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MICRO GR-1, 2, 3

ME Handout #5

CHAPTER TWENTY-ONE APPENDIX

Indifference Curve Analysis

A more advanced explanation of consumer behavior and equilibrium is based on (1) budget lines and (2) so-called indifference curves.

The Budget Line: What Is Attainable

A budget line (or, more technically, the *budget constraint*) is a schedule or curve that shows various combinations of two products a consumer can purchase with a specific money income. If the price of product A is \$1.50 and the price of product B is \$1, a consumer could purchase all the combinations of A and B shown in Table 1 with \$12 of money income. At one extreme, the consumer might spend all of his or her income on 8 units of A and have nothing left to spend on B. Or, by giving up 2 units of A and thereby "freeing" \$3, the consumer could have 6 units of A and 3 of B. And so on to the other extreme, at which the consumer could buy 12 units of B at \$1 each, spending his or her entire money income on B with nothing left to spend on A.

Figure 1 shows the same budget line graphically. Note that the graph is not restricted to whole units of A and B as is the table. Every point on the graph represents a possible combination of A and B, including fractional quantities. The slope of the graphed budget line measures the ratio of the price of B to the price of A; more precisely, the absolute value of the slope is $P_B/P_A = \$1.00/\$1.50 = \frac{2}{3}$. This is the mathematical way of saying that the consumer must forgo 2 units of A (measured on the vertical axis) to buy 3 units of B (measured on the horizontal axis). In moving down the budget or price line, 2 units of A (at \$1.50 each) must be given up to obtain 3 more units of B (at \$1 each). This yields a slope of $\frac{2}{3}$.

FIGURE 1

A consumer's budget line. The budget line shows all the combinations of any two products that can be purchased, given the prices of the products and the consumer's money income.

