for diamond weight and price (Figure 1) and diamond color and price (Figure 2).

Calculating the Correlation Coefficient (Optional Section)

The formula for the (linear) correlation coefficient r can be expressed in several different ways that are all algebraically equivalent, which means that they produce the same value. The following expression has the advantage of relating more directly to the underlying rationale for r :

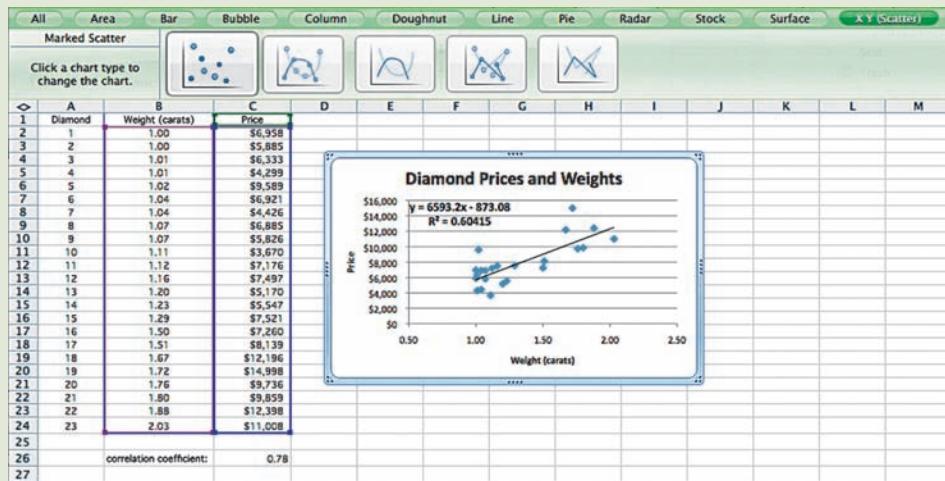
$$r = \frac{\sum \left[\frac{(x - \bar{x})(y - \bar{y})}{s_x s_y} \right]}{n - 1}$$



USING TECHNOLOGY—SCATTERPLOTS AND CORRELATION COEFFICIENTS

Excel The screen shot below shows the process for making a scatterplot like that in Figure 1:

1. Enter the data, which are shown in Columns B (weight) and C (price).
2. Select the columns for the two variables on the scatterplot; in this case, Columns B and C.
3. Choose “XY Scatter” as the chart type, with no connecting lines. You can then use the “chart options” (which comes up with a right-click in the graph) to customize the design, axis range, labels, and more.
4. To calculate the correlation coefficient, shown in row 26, use the built-in function CORREL.
5. [Optional] The straight line on the graph, called a best-fit line, is added by choosing the option to “AddTrendline”; be sure to choose the “linear” option for the trendline. You’ll also find options that add the two items shown in the upper left of the graph: the equation of the line and the value R^2 , which is the square of the correlation coefficient. Best-fit lines and R^2 are discussed in Section 3.



Microsoft Excel 2008 for Mac.

STATDISK Enter the paired data in columns of the STATDISK Data Window. Select **Analysis** from the main menu bar, then select the option **Correlation and Regression**. Select the columns of data to be used, then click on the **Evaluate** button. The STATDISK display will include the value of the linear correlation coefficient r and other. A scatterplot can also be obtained by clicking on the **PLOT** button.

TI-83/84 Plus Enter the paired data in lists L1 and L2, then press **STAT** and select **TESTS**. Using the option of **LinRegTTest** will result in several displayed values, including the value of the linear correlation coefficient r .

To obtain a scatterplot, press **2ND**, then **Y=** (for STAT PLOT). Press **ENTER** **ENTER** to turn Plot 1 on, then select the first graph type, which resembles a scatterplot. Set the **X** list and **Y** list labels to L1 and L2 and press **ZOOM**, then select **ZoomStat** and press **ENTER**.

In the above expression, division by $n - 1$ (where n is the number of pairs of data) shows that r is a type of average, so it does not increase simply because more pairs of data values are included. The symbol s_x denotes the standard deviation of the x values (or the values of the first variable), and s_y denotes the standard deviation of the y values. The expression $(x - \bar{x})/s_x$ is in the same format as the *standard score*. By using the standard scores for x and y , we ensure that the value of r does not change simply because a different scale of values is used. The key to understanding the rationale for r is to focus on the product of the standard scores for x and the standard scores for y . Those products tend to be positive when there is a positive correlation, and they tend to be negative when there is a negative correlation. For data with no correlation, some of the products are positive and some are negative, with the net effect that the sum is relatively close to 0.

The following alternative formula for r has the advantage of simplifying calculations, so it is often used whenever manual calculations are necessary. The following formula is also easy to program into statistical software or calculators:

$$r = \frac{n \times \sum(x \times y) - (\sum x) \times (\sum y)}{\sqrt{n \times (\sum x^2) - (\sum x)^2} \times \sqrt{n \times (\sum y^2) - (\sum y)^2}}$$

This formula is straightforward to use, at least in principle: First calculate each of the required sums, then substitute the values into the formula. Be sure to note that $(\sum x^2)$ and $(\sum x)^2$ are *not* equal: $(\sum x^2)$ tells you to first square all the values of the variable x and then add them; $(\sum x)^2$ tells you to add the x values first and then square this sum. In other words, perform the operation within the parentheses first. Similarly, $(\sum y^2)$ and $(\sum y)^2$ are not the same.

Section 1 Exercises

Statistical Literacy and Critical Thinking

- Correlation.** In the context of correlation, what does r measure, and what is it called?
- Scatterplot.** What is a scatterplot, and how does it help us investigate correlation?
- Correlation.** After computing the correlation coefficient r from 50 pairs of data, you find that $r = 0$. Does it follow that there is no relationship between the two variables? Why or why not?
- Scatterplot.** One set of paired data results in $r = 1$ and a second set of paired data results in $r = -1$. How do the corresponding scatterplots differ?

Does It Make Sense? For Exercises 5–8, decide whether the statement makes sense (or is clearly true) or does not make sense (or is clearly false). Explain clearly; not all of these statements have definitive answers, so your explanation is more important than your chosen answer.

- Births.** A study showed that for one town, as the stork population increased, the number of births in the town also increased. It therefore follows that the increase in the stork population caused the number of births to increase.
- Positive Effect.** An engineer for a car company finds that by reducing the weights of various cars, mileage (mi/gal) increases. Because this is a positive result, we say that there is a positive correlation.

7. Correlation. Two studies both found a correlation between low birth weight and weakened immune systems. The second study had a much larger sample size, so the correlation it found must be stronger.

8. Interpreting r . In investigating correlations between many different pairs of variables, in each case the correlation coefficient r must fall between -1 and 1 .

Concepts and Applications

Types of Correlation. Exercises 9–16, list pairs of variables. For each pair, state whether you believe the two variables are correlated. If you believe they are correlated, state whether the correlation is positive or negative. Explain your reasoning.

9. Weight/Cost. The weights and costs of 50 different bags of apples

10. IQ/Hat Size. The IQ scores and hat sizes of randomly selected adults

11. Weight/Fuel Efficiency. The total weights of airliners flying from New York to San Francisco and the fuel efficiency as measured in miles per gallon

12. Weight/Fuel Consumption. The total weights of airliners flying from New York to San Francisco and the total amounts of fuel that they consume

13. Points and DJIA. The total number of points scored in Super Bowl football games and the changes in the Dow Jones Industrial stock index in the years following those games

14. Altitude/Temperature. The outside air temperature and the altitude of aircraft

15. Height/SAT Score. The heights and SAT scores of randomly selected subjects who take the SAT

16. Golf Score/Prize Money. Golf scores and prize money won by professional golfers

17. Crickets and Temperature. One classic application of correlation involves the association between the temperature and the number of times a cricket chirps in a minute. The scatterplot in Figure 7 shows the relationship for eight different pairs of temperature/chirps data. Estimate the correlation coefficient and determine whether there appears to be a correlation between the temperature and the number of times a cricket chirps in a minute.

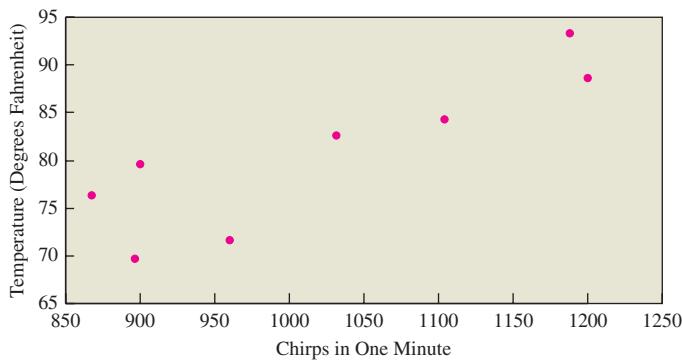


Figure 7 Scatterplot for cricket chirps and temperature.

Source: Based on data from *The Song of Insects* by George W. Pierce, Harvard University Press.

18. Two-Day Forecast. Figure 8 shows a scatterplot in which the actual high temperature for the day is compared with a forecast made two days in advance. Estimate the correlation coefficient and discuss what these data imply about weather forecasts. Do you think you would get similar results if you made similar diagrams for other two-week periods? Why or why not?

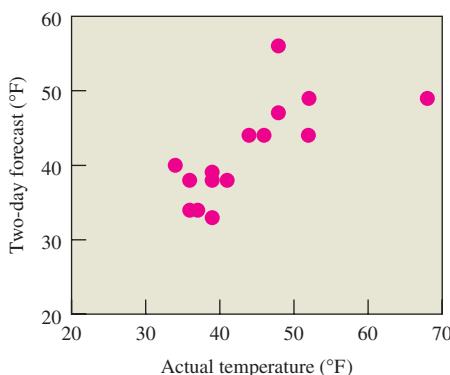


Figure 8

19. Safe Speeds? Consider the following table showing speed limits and death rates from automobile accidents in selected countries.

Country	Death rate (per 100 million vehicle-miles)	Speed limit (miles per hour)
Norway	3.0	55
United States	3.3	55
Finland	3.4	55
Britain	3.5	70
Denmark	4.1	55
Canada	4.3	60
Japan	4.7	55
Australia	4.9	65
Netherlands	5.1	60
Italy	6.1	75

Source: D. J. Rivkin, *New York Times*.

- Construct a scatterplot of the data.
- Briefly characterize the correlation in words (for example, strong positive correlation, weak negative correlation) and estimate the correlation coefficient of the data. (Or calculate the correlation coefficient exactly with the aid of a calculator or software.)
- In the newspaper, these data were presented in an article titled “Fifty-five mph speed limit is no safety guarantee.” Based on the data, do you agree with this claim? Explain.

20. Population Growth. Consider the following table showing percentage change in population and birth rate (per 1,000 of population) for 10 states over a period of 10 years.

State	Percentage change in population	Birth rate
Nevada	50.1%	16.3
California	25.7%	16.9
New Hampshire	20.5%	12.5
Utah	17.9%	21.0
Colorado	14.0%	14.6
Minnesota	7.3%	13.7
Montana	1.6%	12.3
Illinois	0%	15.5
Iowa	-4.7%	13.0
West Virginia	-8.0%	11.4

Source: U.S. Census Bureau and Department of Health and Human Services.

- Construct a scatterplot for the data.
- Briefly characterize the correlation in words and estimate the correlation coefficient.
- Overall, does birth rate appear to be a good predictor of a state’s population growth rate? If not, what other factor(s) may be affecting the growth rate?

21. Brain Size and Intelligence. The table below lists brain sizes (in cm^3) and Wechsler IQ scores of subjects (based on data from “Brain Size, Head Size, and Intelligence Quotient in Monozygotic Twins,” by Tramo et al, *Neurology*, Vol. 50, No. 5). Is there sufficient evidence to conclude that there is

a linear correlation between brain size and IQ score? Does it appear that people with larger brains are more intelligent?

Brain Size	IQ	Brain Size	IQ
965	90	1,077	97
1,029	85	1,037	124
1,030	86	1,068	125
1,285	102	1,176	102
1,049	103	1,105	114

- Construct a scatterplot for the data.
- Briefly characterize the correlation in words and estimate the correlation coefficient.
- Do these data suggest that people with larger brains are more intelligent? Explain.

- 22. Movie Data.** Consider the following table showing total box office receipts and total attendance for all American films.

Year	Total Gross Receipts (billions of dollars)	Tickets Sold (billions)
2001	8.4	1.49
2002	9.2	1.58
2003	9.2	1.53
2004	9.4	1.51
2005	8.8	1.38
2006	9.2	1.41
2007	9.7	1.40
2008	9.6	1.34
2009	10.6	1.41
2010	10.6	1.34

Source: Motion Picture Association of America.

- Construct a scatterplot of the data.
 - Briefly characterize the correlation in words and estimate the correlation coefficient.
- 23. TV Time.** Consider the following table showing the average hours of television watched in households in five categories of annual income.

Household income	Weekly TV hours
Less than \$30,000	56.3
\$30,000–\$40,000	51.0
\$40,000–\$50,000	50.5
\$50,000–\$60,000	49.7
More than \$60,000	48.7

Source: Nielsen Media Research.

- Construct a scatterplot for the data. To locate the dots, use the midpoint of each income category. Use a value of \$25,000 for the category “less than \$30,000,” and use \$70,000 for “more than \$60,000.”
- Briefly characterize the correlation in words and estimate the correlation coefficient.
- Suggest a reason why families with higher incomes watch less TV. Do you think these data imply that you can increase your income simply by watching less TV? Explain.

- 24. January Weather.** Consider the following table showing January mean monthly precipitation and mean daily high temperature for ten Northern Hemisphere cities (National Oceanic and Atmospheric Administration).

City	Mean daily high temperature for January (°F)	Mean January precipitation (inches)
Athens	54	2.2
Bombay	88	0.1
Copenhagen	36	1.6
Jerusalem	55	5.1
London	44	2.0
Montreal	21	3.8
Oslo	30	1.7
Rome	54	3.3
Tokyo	47	1.9
Vienna	34	1.5

Source: *The New York Times Almanac*.

- Construct a scatterplot for the data.
 - Briefly characterize the correlation in words and estimate the correlation coefficient.
 - Can you draw any general conclusions about January temperatures and precipitation from these data? Explain.
- 25. Retail Sales.** Consider the following table showing one year's total sales (revenue) and profits for eight large retailers in the United States.

Company	Total sales (billions of dollars)	Profits (billions of dollars)
Wal-Mart	315.6	11.2
Kroger	60.6	0.98
Home Depot	81.5	5.8
Costco	60.1	1.1
Target	52.6	2.4
Starbuck's	7.8	0.6
The Gap	16.0	1.1
Best Buy	30.8	1.1

Source: Fortune.com.

- Construct a scatterplot for the data.
- Briefly characterize the correlation in words and estimate the correlation coefficient.
- Discuss your observations. Does higher sales volume necessarily translate into greater earnings? Why or why not?

- 26. Calories and Infant Mortality.** Consider the following table showing mean daily caloric intake (all residents) and infant mortality rate (per 1,000 births) for 10 countries.

Country	Mean daily calories	Infant mortality rate (per 1,000 births)
Afghanistan	1,523	154
Austria	3,495	6
Burundi	1,941	114
Colombia	2,678	24
Ethiopia	1,610	107
Germany	3,443	6
Liberia	1,640	153
New Zealand	3,362	7
Turkey	3,429	44
United States	3,671	7

- a. Construct a scatterplot for the data.
- b. Briefly characterize the correlation in words and estimate the correlation coefficient.
- c. Discuss any patterns you observe and any general conclusions that you can reach.

Properties of the Correlation Coefficient. For Exercises 27 and 28, determine whether the given property is true, and explain your answer.

27. Interchanging Variables. The correlation coefficient remains unchanged if we interchange the variables x and y .

28. Changing Units of Measurement. The correlation coefficient remains unchanged if we change the units used to measure x , y , or both.



PROJECTS FOR THE INTERNET & BEYOND

29. Unemployment and Inflation. Use the Bureau of Labor Statistics Web page to find monthly unemployment rates and inflation rates over the past year. Construct a scatterplot for the data. Do you see any trends?

30. Success in the NFL. Find last season's NFL team statistics. Construct a table showing the following for each team: number of wins, average yards gained on offense per game, and average yards allowed on defense per game. Make scatterplots to explore the correlations between offense and wins and between defense and wins. Discuss your findings. Do you think that there are other team statistics that would yield stronger correlations with the number of wins?

31. Statistical Abstract. Explore the "frequently requested tables" at the Web site for the *Statistical Abstract of the United States*. Choose data that are of interest to you and explore at least two correlations. Briefly discuss what you learn from the correlations.

32. Height and Arm Span. Select a sample of at least eight people and measure each person's height and arm span. (When you measure arm span, the person should stand with arms extended like the wings on an airplane.) Using the paired sample data, construct a scatterplot and estimate or calculate the value of the correlation coefficient. What do you conclude?

33. Height and Pulse Rate. Select a sample of at least eight people and record each person's pulse rate by counting the number of heartbeats in 1 minute. Also record each person's height. Using the paired sample data, construct a scatterplot and estimate or calculate the value of the correlation coefficient. What do you conclude?

IN THE NEWS

34. Correlations in the News. Find a recent news report that discusses some type of correlation. Describe the correlation. Does the article give any sense of the strength of the correlation? Does it suggest that the correlation reflects any underlying causality? Briefly discuss whether you believe the implications the article makes with respect to the correlation.

35. Your Own Positive Correlations. Give examples of two variables that you expect to be positively correlated. Explain why the variables are correlated and why the correlation is (or is not) important.

36. Your Own Negative Correlations. Give examples of two variables that you expect to be negatively correlated. Explain why the variables are correlated and why the correlation is (or is not) important.

2

INTERPRETING CORRELATIONS

Statistics show that of those who contract the habit of eating, very few survive.

—Wallace Irwin

Researchers sifting through statistical data are constantly looking for meaningful correlations, and the discovery of a new and surprising correlation often leads to a flood of news reports. You may recall hearing about some of these discovered correlations: dark chocolate consumption correlated with reduced risk of heart disease; musical talent correlated with good grades in mathematics; or eating less correlated with increased longevity. Unfortunately, the task of interpreting such correlations is far more difficult than discovering them in the first place. Long after the news reports have faded, we may still be unsure of whether the correlations are significant and, if so, whether they tell us anything of practical importance. In this section, we discuss some of the common difficulties associated with interpreting correlations.

Beware of Outliers

Examine the scatterplot in Figure 9. Your eye probably tells you that there is a positive correlation in which larger values of x tend to mean larger values of y . Indeed, if you calculate the correlation coefficient for these data, you'll find that it is a relatively high $r = 0.880$, suggesting a very strong correlation.

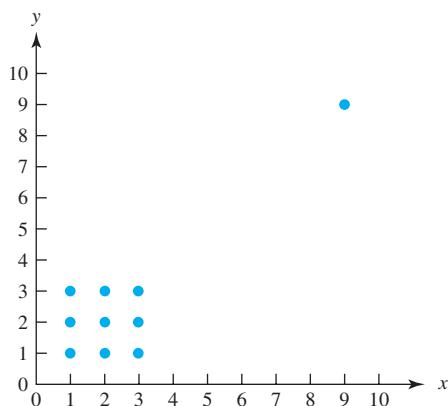


Figure 9 How does the outlier affect the correlation?

However, if you place your thumb over the data point in the upper right corner of Figure 9, the apparent correlation disappears. In fact, without this data point, the correlation coefficient is zero! In other words, removing this one data point changes the correlation coefficient from $r = 0.880$ to $r = 0$.

This example shows that correlations can be very sensitive to outliers. Recall that an *outlier* is a data value that is extreme compared to most other values in a data set. We must therefore examine outliers and their effects carefully before interpreting a correlation. On the one hand, if the outliers are mistakes in the data set, they can produce apparent correlations that are not real or mask the presence of real correlations. On the other hand, if the outliers represent real and correct data points, they may be telling us about relationships that would otherwise be difficult to see.

Note that while we should examine outliers carefully, we should *not* remove them unless we have strong reason to believe that they do not belong in the data set. Even in that case, good research principles demand that we report the outliers along with an explanation of why we thought it legitimate to remove them.

EXAMPLE 1 Masked Correlation

You've conducted a study to determine how the number of calories a person consumes in a day correlates with time spent in vigorous bicycling. Your sample consisted of ten women cyclists, all of approximately the same height and weight. Over a period of two weeks, you asked each woman to record the amount of time she spent cycling each day and what she ate on each of those days. You used the eating records to calculate the calories consumed each day. Figure 10 shows a scatterplot with each woman's mean time spent cycling on the horizontal axis and mean caloric intake on the vertical axis. Do higher cycling times correspond to higher intake of calories?

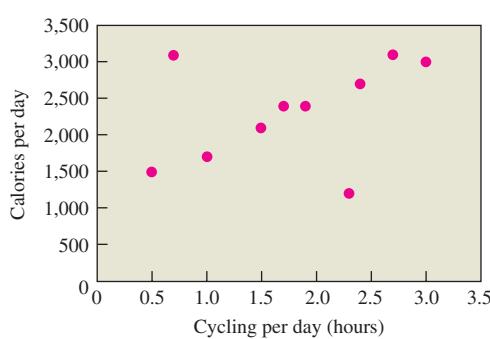


Figure 10 Data from the cycling study.

SOLUTION If you look at the data as a whole, your eye will probably tell you that there is a positive correlation in which greater cycling time tends to go with higher caloric intake. But the correlation is very weak, with a correlation coefficient of $r = 0.374$. However, notice

that two points are outliers: one representing a cyclist who cycled about a half-hour per day and consumed more than 3,000 calories, and the other representing a cyclist who cycled more than 2 hours per day on only 1,200 calories. It's difficult to explain the two outliers, given that all the women in the sample have similar heights and weights. We might therefore suspect that these two women either recorded their data incorrectly or were not following their usual habits during the two-week study. If we can confirm this suspicion, then we would have reason to delete the two data points as invalid. Figure 11 shows that the correlation is quite strong without those two outlier points, and suggests that the number of calories consumed rises by a little more than 500 calories for each hour of cycling. Of course, we should *not* remove the outliers without confirming our suspicion that they were invalid data points, and we should report our reasons for leaving them out.

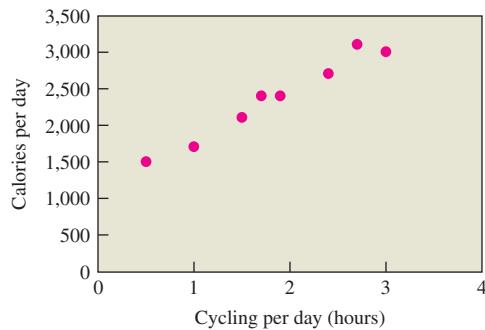


Figure 11 The data from Figure 10 without the two outliers.



Beware of Inappropriate Grouping

Correlations can also be misinterpreted when data are grouped inappropriately. In some cases, grouping data hides correlations. Consider a (hypothetical) study in which researchers seek a correlation between hours of TV watched per week and high school grade point average (GPA). They collect the 21 data pairs in Table 2.

The scatterplot (Figure 12) shows virtually no correlation; the correlation coefficient for the data is about $r = -0.063$. The lack of correlation seems to suggest that TV viewing habits are unrelated to academic achievement. However, one astute researcher realizes that some of the students watched mostly educational programs, while others tended to watch comedies, dramas, and movies. She therefore divides the data set into two groups, one for the students who watched mostly educational television and one for the other students. Table 3 shows her results with the students divided into these two groups.

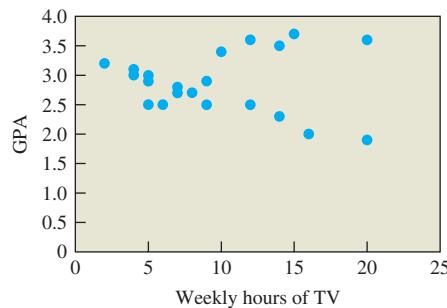


Figure 12 The full set of data concerning hours of TV and GPA shows virtually no correlation.

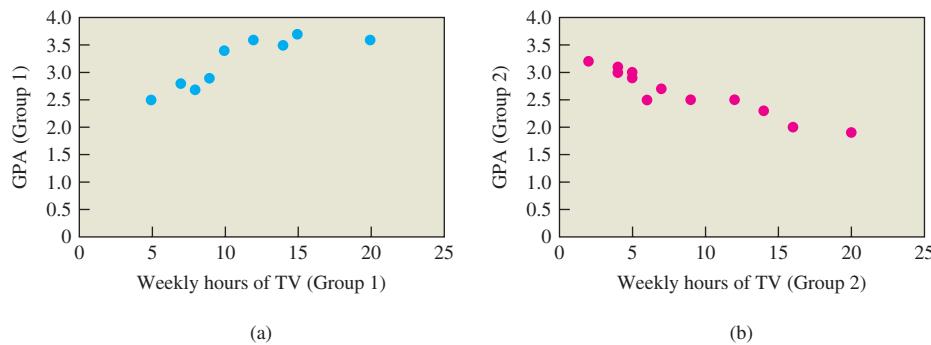
Now we find two very strong correlations (Figure 13): a strong positive correlation for the students who watched educational programs ($r = 0.855$) and a strong negative correlation for the other students ($r = -0.951$). The moral of this story is that the original data set hid an important (hypothetical) correlation between TV and GPA: Watching educational TV correlated positively with GPA and watching non-educational TV correlated negatively with GPA. Only when the data were grouped appropriately could this discovery be made.

TABLE 3 Hours of TV and High School GPA—Grouped Data (hypothetical data)

Group 1: watched educational programs		Group 2: watched regular TV	
Hours per week of TV	GPA	Hours per week of TV	GPA
5	2.5	2	3.2
7	2.8	4	3.0
8	2.7	4	3.1
9	2.9	5	2.9
10	3.4	5	3.0
12	3.6	6	2.5
14	3.5	7	2.7
15	3.7	9	2.5
20	3.6	12	2.5
		14	2.3
		16	2.0
		20	1.9

BY THE WAY

Children ages 2–5 watch an average of 26 hours of television per week, while children ages 6–11 watch an average of 20 hours of television per week (Nielsen Media Research). Adult viewership averages more than 25 hours per week. If the average adult replaced television time with a job paying just \$8 per hour, his or her annual income would rise by more than \$10,000.

**Figure 13** These scatterplots show the same data as Figure 12, separated into the two groups identified in Table 3.

In other cases, a data set may show a stronger correlation than actually exists among subgroups. Consider the (hypothetical) data in Table 4, showing the relationship between the weights and prices of selected cars. Figure 14 shows the scatterplot.

The data set as a whole shows a strong correlation; the correlation coefficient is $r = 0.949$. However, on closer examination, we see that the data fall into two rather distinct categories corresponding to light and heavy cars. If we analyze these subgroups separately, neither shows any correlation: The light cars alone (top six in Table 4) have a correlation coefficient $r = 0.019$ and the heavy cars alone (bottom six in Table 4) have a correlation coefficient $r = -0.022$. You can see the problem by looking at Figure 14. The apparent correlation of the full data set occurs because of the separation between the two clusters of points; there's no correlation within either cluster.

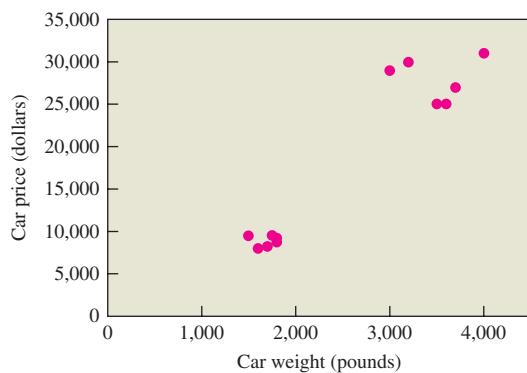
**Figure 14** Scatterplot for the car weight and price data in Table 4.

TABLE 4		Car Weights and Prices (hypothetical data)
Weight (pounds)	Price (dollars)	
1,500	9,500	
1,600	8,000	
1,700	8,200	
1,750	9,500	
1,800	9,200	
1,800	8,700	
3,000	29,000	
3,500	25,000	
3,700	27,000	
4,000	31,000	
3,600	25,000	
3,200	30,000	

TIME OUT TO THINK

Suppose you were shopping for a compact car. If you looked at only the overall data and correlation coefficient from Figure 14, would it be reasonable to consider weight as an important factor in price? What if you looked at the data for light and heavy cars separately? Explain.

**CASE STUDY Fishing for Correlations**

Oxford physician Richard Peto submitted a paper to the British medical journal *Lancet* showing that heart-attack victims had a better chance of survival if they were given aspirin within a few hours after their heart attacks. The editors of *Lancet* asked Peto to break down the data into subsets, to see whether the benefits of the aspirin were different for different groups of patients. For example, was aspirin more effective for patients of a certain age or for patients with certain dietary habits?

Breaking the data into subsets can reveal important facts, such as whether men and women respond to the treatment differently. However, Peto felt that the editors were asking him to divide his sample into too many subgroups. He therefore objected to the request, arguing that it would result in purely coincidental correlations. Writing about this story in the *Washington Post*, journalist Rick Weiss said, "When the editors insisted, Peto capitulated, but among other things he divided his patients by zodiac birth signs and demanded that his findings be included in the published paper. Today, like a warning sign to the statistically uninitiated, the wacky numbers are there for all to see: Aspirin is useless for Gemini and Libra heart-attack victims but is a lifesaver for people born under any other sign."

The moral of this story is that a "fishing expedition" for correlations can often produce them. That doesn't make the correlations meaningful, even though they may appear significant by standard statistical measures.

Correlation Does Not Imply Causality

Perhaps the most important caution about interpreting correlations is one we've already mentioned: ***Correlation does not necessarily imply causality***. In general, correlations can appear for any of the following three reasons.

Possible Explanations for a Correlation

1. The correlation may be a *coincidence*.
2. Both correlation variables might be directly influenced by some *common underlying cause*.
3. One of the correlated variables may actually be a *cause* of the other. But note that, even in this case, it may be just one of several causes.

For example, the correlation between infant mortality and life expectancy in Figure 4 is a case of common underlying cause: Both variables respond to the underlying variable *quality of health care*. The correlation between smoking and lung cancer reflects the fact that smoking causes lung cancer (see the discussion in Section 4). Coincidental correlations are also quite common; Example 2 below discusses one such case.

Caution about causality is particularly important in light of the fact that many statistical studies are designed to look for causes. Because these studies generally begin with the search for correlations, it's tempting to think that the work is over as soon as a correlation is found. However, as we will discuss in Section 4, establishing causality can be very difficult.

EXAMPLE 2 How to Get Rich in the Stock Market (Maybe)

Every financial advisor has a strategy for predicting the direction of the stock market. Most focus on fundamental economic data, such as interest rates and corporate profits. But an alternative strategy might rely on a famous correlation between the Super Bowl winner in January and the direction of the stock market for the rest of the year: The stock market tends to rise when a team from the old, pre-1970 NFL wins the Super Bowl and tends to fall when the winner is not from the old NFL. This correlation successfully matched 28 of the first 32 Super Bowls to the stock market, which made the “Super Bowl Indicator” a far more reliable predictor of the stock market than any professional stock broker during the same period. In fact, detailed calculations show that the probability of such success by pure chance is less than 1 in 100,000. Should you therefore make a decision about whether to invest in the stock market based on the NFL origins of the most recent Super Bowl winner?

SOLUTION The extremely strong correlation might make it seem like a good idea to base your investments on the Super Bowl Indicator, but sometimes you need to apply a bit of common sense. No matter how strong the correlation might be, it seems inconceivable to imagine that the origin of the winning team actually *causes* the stock market to move in a particular direction. The correlation is undoubtedly a coincidence, and the fact that its probability of occurring by pure chance was less than 1 in 100,000 is just another illustration of the fact that you can turn up surprising correlations if you go fishing for them. This fact was borne out in more recent Super Bowls: Following Super Bowl 32, the indicator successfully predicted the stock market direction in only 5 of the next 10 years—exactly the fraction that would be expected by pure chance.

• • •



CASE STUDY Oat Bran and Heart Disease

If you buy a product that contains oat bran, there's a good chance that the label will tout the healthful effects of eating oats. Indeed, several studies have found correlations in which people who eat more oat bran tend to have lower rates of heart disease. But does this mean that everyone should eat more oats?

Not necessarily. Just because oat bran consumption is correlated with reduced risk of heart disease does not mean that it *causes* reduced risk of heart disease. In fact, the question of causality is quite controversial in this case. Other studies suggest that people who eat a lot of oat bran tend to have generally healthful diets. Thus, the correlation between oat bran consumption and reduced risk of heart disease may be a case of a common underlying cause: Having a healthy diet leads people both to consume more oat bran and to have a lower risk of heart disease. In that case, for some people, adding oat bran to their diets might be a *bad* idea because it could cause them to gain weight, and weight gain is associated with *increased* risk of heart disease.

This example shows the importance of using caution when considering issues of correlation and causality. It may be a long time before medical researchers know for sure whether adding oat bran to your diet actually causes a reduced risk of heart disease.

Useful Interpretations of Correlation

In discussing uses of correlation that might lead to wrong interpretations, we have described the effects of outliers, inappropriate groupings, fishing for correlations, and incorrectly concluding that correlation implies causality. But there are many correct and useful interpretations of correlation, some of which we have already studied. So while you should be cautious in interpreting correlations, they remain a valuable tool in any field in which statistical research plays a role.

Section 2 Exercises

Statistical Literacy and Critical Thinking

1. **Correlation and Causality.** In clinical trials of the drug Lisinopril, it is found that increased dosages of the drug correlated with lower blood pressure levels. Based on the correlation, can we conclude that Lisinopril treatments cause lower blood pressure? Why or why not?
2. **SIDS.** An article in the *New York Times* on infant deaths included a statement that, based on the study results, putting infants to sleep in the supine position decreased deaths due to SIDS (sudden infant death syndrome). What is wrong with that statement?
3. **Outliers.** When studying salaries paid to CEOs of large companies, it is found that almost all of them range from a few hundred thousand dollars to several million dollars, but one CEO is paid a salary of \$1. Is that salary of \$1 an outlier? In general, how might outliers affect conclusions about correlation?
4. **Scatterplot.** Does a scatterplot reveal anything about a cause and effect relationship between two variables?

Does It Make Sense? For Exercises 5–8, decide whether the statement makes sense (or is clearly true) or does not make sense (or is clearly false). Explain clearly; not all of these statements have definitive answers, so your explanation is more important than your chosen answer.

5. **Scatterplot.** A set of paired sample data results in a correlation coefficient of $r = 0$, so the scatterplot will show that there is no pattern of the plotted points.
6. **Causation.** If we have 20 pairs of sample data with a correlation coefficient of 1, then we know that one of the two variables is definitely the cause of the other.
7. **Causation.** If we conduct a study showing that there is a strong negative correlation between resting pulse rate and amounts of time spent in rigorous exercise, we can conclude decreases in resting pulse rates are somehow associated with increases in exercise.
8. **Causation.** If we have two variables with one being the direct cause of the other, then there may or may not be a correlation between those two variables.

Concepts and Applications

Correlation and Causality. Exercises 9–16 make statements about a correlation. In each case, state the correlation clearly. (For example, we might state that “there is a positive correlation between variable A and variable B.”) Then state whether the correlation is most likely due to coincidence, a common underlying cause, or a direct cause. Explain your answer.

9. **Guns and Crime Rate.** In one state, the number of unregistered handguns steadily increased over the past several years, and the crime rate increased as well.

10. **Running and Weight.** It has been found that people who exercise regularly by running tend to weigh less than those who do not run, and those who run longer distances tend to weigh less than those who run shorter distances.
11. **Study Time.** Statistics students find that as they spend more time studying, their test scores are higher.
12. **Vehicles and Waiting Time.** It has been found that as the number of registered vehicles increases, the time drivers spend sitting in traffic also increases.
13. **Traffic Lights and Car Crashes.** It has been found that as the number of traffic lights increases, the number of car crashes also increases.
14. **Galaxies.** Astronomers have discovered that, with the exception of a few nearby galaxies, all galaxies in the universe are moving away from us. Moreover, the farther the galaxy, the faster it is moving away. That is, the more distant a galaxy, the greater the speed at which it is moving away from us.
15. **Gas and Driving.** It has been found that as gas prices increase, the distances vehicles are driven tend to get shorter.
16. **Melanoma and Latitude.** Some studies have shown that, for certain ethnic groups, the incidence of melanoma (the most dangerous form of skin cancer) increases as latitude decreases.
17. **Outlier Effects.** Consider the scatterplot in Figure 15.

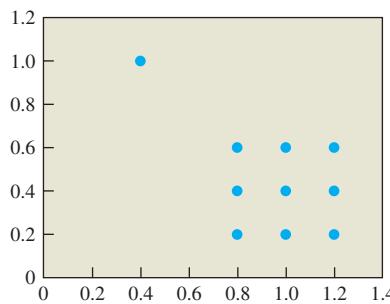


Figure 15

- a. Which point is an outlier? Ignoring the outlier, estimate or compute the correlation coefficient for the remaining points.
- b. Now include the outlier. How does the outlier affect the correlation coefficient? Estimate or compute the correlation coefficient for the complete data set.

- 18. Outlier Effects.** Consider the scatterplot in Figure 16.

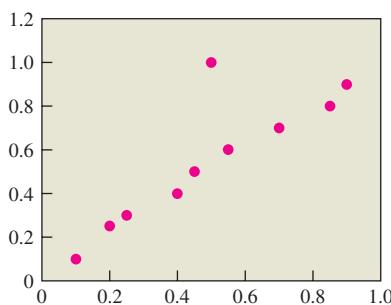


Figure 16

- Which point is an outlier? Ignoring the outlier, estimate or compute the correlation coefficient for the remaining points.
- Now include the outlier. How does the outlier affect the correlation coefficient? Estimate or compute the correlation coefficient for the complete data set.

- 19. Grouped Shoe Data.** The following table gives measurements of weight and shoe size for 10 people (including both men and women).

- Construct a scatterplot for the data. Estimate or compute the correlation coefficient. Based on this correlation coefficient, would you conclude that shoe size and weight are correlated? Explain.

Weight (pounds)	Shoe size
105	6
112	4.5
115	6
123	5
135	6
155	10
165	11
170	9
180	10
190	12

- You later learn that the first five data values in the table are for women and the next five are for men. How does this change your view of the correlation? Is it still reasonable to conclude that shoe size and weight are correlated?

- 20. Grouped Temperature Data.** The following table shows the average January high temperature and the average July high temperature for 10 major cities around the world.

City	January high	July high
Berlin	35	74
Geneva	39	77
Kabul	36	92
Montreal	21	78
Prague	34	74
Auckland	73	56
Buenos Aires	85	57
Sydney	78	60
Santiago	85	59
Melbourne	78	56

- Construct a scatterplot for the data. Estimate or compute the correlation coefficient. Based on this correlation coefficient, would you conclude that January and July temperatures are correlated for these cities? Explain.

- Notice that the first five cities in the table are in the Northern Hemisphere and the next five are in the Southern Hemisphere. How does this change your view of the correlation? Would you now conclude that January and July temperatures are correlated for these cities? Explain.

- 21. Birth and Death Rates.** Figure 17 shows the birth and death rates for different countries, measured in births and deaths per 1,000 population.

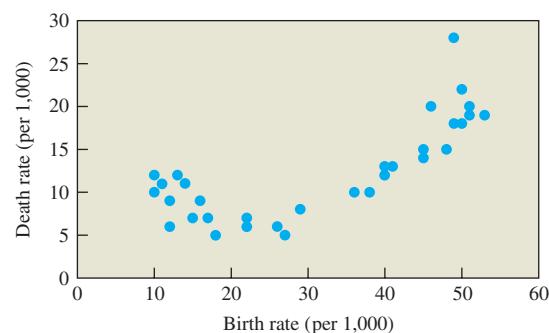


Figure 17 Birth and death rates for different countries.

Source: United Nations.

- Estimate the correlation coefficient and discuss whether there is a strong correlation between the variables.
- Notice that there appear to be two groups of data points within the full data set. Make a reasonable guess as to the makeup of these groups. In which group might you find a relatively wealthy country like Sweden? In which group might you find a relatively poor country like Uganda?
- Assuming that your guess about groups in part b is correct, do there appear to be correlations within the groups? Explain. How could you confirm your guess about the groups?

- 22. Reading and Test Scores.** The following (hypothetical) data set gives the number of hours 10 sixth-graders read per week and their performance on a standardized verbal test (maximum of 100).

Reading time per week	Verbal test score
1	50
1	65
2	56
3	62
3	65
4	60
5	75
6	50
10	88
12	38

- Construct a scatterplot for these data. Estimate or compute the correlation coefficient. Based on this correlation coefficient, would you conclude that reading time and test scores are correlated? Explain.
- Suppose you learn that five of the children read only comic books while the other five read regular books. Make a guess as to which data points fall in which group. How could you confirm your guess about the groups?
- Assuming that your guess in part b is correct, how does it change your view of the correlation between reading time and test scores? Explain.
- Describe a real situation in which there is a positive correlation that is the result of coincidence.
- Describe a real situation in which there is a negative correlation that is the result of a common underlying cause.
- Describe a real situation in which there is a negative correlation that is the result of a direct cause.



PROJECTS FOR THE INTERNET & BEYOND

23. Football-Stock Update. Find data for recent years concerning the Super Bowl winner and the end-of-year change in the stock market (positive or negative). Do recent results still agree with the correlation described in Example 2? Explain.

24. Real Correlations.

- Describe a real situation in which there is a positive correlation that is the result of coincidence.
- Describe a real situation in which there is a positive correlation that is the result of a common underlying cause.

3

BEST-FIT LINES AND PREDICTION

BY THE WAY

The term *regression* comes from an 1877 study by Sir Francis Galton. He found that the heights of boys with short or tall fathers were closer to the mean than were the heights of their fathers. He therefore said that the heights of the children *regress* toward the mean, from which we get the term *regression*. The term is now used even for data that have nothing to do with a tendency to regress toward a mean.

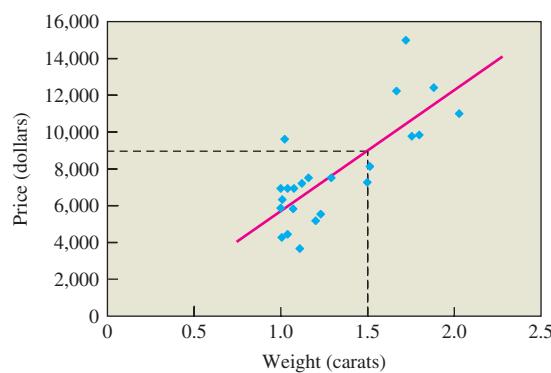


Figure 18 Best-fit line for the data from Figure 1.

Definition

The **best-fit line** (or *regression line*) on a scatterplot is a line that lies closer to the data points than any other possible line (according to a standard statistical measure of closeness).

IN THE NEWS

- 25. Misinterpreted Correlations.** Find a recent news report in which you believe that a correlation may have been misinterpreted. Describe the correlation, the reported interpretation, and the problems you see in the interpretation.
- 26. Well-Interpreted Correlations.** Find a recent news report in which you believe that a correlation has been presented with a reasonable interpretation. Describe the correlation and the reported interpretation, and explain why you think the interpretation is valid.

Of all the possible straight lines that can be drawn on a diagram, how do you know which one is the best-fit line? In many cases, you can make a good estimate of the best-fit line simply by looking at the data and drawing the line that visually appears to pass closest to all the data points. This method involves drawing the best-fit line “by eye.” As you might guess, there are methods for calculating the precise equation of a best-fit line (see the optional topic at the end of this section), and many computer programs and calculators can do these calculations automatically. For our purposes in this text, a fit by eye will generally be sufficient.

Predictions with Best-Fit Lines

We can use the best-fit line in Figure 18 to predict the price of a 1.5-carat diamond. As indicated by the dashed lines in the figure, the best-fit line predicts that the diamond will cost about \$9,000. Notice, however, that two actual data points in the figure correspond to 1.5-carat diamonds, and both of these diamonds cost less than \$9,000. That is, although the predicted price of \$9,000 sounds reasonable, it is certainly not guaranteed. In fact, the degree of scatter among the data points in this case tells us that we should *not* trust the best-fit line to predict accurately the price for any individual diamond. Instead, the prediction is meaningful only in a statistical sense: It tells us that if we examined many 1.5-carat diamonds, their mean price would be about \$9,000.

This is only the first of several important cautions about interpreting predictions with best-fit lines. A second caution is to beware of using best-fit lines to make predictions that go beyond the bounds of the available data. Figure 19 shows a best-fit line for the correlation between infant mortality and longevity from Figure 4. According to this line, a country with a life expectancy of more than about 80 years would have a *negative* infant mortality rate, which is impossible.

It is a capital mistake to theorize before one has data.

—Arthur Conan Doyle

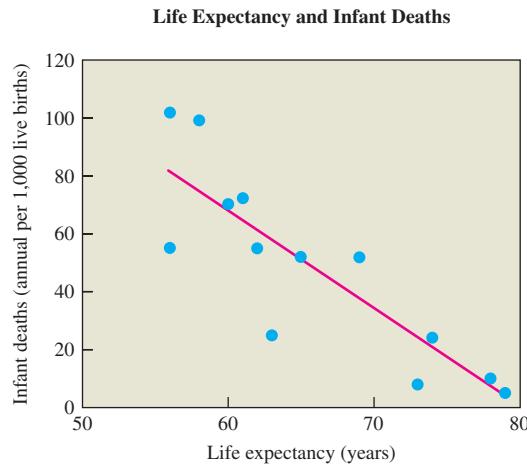


Figure 19 A best-fit line for the correlation between infant mortality and longevity from Figure 4.

Source: United Nations.

