Presentation of the model

Autarky case

Demand

In country n, demand consists in a representative consumer who has to choose his consumption of cereals $q_{c,n}$ and meat $q_{m,n}$ with a CES utility function, and the following problem:

$$\max_{\{q_{c,n},q_{m,n}\}} \left\{ U(q) = \left[q_{c,n}^{\frac{\sigma-1}{\sigma}} + q_{m,n}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \right\}$$

Subject to:

$$q_{c,n} + q_{m,n}p = y_n$$
$$q_{c,n} \geq 0, \quad q_{m,n} \geq 0$$

The price of cereals is normalized to 1, and p stands for the relative price of meat on cereals.

Supply

There are 2 types of producers: cereal producers and meat producers, with the following production functions:

$$Q_{c,n}(L_{c,n}) = L_{c,n}.\theta_c(d_n)$$

and

$$Q_{m,n}(L_{m,n};K_{m,n}) = \min\{a_m L_{m,n}; b_m K_{m,n}\}$$

Where d_n is the distance of country n's temperature compared to the optimal temperature for growing crops, and $\theta_c(d_n)$ the productivity function, decreasing with d_n. Land, L, is the only input used either for growing crops or raising livestock, and $K_{m,n}$ stands for the cereals used as feed into meat production. Land and cereals are perfect complements in meat producers' function.

The perfect competition assumption makes profits negative which yield the optimal price:

$$p^* = \frac{\theta_c(d_n)}{a_m} + \frac{1}{b_m}$$

Equilibrium

The equilibrium must satisfy the following constraints:

$$L_{c,n} + L_{m,n} = \overline{L}_n$$
$$Q_{c,n}(L_{c,n}) = q_{c,n} + K_{m,n}$$
$$Q_{m,n}(L_{c,n};K_{m,n}) = q_{m,n}$$

\end{align} Where $\overline{L}_n$ is the total available arable land in country n.

Subsistence consumption

We check the satisfaction of subsistence consumption ex-post. Denote $\underline{Q}$ the minimal quantity of calories (or any nutritional indicator) to eat for survival, and $M = \begin{pmatrix} \mu_m \\ \mu_c \end{pmatrix}$ the conversion matrix of kgs into calories (or any nutritional indicator). We assume $1 \geq \mu_m \geq \mu_c \geq 0$\ Subsistence consumption is satisfied if:

$$(q_{m,n}^* \quad q_{c,n}^*).M \geq \underline{Q}$$

Notes

The equilibrium and the conditions for the satisfaction of subsistence consumption are solvable by hand. Yet, I programed both the numerical and analytical solutions, whih yield similar result with a tolerance of 1e8. The plots below are computed from the analytical solutions.

Trade case

Demand

We assume agents to be the same across countries, i.e. to have same CES σ. Therefore, the program is the same as under autarky

Production

Under trade, we assume land to be a fully immobile input, while cereals can be freely traded (we assume no trade costs). Production functions are the same as under autarky, and all countries have the same technologies a_m, b_m and $\theta_c(.)$. However, countries may have different land endowments and different temperatures.

Now, the optimal world price is:

$$p^* = \frac{\theta_w}{a_m} + \frac{1}{b_m}$$

Where θ_w denotes the aggregate crop productivity, and depends on the amount of land in the world allocated to crop production.

Equilibrium

Under trade, the equilibrium must satisfy the following constraints:

$$\forall n \in \{R, P\}, \quad L_{c,n} + L_{m,n} = \overline{L}_n$$
$$Q_{c,P} + Q_{c,R} = q_{c,P} + q_{c,R} + K_{m,P} + K_{m,R}$$
$$Q_{m,P} + Q_{m,R} = q_{m,P} + q_{m,R}$$
$$\text{where} \quad q_{i,n} = q_{i,n,n} + q_{i,n^*,n} \quad \text{and} \quad K_{m,n} = K_{m,n,n} + K_{m,n^*,n}$$
$$q_{c,P,R} + K_{m,P,R} + q_{c,R,P} + K_{m,R,P} = 0$$
$$q_{m,P,R} + q_{m,R,P} = 0$$

Subsistence consumption

The conditions for subsistence consumption to be satisfied are the same as under autarky, except equilibrium quantities for cereals and meat are now derived worldwide.

Notes

Under autarky, we analytically solved for the equilibrium by equating $a_m L_{m,n} = b_m K_{m,n}$, which yielded land allocated to crop production as a function of net cereal consumption $L_{c,n}(q_{c,n})$, then meat production as a function of $q_{c,n}$ and ended up resulting with both Relative Demand and optimal levels of $q_{c,n}$ and $q_{m,n}$.

