Lecture 5

Friday, January 19, 2024 11:26 AM

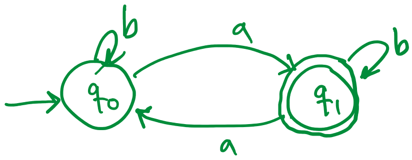
$A = \{x \in \{a,b\}^* \mid \#a(x) \text{ is odd}\}$ (regular, why?)

$B = \{x \in \{a,b\}^* \mid$ the second-to-the-last symbol in x is a and $|x| \geq 2\}$ (reglar, why?)

$C = \{x \in \{a,b\}^* \mid \#a(x) = \#b(x)\}$ (not regular) why?

Regular Set

Set $A \subseteq \Sigma^*$ is said to be regular if there exist a DFA M such that $L(M) = A$.



$L(M) = A$

proof??

$$L(M) = \{\underbrace{x}_{\in \{a,b\}^*} : \hat{\delta}(q_0, x) = q_1\}$$

$$A = \{x \in \{a,b\}^* : \#a(x) \bmod 2 = 1\}$$

Define $f(x) = \#a(x) \bmod 2$

Goal: $\forall x \in \{a,b\}^*, \quad x \in A \iff x \in L(M)$

$$x \in A \iff f(x) = 1 \overset{?}{\underset{1}{\iff}} \hat{\delta}(q_0, x) = q_1 \iff x \in L(M)$$

if we show 1 then we are done.

Another way to state ~~#~~ 1 is the following:

$$\boxed{\forall x \in \{a,b\}^*, \quad \hat{\delta}(q_0, x) = q_{f(x)}}$$

$\forall x \in \{a,b\}^*, \quad \hat{\delta}(q_0, x) = q_{f(x)}$

↳ we are done if we show this.

tells us the exact state that we end up after consuming x. (not just whether it is an accept state)

Induction on length of x.

* Base case: $|x| = 0 \Rightarrow x = \varepsilon$

$$\hat{\delta}(q_0, x) = \hat{\delta}(q_0, \varepsilon) = q_0 = q_{f(\varepsilon)}$$

$f(\varepsilon) = 0$

* Inductive assumption:

$$\boxed{\forall x \in \{a,b\}^*, |x| = n \text{ we have } \hat{\delta}(q_0, x) = q_{f(x)}} \quad (1)$$

* It remains to show that this holds for strings of length $n+1$.

* w.o. loss of generality let $z = xc$, where $|x| = n$, $|c| = 1$, $|z| = n+1$.

$c \in \{a,b\}$

$$\hat{\delta}(q_0, z) \overset{?}{=} q_{f(z)}$$

$$\hat{\delta}(q_0, z) = \hat{\delta}(q_0, xc) = \delta(\hat{\delta}(q_0, x), c)$$

$$= \delta(q_{f(x)}, c) = \begin{cases} q_{[f(x)+1 \bmod 2]} & c = a \\ q_{f(x)} & c = b \end{cases}$$

$$= \delta(\ 'f(x)'\ ) = \{\ q_{f(x)} \quad c=b$$

$$= \begin{cases} q_{f(xa)} & c=a \\ q_{f(xb)} & c=b \end{cases} = q_{f(xc)} = q_{f(z)}$$

$\rightarrow f(xb) = f(x)$

$\rightarrow f(xa) = [f(x)+1] \bmod 2$