A third caution is to avoid using best-fit lines from old data sets to make predictions about current or future results. For example, economists studying historical data found a strong negative correlation between unemployment and the rate of inflation. According to this correlation, inflation should have risen dramatically in the mid-2000s when the unemployment rate fell below 6%. But inflation remained low, showing that the correlation from old data did not continue to hold.

Fourth, a correlation discovered with a sample drawn from a particular population cannot generally be used to make predictions about other populations. For example, we can't expect that the correlation between aspirin consumption and heart attacks in an experiment involving only men will also apply to women.

It's tough to make predictions, especially about the future.

—attributed to Niels Bohr,
Yogi Berra, and others

Fifth, remember that we can draw a best-fit line through any data set, but that line is meaningless when the correlation is not significant or when the relationship is nonlinear. For example, there is no correlation between shoe size and IQ, so we could not use shoe size to predict IQ.

Cautions in Making Predictions from Best-Fit Lines

1. Don't expect a best-fit line to give a good prediction unless the correlation is strong and there are many data points. If the sample points lie very close to the best-fit line, the correlation is very strong and the prediction is more likely to be accurate. If the sample points lie away from the best-fit line by substantial amounts, the correlation is weak and predictions tend to be much less accurate.
2. Don't use a best-fit line to make predictions beyond the bounds of the data points to which the line was fit.
3. A best-fit line based on past data is not necessarily valid now and might not result in valid predictions of the future.
4. Don't make predictions about a population that is different from the population from which the sample data were drawn.
5. Remember that a best-fit line is meaningless when there is no significant correlation or when the relationship is nonlinear.

EXAMPLE 1 Valid Predictions?

State whether the prediction (or implied prediction) should be trusted in each of the following cases, and explain why or why not.

- a. You've found a best-fit line for a correlation between the number of hours per day that people exercise and the number of calories they consume each day. You've used this correlation to predict that a person who exercises 18 hours per day would consume 15,000 calories per day.
- b. There is a well-known but weak correlation between SAT scores and college grades. You use this correlation to predict the college grades of your best friend from her SAT scores.
- c. Historical data have shown a strong negative correlation between national birth rates and affluence. That is, countries with greater affluence tend to have lower birth rates. These data predict a high birth rate in Russia.
- d. A study in China has discovered correlations that are useful in designing museum exhibits that Chinese children enjoy. A curator suggests using this information to design a new museum exhibit for Atlanta-area school children.
- e. Scientific studies have shown a very strong correlation between children's ingesting of lead and mental retardation. Based on this correlation, paints containing lead were banned.
- f. Based on a large data set, you've made a scatterplot for salsa consumption (per person) versus years of education. The diagram shows no significant correlation, but you've drawn a best-fit line anyway. The line predicts that someone who consumes a pint of salsa per week has at least 13 years of education.

SOLUTION

- a. No one exercises 18 hours per day on an ongoing basis, so this much exercise must be beyond the bounds of any data collected. Therefore, a prediction about someone who exercises 18 hours per day should not be trusted.
- b. The fact that the correlation between SAT scores and college grades is weak means there is much scatter in the data. As a result, we should not expect great accuracy if we use this weak correlation to make a prediction about a single individual.
- c. We cannot automatically assume that the historical data still apply today. In fact, Russia currently has a very low birth rate, despite also having a low level of affluence.
- d. The suggestion to use information from the Chinese study for an Atlanta exhibit assumes that predictions made from correlations in China also apply to Atlanta. However, given

the cultural differences between China and Atlanta, the curator's suggestion should not be considered without more information to back it up.

- e. Given the strength of the correlation and the severity of the consequences, this prediction and the ban that followed seem quite reasonable. In fact, later studies established lead as an actual *cause* of mental retardation, making the rationale behind the ban even stronger.
- f. Because there is no significant correlation, the best-fit line and any predictions made from it are meaningless.

BY THE WAY

In the United States, lead was banned from house paint in 1978 and from food cans in 1991, and a 25-year phaseout of lead in gasoline was completed in 1995. Nevertheless, many young children—especially children living in poor areas—still have enough lead in their blood to damage their health. Major sources of ongoing lead hazards include paint in older housing and soil near major roads, which has high lead content from past use of leaded gasoline.

EXAMPLE 2 Will Women Be Faster Than Men?

Figure 20 shows data and best-fit lines for both men's and women's world record times in the 1-mile race. Based on these data, predict when the women's world record will be faster than the men's world record. Comment on the prediction.

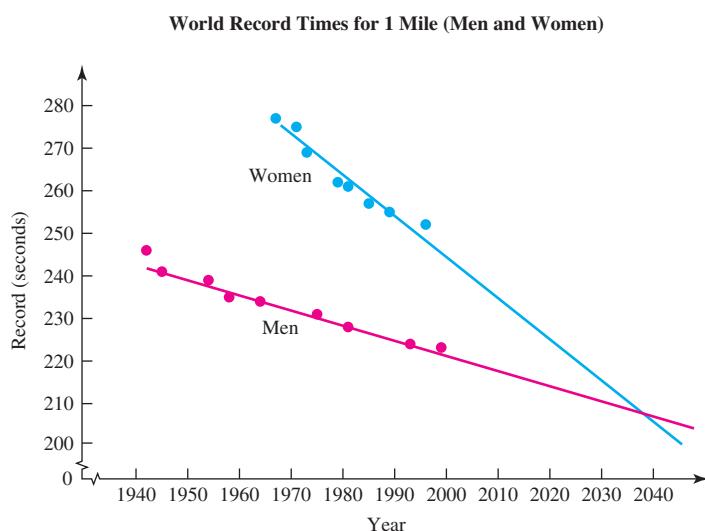


Figure 20 World record times in the mile (men and women).

SOLUTION If we accept the best-fit lines as drawn, the women's world record will equal the men's world record by about 2040. However, this is *not* a valid prediction because it is based on extending the best-fit lines beyond the range of the actual data. In fact, notice that the most recent world records (as of 2011) date all the way back to 1999 for men and 1996 for women, while the best-fit lines predict that the records should have fallen by several more seconds since those dates.

• •

The Correlation Coefficient and Best-Fit Lines

Earlier, we discussed the correlation coefficient as one way of measuring the strength of a correlation. We can also use the correlation coefficient to say something about the validity of predictions with best-fit lines.

For mathematical reasons (not discussed in this text), the *square* of the correlation coefficient, or r^2 , is the proportion of the variation in a variable that is accounted for by the best-fit line (or, more technically, by the linear relationship that the best-fit line expresses). For example, the correlation coefficient for the diamond weight and price data (see Figure 18) turns out to be $r = 0.777$. If we square this value, we get $r^2 = 0.604$ which we can interpret as follows: About 0.6, or 60%, of the variation in the diamond prices is accounted for by the best-fit line relating weight and price. That leaves 40% of the variation in price that must be due to other factors, presumably such things as depth, table, color, and clarity—which is why predictions made with the best-fit line in Figure 18 are not very precise.

A best-fit line can give precise predictions only in the case of a perfect correlation ($r = 1$ or $r = -1$); we then find $r^2 = 1$, which means that 100% of the variation in a variable

can be accounted for by the best-fit line. In this special case of $r^2 = 1$, predictions should be exactly correct, except for the fact that the sample data might not be a true representation of the population data.

TECHNICAL NOTE

Statisticians call r^2 the coefficient of determination.

Best-Fit Lines and r^2

The *square* of the correlation coefficient, or r^2 , is the proportion of the variation in a variable that is accounted for by the best-fit line.

EXAMPLE 3 Retail Hiring

You are the manager of a large department store. Over the years, you've found a strong correlation between your September sales and the number of employees you'll need to hire for peak efficiency during the holiday season; the correlation coefficient is 0.950. This year your September sales are fairly strong. Should you start advertising for help based on the best-fit line?

SOLUTION In this case, we find that $r^2 = 0.950^2 = 0.903$, which means that 90% of the variation in the number of peak employees can be accounted for by a linear relationship with September sales. That leaves only 10% of the variation in the number of peak employees unaccounted for. Because 90% is so high, we conclude that the best-fit line accounts for the data quite well, so it seems reasonable to use it to predict the number of employees you'll need for this year's holiday season.



EXAMPLE 4 Voter Turnout and Unemployment

Political scientists are interested in knowing what factors affect voter turnout in elections. One such factor is the unemployment rate. Data collected in presidential election years since 1964 show a very weak negative correlation between voter turnout and the unemployment rate, with a correlation coefficient of about $r = -0.1$ (Figure 21). Based on this correlation, should we use the unemployment rate to predict voter turnout in the next presidential election?

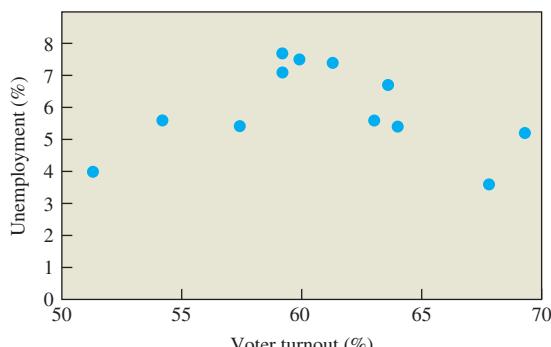


Figure 21 Data on voter turnout and unemployment, 1964–2008.

Source: U.S. Bureau of Labor Statistics.

SOLUTION The square of the correlation coefficient is $r^2 = (-0.1)^2 = 0.01$, which means that only about 1% of the variation in the data is accounted for by the best-fit line. Nearly all of the variation in the data must therefore be explained by other factors. We conclude that unemployment is *not* a reliable predictor of voter turnout.



Multiple Regression

All who drink his remedy recover in a short time, except those whom it does not help, who all die. Therefore, it is obvious that it fails only in incurable cases.

—Galen, Roman “doctor”

If you've ever purchased a diamond, you might have been surprised that we found such a weak correlation between color and price in Figure 2. Surely a diamond cannot be very valuable if it has poor color quality. Perhaps color helps to explain why the correlation between weight and price is not perfect. For example, maybe differences in color explain why two diamonds with the same weight can have different prices. To check this idea, it would be nice to look for a correlation between the price and some combination of *weight and color together*.

TIME OUT TO THINK

Check this idea in Table 1. Notice, for example, that Diamonds 4 and 5 have nearly identical weights, but Diamond 4 costs only \$4,299 while Diamond 5 costs \$9,589. Can differences in their color explain the different prices? Study other examples in Table 1 in which two diamonds have similar weights but different prices. Overall, do you think that the correlation with price would be stronger if we used weight and color together instead of either one alone? Explain.

There is a method for investigating a correlation between one variable (such as price) and a *combination* of two or more other variables (such as weight and color). The technique is called **multiple regression**, and it essentially allows us to find a *best-fit equation* that relates three or more variables (instead of just two). Because it involves more than two variables, we cannot make simple diagrams to show best-fit equations for multiple regression. However, it is still possible to calculate a measure of how well the data fit a linear equation. The most common measure in multiple regression is the *coefficient of determination*, denoted R^2 . It tells us how much of the scatter in the data is accounted for by the best-fit equation. If R^2 is close to 1, the best-fit equation should be very useful for making predictions within the range of the data values. If R^2 is close to zero, then predictions with the best-fit equation are essentially useless.

Definition

The use of **multiple regression** allows the calculation of a best-fit equation that represents the best fit between one variable (such as price) and a *combination* of two or more other variables (such as weight and color). The coefficient of determination, R^2 , tells us the proportion of the scatter in the data accounted for by the best-fit equation.

In this text, we will not describe methods for finding best-fit equations by multiple regression. However, you can use the value of R^2 to interpret results from multiple regression. For example, the correlation between price and *weight and color together* results in a value of $R^2 = 0.79$. This is somewhat higher than the $r^2 = 0.61$ that we found for the correlation between price and weight alone. Statisticians who study diamond pricing know that they can get stronger correlations by including additional variables in the multiple regression (such as depth, table, and clarity). Given the billions of dollars spent annually on diamonds, you can be sure that statisticians play prominent roles in helping diamond dealers realize the largest possible profits.

EXAMPLE 5 **Alumni Contributions**

You've been hired by your college's alumni association to research how past contributions were associated with alumni income and years that have passed since graduation. It is found that $R^2 = 0.36$. What does that result tell us?

SOLUTION With $R^2 = 0.36$, we conclude that 36% of the variation in past contributions can be explained by the variation in alumni income and years since graduation. It follows that 64% of the variation in past contributions can be explained by factors other than alumni income level and years since graduation. Because such a large proportion of the variation can be explained by other factors, it would make sense to try to identify any other factors that might have a strong effect on past contributions.

• •

Finding Equations for Best-Fit Lines (Optional Section)

The mathematical technique for finding the equation of a best-fit line is based on the following basic ideas. If we draw *any* line on a scatterplot, we can measure the *vertical* distance between each data point and that line. One measure of how well the line fits the

BY THE WAY

One study of alumni donations found that, in developing a multiple regression equation, one should include these variables: income, age, marital status, whether the donor belonged to a fraternity or sorority, whether the donor is active in alumni affairs, the donor's distance from the college, and the nation's unemployment rate, used as a measure of the economy (Bruggink and Siddiqui, "An Econometric Model of Alumni Giving: A Case Study for a Liberal Arts College," *The American Economist*, Vol. 39, No. 2).



data is the *sum of the squares* of these vertical distances. A large sum means that the vertical distances of data points from the line are fairly large and hence the line is not a very good fit. A small sum means the data points lie close to the line and the fit is good. Of all possible lines, the best-fit line is the line that minimizes the sum of the squares of the vertical distances. Because of this property, the best-fit line is sometimes called the *least squares line*.

You may recall that the equation of any straight line can be written in the general form

$$y = mx + b$$

where m is the *slope* of the line and b is the *y-intercept* of the line. The formulas for the slope and y-intercept of the best-fit line are as follows:

$$\text{slope} = m = r \times \frac{s_y}{s_x}$$

$$y - \text{intercept} = b = \bar{y} - (m \times \bar{x})$$

In the above expressions, r is the correlation coefficient, s_x denotes the standard deviation of the x values (or the values of the first variable), s_y denotes the standard deviation of the y values, \bar{x} represents the mean of the values of the variable x , and \bar{y} represents the mean of the values of the variable y . Because these formulas are tedious with manual calculations, we usually use a calculator or computer to find the slope and y-intercept of best-fit lines. Statistical software packages and some calculators, such as the TI-83/84 Plus family of calculators, are designed to automatically generate the equation of a best-fit line.

When software or a calculator is used to find the slope and intercept of the best-fit line, results are commonly expressed in the format $y = b_0 + b_1x$, where b_0 is the intercept and b_1 is the slope, so be careful to correctly identify those two values.

Section 3 Exercises

Statistical Literacy and Critical Thinking

- 1. Best-Fit Line.** What is a best-fit line (also called a regression line)? How is a best-fit line useful?
- 2. r^2 .** For a study involving paired sample data, it is found that $r = -0.4$. What is the value of r^2 ? In general, what is r^2 called, what does it measure, and how can it be interpreted? That is, what does its value tell us about the variables?
- 3. Regression.** An investigator has data consisting of heights of daughters and the heights of the corresponding mothers and fathers. She wants to analyze the data to see the effect that the height of the mother and the height of the father has on the height of the daughter. Should she use a (linear) regression or multiple regression? What is the basic difference between (linear) regression and multiple regression?
- 4. R^2 .** Using data described in Exercise 3, it is found that $R^2 = 0.68$. Interpret that value. That is, what does that value tell us about the data?

Does It Make Sense? For Exercises 5–8, decide whether the statement makes sense (or is clearly true) or does not make sense (or is clearly false). Explain clearly; not all of

these statements have definitive answers, so your explanation is more important than your chosen answer.

- 5. r^2 Value.** A value of $r^2 = 1$ is obtained from a sample of paired data with one variable representing the amount of gas (gallons) purchased and the total cost of the gas.
- 6. r^2 Value.** A value of $r^2 = -0.040$ is obtained from a sample of men, with each pair of data consisting of the height in inches and the SAT score for one man.
- 7. Height and Weight.** Using data from the National Health Survey, the equation of the best-fit line for women's heights and weights is obtained, and it shows that a woman 120 inches tall is predicted to weigh 430 pounds.
- 8. Old Faithful.** Using paired sample data consisting of the duration time (in seconds) of eruptions of Old Faithful geyser and the time interval (in minutes) after the eruption, a value

of $r^2 = 0.926$ is calculated, indicating that about 93% of the variation in the interval after eruption can be explained by the relationship between those two variables as described by the best-fit line.

Concepts and Applications

Best-Fit Lines on Scatterplots. For Exercises 9–12, do the following.

- a. Insert a best-fit line in the given scatterplot.
- b. Estimate or compute r and r^2 . Based on your value for r^2 , determine how much of the variation in the variable can be accounted for by the best-fit line.
- c. Briefly discuss whether you could make valid predictions from this best-fit line.
9. Use the scatterplot for color and price in Figure 2.
10. Use the scatterplot for life expectancy and infant mortality in Figure 4.
11. Use the scatterplot for number of farms and size of farms in Figure 5.
12. Use both scatterplots for actual and predicted temperature in Figure 6.

Best-Fit Lines. Exercises 13–20 refer to the tables in the Section 1 Exercises. In each case, do the following.

- a. Construct a scatterplot and, based on visual inspection, draw the best-fit line by eye.
- b. Briefly discuss the strength of the correlation. Estimate or compute r and r^2 . Based on your value for r^2 , identify how much of the variation in the variable can be accounted for by the best-fit line.
- c. Identify any outliers on the scatterplot and discuss their effects on the strength of the correlation and on the best-fit line.
- d. For this case, do you believe that the best-fit line gives reliable predictions outside the range of the data on the scatterplot? Explain.
13. Use the data in Exercise 19 of Section 1.
14. Use the data in Exercise 20 of Section 1.
15. Use the data in Exercise 21 of Section 1.
16. Use the data in Exercise 22 of Section 1.
17. Use the data in Exercise 23 of Section 1. To locate the points, use the midpoint of each income category; use a value of \$25,000 for the category “less than \$30,000,” and use a value of \$70,000 for the category “more than \$60,000.”
18. Use the data in Exercise 24 of Section 1.
19. Use the data in Exercise 25 of Section 1.
20. Use the data in Exercise 26 of Section 1.



PROJECTS FOR THE INTERNET & BEYOND

21. **Lead Poisoning.** Research lead poisoning, its sources, and its effects. Discuss the correlations that have helped researchers understand lead poisoning. Discuss efforts to prevent it.
22. **Asbestos.** Research asbestos, its sources, and its effects. Discuss the correlations that have helped researchers understand adverse health effects from asbestos exposure. Discuss efforts to prevent those adverse health effects.
23. **Worldwide Population Indicators.** The following table gives five population indicators for eleven selected countries. Study these data and try to identify possible correlations. Doing additional research if necessary, discuss the possible correlations you have found, speculate on the reasons for the correlations, and discuss whether they suggest a causal relationship. Birth and death rates are per 1,000 population; fertility rate is per woman.

Country	Birth rate	Death rate	Life expectancy	Percent urban	Fertility rate
Afghanistan	50	22	43	20	6.9
Argentina	21	8	72	88	2.6
Australia	15	7	78	85	1.9
Canada	14	7	78	77	1.6
Egypt	29	8	64	45	3.4
El Salvador	30	6	68	45	3.1
France	13	9	78	73	1.6
Israel	21	7	77	91	2.8
Japan	10	7	79	78	1.5
Laos	45	15	51	22	6.7
United States	16	9	76	76	2.0

Source: *The New York Times Almanac*.

IN THE NEWS

24. **Predictions in the News.** Find a recent news report in which a correlation is used to make a prediction. Evaluate the validity of the prediction, considering all of the cautions described in this section. Overall, do you think the prediction is valid? Why or why not?
25. **Best-Fit Line in the News.** Although scatterplots are rare in the news, they are not unheard of. Find a scatterplot of any kind in a news article (recent or not). Draw a best-fit line by eye. Discuss what predictions, if any, can be made from your best-fit line.
26. **Your Own Multiple Regression.** Come up with an example from your own life or work in which a multiple regression analysis might reveal important trends. Without actually doing any analysis, describe in words what you would look for through the multiple regression and how the answers might be useful.

4

THE SEARCH FOR CAUSALITY

A correlation may *suggest* causality, but by itself a correlation never *establishes* causality. Much more evidence is required to establish that one factor *causes* another. Earlier, we found that a correlation between two variables may be the result of either (1) coincidence, (2) a common underlying cause, or (3) one variable actually having a direct influence on the other. The process of establishing causality is essentially a process of ruling out the first two explanations.

In principle, we can rule out the first two explanations by conducting experiments:

- We can rule out coincidence by repeating the experiment many times (or by using a large number of subjects in the experiment). Because coincidences occur randomly, the same coincidence is unlikely to occur in repeated trials of an experiment.
- We can rule out a common underlying cause by controlling and randomizing the experiment to eliminate the effects of confounding variables. If the controls rule out confounding variables, any remaining effects must be caused by the variables of interest.

Unfortunately, these ideas are often difficult to put into practice. In the case of ruling out coincidence, it may be too time-consuming or expensive to repeat an experiment a sufficient number of times. To rule out a common underlying cause, the experiment must control for *everything* except the variables of interest, and this is often impossible. Moreover, there are many cases in which experiments are impractical or unethical, so we can gather only observational data. Because observational studies cannot definitively establish causality, we must find other ways of trying to establish causality.

Establishing Causality

Suppose you have discovered a correlation and suspect causality. How can you test your suspicion? Let's return to the issue of smoking and lung cancer. The strong correlation between smoking and lung cancer did not by itself prove that smoking causes lung cancer. In principle, we could have looked for proof with a controlled experiment. But such an experiment would be unethical because it would require forcing a group of randomly selected people to smoke cigarettes. So how was smoking established as a cause of lung cancer?

The answer involves several lines of evidence. First, researchers found correlations between smoking and lung cancer among many groups of people: women, men, and people of different races and cultures. Second, among groups of people that seemed otherwise identical, lung cancer was found to be more rare in nonsmokers. Third, people who smoked more and for longer periods of time were found to have higher rates of lung cancer. Fourth, when researchers accounted for other potential causes of lung cancer (such as exposure to radon gas or asbestos), they found that almost all the remaining lung cancer cases occurred among smokers (or people exposed to second-hand smoke).

These four lines of evidence made a strong case, but still did not rule out the possibility that some other factor, such as genetics, predisposes people both to smoking and to lung cancer. However, two additional lines of evidence made this possibility highly unlikely. One line of evidence came from animal experiments. In controlled experiments, animals were divided into randomly chosen treatment and control groups. The experiments still found a correlation between inhalation of cigarette smoke and lung cancer, which seems to rule out a genetic factor, at least in the animals. The final line of evidence came from biologists studying small samples of human lung tissue. The biologists discovered the basic process by which ingredients in cigarette smoke create cancer-causing mutations. This process does not appear to depend in any way on specific genetic factors, making it

BY THE WAY

Statistical methods cannot prove that smoking causes cancer, but statistical methods can be used to identify an association, and physical proof of causation can then be sought by researchers.

Dr. David Sidransky of Johns Hopkins University and other researchers found a direct physical link that involves mutations of a specific gene among smokers. Molecular analysis of genetic changes allows researchers to determine whether cigarette smoking is the cause of a cancer.

(See "Association Between Cigarette Smoking and Mutation of the p53 Gene in Squamous-Cell Carcinoma of the Head and Neck," by Brennan, Boyle et al., *New England Journal of Medicine*, Vol 332, No. 11.)

all but certain that lung cancer is caused by smoking and not by any preexisting genetic factor. The fact that second-hand smoke exposure is also associated with some cases of lung cancer further argues against a genetic factor (since second-hand smoke affects non-smokers) but is consistent with the idea that ingredients in cigarette smoke create cancer-causing mutations.

The following box summarizes these ideas about establishing causality. Generally speaking, the case for causality is stronger when more of these guidelines are met.

Guidelines for Establishing Causality

If you suspect that a particular variable (the suspected cause) is causing some effect:

1. Look for situations in which the effect is correlated with the suspected cause even while other factors vary.
2. Among groups that differ only in the presence or absence of the suspected cause, check that the effect is similarly present or absent.
3. Look for evidence that larger amounts of the suspected cause produce larger amounts of the effect.
4. If the effect might be produced by other potential causes (besides your suspected cause), make sure that the effect still remains after accounting for these other potential causes.
5. If possible, test the suspected cause with an experiment. If the experiment cannot be performed with humans for ethical reasons, consider doing the experiment with animals, cell cultures, or computer models.
6. Try to determine the physical mechanism by which the suspected cause produces the effect.

BY THE WAY

The first four guidelines to the left are called *Mill's methods* after John Stuart Mill (1806–1873). Mill was a leading scholar of his time and an early advocate of women's right to vote. In philosophy, the four methods are called, respectively, the methods of agreement, difference, concomitant variation, and residues.

TIME OUT TO THINK

There's a great deal of controversy concerning whether animal experiments are ethical. What is your opinion of animal experiments? Defend your opinion.



CASE STUDY Air Bags and Children

By the mid-1990s, passenger-side air bags had become commonplace in cars. Statistical studies showed that the air bags saved many lives in moderate- to high-speed collisions. But a disturbing pattern also appeared. In at least some cases, young children, especially infants and toddlers in child car seats, were killed by air bags in low-speed collisions.

At first, many safety advocates found it difficult to believe that air bags could be the cause of the deaths. But the observational evidence became stronger, meeting the first four guidelines for establishing causality. For example, the greater risk to infants in child car seats fit Guideline 3, because it indicated that being closer to the air bags increased the risk of death. (A child car seat sits on top of the built-in seat, thereby putting a child closer to the air bags than the child would be otherwise.)

To seal the case, safety experts undertook experiments using dummies. They found that children, because of their small size, often sit where they could be easily hurt by the explosive opening of an air bag. The experiments also showed that an air bag could impact a child car seat hard enough to cause death, thereby revealing the physical mechanism by which the deaths occurred.

BY THE WAY

Based on these studies, the government now recommends that child car seats *never* be used on the front seat and that children under age 12 (or under 4 feet, 9 inches tall) sit in the back seat whenever possible.





CASE STUDY Cardiac Bypass Surgery

Cardiac bypass surgery is performed on people who have severe blockage of arteries that supply the heart with blood (the coronary arteries). If blood flow stops in these arteries, a patient may suffer a heart attack and die. Bypass surgery essentially involves grafting new blood vessels onto the blocked arteries so that blood can flow around the blocked areas. By the mid-1980s, many doctors were convinced that the surgery was prolonging the lives of their patients.

However, a few early retrospective studies turned up a disconcerting result: Statistically, the surgery appeared to be making little difference. In other words, patients who had the surgery seemed to be faring no better on average than similar patients who did not have it. If this were true, it meant that the surgery was not worth the pain, risk, and expense involved.

Because these results flew in the face of what many doctors thought they had observed in their own patients, researchers began to dig more deeply. Soon, they found confounding variables that had not been accounted for in the early studies. For example, they found that patients getting the surgery tended to have more severe blockage of their arteries, apparently because doctors recommended the surgery more strongly to these patients. Because these patients were in worse shape to begin with, a comparison of longevity between them and other patients was not really valid.

More important, the research soon turned up substantial differences in the results among patients who had the surgery in different hospitals. In particular, a few hospitals were achieving remarkable success with bypass surgery and their patients fared far better than patients who did not have the surgery or had it at other hospitals. Clearly, the surgical techniques used by doctors at the successful hospitals were somehow different and superior. Doctors studied the differences to ensure that all doctors could be trained in the superior techniques.

In summary, the confounding variables of *amount of blockage* and *surgical technique* had prevented the early studies from finding a real correlation between cardiac bypass surgery and prolonged life. Today, cardiac bypass surgery is accepted as a *cause* of prolonged life in patients with blocked coronary arteries. It is now among the most common types of surgery, and it typically adds *decades* to the lives of the patients who undergo it.

BY THE WAY

As you might guess, it is also difficult to define *reasonable doubt*. For criminal trials, the Supreme Court endorsed this guidance from Justice Ruth Bader Ginsburg: "Proof beyond a reasonable doubt is proof that leaves you firmly convinced of the defendant's guilt. There are very few things in this world that we know with absolute certainty, and in criminal cases the law does not require proof that overcomes every possible doubt. If, based on your consideration of the evidence, you are firmly convinced that the defendant is guilty of the crime charged, you must find him guilty. If on the other hand, you think there is a real possibility that he is not guilty, you must give him the benefit of the doubt and find him not guilty."



Hidden Causality

So far we have discussed how to establish causality after first discovering a correlation. However, sometimes a correlation—or the lack of a correlation—can hide an underlying causality. As the next case study shows, such hidden causality often occurs because of confounding variables.

Confidence in Causality

The six guidelines offer us a way to examine the strength of a case for causality, but we often must make decisions before a case of causality is fully established. Consider, for example, the well-known case of global warming. It may never be possible to prove beyond all doubt that the burning of fossil fuels is causing global warming (see the Focus on Environment at the end of this chapter), so we must decide whether to act while we still face some uncertainty about causation. How much must we know before we decide to act?

In other areas of statistics, accepted techniques help us deal with this type of uncertainty by allowing us to calculate a numerical level of confidence or significance. But there are no accepted ways to assign such numbers to the uncertainty that comes with questions of causality. Fortunately, another area of study has dealt with practical problems of causality for hundreds of years: our legal system. You may be familiar with the three broad ways of expressing a legal level of confidence shown on the top of the next page.

Broad Levels of Confidence in Causality

Possible cause: We have discovered a correlation, but cannot yet determine whether the correlation implies causality. In the legal system, possible cause (such as thinking that a particular suspect possibly committed a particular crime) is often the reason for starting an investigation.

Probable cause: We have good reason to suspect that the correlation involves cause, perhaps because some of the guidelines for establishing causality are satisfied. In the legal system, probable cause is the general standard for getting a judge to grant a warrant for a search or wiretap.

Cause beyond reasonable doubt: We have found a physical model that is so successful in explaining how one thing causes another that it seems unreasonable to doubt the causality. In the legal system, cause beyond reasonable doubt is the usual standard for convictions and generally demands that the prosecution have shown how and why (essentially the physical model) the suspect committed the crime. Note that beyond *reasonable* doubt does *not* mean beyond *all* doubt.

While these broad levels remain fairly vague, they give us at least some common language for discussing confidence in causality. If you study law, you will learn much more about the subtleties of interpreting these terms. However, because statistics has little to say about them, we will not discuss them much further in this text.

Section 4 Exercises

Statistical Literacy and Critical Thinking

1. Correlation. Identify three different explanations for the presence of a correlation between two variables.

2. Role of Experiments. In theory, we can use experiments to rule out two of the three different explanations for the presence of a correlation between two variables. Which of the three explanations do we *not* want to rule out? Why would we not want to rule it out?

3. Confounding Variable. What is a confounding variable? How can a confounding variable create a situation in which an underlying causality is hidden?

4. Correlation and Causality. What is the difference between finding a correlation between two variables and establishing causality between two variables?

Does It Make Sense? For Exercises 5–8, decide whether the statement makes sense (or is clearly true) or does not make sense (or is clearly false). Explain clearly; not all of these statements have definitive answers, so your explanation is more important than your chosen answer.

5. Value of r . When analyzing paired sample data, the value of the correlation coefficient r allows us to determine whether one variable has a direct causal effect on the other.

6. Value of r . A variable can have a direct causal effect on another variable only if the correlation coefficient is given by $r = 1$.

7. Smoking and Cotinine. A study showed that there is a correlation between exposure to second-hand smoke and the measured amount of cotinine in the body. We can establish that exposure to second-hand smoke is a cause of cotinine if we can rule out coincidence as a possible explanation of the correlation.

8. Smoking and Cotinine. When the body absorbs nicotine, it converts it into cotinine. Experiments have ruled out coincidence as an explanation for a correlation between

exposure to second-hand smoke and cotinine in the body. The only possible explanations for that correlation are that the exposure causes cotinine or that there is some other underlying cause.

Concepts and Applications

Physical Models. For Exercises 9–12, determine whether the stated causal connection is valid. If the causal connection appears to be valid, provide an explanation.

9. Test Grades. Test grades are affected by the amount of time and effort spent studying and preparing for the test.

10. Magnet Treatment. Heart disease can be cured by wearing a magnetic bracelet on your wrist.

11. Drinking and Reaction Time. Drinking greater amounts of alcohol decreases a person's reaction time.

12. IQ and Pulse Rate. People with higher resting pulse rates (beats per minute) tend to have higher IQ scores.

13. Identifying Causes: Headaches. You are trying to identify the cause of late-afternoon headaches that plague you several days each week. For each of the following tests and observations, explain which of the six guidelines for establishing causality you used and what you concluded. Then summarize your overall conclusion based on all the observations.

a. The headaches occur only on days that you go to work.

b. If you stop drinking Coke at lunch, the headaches persist.

c. In the summer, the headaches occur less frequently if you open the windows of your office slightly. They occur even less often if you open the windows of your office fully.

- 14. Smoking and Lung Cancer.** There is a strong correlation between tobacco smoking and incidence of lung cancer, and most physicians believe that tobacco smoking causes lung cancer. Yet, not everyone who smokes gets lung cancer. Briefly describe how smoking could cause cancer when not all smokers get cancer.
- 15. Other Lung Cancer Causes.** Several things besides smoking have been shown to be probabilistic causal factors in lung cancer. For example, exposure to asbestos and exposure to radon gas, both of which are found in many homes, can cause lung cancer. Suppose that you meet a person who lives in a home that has a high radon level and insulation that contains asbestos. The person tells you, “I smoke, too, because I figure I’m doomed to lung cancer anyway.” What would you say in response? Explain.
- 16. Longevity of Orchestra Conductors.** A famous study in *Forum on Medicine* concluded that the mean lifetime of conductors of major orchestras was 73.4 years, about 5 years longer than that of all American males at the time. The author claimed that a life of music *causes* a longer life. Evaluate the claim of causality and propose other explanations for the longer life expectancy of conductors.
- 17. Older Moms.** A study reported in *Nature* claims that women who give birth later in life tend to live longer. Of the 78 women who were at least 100 years old at the time of the study, 19% had given birth after their 40th birthday. Of the 54 women who were 73 years old at the time of the study, only 5.5% had given birth after their 40th birthday. A researcher stated that “if your reproductive system is aging slowly enough that you can have a child in your 40s, it probably bodes well for the fact that the rest of you is aging slowly too.” Was this an observational study or an experiment? Does the study suggest that later child bearing *causes* longer lifetimes or that later child bearing reflects an underlying cause? Comment on how persuasive you find the conclusions of the report.
- 18. High-Voltage Power Lines.** Suppose that people living near a high-voltage power line have a higher incidence of cancer than people living farther from the power line. Can you conclude that the high-voltage power line is the cause of the elevated cancer rate? If not, what other explanations might there be for it? What other types of research would you like to see before you concluded that high-voltage power lines cause cancer?
- 19. Gun Control.** Those who favor gun control often point to a positive correlation between the availability of handguns and murder rates to support their position that gun control would save lives. Does this correlation, by itself, indicate that handgun availability causes a higher murder rate? Suggest some other factors that might support or weaken this conclusion.
- 20. Vasectomies and Prostate Cancer.** An article titled “Does Vasectomy Cause Prostate Cancer?” (*Chance*, Vol. 10, No. 1) reports on several large studies that found an increased risk of prostate cancer among men with vasectomies. In the absence of a direct cause, several researchers attribute the correlation to *detection bias*, in which men with vasectomies are more

likely to visit the doctor and thereby are more likely to have any prostate cancer found by the doctor. Briefly explain how this detection bias could affect the claim that vasectomies cause prostate cancer.



PROJECTS FOR THE INTERNET & BEYOND

- 21. Air Bags and Children.** Starting from the Web site of the National Highway Traffic Safety Administration, research the latest studies on the safety of air bags, especially with regard to children. Write a short report summarizing your findings and offering recommendations for improving child safety in cars.
- 22. Dietary Fiber and Coronary Heart Disease.** In the largest study of how dietary fiber prevents coronary heart disease (CHD) in women (*Journal of the American Medical Association*, Vol. 281, No. 21), researchers detected a reduced risk of CHD among women who have a high-fiber diet. Find the research paper, summarize its findings, and discuss whether a cause for the correlation is proposed.
- 23. Coffee and Gallstones.** Writing in the *Journal of the American Medical Association* (Vol. 281, No. 22), researchers reported finding a negative correlation between incidence of gallstone disease and coffee consumption in men. Find the research paper, summarize its findings, and discuss whether a cause for the correlation is proposed.
- 24. Alcohol and Stroke.** Researchers reported in the *Journal of the American Medical Association* (Vol. 281, No. 1) that moderate alcohol consumption is correlated with a decreased risk of stroke in people 40 years of age and older. (Heavy consumption of alcohol was correlated with deleterious effects.) Find the research paper, summarize its findings, and discuss whether a cause for the correlation is proposed.
- 25. Tobacco Lawsuits.** Tobacco companies have been the subject of many lawsuits related to the dangers of smoking. Research one recent lawsuit. What were the plaintiffs trying to prove? What statistical evidence did they use? How well do you think they established causality? Did they win? Summarize your findings in one to two pages.

IN THE NEWS

- 26. Causation in the News.** Find a recent news report in which a statistical study led to a conclusion of causation. Describe the study and the claimed causation. Do you think the claim of causation is legitimate? Explain.
- 27. Legal Causation.** Find a news report concerning an ongoing legal case, either civil or criminal, in which establishing causation is important to the outcome. Briefly describe the issue of causation in the case and how the ability to establish or refute causation will influence the outcome of the case.

CHAPTER REVIEW EXERCISES

For Exercises 1–3, refer to the combined city–highway fuel economy ratings (mi/gal) for different cars. The old ratings are based on tests used before 2008 and the new ratings are based on tests that went into effect in 2008.

Old	16	27	17	33	28	24	18	22	20	29	21
New	15	24	15	29	25	22	16	20	18	26	19

1. Construct a scatterplot. What does the result suggest?
2. Estimate the value of the correlation coefficient. What does that value suggest?
3. Can we conclude that the old ratings have a direct causal effect on the new ratings? Explain briefly.
4. In a study of casino size (square feet) and revenue, the value of $r = 0.445$ is obtained. Find the value of r^2 . What does that value tell us?
5. In a study of global warming, assume that we have found a strong positive correlation between carbon dioxide concentration and temperature. Identify three possible explanations for this correlation.
6. For 10 pairs of sample data, the correlation coefficient is computed to be $r = -1$. What do you know about the scatterplot?
7. In a study of randomly selected subjects, it is found that there is a strong correlation between household income and number of visits to dentists. Is it valid to conclude that higher incomes cause people to visit dentists more often? Is it valid to conclude that more visits to dentists cause people to have higher incomes? How might the correlation be explained?

8. You are considering the most expensive purchase that you are likely to make: the purchase of a home. Identify at least five different variables that are likely to affect the actual value of a home. Among the variables that you have identified, which single variable is likely to have the greatest influence on the value of the home? Identify a variable that is likely to have little or no effect on the value of a home.

9. A researcher collects paired sample data and computes the value of the linear correlation coefficient to be 0. Based on that value, he concludes that there is no relationship between the two variables. What is wrong with this conclusion?
10. Examine the scatterplot in Figure 22 and estimate the value of the correlation coefficient.

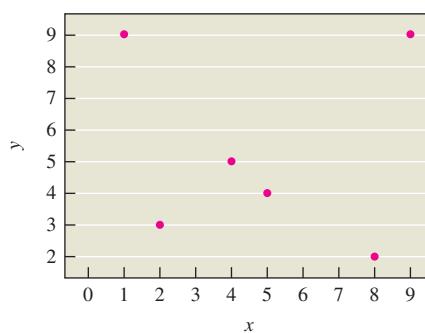


Figure 22

CHAPTER QUIZ

1. Fill in the blanks: Every possible correlation coefficient must lie between the values of _____ and _____.
2. Which of the following are likely to have a correlation?
 - a. SAT scores and weights of randomly selected subjects
 - b. Reaction times and IQ scores of randomly selected subjects
 - c. Height and arm span of randomly selected subjects
 - d. Proportion of seats filled and amount of airline profit for randomly selected flights
 - e. Value of cars owned and annual income of randomly selected car owners
3. For a collection of paired sample data, the correlation coefficient is found to be -0.099 . Which of the following statements best describes the relationship between the two variables?
 - a. There is no correlation.
 - b. There is a weak correlation.
 - c. There is a strong correlation.
 - d. One of the variables is the direct cause of the other variable.
 - e. Neither of the variables is the direct cause of the other variable.
4. Estimate the correlation coefficient for the data in Figure 23.

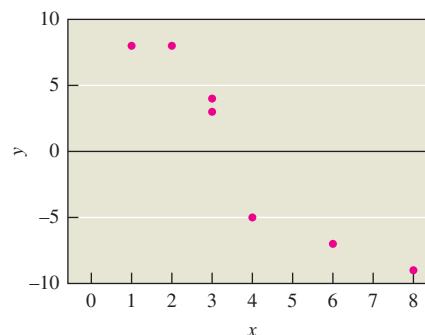


Figure 23

5. Refer again to the scatterplot in Figure 23. Does there appear to be a significant correlation between the two variables?

In Exercises 6–10, determine whether the given statement is true or false.

6. If $r = 0.200$, then $r^2 = 0.040$ and 4% of the plotted points lie on the line of best fit.
7. If $r = 1$ or $r = -1$, then all points in the scatterplot lie directly on the line of best fit.
8. If the value of the correlation coefficient is negative, the value of r^2 must also be negative.
9. A scatterplot is a graph in which the points are scattered throughout, without any noticeable pattern.
10. If the line of best fit is inserted in a scatterplot, it must pass through every point in the graph.



FOCUS ON EDUCATION

What Helps Children Learn to Read?

Everyone has an idea about how best to teach reading to children. Some advocate a phonetic approach, teaching students to “sound out” words. Some advocate a “whole language” approach, teaching students to recognize words from their context. Others advocate a combination of these approaches, or something else entirely. These differing ideas would be unimportant if they were merely opinions. But in a nation that spends more than a *trillion dollars* per year on education, differing approaches to teaching reading involve major political confrontations among groups with different special interests.

The huge stakes involved in teaching reading demand statistics to measure the effectiveness of various approaches. Some of the most important educational statistics are those that come from the National Assessment of Educational Progress (NAEP), often known more simply as “the Nation’s Report Card.” The NAEP is an ongoing survey of student achievement conducted by a government agency, the National Center for Education Statistics, with authorization and funding from the U.S. Congress.

The NAEP uses stratified random sampling to choose representative samples of fourth-, eighth-, and 12th-grade students of varying ethnicity, family income, type of school attended, and so on. Students chosen for the samples are given tests designed to measure their academic achievement in a particular subject area, such as reading, mathematics, or history. Samples are chosen on both state and national levels. Overall, a few thousand students are chosen for each test. Results from NAEP tests inevitably make the news, with articles touting improvements or decrying drops in test scores.

But what really causes improvement in reading performance? Researchers begin by searching for correlations between reading performance and other factors. Sometimes the correlations are clear, but offer no direction for improving reading. For example, parental education is clearly correlated with reading achievement—children with more highly educated parents tend to read more proficiently than those with uneducated parents—but this correlation doesn’t offer much guidance for the schools because children do not choose their parents. Other times the correlations may suggest ways to improve reading. For example, students who report reading more pages daily in school and for homework tend to score higher than students who read fewer pages. This suggests that schools should assign more reading.



Of course, the high stakes involved in education make education statistics particularly prone to misinterpretation or misuse. Consider just a few of the problems that make the NAEP reading tests difficult to interpret:

- They are standardized tests that are mostly multiple choice. Some people believe that such tests are inevitably biased and cannot truly measure reading ability.
- Because the tests generally don’t affect students’ grades, some students may not take the tests seriously, in which case test results may not reflect actual reading ability.
- State-by-state comparisons may not be valid if the makeup of the student population (particularly in its fraction of students for whom English is a second language) varies significantly among states.
- There is some evidence of cheating on the part of the *adults* involved in the NAEP tests by, for example, choosing samples that are not truly representative but instead skewed toward students who read better.

You can probably think of a dozen other problems that make it difficult to interpret NAEP results. So what can you do, as an individual, to help a child to read? Fortunately, the NAEP studies also reveal a few correlations that are uncontroversial and agree with common sense. For example, higher reading performance correlates with each of the following factors:

- more total reading, both for school and for pleasure
- more choice in reading—that is, allowing children to pick their own books to read
- more writing, particularly of extended pieces such as essays or long letters
- more discussion of reading material with friends and family
- less television watching

These correlations give at least some guidance on how to help a child learn to read and should be good starting points for discussions of how to increase literacy.

