COMPSCI 2AC3, Automata and Computability
Assignment 2, Winter 2024

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Due date: Monday, Feb 26, 9pm

Notes. Your solutions **MUST be typeset in Latex** (refer to the first tutorial if you have missed it and not sure how to use Latex). Only upload a single pdf file as your solution to Avenue (avoid compressing your file). For drawing state machines use the the https://finsm.io/ website as discussed in the tutorial (also see the quick guide https://github.com/CSchank/finsm/wiki/QUICKSTART), and export the result to latex.

If you have questions about the assignment, post them in the dedicated Students Questions channel on MS Teams.

1. Consider the regular expression $\alpha = a^*b + (abc)^*$ where the alphabet is $\Sigma = \{a, b, c\}$. [Advice for this question: double-check your answers for each part by trying a few different strings]

   (a) [**15 points**] Draw an NFA N such that $L(N) = L(\alpha)$

   (b) [**15 points**] Draw a DFA M such that $L(M) = \sim L(N) = \sim L(\alpha)$. [Hint: First draw a DFA and then complement it. When turning the NFA to a DFA, avoid drawing unreachable states to make things manageable.]

   (c) [**25 points**] Write a regular expression for $\sim L(\alpha)$ based on M using the recursive approach for construction of regular expressions (that was discussed in the class).

2. [**25 points**] Let A be a set of strings over alphabet $\Sigma = \{a, b\}$. Define

$$B = \{x \in \{a, b\}^* \mid x \text{ is a sub-string of } y \text{ for some } y \in A\}$$

   If A is regular, can we always conclude that B is regular too? Prove your answer.

3. [**20 points**] Let A be a regular language over alphabet $\Sigma = \{a, b\}$. Let M_1 and M_2 be two DFAs, each with 10 states. We want to check if $L(M_1) = A$. For this, we try all strings of length at most 100, and observe that for any such string x, we either have (i) $x \in A$ and $x \in L(M_1)$ or (ii) $x \notin A$ and $x \notin L(M_1)$. Can we conclude that $L(M_1) = A$? Either prove the statement or give a counter-example.