Presentation of the model

Autarky case

Demand

In country n, demand consists in a representative consumer who has to choose his consumption of cereals $q_{c,n}$ and meat $q_{m,n}$ with a CES utility function, and the following problem:

$$\max_{\{q_{c,n}, q_{m,n}\}} \left\{ U(q) = \left[q_{c,n}^{\frac{\sigma-1}{\sigma}} + q_{m,n}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \right\}$$

Subject to:

$$q_{c,n} + q_{m,n} p = y_n$$
$$q_{c,n} \geq 0, \quad q_{m,n} \geq 0$$

The price of cereals is normalized to 1, and p stands for the relative price of meat on cereals.

Supply

There are 2 types of producers: cereal producers and meat producers, with the following production functions:

$$Q_{c,n}(L_{c,n}) = L_{c,n} . \theta_c(d_n)$$

and

$$Q_{m,n}(L_{m,n}; K_{m,n}) = \min \{ a_m L_{m,n} ; b_m K_{m,n} \}$$

Where d_n is the distance of country n's temperature compared to the optimal temperature for growing crops, and $\theta_c(d_n)$ the productivity function, decreasing with d_n. Land, L, is the only input used either for growing crops or raising livestock, and $K_{m,n}$ stands for the cereals used as feed into meat production. Land and cereals are perfect complements in meat producers' function.

Equilibrium

The equilibrium must satisfy the following constraints:

$$L_{c,n} + L_{m,n} = \overline{L}_n$$
$$Q_{c,n}(L_{c,n}) = q_{c,n} + K_{m,n}$$
$$Q_{m,n}(L_{c,n}; K_{m,n}) = q_{m,n}$$

\end{align} Where $\overline{L}_n$ is the total available arable land in country n.

Subsistence consumption

We check the satisfaction of subistence consumption ex-post. Denote $\underline{Q}$ the minimal quantity of calories (or any nutritional indicator) to eat for survival, and $M = \begin{pmatrix} \mu_m \\ \mu_c \end{pmatrix}$ the conversion matrix of kgs into calories (or any nutritional indicator). We assume $1 \geq \mu_m \geq \mu_c \geq 0$\ Subsistence consumption is satisfied if:

$$(q_{m,n}^* \quad q_{c,n}^*) . M \geq \underline{Q}$$

Notes

The equilibrium and the conditions for the satisfaction of subsistence consumption are solvable by hand. Yet, I programed both the numerical and analytical solutions, whih yield similar result with a tolerance of 1e8. The plots below are computed from the analytical solutions.

Trade case

Demand

We assume agents to be the same across countries, i.e. to have same CES σ.

Production

Under trade, we assume land to be a fully immobile input, while cereals can be freely traded (we assume no trade costs). Production functions are the same as under autarky, and all countries have the same technologies a_m, b_m and $\theta_c(.)$. However, countries may have different land endowments and different temperatures.

Equilibrium

Under trade, the equilibrium mus satisfy the following constraint:

$$\forall n \in \{R, P\}, \quad L_{c,n} + L_{m,n} = \overline{L}_n$$
$$Q_{c,P} + Q_{c,R} = q_{c,P} + q_{c,R} + K_{m,P} + K_{m,R}$$
$$Q_{m,P} + Q_{m,R} = q_{m,P} + q_{m,R}$$
$$\text{where} \quad q_{i,n} = q_{i,n,n} + q_{i,n^*,n} \quad \text{and} \quad K_{m,n} = K_{m,n,n} + K_{m,n^*,n}$$
$$q_{c,P,R} + K_{m,P,R} + q_{c,R,P} + K_{m,R,P} = 0$$
$$q_{m,P,R} + q_{m,R,P} = 0$$

Subsistence consumption

The conditions for subsistence consumption to be satisfied are the same as under autarky, except equilibrium quantities for cereals and meat are now derived worldwide.

