COMPSCI 2AC3 Assignment 3

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Question 1

Step 1:
Firstly, we remove all the unreachable states. However, this DFA does not have any so-called unreachable states, so we ignore it and move on. I then make a transition table to assist with the minimization process.

	a	b
q_0	q_1	q_3
q_1	q_2	q_4
q_2	q_1	q_3
q_3	q_4	q_0
q_4	q_4	q_4

Step 2:
Next, we make a 0 equivalence(π_0) of the transition table which is essentially 2 different sets - one for the final states, and one for the non-final states.

$$\pi_0 = \{q_0, q_2\}\{q_1, q_3, q_4\}$$

Step 3:
Next, we make a 1 equivalence(π_1) of the 0 equivalence(π_0) above. This is basically, comparing each state with another and checking if their transitions with a and b are states which are in the same set as above. If they are not, we split them up into different sets. We continue these steps until we get 2 equivalences which are the same ie, no more splitting up of states into different sets. In our case below is the final equivalence.

$$\pi_1 = \{q_0, q_2\}\{q_1\}\{q_3\}\{q_4\}$$

Step 4:
Lastly, we make a DFA using the states and transitions from the original DFA however, after minimization we concur that states (q_0, q_2) are the same so our new DFA will have one reduced state than the original.

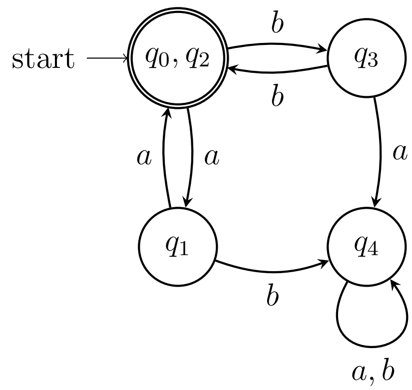

Figure 1: Final DFA after minimization.

Question 2

We define the following production P:

$$
\begin{aligned}
B &\to 1B0 \mid 1\$10 \\
C &\to 1C0 \mid 0D1 \\
A &\to B \mid C \\
D &\to 0D0 \mid 1D1 \mid 1\$1
\end{aligned}
$$

Handling the absence of leading zeros is crutial when working with binary representation of positive integers apart from the reversal part. Issues arrise when we try to factor in the additional digit in the representation of n+1 compared to n, we have to proceed with caution when that happens.

Question 3

$$A = \{a^p \mid p \text{ is a prime number}\}$$

Language A is **NOT** regular, we will prove this using the "Pumping Lemma Theorem".

The theorem states that:
For all k $\geq$ 0, there exists a string x,y,z such that xyz $\in$ A, $|y| \geq$ k, and for all strings u,v,w that satisfy y = uvw, v $\neq \epsilon$, there exists i $\geq$ 0 such that xuv^2wz $\notin$ A.

We start by assuming that A is a regular language:

$$A = \{a, aa, aaaa, \ldots\}$$

Let, $xyz = aaaaaaa \in A$
$x = aa, y = aaa, z = aa$ and $|y| \geq k.$
Let $y = uv^iw$
Then, $aaa = uv^iw$ where $u = a, v^i = a, w = a.$
Therefore, $xuv^iwz = aaaa^iaaa.$
Let $i = 0 \Rightarrow a \cdot a \cdot a \cdot a^i \cdot a \cdot a \cdot \ a$
$\Rightarrow$ a $\cdot a \cdot a \cdot a^0 \cdot a \cdot a \cdot \ a$
$\Rightarrow$ a $\cdot a \cdot a \cdot 1 \cdot a \cdot a \cdot \ a$
$\Rightarrow aaaaaa \notin A.$

Proven by providing a contradiction using the pumping lemma theorem hence, A is not a regular expression.

Question 4

To prove $L(M_1) = L(M_2)$, we will use product construction method to make a new DFA M.

$$M_1 = (Q_1, \Sigma_1, \delta_1, s_1, F_1)$$
$$M_2 = (Q_2, \Sigma_2, \delta_2, s_2, F_2)$$
$$M = M_1 \times M_2 = (Q_1 \times Q_2, \Sigma_1 \times \Sigma_2, \delta_1 \times \delta_2, s_1 \times s_2, F_1 \times F_2)$$

The product DFA M has states (q_1, q_2) from DFA's M_1 and M_2 respectively The starting state of M is a pair consisting of start states from M_1 and M_2. For each transition of states in M_1 and M_2, we add the transitions to M.

The main idea of the arguement is that if a string x belongs to M_1, it will also belong to M_2 and this proves that both DFA's are regular which in turn implies that their product is also regular. This is true for strings of at most 100 characters however, if the length exceeds 100 charcaters, M revisits states due to its finite nature, leading to repitition. This indicates that both M_1 and M_2 must form cycles in their transitions. Therefore, we can prove that the languages are equal.