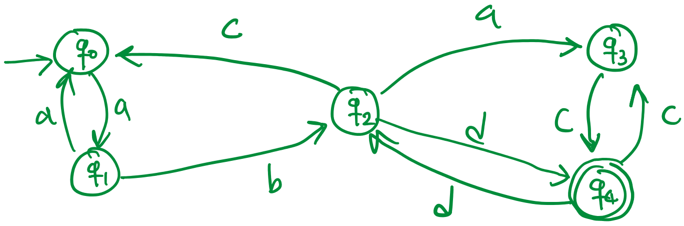
Lecture 14

Friday, February 9, 2024 11:31 AM

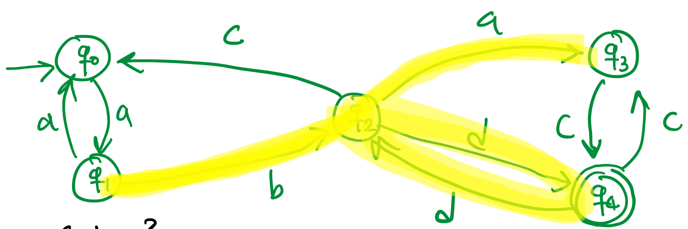


Turning NFAs to regular expressions

NFA $N(Q, \Sigma, \Delta, S, F)$

idea: for any $u, v \in Q$, and any $R \subseteq Q$, define $A^R_{u,v}$ to be the set of all strings that can "take us to v starting from u, and only visiting states in R on the way".

Example:



$$A^{\{q_2\}}_{q_1, q_3} = \{ba\}$$

$$A^{\{q_2, q_4\}}_{q_1, q_3} = L(\; b(dd)^*a + bd(dd)^*c)$$

$$A^{\{q_2, q_3, q_4\}}_{q_1, q_3} = L(\; b(dd)^*a + bd(dd)^*c + ba(cc)^* + \cdots)$$

if $u \notin R$ then you can only visit it in the beginning. Also, if $v \notin R$, you can only visit it at the end.

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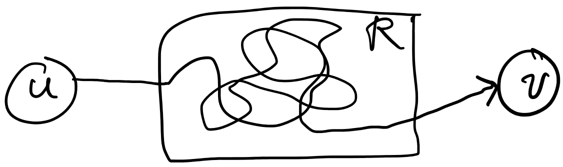
(1) How to find a regular expression $\alpha^R_{u,v}$ "that corresponds to A^R

① How to find a regular expression $\alpha_{u,v}$ that corresponds to $A^R_{u,v}$

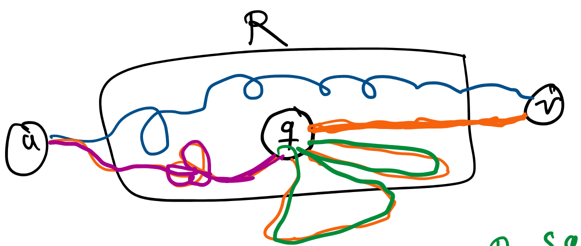
② Using that, how can we find a regular expression for the NFA?

Idea for ①:

$A^R_{u,v}$



take some $q \in R$



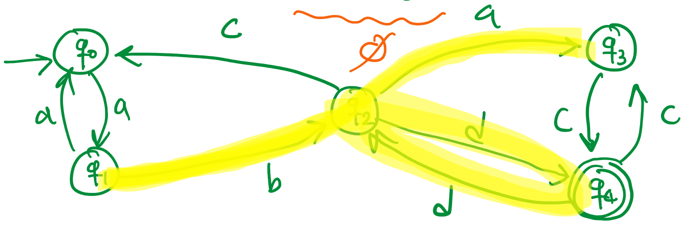
$$A^R_{u,v} = A^{R-\{q\}}_{u,v} \cup A^{R-\{q\}}_{u,q} \left(A^{R-\{q\}}_{q,q}\right)^* A^{R-\{q\}}_{q,v}$$

$$\alpha^R_{u,v} = \alpha^{R-\{q\}}_{u,v} + \alpha^{R-\{q\}}_{u,q} \left(\alpha^{R-\{q\}}_{q,q}\right)^* \alpha^{R-\{q\}}_{q,v}$$

$q = q_2$

$$\alpha^{\{q_2,q_3,q_4\}}_{q_1,q_3} = \alpha^{\{q_3,q_4\}}_{q_1,q_3} + \alpha^{\{q_3,q_4\}}_{q_1,q_2} \left(\alpha^{\{q_3,q_4\}}_{q_2,q_2}\right)^* \alpha^{\{q_3,q_4\}}_{q_2,q_3}$$

$\emptyset$



$$= \emptyset + b\left(ac(cc)^*d + d(cc)^*d\right)^*\left(a(cc)^* + dc(cc)^*\right)$$