Notes

Under autarky, we analytically solved for the equilibrium by equating $a_m L_{m,n} = b_m K_{m,n}$, which yielded land allocated to crop production as a function of net cereal consumption $L_{c,n}(q_{c,n})$, then meat production as a function of $q_{c,n}$ and ended up resulting with both Relative Demand and optimal levels of $q_{c,n}$ and $q_{m,n}$.

Under trade, everything gets more complicated due to the fact that the worldwide productivity of land now depends on the amount of land allocated to cereal production (because countries have different temepratures, hence different crop yields). The model is not solvable by hand, hence the need for computational methods.

However, I get stuck when I need to derive the equilibrium quantities of $q_{c,w}$ and hence $q_{m,w}$ because the function $RD - RS$ is extremely flat around 0, which makes the computation tremendously slow.

Below, I plot both the function $\theta_{c,w}(L_{c,w}$ and the function $RD - RS$.

Endogene temperature

Finally, I make temepratures a function of meat production as the livestock sector is responsible for almost all agricultural GHG emissions. The dynamics of temeprature are modelled that way:

$$e_t = k_1 * q_m(Temp_{t-1})$$
$$GHG_t = k_2 * GHG_{t-1} + e_t$$

$$Temp_t = T0 + k_3 * GHG_t$$

Where e_t represents agricultural GHG emissions, where GHG have a natural dynamic depending on k_2, and temperature depends on the total amount of GHG emissions.

Again, because I only manage to have results for the autarky case, I will only plot the dynamics in autarky.
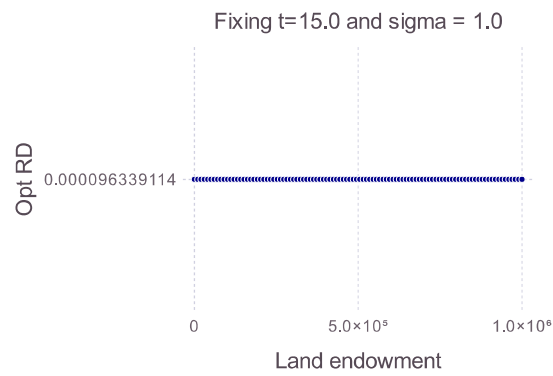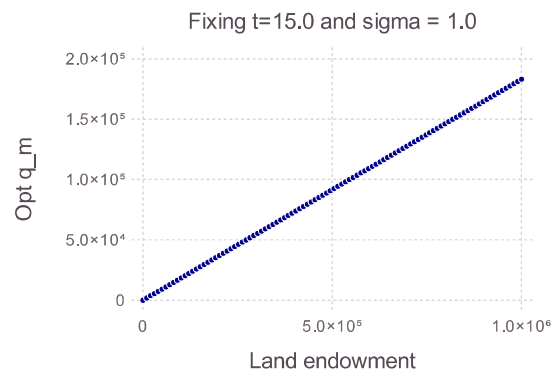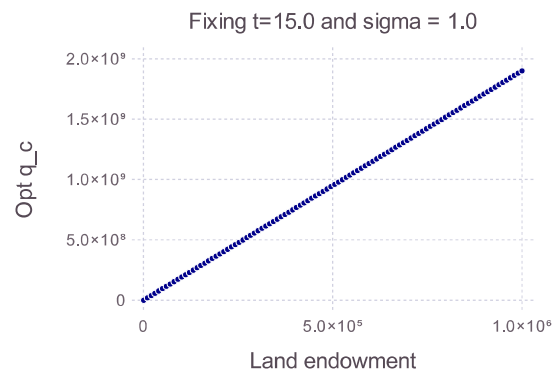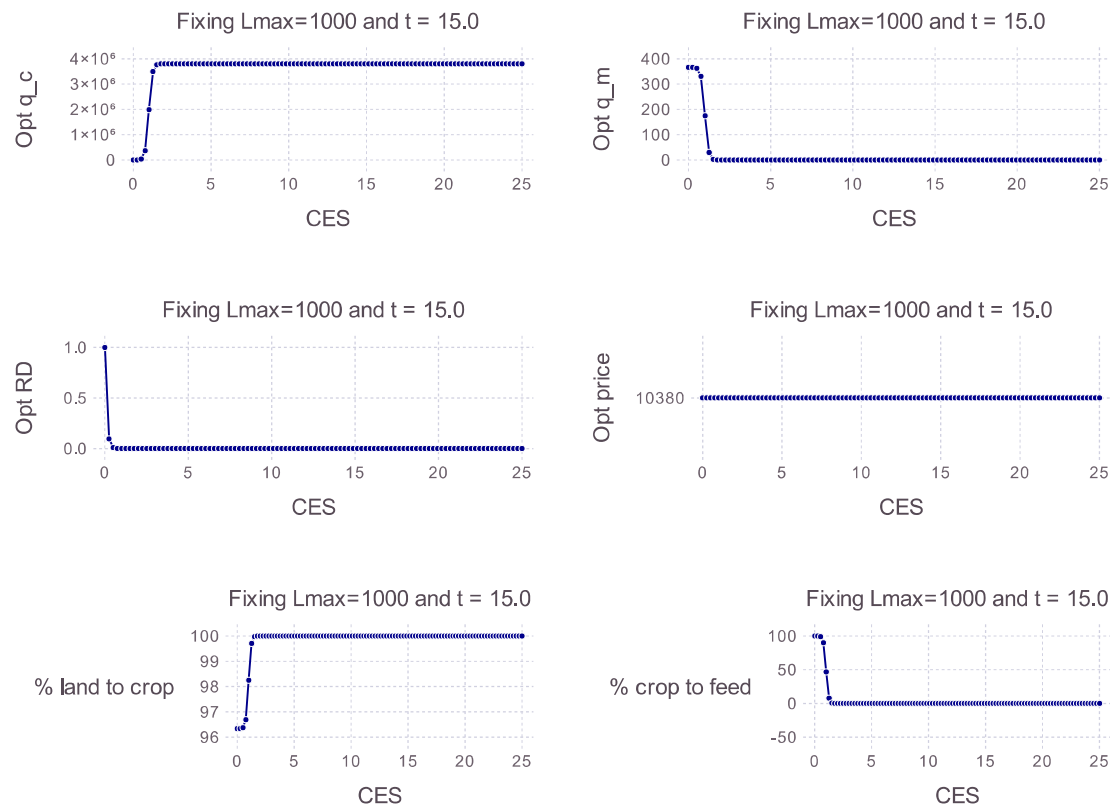
Plots and results

