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UNIVERSITY OF GHANA

SUPPLEMENTARY EXAMINATIONS (Resit) – 2016/2017

LEVEL 100 BACHELOR OF ARTS / SCIENCE

STAT 112: ELEMENATRY PROBABILITY (3 CREDITS)

Two and Half Hours (2½)

ANSWER ALL QUESTIONS IN SECTION A

AND ANY TWO FROM SECTION B

Statistical tables are to be provided

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SECTION A (50 MARKS)

ANSWER ALL QUESTIONS

A1. Consider the experiment of throwing a fair die. Let X be the random variable which assigns 1 if the number that appears is even and 0 if the number that appears is odd.

(a) What is the range of X?

(b) Find P(X=1) and P(X=0)

[10 Marks]

A2. For a discrete r.v X, the c.d.f is given by

$$F(x) = \frac{x^2}{9}, \text{ for } x=1,2,3$$

i. Find $P(X \leq 2)$ ii. P(X=2)

ii. Write down the probability distribution of X and sketch it.

[10 Marks]

A3. A discrete random variable X has probability mass function given by

$$f(x) = \frac{3!}{x!\,(3-x)!}\left(\frac{1}{4}\right)^{x}\left(\frac{1}{4}\right)^{3-x} \; ; x = 0, 1, 2, 3.$$

Determine the mean of X.

[10 Marks]

A4. Consider an experiment of tossing a coin three times.

(a) Find the sample space S_1 if we wish to observe the exact sequence of heads and tails obtained

(b) Find the sample space S_2, if we wish to observe the number of heads in the three tosses.

[10 Marks]

A5.

i. State and prove the finite sample space theorem

ii. A bag contains two red, three green and four black balls of identical size except for colour. Three balls are drawn at random without replacement. Show that the probability is $\frac{55}{84}$ that two balls have the same colour and the third is different

[10 Marks]

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SECTION B (50 MARKS)

ANSWER TWO QUESTIONS

B1. (a) Consider two events associated with a random experiment:

i. What do the following statements mean?

α. A and B are mutually exclusive

β . A and B are independent

ii. If A and B are independent prove that the following pairs of events A and $\bar{B}$ are also independent. **[10 Marks]**

(b) Suppose that $8P(A \cup B) = 5$ and $\frac{2x}{P(A)} = 1$ where $P(B) = x$

i. For what values of x are A and B mutually exclusive? For this value of x, are A and B independent

ii. For what values of x A and B independent?

iii. Determine whether for this value of x, A and B are mutually exclusive

[15 Marks]

B2. (a). The following table gives the cumulative distribution function of a discrete random variable X

X	1	2	4	5	6	8
$P(X \leq x)$	0.1	0.3	0.7	0.8	0.95	1.0

Find i. P(X=5) ii. $P(3 < x \leq 6)$

[8 Marks]

(a) The random variable X has probability mass function (p.m.f) $P(x) = \theta(1-\theta)^x,\ x = 0,1,23, \ldots$ where $0 \leq \theta \leq 1$. Show that the median of the distribution satisfies the equation: $(m+1)\ln(1-\theta) + ln2$ where m is the median.

[8 Marks]

(c) A continuous random variable X has probability density function

$$\begin{cases} \frac{2x}{k^2}, & 0 \leq x \leq k \\ 0, & ortherwise \end{cases}$$

Determine the value of the constant k if $Var(X) = 2$

[9 Marks]

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B3. (a)

i. Define the conditional probability $P(A/B)$ of an event A relative to an event B of positive probability.

ii. Prove that if $P(A/B) \leq P(A)$ then $P(B/A) \leq P(B)$

iii. Suppose that A and B are mutually exclusive events in the sample space of a random experiment. If independent trials of this experiment are performed, show that

$$P(A/A \cup B) = \frac{P(A)}{P(A) + P(B)}$$

[10 marks]

(b) The Coca Cola Company has 150 tonnes tank for sugar storage in the manufacturing of beverages. The monthly demand of sugar in 100 tonnes (denoted by X) shows a relative frequency behaviour that can be modeled by

$$\begin{cases} x, & 0 \leq x \leq 1 \\ 1, & 1 < x < 1.5 \\ 0, & elsewhere \end{cases}$$

i. Show that the above function has the properties of a density function
ii. Find the cumulative distribution function of X
iii. Find $P(0 \leq X < 0.5)$ *and* $P(0.5 \leq x < 1.3)$
iv. Find $P(X \geq 1 | 0.1 \leq X \leq 1.4)$
v. Find the expected value of X

[15 marks]

B4. (a) Consider the sample space associated with the experiment of two tosses of a coin. Define the random variable X as follows: X is the number of heads obtained in the tosses

i. Find the sample space and the range space of X
ii. Derive the probability distribution of the number of heads

[8 Marks]

(b) Suppose X is a geometric r.v. with frequency function

$$P(X = k) = p(q)^{k-1} \quad , k = 1,2, \ldots$$

Derive the expression for $E(X)$ and $Var(X)$

[8 Marks]

(b) Suppose that X is a continuous random variable with pdf:

$$f(x) = \begin{cases} 1 + x , & if -1 \leq x \leq 0 \\ 1 - x , & if \quad 0 \leq x \leq 1 \end{cases}$$

Find its mean and variance.

[9 Marks]