Under trade, everything gets more complicated because worldwide productivity of land now depends on the amount of land allocated to cereal production (because countries have different temepratures, hence different crop yields). The model is not solvable by hand, hence the need for computational methods.

Below, I plot several dataframes corresponding to worlds with different chracteristics.

Endogene temperature

Finally, I make temepratures a function of meat production as the livestock sector is responsible for almost all agricultural GHG emissions. The dynamics of temeprature are modelled that way:

$$e_t = k_1 * q_m(T_{t-1})$$
$$GHG_t = k_2 * GHG_{t-1} + e_t$$

$$T_t = T_0 + k_3 * GHG_t$$

Where e_t represents agricultural GHG emissions, where GHG have a natural dynamic depending on k_2, and temperature depends on the total amount of GHG emissions.

Agents now solve an intertemporal maximization problem:

$$V(T_t) = \max_{q_{m,t+1}} \{u(q_{c,t}, q_{m,t}) + \beta V(T_{t+1})\}$$

Subject to:

$$q_{c,t} = \theta(T_t)\overline{L} - q_{m,t}\left(\frac{\theta(T_t)}{a} + \frac{1}{b}\right)$$
$$T_{t+1} = T_0 + k_3(k_2 . GHG_{t-1} + k_1 . q_{m,t})$$

For now, I have not been able to write a proper code for that part.
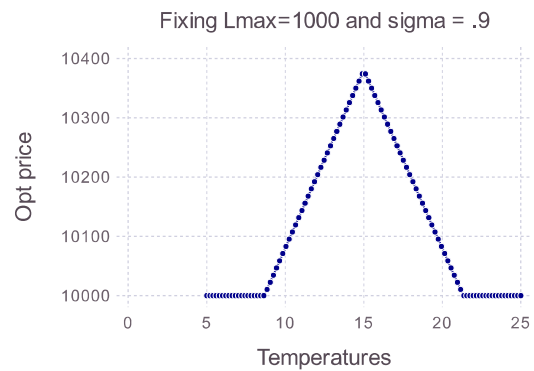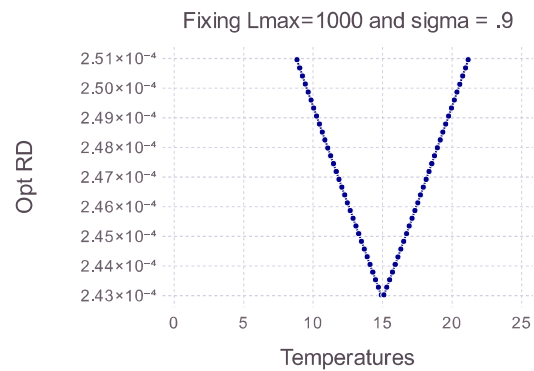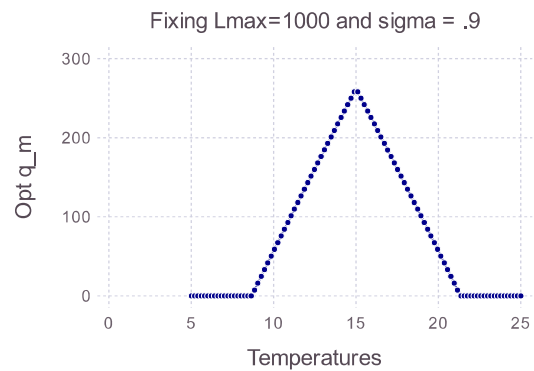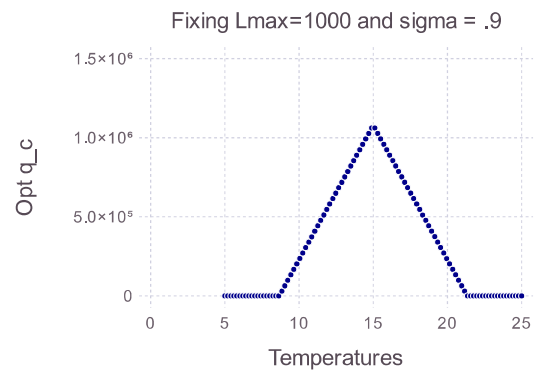
Plots and results