QUESTIONS FOR DISCUSSION

1. One result of the NAEP reading tests is that students in private schools tend to score significantly higher than students in public schools. Does this imply that private schools are “better” than public schools? Defend your opinion.
2. Do you think that standardized tests like those of the NAEP are valid ways to measure academic achievement? Why or why not?
3. Currently, the NAEP tests are given to only a few thousand of the millions of school children in the United States. Some people advocate giving similar tests to all students, on either a voluntary or a mandatory basis. Do you think such “standardized national testing” is a good idea? Why or why not?
4. Have you ever helped a child learn to read? Compare your experiences with those of other classmates who have worked with young children.
5. Read the latest edition of the *NAEP Reading Report Card* (available online). What are some of the latest results with regard to the teaching of reading in the United States?
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FOCUS ON ENVIRONMENT

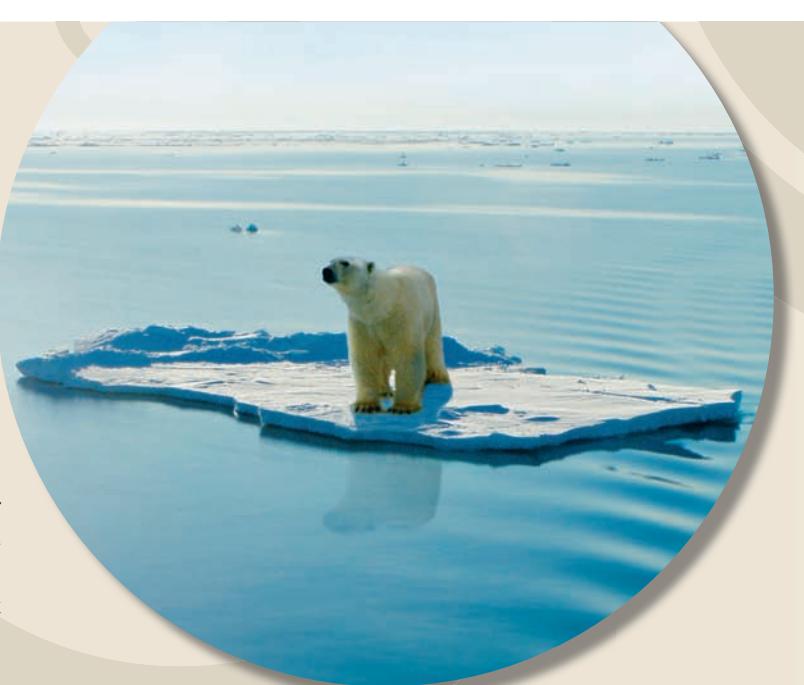
What Is Causing Global Warming?

Global warming is one of the most important issues of our time, yet surveys and media reports suggest that many people doubt that it is real or that humans are responsible for it. In this Focus, we will investigate the evidence that has led the vast majority of climate scientists to conclude that human activity is the cause of global warming.

Measurements clearly show that the atmospheric carbon dioxide concentration is rising rapidly and is now significantly higher than it has been at any time during at least the past 800,000 years. Chemical analysis shows the added carbon dioxide is coming primarily from human activity, especially the burning of fossil fuels. Moreover, data from ice cores show that the carbon dioxide concentration is strongly *correlated* with the global average temperature. The key question, then, is whether this correlation implies causality. To answer it, we must understand how a gas like carbon dioxide can affect the temperature and then investigate whether the recent increase in the concentration is having the expected effects.

The fact that some atmospheric gases—called **greenhouse gases**—can trap heat has been well-known for more than 150 years, ever since Irish physicist John Tyndall measured the heat-absorbing effects of carbon dioxide and water vapor in his laboratory in 1859. Other scientists, most notably Swedish scientist Svante Arrhenius (1859–1927), later pointed out that the burning of fossil fuels releases carbon dioxide, and that this might therefore cause global warming. Today, the mechanism by which carbon dioxide and other greenhouse gases (the most important others being water vapor and methane) warm a planet is called the **greenhouse effect**. It is well understood and summarized in Figure 24.

Scientists can further test this understanding of the greenhouse effect by checking to see whether it successfully accounts for the temperatures of various planets. In the absence of greenhouse gases, a world's average temperature would be determined by only two major factors: its distance from the Sun and the fraction of the incoming sunlight that its surface absorbs (the rest is reflected back into space). There is a simple equation that allows the calculation of the temperature in this case, and it successfully predicts the temperatures of worlds with no atmosphere, such as the Moon and the planet Mercury. For planets with atmospheres, however, scientists can successfully predict their temperatures only by taking the greenhouse effect into account, and the results clearly show that more greenhouse gases mean more excess heating. Our



planetary neighbors vividly demonstrate this fact. Mars has a very thin carbon dioxide atmosphere that gives it a fairly weak greenhouse effect, making the planet about 11°F warmer than it would be otherwise. Venus, which has an extremely dense atmosphere containing nearly 200,000 times as much carbon dioxide as Earth's atmosphere, has a correspondingly extreme greenhouse effect that makes its surface about 850°F hotter than it would be otherwise—giving it a surface hot enough to melt lead.

Earth is the lucky intermediate case. Without the greenhouse effect, Earth's average temperature would be well below freezing, at about -16°C (3°F). But thanks to the carbon

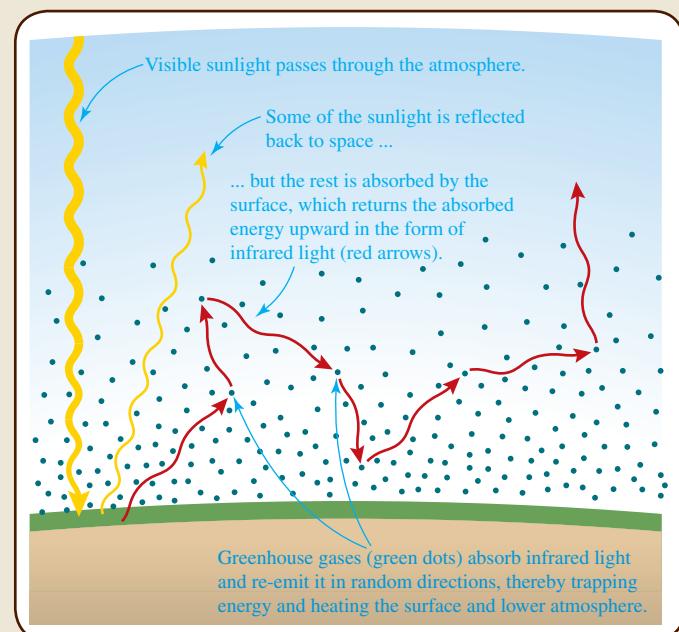


Figure 24 This diagram shows the basic mechanism of the greenhouse effect. The greater the abundance of greenhouse gases, the more the escape of infrared light is slowed and the warmer the planet becomes.

dioxide, methane, and water vapor in our atmosphere, the actual temperature is close to 15°C (60°F). From that standpoint, the greenhouse effect is a very good thing, because our lives would not be possible without it. Just keep in mind that the case of Venus offers proof that it's possible to have too much of this good thing.

The evidence from laboratory measurements and studies of other planets leave no reasonable doubt that carbon dioxide and other greenhouse gases cause a planet's temperature to be hotter than it would be otherwise. Nevertheless, because carbon dioxide is not the only thing that affects our planet's temperature, we might wonder whether any effects due to the recent rise in carbon dioxide might be offset by, say, reductions in the amount of other greenhouse gases or an increase in how much sunlight our planet reflects. Scientists can test this idea in two basic ways. First, they look at data showing changes in Earth's average temperature. As you'll see in Figure 25, Earth's average temperature has indeed been rising, and the data give at least some hint that the rise has been accelerating in recent decades.

The second way to test the idea that human burning of fossil fuels is causing global warming is to conduct experiments. We obviously cannot perform controlled experiments with our entire planet, so scientists instead build *computer models* designed to simulate the way Earth's climate works. Earth's climate is incredibly complex, so the models cannot be perfect. Nevertheless, today's models match real climate data quite well, giving scientists confidence that the models have predictive value. Figure 26 compares real data to models with and without the human contribution to the greenhouse

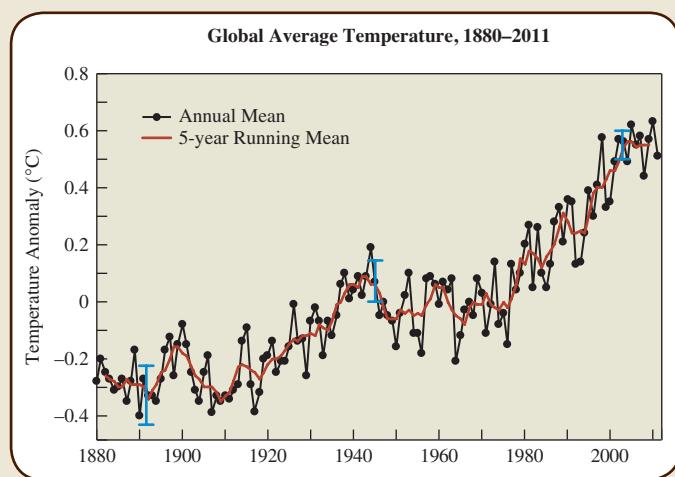


Figure 25 Clear evidence of a warming Earth: The black curve shows the mean global average temperature during each year; the red curve shows a running mean computed over 5-year periods. The vertical scale ("temperature anomaly") shows the difference between each year's actual average temperature and the average during the period 1951–1980. The blue bars represent the uncertainty ranges in the data at three different times; the uncertainty is lower for recent times because measurements have become more precise.

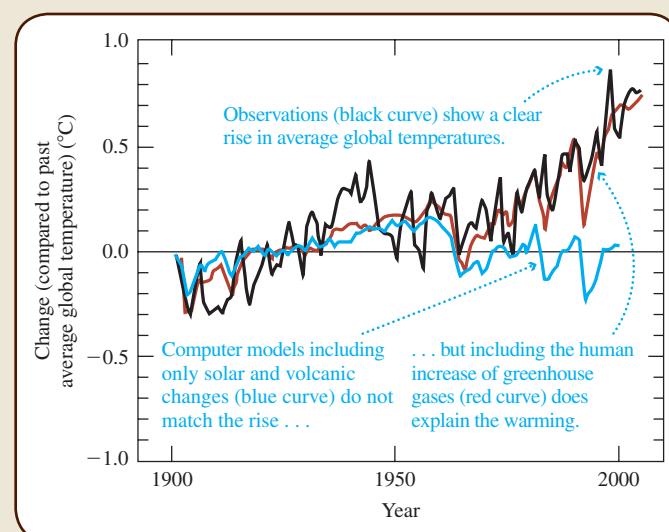


Figure 26 This graph compares observed temperature changes (black curve) with the predictions of climate models that include only natural factors such as changes in the brightness of the Sun and effects of volcanoes (blue curve), and models that also include the human-made increase in the greenhouse gas (red curve). Only the red curve matches the observations well. (The red and blue model curves are each averages of many scientists' independent models of global warming, which generally agree with each other to within $0.1^{\circ}\text{C} - 0.2^{\circ}\text{C}$.)

gas concentration. We see a good match only for models that include the human contribution.

The conclusion is clear: Laboratory measurements of the greenhouse effect, studies of other planets, data for Earth's rising carbon dioxide concentration and temperature, and computer models of the climate all provide evidence in favor of the claim that human activity is causing global warming. It is the fact that so many lines of evidence are all in agreement that makes scientists so confident that the causality is real.

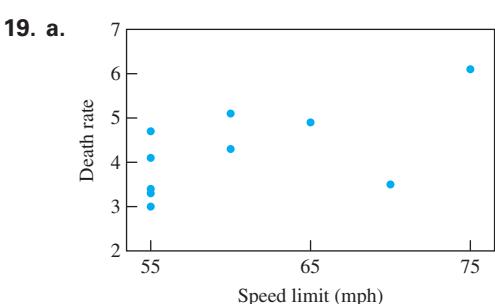
QUESTIONS FOR DISCUSSION

1. Look back at the six guidelines for establishing causality. Discuss whether or how each guideline is met by current data and understanding of global warming.
2. Look back at the legal levels of confidence in causality discussed in Section 4. Would you say that the case for human activity as the cause of global warming is now at the level of possible cause, probable cause, or cause beyond reasonable doubt? Defend your opinion.
3. Investigate some of the likely consequences of global warming. If current trends continue, what changes can you expect in the world by the year 2050? 2100?
4. Based on what you've learned about the cause of global warming and its potential consequences, what do you think we should be doing about it, if anything?

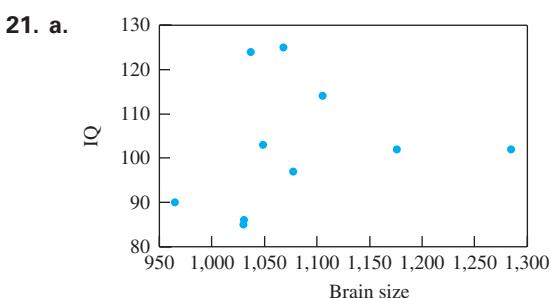
ANSWERS

Section 1

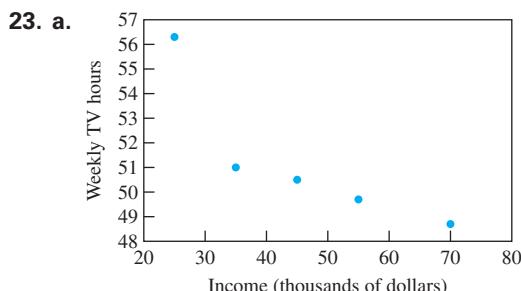
- 1.** r is called the (linear) correlation coefficient, and it measures how well the paired sample data fit the pattern of a straight line.
- 3.** No. It is possible that there is some correlation that corresponds to a scatterplot with a pattern that is not a straight-line pattern.
- 5.** Does not make sense. **7.** Does not make sense.
- 9.** Positive correlation. As the weight goes up, the cost will also go up.
- 11.** Negative correlation. Airliners that weigh more tend to use more fuel, so as weight increases, the fuel consumption in miles per gallon will decrease.
- 13.** The variables are not correlated.
- 15.** The variables are not correlated.
- 17.** There is a strong positive correlation, with the correlation coefficient approximately 0.8 or 0.9.



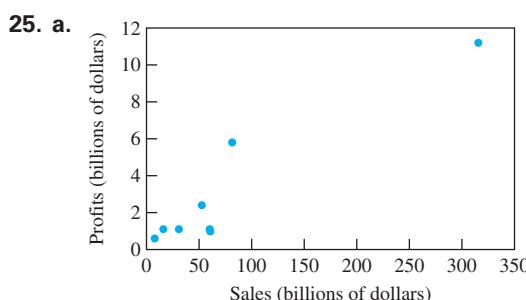
- b.** There is a moderate positive correlation; $r = 0.59$ exactly.
c. With the exception of Britain, the higher speed limits are generally associated with higher death rates. Death rates are also influenced by other factors beside speed limits.



- b.** Because the points are scattered with no obvious pattern, there does not appear to be a correlation between brain size and IQ. The value of the correlation coefficient is approximately 0.2. **c.** The data suggest that there is not a correlation between brain size and IQ.



- b.** There is a strong negative correlation between income and the number of TV hours per week ($r = -0.86$ exactly). **c.** Families with more income have more opportunities to do other things. No.

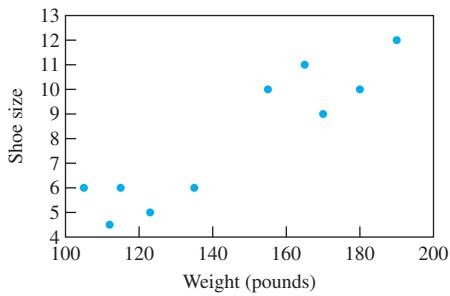


- b.** There is a strong correlation between sales and earnings ($r = 0.92$ exactly). The strong correlation in this case is highly affected by the Wal-Mart data. **c.** Higher sales do not necessarily translate into higher earnings. Some companies have larger expenses, driving earnings down.
- 27.** The variables x and y appear symmetrically in the formula (interchanging x and y does not change the formula).

Section 2

- The correlation is consistent with the possibility that Lisinopril lowers blood pressure, but without further study we cannot be sure this is the case, since the correlation could be due to other factors or coincidence.
- Outliers are values that are very far away from almost all of the other values in a data set, so the salary of \$1 is an outlier. Outliers can have a significant effect on the correlation coefficient and the conclusions we form. Outliers might make it appear that there is a significant correlation that is not real, or they might mask a real correlation.
- Does not make sense.
- Makes sense.

- 9.** There is a positive correlation that is probably due to a common underlying cause. Many crimes are committed with handguns that are not registered.
- 11.** There is a positive correlation that is due to a direct cause. As students study more, they gain a better understanding of the subject and their test scores are likely to be higher.
- 13.** There is a positive correlation that is probably due to a common cause, such as the general increase in the number of cars and traffic.
- 15.** There is a negative correlation that is probably due to a direct cause. As gas prices increase by large amounts, people can't afford to drive as much, so they cut costs by driving less.
- 17. a.** The outlier is the upper left point (0.4, 1.0). Without the outlier, the correlation coefficient is 0.0. **b.** With the outlier, the correlation coefficient is -0.58 .
- 19. a.** The actual correlation coefficient is $r = 0.92$, which is significant at the 0.01 level, so there is a very strong correlation between weight and shoe size.



- b.** Within each of the two groups the correlation is less strong than the overall correlation.
- 21. a.** The actual correlation coefficient is $r = 0.77$. This is significant at the 0.01 level, indicating a strong correlation.
- b.** The 16 points to the right correspond to relatively poor countries, such as Uganda. The remaining points on the left correspond to relative affluent countries, such as Sweden. **c.** There appears to be a negative correlation between the variables for the poorer countries and a positive correlation for the wealthier countries.
- c.** Wealthier countries have a negative correlation; higher birth rates go with lower death rates. Poorer countries have a positive correlation; higher birth rates go with higher death rates.

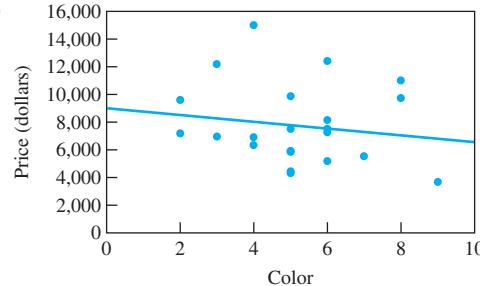
Section 3

- 1.** A best fit line (or regression line) is a line on a scatter diagram that lies closer to the data points than any other possible line (according to the standard statistical measure of closeness). A best fit line is useful for predicting the value of a variable given some value of the other variable.
- 3.** The investigator should use multiple regression. Multiple regression involves determination of the best-fit equation that represents the best fit between one variable and a combination of two or more other variables and, in this case, she wants the best fit between the

variable representing the heights of daughters and the two variables representing heights of mothers and heights of fathers.

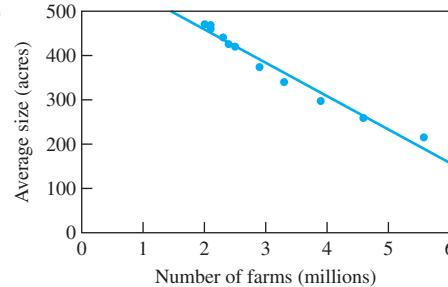
- 5. Makes sense.** **7. Does not make sense.**

- 9. a.**



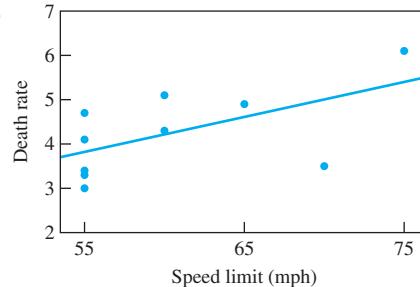
- b.** Actual $r = -0.16$; $r^2 = 0.026$. So about 3% of the variation in price can be explained by the best-fit line. **c.** The best-fit line should not be used to make predictions.

- 11. a.**

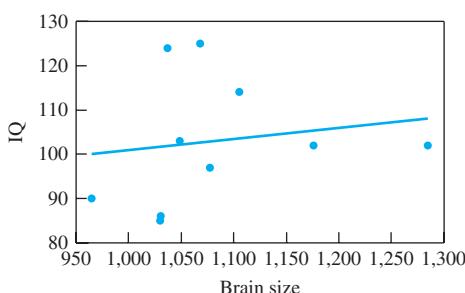


- b.** Actual $r = -0.99$; $r^2 = 0.97$. About 97% of the variation in farm size can be explained by the best-fit line. **c.** The best-fit line could be used to make predictions within the range of the number of farms included in the data. Because there does appear to be a slight curvature to the points, predicting outside that range should not be done.

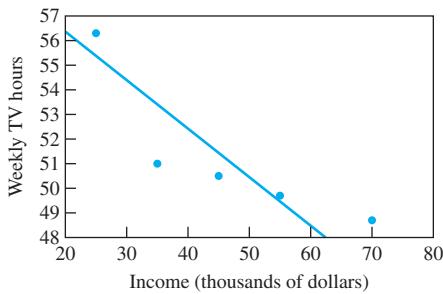
- 13. a.**



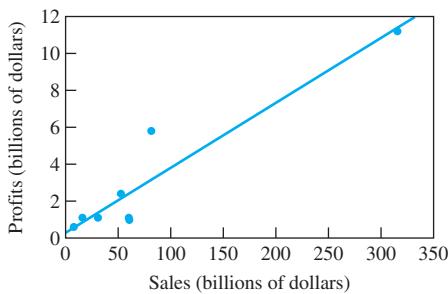
- b.** There is a moderate positive correlation; $r = 0.59$ exactly; $r^2 = 0.35$; 35% of the variation can be accounted for by the best-fit line. **c.** (75, 6.1) and (70, 3.5) are both possible outliers, the first because it is away from most of the data points and the latter because the death rate is lower than might be expected considering the rest of the data. Because one point is above the best-fit line and one is below, the net effect of the two points is probably to cancel each other out since both points will "pull" the line toward themselves. **d.** The value of r is too small to consider predictions based on the best-fit line to be reliable.

15. a.

- b.** The actual $r = 0.179$; $r^2 = 0.032$; only 3% of the variation can be accounted for by the best-fit line. **c.** (1285, 102) could be considered an outlier because it is far from the other data points. Although this outlier is far to the right, it seems to fit the general pattern, so it does not have much of an effect on the strength of the correlation and the best-fit line. **d.** Predictions based on the best-fit line are not reliable.

17. a.

- b.** The actual $r = -0.86$; $r^2 = 0.74$; 74% of the variation can be accounted for by the best-fit line. **c.** (25000, 56.3) is an outlier. The best-fit line would have a less steep downward slope if that point were removed. **d.** Predictions based on the best-fit line could be reliable, but the presence of the outlier and its effect on the best-fit line make the reliability questionable.

19. a.

- b.** The actual $r = 0.941$; and $r^2 = 0.886$; 88.5% of the variation can be accounted for by the best-fit line. **c.** (315.6, 11.2) is an outlier. **d.** Predictions based on the best-fit line could be reliable, but the strong effect of the outlier in determining the best-fit line makes the reliability questionable. More data is needed.

3. A confounding variable is a variable that is not included in the analysis, but it affects the variables that are included in the analysis. Failure to include or account for a confounding variable might cause a researcher to miss an underlying causality by considering only data showing no correlation.

5. Does not make sense.

7. Does not make sense.

9. The causal connection is valid. When students spend more time and effort studying for a test, the test grades tend to be higher.

11. The causal connection is valid. Alcohol is a depressant to the central nervous system, and it has several effects that include decreased reaction time. This is one important reason why drinking and driving is so dangerous.

13. Guideline 1; Guidelines 2 and 5; Guidelines 3 and 5. The headaches are associated with work days in some way. The headaches are not associated with Coke or possibly with caffeine. The headaches are possibly the result of bad ventilation in the building.

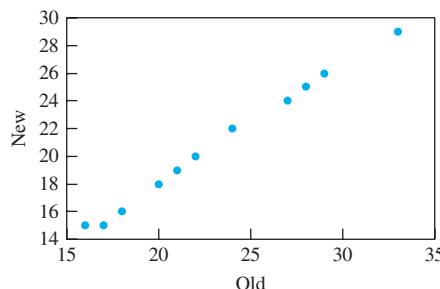
15. Smoking can only increase the risk already present.

17. This was an observational study. Later child bearing reflects an underlying cause. While it's possible that the conclusions are correct, there are other possible explanations for the findings. For example, it's also possible that the younger women lived during a time when having babies after age forty was less likely (by choice). It is still possible for them to live to be 100.

19. Availability is not itself a cause. Social, economic, or personal conditions cause individuals to use the available weapons.

Chapter Review Exercises

1. The scatterplot suggest that there is a very strong positive correlation between the old ratings and the new ratings.



3. No. The presence of a strong positive correlation does not imply that either variable is the direct cause of the other.

5. (1) The correlation is the result of a coincidence; (2) the correlation is due to a common cause; (3) the correlation is due to a direct influence of one of the variables on the other.

7. Correlation alone never implies causation, and, in this case, certainly more trips to the dentist do not cause higher incomes. Households with more disposable income can afford more trips

Section 4

- 1.** (1) The correlation is the result of a coincidence; (2) the correlation is due to a common cause; (3) the correlation is due to a direct influence of one of the variables on the other.

to the dentist or can afford dental insurance that covers the costs of the trips.

- 9.** The data values that were collected were uncorrelated. It's still possible that the variables represented by the data values are related in some non-linear way, i.e., the scatter plot forms a curve instead of a straight line.

Chapter Quiz

- 1.** –1 and 1 **3.** a
5. Yes **7.** True
9. False

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From Samples to Populations

Did you ever wonder how the outcome of a national election can be predicted hours before the polls close? Or how a large retailer can make critical marketing decisions based on a survey of only a few hundred people? These examples illustrate perhaps the most powerful aspect of statistics: the capability to use information gathered from a small sample to make conclusions about a much larger population. This process, called *making inferences*, is the subject of the branch of statistics called *inferential statistics*.

Some people hate the very name statistics, but I find them full of beauty and interest. Whenever they are not brutalized, but delicately handled by the higher methods, and are warily interpreted, their power of dealing with complicated phenomena is extraordinary.

—Sir Francis Galton (1822–1912)

LEARNING GOALS

1 Sampling Distributions

Understand the fundamental ideas of sampling distributions, enabling you to work with both a distribution of sample *means* and a distribution of sample *proportions*.

2 Estimating Population Means

Learn to estimate population means and compute the associated margins of error and confidence intervals.

3 Estimating Population Proportions

Learn to estimate population proportions and compute the associated margins of error and confidence intervals.

FOCUS TOPICS

Focus on History: Where Did Statistics Begin?

Focus on Literature: How Many Words Did Shakespeare Know?

SAMPLING DISTRIBUTIONS

Consider the following statements taken from recent news articles or research reports.

- The mean daily protein consumption by Americans is 67 grams.
- Nationwide, the mean hospital stay after delivery of a baby decreased from 3.2 days in 1980 to the current mean of 2.0 days.
- Thirty percent of high school girls in this country believe they would be happier being married than not being married.
- About 5% of all American children live with a grandparent.

Each statement makes a claim about a population, based on statistics obtained from a much smaller sample. More specifically, the sample statistics were used to make estimates of the corresponding population parameters. In other words, the sample data were used to make *inferences* about the population, which means the methods used are part of the branch of statistics known as *inferential statistics*.

*Public opinion in this country
is everything.*

—Abraham Lincoln

Note also that the four statements are based on two different types of sample statistics. The first two statements give estimates concerning the *mean* of a quantity—mean protein consumption of 67 grams and mean hospital stays of 3.2 days and 2.0 days. The last two statements say something about a *proportion* of the population—30% of high school girls and 5% of American children. The methods for dealing with sample means and sample proportions are slightly different, so we will consider each separately.

Sample Means: The Basic Idea

Consider a family with three children of ages 4, 5 and 9. Imagine that, for some reason, we wanted to study this family alone, so that the three children represented the entire population for our study. Such small populations rarely arise in real statistical applications, but using this small one will make it easier for us to look carefully at the sampling process. Notice that the population mean age for the three children is:

$$\frac{4 + 5 + 9}{3} = 6.0 \text{ years}$$

The smallest possible sample size is $n = 1$, and there are three possible samples that we could draw: the child of age 4, the child of age 5, and the child of age 9. The mean of each sample of size $n = 1$ is simply the age of the child in the sample. Figure 1 shows a histogram of the means of all three samples of size $n = 1$.

The distribution of any sample statistic from *all* possible samples of a particular size is called a **sampling distribution**, and the distribution in Figure 1 is called a **sampling distribution of sample means**, because it shows the means of all three samples of size $n = 1$. Because the three sample means in this case are simply the ages of the three children, we can say that the “mean of the sample means” is equal to the mean of the entire population, or 6.0 years.

TECHNICAL NOTE

In the large populations that we usually deal with in statistics, it makes no significant difference whether we sample with or without replacement. However, independent events are easier to analyze than dependent events, which is why we focus here on sampling with replacement.

Definition

The **sampling distribution of sample means** is the distribution that results when we find the means of *all* possible samples of a given size.

Let’s move on to samples of size $n = 2$, in which each sample consists of two children. Although it may sound a little strange for a population of only three children, we will

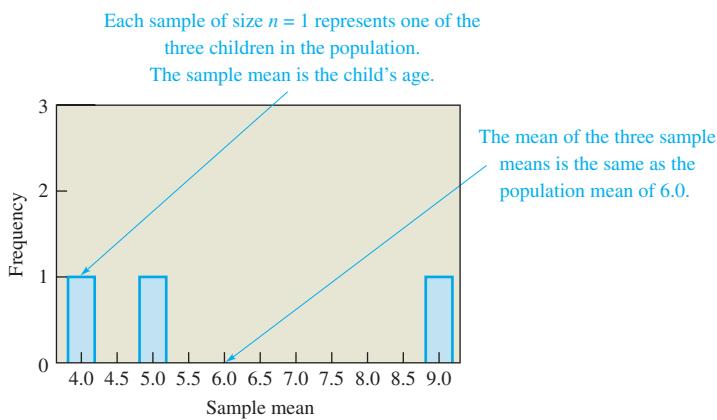


Figure 1 Sampling distribution of sample means for sample size $n = 1$, drawn from a population of three children ages 4, 5, and 9.

assume that it's possible for a sample of size $n = 2$ to have the same child twice. This type of sampling is called sampling *with replacement*, because we choose one child at random as the first member of the sample, then we put that child back in the pool of possible choices before choosing the next member of the sample. As a result, there are three children to choose from for each of the two members of the sample, so the total number of possible samples of size $n = 2$ is $3^2 = 9$. Table 1 lists the nine different possible samples and the mean for each sample. Be sure to notice that each sample mean is simply the mean of the two values in the sample; for example, the sample consisting of the ages 4 and 9 has a sample mean of $(4 + 9)/2 = 6.5$.

To construct the sampling distribution of the sample means, we must look through Table 1 to find the frequency with which each sample mean occurs. For example, a sample mean of 4.0 occurs only once (for the sample $\{4, 4\}$), while a sample mean of 4.5 occurs twice (for the samples $\{4, 5\}$ and $\{5, 4\}$). Table 2 shows the frequency for each of the sample means with sample size $n = 2$, and Figure 2 shows a histogram of this sampling distribution of sample means. You should notice two key facts. First, we can use the data in Table 2 to calculate the mean of the nine individual sample means for sample size $n = 2$:

$$\frac{4.0 + (2 \times 4.5) + 5.0 + (2 \times 6.5) + (2 \times 7.0) + 9.0}{9} = 6.0 \text{ years}$$

TABLE 1		Sample Means for a Three-Child Family of Ages 4, 5, 9; Sample Size $n = 2$
Sample		Sample Mean
4, 4		4.0
4, 5		4.5
4, 9		6.5
5, 4		4.5
5, 5		5.0
5, 9		7.0
9, 4		6.5
9, 5		7.0
9, 9		9.0

TABLE 2		Frequency of Sample Means from Table 1
Sample Mean	Frequency	
4.0	1	
4.5	2	
5.0	1	
6.5	2	
7.0	2	
9.0	1	

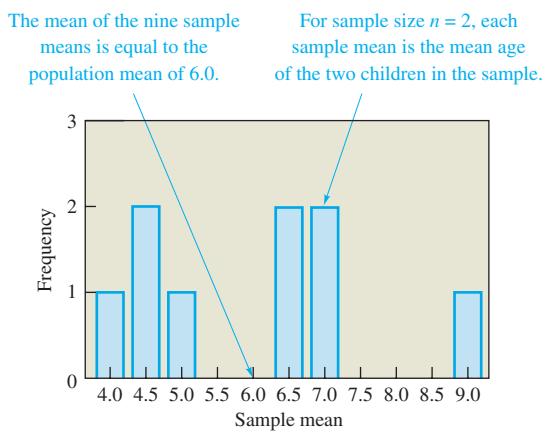


Figure 2 Sampling distribution of sample means for sample size $n = 2$, drawn from a population of three children ages 4, 5, and 9.

In other words, just as we found for our earlier sampling distribution ($n = 1$), the mean of the sample means is equal to the population mean of 6.0 years. Second, although Figure 2 still does not look much like a normal distribution, it looks “more normal” than Figure 1, because we are starting to see some clustering of values near the mean.

In fact, if we used larger sample sizes, we would find that the sampling distribution of sample means would look more and more like a normal distribution (a consequence of the Central Limit Theorem). Figure 3 shows an example in which a computer was used to select 1,000 random samples of size $n = 10$ from our same population of three children with ages 4, 5, and 9. As you can see, the sampling distribution is beginning to look like a normal distribution. Of course, drawing samples of size $n = 10$ from a population of only three children creates the somewhat unrealistic situation in which each sample will always have at least one of the children multiple times. So to continue our discussion, let’s turn to a slightly more realistic situation.

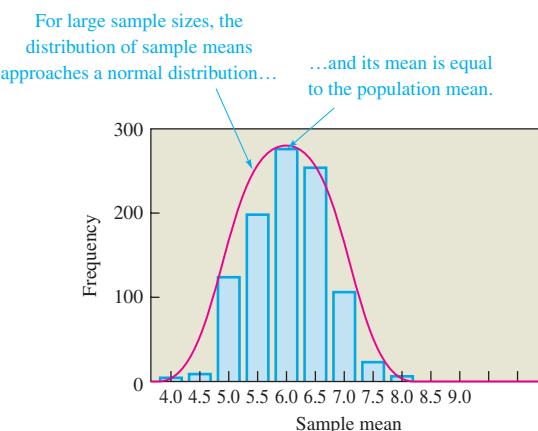


Figure 3 The distribution of sample means for 1,000 randomly selected samples of size $n = 10$, drawn from a population of three children ages 4, 5, and 9.

TIME OUT TO THINK

Figure 3 shows 1,000 random samples of size $n = 10$ drawn from the population of three children, but this is not all of the possible samples. How many samples are possible in this case? (Hint: Remember that there are three children to choose from for each of the 10 selections that you make for the sample.)

Sample Means with Larger Populations

In real statistical studies, we rarely work with populations as small as three individuals. To explore the process of selecting samples and forming distributions of sample means in a more realistic setting, let’s consider a somewhat larger population.

Imagine that you work for the computer services department of a small college. In order to develop networking strategies, you survey all 400 students at the college to determine how many hours per week they spend using a search engine on the Internet. The responses (hours per week) are shown at the top of the next page.

You could, of course, calculate the mean of all 400 responses. You would find that it is 3.88 hours per week. This mean is the true **population mean**, because it is the mean for the entire population of 400 students; we denote the population mean by the Greek letter μ (pronounced “mew”). Similarly, a calculation shows that the population standard deviation is $\sigma = 2.40$ hours.

3.4	6.8	6.7	3.4	0.0	5.0	5.4	1.8	0.7	1.6	2.1	3.5	3.4	6.4	7.2	1.8	7.4	3.0	4.0	5.2
1.2	7.8	7.0	0.4	7.2	4.8	3.6	8.0	5.4	6.4	3.5	5.3	4.7	5.4	5.6	3.8	0.1	2.4	0.5	4.0
4.5	8.0	4.2	1.0	6.2	7.1	3.8	0.7	5.5	1.7	2.6	1.6	0.7	1.3	6.5	2.4	3.0	0.3	2.2	0.4
1.9	5.0	2.0	5.3	7.5	5.0	0.3	7.4	6.0	4.3	1.3	0.8	7.2	6.6	0.2	3.4	1.6	2.2	3.0	4.5
5.5	5.3	6.5	0.1	0.3	4.2	2.2	6.2	7.3	3.1	5.4	1.3	6.3	4.5	7.1	5.8	6.1	0.5	0.4	4.1
7.0	6.0	1.1	0.8	1.4	2.9	7.3	0.8	2.7	0.6	3.0	0.7	2.8	6.5	1.9	3.6	1.6	2.6	2.6	6.6
6.8	6.1	3.6	1.4	7.7	5.2	3.8	6.0	2.2	7.5	6.7	4.4	4.1	7.3	5.2	5.7	6.7	2.4	0.6	6.7
1.0	2.3	0.7	1.2	4.5	3.3	4.2	2.1	5.9	3.0	7.2	7.9	2.5	7.1	8.0	6.7	4.1	4.9	0.0	3.1
6.0	0.5	4.2	2.7	0.1	1.4	2.1	2.5	3.9	5.8	5.9	2.7	2.8	3.7	7.3	0.7	6.9	4.4	0.7	1.6
3.1	2.1	7.4	3.6	6.5	2.9	5.4	3.9	3.0	0.8	0.3	0.8	3.3	0.8	8.0	5.6	7.1	1.3	0.2	5.2
7.8	4.7	7.2	0.9	5.1	0.9	1.7	1.2	0.4	6.9	0.6	3.0	3.6	6.1	1.6	6.0	3.8	0.4	1.1	4.0
3.8	4.0	1.8	0.9	1.1	3.9	1.7	1.7	2.6	0.1	4.0	1.4	1.9	0.9	0.2	4.2	4.7	0.2	5.3	2.2
5.8	7.5	5.8	5.2	3.9	3.4	7.3	4.1	0.5	7.9	7.7	7.7	5.0	2.3	7.8	2.3	5.6	6.5	7.9	5.0
2.0	5.5	5.4	6.6	6.7	4.4	7.2	2.5	4.9	7.0	2.1	7.2	4.1	1.2	6.2	3.3	6.3	2.3	4.9	2.2
6.4	7.2	0.1	5.3	3.0	0.7	1.5	1.2	1.1	7.4	5.1	7.2	7.2	3.0	7.1	4.5	6.7	7.2	7.2	0.9
2.9	4.3	2.5	0.7	7.6	3.9	0.7	5.8	6.6	3.4	0.3	6.5	7.5	0.7	6.1	6.1	4.8	1.9	1.9	5.0
1.1	7.8	6.8	4.9	3.0	6.5	5.2	2.2	5.1	3.4	4.7	7.0	3.8	5.7	6.8	1.2	1.7	6.5	0.1	4.3
6.3	1.2	0.8	0.7	0.6	7.0	4.0	6.6	6.9	0.5	4.3	1.0	0.5	3.1	0.9	2.3	5.7	6.7	7.3	0.5
0.3	0.9	2.4	2.5	7.8	5.6	3.2	0.7	5.4	0.0	5.7	0.3	7.2	5.1	2.5	3.2	3.1	2.8	5.0	5.6
3.1	0.7	0.5	3.9	2.6	7.3	1.4	1.2	7.1	5.5	3.1	5.0	6.8	6.5	1.7	2.1	7.3	4.0	2.2	5.6

In typical statistical applications, populations are huge and it is impractical or too expensive to survey every individual in the population; consequently, we rarely know the true population mean, μ . Therefore, it makes sense to consider using the mean of a *sample* as an estimate of the mean of the entire population. Although a sample is easier to work with, it cannot possibly represent the entire population exactly. Therefore, we should not expect an estimate of the population mean obtained from a sample to be perfect. The error that we introduce by working with a sample is called the **sampling error**. We can explore this idea by considering samples drawn from the 400 responses about Internet use.

Sampling Error

The **sampling error** is the error introduced because a random sample is used to estimate a population parameter. It does not include other sources of error, such as those due to biased sampling, bad survey questions, or recording mistakes.

TIME OUT TO THINK

Would you expect the sampling error to increase or decrease if the sample size were increased? Explain.

Suppose you select a random sample of $n = 32$ responses from the population of 400 responses shown on the previous page and calculate their mean. For example, the random sample might be the following values (in hours):

1.1 7.8 6.8 4.9 3.0 6.5 5.2 2.2 5.1 3.4 4.7 7.0 3.8 5.7 6.5 2.7
2.6 1.4 7.1 5.5 3.1 5.0 6.8 6.5 1.7 2.1 1.2 0.3 0.9 2.4 2.5 7.8

The mean of this sample, which we call the **sample mean**, is $\bar{x} = 4.17$ hours; we use the notation \bar{x} (read as “x-bar”) to denote a sample mean.

Notation for Population and Sample Means

n = sample size

μ = population mean

\bar{x} = sample mean

σ = population standard deviation

s = sample standard deviation

Now, suppose you collect a second sample of size $n = 32$ from the same data set, such as this sample:

1.8	0.4	4.0	2.4	0.8	6.2	0.8	6.6	5.7	7.9	2.5	3.6	5.2	5.7	6.5	1.2
5.4	5.7	7.2	5.1	3.2	3.1	5.0	3.1	0.5	3.9	3.1	5.8	2.9	7.2	0.9	4.0

For this sample, the sample mean is $\bar{x} = 3.98$ hours.

Note that the two sample means found so far ($\bar{x} = 4.17$ and $\bar{x} = 3.98$) do not perfectly match each other, and neither one matches the true population mean ($\mu = 3.88$). This shouldn't be surprising, because we don't expect samples to be exact representations of a population. But suppose you continue to select samples of 32 responses, and each time you calculate the sample mean, \bar{x} . To get a good picture of all these sample means, you could make a histogram showing the number of samples that have various sample means. Figure 4 shows a histogram that results from 100 different samples, each with 32 students. Notice that this histogram is very close to a *normal distribution* and that its mean is very close to the population mean, $\mu = 3.88$ hours.

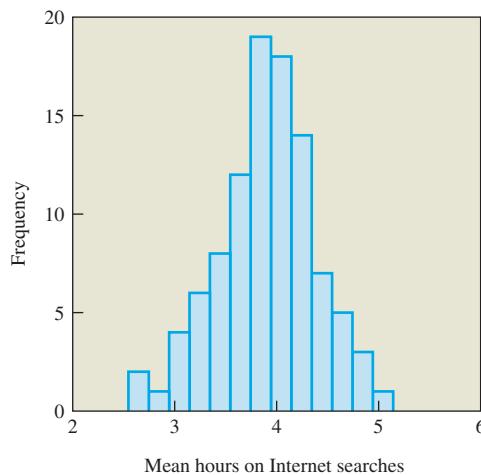


Figure 4 A distribution of 100 sample means, with a sample size of $n = 32$, appears close to a normal distribution with a mean of 3.88 hours.

TIME OUT TO THINK

Suppose you choose only one sample of size $n = 32$. According to Figure 4, are you more likely to choose a sample with a mean less than 2.5 or a sample with a mean less than 3.5? Explain.

What would happen if we made a histogram of the complete distribution of sample means (rather than only of 100 sample means) by analyzing *all* possible samples of size $n = 32$? We can't actually compute all these sample means, because there are too many: The number of possible samples of size $n = 32$ drawn from a population of 400 is 400^{32} (because there are 400 choices for each of the 32 sample selections), which turns out to be a number greater than the total estimated number of atoms in the observable universe!

Nevertheless, if we could compute all of those sample means, we would find that their distribution would be indistinguishable from a perfectly normal distribution, that it would have a mean equal to the population mean (3.88 hours in this case), and that it would have a standard deviation equal to the population standard deviation divided by the square root of the sample size, or $\sigma/\sqrt{n} = 2.40/\sqrt{32} = 0.42$ hours. The following box generalizes these ideas.

Characteristics of the Distribution of Sample Means

For any distribution of sample means:

- The larger the sample size, the more closely the distribution of sample means approximates a normal distribution.
- The mean of the distribution of sample means equals the population mean.
- The standard deviation of the distribution of sample means is given by the expression $\frac{\sigma}{\sqrt{n}}$.

TECHNICAL NOTE

A common guideline is to assume that the distribution of sample means is close to normal if the sample size is greater than 30.

We can use these ideas to make a quantitative statement about how well the mean of a particular sample approximates the population mean. Consider one more sample of 32 responses from the data shown earlier:

5.8 7.5 5.8 5.2 3.9 3.4 7.3 4.1 0.5 7.9 7.7 7.7 5.0 2.3 7.8 2.3
5.0 6.8 6.5 1.7 2.1 7.3 4.0 2.2 5.6 4.7 5.3 3.5 6.5 3.4 6.6 5.0

The mean of this sample is $\bar{x} = 5.01$. Given that the mean of the distribution of sample means is 3.88 and the standard deviation is 0.42, the sample mean of $\bar{x} = 5.01$ has a *standard score* of

$$z = \frac{\text{sample mean} - \text{pop.mean}}{\text{standard deviation}} = \frac{5.01 - 3.88}{0.42} = 2.7$$

In other words, the sample we have selected has a mean that is 2.7 standard deviations above the mean of the sampling distribution. This standard score corresponds to the 99.65th percentile. Therefore, the probability of selecting another sample with a mean *less* than 5.01 is about 0.9965, and the probability of selecting another sample with a mean *greater* than 5.01 is about $1 - 0.9965 = 0.0035$. This low probability tells us that the sample we selected is rather extreme within this distribution.

EXAMPLE 1 Sampling Farms

Texas has roughly 225,000 farms, more than any other state in the United States. The actual mean farm size is $\mu = 582$ acres, and the standard deviation is $\sigma = 150$ acres. For random samples of $n = 100$ farms, find the mean and standard deviation of the distribution of sample means. What is the probability of selecting a random sample of 100 farms with a mean greater than 600 acres?

SOLUTION Because the distribution of sample means is approximately a normal distribution, its mean should be the same as the mean of the entire population, which is 582 acres. The standard deviation of the sampling distribution is $\sigma/\sqrt{n} = 150/\sqrt{100} = 15$. A sample mean of $\bar{x} = 600$ acres therefore has a standard score of

$$z = \frac{\text{sample mean} - \text{pop.mean}}{\text{standard deviation}} = \frac{600 - 582}{15} = 1.2$$

BY THE WAY

The total number of U.S. farms peaked at about 6.8 million in 1935. Today, there are approximately 2 million farms in the United States, but their mean acreage is significantly larger than the mean acreage of the past.



This standard score is in the 88th percentile. Therefore, the probability of selecting a sample with a mean less than 600 acres is about 0.88, and the probability of selecting a sample with a mean greater than 600 acres is about 0.12.

Sample Proportions

Much of what we have learned about distributions of sample means carries over to distributions of *sample proportions*. We can see the parallels by considering another survey of 400 students. Suppose your goal is to determine the proportion (or percentage) of all 400 students who own a car. Each Y (for *yes*) or N (for *no*) below is one person's answer to the question "Do you own a car?"

