MTE 360 Automatic Control Systems: Project 1

Identification, P-Control, and Time and Frequency Responses

Introduction

Setup and Background: A detailed description of the system we will work with is provided in the "Projects→Manuals for the Experimental Setups" document. Please go through and read the description now. This handout lays out some steps you need to follow while performing the project, and clarifies some more involved parts of the background information. **Note:** The project relies on Simulink and MATLAB. Make sure to go through the MATLAB/Simulink Tutorial presented in the beginning of the term and posted on LEARN. If you have further MATLAB/Simulink questions, please contact the TAs and/or the instructor in advance.

Report Submission (due: September 28, Monday): Each project group is required to submit a single project report for Project 1, in the format of a technical report. The report has to be prepared collectively by the members of that group. Communication with other groups is not allowed! Specifically, your report should address all of the questions included in this project instruction document.

Synopsys and Motivation: In Project 1, you will become familiar with the basic principles of control engineering. You will also become acquainted with the experimental setup that will be used in all the three projects: *Quanser* linear motion cart with incremental encoder feedback (IP02 model), shown in Figure 1.

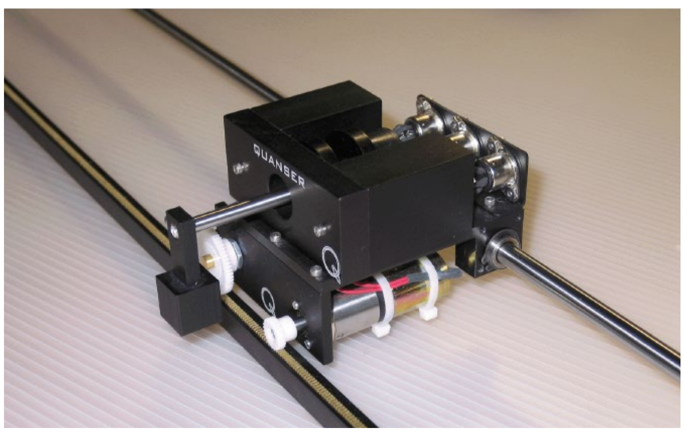

Figure 1: Linear cart setup.

Control engineering practice typically consists of two main tasks:

1) Identifying a model for the system to be controlled
2) Designing and/or tuning an appropriate controller

In many cases, the above tasks are executed iteratively, one after another until the desired performance, stability, and robustness characteristics are achieved. Project 1 is designed to give a flavor of these tasks, which are frequently used in real-life controller design for applications such as robots, CNC machine tools, disk drives, or autopilot systems. The necessary real-time data set has been prepared for the students. This will be explained in context in this manual.

In Project 1, to get an accurate model of the DC motor and disk in the cart setup of Figure 1, you will identify the corresponding system parameters. In this linear cart setup, the inertia and friction components of the dynamics have much larger effect than the armature inductance and resistance. This is because the inductance of the armature is relatively small, and therefore the bandwidth of the electric motor is much greater than that of the inertial dynamics. The result is that we can ignore the faster electric motor dynamics ($L_a \approx 0$) when identifying the system, and work with a simplified system model. Note that we are also ignoring the effects of external disturbances in the identification of the motor parameters.

1. Getting Started

1) Watch the Project-1 videos and try to get more insight into the experimental setup that you will be doing identification for.
2) Download the data file sent to you via email.
3) Load these data sets and visualize them with the help of MATLAB/Simulink tutorial.

2. Details on the Data Sets Collected from the Lab Setup

2.1. Parameter Identification through Step Response Measurement

The velocity response of most servo-drive systems can be represented by the following first order model, as derived in class:

$$V(s) = \frac{K_v}{\tau_v s + 1} U(s), \quad for \quad U(s) = \mathcal{L}\{u(t)\}, V(s) = \mathcal{L}\{v(t)\} \tag{1}$$

where $\boldsymbol{U}(\boldsymbol{s})$ is the Laplace transform of the input voltage signal $\boldsymbol{u}(\boldsymbol{t})$ [V], $\boldsymbol{V}(\boldsymbol{s})$ is the Laplace transform of the axis velocity output $\boldsymbol{v}(\boldsymbol{t})$ [mm/sec], $\boldsymbol{K_v}$ [(mm/sec)/V] is the velocity gain and $\boldsymbol{\tau_v}$ [sec] is the time constant. When a unit step voltage input is applied, the following response would be observed in the ideal case:

$$v(t) = K_v\left(1 - e^{(-t/\tau_v)}\right) \tag{2}$$

A plot of the ideal response (2) is shown in Figure 2.

