# Al Lab2

#### PB18111699

## 魏钊

#### linearClassification:

## 参考博客:

https://zhuanlan.zhihu.com/p/92764814

https://www.cnblogs.com/nowgood/p/lagrangemultipy1.html

https://blog.csdn.net/dou3516/article/details/78795721

$$\mathbf{X} = \begin{bmatrix} 1 & x_{11} & \cdots & x_{1d} \\ \vdots & & \vdots & \\ 1 & x_{N1} & \cdots & x_{Nd} \end{bmatrix}, \qquad \mathbf{y} = \begin{bmatrix} y_1 \\ \vdots \\ y_N \end{bmatrix}, \qquad \mathbf{w} = \begin{bmatrix} \boldsymbol{b} \\ \vdots \\ w_d \end{bmatrix}$$

首先构造 x,w

X 为训练特征数据左接一列 1。

W 为随机生成。

x = np.c\_[np.ones(train\_features.shape[0]), train\_features] # 构建x矩阵
w = np.random.random(train\_features.shape[1] + 1) # 随机生成w, 其值在(0, 1)
# print(w)
w = w.reshape(-1, 1) # 转换为列向量

线性函数:  $f(x)=w_0+w_1x_1+\ldots+w_nx_n=w^Tx$ 

损失函数:  $J(w) = \sum_{i=1}^n \, (w^T x^i - y^i)^2$ 

梯度:  $rac{\partial J(w)}{\partial w} = \sum_{i=1}^n 2(w^Tx^i - y^i) * x_j^{(i)}$ 

更新规则: BGD:  $w_j = w_j - 2lpha \sum_{i=1}^n (w^T x^i - y^i) x_j^i$ 

$$J_{LS}( heta) = rac{1}{2} \|\Phi heta - y\|^2 \ \min(J_{LS}( heta))$$
 约束条件  $\| heta\|^2 < R$ 

该原始问题可以转化对偶问题

$$\max_{\lambda} \min_{\theta} \left[ J_{LS}(\theta) + \frac{\lambda}{2} \left( \|\theta\|^2 - R \right) \right] \quad \text{spread}$$

lagrange 对偶问题的 拉格朗日乘子  $\lambda$  的解由 R 决定. 如果不根据 R 来决定 R, 而是直接指定的话, $l_2$  约束的最小二乘学习法的解  $\hat{\theta}$  可以通过下式求得

$$\hat{ heta} = \arg\min_{ heta} \left[ J_{LS} heta) + rac{\lambda}{2} \| heta\|^2 
ight]$$
 (10)

 $J_{LS} heta)$  表示的是训练样本的拟合程度,与  $rac{\lambda}{2}\|oldsymbol{\theta}\|^2$ 结合求最小值,来防止训练样本的过拟合.  $l_2$ 正则化的最小二乘学习法也称为 岭回归 .

$$\begin{split} \frac{\partial (J_{LS}\theta) + \frac{\lambda}{2} \|\theta\|^2)}{\partial \theta} &= \Phi^T (\Phi\theta - y) + \theta = 0\\ \hat{\theta} &= (\Phi^T \Phi + \lambda I)^{-1} \Phi^T y \end{split}$$

#### 再由上述公式开始讲行迭代。

```
now_epochs = self.epochs # 取训练轮数
while now_epochs > 0:
    xw = x.dot(w) # 计算x*w
    xw_y = xw - train_labels # 计算xw-y
    xw_y_T = xw_y.reshape(xw_y.shape[1], xw_y.shape[0]) # 转置
    gradient = 2*np.dot(xw_y_T, x)/len(train_features) + 2*self.Lambda * w.reshape(1, -1) # 得到梯度
    w -= self.lr * gradient.reshape(-1, 1)
    now_epochs -= 1

self.w = w # 训练结束得到w
```

