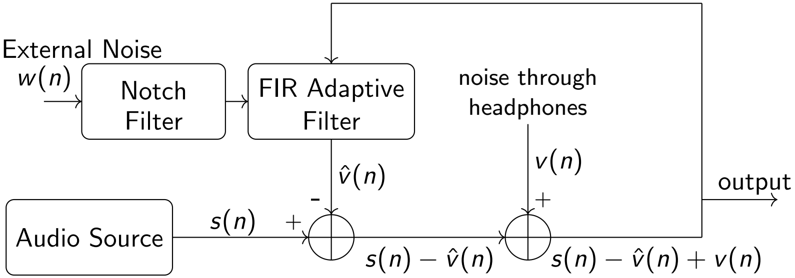
Block Diagram



Design choices and justifications

We chose to perform Normalized Least Mean Squares (NLMS) adaptive filtering to remove the external noise from the “desired” signal. The design choices are as follows:

- Filter type : FIR Adaptive Filter with IIR Notch Filter
- Filter Order : 2 (for less computations)
- $\mu = 0.007$ (for stability of NLMS)
- Bandwidth of Notch Filter : 10 (for narrower notch)

The choice of choosing NLMS over LMS is more stability and over RLS is less computational complexity.

Tradeoffs

- If Filter order is 2 then noise will over adapt
- Sacrificing stability of Notch Filter for a narrow notch

Design

- FIR Filter with Adaptive coefficients are updated using NLMS. This will work as Full suppression mode.
- The IIR Notch Filter removes the specific frequency (tonal frequency) from the external noise, hence our FIR Filter will not be able to adapt to remove the particular frequency in the error signal. This will work as Partial suppression.

Assumptions

- Tonal Noise frequencies are known.
- Noise is reasonably stationary in small time frames.

Pros

- Better adaptability compared to LMS
- Less computations compared to RLS

Cons

- No fixed value for μ, it is completely designer dependent and can cause problems if not chosen properly
- Narrower Bandwidth makes the Notch Filter less stable as the pole is approaching the unit circle

References

- M. H. Hayes, "Statistical Digital Signal Processing and Modeling", John Wiley & Sons, 1996.
- J. G. Proakis and D. G. Manolakis, "Digital Signal Processing"

NLMS

Referring to page 1, the signal of interest is $s(n)$. But, noise from external environment $v(n)$ is being added into the signal. So, the purpose of the adaptive filter is to generate a signal $\hat{v}(n)$, which is subtracted from the signal $s(n)$, so that it may cancel out the noise $v(n)$, such that the output $s(n) + v(n) - \hat{v}(n)$ is very much close to $s(n)$.
Consider the coefficient vector as $\underline{h}$ which is an L dimensional vector such that

$$\underline{h} = [h_1 \ h_2 \ h_3 \ldots h_L]^T$$

The inputs to the filter is the noise signal vector $\underline{w}(n)$ such that

$$\underline{w}(n) = [w(n) \ w(n-1) \ w(n-2) \ldots w(n-L+1)]^T$$

Now,

$$\hat{v}(n) = \underline{h}^T \underline{w}(n)$$

Now, the error signal is

$$e(n) = s(n) + v(n) - \hat{v}(n)$$

We minimize the squared of the error signal

$$\begin{aligned} J(\underline{h}) &= \frac{1}{2} * (e(n))^2 \\ &= \frac{1}{2} * (s(n) + v(n) - \hat{v}(n))^2 \\ &= \frac{1}{2} * (s(n) + v(n) - \underline{h}^T \underline{w}(n))^2 \\ \nabla J(\underline{h}) &= -(s(n) + v(n) - \underline{h}^T \underline{w}(n))(\underline{w}(n)) \end{aligned}$$

Using Gradient descent,

$$\begin{aligned} \underline{h}_{new} &= \underline{h}_{old} + \mu(s(n) + v(n) - \underline{h}^T \underline{w}(n))(\underline{w}(n)) \\ &= \underline{h}_{old} + \mu e(n)\underline{w}(n) \end{aligned}$$

Now , we divide with the l_2 norm square of the noise signal vector to make the gradient vary as the update is progressing

$$\implies \underline{h}_{new} = \underline{h}_{old} + \frac{\mu e(n)\underline{w}(n)}{||\underline{w}(n)||_2^2}$$