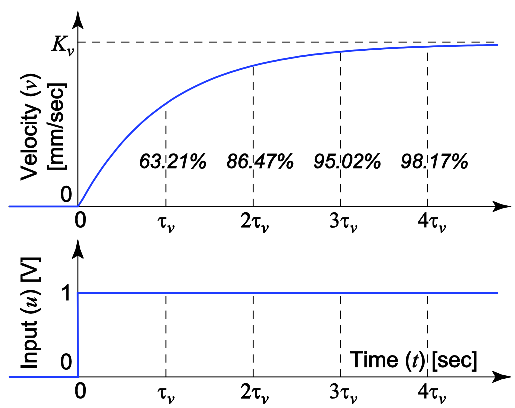

Figure 2: First order system response to a unit step input with zero initial conditions.

2-1a. In the first data set, called "velocity_response_2-1", the data is collected measuring the velocity response of the drive system by applying a square wave input with ±1.5 [V] amplitude and 1 Hz frequency. The Simulink file in Figure 3 is used to run this experiment. To obtain the measured velocity, the position measurements from the encoder on the cart are numerically differentiated with respect to time.

Data Format: [Time, Input, Velocity]

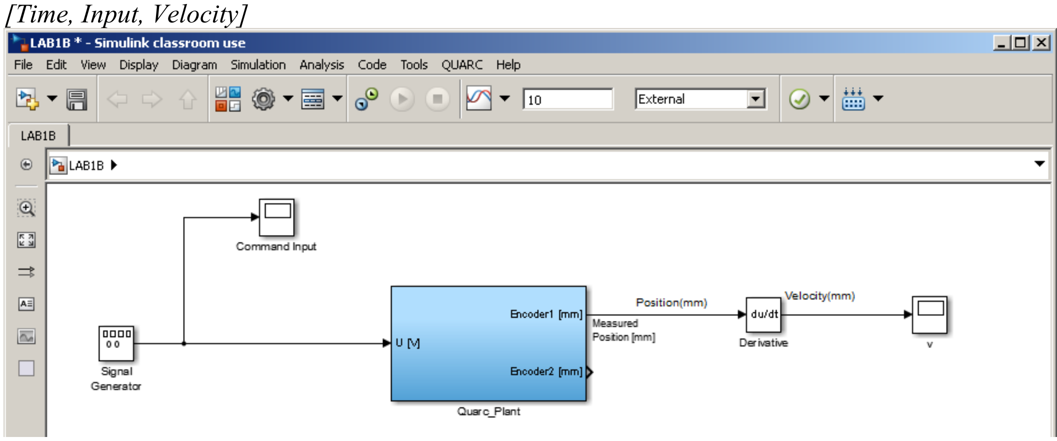

Figure 3: Simulink file for measuring velocity step response.

2.2 Proportional (P) Position Control

A block diagram depicting the proportional (P) position control is shown in Figure 4, where $\boldsymbol{x_r}$ [mm] is the commanded position and $\boldsymbol{x}$ [mm] is the actual cart position. $\boldsymbol{e = x_r - x}$ [mm] is the position error, commonly referred to as "tracking error". $\boldsymbol{K_p}$ [V/(mm/sec)] is the proportional feedback gain which generates the control voltage signal $\boldsymbol{u}$ [V] applied to the amplifier's input based on how large the position error is. In reality, there is also an equivalent disturbance $\boldsymbol{d}$ [V] which originates from the friction in the cart mechanism. The friction disturbance opposes the cart motion; hence it has a negative sign. The effect of the disturbance on the servo performance will be studied in later projects. The cart's position $\boldsymbol{x(t)}$**,** which is integral of the velocity $\boldsymbol{v(t)}$, is measured and fed back into the control loop.

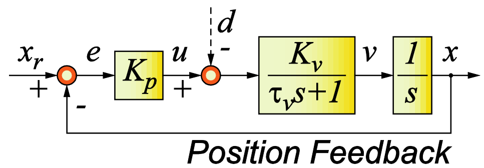

Figure 4: Proportional (P-type) position control.

2-2a. The data set, called "P_Position_Controller_2-2a" is collected by implementing the P position controller using the Simulink model in Figure 5. As position command, apply a ± 5.0 [mm] square wave with 0.5 Hz frequency. The setup is ran for values **0.15**, **0.25**, **0.5** [V/mm] for K_p and corresponding data saved to "P_Position_Controller_(0-15)(0-25)(0-5)**.**

Data Format: [Time, Command, Control Input, Position Measurement]

⇨ In the case of negative values for K_p, the control system can become unstable and the cart can crash to one end! To avoid damages to the drive mechanism, gains that exceed 0.6 [V/mm], and for high feedback gains (e.g. $K_p = 0.5$ [V/mm]), is avoided to run the setup for longer than 10 seconds at a time!

