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1CR18CS028
5-A

ASSIGNMENT - 1

1) Computational methods for specification of a language is represent by 5 tuple:- (Q, Σ, δ, q_0, F)

i) Q is non empty, finite set of states
Eg:- $Q = \{q_0, q_1, q_2\}$

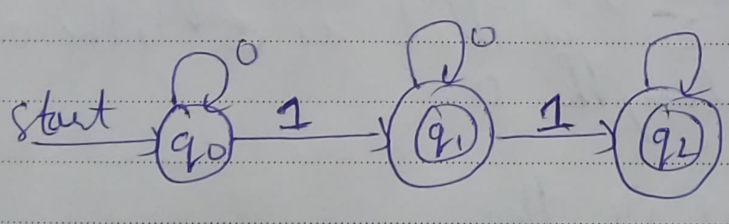
ii) Σ is non empty finite set of input alphabets
Eg:- $\Sigma = \{0, 1\}$ or $\Sigma = \{a, b\}$

iii) δ is a transition function which is a mapping from $Q \times \Sigma$ to Q which is denoted by
$\delta : Q \times \Sigma$ to Q ie $\delta(q_i, a) = q_j$

iv) q_0 is the intial state from where any input is processed ie $q_0 \in Q$.

v) F is a set of final states/state of Q
ie $F \subseteq Q$

Eg:-



$Q = \{q_0, q_1, q_2\}$
$\Sigma = \{0, 1\}$
q_0 is intial state
$q_1, q_2 \rightarrow$ final state

Transition fn
$\delta(q_0, 0) = q_0$
$\delta(q_0, 1) = q_1$
$\delta(q_1, 0) = q_1$
$\delta(q_1, 1) = q_2$
$\delta(q_2, 0) = q_2$
$\delta(q_2, 1) = q_2$

CMR

B) i) Alphabet :- An alphabet is any finite set of symbols Σ

Eg :- $\Sigma = \{a, b, c, d\}$

$\Sigma = \{0, 1\}$ - Binary alphabet

ii) String :- A string is a finite sequence of symbol chosen from some alphabet Σ.

Eg :- 'cabd cab' is a valid string on the alphabet set $\Sigma = \{a, b, c, d\}$, 10111 & 01101 are string from binary alphabet $\Sigma = \{0, 1\}$

Empty string (ε) :- The string with zero occurence of symbol.

iii) Power of an alphabet :- If Σ is an alphabet, we can express the set of all string of certain length from that alphabet by using the exponential notation.

Σ^k : the set of strings of length k, each of whose is in Σ

Eg :- $\Sigma^0 = \{\varepsilon\}$, regardless of what alphabet Σ is That is ε is the only string of length 0.

If $\Sigma = \{0, 1\}$ then

1. $\Sigma^1 = \{0, 1\}$
2. $\Sigma^2 = \{00, 01, 10, 11\}$ and so on...

iv) Language :- A language is a subset of Σ^x for some alphabet Σ it can be finite or infinite.

Eg: If the language takes all possible string of length 2 in alphabet $\Sigma = \{0, 1\}$ then

$L = \{01, 10, 11, 00\}$

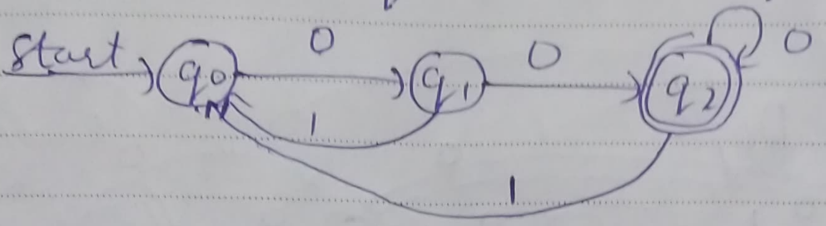
The language of all strings consisting of n 0s followed by n 1s ($n \geq 0$)

$\{\varepsilon, 01, 0011, 000111, \ldots\}$

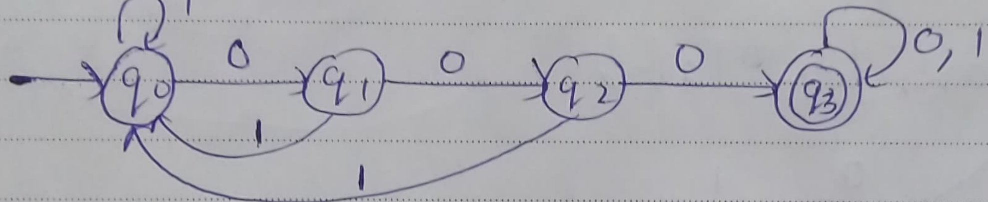
Σ^* is a language for any alphabet Σ

ϕ, the empty language, is a language over any alphabet.

