STA6113

Homework 1

Problem: In this problem we will consider developing a Bayesian model for Poisson data; i.e., our observed data will consist of $Y_1, ..., Y_n \stackrel{iid}{\sim}$ Poisson(λ). Recall, a random variable Y is said to follow a Poisson distribution, with mean parameter λ, if its pmf is given by

$$p(y|\lambda) = \frac{e^{-\lambda}\lambda^y}{y!} I(y \in \{0, 1, 2, ...\}).$$

Note, the Poisson model is often used to analyze count data.

(a) For the Poisson model, identify the conjugate prior. This should be a general class of priors.

(b) Under the conjugate prior, derive the posterior distribution of $\lambda|y$. This should be a general expression based on the choice of the hyper-parameters specified in your prior.

(c) Find the posterior mean and variance of $\lambda|y$. These should be general expressions based on the choice of the hyper-parameters specified in your prior.

(d) Obtain the MLE of λ. Develop and discuss a relationship that exists between the MLE and posterior mean identified in (c).

(e) Write two separate R programs which can be used to find both a $(1-\alpha)100\%$ equal-tailed credible interval and a $(1-\alpha)100\%$ HPD credible interval for the Poisson model. These programs should take as arguments the following inputs: the observed data, prior hyper-parameters, and significance level.

(f) Find a data set which could be appropriately analyzed using the Poisson model. This data set should be of interest to you, and you should discuss, briefly, why the aforementioned model is appropriate; e.g., consider independence, identically distributed, etc. etc. You will also need to provide the source of the data.

(g) Analyze the data set you have selected in (e). Provide posterior point estimates of λ, credible intervals, etc. etc. Your analysis should be accompanied by an appropriate discussion of your findings.

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Problem: An engineer takes a sample of 5 steel I beams from a batch and measures the amount (X) they sag under a standard load. The amounts in mm are 5.19, 4.72, 4.81, 4.87, 4.88. For this dataset, it is known that the sag is normal (μ, σ^2), where the standard deviation σ = .25 is known. Use a normal (0, 1) prior for μ**.**

a. Find the posterior distribution of μ. Show all work!

b. Draw the density of posterior distribution of μ and the density of the prior distribution of μ in the same figure.