⇨ The voltage sent to the motor should never exceed ±5V, otherwise nonlinear effects are introduced, which will corrupt your experimental data, and system response.

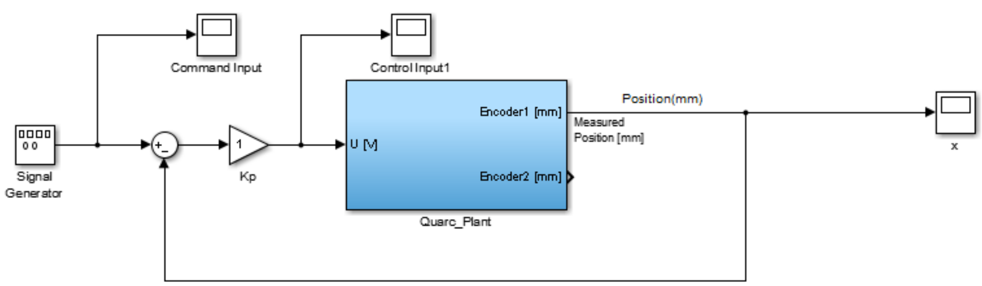

Figure 5: Simulink model for proportional position control.

2.3 Bode Plot Based Identification of Closed-loop Frequency Response

Frequency response of linear dynamic systems can be identified by applying sinusoidal excitations at different frequencies and measuring the relative amplitude and phase shift between the input and the output. For a linear closed-loop system with transfer function $G_{cl}(s)$, input $X_r(s) = \mathcal{L}\{x_r(t)\}$, and output $X(s) = \mathcal{L}\{x(t)\}$, such that

$$X(s) = G_{cl}(s)X_r(s), \tag{1}$$

when a sinusoidal signal $x_r(t)$ is applied to the input, the steady state response ($x(t)$ at steady state) will be as shown in Figure 6. In the figure, Δx_r is the peak-to-peak amplitude of the excitation and Δx is the peak-to-peak amplitude of the output. T [sec] is the period of the excitation and t_ϕ [sec] is the time lag between the input and the output. The gain and phase of the transfer function at the frequency $f = 1/T$ [Hz] can be computed as: $|G_{cl}| = \Delta x / \Delta x_r$ and $\angle G_{cl} = -360° \times t_\phi / T$.

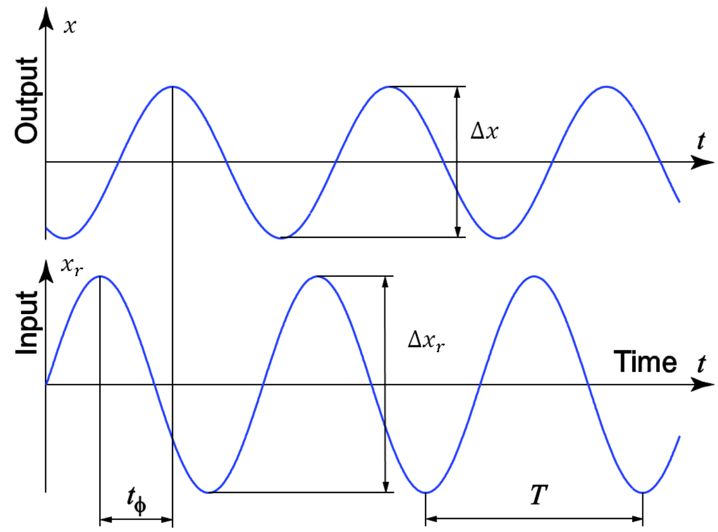

Figure 6: Sinusoidal response of a linear dynamic system.

It is also possible to evaluate the frequency response corresponding to transfer functions analytically, if their mathematical expression is known. The gain and phase are determined by computing the magnitude and angle of $G_{cl}(j\omega) = G_{cl}(s)|_{s=j\omega}$. In either measurement based or analytical calculation, by trying out different values for f (or $\omega = 2\pi f$), it is possible to determine the frequency response of a dynamic system for a wide frequency range. Frequency response analysis provides important insight into issues like tracking performance, disturbance rejection, stability and robustness margins, etc.