2) a) The set of all strings ending in 00

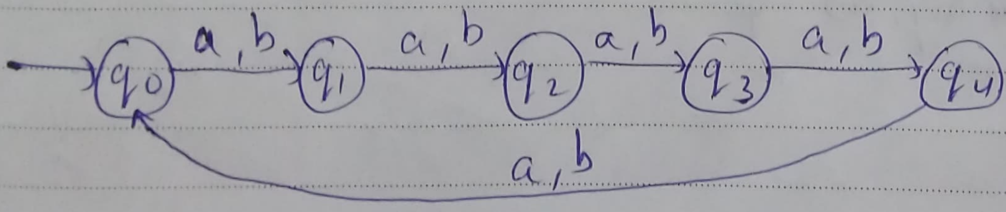


b) The set of all string with three consecutive 0's (not necessarily at end)



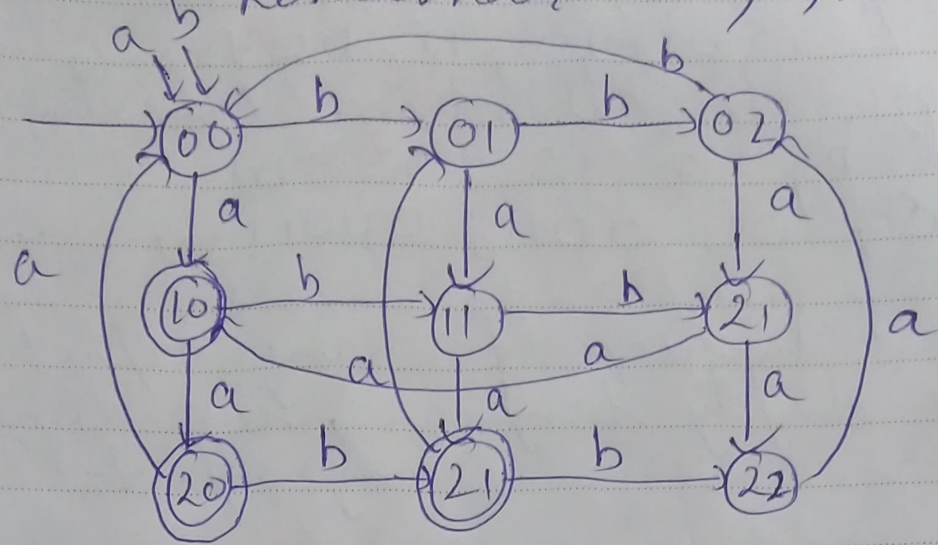
3) a) $L = \{w : |w| \bmod 5 \neq 0\}$

Remainder = 0, 1, 2, 3, 4



b) $L = \{w : n_a(w) \bmod 3 > n_b(w) \bmod 3\}$

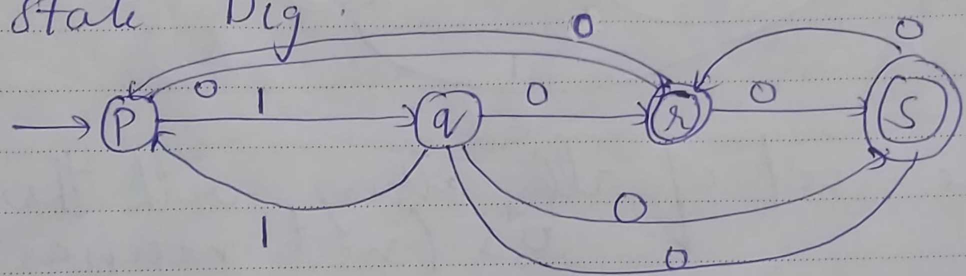
Remainder = 0, 1, 2



Final state = 10, 20, 21

4) NDFSM :-

State Dig :