Autarky

Main results

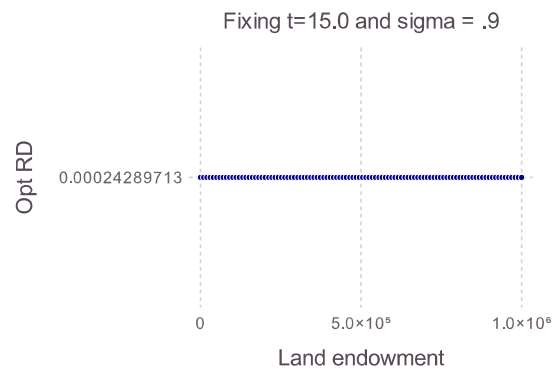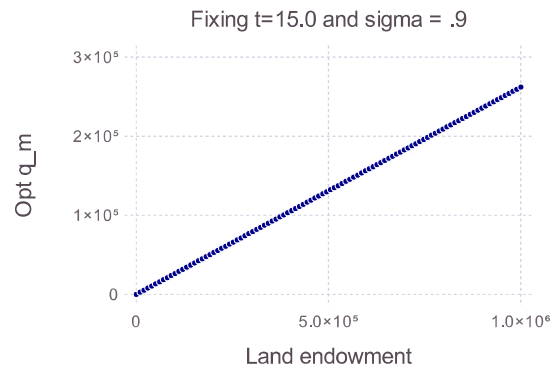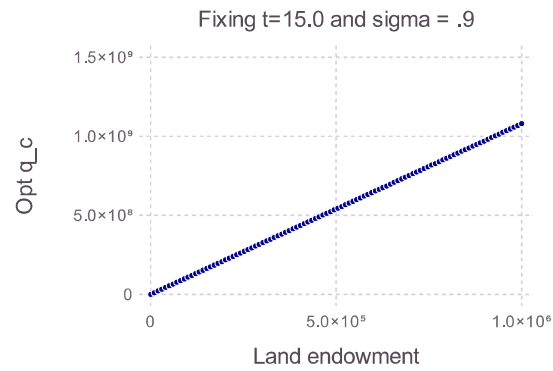I plot here how the results of the analytical solution evolve with the three different characteristics of countries: land endowment, temeprature, and CES.

We obtain as predicted that the consumption of cereals increases with sigma while the consumption of meat decreases with sigma.

In [1]:

```
include("src/autarkyplots.jl")
autarkyplots.plotall()
```
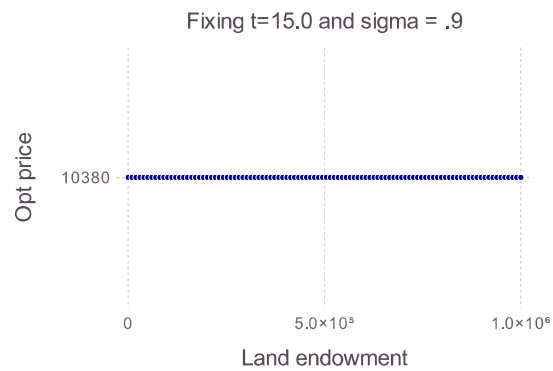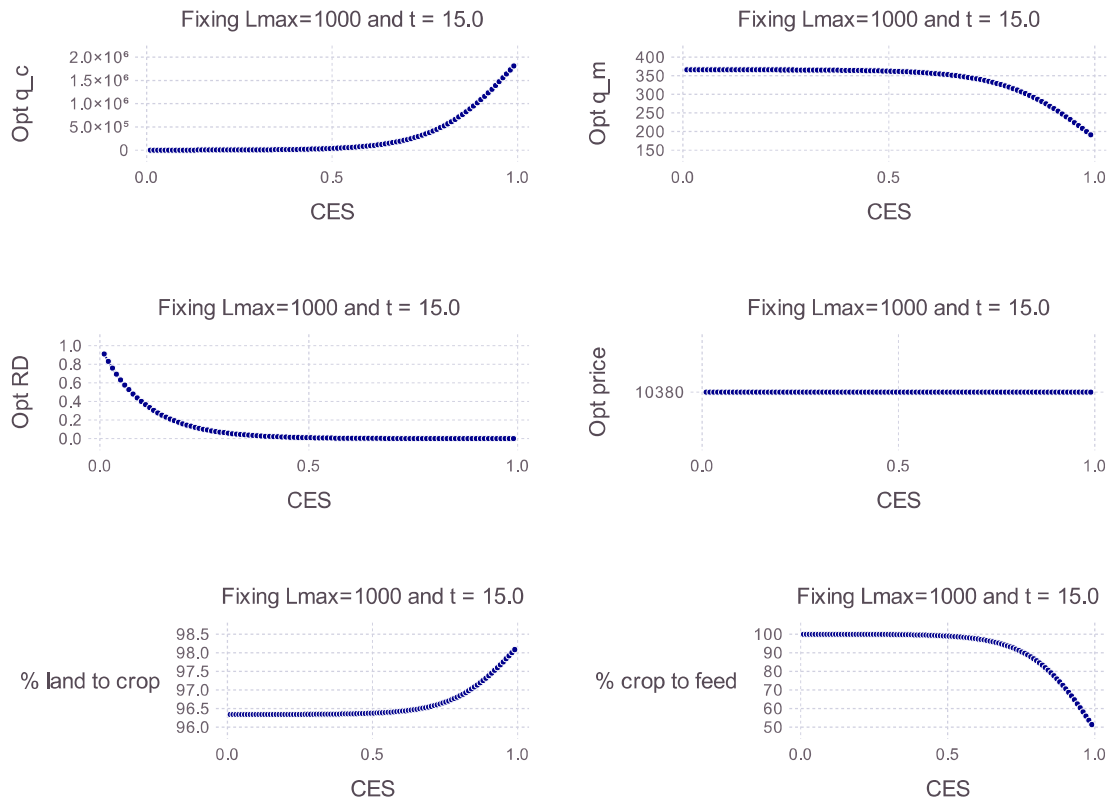
Fixing t=15.0 and sigma = .9

