COMPSCI 2AC3 Assignment 1

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Contents

1 Answer 1

The given DFA can be defined as follows:

- Q = q1, q2, q3, q4, q5
- Σ = 0, 1
- S = q1
- F = q3

This DFA has 5 states, state q1 is the starting state, state q3 is the only final state and state q4 is considered as the garbage state for rejected inputs.

The DFA starts at state q1, it then check the left most bit. If the left most bit is '0' then the input is rejected and the DFA sends it to state q4 which is considered as the garbage state. After entering the garbage state the input cannot exit it and has been rejected as asked in the question.

If the left most bit is '1' then the input is sent to state q2, if the input ends in this state, the remainder is 1 and the input is not divisible by 3. If the input ends at state q5 then, the remainder is 2 and the input is not divisible by 3 either.

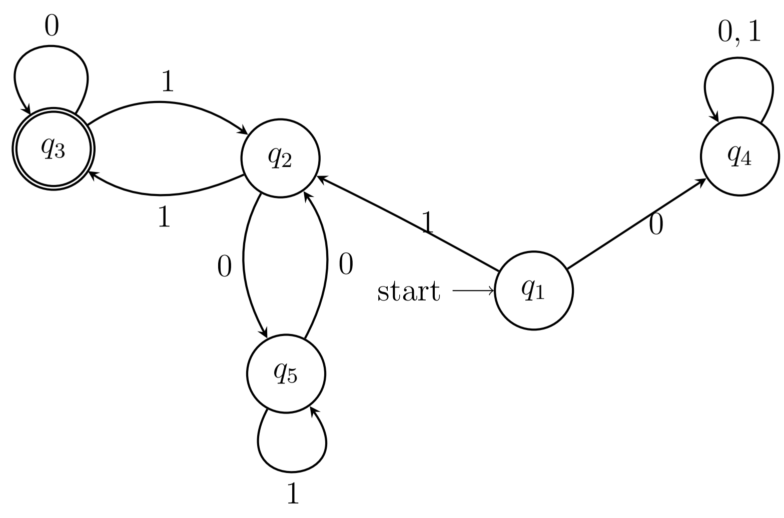

Figure 1: DFA for A

2 Answer 2

Let $N_3 = (Q_3, \Sigma_3, \Delta_3, S_3, F_3)$ then, the NFA of N_3 can be defined as follows:

- States of N_3 will be a union of states from N_1 and N_2
  $Q_3 = Q_1 \cup Q_2$
- All the Σ are the same for N_3
  $\Sigma = \Sigma_1 = \Sigma_2$
- The starting state of N_3 is the starting state of N_1
  $S_3 = S_1$
- The final state of N_3 is the final state of N_2
  $F_3 = F_2$
- The transitions Δ are as follows:
  - For each transition in Δ_1(p,a,q) where, p,q $\in Q_1$ and a $\in \Sigma$, we include that in Δ_3.
  - For each transition in Δ_2(p,a,q) where, p,q $\in Q_2$ and a $\in \Sigma$, we include that in Δ_3.
  - For each transition in Δ_1(p,a,q) where, p $\in F_1$, q $\in S_2$ and a $\in \Sigma$, we include that in Δ_3.

Informal explanation:

The basic idea of N_3 is that it starts at the starting states of S_1 and once is reached and once it reached any F_1, it transitions into S_2 then, it carries out transitions of N_2 until it reaches a state in F_2. This ensures that strings accepted by N_2 are a concatenation of string accepted by N_1 and string accepted by N_2.

3 Answer 3

The statement "$L(N_1) = \sim L(N_2)$" is incorrect. I used the NFA I made for Question 4 to prove that the statement is incorrect using a counterexample.

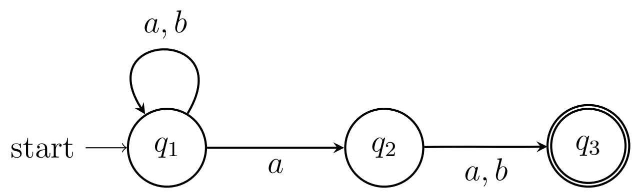

Figure 2: NFA 1

The NFA below accepts strings whose second letter from the last is an 'a'. From that we can see that the example 'abaa' is an accepted input. Therefore, abaa $\in L = N_1$

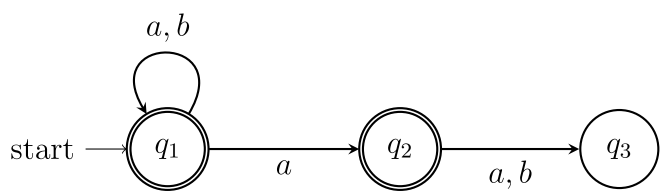

Figure 3: NFA 2 with switched final states.

According to the given statement "$L(N_1) = \sim L(N_2)$", the above NFA should not accept any string whose second last letter is 'a'. However, after simulating the example 'abaa' we see that the NFA does in fact accept it since, state q1 has a self loop and it is an accepted final state. this proves that abaa $\in L = N_2$

Hence, statement is proven to be incorrect.

4 Question 4

4.1 Part A

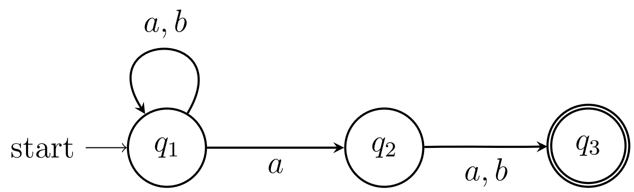

Figure 4: NFA for A

4.2 Part B

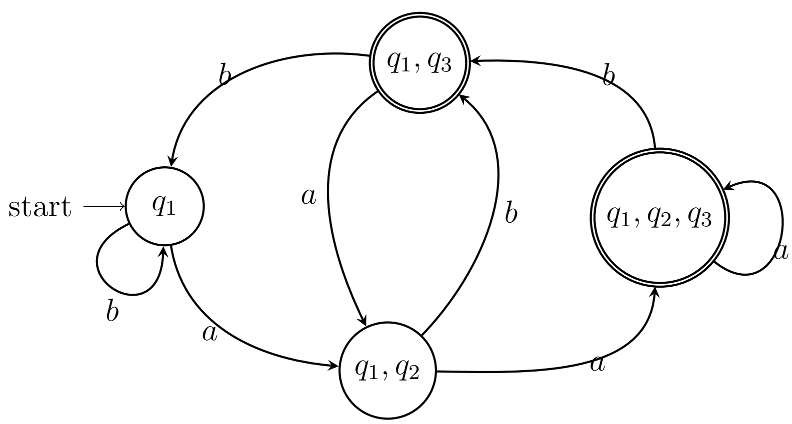

Figure 5: DFA for A

The steps to convert the NFA onto a DFA are:

- After transition b and a state 'q1' goes to 'q1' and state 'q1,q2' respectfully.
- After transition b and a state 'q1,q3' goes to 'q1' and state 'q1,q2' respectfully.

- After transition b and a state 'q1,q2' goes to 'q1,q3' and state 'q1,q2,q3' respectfully.
- After transition b and a state 'q1,q2,q3' goes to 'q1,q3' and state 'q1,q2,q3' respectfully.