Lecture 9

Tuesday, January 30, 2024 11:35 AM

The subset construction game

Thm. If $A = L(N)$ for NFA N, then there exist a DFA M such that $L(M) = L(N) = A$. In other words, A is regular.

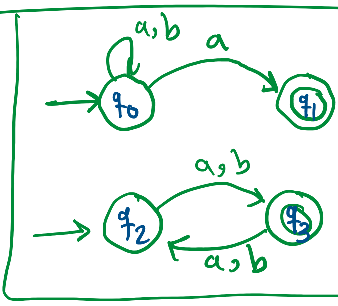
Assume $N = (Q_N, \Sigma, \Delta_N, S_N, F_N)$

Then we take $M = (Q_M, \Sigma, \delta_M, s_M, F_M)$

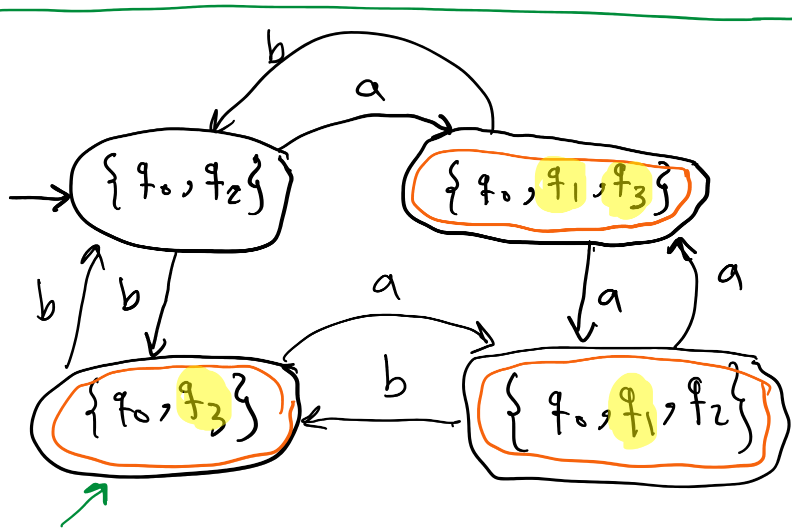
where:

- $Q_M = 2^{Q_N} = \{B \subseteq Q_N\}$
- $\delta_M(q_A, x) = \hat{\Delta}(A, x)$ — $\forall A \subseteq Q_N$, $\forall q_A \in Q_M$, $\forall x \in \Sigma^*$
- $s_M = S_N$
- $F_M = \{q_A \in Q_M : A \cap F_N \neq \emptyset\}$

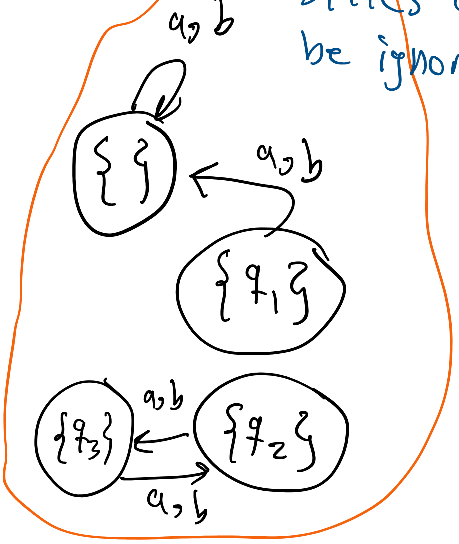
the book might use A and q_A interchangeably.



$\{x \in \{a,b\}^* :$ the last letter of x is a OR $|x| \bmod 2 = 1\}$



unreachable states can be ignored.



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* Lemma 6.1 in the book:

* For any $x, y \in \Sigma^*$, and $A \subseteq Q_N$,

$$\hat{\Delta}(A, xy) = \hat{\Delta}(\hat{\Delta}(A, x), y)$$

proof: induction on $|y|$.

* Lemma 6.3 in the book:

  * For any $A \subseteq Q_N$ and $x \in \Sigma^*$,

$$\hat{\delta}_M(q_A, x) = \hat{\Delta}_N(A, x)$$

M is the DFA that we created using subset construction.

* Thm $L(M) = L(N)$