## 根据下面公式进行预测:

# Prediction for X<sub>0</sub>

$$\hat{y} = \operatorname{sign}\left(\mathbf{w}^{*\top} \begin{bmatrix} 1 \\ \mathbf{x}_0 \end{bmatrix}\right) = \operatorname{sign}\left(\mathbf{y}^{\top} X^{+\top} \begin{bmatrix} 1 \\ \mathbf{x}_0 \end{bmatrix}\right)$$

```
def predict(self, test_features):
    x = np.c_[np.ones(test_features.shape[0]), test_features] # 构造x
    i = test_features.shape[0] # 测试数据的数量
    Prediction = [] # 预测结果
    j = 0
    # 预测类别
    while j < i:
        y = x[j].dot(self.w)
        if y >= 2.5:
            Prediction.append(3)
        elif y >= 1.75:
            Prediction.append(2)
        else:
            Prediction.append(1)
        j += 1

    Prediction = np.array(Prediction).reshape(i, 1) # 格式化
    return Prediction
```

这里 1.75 和 2.5 的选择是经过几次测试后发现,在这两个值的情况下正确率较高。

# 测试结果:

```
C:\Python_Anaconda\envs\myImpl.py\python.exe C:/Users/Lucifer.dark/Desktop/2021春/人工智能/LAB2/src1/linearclassification.py
train_num: 3554
test_num: 983
train_feature's shape:(3554, 8)
test_feature's shape:(983, 8)
Acc: 0.6408952187182095
0.754880694143167
0.5850673194614443
0.6308139534883721
macro-F1: 0.6569206556976611
micro-F1: 0.6408952187182095

Process finished with exit code 0
```

预测成功率可达到 64%左右。

## nBayesClassifier:

依据下图进行训练:

$$\hat{P}(c) = \frac{|D_c| + 1}{|D| + N},$$

$$\hat{P}(x_i|c) = \frac{|D_{c,x_i}| + 1}{|D_c| + N_i},$$

这里对性别当成离散型计算,对其他属性当作连续属性计算,使用高斯分布估计条件概率。

## 先进行预处理:

```
D_c = {} # 统计各类的总数
for i in range(1, 4): # 初始化为0
   D_c[i] = 0
D_c_sex = {} # 统计各类不同性别的数量
for i in range(1, 4): # 初始化为0
   for j in range(1, 4):
       D_c_sex[i, j] = 0
column = {} # 不同类的连续型属性的值集合
for i in range(traindata.shape[0]): # 遍历数据,统计各种数量
   D_c[int(trainlabel[i])] += 1 # 各类的总数
   D_c_sex[(int(trainlabel[i]), int(traindata[i][0]))] += 1 # 各类不同性别的总数
   for j in range(1, 8): # 统计连续型属性
       if (int(trainlabel[i]), j) not in column.keys():
          column[int(trainlabel[i]), j] = np.array(float(traindata[i][j])) # 第一次遇到该类属性
          column[int(trainlabel[i]), j] = np.append(column[int(trainlabel[i]), j],
                                                 float(traindata[i][j])) # 后续遇到进行加入即可
Sum_D = D_c[1] + D_c[2] + D_c[3] # 总数
################预处理完成###############
```

这里统计了给类别各属性的数量。

上图中包含完整注释。

## 预处理完成后,进行概率计算:

特别注意,各种概率都转换为了 log 型。

连续型使用了高斯分布表示。

下面进行预测:

$$h_{nb}(x) = \operatorname{argmax}_{c \in \mathbf{Y}} P(c) \prod_{i=1}^d P(x_i|c)$$

即找到使目标函数最大的参数类别c。

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

根据上面公式计算概率。

因为都转化为了 log 形式,这里进行的是加法而不是乘法。

## 训练结果:

```
C:\Python_Anaconda\envs\myImpl.py\python.exe C:/Users/Lucifer.dark/Desktop/2021春/人工智能/LAB2/src1/nBayesClassifier.py train_num: 3554
test_num: 983
train_feature's shape:(3554, 8)
test_feature's shape:(983, 8)
Acc: 0.5839267548321465
0.7038461538461537
0.4725111441307578
0.6244952893674293
macro-F1: 0.6002841957814469
micro-F1: 0.5929752066115703

Process finished with exit code 0
```

