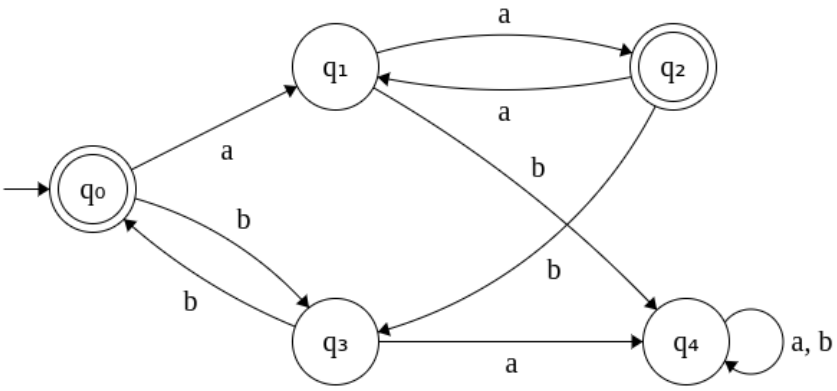
COMPSCI 2AC3, Automata and Computability
Assignment 3, Winter 2024

Hassan Ashtiani, McMaster University

Due date: Monday, March 18, 11pm

Notes. Your solutions **MUST be typeset in Latex** (refer to the first tutorial if you have missed it and not sure how to use Latex). Only upload a single pdf file as your solution to Avenue (avoid compressing your file). For drawing state machines use the the https://finsm.io/ website as discussed in the tutorial (also see the quick guide https://github.com/CSchank/finsm/wiki/QUICKSTART), and export the result to latex. If you have questions about the assignment, post them in the dedicated Students Questions channel on MS Teams.

1. **[25 points]** Consider the following DFA. Draw a new DFA for the same language but with the minimum number of states. Show your work (DFA minimization steps).



2. **[25 points]** Design a CFG for the following language. You don't need to formally prove the correctness.

$$A = \{\text{rev}(f(n))\$f(n+1) \mid n \geq 1\} \subset \{0, 1, \$\}^*$$

where $f(n)$ is the binary representation of n without leading zeros. For example, $f(2) = 10$ or $f(6) = 110$. Also $\text{rev}(x)$ is x in reverse order. For example, $\text{rev}(f(6)) = 011$.

3. **[25 points]** Is the following language regular? Prove your answer.

$$A = \{a^p : p \text{ is a prime number}\}$$

4. **[25 points]** Let M_1 and M_2 be two DFAs each with 10 states. We want to check if $L(M_1) = L(M_2)$. For this, we try all strings of length at most 100, and observe that for any such string x, we either have (i) $x \in L(M_1)$ and $x \in L(M_2)$ or (ii) $x \notin L(M_1)$ and $x \notin L(M_2)$. Can we conclude that $L(M_1) = L(M_2)$? Either prove the statement or give a counter-example.