Question 3

Show that $\log(x) - 8 \cdot x \log(x) + 3 \cdot x^x = O(x^x)$

x^x is the fastest growing power.

$\text{Log}(x) <= x^x, \quad x >= 4$

$X \log(x) <= x^x, \quad x >= 4$

$x^x <= x^x, \quad x >= 0$

so, $\forall x > 4$, we have:

$\log(x) <= x^x$

$- 8 \cdot x \log(x) <= |-8| x \log(x) <= 8.x^x$

$3. x^x <= 3. x^x$

So,

$\text{Log}(x) – 8. X\log(x) + 3.x^x <= x^x + 8.x^x = 3.x^x$

$F(X) = 12. (x^x)$

Therefore $C = 12$, $k = 4$ and $f(x) = O(x^x)$

Question 4

Show that $5 - 4 \cdot \log(x) + 2^x = O(2^x)$

The fastest growing power is 2^x

$1 <= 2^x, \quad x >= 4$

$Log(x) <= 2^x, \quad x >= 4$

$2^x <= 2^x, \quad x >= 0$

So, $\forall x > 4$, we have:

$5 <= 5.\ 2^x$

$-4\ .\ \log(x) <= |-4| \log(x) <= 4.\ 2^x$

$2^x <= 2^x$

So,

$5 – 4.\ Log(x) + 2^x <= 5.\ 2^x + 4.\ 2^x + 2^x$

$F(x) = 10.\ (2^x)$

Therefore $C = 10$, $k = 4$ are witnesses that $F(x) = O(2^x)$

Question 5

Show that $4 - 10 \cdot \log(x) + x^2 = O(x^2)$

Fastest growing power is x^2

$1 \leq x^2$, $x \geq 3$

$\text{Log}(x) \leq x^2$, $x \geq 3$

$x^2 \leq x^2$, $x \geq 0$

So, $\forall x > 3$, we have:

$4 \leq 4.\ x^2$

$-10.\ \text{Log}(x) \leq |-10| \log(x) \leq 10.x^2$

$x^2 \leq x^2$

so,

$4 - 10.\ \text{Log}(x) + x^2 \leq 4.\ x^2 + 10.x^2 + x^2$

$f(x) = 15.\ X^2$

therefore $C = 15$, $k = 3$ are witnesses $f(x) = O(x^2)$

Question 6

Show that $-7 \cdot x - 5 \cdot x \log(x) + 9 \cdot x^2 = O(x^2)$

$x \le x^2, \quad x \ge 3$

$x \log(x) \le x^2, \quad x \ge 3$

$x^2 \le x^2, \quad x \ge 0$

So, $\forall x > 3$, we have:

$-7.x^2 \le |7|x^2 \le 7x^2$

$-5. x.\log(x) \le |-5|. x\log(x) \le 5. x^2$

$9. x^2 \le 9. x^2$

So,

$-7. X - 5. x.\log(x) + 9. x^2 \le 7. x^2 + 5. x^2 + 9. x^2$

$F(x) \le 21. x^2$

Therefore $C = 21$, $k = 3$ are witnesses $f(x) = O(x^2)$