Y N Y Y N Y N N Y N Y Y N N N Y Y Y Y N Y N Y N Y Y N Y Y N Y Y N
 N N Y Y Y N Y N Y N Y N Y Y Y Y Y Y N Y Y N Y N Y N N N Y Y N Y N Y
 N Y N Y Y Y Y N Y N Y N Y N N N Y Y N N N Y Y Y Y N Y N Y N Y N Y
 Y Y Y N Y N Y N Y Y Y N Y N Y N N N Y Y N N N Y Y N Y N Y N Y N Y
 Y N Y Y N Y N N Y N Y Y N N N N Y Y Y N Y N Y N Y N Y Y Y N Y
 Y N Y N N Y N Y Y N N N Y Y Y N Y N Y Y Y N Y N Y N Y Y N Y N Y
 Y N N Y N Y Y N N N Y Y Y N N N Y Y Y N Y N Y N Y N Y Y Y N Y
 N Y N Y Y Y N Y Y N Y Y N Y Y N Y Y N Y Y N N N Y Y Y N N N Y
 Y Y Y N Y N Y N Y Y N Y N Y N Y N Y N Y Y Y Y Y N Y N Y N Y N Y
 Y Y Y Y N Y N Y N Y Y N Y N N N Y Y N N N Y Y Y N N N Y Y Y N Y
 N Y Y Y N Y Y N Y Y Y Y N Y N Y N Y Y N Y N Y N Y N N N Y Y Y N Y
 N Y Y Y N N N Y Y Y N Y N Y N Y N Y N Y N Y N Y N Y N Y N Y N N

If you counted carefully, you would find that 240 of the 400 responses are Y's, so the *exact* proportion of car owners in the 400-student population is

$$p = \frac{240}{400} = 0.6$$

This *population proportion*, $p = 0.6$, is another example of a *population parameter*. Of the 400 students in the population, it is the true proportion of car owners.

TIME OUT TO THINK

Give another survey question that would result in a population proportion rather than a population mean.

Once again, in typical statistical problems, it is impractical or prohibitively expensive to survey every individual in the population. Therefore, it's reasonable to consider the idea of using a random sample of, say, $n = 32$ people to estimate the population proportion. Suppose you randomly draw 32 responses from the list of Y's and N's to generate the following sample:

Y N Y Y N Y Y N Y N Y Y Y N N N Y Y Y N N N Y Y Y N Y Y Y N Y Y

The proportion of Y responses in this list is

$$\hat{p} = \frac{21}{32} = 0.656$$

This proportion is another example of a *sample statistic*. In this case, it is a **sample proportion** because it is the proportion of car owners within a *sample*; we use the symbol \hat{p} (read "*p-hat*") to distinguish this sample proportion from the population proportion, p .

Notation for Population and Sample Proportions

n = sample size

p = population proportion

\hat{p} = sample proportion

Proceeding as we did with sample means, suppose that we next selected many more samples of 32 responses, each time calculating the sample proportion, \hat{p} . If we make a histogram of the many sample proportions, it will show the number of samples that have particular values of \hat{p} from 0 through 1. Figure 5 shows such a histogram, in this case resulting from 100 samples of size $n = 32$. As we found for sample means, this distribution of sample proportions is very close to a normal distribution, and its mean is very close to the population proportion of 0.6.

There are three sorts of opinion: informed, uninformed and inconsequential. Uninformed opinions encompass most of American opinion.

—Paul Talmey, pollster

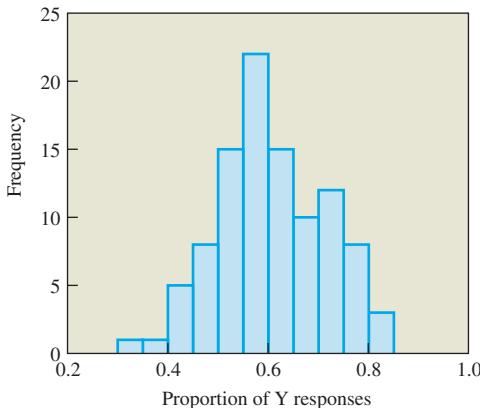


Figure 5 The distribution of 100 sample proportions, with a sample size of 32, appears to be close to a normal distribution.

If it were possible to select *all* possible samples of size $n = 32$, the resulting distribution would be called a **sampling distribution of sample proportions**. As the following box indicates, this distribution shares many similarities with a distribution of sample means and can also be used to determine the probability of a particular sample mean.

The Distribution of Sample Proportions

The **sampling distribution of sample proportions** is the distribution that results when we find the proportions (\hat{p}) in all possible samples of a given size. Note that:

- The larger the sample size, the more closely the distribution of sample proportions approximates a normal distribution.
- The mean of the distribution of sample proportions equals the population proportion.
- The standard deviation of the distribution of sample proportions is given by the expression $\sqrt{\frac{p(1-p)}{n}}$.

EXAMPLE 2 Analyzing a Sample Proportion

Consider the distribution of sample proportions shown in Figure 5. Suppose you randomly select the following sample of 32 responses:

Y Y N Y Y Y N Y Y Y Y Y N Y Y N Y Y N Y Y N Y N Y Y

Compute the sample proportion, \hat{p} , for this sample. How far does it lie from the population mean? What is the probability of selecting another sample with a proportion greater than the one you selected?

SOLUTION The proportion of Y responses in this sample is

$$\hat{p} = \frac{24}{32} = 0.75$$

Using the population mean of 0.6 (given earlier), the standard deviation for the distribution of sample proportions is

$$\sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.6(1-0.6)}{32}} \approx 0.09$$

The sample statistic, $\hat{p} = 0.75$, therefore has a standard score of

$$z = \frac{\text{sample proportion} - \text{pop. proportion}}{\text{standard deviation}} = \frac{0.75 - 0.6}{0.09} = 1.7$$

The sample proportion is 1.7 standard deviations above the mean of the distribution. A standard score of 1.7 corresponds to about the 95th percentile. The probability of selecting another sample with a proportion less than the one we selected is about 0.95, and the probability of selecting another sample with a proportion greater than the one we selected is about 0.05. In other words, if we were to select 100 random samples of 32 responses, we should expect to see only about 5 samples with a higher proportion than the one we selected. •••

Section 1 Exercises

Statistical Literacy and Critical Thinking

1. Sampling Distribution. Pollsters often use randomly selected digits between 0 and 9 to generate parts of telephone numbers to be called. What is the distribution of such randomly selected digits? If we repeat the process of randomly generating 50 digits and finding the mean, what is the distribution of the resulting sample means?

2. Polls. America OnLine published a survey question. Among the 2,300 Internet users who responded, 20% answered “yes” and the others answered “no.” For the “yes” responses, what is the value of \hat{p} ? What is fundamentally wrong with this survey?

3. Notation. What does \bar{x} denote, what does μ denote, and what is the difference between them?

4. Notation. What does \hat{p} denote, what does p denote, and what is the difference between them?

Does It Make Sense? For Exercises 5–8, decide whether the statement makes sense (or is clearly true) or does not make sense (or is clearly false). Explain clearly. Not all of these statements have definitive answers, so your explanation is more important than your chosen answer.

5. Larger Sample Size. Two different surveys will be conducted to estimate the mean salary of employees who have taken a statistics course. If the first survey has a larger sample size than the second survey, the first survey will result in a sample mean that is closer to the population mean

(when compared with the second survey).

6. Larger Sample Size. When a random sample is used to estimate a population mean, the sample mean tends to become a better estimate of the population mean as the sample size increases.

7. Large Sample Size. A researcher for a car dealership wants to estimate the mean age of cars in his county. He goes to the largest shopping mall, he randomly selects 3,000 cars in the parking lot, and he obtains their ages from the registration stickers on the car windows. He then computes the mean age of the sample of 3,000 cars. The sample mean is a good estimate of the population mean, because the large sample size compensates for the fact that he is using a convenience sample.

8. Survey. In an Adecco Staffing survey of 1,000 adults in the United States, 140 (or 14%) said that salary was the most important feature of their job. The sample proportion of 0.14 cannot be a good estimate of the population proportion because this survey is based on such a small proportion of the population of adults.

Concepts and Applications

- 9. Estimating Population Proportions.** In a survey of children 5 to 17 years old, 1,050 children were randomly selected from the nine states in the Northeast, and the proportion who spoke a language other than English in their home was 0.19. Is that sample proportion a good estimate for children in the United States? Why or why not?
- 10. Estimating Population Means.** When 40 women were randomly selected and tested for their cholesterol levels, a mean of 240.9 milligrams was obtained. Would you be more confident of your estimate if the sample included measurements from 500 women? Explain.
- 11. Distribution of Sample Means.** Assume that cans of Coke are filled so that the actual amounts have a mean of 12.00 ounces. A random sample of 36 cans has a mean amount of 12.19 ounces. The distribution of sample means of size 36 is normal with an assumed mean of 12.00 ounces, and those sample means have a standard deviation of 0.02 ounce.
- How many standard deviations is the sample mean from the mean of the distribution of sample means?
 - In general, what is the probability that a random sample of size 36 has a mean of at least 12.19 ounces?
 - Does it appear that consumers are being cheated? Why or why not?
- 12. Distribution of Sample Means.** Assume that the population of heights of men has a normal distribution with a mean of 69.5 in. and random samples of 100 men result in sample mean heights with a mean of 69.5 in. and a standard deviation of 0.24 in.
- If one sample of 100 men results in a mean height of 69.0 in., how many standard deviations is the sample mean from the population mean?
 - What is the probability that a second sample of 100 men would have a mean less than 69.0 in.?
- 13. Sample and Population Proportions.** Suppose that, in a suburb of 12,345 people, 6,523 people moved there within the past five years. You survey 500 people and find that 245 of the people in your sample moved to the suburb in the past five years.
- What is the population proportion of people who moved to the suburb in the past five years?
 - What is the sample proportion of people who moved to the suburb in the past five years?
 - Does your sample appear to be representative of the population of the suburb? Discuss.
- 14. Sample and Population Proportions.** The College of Portland has 2,444 students and 269 of them are left-handed. You conduct a survey of 50 students and find that 8 of them are left-handed.
- What is the population proportion of left-handed students?
 - What is the sample proportion of left-handed students?
 - If the sample mean misses the population mean by a large amount, does that imply that the sampling method is flawed? Explain.
- 15. Estimating Population Proportions.** You select a random sample of 150 people at a medical convention attended by 1,608 people. Within your sample, you find that 73 people have traveled from abroad. Based on this sample statistic, estimate how many people at the convention traveled from abroad. Would you be more confident of your estimate if you sampled 300 people? Explain.
- 16. Estimating Population Proportions.** A random sample of 500 people was selected from the 103,219 people in attendance at the Super Bowl game between the Green Bay Packers and the Pittsburgh Steelers. Within the sample, 290 people supported the Packers. Based on this result, estimate how many people at the game supported the Packers. Would you be more confident of your estimate if the sample size were 2,000 people? Explain.
- 17. Distribution of Sample Proportions.** Suppose you know that the distribution of sample proportions of nonresidents in samples of 200 students is normal with a mean of 0.34 and a standard deviation of 0.03. Suppose you select a random sample of 200 students and find that the proportion of nonresident students in the sample is 0.32.
- How many standard deviations is the sample proportion from the mean of the distribution of sample proportions?
 - What is the probability that a second sample selected would have a proportion less than 0.32?
- 18. Distribution of Sample Proportions.** Suppose you know that the distribution of sample proportions of women employees is normal with a mean of 0.42 and a standard deviation of 0.21. Suppose you select a random sample of employees and find that the proportion of women in the sample is 0.45.
- How many standard deviations is the sample proportion from the mean of the distribution of sample proportions?
 - What is the probability that a second sample selected would have a proportion greater than 0.45?
- 19. Sampling Distribution.** A quarterback threw 1 interception in his first game, 2 interceptions in his second game, 5 interceptions in his third game, and then he retired. Consider the values of 1, 2, and 5 to be a population. Assume that samples of size 2 are randomly selected (with replacement) from the population.
- List the 9 different possible samples and find the mean of each sample.
 - What is the mean of the sample means from part a?
 - Is the mean of the sampling distribution from part b equal to the mean of the population of the three listed values? Are those means *always* equal?

20. Sampling Distribution. Here is the population of all five U.S. presidents who had professions in the military, along with their ages at inauguration: Eisenhower, 62; Grant, 46; Harrison, 68; Taylor, 64; and Washington, 57. Assume that samples of size 2 are randomly selected (with replacement) from the population of five ages.

- List the 25 different possible samples and find the mean of each sample.
- What is the mean of the sample means from part a?
- Is the mean of the sampling distribution (from part b) equal to the mean of the population of the five listed values? Are those means *always* equal?

21. Forming Sampling Distributions. Three states and their areas (in thousands of square miles) are given in the following table. Consider these three states to be the entire population from which samples of size $n = 2$ will be selected (with replacement).

- Find the mean of each of the 9 different possible samples.
- What is the mean of the sample means from part a?
- Is the mean of the sampling distribution from part b equal to the mean of the population of the three listed values? Are those means *always* equal?

State	Area (thousands of square miles)
Connecticut	5
Alabama	52
Georgia	60

22. Forming Sampling Distributions. The ages (years) of the four U.S. presidents when they were assassinated in office are 56 (Lincoln), 49 (Garfield), 58 (McKinley), and 46 (Kennedy). Consider these four ages to be a population.

- Assuming that two of the ages are randomly selected to form samples of size $n = 2$ with replacement, find the mean of each of the 16 possible samples.
- What is the mean of the sample means from part a?
- Is the mean of the sample means equal to the mean of the population of the four ages? Are those means *always* equal?



PROJECTS FOR THE INTERNET & BEYOND

23. Distributions of Sample Means. Consider the large data set of hours students spend using an Internet search engine, listed earlier in this section. Discuss methods for selecting a random sample from this population. Let each person in the class select a random sample of $n = 10$ individuals from the population and find the mean of his or her sample. Find the mean of the individual sample means and compare it to the population mean. Repeat the process with samples of size $n = 20$. How does the mean of the sample means compare to the population mean?

24. Distributions of Sample Proportions. Consider the data set from earlier in this section showing 400 *yes* and *no* responses to a survey question (“Do you own a car?”). Discuss methods for selecting a random sample from this population. Let each person in the class select a random sample of $n = 10$ responses from the population and find the proportion of *yes* responses for his or her sample. Find the mean of the individual sample proportions and compare it to the population proportion of 0.6. Repeat the process with samples of size $n = 20$. How does the mean of the sample proportions compare to the population proportion?

IN THE NEWS

25. Sample Means in the News. Find a news or research report in which a sample mean is cited. Discuss how it is used to estimate a population mean.

26. Sample Proportions in the News. Find a news or research report in which a sample proportion is cited. Discuss how it is used to estimate a population proportion.

2

ESTIMATING POPULATION MEANS

In Section 1, we explored distributions of sample means and sample proportions through cases in which we had data for the entire population. This was useful because it allowed us to compare individual sample statistics (sample mean or sample proportion) to actual population parameters (population mean or population proportion). However, it was also somewhat artificial, because in most real statistical studies, we have *only* the sample data, not the complete population data. In the rest of this chapter, we will consider how we deal with more realistic situations, focusing on population means in this section and population proportions in Section 3.

Estimating a Population Mean: The Basics

Let's begin with an example. We all need protein in our diets, and while different organizations and government agencies make slightly different recommendations, the recommended daily allowance (RDA) for protein is generally around 55 to 60 grams per day for men and 45 to 50 grams per day for women (except when pregnant or nursing, when it is higher). How does the amount of protein that Americans actually eat compare with these recommended daily allowances?

We obviously cannot measure protein intake for the entire population, so we will need to focus on some sample that we hope will be representative of the population. Figure 6 shows the sample we will use; it shows the daily protein intake (in grams) for a sample of $n = 267$ men. The sample data have a mean of $\bar{x} = 77.0$ grams and a standard deviation of $s = 58.6$ grams. Our goal is to use these sample data to make an inference about the average protein intake for the population of all American men. There are two basic steps to this process, summarized in the following box.

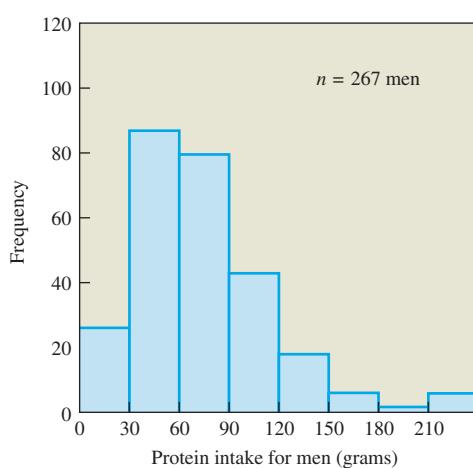


Figure 6 Histogram of daily protein intake for men taken from a sample of $n = 267$ men.

Source: National Center for Health Statistics. This sample comes from the Third National Health and Nutrition Examination Survey, or NHANES III, which surveyed approximately 30,000 participants on different aspects of their health and diet.

BY THE WAY

Why do we need protein?

Proteins are a key building block of all living cells and are therefore necessary for all forms of life on Earth. Our bodies build proteins from smaller building blocks called *amino acids* and therefore need a source of amino acids for this biochemistry. The source is food; our bodies obtain amino acids by breaking down the proteins that were built in plants and animals that we eat.



Using a Sample Mean to Estimate a Population Mean

Suppose that we have computed the sample mean, \bar{x} , for a single sample drawn from a large population. To estimate the population mean:

- Because we have only a single sample mean, we use it as our best (and only) estimate of the population mean.
- Based on the sample size and sample standard deviation (s), we compute a *margin of error* and use it to construct a *confidence interval* around our estimate. We can then make a statement that gives us some idea of how good our estimate is.

In this case, we choose the sample mean of $\bar{x} = 77.0$ grams per day as our best estimate of the population mean. We know that although this is our best (and only) estimate, it may or may not be a *good* estimate. So our next step is to calculate the margin of error and construct the confidence interval.

Finding the Confidence Interval

A **confidence interval** is a range of values likely to contain the true value of the population mean. In this text, we will work only with 95% confidence levels, although other levels of confidence (such as 90% or 99%) are sometimes used.

TECHNICAL NOTE

The 68-95-99.7 rule is an approximation; the precise range within which 95% of the values lie is within 1.96 standard deviations. We will continue to approximate this as 2 standard deviations in this text.

TECHNICAL NOTE

The precise formula for the margin of error uses 1.96 rather than 2, and using the sample standard deviation s is valid only if the sample is large enough (typically $n > 30$, but the size requirement depends on the nature of the actual population distribution). Similar formulas can be used for other confidence levels. For example, for a 90% confidence level, replace the 2 in the formula by 1.645; and for a 99% confidence level, replace the 2 in the formula by 2.575.

The Margin of Error for a Sample Mean

When we use a sample mean as an estimate of the population mean, the **margin of error** for the 95% confidence interval is

$$\text{margin of error} = E \approx \frac{2s}{\sqrt{n}}$$

where s is the standard deviation of the sample.

Continuing our protein example, we are using a sample of $n = 267$ men with a sample mean of $\bar{x} = 77.0$ grams and a sample standard deviation of $s = 58.6$ grams. The margin of error for 95% confidence is therefore

$$E \approx \frac{2s}{\sqrt{n}} = \frac{2 \times 58.6}{\sqrt{267}} = 7.2 \text{ grams}$$

We can now construct the 95% confidence interval. We simply add and subtract the margin of error from the sample mean of $\bar{x} = 77.0$ grams, which gives a 95% confidence interval extending approximately from $77.0 - 7.2 = 69.8$ grams to $77.0 + 7.2 = 84.2$ grams. We can write this result more formally as $69.8 \text{ grams} < \mu < 84.2 \text{ grams}$; note that this suggests that American men are consuming a significantly different amount of protein than the recommended daily allowance of 55 to 60 grams.

More generally, the confidence interval for a population mean always has the sample mean \bar{x} at the center and extends a distance equal to the margin of error in either direction (Figure 7).

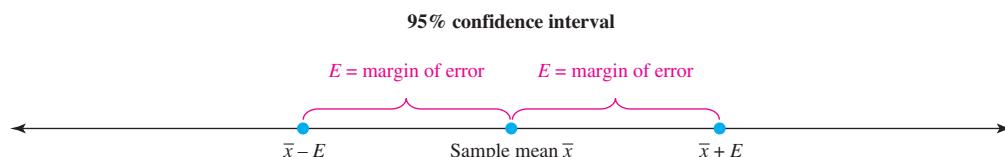


Figure 7 The 95% confidence interval extends a distance equal to the margin of error on either side of the sample mean.

The Confidence Interval for a Population Mean

We find the **95% confidence interval** by adding and subtracting the margin of error from the sample mean. That is, the 95% confidence interval ranges

from $(\bar{x} - \text{margin of error})$ to $(\bar{x} + \text{margin of error})$

We can write this confidence interval formally as $\bar{x} - E < \mu < \bar{x} + E$ or in the format of $\bar{x} \pm E$.

Interpreting the Confidence Interval

Confidence intervals are easy to construct and are very useful, but we need to be very careful in how we interpret them. The proper interpretation requires thinking about the sampling distribution of means. Recall that 95% of all possible samples have a sample mean lying within approximately 2 standard deviations of the population mean. Therefore, if we were to repeat the process of obtaining samples and constructing confidence intervals many times, 95% of the confidence intervals would contain the population mean (μ) and 5% would not include it.

Interpreting a 95% Confidence Interval

When we choose a single sample and construct its 95% confidence interval, we have no way to know whether or not it contains the population mean, (μ). However, if we could repeat the sampling process many times, we would find that 95% of the confidence intervals would contain the population mean and 5% would not.

Figure 8 shows a visual interpretation of this idea. The true population mean (μ) is represented by the blue vertical line running down the middle of the figure. The region to the left of this line represents values below the population mean and the region to the right represents values above the population mean. The figure shows confidence intervals for 20 different samples; in each case, the sample mean (\bar{x}) is the black dot at the center and the red line shows the 95% confidence interval around that sample mean. Notice that 19 of the 20 samples, or 95%, have a confidence interval that actually contains the population mean. One sample (the 6th one from the top), or 5%, has a confidence interval that does not contain the population mean.

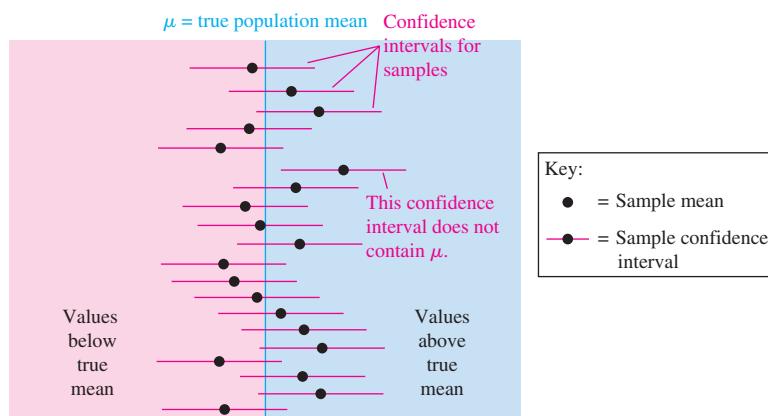


Figure 8 This figure illustrates the idea behind confidence intervals. The central vertical line represents the true population mean, μ . Each of the 20 horizontal lines represents the 95% confidence interval for a particular sample, with the sample mean marked by the dot in the center of the confidence interval. With a 95% confidence interval, we expect that 95% of all samples will give a confidence interval that contains the population mean, as is the case in this figure, for 19 of the 20 confidence intervals do indeed contain the population mean. We expect that the population mean will not be within the confidence interval in 5% of the cases; here, 1 of the 20 confidence intervals (the sixth from the top) does not contain the population mean.



USING TECHNOLOGY—CONFIDENCE INTERVAL ESTIMATE OF A POPULATION MEAN

Excel: Use XLSTAT. Click on **XLSTAT** at the top, click on **Parametric tests**, then select **One sample t test and z test**. In the screen that appears, for the “Data” box enter the range of data, such as A1:A12 for 12 data values in column A. For “Data Format” select **One sample**. Click on **z test**. Click on the **Options** tab, to enter the desired “Significance level (%).” Enter 5 for a 95% confidence interval. Click **OK**. After the results are displayed, look for “confidence interval on the mean.” (The use of Excel’s **CONFIDENCE** tool is not recommended, for a variety of reasons.)

Statdisk. Select **Analysis** and then **Confidence Intervals**. Then select **Mean - One Sample**. In the dialog box that appears, first enter the significance level as a decimal number. Enter 0.95 for a 95% confidence level. Proceed to enter the other required items. Then click on **Evaluate**, and the confidence interval will be displayed.

TI-83/84 PLUS The TI-83/84 Plus calculator can be used to generate confidence intervals for original sample values stored in a list, or you can use the summary statistics n , \bar{x} , and s . Either enter the data in list L1 or have the summary statistics available, then press the **STAT**. Now select **TESTS** and choose **ZInterval**. After making the required entries, the calculator display will include the confidence interval in the format of $(\bar{x} - E, \bar{x} + E)$.

The subtlety arises from the fact that “confidence” and “probability” are not quite the same thing. When we construct a confidence interval around a single sample mean, we have no way to know whether our particular confidence interval is one of the 95% that contain the population mean or one of the 5% that do not. In other words, our confidence interval either does or does not contain the population mean, and any other probability statement would be misleading. All we can say is that 95% of all possible confidence intervals for our sample size would contain the population mean, and 5% would not.

EXAMPLE 1 Protein Intake for Women

Figure 9 shows a histogram for a random sample of $n = 264$ women (drawn from the same NHANES III study used for the sample of men in Figure 6). The mean of these data is $\bar{x} = 59.6$ grams and the standard deviation is $s = 30.5$ grams. Estimate the population mean and give a 95% confidence interval. Comment on how these values compare to the recommended daily allowance (RDA) for women of 45–50 grams.

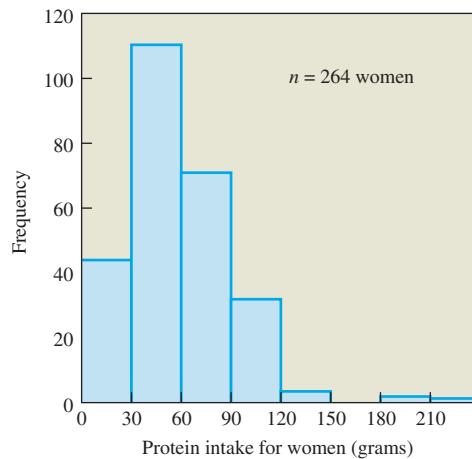


Figure 9 Histogram for daily protein intake for 264 women.

Source: National Center for Health Statistics.

SOLUTION We use the sample mean, $\bar{x} = 59.6$ grams, as our best (and only) estimate of the population mean. To find the 95% confidence interval, we must first find the margin of error using the sample size ($n = 264$) and sample standard deviation $s = 30.5$ grams:

$$E \approx \frac{2s}{\sqrt{n}} = \frac{2 \times 30.5 \text{ grams}}{\sqrt{264}} = 3.8 \text{ grams}$$

The 95% confidence interval therefore ranges from $59.6 - 3.8 = 55.8$ grams to $59.6 + 3.8 = 63.4$ grams, or

$$55.8 \text{ grams} < \mu < 63.4 \text{ grams}$$

We have 95% confidence that the interval from 55.8 grams to 63.4 grams contains the true value of the population mean. This means that if we were to repeat this process many times, 95% of the resulting confidence intervals should contain the true value of the population mean. We conclude that protein consumption by women is very likely to be different than the recommended daily allowance of 45–50 grams for women.



TIME OUT TO THINK

Recall that the standard deviation of the data on protein intake for the sample of men was $s = 58.6$ grams—almost double the standard deviation for the sample of women ($s = 30.5$ grams). How does this difference affect the margins of error? Speculate on why the standard deviations are different.

EXAMPLE 2 Garbage Production

A study conducted by the Garbage Project at the University of Arizona analyzed the contents of garbage discarded by $n = 62$ households; the households ranged in size from 2 to 11 members. The histogram in Figure 10 shows the total weekly garbage production (in pounds) for the households in the sample. (The complete study gives the breakdown of garbage by various categories.) The mean for the sample is $\bar{x} = 27.4$ pounds and the standard deviation is $s = 12.5$ pounds. Estimate the population mean for weekly garbage production with a 95% confidence interval.

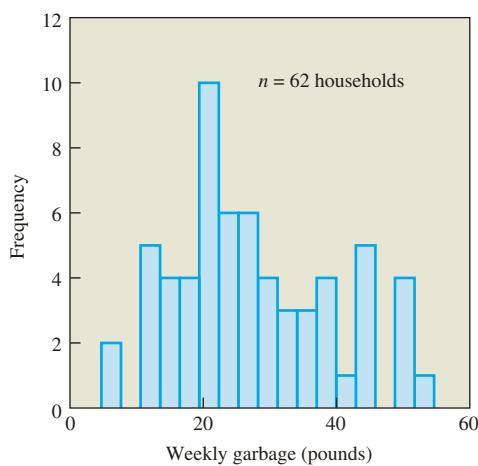


Figure 10 Histogram for total garbage production for $n = 62$ households.

SOLUTION We use the sample mean, $\bar{x} = 27.4$ pounds, as our best (and only) estimate of the population mean. We use the margin of error formula to find that

$$E \approx \frac{2s}{\sqrt{n}} = \frac{2 \times 12.5 \text{ grams}}{\sqrt{62}} = 3.2 \text{ pounds}$$

BY THE WAY

According to the Garbage Project data, the leading component of most household garbage is paper (comprising a third to a half of the total garbage), followed by food and glass. That is why recycling and composting can drastically reduce the amount of waste going to landfills.



The 95% confidence interval therefore ranges from $27.4 - 3.2 = 24.2$ pounds to $27.4 + 3.2 = 30.6$ pounds, or

$$24.2 \text{ pounds} < \mu < 30.6 \text{ pounds}$$

We have 95% confidence that the interval from 24.2 pounds to 30.6 pounds contains the mean amount of garbage discarded by American households in a week.



EXAMPLE 3 Mean Body Temperature

A study by University of Maryland researchers investigated the body temperatures of $n = 106$ subjects. The sample mean of the data set is $\bar{x} = 98.20^\circ\text{F}$ and the standard deviation for the sample is $s = 0.62^\circ\text{F}$. Estimate the population mean body temperature with a 95% confidence interval.

SOLUTION The sample mean, $\bar{x} = 98.20^\circ\text{F}$, is our best (and only) estimate of the population mean body temperature. For a 95% confidence interval, the margin of error is

$$E \approx \frac{2s}{\sqrt{n}} = \frac{2 \times 0.62^\circ\text{F}}{\sqrt{106}} = 0.12^\circ\text{F}$$

The 95% confidence interval therefore ranges from $98.20^\circ\text{F} - 0.12^\circ\text{F} = 98.08^\circ\text{F}$ to $98.20^\circ\text{F} + 0.12^\circ\text{F} = 98.32^\circ\text{F}$, or

$$98.08^\circ\text{F} < \mu < 98.32^\circ\text{F}$$

We have 95% confidence that the interval from 98.08°F to 98.32°F contains the true population mean. This means that if we were to select many different samples of size $n = 106$ and compute confidence intervals for all of the samples, we expect that 95% of the confidence intervals would contain the true population mean. Notice that the commonly cited mean body temperature for humans (98.6°F) is *not* contained in the confidence interval. Based on this sample, it is likely that the accepted mean body temperature of 98.6°F is wrong.



Choosing Sample Size

In planning statistical surveys and experiments, we often know in advance the margin of error we would like to achieve. For example, we might want to estimate the mean cost of a new car to within \$200. We can estimate the sample size, n , needed to ensure this margin of error by solving the margin of error formula ($E \approx 2s/\sqrt{n}$) for n . With a little bit of algebra, we find

$$n \approx \left(\frac{2s}{E} \right)^2$$

The exact sample size formula uses the population standard deviation, σ , in place of s . In practice, we rarely know the population standard deviation, because we study only samples. Therefore, to use the exact sample size formula, we usually estimate the population standard deviation based on previous studies, pilot studies, or educated guesses. The size of any actual sample must be a whole number, so we round the result of the sample size formula *up* to the nearest whole number. Any sample larger than this size will give us a margin of error as small as or smaller than the one we seek.

TECHNICAL NOTE

The given formula for sample size assumes we are working with a 95% level of confidence; a more accurate formula would use 1.96 instead of 2, as discussed earlier. For the methods of this section, if the population standard deviation is not known, we can use the sample standard deviation, s , as an estimate of that value. However, the results will be good only if the sample size is large. If a small sample is taken from a non-normal population, the use of s in place of σ may lead to very poor results.

Choosing the Correct Sample Size

In order to estimate the population mean with a specified margin of error of at most E , the size of the sample should be at least

$$n = \left(\frac{2\sigma}{E} \right)^2$$

where σ is the population standard deviation (often estimated by the sample standard deviation s).

EXAMPLE 4 Mean Housing Costs

You want to study housing costs in the country by sampling recent house sales in various (representative) regions. Your goal is to provide a 95% confidence interval estimate of the housing cost. Previous studies suggest that the population standard deviation is about \$7,200. What sample size (at a minimum) should be used to ensure that the sample mean is within

- \$500 of the true population mean?
- \$100 of the true population mean?

SOLUTION

- a. With $E = \$500$ and σ estimated as \$7,200, the minimum sample size that meets the requirements is

$$n = \left(\frac{2\sigma}{E}\right)^2 = \left(\frac{2 \times 7,200}{500}\right)^2 = 28.8^2 = 829.4$$

Because the sample size must be a whole number, we conclude that the sample should include *at least* 830 prices.

- b. With $E = \$100$ and $\sigma = \$7,200$, the minimum sample size that meets the requirements is

$$n = \left(\frac{2\sigma}{E}\right)^2 = \left(\frac{2 \times 7,200}{100}\right)^2 = 144^2 = 20,736$$

Notice that to decrease the margin of error by a factor of 5 (from \$500 to \$100), we must increase the sample size by a factor of 25. That is why achieving greater accuracy generally comes with a high cost.

• • •

TIME OUT TO THINK

If you decide you want a smaller margin of error for a confidence interval, should you increase or decrease the sample size? Explain.

Section 2 Exercises**Statistical Literacy and Critical Thinking**

- 1. Confidence Interval.** From a random sample of weights of dollar coins, we construct this 95% confidence interval estimate of the mean:

$$8.0518 \text{ grams} < \mu < 8.0902 \text{ grams}$$

Interpret this confidence interval.

- 2. Margin of Error.** Based on a random sample of hospital costs for car crash victims, the sample mean is \$9,004 and the margin of error for a 95% confidence interval is \$266. Identify the confidence interval.

- 3. Confidence Intervals in the Media.** Here is a typical statement made by the media: “Based on a recent study, pennies weigh an average of 2.5 grams with a margin of error of 0.006 gram.” What important and relevant piece of information is omitted from that statement? Is it OK to use the word “average”?

- 4. Sample Size.** The National Health Examination involves measurements from about 25,000 people, and the results are used to estimate values of various population means.

Is it valid to criticize this survey because the sample size is only about 0.01% of the population of all Americans? Explain.

Does It Make Sense? For Exercises 5–8, decide whether the statement makes sense (or is clearly true) or does not make sense (or is clearly false). Explain clearly. Not all of these statements have definitive answers, so your explanation is more important than your chosen answer.

- 5. Interpreting a Confidence Interval.**

For the confidence interval given in Exercise 1, we can interpret that result

by saying that “95% of sample means will fall between 8.0518 grams and 8.0902 grams.”

6. Margin of Error. The mean income of high school mathematics teachers was estimated to be \$48,213 with a margin of error of five percentage points.

7. Margin of Error. When sample data are used to estimate the value of a population mean, the margin of error decreases as the sample size increases.

8. Best Estimate. When sample data were used to estimate the value of the mean weight of all pennies, this 95% confidence interval was obtained:

$$2.512 \text{ grams} < \mu < 2.512 \text{ grams}$$

Concepts and Applications

Finding Margins of Error and Confidence Intervals. For Exercises 9–12, assume that population means are to be estimated from the samples described. In each case, use the sample results to approximate the margin of error and 95% confidence interval.

9. sample size = 49, sample mean = 25.2 cm, sample standard deviation = 2.2 cm

10. Sample size = 81, sample mean = 4.5 km, sample standard deviation = 3.1 km

11. $n = 100$, $\bar{x} = 8.0$ ft, $s = 2.0$ ft

12. $n = 64$, $\bar{x} = \$550$, $s = \$60$

Sample Sizes. For Exercises 13–16, assume that you want to construct a 95% confidence interval estimate of a population mean. Find an estimate of the sample size needed to obtain the specified margin of error for the 95% confidence interval. The sample standard deviation is given.

13. Margin of error = \$5, standard deviation = \$20

14. Margin of error = 18.2 cm, standard deviation = 95.2 cm

15. Margin of error = 3.5 ml, standard deviation = 155.2 ml

16. Margin of error = 0.5 g, standard deviation = 8.7 g

17. Sample Size for TV Survey. Nielsen Media Research wishes to estimate the mean number of hours that high school students spend watching TV on a weekday. A margin of error of 0.25 hour is desired. Past studies suggest that a population standard deviation of 1.7 hours is reasonable. Estimate the minimum sample size required to estimate the population mean with the stated accuracy.

18. Sample Size for Housing Prices. A government survey conducted to estimate the mean price of houses in a large metropolitan area is designed to have a margin of error of \$10,000. Pilot studies suggest that the population standard deviation is \$65,500. Estimate the minimum sample size needed to estimate the population mean with the stated accuracy.

19. Sample Size for Mean IQ of Statistics Students. The Wechsler IQ test is designed so that the mean is 100 and the standard deviation is 15 for the population of normal adults. Find the sample size necessary to estimate the mean IQ score of Delaware residents. We want to be 95% confident that our sample mean is within 3 IQ points of the true mean. Assume that $\sigma = 15$ and determine the required sample size.

20. Sample Size for Estimating Income. An economist wants to estimate mean annual income from the first year of work for college graduates who have had the profound wisdom to take a statistics course. How many such incomes must be found if she wants to be 95% confident that the sample mean is within \$750 of the true population mean? Assume that a previous study has revealed that for such incomes, $\sigma = \$6,250$.

21. Weight of Quarters. You want to estimate the mean weight of quarters in circulation. A sample of 40 quarters has a mean weight of 5.639 grams and a standard deviation of 0.062 gram. Use a single value to estimate the mean weight of all quarters. Also, find the 95% confidence interval.

22. Weights of Babies. A sample of 186 newborn babies has a mean weight of 3103 g and a standard deviation of 696 g. Use a single value to estimate the mean weight of a newborn baby. Also, find the 95% confidence interval.

23. Time to Graduation. Data from the National Center for Education Statistics on 4,400 college graduates show that the mean time required to graduate with a bachelor’s degree is 5.15 years with a standard deviation of 1.68 years. Use a single value to estimate the mean time required to graduate for all college graduates. Also, find the 95% confidence interval.

24. Garbage Production. Based on a sample of 62 households, the mean weight of discarded plastic is 1.91 pounds and the standard deviation is 1.07 pounds (data from the Garbage Project at the University of Arizona). Use a single value to estimate the mean weight of discarded plastic for all households. Also, find the 95% confidence interval.

25. Weight of Bears. The health of the bear population in Yellowstone National Park is monitored by periodic measurements taken from anesthetized bears. A sample of the weights (pounds) of such bears is given below. Find a 95% confidence interval estimate of the mean of the population of all such bear weights.

80	344	416	348	166	220	262	360	204
144	332	34	140	180	105	166	204	26
120	436	125	132	90	40	220	46	154
116	182	150	65	356	316	94	86	150

26. Cotinine Levels of Smokers. When people smoke, the nicotine they absorb is converted to cotinine, which can

be measured. A sample of cotinine levels of 40 smokers is listed below. Find a 95% confidence interval estimate of the mean cotinine level of all smokers.

1	0	131	173	265	210	44	277
32	3	35	112	477	289	227	103
222	149	313	491	130	234	164	198
17	253	87	121	266	290	123	167
250	245	48	86	284	1	208	173

- 27. Family Size.** You select a random sample of $n = 31$ families in your neighborhood and find the following family sizes (number of people in the family):

2	3	6	5	4	2	3	3	1	2	3
2	3	4	5	3	1	3	3	4	7	3
2	3	2	2	3	4	1	5	2		

- a. What is the mean family size for the sample?
 - b. What is the standard deviation for the sample?
 - c. What is the best estimate for the mean family size for the population of all American families?
 - d. What is the 95% confidence interval for the estimate?
 - e. Comment on the reliability of the estimate.
- 28. TV Sets.** A random sample of $n = 31$ households is asked the number of TV sets in the household. The responses are as follows:

1	0	2	3	2	3	4	2	1	1	2
4	3	2	3	3	0	1	0	1	3	2
4	3	2	1	4	0	1	2	3		

- a. What is the mean number of TVs for the sample?
- b. What is the standard deviation for the sample?
- c. What is the best estimate for the mean number of TVs for the population of all American households?
- d. What is the 95% confidence interval for the estimate?
- e. Comment on the reliability of the estimate.



PROJECTS FOR THE INTERNET & BEYOND

- 29. Car Ages.** Assume that you want to estimate the mean age of cars driven by students at your college. A previous study shows that the standard deviation of those ages is approximately 3.7 years. How many car ages must you randomly select in order to be 95% confident that your sample mean is within 1 year of the population mean? Using that sample size, collect your own sample data, consisting of the ages of cars driven by students at your college. Then use the methods of this section to construct a 95% confidence interval. Write a statement summarizing your results.
- 30. Polling Organizations.** Three leading public polling organizations are the Gallup Organization, Harris Poll, and Yankelovich Partners. Visit their Web sites. Describe the history of each organization and the polling services it provides. Which organization has the best description on its Web site of its polling methods?
- 31. Network Polls.** All of the major television networks conduct regular polls on a variety of issues. Visit the Web site of at least one major network and gather the results of a particular poll that involves estimation of a population mean. Be sure to include all information that is given about the sample size, margin of error, and confidence intervals.
- 32. International Corruption.** Transparency International uses surveys to determine a Bribe Payer's Index and Corruption Perception Index that measure the degree of corruption in many countries worldwide. Visit the Transparency International Web site and review the extensive documentation describing the methods used by this organization. Discuss the results and their validity.
- 33. Estimating Population Means.** Find a news article or report in which a population mean is estimated from a sample. The article should include a margin of error and/or a confidence interval. Discuss the methods used in the study and how the conclusions were reached.

IN THE NEWS

3

ESTIMATING POPULATION PROPORTIONS

We are now ready to turn our attention to estimates of population *proportions*. Many well-known polls and surveys rely on the techniques that we will discuss. For example, the Nielsen ratings estimate the proportion of the population tuned in to certain radio and television shows, the monthly unemployment figures released by the Bureau of Labor Statistics are estimates of the proportion of Americans who are unemployed, and the opinion polls that dominate American politics estimate the proportion of the population that supports a particular candidate.

The Basics of Estimating a Population Proportion

The Bureau of Labor Statistics estimates the unemployment rate from a monthly survey of 60,000 households. The unemployment rate for this sample is the *sample proportion* (the proportion of people in the sample who are unemployed), denoted \hat{p} . As we did with means, we use the sample proportion as our best and only estimate of *population proportion* (the proportion of people in the population who are unemployed), denoted p .

Just as with population means, an estimate of the population proportion can be better understood if we describe its accuracy using margins of error and confidence intervals. The only change from estimating population means is in the definition of the margin of error, which is given in the following box.

TECHNICAL NOTE

The precise formula for the margin of error uses 1.96 rather than 2, and the formula is valid only when $np \geq 5$ and $n(1 - p) \geq 5$, conditions that are usually met easily in practice. (More advanced texts provide techniques for smaller sample sizes.)

The Margin of Error for a Population Proportion

For a population proportion, the **margin of error** for the 95% confidence interval is

$$E \approx 2\sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

where \hat{p} is the sample proportion.

Again, once we have the margin of error, we construct the 95% confidence interval simply by adding and subtracting it from the sample proportion, as illustrated in Figure 11.

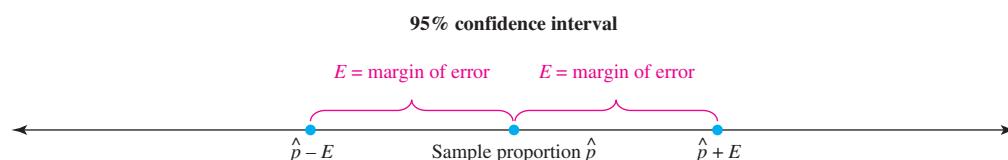


Figure 11 The confidence interval extends a distance equal to the margin of error on either side of the sample proportion, \hat{p} .

The Confidence Interval for a Population Proportion

We find the **95% confidence interval** by adding and subtracting the margin of error from the sample proportion. That is, the 95% confidence interval ranges

from $\hat{p} - \text{margin of error}$ to $\hat{p} + \text{margin of error}$

We can write this confidence interval more formally as

$$\hat{p} - E < p < \hat{p} + E$$

EXAMPLE 1 Unemployment Rate

The Bureau of Labor Statistics finds 5,160 unemployed people in a sample of $n = 60,000$ people. Estimate the population unemployment rate and give a 95% confidence interval.

SOLUTION The sample proportion is the unemployment rate for the sample:

$$\hat{p} = \frac{5,160}{60,000} = 0.086$$

We use this as the best estimate for the population unemployment rate. The margin of error is

$$E \approx 2\sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} = 2\sqrt{\frac{0.086(1 - 0.086)}{60,000}} = 0.0023$$

(The approximation is valid because of the large sample size.) The 95% confidence interval ranges from $0.0860 - 0.0023 = 0.0837$ to $0.0860 + 0.0023 = 0.0883$ or

$$0.0837 < p < 0.0883$$

We have 95% confidence that the interval from 8.37% to 8.83% contains the true unemployment rate for the population. We interpret this result as follows: If we computed confidence intervals for many samples of size $n = 60,000$, we should expect 95% of the confidence intervals to contain the true population proportion.

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EXAMPLE 2 TV Nielsen Ratings

The Nielsen ratings for television use a random sample of households. A Nielsen survey results in an estimate that a women's World Cup soccer game had 72.3% of the entire viewing audience. Assuming that the sample consists of $n = 5,000$ randomly selected households, find the margin of error and the 95% confidence interval for this estimate.

SOLUTION The sample proportion, $\hat{p} = 72.3\% = 0.723$, is the best estimate of the population proportion. The margin of error is

$$E \approx 2\sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} = 2\sqrt{\frac{0.723(1 - 0.723)}{5,000}} = 0.013$$



USING TECHNOLOGY—CONFIDENCE INTERVAL ESTIMATE OF A POPULATION PROPORTION

Excel: Use XLSTAT. Click on **XLSTAT** at the top, click on **Parametric Tests**, then select **Tests for one proportion**. In the screen that appears, enter the sample proportion in the “Proportion” box, enter the sample size in the “Sample size” box, select the “Data Format” of **Proportion**, and be sure that the box next to “z test” is checked. For the “Range” box, enter A1 so that the results will start at cell A1. Click on the **Options** tab, to enter the desired “Significance level (%).” Enter 5 for a 95% confidence interval. For the “Variance (confidence interval)” options, select **Sample** (so that the sample variance is used in the computation of the confidence interval). There are four options for the type of confidence interval; accept the default of **Wald**. Click **OK**. After the results are displayed, look for “confidence interval on the proportion (Wald).”

Statdisk. Select **Analysis** and then **Confidence Intervals**. Then select **Proportion – One Sample**. In the dialog box that appears, first enter the significance level as a decimal number. Enter 0.95 for a 95% confidence level. Proceed to enter the other required items. Then click on **Evaluate** and the confidence interval will be displayed.

TI-83/84 Plus Press **STAT**, select **TESTS**, then select **1-PropZInt** and enter the required items. Like many technologies, the TI-83/84 Plus calculator requires entry of the number of successes. Also like many technologies, the confidence interval limits are expressed in the format of $(\hat{p} - E, \hat{p} + E)$.

BY THE WAY

Here's how the Bureau of Labor Statistics describes uncertainty in its unemployment survey: “A sample is not a total count and the survey may not produce the same results that would be obtained from interviewing the entire population. But the chances are 90 out of 100 that the monthly estimate of unemployment from the sample is within 290,000 of the figure obtainable from a total census. Since monthly unemployment totals have ranged between about 7 and 11 million in recent years, the possible error resulting from sampling is not large enough to distort the total unemployment picture.”



The 95% confidence interval is $0.723 - 0.013 < p < 0.723 + 0.013$, or

$$0.710 < p < 0.736$$

With 95% confidence, we conclude that between 71.0% and 73.6% of the entire viewing audience watched the women's World Cup soccer game.



EXAMPLE 3 Pete Rose Poll

The Gallup Organization conducted a survey of 1,016 randomly selected adults who were asked:

As you may know, former major league player Pete Rose is ineligible for baseball's Hall of Fame due to charges that he had gambled on baseball games. Do you think he should or should not be eligible for admission to the Hall of Fame?

Among those surveyed, 59% believed that Pete Rose should be eligible. Find the margin of error and confidence interval for this survey. The reported results cited a margin of error of "no more than 5 percentage points." Is this claim consistent with the sample size?

SOLUTION A sample with $n = 1,016$ respondents and a sample proportion of $\hat{p} = 0.59$ has a margin of error of

$$E \approx 2\sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} = 2\sqrt{\frac{0.59(1 - 0.59)}{1,016}} = 0.031$$

This margin of error is about 3 percentage points, so the confidence interval ranges from approximately $59 - 3 = 56\%$ to $59 + 3 = 62\%$. Note that the cited margin of error of "no more than 5 percentage points" is, in fact, an overestimate.



Choosing Sample Size

Designers of surveys and polls often specify a certain level of accuracy for the results. For example, it might be desirable to estimate a population proportion with a 95% confidence interval and a margin of error of no more than 1.5 percentage points. In such situations, it's necessary to determine how large the sample must be to guarantee this accuracy. As long as we use a 95% level of confidence, we can work with this simplified, approximate formula for the margin of error:

$$E \approx \frac{1}{\sqrt{n}}$$

This formula gives a conservative (higher than necessary) estimate for the margin of error. Solving for n yields the sample size needed to achieve a margin of error E :

$$n \approx \frac{1}{E^2}$$

Any sample size equal to or larger than this value will suffice.

TECHNICAL NOTE

You can derive the $E \approx 1/\sqrt{n}$ formula from the more precise formula given earlier for the margin of error by replacing the product $\hat{p}(1 - \hat{p})$ by its maximum possible value of 0.25. This approximation overestimates the actual margin of error and is most accurate when p is near 0.5.

Choosing the Correct Sample Size

In order to estimate a population proportion with a 95% degree of confidence and a specified margin of error of E , the size of the sample should be at least

$$n = \frac{1}{E^2}$$

EXAMPLE 4 Minimum Sample Size for Survey

You plan a survey to estimate the proportion of students on your campus who carry an iPad regularly. How many students should be in the sample if you want (with 95% confidence) a margin of error of no more than 4 percentage points?

SOLUTION Note that 4 percentage points means a margin of error of 0.04. From the given formula, the minimum sample size is

$$n = \frac{1}{E^2} = \frac{1}{0.04^2} = 625$$

You should survey at least 625 students.

• •

EXAMPLE 5 Yankelovich Poll

A poll by Yankelovich Partners concluded that 61% of all households have a computer, with a margin of error of 3.5 percentage points. Approximately what sample size must have been used in this poll?

SOLUTION A margin of error of 3.5 percentage points (or 0.035) could be achieved with a sample size of

$$n = \frac{1}{E^2} = \frac{1}{0.035^2} = 816.3$$

Because we must round up to the next larger whole number, we conclude that approximately 817 households were surveyed.

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Section 3 Exercises

Statistical Literacy and Critical Thinking

- 1. Confidence Interval.** In an Accountemps survey, senior executives were asked to identify the most common error made in interviews of job applicants. The following 95% confidence interval estimates the population proportion p for “little or no knowledge of the company.” Interpret that confidence interval.

$$0.393 < p < 0.553$$

- 2. Margin of Error.** In a study of 1,228 randomly selected medical malpractice lawsuits, it is found that the proportion that were dropped or dismissed is 0.697. When a 95% confidence interval is constructed for the population proportion of all lawsuits, the margin of error is found to be 0.026. Identify the confidence interval.

- 3. Confidence Intervals in the Media.** Here is a typical statement made by the media: “Based on a survey of 1050 likely voters, 44% plan to vote for the Republican candidate, and this survey has a margin of error of 3 percentage points.” What important and relevant piece of information is omitted from that statement?

- 4. Small Sample.** In a Pew Research Center poll, 73% of 3011 adults surveyed said that they use the Internet. A reporter claims that the results are not very good because

they are based on a survey of only 0.001% of the adults in the United States, and that percentage is too small to be meaningful. Is that claim valid? Why or why not?

Does It Make Sense? For Exercises 5–8, decide whether the statement makes sense (or is clearly true) or does not make sense (or is clearly false). Explain clearly. Not all of these statements have definitive answers, so your explanation is more important than your chosen answer.

5. Interpreting a Confidence Interval.

We can interpret the confidence interval given in Exercise 1 by saying that there is a 95% chance that the true population proportion will fall between 0.393 and 0.553.

6. Confidence Interval in the Media. The

Kingston Chronicle publishes an article stating that, based on survey results, 82% of Orange County residents oppose an increase in the sales tax, with a margin of error of 4 percentage points. A reader says that this can be expressed as the confidence interval $0.78 < p < 0.86$.

7. Sample Size. The 95% confidence interval given in Example 1 is based on a sample size of 150. If the sample size is increased, the confidence interval will become smaller (or narrower).

8. Sample Size. A reporter for the *Kingston Chronicle* claims that any good confidence interval should be based on a sample that is at least 5% of the population size.

Concepts and Applications

Margins of Error and Confidence Intervals. For Exercises 9–12, assume that population proportions are to be estimated from the samples described. In each case, find the approximate margin of error and 95% confidence interval.

9. Sample size = 1,000, sample proportion = 0.4

10. Sample size = 800, sample proportion = 0.25

11. $n = 550$, $\hat{p} = 0.1$

12. $n = 780$, $\hat{p} = 0.160$

Sample Size. For Exercises 13–16, estimate the minimum sample size needed to achieve the given margin of error.

13. $E = 0.04$

14. $E = 0.05$

15. $E = 0.015$

16. $E = 0.035$

17. Nielsen Ratings. Nielsen Media Research uses samples of 5,000 households to rank TV shows. Nielsen reported that *60 Minutes* had 15% of the TV audience. What is the 95% confidence interval for this result?

18. Nielsen Ratings. Repeat Exercise 17 assuming that the sample size is doubled to 10,000. Given that the large cost and effort of conducting the Nielsen survey would be doubled, does this increase in sample size appear to be justified by the increased reliability?

19. Hazing of Athletes. A study done by researchers at Alfred University concluded that 80% of all student athletes in this country have been subjected to some form of hazing. The study is based on responses from 1,400 athletes. What are the margin of error and 95% confidence interval for the study?

20. Student Opinions. An annual survey of first-year college students, conducted by the Higher Education Research Institute at UCLA, asks approximately 276,000 students about their attitudes on a variety of subjects. According to a recent survey, 51% of first-year students believe that abortion should be legal (down from 65% in 1990) and 40% believe that casual sex is acceptable (down from 50% in 1975). What are the margins of error and 95% confidence intervals for these estimates?

21. Gender Selection. The Genetics and IVF Institute conducted clinical trials of the YSORT method designed to increase the probability of conceiving a boy. Among 152 babies born to parents using the YSORT method, 127

were boys. Identify the margin of error and the 95% confidence interval for these clinical trials.

22. Global Warming. A Pew Research Center poll included 1708 randomly selected adults who were asked whether “global warming is a problem that requires immediate government action.” Results showed that 939 of those surveyed indicated that immediate government action is required. A news reporter wants to determine whether these survey results constitute strong evidence that the majority (more than 50%) of people believe that immediate government action is required.

- What is the best estimate of the proportion of adults who believe that immediate government action is required?
- Construct a 95% confidence interval estimate of the proportion of adults believing that immediate government action is required.
- Is there strong evidence supporting the claim that the majority is in favor of immediate government action? Why or why not?

23. Drugs in Movies. A study by Stanford University researchers for the Office of National Drug Control Policy and the Department of Health and Human Services concluded that 98% of the top rental films involve drugs, drinking, or smoking. Assume that this study is based on the top 400 rental films.

- Use the results of this sample to estimate the proportion of all films that involve drugs, drinking, or smoking.
- What is the 95% confidence interval?
- Do you believe that the top 400 films represent a random sample? Explain.

24. Teen Pressure. A study commissioned by the U.S. Department of Education concluded that 44% of teenagers cite grades as their greatest source of pressure. The study was based on responses from 1,015 teenagers. What is the 95% confidence interval?

25. Pre-Election Polls. Prior to a statewide election for the U.S. Senate, three polls are conducted. In the first poll, 780 of 1,500 voters favor candidate Martinez. In the second poll, 1,285 of 2,500 voters favor Martinez. In the third poll, 1,802 of 3,500 voters favor Martinez. Find the 95% confidence intervals for all three polls. Discuss Martinez’s prospects for victory based on these polls.

26. Unemployment Survey. The Bureau of Labor Statistics estimates the unemployment rate in the United States monthly by surveying 60,000 individuals.

- In one month, 3.4% of the 60,000 individuals surveyed are found to be unemployed. Find the margin of error for this estimate. Is the precision (nearest tenth of a percent) reasonable? Explain.
- Suppose that the number of individuals surveyed were increased by a factor of four (to 240,000). By how much would the margin of error change?

- c. Suppose that the number of individuals surveyed were decreased by a factor of one-fourth (to 15,000). By how much would the margin of error change?
- 27. Opinion Poll.** A poll finds that 54% of the population approves of the job that the President is doing; the poll has a margin of error of 4% (assuming a 95% degree of confidence).
- What is the 95% confidence interval for the true population percentage that approves of the President's performance?
 - What was the size of the sample for this poll?
- 28. Concealed Weapons.** Two-thirds (or 66.6%) of 626 Colorado residents polled by Talmey-Drake Research & Strategy Inc. said they backed a bill pending in the legislature that would standardize laws on granting concealed-weapon permits to gun owners. The bill would force local law enforcement to grant such permits to anyone who can legally carry a gun. The margin of error in the poll was reported as 4 percentage points.
- Is the reported margin of error consistent with the sample size for this estimate?
 - What sample size would be needed to give a margin of error of 2 percentage points?

PROJECTS FOR THE INTERNET & BEYOND

- 29. Who's the Vice President?** Assume that you want to estimate the proportion of students at your college who can correctly identify the Vice President of the United States.

How many students must you randomly select in order to be 95% confident that your sample proportion is within 0.1 of the population proportion? Using that sample size, collect your own sample data by randomly selecting and surveying students at your college. Then use the methods of this section to construct a 95% confidence interval. Write a statement summarizing your results.

- 30. Nielsen Methods.** Visit the Nielsen Media Research Web site and report on the actual methods used to estimate population proportions and confidence intervals in Nielsen ratings.
- 31. Network Polls.** All of the major television networks conduct regular polls on a variety of issues. Visit the Web sites of the major networks and gather the results of a particular poll that involves estimation of a population proportion. Be sure to include all information that is given about the sample size, margin of error, and confidence intervals. Include any details about the actual polling procedure.

IN THE NEWS

- 32. Estimating Population Proportions.** Find a news article or report in which a population proportion is estimated from a sample. The article should include a margin of error and/or a confidence interval. Discuss the methods used in the study and how the conclusions were reached.

CHAPTER REVIEW EXERCISES

1. In a Pew Research Center poll of 745 randomly selected adults, 589 said that it is morally wrong to not report all income on tax returns.

 - a. What single value is the best estimate of the population proportion of all adults who say that it is morally wrong to not report all income on tax returns?
 - b. Use the sample results to construct a 95% confidence interval estimate of the population proportion of adults who say that it is morally wrong to not report all income on tax returns.
 - c. What is the margin of error?
 - d. Write a statement that correctly interprets the confidence interval found in part b.
 - e. The sample proportion of 0.791 was obtained from one specific sample of 745 adults. Suppose many different samples of 745 adults are obtained and the sample proportion is found for each sample. What do we know about the shape of the distribution of the sample proportions?
 - f. Assume that we plan to conduct another poll to determine whether the given sample results are accurate. How many adults would we need to poll in order to have 95% confidence that the margin of error is 0.02?
2. We want to estimate the mean IQ score on the Stanford-Binet test for the population of college students. We know that for people randomly selected from the general population, the standard deviation of IQ scores on the Stanford-Binet test is 16.

 - a. Using a standard deviation of 16, how many college students must we randomly select for IQ tests if we want to have 95% confidence that the sample mean is within 3 IQ points of the population mean?
3. How is our estimate of the mean IQ of college students affected if the sample size is larger than necessary? Smaller than necessary?
4. Is the actual standard deviation of IQ scores for college students likely to be equal to 16, more than 16, or less than 16? Explain. If we used the actual value instead of 16 in part a, how would the answer to part a be affected? Would it be the same, smaller, or larger?
5. A sample of 40 randomly selected women is obtained, and the blood platelet count of each subject is measured. The mean is 279.5 and the standard deviation is 65.2.

 - a. Use these sample results to construct a 95% confidence interval estimate of the population mean.
 - b. What is the margin of error?
 - c. Write a statement that correctly interprets the confidence interval found in part a.
 - d. If the standard deviation is 65.2, how many subjects must be included if we want 95% confidence that the sample mean is in error by at most 10?
6. a. You have been hired by Intel to determine the proportion of computer owners who plan to upgrade to a new operating system. Assuming that you want to be 95% confident that your sample proportion is within 0.02 of the true population proportion, how many people must you survey?

b. Suppose that, in conducting the survey described in part a, you find that half of the people called refuse to answer the survey questions because they believe that you are trying to sell them something. If you proceed by calling twice as many people so that your sample size is large enough, will your results be good? Explain.



CHAPTER QUIZ

1. If many different random samples of size 100 are selected from the population of pulse rates of adult women, what is the shape of the distribution of the sample means?
 2. If many different random samples of size 500 are selected from the population of college students, what is the shape of the distribution of the proportions of women?
 3. What does the notation \hat{p} represent?
 4. A journal article provides a confidence interval for a proportion in the format 0.60 ± 0.08 . Express this confidence interval in the format $a < p < b$. (That is, rewrite $a < p < b$ using specific values in place of a and b .)
 5. Assume that we want to estimate the mean grip strength of adult males in the United States. If a random sample of grip strengths is obtained, which of the following is the best estimate of the population mean?
 - a. Median of the sample
 - b. Mean of the sample
 - c. Standard deviation of the sample
 - d. Range of the sample
 - e. The sample proportion
6. Find the margin of error corresponding to this 95% confidence interval: $0.440 < p < 0.500$.
 7. Find the margin of error corresponding to this 95% confidence interval: $98.0 < \mu < 98.6$.
 8. When a 95% confidence interval is constructed for the population mean, a sample mean is found to be 69.6 and the margin of error is found to be 1.2. Identify the 95% confidence interval.
 9. When a 95% confidence interval is constructed for the population proportion, a sample proportion is found to be 0.720 and the margin of error is found to be 0.025. Identify the 95% confidence interval.
 10. Identify what is wrong with this 95% confidence interval for the population proportion: $0.950 < p < 1.250$.



FOCUS ON HISTORY

Where Did Statistics Begin?

The origins of many disciplines are lost in antiquity, but the roots of statistics can be identified with some certainty. Systematic record keeping began in London in 1532 with weekly data collection on deaths. Later in the same decade, official data collection on baptisms, deaths, and marriages began in France. In 1608, the collection of similar vital statistics began in Sweden. Canada conducted the first official census in 1666.

Of course, statistics is more than the collection of data. If there is a founder of statistics, that person must be someone who worked with the data in clever and systematic ways and who used the data to reach conclusions that were not previously evident. Many experts believe that an Englishman named John Graunt deserves the title of the founder of statistics.*

John Graunt was born in London in 1620. As the eldest child in a large family, he took up his father's business as a draper (a dealer in clothing and dry goods). He spent most of his life as a prominent London citizen, until he lost his house and possessions in the Fire of London in 1666. Eight years later, he died in poverty.

It's not clear how John Graunt became interested in the weekly records of baptisms and burials—known as bills of mortality—that had been kept in London since 1563. In the preface of his book *Natural and Political Observations on the Bills of Mortality*, he noted that others “made little other use of them” and wondered “what benefit the knowledge of the same would bring to the World.” He must have worked on his statistical projects for many years before his book was first published in 1662.

Graunt worked primarily with the annual bills, which were year-end summaries of the weekly bills of mortality. Figure 12 shows the annual bill for 1665 (the year of the Great Plague). The top third of the bill shows the numbers of burials and baptisms (christenings) in each parish. Total burials and baptisms are noted in the middle of the bill, with deaths due to the plague recorded separately. The lower third of the bill

*Some historians claim that Graunt's book was actually written by his life-long friend and collaborator William Petty, though most statisticians believe Graunt wrote his own book. Either way, we know that Petty continued Graunt's work, publishing later editions of Graunt's book and creating the field of “political arithmetic,” which we now call *demography*.



shows deaths due to a variety of other causes, with totals given for males and females.

Graunt was aware of rough estimates of the population of London that were made periodically for taxation purposes, but he must have been skeptical of one estimate that put the population of London at 6 or 7 million in 1661. Using the annual bills, comparing burials and baptisms, and estimating the density of families in London (with an average family size of eight), he arrived at a population estimate of 460,000 by three different methods—quite a change from 6 or 7 million! He also found that the population of London was increasing while the populations of towns in the countryside were decreasing, showing an early trend toward urbanization. He raised awareness of the high rates of infant mortality. He also refuted a popular theory that plagues arrive with new kings.

Graunt's most significant contribution may have been his construction of the first life table. Although detailed data on age at death were not available, Graunt knew that of 100 new babies, “36 of them die before they be six years old, and that perhaps but one surviveth 76.” With these two data points, he filled in the intervening years as shown in Table 5, using methods that he did not fully explain.

With estimates of deaths for various ages, he was able to make the companion table of survivors shown in Table 6. Although some modern statisticians have doubted the methods used to construct these tables, Graunt appears to have appreciated their value and anticipated the actuarial tables now used by life insurance companies. It wasn't until 1693 that Edmund Halley, of comet fame, constructed life tables using age-based mortality rates.

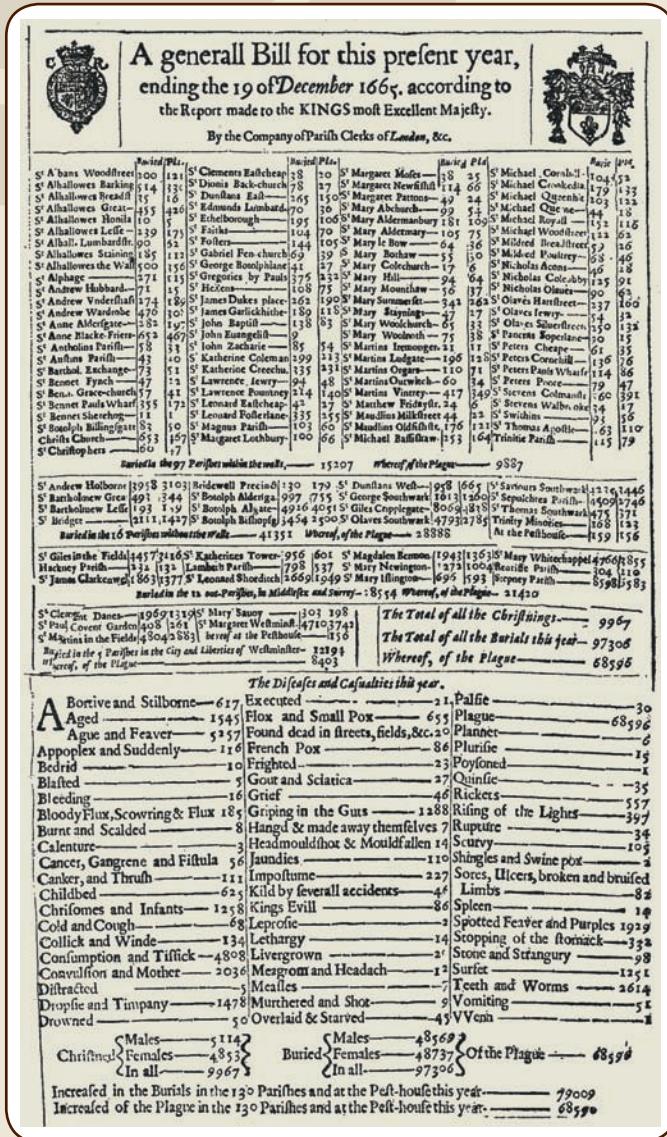


Figure 12 Reproduction of 1665 annual bill of mortality.

Source: From the Wellcome Historical Medical Library, reprinted in *Journal of the Royal Statistical Society*, Vol. 126, part 4, 1963, pp. 537-557.

TABLE 5 **Graunt's Life Table:**
Number of Deaths by
Age for 100 People

Age	Deaths
Within the first six years	36
The next ten years	24
The second decade	15
The third decade	9
The fourth	6
The next	4
The next	3
The next	2
The next	1

TABLE 6 Graunt's Table of Survivors at Various Ages

Age	Survivors
At sixteen years end	40
At twenty six	25
At thirty six	16
At forty six	10
At fifty six	6
At sixty six	3
At seventy six	1
At eighty	0

QUESTIONS FOR DISCUSSION

1. Is it surprising that accurate estimates of the population of London were not available in 1660? How would you have suggested making such estimates at that time?
 2. Do you think that records of burials and baptisms would have given accurate counts of actual births and deaths? Why or why not?
 3. Assuming that the estimates of deaths in Table 5 are accurate, are the estimates of survivors in Table 6 consistent? Note that the numbers in Table 6 do not total to 100; should they? Explain.
 4. Based on Graunt's tables, estimate the average life expectancy in 1660. Explain your reasoning.



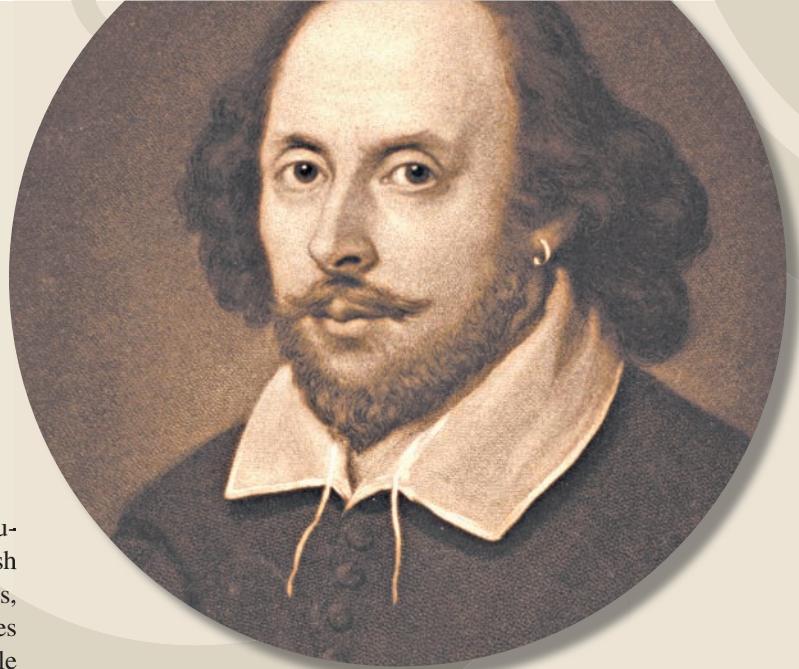
FOCUS ON LITERATURE

How Many Words Did Shakespeare Know?

Imagine that you go to an orientation party for international students. During the course of the evening, you meet 12 Swedish people, 9 Chinese people, 6 French people, 4 Israelis, 3 Koreans, and 1 Iranian. You know there are people of other nationalities whom you did *not* meet at the party. Based only on the people you met, is it possible to estimate the *total* number of nationalities represented at the party—including those of the people you did not meet? Thanks to ideas of sampling, the answer is yes.

The party problem may be a bit frivolous. However, essentially the same question arose when Oxford University marine biologist Charles Paxton wondered how many “sea monsters” (creatures more than 2 meters in length) remain to be discovered. In this case, the nationalities at the party correspond to species of sea monsters. Using statistical methods, Paxton was able to estimate that, in addition to the roughly 220 sea monsters already known (as of 1998), another 47 wait to be discovered.

Similar methods have been used to analyze the works of Shakespeare. Statisticians Bradley Efron and Ronald Thisted wondered about the number of words Shakespeare actually knew, which must have been larger than the number he used in his writings. Here, the nationalities at the party correspond to different words in Shakespeare’s plays and poems. The data collected at the party can be regarded as the *first sample*. For the Shakespeare question, the first sample consists of the complete known works of Shakespeare—specifically, the numbers of words that are used in these works once, twice, three times, and so forth. Table 7 shows a (small) part of the first sample. For example, the table says that in the works of Shakespeare,



14,376 words were used exactly once, 4,343 words were used exactly twice, and so forth. (The full table is much larger and continues far beyond 10 occurrences.)

Given the full table for the first sample, we can now ask a hypothetical question. Suppose a second, new and different sample of Shakespeare’s works of the same size as the first sample were discovered. How many words could we expect to find in the second sample that were *not* used in the first sample? We would expect there to be fewer new words in the second sample, because in the first sample *every* first occurrence of a word is new, even a common word like “the”; in the second sample, those common words are no longer new. Efron and Thisted estimated that 11,430 words would appear in the second sample that did not appear in the first sample.

They repeated this argument with a third sample, fourth sample, fifth sample, and so on. With each new sample, the number of new words decreases, but the total number of words used (among all samples) increases. Efron and Thisted eventually found that the number of new words approached about 35,000. This means that in addition to the 31,534 words that Shakespeare knew and used, there were approximately 35,000 words that he knew but didn’t use. Thus, they estimated that Shakespeare knew approximately 66,500 words.

The analysis of Efron and Thisted, which required advanced methods, was done in 1976. More than 10 years later, the ideas were put to practical use when a new sonnet by an unknown author of Shakespeare’s time period was discovered. As before, the complete volume of Shakespeare’s works was considered the first sample. Now, the second sample was the new sonnet with 429 words. The same statistical method was used to predict that, if Shakespeare was the author, the new sonnet should have seven new words that did not appear in the complete works of Shakespeare. In fact, the new sonnet had nine new words that did not previously appear. Similarly, the method predicted that the new sonnet should have four words that were used exactly once in the complete works. In fact, there were seven words that were used exactly once in the complete works. And the number of words in the sonnet that

TABLE 7

Numbers of Words in Complete Works of Shakespeare Used from One to 10 Times

Occurrences	Number of words
1	14,376
2	4,343
3	2,292
4	1,463
5	1,043
6	837
7	638
8	519
9	430
10	364

From Samples to Populations

were used exactly twice in the complete works was predicted to be three, when in fact there were five such words. The authors concluded that the agreement between the predictions and the actual word frequencies in the sonnet was good enough to attribute authorship to Shakespeare.

These statistical methods have been used to distinguish the works of Shakespeare from those of other Elizabethan writers such as Marlowe, Donne, and Jonson. They have been used to strengthen the belief that James Madison wrote certain of the Federalist Papers whose authorship was in doubt. They have even been used to establish the order of Plato's works.

This example illustrates the remarkable applicability of statistics to problems in a seemingly unrelated discipline. Perhaps more important, it demonstrates how two disciplines

as distant from each other as literature and marine biology can be related by a common statistical thread.

QUESTIONS FOR DISCUSSION

1. Comment on whether you believe that literature is enhanced by statistical analysis. For example, is your appreciation of Shakespeare improved by knowing how many words Shakespeare knew?
 2. Do you believe that statistical analysis is useful for identifying authors of “lost works”?
 3. Suggest another discipline, besides biology and literature, in which the ideas described in this Focus could be used to estimate an unknown quantity.
- • • • • • • • • • •

ANSWERS

Section 1

1. Uniform; approximately normal.
3. \bar{x} denotes the mean of a sample and μ denotes the mean of a population. \bar{x} is the mean of a sample, but μ is the mean of a population.
5. Does not make sense.
7. Does not make sense.
9. The given sample proportion is not likely to be a good estimate of the proportion for the population of children in the United States. There are regional factors that could have a strong influence on that proportion in different parts of the country.
11. a. 9.5 b. The probability is very small, such as 0.01%.
c. No. It appears that the cans are being filled with an amount that is greater than 12.00 ounces.
13. a. 0.528 b. 0.49 c. The sample proportion is a little too low.
15. 783 people. We would be more confident of the estimate if we sampled 300 people.
17. a. -0.67 standard deviations below the mean ($z = -0.67$)
b. 0.253
19. a. 1.0, 1.5, 3.0, 1.5, 2.0, 3.5, 3.0, 3.5, 5.0 b. 2.7 c. Yes; yes
21. a. 5.0, 28.5, 28.5, 32.5, 32.5, 52.0, 56.0, 56.0, 60.0 b. 39.0
c. Yes; yes

Section 2

1. We have 95% confidence that the limits of 8.0518 g and 8.0902 g actually do contain the true population mean weight of all dollar coins. We expect that 95% of such samples will result in confidence interval limits that do contain the population mean.
3. The media often omit reference to the confidence level, which is typically 95%. The word “mean” should be used instead of the word “average.”
5. Does not make sense.
7. Makes sense.
9. Margin of error: 0.6. Confidence interval: $24.6 \text{ cm} < \mu < 25.8 \text{ cm}$
11. Margin of error: 0.4 ft. Confidence interval: $7.6 \text{ ft} < \mu < 8.4 \text{ ft}$
13. 64 15. 7866 17. 185 19. 100
21. $5.639 \text{ g}; 5.619 \text{ g} < \mu < 5.659 \text{ g}$
23. $5.15; 5.10 < \mu < 5.20$
25. $146 \text{ lb} < \mu < 221 \text{ lb}$
27. a. 3.1 b. 1.4 c. 3.1 d. $2.6 < \mu < 3.6$ e. The sample is small and was concentrated in one neighborhood, so it is unlikely to be very representative of all American families.

Section 3

1. We have 95% confidence that the limits of 0.393 and 0.553 actually do contain the true population proportion. We expect that 95% of such samples will result in confidence interval limits that do contain the population proportion.
3. The media often omit reference to the confidence level, which is typically 95%.
5. Does not make sense.
7. Makes sense.
9. Margin of error: 0.0310. $0.369 < p < 0.431$
11. Margin of error: 0.0256. $0.074 < p < 0.126$
13. 625 15. 4445 17. $0.140 < p < 0.160$
19. $E = 0.021; 0.779 < p < 0.821$
21. Margin of error: 0.060. $0.775 < p < 0.896$
23. a. 0.98 b. $0.966 < p < 0.994$ c. To be a random sample, all films must have an equal chance of being chosen.
25. Poll 1: $0.494 < p < 0.546$; Poll 2: $0.494 < p < 0.534$; Poll 3: $0.498 < p < 0.532$. Because all of these confidence intervals include values less than 0.5, Martinez cannot be confident of winning a majority.
27. a. $0.50 < p < 0.58$ b. $n = 625$

Chapter Review Exercises

1. a. 0.791 b. $0.761 < p < 0.820$ c. 0.030 d. There is 95% confidence that the limits of 0.754 and 0.827 contain the true value of the population proportion. If such samples of size 745 were randomly selected many times, the resulting confidence interval limits would contain the true population proportion in 95% of those samples. e. The distribution will be approximately normal, or bell-shaped. f. 2500
3. a. $258.9 < \mu < 300.1$ b. 20.6 c. There is 95% confidence that the limits of 258.9 and 300.1 contain the true value of the population mean. If such samples of size 40 were randomly selected many times, the resulting confidence interval limits would contain the true population mean in 95% of those samples. d. 171

Chapter Quiz

1. Approximately normal or bell-shaped
3. The sample proportion
5. b: mean of the sample
7. Margin of error is 0.3
9. $0.695 < p < 0.745$

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Hypothesis Testing



LEARNING GOALS

1 Fundamentals of Hypothesis Testing

Understand the goal of hypothesis testing and the basic structure of a hypothesis test.

2 Hypothesis Tests for Population Means

Understand and interpret one- and two-tailed hypothesis tests for claims made about population means.

3 Hypothesis Tests for Population Proportions

Understand and interpret hypothesis tests for claims made about population proportions.

FOCUS TOPICS

Focus on Health & Education: Will Your Education Help You Live Longer?

Focus on Agriculture: Are Genetically Modified Foods Safe?

Hypothesis Testing

Everyone makes claims. Advertisers make claims about their products. Universities claim their programs are superb. Governments claim their programs are effective. Lawyers make claims about a suspect's guilt or innocence. Medical diagnoses are claims about the presence or absence of disease. Pharmaceutical companies make claims about the effectiveness of their drugs. But how do we know whether any of these claims are true? Statistics offers a way to test many claims, through a powerful set of techniques called *hypothesis testing*.

It is the mark of an educated mind to be able to entertain a thought without accepting it.
—Aristotle

1

FUNDAMENTALS OF HYPOTHESIS TESTING

A company called ProCare Industries, Ltd. once claimed that its product, called Gender Choice, could increase a woman's chance of giving birth to a baby girl. The company claimed that the chance of a baby girl could be increased "up to 80%," but let's focus simply on the claim that the chance of having a baby girl is greater than the approximately 50%, or 0.5, expected under ordinary conditions. How could we test whether the Gender Choice claim is true?

One way would be to study a random sample of, say, 100 babies born to women who used the Gender Choice product. If the product does not work, we would expect about half of these babies to be girls. If it does work, we would expect significantly more than half of the babies to be girls. The key question, then, is what constitutes "significantly more." If there were 97 girls among the 100 births in the sample, we would all agree that this constituted significantly more than half and that the product is probably effective. If there were only 52 girls among the 100 births, we'd probably agree that 52 was so close to half that we had no reason to think the product had any effect. But would we consider, say, 64 girls in the sample of 100 babies to be "significantly more" than half, and therefore think that the product might really work?

In statistics, we answer such questions through *hypothesis testing*. Before we discuss the specific terminology and procedures of hypothesis testing, let's look at the procedure we would use to draw a conclusion about Gender Choice based on a sample in which there are 64 girls among 100 babies born to women who used the product.

- We begin by assuming that Gender Choice does *not* work; that is, it does *not* increase the percentage of girls. If this is true, then we should expect about 50% girls among the *population* of all births to women using the product. That is, if Gender Choice does not work, then the proportion of girls should be 0.50.
- We now use our *sample* (with 64 girls among 100 births) to test the above assumption. We conduct this test by calculating the likelihood of drawing a random sample of 100 births in which 64% (or more) of the babies are girls from a population in which the overall proportion of girls is only 50%.
- If we find that a random sample of births is fairly likely to have 64% (or more) girls, then we do *not* have evidence that Gender Choice works. However, if we find that a random sample of births is *unlikely* to have at least 64% girls, then we conclude that the result in the Gender Choice sample is probably due to something other than chance—meaning that the product may be effective. Note, however, that even in this case we will not have *proved* that the product is effective, as there could still be other explanations for the result (such as that we happened to choose an unusual sample or that there were confounding variables we did not account for).

TIME OUT TO THINK

Suppose Gender Choice is effective and you select a new random sample of 1,000 babies born to women who used the product. Based on the first sample (with 64 girls among 100 births), how many baby girls would you expect in the new sample? Next, suppose Gender Choice is *not* effective; how many baby girls would you expect in a random sample of 1,000 babies born to women who used the product?

Formulating the Hypotheses

The key remaining issues in our test of the effectiveness of Gender Choice are that we have not yet described how to calculate the likelihood of drawing a random sample like the one with 64% girls, and we have not yet defined exactly what we mean when we ask whether such a sample is "fairly likely" or "unlikely" to be drawn at random. Most of this chapter is devoted to the formal procedures used to resolve these issues. The first step in resolving them is to define exactly what we are testing.

BY THE WAY

The Gender Choice product was based on the principle that the gender of a baby can be determined through careful timing of conception. There was no evidence that the product actually worked, and it is no longer available. However, newer technologies have proven successful in allowing gender selection (with artificial insemination), raising ethical questions about whether or when gender selection should be an option for parents.



In the context of statistics, a **hypothesis** is a claim about a population parameter or some other characteristic of a population. For example, in the Gender Choice case, the population parameter is the proportion of girls born to all women who use the product. A **hypothesis test**, then, is a test of whether a particular claim about a population is supported or unsupported by the available sample evidence.

Definitions

A **hypothesis** is a claim about a population parameter (such as a population proportion p or population mean μ) or some other characteristic of a population.

A **hypothesis test** is a standard procedure for testing a claim about a population parameter or some characteristic of a population.

There are always at least two hypotheses in any hypothesis test. In the Gender Choice case, the two hypotheses are essentially: (1) that Gender Choice works in raising the population proportion of girls from the 50% that we would normally expect; and (2) that Gender Choice does *not* work in raising the population proportion of girls. The starting point for the hypothesis test is the second of those hypotheses—that it does *not* raise the proportion of baby girls. We call this starting point the **null hypothesis**, denoted by H_0 (read “*H-naught*”). The other hypothesis (1) is called the **alternative hypothesis**, denoted by H_a (read “*H-a*”). To summarize, for the Gender Choice example:

- The *null hypothesis* is the claim that Gender Choice does *not* work, in which case the population proportion of girls born to women who use the product should be 50%, or 0.50. Using p to represent the population proportion, we write this null hypothesis as

$$H_0(\text{null hypothesis}): p = 0.50$$

- The *alternative hypothesis* is the claim that Gender Choice *does* work, in which case the population proportion of girls born to women who use the product should be *greater than* 0.50. We write this alternative hypothesis as

$$H_a(\text{alternative hypothesis}): p > 0.50$$

In this chapter, the null hypothesis will always include the condition of equality, just as the null hypothesis for the Gender Choice example is the equality $p = 0.50$. However, the alternative hypothesis for the Gender Choice case ($p > 0.50$) is only one of three general forms of alternative hypotheses that we will encounter in this chapter:

population parameter $<$ claimed value
 population parameter $>$ claimed value
 population parameter \neq claimed value

As we will see in Section 2, these three different types of alternative hypotheses require slightly different calculations in the hypothesis test; for that reason, it is useful to give them names. The first form (“less than”) leads to a **left-tailed** hypothesis test, because it requires testing whether the population parameter lies to the *left* (lower values) of the claimed value. Similarly, the second form (“greater than”) leads to a **right-tailed** hypothesis test, because it requires testing whether the population parameter lies to the *right* (higher values) of the claimed value. The third form (“not equal to”) leads to a **two-tailed** hypothesis test, because it requires testing whether the population parameter lies significantly far to *either* side of the claimed value.

BY THE WAY

The word *null* comes from the Latin *nullus*, meaning “nothing.” A null hypothesis usually states that there is no special effect or difference.

Null and Alternative Hypotheses

The **null hypothesis**, or H_0 , is the starting assumption for a hypothesis test. For the types of hypothesis tests in this chapter, the null hypothesis always claims a specific value for a population parameter and therefore takes the form of an equality:

$$H_0 \text{ (null hypothesis): population parameter} = \text{claimed value}$$

The **alternative hypothesis**, or H_a , is a claim that the population parameter has a value that differs from the value claimed in the null hypothesis. It may take one of the following forms:

- (left-tailed) H_a : population parameter $<$ claimed value
- (right-tailed) H_a : population parameter $>$ claimed value
- (two-tailed) H_a : population parameter \neq claimed value

EXAMPLE 1 Identifying Hypotheses

In each case, identify the population parameter about which a claim is made, state the null and alternative hypotheses for a hypothesis test, and indicate whether the hypothesis test will be left-tailed, right-tailed, or two-tailed.

- Nissan claims that the Leaf, an electric car, has a mean range of 110 miles between charges. A consumer group claims that the mean range is less than 110 miles.
- The Ohio Department of Health claims that the average stay in Ohio hospitals after childbirth is greater than the national mean of 2.0 days.
- A wildlife biologist working in the African savanna claims that the actual proportion of female zebras in the region is different from the accepted proportion of 50%.

SOLUTION

- The population parameter about which a claim is made is a population *mean* (μ)—the mean range of the car between charges. The null hypothesis must state that this population mean is *equal* to some specific value. We therefore recognize the null hypothesis as the advertised claim that the mean range for the cars is 110 miles. The alternative hypothesis is the consumer group's claim that the true range is *less* than advertised. To summarize:

$$\begin{aligned} H_0 \text{ (null hypothesis): } & \mu = 110 \text{ miles} \\ H_a \text{ (alternative hypothesis): } & \mu < 110 \text{ miles} \end{aligned}$$

Because the alternative hypothesis has a “less than” form, the hypothesis test will be left-tailed.

- The population is all women in Ohio who have recently given birth, and the population parameter is their mean (μ) stay after childbirth in Ohio hospitals. The null hypothesis must be an equality, so in this case it is the claim that the mean hospital stay equals the national average of 2.0 days. The alternative hypothesis is the Health Department's claim that the mean stay in Ohio is *greater than* this national average. To summarize:

$$\begin{aligned} H_0 \text{ (null hypothesis): } & \mu = 2.0 \text{ days} \\ H_a \text{ (alternative hypothesis): } & \mu > 2.0 \text{ days} \end{aligned}$$

Because the alternative hypothesis has a “greater than” form, the hypothesis test will be right-tailed.

- In this case, the claim is about a population *proportion* (p)—the proportion of female zebras among the zebra population of the region. The accepted population proportion is $p = 0.5$, which becomes the null hypothesis. The wildlife biologist claims that the actual population proportion is different from the value in the null hypothesis. Because “different” can be either greater or less than, the alternative hypothesis has a “not equal to” form:

$$\begin{aligned} H_0 \text{ (null hypothesis): } & p = 0.5 \\ H_a \text{ (alternative hypothesis): } & p \neq 0.5 \end{aligned}$$

The “not equal to” form of this alternative hypothesis leads to a two-tailed test.



Possible Outcomes of a Hypothesis Test

A hypothesis test always begins with the assumption that the null hypothesis is true. We then test to see whether the data give us reason to think otherwise. As a result, there are generally only two possible outcomes to a hypothesis test, summarized in the following box.

Two Possible Outcomes of a Hypothesis Test

There are two possible outcomes to a hypothesis test:

1. *Reject* the null hypothesis, H_0 , in which case we have evidence in support of the alternative hypothesis.
2. *Not reject* the null hypothesis, H_0 , in which case we do not have enough evidence to support the alternative hypothesis.

Notice that “accepting the null hypothesis” is *not* a possible outcome, because the null hypothesis is always the starting assumption. The hypothesis test may not give us reason to reject this starting assumption, but it cannot by itself give us reason to conclude that the starting assumption is true.

The fact that there are only two possible outcomes makes it extremely important that the null and alternative hypotheses be chosen in an unbiased way. In particular, both hypotheses must always be formulated *before* a sample is drawn from the population for testing. Otherwise, data from the sample might inappropriately bias the selection of the hypotheses to test.