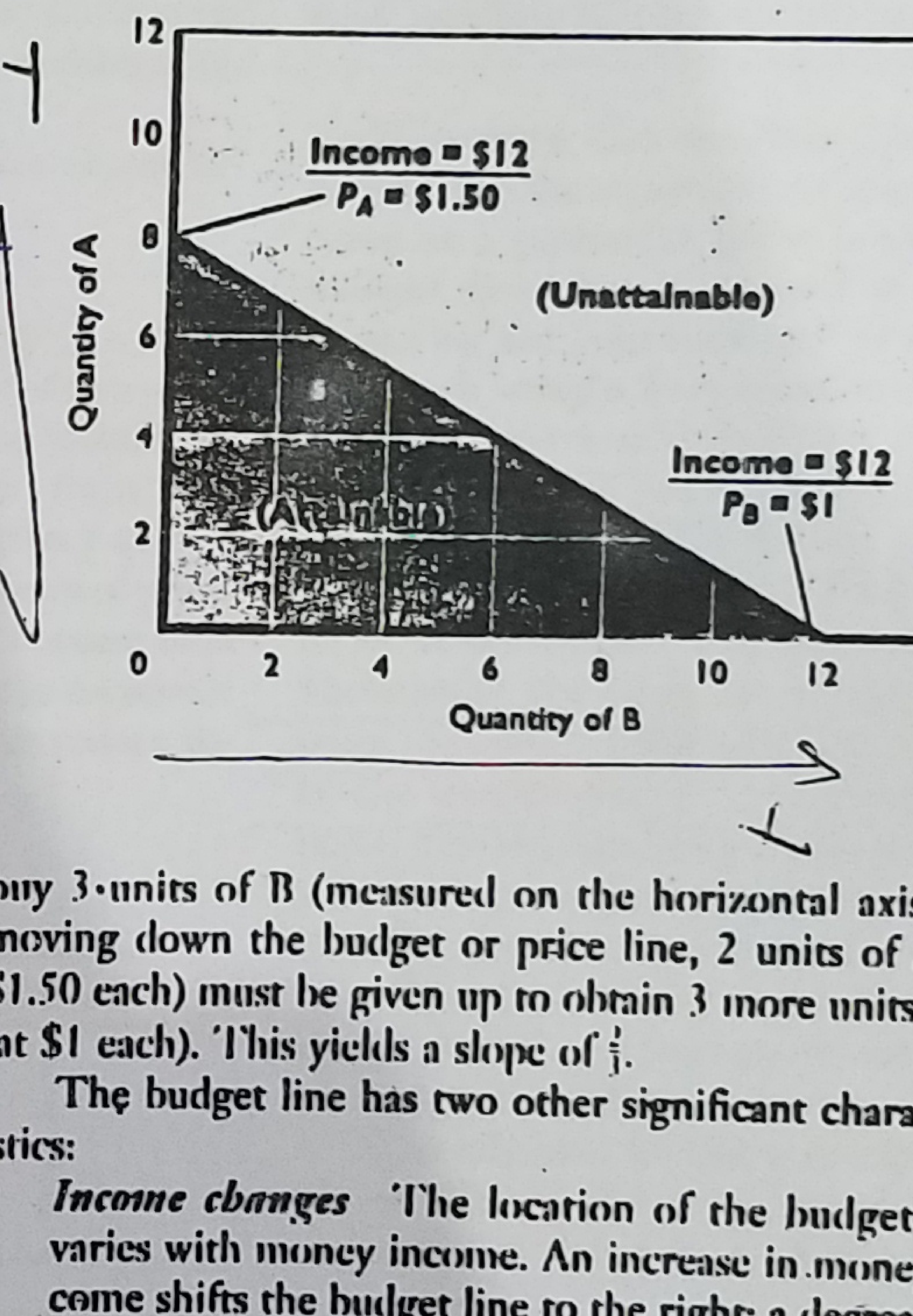

TABLE 1

The Budget Line: Whole-Unit Combinations of A and B Attainable with an Income of \$12

Units of A (Price = \$1.50)	Units of B (Price = \$1)	Total Expenditure
8	0	\$12 (= \$12 + \$0)
6	3	\$12 (= \$9 + \$3)
4	6	\$12 (= \$6 + \$6)
2	9	\$12 (= \$3 + \$9)
0	12	\$12 (= \$0 + \$12)

The budget line has two other significant characteristics:

- ***Income changes*** The location of the budget line varies with money income. An increase in money income shifts the budget line to the right; a decrease in money income shifts it to the left. To verify this, recalculate Table 1, assuming that money income is (a) \$24 and (b) \$6, and plot the new budget lines in Figure 1.
- ***Price changes*** A change in product prices also shifts the budget line. A decline in the prices of both products—the equivalent of an increase in real income—shifts the curve to the right. (You can verify this by recalculating Table 1 and replotting Figure 1 assuming that P_A = \$.75 and P_B = \$.50.) Conversely, an increase in the prices of A and B shifts the curve to the left. (Assume P_A = \$3 and P_B = \$2, and rework Table 1 and Figure 1 to substantiate this statement.)

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Note what happens if P_B changes while P_A and money income remain constant. In particular, if P_B drops, say, from \$1 to \$.50, the lower end of the budget line fans outward to the right. Conversely, if P_B increases, say, from \$1 to \$1.50, the lower end of the line fans inward to the left. In both instances the line remains "anchored" at 8 units on the vertical axis because P_A has not changed.

Indifference Curves: What Is Preferred

Budget lines reflect "objective" market data, specifically income and prices. They reveal combinations of products A and B that can be purchased, given current money income and prices.

Indifference curves, on the other hand, reflect "subjective" information about consumer preferences for A and B. An indifference curve shows all the combinations of two products A and B that will yield the same total satisfaction or total utility to a consumer. Table 2 and Figure 2 present a hypothetical indifference curve for products A and B. The consumer's subjective preferences are such that he or she will realize the same total utility from each combination of A and B shown in the table or on the curve. So the consumer will be indifferent (will not care) as to which combination is actually obtained.

21.4 Indifference curves

Indifference curves have several important characteristics.

Indifference Curves Are Downsloping An indifference curve slopes downward because more of one product means less of the other if total utility is to remain unchanged. Suppose the consumer moves from one combination of A and B to another, say, from *j* to *k* in Figure 2. In so doing, the consumer obtains more of product B, increasing his or her total utility. But because total utility is the same everywhere on the curve, the consumer must give up some of the other product, A, to reduce total utility by a precisely offsetting amount. Thus "more of B" necessitates "less of A," and the quantities of A and B are inversely related. A curve that reflects inversely related variables is downward-sloping.