Transition Table

	0	1
{p}	{p, r}	{q}
{q}	{r, s}	{p}
{r}	{p, s}	{r}
{s}	{q, r}	ϕ

Here final state r, s

DFSM

	0	1
$\{p\}$	$\{p,r\}$	$\{q\}$
$\{p,r\}$	$\{p,r,s\}$	$\{q,r\}$
$\{q\}$	$\{r,s\}$	$\{p\}$
$\{p,r,s\}$	$\{p,q,r,s\}$	$\{q,r\}$
$\{q,r\}$	$\{p,r,s\}$	$\{p,r\}$
$\{r,s\}$	$\{p,q,r,s\}$	$\{r\}$
$\{p,q,r,s\}$	$\{p,q,r,s\}$	$\{p,q,r\}$
$\{r\}$	$\{p,s\}$	$\{r\}$
$\{p,q,r\}$	$\{p,r,s\}$	$\{p,q,r\}$
$\{p,s\}$	$\{p,q,r\}$	$\{q\}$

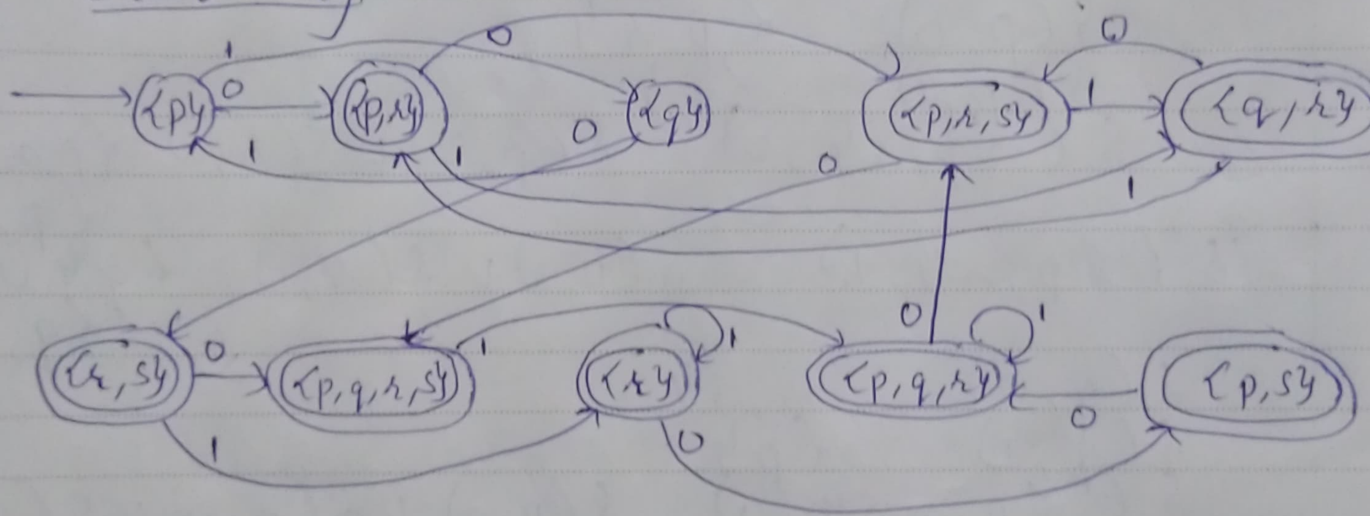
$$\delta(p,0) = \{p,r\}$$
$$\delta(p,1) = \{q\}$$
$$\delta(\{p,r\},0) = \delta(p,0) \cup \delta(r,0) = \{p,r\} \cup \{r,s\} = \{p,r,s\}$$
$$\delta(\{p,r\},1) = \delta(p,1) \cup \delta(r,1) = \{q\} \cup \{r\} = \{q,r\}$$
$$\delta(q,0) = \{r,s\}$$
$$\delta(q,1) = \{p\}$$
$$\delta(\{p,r,s\},0) = \delta(p,0) \cup \delta(r,0) \cup \delta(s,0) = \{p,q,r,s\}$$
$$\delta(\{p,r,s\},1) = \delta(p,1) \cup \delta(r,1) \cup \delta(s,1) = \{q,r\}$$
$$\delta(\{q,r\},1) = \delta(q,1) \cup \delta(r,0) = \{r,s\} \cup \{p,s\} = \{p,r,s\}$$
$$\delta(\{q,r\},1) = \delta(q,1) \cup \delta(r,1) = \{p\} \cup \{r\} = \{p,r\}$$
$$\delta(\{r,s\},0) = \delta(r,0) \cup \delta(s,0) = \{p,s\} \cup \{q,r\} = \{q,q,r,s\}$$
$$\delta(\{r,s\},1) = \delta(r,1) \cup \delta(s,1) = \{r\}$$
$$\delta(\{p,q,r,s\},0) = \delta(p,0) \cup \delta(q,0) \cup \delta(r,0) \cup \delta(s,0) = \{p,q,r,s\}$$

