

\*  $\{a^n b^n c^n : n \geq 0\}$  is not CFL (why?)

\* write a program that checks whether a given  $x \in \{[, ], (, )\}^*$  is a valid parenthesization.

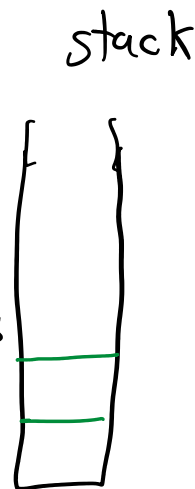
$[ ] ) ($  ✓

$( [ ] ]$  not valid

$(( )$  not valid

$) ($  not valid

\* we can use a stack and push any open brackets that we see, and pop/match them with close brackets (or reject) and the stack should become empty at the end.




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Push Down Automata (PDAs)



The state gets updated based on the previous state, the current input symbol, and the top of the stack,  
 [Also with each transition, we can push or pop an element to the stack]

We will introduce

Deterministic PDAs ↗ balanced parenthesis  
 Non deterministic PDAs ↙ palindromes

\* NPDA's are actually more powerful than PDAs.

\* Being a CFL is equivalent to being a NPDA

\* Being a ...  
to having a NPDA

Given a string, check whether  
it can be generated by a given  
CFG?

$$S \rightarrow [S] \mid SS \mid \epsilon$$

$$S \rightarrow SS \rightarrow SSS \rightarrow SSSS \rightarrow SSSSS \rightarrow$$

$$\rightarrow SSSS \rightarrow SSS \rightarrow SS \rightarrow S \rightarrow [S] \rightarrow [ ]$$

we have a long sequence of derivations  
but the final string is short. It seems  
that some of the transitions are useless  
or they don't make any progress towards  
generating  $x$ .

$\epsilon$ -productions (e.g.  $A \rightarrow \epsilon$ ) are problematic.

$$S \rightarrow A \mid a$$

$$A \rightarrow B$$

$$B \rightarrow S$$

$$S \rightarrow A \rightarrow B \rightarrow S \rightarrow A \rightarrow B \rightarrow S \rightarrow a$$

unit productions (e.g.  $A \rightarrow B$ ) are  
problematic.

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A grammar  $G = (N, \Sigma, P, S)$  is in Chomsky Normal Form (CNF) if every production in  $P$  is in one of these forms:

$$A \rightarrow BC$$

$$D \rightarrow a$$

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Question: Given a CFG  $G$ , can we always find  $G'$  that is in CNF and  $L(G) = L(G')$ ?

No, since we cannot generate  $\epsilon$ .

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Thm.  $\forall$  CFG  $G$ ,  $\exists$  CFG  $G'$  that is in CNF and

$$L(G') = L(G) - \{\epsilon\}$$

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$$S \rightarrow [S] \mid SS \mid \epsilon$$

CNF?

$$S \rightarrow aSb \mid \epsilon$$

CNF?