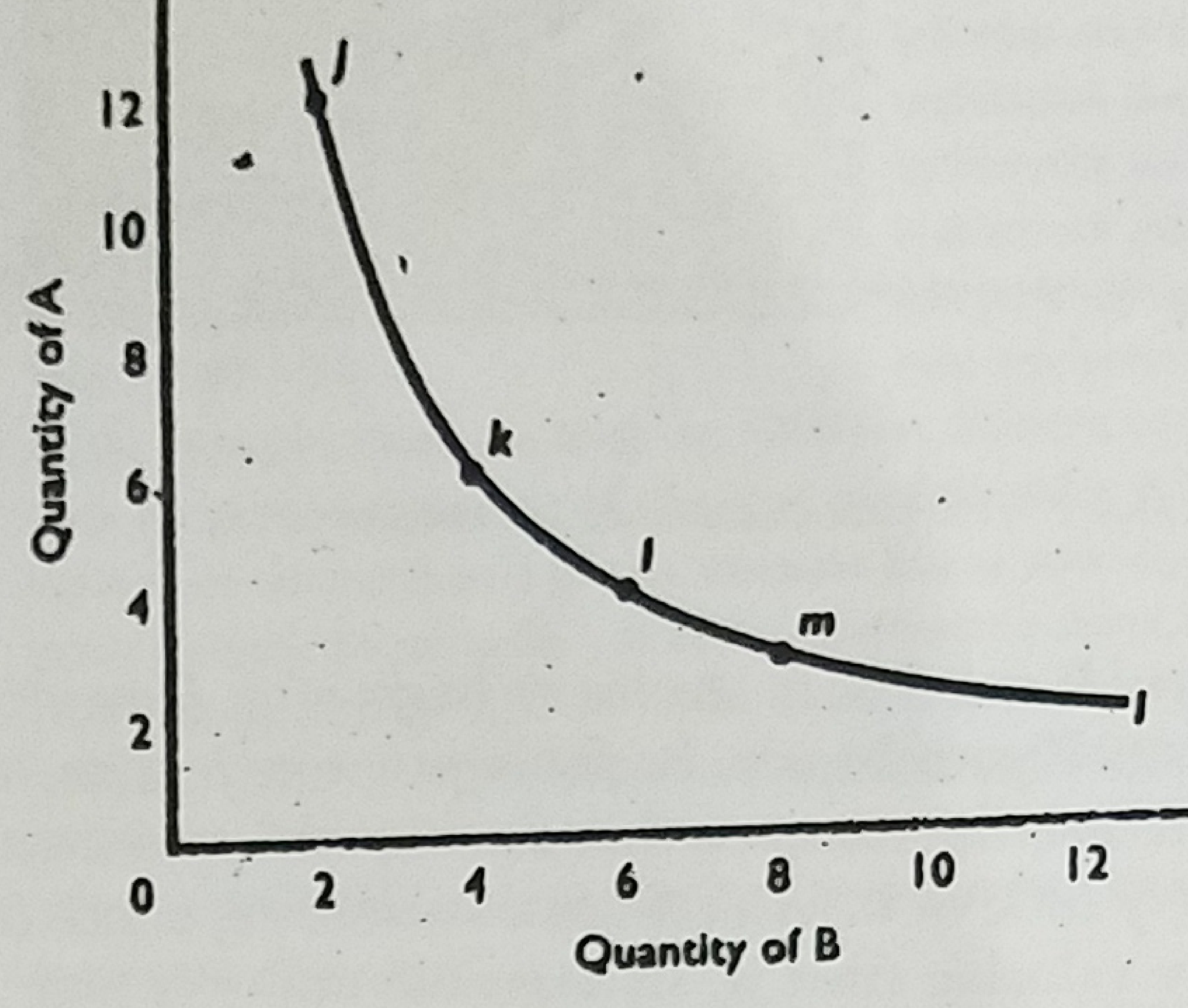
TABLE 2

An Indifference Schedule (Whole Units)

Combination	Units of A	Units of B
j	12	2
k	6	4
l	4	6
m	3	8

FIGURE 2

A consumer's indifference curve. Every point on indifference curve *I* represents some combination of products A and B, and all those combinations are equally satisfactory to the consumer. That is, each combination of A and B on the curve yields the same total utility.



Indifference Curves Are Convex to the Origin Recall from the appendix to Chapter 1 that the slope of a curve at a particular point is measured by drawing a straight line that is tangent to that point and then measuring the "rise over run" of the straight line. If you drew such straight lines for several points on the curve in Figure 2, you would find that their slopes decline (in absolute terms) as you move down the curve. An indifference curve is therefore convex (bowed inward) to the origin of the graph. Its slope diminishes or becomes flatter as we move down the curve from *j* to *k* to *l*, and so on. Technically, the slope of an indifference curve at each point measures the marginal rate of substitution (MRS) of the combination of two goods represented by that point. The slope or MRS shows the rate at which the consumer who possesses the combination will substitute one good for the other (say, B for A) to remain equally satisfied. The diminishing slope of the indifference curve means that the willingness to substitute B for A diminishes as more of B is obtained.

The rationale for this convexity—that is, for a diminishing MRS—is that a consumer's subjective willingness to substitute B for A (or A for B) will depend on the amounts of B and A he or she has to begin with. Consider Table 2 and Figure 2 again, beginning at point j. Here, in relative terms, the consumer has a substantial amount of A and very little of B. Within this combination, a unit of B is very valuable (that is, its marginal utility is high), while a unit of A is less valuable (its marginal utility is low). The consumer will then be willing to give up a substantial amount of A to get, say, 2 more units of B. In this case, the consumer is willing to forgo 6 units of A to get 2 more units of B; the MRS is $\frac{6}{2}$, or 3, for the jk segment of the curve.