Autarky

Main results

I plot here how the results of the analytical solution evolve with the three different characteristics of countries: land endowment, temeprature, and CES.

In [3]:

```
cd("C:/Users/lucil/OneDrive/Documents/GitHub/Master_thesis")
include("src/autarkyeq.jl")
autarkyeq.plotall()
```

Fixing t=15.0 and sigma = 1.0

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Fixing Lmax=1000 and sigma = 1.0

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Fixing Lmax=1000 and sigma = 1.0

Fixing Lmax=1000 and t = 15.0

Opt q_c

CES

Fixing Lmax=1000 and t = 15.0

Opt q_m

CES

Fixing Lmax=1000 and t = 15.0

Opt RD

CES

Fixing Lmax=1000 and t = 15.0

Opt price

10380

CES

Fixing Lmax=1000 and t = 15.0

% land to crop

CES

Fixing Lmax=1000 and t = 15.0

% crop to feed

CES

```
WARNING: replacing module autarkyeq
```

Subsistence consumption under autarky

Here we plot how the satisfaction of the subsistence consumption moves with the characteristics of the country. In order to make plots out of these, the function yields a 1 when the condition is satisfied, and 0 if not. Note that this highly depends on the calibration of the parameters, that we have not done properly yet.

In [25]:

```
cd("C:\\Users\\lucil\\OneDrive\\Documents\\GitHub\\Master_thesis")
include("src/subsistconst.jl")
subsistconst.plotall()
```
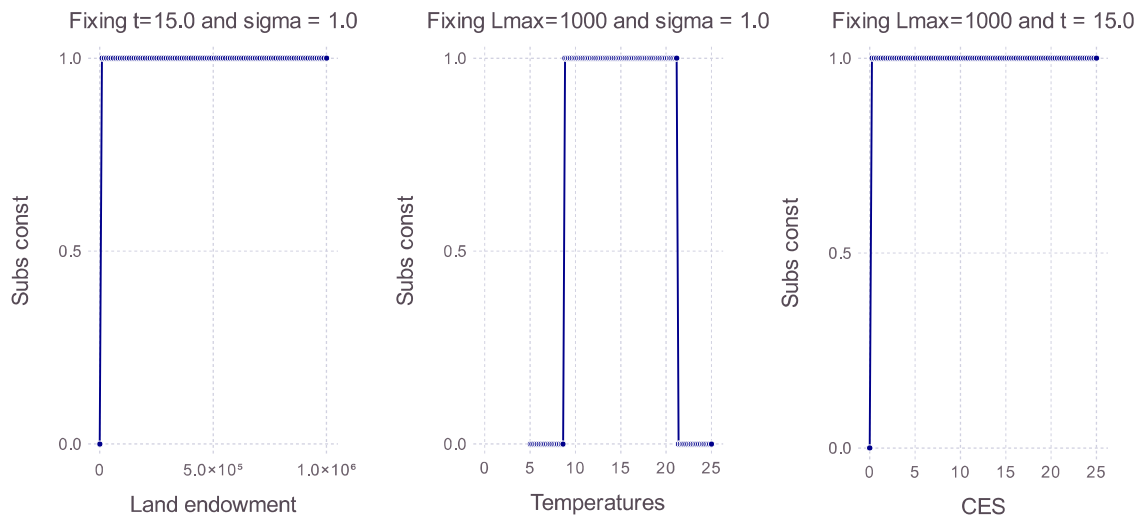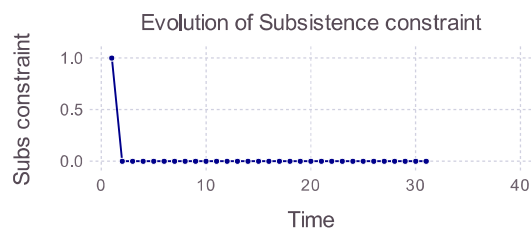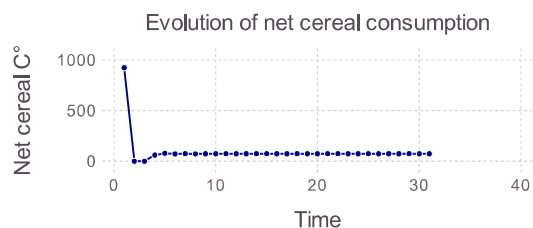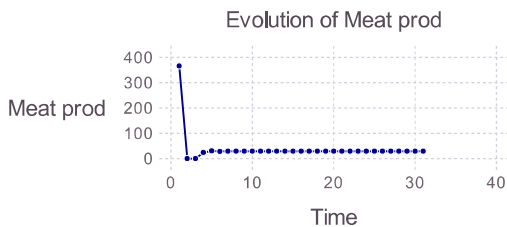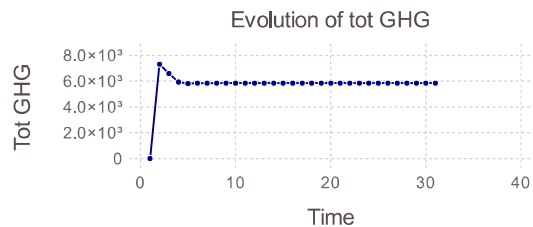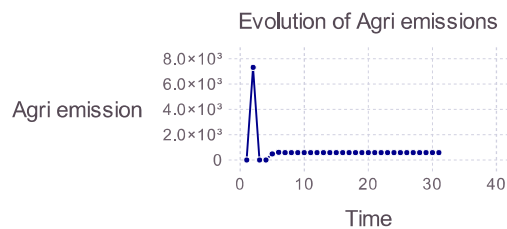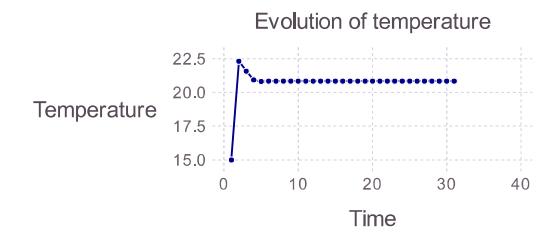


```
WARNING: replacing module subsistconst
```

