Lecture 4

January 18, 2024 10:50 AM

Quiz: $A \cap (B \cup C) \stackrel{?}{=} AB \cup AC$ ✓

$A \cap (B \cap C) \stackrel{?}{=} AB \cap AC$ ✗

↳ $A = \{a, aa\}$, $B = \{b\}$, $C = \{ab\}$

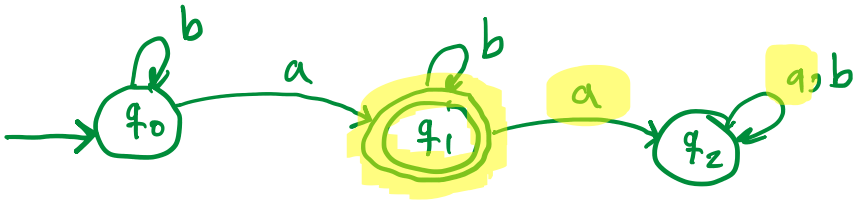
Finite State Automata

A finite state machine (automaton) has a finite number of "states". The machine is initialized at a certain state, and "consumes" a sequence of characters (input string) one-by-one. The state gets updated in each step based on the current state and the input symbol. Once the whole string is consumed, the machine terminates and accepts / rejects based on the final state.

$A = \{ x \in \{a,b\}^* \mid \#a(x) = 1 \}$

↳ a set of strings or a language

a^b a^b



$x = a\,b\,b\,a\,b$

$q_0 \xrightarrow{a} q_1 \xrightarrow{b} q_1 \xrightarrow{b} q_1 \xrightarrow{a} q_2 \xrightarrow{b} q_2$

rejects x since q_2 is not an accept state.

Deterministic Finite Automaton (DFA)

A DFA is a structure

$M = (Q, \Sigma, \delta, s, F)$ where:

* Q: set of states,
  e.g: $Q = \{q_0, q_1, q_2\}$

* Σ: alphabet
  e.g.: $\Sigma = \{a, b\}$

* $\delta : Q \times \Sigma \longrightarrow Q$ transition function

e.g., $\delta(q_0, b) = q_0$
$\delta(q_0, a) = q_1$
$\delta(q_1, a) = q_2$
⋮

Q에서 a b

Q \ Σ	a	b
q_0	q_1	q_0
q_1	q_2	q_1
q_2	q_2	q_2

* $s \in Q$: initial state

  e.g.: $s = q_0$

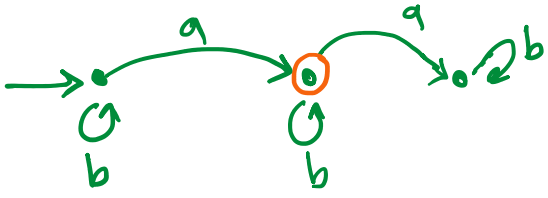
* $F \subseteq Q$: accept states

  e.g., $F = \{q_1\}$

informally: M accepts x if the final state after consuming x belongs to F (o.w. rejects)

Book's notation:

we don't use this notation.



Formal definition of what is accepted by M

* Muti-step transition function $\hat{\delta} : Q \times \Sigma^* \to Q$

where $\hat{\delta}$ is defined:

$\forall q \in Q, \quad \hat{\delta}(q, \varepsilon) \triangleq q$

$\forall q \in Q$
$\forall x \in \Sigma^*$
$\forall a \in \Sigma$

$\hat{\delta}(q, xa) = \delta(\hat{\delta}(q, x), a)$

$\hat{\delta}(q, ax) = \hat{\delta}(\delta(q, a), x)$

alternatively, we could have defined it like this

e.g., $\hat{\delta}(q_0, bbbabb) = q_1$

$\hat{\delta}(q_1, abab) = q_2$

An input string x is accepted by $M = (Q, \Sigma, \delta, s, F)$ iff $\hat{\delta}(s, x) \in F$

Language corresponding to M:

$L(M) = \{x \in \Sigma^* : \hat{\delta}(s, x) \in F\}$

Draw a DFA for:

$A = \{x \in \{a, b\}^* : \#(a) \text{ is odd}\}$