Fixing t=15.0 and sigma = .9

Fixing t=15.0 and sigma = .9

Fixing t=15.0 and sigma = .9

Fixing Lmax=1000 and sigma = .9

Fixing Lmax=1000 and sigma = .9

Fixing Lmax=1000 and sigma = .9

Fixing Lmax=1000 and sigma = .9

Fixing Lmax=1000 and t = 15.0 (Opt q_c vs CES)

Fixing Lmax=1000 and t = 15.0 (Opt q_m vs CES)

Fixing Lmax=1000 and t = 15.0 (Opt RD vs CES)

Fixing Lmax=1000 and t = 15.0 (Opt price vs CES)

Fixing Lmax=1000 and t = 15.0 (% land to crop vs CES)

Fixing Lmax=1000 and t = 15.0 (% crop to feed vs CES)

```
WARNING: Ipopt finished with status Restoration_Failed
WARNING: Not solved to optimality, status: Error
```

Trade

Below, we plot results in a world with 2 countries with same CES = .5, and we make land endowment and temperature vary. We also plot the results for a world with 3 countries and differences in both land endowment and temperatures. For each world I plot one table with results at the country scale, and one table with global results.

When the 2 countries are identical, we obtain as expected that the countries produce the same amount of cereals and meat, and that none of them exports or imports ("trade_in_cereals" and "Trade_in_meat" close to 0). We also find that the country that has a higher endowment in land has a higher share in total cereal production, produces all the meat and exports both products. As predicted again, the country with the highest crop productivity exports both products.

Note1: exports and imports are only roughly equal, due to the approximations to find the trade equilibrium.

Note2: so far, I have not been able to code cases where there are more than 2 countries among which at least 2 have same land endowment or same temperatures.

In [2]:

```
include("src/tradeeqdf.jl")
tradeeqdf.runall()
```

2 indentical countries
2×9 DataFrames.DataFrame

Row	Land_endowment	Temperature	CES	Crop_yields	Share_land_to_crops
1	1500.0	13.75	0.9	3050.0	0.5
2	1500.0	13.75	0.9	3050.0	0.5

Row	Share_cereal_prod	Share_meat_prod	Trade_in_cereals	Trade_in_meat
1	0.5	0.5	-0.00470488	4.56564e-7
2	0.5	0.5	-0.00470488	4.56564e-7

1×4 DataFrames.DataFrame

Row	Net_cereal_cons	Meat_cons	Tot_land_to_crops	Per_land_to_crops
1	89256.5	879.257	2912.07	97.0691

---

2 countries with different land endowments but same temperature
2×9 DataFrames.DataFrame

Row	Land_endowment	Temperature	CES	Crop_yields	Share_land_to_crops
1	1500.0	13.75	0.9	3050.0	0.437848
2	2000.0	13.75	0.9	3050.0	0.562152