Endogene temperature

Now we plot the evolution of the results over 30 periods, for 3 different cases of CES, keeping temepratures and land endowment constant.
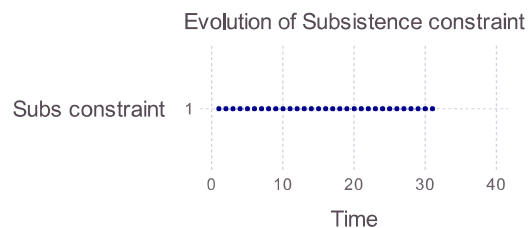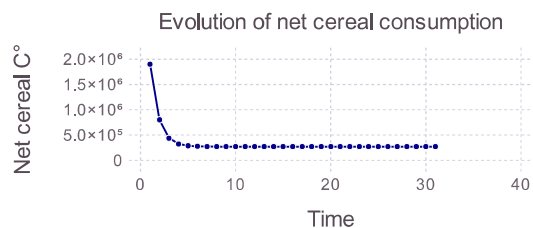
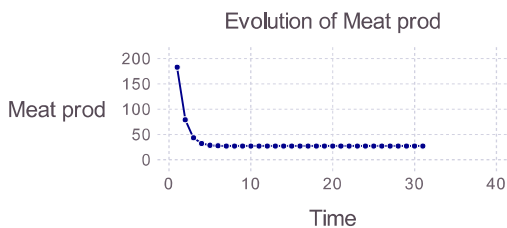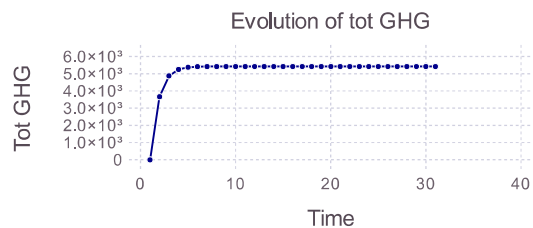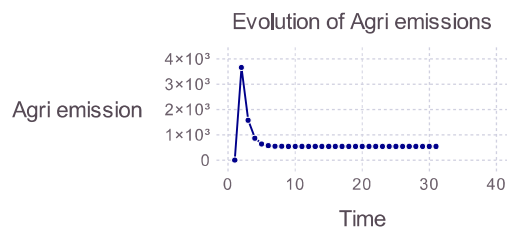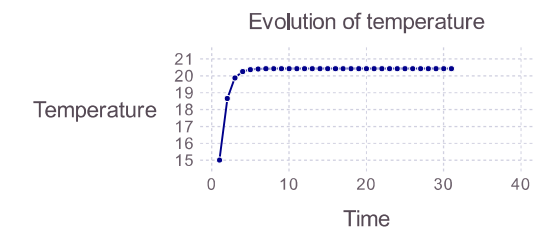In [24]:

```julia
cd("C:\\Users\\lucil\\OneDrive\\Documents\\GitHub\\Master_thesis")
include("src/dynamictemp.jl")
dynamictemp.plotall(31, 1000.0, 15.0, 1e-1)
dynamictemp.plotall(31, 1000.0, 15.0, 1.0)
dynamictemp.plotall(31, 1000.0, 15.0, 1e1)
```