2.3a. In Simulink, the setup shown in Figure 7 is used to collect the data set needed for this part of the project. The Proportional Gain K_p is set to **1.0** [V/mm] and the frequency response of the closed-loop position control system is measured by applying sine wave position commands with **±2.0** [mm] amplitude. The following frequencies are used: $f = 0.5,\ 1,\ 2.5,\ 5,\ 10,\ 25$ (Hz).
Data Format: [Time, Command, Control Input, Position Measurement]

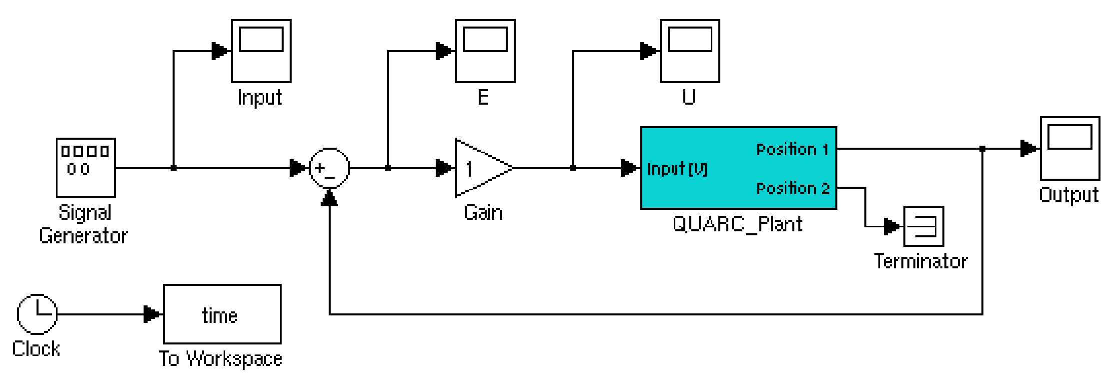

Figure 7: Simulink block diagram for frequency response test.

Command (input) signal (x_r), output signal (x), and control input signal (u) measurements are saved under "frequency_4-3_f" (f being the frequencies mentioned above.

⇨ **In testing frequencies over 5 Hz, the setup is not being ran for longer than 5 seconds at a time!**

3. Project Instructions

For Parameter Identification through Step Response Measurement

3-1a. Using the measured data in Part 2-1a, determine the values of the velocity gain $\boldsymbol{K_v}$ and the time constant $\boldsymbol{\tau_v}$. Clearly explain how you obtained/identified the values plant parameters $\boldsymbol{K_v}$ and $\boldsymbol{\tau_v}$.

3-1b. Construct a Simulink model of the system, using identified gain ($\boldsymbol{K_v}$) and time constant ($\boldsymbol{\tau_v}$) parameters obtained in Part 3-1a. Simulate the theoretical velocity response using this Simulink model, applying the input signal profile (u) captured during the experiment.

3-1c. Provide a plot of the measured and simulated step response graphs overlaid on top of each other (i.e. v_{meas} (from Part 2-1a) and v_{ave} (from Part 3-1b) vs. time t,). Comment on the similarities and/or discrepancies between the two, reflecting your engineering judgment. Also plot the input profile (u vs. t) underneath the velocity response graph. The time axes should be identical.

For Proportional (P-type) Position Control:

3-2a. The P-controlled closed loop system can be represented with an equivalent 2nd order model, as shown in Figure 8. In the simplified model, ω_n [rad/sec] represents natural frequency and ζ represents damping ratio of the closed-loop poles. Derive the expressions for ω_n andζ, and compute their values for $K_p = 0.15, 0.25$ and 0.5 [V/mm].

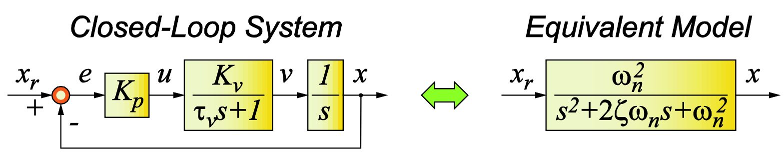

Figure 8: Equivalent 2nd order model for P-controlled servo system.

3-2b. Plot the closed-loop position step response for different values of K_p (i.e. 0.15, 0.25, and 0.5 [V/mm]) in the format shown in Figure 9. Comment on how the rise time, overshoot, steady state error, and control signal in the step response change as the proportional gain is varied. *Refer to control theory from your textbook (or additional references) in coming up with appropriate explanations that account for your experimental observations*.

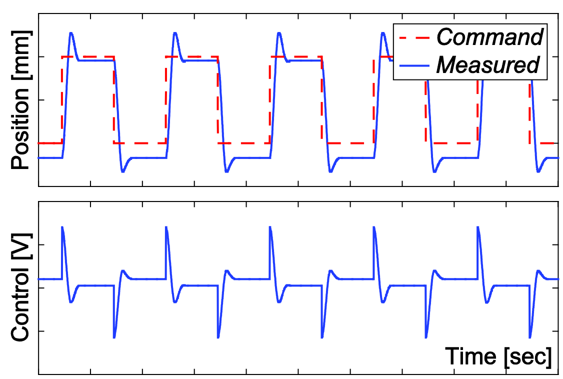

Figure 9: Sample step response for P-control.