But at point k the consumer has less A and more B. Here A is somewhat more valuable, and B less valuable, "at the margin." In a move from point k to point l, the consumer is willing to give up only 2 units of A to get 2 more units of B, so the MRS is only $\frac{2}{2}$, or 1. Having still less of A and more of B at point l, the consumer is willing to give up only 1 unit of A in return for 2 more units of B and the MRS falls to $\frac{1}{2}$ between l and m.[1]

In general, as the amount of B *increases*, the marginal utility of additional units of B *decreases*. Similarly, as the quantity of A *decreases*, its marginal utility *increases*. In Figure 2 we see that in moving down the curve, the consumer will be willing to give up smaller and smaller amounts of A to offset acquiring each additional unit of B. The result is a curve with a diminishing slope, a curve that is convex to the origin. The MRS declines as one moves southeast along the indifference curve.

The Indifference Map

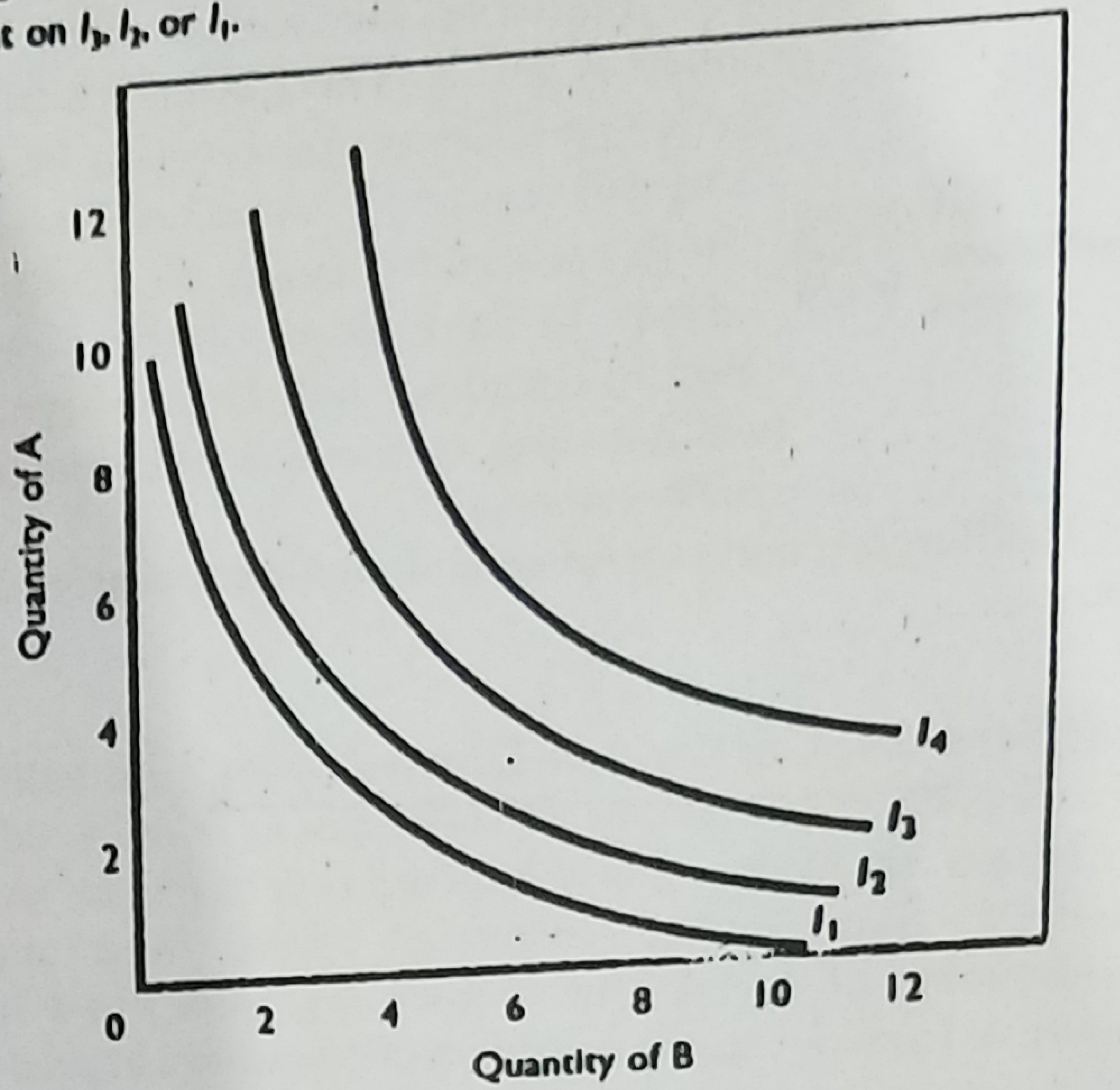
The single indifference curve of Figure 2 reflects some constant (but unspecified) level of total utility or satisfaction. It is possible and useful to sketch a whole series of indifference curves or an **indifference map**, as shown in Figure 3. Each curve reflects a different level of total utility. Specifically, each curve to the right of our original curve (labeled I_1 in Figure 3) reflects combinations of A and B that yield more utility than I_1. Each curve to the left of I_1 reflects less total utility than I_1. As we move out from the origin, each successive indifference curve represents a higher level of utility. To demonstrate this fact, draw a line in a northeasterly direction from the origin; note that its points of intersection with successive curves entail larger amounts of both A and B and therefore higher levels of total utility.