Evolution of temperature

Evolution of Agri emissions

Evolution of tot GHG

Evolution of Meat prod

Evolution of net cereal consumption

Evolution of Subsistence constraint

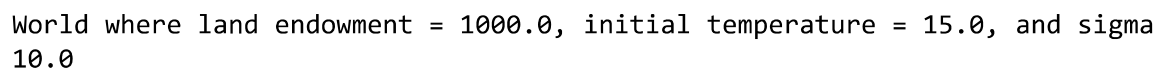World where land endowment = 1000.0, initial temperature = 15.0, and sigma
0.1

WARNING: replacing module dynamictemp

Evolution of temperature

Evolution of Agri emissions

Evolution of tot GHG

Evolution of Meat prod

Evolution of net cereal consumption

Evolution of Subsistence constraint

World where land endowment = 1000.0, initial temperature = 15.0, and sigma
1.0

Evolution of temperature

Evolution of Agri emissions

Evolution of tot GHG

Evolution of Meat pr[...]

Evolution of net cereal consumption

### Evolution of Subsistence constraint


```
World where land endowment = 1000.0, initial temperature = 15.0, and sigma
10.0
```

Trade

As said before, I didn't manage to find results in the trade case. Below, I plot world productivity (world with 4 countries here), which shows kinks at each new country, sorted by decreasing thetas. I also plot the function $RD - RS$ that is too flat around 0 so that the `fzeros()` function can yield a result quickly.

In [29]:

```
cd("C:\\Users\\lucil\\OneDrive\\Documents\\GitHub\\Master_thesis")
include("src/tradeeqdraft.jl")
Lmax = [1000.0, 1300.0, 750.0, 300.0]
t = [13.5, 16.7, 15.0, 12.3]
sigma = ones(4)
tradeeqdraft.plotall(Lmax, t, sigma)
```

Theta_w : world crop productivity

RD = RS, flat curve
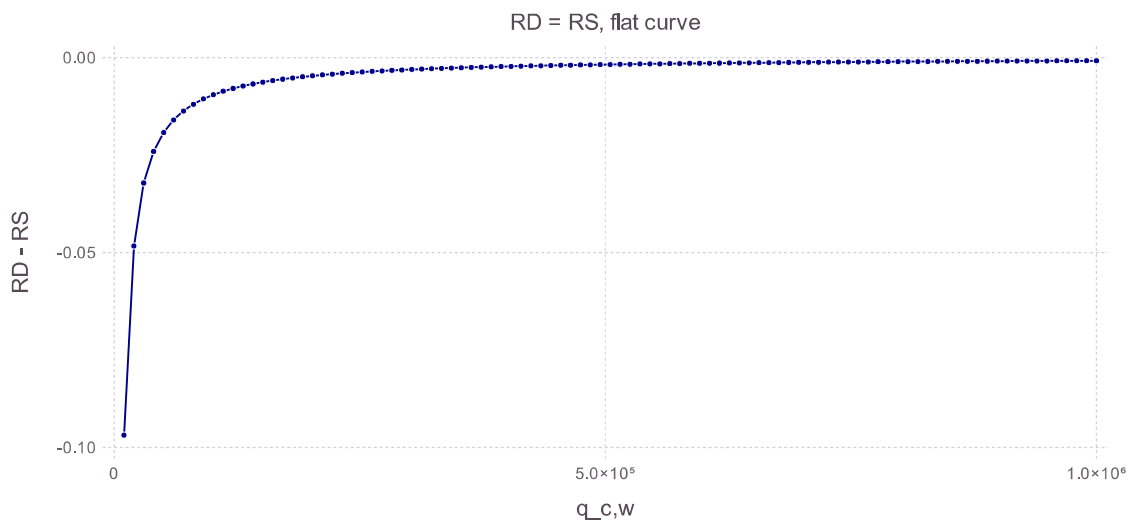

```
WARNING: replacing module tradeeqdraft
```

In [ ]: