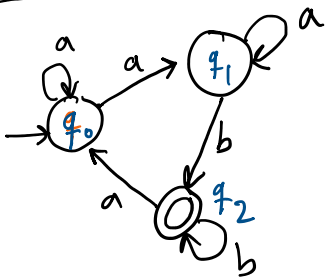


DFA minimization:  
next tutorial, excluded from  
the midterm.

- \* Pumping lemma
- \* closure properties
- \* DFA/NFA to regular expression.



$$\alpha_{q_0, q_2}^Q = \alpha_{q_0, q_2}^{Q - \{q_1\}} + \alpha_{q_0, q_1}^{Q - \{q_1\}} (\alpha_{q_1, q_1}^{Q - \{q_1\}})^* \alpha_{q_1, q_2}^{Q - \{q_1\}}$$

$$= \emptyset + a^* a (\alpha_{q_1, q_1}^{Q - \{q_1\}})^* b b^*$$

$$\alpha_{u,v}^\emptyset = \begin{cases} \varepsilon & u=v, k=0 \\ \emptyset & u \neq v, k=0 \\ \varepsilon + a_1 + \dots + a_k & u=v, k > 0 \\ a_1 + \dots + a_k & u \neq v, k > 0 \end{cases}$$

$$B = Q - \{q_1, q_2\} = \{q_0\}$$

$$\alpha_{q_1, q_1}^B = \alpha_{q_1, q_1}^{Q - \{q_1, q_2\}} = \emptyset$$

$$\alpha_{q_1, q_2}^B (\alpha_{q_2, q_2}^B)^* \alpha_{q_2, q_1}^B = a (b)^* a$$

$$= a + b (b)^* \underline{a a^* a}$$

$$\alpha_{u,v}^R = \alpha_{u,v}^{R - \{q\}} + \alpha_{u,q}^{R - \{q\}} (\alpha_{q,q}^{R - \{q\}})^* \alpha_{q,v}^{R - \{q\}}$$

which of these sets are regular:

- \*  $A = \{x \in \{a,b\}^*, |x| < 10\}$  ✓
- \*  $B = \{x \in \{a,b\}^*, |x| > 10\}$  ✓
- \*  $C = \{x \in \{a,b\}^* : \#a(x) = \#b(x)\}$  ✗
- \*  $A \cap C$  ✓

\*  $B \cap C$  ✗

\*  $D = \{ x : \{ a, b \}^* : [\#a(x) + \#b(x)] \bmod 3 = 1 \}$  ✓

\*  $(B \cap C) \cup (C - B) = C$  → not regular

cannot be regular  
otherwise the  
right side would  
be regular.

finite set  
so regular