[1]MRS declines continuously between j and k, k and l, and l and m. Our numerical values for MRS relate to the curve segments between points and are not the actual values of the MRS at each point. For example, the MRS *at* point l is $\frac{2}{3}$.

Q ∝ 1/Marginal Utility

FIGURE 3

An indifference map. An indifference map is a set of indifference curves. Curves farther from the origin indicate higher levels of total utility. Thus any combination of products A and B represented by a point on I_4 has greater total utility than any combination of A and B represented by a point on I_3, I_2, or I_1.



Equilibrium at Tangency

Since the axes in Figures 1 and 3 are identical, we can superimpose a budget line on the consumer's indifference map, as shown in Figure 4. By definition, the budget line indicates all the combinations of A and B that the consumer can attain with his or her money income, given the prices of A and B. Of these attainable combinations, the consumer will prefer the combination that yields the greatest satisfaction or utility. Specifically, the utility-maximizing combination will be the combination lying on the highest attainable indifference curve. It is called the **consumer's equilibrium position.**

In Figure 4 the consumer's equilibrium position is at point X, where the budget line is *tangent* to I_3. Why not point Y? Because Y is on a lower indifference curve, I_2. By moving "down" the budget line—by shifting dollars from purchases of A to purchases of B—the consumer can attain an indifference curve farther from the origin and thereby increase the total utility derived from the same income. Why not point Z? For the same reason: Point Z is on a lower indifference curve, I_1. By moving "up" the budget line—by reallocating dollars from B to A—the consumer can get on higher indifference curve I_3 and increase total utility.

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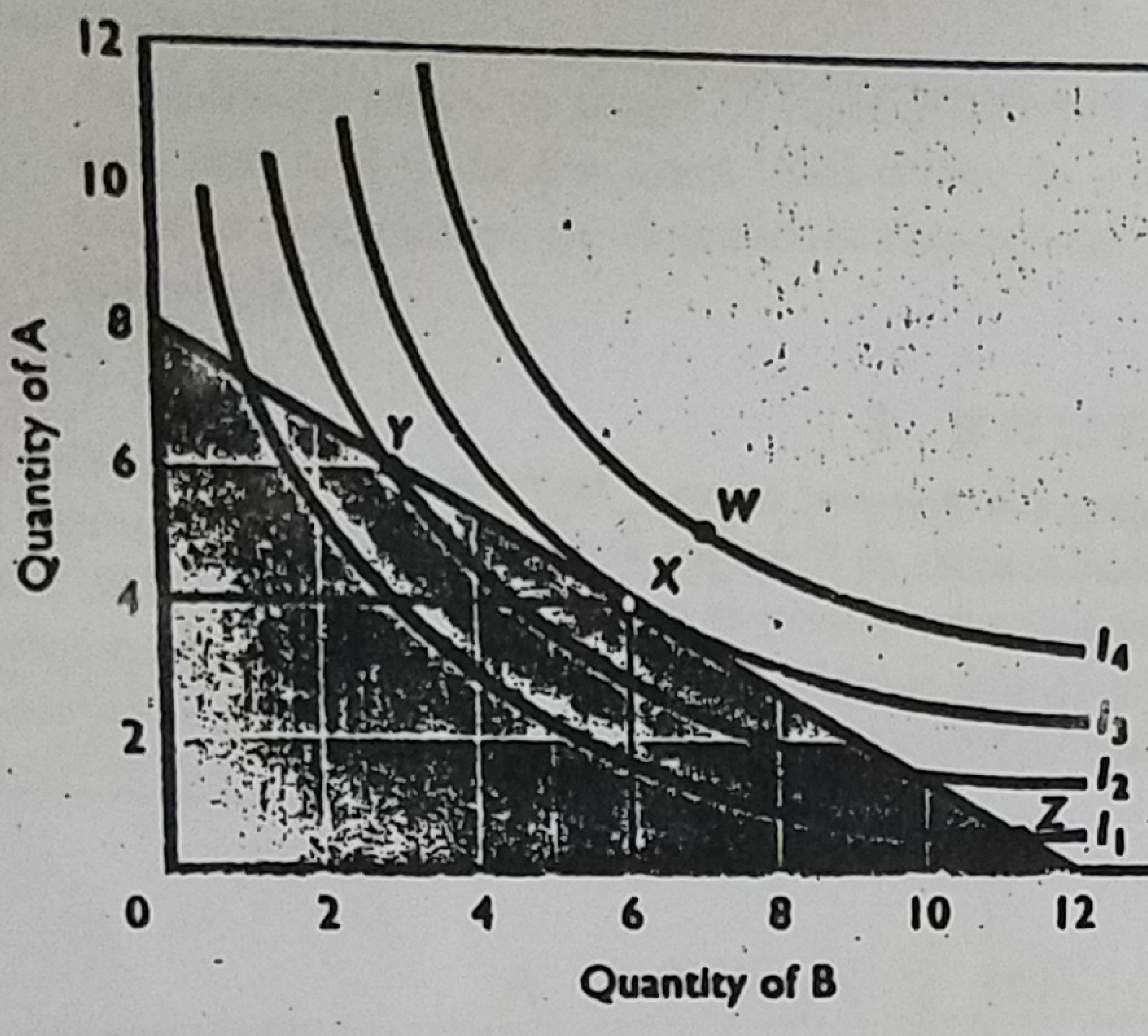
FIGURE 4

