Lecture 24

Tuesday, March 19, 2024 11:29 AM

Pumping Lemma for CFLs

How do we prove that $A = \{a^n b^n c^n : n \geq 0\}$ is not a CFL?

$L = \{a^n b^n : n \geq 1\}$

$S \to AC \mid AB$

$C \to SB$

$A \to a$

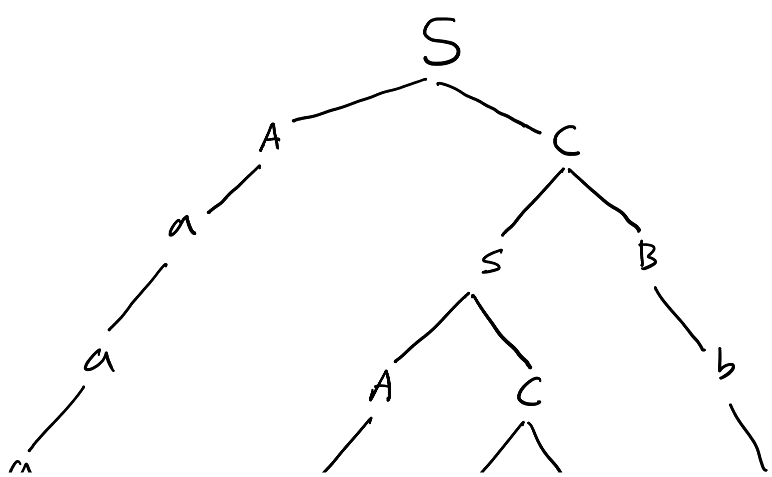
$B \to b$

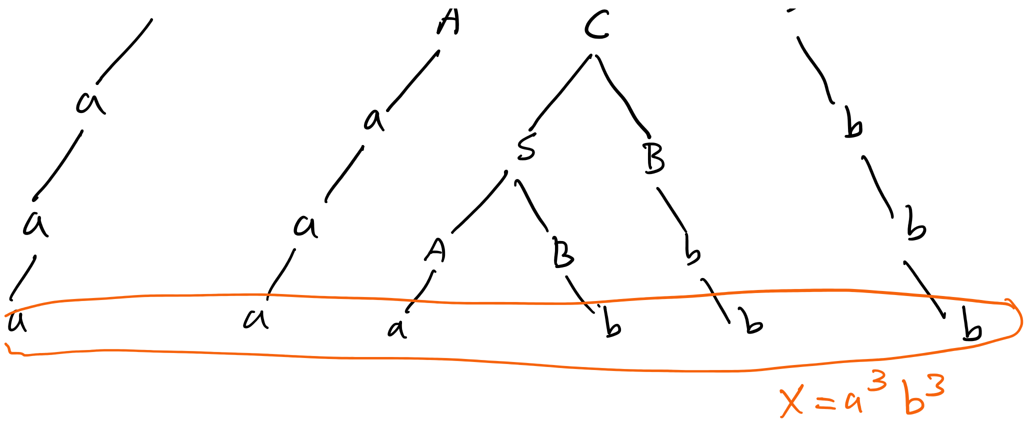
$a^3 b^3 \in L$?

left-most derivation $S \to AC \to aC \to aSB \to aACB$
$\to aaCB \to aaSBB \to$
$\to aaABBB \to \cdots\cdots \to a^3 b^3$

$\to S \to AC \to ASB \to ASb \to \cdots$
right-most derivation $\to a^3 b^3$

The order does not seem to matter, so we use a parse tree.

