Example Assignment: Dependency Theory

COMPSCI 2DB3: Databases

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Description

The Professor S. Marty Pants, a recent faculty hire of the University, is convinced that they are the smartest database person of all times. To impress people, the Professor often states problems that the Professor then claims are *almost impossible* to solve and then shows how to solve them. Next, we will take a look at a few of these problems from three categories:

D1 Reasoning with dependencies

1. Prove that $\{AB \longrightarrow C, A \longrightarrow D, CD \longrightarrow EF\} \models AB \longrightarrow F$ holds using only the Armstrong Axioms.
2. Prove the soundness of the following inference rule directly from the definition of functional dependencies (without using any inference rules):

$$\text{if } X \longrightarrow Y \text{ and } YW \longrightarrow Z, \text{ then } XW \longrightarrow Z.$$

3. Prove the soundness of the following inference rule for inclusion dependencies:

$$\text{if } R[X] \subseteq S[Y] \text{ and } S[Y] \subseteq T[Z], \text{ then } R[X] \subseteq T[Z].$$

4. Prove the soundness of the following inference rule

$$\text{if } X \twoheadrightarrow Y \text{ and } XY \longrightarrow Z, \text{ then } X \longrightarrow Z \setminus (X \cup Y).$$

5. Prove that the following inference rule *is not sound*:

$$\text{if } XW \longrightarrow Y \text{ and } XY \longrightarrow Z, \text{ then } X \longrightarrow Z.$$

   Hint: Look for a counterexample by constructing a table in which $XW \longrightarrow Y$ and $XY \longrightarrow Z$ hold, but $X \longrightarrow Z$ does not hold.

D2 A completeness proof for Armstrong

Consider the following inference rules.

R1. *Reflexivity*. If $Y \subseteq X$, then $X \longrightarrow Y$.

R2. *Augmentation*. If $X \longrightarrow Y$ then $XZ \longrightarrow YZ$ for any Z.

R3. *Transitivity*. If $X \longrightarrow Y$ and $Y \longrightarrow Z$, then $X \longrightarrow Z$.

6. Prove that the inference rules R1, R2, and R3 are *complete*: prove that if $\mathfrak{S} \models X \longrightarrow Y$ holds for some set of functional dependencies $\mathfrak{S}$, then we can derive $X \longrightarrow Y$ from $\mathfrak{S}$ using only the inference rules R1, R2, and R3.

   Hint: Use the fact that the Closure algorithm is complete. Can you prove that any *sound* derivation made by the Closure algorithm can also be derived using the inference rules R1–R3? You may use the Union rule and the Decomposition rule (as we have derived them using the inference rules R1–R3).

D3 The human computer

Consider the relational schema $\mathbf{r}(A, B, C, D, E)$ and the following set of functional dependencies:

$$\mathfrak{S} = \{AB \longrightarrow D, AC \longrightarrow E, BC \longrightarrow D, C \longrightarrow A, D \longrightarrow A, E \longrightarrow B\}.$$

Answer the following questions:

7. Provide the attribute closure of set of attributes C (hence, C^+) and of set of attributes EA (hence, $(EA)^+$) with respect to $\mathfrak{S}$. Explain your steps.

8. Compute the closure $\mathfrak{S}^+$. Explain your steps. Based on the closure, indicate which attributes are *superkeys* and which attributes are *keys*.

   Hint: A *superkey* is a set of attributes that determines *all* attributes from $\mathbf{r}$. A *key* k is a superkey of minimal size (if we remove any attribute from k, it is no longer a key).

9. Provide a minimal cover for $\mathfrak{S}$. Explain your steps.

Assignment

The goal of the assignment is to show that Professor S. Marty Pants is just an average genius by showing that many people can solve its “problems”. To do so, solve each of the above problems.