The consumer's equilibrium position. The consumer's equilibrium position is represented by point X, where the black budget line is tangent to indifference curve I_3. The consumer buys 4 units of A at $1.50 per unit and 6 of B at $1 per unit with a $12 money income. Points Z and Y represent attainable combinations of A and B but yield less total utility, as is evidenced by the fact that they are on lower indifference curves. Point W would entail more utility than X, but it requires a greater income than the $12 represented by the budget line.



How about point W on indifference curve I_4? While it is true that W would yield a greater total utility than X, point W is beyond (outside) the budget line and hence is *not* attainable by the consumer. Point X represents the optimal *attainable* combination of products A and B. Note that, according to the definition of tangency, the slope of the highest attainable indifference curve equals the slope of the budget line. Because the slope of the indifference curve reflects the MRS (marginal rate of substitution) and the slope of the budget line is P_B/P_A, the consumer's optimal or equilibrium position is the point where

$$\text{MRS} = \frac{P_B}{P_A}$$

(You may benefit by trying Appendix Key Question 3 at this time.)

CONSIDER THIS . . .

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Indifference Maps and Topographical Maps

The familiar topographical map may help you understand the idea of indifference curves and indifference maps. Each line on a topographical map represents a particular elevation above sea level, say, 4000 feet. Similarly, an indifference curve represents a particular level of total utility. When you move from one point on a specific elevation line to another, the elevation remains the same. So it is with an indifference curve. A move from one position to another on the curve leaves total utility unchanged. Neither elevation lines nor indifference curves can intersect. If they did, the meaning of each line or curve would be violated. An elevation line is "an equal-elevation line"; an indifference curve is "an equal-total-utility curve."

Like the topographical map, an indifference map contains not just one line but a series of lines. That is, the topographical map may have elevation lines representing successively higher elevations of 1000, 2000, 3000, 4000, and 5000 feet. Similarly, the indifference curves on the indifference map represent successively higher levels of total utility. The climber whose goal is to maximize elevation wants to get to the highest possible elevation line; the consumer desiring to maximize total utility wants to get to the highest possible indifference curve.

Finally, both topographical maps and indifference maps show only a few of the many such lines that could be drawn. The topographical map, for example, leaves out the elevation lines for 1001 feet, 1002, 1003, and so on. The indifference map leaves out all the indifference curves that could be drawn between those illustrated.

The Measurement of Utility

There is an important difference between the marginal-utility theory of consumer demand and the indifference curve theory. The marginal-utility theory assumes that utility is *numerically* measurable, that is, that the consumer can say how much extra utility he or she derives from each extra unit of A or B. The consumer needs that information to realize the utility-maximizing (equilibrium) position, as indicated by

$$\frac{\text{Marginal utility of A}}{\text{Price of A}} = \frac{\text{Marginal utility of B}}{\text{Price of B}}$$

The indifference curve approach imposes a less stringent requirement on the consumer. He or she need only specify whether a particular combination of A and B will yield more than, less than, or the same amount of utility as some other combination of A and B will yield. The consumer need only say, for example, that 6 of A and 7 of B will yield more (or less) satisfaction than will 4 of A and 9 of B. Indifference curve theory does not require that the consumer specify *how much* more (or less) satisfaction will be realized.