准确率在60%左右。

#### SVM:

## 参考博客:

https://blog.csdn.net/QW sunny/article/details/79793889

https://blog.csdn.net/weixin\_35755640/article/details/113660632

## 原问题等价于如下:

$$egin{aligned} \min_{lpha} rac{1}{2} \sum_{i=1}^m \sum_{j=1}^m lpha_i lpha_j y_i y_j x_i^T x_j - \sum_{i=1}^m lpha_i \ &s.t. \quad \sum_{i=1}^m lpha_i y_i = 0 \ &0 \leq lpha_i \leq C, i = 1, 2, \ldots, m \end{aligned}$$

## 将其化为如下形式即可求解:

minimize 
$$(1/2)x^TPx + q^Tx$$
  
subject to  $Gx \leq h$   
 $Ax = b$ 

# 首先进行预处理计算所需的 p,q,G,h,A,b:

上述代码有详尽注释。

# 之后利用线性规划求解器求解 alpha:

## 利用如下公式恢复 b:

$$b = y_i - \sum_{j=1}^n \alpha_j y_j K(\mathbf{x}_i, \mathbf{x}_j) \quad \text{for any } i \text{ that } \alpha_i \neq 0$$

```
index = np.where(alpha >= self.Epsilon)[0] # 找到所有值不低于阈值的index

# 利用alpha计算b
b = np.mean(
    [train_label[i] - sum(
        [train_label[i] * alpha[i] * self.KERNEL(x, train_data[i], self.kernel) for x in train_data[index]])
    for i in index])
```

## 最后利用如下公式进行预测:

```
y^* = \operatorname{sign}\left(\sum_{i \in SV} \alpha_i y_i K\left(\mathbf{x}_i, \mathbf{x}'\right) + b\right)
```

```
####进行预测####

predictions = []

for j in range(test_data.shape[0]):

    y = b + sum(
        [train_label[i] * alpha[i] * self.KERNEL(test_data[j], train_data[i], self.kernel) for i in index])
    predictions.append(y)

y = np.array(predictions).reshape(test_data.shape[0], 1)

return y
```

## 测试结果如下:

```
C:\Python_Anaconda\envs\myImpl.py\python.exe C:/Users/Lucifer.dark/Desktop/2021春/人工智能/LAB2/src1/SVM.py
test_num: 983
test_feature's shape: (983, 8)
                                  1e-01
9: -1.1162e+03 -1.1438e+03 3e+01 4e-04 2e-13
14: -1.1266e+03 -1.1275e+03 9e-01 2e-06 2e-13
Optimal solution found.
4: -2.6536e+03 -2.9271e+03 3e+02 4e-03 3e-13
                                  2e-03
9: -2.7432e+03 -2.7953e+03 5e+01 4e-04 4e-13
                                  5e-06
14: -2.7648e+03 -2.7655e+03 7e-01 1e-06 4e-13
15: -2.7651e+03 -2.7652e+03 7e-02 6e-08 4e-13
17: -2.7651e+03 -2.7651e+03 6e-04 4e-10 4e-13
4: -1.8490e+03 -2.0498e+03 2e+02 3e-03
5: -1.8550e+03 -2.0397e+03 2e+02 2e-03 3e-13
9: -1.9125e+03 -1.9453e+03 3e+01 8e-05
10: -1.9211e+03 -1.9341e+03 1e+01 1e-05 4e-13
11: -1.9252e+03 -1.9293e+03 4e+00 3e-06 4e-13
12: -1.9267e+03 -1.9276e+03 9e-01 4e-07 4e-13
                           9e-02
14: -1.9271e+03 -1.9271e+03 4e-03 2e-09 4e-13
15: -1.9271e+03 -1.9271e+03 4e-05 2e-11 4e-13
Acc: 0.6581892166836215
0.568733153638814
Process finished with exit code 0
```

