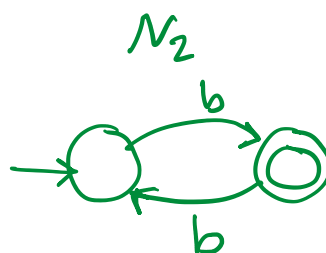
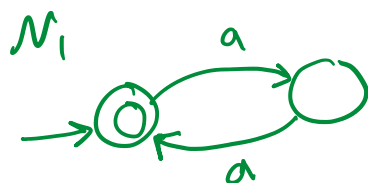


# Lecture 10

Thursday, February 1, 2024 11:30 AM



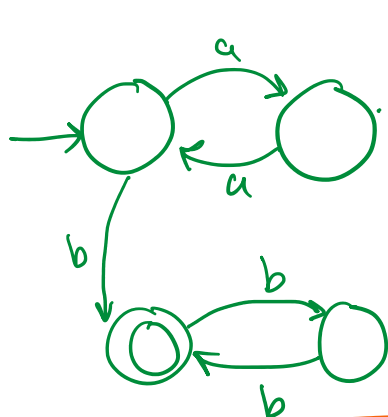
\* Draw an NFA  $N_3$  such that

$$L(N_3) = L(N_1) \cdot L(N_2)$$

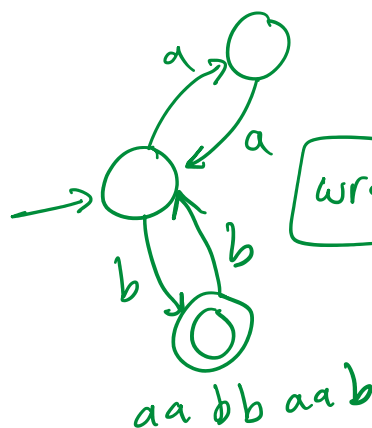
.....      ↓      .....  
set concat

$aa bbb \in L(N_1) \cdot L(N_2)$   
 $aa bb \notin L(N_1) \cdot L(N_2)$   
 $bb aaa \notin L(N_1) \cdot L(N_2)$   
 $bbbaa \notin L(N_1) \cdot L(N_2)$   
 $b \in L(N_1) \cdot L(N_2)$

$$AB = \{xy : x \in A, \underline{y \in B}\}$$



right



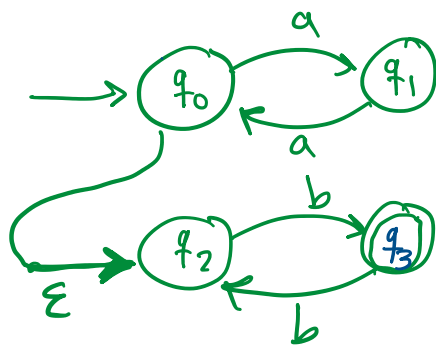
wrong

NFAs with  $\epsilon$ -transitions:

We extend  $\Delta: Q \times \Sigma \rightarrow 2^Q$  to

$\Delta: Q \times (\Sigma \cup \{\epsilon\}) \rightarrow 2^Q$  so that

the NFA can change its state without consuming any symbol.



$aab$   
 $q_0 \xrightarrow{a} q_1 \xrightarrow{a} q_0 \xrightarrow{\epsilon} q_2 \xrightarrow{b} q_3 \in F$

Thm. Any NFA with  $\epsilon$ -transitions can be turned into an equivalent NFA (or DFA)  
 (if you're interested, check the book but we don't cover it)

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Example: Assume you are given an NFA  $N_1$ . Describe an NFA  $N_2$  such that

$$(a) L(N_2) = L(N_1)^+$$

$$(b) L(N_2) = L(N_1)^*$$

$$\begin{aligned}
 \text{Recall: } A^* &= A^0 \cup A^1 \cup A^2 \cup A^3 \cup \dots \\
 &= \{x_1 x_2 x_3 \dots : x_i \in A\} \cup \{\epsilon\}
 \end{aligned}$$

$$\begin{aligned}
 A^+ &= A^1 \cup A^2 \cup A^3 \cup \dots \\
 &= \{x_1 x_2 x_3 \dots : x_i \in A\}
 \end{aligned}$$

$$\text{e.g. } A = \{aa, bb\}$$

$$aa aa bb aa bb \in A^*$$

$$\epsilon \in A^*$$

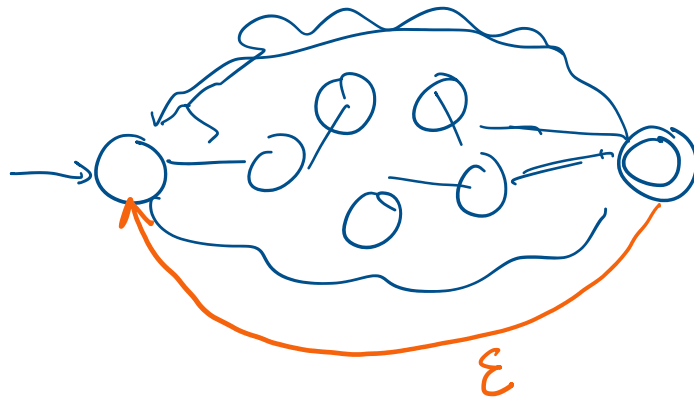
$$\epsilon \notin A^+$$

$$\epsilon \in A'$$

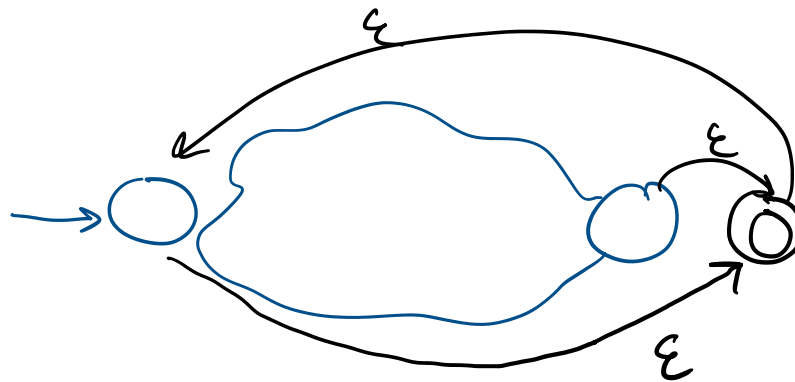
$$\epsilon \notin A^+$$

$$B = \{ \epsilon, aa, bb \}$$

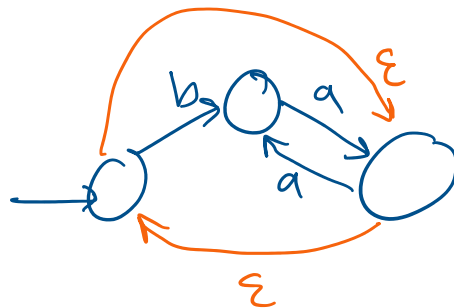
$$B^+ = B^*$$



↓  
 $L(N_1)^+$   
 ✓



$L(N_1)^*$   
 ←



counter example  
 "aa"

\* what if we have more than  
 one start or accept states

\* what about without using  $\epsilon$ -transitions.