Lecture 7

Thursday, January 25, 2024 11:28 AM

Quiz: $A \cup B \cup C$ → criterion 3

$A \cup B$ is regular if A and B regular?

A is regular → $\sim A$ regular? → $(\sim A \cap \sim B)$ regular → $\sim(\sim A \cap \sim B)$ regular
→ $\sim B$

DFA for union:

$$M_3 = (Q_1 \times Q_2, \Sigma, \delta_3, (s_1, s_2), F_1 \times Q_2 \cup Q_1 \times F_2)$$

* what about AB? $AB = \{xy : x \in A, y \in B\}$

Product Construction for $A \cap B$

$$M_1 = (Q_1, \Sigma, \delta_1, s_1, F_1)$$

$$M_2 = (Q_2, \Sigma, \delta_2, s_2, F_2)$$

$$M_3 = (Q_1 \times Q_2, \Sigma, \delta_3, (s_1, s_2), F_1 \times F_2)$$

where $\forall p \in Q_1$, $\forall q \in Q_2$, $\forall c \in \Sigma$

$$\delta_3((p,q),c) = (\delta_1(p,c), \delta_2(q,c))$$

$$L(M_3) = L(M_1) \cap L(M_2) \text{ ?!}$$

Observation: $\forall x \in \Sigma^*$, $\forall p \in Q_1$, $\forall q \in Q_2$

$$\hat{\delta}_3((p,q),x) = (\hat{\delta}_1(p,x), \hat{\delta}_2(q,x))$$

(use induction to prove it)

Rest of the proof:

Rest of the proof:

$x \in L(M_3) \iff \hat{\delta}_3((s_1,s_2),x) \in F_1 \times F_2$

$\iff (\hat{\delta}_1(s_1,x), \hat{\delta}_2(s_2,x)) \in F_1 \times F_2$

$\iff \hat{\delta}_1(s_1,x) \in F_1$ and $\hat{\delta}_2(s_2,x) \in F_2$

$\iff x \in L(M_1) \cap L(M_2)$

Example:

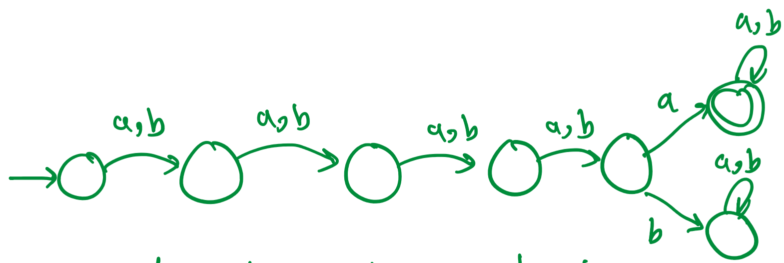
$A = \{ x \in \{a,b\}^* \mid |x| \geq 5$ and the 5th symbol from left is an a $\}$

$B = \{ x \in \{a,b\}^* \mid |x| \geq 5$ and the 5th symbol from right is an a $\}$

bbaabaa $\notin A$

$\in B$

For A:



For B: we need to keep track of the last 5 symbols ... for which we would need $\geq 2^5 = 32$ states

---

Non deterministic Finite Automaton (NFA)

We want to make DFAs "stronger" so we "relax" DFAs in mulliple ways:

* Multiple start states:

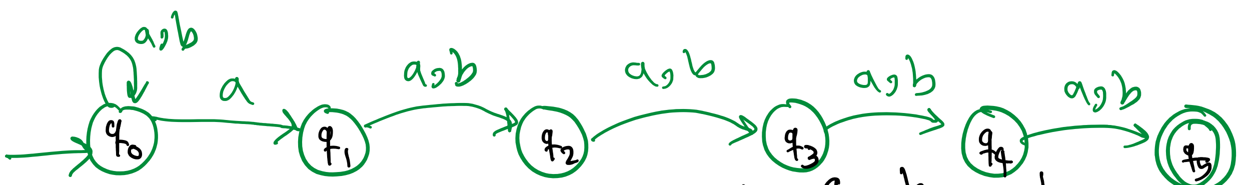
* Allow a <u>set</u> of next states:

In each state, given an input symbol, the next state is chosen from a set of possible states (0, 1, 2, or more)

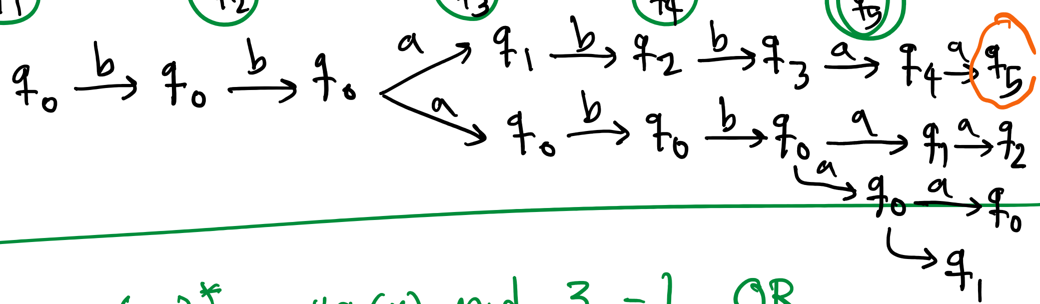
* There could be more than one "computational path" (ways of processing) the same string.

* NFA accepts a string if **at least** one computational path ends up in an accept state.

NFA for B:



bbabbaa



$C = \{ x \in \{a\}^* : \#a(x) \bmod 3 = 1$ OR $\#a(x) \bmod 5 = 1 \}$

NFA: