COMPSCI 2AC3, Automata and Computability
Assignment 1, Winter 2024

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Due date: Saturday, Feb 10, 9pm

Notes. Your solutions **MUST be typeset in Latex** (refer to the first tutorial if you have missed it and not sure how to use Latex). Only upload a single pdf file as your solution to Avenue (avoid compressing your file). For drawing state machines use the the https://finsm.io/ website as discussed in the tutorial (also see the quick guide https://github.com/CSchank/finsm/wiki/QUICKSTART), and export the result to latex.

If you have questions about the assignment, post them in the dedicated Students Questions channel on MS Teams.

1. [**25 points**] Recall that a string consisting of only 0's and 1's can be thought as a number in base 2. For example, the decimal numbers 9 and 10 can be respectively represented by 1001 and 1010 in base 2. Let A be the set of all strings x over alphabet $\{0,1\}$ whose first (i.e. left-most) symbol is 1, AND x is a base-2 representation of a number that is divisible by 3. For example, $11 \in A$, $110 \in A$, $1001 \in A$ but $10 \notin A$, $111 \notin A$, $1 \notin A$. Draw a DFA for A. Also, describe how your DFA works, and what each state represents (but you don't need to formally prove the correctness of your DFA).

2. [**25 points**] Assume we have two NFAs, $N_1 = (Q_1, \Sigma, \Delta_1, S_1, F_1)$ and $N_2 = (Q_2, \Sigma, \Delta_2, S_2, F_2)$. Describe an NFA N_3 that satisfies $L(N_3) = L(N_1)L(N_2)$ (recall that for two sets A and B, their concatenation is defined by $AB = \{xy \mid x \in A, y \in B\}$). Include both a formal definition of N_3, and also an informal explanation of why it works (but you don't need to "prove" that it works). Note: in this question, we assume the NFAs don't have ε-transitions.

3. [**25 points**] Is the following statement true? "Let $N_1 = (Q, \Sigma, \Delta, S, F)$ be any non-deterministic finite state machine. Let $N_2 = (Q, \Sigma, \Delta, S, Q \setminus F)$. Then $L(N_1) =\sim L(N_2)$". If you think it is true, then prove it. Otherwise, provide a counterexample.

4. (a) [**5 points**] Let $A = \{x \in \{a,b\}^* : |x| \geq 2$ and the 2nd symbol of x from the right is an $a\}$. Draw an NFA for A with 3 states (no proof is required).

   (b) [**20 points**] Use the subset construction game to create a DFA for A out of its NFA. Label each state with subsets of the states of the NFA, and show your work.