Predicate logic

Exercise 3

Prove or disprove ∀x ∈ N, ∃y ∈ N: x – y^3 = 0

Quantifiers - ∀, ∃

Predicates – P(x) ≡ ∃y ∈ N: x – y 3 = 0

Q (x, y) ≡ x – y 3 = 0

Variables – x and y.

Domain - ∈ N.

This statement is false.

X is a universal quantifier. To prove it false, we need to provide a counter example which makes f(x) false. We can take x=2.

Y is an existential quantifier. To prove it false, we need to provide a general argument. Q(2,y) must be false in general. This is the case because $2 - y^3 = 0$.

$2\text{-}y^3 = 0$

$y^3 = 2$

$y = \sqrt[3]{2}$

$\sqrt[3]{2} \neq N$

Exercise 7

Prove or disprove $\exists y \in N, \exists x \in N: x - y = 0$

Quantifiers - $\exists$, $\exists$

Predicates - $P(y) \equiv \exists x \in N: x - y = 0$

$Q(x, y) \equiv x - y = 0$

Variables – x, y

Domain – $\in N$

This is a true statement

Y is an existential quantifier. To prove it true, we need to provide an example. To prove p(y) is true, we can take y = 2.

X is an existential quantifier. To prove it true, we need to provide an example. Now Q(x,2) which already proves that it is true. We can take x = 2.

$Q(X,2) \equiv x - y = 0$

$x - 2 = 0$

$x = 2$

now both x and y are equal and are natural numbers.

Exercise 9

Prove or disprove $\forall y \in N, \forall x \in N: x - y = 0$

Quantifiers - $\forall$, $\forall$

Predicates - $P(y) \equiv \forall x \in N: x - y = 0$

$Q(x, y) \equiv x - y = 0$

Domain - $\in N$

Variables – x, y

This is a false statement

Y is a universal quantifier. To prove it false, we need to give an example. To prove p(y) is false, we can take y = 2.

X is a universal quantifier. To prove it false, we need to give an example. To prove Q(x,2) is false, we can take x =3.

If both the x and y are same then the statement might be true, but since it says it has universal quantifiers, and if we give different natural numbers, then the statement is false.

$x - y = 0$

$3 - 2 = 0$

$3 \neq 2$

Exercise 11

Prove or disprove ∃y ∈ R, ∃x ∈ N: x − y = 0

Quantifier - ∃, ∃

Predicates - P(y) ≡ ∃x ∈ N: x − y = 0

Q (x, y) ≡ x − y = 0

Domains - ∈ R, ∈ N

Variables – x, y

This is a true statement

Y is an existential quantifier. To prove p(y) is true, we need to provide an example. We can take y = $\sqrt{4}$ as it is a real number. Where as $\sqrt{4} = 2$

X is an existential quantifier. To prove Q(x,$\sqrt{4}$) is true, we need to provide an example. We can take x = 2.

Q (x, y) ≡ x – y = 0

Q (x, $\sqrt{4}$) ≡ x - $\sqrt{4}$ = 0

X = $\sqrt{4}$

X = 2