When we compare the equilibrium situations in the two theories, we find that in the indifference curve analysis the MRS equals P_B/P_A at equilibrium; however, in the marginal-utility approach the ratio of marginal utilities equals P_B/P_A. We therefore deduce that at equilibrium the MRS is equivalent in the marginal-utility approach to the ratio of the marginal utilities of the last purchased units of the two products.[2]

The Derivation of the Demand Curve

We noted earlier that with a fixed price for A, an increase in the price of B will cause the bottom of the budget line to fan inward to the left. We can use that fact to derive a demand curve for product B. In Figure 5a we reproduce the part of Figure 4 that shows our initial consumer equilibrium at point X. The budget line determining this equilibrium position assumes that money income is $12 and that P_A = $1.50 and P_B = $1. Let's see what happens to the equilibrium position when we increase P_B to $1.50 and hold both money income and the price of A constant.

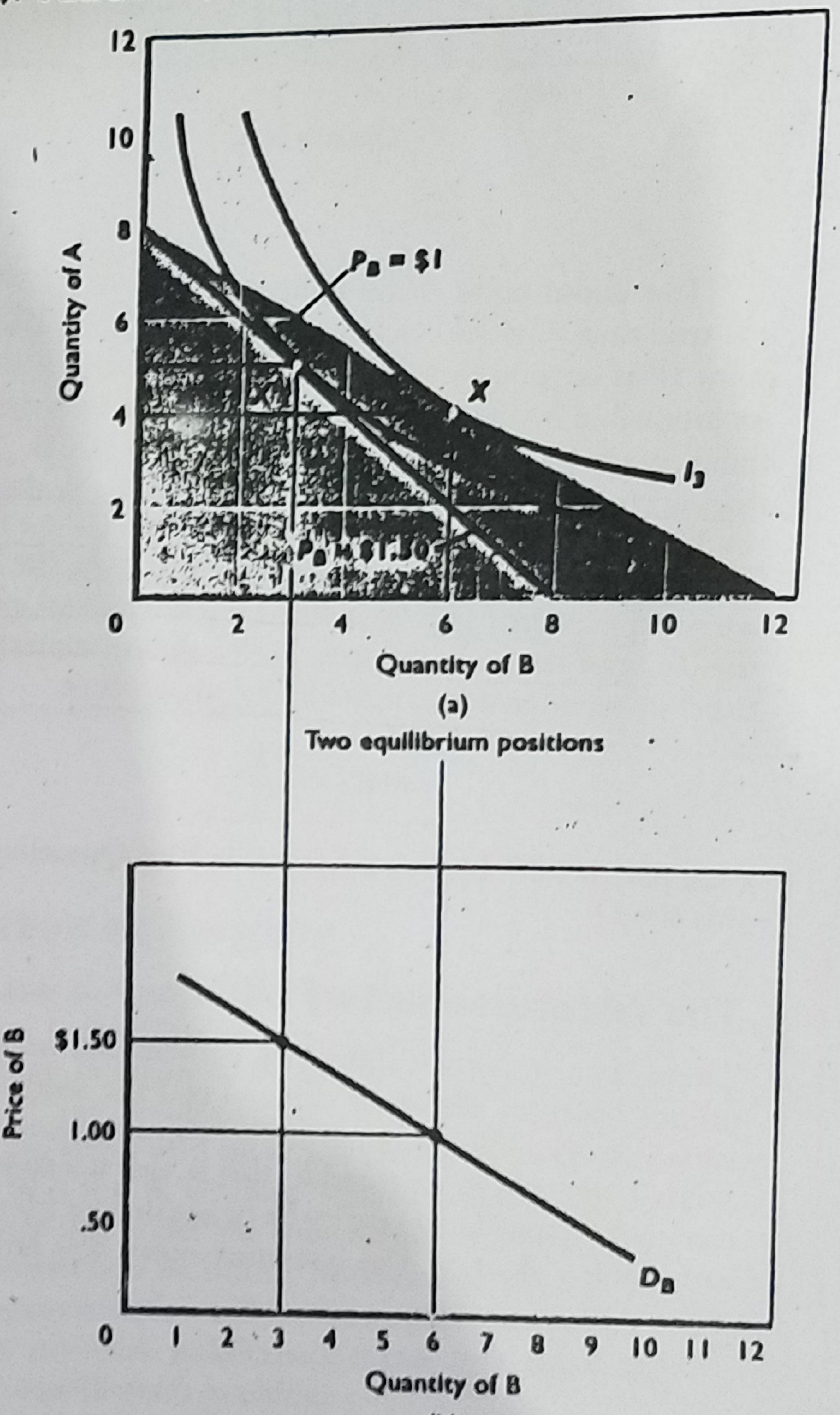
The result is shown in Figure 5a. The budget line fans to the left, yielding a new equilibrium point X′ where it is tangent to lower indifference curve I_2. At X′ the consumer buys 3 units of B and 5 of A, compared with 4 of A and 6 of B at X. Our interest is in B, and we now have sufficient information to locate two points on the demand curve for product B. We know that at equilibrium point X the price of B is $1 and 6 units are purchased; at equilibrium point X′ the price of B is $1.50 and 3 units are purchased.

