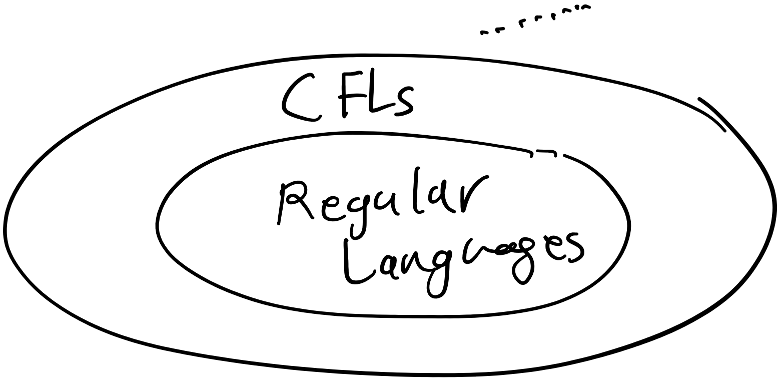
Lecture 20

Friday, March 8, 2024 11:33 AM

CFLs and CFGs



valid parenthesizations:

$S \to \varepsilon \mid (S)S$

$()()(())(())((())())$

$S \to \varepsilon \mid SS \mid (S)$

$S \to SS \to SSS \to SSSS \to SSSSS$

$\to (S)SSSS \to ()SSSS \to$

What about a CFG for palindromes? (over $\Sigma = \{a,b\}$)

$S \to \varepsilon \mid aSa \mid bSb \mid a \mid b$

Formal Definition of CFLs/CFGs

A CFG is quadruple

$G = (N, \Sigma, P, S)$ where:

* N : finite set of non-terminal symbols
* Σ : " " " terminal symbols
* $P \subseteq N \times (N \cup \Sigma)^*$: finite set of productions
* $S \in N$: The start symbol

Note: $S \to \varepsilon$, $\varepsilon \notin \Sigma$, $\varepsilon \notin N$ $\quad \varepsilon \in (N \cup \Sigma)^*$

$A = L(a\ b^* c)$

$S \to A B C$

$A \to a$

$C \to c$

$B \to bB \mid \varepsilon$

We can find a CFG for every regular expression!

Notes:

* We use small letters for terminals
* " " capital " " non-terminals
* " " $\alpha, \beta, \gamma, \ldots$ for sentential forms (strings in $(N \cup \Sigma)^*$)

sentential forms (strings in
that we define later.

Defining the CFL corresponding to a CFG

$\alpha \xrightarrow[G]{1} \beta$ if β can be derived from α by replacing one occurance of one non-terminal A with γ where $A \to \gamma$ is a production

$\alpha \xrightarrow[G]{0} \alpha$

$\alpha \xrightarrow[G]{n+1} \beta$ if $\exists \gamma$ s.t. $\alpha \xrightarrow[G]{n} \gamma$ and $\gamma \xrightarrow[G]{1} \beta$

palindromes:

* $S \xrightarrow[G]{1} aSa \xrightarrow[G]{1} abSba \xrightarrow[G]{1} abba$
* $S \xrightarrow[G]{3} abba$
* $abSba \xrightarrow[G]{2} abbbba$

$\alpha \xrightarrow[G]{*} \beta$; if there exist $n \geq 0$ s.t $\alpha \xrightarrow[G]{n} \beta$

$$L(G) = \{ x \in \Sigma^* : S \xrightarrow[G]{*} x \}$$

$$L(G) = \{ x \in \Sigma^* : S \xrightarrow[G]{*} x \}$$

* $\alpha \in (N \cup \Sigma)^*$ is called a sentential form if $S \xrightarrow[G]{*} \alpha$.

* A sentential form is called a sentence if it consists of only terminals.

$L(G)$ = set of all sentences of G.

---

* The set of all syntactically valid strings for python language forms a CFL.

* <html> .... </html>

<stmt> → <if-stmt> | <loop-stmt> | | <begin-end> | assign-stmt> |

<if-stmt> → if <bool-expr> then <stmt> else <stmt>

<bool-expr> $\rightarrow$ - - - - - -