Question 2

Proof by cases

Case 1: $7n^2 + 5n - 1$ is odd. Let $n = (2k + 1)$

$= 7(2k + 1)^2 + 5(2k + 1) - 1$

$= 7(4k^2 + 4k + 1) + 10k + 5 - 1$

$= 28k^2 + 28k + 7 + 10k + 5 - 1$

$= 28k^2 + 38k + 11$

$= 2(14k^2 + 19k) + 11$

So, this is odd in case 1

Case 2: $7n^2 + 5n - 1$ is even. Let $n = 2k$

$= 7(2k)^2 + 5(2k) - 1$

$= 7(4k^2) + 10k - 1$

$= 28k^2 + 10k - 1$

This is still odd because two even number and one odd number in this equation sum up to an odd number, so the statement $7n^2 + 5n - 1$ is odd for any n € N is true.

Question 4

Proof by equivalence

p: n is even

q: $2n^2 + 7n + 8$ is even

p -> q Direct Proof: Assume n is even. Then there is a number m with $n = 2m$

so $= 2(2m)^2 + 7(2m) + 8$

$= 2(4m^2) + 14m + 8$

$= 8m^2 + 14m + 8$

$= 2(4m^2 + 7m + 8)$

Therefore, q is true.

¬p: n is odd

¬q: $2n^2 + 7n + 8$ is odd

q -> p Indirect proof: Assume n is odd. Then there is a number m with

$n = 2m + 1.$

$= 2(2m + 1)^2 + 7(2m + 1) + 8$

$= 2(4m^2 + 4m + 1) + 7(2m + 1) + 8$

$= 8m^2 + 8m + 2 +14m + 7 + 8$

$= 8m^2 + 22m + 17$

So, this final equation is odd as a sum of two even numbers and one odd number. Therefore ¬q is true.

Question 5

p: n is odd

q: $8n^2 - n - 3$ is even

we have to prove p -> q

so, for n is odd, there is $n = 2m + 1$

$= 8n^2 - n - 3$

$= 8(2m + 1)^2 - (2m + 1) - 3$

$= 8(4m^2 + 4m + 1) - 2m - 1 - 3$

$= 32m^2 + 32m + 8 - 2m - 4$

$= 32m^2 + 30m + 4$

$= 2(16m^2 + 15m + 4)$

So, q is true and the statement is true.