## MLP\_manual:

首先随机生成各种所需的参数:

```
# 输入数据,随机生成
x = torch.rand(size=(100, 5), requires_grad=True)
y = torch.randint(3, size=(100, 1))
w1 = torch.rand(size=(4, 5), requires_grad=True)
w2 = torch.rand(size=(4, 4), requires_grad=True)
w3 = torch.rand(size=(3, 4), requires_grad=True)
```

#### 初始化:

```
def __init__(self, x, y, w1, w2, w3, lr=0.01, epochs=500):
    self.x = x
    self.y = y
    self.lr = lr
    self.epochs = epochs
    self.w1 = w1
    self.w2 = w2
    self.w3 = w3
```

## 激活函数如下:

Sigmoid函数由下列公式定义

$$S\left( x
ight) =rac{1}{1+e^{-x}}$$

## 前向传播部分:

第一层到第二次(5-4):

首先计算: WX

```
wx = torch.mm(self.w1, torch.transpose(self.x, 0, 1)) # 计算 wx12
```

## 使用激活函数计算 y2:

```
self.y2 = torch.transpose(torch.div(1, 1 + torch.exp(-wx)), 0, 1) # 第二层的激活函数输出 4-4:
```

```
####4-4####
wx = torch.mm(self.w2, torch.transpose(self.y2, 0, 1)) # 计算 wx23
self.y3 = torch.transpose(torch.div(1, 1 + torch.exp(-wx)), 0, 1) # 第三层的激活函数输出
```

特别注意最后一次 Softmax 函数如下:

$$s_3(x_1, x_2, x_3) = Softmax(x_1, x_2, x_3)$$

$$= \frac{1}{e^{x_1} + e^{x_2} + e^{x_3}} (e^{x_1}, e^{x_2}, e^{x_3})$$

```
####4-3####
wx = torch.mm(self.w3, torch.transpose(self.y3, 0, 1)) # 计算 wx34
y4 = torch.exp(torch.transpose(wx, 0, 1))
s = y4.sum(1) # 求分母上的和
self.y4 = torch.div(y4, s.reshape(-1, 1)) # 输出结果
```

Loss 的计算如下:

$$\ell(y, \hat{y}) = CrossEntropy(y, \hat{y}) = -\log \hat{y}_i$$
,  $i = y$ 

```
####求loss###

self.loss = torch.zeros(1) # 初始化

for j in range(self.x.shape[0]):
    self.loss = self.loss - torch.log(self.y4[j][self.y[j]])

self.loss = self.loss / self.x.shape[0]
```

## 反向传播部分:

从 W3 开始一直到 W1, 根据以下公式完成计算:

$$(\ell' \boldsymbol{s}_3')_i = \begin{cases} \hat{y}_i - 1, i = y \\ \hat{y}_i, i \neq y \end{cases}$$

$$\frac{\partial L}{\partial \mathbf{W}_{1}} = (\mathbf{W}_{2}^{\mathrm{T}} (\mathbf{W}_{3}^{\mathrm{T}} (\ell' \mathbf{s}_{3}') \odot \mathbf{s}_{2}') \odot \mathbf{s}_{1}') \mathbf{x}^{\mathrm{T}}$$

$$\frac{\partial L}{\partial \mathbf{W}_{2}} = (\mathbf{W}_{3}^{\mathrm{T}} (\ell' \mathbf{s}_{3}') \odot \mathbf{s}_{2}') \mathbf{h}_{1}^{\mathrm{T}}$$

$$\frac{\partial L}{\partial \mathbf{W}_{3}} = (\ell' \mathbf{s}_{3}') \mathbf{h}_{2}^{\mathrm{T}}$$

$$f'(\mathbf{u}_l) = sigmoid'(\mathbf{u}_l) = sigmoid(\mathbf{u}_l)(1 - sigmoid(\mathbf{u}_l)) = \mathbf{y}_l(1 - \mathbf{y}_l)$$

```
self.WL3 = self.y4
for j in range(self.y4.shape[0]):
    self.WL3[j][self.y[j]] = self.WL3[j][self.y[j]] - 1
self.WL2 = torch.mm(self.WL3, self.w3)
self.WL3 = torch.mm(torch.transpose(self.WL3, 0, 1), self.y3)
```