3-2c. Include a plot in your report showing the experimental P-Control closed loop system output overlaid with the simulated one, and accompanied by a subplot showing actual control signal and simulated control signal. Comment on the similarities and/or differences between the two for each K_p value, reflecting your engineering judgement.

For Bode Plot Based Identification of Closed-loop Frequency Response

3-3a. Summarize the measurements from 2.3a **in the table provided at the end of the document**.
Hint: Use the "Zoom In" and "Data Cursor" tools located in the toolbar of the plot to read values accurately.

3-3b. Using the table, plot the gain and phase values versus frequency (**Bode Plots**), in the format shown in Figure 10. Use logarithmic scales for gain $|G_{cl}|$ and frequency ω (rad/sec), and a linear scale for the phase shift $\angle G_{cl}$ (deg).

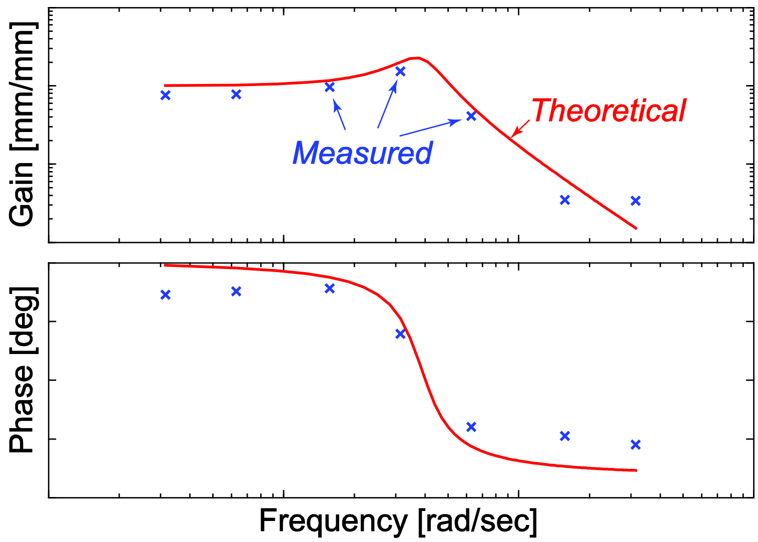

Figure 10: Closed-loop frequency response.

3.3c. Compute the theoretical (equivalent model in Figure 11) frequency response for the frequency $f = \frac{\omega}{2\pi}$ range 0, 1, 2, …, 25 [Hz]. (Use K_v, τ_v values from part 3-1a results to find ω_n, ζ) Overlay the theoretical magnitude and phase values on top of the measured ones, as shown in Figure 10. Comment on the similarities and/or discrepancies between the two. Explain various features you observe in the two graphs (DC gain, resonance magnitude, gain attenuation (roll-off), and change in phase angle), and how some of these features relate to the closed-loop servo performance. Use your engineering judgment, and research from your textbook and/or additional references, in preparing your comments.

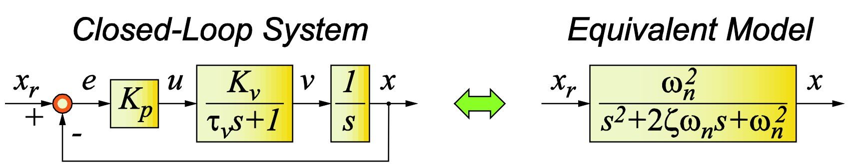

Figure 11: Closed Loop P-Controlled System
(for designing theoretical figure of Bode gain and phase response)

Hint: To obtain the theoretical (model-based) Bode plot as shown in Figure 10, you can use the following sample MATLAB commands:

```
>> num = [1 1]; den = [1 1 1];
>> sysD = tf(num,den);
>> w = logspace(-1,2);
>> [mag, phase] = bode(sysD,w);
>> loglog(w,squeeze(mag)),grid;
>> semilogx(w,squeeze(phase)),grid;
```

The transfer function here is defined by "num" and "den" variables representing the numerator and denominator coefficients respectively. (i.e. the transfer function defined here is $(s + 1) / (s^2 + s + 1)$)

Group No:

Frequency f [Hz]	Input Δx_r [mm]	Output Δx [mm]	Time Lag t_ϕ [sec]	Gain $\lvert G_{cl} \rvert$ [mm/mm]	Phase $\angle G_{cl}$ [deg]