1. In this problem we will explore an alternate class of priors.

   (a) Find a resource and write a basic description of what a Jeffreys prior is. Be thorough, your exposition here needs to be such that someone who knows nothing of what a Jeffreys prior is could learn the salient features. Identify why people like Jefferys priors and discuss drawbacks.

   (b) Derive the Jefferys prior for the Bernoulli model (i.e., assuming Y_i iid Bernoulli (p)), and find the posterior distribution of p under this prior. Is it a known distribution? Discuss how you would proceed to conduct posterior inference.

   (c) Derive the Jefferys prior for the Poisson model with parameter λ (i.e., assuming Y_i iid Poisson(λ)), and find the posterior distribution of λ under this prior. Is it a known distribution? Discuss how you would proceed to conduct posterior inference.

   (d) Derive the Jefferys prior for the exponential model with rate parameter λ (i.e., assuming Y_i iid Exponential(λ)), and find the posterior distribution of λ under this prior. Is it a known distribution? Discuss how you would proceed to conduct posterior inference.

2. Let X_i for $j = 1, \cdots, n$ be independent and identically distributed (iid) variables drawn from the normal distribution with mean μ and variance σ^2, denoted by

$$p(x_i \mid \mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{ -\frac{1}{2\sigma^2}(x_i - \mu)^2 \right\}, \quad i = 1, \cdots, n, \tag{1}$$

   where μ and σ^2 are unknown parameters.

   (a) Under the noninformative prior for μ and σ^2 given by

$$\pi(\mu, \sigma^2) \propto \frac{1}{\sigma^2}, \tag{2}$$

   Write down the joint posterior distribution of μ and σ^2, denoted by $\pi(\mu, \sigma^2 \mid \mathbf{x})$, where $\mathbf{x} = (x_1, \cdots, x_n)'$.

   (b) Based on Part (a), obtain the conditional posterior distribution of $\mu \mid \sigma^2, \mathbf{x}$.

   (c) Based on Part (b), obtain the conditional posterior distribution of $\sigma^2 \mid \mu, \mathbf{x}$.

   (d) Clearly write down the steps for implementing Gibbs sampling to generate posterior draws of μ and σ^2 based on parts (b) and (c)?

   (e) List at least two drawbacks of using the Gibbs sampling algorithm for posterior sampling.

   (f) The local consumer watchdog group was concerned about the cost of electricity to residential customers over the New Zealand winter months (Southern Hemisphere). They took a random sample of 25 residential electricity accounts and looked at the total cost of electricity used over the three months of June, July, and August. The costs were: Use

514	536	345	440	427
443	386	418	364	483
506	385	410	561	275
306	294	402	350	343
480	334	324	414	296

   the Gibbs sampling algorithm in part (e)to generate $M = 10,000$ posterior samples for

μ and σ^2, respectively and then construct 95% Bayesian credible intervals for μ and σ^2, respectively.