TIME OUT TO THINK

The idea that a hypothesis test cannot lead us to accept the null hypothesis is an example of the old dictum that “absence of evidence is not evidence of absence.” As an illustration of this idea, explain why it would be easy in principle to prove that some legendary animal (such as Bigfoot or the Loch Ness Monster) exists, but it is nearly impossible to prove that it does *not* exist.

EXAMPLE 2 Hypothesis Test Outcomes

For each of the three cases from Example 1, describe the possible outcomes of a hypothesis test and how we would interpret these outcomes.

BY THE WAY

The first recorded example of hypothesis testing is attributed to the Scotsman John Arbuthnot (1667–1735). Using 82 years of data, he noticed that the annual number of male baptisms was consistently higher than the annual number of female baptisms. Knowing that there was no bias in baptisms based on gender, he argued that such a regular pattern could not be explained by chance and had to be due to “divine providence,” meaning boys must represent slightly more than 50% of all births. In modern terminology, he rejected the null hypothesis that the data could be explained by chance alone.

SOLUTION

- a. Recall that the null hypothesis is the advertised claim that the mean range for the cars is $\mu = 110$ miles. The alternative hypothesis is the consumer group’s claim that the true range is *less* than advertised, or $\mu < 110$ miles. The possible outcomes are
 - Reject the null hypothesis of $\mu = 110$ miles, in which case we have evidence in support of the consumer group’s claim that the range is less than advertised.
 - Do not reject the null hypothesis, in which case we lack evidence to support the consumer group’s claim. Note, however, that this option does not imply that the advertised claim is true.
- b. The null hypothesis is that the mean hospital stay is the national average of 2.0 days. The alternative hypothesis is the Health Department’s claim that the mean stay in Ohio is *greater than* the national average. The possible outcomes are
 - Reject the null hypothesis of $\mu = 2.0$ days, in which case we have evidence in support of the Health Department’s claim that the mean stay in Ohio is greater than the national average.
 - Do not reject the null hypothesis, in which case we lack evidence to support the Health Department’s claim. Note, however, that this option does not imply that the Ohio average stay is actually equal to the national average stay of 2.0 days.

- c. The null hypothesis is that the proportion of female zebras is the accepted population proportion of 50% ($p = 0.5$). The alternative hypothesis is the biologist's claim that the accepted value is wrong, meaning that the actual proportion of female zebras is not 50% (it may be either more or less than 50%). The possible outcomes are
- Reject the null hypothesis of $p = 0.5$, in which case we have evidence in support of the biologist's claim that the accepted value is wrong.
 - Do not reject the null hypothesis, in which case we lack evidence to support the biologist's claim. Note, however, that this does not imply that the accepted value is correct.
- • •

Drawing a Conclusion from a Hypothesis Test

Let's return to the Gender Choice example in which we imagine drawing a random sample of 100 babies (born to women using the Gender Choice product) and find that 64 of these babies are girls. How do we decide whether this sample result should lead us to reject or not reject the null hypothesis? The answer comes down to deciding whether the sample result was *likely* or *unlikely* to have occurred by chance if the null hypothesis is true.

Remember that the null hypothesis for this case is that the true proportion of baby girls among the population of Gender Choice users is 50%, or $p = 0.50$. The sample we are studying has a sample size of $n = 100$ and a sample proportion of $\hat{p} = 0.64$. The precise question, then, is this: If the true population proportion is $p = 0.50$ (as the null hypothesis claims), what is the probability that by chance alone a sample of size $n = 100$ would have a sample proportion of at least $\hat{p} = 0.64$? If the probability is low, then it is highly unlikely that we would have found such a sample by chance; we therefore have reason to reject the null hypothesis. If the probability of observing the sample result is moderate or high, then it is fairly likely that we could have found such a result by chance, so we cannot reject the null hypothesis.

There are multiple ways to make the decision about rejecting or not rejecting the null hypothesis. Here, we'll look at two closely related options: making the decision based on the statistical significance of the result and making the decision based on the actual probability, or "P-value," of the test result.

Statistical Significance

If the probability of a particular result is 0.05 or less, we say that the result is statistically significant at the 0.05 level; if the probability is 0.01 or less, the result is statistically significant at the 0.01 level. The 0.01 level therefore means stronger significance than the 0.05 level. The following box summarizes how we can apply these ideas directly to hypothesis tests.

Probable truth relies on statistical arguments to weigh which of several possibilities is more likely to be true. When something must be proven beyond a reasonable doubt, just what does that mean? What level of doubt is acceptable? One in twenty? One in a trillion?

—K. C. Cole

Hypothesis Test Decisions Based on Levels of Statistical Significance

We decide the outcome of a hypothesis test by comparing the actual sample statistic (mean or proportion) to the result expected if the null hypothesis is true. We must choose a significance level for the decision.

- If the chance of a sample statistic at least as extreme as the observed statistic is less than 1 in 100 (or 0.01), then the test is statistically significant at the 0.01 level and offers strong evidence for rejecting the null hypothesis.
- If the chance of a sample statistic at least as extreme as the observed statistic is less than 1 in 20 (or 0.05), then the test is statistically significant at the 0.05 level and offers moderate evidence for rejecting the null hypothesis.
- If the chance of a sample statistic at least as extreme as the observed statistic is greater than the chosen level of significance (0.05 or 0.01), then we do not reject the null hypothesis.

EXAMPLE 3 Statistical Significance in Hypothesis Testing

Consider the Gender Choice example in which the sample size is $n = 100$ and the sample proportion is $\hat{p} = 0.64$. Using techniques that we will discuss later in this chapter, it is possible to calculate the probability of randomly choosing such a sample proportion (or a more extreme proportion with $\hat{p} > 0.64$) under the assumption that the null hypothesis ($p = 0.50$) is true; the result is that the probability is 0.0026. (The result is 0.0033 if technology is used to obtain very accurate results.) Based on this result, should you reject or not reject the null hypothesis?

SOLUTION The probability of 0.0026 means that if the null hypothesis is true (meaning that the true population proportion is 50%), the probability of drawing a random sample with a sample proportion of at least $\hat{p} = 0.64$ is less than 1 in 100, so this result is statistically significant at the 0.01 level. It therefore gives us good reason to reject the null hypothesis, which means it provides support for the alternative hypothesis that Gender Choice raises the proportion of girls to more than 50%. •••

P-Values

In Example 3 above, we concluded that the sample result gave us reason to reject the null hypothesis, because the sample proportion was statistically significant at the 0.01 level. In fact, the result was even better than that: The computed probability of 0.0026 is called a **P-value** (short for *probability value*), and it gives us more information than simply stating a level of statistical significance; notice the capital P , used to avoid confusion with the lowercase p that stands for a population proportion. In other words, for Example 3 we say that the P -value for the hypothesis test is 0.0026. We will discuss the actual calculation of P -values in Sections 2 and 3; here, we focus only on their interpretation.

Hypothesis Test Decisions Based on P-Values

The **P-value** (probability value) for a hypothesis test of a claim about a population parameter is the probability of selecting a sample with a statistic that is at least as extreme as the observed statistic, assuming that the null hypothesis is true:

- A small P -value (such as less than or equal to 0.05) indicates that the sample result is unlikely and therefore provides reason to reject the null hypothesis.
- A large P -value (such as greater than 0.05) indicates that the sample result could easily occur by chance, so we cannot reject the null hypothesis.

EXAMPLE 4 Fair Coin?

You suspect that a coin may have a bias toward landing tails more often than heads, and decide to test this suspicion by tossing the coin 100 times. The result is that you get 40 heads (and 60 tails). A calculation (not shown here) indicates that the probability of getting 40 or fewer heads in 100 tosses with a fair coin is 0.0284. Find the P -value and level of statistical significance for your result. Should you conclude that the coin is biased against heads?

SOLUTION The null hypothesis is that the coin is fair, in which case the proportion of heads should be about 50% ($H_0: p = 0.50$). The alternative hypothesis is your suspicion that the coin is biased against heads, in which case the proportion of heads would be less than 50% ($H_a: p < 0.50$). Your 100 coin tosses represent a random sample of size $n = 100$, and the result of 40 heads is the sample proportion ($\hat{p} = 0.40$) for the hypothesis test. The P -value for the test is the probability of getting a sample proportion at least as extreme as the one you found ($\hat{p} \leq 0.40$), *assuming* that the coin is fair and the population proportion of heads is 0.5. The given probability for this occurrence is 0.0284, which is the P -value for the test. Because this P -value is smaller than 0.05, the result is statistically significant at the 0.05 level. Because it is not smaller than 0.01, the result is not significant at the 0.01 level. Statistical significance at the 0.05 level gives you moderate reason to reject the null hypothesis and conclude that the coin is biased against heads. •••

Putting It All Together

We have now covered all the basic ideas of hypothesis testing, except for the actual calculations that are required. We will discuss these calculations for hypothesis tests of population means in Section 2 and of population proportions in Section 3. The following box summarizes the steps that go into a hypothesis test.

The Hypothesis Test Process

- Step 1. Formulate the null and alternative hypotheses, each of which must make a claim about a *population* parameter, such as a population mean (μ) or a population proportion (p), be sure this is done before drawing a sample or collecting data. Based on the form of the alternative hypothesis, decide whether you will need a left-, right-, or two-tailed hypothesis test.
- Step 2. Draw a sample from the population and measure the sample statistics, including the sample size (n) and the relevant sample statistic, such as the sample mean (\bar{x}) or sample proportion (\hat{p}).
- Step 3. Determine the likelihood of observing a sample statistic (mean or proportion) at least as extreme as the one you found *under the assumption that the null hypothesis is true*. The precise probability of such an observation is the P -value (probability value) for your sample result.
- Step 4. Decide whether to reject or not reject the null hypothesis, based on your chosen level of significance (usually 0.05 or 0.01, but other significance levels are sometimes used).

Again, be sure to avoid confusion among the three different uses of the letter p :

- A lowercase p represents a *population proportion*—that is, the true proportion among a complete population.
- A lowercase \hat{p} (“ p -hat”) represents a *sample proportion*—that is, the proportion found in a sample drawn from a population.
- An uppercase P stands for probability in a P -value.

EXAMPLE 5 Mean Rental Car Mileage

In the United States, the average car is driven about 12,000 miles each year. The owner of a large rental car company suspects that for his fleet, the mean distance is greater than 12,000 miles each year. He selects a random sample of $n = 225$ cars from his fleet and finds that the mean annual mileage for this sample is $\bar{x} = 12,375$ miles. A calculation shows that if you assume the fleet mean is the national mean of 12,000 miles, then the probability of selecting a 225-car sample with a mean annual mileage of at least 12,375 miles is 0.01. Based on these data, describe the process of conducting a hypothesis test and drawing a conclusion.

SOLUTION We follow the four steps listed in the box above:

- Step 1. The population parameter of interest is a *population mean* (μ)—the mean annual mileage for the population of all cars in the rental car fleet. The null hypothesis is that the population mean is the national average of 12,000 annual miles per car. The alternative hypothesis is the owner’s claim that the population mean for his fleet is greater than this national average. That is,

$$\begin{aligned} H_0: \mu &= 12,000 \text{ miles} \\ H_a: \mu &> 12,000 \text{ miles} \end{aligned}$$

Because the alternative hypothesis has a “greater than” form, the hypothesis test will be right-tailed.

- Step 2. This step asks that we select the sample and measure the sample statistics; here, we are given the sample size of $n = 225$ and the sample mean of $\bar{x} = 12,375$ miles.
- Step 3. This step asks us to determine the likelihood that, under the assumption that the fleet average is really 12,000 miles (the null hypothesis), we would by chance select a sample with a mean of at least 12,375 miles. Here, we don’t need to do the calculation,

because we are told that the probability is 0.01. (The calculation method is given in Section 2.) This probability is the *P*-value; it tells us that if the null hypothesis were true, there would be only a 0.01 probability of selecting a sample as extreme as the one observed.

- Step 4. The *P*-value of 0.01 tells us that the result is significant at the 0.01 level. We therefore reject the null hypothesis and conclude that the test provides strong support for the alternative hypothesis, implying that the mean annual mileage for the rental car fleet is greater than the national average of 12,000 miles.
- • •



A Legal Analogy of Hypothesis Testing

A legal analogy might help clarify the idea of hypothesis testing. In American courts of law, the fundamental principle is that a defendant is presumed innocent until proven guilty. Because the starting assumption is innocence, this represents the null hypothesis:

H_0 : The defendant is innocent.

H_a : The defendant is guilty.

The job of the prosecutor is to present evidence so compelling that the jury is persuaded to reject the null hypothesis and find the defendant guilty. If the prosecutor does not make a sufficiently compelling case, then the jury will not reject H_0 and the defendant will be found “not guilty.” Note that finding a person innocent (accepting H_0) is not an option: A verdict of not guilty means the evidence is not sufficient to establish guilt, but it does not prove innocence.

TIME OUT TO THINK

Consider two situations. In one, you are a juror in a case in which the defendant could be fined a maximum of \$2,000. In the other, you are a juror in a case in which the defendant could receive the death penalty. Compare the significance levels you would use in the two situations. In each situation, what are the consequences of wrongly rejecting the null hypothesis?

Section 1 Exercises

Statistical Literacy and Critical Thinking

- Hypothesis Test.** What is a hypothesis test?
- Hypotheses.** What is a null hypothesis? What notation is used for a null hypothesis? What is an alternative hypothesis? What notation is used for an alternative hypothesis?
- Alternative Hypotheses.** A researcher wants to test the claim that the mean weight of male airline passengers, including carry-on baggage, is 195 lb. Identify the three different possible expressions that could be used for the alternative hypothesis.
- P-Value.** What is a *P*-value for a hypothesis test?

Does It Make Sense? For Exercises 5–12, decide whether the statement makes sense (or is clearly true) or does not make sense (or is clearly false). Explain clearly. Not all of these statements have definitive answers, so your explanation is more important than your chosen answer.

- Interpreting a Hypothesis Test.** To test the claim that a particular drug lowers blood pressure, sample data are

collected and a hypothesis test is used, and we form this conclusion: “The study proves that the drug lowers blood pressure in everyone.”

- P-Value.** A researcher is convinced that she can show that a new drug is effective in lowering LDL cholesterol. She claims that the *P*-value of 0.20 supports her claim of a lower mean level of LDL cholesterol.
- Null Hypothesis.** In testing a claim that the mean LDL cholesterol level is less than 130 mg/dL, the researcher states the alternative hypothesis as $\mu < 130$ mg/dL.
- Null Hypothesis.** After conducting a hypothesis test, a researcher concludes that there is sufficient sample evidence to support the null hypothesis that $\mu = 75$.

- 9. Drug Treatment.** In a test of the claim that, among patients treated with Ziac, the proportion who experience dizziness is less than 0.06, the null hypothesis is $p = 0.06$.
- 10. Sample Mean.** A study is designed to determine the proportion of men who weigh more than 195 pounds, so the null hypothesis is $\mu = 195$ pounds and the alternative hypothesis is $\mu > 195$ pounds.
- 11. P-Value.** In interpreting a P -value of 0.45, a researcher states that the results are statistically significant because the P -value is less than 0.5, indicating that the results are not likely to occur by chance.
- 12. P-Value.** In interpreting a P -value of 0.001, a researcher states that the results are statistically significant because the P -value is very small, indicating that the results are not likely to occur by chance.

Concepts and Applications

- 13. What Is Significant?** In testing a method of gender selection, 40 couples are given a treatment designed to increase the likelihood of a girl, and each couple has one baby.
- If the 40 babies include exactly 22 girls, would you consider this result statistically significant or would you attribute it to random fluctuations?
 - If the 40 babies include exactly 35 girls, would you consider this result statistically significant or would you attribute it to random fluctuations?
 - The P -value of 0.0000007 is obtained for the results of either part a or b. Which is it? Why?
- 14. What Is Significant?** In testing a method of gender selection, 200 couples are given a treatment designed to increase the likelihood of a girl, and each couple has one baby.
- If the 200 babies include exactly 130 girls, would you consider this result statistically significant or would you attribute it to random fluctuations?
 - If the 200 babies include exactly 105 girls, would you consider this result statistically significant or would you attribute it to random fluctuations?
 - The P -value of 0.262 is obtained for the results of either part a or b. Which is it? Why?

Formulating Hypotheses. In Exercises 15–22, formulate the null and alternative hypotheses for a hypothesis test. State clearly the two possible conclusions that address the given claim.

- 15. Calories.** Package labeling indicates that Twix candy bars have an amount of caloric content equal to 250 calories.
- 16. SAT Scores.** A high school principal claims that the mean SAT score of seniors at his school is less than the national average of 1518.
- 17. Gender Selection.** The Chief Operations Officer of a medical facility claims that treatments can increase the probability that a baby will be a girl so that the proportion of girls is greater than 0.7.
- 18. Quality Control.** The quality control manager at a manufacturing company claims that the proportion of defective carbon monoxide detectors is less than 0.04.
- 19. Vending Machines.** A sales representative claims that her vending machines dispense coffee so that the mean amount supplied is equal to 10 ounces.
- 20. Aspirin.** The Food and Drug Administration claims that a pharmaceutical company is producing aspirin tablets with a mean amount of aspirin that is less than 350 milligrams.
- 21. Holocaust.** A high school teacher claims that the majority of her students do not know what the term *Holocaust* refers to.
- 22. Smoking.** An educator claims that less than 20% of college graduates smoke.

P-Values and Births. Assume that male births and female births are equally likely. The following table shows the probabilities of various numbers of male babies in a random sample of 100 births. Use this information for Exercises 23–28, and assume that we are testing for a bias against males.

Number of males among 100	Probability
35 or fewer	0.002
40 or fewer	0.028
45 or fewer	0.184
48 or fewer	0.382

- 23.** A random sample of 100 births has 48 male babies. Is this result significant at the 0.05 level? What is the P -value for this result?
- 24.** A random sample of 100 births has 45 male babies. Is this result significant at the 0.01 level? What is the P -value for this result?
- 25.** A random sample of 100 births has 40 male babies. Is this result significant at the 0.01 level? What is the P -value for this result?
- 26.** A random sample of 100 births has 40 male babies. Is this result significant at the 0.05 level? What is the P -value for this result?
- 27.** A random sample of 100 births has 35 male babies. Is this result significant at the 0.01 level? What is the P -value for this result?
- 28.** A random sample of 100 births has 32 male babies. Is this result significant at the 0.01 level? What is the P -value for this result?

- 
- PROJECTS FOR THE INTERNET & BEYOND**
- 29. Professional Journals.** Many professional journals, such as *Journal of the American Medical Association*, contain articles that include information about formal tests of hypotheses. Find such an article and identify the null

hypothesis and alternative hypothesis. In simple terms, state the objective of the hypothesis test and the conclusion that was reached.

- 30. Coin Activity.** Select a particular quarter and test the claim that it favors heads when flipped. State the null and alternative hypotheses; then flip the quarter 100 times. Applying only common sense, what do you conclude about the claim that the quarter favors heads?

IN THE NEWS

- 31. Hypothesis Testing in the News.** Find a news article or research report that describes (perhaps not explicitly) a hypothesis test for a population mean or proportion. Attach the article and summarize the method used.

2

HYPOTHESIS TESTS FOR POPULATION MEANS

In Section 1, we sketched the basic outline of the hypothesis testing process. Remember that in all cases, we need to make a decision about whether to reject or not reject the null hypothesis, which is the starting assumption for the test. In this section, we describe the calculations used to make that decision for hypothesis tests with population means. We first investigate the procedure for one-tailed (left-tailed or right-tailed) tests and then discuss the small differences in procedure necessary for two-tailed tests.

One-Tailed Hypothesis Tests

Consider the following hypothetical situation. Columbia College advertises that the mean starting salary of its graduates is \$39,000. The Committee for Truth in Advertising, an independent organization, suspects that this claim is exaggerated and decides to conduct a hypothesis test to seek evidence to support its suspicion.

The parameter of interest is the mean starting salary of the population of all Columbia College graduates, so the hypothesis test will concern a *population mean* (μ). The null hypothesis is the college's claim that the mean starting salary is \$39,000. The alternative hypothesis is the committee's claim that the college has exaggerated the mean starting salary, in which case the mean starting salary is *less than* \$39,000. The null and alternative hypotheses are

$$H_0: \mu = \$39,000$$

$$H_a: \mu < \$39,000$$

Because the alternative hypothesis has a "less than" form, we are dealing with a left-tailed hypothesis test. The general procedure for left- and right-tailed tests is the same, so we will consider both together as *one-tailed* tests.

Having formed the hypotheses, the Committee for Truth in Advertising selects a random sample of 100 recent graduates from the college. The mean salary of the graduates in the sample turns out to be \$37,000. The sample size is $n = 100$ and the sample mean is $\bar{x} = \$37,000$.

If you look back at the four-step hypothesis test process, you'll see that the first two steps are already complete: The mean starting salary has been identified as the population parameter of interest, the null and alternative hypotheses have been stated, and the sample has been drawn and measured to determine its sample size and sample mean. We are therefore ready for Steps 3 and 4, in which we assume the null hypothesis is true and then determine whether the sample statistics give us reason to reject this assumption.

The Sampling Distribution

Step 3 of the hypothesis test process is finding the likelihood of observing a sample mean as extreme as the one found, under the assumption that the null hypothesis is true. For the Columbia College example, the question becomes this: How likely are we to select a sample (of size $n = 100$) with a mean of \$37,000 or less when the mean for the whole population is \$39,000? To answer this question, we need a crucial observation: *The observed sample mean ($\bar{x} = \$37,000$) is just one point in a distribution of sample means.*

BY THE WAY

As of 2010, data from *The New York Times* indicate that the median starting salary for *all* college graduates with a bachelor's degree is about \$27,000. The median is given rather than the mean, because salary distributions tend to be very skewed, with great variations by major. Graduates in engineering and business generally have the highest starting salaries.

To understand the importance of this fact, imagine that instead of drawing just one sample of size $n = 100$, the Committee for Truth in Advertising drew many samples of that size. Each sample would have a unique sample mean, \bar{x} , so we could construct a graph of the distribution of all these sample means. For a reasonably large sample size such as $n = 100$, the resulting sampling distribution will be approximately normal with a mean that is equal to the population mean. In this case, the null hypothesis claims that the population mean is $\mu = \$39,000$, so the sampling distribution will peak at this value if the null hypothesis is correct.

Figure 1 shows the idea graphically. The red curve represents the sampling distribution under the assumption that the null hypothesis is true. Remember that this curve is what we would expect to find if we plotted the distribution of means from *many* samples. When we have only a single sample mean (such as the sample mean $\bar{x} = \$37,000$ for the Columbia College example), it represents just one point along this curve. If this point lies close to the peak of the curve, it tells us that the sample mean is near the population mean expected with the null hypothesis. In that case, the probability of finding such a sample mean is not small and there is no reason to reject the null hypothesis. In contrast, if the sample mean lies far from the population mean claimed by the null hypothesis, then the probability of finding such a sample mean is small if the null hypothesis is true. We would therefore conclude that the true population mean probably is *not* what the null hypothesis claims, in which case we have reason to reject the null hypothesis.

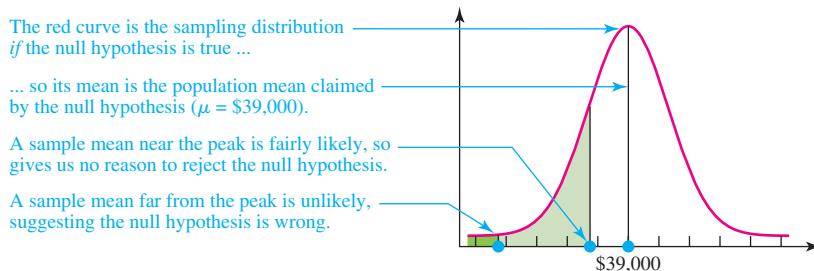


Figure 1 Graphical interpretation of the hypothesis test for the Columbia College example. If the null hypothesis is true, then the population mean is $\mu = \$39,000$. In that case, if we took many samples from the population, the distribution of sample means would be approximately normal with a mean of $\$39,000$. Under this assumption, the distance of a single sample mean from the population mean allows us to decide whether to reject or not reject the null hypothesis.

Finding the Standard Score

As illustrated in Figure 1, the decision of whether to reject the null hypothesis depends on whether the sample mean is “near” to or “far” from the population mean claimed by the null hypothesis. Because the sampling distribution is a (nearly) normal distribution, the *standard score* of the sample mean represents a quantitative measure of the distance between the sample mean and the claimed population mean. The standard score (or *z-score*) of a data value in a normal distribution is the number of standard deviations that it lies above or below the mean of the distribution. From the Central Limit Theorem, the standard deviation of a distribution of sample means is σ/\sqrt{n} , where σ is the population standard deviation and n is the sample size. Putting these ideas together, we find the following formula for the standard score of a sample mean \bar{x} :

$$z = \frac{\text{sample mean} - \text{population mean}}{\text{standard deviation of sampling distribution}} = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

The only remaining problem is that we generally do not know the population standard deviation, σ . For now, let’s assume that we can approximate the population standard deviation with the sample standard deviation, s , and that $s = \$6,150$ for the 100 salaries in the sample. In that case, we set $\sigma = \$6,150$ and the standard deviation for the distribution of sample means is

$$\frac{\sigma}{\sqrt{n}} = \frac{\$6,150}{\sqrt{100}} = \$615$$

Using this value in the previous equation tells us that the standard score for the sample mean of $\bar{x} = \$37,000$ in a sampling distribution with a population mean of $\mu = \$39,000$ is

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{\$37,000 - \$39,000}{\$615} = -3.25$$

In other words, the sample mean of $\bar{x} = \$37,000$ lies 3.25 standard deviations below the mean of the sampling distribution.

Figure 2 shows that the standard score of -3.25 places the sample result far out on the left tail of the distribution, indicating that it would be a rare sample result *if* the population mean really were the $\$39,000$ claimed by the null hypothesis. This suggests that the null hypothesis is incorrect and should be rejected. However, rather than rejecting the null hypothesis for this visual reason alone, let's look at a formal procedure for analyzing the standard score and making the hypothesis test decision.

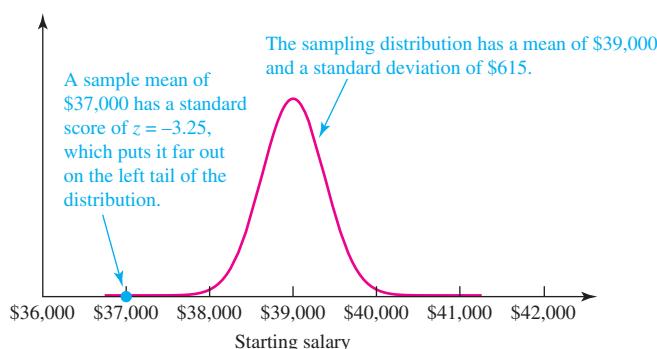


Figure 2 Assuming that the null hypothesis is true, the distribution of sample means for the Columbia College example has a mean of $\$39,000$ and a standard deviation of $\$615$. In that case, a sample mean of $\bar{x} = \$37,000$ has a standard score of -3.25 .

Computing the Standard Score for the Sample Mean in a Hypothesis Test

When we draw a random sample for a hypothesis test, we can consider it to be one of many possible samples in the sampling distribution. Given the sample size (n), the sample mean (\bar{x}), the population standard deviation (σ), and the claimed population mean (μ), we make the following computations:

$$\text{standard deviation for the distribution of sample means} = \frac{\sigma}{\sqrt{n}}$$

$$\text{standard score for the sample mean, } z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

Note: In reality, it is rare that we know the population standard deviation σ , so we often approximate it with the sample standard deviation s .

TECHNICAL NOTE

The standard score for the sample mean is often called the *test statistic*.

Critical Values for Statistical Significance

Recall that the hypothesis test is significant at the 0.05 level if the probability of finding a result as extreme as the one actually observed is 0.05 or less (assuming the null hypothesis is true). For a left-tailed test, then, we are looking for a standard score that is at or below the 5th percentile of the sampling distribution. The 5th percentile has a standard score between $z = -1.6$ and $z = -1.7$; a precise standard score table shows that the 5th percentile has a standard score of $z = -1.645$. Therefore, a left-tailed hypothesis test is significant at the 0.05 level if the standard score of the sample mean is less than or equal to $z = -1.645$. This standard score represents the **critical value** for significance at the 0.05 level in a left-tailed hypothesis test.

A similar argument applies to right-tailed tests with alternative hypotheses of the form $H_a: \mu > \text{claimed value}$. In such cases, significance at the 0.05 level requires that the sample mean lie at or above the 95th percentile—which requires a standard score greater than or equal

to $z = 1.645$. Figure 3 illustrates these critical values for both left- and right-tailed tests. To find critical values for significance at the 0.01 level, we look for the standard scores of the 1st and 99th percentiles (rather than the 5th and 95th); these are -2.33 and 2.33 , respectively.

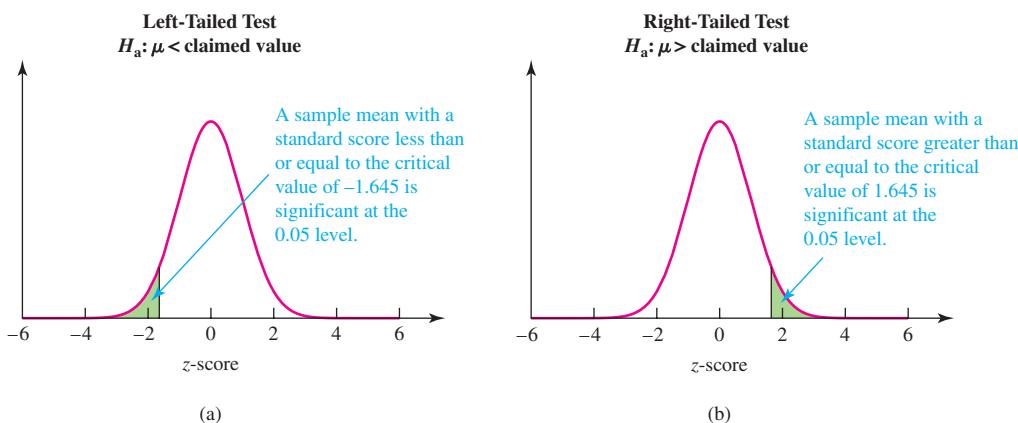


Figure 3 These graphs illustrate the meaning of critical values for one-tailed hypothesis tests when testing for significance at the 0.05 level. The locations of the critical values correspond to the 5th percentile for left-tailed tests and the 95th percentile for right-tailed tests.

Decisions Based on Statistical Significance for One-Tailed Hypothesis Tests

We decide whether to reject or not reject the null hypothesis by comparing the standard score (z) for a sample mean to critical values for significance at a given level. Table 1 summarizes the decisions for one-tailed hypothesis tests at the 0.05 and 0.01 levels of significance.

TABLE 1 Testing for Significance at the 0.05 and 0.01 Levels

Type of test	Form of H_a	For 0.05 level: Reject H_0 if standard score is	For 0.01 level: Reject H_0 if standard score is
Left-tailed test	$H_a: \mu < \text{claimed value}$	$z \leq -1.645$	$z \leq -2.33$
Right-tailed test	$H_a: \mu > \text{claimed value}$	$z \geq 1.645$	$z \geq 2.33$

TIME OUT TO THINK

Suppose that, in right-tailed tests, one study finds a sample mean with $z = 3$ and another study finds a sample mean with $z = 10$. Both are significant at the 0.01 level, but which result provides stronger evidence for rejecting the null hypothesis? Explain.

EXAMPLE 1 Columbia College Test Significance

Assuming that the null hypothesis is true and the mean starting salary for Columbia College graduates is \$39,000, is it statistically significant to find a sample in which the mean is only \$37,000? Based on your answer, should we reject or not reject the null hypothesis?

SOLUTION The hypothesis test is left-tailed (because the alternative hypothesis has the “less than” form of $\mu < \$39,000$) and we already found that a sample mean of $\bar{x} = \$37,000$ has a standard score of -3.25 . This result is significant at the 0.05 level because the standard score is less than the critical value of $z = -1.645$. In fact, it is also significant at the 0.01 level, because the standard score is less than -2.33 . We therefore have strong reason to reject the null hypothesis and conclude that Columbia College officials did indeed exaggerate the mean starting salary of its graduates.



Finding the P-Value

We can use P -values to be more precise about the significance of a result. Recall that the P -value is the probability of finding a sample mean as extreme (or more extreme) as the one found, under the assumption that the null hypothesis is true. For the hypothesis tests we discuss in this chapter, we generally find the P -value from the standard score of the sample mean. Figure 4 shows the idea. The total area under the curve for the distribution of sample means is 1, so we can interpret areas under the curve as probabilities. For a left-tailed test (Figure 4a), the probability of finding a sample mean less than or equal to some particular value is simply the area under the curve to the *left* of the sample mean. For a right-tailed test (Figure 4b), the probability is the area under the curve to the *right* of the sample mean.

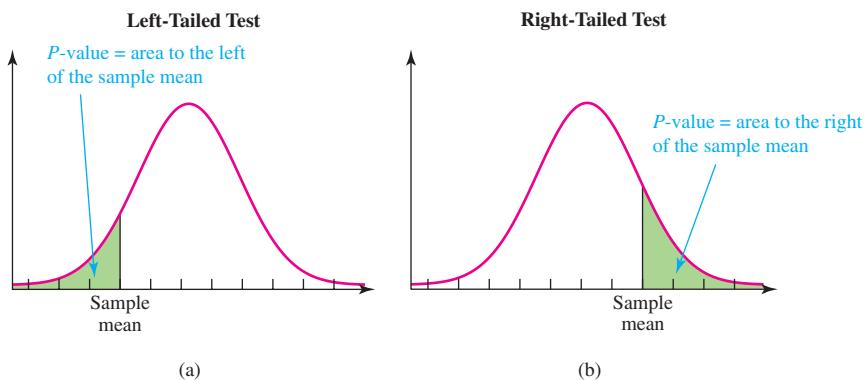


Figure 4 The P -value for one-tailed hypothesis tests corresponds to an area under the sampling distribution curve. Once we compute the standard score for a particular sample mean, we can find the corresponding area from standard score tables.

The P-value ... is often used as a gauge of the degree of contempt in which the null hypothesis deserves to be held.

—Robert Abelson, *Statistics as a Principled Argument*

As an example, let's find the P -value for the Columbia College hypothesis test. We have already determined that the sample mean ($\bar{x} = 37,000$) has a standard score of $z = -3.25$. This standard score corresponds to a "cumulative area from the left" of 0.0006—so this is the P -value we are seeking. This small P -value provides strong evidence against the null hypothesis. Be sure to notice that this P -value is less than 0.01, which is why the result is significant at the 0.01 level; the fact that it is much less than 0.01 tells us that we have very strong evidence for rejecting the null hypothesis.

Summary of One-Tailed Tests for Population Means

We began this section with the goal of learning how to carry out Steps 3 and 4 of the four-step hypothesis test process. We have now succeeded in doing this for one-tailed hypothesis tests for population means. To summarize:

- Because we are dealing with population means, the null hypothesis has the form $\mu = \text{claimed value}$. To decide whether to reject or not reject the null hypothesis, we must determine whether a sample as extreme as the one found in the hypothesis test is likely or unlikely to occur if the null hypothesis is true.
- We determine this likelihood from the standard score (z) of the sample mean, which we compute from the formula

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

where n is the sample size, \bar{x} is the sample mean, μ is the population mean claimed by the null hypothesis, and σ is the population standard deviation.

- We can then assess the standard score in two ways:
 1. We can assess its level of statistical significance by comparing it to the critical values given in Table 1.
 2. We can determine its P -value with standard score tables. For a left-tailed test, the P -value is the area under the normal curve to the left of the standard score;

for a right-tailed test, it is the area under the normal curve to the right of the standard score.

- If the result is statistically significant at the chosen level (usually either the 0.05 or the 0.01 significance level), we reject the null hypothesis. If it is not statistically significant, we do not reject the null hypothesis.

EXAMPLE 2 Mean Rental Car Mileage (Revisited)

Recall the case of the rental car fleet owner (Example 5 of Section 1) who suspects that the mean annual mileage of his cars is greater than the national mean of 12,000 miles. He selects a random sample of $n = 225$ cars and calculates the sample mean to be $\bar{x} = 12,375$ miles and the sample standard deviation to be $s = 2,415$ miles. Determine the level of statistical significance and P -value for this hypothesis test, and interpret your findings.

SOLUTION To determine the significance and P -value we must first calculate the standard score for the sample mean of $\bar{x} = 12,375$ miles. We are given the claimed population mean ($\mu = 12,000$ miles) and the sample size ($n = 225$), we do not know the population standard deviation, σ , but we will assume it is the same as the sample standard deviation and set $\sigma = 2,415$ miles. We find

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{12,375 - 12,000}{2,415/\sqrt{225}} = 2.33$$

This standard score is greater than the critical value of $z = 1.645$ for significance at the 0.05 level and equal to the value of $z = 2.33$ for significance at the 0.01 level. The area to the left of a standard score of $z = 2.33$ is 0.9901; because this is a right-tailed test, the probability is the area to the right (see Figure 4b), which is $1 - 0.9901 = 0.0099$. Therefore, the P -value is 0.0099, which is very close to 0.01. We therefore have strong reason to reject the null hypothesis and conclude that the rental car fleet mean really is greater than the national mean. •••

Two-Tailed Tests

The same basic ideas apply to two-tailed hypothesis tests in which the alternative hypothesis has the “not equal to” form of $H_a: \mu \neq$ claimed value. However, the critical values and the calculations for P -values are slightly different for two-tailed tests.

As always, the hypothesis test is significant at the 0.05 level if the probability of finding a result as extreme as the one actually found is 0.05 or less. For one-tailed tests, a probability of 0.05 corresponds to standard scores at the 5th percentile for left-tailed tests and 95th percentile for right-tailed tests (see Figure 3). For two-tailed tests, however, a value “at least as extreme as the one actually found” can lie *either* on the left or on the right of the sampling distribution (Figure 5). A probability of 0.05, or 5%, therefore corresponds to standard scores either in the first 2.5% of the sampling distribution on the left or in the last 2.5% on the right. The 2.5th percentile corresponds to a standard score of -1.96 and the 97.5th

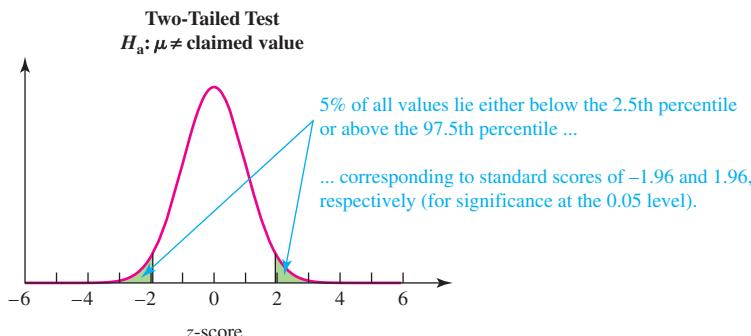


Figure 5 For two-tailed tests, the critical values for significance at the 0.05 level correspond to the 2.5th and 97.5th percentiles (as opposed to the 5th or 95th percentiles for one-tailed tests).

percentile corresponds to a standard score of 1.96. These standard scores become the critical values for two-tailed tests to be significant at the 0.05 level. Similarly, the two-tailed critical values for significance at the 0.01 level correspond to standard scores for the 0.5th and 99.5th percentiles; these values are -2.575 and 2.575 , respectively.

Similar considerations apply to P -values for two-tailed tests. Recall that for a one-tailed test, the P -value is the area under the curve to the left of the sample mean for a left-tailed test or to the right of the sample mean for a right-tailed test. Because a two-tailed test asks us to consider extremes on both sides of the claimed mean, the P -value for a two-tailed test must be twice what it would be if the test were one-tailed.

Two-Tailed Test ($H_a: \mu \neq \text{claimed value}$)

Statistical significance: A two-tailed test is significant at the 0.05 level if the standard score of the sample mean is at or below a critical value of -1.96 or at or above a critical value of 1.96 . For significance at the 0.01 level, the critical values are -2.575 and 2.575 .

P -value: To find the P -value for a sample mean in a two-tailed test, first use the standard score of the sample mean to find the P -value assuming the test is one-tailed; then *double* this value to find the P -value for the two-tailed test.

An example should help clarify these ideas. Consider a drug company that seeks to be sure that its “500-milligram” aspirin tablets really contain 500 milligrams of aspirin. If the tablets contain less than 500 milligrams, consumers are not getting the advertised dose. If the tablets contain more than 500 milligrams, consumers are getting too much of the drug. The null hypothesis says that the population mean of the aspirin content is 500 milligrams:

$$H_0: \mu = 500 \text{ milligrams}$$

The drug company is interested in the possibility that the mean weight is *either* less than *or* greater than 500 milligrams. Because the company is interested in possibilities on both sides of the claimed mean of 500 milligrams, we have a two-tailed test in which the alternative hypothesis is

$$H_a: \mu \neq 500 \text{ milligrams}$$

Suppose the company selects a random sample of $n = 100$ tablets and finds that they have a mean weight of $\bar{x} = 501.5$ milligrams; further suppose that the population standard deviation is $\sigma = 7.0$ milligrams. Then the standard score (z) for this sample mean is

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{501.5 - 500}{7/\sqrt{100}} = 2.14$$

This standard score is above the critical value of 1.96 for a two-tailed test, so it is significant at the 0.05 level. (It is not significant at the 0.01 level, because it is not above the critical value of 2.575.) This gives us good reason to reject the null hypothesis and conclude that the mean weight of the “500-milligram” aspirin tablets is different from 500 milligrams.

The area to the *left* of a standard score of $z = 2.14$ is 0.9838. Therefore, if this were a one-tailed test, the P -value would be the area to the *right*, or $1 - 0.9838 = 0.0162$. But because this is a two-tailed test, we *double* this number to find that the P -value is $2 \times 0.0162 = 0.0324$. In other words, *if* the null hypothesis is true, the probability of drawing a sample with a mean at least as extreme as the one found is 0.0324.

EXAMPLE 3 What Is Human Mean Body Temperature?

Consider the study in which University of Maryland researchers measured body temperatures in a sample of $n = 106$ healthy adults, finding a sample mean body temperature of $\bar{x} = 98.20^\circ\text{F}$ with a sample standard deviation of $s = 0.62^\circ\text{F}$. We will assume that the population standard deviation is the same as the sample standard deviation. Determine whether this sample provides enough evidence for rejecting the common belief that mean human body temperature is $\mu = 98.6^\circ\text{F}$.

SOLUTION The null hypothesis is the claim that mean human body temperature is 98.6°F, or $H_0: \mu = 98.6$. We are asked to test the alternative hypothesis that mean body temperature is *not* 98.6°F, or $H_a: \mu \neq 98.6$. The “not equal to” form of the alternative hypothesis tells us that we need a two-tailed test. We are given the sample size $n = 106$, the sample mean $\bar{x} = 98.2$, and the assumed population standard deviation $\sigma = 0.62$. The standard score of the sample mean is

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{98.20 - 98.60}{0.62/\sqrt{106}} = -6.64$$

This standard score is much less than the critical values of -1.96 for significance at the 0.05 level and -2.575 for significance at the 0.01 level (Figure 6). Because the result is significant at the 0.01 level, it provides strong evidence against the null hypothesis. We therefore reject the null hypothesis and conclude that human mean body temperature is *not* equal to 98.6°F. (In this case, the standard score of -6.64 is very extreme; a calculation shows that the P -value is about 3×10^{-11} .)

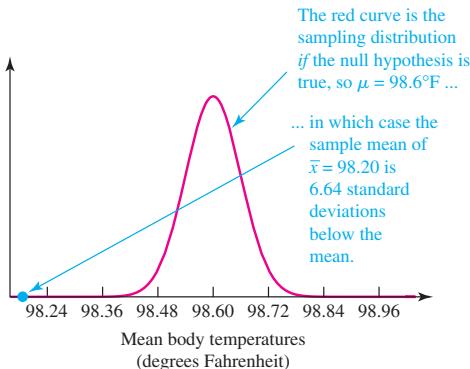


Figure 6 The null hypothesis states that human mean body temperature is 98.6°F. If this is true, then the distribution of sample means has a mean of 98.6°F. Based on the sample size and standard deviation, the sample mean of 98.20°F is more than 6 standard deviations below the claimed mean temperature—which makes it seem likely that the claim is wrong and human mean body temperature is *not* equal to 98.6°F.



Common Errors in Hypothesis Testing

We have now covered all the fundamentals of hypothesis testing, as well as specific methods used for hypothesis tests for population means. However, even if a hypothesis test is carried out correctly, two common types of error may affect the conclusions. To understand these errors, recall the legal analogy at the end of Section 1, in which the null hypothesis is H_0 : The defendant is innocent. One type of error occurs if we conclude that the defendant is guilty when, in reality, he or she is innocent. In this case, the null hypothesis (innocence) has been wrongly rejected. The other type of error occurs if we find the defendant *not guilty* when he or she actually is guilty. In this case, we have wrongly failed to reject the null hypothesis.

For a statistical example, consider again the pharmaceutical company testing the claim that the mean amount of aspirin in its tablets is 500 milligrams ($H_0: \mu = 500$ milligrams). In drawing a conclusion from the test, the company could make the following two types of errors:

- The company might reject the null hypothesis and conclude that the mean amount is not 500 milligrams when it really is. This error can lead to wasting time and money trying to fix a process that isn’t broken. An error of this type, in which H_0 is *wrongly rejected*, is called a **type I error**.
- The company might fail to reject the null hypothesis when, in fact, the mean amount of aspirin is not 500 milligrams. In this case, the company will distribute tablets that have too much or too little aspirin; consumers could suffer or sue. An error of this type, in which we *wrongly fail to reject* H_0 , is called a **type II error**.

