

Homework 2

1. Game of chance, the game of craps and a betting strategy are going to be explored in this Monte Carlo simulation experiment.
 - a. Do a search and outline the rules of craps.
 - b. Your initial bet will be 10 dollars. The return on any bet will be the amount that you bet; e.g., if you bet 10 dollars and win, then you win 10 dollars. You are going to play exactly 10 games of craps. If you lose on the previous game, your next bet requires you to “double down,” that is, if on one game you bet X dollars and lose, on the next game you have to bet $2X$ dollars. If you win on the previous game, your next bet will be 10 dollars.
 - c. Write a Monte Carol simulation to find your expected winnings after the 10 games. Also examine the distribution of your expected winnings. Does it appear that this is a good betting strategy?
2. A manufacturer of microwave ovens is trying to determine the length of warranty period it should attach to its magnetron tube, the most critical component in the oven. Preliminary testing has shown that the length of life (in years), x , of a magnetron tube has an exponential distribution probability distribution with a rate parameter β . The manufacturer has collected 100 observations about the length of life of magnetron tubes (data set is available on blackboard).
 - a. Write down the likelihood function of rate parameter β and draw the likelihood function of β based on the given dataset.
 - b. Based on part a) and then find the likelihood estimate (MLE) of β .
 - c. Based on the uniform prior: $p(\beta) = 1$, find the posterior distribution of β and then find the posterior mean of β . Draw the posterior density function of β using R-software.
 - d. Based on the gamma distribution $p(\beta) \propto \beta^{a-1} \exp(-\beta b)$, find the posterior distribution of β and then find the posterior mean of β .
 - e. What is a 95% Bayesian credible interval for the average length of life of a magnetron tube under the uniform prior on part c)?
 - f. What is the posterior probability that the average length of life of a magnetron tube for the average length of life of a magnetron tube is less than 10 years under the uniform prior on part c)?
 - g. Under the uniform prior, find Bayesian credible interval of β and Bayesian credible interval $1/\beta$? Is Bayesian credible interval is invariant?