Zadaća 01.

Zadatak 8.

a) ~ FORMULE ZA RAČUNANJE SJECIŠTA ZRAKE I SFERE ~

$S(c, R)$ sfera u centru $c = (x_c, y_c, z_c)$ radijusa R

jednadžba: $(p-c)\cdot(p-c) - R^2 = 0$

$p = e + td$

$(e + td - c)\cdot(e + td - c) - R^2 = 0$ dobivamo:

$(d\cdot d)t^2 + 2d\cdot(e-c) + (e-c)\cdot(e-c) - R^2 = 0$

$$t_{1,2} = \frac{-d\cdot(e-c) \pm \sqrt{(d\cdot(e-c))^2 - (d\cdot d)((e-c)\cdot(e-c) - R^2)}}{d\cdot d}$$

Ako je DISKRIMINANTA $d \geq 0$ zraka sijeće sferu.

~ FORMULE ZA RAČUNANJE SJECIŠTA ZRAKE I TROKUTA ~

trokut Δabc, zraka $e + td$, baricentrične koordinate $\alpha + \beta + \gamma = 1$

$e + td = \alpha a + \beta b + \gamma c = a + \beta(b-a) + \gamma(c-a)$

$$\begin{bmatrix} x_a - x_b & x_a - x_c & x_d \\ y_a - y_b & y_a - y_c & y_d \\ z_a - z_b & z_a - z_c & z_d \end{bmatrix} \begin{bmatrix} \beta \\ \gamma \\ t \end{bmatrix} = \begin{bmatrix} x_a - x_e \\ y_a - y_e \\ z_a - z_e \end{bmatrix}$$

- 3×3 sustav $Ax = b$

$$A = \begin{bmatrix} x_a - x_b & x_a - x_c & x_d \\ y_a - y_b & y_a - y_c & y_d \\ z_a - z_b & z_a - z_c & z_d \end{bmatrix} = \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$$

$$x = \begin{bmatrix} \beta \\ \gamma \\ t \end{bmatrix} \quad b = \begin{bmatrix} x_a - x_e \\ y_a - y_e \\ z_a - z_e \end{bmatrix} = \begin{bmatrix} j \\ k \\ l \end{bmatrix}$$

- Cramerovo pravilo:

$$\beta = \frac{\det[b A_2 A_3]}{\det A}, \quad \gamma = \frac{\det[A_1 b A_3]}{\det A}, \quad t = \frac{\det[A_1 A_2 b]}{\det A}$$

raspisano:

$$\beta = \frac{j(ei - hf) + k(gf - di) + l(dh - eg)}{a(ei - hf) + b(gf - di) + c(dh - eg)}$$

$$\gamma = \frac{i(ak - jb) + h(jc - al) + g(bl - kc)}{a(ei - hf) + b(gf - di) + c(dh - eg)}$$

$$t = \frac{f(ak - jb) + e(jc - al) + d(bl - kc)}{a(ei - hf) + b(gf - di) + c(dh - eg)}$$

- ako $\alpha, \beta, \gamma \in (0,1)$; $t > 0$ onda zraka sijeće trokut u točki $p = e + td$

BRUNNEN

Skenirano CamScannerom