USING TECHNOLOGY

HYPOTHESIS TEST FOR A POPULATION MEAN

Excel Use XLSTAT. First enter the list of original sample values in a list. Click on **XLSTAT** at the top, click on **Parametric tests**, then select **One sample t test and z test**. In the screen that appears, for the “Data” box enter the range of data, such as A1:A11 for 11 data values in column A. For “Data Format” select **One sample**. Click on the “z test” box. Click on the **Options** tab to select the type of test; select the option including \neq for a two-tailed test, select the option including $<$ for a left-tailed test, or select the option including $>$ for a right-tailed test. For the “Theoretical mean” box, enter the claimed value of the population mean, which is the same value used in the statement of the null hypothesis. Enter the desired “Significance level (%).” For example, enter 5 for a 0.05 significance level. Click **OK**. After the results are displayed, look for the test statistic identified as “z (Observed value).” The *P*-value will also be displayed. See the accompanying display that results from Example 3. See that the *P*-value in this display is so small that it is given as less than 0.0001.

XLSTAT

Difference	-0.4000
z (Observed value)	-6.6423
z (Critical value)	1.9600
p-value (Two-tailed)	< 0.0001
alpha	0.05

STATDISK If working with a list of the original sample values, first find the sample size, sample mean, and sample standard deviation by selecting **Analysis**, then **Descriptive Statistics**. After finding the values of n , \bar{x} , and s , select **Analysis** from the main menu, then select **Hypothesis Testing**, followed by **Mean-One Sample**. In the dialog box that appears, select the format of the claim being tested and proceed to enter the other required items, then click on **Evaluate**.

TI-83/84 PLUS If using a TI-83/84 Plus calculator, press **STAT**, then select **TESTS** and choose the menu item of **Z-Test**. You can use the original data (**Data**) or the summary statistics (**Stats**) by providing the entries indicated in the window display. The first three items of the TI-83/84 Plus calculator results will include the alternative hypothesis, the test statistic, and the *P*-value.

Table 2 summarizes the four possible cases.

		Decision Table for H_0 and H_a	
		Reality	
		H_0 true	H_a true
Decision	Reject H_0	Type I error	Correct decision
	Do not reject H_0	Correct decision	Type II error

If you think about it, you’ll realize that there is an important connection between the significance level of a hypothesis test and type I errors (wrongly rejecting H_0): *The significance level is the probability of making a type I error*. For example, if we conduct a hypothesis test using a 0.05 significance level, there is a 0.05 probability of making the mistake of rejecting the null hypothesis when it is actually true. If we conduct the test at a 0.01 significance level, there is a 0.01 probability of wrongly rejecting the null hypothesis. (The probability of a type II error can also be quantified but is beyond the scope of this text.)

EXAMPLE 4 Errors in the Body Temperature Test

Consider the null hypothesis from Example 3—that mean body temperature equals 98.6°F ($H_0: \mu = 98.6^\circ\text{F}$).

- What correct decisions are possible with this null hypothesis?
- Explain the meaning of type I and type II errors in this case.
- In Example 3, we rejected the null hypothesis. What is the probability that we made a type I error in doing so?

SOLUTION

- Any hypothesis test has two possible correct decisions. In this case, one correct decision occurs if the mean body temperature really is 98.6°F and we do *not* reject H_0 . The other correct decision occurs if the mean body temperature is *not* 98.6°F and we reject H_0 .
- A type I error occurs if we reject H_0 when it is actually true. In this case, a type I error occurs if mean body temperature really is 98.6°F but we conclude that it is not. A type II error occurs if we do not reject H_0 when it is actually false. In this case, a type II error occurs if the mean body temperature is not 98.6°F but we fail to reach this conclusion.
- We rejected the null hypothesis at a 0.01 significance level. Therefore, there is a 0.01 probability that we made a type I error of rejecting a null hypothesis that is actually true. •••

Bias in Choosing Hypotheses

As we have seen, a well-conducted hypothesis test follows a fairly strict process that tends to minimize the possibility of tainting the test with bias. Nevertheless, bias can be introduced in many ways, and in particular it can occur in the initial selection of the hypotheses. Consider a situation in which a factory is investigated for releasing pollutants into a nearby stream at a level that may (or may not) exceed the maximum level allowed by the government. The logical choice for the null hypothesis is that the actual mean level of pollutants equals the maximum allowed level:

$$H_0: \text{mean level of pollutants} = \text{maximum level allowed}$$

However, this leaves us with two reasonable choices for the alternative hypothesis, each of which introduces some bias into the ultimate conclusions.

If *factory officials* conducted the test, they would be inclined to claim that the mean level of pollutants is *less than* the maximum allowed level. That is, they would choose

$$H_a: \text{mean level of pollutants} < \text{maximum level allowed}$$

With this alternative hypothesis, the two possible outcomes are

- Reject H_0 , in which case we conclude that the mean level of pollutants is less than the allowed maximum. This outcome would please the factory officials.
- Not reject H_0 , in which case the test is inconclusive.

In other words, by choosing the left-tailed (“less than”) hypothesis test, the factory officials ensure that at worst we end up with no evidence that they have exceeded the allowed maximum, while it’s possible that the conclusion will be in their favor.

Now suppose an *environmental group* conducts the test. Because the group suspects that the factory is violating the government standards, its choice for the alternative hypothesis would be that the mean level of pollutants is *greater than* the maximum allowed level, or

$$H_a: \text{mean level of pollutants} > \text{maximum level allowed}$$

With this choice of alternative hypothesis, rejection of H_0 implies that the factory has violated the standards, while not rejecting H_0 is again inconclusive. In other words, choosing the right-tailed (“greater than”) test creates a situation in which the factory may be found in violation but cannot be proved to be in compliance.

EXAMPLE 5 Mining Gold

The success of precious metal mines depends on the purity (or grade) of ore removed and the market price for the metal. Suppose the purity of gold ore must be at least 0.5 ounce of gold per ton of ore in order to keep a particular mine open. Samples of gold ore are used to estimate the purity of the ore for the entire mine. Discuss the impact of type I and type II errors on two of the possible alternative hypotheses:

$$H_a: \text{purity} < 0.5 \text{ ounce per ton}$$

$$H_a: \text{purity} > 0.5 \text{ ounce per ton}$$



SOLUTION For the left-tailed case, the null and alternative hypotheses are

$$H_0: \text{purity} = 0.5 \text{ ounce per ton}$$

$$H_a: \text{purity} < 0.5 \text{ ounce per ton}$$

The two possible decisions in this case are

- Reject H_0 , which means concluding that the purity of the ore is less than that needed to keep the mine open—so the mine is closed.
- Not reject H_0 , which means we have insufficient evidence to conclude that the purity of the ore is less than that needed—so the mine stays open.

A type I error (wrongly rejecting a true H_0) means the mine is closed when, in fact, the purity of the gold ore is sufficient to operate the mine. For the mine's operators, this means the loss of potential profits from the mine; for the employees, it means unnecessary loss of jobs. A type II error (failing to reject a false H_0) means that the mine continues to operate but is actually unprofitable.

Now suppose that the mine will be kept open only if the purity is greater than 0.5 ounce per ton. For this right-tailed case, the null and alternative hypotheses are

$$H_0: \text{purity} = 0.5 \text{ ounce per ton}$$

$$H_a: \text{purity} > 0.5 \text{ ounce per ton}$$

The two possible decisions are

- Reject H_0 , which means concluding that the purity of the ore is *more* than that needed to keep the mine open—so the mine stays open.
- Not reject H_0 , which means there is insufficient evidence to conclude that the purity of the ore is high enough to keep the mine open—so the mine is closed.

A type I error (wrongly rejecting a true H_0) means the mine is left open when it is actually unprofitable. A type II error (failing to reject a false H_0) means closing the mine when it is actually profitable—putting the employees out of work for no reason.

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Section 2 Exercises

Statistical Literacy and Critical Thinking

1. Notation. Briefly describe what each of the variables n , \bar{x} , s , σ , and μ represent in hypothesis tests of a claim made about a population mean.

2. Waiting Times. A student in a statistics class conducts a project designed to test the claim that wait times at McDonald's drive-up windows have a mean that is less than 5 minutes. She collects sample data from wait times at five randomly selected McDonald's locations that are within 30 miles of her home. What is wrong with this project?

3. Types of Errors. What is a type I error? What is a type II error?

4. Statistical Significance. Consider a test of the claim that with a new device installed, the mean fuel consumption of cars is reduced by 0.08 miles per gallon. The resulting P -value is 0.009. Is the mean fuel consumption reduction statistically significant? Does the mean fuel consumption reduction of 0.08 miles per gallon have practical significance? Why or why not?

Does It Make Sense? For Exercises 5–12, decide whether the statement makes sense (or is clearly true) or does not make sense

(or is clearly false). Explain clearly. Not all of these statements have definitive answers, so your explanation is more important than your chosen answer.

5. Hypothesis Test. In hypothesis tests, the P -value is always the same as the significance level.

6. P-Value. In testing a claim about a population mean, a larger z test statistic always results in a larger P -value.

7. P-Value. In a hypothesis test, a P -value of 0.001 indicates that you should support the alternative hypothesis.

8. Alternative Hypothesis. In testing the claim that the mean IQ score of statistics students is greater than 100, the alternative hypothesis is expressed as $\mu > 100$.

9. Significance. A significance level of 0.05 indicates that the probability of making a type I error is 0.05.

10. Type I Error. When a consumer group is testing the claim that the mean amount of aspirin in tablets is 350 milligrams, it is extremely important not to reject a true null hypothesis wrongly. Thus, it is better to choose 0.01 than 0.05 for the significance level.

11. Significance Level. Because the significance level is the probability of making a type I error, it is wise to select a significance level of zero so that there is no probability of making that error.

12. Mnemonic. A handy mnemonic for interpreting the P -value in a hypothesis test is this: “If the P (value) is low, then the null must go.”

Concepts and Applications

Alternative Hypothesis Supported? In Exercises 13–20, find the value of the standard score, z , and determine whether the alternative hypothesis is supported at a 0.05 significance level.

13. $H_a: \mu < 75, n = 100, \bar{x} = 70, \sigma = 15$.

14. $H_a: \mu < 75, n = 36, \bar{x} = 72, \sigma = 15$.

15. $H_a: \mu > 12, n = 64, \bar{x} = 14, \sigma = 2$.

16. $H_a: \mu > 1007, n = 225, \bar{x} = 1021, \sigma = 35$.

17. $H_a: \mu \neq 2.55, n = 100, \bar{x} = 2.58, \sigma = 0.29$.

18. $H_a: \mu \neq 156.2, n = 225, \bar{x} = 155.5, \sigma = 29$.

19. $H_a: \mu \neq 0.88, n = 50, \bar{x} = 0.75, \sigma = 0.18$.

20. $H_a: \mu \neq 877, n = 90, \bar{x} = 921, \sigma = 52$.

Finding P -Values. In Exercises 21–34, use a standard score table to find the P -value that corresponds to the standard z -score, and determine whether the alternative hypothesis is supported at the 0.05 significance level.

21. $z = -0.4$ for $H_a: \mu < 25$

22. $z = -2.4$ for $H_a: \mu < 727$

23. $z = 1.9$ for $H_a: \mu > 36.35$

24. $z = 1.5$ for $H_a: \mu > 0.227$

25. $z = -1.6$ for $H_a: \mu \neq 0.389$

26. $z = -2.0$ for $H_a: \mu \neq 172$

27. $z = 1.7$ for $H_a: \mu \neq 75$

28. $z = 1.9$ for $H_a: \mu \neq 25.7$

29. $z = 2.7$ for $H_a: \mu > 19.4$

30. $z = 3.5$ for $H_a: \mu > 75$

31. $z = -2.1$ for $H_a: \mu < 1,007$

32. $z = -1.1$ for $H_a: \mu < 149.6$

33. $z = 0.15$ for $H_a: \mu \neq 90.3$

34. $z = 3.5$ for $H_a: \mu \neq 1,022$

35. Interpreting P -Values. Assume that you are testing an alternative hypothesis of the form $H_a: \mu >$ claimed value. If the sample mean has a standard score of $z = -1.0$, what

do you conclude? Why is it not necessary to actually conduct the formal hypothesis test?

36. Interpreting P -Values. Assume that you are testing an alternative hypothesis of the form $H_a: \mu <$ claimed value. If the sample mean has a standard score of $z = 0.5$, what do you conclude? Why is it not necessary to actually conduct the formal hypothesis test?

Hypothesis Tests for Means. For Exercises 37–50, use a 0.05 significance level and conduct a full hypothesis test using the four-step process described in the text. Be sure to state your conclusion.

37. Roper Poll. A Roper poll used a sample of 100 randomly selected car owners. Within the sample, the mean time of ownership for a single car was 7.01 years with a standard deviation of 3.74 years. Test the claim by the owner of a large dealership that the mean time of ownership for all cars is less than 7.5 years.

38. Hospital Time. According to a study by the Centers for Disease Control, the national mean hospital stay after childbirth is 2.0 days. Reviewing records at her own hospital, a hospital administrator calculates that the mean hospital stay for a sample of 81 women after childbirth is 2.2 days with a standard deviation of 1.2 days. Assuming that the patients represent a random sample of the population, test the claim that this hospital keeps new mothers longer than the national average.

39. Cola. A random sample of 36 cans of regular Coke is obtained and the contents are measured. The sample mean is 12.19 oz and the standard deviation is 0.11 oz. Test the claim that the contents of all such cans have a mean different from 12.00 oz, as indicated by the label.

40. Motorcycle Manufacturing. The manufacturers of motorcycles must produce axles that meet specified dimensions. In particular, the diameters of the axles must be 8.50 centimeters. The axles in a sample of $n = 64$ axles have a mean diameter of 8.56 centimeters with a standard deviation of 0.24 centimeter. Test the claim that the axles actually have the specified mean diameter.

41. Drug Amounts. The cold medicine Dozenol lists 600 milligrams of acetaminophen per fluid ounce as an active ingredient. The Food and Drug Administration tests 65 one-ounce samples of the medicine and finds that the mean amount of acetaminophen for the sample is 589 milligrams with a standard deviation of 21 milligrams. Test the claim of the FDA that the medicine does not contain the required amount of acetaminophen.

42. Red Blood Cell Count. A simple random sample of 50 adults is obtained, and each person’s red blood cell count (in cells per microliter) is measured. The sample mean is 5.23. The population standard deviation for red blood cell counts is 0.54. Test the claim that the sample is from a population with a mean less than 5.4, which is a value often used for the upper limit of the range of normal values. What do the results suggest about the sample group?

43. Fuel Consumption. According to the Energy Information Administration (Federal Highway Administration data), the average gas mileage of all automobiles is 21.4 miles per gallon. For a random sample of 40 sport utility vehicles (SUVs), the mean gas mileage is 19.8 miles per gallon with a standard deviation of 3.5 miles per gallon. Test the claim that the mean mileage of all SUVs is less than 21.4 miles per gallon.

44. Baseballs. A random sample of 40 new baseballs is obtained. Each ball is dropped onto a concrete surface, and the bounce heights have a mean of 92.67 inches and a standard deviation of 1.79 inches (based on data from *USA Today*). Test the claim that the new baseballs have a mean bounce height that is less than the mean bounce height of 92.84 inches found for older baseballs.

45. Compulsive Buyers. Researchers developed a questionnaire to identify compulsive buyers. A random sample of 32 subjects who identified themselves as compulsive buyers was obtained, and they had a mean questionnaire score of 0.83 with a standard deviation of 0.24 (based on data from “A Clinical Screener for Compulsive Buying,” by Faber and Guinn, *Journal of Consumer Research*, Vol. 19). Test the claim that the population of self-identified compulsive buyers has a mean greater than the mean of 0.21 for the general population.

46. Coin Weight. According to the U.S. Department of the Treasury, the mean weight of a quarter is 5.670 grams. A random sample of 50 quarters has a mean weight of 5.622 grams with a standard deviation of 0.068 gram. Test the claim that the mean weight of quarters in circulation is 5.670 grams.

47. Birth Weight. The mean birth weight of male babies born to 121 mothers taking a vitamin supplement is 3.67 kilograms with a standard deviation of 0.66 kilogram (based on data from the New York State Department of Health). Test the claim that the mean birth weight of all babies born to mothers taking the vitamin supplement is equal to 3.39 kilograms, which is the mean for the population of all male babies.

48. Weights of Bears. The health of the bear population in Yellowstone National Park is monitored by periodic measurements taken from anesthetized bears. A sample of 54 bears has a mean weight of 182.9 lb. Assuming that σ is known to be 121.8 lb, test the claim that the population mean of all such bear weights is greater than 150 lb.

49. Convicted Embezzlers. When 70 convicted embezzlers were randomly selected, the mean length of prison terms was found to be 22.1 months and the standard deviation was 8.6 months (based on data from the U.S. Department of Justice). Jane Fleming is running for political office on a platform of tougher treatment of convicted criminals. Test her claim that prison terms for convicted embezzlers have a mean of less than 24 months.

50. NCAA Football Coach Salaries. A simple random sample of 40 salaries of NCAA football coaches in the NCAA has a mean of \$415,953. The standard deviation of all salaries

of NCAA football coaches is \$463,364. Test the claim that the mean salary of a football coach in the NCAA is less than \$500,000.

Type I and Type II Errors. In Exercises 51–54, a null and alternative hypothesis are given. Without using the terms “null hypothesis” and “alternative hypothesis,” identify the type I error and identify the type II error.

51. H_0 : The patient is free of a particular disease.

H_a : The patient has the disease.

52. H_0 : The defendant is not guilty.

H_a : The defendant is guilty.

53. H_0 : The lottery is fair.

H_a : The lottery is biased.

54. H_0 : The mean length of a bolt in the suspension system of new Audi cars is 3.456 centimeters.

H_a : The mean length of a bolt in the suspension system of new Audi cars is not equal to 3.456 centimeters.



PROJECTS FOR THE INTERNET & BEYOND

55. Comparisons with National Averages. Choose several variables that are relatively easy to measure in a class or sample of students. The variables should involve a quantity that can be averaged (for example, height, weight, family size, blood pressure, heart rate, reaction time). Use the Internet or other references to determine national averages (means) for these variables (by age categories, if appropriate). Collect data on the variables, using a random sample of at least 50 individuals. Carry out the relevant hypothesis test to determine whether the sample mean differs significantly from the population mean.

56. County Data. The *Statistical Abstract of the United States* and the Current Population Survey provide an extensive supply of social, economic, and vital statistics at the county, state, and local levels. Use their Web sites to compare state data to national data in the following way.

a. Choose a variable of interest that involves a mean for a particular state (for example, the mean household size in Illinois).

b. Find the current national value for that variable (for example, the national mean household size).

c. Choose a particular county within the state and obtain the corresponding data for the county, as well as the sample size (for example, the mean household size in Cook County, Illinois).

d. Assuming that the county is a random sample of the state, test the claim that the state is above or below the national level in terms of that variable.

e. Discuss and interpret your results. The hypothesis test depends on the sample (the county) being a random sample of the population (the state). Be sure to discuss this factor in your conclusions.

- 57. Hypothesis Test Applet.** Using a search engine such as Google, search for “hypothesis testing” and “applet.” Find an applet and run it. Describe how the applet works and what it illustrates.

- 58. Power of a Test.** Using a search engine such as Google, search for “power” of a hypothesis test. Describe what the power of a hypothesis test is.

3**HYPOTHESIS TESTS FOR POPULATION PROPORTIONS**

We now consider hypothesis testing with *proportions*. All the ideas from previous sections apply, except we need a different method for calculating the standard deviation of the sampling distribution. An example will illustrate the process.

Suppose a political candidate commissions a poll in advance of a close election. Using a random sample of $n = 400$ likely voters, the poll finds that 204 people support the candidate. Should the candidate be confident of winning? We now cast the question as a hypothesis test.

For a hypothesis test, we ask whether the poll results (which are the sample statistics) support the hypothesis that the candidate has more than 50% of the vote. As usual, we let p represent the proportion of people in the voting *population* who favor the candidate, and we let \hat{p} denote the proportion of people in the *sample* who favor the candidate. Because 204 of the 400 people in the sample support the candidate, the sample proportion is

$$\hat{p} = \frac{204}{400} = 0.510$$

We can now formulate the null and alternative hypotheses. As usual, we set up our null hypothesis as an equality:

$$H_0: p = 0.5 \text{ (50\% of voters favor the candidate)}$$

The candidate wants to know if she has majority support, so the alternative hypothesis is right-tailed:

$$H_a: p > 0.5 \text{ (more than 50\% of voters favor the candidate)}$$

Calculations for Hypothesis Tests with Proportions

How do we determine whether there is enough evidence in the sample to reject the null hypothesis? Following the hypothesis test process, we see that we have completed the first two steps (formulating the hypotheses and collecting the sample data). Now, just as we did for sample means in Section 2, we must determine the likelihood that the sample result could have arisen by chance assuming that the null hypothesis is true.

Proceeding as we did with sample means, we imagine selecting many samples of size $n = 400$. For each sample, we compute the proportion of people who favor the candidate. Because we have a reasonably large sample size, the distribution of sample proportions should be very close to a normal distribution. Under the starting assumption that the null hypothesis is true (that the proportion of people in the population who favor the candidate is 0.5), the peak of this distribution will be the population proportion claimed by the null hypothesis, $p = 0.5$. The standard deviation of the sampling distribution is given by the following formula (the derivation is beyond the scope of this text):

$$\text{standard deviation of distribution of sample proportions} = \sqrt{\frac{p(1 - p)}{n}}$$

For this case, the standard deviation is

$$\sqrt{\frac{p(1 - p)}{n}} = \sqrt{\frac{0.5(1 - 0.5)}{400}} = 0.025$$

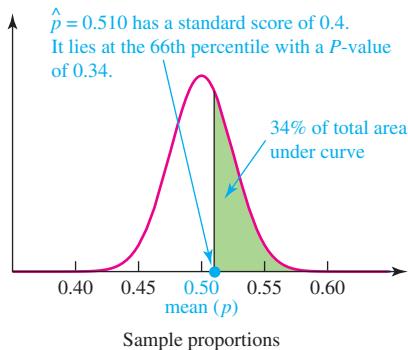


Figure 7 The distribution of sample proportions for an election poll with a mean of $p = 0.5$ and a standard deviation of 0.025. The sample proportion $\hat{p} = 0.510$ has a standard score of 0.4 and a P -value of 0.34.

Figure 7 shows the distribution of sample proportions. We consider the sample proportion from the poll ($\hat{p} = 0.510$) as a single point in this sampling distribution. Much as in hypothesis tests with means (see Figure 1), we observe the following:

- If the sample result is close to the peak of the sampling distribution, then we have no reason to think the null hypothesis is wrong and we do not reject the null hypothesis.
- If the sample result is far from the peak of the sampling distribution, then the more likely explanation is that the sampling distribution does not really peak where the null hypothesis claims, in which case we reject the null hypothesis.

Again as we did for sample means, we decide whether the sample proportion is “near” to or “far” from the peak of the sampling distribution (assuming the null hypothesis is true) by quantifying the distance with a standard score. The formula for the standard score (z) always has the same general form; for the distribution of sample proportions, the formula becomes

$$z = \frac{\text{sample proportion} - \text{population proportion}}{\text{standard deviation of sampling distribution}} = \frac{\hat{p} - p}{\sqrt{p(1-p)/n}}$$

We can now compute and interpret the standard score for our current example. The sample size is the $n = 400$ people interviewed, the sample proportion is the 51% of the sample that supports the candidate ($\hat{p} = 0.510$), and the population proportion is the $p = 0.5$ claimed by the null hypothesis. Therefore, the standard score for the sample proportion is

$$z = \frac{\hat{p} - p}{\sqrt{p(1-p)/n}} = \frac{0.510 - 0.5}{\sqrt{0.5(1-0.5)/400}} = 0.4$$

In other words, the sample used in the poll has a proportion that is 0.4 standard deviation away from the peak of the distribution of sample proportions. As you can see in Figure 7, this sample result is *not* very extreme, indicating that we should not reject the null hypothesis. That is, the candidate cannot be confident of majority support.

BY THE WAY

In the 1948 presidential election, most pollsters and newspapers predicted a large victory for the Republican Dewey over the Democrat Truman. In fact, Truman won by a popular vote margin of 49.5% to 45.1%.

Standard Score for the Sample Proportion in a Hypothesis Test

Given the sample size (n), the sample proportion (\hat{p}), and the claimed population proportion (p), the standard score for the sample proportion is

$$z = \frac{\hat{p} - p}{\sqrt{p(1-p)/n}}$$

Significance Levels and *P*-Values

We can be more quantitative by using a significance level or *P*-value. Let's start with significance. Because the election poll example uses a right-tailed test, we find the critical values for significance in Table 1. The standard score of $z = 0.4$ is *not* greater than the critical value of $z = 1.645$ for significance at the 0.05 level, confirming that the result should not cause us to reject the null hypothesis. The area to the *left* of a standard score of $z = 0.4$ is 0.6554. Because this is a right-tailed test, we subtract this value from 1 to find the area to the right (shown in Figure 7), which is the *P*-value for this hypothesis test. That is, the *P*-value is $1 - 0.6554 = 0.3446$, telling us that if the null hypothesis is true, there is a more than 0.34 chance of randomly selecting a sample as extreme as the one found in this poll. With such a high probability of drawing such a sample by chance, we have no reason to reject the null hypothesis, so the candidate cannot assume that she has the support of more than 50% of voters.

Summary of Hypothesis Tests with Proportions

We can now summarize the procedure for doing a hypothesis test with a population proportion. Be sure to notice that we are still following the general four-step hypothesis test process, and here focus only on the specifics of dealing with proportions.

- Because we are dealing with population proportions, the null hypothesis has the form $p = \text{claimed value}$. To decide whether to reject or not reject the null hypothesis, we must determine whether a sample as extreme as the one found in the hypothesis test is likely or unlikely to occur if the null hypothesis is true.
- We determine this likelihood from the standard score (z) of the sample proportion, which we compute from the formula

$$z = \frac{\hat{p} - p}{\sqrt{p(1-p)/n}}$$

where n is the sample size, \hat{p} is the sample proportion, and p is the population proportion claimed by the null hypothesis.

- We then use the standard score in the same two ways it was used for hypothesis tests with means:
 - We can assess its level of statistical significance by comparing the standard score to the critical values given in Table 1 for one-tailed tests and in the box in Section 2 for two-tailed tests.
 - We can determine its *P*-value with standard score tables. For a left-tailed test, the *P*-value is the area under the normal curve to the left of the standard score; for a right-tailed test, it is the area under the normal curve to the right of the standard score; and for a two-tailed test, it is double the value we would find if we were calculating the *P*-value for a one-tailed test.
- If the result is statistically significant at the chosen level (usually either the 0.05 or the 0.01 significance level), we reject the null hypothesis. If it is not statistically significant, we do not reject the null hypothesis.

EXAMPLE 1 Local Unemployment Rate

Suppose the national unemployment rate is 9.5%. In a survey of $n = 450$ people in a rural Wisconsin county, 54 people are found to be unemployed. County officials apply for state aid based on the claim that the local unemployment rate is higher than the national average. Test this claim at a 0.05 significance level.



USING TECHNOLOGY

HYPOTHESIS TEST FOR A POPULATION PROPORTION

EXCEL Use XLSTAT. Click on **XLSTAT** at the top, click on **Parametric Tests**, then select **Tests for one proportion**. In the screen that appears, enter the sample proportion in the "Proportion" box, enter the sample size in the "Sample size" box, and enter the claimed value of the population proportion in the "Test proportion" box. (This is the same proportion used in the null hypothesis.) Select the "Data Format" of **Proportion**, and be sure that the box next to "z test" is checked. For the "Range" box, enter A1 so that the results will start at cell A1. Click on the **Options** tab to select the type of test; select the option including \neq for a two-tailed test, select the option including $<$ for a left-tailed test, or select the option including $>$ for a right-tailed test. Enter the desired "Significance level (%)." For example, enter 5 for a 0.05 significance level. Click **OK**. After the results are displayed, look for the test statistic identified as "z (Observed value)" and the *P*-value. Critical values will also be displayed.

STATDISK Select **Analysis**, then select **Hypothesis Testing**. For the methods discussed in this section, select **Proportion–One Sample**. A dialog box will appear. Click on the box in the upper left corner and select the item that matches the claim being tested. Proceed to enter the other items in the dialog box, then click on **Evaluate**. The results will include the test statistic and *P*-value.

TI-83/84 PLUS Press **STAT**, select **TESTS**, and then select **1-PropZTest**. Enter the claimed value of the population proportion for p_0 , then enter the values for x and n , and then select the type of test. Highlight **Calculate**, then press **ENTER**.

SOLUTION

Let's follow the original four-step process.

- Step 1. The unemployment rate is a population proportion (as opposed to a population mean). The null hypothesis is the assumption that the local unemployment rate is equal to the national rate, or $H_0: p = 0.095$. The alternative hypothesis is the county's claim that the local unemployment rate is higher than the national average, or $H_a: p > 0.095$. This is a right-tailed test.
- Step 2. The sample statistics are the sample size, $n = 450$, and sample proportion, \hat{p} . From the given data, the sample proportion is

$$\hat{p} = \frac{54}{450} = 0.12$$

- Step 3. We now determine the likelihood that, *under the assumption that the null hypothesis is true*, chance alone would yield a sample proportion at least as extreme as the $\hat{p} = 0.12$ found for this particular sample. To find this likelihood, we start by calculating the standard score for this sample proportion:

$$z = \frac{\hat{p} - p}{\sqrt{p(1-p)/n}} = \frac{0.12 - 0.095}{\sqrt{0.095(1-0.095)/450}} \approx 1.81$$

This standard score is above the critical value of $z = 1.645$ for a right-tailed test, so the result is significant at the 0.05 level. The standard score of 1.81 has a *P*-value of $1 - 0.9649 = 0.0351$, confirming that there is a smaller than 0.05 probability that this sample would arise by chance if the null hypothesis is true. Figure 8 shows these ideas on the sampling distribution.

- Step 4. Because the test meets the criterion for significance at the 0.05 level, we reject the null hypothesis. In other words, this sample provides some evidence to support the alternative hypothesis that the county unemployment rate is above the national average.

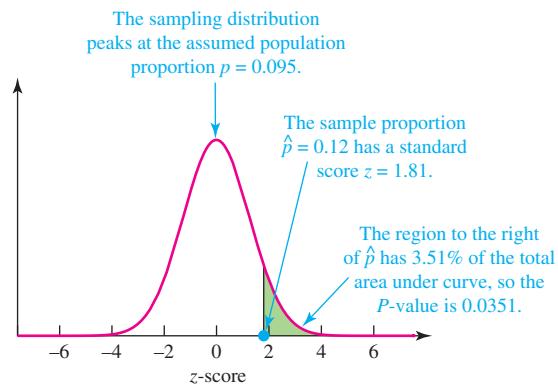


Figure 8 The distribution of sample proportions for Example 1. The sample proportion has a standard score of 1.81, which is above the critical value $z = 1.645$ for significance at the 0.05 level.

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EXAMPLE 2 Left-Handed Population

A random sample of $n = 750$ people is selected, of whom 92 are left-handed. Use these sample data to test the claim that 10% of the population is left-handed.

SOLUTION We again follow the four-step process.

- Step 1. The claim concerns the *proportion* of the population that is left-handed, so this is a test with a population proportion. The null hypothesis is the claim that 10% of the population is left-handed, or $H_0: p = 0.1$. To test this claim, we need to account for the possibility that the actual population proportion is either less than *or* greater than 10%. Therefore, the alternative hypothesis is $H_a: p \neq 0.1$, which calls for a two-tailed test.
- Step 2. The sample statistics are the sample size, $n = 750$, and the proportion of left-handed people in the sample:

$$\hat{p} = \frac{92}{750} = 0.123$$

- Step 3. The standard score for this sample proportion ($\hat{p} = 0.123$) is

$$z = \frac{\hat{p} - p}{\sqrt{p(1-p)/n}} = \frac{0.123 - 0.1}{\sqrt{0.1(1-0.1)/750}} = 2.1$$

From the box in Section 2, the critical values for significance at the 0.05 level in a two-tailed test are standard scores less than -1.96 or greater than 1.96 . The standard score of 2.1 for this test is greater than 1.96, so we conclude that the test is significant at the 0.05 level. The area to the right of a standard score of 2.1 is $1 - 0.9821 = 0.0179$. This would be the *P*-value if we were conducting a one-tailed test, but because we have a two-tailed test, we *double* it to find that the *P*-value is $2 \times 0.0179 = 0.0358$. Figure 9 shows the meaning of this *P*-value on the sampling distribution.

- Step 4. Because the test is significant at the 0.05 level, we reject the null hypothesis and conclude that the proportion of the population that is left-handed is *not* equal to 10%. Remembering that the significance level is the probability of a type I error, we recognize that there is a 0.05 probability that we made a type I error of rejecting a hypothesis that is actually true.

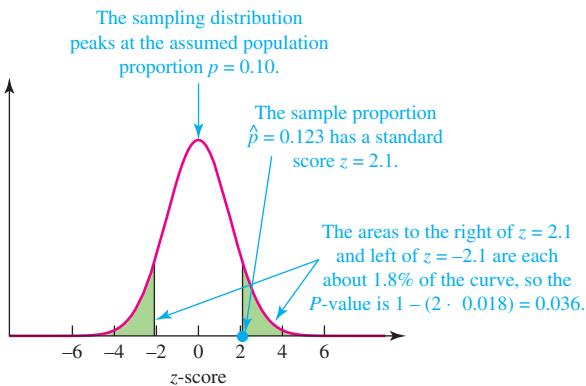


Figure 9 This graph shows the position of the sample proportion ($\hat{p} = 0.123$) on the distribution of sample proportions for Example 2. Because this is a two-tailed test, the P -value corresponds to the area under the curve more than 2.1 standard deviations from the peak in either direction.

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Section 3 Exercises

Statistical Literacy and Critical Thinking

- Notation.** What do p , \hat{p} , and P -value represent?
- Distribution.** In conducting a hypothesis test as described in this section, which term best describes the particular distribution that is used: uniform, normal, probability, weighted?
- Scream.** A survey of 61,647 people included several questions about office relationships. Of the respondents, 26% reported that bosses scream at employees. The survey is an *Elle/MSNBC.COM* survey in which Internet users chose whether to respond. Is it valid to use the sample results for testing the claim that less than 50% of employees have their bosses scream at them? Why or why not?
- P -Value.** A P -value of 0.00001 is obtained when sample data are used to test the claim that the majority of car crashes occur within 5 miles of home. What does this P -value tell us?

Does It Make Sense? For Exercises 5–8, decide whether the statement makes sense (or is clearly true) or does not make sense (or is clearly false). Explain clearly. Not all of these statements have definitive answers, so your explanation is more important than your chosen answer.

- Null Hypothesis.** In a test of the claim that a majority of Americans favor registration of all handguns, the null hypothesis is $p = 0.5$ and the alternative hypothesis is $p > 0.5$.
- Null and Alternative Hypotheses.** In a hypothesis test involving a claim made about a population proportion, if there is sufficient evidence to reject the null hypothesis, then there is sufficient evidence to support the alternative hypothesis.
- Alternative Hypothesis.** In a two-tailed hypothesis test of a claim about a proportion, the P -value is the area to the right of the standard score, z .

- Alternative Hypothesis.** The claim of $p < 0.25$ can never be supported if the sample proportion \hat{p} is greater than 0.25.

Concepts and Applications

Hypothesis Tests. For Exercises 9–18, use a 0.05 significance level to conduct a hypothesis test using the four-step procedure described in the text. Be sure to state your conclusion.

- Voter Poll.** In a pre-election poll, a candidate for district attorney receives 205 of 400 votes. Assuming that the people polled represent a random sample of the voting population, test the claim that a majority of voters support the candidate.
- Smoking and College Education.** A survey showed that among 785 randomly selected subjects who completed four years of college, 144 smoke and the others do not smoke (based on data from the American Medical Association). Test the claim that the rate of smoking among those with four years of college is less than the 27% rate for the general population.
- Grade Pressure.** A study commissioned by the U.S. Department of Education, based on responses from 1,015 randomly selected teenagers, concluded that 44% of teenagers cite grades as their greatest source of pressure. Test the claim that fewer than half of all teenagers in the population feel that grades are their greatest source of pressure.

- 12. Married Adults.** In a recent year, 125.8 million adults, or 58.6% of the adult American population, were married. In a New England town, a simple random sample of 1,445 adults includes 56.0% who are married. Test the claim that this sample comes from a population with a married percentage of less than 58.6%.
- 13. Drug Use.** A Department of Health and Human Services study of illegal drug use among 12- to 17-year-olds reported a decrease in use (from 11.4% in 1997) to 9.9% now. Suppose a survey in a large high school reveals that, in a random sample of 1,050 students, 98 report using illegal drugs. Test the principal's claim that illegal drug use in her school is below the current national average.
- 14. Poverty.** According to recent estimates, 12.1% of the 4,342 people in Custer County, Idaho, live in poverty. Assume that the people in this county represent a random sample of all people in Idaho. Based on this sample, test the claim that the poverty rate in Idaho is less than the national rate of 13.3%.
- 15. Abortion Survey.** An annual survey of first-year college students, conducted by the Higher Education Research Institute at UCLA, asks approximately 276,000 students about their attitudes on a variety of subjects. According to a recent survey, 51% of first-year students believe that abortion should be legal (down from 65% in 1990). Test the claim that more than half of all first-year students believe that abortion should be legal.
- 16. Internet Use.** When 3011 adults were surveyed in a Pew Research Center poll, 2198 said that they use the Internet. Is it OK for a newspaper reporter to write that "3/4 of all adults use the Internet"? Why or why not?
- 17. Natural Gas Use.** According to the Energy Information Administration, 53.0% of households nationwide used natural gas for heating in 1997. A recent survey of 3,600 randomly selected households showed that 54.0% used natural gas. Use a 0.05 significance level to test the claim that the 53.0% national rate has changed.
- 18. Clinical Test.** In clinical tests of the drug Lipitor, 863 patients were treated with the drug and 19 of them experienced flu symptoms (based on data from Parke-Davis). Test the claim that the percentage of treated patients with flu symptoms is greater than the 1.9% rate for patients not given treatments.
- 20. Smoking.** Use the Internet or library references to determine the proportion of Americans who smoke. Test the claim that the proportion of students at your college who smoke is different from the proportion of all Americans. Collect sample data from at least 50 randomly selected students.
- 21. Women College Students.** Use the Internet or library references to find the proportion of college students in the United States who are women. Test the claim that the proportion of women students at your college is different from the proportion of all U.S. college students. Collect sample data from at least 100 randomly selected students.
- 22. County Data.** The *Statistical Abstract of the United States* and the Current Population Survey provide an extensive supply of social, economic, and vital statistics at the county, state, and local levels. Use their Web sites to compare state data to national data in the following way.
- Choose a variable of interest that involves a proportion for a particular state (for example, the percentage of people living in poverty in Arizona).
 - Find the current national value for that variable (for example, the national poverty rate).
 - Choose a particular county within the state and obtain the corresponding data for the county (for example, the poverty rate in Pima County, Arizona).
 - Assuming that the county is a random sample of the state, test the claim that the state is above or below the national level in terms of that variable.
 - Discuss and interpret your results. The hypothesis test depends on the sample (the county) being a random sample of the population (the state). Be sure to discuss this factor in your conclusions.

IN THE NEWS

- 23. Hypothesis Testing in the News.** Find a news article or research report that describes (perhaps not explicitly) a hypothesis test for a population proportion. Attach the article and summarize the method used.



- 19. Left-Handedness.** Given the claim that 10% of Americans are left-handed, randomly select at least 50 students at your college and determine whether they are left-handed. Test the claim with a formal hypothesis test.

CHAPTER REVIEW EXERCISES

1. Randomly selected cans of Coke are measured for the amount of cola, in ounces. The sample values listed below have a mean of 12.19 ounces and a standard deviation of 0.11 ounce. Assume that we want to use a 0.05 significance level to test the claim that cans of Coke have a mean amount of cola greater than 12 ounces. Assume that the population has a standard deviation of $\sigma = 0.115$ ounce.

12.3 12.1 12.2 12.3 12.2 12.3 12.0 12.1 12.2
12.1 12.3 12.3 11.8 12.3 12.1 12.1 12.0 12.2
12.2 12.2 12.2 12.2 12.2 12.4 12.2 12.2 12.3
12.2 12.2 12.3 12.2 12.2 12.1 12.4 12.2 12.2

- a. What is the null hypothesis?
- b. What is the alternative hypothesis?
- c. What is the value of the standard score for the sample mean of 12.19 ounces?
- d. What is the critical value?
- e. What is the P -value?
- f. What do you conclude? (Be sure to address the original claim that the mean is greater than 12 ounces.)
- g. Describe a type I error for this test.
- h. Describe a type II error for this test.
- i. Find the P -value if the test is modified to test the claim that the mean is *different from* 12 ounces (instead of being greater than 12 ounces).

2. In a study of smokers who tried to quit smoking with nicotine patch therapy, 39 were smoking one year after the treatment, and 32 were not smoking one year after the treatment (based on data from “High Dose Nicotine Patch Therapy,” by Dale et al., *Journal of the American Medical Association*, Vol. 274, No. 17). We want to use a 0.05 significance level to test the claim that among smokers who try

to quit with nicotine patch therapy, the majority are smoking a year after the treatment.

- a. What is the null hypothesis?
- b. What is the alternative hypothesis?
- c. What is the value of the standard score for the sample proportion?
- d. What is the critical value?
- e. What is the P -value?
- f. What do you conclude? (Be sure to address the original claim that among smokers who try to quit with nicotine patch therapy, the majority are smoking a year the treatment.)
- g. Describe a type I error for this test.
- h. Describe a type II error for this test.
- i. What is the P -value if the claim is modified to state that the proportion is *equal to* 0.5?

3. **Finding a Job Through Networking.** In a survey of 703 randomly selected workers, 429 got their jobs through networking (based on data from Taylor Nelson Sofres Research).

- a. Use the sample data with a 0.05 significance level to test the claim that most (more than 50% of) workers get their jobs through networking.
- b. If given a claim that less than 50% of workers get their jobs through networking, why is it not necessary to go through the steps of conducting a formal hypothesis test? What should we conclude?
- 4. A medical student wants to test the claim that males who smoke have pulse rates with a mean greater than 70. She collects sample data by surveying her fellow medical students. What is the fundamental flaw in his procedure?



CHAPTER QUIZ

1. What is the alternative hypothesis that results from the claim that the mean brain volume of adults is larger than $1,126 \text{ cm}^3$?
2. What is the null hypothesis that is used for testing the claim that the proportion of college graduates who voted in the last election is greater than 0.5?
3. What is the alternative hypothesis that results from the claim that the proportion of convicted felons who serve time in prison is equal to 0.6?
4. Is a test of the claim that $p \neq 0.75$ left-tailed, right-tailed, or two-tailed?
5. What is the null hypothesis?
6. What is the alternative hypothesis?
7. If the test results in a P -value of 0.007, what do you conclude about the given claim?
8. What are the two possible conclusions that can be reached about the null hypothesis?
9. What are the two possible conclusions that can be reached about the claim being tested?
10. If you incorrectly conclude that professional comedians have a mean IQ score greater than 110 when their actual mean IQ score is 100, have you made a type I error or a type II error?

In Exercises 5–10, assume that we want to use a 0.05 significance level to test the claim that the mean IQ score of professional comedians is greater than 110.



FOCUS ON

HEALTH & EDUCATION

Will Your Education Help You Live Longer?

In this chapter, we focused on hypothesis testing in its simplest form, in which we test a single claim about a population and determine whether it is supported by evidence collected from a single sample. Not surprisingly, many of the most interesting statistical problems require much more complex analysis. Consider, for example, the question of what you can do to help you live longer.

Although this question is much more difficult to answer than anything we encountered in this chapter, it can be addressed through the same basic principles. Suppose, for example, that a researcher suspects that eating oats can make you live longer. To test this suspicion, the researcher conducts a hypothesis test. He or she starts with the null hypothesis that oat consumption has no effect on life span, then examines data to look for evidence that would support rejecting this null hypothesis and concluding that oat consumption really does increase life span. The difficult parts of this research are collecting the data—for example, finding people who consume oats to be



compared to people who don't—and then finding a way to separate the effects of oat consumption from those of the huge number of other variables that may also affect life span.

Over the past few decades, researchers have identified many factors that appear to contribute to longer lives. For example, greater wealth is correlated with longer life. Race plays a role in life span. Diet has numerous health effects. Exercise is generally a positive factor for longer life. Somewhat surprisingly, however, one factor appears to be more important than all the others: years of education. The longer you stay in school, the longer you'll live, at least on average. Figure 10 shows some of the data that support this conclusion.

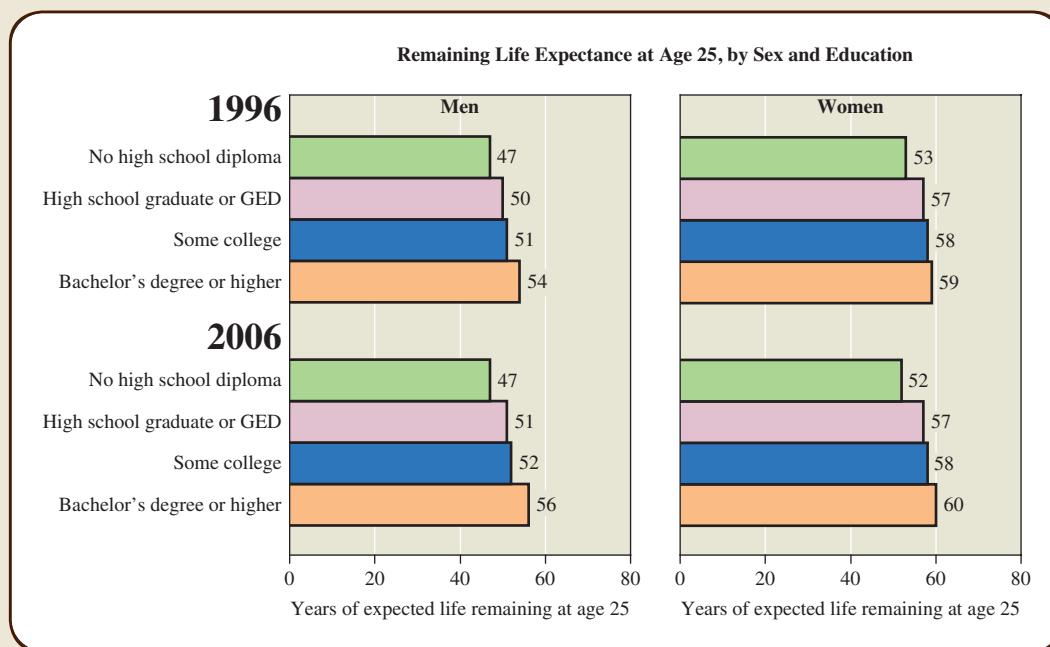


Figure 10 These bar charts show how the remaining life expectancy for 25-year-olds in the United States varies with education level for both men and women, and compares the results from 1996 and 2006. Notice the significant increase in life expectancy with education in all cases. Also notice that from 1996 to 2006, life expectancy remained stagnant or declined for the least educated people, while it increased for the most educated. *Source: US Department of Health and Human Services, "Health, United States, 2011."*

Why might more education lead to longer life? Researchers have suggested many possible reasons. For example, people with more education tend to be less likely to engage in life-shortening behaviors such as smoking or excessive drinking, while also undertaking exercise and other activities that can increase life span. One particularly interesting hypothesis is that getting an education involves some measure of short-term sacrifice for longer-term gain (such as paying for your education now in hopes of getting a higher-paying job later), and this willingness to accept delayed gratification helps people make all kinds of decisions that contribute to longer life. Of course, no one yet knows for sure—so you can be sure there is much more hypothesis testing to be done.

Nevertheless, based on the current data, we can draw at least one very interesting conclusion directly relevant to your reading of this text: You may be taking a statistics class because it is required, but it may also help you live longer.

QUESTIONS FOR DISCUSSION

1. Figure 10 presents a wealth of data. Interpret it carefully. How does it support the hypothesis that more education leads to longer life? Explain.

2. Why do *you* think education contributes to longer life? For example, do you think the mere fact of staying in school makes people live longer, or is it the extra learning associated with more years of schooling that makes the difference? How would you test your hypothesis?

Smoking is well known to be one of the biggest risk factors in disease and early death. As a result, the government and health care groups have often undertaken expensive advertising campaigns in hopes of convincing people to stop smoking (or never start). But the smoking rate drops dramatically with years of education. Therefore, if the goal is to reduce smoking, one might argue that it would make more sense to spend money on programs to help people stay in school than to spend money on anti-smoking campaigns. Do you think this is a good idea? Defend your opinion.

3. Education correlates with healthier lives as well as longer lives, which means that more educated people tend to have lower health care costs. Can you think of any ways in which this fact could be used to help get health care costs under control? Discuss with friends or classmates.
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FOCUS ON AGRICULTURE

Are Genetically Modified Foods Safe?

British newspapers call them “Frankenfoods” (a word play on *Frankenstein*). Europeans, by and large, don’t eat them at all. But today, many foods sold in the United States already fall in this category, including an estimated 65% of American corn and 94% of American soybeans. We are speaking of “genetically modified foods,” or GM foods. GM foods are a fairly recent agricultural invention, dating only from about the mid-1990s.* Nevertheless, they are at the forefront of one of the biggest debates in agricultural history.

A *genetically modified organism* is an organism into which scientists have inserted a gene that does not naturally exist in the organism or its close relatives. Genetically modified organisms are being tested and used for many purposes. For example, scientists have developed bacteria containing genes that produce drugs like insulin.

In agriculture, genetic modification allows scientists to create in crops traits that would be difficult or impossible to achieve by traditional breeding techniques. One of the first widely used genetically modified crops, often called “Bt corn,” illustrates the idea behind genetic modification. Corn is usually susceptible to destruction by a variety of insect pests. As a result, farmers usually spray crops with pesticides. Unfortunately, many pesticides are toxic to animals besides the insect pests and hence can cause environmental damage (and may not be ideal for humans to consume).

Here’s where Bt, a bacterium known as *bacillus thuringiensis*, comes in. Bt lives naturally in soil. As early as 1911, scientists discovered that Bt produces a toxin that kills certain types of insects. Different strains of Bt kill different insects, but are generally harmless to other animals and humans. By the 1960s, these traits had led to the use of Bt as an “environmentally friendly” pesticide that could be sprayed on crops (in the form of killed bacteria). Unfortunately, Bt pesticide proved to be expensive and often ineffective, mainly because it works only if insects eat it (more effective pesticides kill on contact). And, because it breaks down rapidly in sunlight and washes off plants in rain, it kills pests only if its application is timed just right.

*More than 50 new GM crops have been approved for sale in the United States, including corn and soybeans that produce their own pesticide and tomatoes engineered for longer shelf life. GM crops under development include potatoes that resist bruising, better-tasting soybeans, and grains containing vitamins and other nutrients that could improve nutrition in impoverished countries.



Genetic modification solves the problems inherent in Bt pesticide. The pesticide action of Bt bacteria arises from particular proteins that the bacteria produce. Scientists identified the genes responsible for these proteins, then transferred these genes into plants such as corn. Once the corn contains the necessary genes, it produces the same pest-killing proteins as the Bt bacteria. The application of a sprayed pesticide is no longer necessary because the corn itself is now toxic to the pests. Moreover, because the corn continually produces the pest-killing proteins, there are no more concerns about the pesticide breaking down or washing away.

The advantages of the Bt corn over traditional strains of corn are clear, but are there any disadvantages? This is where the great debate over GM foods begins. On one side, many scientists argue that GM foods are completely safe and that their benefits will help improve nutrition for people throughout the world. On the other side stand the people, including some scientists, who label GM foods as “Frankenfoods” and argue that they are one of the most dangerous technologies ever invented.

Broadly speaking, the issue of GM food safety can be broken down into three major questions:

1. Do GM foods have any toxic effects in humans?
2. Do the new proteins contained in GM foods cause allergic reactions in some people?
3. Can the GM crops cause any unforeseen environmental damage, such as transferring their genes into weeds (thereby making “superweeds”) or killing animals besides the insect pests?

These questions can be addressed through hypothesis testing. In each case, we begin with a null hypothesis that states that there is no safety difference between traditional foods and GM foods. For example, the null hypothesis for Question 1 says that GM foods are no more toxic than traditional foods (which often contain low levels of toxic chemicals). The alternative hypothesis states that there is a difference in toxicity. Scientists then develop experiments to test the hypotheses. If the evidence provided by the experiments reveals a significant difference in toxicity between the food groups (beyond what would be expected by chance alone), there is reason to reject the null hypothesis.

Unfortunately, experiments to date have been unable to resolve the controversy. For example, some genetically modified crops *do* contain products toxic to humans—and therefore never receive approval for sale. To proponents of GM foods, this fact provides an argument in their favor, because it appears to show that current regulatory procedures (for example, requiring approval of GM foods by the U.S. Food and Drug Administration) adequately ensure that only safe foods enter the marketplace. Similarly, while some of the proteins in GM foods undoubtedly can cause allergic reactions, many scientists think they understand these reactions well enough to ensure that only safe foods are approved.

Opponents of GM foods use the very same experimental results to support their case. They argue that even if scientists have identified toxicity that is obvious in experiments, they may still be missing long-term effects that might not show up in the population for many years. Similarly, they claim that we cannot be sure we understand *all* allergic reactions and therefore might inadvertently approve GM foods that could have severe allergic consequences in at least some people.

The environmental issues are even more difficult to study. For example, one study has shown that Bt corn is toxic

to the Monarch butterfly, a species that is not a pest and that no one wants to kill. But the study was conducted in a laboratory and may not accurately represent what occurs in the real environment. Similarly, the possibility of “gene jumping,” in which the Bt genes might spread from corn to other plants, is not well understood and therefore is very difficult to study.

The debate over GM foods is likely to continue, and even to intensify, for many years to come. After all, when it comes to food, everyone has an interest.

QUESTIONS FOR DISCUSSION

1. Propose an experiment that could be used to test the safety of a GM food product, such as corn. Describe your experiment in detail, and discuss any practical difficulties that might be involved in carrying it out or interpreting its results.
 2. Ignoring any safety issues of GM foods, make a list of as many ways as you can think of in which GM foods might be beneficial to humanity. Then, ignoring any benefits of GM foods, make a list of as many ways as you can think of in which GM foods might be dangerous. Overall, do you think that further use of GM foods should be encouraged or discouraged? Defend your opinion.
 3. Some people advocate giving the choice about GM foods to consumers by requiring labeling on all products that contain genetically modified ingredients. Do you think this is a good idea? Why or why not?
 4. Investigate recent developments in the debate over GM foods. Does the new information alter any of the major arguments in the debate? Explain.
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ANSWERS

Section 1

1. A hypothesis test is a standard procedure for testing a claim about the value of a population parameter.
3. $\mu \neq 195$ lb, $\mu < 195$ lb, $\mu > 195$ lb
5. Does not make sense.
7. Makes sense.
9. Makes sense.
11. Does not make sense.
13. a. 22 girls in 40 births does not appear to be statistically significant. b. 35 girls in 40 births does appear to be statistically significant. c. Part (b), because 35 girls in 40 births is statistically significant, so the P -value must be relatively small number.
15. $H_0: \mu = 250$ calories. $H_a: \mu \neq 250$ calories. There is sufficient sample evidence to warrant rejection of the claim that the mean caloric content is equal to 250 calories. There is not sufficient sample evidence to warrant rejection of the claim that the mean caloric content is equal to 250 calories.
17. $H_0: p = 0.7$. $H_a: p > 0.7$. There is not sufficient sample evidence to support the claim that the proportion of girls is greater than 0.7. There is sufficient sample evidence to support the claim that the proportion of girls is greater than 0.7.
19. $H_0: \mu = 10$ oz. $H_a: \mu \neq 10$ oz. There is not sufficient sample evidence to warrant rejection of the claim that the mean is equal to 10 oz. There is sufficient sample evidence to warrant rejection of the claim that the mean amount supplied is equal to 10 oz.
21. $H_0: p = 0.5$. $H_a: p > 0.5$. There is not sufficient sample evidence to support the claim that the proportion of students is greater than 0.5. There is sufficient sample evidence to support the claim that the proportion of students is greater than 0.5.
23. No. The P -value of 0.382 is greater than the significance level of 0.05, and this high value suggests that there is a high likelihood of getting 48 or fewer males by chance.
25. No. The P -value of 0.028 is greater than the significance level of 0.01, and this high value suggests that there is a high likelihood of getting 40 or fewer males by chance.
27. Yes. The P -value of 0.002 is less than the significance level of 0.01, and this low value suggests that there is a small likelihood of getting 35 or fewer males by chance.

Section 2

1. n represents the sample size, \bar{x} represents the mean of the sample values, s represents the standard deviation of the sample values, σ represents the standard deviation of the population values, and μ represents the mean of the population.
3. A type I error is the mistake of rejecting the null hypothesis when that null hypothesis is actually true. A type II error is the mistake

of failing to reject the null hypothesis when that null hypothesis is actually false.

5. Does not make sense.
7. Makes sense. 9. Makes sense.
11. Does not make sense.
13. $z = -3.33$. The alternative hypothesis is supported.
15. $z = 8.00$. The alternative hypothesis is supported.
17. $z = 1.03$. The alternative hypothesis is not supported.
19. $z = -5.11$. The alternative hypothesis is supported.
21. 0.3446. H_a is not supported.
23. 0.0287. H_a is supported.
25. 0.1096. H_a is not supported.
27. 0.0892. H_a is not supported.
29. 0.0035. H_a is supported.
31. 0.0179. H_a is supported.
33. 0.8808. H_a is not supported.
35. The test is right-tailed, but the standard score is to the left of the center of the distribution, so the P -value must be greater than 0.5. The alternative hypothesis is not supported. The formal hypothesis test is not necessary because there is no way that we could support a claim that the mean is greater than some value, when the sample mean is less than that value.
37. $H_0: \mu = 7.5$ years, $H_a: \mu < 7.5$ years; $n = 100$, $\bar{x} = 7.01$ years, $s = 3.74$ years, $\sigma = 0.374$ years, $z = -1.3$; not significant at the 0.05 level, P -value = 0.0968; there is not sufficient evidence to support the claim that the mean time of ownership is less than 7.5 years.
39. $H_0: \mu = 12$ oz, $H_a: \mu \neq 12$ oz; $n = 36$, $\bar{x} = 12.19$ oz, $s = 0.11$ oz, $\sigma \approx s$, $z = 10.36$; significant at the 0.05 level, P -value is less than 0.0001; there is sufficient evidence to support the claim that the mean weight of the packages is different from 12.00 oz.
41. $H_0: \mu = 600$ mg, $H_a: \mu \neq 600$ mg; $n = 65$, $\bar{x} = 589$ mg, $s = 21$ mg, $\sigma \approx s$, $z = -4.2$; significant at the 0.05 level, P -value = 0.0004; there is sufficient evidence to support the claim that the mean amount of acetaminophen is different from the listed 600 mg.
43. $H_0: \mu = 21.4$ mpg, $H_a: \mu < 21.4$ mpg; $n = 40$, $\bar{x} = 19.8$ mpg, $s = 3.5$ mpg, $\sigma \approx s$, $z = -2.9$; significant at the 0.05 level, P -value = 0.0019; there is sufficient evidence to support the claim that the mean gas mileage for SUVs is less than 21.4 miles per gallon.
45. $H_0: \mu = 0.21$, $H_a: \mu > 0.21$; $n = 32$, $\bar{x} = 0.83$, $s = 0.24$, $\sigma \approx s$, $z = 14.6$; significant at the 0.05 level, P -value < 0.0002; there is sufficient evidence to support the claim that the mean is greater than 0.21.

- 47.** $H_0: \mu = 3.39 \text{ kg}$, $H_a: \mu \neq 3.39 \text{ kg}$; $n = 121$, $\bar{x} = 3.67 \text{ kg}$, $s = 0.66 \text{ kg}$, $\sigma \approx s$, $z = 4.7$; significant at the 0.05 level, $P\text{-value} < 0.0004$; there is sufficient evidence to support the claim that the mean birth weight of male babies born to mothers on a vitamin supplement is different from the national average for all male babies.
- 49.** $H_0: \mu = 24 \text{ mo}$, $H_a: \mu < 24 \text{ mo}$; $n = 70$, $\bar{x} = 22.1 \text{ mo}$, $s = 8.6 \text{ mo}$, $\sigma \approx s$, $z = -1.85$; significant at the 0.05 level, $P\text{-value} = 0.0322$; there is sufficient evidence to support the claim that the mean is less than 24 months.
- 51.** Type I error: Reject the claim that the patient is free of a disease when the patient is actually free of the disease. Type II error: Fail to reject the claim that the patient is free of a disease when the patient is not free of the disease.
- 53.** Type I error: Reject the claim that the lottery is fair when the lottery is fair. Type II error: Fail to reject the claim that the lottery is fair when the lottery is biased.

Section 3

1. The symbol p represents the proportion in a population, \hat{p} represents the proportion in a sample, and a P -value is the probability of getting a sample proportion that is at least as extreme as the sample being considered.
3. No. The sample is a voluntary response sample, not a simple random sample. It is likely that those who responded are not representative of the general population.
5. Makes sense.
7. Does not make sense.
9. $H_0: p = 0.5$, $H_a: p > 0.5$; $n = 400$, $\hat{p} = 0.5125$, standard deviation of distribution of sample proportions is 0.025, $z = 0.5$; not significant at the 0.05 level, $P\text{-value} = 0.3085$; there is not sufficient evidence to support the claim that a majority of the voters support the candidate.
11. $H_0: p = 0.50$, $H_a: p < 0.50$; $n = 1015$, $\hat{p} = 0.44$, standard deviation of distribution of sample proportions is 0.0157, $z = -3.8$; significant at the 0.05 level, $P\text{-value} < 0.0002$; there is sufficient evidence to support the claim that fewer than half of all teenagers in the population feel that grades are the greatest source of pressure.
13. $H_0: p = 0.099$, $H_a: p < 0.099$; $n = 1050$, $\hat{p} = 0.0933$, standard deviation of distribution of sample proportions is 0.0092,

$z = -0.6$; not significant at the 0.05 level, $P\text{-value} = 0.2743$; there is not sufficient evidence to support the claim that illegal drug use is less than the national average.

- 15.** $H_0: p = 0.5$, $H_a: p > 0.5$; $n = 276,000$, $\hat{p} = 0.51$, standard deviation of distribution of sample proportions is 0.000952, $z = 10.5$; significant at the 0.05 level, $P\text{-value} < 0.0002$; there is sufficient evidence to support the claim that over half of all freshmen believe that abortion is should be legal.
- 17.** $H_0: p = 0.53$, $H_a: p \neq 0.53$; $n = 3600$, $\hat{p} = 0.54$, standard deviation of distribution of sample proportions is 0.0083, $z = 1.2$; not significant at the 0.05 level, $P\text{-value} = 0.2302$; there is not sufficient evidence to support the claim that the percentage of households using natural gas has changed.

Chapter Review Exercises

1. **a.** $H_0: \mu = 12 \text{ ounces}$ **b.** $H_a: \mu > 12 \text{ ounces}$ **c.** $z = 10.4$ **d.** $z = 1.645$ **e.** Less than 0.0002 **f.** Reject the null hypothesis and claim that cans of Coke have a mean amount of cola greater than 12 ounces. **g.** A Type I error results if we concluded that the population mean was greater than 12 ounces when, in fact, it was not. **h.** A type II error results if we did not conclude that the population mean was greater than 12 ounces when, in fact, it is greater than 12 ounces. **i.** The P -value would be the probability that z is greater than 10.4 or less than -10.4 . Since the probability that z is greater than 10.4 is less than 0.0002, the P -value is less than $2 \times 0.0002 = 0.0004$.
3. **a.** $H_0: p = 0.5$, $H_a: p > 0.5$; $n = 703$, $\hat{p} = 0.610$, standard deviation of distribution of sample proportions is 0.0184, $z = 5.85$; significant at the 0.05 level, $P\text{-value} < 0.0001$; there is sufficient evidence to support the claim that most workers get their jobs through networking. **b.** With $\hat{p} = 0.610$, there is no way that the sample data could support a claim that $p < 0.5$. We would need a value of \hat{p} less than 0.5.

Chapter Quiz

1. $\mu > 1126 \text{ cm}^3$
3. $p \neq 0.6$
5. $\mu = 110$
7. Support the claim that the mean is greater than 110.
9. Support the claim that the mean is greater than 110. Do not support the claim that the mean is greater than 110.

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Appendix: z-Score Tables

APPENDIX: Z-SCORE TABLES

Note that the areas under the curve shown here correspond to *percentiles*. To read this table, find the first two digits of the *z*-score in the left column, then read across the rows for third digit. Negative *z*-scores are on the first page and positive *z*-scores are on the second page.

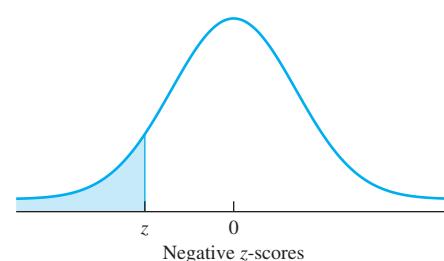


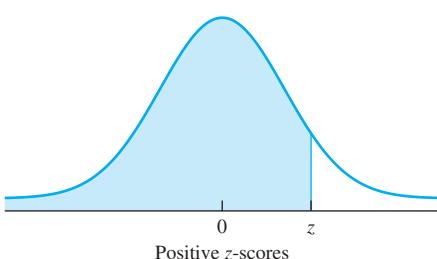
TABLE 1 Standard Normal (*z*) Distribution: Cumulative Area from the LEFT

<i>z</i>	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.50 and lower	.0001									
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	* .0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	* .0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	* .0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	* .0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	* .0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	* .0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	* .0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	* .0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	* .0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	* .0495	.0485	.0475	* .0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	* .0606	.0594	.0582	* .0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	* .0735	.0721	.0708	* .0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	* .0885	.0869	.0853	* .0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	* .1056	.1038	.1020	* .1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	* .1251	.1230	.1210	* .1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	* .1469	.1446	.1423	* .1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	* .1711	.1685	.1660	* .1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	* .1977	.1949	.1922	* .1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	* .2266	.2236	.2206	* .2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	* .2578	.2546	.2514	* .2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	* .2912	.2877	.2843	* .2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	* .3264	.3228	.3192	* .3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	* .3632	.3594	.3557	* .3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	* .4013	.3974	.3936	* .3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	* .4404	.4364	.4325	* .4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	* .4801	.4761	.4721	* .4681	.4641

Note: For values of *z* below -3.49, use 0.0001 for the area.

*Use these common values that result from interpolation:

z-score	Area
-1.645	0.0500 ←
-2.575	0.0050 ←

**TABLE 1 (continued) Cumulative Area from the LEFT**

<i>z</i>	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	* .9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	* .9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.50	.9999									
and up										

Note: For values of *z* above 3.49, use 0.9999 for the area.

*Use these common values that result from interpolation:

z-score	Area
1.645	0.9500 ←
2.575	0.9950 ←

TABLE 2 Select Critical Values of z			
	Left-tailed test	Right-tailed test	Two-tailed test
0.05 significance level	– 1.645	1.645	– 1.96 and 1.96
0.01 significance level	– 2.33	2.33	– 2.576 and 2.576

APPENDIX: TABLE OF RANDOM NUMBERS

For many statistical applications it is useful to generate a set of randomly chosen numbers. You can generate such numbers with most calculators and computers, but sometimes it is easier to use a table such as the one given below. This table was generated by randomly selecting one of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, or 9 for each position in the table; that is, each of these digits is equally likely to appear in any position. Thus, you can generate a sequence of random digits simply by starting at any point in the table and taking the digits in the order in which they appear. A larger set of random numbers is available on the text Web site (www.aw.com/bbt).

Example 1: Generate a random list of yes/no responses.

Solution: Start at an arbitrary point in the table. If the digit is 0, 1, 2, 3, or 4, call it a *yes* response. If the digit is 5, 6, 7, 8, or 9, call it a *no* response. Continue through the table from your starting point, using each digit shown to determine either a *yes* or a *no* response for your list.

Example 2: Generate a random list of letter grades A, B, C, D, or F.

Solution: Let 0 or 1 be an A, 2 or 3 be a B, 4 or 5 be a C, 6 or 7 be a D, and 8 or 9 be an F. Start at an arbitrary point in the table and use each digit shown to determine a grade for your list.

9	9	3	2	7	5	6	0	8	1	6	0	2	3	2	8	3	3	1	2	4	7	6	3	4
9	7	1	8	1	6	6	7	6	6	5	4	4	7	7	6	8	1	7	1	0	8	4	9	9
8	1	7	5	0	7	8	5	2	0	9	4	3	9	0	7	6	1	9	1	0	8	7	3	4
0	5	1	0	2	8	7	4	0	3	9	2	6	2	5	8	4	2	2	5	1	9	8	3	4
2	7	8	0	8	1	8	5	6	6	4	4	5	5	4	9	3	5	2	8	6	5	5	4	3
4	8	8	3	3	8	4	6	9	1	8	2	5	7	6	9	7	1	2	3	6	5	1	8	2
5	4	5	8	7	3	8	4	5	7	4	5	2	2	2	7	7	0	2	3	0	2	4	8	6
4	1	9	4	3	0	7	1	9	0	7	3	1	4	0	8	3	2	8	0	5	0	1	0	1
2	8	7	4	6	5	7	7	6	0	0	8	9	5	5	4	0	7	3	9	1	6	3	3	2
5	8	6	8	6	9	6	7	7	5	1	3	5	2	9	7	6	6	3	5	9	4	6	0	5
8	0	9	4	8	5	0	5	6	9	1	0	6	9	5	9	0	7	8	9	9	4	8	3	7
6	1	0	4	1	7	4	0	0	3	5	6	4	2	1	5	1	1	9	0	0	2	5	0	7
3	4	0	9	1	9	3	9	5	1	0	7	4	8	1	1	9	7	0	7	1	4	5	2	6
9	9	6	5	0	7	8	6	1	0	4	9	8	7	7	4	4	7	4	0	7	8	6	4	9
1	5	0	7	8	1	6	3	6	9	5	4	9	5	4	2	4	6	0	4	3	2	6	8	4
5	7	2	8	5	8	3	1	6	4	4	2	2	3	7	0	6	6	3	2	3	5	0	4	6
1	2	3	5	7	1	9	7	7	6	5	5	3	1	9	6	0	5	1	6	3	8	0	3	4
3	4	9	3	5	0	3	7	4	6	2	1	8	5	1	4	1	7	0	2	1	4	9	4	9
2	4	2	6	6	9	9	7	2	1	7	7	3	2	0	6	7	0	7	3	9	3	3	7	5
9	9	1	5	2	8	3	9	9	4	8	9	6	0	6	7	4	6	3	1	3	6	9	8	3
2	6	3	3	9	3	4	1	9	3	0	6	9	1	2	4	1	9	9	8	7	3	2	6	9
1	3	7	8	0	1	0	6	1	6	5	4	9	3	0	7	0	7	2	3	1	7	8	9	2
8	8	4	8	4	0	2	8	5	5	7	3	7	1	2	1	8	3	5	2	0	1	5	3	2
4	2	9	2	8	8	8	9	1	2	4	6	7	1	7	5	1	5	1	9	3	2	2	8	0
1	3	7	2	1	0	6	4	7	6	1	0	8	4	8	9	7	6	3	5	5	1	2	2	9

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Suggested Readings

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SUGGESTED READINGS

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Glossary

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GLOSSARY

absolute change The actual increase or decrease from a reference value to a new value:

$$\text{absolute change} = \text{new value} - \text{reference value}$$

absolute difference The actual difference between the compared value and the reference value:

$$\text{absolute difference} = \text{compared value} - \text{reference value}$$

absolute error The actual amount by which a measured value differs from the true value:

$$\text{absolute error} = \text{measured value} - \text{true value}$$

accident rate The number of accidents due to some particular cause, expressed as a fraction of all people at risk for the same cause. For example, an accident rate of “5 per 1,000 people” means that an average of 5 in 1,000 people suffer an accident from this particular cause.

accuracy How closely a measurement approximates a true value. An accurate measurement is very close to the true value.

alternative hypothesis (H_a) A statement that can be supported only if the null hypothesis is rejected.

analysis of variance (ANOVA) A method of testing the equality of three or more population means by analyzing sample variances.

and probability The probability that event A *and* event B will both occur. How it is calculated depends on whether the events are independent or dependent. Also called *joint probability*.

ANOVA See *analysis of variance*.

a priori method See *theoretical method*.

bar graph A diagram consisting of bars representing the frequencies (or relative frequencies) for particular categories. The bar lengths are proportional to the frequencies.

best-fit line The line on a scatter diagram that lies closer to the data points than all other possible lines (according to a standard statistical measure of closeness). Also called *regression line*.

bias In a statistical study, any problem in the design or conduct of the study that tends to favor certain results. See also *participation bias; selection bias*.

bimodal distribution A distribution with two peaks, or modes.

binning Grouping data into categories (bins), each of which covers a range of possible data values.

blinding The practice of keeping experimental subjects and/or experimenters in the dark about who is in the treatment group and who is in the control group. See also *double-blind experiment; single-blind experiment*.

boxplot A graphical display of a five-number summary. A number line is used for reference, the values from the lower to the upper quartiles are enclosed in a box, a line is drawn through the box for the median, and two “whiskers” are extended to the low and high data values. Also called *box-and-whisker plot*.

case-control study An observational study that resembles an experiment because the sample naturally divides into two (or more) groups. The participants who engage in the behavior under study form the cases, like the treatment group in an experiment. The participants who do not engage in the behavior are the *controls*, like the control group in an experiment.

causality A relationship present when one variable is a cause of another.

census The collection of data from every member of a population.

Central Limit Theorem Theorem stating that, for random samples (all of the same size) of a variable with any distribution (not necessarily a normal distribution), the distribution of the means of the samples will, as the sample size increases, tend to be approximately a normal distribution.

chi-square statistic (χ^2) A number used to determine the statistical significance of a hypothesis test in a contingency table (or two-way table). If it is less than a critical value (which depends on the table size and the desired significance level), the differences between the observed frequencies and the expected frequencies are not significant.

cluster sampling Dividing the population into groups, or clusters; selecting some of these clusters at random; and then obtaining the sample by choosing all the members within each cluster.

coefficient of determination (R^2) A number that describes how well data fit a best-fit equation found through multiple regression.

compared value A number that is compared to a reference value in computing a relative difference.

complement For an event A, all outcomes in which A does *not* occur, expressed as \bar{A} . Its probability is $P(\bar{A}) = 1 - P(A)$.

conditional probability The probability of one event given the occurrence of another event, written $P(B \text{ given } A)$ or $P(B|A)$.

confidence interval A range of values associated with a confidence level, such as 95%, that is likely to contain the true value of a population parameter.

confounding Confusion in the interpretation of statistical results that occurs when the effects of different factors are mixed such that the effects of the individual factors being studied cannot be determined.

confounding factors Any factors or variables in a statistical study that can lead to confounding. Also called *confounding variables*.

Consumer Price Index (CPI) An index number designed to measure the rate of inflation. It is computed and reported monthly, based on a sample of more than 60,000 goods, services, and housing costs.

contingency table See *two-way table*.

continuous data Quantitative data that can take on any value in a given interval.

contour map A map that uses curves (contours) to connect geographical regions with the same data values.

control group The group of subjects in an experiment who do not receive the treatment being tested.

convenience sampling Selecting a sample that is readily available.

correlation A statistical relationship between two variables. See also *negative correlation; no correlation; positive correlation*.

correlation coefficient (r) A measure of the strength of the relationship between two variables. Its value is always between -1 and 1 (that is, $-1 \leq r \leq 1$).

cumulative frequency For any data category, the number of data values in that category and all preceding categories.

death rate The number of deaths due to some particular cause, expressed as a fraction of all people at risk for the same cause. For example, a death rate of “5 per 1,000 people” means that an average of 5 in 1,000 people die from this particular cause.

degrees of freedom (for a *t* distribution) The sample size minus one ($n - 1$).

dependent events Two events for which the outcome of one affects the probability of the other.

deviation How far a particular data value lies from the mean of a data set, used to compute standard deviation.

discrete data Quantitative data that can take on only particular values and not other values in between (for example, the whole numbers 0, 1, 2, 3, 4, 5).

distribution The way the values of a variable are spread over all possible values. It can be displayed with a table or with a graph.

distribution of sample means The distribution that results when the means (\bar{x}) of all possible samples of a given size are found.

distribution of sample proportions The distribution that results when the proportions (\hat{p}) in all possible samples of a given size are found.

dotplot A diagram similar to a bar graph except that each individual data value is represented with a dot.

double-blind experiment An experiment in which neither the participants nor the experimenters know who belongs to the treatment group and who belongs to the control group.

either/or probability The probability that *either* event A *or* event B will occur. How it is calculated depends on whether the events are overlapping or non-overlapping.

empirical method See *relative frequency method*.

event In probability, a collection of one or more outcomes that share a property of interest. See also *outcome*.

expected frequency In a two-way table, the frequency one would expect in a given cell of the table if the row and column variables were independent of each other.

expected value The mean value of the outcomes for some random variable.

experiment A study in which researchers apply a treatment and then observe its effects on the subjects.

experimenter effect An effect that occurs when a researcher or experimenter somehow influences subjects through such factors as facial expression, tone of voice, or attitude.

five-number summary A description of the variation of a data distribution in terms of the minimum value, lower quartile, median, upper quartile, and maximum value.

frequency For a data category, the number of times data values fall within that category.

frequency table A table that lists all the categories of data in one column and the frequency for each category in another column.

gambler’s fallacy The mistaken belief that a streak of bad luck makes a person “due” for a streak of good luck.

geographical data Data that can be assigned to different geographical locations.

histogram A bar graph showing a distribution for quantitative data (at the interval or ratio level of measurement). The bars have a natural order, and the bar widths have specific meaning.

hypothesis In statistics, a claim about a population parameter, such as a population proportion, p , or population mean, μ . See also *alternative hypothesis; null hypothesis*.

hypothesis test A standard procedure for testing a claim about the value of a population parameter.

independent events Two events for which the outcome of one does not affect the probability of the other.

index number A number that provides a simple way to compare measurements made at different times or in different places. The value at one particular time (or place) must be chosen to be the reference value (or base value). The index number for any other time (or place) is

$$\text{index number} = \frac{\text{value}}{\text{reference value}} \times 100$$

inflation The increase over time in prices and wages. Its overall rate is measured by the CPI.

interval level of measurement A level of measurement for quantitative data in which differences, or intervals, are meaningful but ratios are not. Data at this level have an arbitrary starting point.

joint probability See *and probability*.

law of large numbers An important result in probability that applies to a process for which the probability of an event A is $P(A)$ and the results of repeated trials are independent. It states: If the process is repeated through many trials, the larger the number of trials, the closer the proportion should be to $P(A)$. Also called *law of averages*.

left-skewed distribution A distribution in which the values are more spread out on the left side.

left-tailed test A hypothesis test that involves testing whether a population parameter lies to the left (lower values) of a claimed value.

level of measurement See *nominal level of measurement; ordinal level of measurement; interval level of measurement; ratio level of measurement*.

life expectancy The number of years a person of a given age today can be expected to live, on average. It is based on current health and medical statistics and does not take into account future changes in medical science or public health.

line chart A graph showing the distribution of quantitative data as a series of dots connected by lines. The horizontal position of each dot corresponds to the center of the bin it represents and the vertical position corresponds to the frequency value for the bin.

lower quartile See *quartile, lower*.

margin of error The maximum likely difference between an observed sample statistic and the true value of a population parameter. Its size depends on the desired level of confidence.

mean The sum of all values divided by the total number of values. It’s what is most commonly called the average value.

median The middle value in a sorted data set (or halfway between the two middle values if the number of values is even).

median class For binned data, the bin into which the median data value falls.

meta-analysis A study in which researchers analyze many individual studies (on a particular topic) as a combined group, with the aim of finding trends that were not evident in the individual studies.

middle quartile See *quartile, middle*.

mode The most common value (or group of values) in a distribution.

multiple bar graph A simple extension of a regular bar graph, in which two or more sets of bars allow comparison of two or more data sets.

multiple line chart A simple extension of a regular line chart, in which two or more lines allow comparison of two or more data sets.

multiple regression A technique that allows the calculation of a best-fit equation that represents the best fit between one variable (such as price) and a *combination* of two or more other variables (such as weight and volume).

negative correlation A correlation in which the two variables tend to change in opposite directions, with one increasing while the other decreases.

no correlation Absence of any apparent relationship between two variables.

nominal level of measurement A level of measurement for qualitative data that consist of names, labels, or categories only and cannot be ranked or ordered.

nonlinear relationship A relationship between two variables that cannot be expressed with a linear (straight-line) equation.

non-overlapping events Two events for which the occurrence of one precludes the occurrence of the other.

normal distribution A special type of symmetric, bell-shaped distribution with a single peak that corresponds to the mean, median, and mode of the distribution. Its variation can be characterized by the standard deviation. See also *68-95-99.7 rule*.

null hypothesis (H_0) A specific claim (such as a specific value for a population parameter) against which an alternative hypothesis is tested.

observational study A study in which researchers observe or measure characteristics of the sample members, but do not attempt to influence or modify these characteristics.

of versus more than (less than) rule A rule for comparisons. It states: If the compared value is $P\%$ *more than* the reference value, then it is $(100 + P)\%$ of the reference value. If the compared value is $P\%$ *less than* the reference value, then it is $(100 - P)\%$ of the reference value.

one-tailed test See *left-tailed test; right-tailed test*.

ordinal level of measurement A level of measurement for qualitative data that can be arranged in some order. It generally does not make sense to do computations with the data.

outcome In probability, the most basic possible result of an observation or experiment. See also *event*.

outlier A value in a data set that is much higher or much lower than almost all other values.

overlapping events Two events that could possibly both occur.

Pareto chart A bar graph of data at the nominal level of measurement, with the bars arranged in descending order.

participants People (as opposed to objects) who are the subjects of a study.

participation bias Bias that occurs any time participation in a study is voluntary.

peer review A process by which several experts in a field evaluate a research report before it is published.

percentiles Values that divide a data distribution into 100 segments, each representing about 1% of the data values.

pictograph A graph embellished with artwork.

pie chart A circle divided so that each wedge represents the relative frequency of a particular category. The wedge size is proportional to the relative frequency, and the entire pie represents the total relative frequency of 100%.

placebo Something that lacks the active ingredients of a treatment but is identical in appearance to the treatment. Thus, participants in a study cannot distinguish the placebo from the real treatment.

placebo effect An effect in which patients improve simply because they believe they are receiving a useful treatment, when in fact they may be receiving only a placebo.

population The complete set of people or things being studied.

population mean The true mean of a population, denoted by the Greek letter μ (pronounced “mew”).

population parameters Specific characteristics of the population that a statistical study is designed to estimate.

population proportion The true proportion of some characteristic in a population, denoted by p .

positive correlation A type of correlation in which two variables tend to increase (or decrease) together.

practical significance In a statistical study, significance in the sense that the result is associated with some meaningful course of action.

precision The amount of detail in a measurement.

probability For an event, the likelihood that the event will occur. The probability of an event, written as $P(\text{event})$, is always between 0 and 1 inclusive. A probability of 0 means the event is impossible and a probability of 1 means the event is certain. See also *relative frequency method; subjective method; theoretical method*.

probability distribution The complete distribution of the probabilities of all possible events associated with a particular variable. It may be shown as a table, graph, or formula.

P-value In a hypothesis test, the probability of selecting a sample at least as extreme as the observed sample, assuming that the null hypothesis is true.

qualitative data Data consisting of values that describe qualities or nonnumerical categories.

quantitative data Data consisting of values representing counts or measurements.

quartile, lower The median of the data values in the lower half of a data set. Also called *first quartile*.

quartile, middle The overall median of a data set. Also called *second quartile*.

quartile, upper The median of the data values in the upper half of a data set. Also called *third quartile*.

quartiles Values that divide a data distribution into four parts with approximately 25% of the data values in each part.

random errors Errors that occur because of random and inherently unpredictable events in the measurement process.

randomization The process of ensuring that the subjects of an experiment are assigned to the treatment or control group at random and in such a way that each subject has an equal chance of being assigned to either group.

range For a distribution, the difference between the lowest and highest data values.

range rule of thumb A guideline stipulating that, for a data set with no outliers, the standard deviation is approximately equal to range/4.

rare event rule Rule stating that it is appropriate to conclude that a given assumption (such as the null hypothesis) is probably not correct if the probability of a particular event at least as extreme as the observed event is very small.

ratio level of measurement A level of measurement for quantitative data in which both intervals and ratios are meaningful. Data at this level have a true zero point.

raw data The actual measurements or observations collected from a sample.

reference value The number that is used as the basis for a comparison.

regression line See *best-fit line*.

relative change The size of an absolute change in comparison to the reference value, expressed as a percentage:

$$\text{relative change} = \frac{\text{new value} - \text{reference value}}{\text{reference value}} \times 100\%$$

relative difference The size of an absolute difference in comparison to the reference value, expressed as a percentage:

$$\text{relative difference} = \frac{\text{compared value} - \text{reference value}}{\text{reference value}} \times 100\%$$

relative error The relative amount by which a measured value differs from the true value, expressed as a percentage:

$$\text{relative error} = \frac{\text{measured value} - \text{true value}}{\text{true value}} \times 100\%$$

relative frequency For any data category, the fraction or percentage of the total frequency that falls in that category:

$$\text{relative frequency} = \frac{\text{frequency in category}}{\text{total frequency}}$$

relative frequency method A method of estimating a probability based on observations or experiments by using the observed or measured relative frequency of the event of interest. Also called *empirical method*.

representative sample A sample in which the relevant characteristics of the members are generally the same as the characteristics of the population.

right-skewed distribution A distribution in which the values are more spread out on the right side.

right-tailed test A hypothesis test that involves testing whether a population parameter lies to the right (higher values) of a claimed value.

rounding rule For statistical calculations, the practice of stating answers with one more decimal place of precision than is found in the raw data. For example, the mean of 2, 3, and 5 is 3.3333 . . . , which would be rounded to 3.3.

sample A subset of the population from which data are actually obtained.

sample mean The mean of a sample, denoted \bar{x} ("x-bar").

sample proportion The proportion of some characteristic in a sample, denoted \hat{p} ("p-hat").

sample statistics Characteristics of the sample that are found by consolidating or summarizing the raw data.

sampling The process of choosing a sample from a population.

sampling distribution The distribution of a sample statistic, such as a mean or proportion, taken from all possible samples of a particular size.

sampling error Error introduced when a random sample is used to estimate a population parameter; the difference between a sample result and a population parameter.

sampling methods See *cluster sampling*; *convenience sampling*; *simple random sampling*; *stratified sampling*; *systematic sampling*.

scatterplot A graph, often used to investigate correlations, in which each point corresponds to the values of two variables. Also called *scatter diagram*.

selection bias Bias that occurs whenever researchers select their sample in a biased way. Also called *selection effect*.

self-selected survey A survey in which people decide for themselves whether to be included. Also called *voluntary response survey*.

simple random sampling A sample of items chosen in such a way that every possible sample of the same size has an equal chance of being selected.

Simpson's paradox A statistical paradox that arises when the results for a whole group seem inconsistent with those for its subgroups; it can occur whenever the subgroups are unequal in size.

single-blind experiment An experiment in which the participants do not know whether they are members of the treatment group or the control group but the experimenters do know—or, conversely, the participants do know but the experimenters do not.

single-peaked distribution A distribution with a single mode. Also called *unimodal distribution*.

68-95-99.7 rule Guideline stating that, for a normal distribution, about 68% (actually, 68.3%) of the data values fall within 1 standard deviation of the mean, about 95% (actually, 95.4%) of the data values fall within 2 standard deviations of the mean, and about 99.7% of the data values fall within 3 standard deviations of the mean.

skewed See *left-skewed distribution*; *right-skewed distribution*.

stack plot A type of bar graph or line chart in which two or more different data sets are stacked vertically.

standard deviation A single number commonly used to describe the variation in a data distribution, calculated as

$$\text{standard deviation} = \sqrt{\frac{\text{sum of all (deviations from the mean)}^2}{\text{total number of data values} - 1}}$$

standard score For a particular data value, the number of standard deviations (usually denoted by z) between it and the mean of the distribution:

$$z = \text{standard score} = \frac{\text{data value} - \text{mean}}{\text{standard deviation}}$$

Also called *z-score*.

statistical significance A measure of the likelihood that a result is meaningful.

statistically significant result A result in a statistical study that is unlikely to have occurred by chance. The most commonly quoted levels of statistical significance are the 0.05 level (the probability of the result's having occurred by chance is 5% or less, or less than 1 in 20) and the 0.01 level (the probability of the result's having occurred by chance is 1% or less, or less than 1 in 100).

statistics (plural) The data that describe or summarize something.

statistics (singular) The science of collecting, organizing, and interpreting data.

stemplot A graph that looks much like a histogram turned sideways, with lists of the individual data values in place of bars. Also called *stem-and-leaf plot*.

stratified sampling A sampling method that addresses differences among subgroups, or strata, within a population. First the strata are

identified, and then a random sample is drawn within each stratum. The total sample consists of all the samples from the individual strata.

subjective method A method of estimating a probability based on experience or intuition.

subjects In a statistical study, the people or objects chosen for the sample. See also *participants*.

symmetric distribution A distribution in which the left half is a mirror image of the right half.

systematic errors Errors that occur when there is a problem in the measurement system that affects all measurements in the same way.

systematic sampling Using a simple system to choose the sample, such as selecting every 10th or every 50th member of the population.

t distribution A distribution that is very similar in shape and symmetry to the normal distribution but that accounts for the greater variability expected with small samples. It approaches the normal distribution for large sample sizes.

theoretical method A method of estimating a probability based on a theory, or set of assumptions, about the process in question. Assuming that all outcomes are equally likely, the theoretical probability of a particular event is found by dividing the number of ways the event can occur by the total number of possible outcomes. Also called *a priori method*.

time-series diagram A histogram or line chart in which the horizontal axis represents time.

treatment Something given or applied to the members of the treatment group in an experiment.

treatment group The group of subjects in an experiment that receive the treatment being tested.

two-tailed test A hypothesis test that involves testing whether a population parameter lies to either side of a claimed value.

two-way table A table showing the relationship between two variables by listing the values of one variable in its rows and the values of the other variable in its columns. Also called *contingency table*.

type I error In a hypothesis test, the mistake of rejecting the null hypothesis, H_0 , when it is true.

type II error In a hypothesis test, the mistake of failing to reject the null hypothesis, H_0 , when it is false.

uniform distribution A distribution in which all data values have the same frequency.

unimodal distribution See *single-peaked distribution*.

unusual values In a data distribution, values that are not likely to occur by chance, such as those values that are more than 2 standard deviations away from the mean.

upper quartile See *quartile, upper*.

variable Any item or quantity that can vary or take on different values.

variables of interest In a statistical study, the items or quantities that the study seeks to measure.

variation How widely data are spread out about the center of a distribution. See also *five-number summary; range; standard deviation*.

vital statistics Data concerning births and deaths of people.

voluntary response survey See *self-selected survey*.

weighted mean A mean that accounts for differences in the relative importance of data values. Each data value is assigned a weight, and then

$$\text{weighted mean} = \frac{\text{sum of (each data value} \times \text{its weight)}}{\text{sum of all weights}}$$

z-score See *standard score*.

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