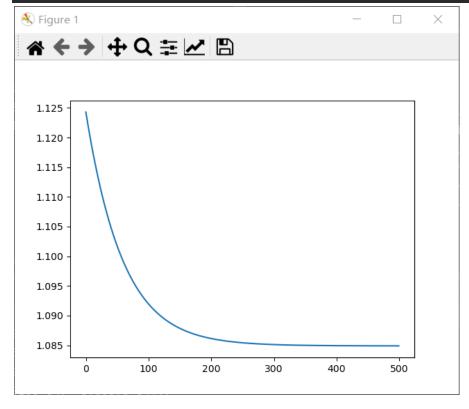
```
self.WL1 = torch.mm(self.WL2 * (self.y3 * (1 - self.y3)), self.w2)
self.WL2 = torch.mm(torch.transpose(self.WL2 * (self.y3 * (1 - self.y3)), 0, 1),
self.y2)
self.WL1 = torch.mm(torch.transpose(self.WL1 * (self.y2 * (1 - self.y2)), 0, 1),
self.x)
```

# 梯度下降部分:

## 梯度下降算法

$$\boldsymbol{W_i} = \boldsymbol{W_i} - \eta \frac{\partial L}{\partial \boldsymbol{W_i}}$$

```
loss = []
for i in range(self.epochs):
    self.Forwarding()
    temp = 0
    for j in range(self.x.shape[0]):
        temp = temp - math.log(self.y4[j][self.y[j]])
    temp = temp / self.x.shape[0]
    loss.append(temp)
    self.Backwarding()
    # 梯度下降
    self.w1 = self.w1 - self.lr * (self.WL1 / self.x.shape[0])
    self.w2 = self.w2 - self.lr * (self.WL2 / self.x.shape[0])
    self.w3 = self.w3 - self.lr * (self.WL3 / self.x.shape[0])
plt.plot(loss)
plt.show()
```



以上为 loss 曲线。

## 各参数矩阵的梯度如下(方便起见进行了一轮比较):

```
C:\Python_Anaconda\python.exe C:/Users/Lucifer.dark/Desktop/2021春/人工智能/LAB2/src2/MLP_manual.py
自动计算W3:
tensor([[ 0.0201, 0.0184, 0.0174, 0.0207],
       [ 0.0836, 0.0772, 0.0711, 0.0851],
       [-0.1037, -0.0955, -0.0884, -0.1057]])
手动计算W3:
tensor([[ 0.0201, 0.0184, 0.0174, 0.0207],
       [ 0.0836, 0.0772, 0.0711, 0.0851],
       [-0.1037, -0.0955, -0.0884, -0.1057]], grad_fn=<DivBackward0>)
自动计算W2:
tensor([[ 0.0034, 0.0030, 0.0032, 0.0030],
       [ 0.0014, 0.0012, 0.0013, 0.0012],
       [-0.0067, -0.0059, -0.0062, -0.0059],
       [ 0.0028, 0.0025, 0.0026, 0.0025]])
手动计算W3:
tensor([[ 0.0034, 0.0030, 0.0032, 0.0030],
       [ 0.0014, 0.0012, 0.0013, 0.0012],
       [-0.0067, -0.0059, -0.0062, -0.0059],
       [ 0.0028, 0.0025, 0.0026, 0.0025]], grad_fn=<DivBackward0>)
自动计算W1:
tensor([[ 2.8901e-04, 8.6058e-05, 2.8591e-04, 5.0822e-04, 5.2968e-04],
       [-1.6404e-04, -8.9121e-05, -1.7803e-04, -1.8509e-04, -1.7736e-04],
       [ 2.7710e-04, 1.4336e-04, 3.0345e-04, 3.8993e-04, 4.5165e-04],
       [ 5.7307e-04, 2.5546e-04, 5.9403e-04, 8.0813e-04, 8.8359e-04]])
手动计算W1:
tensor([[ 2.8901e-04, 8.6058e-05, 2.8591e-04, 5.0822e-04, 5.2968e-04],
       [-1.6404e-04, -8.9121e-05, -1.7803e-04, -1.8509e-04, -1.7736e-04],
       [ 2.7710e-04, 1.4336e-04, 3.0345e-04, 3.8993e-04, 4.5165e-04],
       [ 5.7307e-04, 2.5546e-04, 5.9403e-04, 8.0813e-04, 8.8359e-04]],
      grad_fn=<DivBackward0>)
Process finished with exit code 0
```

