Lecture 3

Thursday, January 11, 2024 11:33 AM

In this course we are interested in "decision problems" that have binary output where the input is a string.

* Is the given word is a palindrome?
* Is the " number even?

Notations

* a, b, c, d, ... used for symbols/letters
* u, v, w, x, y, z for strings
* $\alpha, \beta, \gamma, \ldots$ for patterns
* A, B, C, D, ... for sets

Alphabet, Σ: a finite set of symbols

string: a finite sequence of symbols

$\Sigma = \{0, 1\}$, $\Sigma = \{a, b, c, d\}$

$\Sigma = \{0, 1, 2, \ldots, 9\}$

$x = abcab$

null string, with length 0 $\quad x = \varepsilon$

length of a string: $|x|$

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$$|aba| = 3$$
$$|\varepsilon| = 0$$

* concatenation of two strings:

$$x = abc \quad , \quad y = ab$$
$$xy = abcab$$

new string

$$|xy| = |x| + |y|$$
$$\varepsilon x = x\varepsilon = x$$

by definition

* power of string: $x^0 \triangleq \varepsilon$

$$x^{n+1} \triangleq x^n x$$

$$x^1 = x$$
$$x = ab \rightarrow x^2 = abab$$

* $\#a(x)$: the total number of a in x

$$\#b(abccbbbb) = 5$$

* prefix: we say x is a prefix of y if there exist string z such that $y = xz$.

$x = abbca$
$y = abbcada$
x is a prefix of y.

* ε is a prefix of any string.

* ε is a prefix of any string.
* any string is a prefix of itself.

* proper prefix: a prefix that is not the string itself.
  * ab is a prefix of ab, but not a proper prefix of ab.

Sets of strings

* Σ^*: the set of all strings that can be generated from Σ.
  * $\Sigma = \{a, b\}$
    $abbab \in \Sigma^*$, $\varepsilon \in \Sigma^*$, $a \in \Sigma^*$, ...

* $\emptyset$: empty set: $\emptyset = \{\}$
* $A = \{ab, a, bba\} \subseteq \Sigma^*$
* $A = \{x : \Sigma^*, \#a(x) = 1\}$ where $\Sigma = \{a, b\}$.
  $A = \{a, ab, ba, abb, bab, bba, \ldots\ldots\}$
  A is an infinite set, but every string in it has finite length (by definition)

* Usual operations on sets:

  $A \cup B$, $A \cap B$,

  $\{x \in \Sigma^*, x \notin A\}$

$A \cup B$, $A \cap B$,

$\sim A = \Sigma^* \setminus A = \{x \in \Sigma^*, x \notin A\}$

* Concatenation of sets:

$$AB \triangleq \{xy : x \in A, y \in B\}$$

* $A = \{a, aa, b\}$, $B = \{a, b\}$

$|AB| = ?$

$AB = \{aa, ab, aaa, aab, ba, bb\}$

* $A = \{a, aa\}$, $B = \{\varepsilon, a\}$

$AB = \{a, aa, \cancel{aa}, aaa\}$

* powers of sets:

$$A^0 \triangleq \{\varepsilon\}$$

$$A^{n+1} \triangleq A^n A$$

* $A^1 = A$

* $A^* \triangleq A^0 \cup A^1 \cup A^2 \dots = \bigcup_{i=0}^{\infty} A^i$

* Σ^*

* $A = \{a, ab\}$

$\to A^* = \{\underbrace{\varepsilon}_{\in A^0}, \underbrace{a, ab}_{\in A^1}, aa, abab, aab, aba, \dots\}$

Q: is $A^* = \{x \in \{a,b\}^*, a \text{ is a prefix of } x\}$?

$abb \notin A^*$

Q: is $A^* = \{x \in \{a,b\}^*,$ a is a prefix of x\}?

no, since $abbbbb \notin A^*$

* $\emptyset^* \triangleq \{\varepsilon\}$: This definition makes the notations cleaner.

* We say a binary operation $\otimes$ is associative if $\forall A, B, C$, we have

$$(A \otimes B) \otimes C = A \otimes (B \otimes C)$$

* We say a binary operation $\otimes$ is commutative if $\forall A, B$;

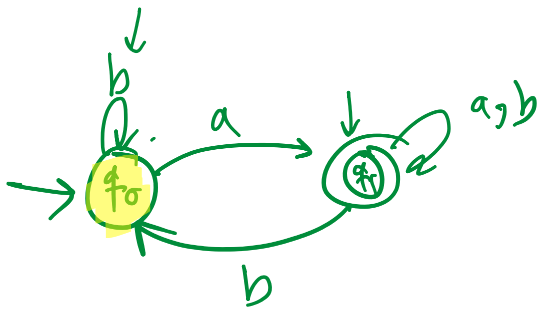
$$A \otimes B = B \otimes A$$

operation	Associative	Commutative	identity
Union	✓	✓	$\emptyset$
intersection	✓	✓	Σ^*
Concatanation	✓	✗	$\{\varepsilon\} = \emptyset^*$

* A is an identity element of binary operation $\otimes$ if $\forall B$,

$$A \otimes B = B \otimes A = B$$

* $B \cup \emptyset = \emptyset \cup B = B$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