COMPSCI 2DB3
Assignment 5
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P1.1)

If X –> Y and V ⊆ (W ∪ Y), then XW → V.

1. Given X → Y
2. Using Augmentation with rule X → Y to obtain XW → YW.
3. Given V ⊆ (W ∪ Y).
(Optional step -> Reflexivity of V ⊆ (W ∪ Y) to W → V and Y → V)
4. Using decomposition we decompose XW → YW to XW → V.

P1.2)

If X → Y and Y → Z, then X → XYZ.

Solution:

Let R be any relational schema that satisfies both X → Y and Y → Z. By definition, relational schema R satisfies X → XYZ if we have r1[X] = r2[X] ⇒ r1[XYZ] = r2[XYZ] for every instance I of R and every pair of rows $r1$, $r2$ ∈ I.
Assume we have rows $r1$,$r2$ ∈ I of instance I of R with r1[X] = r2[X].

- Through the description above, we conclude r1[X] = r2[X].
- By r1[X] = r2[X] and X → Y, we conclude r1[Y] = r2[Y].
- By r1[XYZ] = r2[XYZ] and r1[Y] = r2[Y], we conclude r1[Z] = r2[Z].

P1.3)

$$\emptyset \rightarrow \emptyset$$

In general, we know that every set is a subset of its own, therefore, $\emptyset \subseteq \emptyset$. If we apply reflexivity to $\emptyset \subseteq \emptyset$, we get $\emptyset \rightarrow \emptyset$. Hence proved.

P1.4)

1. If X –> Y and V ⊆ (W ∪ Y), then XW → V.
2. If X → Y and Y → Z, then X → XYZ.
3. $\emptyset \rightarrow \emptyset$

A set of inference rules is complete if we can derive any functional dependency D from any set of functional dependencies $\mathfrak{S}$ whenever $\mathfrak{S} \models D$ holds. To show that the rules are complete, we use the fact that the Armstrong Axioms are complete: using the Armstrong Axioms, we can derive D from $\mathfrak{S}$ whenever $\mathfrak{S} \models D$ holds. Hence, if we can derive the Armstrong Axioms using our three rules, then any derivation we can make with the Armstrong Axioms can also be done using our three rules (by replacing the Armstrong Axioms by our derivation of these Axioms). Next, we shall derive the Armstrong Axioms using our three rules.

Reflexivity If Y ⊆ X , then X → Y
Assume Y ⊆ X

- Use rule 3 to derive $\varnothing \rightarrow \varnothing$.
- Use rule 1 with $\varnothing \rightarrow \varnothing$ and $Y \subseteq X$ to derive $X \rightarrow Y$.

Augmentation if $X \rightarrow Y$ and $XZ \rightarrow YZ$ for any Z

Assume we have $X \rightarrow Y$

- Notice that $Z \subseteq Z$

P3.1)

$$\mathfrak{S} = \{ A \rightarrow B, BC \rightarrow D, CD \rightarrow E, AC \rightarrow DE, ABD \rightarrow C\}$$
$$A^{+} = \{A,B\}$$
$$AC^{+} = \{A,B,C,D,E\}$$

Finding an attribute closure for A, it will be included in the set directly. From the equation $A \rightarrow B$, we derive B. For finding a set for A, C it will also be included in the set directly. We derive B from the same functional dependency - $A \rightarrow B$. From the functional dependency $BC \rightarrow D$, we can include D. And lastly, from $CD \rightarrow E$, we get E.

P3.2)

Attributes	**Closure**	**From**
A	{A,B}	$A \rightarrow A$, $A \rightarrow B$, $A \rightarrow AB$
B	{B}	$B \rightarrow B$
C	{C}	$C \rightarrow C$
D	{D}	$D \rightarrow D$
E	{E}	$E \rightarrow E$
AD	{A,D}	$AD \rightarrow A$, $AD \rightarrow D$, $AD \rightarrow AD$
AB	{A,B}	$AB \rightarrow A$, $AB \rightarrow B$, $AB \rightarrow AB$
AE	{A,E}	$AE \rightarrow A$, $AE \rightarrow E$, $AE \rightarrow AE$
AC	{A,B,C,D,E}	$AC \rightarrow Y$ for all $Y \subseteq \{A,B,C,D,E\}$
BE	{B,E}	$BE \rightarrow B$, $BE \rightarrow E$, $BE \rightarrow BE$
BC	{B,C,D,E}	$BC \rightarrow Y$, for all $Y \subseteq \{A,B,D\}$
BD	{B,D}	$BD \rightarrow D$, $BD \rightarrow B$, $BD \rightarrow BD$

DE	{D,E}	DE → E, DE → D, DE → DE
CE	{C,E}	CE → C, CE → E, CE → CE
ABE	{A,B,E}	ABE → Y for all Y ⊆ {A,B,E}