可以看到手动计算的和自动计算得到的一样。

#### **MLP Mixer:**

## 参考:

https://blog.csdn.net/u013468614/article/details/117220561?ops\_request\_misc=%2
578%2522request%255Fid%2522%253A%2522162604716016780271527677%2522%252C%
2522scm%2522%253A%252220140713.130102334.pc%255Fall.%2522%257D&request\_id=
162604716016780271527677&biz\_id=0&utm\_medium=distribute.pc\_search\_result.non
e-task-blog-2~all~first\_rank\_v2~rank\_v29-7117220561.first\_rank\_v2\_pc\_rank\_v29&utm\_term=mlpmixer&spm=1018.2226.3001.4187
https://github.com/lucidrains/mlp-mixerpytorch/blob/main/mlp\_mixer\_pytorch/mlp\_mixer\_pytorch.py
https://github.com/rishikksh20/MLP-Mixer-pytorch/blob/master/mlp-mixer.py

https://github.com/920232796/MlpMixer-pytorch/blob/master/MlpMixer/model.py

# mixer\_layer 部分:

### MLPMixer 部分:

## 利用卷积实现:

### train:

```
optimizer.zero_grad()
loss = criterion(model(data), target)
loss.backward()
optimizer.step()
```

#### test:

#### main:

```
optimizer = torch.optim.Adam(model.parameters(), lr=learning_rate)
criterion = torch.nn.CrossEntropyLoss()
```

### 测试:

```
C:\Python_Anaconda\python.exe C:/Users/Lucifer.dark/Desktop/2021春/人工智能/LAB2/src2/MLP_Mixer.py
Train Epoch: 0/5 [0/60000] Loss: 2.356595
Train Epoch: 0/5 [12800/60000] Loss: 1.010329
Train Epoch: 0/5 [25600/60000] Loss: 0.561464
Train Epoch: 0/5 [38400/60000] Loss: 0.439387
Train Epoch: 0/5 [51200/60000] Loss: 0.262845
Train Epoch: 1/5 [0/60000] Loss: 0.214819
Train Epoch: 1/5 [12800/60000] Loss: 0.155095
Train Epoch: 1/5 [25600/60000] Loss: 0.189344
Train Epoch: 1/5 [38400/60000] Loss: 0.253254
Train Epoch: 1/5 [51200/60000] Loss: 0.106105
Train Epoch: 2/5 [0/60000] Loss: 0.179545
Train Epoch: 2/5 [12800/60000] Loss: 0.091536
Train Epoch: 2/5 [25600/60000] Loss: 0.189698
Train Epoch: 2/5 [38400/60000] Loss: 0.159349
Train Epoch: 2/5 [51200/60000] Loss: 0.123501
Train Epoch: 3/5 [0/60000] Loss: 0.266544
Train Epoch: 3/5 [12800/60000] Loss: 0.154652
Train Epoch: 3/5 [25600/60000] Loss: 0.125598
Train Epoch: 3/5 [38400/60000] Loss: 0.103392
Train Epoch: 3/5 [51200/60000] Loss: 0.107305
Train Epoch: 4/5 [0/60000] Loss: 0.114238
Train Epoch: 4/5 [12800/60000] Loss: 0.148439
Train Epoch: 4/5 [25600/60000] Loss: 0.102706
Train Epoch: 4/5 [38400/60000] Loss: 0.132604
Train Epoch: 4/5 [51200/60000] Loss: 0.237750
Test set: Average loss: 0.0011 Acc 0.96
```

准确率达到了96%。