Assignment 2 CompSci2AC3

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1 question 1

1.1 part a

1.2 part b

1.3 part c

2 Question 2:

Assumption: If A is regular, then B is also regular.

Proof:

Since A is regular, meaning there must exists a finite automaton M that recognizes A, We can construct a NFA N that recognizes B by using M which prove B is regular :

- N starts in an initial state where it non-deterministically decides to enter M at any point.
- N reads a substring of the input and simulates M.
- If the simulation of M enters an accept state, N non-deterministically decides to exit M and transitions to an accept state itself.

This NFA can start and end its simulation of M at any point in the input, effectively checking for substrings that are in A. Since any string in B is a substring of some string in A, this NFA will accept exactly the set B.

Since NFAs and DFAs are equivalent in the sense that for every NFA, there is a DFA that recognizes the same language, there exists some DFA that recognizes B. Therefore, B is regular.

3 Question 3:

No, we cannot conclude that $L(M_1) = A$. Here is a counter-example:

Let A be a language that includes all strings over the alphabet $\{a, b\}$ that have an even number of a's. A DFA for this language would switch between two states: one for an even number of a's and one for an odd number of a's.

Now, suppose M_1 is constructed incorrectly so that it only checks if the last 100 characters of the input string have an even number of a's. For all strings of length 100 or less, M_1 would correctly determine if the string has an even number of a's. However, for strings longer than 100 characters, M_1 might fail. For example, the string $a^{101}b$ (which means 101 consecutive 'a's followed by a 'b') is not in A because it has an odd number of a's, but if M_1 only looks at the last 100 characters, it would incorrectly accept this string as being in $L(M_1)$.

Therefore, the observation that M_1 and A agree on all strings up to length 100 is not sufficient to conclude that $L(M_1) = A$.