Lecture 28

Thursday, March 28, 2024 11:40 AM

Acceptance by NPDAs

Configuration:

A configuration is an element $\underline{Q \times \Sigma^* \times \Gamma^*}$ and tells us "everything" about the current situation.

The start configuration is always $(s, x, \perp)$

start state (s) input string (x)

The next config. relation denoted by $\xrightarrow[M]{1}$ describes how the machine can move from one config to another.

If $((q_1, a, A), (q_2, \gamma)) \in \delta$ then

$\forall y \in \Sigma^*, \beta \in \Gamma^*$

$$(q_1, ay, A\beta) \xrightarrow[M]{1} (q_2, y, \gamma\beta)$$

If $((q_1, \varepsilon, A), (q_2, \gamma)) \in \delta$ then

$\forall y \in \Sigma^*, \forall \beta \in \Gamma^*$

$$(q_1, y, A\beta) \xrightarrow[M]{1} (q_2, y, \gamma\beta)$$

$$C \xrightarrow[M]{0} C$$

$$C \xrightarrow[M]{n+1} D \iff \exists E, \ C \xrightarrow[M]{n} E, \ E \xrightarrow[M]{1} D$$

$$C \xrightarrow[M]{*} D \iff \exists n \geq 0 \text{ s.t } C \xrightarrow[M]{n} D$$

M accepts x $\iff$ $(s, x, \perp) \xrightarrow[M]{*} (q, \varepsilon, \gamma)$

for some $q \in F$

There is another way / convention for acceptance which is defined based on "empty stack" rather than accept state. In that convention, we don't need F.

M accept x $\iff$ $(s, x, \perp) \xrightarrow[M]{*} (q, \varepsilon, \varepsilon)$

$q \in Q$

These two definitions are "equivalent" so we use the former convention.

CFGs vs NPDAs

* If $A = L(G)$ for some CFG G, then we can always find an NPDA

then we can always find an NPDA M such that $L(M) = L(G)$.

* In fact, we can find M with only 2 states!

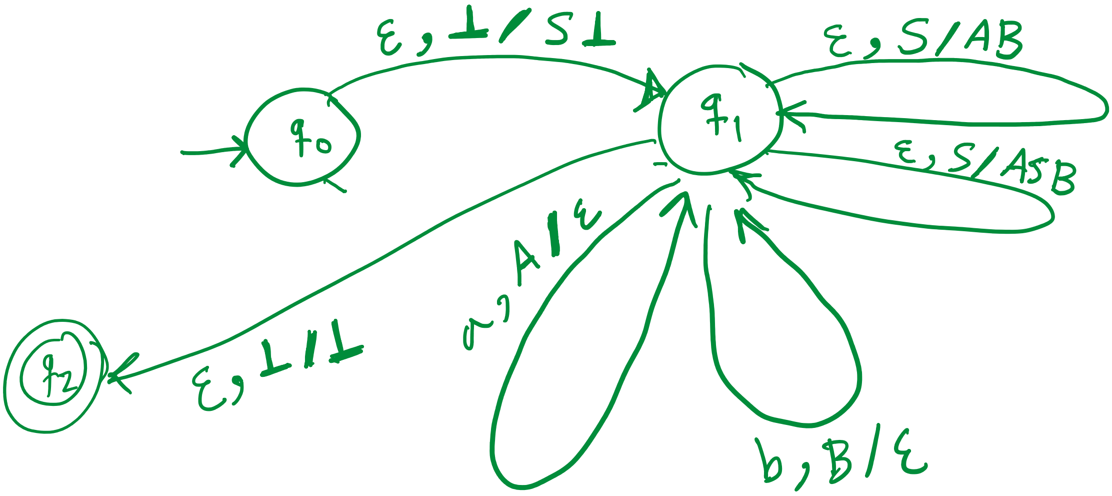
$S \to \underset{\sim\sim\sim}{ASB} \mid AB$

$A \to a$

$B \to b$

$L(G) = \{a^n b^n : n \geqslant 1\}$

$(q_0, \varepsilon, \bot), (q_1, S\bot)$



* For every NPDA M, $\exists$ a CFG G such that $L(M) = L(G)$. (proof?)

$$\boxed{\text{CFG} \approx \text{NPDAs}}$$

Deterministic PDAs (DPDA)

* no ε-transitions

- δ is a function (rather than a relation)

- The end of the input string is marked by $\dashv$

$$x = abca\dashv$$

- It is very easy to check if x is accepted by a DPDA. (no need for CKY).

- Some languages can be described by CFGs/NPDAs but not with DPDAs.

- DPDAs are closed under complementation.

$$A \longleftrightarrow \Sigma^* - A$$

[we can switch accept with non-accept states]