Lecture 13

February 8, 2024 11:15 AM

Theorem. The following statements are equivalent for every set $A \subseteq \Sigma^*$:

(i) A is regular (has a DFA)

(ii) $\exists$ NFA N, s.t. $L(N) = A$

(iii) " " N with ε-transitions s.t. $L(N) = A$

(iv) $\exists$ pattern α s.t. $L(\alpha) = A$

(v) $\exists$ regular expression β s.t. $L(\beta) = A$

(i) $\Longrightarrow$ (ii) : trivial

(ii) $\Longrightarrow$ (i) : subset construction

(ii) $\Longrightarrow$ (iii) : trivial

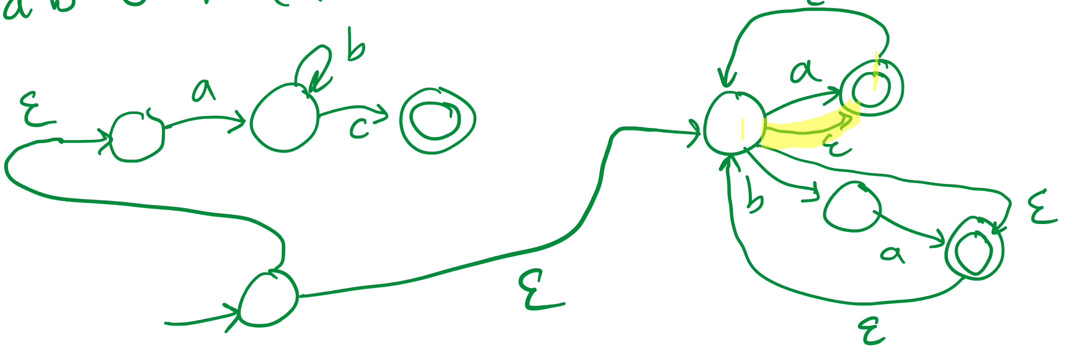
(iii) $\Longrightarrow$ (ii) : we had a theorem (didn't prove)

(v) $\Longrightarrow$ (iv) : trivial

(iv) $\Longrightarrow$ (v) : we discussed how to turn patterns into regular expressions (except for removing $\sim$)

(v) $\Longrightarrow$ (iii) :

$ab^*c + (a+ba)^*$ where $\Sigma = \{a, b, c\}$



If the regular expression is atomic

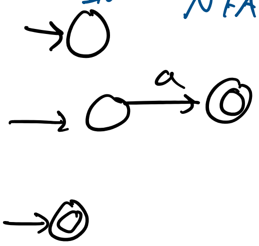
If the regular expression is atomic then finding an NFA is trivial:

expressions $\qquad$ N $\qquad$ NFA $\qquad$ $L(N) = L(\alpha)$

$\alpha = \emptyset$

$\alpha = a$

$\alpha = \varepsilon$



Otherwise we use structural induction:

Assume $L(\alpha)$ and $L(\beta)$ are regular (we have NFAs for them). Then

$$\begin{cases} L(\alpha\beta) = L(\alpha)L(\beta) & \text{regular,} \\ L(\alpha+\beta) = L(\alpha) \cup L(\beta) & \text{regular} \\ L(\alpha^*) = L(\alpha)^* & \text{regular} \end{cases}$$

closure properties of regular sets

we also know how to combine the two basic NFAs/DFAs to create an NFA for the combination.

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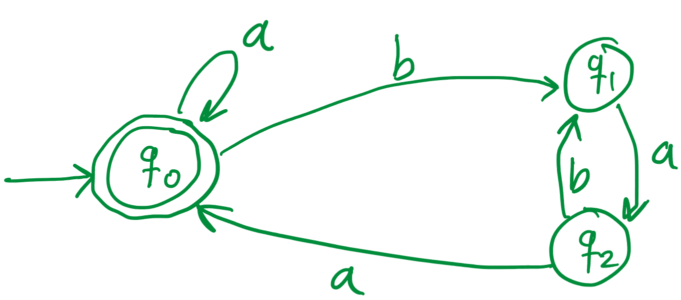
ii ⟹ v

Turning NFAs into regular expressions



$X(a^* \ (ba)^*)^*$ $\quad X = ba$

$X = ba$

$X a^* + b(ab)^* a$

$X = abaa$

$X a^* + ba(ba)^* a$

$X(a^* + a^* b(ab)^* a)^*$

$X = ba$

$X(a^* ba(ba)^* a)^*$ $\quad X = a$

$x = ba$

$\times\ (a^* ba (ba)^* a)^*$ $\quad x = a$

$(a^* + a^* ba (ba)^* a)^*$

$(a^* (ba (ba)^* a)^*)^*$

$(a + ba (ba)^* a)^*$