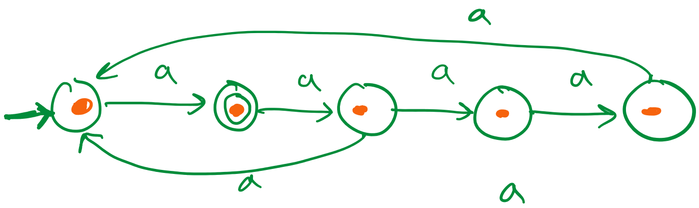
Lecture 8

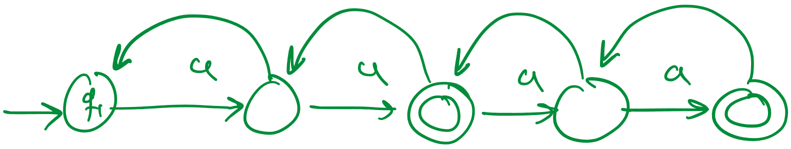
Friday, January 26, 2024 11:39 AM

$C = \{ x \in \{a\}^* : \#a(x) \bmod 3 = 1$ **OR** $\#a(x) \bmod 5 = 1 \}$

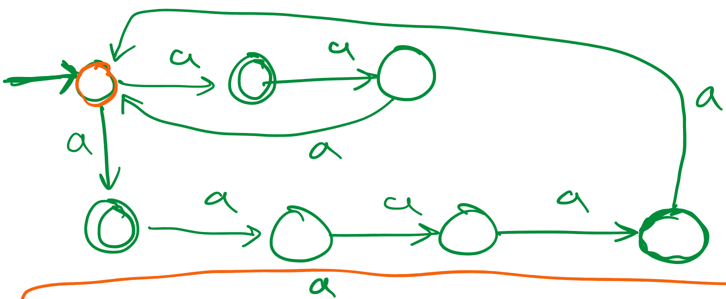
NFA:



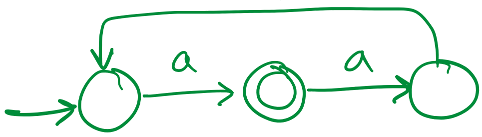
✗ a^9 ✗



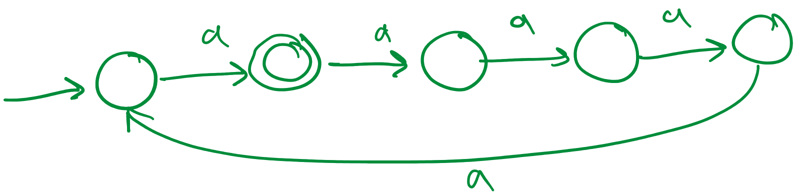
a^2 ✗



a^9 ✗



NFA
allow two start state



Formal Definition of NFAs

An NFA is a five tuple

$N = (Q, \Sigma, \Delta, S, F)$ where:

* Q : set of states
* Σ : alphabet
* $\Delta : Q \times \Sigma \to 2^Q$

2^Q = power set of Q

= all subsets of Q

$= \{ A \subseteq Q \}$

* $S \subseteq Q$: set of start states
* $F \subseteq Q$: set " accept "

Define $\hat{\Delta} : 2^Q \times \Sigma^* \to 2^Q$

$\forall A \subseteq Q$, $\forall x \in \Sigma^*$

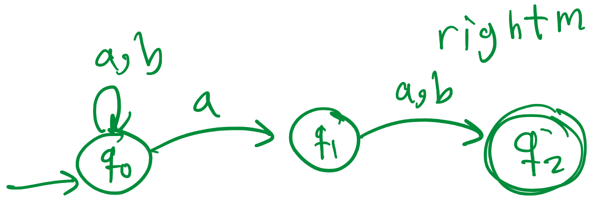
$$\hat{\Delta}(A, \varepsilon) = A$$

$$\hat{\Delta}(A, xa) = \bigcup_{q \in \hat{\Delta}(A,x)} \Delta(q,a)$$

intuitively $\hat{\Delta}$ represents all the states that we can end up with by starting from some state in A, and consuming x.

* N accepts x if $\hat{\Delta}(S, x) \cap F \neq \emptyset$
* $L(N) = \{ x \in \Sigma^* : F \cap \hat{\Delta}(S, x) \neq \emptyset \}$

Example: $A = \{ x \in \{a,b\}^* :$ second rightmost symbol of x is $a \}$



$$\hat{\Delta}(\{q_0\}, \varepsilon) = \{q_0\}$$

$$\hat{\Delta}(\{q_0\}, a) = \{q_0, q_1\}$$

$$\hat{\Delta}(\{q_0\}, ab) = \{q_0, q_2\} = \hat{\Delta}(\{q_0, q_1\}, b)$$

$\hat{\Delta}(\{q_0\}, ab) = \{q_0, q_2\} = \Delta(\{q_0, q_1\}, b)$

$\{q_0, q_2\} \cap F \neq \emptyset$

it worked for $x = ab$

$\hat{\Delta}(\{q_0\}, abb) = \hat{\Delta}(\{q_0, q_1\}, bb)$

$= \hat{\Delta}(\{q_0, q_2\}, b) = \{q_0\}$

$abb \notin L(N)$ ✓

$\{q_0\} \cap F = \emptyset$ ✓

Thm. Set A is regular if and only if there is an NFA N such that $L(N) = A$.