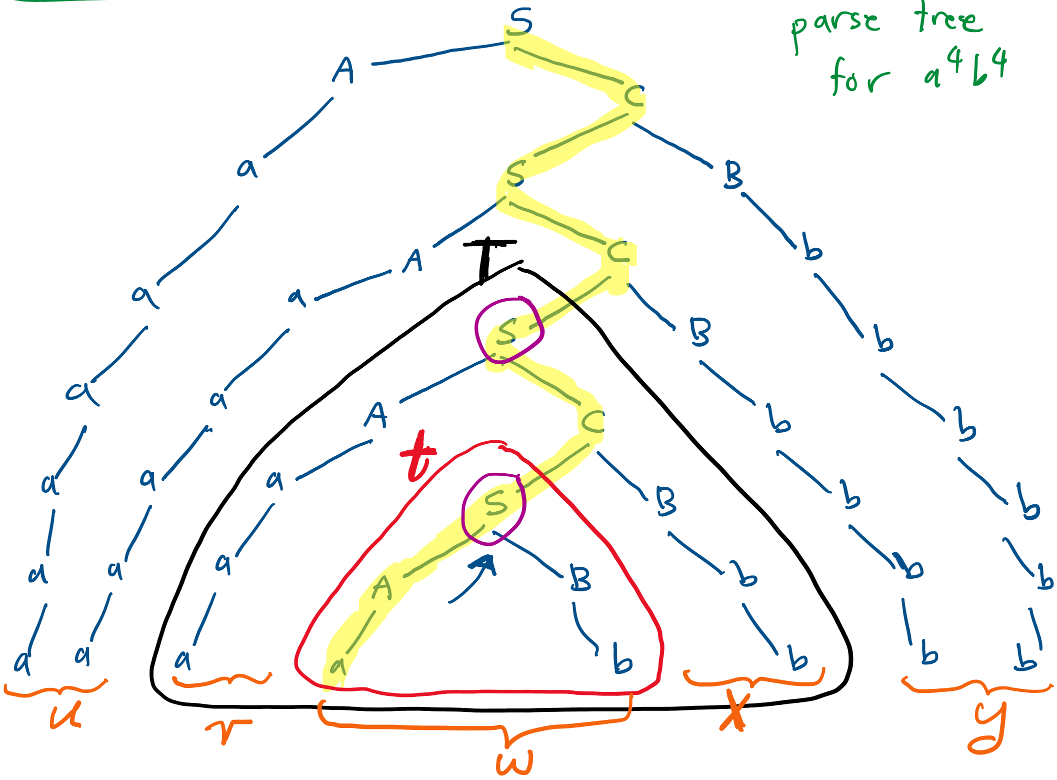




* Every node has at most two children (CNF grammar)
* The number of symbols in each depth is at most twice of those in the previous depth.
* Long strings require larger parse trees.

[the length of the string gives an upper and lower bound on the depth of the tree]

parse tree for $a^4 b^4$



non-terminal

$\underbrace{\qquad\qquad}_{w}$

* The highlighted path has a non-terminal until the second to last layer (we chose the left most such path)
* If the string (e.g., a^4b^4) is long enough, then the path will be long too (longer than the number of non-terminals)
* By pigeon hole principle, there is a repetative non terminal on the path.
* we pick the lowest/deepest repetition.
* We call the corresponding subtrees t and T.
* $uv^iwx^iy \in L$ for any i.

  [e.g., by replacing t with T repeatedly]

Pumping Lemma for CFLs (contra positive form)

A language A is not CFL if:

$\forall K \geq 0$ (demon chooses K)

$\exists z \in A, |z| \geq K$ (we choose z) such that for every way of breaking

$z = uvwxy$ with $vx \neq \varepsilon$ and $|vwx| \leq K$

(demon chooses u, v, w, x, y)

$\exists\ i \geq 0$ such that $uv^iwx^iy \notin A$.

(we choose i)

[if we have a winning strategy against the demon then A is not CFL]

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prove $A = \{a^n b^n c^n : n \geq 0\}$ is not CFL.

Given K, pick $z = a^K b^K c^K$,

$|z| \geq K$ ✓

For every $uvwxy = z$, $|vx| \neq 0$, $|vwx| \leq K$,

pick $i = 2$.

$$z = \overbrace{aaa\cdots aaa}^{K}\ \overbrace{bbb\cdots bb}^{K}\ \overbrace{cc\cdots cc}^{K}$$

$u \quad \underbrace{v \quad w \quad x}\quad y$

since $|vwx| \leq K$, vwx consists of only two types of symbols (e.g., $a^m b^n$, or $b^m c^n$, ..)

so $uv^2wx^2y \notin A$ [will have more of those two symbols compared to the third]