Row	Share_cereal_prod	Share_meat_prod	Trade_in_cereals	Trade_in_meat
1	0.437848	0.0	-2.16654e5	-52.9691
2	0.562152	1.0	2.16654e5	52.9691

1×4 DataFrames.DataFrame

Row	Net_cereal_cons	Meat_cons	Tot_land_to_crops	Per_land_to_crops
1	1.04133e5	1025.8	3397.42	97.0691

---

2 countries with same land endowment but different temperatures
2×9 DataFrames.DataFrame

Row	Land_endowment	Temperature	CES	Crop_yields	Share_land_to_crops

Row				
1	1500.0	15.0	0.9	3800.0	0.512174
2	1500.0	16.25	0.9	3050.0	0.487826

Row	Share_cereal_prod	Share_meat_prod	Trade_in_cereals	Trade_in_meat
1	0.566741	0.0	155484.0	36.6353
2	0.433259	1.0	-1.63328e5	-38.7357

1×4 DataFrames.DataFrame

Row	Net_cereal_cons	Meat_cons	Tot_land_to_crops	Per_land_to_crops
1	1.00417e5	987.344	2901.27	96.7089

----
3 countries with different land endowments and temperatures
3×9 DataFrames.DataFrame

Row	Land_endowment	Temperature	CES	Crop_yields	Share_land_to_crops
1	3000.0	15.0	0.9	3800.0	0.438064
2	1500.0	13.75	0.9	3050.0	0.219032
3	2500.0	17.5	0.9	2300.0	0.342904

Row	Share_cereal_prod	Share_meat_prod	Trade_in_cereals	Trade_in_meat
1	0.533305	0.0	1.16815e6	280.765
2	0.214024	0.0	-7.69393e5	-187.792
3	0.252671	1.0	-4.34679e5	-103.322

1×4 DataFrames.DataFrame

| Row	Net_cereal_cons	Meat_cons	Tot_land_to_crops	Per_land_to_crops
1	6.20794e6	1516.82	6848.32	97.8331

Subsistence consumption constraint

Here we plot how the satisfaction of the subsistence consumption moves with the characteristics of the country (resp. world). In order to make plots out of these, the function yields a 1 when the condition is satisfied, and 0 if not. Note that this highly depends on the calibration of the parameters, that we have not done properly yet.

In [3]:

```
include("src/subsistconst.jl")
subsistconst.plotall()
```

WARNING: Ipopt finished with status Restoration_Failed
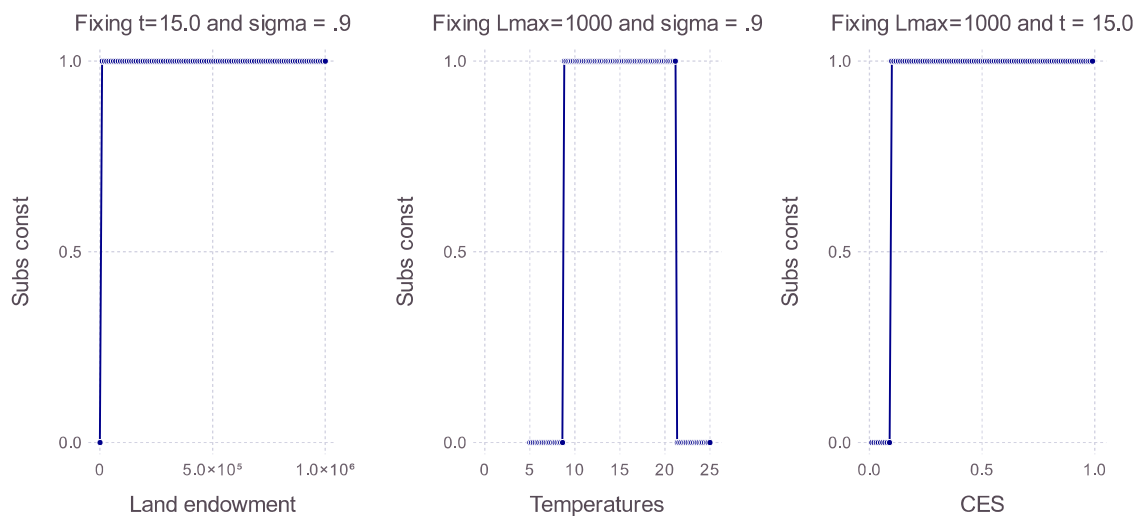
WARNING: Not solved to optimality, status: Error

Endogene temperature

So far, I have not been able to write a proper code for dynamic programming