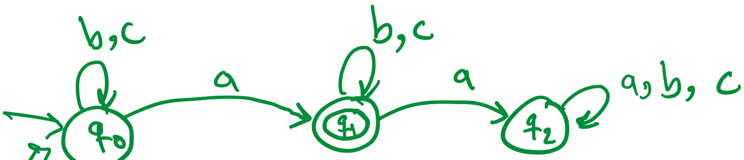
Lecture 6

January 23, 2024 11:27 AM

Closure Properties of Regular Sets

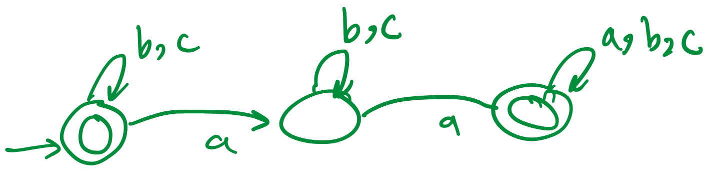
* if A and B are regular sets, can we always say $\sim A$, $A \cap B$, $A \cup B$, ... are regular too?

Thm. $\forall A \subseteq \Sigma^*$, if A is regular then so is $\sim A$.



why are they equal?

$A = \{ x \in \{a, b, c\}^* \mid \#a(x) = 1 \}$



$L(M) = \sim A$

Proof: assume A is regular theorem.

Then $A = L(M)$ for some $M = (Q, \Sigma, \delta, s, F)$. We claim that for $M' = (Q, \Sigma, \delta, s, Q \setminus F)$ we have $L(M') = \sim A$.

$$x \in L(M') \iff \hat{\delta}(s, x) \in Q \setminus F \iff \hat{\delta}(s, x) \notin F \iff x \notin L(M)$$

Thm. $\forall A \subseteq \Sigma^*, \forall B \subseteq \Sigma^*$, if A and B

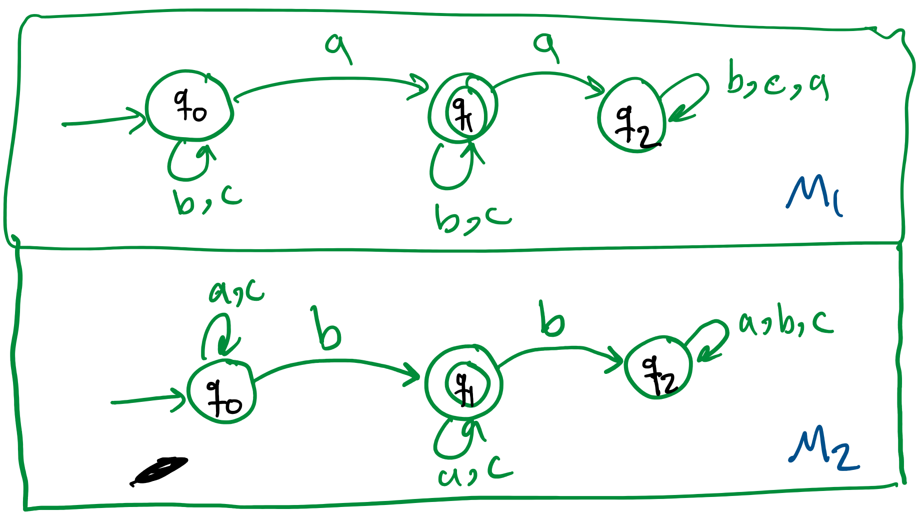
Thm. $\forall A \subseteq \Sigma^*$, $\forall B \subseteq \Sigma^*$, if A and B are regular then $A \cap B$ is also regular.

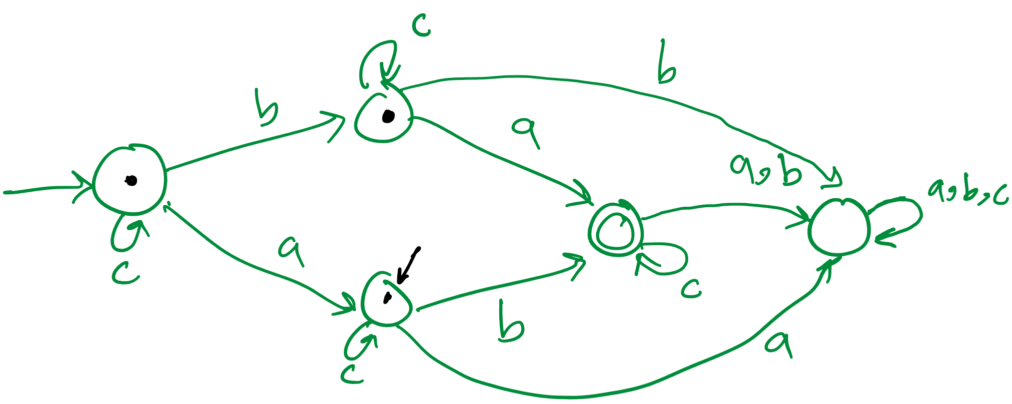
$A = L(M_1)$ where $M_1 = (Q_1, \Sigma, \delta_1, s_1, F_1)$

