

# Graph Theory (Core)

Click on a question number to see how your answers were marked and, where available, full solutions.

Question Number	Score
<b>Trees</b>	
Question 1	3 / 3
Question 2	2 / 2
<b>Euler Paths and Circuits</b>	
Question 3	4 / 4
Question 4	4 / 4
<b>Total</b>	<b>13 / 13 (100%)</b>

Congratulations, you passed this quiz with a sufficient score. You may include this attempt as part of your self-assessment evidence.

Make sure that you click on "Print this results summary" and save to pdf, so that everything can be read clearly. Do not navigate away from this page before you have saved your result.

## Performance Summary

<b>Exam Name:</b>	Graph Theory (Core)
<b>Session ID:</b>	110911607444
<b>Exam Start:</b>	Tue Dec 26 2023 12:07:29
<b>Exam Stop:</b>	Tue Dec 26 2023 12:25:09
<b>Time Spent:</b>	0:17:39

# Question 1

Which of the following graphs are trees over the vertices  $\{a, b, c, d, e, f\}$ ?

*Please note that you will receive 1 mark for each correct answer and lose 1 mark for each incorrect one.*

$[[a, c], [a, d], [b, d], [c, d], [e, f]]$

☐ True ☒ False

Expected answer:

☐ True ☒ False



$[[a, b], [c, b], [d, a], [e, c], [f, d]]$

☒ True ☐ False

Expected answer:

☒ True ☐ False



$[[c, e], [e, f], [a, b], [b, c], [a, d]]$

☒ True ☐ False

Expected answer:

☒ True ☐ False



**Gap 0**

✓ You chose a correct answer. You were awarded 1 mark.

**Gap 1**

✓ You chose a correct answer. You were awarded 1 mark.

**Gap 2**

✓ You chose a correct answer. You were awarded 1 mark.  
You scored 3 marks for this part.

**Score: 3/3** ✓

## Advice

1. The first graph is not a tree as it contains a loop.
2. The second graph is a tree.
3. The third graph is a tree.

## Question 2

Find spanning trees for the following graphs over the vertices  $\{a, b, c, d, e, f\}$ . Enter your solution as the list of edges, using square brackets - such as  $[[a,b],[c,d]]$ .

### First Graph

Enter a spanning tree for the graph with edges  $[[a, b], [a, c], [a, d], [a, f], [b, e], [b, f], [c, d], [c, f], [d, e], [d, f], [e, f]]$ :

$[[a, b], [a, c], [a, d], [a, f], [b, e]]$

$[[a, b], [a, c], [a, d], [a, f], [b, e]]$  ✓

Expected answer:

$[[a, b], [a, c], [a, d], [a, f], [b, e]]$

✓ The graph you entered forms a spanning tree of the original graph. You were awarded 1 mark.  
You scored 1 mark for this part.

Score: 1/1 ✓

## Second Graph

Enter a spanning tree for the graph with edges  $[[a, d], [a, e], [a, f], [b, c], [b, d], [c, e], [c, f], [d, e], [d, f], [e, f]]$ :

$[[a, d], [d, e], [b, d], [b, c], [d, f]]$

$[[a, d], [d, e], [b, d], [b, c], [d, f]]$  ✓

Expected answer:

$[[a, d], [d, e], [b, d], [b, c], [d, f]]$

✓ The graph you entered forms a spanning tree of the original graph. You were awarded 1 mark.

You scored 1 mark for this part.

Score: 1/1 ✓

## Advice

1. A possible spanning tree for the first graph is  $[[a, b], [a, c], [a, d], [a, f], [f, e]]$ .

## Question 3

Consider the following graphs over the vertices  $\{a, b, c, d, e, f\}$ .

Which of them contain Euler paths?

a)

$\{[a, b], [a, c], [c, d], [d, e], [d, f]\}$

- ☐ Contains a Euler path   
 ☐ Contains a Euler Circuit   
 ☐ Contains both  
☒ Contains neither.



Expected answer:

- ☐ Contains a Euler path   
 ☐ Contains a Euler Circuit  
☐ Contains both   
☒ Contains neither.

You chose a correct answer. You were awarded **2** marks.  
 You scored **2** marks for this part.

Score: 2/2

b)

$\{[a, b], [a, d], [b, c], [b, e], [b, f], [c, d], [c, f]\}$

- ☒ Contains a Euler path   
 ☐ Contains a Euler Circuit   
 ☐ Contains both  
☐ Contains neither.



Expected answer:

- ☒ Contains a Euler path   
 ☐ Contains a Euler Circuit  
☐ Contains both   
☐ Contains neither.

You chose a correct answer. You were awarded **2** marks.  
 You scored **2** marks for this part.

## Advice

1. The degrees of the vertices in the first graph are  $[2, 1, 2, 3, 1, 1]$ . The number of vertices with an odd degree is 4. Since  $4 \geq 2$ , it doesn't contain Euler paths, Since  $4 \neq 0$ , it doesn't contain Euler circuits,

2. The degrees of the vertices in the first graph are  $[2, 4, 3, 2, 1, 2]$ . The number of vertices with an odd degree is 2. Since  $2 \leq 2$ , it contains Euler paths, Since  $2 \neq 0$ , it doesn't contain Euler circuits,

## Question 4

Consider the following graphs over the vertices  $\{a, b, c, d, e, f\}$ .

Which of them contain Euler paths?

a)

$[[a, b], [a, c], [a, e], [b, e], [b, f], [c, d], [c, e], [d, f], [e, f]]$

☐ Contains a Euler path    ☐ Contains a Euler Circuit    ☐ Contains both

☒ Contains neither.



Expected answer:

☐ Contains a Euler path    ☐ Contains a Euler Circuit

☐ Contains both    ☒ Contains neither.

✓ You chose a correct answer. You were awarded **2** marks.

You scored **2** marks for this part.

Score: 2/2 ✓

b)

$$[[a, b], [a, c], [a, e], [a, f], [b, d], [b, e], [c, e], [d, e], [d, f], [e, f]]$$

- ☐ Contains a Euler path   
 ☐ Contains a Euler Circuit   
 ☐ Contains both  
☒ Contains neither.



Expected answer:

- ☐ Contains a Euler path   
 ☐ Contains a Euler Circuit  
☐ Contains both   
 ☒ Contains neither.

✓ You chose a correct answer. You were awarded **2** marks.  
 You scored **2** marks for this part.

Score: 2/2 ✓

## Advice

1. The degrees of the vertices in the first graph are  $[3, 3, 3, 2, 4, 3]$ . The number of vertices with an odd degree is 4. Since  $4 \geq 2$ , it doesn't contain Euler paths, Since  $4 \neq 0$ , it doesn't contain Euler circuits,
2. The degrees of the vertices in the first graph are  $[4, 3, 2, 3, 5, 3]$ . The number of vertices with an odd degree is 4. Since  $4 \geq 2$ , it doesn't contain Euler paths, Since  $4 \neq 0$ , it doesn't contain Euler circuits,