Lecture 30

April 4, 2024 10:50 AM

Configuration of a TM

Notations:

* $\sqcup^\omega$: $\sqcup\sqcup\sqcup$-----
  infinit sequence of blank symbols
* z_n where $z \in \Gamma^*$, $n \in \{1, 2, 3, \cdots\}$
  is the n-th symbol of z
* $S_b^n(z)$ where $n \in \{1, 2, \cdots\}$, $b \in \Gamma$, $z \in \Gamma^*$
  is a string that is the result of substituting n-th symbol of z with b

  $z = abbca$

  $S_a^2(z) = aabca$
* A configuration $\alpha \in Q \times \{ y\sqcup^\omega \mid y \in \Gamma^* \} \times \mathbb{N}$
  is a snapshot of all relevant info.

* start config: $(s, \vdash x \sqcup^\omega, 0)$
  start state input string head location

* Next config relation $\xrightarrow[M]{1}$:

$$\frac{}{(q_1, z, n) \xrightarrow[M]{1} (q_2, S_b^n(z), n-1)}$$

$$(q_1, z, n) \xrightarrow[M]{1} (q_2, s_b^{n}(z), n-1)$$
$$\text{if} \quad \delta(q_1, z_n) = (q_2, b, L)$$

$$(q_1, z, n) \xrightarrow[M]{1} (q_2, s_b^{n}(z), n+1)$$
$$\text{if} \quad \delta(q_1, z_n) = (q_2, b, R)$$

$$\alpha \xrightarrow[M]{0} \alpha$$

$$\alpha \xrightarrow[M]{n+1} \beta \iff \exists \gamma : \alpha \xrightarrow[M]{n} \gamma, \ \gamma \xrightarrow[M]{1} \beta$$

$$\alpha \xrightarrow[M]{*} \beta \iff \exists n \geq 0 \text{ s.t. } \alpha \xrightarrow[M]{n} \beta$$

TM M accepts (reject) x $\iff$ $(s, \vdash x \sqcup^{\omega}, 0) \xrightarrow[M]{*} (q_t, y, n)$ (q_r) for some $y \in \Gamma^*$ and $n \in \mathbb{N}$.

Note: sometimes M does not accept nor reject x.

Example: $A = \{a^n b^n c^n : n \geq 0\}$

	$\vdash$	a	b	c	$\sqcup$	$\#$
s	$(s, \vdash, R)$	(s, a, R)	(q_1, b, R)	(q_2, c, R)	$(q_3, \sqcup, L)$	—
q_1	—	$(q_r, -, -)$	(q_1, b, R)	(q_2, c, R)	$(q_3, \sqcup, L)$	—
q_2	—	$(q_r, -, -)$	$(q_r, -, -)$	(q_2, c, R)	$(q_3, \sqcup, L)$	—
q_3	$(q_t, -, -)$	$(q_r, -, -)$	$(q_r, -, -)$	$(q_4, \#, L)$	—	$(q_3, \# L)$
q_4	$(q_r, -, -)$	$(q_r, -, -)$	$(q_5, \#, L)$	(q_4, c, L)	—	$(q_4, \#, L)$
q_5	?	$(q_6, \#, R)$	(q_5, b, L)	?	?	?
q_6	—	—	(q_6, b, R)	(q_6, c, R)	$(q_3, \sqcup, L)$	$(q_6, \#, R)$

— → don't care

$$(s, \vdash abc \sqcup^{\omega}, 0) \xrightarrow[M]{1} (s, \vdash abc \sqcup^{\omega}, 1)$$

$(s, \vdash abc \sqcup^{\omega}, 0) \xrightarrow[M]{1} (\ldots$

$\xrightarrow[M]{1} (s, \vdash abc \sqcup^{\omega}, 2) \xrightarrow[M]{1} (q_1, \vdash abc \sqcup^{\omega}, 3)$

$\xrightarrow[M]{1} (q_2, \vdash abc \sqcup^{\omega}, 4) \xrightarrow[M]{1} (q_3, \vdash abc \sqcup^{\omega}, 3)$

$\rightarrow \ldots$

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* $L(M)$: all the strings that M accepts

* A TM M is called a Total TM if it either accepts or rejects every string.

  Example: the above TM.

* A set A of strings is

  * Recursive Enumerable if $\exists$ TM M, $L(M) = A$

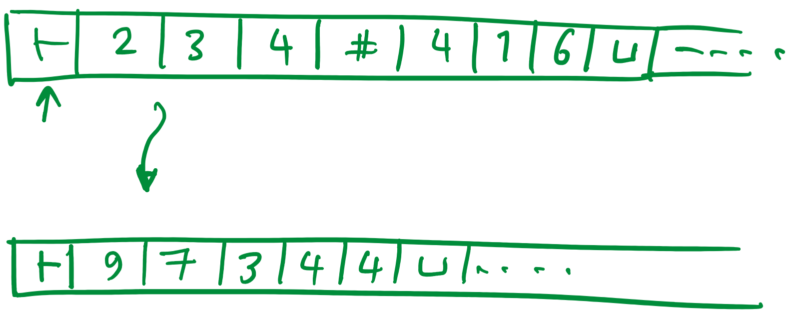
  * Recursive if $\exists$ a total TM M, $L(M) = A$

  * co-recursively Enumerable if $\exists$ TM M, $L(M) = \Sigma^* - A = \sim A$

Claim: if A is recursive, then $\sim A$ is also recursive.

* But if A is r.e., then
$\sim A$ may not be r.e.
(but it is co-r.e.)

We say a function $f: \Omega \to R$
is computable if $\exists$ a Total TM M
that "computes" it: if you write the
description of $x \in \Omega$ on the tape,
M will run and halt at the end,
where $f(x)$ is written on the tape.



* For binary-valued functions $f: \Omega \to \{0,1\}$,
we are dealing with decision problem.
A decision problem is said to be
decidable if $\exists$ a Total TM M
s.t. $f(x) = 1 \iff M$ accepts x.