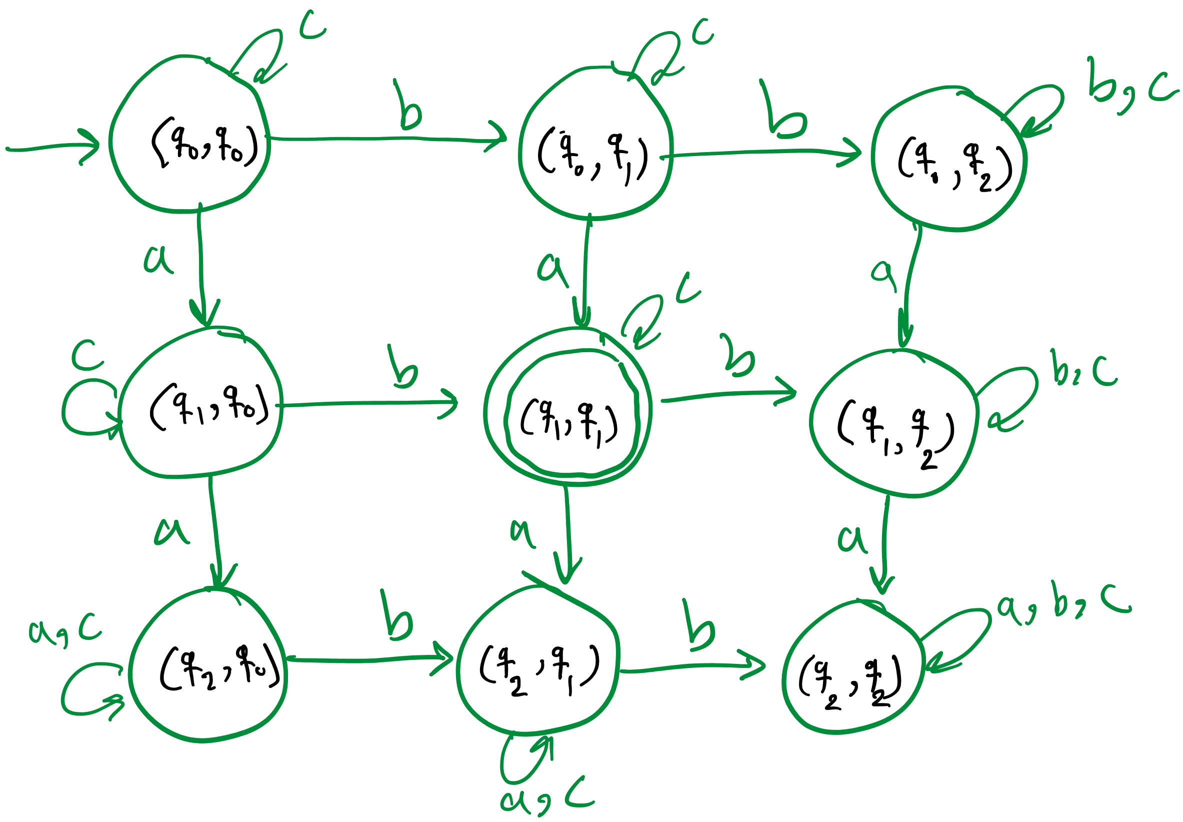
$B = L(M_2)$ " $M_2 = (Q_2, \Sigma, \delta_2, s_2, F_2)$

$$M_3 = (Q_1 \times Q_2, \Sigma, \delta_3, (s_1, s_2), F_1 \times F_2)$$

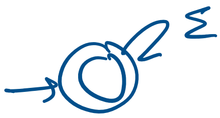
where $\delta_3((p,q),d) = (\delta_1(p,d), \delta_2(q,d))$ $\forall p \in Q_1, \forall q \in Q_2, \forall d \in \Sigma$







Σ^* is regular



what about $A \cup B$?