These data are shown graphically in Figure 5b as points on the consumer's demand curve for B. Note that the horizontal axes of Figure 5a and 5b are identical; both measure the quantity demanded of B. We can therefore drop vertical reference lines from Figure 5a down to the horizontal axis of Figure 5b. On the vertical axis of Figure 5b we locate the two chosen prices of B. Knowing that these prices yield the relevant quantities demanded, we locate two points on the demand curve for B. By simple manipulation of the price of B in an indifference curve–budget line context, we have obtained a downward-sloping demand curve for B. We have thus again derived the law of demand assuming "other things equal," since only the price of B was changed (the price of A and the consumer's money income and tastes remained constant). But, in this case, we have derived the demand curve without resorting to the questionable assumption that consumers can measure utility in units called "utils." In this indifference curve approach, consumers simply compare combinations of products A and B and determine which combination they prefer, given their incomes and the prices of the two products.

[2]Technical footnote: If we begin with the utility-maximizing rule, $MU_A/P_A = MU_B/P_B$, and then multiply through by P_B and divide through by MU_A, we obtain $P_B/P_A = MU_B/MU_A$. In indifference curve analysis we know that at the equilibrium position MRS = P_B/P_A. Hence, at equilibrium, MRS also equals MU_B/MU_A.

FIGURE 5

Deriving the demand curve. (a) When the price of product B is increased from $1 to $1.50, the equilibrium position moves from X to X′, decreasing the quantity of product B demanded from 6 to 3 units. (b) The demand curve for product B is determined by plotting the $1–6-unit and the $1.50–3-unit price-quantity combinations for product B.



APPENDIX SUMMARY

1. The indifference curve approach to consumer behavior is based on the consumer's budget line and indifference curves.
2. The budget line shows all combinations of two products that the consumer can purchase, given product prices and his or her money income.
3. A change in either product prices or money income moves the budget line.
4. An indifference curve shows all combinations of two products that will yield the same total utility to a consumer. Indifference curves are downward-sloping and convex to the origin.
5. An indifference map consists of a number of indifference curves; the farther from the origin, the higher the total utility associated with a curve.
6. The consumer is in equilibrium (utility is maximized) at the point on the budget line that lies on the highest attainable indifference curve. At that point the budget line and indifference curve are tangent.
7. Changing the price of one product shifts the budget line and determines a new equilibrium point. A downsloping demand curve can be determined by plotting the price-quantity combinations associated with two or more equilibrium points.

APPENDIX TERMS AND CONCEPTS

budget line
indifference curve
marginal rate of substitution (MRS)
indifference map
equilibrium position

APPENDIX STUDY QUESTIONS

1. What information is embodied in a budget line? What shifts occur in the budget line when money income (*a*) increases and (*b*) decreases? What shifts occur in the budget line when the price of the product shown on the vertical axis (*a*) increases and (*b*) decreases?
2. What information is contained in an indifference curve? Why are such curves (*a*) downward-sloping and (*b*) convex to the origin? Why does total utility increase as the consumer moves to indifference curves farther from the origin? Why can't indifference curves intersect?
3. ***Appendix Key Question*** Using Figure 4, explain why the point of tangency of the budget line with an indifference curve is the consumer's equilibrium position. Explain why any point where the budget line intersects an indifference curve is not equilibrium. Explain: "The consumer is in equilibrium where MRS = P_B/P_A."
4. Assume that the data in the accompanying table give an indifference curve for Mr. Chen. Graph this curve, putting A on the vertical axis and B on the horizontal axis. Assuming that the prices of A and B are \$1.50 and \$1, respectively, and that Mr. Chen has \$24 to spend, add his budget line to your graph. What combination of A and B will Mr. Chen purchase? Does your answer meet the MRS = P_B/P_A rule for equilibrium?

Units of A	Units of B
16	6
12	8
8	12
4	24

5. Explain graphically how indifference analysis can be used to derive a demand curve.
6. ***Advanced Analysis*** Demonstrate mathematically that the equilibrium condition MRS = P_B/P_A is the equivalent of the utility-maximizing rule $MU_A/P_A = MU_B/P_B$.