$\delta(\{p,q,r,s\},1) = \delta(p,1) \cup \delta(q,1) \cup \delta(r,1) \cup \delta(s,1)$
$= \{p,q,r\}$

$\delta\{r,0\} = \{p,s\}$
$\delta\{r,1\} = \{r\}$
$\delta(\{p,q,r\},0) = \delta(p,0) \cup \delta(q,0) \cup \delta(r,0)$
$= \{p,r,s\}$
$\delta(\{p,s\},0) = \delta(p,0) \cup \delta(s,0) = \{p,q,r\}$
$\delta(\{p,s\},1) = \delta(p,1) \cup \delta(s,1) = \{q\}$

Final State : $\{p,r\}$, $\{p,r,s\}$ $\{q,r\}$ $\{r,s\}$
$\{p,q,r,s\}$ $\{r\}$ $\{p,q,r\}$ $\{p,s\}$

State Dig :



5) Theorem :
Let language $L \leq \Sigma^*$ & suppose L is accepted by NFA $N = (Q, \Sigma, \delta, q_0, F)$ There exist a DFA $D = (Q', \Sigma, \delta', q_0', F')$ that also accepts L i.e. $L(N) = L(D)$
Parameters of D are defined as follows
→ $Q' = 2^Q$
→ $q_0' = \{q_0\}$

→ F' is the set of states in Q' that contain any element of F, $F' = \{q \in Q' \mid q \cap F \neq \phi\}$

→ δ' is the transition fn for D $\delta'(q, a) = \bigcup_{p \in q} \delta(p, a)$ for $q \in Q'$ & $a \in \Sigma$

Now we will prove that $\hat{\delta}'(q_0', x) = \hat{\delta}(q_0, x)$ for every x i.e $L(D) = L(N)$

<u>Basic Step:-</u>

Let x be the empty string ϵ.

$$\hat{\delta}'(q_0', x) = \hat{\delta}'(q_0', \epsilon) = q_0' = \{q_0\}$$

$$\hat{\delta}(q_0, x) = \hat{\delta}(q_0, \epsilon) = \{q_0\}$$

So prove $\hat{\delta}(q_0', x) = \hat{\delta}(q_0, x)$ for $x = \epsilon$

<u>Inductive hypothesis:</u>

Assume that for any y with $|y| \geq 0$ & $|y| = n$

$$\hat{\delta}(q_0', y) = \hat{\delta}(q_0, y) = \{P_0, P_1, P_2, \ldots P_r\} \quad \text{— (1)}$$

is true

<u>Inductive step:-</u>

We have assumed that $|y| = n$ it is true

Now for $|z| = n+1$, we have to prove that $\hat{\delta}'(q_0', z) = \hat{\delta}(q_0, z)$. So z can be represented as $z = ya$ where $|y| = n$

LHS

$\hat{\delta}'(q_0', z) = \hat{\delta}'(q_0', ya)$

$= \delta'(\hat{\delta}'(q_0', y), a)$

$= \delta'(\{P_0, P_1, P_2, \dots P_x\}, a)$

– from eqn ①

$= \delta'(P_0, a) \cup \delta'(P_1, a) \cup \dots \cup \delta'(P_K, a)$

$= \{R_0, R_1 \dots R_K\}$ — ②

RHS :

$\hat{\delta}(q_0, z) = \hat{\delta}(q_0, ya)$

$= \delta(\hat{\delta}(q_0, y), a) = \delta(\{P_0, P_1, P_2 \dots P_K\}, a)$

from eqn ①

$= \delta(P_0, a) \cup \delta(P_1, a) \cup \dots \cup \delta(P_K, a)$

$= \{R_0, R_1, \dots R_K\}$ — ③

From eqn ② & ③ it is proved that LHS = RHS

So $\hat{\delta}'(q_0', z) = \hat{\delta}(q_0, z)$

Hence Proved

So a string is accepted by DFA D iff $\hat{\delta}'(q_0', x) \in F$

From defn it follows that D accepts x iff $\hat{\delta}(q_0, x) \cap F \neq \phi$. So a string is accepted by DFA D, iff it is accepted by NFA N.